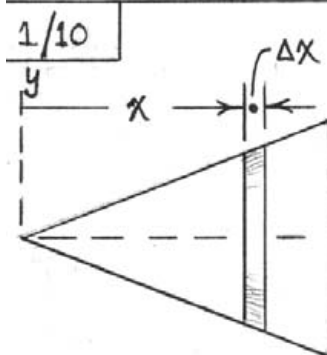

$$\boxed{1/1} \quad V = \sqrt{V_x^2 + V_y^2} = \sqrt{36^2 + 15^2} = 39$$

$$\cos \theta_x = \frac{V_x}{V} = \frac{-36}{39}, \quad \theta_x = \underline{157.4^\circ}$$

$$\cos \theta_y = \frac{V_y}{V} = \frac{15}{39}, \quad \theta_y = \underline{67.4^\circ}$$

$$\underline{n} = \frac{\underline{V}}{V} = \frac{-36\underline{i} + 15\underline{j}}{39} = \underline{-0.923\underline{i} + 0.385\underline{j}}$$

1/10



$$y(x) = \frac{r}{h} x$$

$$y(x + \Delta x) = \frac{r}{h} (x + \Delta x)$$

$$\Delta V = V(x + \Delta x) - V(x)$$

$$= \frac{1}{3} \pi [y(x + \Delta x)]^2 [x + \Delta x] - \frac{1}{3} \pi [y(x)]^2 [x]$$

$$= \frac{1}{3} \pi \left[\frac{r}{h} (x + \Delta x) \right]^2 [x + \Delta x] - \frac{1}{3} \pi \left[\frac{r}{h} x \right]^2 [x]$$

$$= \frac{1}{3} \pi \frac{r^2}{h^2} [(x + \Delta x)^3 - x^3]$$

$$= \frac{1}{3} \pi \frac{r^2}{h^2} [x^3 + 3x^2 \Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3]$$

$$= \frac{\pi r^2}{h^2} \left[x^2 \Delta x + x(\Delta x)^2 + \frac{1}{3} (\Delta x)^3 \right]$$

In the limit as $\Delta x \rightarrow dx$, the higher-order terms drop out.

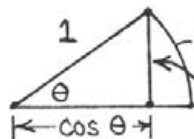
$$\boxed{1/11} \quad 20^\circ \left(\frac{\pi \text{ rad}}{180^\circ} \right) = 0.3491 \text{ rad}$$

$$\sin 20^\circ = 0.3420$$

$$\text{Percent error is } \frac{0.3420 - 0.3491}{0.3420} (100) = \underline{2.06\%}$$

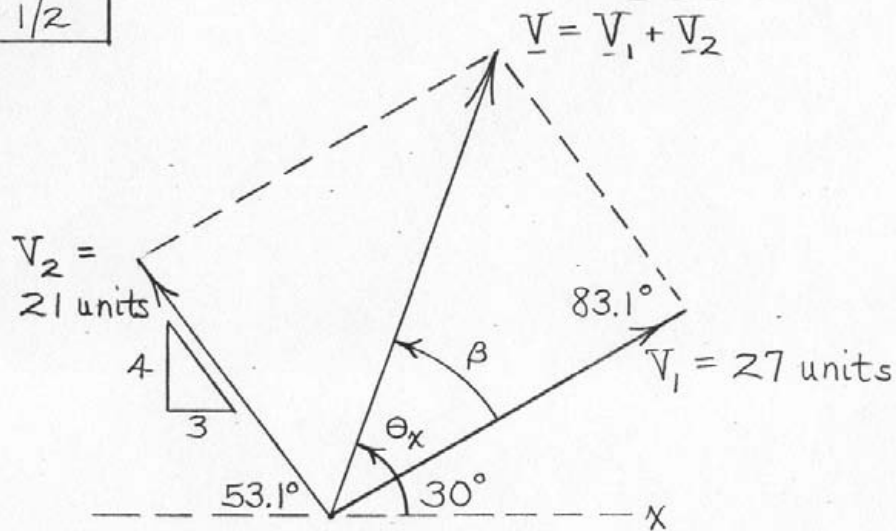
$$\tan 20^\circ = 0.3640$$

$$\text{Percent error is } \frac{0.3640 - 0.3491}{0.3640} (100) = \underline{4.09\%}$$



The approximation $\sin \theta \approx \theta$ involves the approximation that the arclength $s = \theta$ is the vertical side of the triangle. The approximation that $\tan \theta \approx \theta$ involves, in addition, the approximation that 1 is the horizontal side of the triangle.

1/2



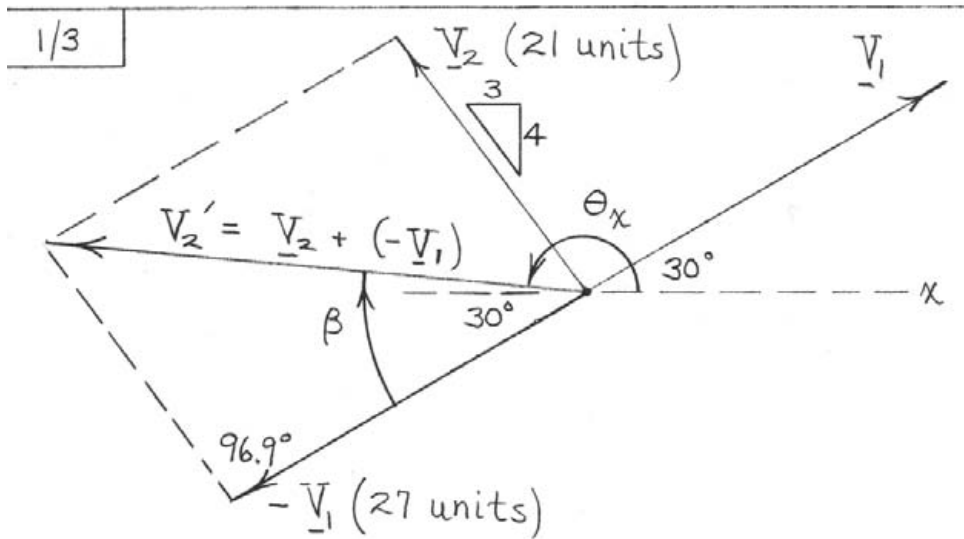
Graphically, $V = \underline{32 \text{ units}}$, $\theta_x = \underline{70^\circ}$

Algebraically, $V^2 = 27^2 + 21^2 - 2(27)(21)\cos 83.1^\circ$

$$V = \underline{32.2 \text{ units}}$$

$$\frac{\sin \beta}{21} = \frac{\sin 83.1^\circ}{32.2}, \quad \beta = 40.4^\circ$$

$$\theta_x = \beta + 30^\circ = 40.4^\circ + 30^\circ = \underline{70.4^\circ}$$



Graphically, $\underline{V}' = 36 \text{ units}$, $\theta_x = 175^\circ$

Algebraically, $V'^2 = 27^2 + 21^2 - 2(27)(21)\cos 96.9^\circ$

$$\underline{V' = 36.1 \text{ units}}$$

$$\frac{\sin \beta}{21} = \frac{\sin 96.9^\circ}{36.1}, \quad \beta = 35.2^\circ$$

$$\theta_x + \beta = 210^\circ, \quad \theta_x = 210 - 35.2^\circ = \underline{174.8^\circ}$$

$$\frac{1}{4} \quad F = \sqrt{160^2 + 80^2 + 120^2} = 215 \text{ lb}$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{160}{215} = 0.743, \quad \underline{\theta_x = 42.0^\circ}$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{80}{215} = 0.371, \quad \underline{\theta_y = 68.2^\circ}$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{-120}{215} = -0.557, \quad \underline{\theta_z = 123.9^\circ}$$

$$\frac{1/5}{m} = \frac{W}{g} = \frac{1000}{32.2} = \underline{31.1 \text{ slugs}}$$

$$m = 31.1 \text{ slugs} \left(\frac{14.594 \text{ kg}}{\text{slug}} \right) = \underline{453 \text{ kg}}$$

1/6 | $F = W = \frac{Gm_1m_2}{r^2}$

where $G = 6.673 (10^{-11}) \text{ m}^3/(\text{kg} \cdot \text{s}^2)$

$m_1 = 80 \text{ kg}$

$m_2 = 5.976 (10^{24}) \text{ kg}$

and $r = (6371 + 250) (10^3) \text{ m}$

Substitute these numbers & obtain $W = 728 \text{ N}$

U.S. units : $W = 728 \text{ N} \left(\frac{1 \text{ lb}}{4.4482 \text{ N}} \right) = \underline{163.6 \text{ lb}}$

$$\boxed{1/7} \quad W = (125 \text{ lb}) \left(\frac{4.4482 \text{ N}}{\text{lb}} \right) = \underline{556 \text{ N}}$$

$$m = \frac{W}{g} = \frac{125}{32.2} = \underline{3.88 \text{ slugs}}$$

$$m = \frac{W}{g} = \frac{556}{9.81} = \underline{56.7 \text{ kg}}$$

1/8

$$A = 8.67, \quad B = 1.429$$

$$(A+B) = 8.67 + 1.429 = \underline{10.10}$$

$$(A-B) = 8.67 - 1.429 = \underline{7.24}$$

$$(AB) = (8.67)(1.429) = \underline{12.39}$$

$$(A/B) = 8.67 / 1.429 = \underline{6.07}$$

1/9

$$F = \frac{G m_1 m_2}{d^2} = \frac{3.439 (10^{-8}) (1) (333,000) (4.095 \cdot 10^{23})^2}{(92.96 \cdot 10^6 \cdot 5280)^2}$$

$$= 7.97 (10^{21}) \text{ lb}$$

$$F = 7.97 (10^{21}) \text{ lb} \left(\frac{4.4482 \text{ N}}{\text{lb}} \right) = \underline{3.55 (10^{22}) \text{ N}}$$

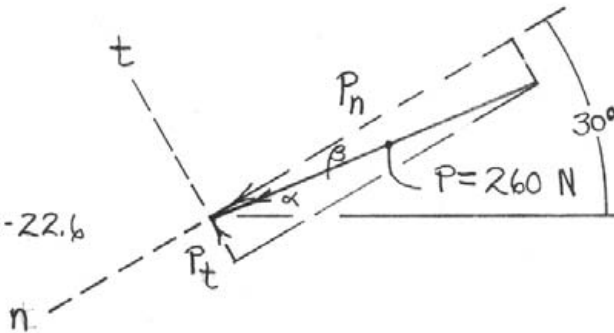
$$\begin{array}{|l} 2/1 \end{array} \left\{ \begin{array}{l} F_x = -800 \sin 35^\circ = \underline{-459 \text{ N}} \\ F_y = 800 \cos 35^\circ = \underline{655 \text{ N}} \end{array} \right.$$

$$\underline{\underline{\underline{F = -459\hat{i} + 655\hat{j} \text{ N}}}}}$$

2/10

$$\alpha = \tan^{-1} \frac{5}{12}$$
$$= 22.6^\circ$$

$$\beta = 30^\circ - \alpha = 30^\circ - 22.6^\circ$$
$$= 7.38^\circ$$



$$P_n = P \cos \beta = 260 \cos 7.38^\circ = \underline{258 \text{ N}}$$

$$P_t = P \sin \beta = 260 \sin 7.38^\circ = \underline{33.4 \text{ N}}$$

$$\boxed{2/100} \quad \underline{T} = T \underline{n}_{BA}$$

$$\underline{T} = 12 \left[\frac{-35\underline{i} + 25\underline{j} + 60\underline{k}}{\sqrt{35^2 + 25^2 + 60^2}} \right]$$

$$= \underline{-5.69\underline{i} + 4.06\underline{j} + 9.75\underline{k} \text{ kN}}$$

2/101

$$\begin{aligned}\underline{F} &= F \underline{n}_{AB} \\ &= 750 \left[\frac{-40\underline{i} + 70\underline{j} + 65\underline{k}}{\sqrt{40^2 + 70^2 + 65^2}} \right] \\ &= -290\underline{i} + 507\underline{j} + 471\underline{k} \text{ lb}\end{aligned}$$

$$\cos \theta_x = \frac{F_x}{F} = \frac{-290}{750}, \quad \underline{\theta_x = 112.7^\circ}$$

$$\cos \theta_y = \frac{F_y}{F} = \frac{507}{750}, \quad \underline{\theta_y = 47.5^\circ}$$

$$\cos \theta_z = \frac{F_z}{F} = \frac{471}{750}, \quad \underline{\theta_z = 51.1^\circ}$$

$$\underline{2/102} \quad F_z = 5 \cos 40^\circ = 3.83 \text{ kN}$$

$$F_h = 5 \sin 40^\circ = 3.21 \text{ kN}$$

$$F_x = -3.21 \sin 35^\circ = -1.843 \text{ kN}$$

$$F_y = 3.21 \cos 35^\circ = 2.63 \text{ kN}$$

$$\text{So } \underline{\underline{F = -1.843\hat{i} + 2.63\hat{j} + 3.21\hat{k} \text{ kN}}}$$

$$F_{OA} = F \cdot n_{OA}$$

$$= (-1.843\hat{i} + 2.63\hat{j} + 3.21\hat{k}) \cdot (\cos 30^\circ\hat{i} + \sin 30^\circ\hat{j})$$

$$= -0.280 \text{ kN (as a scalar)}$$

$$\underline{F}_{OA} = F_{OA} n_{OA}$$

$$= -0.280 (\cos 30^\circ\hat{i} + \sin 30^\circ\hat{j})$$

$$= -0.243\hat{i} - 0.140\hat{j} \text{ kN}$$

$$\begin{aligned} \underline{2/103} \quad \underline{\underline{F = F_n = 300 \left[\frac{4\mathbf{i} - 8\mathbf{j} - 8\mathbf{k}}{\sqrt{4^2 + 8^2 + 8^2}} \right]}} \\ \underline{\underline{= 300 \left[\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k} \right] \text{ lb}}} \end{aligned}$$

$$\underline{\underline{F_x = 100 \text{ lb}, \quad F_y = -200 \text{ lb}, \quad F_z = -200 \text{ lb}}}$$

$$\underline{2/104} \quad \underline{T = T \underline{n}_{AB} = 2.4 \left(\frac{2\underline{i} + \underline{j} - 5\underline{k}}{\sqrt{2^2 + 1^2 + 5^2}} \right)}$$

$$= 0.876 \underline{i} + 0.438 \underline{j} - 2.19 \underline{k} \text{ kN}$$

$$\text{Projection } \underline{T_{AC}} = \underline{T} \cdot \underline{n}_{AC}$$

$$= (0.876 \underline{i} + 0.438 \underline{j} - 2.19 \underline{k}) \cdot \left(\frac{2\underline{i} - 2\underline{j} - 5\underline{k}}{\sqrt{2^2 + 2^2 + 5^2}} \right)$$

$$= \underline{2.06 \text{ kN}}$$

$$\begin{aligned}
 \boxed{2/105} \quad \underline{\underline{T}} &= T \frac{\underline{\underline{CD}}}{\underline{\underline{CD}}} = 1.2 \frac{1.5\underline{i} + 3\underline{j} - 4.5\underline{k}}{\sqrt{1.5^2 + 3^2 + 4.5^2}} \\
 &= \underline{\underline{0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k} \text{ kN}}}
 \end{aligned}$$

The two indicated coordinate systems are equivalent for the question at hand.

2/106

$$T_{GF} = \underline{T} \cdot \underline{n}_{GF}$$

$$= (0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}) \cdot \frac{2\underline{i} - 3\underline{k}}{\sqrt{2^2 + 3^2}}$$

$$= \underline{0.978 \text{ kN}}$$

$$\begin{aligned} \underline{2/107} \quad \underline{\underline{T}} &= T \underline{n}_{AB} = 10 \left[\frac{4\underline{i} - 7.5\underline{j} + 5\underline{k}}{(4^2 + (-7.5)^2 + 5^2)^{1/2}} \right] \\ &= \underline{10 (0.406 \underline{i} - 0.761 \underline{j} + 0.507 \underline{k}) \text{ kN}} \end{aligned}$$

$$\cos \theta_x = 0.406, \quad \theta_x = 66.1^\circ$$

$$\cos \theta_y = -0.761, \quad \theta_y = 139.5^\circ$$

$$\cos \theta_z = 0.507, \quad \underline{\theta_z = 59.5^\circ}$$

2/108 The coordinates of point B are

$$(x_B, y_B, z_B) = (1.6, -0.8 \sin 30^\circ, 0.8 \cos 30^\circ)$$

$$= (1.6, -0.4, 0.693) \text{ m}$$

The position vector \underline{BC} is

$$\underline{BC} = (0-1.6)\underline{i} + (0.7-(-0.4))\underline{j} + (1.2-0.693)\underline{k}$$

$$= -1.6\underline{i} + 1.1\underline{j} + 0.507\underline{k} \text{ m}$$

The unit vector which characterizes \underline{BC} is

$$\underline{n}_{BC} = \frac{-1.6\underline{i} + 1.1\underline{j} + 0.507\underline{k}}{\sqrt{1.6^2 + 1.1^2 + 0.507^2}}$$

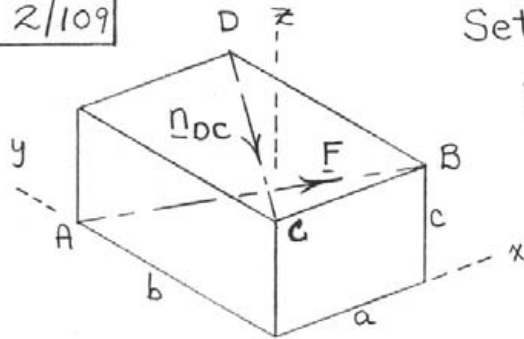
$$= -0.797\underline{i} + 0.548\underline{j} + 0.253\underline{k}$$

Then $\underline{I} = T \underline{n}_{BC}$

$$= 750(-0.797\underline{i} + 0.548\underline{j} + 0.253\underline{k})$$

$$= \underline{-598\underline{i} + 411\underline{j} + 189.5\underline{k} \text{ N}}$$

2/109



Set up x-y-z axes shown

$$\underline{n}_{DC} = \frac{\underline{DC}}{DC}$$

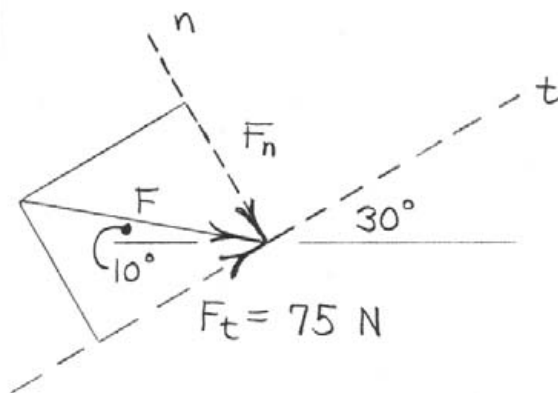
$$= \frac{-a\mathbf{i} - b\mathbf{j}}{\sqrt{a^2 + b^2}}$$

$$\underline{F} = F \underline{n}_{AB} = F \frac{\underline{AB}}{AB}$$

$$= F \frac{a\mathbf{i} - b\mathbf{j} + c\mathbf{k}}{\sqrt{a^2 + b^2 + c^2}}$$

$$F_{DC} = \underline{F} \cdot \underline{n}_{DC} = \frac{(b^2 - a^2)F}{\sqrt{(a^2 + b^2)} \sqrt{(a^2 + b^2 + c^2)}}$$

2/11



$$\frac{|F_n|}{F_t} = \tan 40^\circ, \quad |F_n| = F_t \tan 40^\circ = 75 \tan 40^\circ$$

$$F_n = \underline{-62.9 \text{ N}}$$

$$\frac{F_t}{F} = \cos 40^\circ, \quad F = \frac{F_t}{\cos 40^\circ} = \frac{75}{\cos 40^\circ}$$

$$= \underline{97.9 \text{ N}}$$

2/110

$$\underline{F} = F \underline{n}_{OM}$$

$$= F \left[\frac{\frac{a}{2} \underline{i} + b \underline{j} + \frac{c}{2} \underline{k}}{\sqrt{\frac{a^2}{4} + b^2 + \frac{c^2}{4}}} \right]$$

$$= F \left[\frac{a \underline{i} + 2b \underline{j} + c \underline{k}}{\sqrt{a^2 + 4b^2 + c^2}} \right]$$

$$F_{AE} = \underline{F} \cdot \underline{n}_{AE}$$

$$= F \left[\frac{a \underline{i} + 2b \underline{j} + c \underline{k}}{\sqrt{a^2 + 4b^2 + c^2}} \right] \cdot \left[\frac{-a \underline{i} - b \underline{j}}{\sqrt{a^2 + b^2}} \right]$$

$$= - \frac{F(a^2 + 2b^2)}{\sqrt{a^2 + b^2} \sqrt{a^2 + 4b^2 + c^2}}$$

$$2/111 \quad \underline{F} = F_{\underline{n}_{AB}} = 200 \left[\frac{-12\underline{i} + 24\underline{j} + 8\underline{k}}{\sqrt{12^2 + 24^2 + 8^2}} \right]$$

$$= -85.7\underline{i} + 171.4\underline{j} + 57.1\underline{k} \quad \text{lb}$$

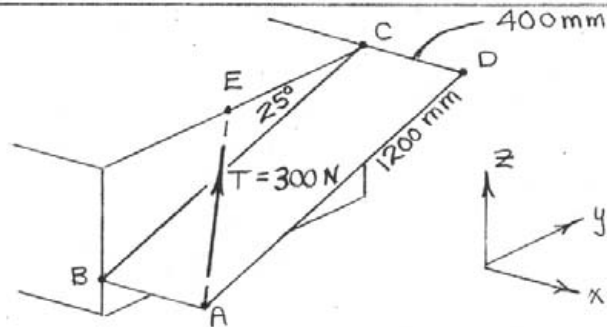
$$\underline{OC} = 12\underline{i} + 24\underline{j} \quad \text{in.}$$

The angle θ between \underline{F} and \underline{OC} is

$$\theta = \cos^{-1} \frac{\underline{F} \cdot \underline{OC}}{F(\underline{OC})} = \cos^{-1} \left[\frac{-85.7(12) + 171.4(24)}{200(\sqrt{12^2 + 24^2})} \right]$$

$$= \underline{54.9^\circ}$$

2/112



$$\begin{aligned}\underline{T} &= T \underline{n}_{AE} = 300 \left[\frac{-400\underline{i} + 544\underline{j} + 507\underline{k}}{\sqrt{400^2 + 544^2 + 507^2}} \right] \\ &= 300 [-0.474\underline{i} + 0.644\underline{j} + 0.601\underline{k}] \text{ N}\end{aligned}$$

$$\underline{n}_{BC} = \cos 25^\circ \underline{j} + \sin 25^\circ \underline{k}$$

Carry out $T_{BC} = \underline{T} \cdot \underline{n}_{BC}$ to obtain

$$\underline{T_{BC}} = 251 \text{ N}$$

2/113 | $T = 150 \text{ N}$

$$T = 150 \text{ N}$$

Coordinates of A are

$$(0.9 \cos 30^\circ, 0, 0.9 \sin 30^\circ)$$

or $(0.779, 0, 0.45) \text{ m}$

$$\overline{AB} = \sqrt{0.779^2 + 1.2^2 + (0.9 - 0.45)^2}$$

$$= 1.50 \text{ m}$$

$$l = -0.779/1.5, m = 1.2/1.5, n = \frac{0.45}{1.5}$$

$$\underline{T} = \frac{100}{1.5} (-0.779 \underline{i} + 1.2 \underline{j} + 0.45 \underline{k}) \text{ N}$$

$\lambda = \text{unit vector along } CD$

$$\overline{CD} = \sqrt{0.9^2 + 1.2^2} = 1.5 \text{ m}$$

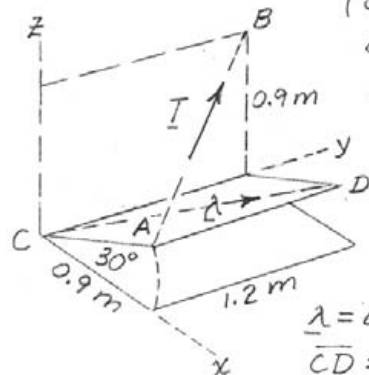
$$\ell = 0.779/1.5, m = 1.2/1.5, n = 0.45/1.5$$

$$\lambda = \frac{1}{1.5} (0.779 \underline{i} + 1.2 \underline{j} + 0.45 \underline{k})$$

$$\underline{T}_{CD} = \underline{T} \cdot \underline{\lambda} = \frac{100}{1.5} (-0.779 \underline{i} + 1.2 \underline{j} + 0.45 \underline{k}) - \frac{1}{1.5} (0.779 \underline{i} + 1.2 \underline{j} + 0.45 \underline{k})$$

$$= \frac{100}{2.25} (-0.779^2 + 1.2^2 + 0.45^2) = \frac{100}{2.25} (-0.6075 + 1.44 + 0.2025)$$

$$= \frac{400}{9} (1.035) = \underline{46.0 \text{ N}}$$



► 2/114 The position of point A is

$$\begin{aligned}\underline{r}_A &= 10\cos 15^\circ \underline{i} + L\underline{j} + 10\sin 15^\circ \underline{k} \\ &= 9.66\underline{i} + L\underline{j} + 2.59\underline{k} \text{ in.} \quad \left[\begin{array}{l} L = \text{distance from } O \\ \text{to disk center} \end{array} \right.\end{aligned}$$

$$\begin{aligned}\underline{r}_B &= 8\cos 30^\circ \underline{i} + (L+36)\underline{j} - 8\sin 30^\circ \underline{k} \\ &= 6.93\underline{i} + (L+36)\underline{j} - 4\underline{k} \text{ in.}\end{aligned}$$

$$\underline{r}_{AB} = \underline{r}_B - \underline{r}_A = -2.73\underline{i} + 36\underline{j} - 6.59\underline{k} \text{ in.}$$

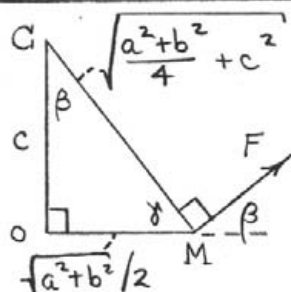
$$r_{AB} = \sqrt{2.73^2 + 36^2 + 6.59^2} = 36.7 \text{ in.} \quad \left| \begin{array}{l} \text{unstretched length} \\ \text{is } \sqrt{(8-10)^2 + 36^2} \\ = 36.1 \text{ in.} \end{array} \right.$$

The spring force is $F = k\delta = 15(36.7 - 36.1) = 9.66 \text{ lb}$

As a vector: $\underline{F} = F\underline{n}_{AB} = F \frac{\underline{r}_{AB}}{r_{AB}}$

$$\begin{aligned}\underline{F} &= 9.66 \left[\frac{-2.73\underline{i} + 36\underline{j} - 6.59\underline{k}}{36.7} \right] \\ &= \underline{-0.719\underline{i} + 9.48\underline{j} - 1.734\underline{k} \text{ lb}}\end{aligned}$$

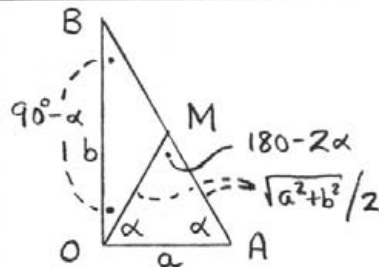
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$$\tan \gamma = \frac{c}{\sqrt{a^2 + b^2}/2} = \frac{2c}{\sqrt{a^2 + b^2}}$$

$$\gamma + 90^\circ + \beta = 180^\circ$$

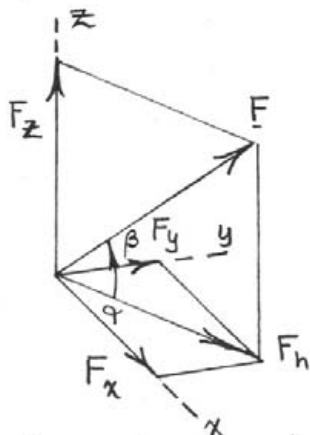
$$\beta = 90^\circ - \gamma = 90^\circ - \tan^{-1} \frac{2c}{\sqrt{a^2 + b^2}}$$



$$\tan \alpha = \frac{b}{a}$$

$$\cos \alpha = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\sin \alpha = \frac{b}{\sqrt{a^2 + b^2}}$$



$$\begin{cases} F_z = F \sin \beta \\ F_h = F \cos \beta \\ F_x = F_h \cos \alpha = F \cos \beta \cos \alpha \\ F_y = F_h \sin \alpha = F \cos \beta \sin \alpha \end{cases}$$

Now simplify $\sin \beta$ & $\cos \beta$ expressions:

$$\begin{aligned} \sin \beta &= \sin \left[90^\circ - \tan^{-1} \frac{2c}{\sqrt{a^2 + b^2}} \right] \\ &= \sin 90^\circ \cos \left[\tan^{-1} \frac{2c}{\sqrt{a^2 + b^2}} \right] - \cos 90^\circ \sin \left[\tan^{-1} \frac{2c}{\sqrt{a^2 + b^2}} \right] \\ &= \frac{\sqrt{a^2 + b^2}}{\sqrt{a^2 + b^2 + 4c^2}} \quad (\text{Also see that } \sin \beta = \cos \gamma!) \end{aligned}$$

$$\cos \beta = \sin \gamma = \frac{c}{\sqrt{\frac{a^2 + b^2}{4} + c^2}} = \frac{2c}{\sqrt{a^2 + b^2 + 4c^2}}$$

Finally,

$$F_x = F \frac{2c}{\sqrt{a^2 + b^2 + 4c^2}} \frac{a}{\sqrt{a^2 + b^2}} = \frac{2acF}{\sqrt{a^2 + b^2} \sqrt{a^2 + b^2 + 4c^2}}$$

$$F_y = F \frac{2c}{\sqrt{a^2 + b^2 + 4c^2}} \frac{b}{\sqrt{a^2 + b^2}} = \frac{2bcF}{\sqrt{a^2 + b^2} \sqrt{a^2 + b^2 + 4c^2}}$$

$$F_z = F \sqrt{\frac{a^2 + b^2}{a^2 + b^2 + 4c^2}}$$

►2/116

$$F_x = F_{xy} \cos \theta, \quad F_y = F_{xy} \sin \theta$$

$$F_z = F \sin \beta, \quad F_{xy} = F \cos \beta$$

$$\tan \beta = \frac{R \cos \phi}{R \sin \phi - \frac{R}{2}} = \frac{2 \cos \phi}{2 \sin \phi - 1}$$

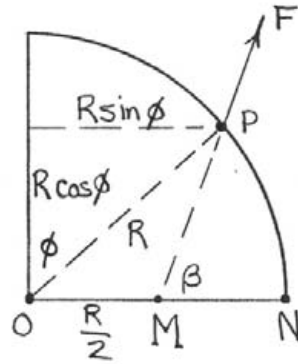
$$\text{So } \sin \beta = \frac{2 \cos \phi}{\sqrt{(2 \sin \phi - 1)^2 + (2 \cos \phi)^2}}$$

$$\cos \beta = \frac{2 \sin \phi - 1}{\sqrt{(2 \sin \phi - 1)^2 + (2 \cos \phi)^2}}$$

$$\text{Note that } \sqrt{(2 \sin \phi - 1)^2 + (2 \cos \phi)^2} = \sqrt{5 - 4 \sin \phi}$$

$$\text{So } \underline{F} = F [\cos \theta \cos \beta \underline{i} + \sin \theta \cos \beta \underline{j} + \sin \beta \underline{k}]$$

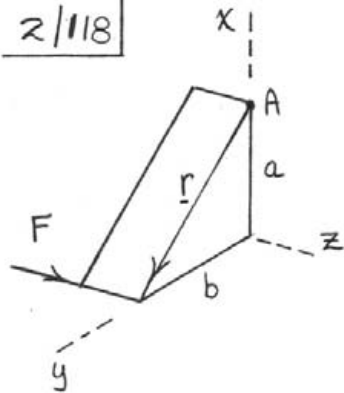
$$= \frac{F}{\sqrt{5 - 4 \sin \phi}} [(2 \sin \phi - 1)(\cos \theta \underline{i} + \sin \theta \underline{j}) + 2 \cos \phi \underline{k}]$$



2/117 By inspection, $\underline{M}_o = F(\underline{c}_j - \underline{b}_k)$

$$\begin{aligned}\text{Or, } \underline{M}_o &= \underline{r} \times \underline{F} = (\underline{b}_j + \underline{c}_k) \times F_i \\ &= \underline{F(\underline{c}_j - \underline{b}_k)} \quad \checkmark\end{aligned}$$

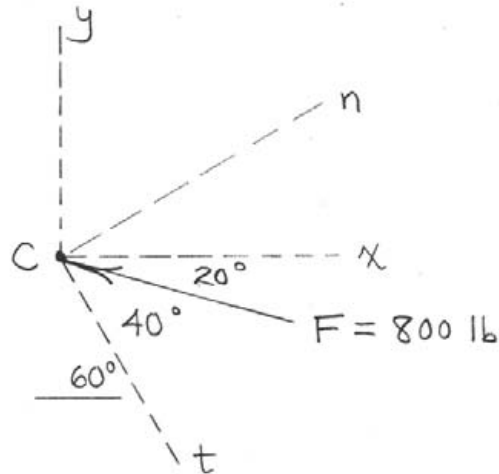
2/118



$$\begin{aligned}\underline{M}_A &= \underline{r} \times \underline{F} \\ &= (-a\underline{i} + b\underline{j}) \times F\underline{k} \\ &= \underline{F(b\underline{i} + a\underline{j})}\end{aligned}$$

$$\begin{array}{l} 2/119 \quad (a) \quad \underline{M}_0 = \underline{FLi} \\ (b) \quad \underline{M}_0 = \underline{FLi} + \underline{FDk} = \underline{F(Li + Dk)} \end{array}$$

2/12



$$\begin{cases} F_x = -800 \cos 20^\circ = \underline{-752 \text{ lb}} \\ F_y = 800 \sin 20^\circ = \underline{274 \text{ lb}} \end{cases}$$

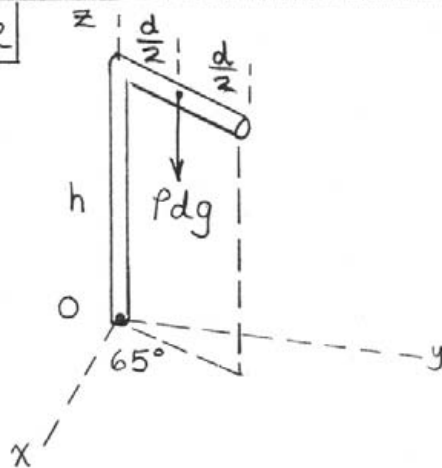
$$\begin{cases} F_n = -800 \sin 40^\circ = \underline{-514 \text{ lb}} \\ F_t = -800 \cos 40^\circ = \underline{-613 \text{ lb}} \end{cases}$$

2/120

$$\begin{aligned} M_z &= (P \cos 30^\circ) d \quad k \\ &= (6 \cos 30^\circ) (40) \quad k \\ &= \underline{208 \quad k \quad lb-in.} \end{aligned}$$

$$\begin{aligned} \underline{2/12} \quad \underline{M} &= -150(0.250 + 0.250)\underline{i} + 150(0.150)\underline{j} \\ &= -75\underline{i} + 22.5\underline{j} \text{ N}\cdot\text{m} \end{aligned}$$

2/122



$$\begin{aligned}
 \underline{M}_O &= -pdg \left(\frac{d}{2} \sin 65^\circ \right) \underline{i} + pdg \left(\frac{d}{2} \cos 65^\circ \right) \underline{j} \\
 &= pgd^2 (-0.453 \underline{i} + 0.211 \underline{j})
 \end{aligned}$$

2/123

$$\underline{M}_o = \underline{r} \times \underline{F}$$

$$= (-6\underline{i} + 0.8\underline{j} + 1.2\underline{k}) \times (-400\underline{j})$$

$$= \underline{480\underline{i} + 2400\underline{k} \text{ N}\cdot\text{m}}$$

2/124

$$\underline{M}_O = \underline{r}_{OA} \times \underline{F}$$

$$= (1.5\underline{j} + 0.75\underline{k}) \times 4(-\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j})$$

$$= -1.5\underline{i} - 2.60\underline{j} + 5.20\underline{k} \text{ lb-in.}$$

2/125 From the solution to Prob. 2/108, the force is $\underline{R} = \underline{T} = -598\underline{i} + 411\underline{j} + 189.5\underline{k}$ N

The moment associated with the couple is $\underline{M}_o = \underline{r}_{oc} \times \underline{T}$, where $\underline{r}_{oc} = 0.7\underline{j} + 1.2\underline{k}$ m

Carry out the cross product to obtain

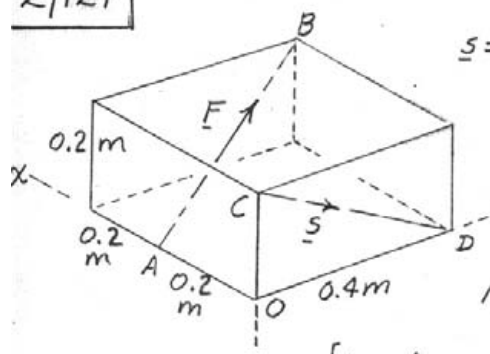
$$\underline{M}_o = -361\underline{i} - 718\underline{j} + 419\underline{k} \text{ N}\cdot\text{m}$$

$$\boxed{2/126} \quad \underline{M} = \underline{r} \times \underline{F}$$

$$= -0.5 \underline{i} \times 400 (\cos 15^\circ \underline{j} + \sin 15^\circ \underline{k})$$

$$= \underline{51.8 \underline{j} - 193.2 \underline{k} \text{ N}\cdot\text{m}}$$

2/127



$$\underline{s} = \frac{1}{\sqrt{5}}(2\underline{j} + \underline{k})$$

$$\underline{F} = \frac{F}{\sqrt{6}}(\underline{i} + 2\underline{j} - \underline{k})$$

Take $\underline{r} = \underline{DA}$
 $= 0.2\underline{i} - 0.4\underline{j} \text{ m}$

$$M_{CD} = \underline{r} \times \underline{F} \cdot \underline{s}$$

$$= [(0.2\underline{i} - 0.4\underline{j}) \times \frac{F}{\sqrt{6}}(\underline{i} + 2\underline{j} - \underline{k})] \cdot \frac{1}{\sqrt{5}}(2\underline{j} + \underline{k})$$

so

$$50 = \frac{F}{\sqrt{30}} \begin{vmatrix} 0.2 & -0.4 & 0 \\ 1 & 2 & -1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= \frac{F}{\sqrt{30}}(0.8 + 0.4) = \frac{1.2F}{\sqrt{30}}, F = \frac{50\sqrt{30}}{1.2} = 228 \text{ N}$$

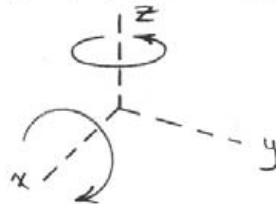
2/128

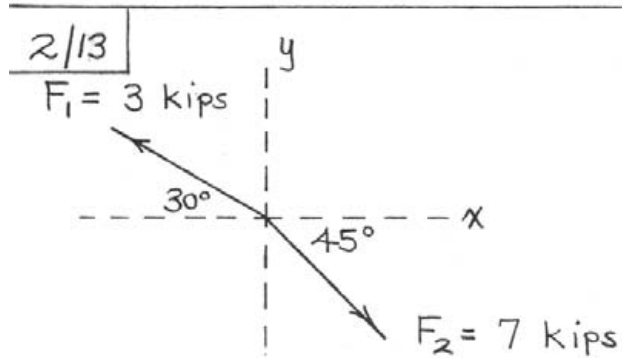
$$\begin{aligned}\underline{M}_O &= 100 (0.185 \sin 15^\circ) \underline{i} - 100 (0.185 \cos 15^\circ) \underline{j} \\ &= \underline{4.79 \underline{i} - 17.87 \underline{j} \text{ N}\cdot\text{m}}\end{aligned}$$

2/129

$$\underline{M} = 1.2(40)\underline{k} - 1.2(50)\underline{i}$$
$$= -60\underline{i} + 48\underline{k} \text{ lb-in.}$$

The spacecraft will begin to rotate about its
x- and z axes.





$$R_x = \sum F_x = -3 \cos 30^\circ + 7 \cos 45^\circ = 2.35 \text{ kips}$$

$$R_y = \sum F_y = 3 \sin 30^\circ + 7 \sin 45^\circ = -3.45 \text{ kips}$$

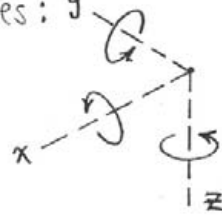
$$\underline{R} = 2.35 \underline{i} - 3.45 \underline{j} \text{ kips}$$

2/130

$$\begin{aligned} M_o &= (250 \sin 60^\circ)12 + (250 \cos 60^\circ) \sin 40^\circ (8 - 4.2) \\ &= 2900 \text{ lb-in.} \end{aligned}$$

$$\begin{aligned} \underline{2/131} \quad \underline{\underline{M}} &= (1700)(2)\underline{i} - (1700)(30)\underline{j} - (1700)(30)\underline{k} \\ &= 3400\underline{i} + 51000\underline{j} - 51000\underline{k} \text{ N}\cdot\text{m} \end{aligned}$$

The orbiter would acquire rotational motion about all three axes:



$$2/132 \quad \underline{M}_O = \underline{r}_{OB} \times \underline{T}, \quad \underline{r}_{OB} = 6\underline{i} + 13\underline{j} \text{ m}$$

$$\underline{T} = T \underline{n}_{AB} = 24 \left[\frac{6\underline{i} - 5\underline{j} - 30\underline{k}}{\sqrt{6^2 + 5^2 + 30^2}} \right]$$

$$= 4.65\underline{i} - 3.87\underline{j} - 23.2\underline{k} \text{ kN}$$

Carry out $\underline{r}_{OB} \times \underline{T}$ to obtain

$$\underline{M}_O = -302\underline{i} + 139.4\underline{j} - 83.6\underline{k} \text{ N}\cdot\text{m}$$

2/133

$$\begin{aligned}\underline{M}_O &= 8(12)\underline{i} - 8(9)\underline{k} \\ &= \underline{96i - 72k \text{ lb-in.}}\end{aligned}$$

The large moment about the x -axis
is an undesirable characteristic of this
wrench.

2/134 | $F = 16 \text{ lb (up)}$ will make $(M_o)_x$ zero.

The net moment about O is then

$$\underline{M_o} = [16(4) - 8(9)] \underline{k} = \underline{-8k \text{ lb-in.}}$$

Comment: This wrench should be used only when access considerations make its use absolutely necessary!

2/135

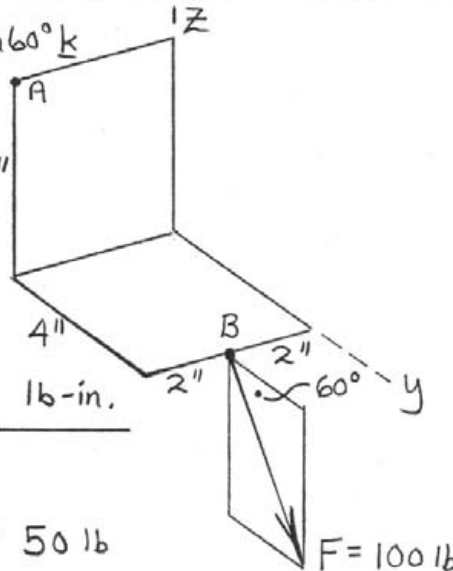
$$\underline{F} = 100 \cos 60^\circ \underline{j} - 100 \sin 60^\circ \underline{k}$$

$$(a) \quad = 50 \underline{j} - 86.6 \underline{k} \text{ lb}$$

$$\underline{r}_{AB} = -2 \underline{i} + 4 \underline{j} - 5 \underline{k} \text{ in. } 5''$$

Carry out $\underline{M}_A = \underline{r}_{AB} \times \underline{F}$
to obtain

$$\underline{M}_A = -96.4 \underline{i} - 173.2 \underline{j} - 100 \underline{k} \text{ lb-in.}$$



$$(b) \quad \begin{cases} F_x = 0 \\ F_y = 100 \cos 60^\circ = 50 \text{ lb} \\ F_z = -100 \sin 60^\circ = -86.6 \text{ lb} \end{cases}$$

$$(M_A)_x = 50(5) - 86.6(4) = -96.4 \text{ lb-in.}$$

$$(M_A)_y = -86.6(2) = -173.2 \text{ lb-in.}$$

$$(M_A)_z = -50(2) = -100 \text{ lb-in.}$$

2/136

$$\begin{aligned}\underline{M}_o &= 0\underline{i} - (200)(0.2 + 0.125 \sin 20^\circ)\underline{j} \\ &\quad - 200(0.125 \cos 20^\circ - 0.070)\underline{k} \\ &= -48.6\underline{j} - 9.49\underline{k} \quad \text{N}\cdot\text{m}\end{aligned}$$

There would be no z-component of \underline{M}_o if
 $d \cos 20^\circ - 70 = 0, \quad \underline{d = 74.5 \text{ mm}}$

2/137

Wheel detail:

$$\sin \alpha = \frac{BC}{BE} = \frac{100}{400}$$

$$\alpha = 14.48^\circ$$

$$\text{So } \underline{T} = T \underline{n}_{CE}$$

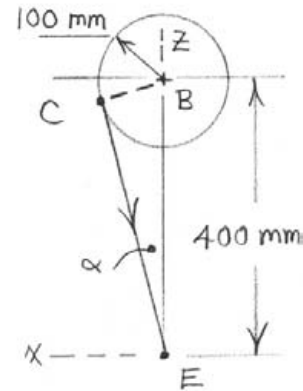
$$= 80 [-\cos \alpha \underline{k} - \sin \alpha \underline{i}]$$

$$= -20 \underline{i} - 77.5 \underline{k} \text{ N}$$

$$\underline{M}_O = \underline{r}_{OE} \times \underline{T}, \text{ where } \underline{r}_{OE} = 0.2 \underline{j} \text{ m}$$

Carry out to obtain

$$\underline{M}_O = -15.49 \underline{i} + 4 \underline{k} \text{ N}\cdot\text{m}$$



$$\underline{2/138} \quad \underline{M_o} = \underline{r_{OB}} \times \underline{F}, \text{ where}$$

$$\underline{r_{OB}} = \underline{OA} + \underline{AB}$$

$$= 4\underline{k} + 16 \cos 10^\circ (\cos 20^\circ \underline{i} + \sin 20^\circ \underline{j}) \\ + 16 \sin 10^\circ \underline{k}$$

$$= 14.81\underline{i} + 5.39\underline{j} + 6.78\underline{k} \text{ in.}$$

$$\underline{F} = 80(-\cos 20^\circ \underline{j} + \sin 20^\circ \underline{i})$$

$$= 27.4\underline{i} - 75.2\underline{j} \text{ lb}$$

Carry out $\underline{r_{OB}} \times \underline{F}$ to obtain

$$\underline{M_o} = 510\underline{i} + 185.5\underline{j} + 1261\underline{k} \text{ lb-in.}$$

2/139

$$\underline{T} = T \left[\frac{-0.35\mathbf{i} - 0.45\cos 20^\circ \mathbf{j} + (0.4 + 0.45\sin 20^\circ)\mathbf{k}}{\sqrt{(0.35)^2 + (0.45\cos 20^\circ)^2 + (0.4 + 0.45\sin 20^\circ)^2}} \right]$$

$$= 143.4 [-0.449\mathbf{i} - 0.542\mathbf{j} + 0.710\mathbf{k}] \text{ N}$$

Moment of this force about the x -axis is

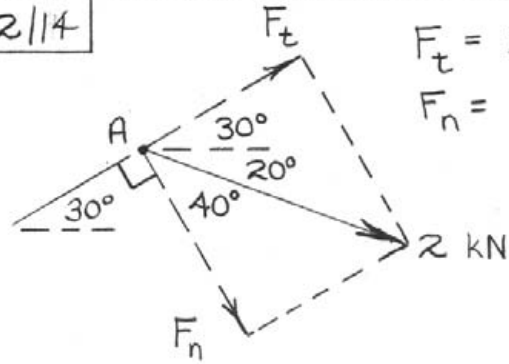
$$M_{O_x} = (0.710)(143.4)(0.45\cos 20^\circ) - 0.542(143.4)(0.45\sin 20^\circ) = \underline{31.1 \text{ N}\cdot\text{m}}$$

The moment of the weight W of the 15-kg plate about the x -axis is

$$(M_{O_x})_W = -15(9.81) \frac{0.45\cos 20^\circ}{2} = \underline{-31.1 \text{ N}\cdot\text{m}}$$

The moment of \underline{T} about the line OB is zero, because \underline{T} intersects OB .

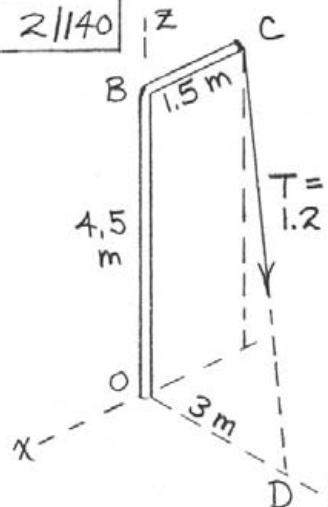
2/14



$$F_t = 2 \cos 50^\circ = \underline{1.286 \text{ kN}}$$

$$F_n = 2 \sin 50^\circ = \underline{1.532 \text{ kN}}$$

2/140



From the solution to Prob. 2/95,
 $\underline{T} = 0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}$ kN

(a) $\underline{M}_O = \underline{r}_{OO} \times \underline{T}$, $\underline{r}_{OO} = 3\underline{j}$ m
 $\underline{M}_O = -2.89\underline{i} - 0.962\underline{k}$ kN·m

Could also use
 $\underline{M}_O = \underline{r}_{OC} \times \underline{T}$, where
 $\underline{r}_{OC} = -1.5\underline{i} + 4.5\underline{k}$ N
 Result for \underline{M}_O is the same.

(b) About z -axis:

$$\underline{M}_z = -1.5 T_y \underline{k} = -1.5 (0.641) \underline{k} = -0.962 \underline{k} \text{ kN} \cdot \text{m}$$

$$\text{Also, } \underline{M}_z = \underline{M}_O \cdot \underline{k} \underline{k} = (-2.89\underline{i} - 0.962\underline{k}) \cdot \underline{k} \underline{k} \\ = -0.962 \underline{k} \text{ kN} \cdot \text{m}$$

2/141 Using the coordinates of the figure:

$$\underline{M}_A = \underline{r} \times \underline{F}, \quad \underline{F} = -1.8 \underline{k} \text{ lb}$$

$$\underline{r} = [(2+1) \cos 30^\circ] \underline{i} + 3 \underline{j} + [(2+1) \sin 30^\circ] \underline{k} \text{ in.}$$

$$\therefore \underline{M}_A = -5.40 \underline{i} + 4.68 \underline{j} \text{ lb-in.}$$

$$\underline{M}_{AB} = (\underline{M}_A \cdot \underline{n}_{AB}) \underline{n}_{AB}, \quad \underline{n}_{AB} = \cos 30^\circ \underline{i} + \sin 30^\circ \underline{k}$$

$$\therefore \underline{M}_{AB} = -4.05 \underline{i} - 2.34 \underline{k} \text{ lb-in.}$$

2/142

$$\underline{M}_O = \underline{r}_{OA} \times \underline{F}$$

$$\begin{aligned}\underline{r}_{OA} &= (0.050 + 0.130 \sin 60^\circ) \underline{i} \\ &\quad + (-0.140 - 0.130 \cos 60^\circ) \underline{j} + 0.150 \underline{k} \\ &= 0.1626 \underline{i} - 0.205 \underline{j} + 0.150 \underline{k} \text{ m}\end{aligned}$$

$$\begin{aligned}\underline{F} &= 600 (\cos 45^\circ \sin 60^\circ \underline{i} - \cos 45^\circ \cos 60^\circ \underline{j} + \sin 45^\circ \underline{k}) \\ &= 600 (0.612 \underline{i} - 0.354 \underline{j} + 0.707 \underline{k}) \\ &= 367 \underline{i} - 212 \underline{j} + 424 \underline{k} \text{ N}\end{aligned}$$

Carry out $\underline{M}_O = \underline{r}_{OA} \times \underline{F}$ to obtain

$$\underline{M}_O = -55.2 \underline{i} - 13.86 \underline{j} + 40.8 \underline{k} \text{ N}\cdot\text{m}$$

$$\begin{cases} R_x = \sum F_x = -7 \text{ kN} \\ R_y = \sum F_y = 4 - F_3 \cos \theta = -5 \text{ kN} \quad (1) \\ R_z = \sum F_z = F_3 \sin \theta = 6 \text{ kN} \quad (2) \end{cases}$$

$$(1): F_3 \cos \theta = 9$$

$$(2): F_3 \sin \theta = 6$$

$$\text{Divide Eq. (2) by Eq. 1: } \tan \theta = \frac{2}{3}$$

$$\theta = 33.7^\circ$$

$$\text{Then } F_3 = 10.82 \text{ kN}$$

$$R = \sqrt{7^2 + 5^2 + 6^2} = 10.49 \text{ kN}$$

$$\boxed{2/144} \left\{ \begin{array}{l} \underline{R} = -3F\underline{k} \\ \underline{M}_0 = -\frac{\sqrt{3}}{2} bF\underline{i} \end{array} \right.$$

$$\underline{R} \cdot \underline{M}_0 = 0 \quad \text{so} \quad \underline{R} \perp \underline{M}_0$$

2/145 The given loads form two couples, each of which has an associated moment which is in the x -direction. So

$$\underline{R} = \underline{\Sigma F} = \underline{0}$$

$$\begin{aligned}\underline{M}_0 &= Fb\underline{i} + F\left(b\frac{\sqrt{3}}{2}\right)\underline{i} \\ &= \underline{Fb\left(1 + \frac{\sqrt{3}}{2}\right)\underline{i}}\end{aligned}$$

The resultant of the system is a couple.

$$\underline{2/146} \quad \underline{R} = (1.2 - 1.2 - 1.2)\underline{j} = -1.2\underline{j} \text{ lb}$$

$$\underline{M_G} = 1.2(3)(20)\underline{k} + (1.2 - 1.2 - 1.2)(25)\underline{i}$$

$$\underline{M_G} = -30\underline{i} + 72\underline{k} \text{ lb-in.}$$

$$\underline{2/147} \quad \underline{\underline{R = \sum \underline{F} = -8 \underline{i} \text{ kN}}}$$

$$\begin{aligned} \underline{M_g} &= 50(10) \underline{k} + 8(6) \underline{j} + 8(40) \underline{k} \\ &= \underline{48 \underline{j} + 820 \underline{k} \text{ kN}\cdot\text{m}} \end{aligned}$$

$$\begin{aligned} \underline{2/148} \quad \underline{\underline{R}} &= (200 + 800) \underline{i} + 1200 (\cos 10^\circ \underline{j} - \sin 10^\circ \underline{i}) \\ &= \underline{792 \underline{i} + 1182 \underline{j} \text{ N}} \end{aligned}$$

$$\begin{aligned} \underline{M_o} &= [(200 - 800)(0.1) + (1200 \cos 10^\circ)(0.075)] \underline{k} \\ &\quad + [-(200 + 800)(0.220 + 0.330) + 1200 \sin 10^\circ (0.220)] \underline{j} \\ &\quad + [1200 \cos 10^\circ (0.220)] \underline{i} \\ &= \underline{260 \underline{i} - 504 \underline{j} + 28.6 \underline{k} \text{ N}\cdot\text{m}} \end{aligned}$$

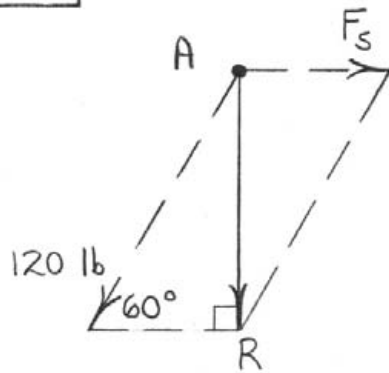
$$\begin{aligned}
 \underline{2/149} \quad \underline{\underline{R}} &= \underline{\Sigma F} = 600(\sin 30^\circ \underline{j} + \cos 30^\circ \underline{k}) \\
 &\quad + 800(-\sin 45^\circ \underline{j} + \cos 45^\circ \underline{k}) \\
 &= \underline{-266 \underline{j} + 1085 \underline{k} \text{ N}}
 \end{aligned}$$

$$\begin{aligned}
 \underline{M_o} &= -0.080 \underline{i} \times 600(\sin 30^\circ \underline{j} + \cos 30^\circ \underline{k}) \\
 &\quad + 0.160 \underline{i} \times 800(-\sin 45^\circ \underline{j} + \cos 45^\circ \underline{k}) \\
 &= \underline{-48.9 \underline{j} - 114.5 \underline{k} \text{ N}\cdot\text{m}}
 \end{aligned}$$

\underline{R} is not perpendicular to $\underline{M_o}$, because

$$\underline{R} \cdot \underline{M_o} \neq 0.$$

2/15



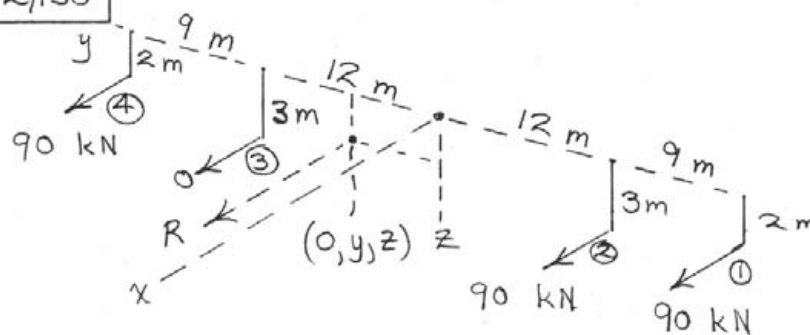
$$\cos 60^\circ = \frac{F_s}{120}$$

$$F_s = 60 \text{ lb}$$

$$\sin 60^\circ = \frac{R}{120}$$

$$R = 103.9 \text{ lb}$$

2/150



$$R = \Sigma F = 3(90) = 270 \text{ kN}$$

$$\Sigma M_z = -R_y : 90(21) + 90(12) - 90(21) = -270y$$

$$y = -4 \text{ m}$$

$$\Sigma M_y = R_z : 2(90)(2) + 90(3) = 270z$$

$$z = 2.33 \text{ m}$$

2/151 The two 160-N forces constitute a couple $160(0.250)\underline{j} = 40\underline{j} \text{ N}\cdot\text{m}$

$$\underline{R} = \sum \underline{F} = 120\underline{i} - 180\underline{j} - 100\underline{k} \text{ N}$$

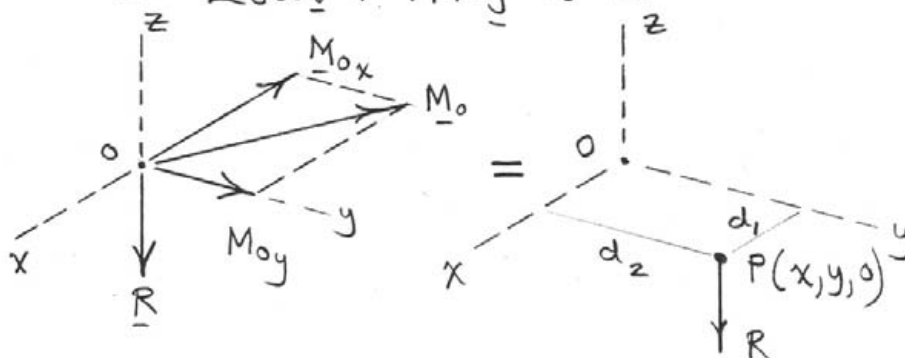
$$\underline{M} = \sum \underline{M}_A = [120(0.25) + 100(0.3) + 40]\underline{j} + 50\underline{k} \text{ N}\cdot\text{m}$$
$$= 100\underline{j} + 50\underline{k} \text{ N}\cdot\text{m}$$

2/152 At O: $\underline{R} = \Sigma \underline{F} = (200 + 400)\underline{j} = 600\underline{j} \text{ lb}$
 $\underline{M}_O = 600(8)\underline{k} + 400(3)\underline{i} = 1200\underline{i} + 4800\underline{k} \text{ lb-ft}$
 $\underline{R} \cdot \underline{M}_O = 0 \Rightarrow \underline{R} \perp \underline{M}_O$ (Loading system can be represented by single force)
 Let P have coordinates $(x, 0, z)$ and let \underline{R} act at P.
 $\underline{r}_{OP} \times \underline{R} = \underline{M}_O: (x\underline{i} + z\underline{k}) \times 600\underline{j} = 1200\underline{i} + 4800\underline{k}$
 $600x\underline{k} - 600z\underline{i} = 1200\underline{i} + 4800\underline{k}$
 $\Rightarrow \underline{x = 8 \text{ ft}, z = -2 \text{ ft}}$

$$2/153 \quad \underline{R} = \sum \underline{F} = -50\underline{k} - 20\underline{k} = \underline{-70\underline{k} \text{ lb}}$$

$$\underline{M}_o = 20(12\underline{j} - 20\underline{i}) + 50(24\underline{j} - 48\underline{i})$$

$$= -2800\underline{i} + 1440\underline{j} \text{ lb-ft}$$



Conditions on P:

$$Rd_1 = |M_{oy}|, \quad d_1 = \frac{1440}{70} = 20.6 \text{ in.}$$

$$Rd_2 = |M_{ox}|, \quad d_2 = \frac{2800}{70} = 40 \text{ in.}$$

So the coordinates of P are

$$(x, y) = \underline{(20.6, 40) \text{ in.}} \quad \left(\begin{array}{l} \text{Resultant is a} \\ \text{standalone force} \end{array} \right)$$

$$2/154 \quad R_x = -120 \text{ N}, \quad R_y = 0, \quad R_z = -160 \text{ N}$$

$$R = \sqrt{120^2 + 160^2} = 200 \text{ N}, \quad \underline{R} = -120\underline{i} - 160\underline{k} \text{ N}$$

$$M_x = 25 - 160(0.2) = -7 \text{ N}\cdot\text{m}$$

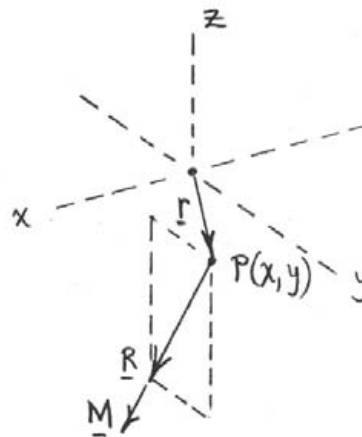
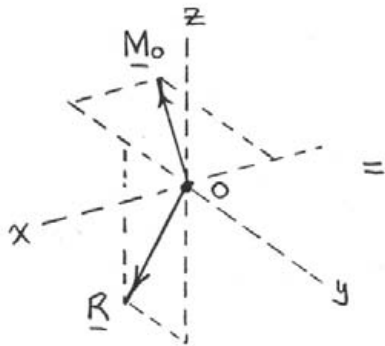
$$M_y = 160(0.075) - 120(0.100 - 0.075) = 9 \text{ N}\cdot\text{m}$$

$$M_z = 120(0.2) = 24 \text{ N}\cdot\text{m}$$

$$M = \sqrt{7^2 + 9^2 + 24^2} = 25.5 \text{ N}\cdot\text{m}$$

$$\underline{M} = -7\underline{i} + 9\underline{j} + 24\underline{k} \text{ N}\cdot\text{m}$$

$$\frac{2}{155} \left\{ \begin{array}{l} \underline{R} = -20\underline{j} - 40\underline{k} \text{ lb} \quad (= 44.7(-0.447\underline{j} - 0.894\underline{k})) \\ \underline{M}_o = -40(1.4)\underline{i} - 40(8)\underline{j} \\ \quad = -56\underline{i} - 320\underline{j} \text{ lb-in.} \end{array} \right.$$



$$\underline{M}_o: -56\underline{i} - 320\underline{j} = \underline{r} \times \underline{R} + \underline{M} = (x\underline{i} + y\underline{j}) \times (-20\underline{j} - 40\underline{k}) + M(-0.447\underline{j} - 0.894\underline{k})$$

Equate coefficients:

$$\begin{cases} \underline{i}: -56 = -40y \\ \underline{j}: -320 = 40x - 0.447M \\ \underline{k}: 0 = -20x - 0.894M \end{cases}$$

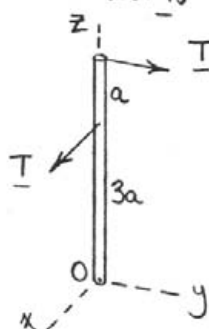
Solution:

$$x = -6.4 \text{ in.}$$

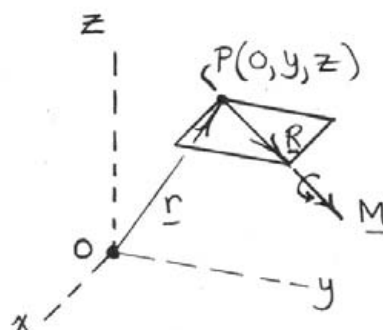
$$y = 1.4 \text{ in.}$$

$$2/156 \quad \underline{R} = \Sigma \underline{F} = T \underline{i} + T \underline{j} = \sqrt{2}T \left[\frac{1}{\sqrt{2}} \underline{i} + \frac{1}{\sqrt{2}} \underline{j} \right]$$

$$\Sigma \underline{M}_o = 3aT \underline{j} - 4aT \underline{i}$$



=



$$\Sigma \underline{M}_o = \underline{r} \times \underline{R} + \underline{M}$$

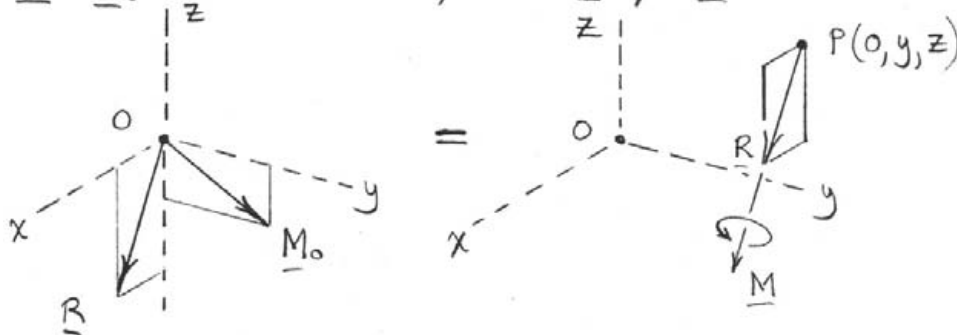
$$3aT \underline{j} - 4aT \underline{i} = (y \underline{j} + z \underline{k}) \times (T \underline{i} + T \underline{j}) + M \left(\frac{1}{\sqrt{2}} \underline{i} + \frac{1}{\sqrt{2}} \underline{j} \right)$$

$$\Rightarrow \begin{cases} -4aT = -zT + \frac{M}{\sqrt{2}} \\ 3aT = zT + \frac{M}{\sqrt{2}} \\ 0 = -yT \end{cases} \quad \text{So } \begin{cases} y = 0 \\ z = \frac{7}{2}a \\ M = -\frac{\sqrt{2}}{2}aT \end{cases}$$

$$\text{So } \underline{M} = -\frac{\sqrt{2}}{2}aT \left(\frac{1}{\sqrt{2}} \underline{i} + \frac{1}{\sqrt{2}} \underline{j} \right) \\ = \underline{\underline{-\frac{aT}{2} (\underline{i} + \underline{j})}} \quad (\text{a negative wrench})$$

2/157 At 0:
$$\begin{cases} \underline{R} = \sum \underline{F} = F\underline{i} - 3F\underline{k} = \sqrt{10} F \left(\frac{1}{\sqrt{10}} \underline{i} - \frac{3}{\sqrt{10}} \underline{k} \right) \\ \underline{M}_0 = F(2a)\underline{j} - F(a)\underline{k} = Fa(2\underline{j} - \underline{k}) \end{cases}$$

$\underline{R} \cdot \underline{M}_0 = 3F^2a \neq 0, \text{ so } \underline{R} \nparallel \underline{M}_0.$



$$\underline{M}_0 = \underline{M} + \underline{r}_{OP} \times \underline{R}$$

$$Fa(2\underline{j} - \underline{k}) = M \left(\frac{1}{\sqrt{10}} \underline{i} - \frac{3}{\sqrt{10}} \underline{k} \right) + (y\underline{j} + z\underline{k}) \times (F\underline{i} - 3F\underline{k})$$

Equate coefficients:

$$\left. \begin{aligned} \underline{i}: 0 &= -3Fy + \frac{M}{\sqrt{10}} \\ \underline{j}: 2Fa &= Fz \\ \underline{k}: -Fa &= -Fy - \frac{3M}{\sqrt{10}} \end{aligned} \right\}$$

Solution:

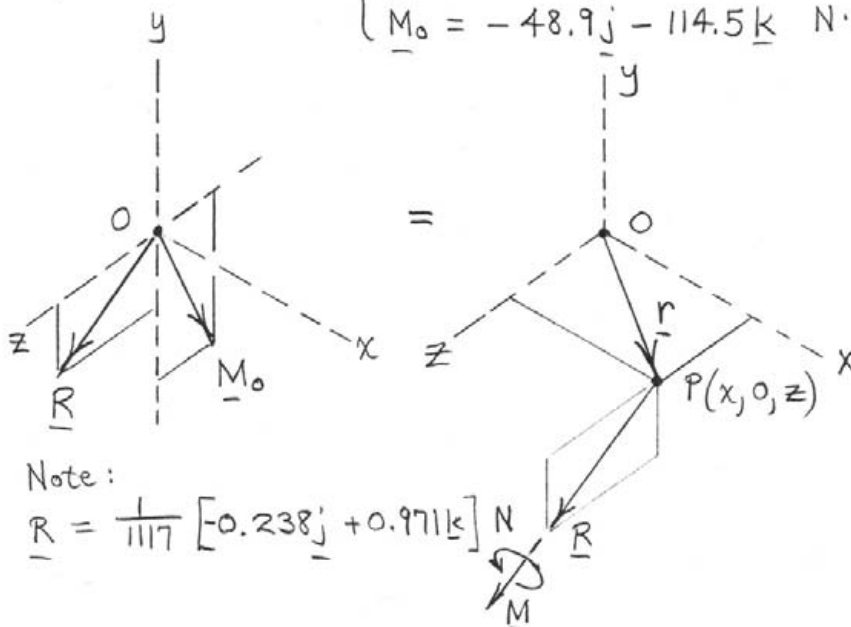
$$y = \frac{a}{10}$$

$$z = 2a$$

$$M = 3\sqrt{10}aF/10 = 0.949Fa$$

$$\underline{M} = \frac{3\sqrt{10}aF}{10} \left(\frac{1}{\sqrt{10}} \underline{i} - \frac{3}{\sqrt{10}} \underline{k} \right) = \frac{3aF}{10} (\underline{i} - 3\underline{k})$$

2/158 From Prob. 2/149, The force-couple system at O is

$$\begin{cases} \underline{R} = -266\underline{j} + 1085\underline{k} \text{ N} \\ \underline{M}_O = -48.9\underline{j} - 114.5\underline{k} \text{ N}\cdot\text{m} \end{cases}$$


Note:

$$\underline{R} = \frac{1}{1117} [-0.238\underline{j} + 0.971\underline{k}] \text{ N}$$

$$\underline{M}_O = \underline{r} \times \underline{R} + \underline{M} = (x\underline{i} + z\underline{k}) \times (-266\underline{j} + 1085\underline{k}) + M(-0.238\underline{j} + 0.971\underline{k})$$

Equate coefficients:

$$\underline{i}: 0 = 266z$$

$$\underline{j}: -48.9 = -1085x - 0.238M$$

$$\underline{k}: -114.5 = -266x + 0.971M$$

Solution:

$$x = 0.0669 \text{ m (66.9 mm)}$$

$$z = 0$$

$$M = -99.6 \text{ N}\cdot\text{m}$$

► 2/159

$$\underline{R} = \Sigma \underline{F} = 100 \underline{i} + 100 \underline{j} \text{ N}$$

Direction cosines $l = \frac{1}{\sqrt{2}}, m = \frac{1}{\sqrt{2}}, n = 0$

Let $P = P(x, 0, z)$

$$\underline{M}_P = 100z \underline{i} + 100(0.4-x) \underline{k} + 100(0.4-z) \underline{j} \\ - 100(0.3) \underline{k} - 20 \underline{j}$$

$$= 100z \underline{i} + 100(0.2-z) \underline{j} + 100(0.1-x) \underline{k} \text{ N}\cdot\text{m}$$

Let $M = |\underline{M}_P|$. Equate direction cosines of \underline{R} & \underline{M}_P to obtain

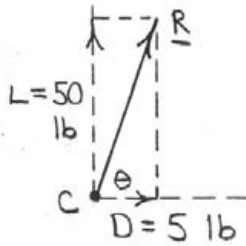
$$\frac{100z}{M} = \frac{1}{\sqrt{2}}, \quad \frac{100(0.2-z)}{M} = \frac{1}{\sqrt{2}}, \quad \frac{100(0.1-x)}{M} = 0$$

$$\text{Solution: } \begin{cases} x = 0.1 \text{ m} \\ z = 0.1 \text{ m} \\ M = 10\sqrt{2} \text{ N}\cdot\text{m} \end{cases}$$

$$\underline{M} = 10\sqrt{2} \left(\frac{1}{\sqrt{2}} \underline{i} + \frac{1}{\sqrt{2}} \underline{j} \right) = \underline{10 \underline{i} + 10 \underline{j}} \text{ N}\cdot\text{m}$$

2/16

$$\frac{L}{D} = \frac{50}{5} = 10 ; D = 5 \text{ lb}$$



$$R = \sqrt{L^2 + D^2} = \sqrt{50^2 + 5^2}$$
$$= \underline{50.2 \text{ lb}}$$

$$\theta = \tan^{-1} \left(\frac{L}{D} \right) = \tan^{-1} \left(\frac{50}{5} \right)$$
$$= \underline{84.3^\circ}$$

► 2/160 $\underline{R} = \sum \underline{F} = 400(-\cos 24^\circ \underline{j} + \sin 24^\circ \underline{i}) + 800(-\cos 24^\circ \underline{j} - \sin 24^\circ \underline{i})$

$$\underline{R} = -162.7 \underline{i} - 1096 \underline{j} \text{ N}, \quad R = \sqrt{R_x^2 + R_y^2} = 1108 \text{ N}$$

Dir. cosines: $l = -0.1468, m = -0.9892, n = 0$

$$\underline{M}_A = [(-400(0.2) - 800(0.6)) \cos 24^\circ] \underline{i} + [(-400 + 800)(0.2) \cos 24^\circ - 200] \underline{k} + [800(0.6) - 400(0.2)] \sin 24^\circ \underline{j} = -512 \underline{i} + 162.7 \underline{j} - 126.9 \underline{k} \text{ N}\cdot\text{m}$$

$$\underline{R} \cdot \underline{M}_A = -95100 \text{ N}^2\cdot\text{m} \neq 0 \Rightarrow \underline{R} \nparallel \underline{M}_A$$

Now move system to $P(x, 0, z)$ in form wrench

$$\underline{R} = -162.7 \underline{i} - 1096 \underline{j} \text{ N}$$

$$\underline{M}_P = \underline{M}_A + \underline{r}_{PA} \times \underline{R}, \text{ where } \underline{r}_{PA} = -x \underline{i} - z \underline{k}$$

$$= (-512 - 1096z) \underline{i} + (162.7z + 162.7) \underline{j} + (-126.9 + 1096x) \underline{k}$$

Let $M = |\underline{M}_P|$ & equate dir. cosines of \underline{R} and \underline{M}_P :

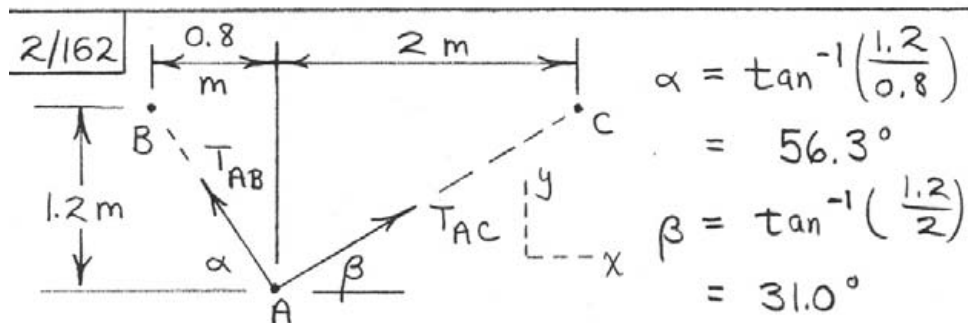
$$\frac{-512 - 1096z}{M} = -0.1468; \quad \frac{162.7z + 162.7}{M} = -0.9892; \quad \frac{-126.9 + 1096x}{M} = 0$$

Solve to obtain

$$\begin{cases} M = -85.8 \text{ N}\cdot\text{m} \\ x = 0.1158 \text{ m} \\ z = -0.478 \text{ m} \end{cases}$$

2/161

$M = Fd = 15(10) = 2F(0.25), \underline{F = 300 \text{ lb}}$



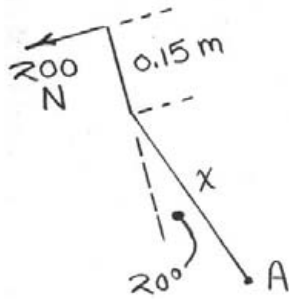
$$\begin{aligned}\underline{T}_{AB} &= T_{AB} \underline{n}_{AB} = 0.858(60)(9.81) [-\cos 56.3^\circ \underline{i} + \sin 56.3^\circ \underline{j}] \\ &= \underline{-280 \underline{i} + 420 \underline{j} \text{ N}}\end{aligned}$$

$$\begin{aligned}\underline{T}_{AC} &= T_{AC} \underline{n}_{AC} = 0.555(60)(9.81) [\cos 31.0^\circ \underline{i} + \sin 31.0^\circ \underline{j}] \\ &= \underline{280 \underline{i} + 168.1 \underline{j} \text{ N}}\end{aligned}$$

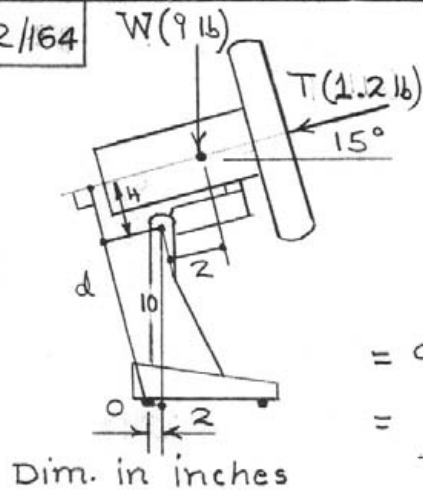
2/163

$$M_A = Fd : 80 = 200(0.15 + x \cos 20^\circ)$$

$$x = 0.266 \text{ m or } \underline{266 \text{ mm}}$$



2/164



Dim. in inches

$$M_o = Fd$$

$$= 1.2(4 + 10 \cos 15^\circ - 2 \sin 15^\circ)$$

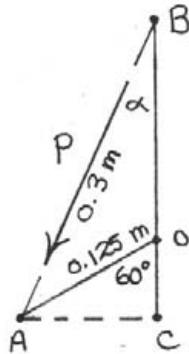
$$= 15.77 \text{ lb-in. } (\checkmark)$$

$$M_{o_w} = W d_w$$

$$= 9(2 + 2 \cos 15^\circ - 4 \sin 15^\circ)$$

$$= 26.1 \text{ lb-in. } (2)$$

2/165



$$AC = 0.125 \sin 60^\circ = 0.1083 \text{ m}$$

$$\alpha = \sin^{-1} \frac{0.1083}{0.300} = 21.2^\circ$$

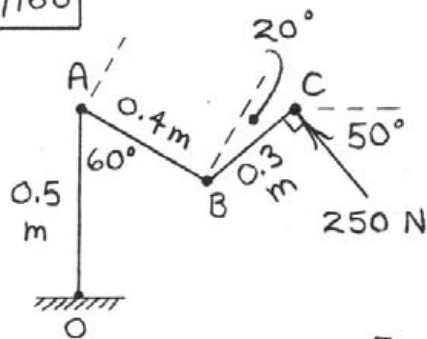
$$BC = 0.300 \cos \alpha = 0.280 \text{ m}$$

$$BO = 0.280 - 0.125 \cos 60^\circ = 0.217 \text{ m}$$

$$\curvearrowright M_O = 720 = P \sin \alpha (BO) = P \sin 21.2^\circ (0.217)$$

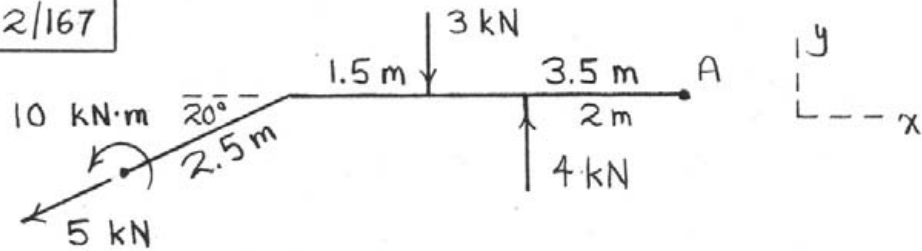
$$\underline{P = 9.18 \text{ kN}}$$

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$$\begin{aligned}
 \curvearrowright M_O &= 250 \cos 50^\circ [0.5 - 0.4 \cos 60^\circ + 0.3 \sin 40^\circ] \\
 &\quad + 250 \sin 50^\circ [0.4 \sin 60^\circ + 0.3 \cos 40^\circ] \\
 &= \underline{189.6 \text{ N}\cdot\text{m} \text{ CCW}}
 \end{aligned}$$

2/167



$$R_x = \sum F_x = -5 \cos 20^\circ = -4.70 \text{ kN}$$

$$R_y = \sum F_y = -5 \sin 20^\circ + 4 - 3 = -0.710 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{4.75 \text{ kN}}$$

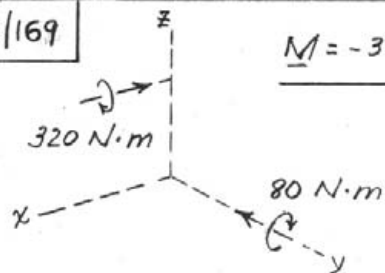
$$\begin{aligned} \sum M_A &= 10 + 5 \sin 20^\circ (5) + 3(3.5) - 4(2) \\ &= \underline{21.1 \text{ kN}\cdot\text{m} \text{ CCW}} \end{aligned}$$

$$\underline{2/168} \quad \text{At } A: R = \Sigma F = 200 + 180 - 300 = \underline{80 \text{ lb } (\downarrow)}$$

$$\Sigma M_A = 200(8) + 180(28) - 300(18) = \underline{1240 \text{ lb-in.}}$$

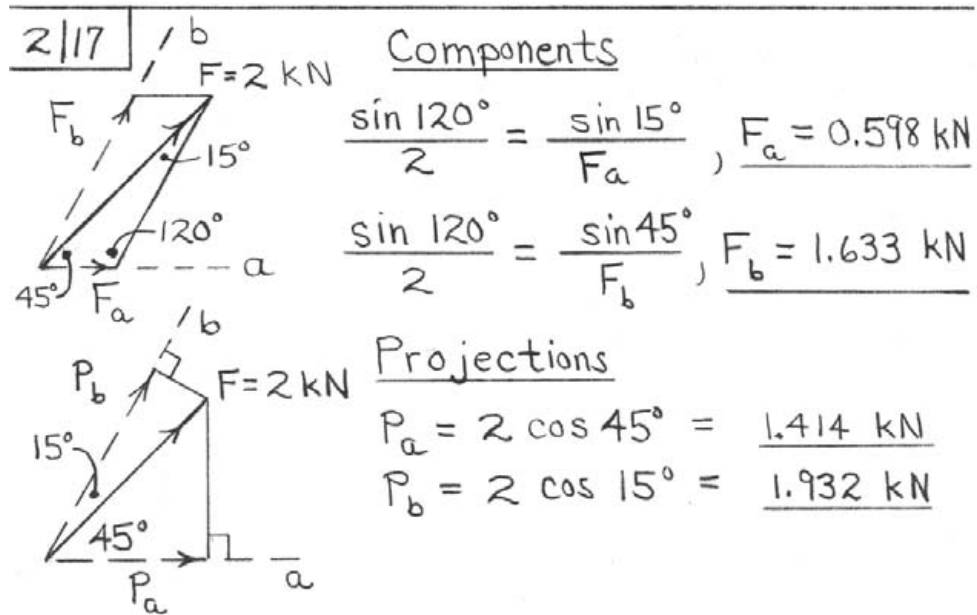
$$\begin{array}{c} \downarrow 80 \text{ lb} \\ \curvearrowleft A \text{ --- } x \\ \quad \quad \quad 1240 \text{ lb-in.} \end{array} = \begin{array}{c} \downarrow 80 \text{ lb} \\ A \text{ --- } x \text{ --- } \downarrow \\ \quad \quad \quad 80x = 1240, \quad \underline{x = 15.5 \text{ in.}} \end{array}$$

2/169



$$\underline{M} = -320\underline{i} - 80\underline{j} \text{ N}\cdot\text{m}$$

$$\cos \theta_x = \frac{M_x}{|\underline{M}|} = \frac{-320}{\sqrt{320^2 + 80^2}} = -0.970$$



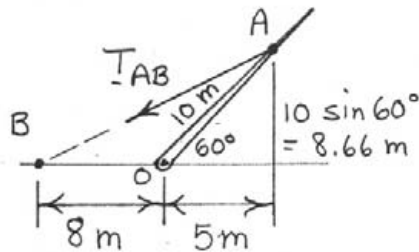
$$\underline{2/170} \quad \underline{P} = P \left(\frac{4}{5} \underline{i} + \frac{3}{5} \underline{j} \right) ; \quad \underline{r}_{AB} = b(-\underline{i} + \underline{j} + \underline{k})$$

Carry out $\underline{M}_A = \underline{r}_{AB} \times \underline{P}$ to obtain

$$\underline{M}_A = \frac{Pb}{5} (-3\underline{i} + 4\underline{j} - 7\underline{k})$$

$$\overline{AB}^2 = 8^2 + 10^2 - 2(8)(10) \cos 120^\circ$$

$$\overline{AB} = 15.62 \text{ m}$$



$$\begin{aligned} \text{(a)} \quad \underline{T}_{AB} &= 3 \left[-\frac{13}{15.62} \cos 35^\circ \underline{i} - \frac{13}{15.62} \sin 35^\circ \underline{j} \right. \\ &\quad \left. - \frac{8.66}{15.62} \underline{k} \right] \\ &= -2.05 \underline{i} - 1.432 \underline{j} - 1.663 \underline{k} \text{ kN} \end{aligned}$$

(b) Carry out $\underline{r}_{OB} \times \underline{T}_{AB}$, where $\underline{r}_{OB} = 8(-\cos 35^\circ \underline{i} - \sin 35^\circ \underline{j}) \text{ m}$ to obtain

$$\underline{M}_O = 7.63 \underline{i} - 10.90 \underline{j} \text{ kN}\cdot\text{m}$$

$$\therefore \underline{M}_{Ox} = 7.63 \text{ kN}\cdot\text{m}, \underline{M}_{Oy} = -10.90 \text{ kN}\cdot\text{m}, \underline{M}_{Oz} = 0$$

$$\text{(c)} \quad T_{A0} = \underline{T}_{AB} \cdot \underline{n}_{A0}$$

With $\underline{n}_{A0} = -\cos 60^\circ \cos 35^\circ \underline{i} - \cos 60^\circ \sin 35^\circ \underline{j} - \sin 60^\circ \underline{k}$,

we obtain $\underline{T}_{A0} = 2.69 \text{ kN}$

2/172 Coordinates of A: $(x_A, y_A, z_A) = (0, r, 0)$
 Coordinates of B: $(x_B, y_B, z_B) = (h, r \cos \theta, r \sin \theta)$
 So $\underline{r}_{AB} = h \underline{i} + (r \cos \theta - r) \underline{j} + r \sin \theta \underline{k}$
 and $\underline{F} = F \underline{n}_{AB} = F \left[\frac{h \underline{i} + r (\cos \theta - 1) \underline{j} + r \sin \theta \underline{k}}{\sqrt{h^2 + [r (\cos \theta - 1)]^2 + [r \sin \theta]^2}} \right]$

$$= F \left[\frac{h \underline{i} + r (\cos \theta - 1) \underline{j} + r \sin \theta \underline{k}}{\sqrt{h^2 + 2r^2 (1 - \cos \theta)}} \right]$$

$$\underline{2/173} \quad \underline{M}_{100} = -100 (0.200) \underline{i} = -20 \underline{i} \text{ N}\cdot\text{m}$$

$$\begin{aligned} \underline{M}_{80} &= 80 (0.180 \cos 20^\circ) (-\underline{j} \sin 30^\circ - \underline{k} \cos 30^\circ) \\ &= -6.77 \underline{j} - 11.72 \underline{k} \text{ N}\cdot\text{m} \end{aligned}$$

$$\underline{M}_{120} = -120 (0.300 \cos 45^\circ) \underline{k} = -25.5 \underline{k} \text{ N}\cdot\text{m}$$

$$\text{So } \underline{M} = -20 \underline{i} - 6.77 \underline{j} - 37.2 \underline{k} \text{ N}\cdot\text{m}$$

$$\begin{aligned} \underline{2/174} \quad \underline{\underline{R}} &= 800 \left[-\sin 30^\circ \cos 20^\circ \underline{i} + \sin 30^\circ \sin 20^\circ \underline{j} \right. \\ &\quad \left. + \cos 30^\circ \underline{k} \right] \\ &= \underline{-376 \underline{i} + 136.8 \underline{j} + 693 \underline{k} \quad \text{N}} \end{aligned}$$

$$\underline{M}_O = \underline{r}_{OB} \times \underline{F}$$

$$\underline{r}_{OB} = [300 \sin 20^\circ \underline{i} + 300 \cos 20^\circ \underline{j} + 250 \underline{k}] \text{ mm}$$

$$\underline{M}_O = 161.1 \underline{i} - 165.1 \underline{j} + 120 \underline{k} \quad \text{N}\cdot\text{m}$$

$$\underline{2/175} \quad \underline{R = \Sigma F = 500 \cos 45^\circ (\underline{i}) + 400 \underline{j} - (600 + 500 \sin 45^\circ) \underline{k}}$$

$$= 354 \underline{i} + 400 \underline{j} - 954 \underline{k} \quad \text{lb}$$

$$R = \sqrt{354^2 + 400^2 + 954^2} = \underline{1093 \text{ lb}}$$

$$\underline{M} = [500 \cos 45^\circ (3) - 600(3) - 400(10)] \underline{i}$$

$$+ [500 \cos 45^\circ (6) + 500 \sin 45^\circ (7) + 600(6)] \underline{j}$$

$$+ [500 \sin 45^\circ (3) + 400(3)] \underline{k}$$

$$= -4739 \underline{i} + 8196 \underline{j} + 2261 \underline{k} \quad \text{lb-ft}$$

$$M = \sqrt{4739^2 + 8196^2 + 2261^2} = \underline{9730 \text{ lb-ft}}$$

*2/176

$$\Sigma F_x = 0: -360 - 240 \sin \theta + T \sin 30^\circ + 400 \cos 30^\circ = 0 \quad (1)$$

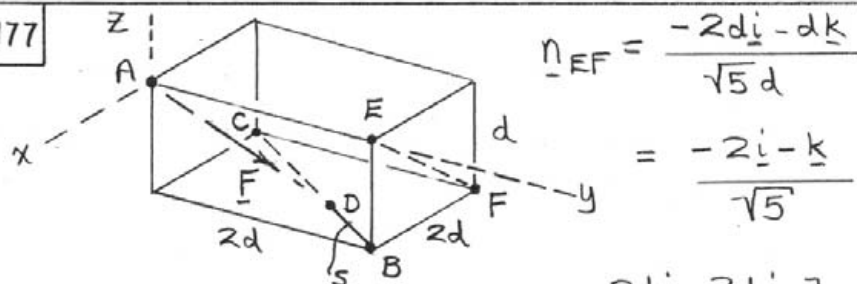
$$\Sigma F_y = 600: 240 \cos \theta + T \cos 30^\circ + 400 \sin 30^\circ = 600 \quad (2)$$

Numerical solution of Eqs. (1) & (2):

$$\theta = 21.7^\circ, \quad T = 204 \text{ lb}$$

(We could eliminate T between Eqs. (1) & (2),
but the resulting equation is still transcendental.)

*2/177



$$\underline{AD} = \underline{AB} + \underline{BD} = 2d\mathbf{j} - d\mathbf{k} + s \left[\frac{-2d\mathbf{i} - 2d\mathbf{j}}{\sqrt{8}d} \right]$$

$$= -\frac{s}{\sqrt{2}}\mathbf{i} + \left(2d - \frac{s}{\sqrt{2}}\right)\mathbf{j} - d\mathbf{k}$$

$$|\underline{AD}| = \sqrt{\frac{s^2}{2} + \left(2d - \frac{s}{\sqrt{2}}\right)^2 + d^2} = \sqrt{s^2 + 5d^2 - 2\sqrt{2}ds}$$

$$\underline{F} = F \frac{\underline{AD}}{|\underline{AD}|} = F \frac{-\frac{s}{\sqrt{2}}\mathbf{i} + \left(2d - \frac{s}{\sqrt{2}}\right)\mathbf{j} - d\mathbf{k}}{\sqrt{s^2 + 5d^2 - 2\sqrt{2}ds}}$$

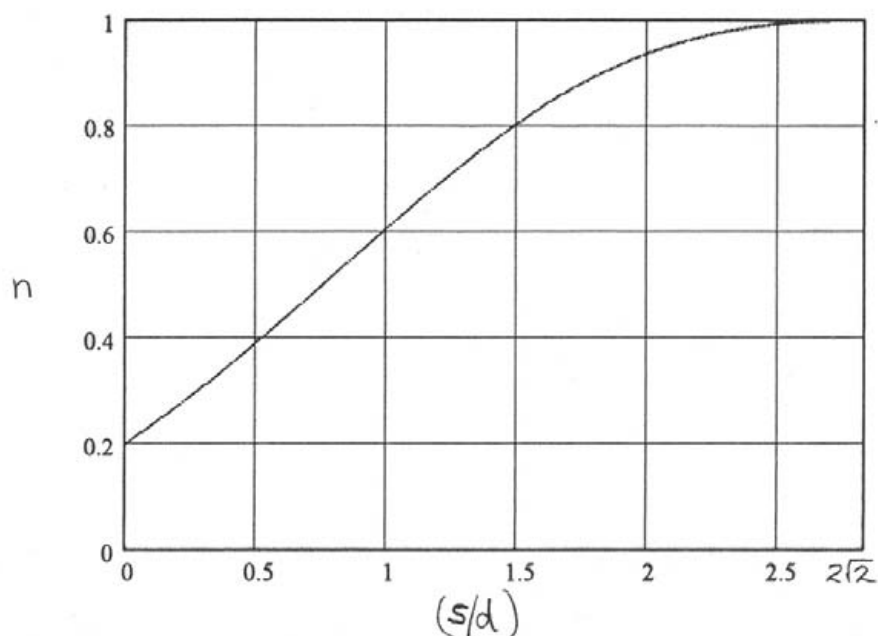
Carry out $\underline{F} \cdot \underline{n}_{EF}$ to obtain the projection

$$\underline{F} \cdot \underline{n}_{EF} = \frac{F(s\sqrt{2} + d)}{\sqrt{5} \sqrt{s^2 + 5d^2 - 2\sqrt{2}ds}}$$

The nondimensionalized fraction n of the magnitude F projected is then

$$n = \frac{F \cdot n_{EF}}{F} = \frac{\sqrt{2} \frac{s}{d} + 1}{\sqrt{5} \sqrt{\left(\frac{s}{d}\right)^2 + 5} - 2\sqrt{2} \frac{s}{d}}$$

We let $\frac{s}{d}$ vary from 0 to $2\sqrt{2}$ as D moves from B to C. Resulting plot:



*2/178

$$\underline{T} = T \underline{n}_{AB}$$

y
x

$$\underline{T} = T \left[\frac{(d + 40 \cos \beta) \underline{i} + 40(1 - \sin \beta) \underline{j}}{\sqrt{(d + 40 \cos \beta)^2 + 40^2(1 - \sin \beta)^2}} \right]$$

$$\underline{r}_{OB} = (d \underline{i} + 40 \underline{j})$$

$$(\beta = \theta + \frac{\pi}{4})$$

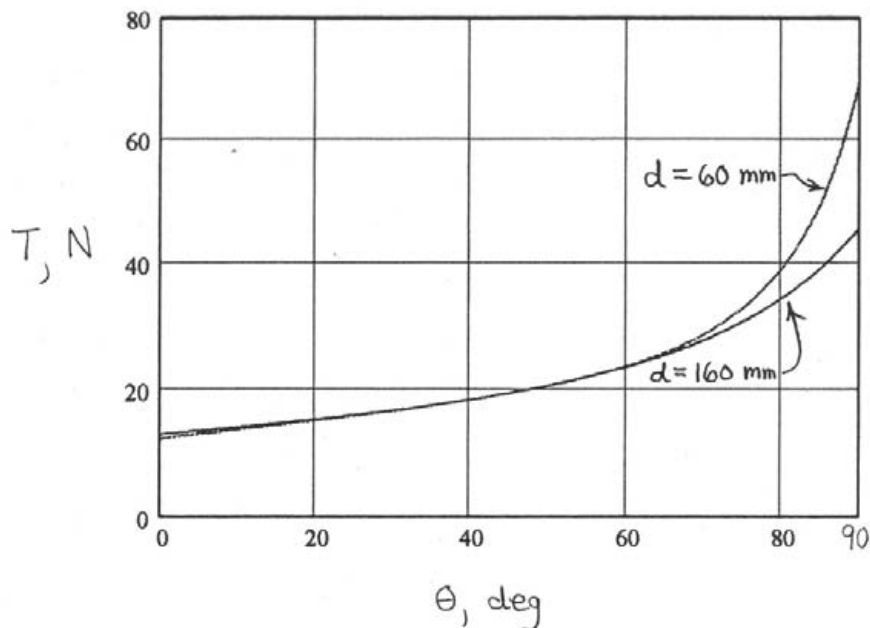
$$\sum \underline{M}_O = \underline{0}: \quad \underline{r}_{OB} \times \underline{T} + K \beta \underline{k} = \underline{0}$$

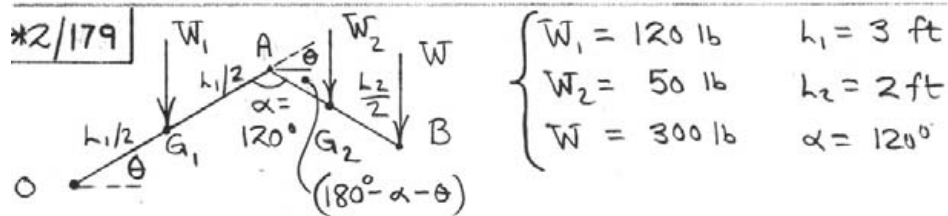
Carry out the cross product, consider the z-component, and solve for T to obtain

$$T = \frac{12.5(\theta + \frac{\pi}{4}) \sqrt{d^2 + 80d \cos(\theta + \frac{\pi}{4}) - 3200 \sin(\theta + \frac{\pi}{4}) + 3200}}{\left[d \sin(\theta + \frac{\pi}{4}) + 40 \cos(\theta + \frac{\pi}{4}) \right]}$$

(T in N, θ in radians)

Plot of T versus θ for $d = 60$ mm and
for $d = 160$ mm:





$$+2 M_o = W_1 \frac{l_1}{2} \cos \theta + W_2 \left(l_1 \cos \theta + \frac{l_2}{2} \cos (180^\circ - \alpha - \theta) \right) + W \left(l_1 \cos \theta + l_2 \cos (180^\circ - \alpha - \theta) \right)$$

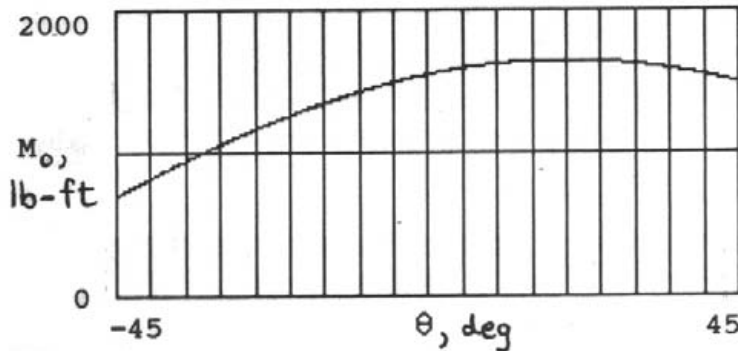
With the above numbers :

$$M_o = 1230 \cos \theta + 650 \cos (60^\circ - \theta) \quad (\text{in lb-ft})$$

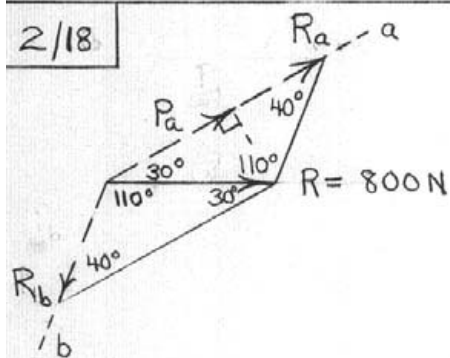
(see plot below)

For $(M_o)_{\max}$: $\frac{dM_o}{d\theta} = -1230 \sin \theta + 650 \sin (60^\circ - \theta) = 0$

Numerical solution : $\theta = 19.90^\circ$, $(M_o)_{\max} = 1654 \text{ lb-ft}$



2/18



Law of sines :

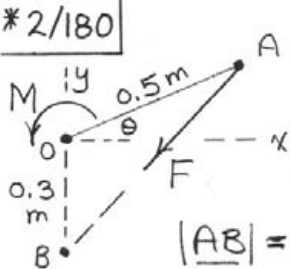
$$\frac{800}{\sin 40^\circ} = \frac{R_a}{\sin 110^\circ} = \frac{R_b}{\sin 30^\circ}$$

$$\underline{R_a = 1170 \text{ N}}$$

$$\underline{R_b = 622 \text{ N}}$$

Projection $P_a = R \cos 30^\circ = 800 \cos 30^\circ = \underline{693 \text{ N}}$

*2/180



$$\underline{F} = F \underline{n}_{AB} = F \frac{\underline{AB}}{|\underline{AB}|}$$

$$\underline{AB} = -0.5 \cos \theta \underline{i} - (0.5 \sin \theta + 0.3) \underline{j} \text{ m}$$

$$|\underline{AB}| = \sqrt{(0.5 \cos \theta)^2 + (0.5 \sin \theta + 0.3)^2}$$

$$= \sqrt{0.34 + 0.3 \sin \theta} \text{ m}$$

$$F = k\delta = 600 \left[\sqrt{0.34 + 0.3 \sin \theta} - 0.65 \right] \text{ N}$$

So

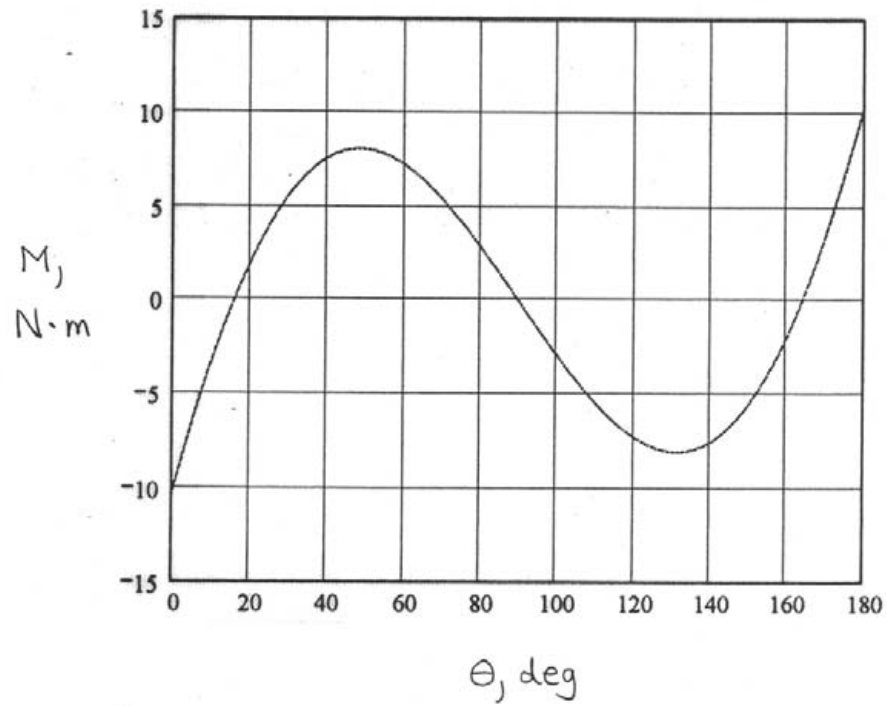
$$\underline{F} = \frac{600 \left[\sqrt{0.34 + 0.3 \sin \theta} - 0.65 \right]}{\sqrt{0.34 + 0.3 \sin \theta}} \left[-0.5 \cos \theta \underline{i} - (0.5 \sin \theta + 0.3) \underline{j} \right]$$

Now form $\underline{r}_{OB} \times \underline{F}$, where $\underline{r}_{OB} = -0.3 \underline{j} \text{ m}$,
to obtain $-\frac{90 \cos \theta (\sqrt{0.34 + 0.3 \sin \theta} - 0.65)}{\sqrt{0.34 + 0.3 \sin \theta}} \underline{k}$

With the above moment plus $M \underline{k}$ summing to zero, we obtain the scalar

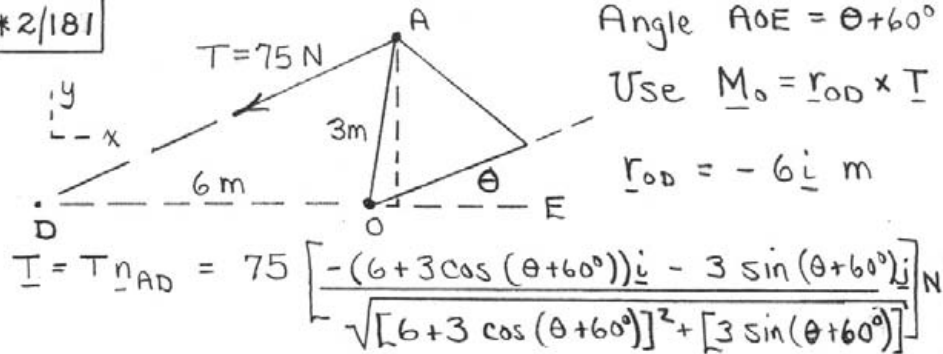
$$M = \frac{90 \cos \theta \left[\sqrt{0.34 + 0.3 \sin \theta} - 0.65 \right]}{\sqrt{0.34 + 0.3 \sin \theta}}$$

Plot:



(Note: $M(0) = -10.33 \text{ N}\cdot\text{m}$
 $M(180^\circ) = 10.33 \text{ N}\cdot\text{m}$)

*2/181



Angle $\angle AOE = \theta + 60^\circ$

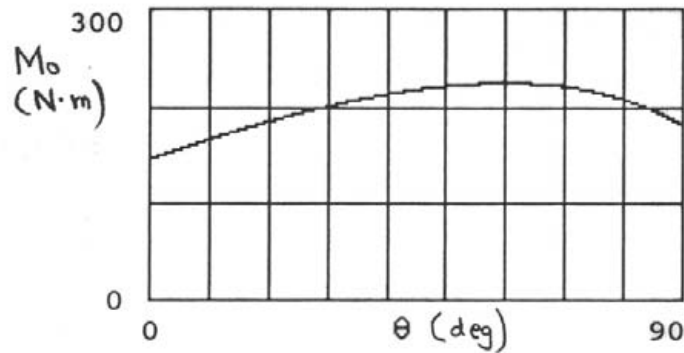
Use $\underline{M}_O = \underline{r}_{OD} \times \underline{T}$

$$\underline{r}_{OD} = -6 \underline{i} \text{ m}$$

$$\underline{T} = T \underline{n}_{AD} = 75 \left[\frac{-(6 + 3 \cos(\theta + 60^\circ)) \underline{i} - 3 \sin(\theta + 60^\circ) \underline{j}}{\sqrt{[6 + 3 \cos(\theta + 60^\circ)]^2 + [3 \sin(\theta + 60^\circ)]^2}} \right] \text{ N}$$

$$\underline{M}_O = \underline{r}_{OD} \times \underline{T} = \frac{1350 \sin(\theta + 60^\circ) \underline{k}}{\sqrt{45 + 36 \cos(\theta + 60^\circ)}} \text{ N}\cdot\text{m}$$

M_O is a max @ $\theta = 60^\circ$: $M_O = 225 \text{ N}\cdot\text{m}$



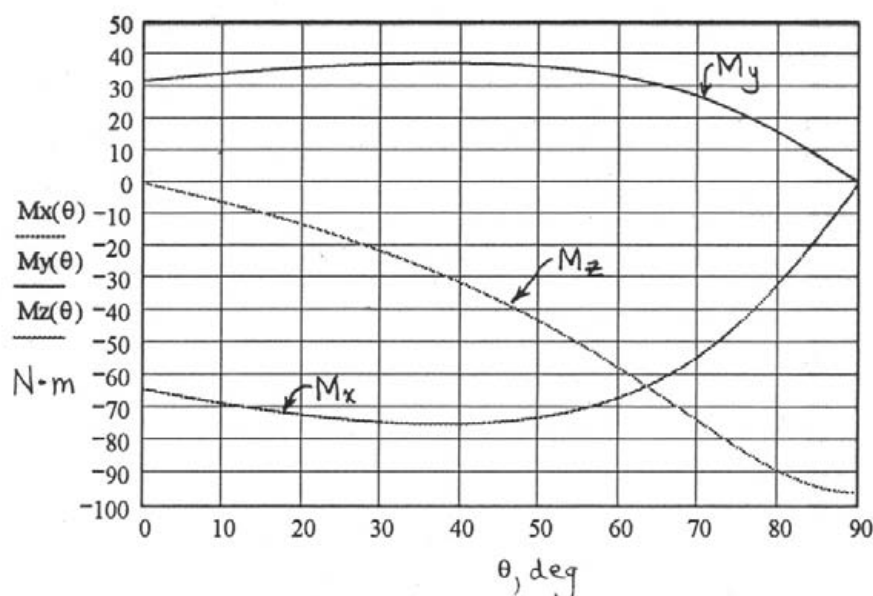
$$\begin{aligned}
 \text{*2/182} \quad \underline{T} &= T \underline{n}_{AB} = T \frac{\underline{AB}}{|\underline{AB}|} \\
 &= 120 \left[\frac{0.4\underline{i} + 0.8(1-\sin\theta)\underline{j} - 0.8\cos\theta\underline{k}}{\sqrt{0.4^2 + 0.8^2(1-\sin\theta)^2 + 0.8^2\cos^2\theta}} \right] \\
 &= \frac{48\underline{i} + 96(1-\sin\theta)\underline{j} - 96\cos\theta\underline{k}}{\sqrt{1.44 - 1.28\sin\theta}} \text{ N}
 \end{aligned}$$

$$\underline{r}_{OB} = 0.4\underline{i} + 0.8\underline{j} \text{ m}$$

Carry out $\underline{M}_O = \underline{r}_{OB} \times \underline{T}$ to obtain

$$\underline{M}_O = \frac{-76.8\cos\theta\underline{i} + 38.4\cos\theta\underline{j} - 38.4\sin\theta\underline{k}}{\sqrt{1.44 - 1.28\sin\theta}}$$

The \underline{i} , \underline{j} , & \underline{k} - components of \underline{M}_O are shown in the following plot:



$$\text{*2/183} \quad \underline{BC} = -1.2 \cos \theta \underline{i} + 1.2(1 - \sin \theta) \underline{j} - 2.4 \underline{k} \text{ ft}$$

$$|\underline{BC}| = \sqrt{(1.2 \cos \theta)^2 + 1.2^2(1 - \sin \theta)^2 + 2.4^2}$$

$$= \sqrt{8.64 - 2.88 \sin \theta} \text{ ft}$$

$$F = k\delta = 180 (\sqrt{8.64 - 2.88 \sin \theta} - 2.4)$$

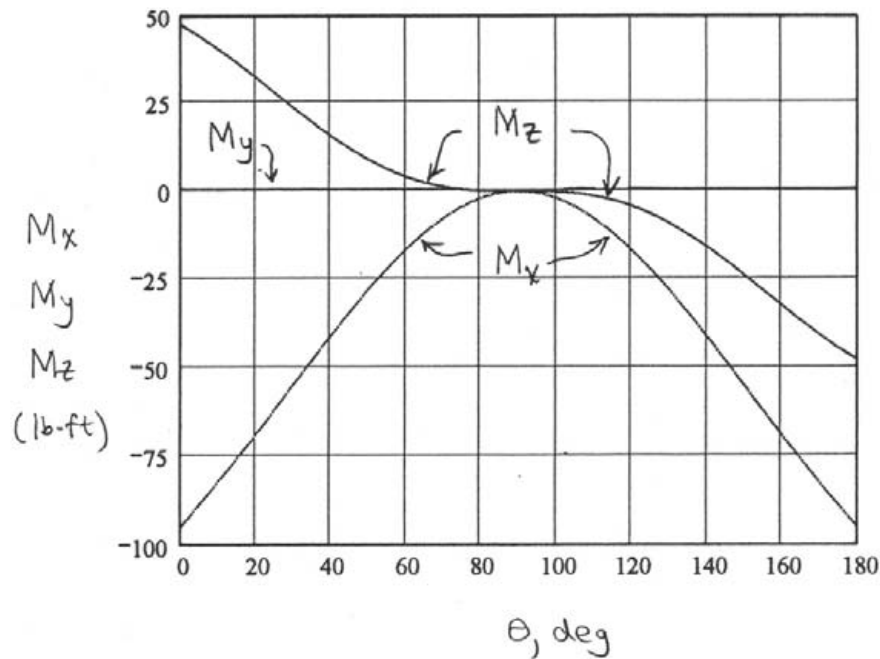
$$\underline{F} = F \underline{n}_{BC} = F \frac{\underline{BC}}{|\underline{BC}|}$$

$$= \frac{180 [\sqrt{8.64 - 2.88 \sin \theta} - 2.4]}{\sqrt{8.64 - 2.88 \sin \theta}} \left[-1.2 \cos \theta \underline{i} + 1.2(1 - \sin \theta) \underline{j} - 2.4 \underline{k} \right] \text{ lb}$$

Now, carry out $\underline{M}_O = \underline{r}_{Oc} \times \underline{F}$, where $\underline{r}_{Oc} = 1.2 \underline{j}$ ft, to obtain

$$\underline{M}_O = \frac{\sqrt{8.64 - 2.88 \sin \theta} - 2.4}{\sqrt{8.64 - 2.88 \sin \theta}} \left[-518 \underline{i} + 259 \cos \theta \underline{k} \right] \text{ lb-ft}$$

Plots of the moment components:



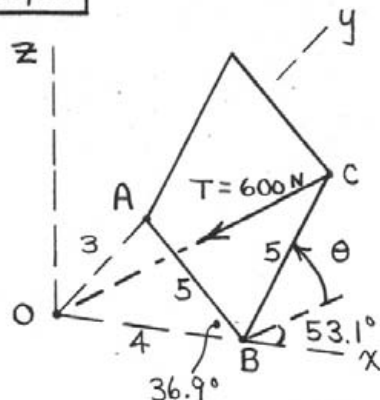
Maximum absolute values:

For M_x : 95.1 lb-ft @ $\theta = 0$ & $\theta = \pi$

For M_z : 47.6 lb-ft @ $\theta = 0$ & $\theta = \pi$

($M_y = 0$ for all θ)

* 2/184



$$C = C(4 + 5 \cos 53.1^\circ \cos \theta, 5 \sin 53.1^\circ \cos \theta, 5 \sin \theta) \text{ m}$$

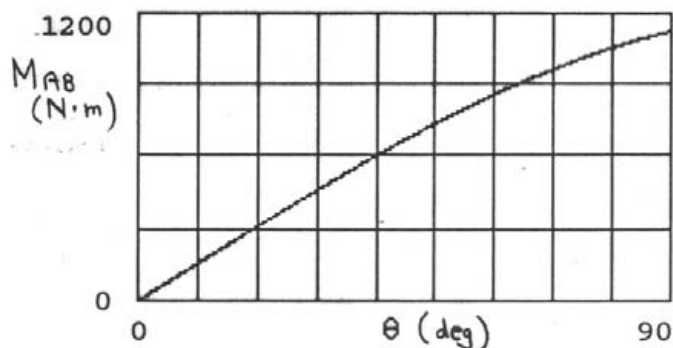
$$\underline{n}_{AB} = 0.8 \underline{i} - 0.6 \underline{j}$$

$$\underline{T} = T \underline{n}_{co} = 600 \left[\frac{-(4 + 3 \cos \theta) \underline{i} - (4 \cos \theta) \underline{j} - (5 \sin \theta) \underline{k}}{\sqrt{(4 + 3 \cos \theta)^2 + (4 \cos \theta)^2 + (5 \sin \theta)^2}} \right]$$

$$= \frac{600 [-(4 + 3 \cos \theta) \underline{i} - (4 \cos \theta) \underline{j} - (5 \sin \theta) \underline{k}]}{\sqrt{41 + 24 \cos \theta}} \text{ N}$$

$$\underline{M}_B = \underline{r}_{OB} \times \underline{T} = \frac{600}{\sqrt{41 + 24 \cos \theta}} (-20 \sin \theta \underline{j} + 16 \cos \theta \underline{k}) \text{ N}\cdot\text{m}$$

$$\text{Now, } M_{AB} = \underline{M}_B \cdot \underline{n}_{AB} = \frac{7200 \sin \theta}{\sqrt{41 + 24 \cos \theta}} \text{ N}\cdot\text{m}$$



2/19

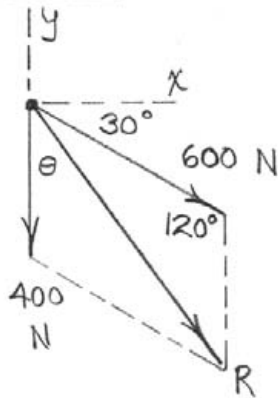
Law of cosines:

$$R^2 = 400^2 + 600^2 - 2(400)(600)\cos 120^\circ$$

$$R = 872 \text{ N}$$

Law of sines:

$$\frac{600}{\sin \theta} = \frac{872}{\sin 120^\circ}, \quad \theta = 36.6^\circ$$



$$R_x = \sum F_x = 600 \cos 30^\circ = 520 \text{ N}$$

$$R_y = \sum F_y = -600 \sin 30^\circ - 400 = -700 \text{ N}$$

$$\text{So } \underline{R} = 520\mathbf{i} - 700\mathbf{j} \text{ N}$$

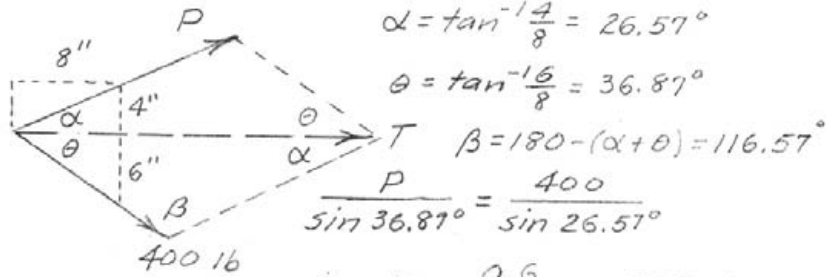
$$\left(\begin{array}{l} \text{Check: } R = \sqrt{520^2 + 700^2} = 872 \text{ N } \checkmark \\ \theta = \tan^{-1} \frac{520}{700} = 36.6^\circ \checkmark \end{array} \right)$$

$$\begin{aligned} \underline{2/2} \quad \underline{\underline{F}} &= 600 (\cos 30^\circ \underline{i} - \sin 30^\circ \underline{j}) \\ &= \underline{520 \underline{i} - 300 \underline{j} \text{ lb}} \end{aligned}$$

$$\text{Scalar components: } \begin{cases} F_x = 520 \text{ lb} \\ F_y = -300 \text{ lb} \end{cases}$$

$$\text{Vector components: } \begin{cases} \underline{F_x} = 520 \underline{i} \text{ lb} \\ \underline{F_y} = -300 \underline{j} \text{ lb} \end{cases}$$

2/20

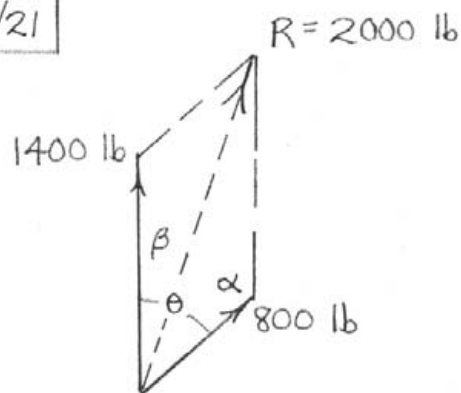


$$\frac{P}{\sin 36.87^\circ} = \frac{400}{\sin 26.57^\circ}$$

$$P = 400 \frac{0.6}{0.4472} = 537 \text{ lb}$$

$$\frac{T}{\sin 116.56^\circ} = \frac{400}{\sin 26.57^\circ}, T = 400 \frac{0.8944}{0.4472} = 800 \text{ lb}$$

2/21



Law of cosines: $2000^2 = 1400^2 + 800^2 - 2(1400)(800)\cos\alpha$

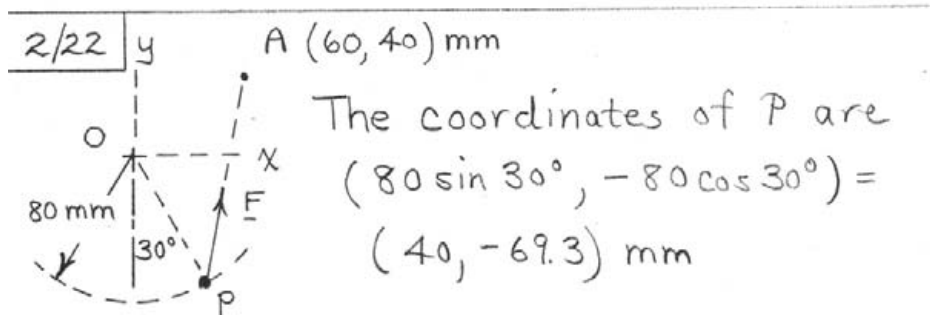
With $\alpha = 180^\circ - \theta$ and $\cos(180^\circ - \theta) = -\cos\theta$:

$$2000^2 = 1400^2 + 800^2 + 2(1400)(800)\cos\theta$$

$$\theta = 51.3^\circ$$

Law of sines: $\frac{800}{\sin\beta} = \frac{2000}{\sin(180^\circ - 51.3^\circ)}$

$$\beta = 18.19^\circ$$



$$\underline{n}_{PA} = \frac{\underline{PA}}{\overline{PA}} = \frac{(60-40)\underline{i} + (40 - (-69.3))\underline{j}}{\sqrt{20^2 + 109.3^2}}$$

$$= 0.1800\underline{i} + 0.984\underline{j}$$

The magnitude of \underline{F} is

$$F = kx = 1200 \left[\frac{\overline{PA} - 100}{1000} \right] = 1200 \left[\frac{111.1 - 100}{1000} \right]$$

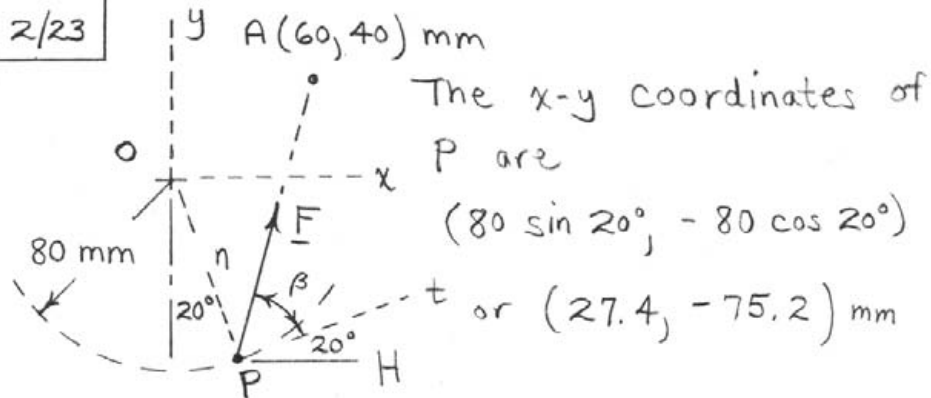
$$= 13.32 \text{ N}$$

$$\text{Then } \underline{F} = F \underline{n}_{PA} = 13.32 (0.1800\underline{i} + 0.984\underline{j})$$

$$= 2.40\underline{i} + 13.10\underline{j} \text{ N}$$

$$\text{So } \begin{cases} F_x = 2.40 \text{ N} \\ F_y = 13.10 \text{ N} \end{cases}$$

2/23



$$\overline{PA} = [(60 - 27.4)^2 + (40 - (-75.2))^2]^{1/2} = 119.7 \text{ mm}$$

$$F = kx = 1200 \left[\frac{119.7 - 100}{1000} \right] = 23.7 \text{ N}$$

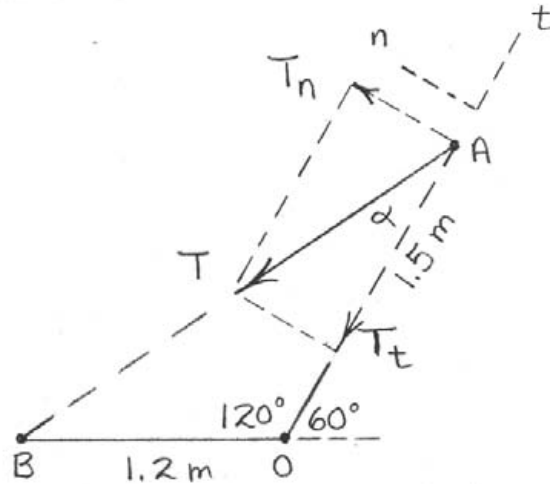
The angle APH is

$$\alpha = \tan^{-1} \left[\frac{40 + 75.2}{60 - 27.4} \right] = 74.2^\circ$$

$$\text{Then } \beta = \alpha - 20^\circ = 74.2 - 20 = 54.2^\circ$$

$$\begin{cases} F_t = F \cos \beta = 23.7 \cos 54.2^\circ = \underline{13.84 \text{ N}} \\ F_n = F \sin \beta = 23.7 \sin 54.2^\circ = \underline{19.18 \text{ N}} \end{cases}$$

2/24	n	t
------	---	---



$$\overline{AB}^2 = 1.2^2 + 1.5^2 - 2(1.2)(1.5) \cos 120^\circ$$

$$\overline{AB} = 2.34 \text{ m}$$

$$\frac{\sin \alpha}{1.2} = \frac{\sin 120^\circ}{2.34} \quad \alpha = 26.3^\circ$$

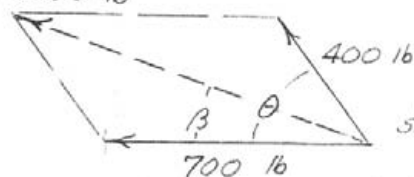
$$T_n = T \sin \alpha = 750 \sin 26.3^\circ = 333 \text{ N}$$

$$T_t = -T \cos \alpha = -750 \cos 26.3^\circ = -672 \text{ N}$$

2/25

Law of cosines: $1000^2 = 400^2 + 700^2 + 2(400)(700)\cos\theta$

$$\cos\theta = 0.6250, \theta = 51.3^\circ$$

 $R = 1000 \text{ lb}$ 

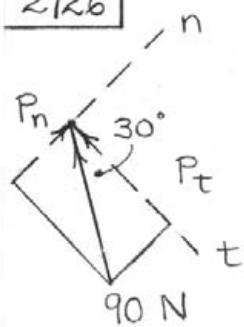
Law of sines:

$$\frac{1000}{\sin(180 - 51.3)^\circ} = \frac{400}{\sin\beta}$$

$$\sin\beta = \frac{400}{1000} 0.7806 = 0.312$$

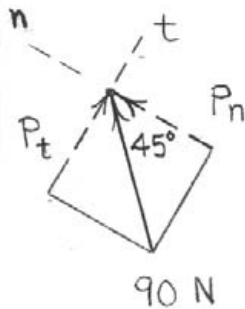
$$\beta = 18.19^\circ$$

2/26

BC

$$P_t = -90 \cos 30^\circ = \underline{-77.9 \text{ N}}$$

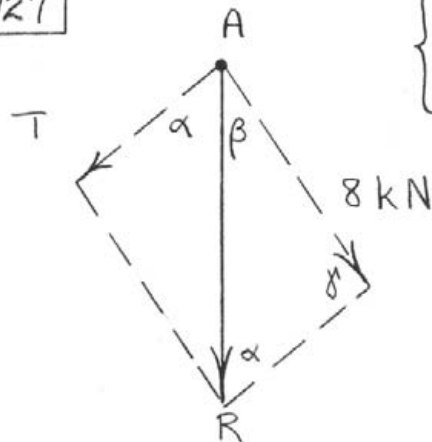
$$P_n = 90 \sin 30^\circ = \underline{45.0 \text{ N}}$$

AB

$$P_t = 90 \sin 45^\circ = \underline{63.6 \text{ N}}$$

$$P_n = 90 \cos 45^\circ = \underline{63.6 \text{ N}}$$

2/27



$$\begin{cases} \alpha = \tan^{-1} \frac{50}{40} = 51.3^\circ \\ \beta = \tan^{-1} \frac{40}{60} = 33.7^\circ \end{cases}$$

$$\gamma = 180 - \alpha - \beta = 95.0^\circ$$

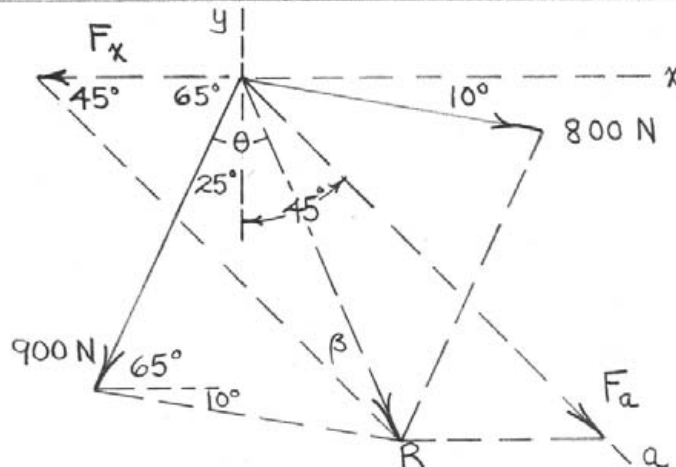
$$\frac{\sin \beta}{T} = \frac{\sin \alpha}{8}$$

$$\underline{T = 5.68 \text{ kN}}$$

$$\frac{\sin \gamma}{R} = \frac{\sin \alpha}{8}$$

$$\underline{R = 10.21 \text{ kN}}$$

2/28



Law of cosines: $R = \sqrt{900^2 + 800^2 - 2(900)(800)\cos 75^\circ} = 1038 \text{ N}$

Law of sines: $\frac{1038}{\sin 75^\circ} = \frac{800}{\sin \theta}$, $\theta = 48.1^\circ$

$\beta = 180^\circ - 45^\circ - (65^\circ + 48.1^\circ) = 21.9^\circ$

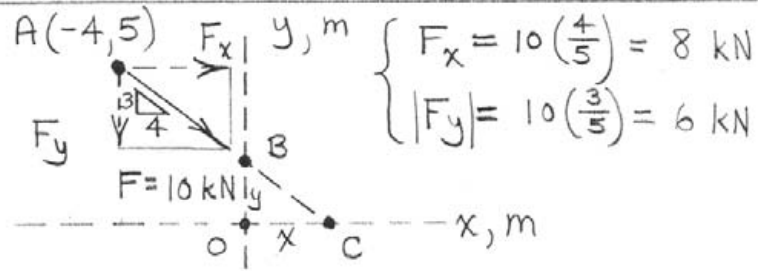
$\frac{1038}{\sin 45^\circ} = \frac{F_x}{\sin 21.9^\circ}$

$F_x = 547 \text{ N}$

$\frac{F_a}{\sin(65^\circ + 48.1^\circ)} = \frac{1038}{\sin 45^\circ}$

$F_a = 1350 \text{ N}$

2/29



$$\rightarrow M_O = 8(5) - 6(4) = \underline{16 \text{ kN}\cdot\text{m} \text{ CW}}$$

By similar triangles, $\frac{4+x}{5} = \frac{4}{3}$, $x = 2.67 \text{ m}$

$$\frac{y}{2.67} = \frac{3}{4}, \quad y = 2 \text{ m}$$

So the two intercept points are $(x, y) =$
 $(2.67, 0) \text{ m}$ and $(0, 2) \text{ m}$.

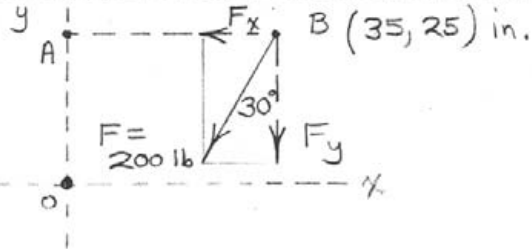
2/3

$$\begin{aligned}\underline{F} &= 4.8 \left(-\frac{3}{5} \underline{i} - \frac{4}{5} \underline{j} \right) \\ &= -2.88 \underline{i} - 3.84 \underline{j} \text{ kN}\end{aligned}$$

2/30

$$|F_x| = 200 \sin 30^\circ \\ = 100 \text{ lb}$$

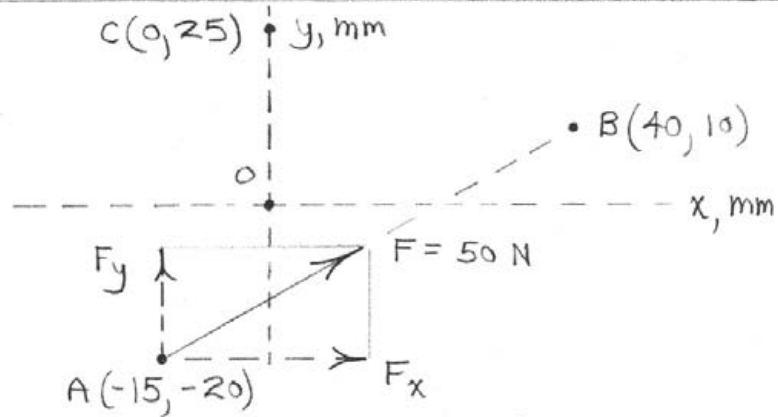
$$|F_y| = 200 \cos 30^\circ \\ = 173.2 \text{ lb}$$



$$+\curvearrowright M_A = 173.2(35) = 6060 \text{ lb-in. (505 lb-ft)} \\ \text{CW}$$

$$+\curvearrowright M_O = 173.2(35) - 100(25) \\ = 3560 \text{ lb-in. (297 lb-ft) CW}$$

2/31



$$\underline{F} = F \underline{n}_{AB} = 50 \left[\frac{55\underline{i} + 30\underline{j}}{\sqrt{55^2 + 30^2}} \right]$$

$$= 43.9\underline{i} + 23.9\underline{j} \text{ N}$$

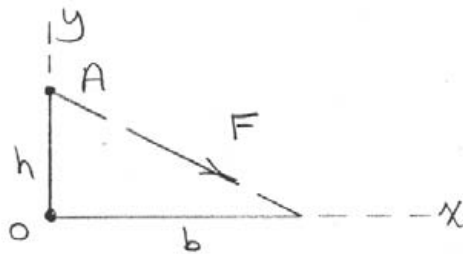
$$\checkmark M_o = 43.9(15) - 23.9(20) = \underline{179.6 \text{ N}\cdot\text{mm CCW}}$$

$$\underline{M}_c = \underline{r}_{CB} \times \underline{F}$$

$$= (40\underline{i} - 15\underline{j}) \times (43.9\underline{i} + 23.9\underline{j})$$

$$= \underline{1616 \underline{k} \text{ N}\cdot\text{mm}} \text{ or } \underline{1.616 \underline{k} \text{ N}\cdot\text{m}}$$

2/32



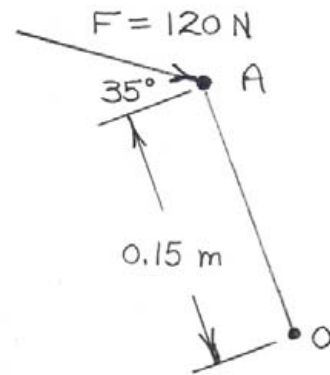
$$\underline{F} = F \left[\frac{b\underline{i} - h\underline{j}}{\sqrt{b^2 + h^2}} \right]$$

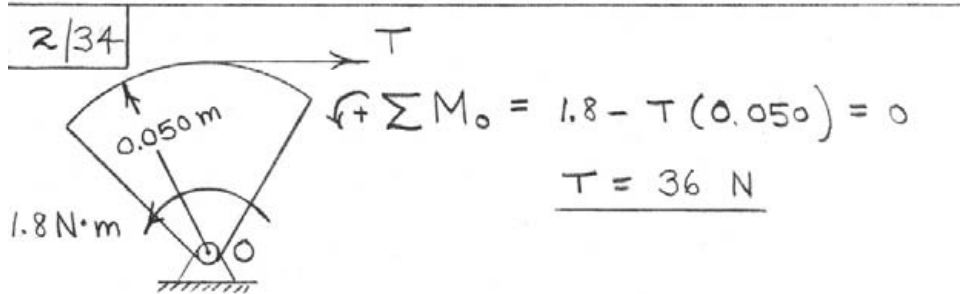
Acting at A:

$$+ \circlearrowleft M_O = \frac{Fb}{\sqrt{h^2 + b^2}} (h) = \frac{Fbh}{\sqrt{h^2 + b^2}} \quad \text{CW}$$

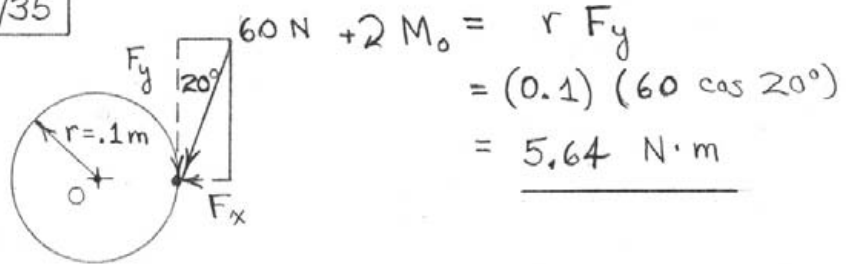
2/33

$$\begin{aligned} +\curvearrowright M_o &= 120 \cos 35^\circ (0.15) \\ &= \underline{14.74 \text{ N}\cdot\text{m CW}} \end{aligned}$$

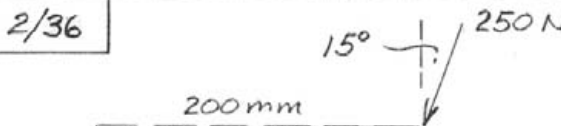




2/35



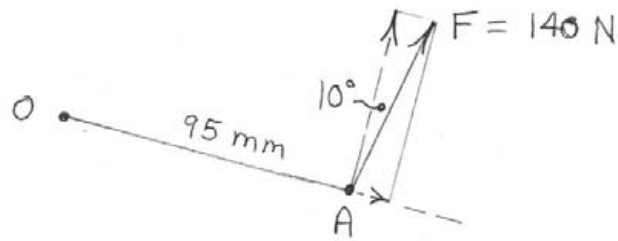
2/36



A diagram showing a horizontal beam of length 200 mm. At the right end of the beam, a force of 250 N is applied at an angle of 15° below the horizontal. A vertical dashed line indicates the 15° angle. At the left end of the beam, a point is labeled 'O'. A vertical dimension of 30 mm is shown from point O to the horizontal line of the beam.

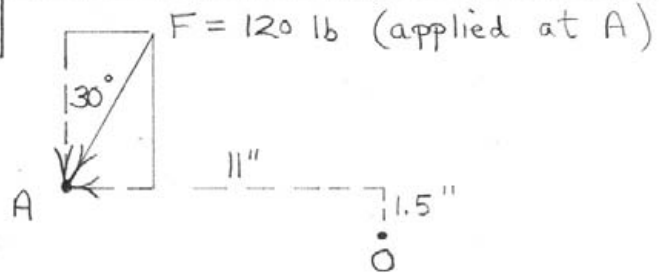
$$\begin{aligned} +\circlearrowleft M_O &= 250 \cos 15^\circ (0.200) - 250 \sin 15^\circ (0.030) \\ &= 48.30 - 1.941 = \underline{46.4 \text{ N}\cdot\text{m}} \end{aligned}$$

2/37



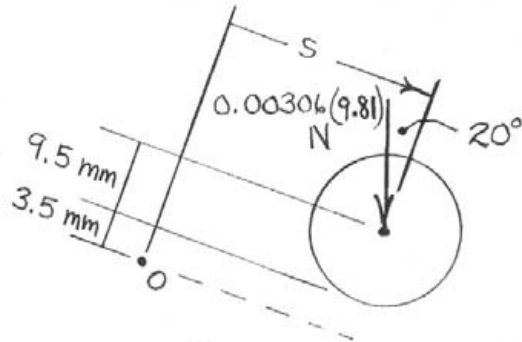
$$\curvearrowleft M_O = (140 \cos 10^\circ)(0.095) = \underline{13.10 \text{ N}\cdot\text{m} \text{ CCW}}$$

2/38



$$\begin{aligned} \sum M_O &= 120 \cos 30^\circ (11) + 120 \sin 30^\circ (1.5) \\ &= 1233 \text{ lb-in. or } 102.8 \text{ lb-ft CCW} \end{aligned}$$

2/39



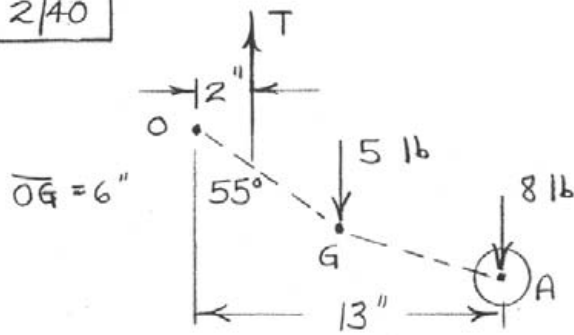
$$\begin{aligned} \rightarrow M_O &= 0.00306(9.81) [s \cos 20^\circ + (9.5 + 3.5) \sin 20^\circ] \\ &= 0.1335 + 0.0282s \text{ N}\cdot\text{mm} \quad (s \text{ in mm}) \end{aligned}$$

$$\boxed{2/4} \quad \underline{F} = F \underline{n}_{AB} = 4800 \left[\frac{45\underline{i} + 30\underline{j}}{\sqrt{45^2 + 30^2}} \right]$$

$$= 3990\underline{i} + 2660\underline{j} \text{ lb}$$

$$\text{Scalar components: } \begin{cases} F_x = 3990 \text{ lb} \\ F_y = 2660 \text{ lb} \end{cases}$$

2/40



The combined moment about O of the 5-lb and 8-lb weights is

$$\uparrow \curvearrowright M_o = 5(6 \sin 55^\circ) + 8(13) = 128.6 \text{ lb-in. (CW)}$$

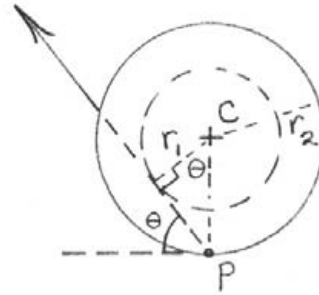
$$\uparrow \curvearrowright \Sigma M_o = 0 : -T(2) + 128.6 = 0$$

$$\underline{T = 64.3 \text{ lb}}$$

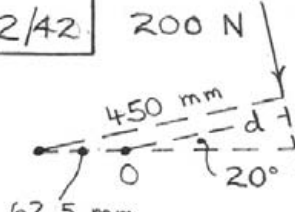
2/41

$$T = 32 \text{ lb}$$

$$\begin{aligned}\curvearrowright M_c &= Tr_1 = 32(5) \\ &= 160 \text{ lb-in.} \quad \text{CW} \\ \cos \theta &= r_1/r_2 = 5/8 \\ \theta &= 51.3^\circ\end{aligned}$$



2/42



200 N

450 mm

62.5 mm

20°

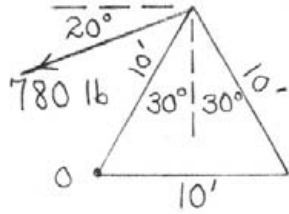
$d = 450 - 62.5 \cos 20^\circ$

$= 391 \text{ mm}$

$\circlearrowright M = Fd = 200(0.391)$

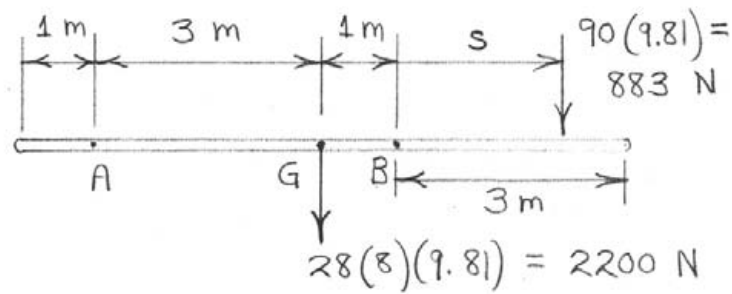
$= \underline{78.3 \text{ N}\cdot\text{m}}$

2/43



$$\begin{aligned} \curvearrowright M_o &= 780 \cos 20^\circ (10 \cos 30^\circ) \\ &\quad - 780 \sin 20^\circ (5) = \underline{5010 \text{ lb-ft}} \end{aligned}$$

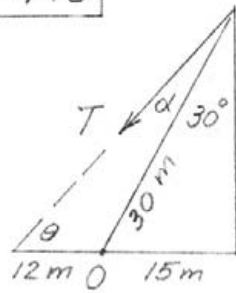
2/44



$$28(8)(9.81) = 2200 \text{ N}$$

$$\curvearrowright M_B = 2200(1) - 883s = 0, \quad \underline{s = 2.49 \text{ m}}$$

2/45



$$\theta = \tan^{-1} \frac{30(0.866)}{12 + 15} = 43.90^\circ$$

$$\alpha = 90^\circ - (30^\circ + 43.90^\circ) = 16.10^\circ$$

$$M_o = 72 \text{ kN}\cdot\text{m}$$

$$= T \sin 16.10^\circ (30) = 8.32 T$$

$$T = \frac{72}{8.32} = \underline{8.65 \text{ kN}}$$

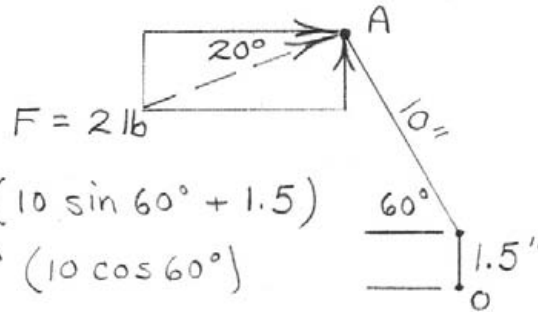
2/46 (Dim. in mm)

$AB = \sqrt{400^2 + 100^2}$
 $= 412 \text{ mm}$

$\circlearrowright \sum M_o = \left(\frac{400}{412} \cdot 40 \right) (75) + \left(\frac{100}{412} \cdot 40 \right) (425)$
 $= 7030 \text{ N} \cdot \text{mm} \text{ or } \underline{7.03 \text{ N} \cdot \text{m}}$

$\circlearrowright \sum M_o = 0 : -F_C (825) + 7030 = 0$
 $\underline{F_C = 8.53 \text{ N}}$

2/47



$$\begin{aligned} +\curvearrowright M_O &= 2 \cos 20^\circ (10 \sin 60^\circ + 1.5) \\ &\quad + 2 \sin 20^\circ (10 \cos 60^\circ) \\ &= \underline{22.5 \text{ lb-in, CW}} \end{aligned}$$

2/48



$$(1) M_A = 200 \cos 15^\circ (0.280) + 200 \sin 15^\circ (0.400) = \underline{74.8 \text{ N}\cdot\text{m}}$$

$$(2) AE = 0.280 \cos 15^\circ + 0.400 \sin 15^\circ$$

$$= 0.374 \text{ m}; M_A = 200(0.374) = \underline{74.8 \text{ N}\cdot\text{m}}$$

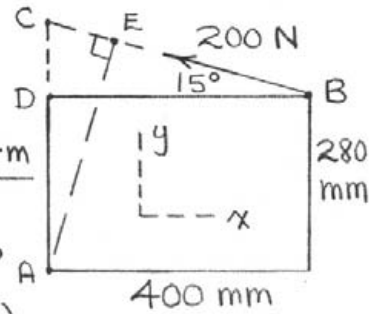
(3) Apply force at C

$$CD = 0.400 \tan 15^\circ = 0.1072 \text{ m}$$

$$CA = 0.280 + 0.1072 = 0.387 \text{ m}$$

$$M_A = (200 \cos 15^\circ)(0.387) = \underline{74.8 \text{ N}\cdot\text{m}}$$

$$(4) \underline{M}_A = \underline{r} \times \underline{F} = (0.400\underline{i} + 0.280\underline{j}) \times 200(-\cos 15^\circ\underline{i} + \sin 15^\circ\underline{j}) = \underline{74.8 \text{ N}\cdot\text{m}}$$



2/49

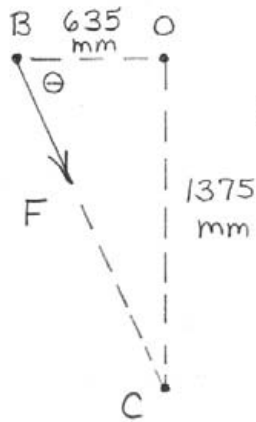
$$F = kx = 60 (\sqrt{0.635^2 + 1.375^2} - 0.740)$$

$$= 46.5 \text{ N}$$

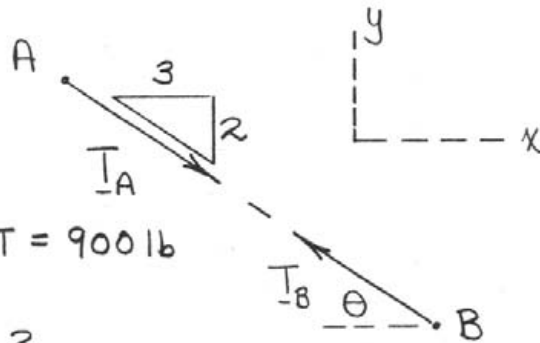
$$\theta = \tan^{-1} \frac{1.375}{0.635} = 65.2^\circ$$

$$\curvearrowleft M_o = 46.5 \sin 65.2^\circ (0.635)$$

$$= \underline{26.8 \text{ N}\cdot\text{m} \text{ CCW}}$$



2/5



$$T_A = T_B = T = 900 \text{ lb}$$

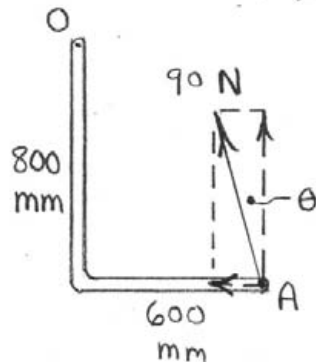
$$\sin \theta = \frac{2}{\sqrt{2^2 + 3^2}} = 0.555$$

$$\cos \theta = \frac{3}{\sqrt{2^2 + 3^2}} = 0.832$$

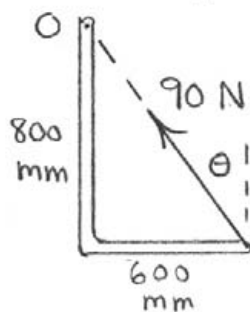
$$\underline{T}_A = 900(0.832\underline{i} - 0.555\underline{j}) = \underline{749\underline{i} - 499\underline{j} \text{ lb}}$$

$$\underline{T}_B = -\underline{T}_A = \underline{-749\underline{i} + 499\underline{j} \text{ lb}}$$

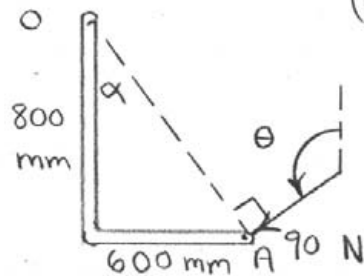
(a) $\theta = 15^\circ$



$$\begin{aligned} \curvearrowright M_O &= 90 \cos 15^\circ (0.6) - 90 \sin 15^\circ (0.8) \\ &= \underline{33.5 \text{ N}\cdot\text{m}} \end{aligned}$$



(b) $\theta = \tan^{-1}\left(\frac{600}{800}\right) = \underline{36.9^\circ}$
(or 217°)

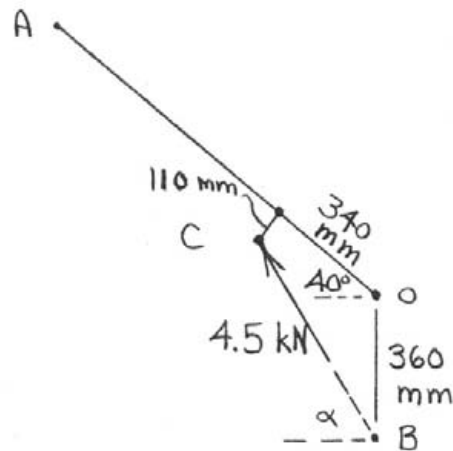


(c) $F \perp OA$, so

$$\theta = 90^\circ + \alpha, \quad \alpha = \tan^{-1} \frac{600}{800} = 36.9^\circ$$

So $\theta = \underline{126.9^\circ}$ (or $\underline{307^\circ}$)

2/51

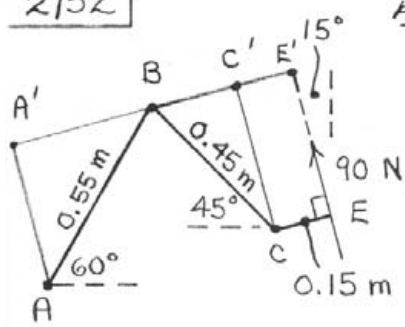


$$\alpha = \tan^{-1} \left[\frac{360 + 340 \sin 40^\circ - 110 \sin 50^\circ}{340 \cos 40^\circ + 110 \cos 50^\circ} \right]$$

$$= 56.2^\circ$$

$$\rightarrow M_o = 4.5 (0.360 \cos 56.2^\circ) = \underline{0.902 \text{ kN} \cdot \text{m CW}}$$

2/52



$$\uparrow M_C = F(\overline{CE}) = 90(0.15) = \underline{13.50 \text{ N}\cdot\text{m}}$$

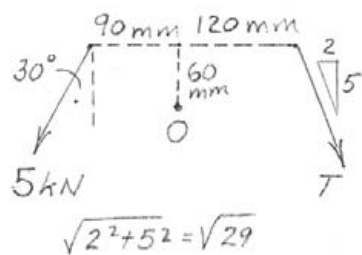
$$\begin{aligned} M_B &= F(\overline{BE'}) \\ &= 90(0.15 + 0.45 \sin 30^\circ) \\ &= \underline{33.8 \text{ N}\cdot\text{m}} \end{aligned}$$

$$\begin{aligned} M_A &= F(\overline{A'E'}) \\ &= 90(0.15 + 0.45 \sin 30^\circ + 0.55 \sin 45^\circ) \\ &= \underline{68.8 \text{ N}\cdot\text{m}} \end{aligned}$$

2/53

$$M_O = 5[(\cos 30^\circ)90 + (\sin 30^\circ)60]$$

$$- T \left[\frac{5}{\sqrt{29}}(120) + \frac{2}{\sqrt{29}}(60) \right] = 0$$

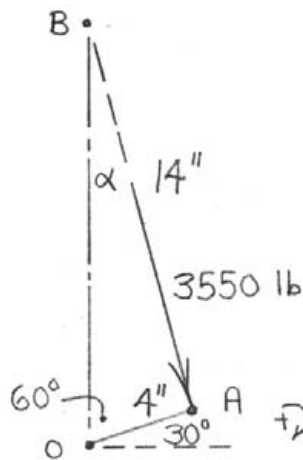


$$539.7 - 133.7T = 0, \underline{T = 4.04 \text{ kN}}$$

2/54

$$\text{Law of sines: } \frac{4}{\sin \alpha} = \frac{14}{\sin 60^\circ}$$

$$\alpha = 14.33^\circ$$



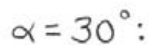
$$\begin{aligned} \overline{BO} &= 14 \cos 14.33^\circ + 4 \cos 60^\circ \\ &= 15.56 \text{ in.} \end{aligned}$$

Consider 3550 lb acting at B:

$$\begin{aligned} +\circlearrowleft M_o &= (3550 \sin 14.33^\circ)(15.56) \\ &= 13,670 \text{ lb-in.} \end{aligned}$$

$$(\text{or } M_o = \underline{1139 \text{ lb-ft}})$$

Elements of wrench:



For maximum M_o :

$$(M_o)_{\max} = 120 \sqrt{(25+70+25+70)^2 + (70+150+70)^2}$$

$$= 41600 \text{ N}\cdot\text{mm} \text{ or } 41.6 \text{ N}\cdot\text{m CW}$$

► 2/56

$$d = \frac{4}{\tan 60^\circ} = 2.31 \text{ in.}$$

$$\alpha = \tan^{-1}\left(\frac{8}{d+8}\right) = 37.8^\circ$$

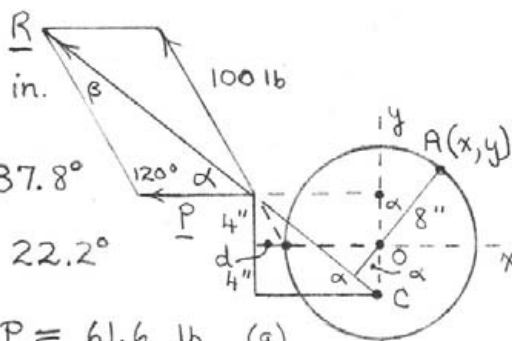
$$\beta = 180^\circ - (120^\circ + \alpha) = 22.2^\circ$$

$$\frac{P}{\sin \beta} = \frac{100}{\sin \alpha}, \quad P = 61.6 \text{ lb} \quad (a)$$

$$\frac{R}{\sin 120^\circ} = \frac{100}{\sin \alpha}, \quad R = 141.3 \text{ lb} \quad 37.8^\circ \quad (b)$$

$$x = 8 \sin \alpha = 4.90 \text{ in.}, \quad y = 8 \cos \alpha = 6.32 \text{ in.} \quad (c)$$

$$+ \circlearrowleft M_A = R d = 141.3 (8 + 4 \cos \alpha) = 1577 \text{ lb-in.} \quad (d)$$

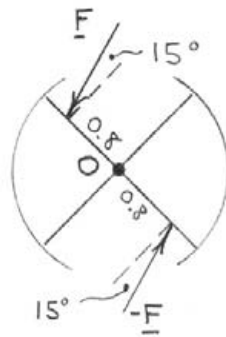


$$\begin{array}{|l} 2/57 \end{array} \quad \begin{array}{l} \rightarrow M = Fd = 80(1.4) = 112 \text{ lb-in. CW} \\ \text{(or 9.33 lb-ft CW)} \end{array}$$

$$\boxed{2/58} \quad \underline{\underline{F = 60\left(-\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}\right) = -36\underline{i} + 48\underline{j} \text{ N}}}$$

$$\curvearrowright M_C = 48(50) = 2400 \text{ N}\cdot\text{mm} = \underline{\underline{2.4 \text{ N}\cdot\text{m} \text{ CCW}}}$$

2/59



$$\curvearrowright M_o = \Sigma Fd$$

$$25 = 2 F(\cos 15^\circ)(0.8)$$

$$F = \underline{16.18 \text{ N}}$$

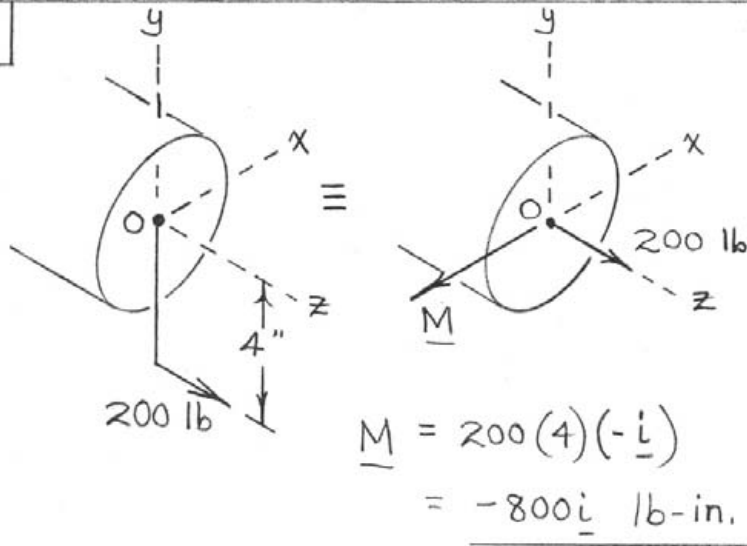
2/6

$$\underline{F} = 1800 \left(-\frac{3}{5} \underline{i} - \frac{4}{5} \underline{j} \right) = -1080 \underline{i} - 1440 \underline{j} \text{ N}$$

$$\begin{array}{|l} 2/60 \\ \hline \end{array} \quad \underline{R = 6 \text{ j kN}} \quad @ \quad x = \frac{400}{6000} = 0.0667 \text{ m}$$

or $x = 66.7 \text{ mm}$

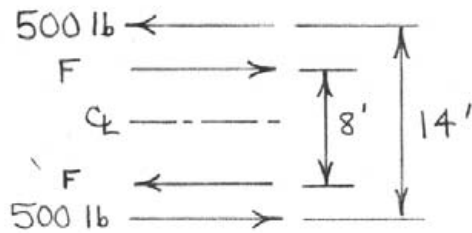
2/61



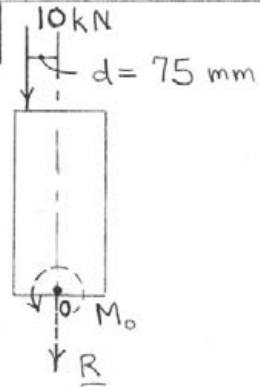
2/62

$$\curvearrowright \sum M = 500(14) - F(8) = 0$$

$$F = 875 \text{ lb}$$

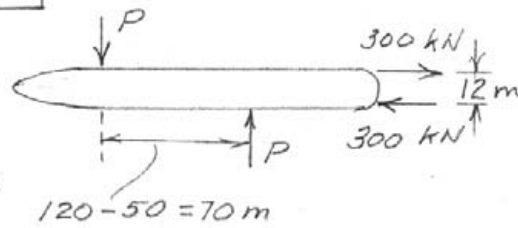


2/63



$$\begin{cases} R = 10 \text{ kN } (\downarrow) \\ M_o = Fd = (10)(0.075) \\ = 0.75 \text{ kN}\cdot\text{m } (\curvearrowleft) \end{cases}$$

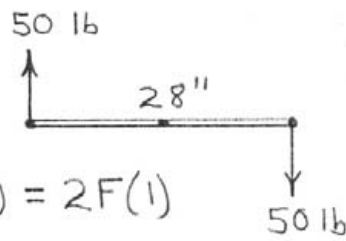
2/64



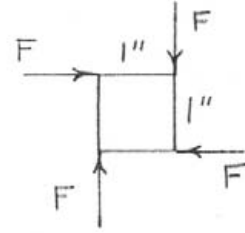
$$70P = 300(12)$$

$$\underline{P = 51.4 \text{ kN}}$$

2/65



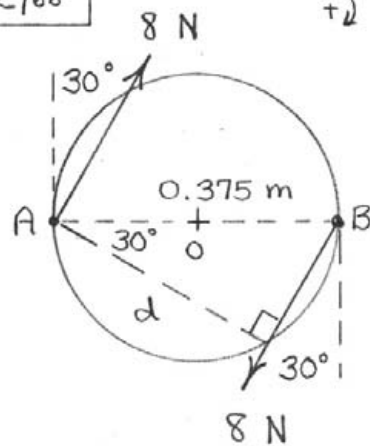
=



$$M = 50(28) = 2F(1)$$

$$\underline{F = 700 \text{ lb}}$$

2/66



$$\begin{aligned}\uparrow \curvearrowright M &= Fd = 8 (0.375 \cos 30^\circ) \\ &= \underline{2.60 \text{ N}\cdot\text{m} \text{ CW}}\end{aligned}$$

2/67

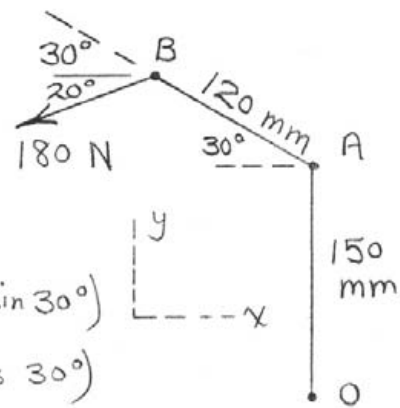
$$\underline{F} = 180(-\cos 20^\circ \underline{i} - \sin 20^\circ \underline{j})$$

$$= \underline{-169.1 \underline{i} - 61.6 \underline{j} \text{ N}}$$

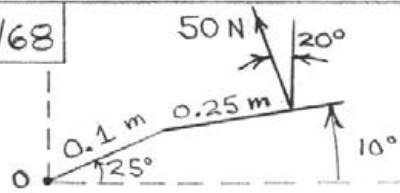
$$\checkmark M_o = 180 \cos 20^\circ (150 + 120 \sin 30^\circ)$$

$$+ 180 \sin 20^\circ (120 \cos 30^\circ)$$

$$= 41900 \text{ N}\cdot\text{mm} \text{ or } \underline{41.9 \text{ N}\cdot\text{m CCW}}$$



2/68



Use principle of moments.

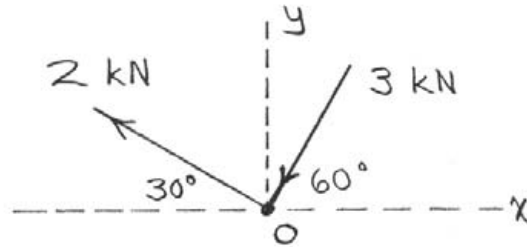
$$\begin{aligned}
 \curvearrowleft \Sigma M_o &= 50 \cos 20^\circ [0.1 \cos 25^\circ + 0.25 \cos 10^\circ] \\
 &\quad + 50 \sin 20^\circ [0.1 \sin 25^\circ + 0.25 \sin 10^\circ] \\
 &= 17.29 \text{ N}\cdot\text{m}
 \end{aligned}$$

Force - Couple System at O:

$$\begin{cases} R = 50 \text{ N} \nearrow 110^\circ \\ M_o = 17.29 \text{ N}\cdot\text{m} \curvearrowleft \end{cases}$$

$$\begin{aligned}
 \underline{2/69} \quad \underline{\underline{\underline{F}}} &= 250 (\sin 10^\circ \underline{i} + \cos 10^\circ \underline{j}) \\
 &= 43.4 \underline{i} + 246 \underline{j} \text{ N} \\
 \rightarrow \underline{M_o} &= 250 [\cos 10^\circ (0.235) + \sin 10^\circ (0.050)] \\
 &= \underline{60.0 \text{ N}\cdot\text{m} \text{ CW}}
 \end{aligned}$$

2/7



$$R_x = \sum F_x = -2 \cos 30^\circ - 3 \cos 60^\circ = -3.23 \text{ kN}$$

$$R_y = \sum F_y = 2 \sin 30^\circ - 3 \sin 60^\circ = -1.598 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = 3.61 \text{ kN}$$

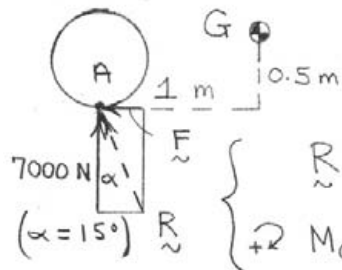
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-1.598}{-3.23}\right) = 206^\circ$$

2/70

$$\tan 15^\circ = \frac{F}{7000}$$

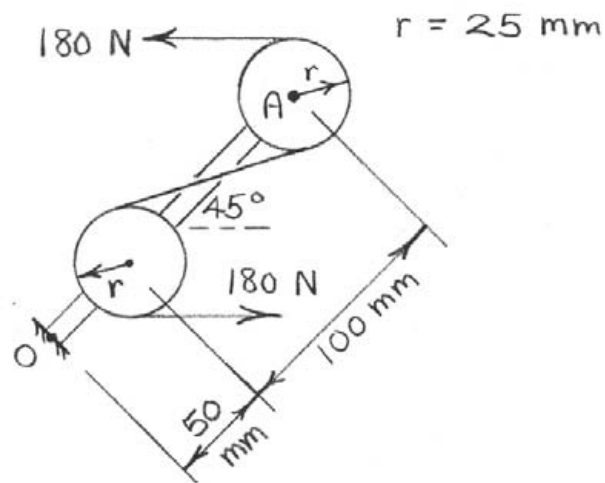
$$F = 1876 \text{ N}$$

At G :



$$\left\{ \begin{array}{l} \vec{R} = \sum \vec{F} = \underline{7250 \text{ N}} \\ +\circlearrowleft M_G = 7000(1) + 1876(.5) \\ = \underline{7940 \text{ N}\cdot\text{m}} \end{array} \right.$$

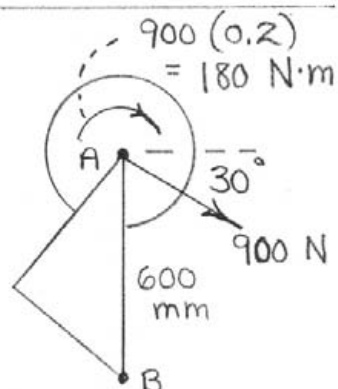
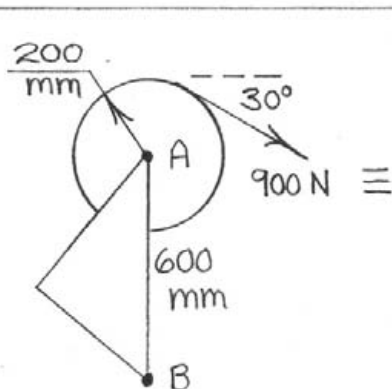
$\nearrow 105^\circ$



The system at O is a couple.

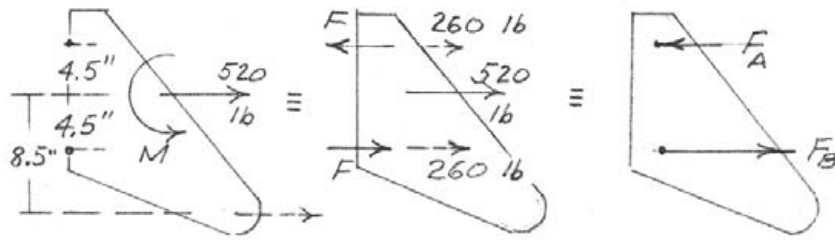
$$\begin{aligned}\hat{+} M &= Fd = 180(100 \sin 45^\circ + 25 + 25) \\ &= 21\,700 \text{ N}\cdot\text{mm} \text{ or } \underline{21.7 \text{ N}\cdot\text{m CCW}}\end{aligned}$$

2/72



$$+\circlearrowleft M_B = 180 + 900 \cos 30^\circ (0.6) = \underline{648 \text{ N}\cdot\text{m CW}}$$

2/73



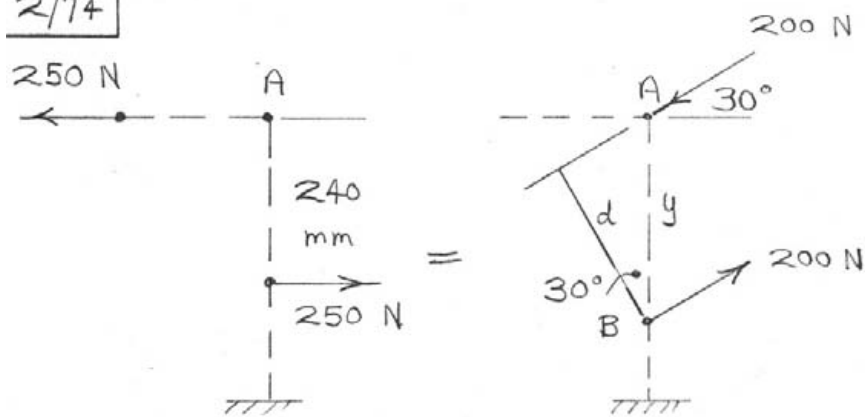
$$M = Fd = 520(8.5) = 4420 \text{ lb-in.}$$

$$F(9) = 4420, \quad F = 491 \text{ lb}$$

$$F_A = 491 - 260 = \underline{231 \text{ lb}}$$

$$F_B = 491 + 260 = \underline{751 \text{ lb}}$$

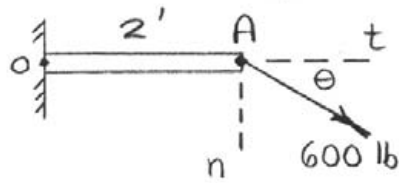
2/74



Equal couples :

$$\begin{aligned} \curvearrowright 250(240) &= 200(y \cos 30^\circ) \\ y &= 346 \text{ mm} \end{aligned}$$

2/75



The equivalent force-couple system at O is

$$R_t = 600 \cos \theta \quad (lb)$$

$$R_n = 600 \sin \theta \quad (lb)$$

$$\uparrow M_o = 2(600 \sin \theta) = 1200 \sin \theta \quad (lb-ft)$$

Constraints:

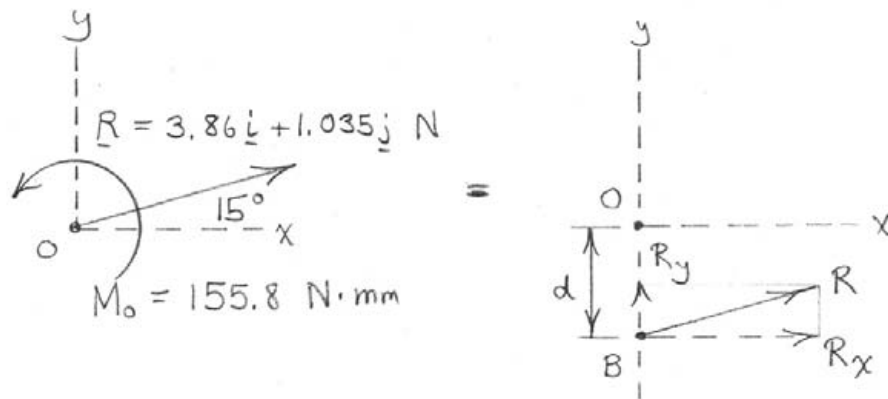
$$\begin{cases} 600 \cos \theta \leq 550, & \theta \geq 23.6^\circ \\ 600 \sin \theta \leq 550, & \theta \leq 66.4^\circ \\ 1200 \sin \theta < 1000, & \theta \leq 56.4^\circ \end{cases}$$

All considered, $23.6^\circ \leq \theta \leq 56.4^\circ$

2/76 At O:

$$\underline{R} = 4 (\cos 15^\circ \underline{i} + \sin 15^\circ \underline{j}) = 3.86 \underline{i} + 1.035 \underline{j} \text{ N}$$

$$\begin{aligned} \curvearrowright M_o &= 300 - 4 \cos 15^\circ (40) + 4 \sin 15^\circ (10) \\ &= 155.8 \text{ N}\cdot\text{mm} \text{ CCW} \end{aligned}$$



$$\text{Condition: } R_x d = M_o$$

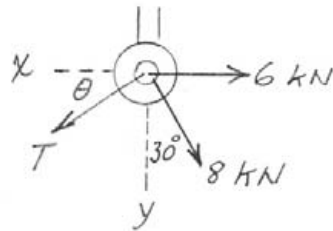
$$3.86 d = 155.8, \quad d = 40.3 \text{ mm}$$

$$\text{So } \underline{y = -40.3 \text{ mm}}$$

2/77

$$R = R_y = 15 = T \sin \theta + 8 \cos 30^\circ$$

$$R_x = 0 = T \cos \theta - 6 - 8 \sin 30^\circ$$



$$\text{So } T \sin \theta = 8.07$$

$$T \cos \theta = 10$$

$$\text{Divide \& set } \theta = \tan^{-1} \frac{8.07}{10}$$

$$\theta = 38.9^\circ$$

$$T = \frac{10}{\cos 38.9^\circ} = 12.85 \text{ kN}$$

$$\begin{aligned} 2/78 \quad R_x = \sum F_x &= 60 \cos 40^\circ + 50 \sin 20^\circ - 30 \cos 20^\circ \\ &= 34.87 \text{ kN} \end{aligned}$$

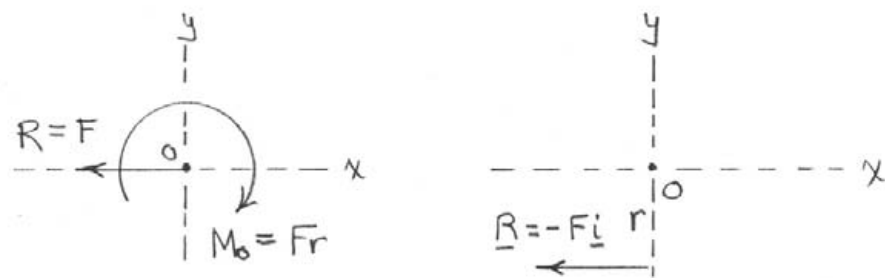
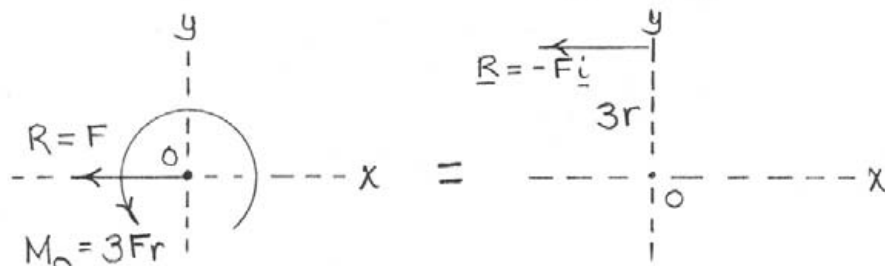
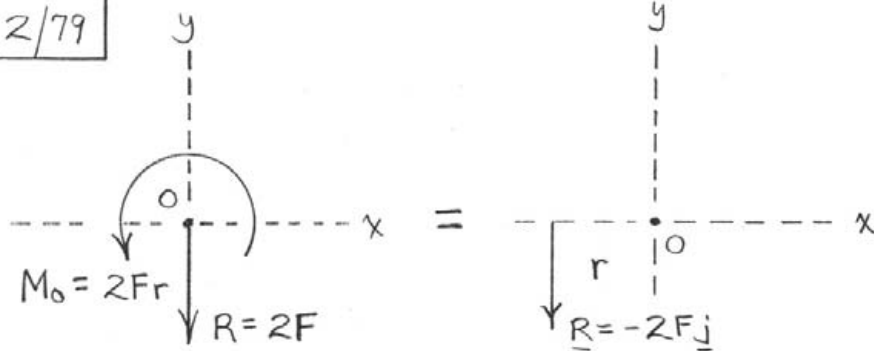
$$\begin{aligned} R_y = \sum F_y &= 60 \sin 40^\circ + 40 - 50 \cos 20^\circ + 30 \sin 20^\circ \\ &= 41.84 \text{ kN} \end{aligned}$$

$$R = \sqrt{34.87^2 + 41.84^2} = 54.5 \text{ kN}$$

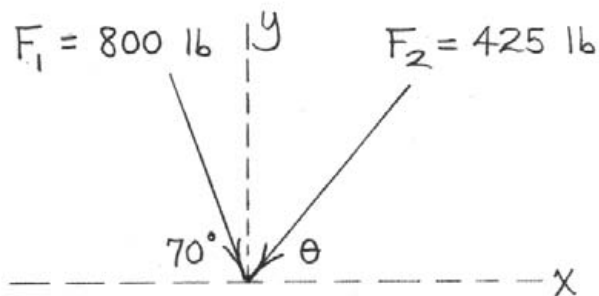
$$\theta_x = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{41.84}{34.87} = 50.2^\circ$$

$$\underline{R = 34.9\hat{i} + 41.8\hat{j} \text{ kN}}$$

2/79



2/8



$$R_x = \sum F_x = 800 \cos 70^\circ - 425 \cos \theta = 0$$

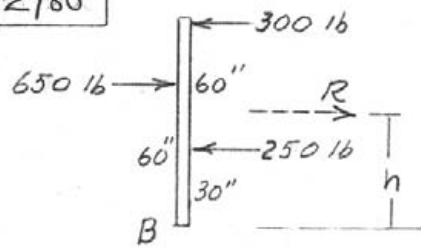
$$\theta = \underline{49.9^\circ}$$

$$R_y = \sum F_y = -800 \sin 70^\circ - 425 \sin 49.9^\circ$$

$$= -1077 \text{ lb}$$

$$\text{So } \underline{R = 1077 \text{ lb}}$$

2/80



$$R = \Sigma F = 650 - 250 - 300 \\ = 100 \text{ lb}$$

$$Rh = \Sigma M_B;$$

$$100h = 650(60) - 300(90) \\ - 250(30)$$

$$h = 45 \text{ in.}$$

2/81

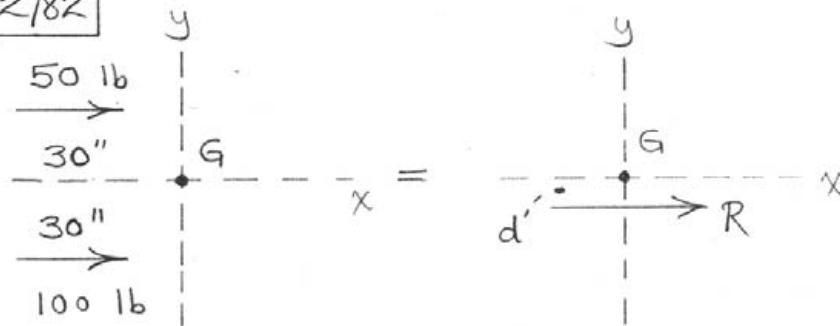
$$R = \Sigma F = 680 - 660 = 20 \text{ N}$$

$$Rd = \Sigma M_A$$

$$20d = 680(0.8) - 660(0.5)$$

$$d = 10.70 \text{ m to the left of A}$$

2/82



$$\rightarrow R = \sum F = 50 + 100 = 150 \text{ lb (to the right)}$$

$$\curvearrowleft M_G = 100(30) - 50(30) = 1500 \text{ lb-ft CCW}$$

$$= R d = 150 d, \quad d = 10 \text{ in.}$$

So the y-intercept of the standalone force resultant is y = -10 in. The effect is to propel the truck forward and rotate it CCW about G.

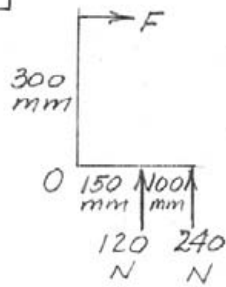
2/83

$$M_o = 0, \text{ so}$$

$$\curvearrowright_+ M - 400(0.150 \cos 30^\circ) - 320(0.300) = 0$$

$$\underline{M = 148.0 \text{ N}\cdot\text{m}}$$

2/84

 $\sum M_O = 0$ for R to pass through O

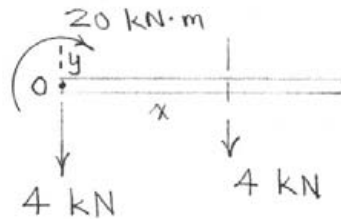
$$300F - 120(150) - 240(250) = 0$$

$$\underline{F = 260 \text{ N}}$$

2/85

Force-Couple system at point O:

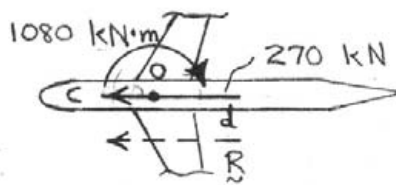
$$\left. \begin{aligned} \underline{R} &= \sum \underline{F} = (6-10) \underline{j} = -4 \underline{j} \text{ kN} \\ \curvearrowright M_o &= 6(3) - 10(9) + 52 = -20 \text{ kN}\cdot\text{m} \end{aligned} \right\}$$



$$\begin{aligned} x &= \frac{M_o}{R} = \frac{20}{4} \\ &= \underline{5 \text{ m}} \quad (\text{on beam!}) \end{aligned}$$

2/86 Force - Couple system at point O:

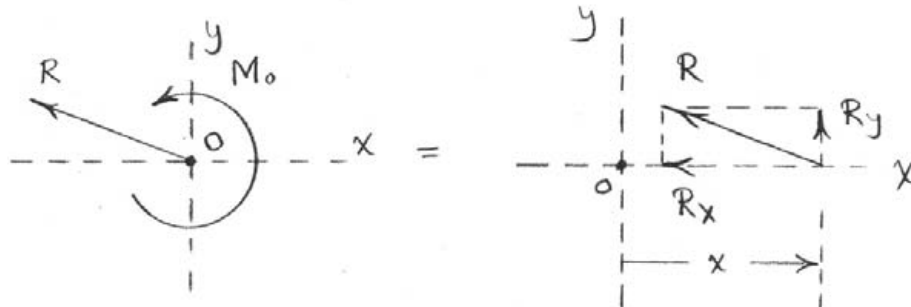
$$\begin{cases} R = 3(90) = 270 \text{ kN } (\leftarrow) \\ +2 M_o = 12(90) = 1080 \text{ kN} \cdot \text{m} \end{cases}$$



$$d = \frac{M_o}{R} = \frac{1080}{270} = 4 \text{ m}$$

$$2/87 \quad \underline{R} = -50\mathbf{i} + 20\mathbf{j} \text{ lb}$$

$$\curvearrowright M_o = -40(10) + 60(20) + 50(10) = 1300 \text{ lb-in.}$$



$$R_y x = M_o, \quad x = \frac{1300}{20} = \underline{65 \text{ in. (off pipe)}}$$

2/88

$$\underline{R} = \Sigma \underline{F} = T \underline{i} + T (\cos 15^\circ \underline{i} + \sin 15^\circ \underline{j})$$

$$= 1.966 T \underline{i} + 0.259 T \underline{j}$$

$$+ 2 M_o = 3T - T \cos 15^\circ (3)$$

$$+ T \sin 15^\circ (10) = \underline{2.69 T}$$

$$- R_y x = M_o: - 0.259 T (x) = 2.69 T$$

$$\underline{x = -10.39 \text{ m}}$$

2/89 Equivalent force - couple system at point O:

$$\underline{R} = \Sigma \underline{F} = (-25 + 20 \sin 30^\circ) \underline{i} + (-30 - 20 \cos 30^\circ) \underline{j} = \underline{-15 \underline{i} - 47.3 \underline{j} \text{ kN}}$$

$$\begin{aligned} \curvearrowright M_o &= 25(5) - 30(9) - (20 \cos 30^\circ) 9 \\ &\quad - (20 \sin 30^\circ) 5 = -351 \text{ kN}\cdot\text{m} \end{aligned}$$

For final location of \underline{R} :

$$\begin{aligned} \underline{r} \times \underline{R} &= \underline{M}_o, (x \underline{i} + y \underline{j}) \times (-15 \underline{i} - 47.3 \underline{j}) \\ &= -351 \underline{k} \end{aligned}$$

$$-47.3x + 15y = -351$$

$$\text{Axis intersections: } \underline{x = 7.42 \text{ m}, y = -23.4 \text{ m}}$$

2/9	$P_x = -260 \left(\frac{12}{13} \right) = -240 \text{ N}$ $P_y = -260 \left(\frac{5}{13} \right) = \underline{-100 \text{ N}}$
-----	--

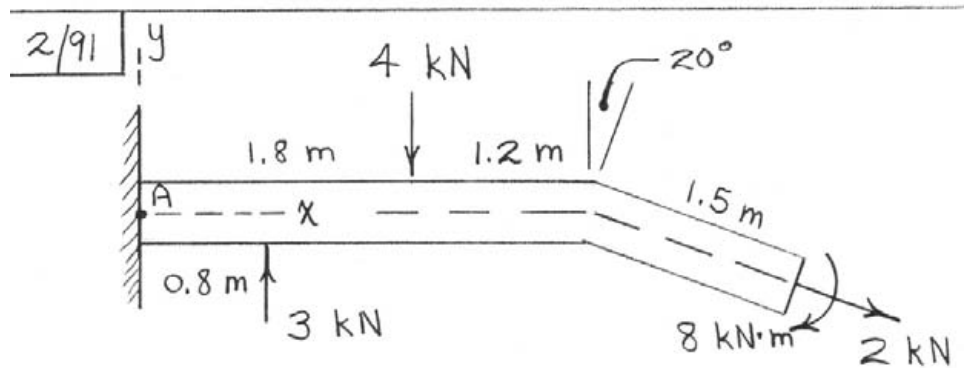
2/90

$R_x = \sum F_x = (800 + 450) \cos 15^\circ - 1600 \sin(30^\circ - 20^\circ)$
 $= 1207 - 278 = 929.6 \text{ N}$

$R_y = \sum F_y = 1600 \cos 10^\circ + (800 - 450) \sin 15^\circ$
 $= 1576 + 90.6 = 1666 \text{ N}$

$\underline{R} = 930 \underline{i} + 1666 \underline{j} \text{ N}$

$M = \sum M_O \uparrow; M = 1600 \cos 20^\circ (0.150) + (450 - 800) 0.280$
 $= 225.5 - 98.0 = 127.5 \text{ N}\cdot\text{m CCW}$
 so unit is speeding up in CCW dir.



$$R_x = \sum F_x = 2 \cos 20^\circ = 1.879 \text{ kN}$$

$$R_y = \sum F_y = 3 - 4 - 2 \sin 20^\circ = -1.684 \text{ kN}$$

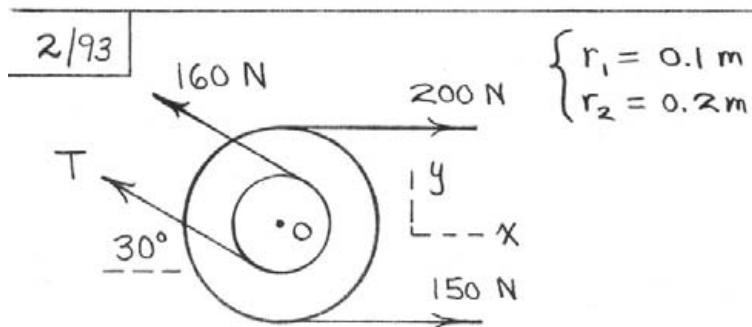
$$\underline{R = 1.879\mathbf{i} - 1.684\mathbf{j} \text{ kN}}, \quad R = \sqrt{1.879^2 + 1.684^2} = 2.52 \text{ kN}$$

$$\begin{aligned} \circlearrowleft M = \sum M_A &= 4(1.8) - 3(0.8) + 2 \sin 20^\circ (3.0) + 8 \\ &= \underline{14.85 \text{ kN}\cdot\text{m} \text{ CW}} \end{aligned}$$

2/92 $\Sigma M_O = 0$ since \underline{R} passes through O .

$$40(8) + 60(4) - 5P \cos 20^\circ = 0, \quad \underline{P = 119.2 \text{ lb}}$$

Moment of 40-lb & 60-lb forces unaffected by θ
So result for P is not dependent on θ .



$$+\circlearrowleft M_O = 0 : 200(0.2) - 150(0.2) - 160(0.1) + (0.1)T = 0$$

$$\underline{T = 60 \text{ N}}$$

$$R_x = \sum F_x = 200 + 150 - (160 + 60) \cos 30^\circ = 159.5 \text{ N}$$

$$R_y = \sum F_y = (160 + 60) \sin 30^\circ = 110 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{193.7 \text{ N}}$$

$$\theta = \tan^{-1}(R_y/R_x) = \underline{34.6^\circ}$$

$$\boxed{2/94} \quad \underline{R} = 45\underline{i} - 15\underline{j} \text{ lb}$$

$$\Rightarrow \underline{M}_A = 25(30) + 15(60) = 1650 \text{ lb-in.}$$

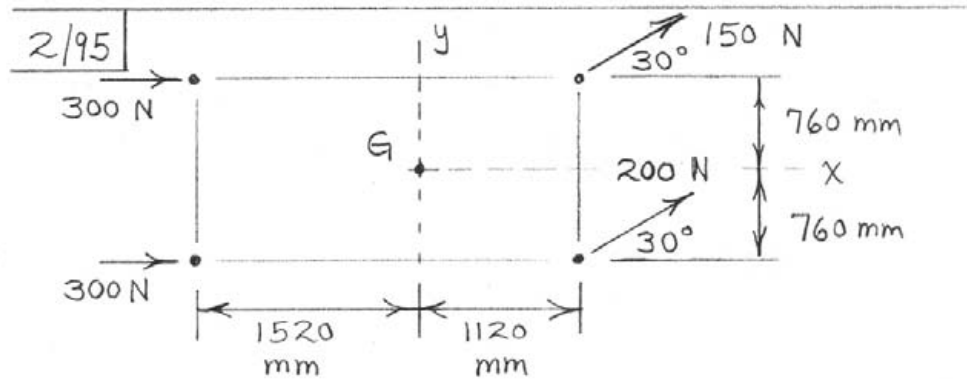
$$\text{or } \underline{M}_A = -1650\underline{k} \text{ lb-in.}$$

For final line of action, $\underline{r} \times \underline{R} = \underline{M}_A$

$$(\underline{x}\underline{i} + \underline{y}\underline{j}) \times (45\underline{i} - 15\underline{j}) = -1650\underline{k}$$

$$\Rightarrow -15x - 45y = -1650 \text{ or } \underline{y = -\frac{1}{3}x + \frac{110}{3} \text{ in.}}$$

$$(\text{Axis intercepts: } x = 110", y = 110/3")$$



$$\underline{R} = \sum \underline{F} = 2(300\underline{i}) + 150(\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j}) + 200(\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j}) = 903\underline{i} + 175\underline{j} \text{ N}$$

$$\begin{aligned} \sum M_G &= -150 \cos 30^\circ (760) + 150 \sin 30^\circ (1120) \\ &\quad + 200 \cos 30^\circ (760) + 200 \sin 30^\circ (1120) \\ &= 229000 \text{ N}\cdot\text{mm} \text{ or } 229 \text{ N}\cdot\text{m} \text{ CCW} \end{aligned}$$

Condition: $\underline{r} \times \underline{R} = M_G$

$(x\underline{i} + y\underline{j}) \times (903\underline{i} + 175\underline{j}) = +229\underline{k}$

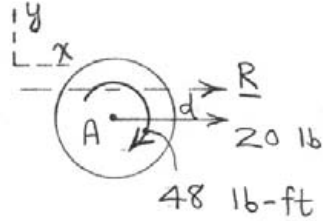
$175x - 903y = +229$

$\begin{cases} y=0: & x = 1.308 \text{ m} \\ x=0: & y = -0.253 \text{ m} \end{cases}$

2/96 Force - Couple System at point A:

$$\underline{R} = \sum \underline{F} = -500\underline{j} + 60\underline{i} - 100\underline{j} - 40\underline{i} + 600\underline{j}$$
$$= \underline{20\underline{i} \text{ lb}}$$


$$\curvearrowright M_A = 2 - 40\left(\frac{15}{12}\right) = -48 \text{ lb-ft}$$



$$Rd = M_A$$

$$d = \frac{M_A}{R} = \frac{48}{20}$$

$$= \underline{2.40 \text{ ft}}$$

2/97 Use  system at G:

$$\begin{aligned}\underline{R} = \Sigma \underline{F} &= (80 + 40 + 40 + 50 \sin 30^\circ) \underline{i} \\ &\quad + (50 \cos 30^\circ + 70) \underline{j} \\ &= \underline{185 \underline{i} + 113.3 \underline{j} \text{ lb}}\end{aligned}$$

$$\begin{aligned}M_G &= 70(66) + 50 \sin 30^\circ (36) = 5520 \text{ lb-in.} \\ &= \underline{460 \text{ lb-ft}} \quad (\curvearrowright)\end{aligned}$$

For line of action of resultant:

$$\underline{r} \times \underline{R} = \underline{M}_G$$

$$(x \underline{i} + y \underline{j}) \times (185 \underline{i} + 113.3 \underline{j}) = 460 \underline{k}$$

$$113.3x - 185y = 460$$

$$\underline{x = 4.06 \text{ ft}} \quad \text{when } y = 0.$$

2/98

For a zero force-couple system
at point O:

$$\underline{R} = \Sigma \underline{F} = (-F_C \sin 30^\circ + F_D \sin 30^\circ) \underline{i} + (50 - 10 - 100 - 50 + F_B + F_C \cos 30^\circ + F_D \cos 30^\circ) \underline{j} = \underline{0}$$

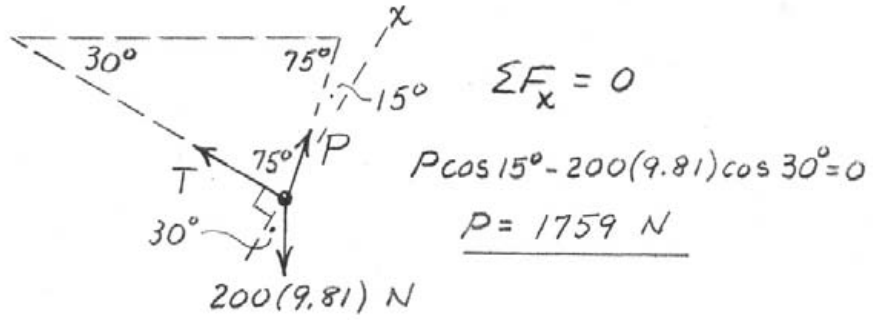
$$\Rightarrow F_C = F_D = F$$

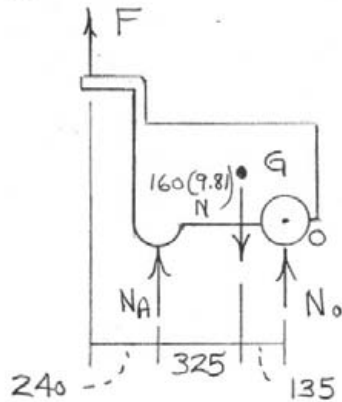
$$\Sigma M_O = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) - 50(2.5) + 2F \cos 30^\circ (2.9) = 0$$

$$\underline{F = F_C = F_D = 6.42 \text{ N}}, \quad \underline{F_B = 98.9 \text{ N}}$$

$$\begin{aligned}
 \boxed{2/99} \quad \underline{F} &= F \underline{n} \\
 &= 60 \left[\frac{40\underline{i} - 50\underline{j} + 110\underline{k}}{\sqrt{40^2 + 50^2 + 110^2}} \right] \\
 &= \underline{18.86\underline{i} - 23.6\underline{j} + 51.9\underline{k} \text{ N}} \\
 \cos \theta_y &= \frac{F_y}{F} = \frac{-23.6}{60}, \quad \underline{\theta_y = 113.1^\circ}
 \end{aligned}$$

3/1





With $F = 0$:

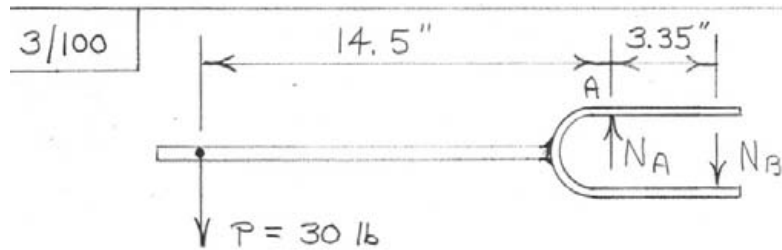
$$\sum M_o = 0: 160(9.81)(135) - N_A(460) = 0$$

$$N_A = 461 \text{ N}$$

For $\frac{1}{2} N_A$:

$$\sum M_o = 0: 160(9.81)(135) - \frac{461}{2}(460) - F(700) = 0$$

$$\underline{F = 151.4 \text{ N}}$$



$$\sum M_A = 0: 30(14.5) - N_B(3.35) = 0$$

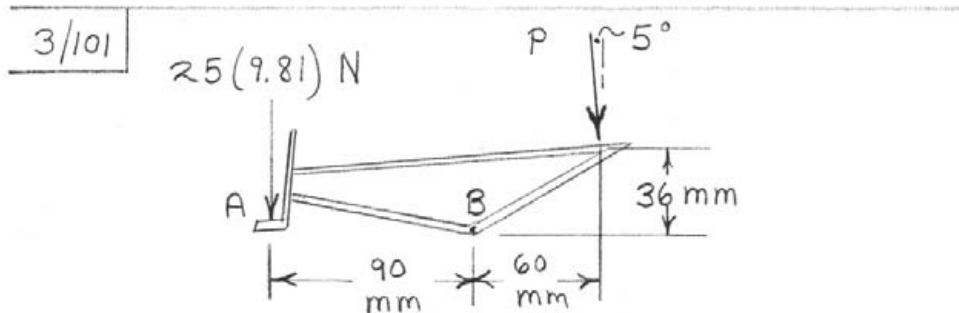
$$N_B = 129.9 \text{ lb}$$

$$\sum F = 0: -30 + N_A - 129.9 = 0$$

$$N_A = 159.9 \text{ lb}$$

So the forces applied to the stud are

$$\begin{cases} N_A = 159.9 \text{ lb} \downarrow \\ N_B = 129.9 \text{ lb} \uparrow \end{cases}$$

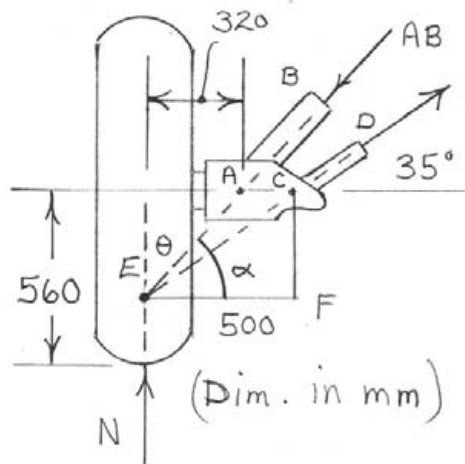


$$\begin{aligned} \sum M_B = 0: & \quad 25(9.81)(90) - P \cos 5^\circ (60) \\ & \quad - P \sin 5^\circ (36) = 0 \\ & \quad \underline{P = 351 \text{ N}} \end{aligned}$$

Assumptions: The weight of the panel acts at the 90-mm dimension as shown above; panel does not slip.

3/102

The three forces must be concurrent at point E.



$$\overline{CF} = 500 \tan 35^\circ$$

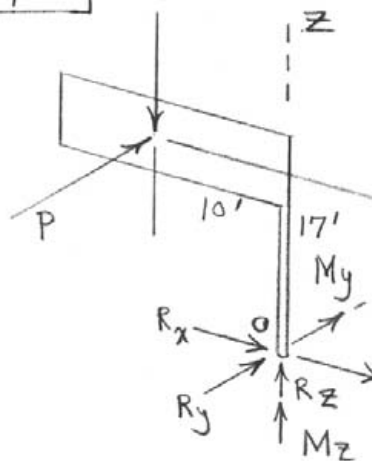
$$= 350 \text{ mm}$$

$$\tan \theta = \frac{320}{350}$$

$$\theta = 42.4^\circ$$

3/103

600 lb

Wind force $P = pA$

$$= 17.5(6)(12) = 1260 \text{ lb}$$

$$\sum F_x = 0 \Rightarrow R_x = 0$$

$$\sum F_y = 0 \Rightarrow R_y = -1260 \text{ lb}$$

$$\sum F_z = 0 \Rightarrow R_z = 600 \text{ lb}$$

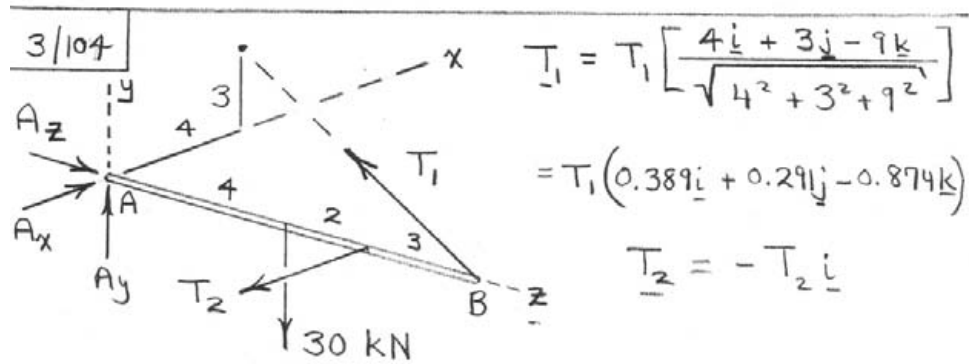
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= 1396 \text{ lb}$$

$$\sum \underline{M}_o = \underline{0} : \underline{M} + (-10\underline{i} + 17\underline{k}) \times (1260\underline{j} - 600\underline{k}) = \underline{0}$$

$$\Rightarrow \underline{M} = 21,400 \underline{i} + 6000 \underline{j} + 12,600 \underline{k} \text{ lb-ft}$$

$$M = 25,600 \text{ lb-ft}$$



$$\sum M_x = 0: 30(4) - 0.291 T_1(9) = 0, \quad \underline{T}_1 = 45.8 \text{ kN}$$

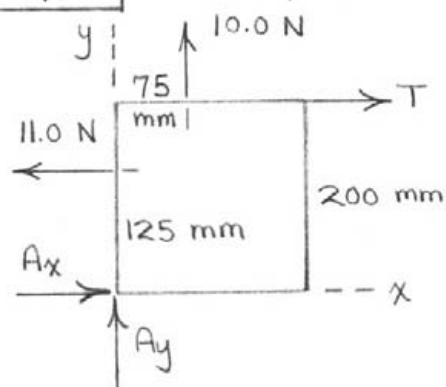
$$\sum M_y = 0: -T_2(6) + 0.389 T_1(9) = 0, \quad \underline{T}_2 = 26.7 \text{ kN}$$

$$\sum \underline{F} = \underline{0}: \underline{A} + 45.8(0.389\underline{i} + 0.291\underline{j} - 0.874\underline{k}) - 26.7\underline{i} - 30\underline{j} = \underline{0}$$

$$\underline{A} = 8.89\underline{i} + 16.67\underline{j} + 40.0\underline{k} \quad \text{kN}$$

$$\underline{A} = 44.2 \text{ kN}$$

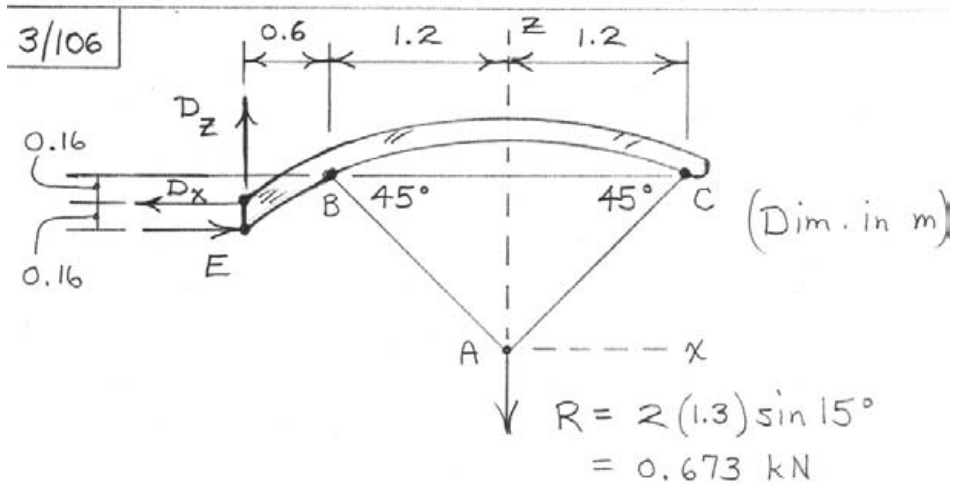
3/105



Isolate entire system
of plate, pulleys, & tape.

$$+\circlearrowleft \sum M_A = 0: T(200) - 10.0(75) - 11.0(125) = 0$$

$$\underline{T = 10.62 \text{ N}}$$



$$\sum F_z = 0: D_z - 0.673 = 0, \quad D_z = 0.673 \text{ kN}$$

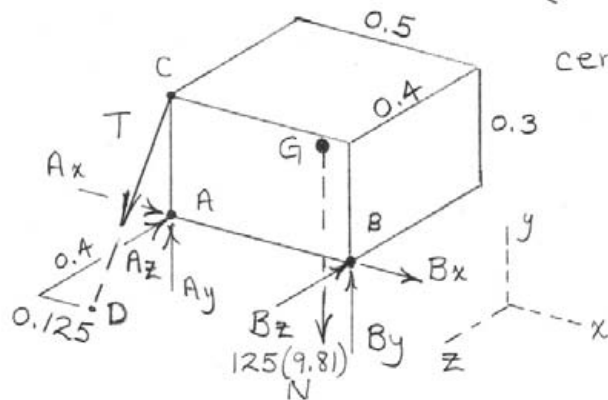
$$\sum M_E = 0: 0.16 D_x - 0.673(1.8) = 0$$

$$D_x = 7.57 \text{ kN}$$

$$D = \sqrt{0.673^2 + 7.57^2} = \underline{7.60 \text{ kN}}$$

3/107

(Dim. in m, G is centered in body)



$$\underline{T} = T \underline{n}_{CD} = T \frac{\underline{CD}}{CD}$$

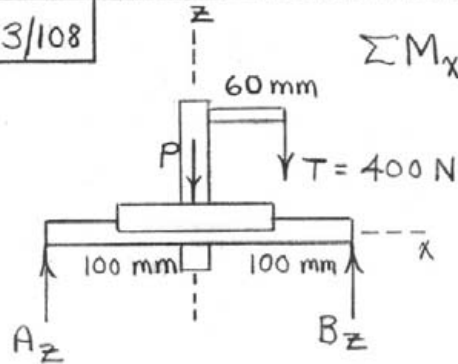
$$= T \left[\frac{0.125 \underline{i} - 0.3 \underline{j} + 0.4 \underline{k}}{\sqrt{0.125^2 + 0.3^2 + 0.4^2}} \right]$$

$$= T (0.243 \underline{i} - 0.582 \underline{j} + 0.776 \underline{k})$$

$$\sum M_{Ax} = 0 : 0.776T(0.3) - 125(9.81)(0.2) = 0$$

$$\underline{T = 1053 \text{ N}}$$

3/108



$$\sum M_x = 0: P(200) - 400(120 \cos 30^\circ) = 0$$

$$P = 208\text{ N}$$

$$\sum M_{Ay} = 0: 208(100) + 400(160) - B_z(200) = 0$$

$$B_z = 424\text{ N}$$

$$\sum F_z = 0: A_z + 424 - 208 - 400 = 0, A_z = 183.9\text{ N}$$

$$\text{Because } A_y = B_y = 0, \underline{A = A_z = 183.9\text{ N}}, \underline{B = B_z = 424\text{ N}}$$

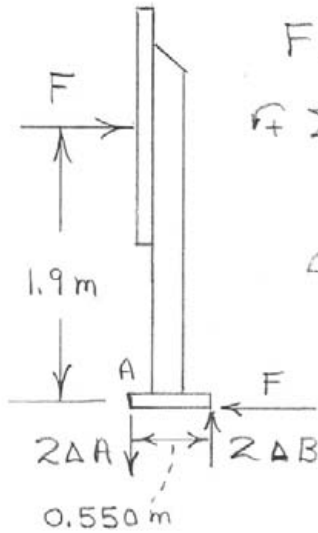
3/109

$$F = pA = 175 (1.6 \cdot 3) = 840 \text{ N}$$

$$\text{From } \uparrow \Sigma F = 0, \Delta A = \Delta B$$

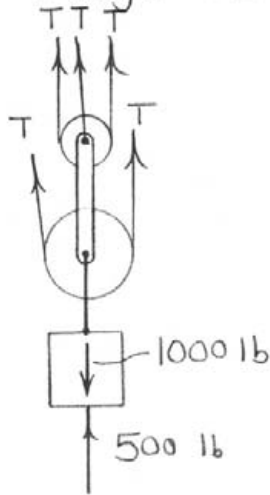
$$\curvearrowright \Sigma M_A = 0: -840(1.9) + 2\Delta B(0.550) = 0$$

$$\Delta B = \Delta A = \underline{1451 \text{ N}}$$



3/11

FBD of 1000-lb weight and lower pair of pulleys:



$$\uparrow \Sigma F = 0: 5T + 500 - 1000 = 0, \quad \underline{T = 100 \text{ lb}}$$

(We assume that the nonverticality of some of the cables is negligible.)

3/110

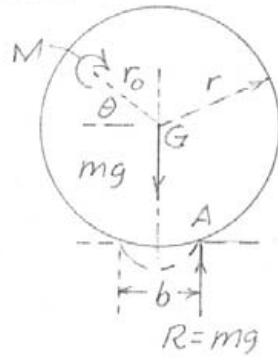
$W = 20(9.81) = 196.2 \text{ N}$

$\Sigma M_x = 0;$
 $1200 T_C - 2(196.2)600 = 0$
 $T_C = 196.2 \text{ N}$

$\Sigma M_y = 0;$
 $196.2(600 + 600 \cos 60^\circ) - 1200 T_A = 0$
 $T_A = 147.2 \text{ N}$

$\Sigma F_z = 0; T_B + 147.2 + 196.2 - 3(196.2) = 0, T_B = 245 \text{ N}$

3/111

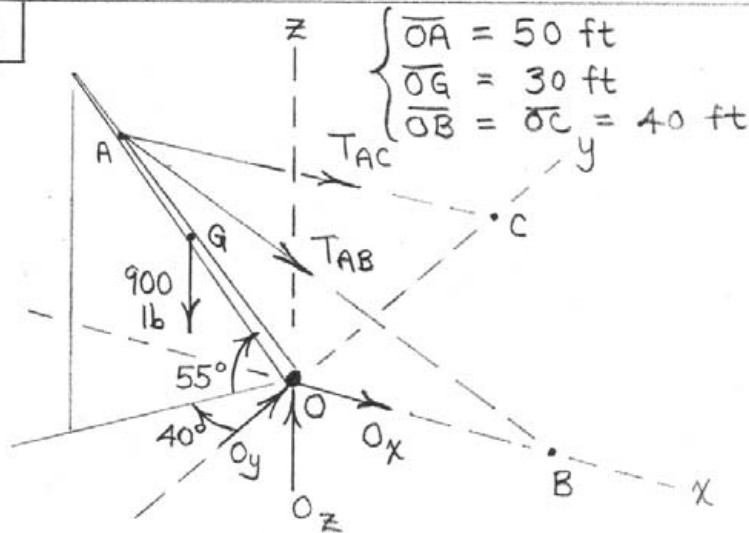


$$\sum M_A = 0; M - mg \frac{b}{2} = 0$$

$$M = \frac{1}{2} mg b$$

Independent of r_0, r, θ

3/112



Coordinates of A: $50(-\cos 55^\circ \sin 40^\circ, -\cos 55^\circ \cos 40^\circ, \sin 55^\circ) = (-18.43, -22.0, 41.0)$ ft

Coordinates of G: $30(-\cos 55^\circ \sin 40^\circ, -\cos 55^\circ \cos 40^\circ, \sin 55^\circ) = (-11.06, -13.18, 24.6)$ ft

$$\begin{aligned} \underline{T}_{AB} &= T_{AB} \left[\frac{(18.43 + 40)\underline{i} + 22.0\underline{j} - 41.0\underline{k}}{\sqrt{(18.43 + 40)^2 + 22.0^2 + 41.0^2}} \right] \\ &= T_{AB} [0.783\underline{i} + 0.294\underline{j} - 0.549\underline{k}] \end{aligned}$$

$$\underline{T}_{AC} = T_{AC} \left[\frac{18.43\underline{i} + (22.0 + 40)\underline{j} - 41.0\underline{k}}{\sqrt{18.43^2 + (22.0 + 40)^2 + 41.0^2}} \right]$$

$$= T_{AC} [0.241\mathbf{i} + 0.810\mathbf{j} - 0.535\mathbf{k}]$$

$$\Sigma F_x = 0: 0.783T_{AB} + 0.241T_{AC} + O_x = 0 \quad (1)$$

$$\Sigma F_y = 0: 0.294T_{AB} + 0.810T_{AC} + O_y = 0 \quad (2)$$

$$\Sigma F_z = 0: -0.549T_{AB} - 0.535T_{AC} + O_z - 900 = 0 \quad (3)$$

$$\Sigma M_{BC} = 0: \Sigma \mathbf{M}_B \cdot \mathbf{n}_{BC} = 0:$$

$$\left\{ -40\mathbf{i} \times 0_z\mathbf{k} + [(-40 - 11.06)\mathbf{i} - 13.18\mathbf{j}] \times [-900\mathbf{k}] \right\} \cdot$$

$$\left(-\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} \right) = 0$$

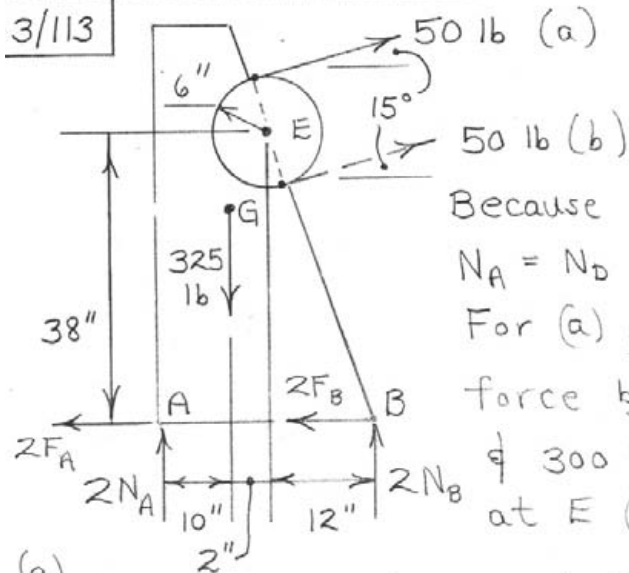
$$\text{or } (40O_z - 46,000)\frac{\sqrt{2}}{2} - 11,860\frac{\sqrt{2}}{2} = 0 \quad (4)$$

$$\Sigma M_{Oy} = 0: 0.549T_{AB}(40) - 900(11.06) = 0 \quad (5)$$

Solve Eqs. (1)-(5) in reverse order to

$$\text{obtain } \begin{cases} O_x = -489 \text{ lb} & T_{AB} = 454 \text{ lb} \\ O_y = -582 \text{ lb} & T_{AC} = 554 \text{ lb} \\ O_z = 1445 \text{ lb} \end{cases}$$

3/113



Because of symmetry,

$$N_A = N_D \text{ \& } N_B = N_C.$$

For (a), replace 50-lb

force by a 50-lb force

\& 300 lb-in. CW couple at E (CCW for (b)).

(a)

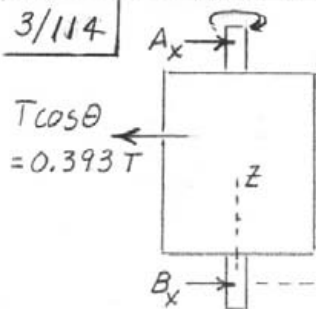
$$\begin{aligned} \sum M_A = 0: & -325(10) + 2N_B(24) - 50 \cos 15^\circ (38) \\ & + 50 \sin 15^\circ (12) - 300 = 0, \quad N_B = N_C = 109.0 \text{ lb} \\ \uparrow \sum F = 0: & 2(109.0) + 2N_A - 325 + 50 \sin 15^\circ = 0 \\ & N_A = N_D = 47.1 \text{ lb} \end{aligned}$$

(b) Reverse the sign on the 300 lb-in. couple

\& obtain

$$\begin{cases} N_B = N_C = 96.5 \text{ lb} \\ N_A = N_D = 59.6 \text{ lb} \end{cases}$$

3/114



$$\sum M_z = 0; 120 - 0.150T = 0, T = 800 \text{ N}$$

$$x-z; \sum M_B = 0; 0.393(800)(0.360) - 0.700A_x = 0; A_x = 161.6 \text{ N}$$

$$\sum F_x = 0; B_x + 161.6 - 0.393(800) = 0$$

$$B_x = 152.6 \text{ N}$$

$$y-z; \sum M_B = 0; 0.7A_y - 0.920(800)(0.360) - 50(9.81)(0.300) = 0; A_y = 588.6 \text{ N}$$

$$\sum F_y = 0; W + T \sin \theta - A_y - B_y = 0,$$

$$B_y = 637.6 \text{ N}$$

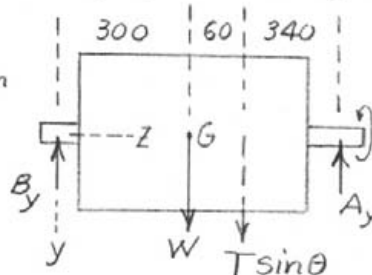
$$W = 50(9.81) \text{ N}$$



$$\beta = \tan^{-1} \frac{180}{240} = 36.9^\circ$$

$$\alpha = \sin^{-1} \frac{150}{300} = 30^\circ$$

$$\theta = \alpha + \beta = 66.9^\circ$$



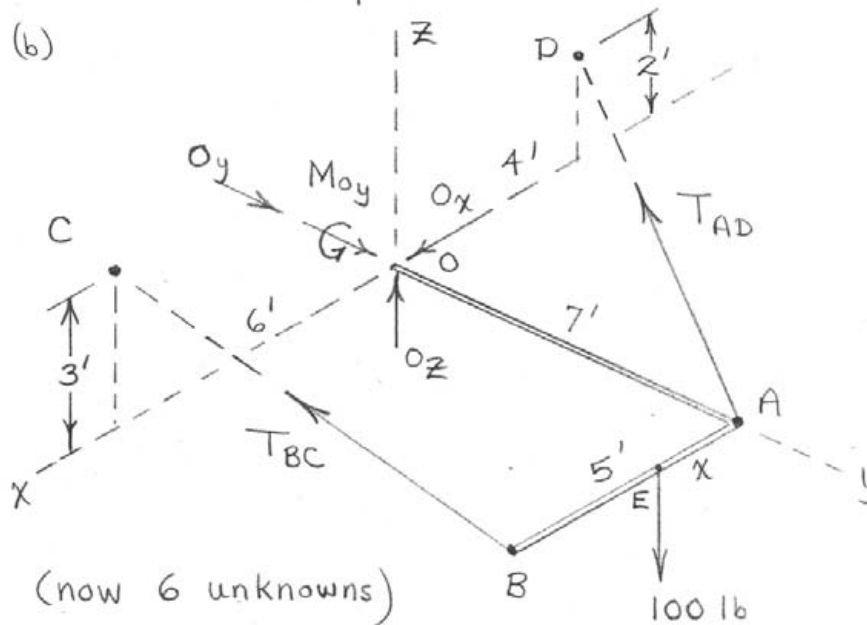
$$A = \sqrt{(161.6)^2 + (588.6)^2} = 610 \text{ N}$$

$$B = \sqrt{(152.6)^2 + (637.6)^2} = 656 \text{ N}$$

3/115

(a) There are 5 unknown constraint forces. The bar is free to rotate about a line which passes through point O and through which the lines of action of both tension forces pass.

(b)



$$\underline{T}_{AD} = T_{AD} \left[\frac{-4\mathbf{i} - 7\mathbf{j} + 2\mathbf{k}}{\sqrt{69}} \right]$$

$$\underline{T}_{BC} = T_{BC} \left[\frac{\mathbf{i} - 7\mathbf{j} + 3\mathbf{k}}{\sqrt{59}} \right]$$

$$\left\{ \begin{array}{l} \sum F_x = 0: O_x - \frac{4}{\sqrt{69}} T_{AD} + \frac{1}{\sqrt{59}} T_{BC} = 0 \end{array} \right. \quad (1)$$

$$\sum F_y = 0: O_y - \frac{7}{\sqrt{69}} T_{AD} - \frac{7}{\sqrt{59}} T_{BC} = 0 \quad (2)$$

$$\sum F_z = 0: O_z + \frac{2}{\sqrt{69}} T_{AD} + \frac{3}{\sqrt{59}} T_{BC} - 100 = 0 \quad (3)$$

$$\sum M_{O_x} = 0: 7 \left(\frac{2}{\sqrt{69}} T_{AD} \right) + 7 \left(\frac{3}{\sqrt{59}} T_{BC} \right) - 7(100) = 0 \quad (4)$$

$$\sum M_{O_y} = 0: -5 \left(\frac{3}{\sqrt{59}} T_{BC} \right) + M_{O_y} + 100x = 0 \quad (5)$$

$$\sum M_{O_z} = 0: 7 \left(\frac{4}{\sqrt{69}} T_{AD} \right) - 7 \left(\frac{1}{\sqrt{59}} T_{BC} \right) - 5 \left(\frac{7}{\sqrt{59}} T_{BC} \right) = 0 \quad (6)$$

Solve Eqs. (1)-(6) over $0.5 \leq x \leq 4.5$ ft

and discover that three of the requested quantities are constant:

$$\underline{T_{AD} = 208 \text{ lb} = \text{constant}}$$

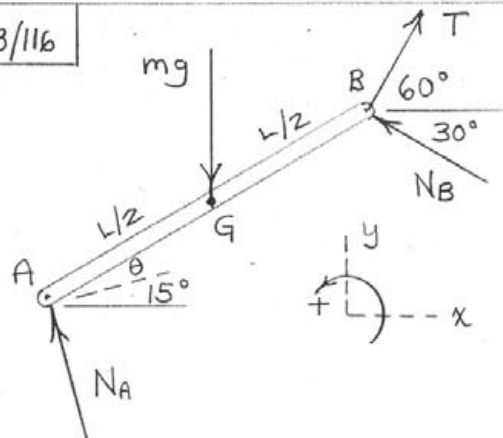
$$\underline{T_{BC} = 128.0 \text{ lb} = \text{constant}}$$

$$\underline{O = \sqrt{O_x^2 + O_y^2 + O_z^2} = 303 \text{ lb} = \text{constant}}$$

$$\text{and } \underline{M_{O_y} = -100x + 250} \text{ (in lb-ft if } x \text{ in ft)}$$

(Note that O_y could have been obtained from $\sum M_{CD} = 0$ & O_z from $\sum M_{AB} = 0$)

*3/116

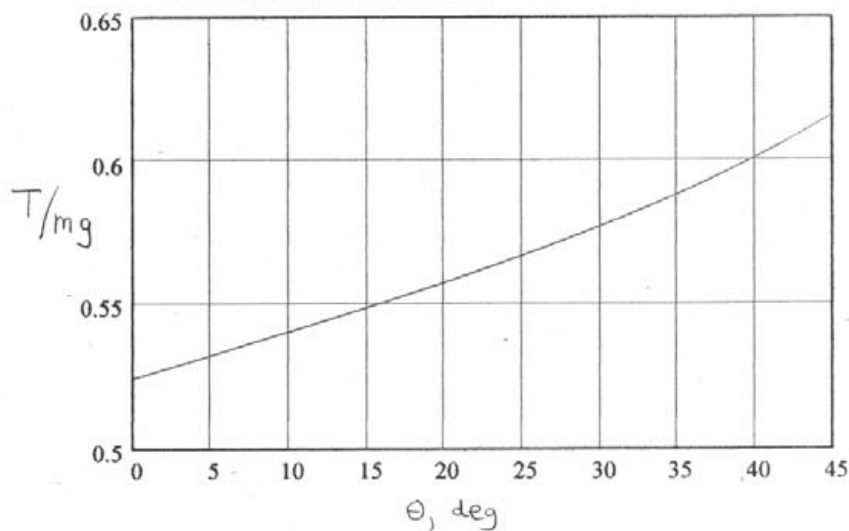


$$\sum F_x = 0: -N_A \sin 15^\circ - N_B \cos 30^\circ + T \sin 30^\circ = 0$$

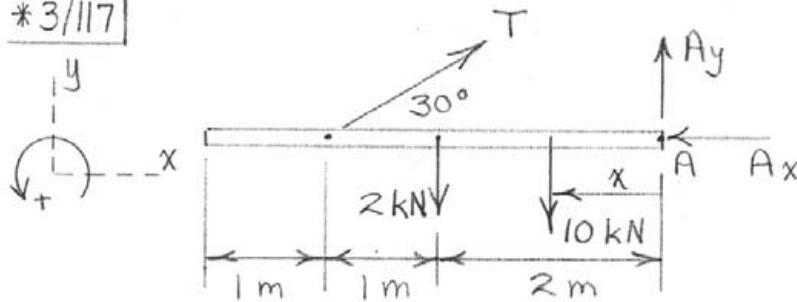
$$\sum F_y = 0: N_A \cos 15^\circ + N_B \sin 30^\circ + T \cos 30^\circ - mg = 0$$

$$\sum M_B = 0: mg \frac{L}{2} \cos (\theta + 15^\circ) - N_A \cos \theta (L) = 0$$

$$\text{Solving, } T = mg \frac{\frac{\sqrt{3}}{2} \cos \theta - \frac{\sqrt{2}}{4} \cos (\theta + 15^\circ)}{\cos \theta}$$



*3/117



(Weight of beam = $200(10)/1000 = 2 \text{ kN}$)

$$\sum M_A = 0: 10x + 2(2) - T \sin 30^\circ (3) = 0$$

$$T = \frac{2}{3} (10x + 4) \quad (\text{in kN})$$

$$\sum F_x = 0: T \cos 30^\circ - A_x = 0$$

$$A_x = \frac{1}{\sqrt{3}} (10x + 4)$$

$$\sum F_y = 0: T \sin 30^\circ - 2 - 10 + A_y = 0$$

$$A_y = \frac{1}{3} (-10x + 32)$$

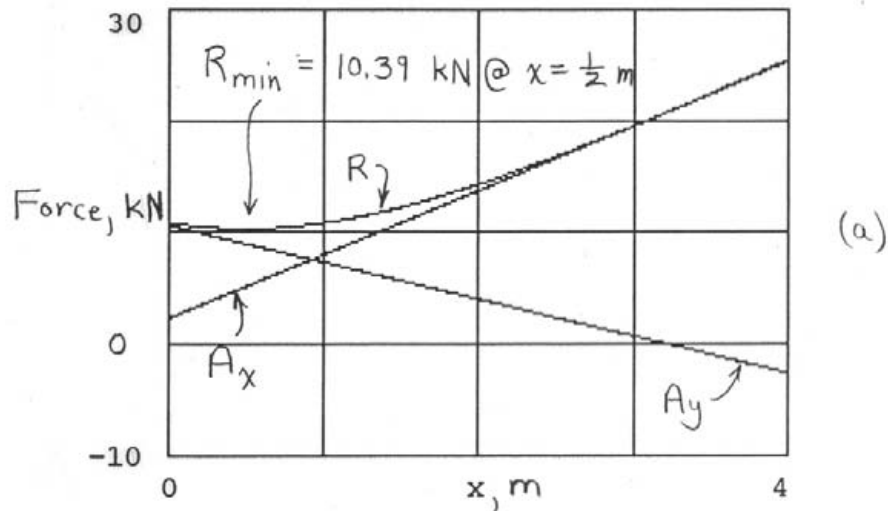
$$R = \{A_x^2 + A_y^2\}^{1/2} = \frac{1}{3} \{400x^2 - 400x + 1072\}^{1/2}$$

Set $\frac{dR^2}{dx} = 0$: $800x - 400 = 0$
 $x = \frac{1}{2} \text{ m}$

$$R_{\min} = \frac{1}{3} \left\{ 400 \left(\frac{1}{2} \right)^2 - 400 \left(\frac{1}{2} \right) + 1072 \right\}^{1/2}$$

$$= \underline{10.39 \text{ kN}} \quad (b)$$

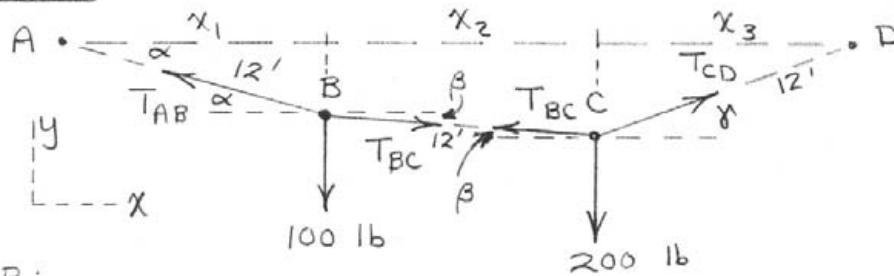
Plot of A_x , A_y , and R :



(c)

$R_{\max} = 24.3 \text{ kN} @ x = 3.8 \text{ m}$ is the value of R which must be used for the design of the pin at A.

*3/118



B:

$$\sum F_x = 0: -T_{AB} \cos \alpha + T_{BC} \cos \beta = 0 \quad (1)$$

$$\sum F_y = 0: T_{AB} \sin \alpha - T_{BC} \sin \beta - 100 = 0 \quad (2)$$

C:

$$\sum F_x = 0: -T_{BC} \cos \beta + T_{CD} \cos \gamma = 0 \quad (3)$$

$$\sum F_y = 0: T_{BC} \sin \beta + T_{CD} \sin \gamma - 200 = 0 \quad (4)$$

$$\cos \alpha = \frac{x_1}{12}, \quad \cos \beta = \frac{x_2}{12}, \quad \cos \gamma = \frac{x_3}{12}$$

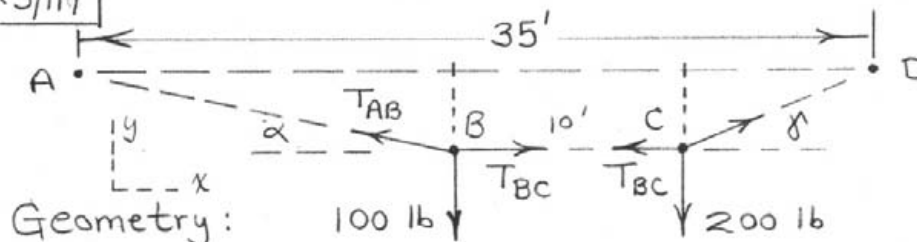
$$\text{So } 12 \cos \alpha + 12 \cos \beta + 12 \cos \gamma = 35 \quad (5)$$

$$\sin \alpha + \sin \beta = \sin \gamma \quad (\text{from figure}) \quad (6)$$

Solve numerically:

$$\begin{cases} \alpha = 14.44^\circ \\ \beta = 3.57^\circ \\ \gamma = 18.16^\circ \end{cases} \quad \begin{cases} T_{AB} = 529 \text{ lb} \\ T_{BC} = 513 \text{ lb} \\ T_{CD} = 539 \text{ lb} \end{cases}$$

*3/119



Geometry:

$$\overline{AB} + \overline{BC} + \overline{CD} = 36 \text{ ft} \quad (1)$$

$$\overline{AB} \cos \alpha + \overline{BC} + \overline{CD} \cos \gamma = 35 \text{ ft} \quad (2)$$

$$\overline{AB} \sin \alpha = \overline{CD} \sin \gamma \quad (3)$$

Equilibrium:

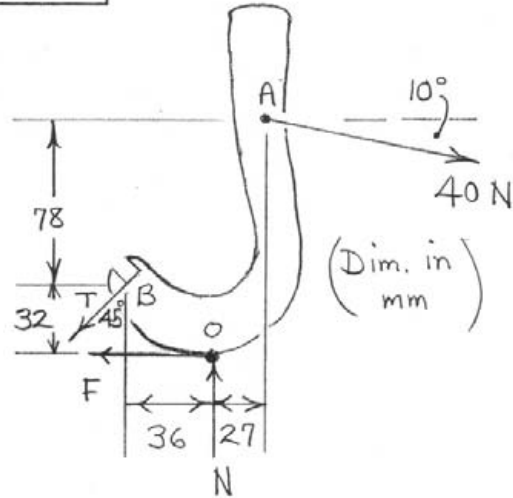
$$\textcircled{B} \begin{cases} \sum F_x = 0: -T_{AB} \cos \alpha + T_{BC} = 0 & (4) \\ \sum F_y = 0: T_{AB} \sin \alpha - 100 = 0 & (5) \end{cases}$$

$$\textcircled{C} \begin{cases} \sum F_x = 0: -T_{BC} + T_{CD} \cos \gamma = 0 & (6) \\ \sum F_y = 0: T_{CD} \sin \gamma - 200 = 0 & (7) \end{cases}$$

With \overline{BC} set to 10 ft, solve 7 equations in 7 unknowns & obtain

$\overline{AB} = 17.01 \text{ ft}$	$\alpha = 11.47^\circ$	$T_{AB} = 503 \text{ lb}$
$\overline{CD} = 8.99 \text{ ft}$	$\gamma = 22.1^\circ$	$T_{BC} = 493 \text{ lb}$
		$T_{CD} = 532 \text{ lb}$

3/12



$$\begin{aligned} \sum M_O = 0: & -40 \cos 10^\circ (110) - 40 \sin 10^\circ (27) \\ & + T \cos 45^\circ (32) + T \sin 45^\circ (36) = 0 \\ & \underline{T = 94.0 \text{ N}} \end{aligned}$$

*3/120

$$W = mg = 20(10) = 200 \text{ N}$$

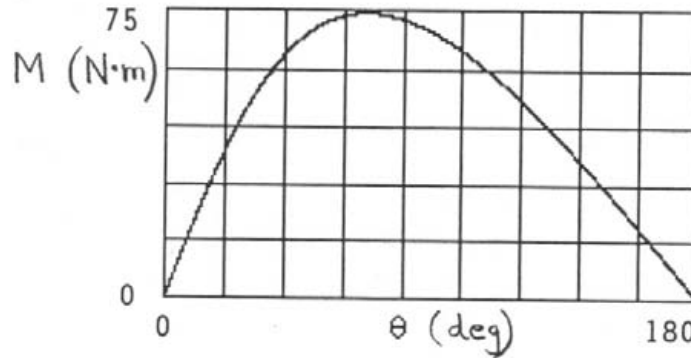


Top view,

$$\sum M = 0 : M = 2W \sin \beta (r \cos \frac{\theta}{2})$$

where $\sin \beta = \frac{2r \sin \frac{\theta}{2}}{\sqrt{(2r \sin \frac{\theta}{2})^2 + h^2}}$

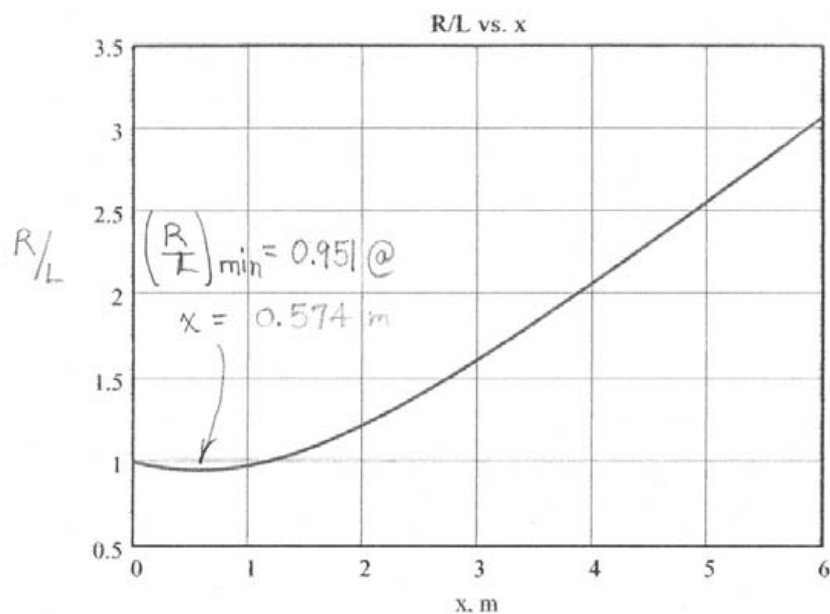
With $h=r$: $M = 2Wr \frac{\sin \theta}{\sqrt{4 \sin^2 \frac{\theta}{2} + 1}} = 120 \frac{\sin \theta \text{ N}\cdot\text{m}}{\sqrt{4 \sin^2 \frac{\theta}{2} + 1}}$



$M_{\max} = 74.2 \text{ N}\cdot\text{m} @ \theta = 67.5^\circ$

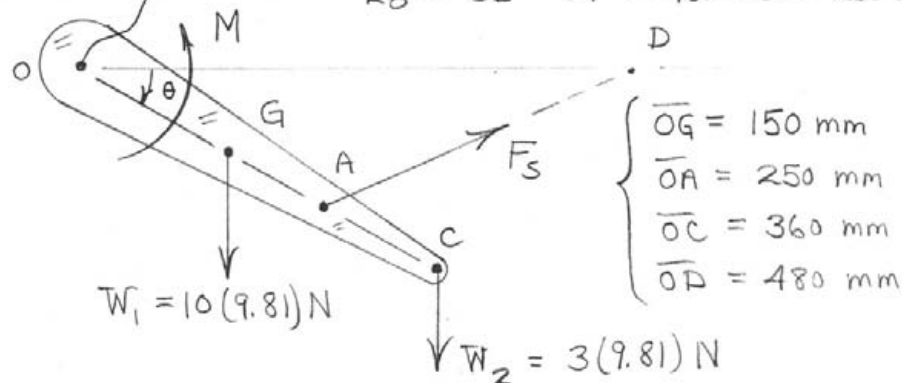
*3/121

$AB = \sqrt{6^2 + 2^2 + 3^2} = 7 \text{ m}$, $CD = \sqrt{3^2 + 4^2} = 5 \text{ m}$
 $\underline{T}_1 = \frac{T_1}{7}(-6\underline{i} - 2\underline{j} + 3\underline{k})$, $\underline{T}_2 = \frac{T_2}{5}(-3\underline{i} + 4\underline{j})$, $\underline{L} = -L\underline{k}$
 $\underline{r}_1 = 6\underline{i}$, $\underline{r}_2 = 3\underline{i}$, $\underline{r} = x\underline{i}$
 $\Sigma \underline{M}_O = 0: \underline{r}_1 \times \underline{T}_1 + \underline{r}_2 \times \underline{T}_2 + \underline{r} \times \underline{L} = 0$
 $6\underline{i} \times \frac{T_1}{7}(-6\underline{i} - 2\underline{j} + 3\underline{k}) + 3\underline{i} \times \frac{T_2}{5}(-3\underline{i} + 4\underline{j}) + x\underline{i} \times (-L\underline{k}) = 0$
Expand: $\frac{6}{7}T_1(-2\underline{k} - 3\underline{j}) + \frac{3}{5}T_2(4\underline{k}) + Lx\underline{j} = 0$
 $-\frac{12}{7}T_1 + \frac{12}{5}T_2 = 0$, $-\frac{18}{7}T_1 + Lx = 0$, so $T_1 = \frac{7}{18}Lx$, $T_2 = \frac{5}{18}Lx$
 $\Sigma F_x = 0; R_x - \frac{3}{5}T_2 - \frac{6}{7}T_1 = 0$, $R_x = \frac{3}{5} \cdot \frac{5}{18}Lx + \frac{6}{7} \cdot \frac{7}{18}Lx = \frac{1}{2}Lx$
 $\Sigma F_y = 0; R_y + \frac{4}{5}T_2 - \frac{2}{7}T_1 = 0$, $R_y = -\frac{4}{5} \cdot \frac{5}{18}Lx + \frac{2}{7} \cdot \frac{7}{18}Lx = -\frac{1}{9}Lx$
 $\Sigma F_z = 0; R_z - L + \frac{3}{7}T_1 = 0$, $R_z = L - \frac{3}{7} \cdot \frac{7}{18}Lx = L(1 - \frac{1}{6}x)$
 $R^2 = L^2(\frac{x^2}{4} + \frac{x^2}{81} + 1 - \frac{x}{3} + \frac{x^2}{36})$
 $R/L = \sqrt{(47x^2/162) - x/3 + 1}$
 $\frac{dR^2/L^2}{dx} = \frac{47x}{81} - \frac{1}{3} = 0$ for min.; $x = 0.574 \text{ m}$



*3/122

Unstretched spring length is
 $L_0 = \overline{OD} - \overline{OA} = 480 - 250 = 230 \text{ mm}$



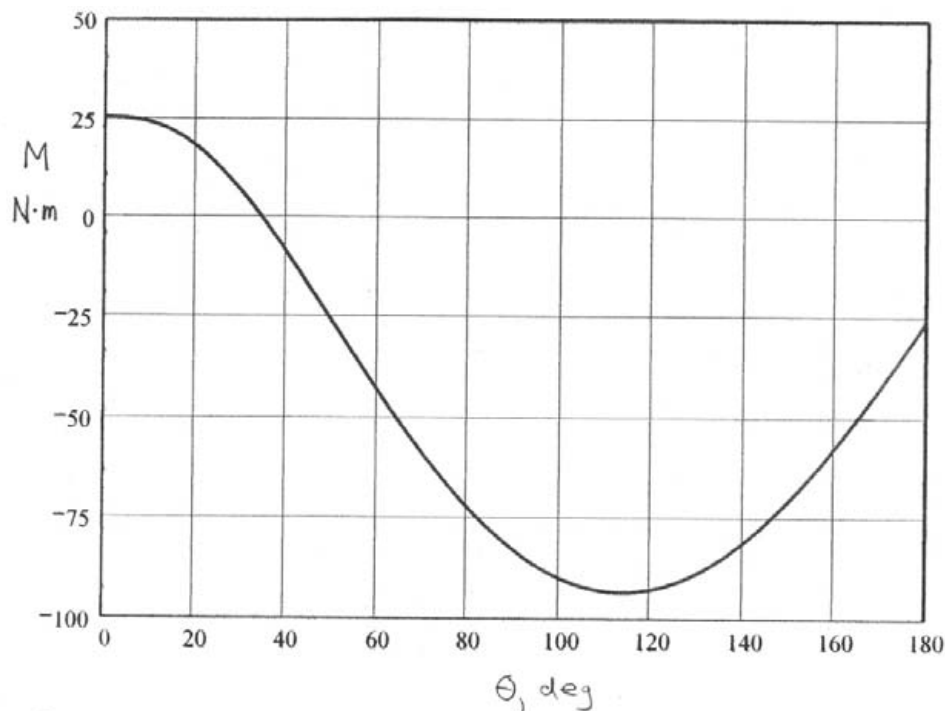
$$\overline{AD} = (\overline{OD} - \overline{OA} \cos \theta) \underline{i} + \overline{OA} \sin \theta \underline{j}$$

$$\overline{AD} = \left\{ [\overline{OD} - \overline{OA} \cos \theta]^2 + [\overline{OA} \sin \theta]^2 \right\}^{1/2}$$

$$\underline{F}_S = k(\overline{AD} - L_0) \frac{\overline{AD}}{\overline{AD}} ; \underline{r}_{OA} = \overline{OA}(\cos \theta \underline{i} - \sin \theta \underline{j})$$

$$\sum M_O = 0: M - (W_1 \overline{OG} + W_2 \overline{OC}) \cos \theta + (\underline{r}_{OA} \times \underline{F}_S)_z = 0$$

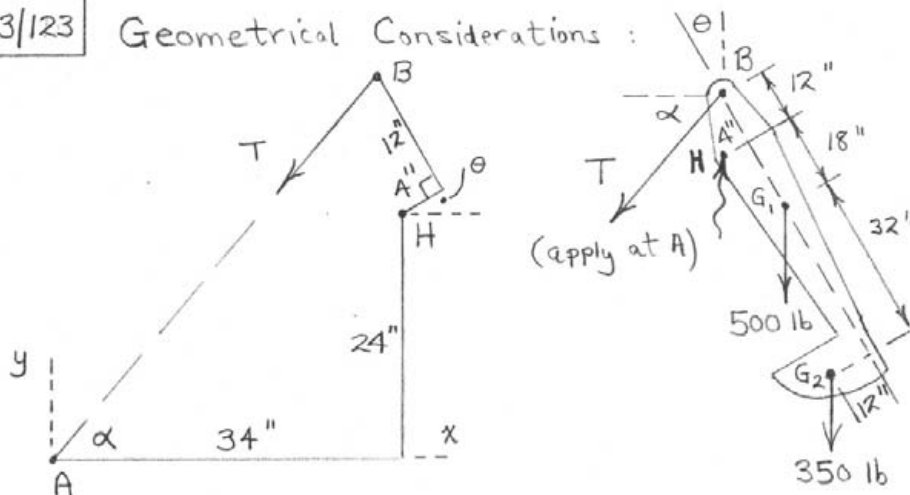
Carry out algebra, solve for M as a function of θ , & plot to obtain



$$\begin{cases} M = 0 @ \theta = 34.6^\circ \\ M_{\min} = -93.4 \text{ N}\cdot\text{m} @ \theta = 113.9^\circ \end{cases}$$

*3/123

Geometrical Considerations :



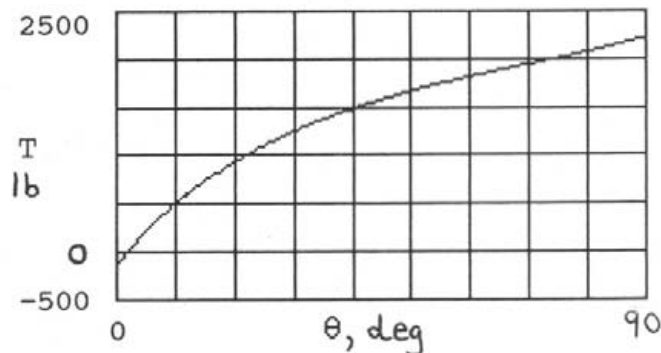
$$\text{Coordinates of B : } \begin{cases} x = 34 + 4 \cos \theta - 12 \sin \theta & (\text{in.}) \\ y = 24 + 4 \sin \theta + 12 \cos \theta & (1) \end{cases}$$

$$\therefore \alpha = \tan^{-1} \frac{y}{x} \quad (\text{aims } \perp) \quad (2)$$

$$\sum M_H = 0 : -(T \cos \alpha)(24) + (T \sin \alpha)(34) - 500(18 \sin \theta + 4 \cos \theta) - 350(4 \cos \theta + (18 + 32) \sin \theta - 12 \cos \theta) = 0$$

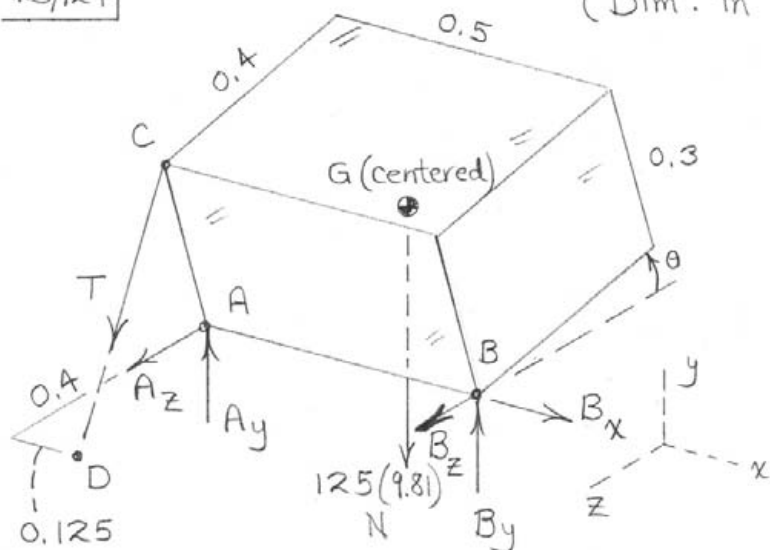
$$\Rightarrow T = \frac{26,500 \sin \theta - 800 \cos \theta}{34 \sin \alpha - 24 \cos \alpha} \quad (\text{lb; positive is tension in AB})$$

Solve (1), (2), & (3) & plot: (Note $T=0$ @ $\theta = 1.729^\circ$)



*3/124

(Dim. in m)



$$\underline{CD} = 0.125\mathbf{i} - 0.3\cos\theta\mathbf{j} + (0.4 - 0.3\sin\theta)\mathbf{k}$$

$$\text{Then } \underline{I} = T\underline{n}_{cd} = T \frac{\underline{CD}}{|\underline{CD}|}, \text{ or}$$

$$\underline{I} = T \left[\frac{0.125\mathbf{i} - 0.3\cos\theta\mathbf{j} + (0.4 - 0.3\sin\theta)\mathbf{k}}{\sqrt{0.266 - 0.24\sin\theta}} \right]$$

$$\sum F_x = 0: \frac{0.125T}{\sqrt{0.266 - 0.24\sin\theta}} + B_x = 0 \quad (1)$$

$$\sum F_y = 0: \frac{-0.3T\cos\theta}{\sqrt{0.266 - 0.24\sin\theta}} + A_y + B_y - 125(9.81) = 0 \quad (2)$$

$$\Sigma F_z = 0: \frac{(0.4 - 0.3 \sin \theta) T}{\sqrt{0.266 - 0.24 \sin \theta}} + A_z + B_z = 0 \quad (3)$$

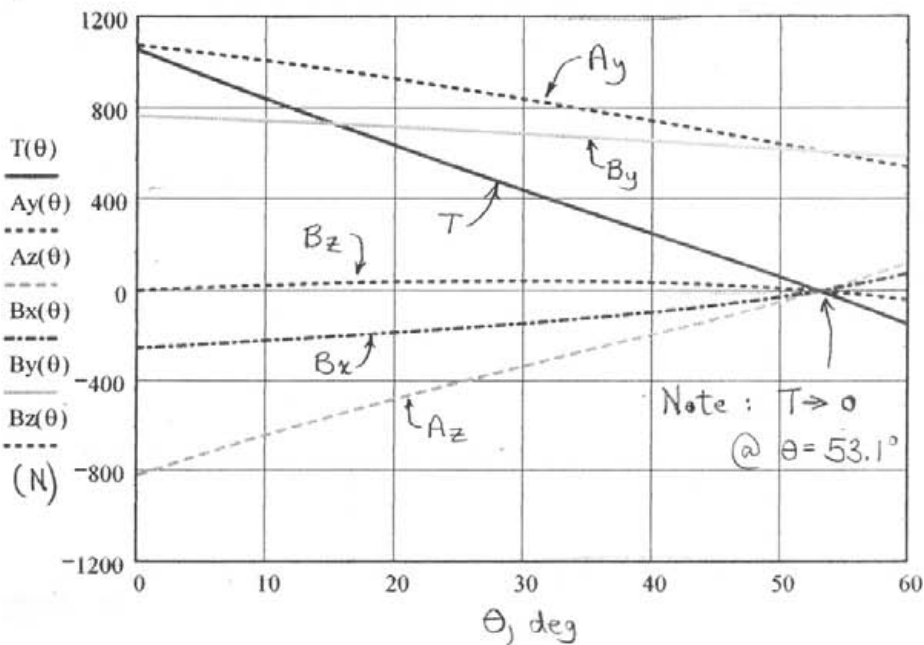
$$\Sigma M_{D_x} = 0: (A_y + B_y)(0.4) - 125(9.81)(0.4 + 0.2 \cos \theta - 0.15 \sin \theta) = 0 \quad (4)$$

$$\Sigma M_{D_y} = 0: -B_x(0.4) + A_z(0.125) - B_z(0.375) = 0 \quad (5)$$

$$\Sigma M_{D_z} = 0: -A_y(0.125) + B_y(0.375) - 125(9.81)(0.125) = 0 \quad (6)$$

Computer Solution:

$$\begin{aligned} T &= -12.77 \frac{-4 \cos \theta + 3 \sin \theta}{\cos \theta} \sqrt{425 - 384 \sin \theta} \\ A_y &= 613 + 460 \cos \theta - 345 \sin \theta \\ A_z &= -12.77 \frac{64 \cos \theta - 48 \sin \theta - 36 \sin \theta \cos \theta + 27 \cos^2 \theta}{\cos \theta} \\ B_x &= 63.9 \frac{-4 \cos \theta + 3 \sin \theta}{\cos \theta} \\ B_y &= 613 + 153.3 \cos \theta - 115.0 \sin \theta \\ B_z &= -38.3 \frac{-4 \cos \theta + 3 \sin \theta}{\cos \theta} \sin \theta \end{aligned}$$



*3/125 With reference to the FBD and solution to Prob. 3/96, the various tension vectors are $\underline{T}_2 = 1000 (0 \underline{i} - \cos 10^\circ \underline{j} - \sin 10^\circ \underline{k})$ N

$$\underline{T}_1 = T_1 (-\sin 30^\circ \cos 10^\circ \underline{i} + \cos 30^\circ \cos 10^\circ \underline{j} - \sin 10^\circ \underline{k})$$

$$\underline{T}_{BE} = \frac{T (10 \cos \theta \underline{i} + 10 \sin \theta \underline{j} - 11 \underline{k})}{\sqrt{(10 \cos \theta)^2 + (10 \sin \theta)^2 + 11^2}}$$

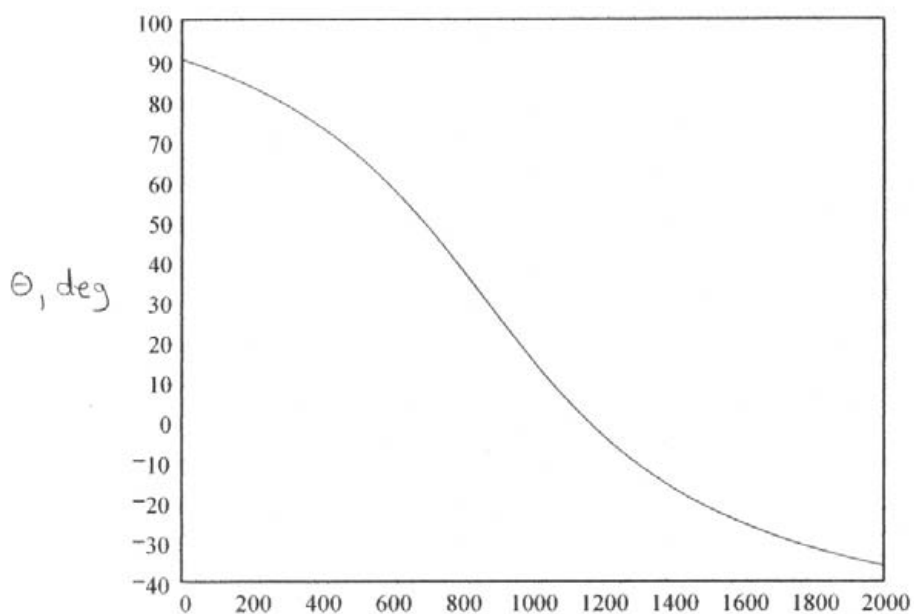
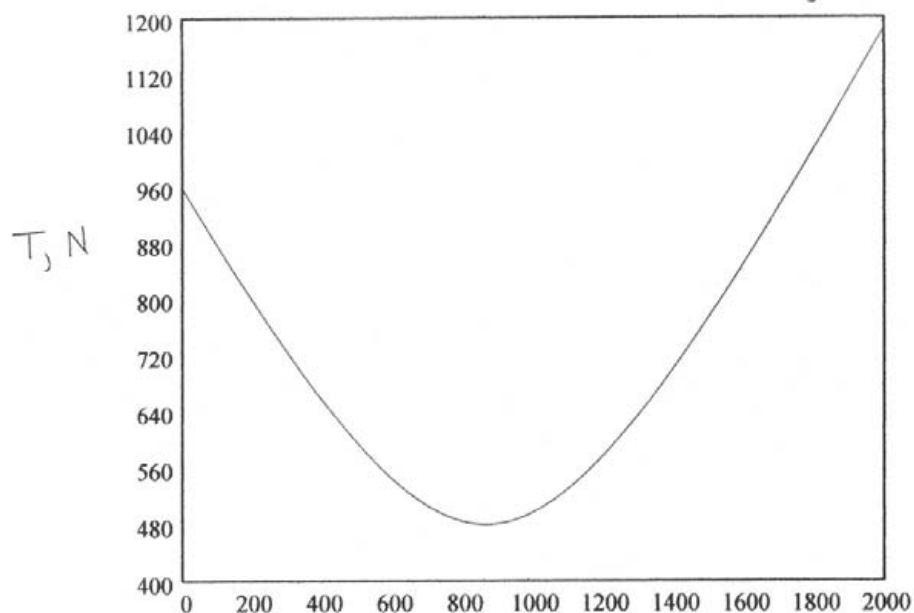
$$\underline{T}_{AD} = \frac{T (8 \cos \theta \underline{i} + 8 \sin \theta \underline{j} - 9 \underline{k})}{\sqrt{(8 \cos \theta)^2 + (8 \sin \theta)^2 + 9^2}}$$

Needed position vectors are $\underline{r}_{oc} = 13 \underline{k}$ m, $\underline{r}_{ob} = 11 \underline{k}$ m, & $\underline{r}_{oa} = 9 \underline{k}$ m.

$$\Sigma \underline{M}_o = \underline{0}: \underline{r}_{oc} \times (\underline{T}_1 + \underline{T}_2) + \underline{r}_{ob} \times \underline{T}_{BE} + \underline{r}_{oa} \times \underline{T}_{AD} = \underline{0}$$

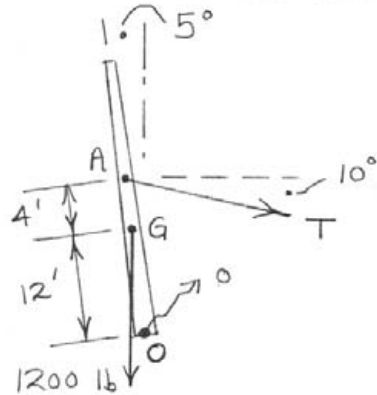
Carry out cross products, collect terms, & set the \underline{i} - & \underline{j} -components to zero, solving

for T and θ for each value of T_1 . Resulting plots:



For $T_1 = 1000 N$:
 $T = 495 N, \theta = 15^\circ$

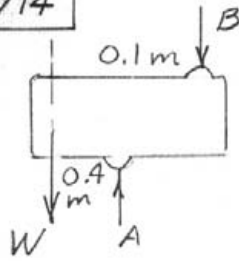
3/13



$$\curvearrowright \sum M_O = 0: 1200(12 \sin 5^\circ) - T \cos 15^\circ(16) = 0$$

$$\underline{T = 81.2 \text{ lb}}$$

3/14



$$W = 300(9.81) = 2943 \text{ N}$$

$$\sum M_A = 0; 2943(0.4) - B(0.6) = 0$$

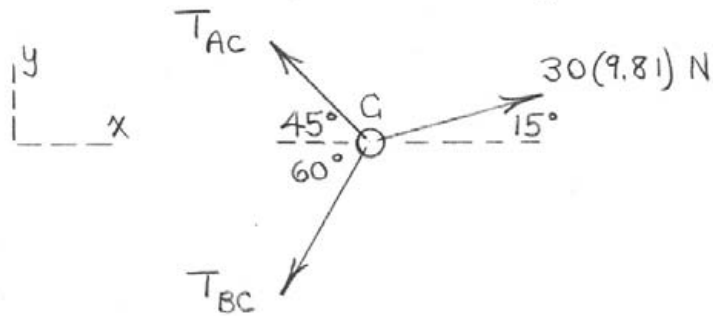
$$B = 1962 \text{ N or } 1.962 \text{ kN}$$

$$\sum F = 0; A = 2943 + 1962$$

$$= 4910 \text{ N or } 4.91 \text{ kN}$$

3/15

FBD of junction ring C:



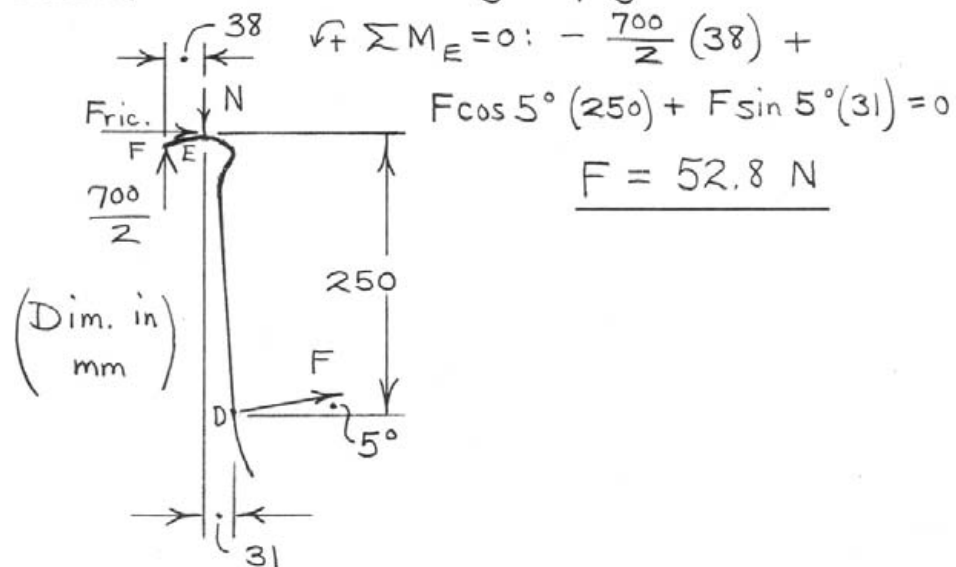
$$\begin{cases} \sum F_x = 0 : -T_{AC} \cos 45^\circ - T_{BC} \cos 60^\circ + 30(9.81) \cos 15^\circ = 0 \\ \sum F_y = 0 : T_{AC} \sin 45^\circ - T_{BC} \sin 60^\circ + 30(9.81) \sin 15^\circ = 0 \end{cases}$$

Solve simultaneously to obtain

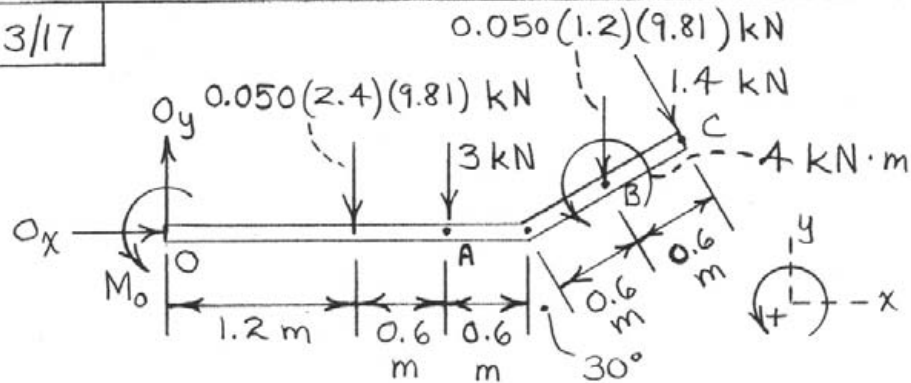
$$\begin{cases} T_{AC} = 215 \text{ N} \\ T_{BC} = 264 \text{ N} \end{cases}$$

3/16

Consider the right pry bar:



3/17



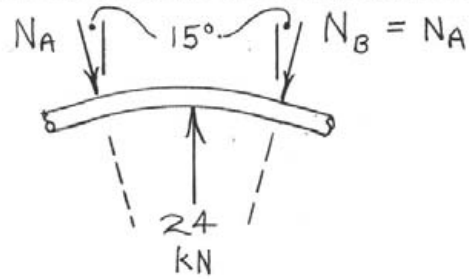
$$\sum F_x = 0 : O_x + 1.4 \sin 30^\circ = 0$$

$$O_x = -0.7 \text{ kN}$$

$$\sum F_y = 0 : O_y - 0.050(2.4)(9.81) - 3 - 1.4 \cos 30^\circ - 0.050(1.2)(9.81) = 0, \quad O_y = 5.98 \text{ kN}$$

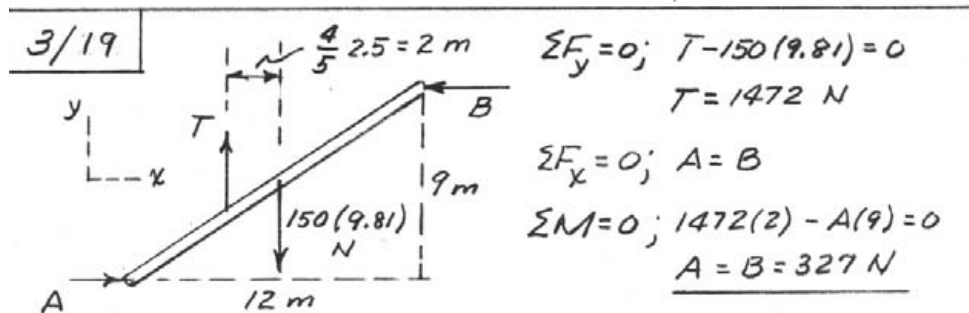
$$\begin{aligned} \sum M_o = 0 : & M_o - 0.050(2.4)(9.81)(1.2) - 3(1.8) \\ & - 0.050(1.2)(9.81)(2.4 + 0.6 \cos 30^\circ) + 4 \\ & - 1.4(2.4 \cos 30^\circ + 1.2) = 0, \quad M_o = 9.12 \text{ kN}\cdot\text{m} \end{aligned}$$

3/18

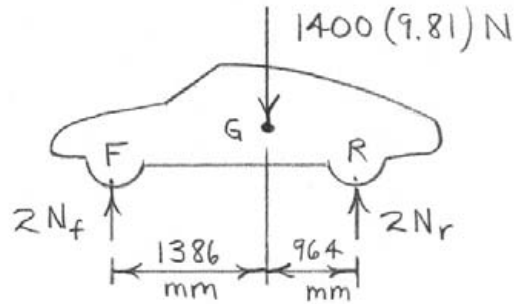


$$\uparrow \sum F = 0: 24 - 2N_A \cos 15^\circ = 0$$

$$\underline{N_A = N_B = 12.42 \text{ kN}}$$



3/2

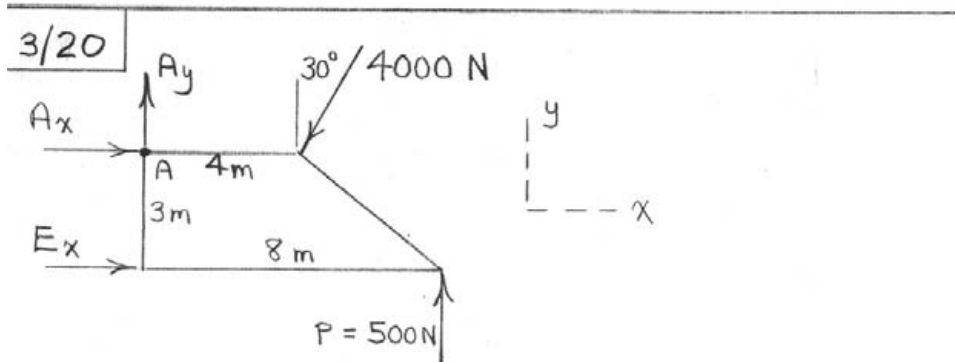


$$\uparrow \Sigma F = 0 : 2N_f + 2N_r - 1400(9.81) = 0$$

$$\curvearrowright \Sigma M_F = 0 : -1400(9.81)(1386) + 2N_r(1386 + 964) = 0$$

$$\text{Solution : } \begin{cases} N_f = 2820 \text{ N} \\ N_r = 4050 \text{ N} \end{cases}$$

Assumes G midway between left and right wheels.



$$\sum F_x = 0: A_x + E_x - 4000 \sin 30^\circ = 0$$

$$\sum F_y = 0: A_y - 4000 \cos 30^\circ + 500 = 0$$

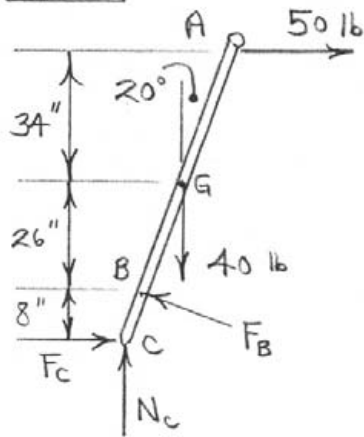
$$\sum M_A = 0: E_x(3) + 500(8) - 4000 \cos 30^\circ(4) = 0$$

$$\Rightarrow \underline{A_x = -1290 \text{ N}}, \quad \underline{A_y = 2960 \text{ N}}, \quad \underline{E_x = 3290 \text{ N}}$$

For maximum P : $E_x = 0$ and $\sum M_A = 0$:

$$P(8) - 4000 \cos 30^\circ(4) = 0, \quad \underline{P = 1732 \text{ N}}$$

3/21



(a) Including 40-lb weight:

$$\curvearrowright \sum M_C = 0: 50(68) + 40(34 \tan 20^\circ) - F_B \frac{8}{\cos 20^\circ} = 0$$

$$F_B = 458 \text{ lb}$$

$$\rightarrow \sum F = 0: F_C - 458 \cos 20^\circ + 50 = 0$$

$$F_C = 380 \text{ lb}$$

(b) Exclude 40-lb weight:

$$\curvearrowright \sum M_C = 0: 50(68) - F_B \frac{8}{\cos 20^\circ} = 0$$

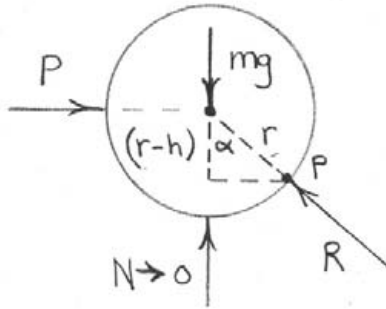
$$F_B = 399 \text{ lb}$$

$$\rightarrow \sum F = 0: F_C - 399 \cos 20^\circ + 50 = 0$$

$$F_C = 325 \text{ lb}$$

3/22

$$\cos \alpha = \frac{r-h}{r}$$

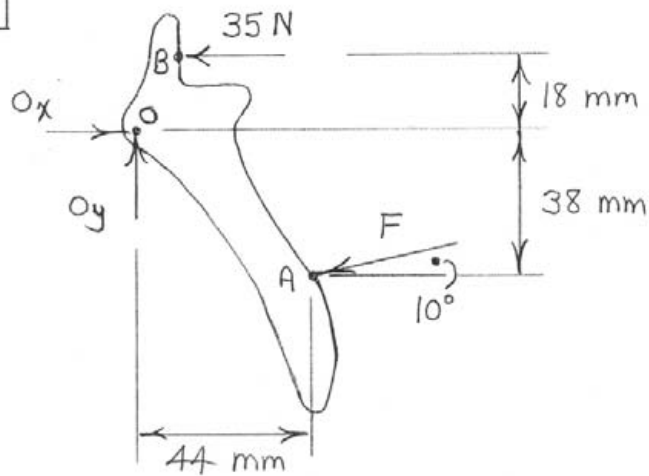


$$\begin{array}{c} r \\ \alpha \\ (r-h) \end{array} \quad \begin{array}{c} \sqrt{r^2 - (r-h)^2} \\ = \sqrt{2rh - h^2} \end{array}$$

$$\sin \alpha = \sqrt{2rh - h^2} / r$$

$$\begin{aligned} +2) \sum M_P = 0 : & \quad P(r-h) - mgr \sin \alpha = 0 \\ \Rightarrow P = & \quad \frac{mg \sqrt{2rh - h^2}}{r-h} \end{aligned}$$

3/23



$$\curvearrowright + \Sigma M_O = 0: 35(18) - (F \cos 10^\circ)(38) - (F \sin 10^\circ)(44) = 0$$

$$F = 13.98 \text{ N}$$

$$\Sigma F_x = 0: O_x - 35 - 13.98 \cos 10^\circ = 0$$

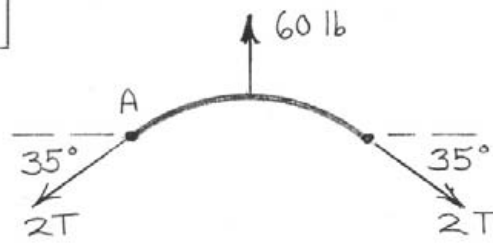
$$O_x = 48.8 \text{ N}$$

$$\Sigma F_y = 0: O_y - 13.98 \sin 10^\circ = 0$$

$$O_y = 2.43 \text{ N}$$

$$O = \sqrt{48.8^2 + 2.43^2} = 48.8 \text{ N}$$

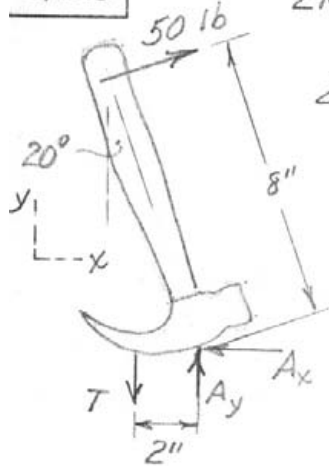
3/24



$$+\uparrow \sum F = 0: 60 - 4T \sin 35^\circ = 0$$

$$\underline{T = 26.2 \text{ lb}}$$

3/25



$$\sum M_A = 0; 50(8) - 2T = 0, \underline{T = 200 \text{ lb}}$$

$$\sum F_x = 0; 50 \cos 20^\circ - A_x = 0$$

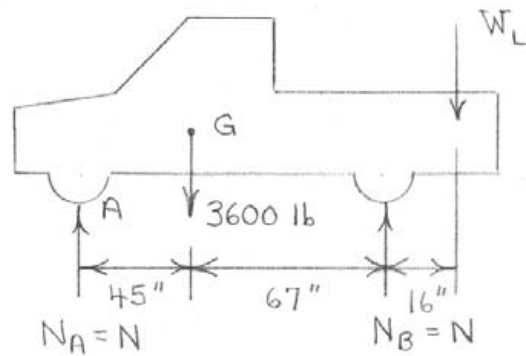
$$A_x = 46.98 \text{ lb}$$

$$\sum F_y = 0; A_y + 50 \sin 20^\circ - 200 = 0$$

$$A_y = 182.9 \text{ lb}$$

$$A = \sqrt{(46.98)^2 + (182.9)^2} = \underline{188.8 \text{ lb}}$$

3/26



$$+\circlearrowleft \sum M_A = 0: 3600(45) - N(112) + W_L(128) = 0$$

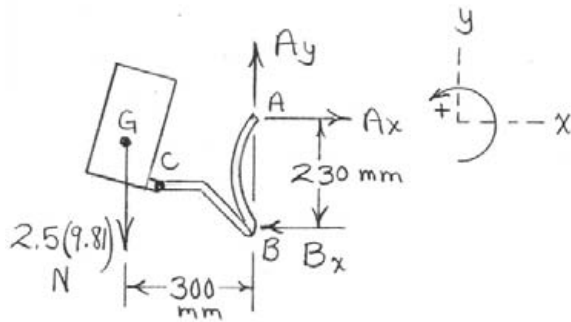
$$+\uparrow \sum F = 0: 2N - 3600 - W_L = 0$$

Solve to obtain $N = 2075 \text{ lb}$

$W_L = 550 \text{ lb}$

3/27

Entire unit:



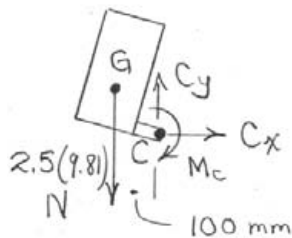
$$\sum M_A = 0: 2.5(9.81)(300) - B_x(230) = 0$$

$$\underline{B_x = 32.0 \text{ N}}$$

$$\sum F_x = 0: A_x - 32.0 = 0, \quad \underline{A_x = 32.0 \text{ N}}$$

$$\sum F_y = 0: A_y - 2.5(9.81) = 0, \quad \underline{A_y = 24.5 \text{ N}}$$

Fixture only:

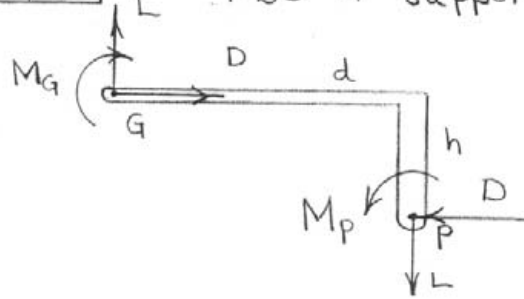


$$\sum M_C = 0: 2.5(9.81)(100) - M_c = 0$$

$$\underline{M_c = 2.45 \text{ N}\cdot\text{m CW}}$$

3/28

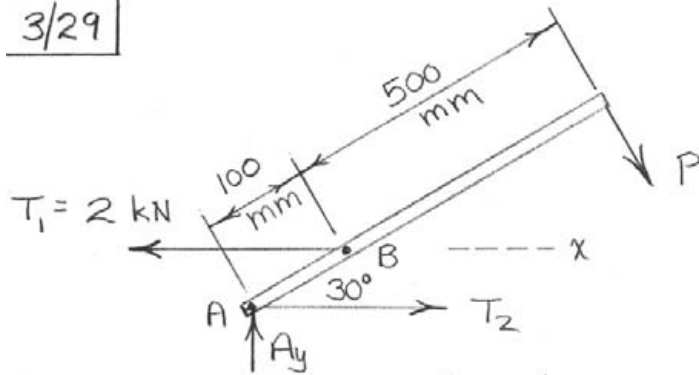
FBD of support & model:



$$\sum M_P = 0: M_P - M_G - Ld - Dh = 0$$

$$M_G = M_P - Ld - Dh$$

3/29



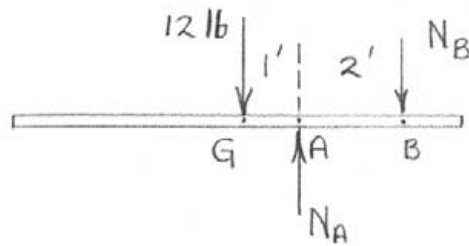
$$\sum M_A = 0: P(500 + 100) - 2(100 \sin 30^\circ) = 0$$

$$P = 0.1667 \text{ kN} \quad \text{or} \quad \underline{P = 166.7 \text{ N}}$$

$$\sum F_x = 0: 0.1667 \sin 30^\circ + T_2 - 2 = 0$$

$$\underline{T_2 = 1.917 \text{ kN}}$$

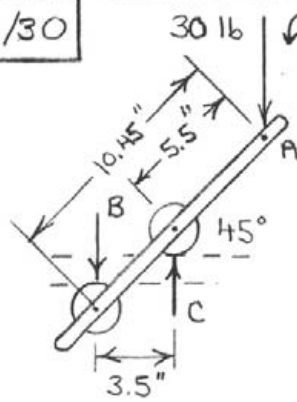
3/3



$$\sum M_B = 0 : 12(3) - N_A(2) = 0$$

$$\underline{N_A = 18 \text{ lb}}$$

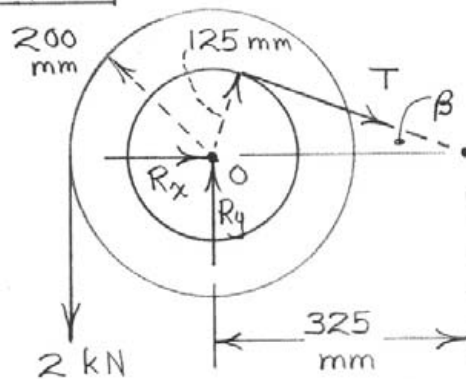
3/30



$$\sum M_C = 0: B(3.5) - (30 \cos 45^\circ)(5.5) = 0, \quad \underline{B = 33.3 \text{ lb}}$$

$$\sum M_B = 0: C(3.5) - (30 \cos 45^\circ)(10.45) = 0, \quad \underline{C = 63.3 \text{ lb}}$$

3/31



$$\begin{cases} \sin \beta = \frac{125}{325} = \frac{5}{13} \\ \cos \beta = \frac{12}{13} \end{cases}$$



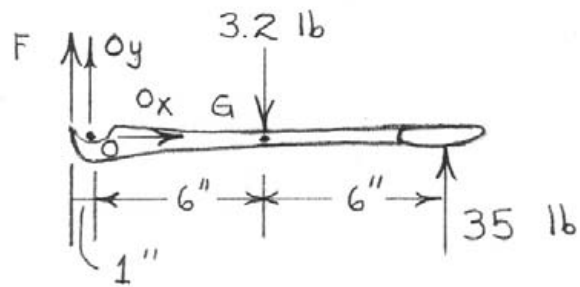
$$\sum M_O = 0: 2(200) - T(125) = 0, T = 3.2 \text{ kN}$$

$$\sum F_x = 0: 3.2\left(\frac{12}{13}\right) + R_x = 0, R_x = -2.95 \text{ kN}$$

$$\sum F_y = 0: R_y - 2 - 3.2\left(\frac{5}{13}\right) = 0, R_y = 3.23 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{2.95^2 + 3.23^2} = \underline{4.38 \text{ kN}}$$

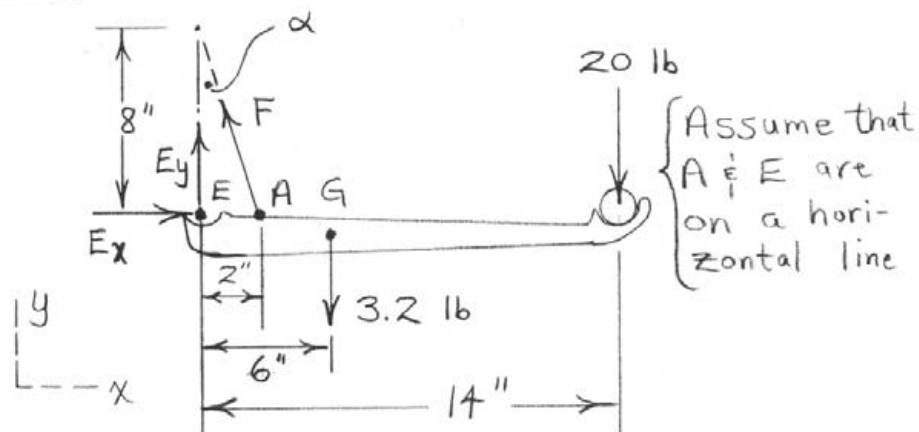
3/32



$$\begin{aligned} \curvearrowright + \sum M_O = 0: & -F(1) - 3.2(6) + 35(12) = 0 \\ & \underline{F = 401 \text{ lb}} \end{aligned}$$

3/33

$$\alpha = \tan^{-1} \frac{2}{8} = 14.04^\circ$$



$$\sum M_E = 0: F \cos 14.04^\circ (2) - 3.2(6) - 20(14) = 0$$

$$F = 154.2 \text{ lb}$$

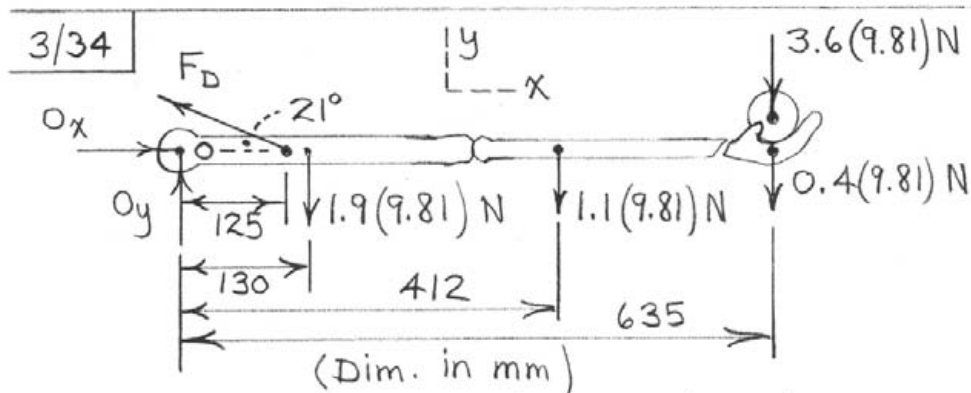
$$\sum F_x = 0: -154.2(\sin 14.04^\circ) + E_x = 0$$

$$E_x = 37.4 \text{ lb}$$

$$\sum F_y = 0: 154.2 \cos 14.04^\circ - 3.2 - 20 + E_y = 0$$

$$E_y = -126.4 \text{ lb}$$

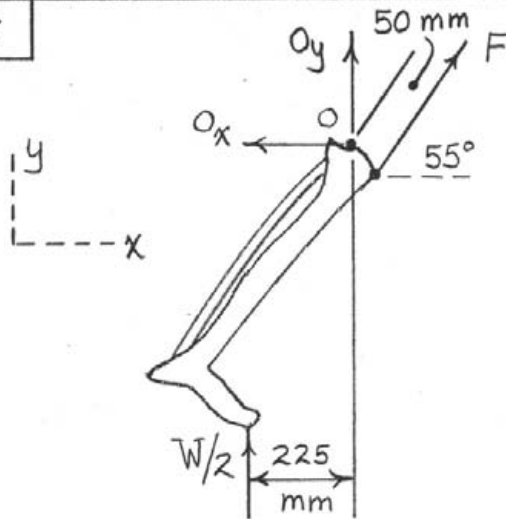
$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{37.4^2 + 126.4^2} = 131.8 \text{ lb}$$



$$\begin{aligned} \curvearrowright \sum M_O = 0: & F_D \sin 21^\circ (125) - 1.9(9.81)(130) \\ & - 1.1(9.81)(412) - (3.6 + 0.4)(9.81)(635) = 0 \\ & \underline{F_D = 710 \text{ N}} \end{aligned}$$

$$\rightarrow \sum F_x = 0: O_x - 710 \cos 21^\circ = 0, \quad \underline{O_x = 662 \text{ N}}$$

$$\begin{aligned} \uparrow \sum F_y = 0: & O_y + 710 \sin 21^\circ - (1.9 + 1.1 + 3.6 + 0.4) 9.81 \\ & = 0, \quad \underline{O_y = -185.6 \text{ N}} \end{aligned}$$



$$\sum M_O = 0: F(50) - \frac{W}{2}(225) = 0, \quad \underline{F = 2.25W}$$

$$\sum F_x = 0: -O_x + 2.25W \cos 55^\circ = 0, \quad O_x = 1.291W$$

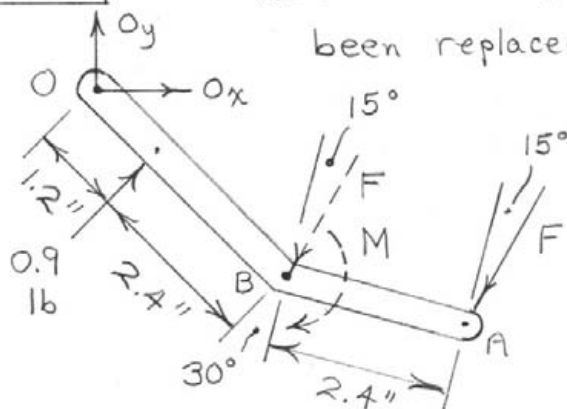
$$\sum F_y = 0: \frac{W}{2} + O_y + 2.25W \sin 55^\circ = 0$$

$$O_y = -2.34W$$

$$O = \sqrt{O_x^2 + O_y^2} = \sqrt{(1.291W)^2 + (2.34W)^2} = \underline{2.67W}$$

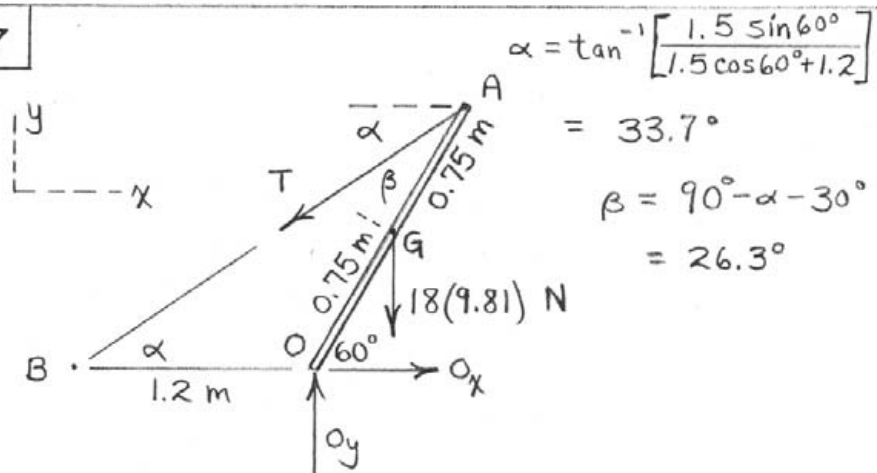
3/36

Note that the force F has been replaced by a force-couple system at B (dashed), where $M = F(2.4 \cos 15^\circ)$



$$\begin{aligned} \sum M_O = 0: & \quad 0.9(1.2) - F(2.4 \cos 15^\circ) \\ & \quad - F(3.6 \cos 15^\circ) = 0 \\ & \quad \underline{F = 0.1863 \text{ lb}} \end{aligned}$$

3/37



$$\sum M_O = 0: T \sin 33.7^\circ (1.2) - 18(9.81)(0.75) \cos 60^\circ = 0$$

$$T = 99.5 \text{ N}$$

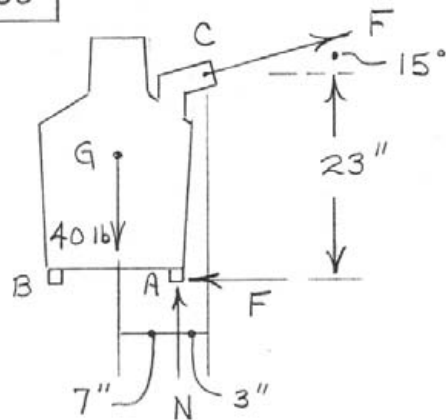
$$\sum F_x = 0: -99.5 \cos 33.7^\circ + O_x = 0$$

$$O_x = 82.8 \text{ N}$$

$$\sum F_y = 0: -99.5 \sin 33.7^\circ - 18(9.81) + O_y = 0$$

$$O_y = 232 \text{ N}$$

$$\text{So } O = 246 \text{ N @ } 70.3^\circ \text{ CCW from } +x\text{-axis}$$

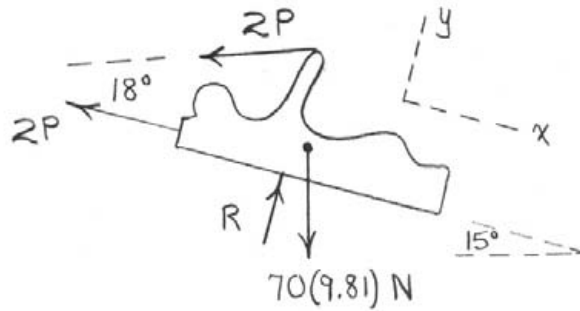


For impending tip,
 $N_B \rightarrow 0$.

$$\sum M_A = 0: 40(7) - F \cos 15^\circ (23) + F \sin 15^\circ (3) = 0$$

$$F = 13.06 \text{ lb}$$

3/39

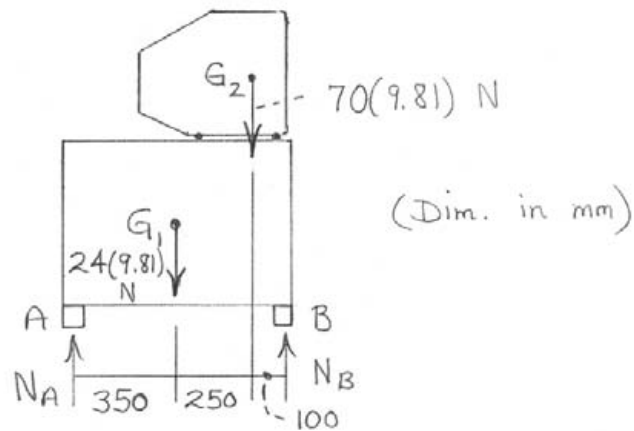


$$\Sigma F_x = 0: 70(9.81) \sin 15^\circ - 2P - 2P \cos 18^\circ = 0$$

$$\underline{P = 45.5 \text{ N}}$$

$$\Sigma F_y = 0: R - 70(9.81) \cos 15^\circ - 2(45.5) \sin 18^\circ = 0$$

$$\underline{R = 691 \text{ N}}$$



$$\curvearrowright \sum M_A = 0: -24(9.81)(350) - 70(9.81)(600) + N_B(700) = 0$$

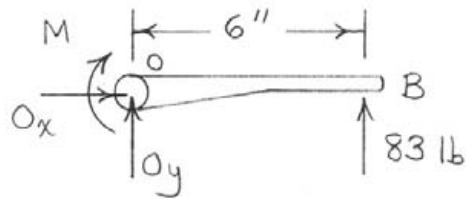
$$N_B = 706 \text{ N}$$

$$\uparrow \sum F = 0: N_A + 706 - (70 + 24)(9.81) = 0$$

$$N_A = 216 \text{ N}$$

3/40

Arm OB:

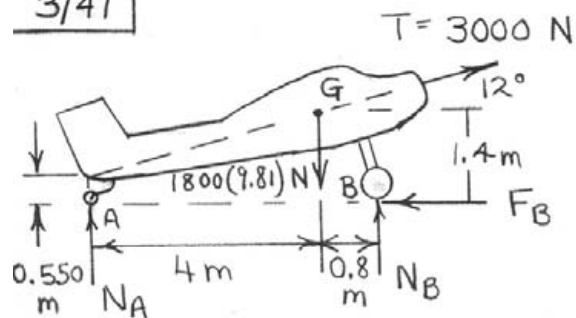


$$\curvearrowleft \sum M_O = 0 : 83(6) - M = 0$$

$$M = 498 \text{ lb-in. or } \underline{41.5 \text{ lb-ft}}$$

$$F \cos 20^\circ (15) = M = 498$$

$$\underline{F = 35.3 \text{ lb}}$$



Engine off : $T = 0$, $F_B = 0$

$$\left\{ \begin{array}{l} \sum M_A = 0: 1800(9.81)4 - N_B(4.8) = 0 \quad N_B = 14720 \text{ N} \\ \sum F_y = 0: N_A + 14720 - 1800(9.81) = 0, \quad N_A = 2940 \text{ N} \end{array} \right.$$

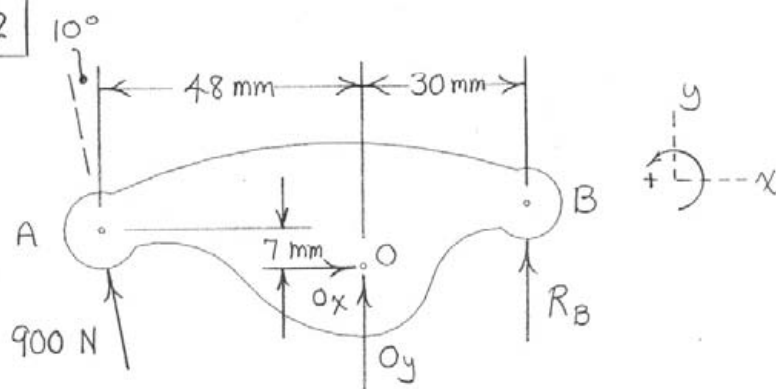
$$\sum M_A = 0: 1800(9.81)4 - N'_B(4.8) + 3000 \cos 12^\circ (0.550) = 0$$

$$N'_B = 15,050 \text{ N}$$

$$\sum F_y = 0: N'_A + 15,050 - 1800(9.81) + 3000 \sin 12^\circ = 0, \quad N'_A = 1983 \text{ N}$$

$$n_A = \frac{N'_A - N_A}{N_A} (100) = -32.6\%, \quad n_B = \frac{N'_B - N_B}{N_B} = 2.28\%$$

3/42



$$\sum M_O = 0: -900 \cos 10^\circ (48) + 900 \sin 10^\circ (7) + R_B (30) = 0, \quad \underline{R_B = 1382 \text{ N}}$$

$$\sum F_x = 0: O_x - 900 \sin 10^\circ = 0, \quad O_x = 156.3 \text{ N}$$

$$\sum F_y = 0: 900 \cos 10^\circ + 1382 + O_y = 0$$

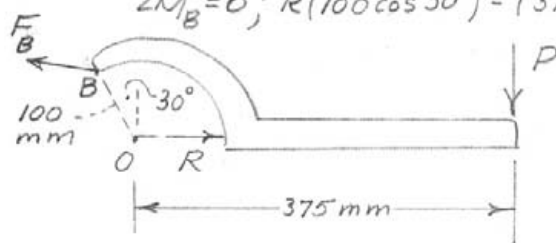
$$O_y = -2270 \text{ N}$$

$$O = \sqrt{O_x^2 + O_y^2} = \underline{2270 \text{ N}}$$

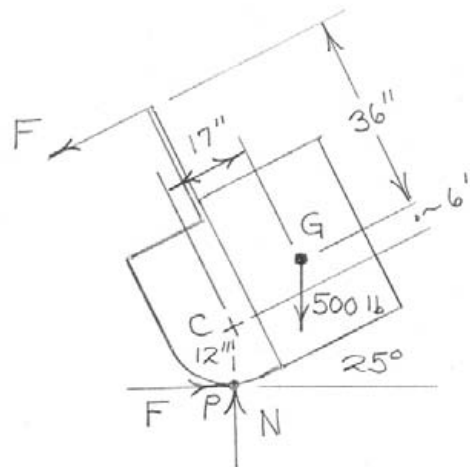
3/43

$$M = Fd; 80 = P(0.375), P = 213 \text{ N}$$

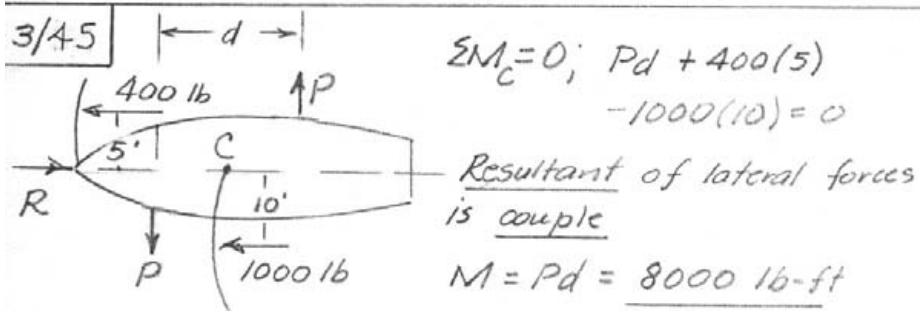
$$\sum M_B = 0; R(100 \cos 30^\circ) - (375 + 100 \sin 30^\circ)(213) = 0$$



$$R = 1047 \text{ N}$$

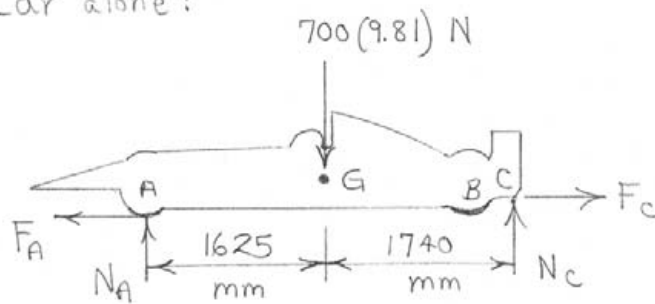


$$\begin{aligned}
 \curvearrowright \sum M_P = 0: & F(36 + 6 + 12 \cos 25^\circ) - \\
 & 500 [17 \cos 25^\circ] + 500 [6 \sin 25^\circ] = 0 \\
 & \underline{F = 121.7 \text{ lb}}
 \end{aligned}$$



3/46

Car alone:

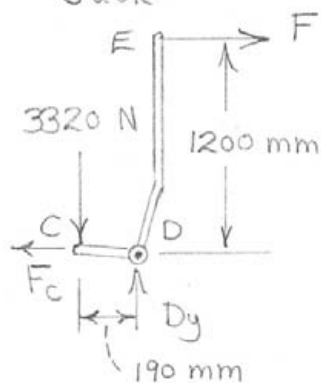


$$\uparrow \sum M_A = 0: -700(9.81)(1625) + N_C(1625 + 1740) = 0$$

$$N_C = 3320 \text{ N}$$

(assumes moment of F_C about A is negligible)

Jack

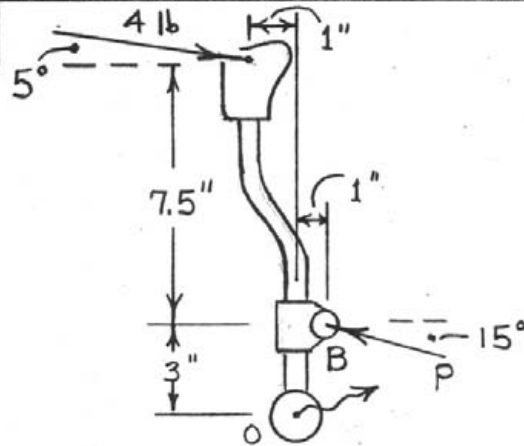


$$\uparrow \sum M_D = 0:$$

$$3320(190) - F(1200) = 0$$

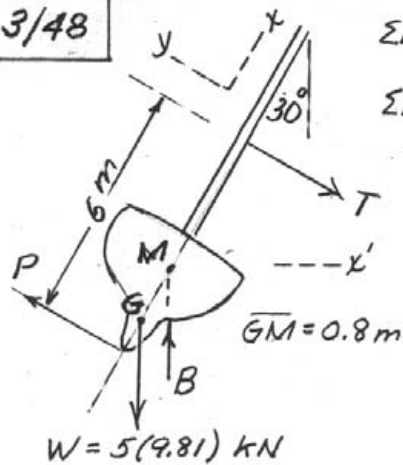
$$\underline{F = 525 \text{ N}}$$

3/47



$$\begin{aligned}
 \sum M_o = 0: & -4 \cos 5^\circ (10.5) + 4 \sin 5^\circ (1) \\
 & + P \cos 15^\circ (3) + P \sin 15^\circ (1) = 0 \\
 & \underline{P = 13.14 \text{ lb}}
 \end{aligned}$$

3/48



$$\Sigma F_x = 0; B = 5(9.81) = 49.05 \text{ kN}$$

$$\Sigma F_{x'} = 0; P = T$$

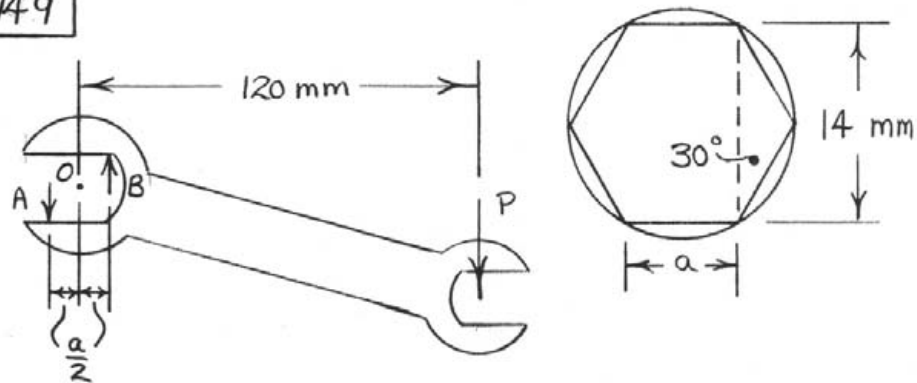
Two couples, so

$$\Sigma M = 0; 6T - 5(9.81)(0.8) \sin 30^\circ = 0$$

$$T = \frac{5(9.81)(0.8)(0.5)}{6}$$

$$= \underline{3.27 \text{ kN}}$$

3/49



$$2a \cos 30^\circ = 14, \quad \frac{a}{2} = 4.04 \text{ mm}$$

$$\curvearrowright \sum M_O = 0: 0.120P - 24 = 0, \quad \underline{P = 200 \text{ N}}$$

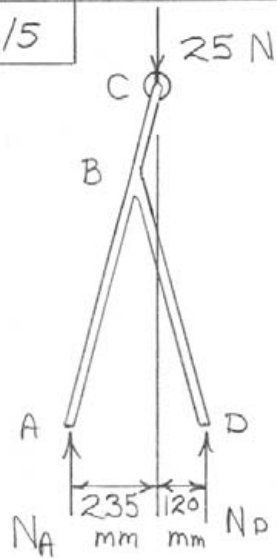
(for wrench and bolt)

For wrench alone,

$$\curvearrowright \sum M_A = 0: 200(0.120 + 0.00404) - B(2 \cdot 0.00404) = 0, \quad \underline{B = 3070 \text{ N}}$$

$$+\uparrow \sum F = 0: -A + 3070 - 200 = 0, \quad \underline{A = 2870 \text{ N}}$$

3/5



$$\curvearrowright \sum M_A = 0 :$$

$$N_D (355) - 25 (235) = 0$$

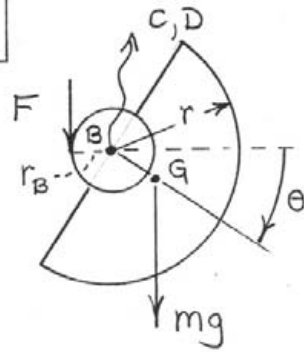
$$N_D = 16.55 \text{ N}$$

$$\uparrow \sum F = 0 :$$

$$N_A + 16.55 - 25 = 0$$

$$N_A = 8.45 \text{ N}$$

3/50



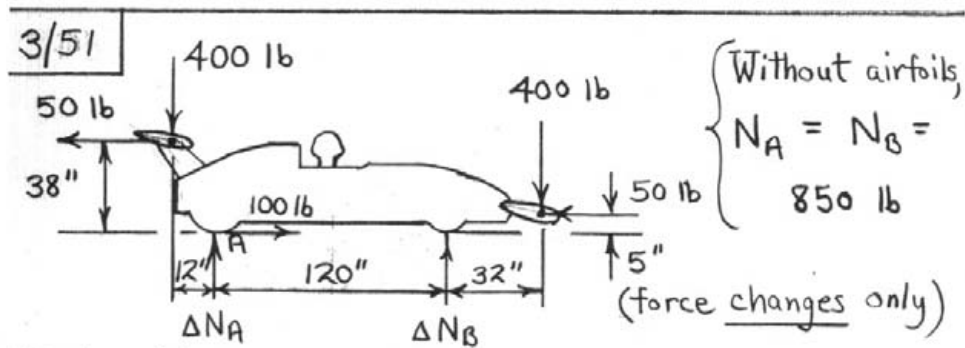
$$\overline{BG} = \bar{r} = \frac{3}{8} r \text{ (from Table D/4)}$$

FBD is of hemisphere,
shaft, and gear B.

$$\begin{aligned} \uparrow \Sigma M_B = 0: & Fr_B - mg \left(\frac{3}{8} r \cos \theta \right) \\ F &= \frac{3}{8} mg \frac{r}{r_B} \cos \theta \end{aligned}$$

Gear A:

$$\begin{aligned} \uparrow \Sigma M_B = 0: & Fr_A - M = 0 \\ M &= \left(\frac{3}{8} mg \frac{r}{r_B} \cos \theta \right) r_A \\ &= \frac{3mgr}{8} \frac{r_A}{r_B} \cos \theta \end{aligned}$$



With airfoils,

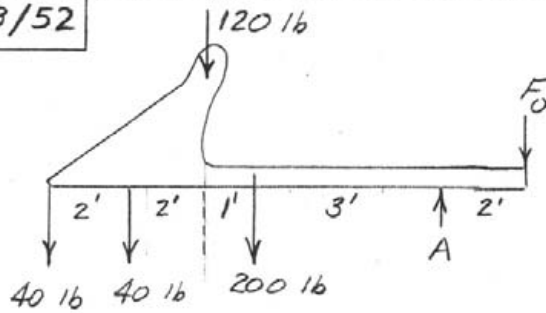
$$+\uparrow \sum F_y = 0 : \Delta N_A + \Delta N_B - 2(400) = 0$$

$$+\circlearrowleft \sum M_A = 0 : 50(38) + 400(12) + \Delta N_B(120) + 50(5) - 400(152) = 0$$

$$\left. \begin{array}{l} \Delta N_A = 351 \text{ lb} \\ \Delta N_B = 449 \text{ lb} \end{array} \right\} \Rightarrow \begin{array}{l} N_A = 850 + 351 = 1201 \text{ lb} \quad (48.0\%) \\ N_B = 850 + 449 = 1299 \text{ lb} \quad (52.0\%) \end{array}$$

Note that a 100-lb propulsive force has been added (at A) to maintain equilibrium.

3/52



$$\sum M_A = 0;$$

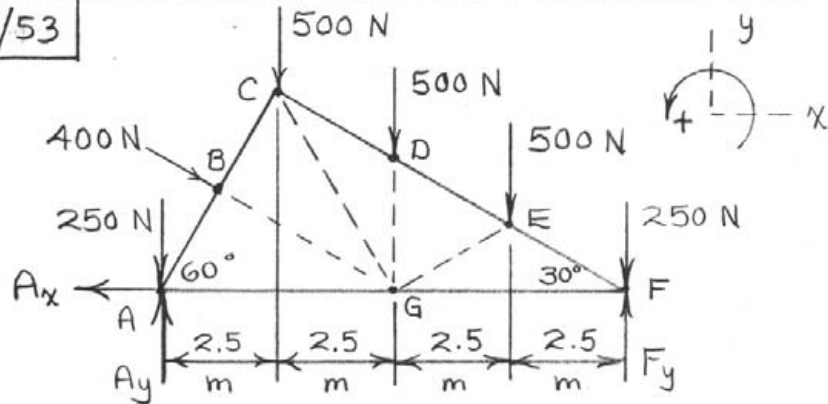
$$40(8) + 40(6)$$

$$+ 120(4) + 200(3)$$

$$- 2F_0 = 0$$

$$F_0 = 820 \text{ lb}$$

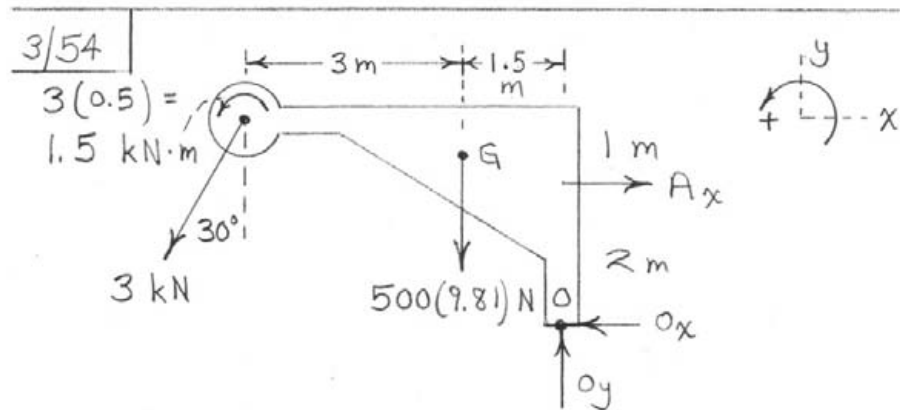
3/53



$$\sum F_x = 0: -A_x + 400 \cos 30^\circ = 0, \quad \underline{A_x = 346 \text{ N}}$$

$$\begin{aligned} \sum M_A = 0: & 400 \left(\frac{10}{4} \right) + 500(2.5) + 500(5) \\ & + 500(7.5) + 250(10) - 10 F_y = 0 \\ & \underline{F_y = 1100 \text{ N}} \end{aligned}$$

$$\begin{aligned} \sum F_y = 0: & -250 - 400 \sin 30^\circ - 500(3) - 250 \\ & + 1100 + A_y = 0, \quad \underline{A_y = 1100 \text{ N}} \end{aligned}$$



$$\begin{aligned}\sum M_o = 0: & -2A + 500(9.81)(1.5) + 1500 \\ & + 3000 \cos 30^\circ (4.5) + 3000 \sin 30^\circ (3) = 0 \\ A = & 12520 \text{ N}\end{aligned}$$

$$\begin{aligned}\sum F_x = 0: & -3000 \sin 30^\circ + 12520 - O_x = 0 \\ O_x = & 11020 \text{ N}\end{aligned}$$

$$\begin{aligned}\sum F_y = 0: & -3000 \cos 30^\circ - 500(9.81) + O_y = 0 \\ O_y = & 7500 \text{ N}\end{aligned}$$

$$O = \sqrt{11020^2 + 7500^2} = 13340 \text{ N or } \underline{13.34 \text{ kN}}$$

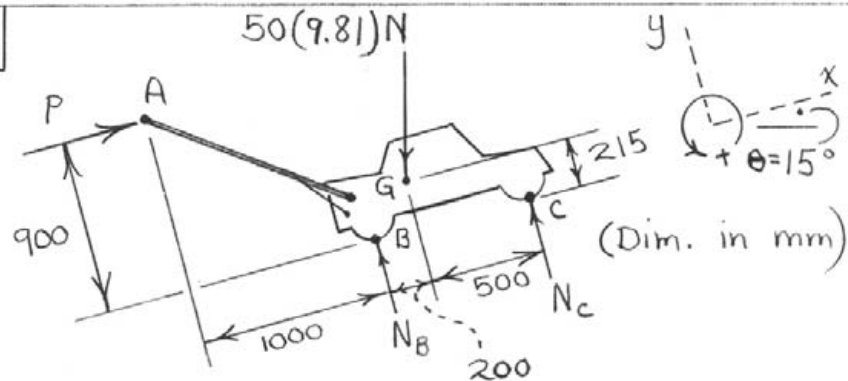
3/55

Law of Cosines: $7^2 = 22^2 + 24^2 - 2(22)(24) \cos \alpha$

$\alpha = 16.79^\circ$

$\sum M_O = 0: 10(45) - 2F(22 \sin \alpha) + 90(22 \cos(30^\circ - \alpha)) = 0,$

$F = \underline{\underline{187.1 \text{ lb}}}$



$$\sum F_x = 0: P - 50(9.81) \sin 15^\circ = 0 \quad (1)$$

$$\sum F_y = 0: N_B + N_C - 50(9.81) \cos 15^\circ = 0 \quad (2)$$

$$\sum M_C = 0: -P(900) - N_B(700) + 50(9.81)[500 \cos 15^\circ + 215 \sin 15^\circ] = 0 \quad (3)$$

Solution to Eqs. (1) - (3):

$$P = 127.0 \text{ N}$$

$$N_B = 214 \text{ N}$$

$$N_C = 260 \text{ N}$$

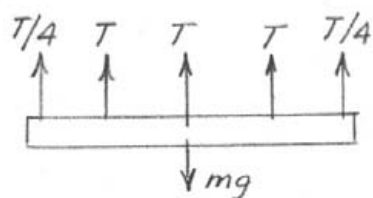
With $\theta = P = 0$:

$$P = 0$$

$$N_B = 350 \text{ N}$$

$$N_C = 140.1 \text{ N}$$

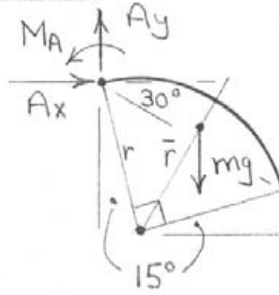
3/57



$$\Sigma F = 0; \left(3 + \frac{1}{4} + \frac{1}{4}\right)T - mg = 0$$

$$T = \frac{2}{7}mg$$

3/58



$$\begin{cases} \text{From Table D/3, } \bar{r} = \frac{r \sin \alpha}{\alpha} \\ \text{For } \alpha = \pi/4, \quad \bar{r} = 2\sqrt{2} r / \pi \end{cases}$$

$$\sum M_A = 0 :$$

$$M_A - mg(r \sin 15^\circ + \bar{r} \cos 60^\circ) = 0$$

$$M_A = 0.709 mgr$$

3/59 Torque $M = 100 \text{ N}\cdot\text{m} = (600 - T)(0.225)$, $T = 155.6 \text{ N}$

Dimensions in mm

Replace T by force at E and a couple, so that moment of T about D in CW sense becomes

$$155.6(0.2 \cos 30^\circ - 0.125 \sin 30^\circ) + 155.6(0.075) = 28.89 \text{ N}\cdot\text{m}$$

$$\sum M_D = 0; 28.89 + 981(0.125) + 600(0.2 - 0.075) - 0.250 Q_y = 0$$

$$Q_y = 906 \text{ N}$$

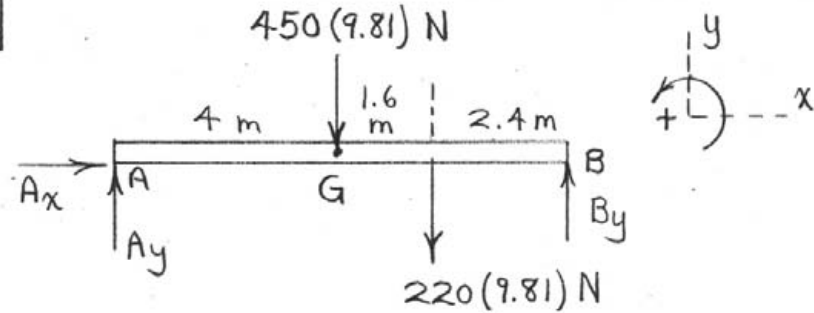
$$\sum F_x = 0; 155.6 \cos 30^\circ + 600 - Q_x = 0$$

$$Q_x = 735 \text{ N}$$

$$R = \sqrt{Q_x^2 + Q_y^2} = \sqrt{(735)^2 + (906)^2} = 1167 \text{ N or } \underline{1.167 \text{ kN}}$$

$W = 100(9.81) \text{ N}$

3/6



From $\Sigma F_x = 0$, $A_x = 0$

$$\Sigma M_A = 0: -450(9.81)4 - 220(9.81)(5.6)$$

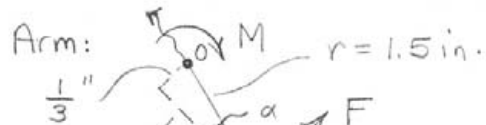
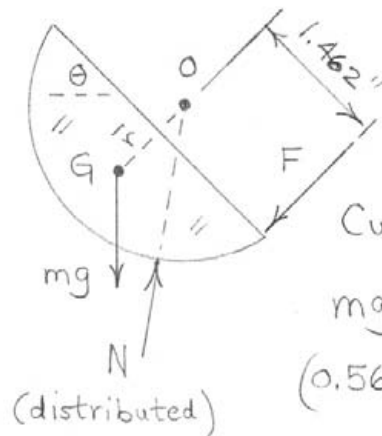
$$+ B_y(8) = 0, \quad \underline{B_y = 3720 \text{ N}}$$

$$\Sigma F_y = 0: A_y - 450(9.81) - 220(9.81) + 3720 = 0$$

$$\underline{A_y = 2850 \text{ N}}$$

3/60

Cube:



$$\alpha = \sin^{-1} \frac{1/3}{1.5} = 12.84^\circ$$

Cube: $\sum M_O = 0$:

$$mg \bar{r} \sin \theta - F(1.462) = 0$$

$$(0.56)(0.55 \cdot 1.5) \sin \theta - F(1.462) = 0$$

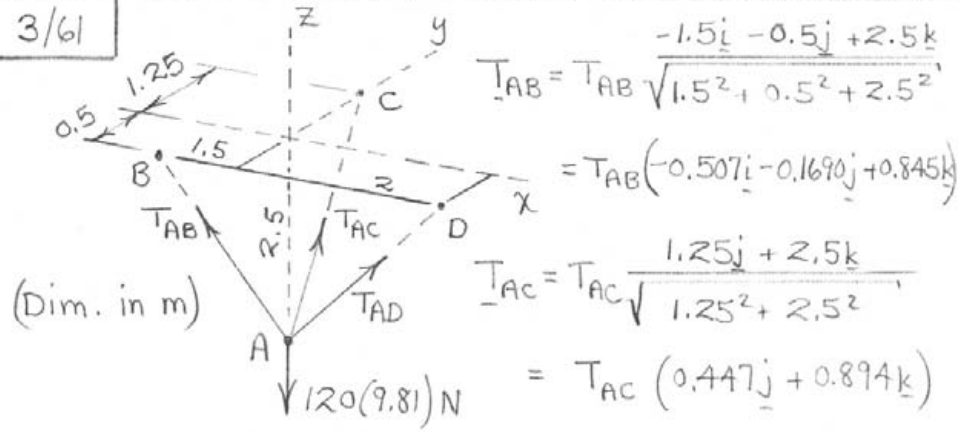
$$F = 0.316 \sin \theta \quad (\text{in lb})$$

Arm: $\sum M_O = 0$:

$$-M + 0.316 \sin \theta (1.462) = 0$$

$$M = 0.462 \sin \theta \quad \text{lb-in.}$$

3/61



$$T_{AD} = T_{AD} \frac{2\mathbf{i} - 0.5\mathbf{j} + 2.5\mathbf{k}}{\sqrt{2^2 + 0.5^2 + 2.5^2}} = T_{AD}(0.617\mathbf{i} - 0.1543\mathbf{j} + 0.772\mathbf{k})$$

$$\sum F_x = 0: -0.507T_{AB} + 0.617T_{AD} = 0$$

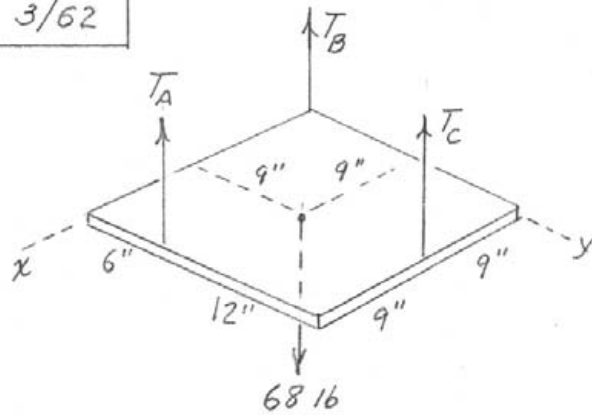
$$\sum F_y = 0: -0.169T_{AB} + 0.447T_{AC} - 0.1543T_{AD} = 0$$

$$\sum F_z = 0: 0.845T_{AB} + 0.894T_{AC} + 0.772T_{AD} - 120(9.81) = 0$$

Solution :

$$\begin{cases} T_{AB} = 569 \text{ N} \\ T_{AC} = 376 \text{ N} \\ T_{AD} = 467 \text{ N} \end{cases}$$

3/62



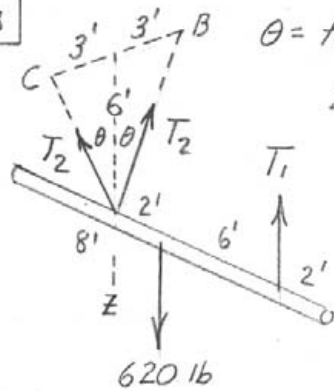
$$\sum M_x = 0; 6T_A + 18T_C - 68(9) = 0, \quad T_A + 3T_C = 102$$

$$\sum M_y = 0; -18T_A - 9T_C + 68(9) = 0, \quad 2T_A + T_C = 68$$

$$\text{Solve \& get } \underline{T_A = 20.4 \text{ lb}}, \quad \underline{T_C = 27.2 \text{ lb}}$$

$$\sum F_z = 0; 20.4 + T_B + 27.2 - 68 = 0, \quad \underline{T_B = 20.4 \text{ lb}}$$

3/63



$$\theta = \tan^{-1} 3/6 = 26.57^\circ$$

$$\sum M_{CB} = 0; 8T_1 - 620(2) = 0$$

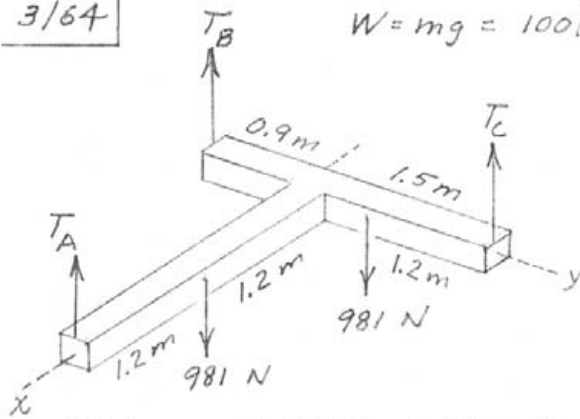
$$T_1 = 155 \text{ lb}$$

$$\sum F_z = 0; 620 - 155 - 2T_2 \cos \theta = 0$$

$$T_2 = \frac{465}{2 \cos 26.57^\circ} = 260 \text{ lb}$$

3/64

$$W = mg = 100(9.81) = 981 \text{ N}$$



$$\sum M_y = 0; 981(1.2) - 2.4T_A = 0; \underline{T_A = 490 \text{ N}}$$

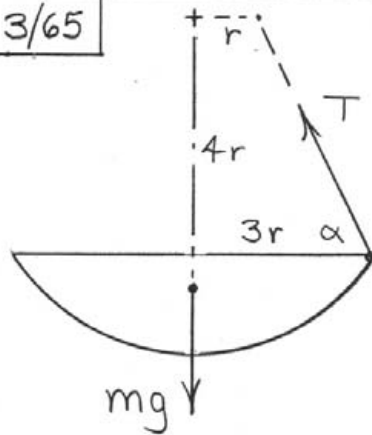
$$\sum M_x = 0; 1.5T_C - 0.9T_B - 981(0.3) = 0$$

$$\sum F_z = 0; 491 + T_B + T_C - 2(981) = 0, T_B + T_C = 1472 \text{ N}$$

$$\text{so } 1.5T_C - 0.9(1472 - T_C) = 294.3, \underline{T_C = 674 \text{ N}}$$

$$\underline{T_B = 797 \text{ N}}$$

3/65

 T (one of three)

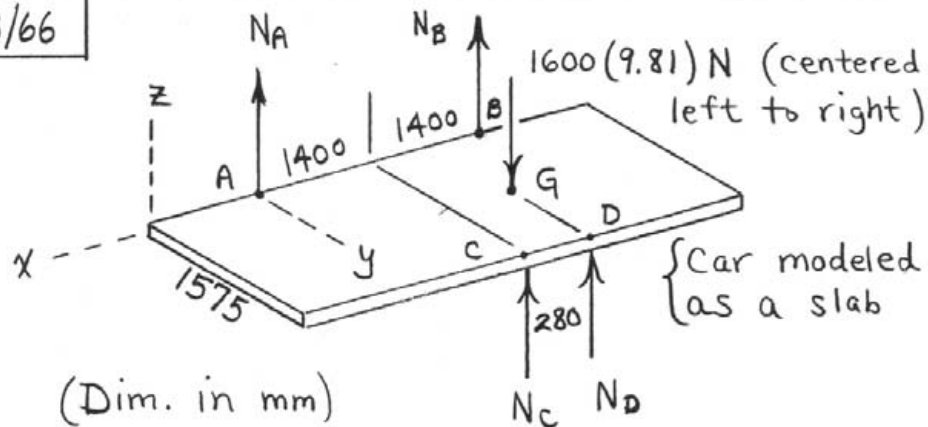
$$\alpha = \tan^{-1} \frac{4r}{3r} = 63.4^\circ$$

$$+\uparrow \Sigma F = 0 :$$

$$3(T \sin \alpha) - mg = 0$$

$$\underline{T = 0.373mg}$$

3/66



Jacking at C ($N_D = 0$):

$$\sum M_x = 0 : -1600(9.81) \left(\frac{1575}{2} \right) + N_C (1575) = 0$$

$$\underline{N_C = 7850 \text{ N}}$$

$$\sum M_y = 0 : -1600(9.81)(1680) + N_B(2800) + N_C(1400) = 0$$

$$\sum F_z = 0 : N_A + N_B + N_C - 1600(9.81) = 0$$

$$\Rightarrow \underline{N_A = 2350 \text{ N}}, \quad \underline{N_B = 5490 \text{ N}}$$

Jacking at D ($N_C = 0$): $\underline{N_D = 7850 \text{ N}}$ { Same as for N_C

$$\sum M_y = 0 : -1600(9.81)(1680) + N_B(2800) + N_D(1680) = 0$$

$$\sum F_z = 0 : N_A + N_B + N_D - 1600(9.81) = 0$$

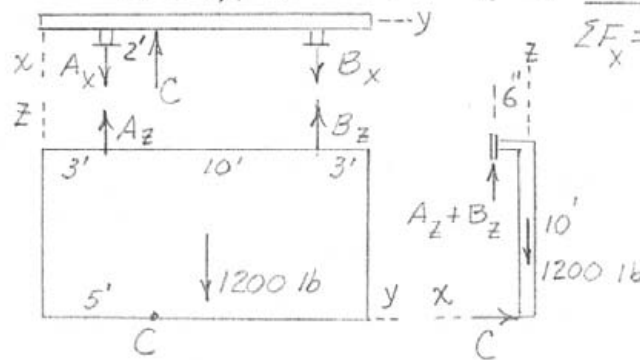
$$\Rightarrow \underline{N_A = 3140 \text{ N}}, \quad \underline{N_B = 4710 \text{ N}}$$

3/67 $x-z; \sum M_{AB} = 0; 10C - 1200(6/12) = 0, C = 60 \text{ lb}$

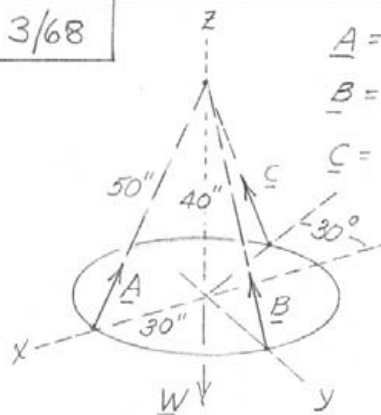
$x-y; \sum M_A = 0; 2(60) - 10B_x = 0; B_x = 12 \text{ lb}$

$\sum F_x = 0; A_x + 12 - 60 = 0$

$A_x = 48 \text{ lb}$



3/68



$$\underline{A} = \frac{A}{5}(-3\hat{i} + 4\hat{k})$$

$$\underline{B} = \frac{B}{5}(-3\hat{j} + 4\hat{k})$$

$$\underline{C} = \frac{C}{5}(3\hat{i} \cos 30^\circ + 3\hat{j} \sin 30^\circ + 4\hat{k})$$

$$\underline{W} = -600\hat{k} \text{ lb}$$

$$\Sigma \underline{F} = 0; \quad \frac{1}{5}(-3A + \frac{3\sqrt{3}}{2}C)\hat{i} + \frac{1}{5}(-3B + \frac{3}{2}C)\hat{j} + \frac{1}{5}(4A + 4B + 4C - 5[600])\hat{k} = 0$$

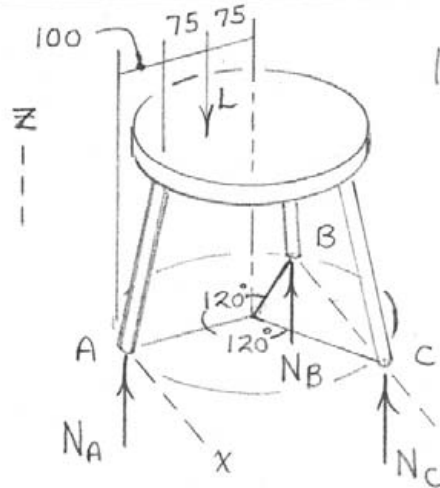
$$\text{Thus } A = \frac{\sqrt{3}}{2}C, \quad B = \frac{C}{2}, \quad 4(A + B + C) = 3000$$

$$4\left(\frac{\sqrt{3}}{2} + \frac{1}{2} + 1\right)C = 3000, \quad C = \frac{3000}{2(3 + \sqrt{3})} = 317 \text{ lb}$$

$$B = 158 \text{ lb}$$

$$A = 275 \text{ lb}$$

3/69



Note: x -axis is
 $\parallel BC$.

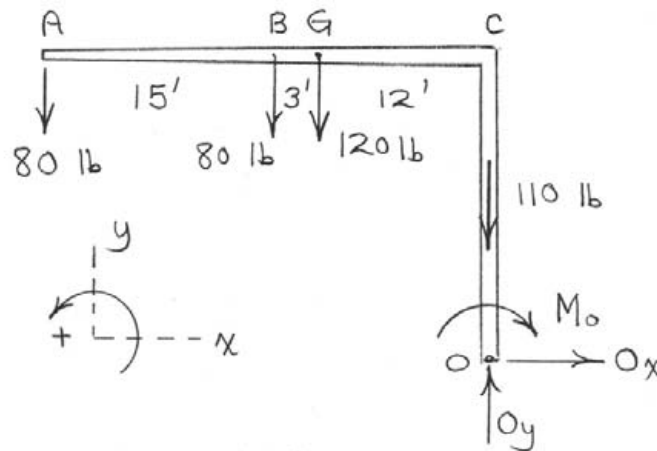
$N_B = N_C = N$,
 by symmetry.

$$\sum M_x = 0: -L(175) + 2N(250 + 250 \cos 60^\circ) = 0$$

$$N = 0.233L = N_B = N_C$$

$$\sum F_z = 0: N_A + 2(0.233L) - L = 0$$

$$N_A = 0.533L$$



$$\sum F_x = 0 : \quad \underline{O_x = 0}$$

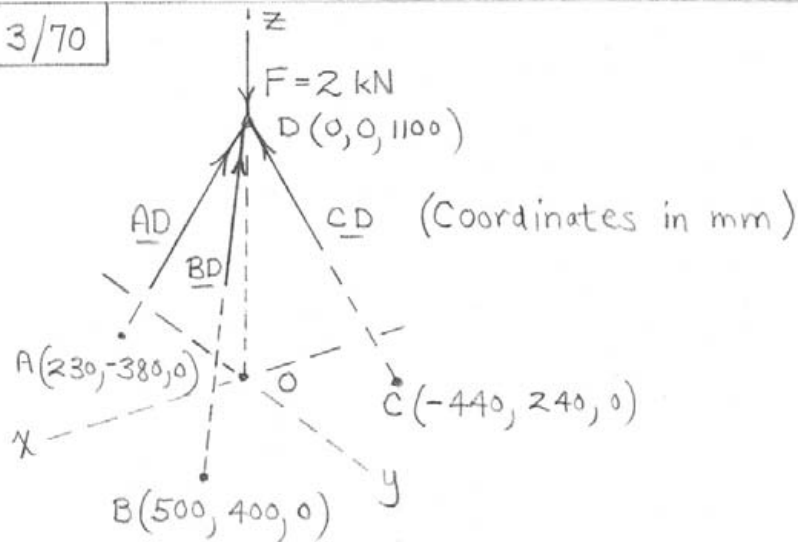
$$\sum F_y = 0 : \quad O_y - 80 - 80 - 120 - 110 = 0$$

$$\underline{O_y = 390 \text{ lb}}$$

$$\sum M_o = 0 : \quad 80(30) + 80(15) + 120(12) - M_o = 0$$

$$\underline{M_o = 5040 \text{ lb-ft CW}}$$

3/70



$$\underline{AD} = AD \frac{-230\mathbf{i} + 380\mathbf{j} + 1100\mathbf{k}}{\sqrt{230^2 + 380^2 + 1100^2}} = AD(-0.1939\mathbf{i} + 0.320\mathbf{j} + 0.927\mathbf{k})$$

$$\underline{BD} = BD \frac{-500\mathbf{i} - 400\mathbf{j} + 1100\mathbf{k}}{\sqrt{500^2 + 400^2 + 1100^2}} = BD(-0.393\mathbf{i} - 0.314\mathbf{j} + 0.864\mathbf{k})$$

$$\underline{CD} = CD \frac{440\mathbf{i} - 240\mathbf{j} + 1100\mathbf{k}}{\sqrt{440^2 + 240^2 + 1100^2}} = CD(0.364\mathbf{i} - 0.1985\mathbf{j} + 0.910\mathbf{k})$$

$$\text{From } \Sigma \underline{F} = \underline{0} : \begin{cases} -0.1939AD - 0.393BD + 0.364CD = 0 \\ 0.320AD - 0.314BD - 0.1985CD = 0 \\ 0.927AD + 0.864BD + 0.910CD - 2 = 0 \end{cases}$$

$$\text{Solution : } \underline{AD = 0.925 \text{ kN}}, \underline{BD = 0.376 \text{ kN}}, \underline{CD = 0.898 \text{ kN}} \\ (\text{all compression})$$

3/71

$\underline{R} = R(\underline{i} \cos 30^\circ + \underline{j} \sin 30^\circ) = \frac{R}{2}(\sqrt{3}\underline{i} + \underline{j})$
 $\underline{W} = m\underline{g} = 200(9.81)(-\underline{k}) = -1962\text{ k N}$
 $h = \sqrt{7^2 - 6^2 - 2^2} = 3\text{ m}$

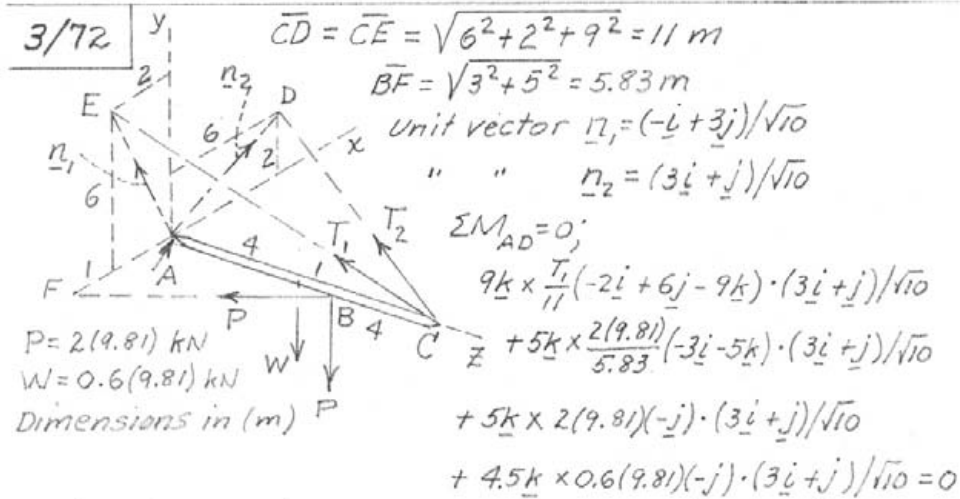
$\sum \underline{M}_A = 0; \underline{r}_{AG} \times \underline{W} + \underline{r}_{AB} \times (\underline{P} + \underline{R}) = 0$
 $(-1\underline{i} - 3\underline{j} + 1.5\underline{k}) \times (-1962\underline{k})$
 $+ (-2\underline{i} - 6\underline{j} + 3\underline{k}) \times (P\underline{j} + \frac{R\sqrt{3}}{2}\underline{i} + \frac{R}{2}\underline{j}) = 0$

Simplify & get

$(5886 - 3P - 3R/2)\underline{i}$
 $+ (-1962 + 3\sqrt{3}R/2)\underline{j}$
 $+ (-2P + R + 3\sqrt{3}R)\underline{k} = 0$

$R = \frac{2(1962)}{3\sqrt{3}} = 755\text{ N}$
 $3P = 5886 - \frac{3}{2}755, P = 1584\text{ N}$

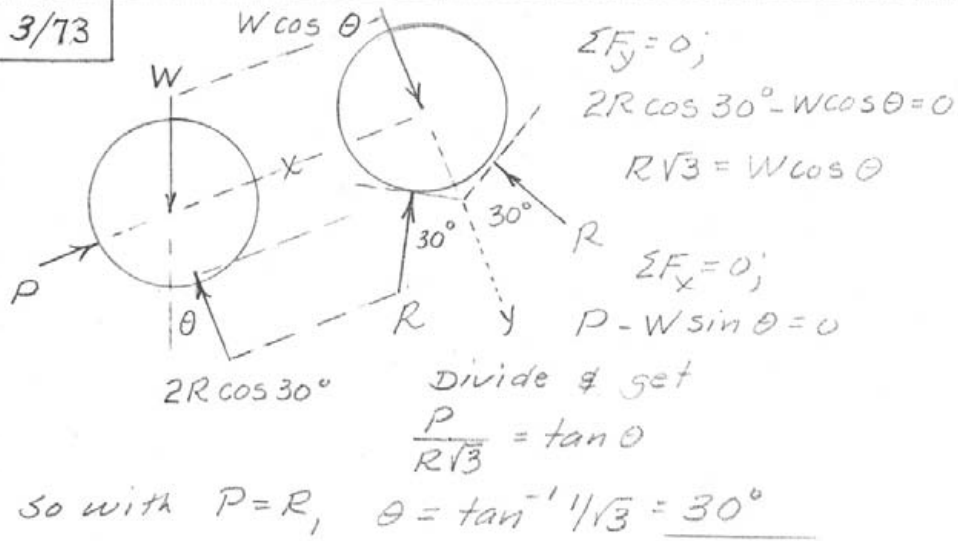
3/72



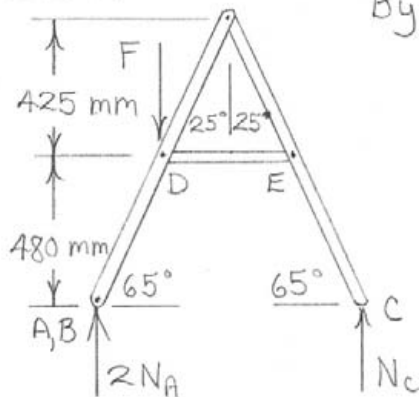
Simplify & get $\frac{180T_1}{11\sqrt{10}} + \frac{294.3}{5.83\sqrt{10}} = \frac{373.8}{\sqrt{10}}, T_1 = 19.76 \text{ kN}$

(Similarly, if T_2 is desired, $\Sigma M_{AE} = 0$ will give $T_2 = 16.86 \text{ kN}$)

3/73



3/74

By inspection, $N_A = N_B$.

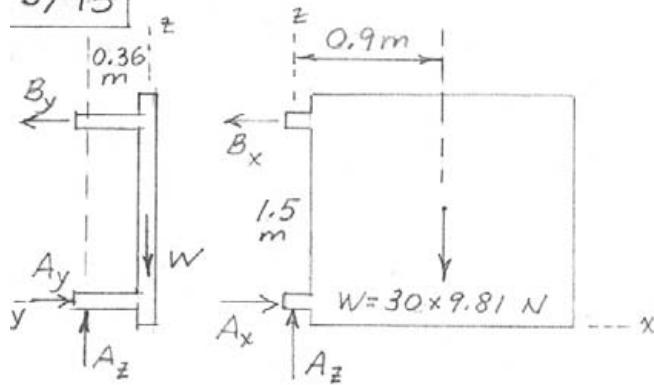
$$\begin{aligned} \uparrow \sum M_C = 0: & -2N_A(2.905 \tan 25^\circ) \\ & + F(905 \tan 25^\circ + 425 \tan 25^\circ) = 0 \end{aligned}$$

$$\underline{N_A = 0.367F = N_B}$$

$$\uparrow \sum F = 0: 2(0.367F) + N_C - F = 0$$

$$\underline{N_C = 0.265F}$$

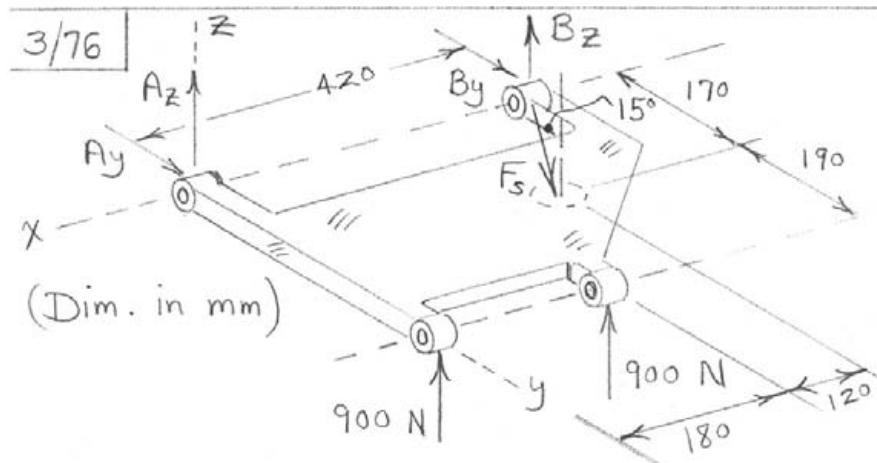
3/75



$$x-z; \sum M_A = 0; 1.5 B_x - 0.9(30)(9.81) = 0, B_x = 176.6 \text{ N}$$

$$y-z; \sum M_A = 0; 1.5 B_y - 0.36(30)(9.81) = 0, B_y = 70.6 \text{ N}$$

$$B = \sqrt{176.6^2 + 70.6^2} = \underline{190.2 \text{ N}}$$



$$\sum M_x = 0: 2(900)(360) - F_5 \cos 15^\circ (170) = 0, \underline{F_5 = 3950 \text{ N}}$$

$$\sum M_y = 0: B_z (420) + 900(180) - 3950 \cos 15^\circ (300) = 0$$

$$B_z = 2340 \text{ N}$$

$$\sum M_z = 0: -B_y (420) - 3950 \sin 15^\circ (300) = 0, B_y = -730 \text{ N}$$

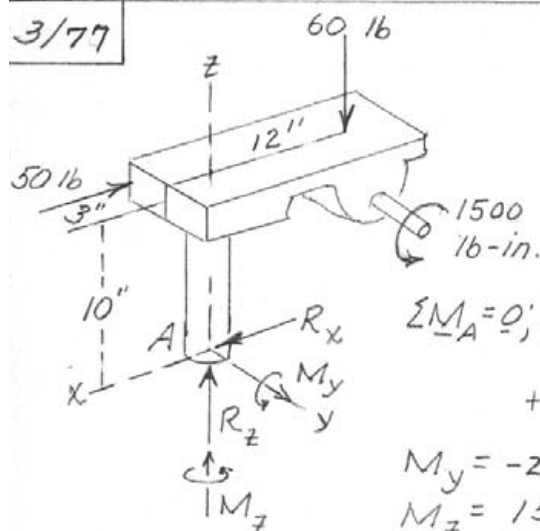
$$\sum F_y = 0: A_y - 730 + 3950 \sin 15^\circ = 0, A_y = -292 \text{ N}$$

$$\sum F_z = 0: A_z + 2340 - 3950 \cos 15^\circ + 1800 = 0, A_z = -325 \text{ N}$$

$$F_A = \sqrt{A_y^2 + A_z^2} = \underline{437 \text{ N}}$$

$$F_B = \sqrt{B_y^2 + B_z^2} = \underline{2450 \text{ N}}$$

3/77



$$\sum \underline{M}_A = \underline{0}; [-60(12) - 50(10) + 1500 + M_y] \underline{j} + [-50(3) + M_z] \underline{k} = \underline{0}$$

$$M_y = -280 \text{ lb-in.}$$

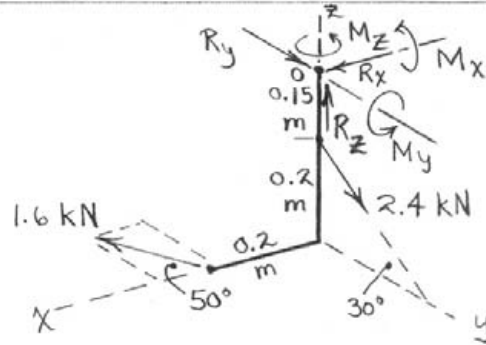
$$M_z = 150 \text{ lb-in.}$$

$$\underline{M} = -280 \underline{j} + 150 \underline{k} \text{ lb-in.}$$

$$\sum \underline{F} = \underline{0}; (R_x - 50) \underline{i} + (R_z - 60) \underline{k} = \underline{0}$$

$$R_x = 50 \text{ lb}, R_z = 60 \text{ lb}$$

$$\underline{R} = 50 \underline{i} + 60 \underline{k} \text{ lb}$$



$$\sum F_x = 0 : R_x + 1.6 \cos 50^\circ, \quad R_x = -1.028 \text{ kN}$$

$$\sum F_y = 0 : R_y + 2.4 \cos 30^\circ - 1.6 \sin 50^\circ = 0, \quad R_y = -0.853 \text{ kN}$$

$$\sum F_z = 0 : R_z - 2.4 \sin 30^\circ = 0, \quad R_z = 1.2 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \underline{1.796 \text{ kN}}$$

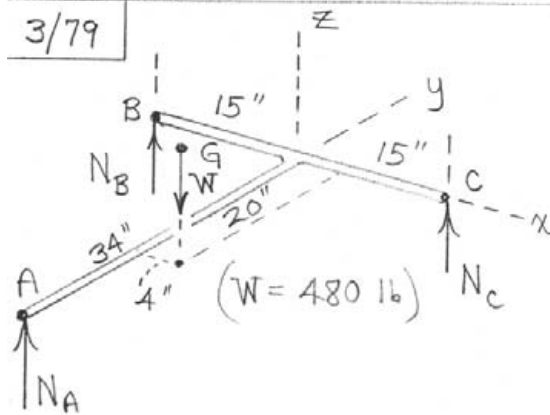
$$\begin{aligned} \sum M_{Ox} = 0 : M_x + 2.4 \cos 30^\circ (0.15) - 1.6 \sin 50^\circ (0.35) &= 0 \\ M_x &= 0.1172 \text{ kN}\cdot\text{m} \end{aligned}$$

$$\sum M_{Oy} = 0 : M_y - 1.6 \cos 50^\circ (0.35) = 0, \quad M_y = 0.360 \text{ kN}\cdot\text{m}$$

$$\sum M_{Oz} = 0 : M_z - 1.6 \sin 50^\circ (0.2) = 0, \quad M_z = 0.245 \text{ kN}\cdot\text{m}$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2} = \underline{0.451 \text{ kN}\cdot\text{m}}$$

3/79



$$\sum F_z = 0: N_A + N_B + N_C - 480 = 0$$

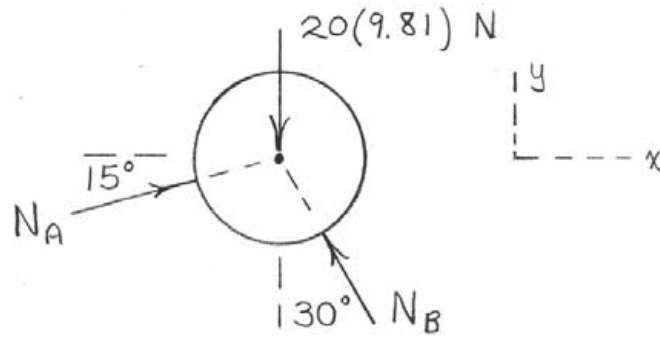
$$\sum M_x = 0: 480(20) - N_A(34) = 0$$

$$\sum M_y = 0: N_B(15) - N_C(15) + 480(4) = 0$$

Solution :

$$\begin{cases} N_A = 282 \text{ lb} \\ N_B = 34.8 \text{ lb} \\ N_C = 162.8 \text{ lb} \end{cases}$$

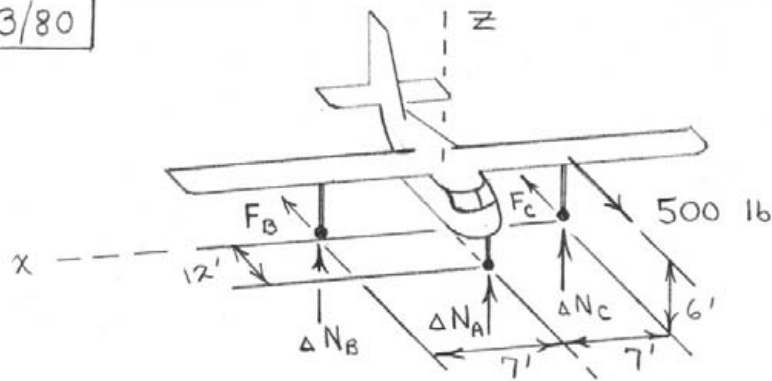
3/8



$$\left\{ \begin{array}{l} \sum F_x = 0: N_A \cos 15^\circ - N_B \sin 30^\circ = 0 \end{array} \right. \quad (1)$$

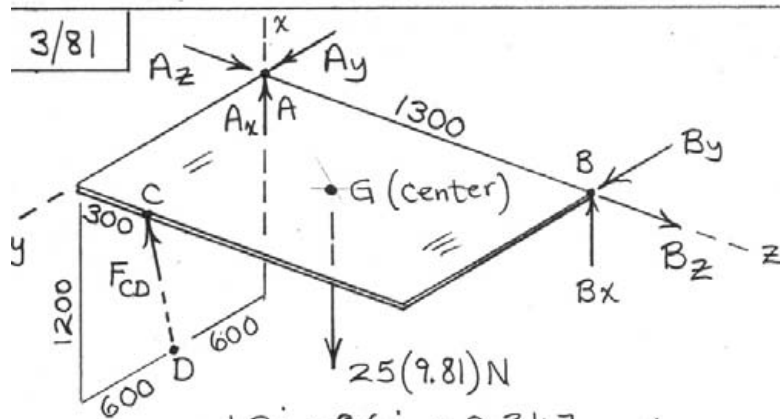
$$\left\{ \begin{array}{l} \sum F_y = 0: N_A \sin 15^\circ + N_B \cos 30^\circ - 20(9.81) = 0 \end{array} \right. \quad (2)$$

$$\text{Solution: } \begin{cases} N_A = 101.6 \text{ N} \\ N_B = 196.2 \text{ N} \end{cases}$$



$$\begin{aligned}
 \sum M_x = 0 : \quad & \Delta N_A (12) - 500(6) = 0 \quad \Delta N_A = 250 \text{ lb} \\
 \sum F_z = 0 : \quad & \Delta N_A + \Delta N_B + \Delta N_C = 0 \\
 \sum M_y = 0 : \quad & \Delta N_C (7) - \Delta N_B (7) = 0 \quad \Delta N_B = \Delta N_C = -125 \text{ lb}
 \end{aligned}$$

More information would be required to determine F_B and F_C . x-components of friction at B and C are possible.



$$\underline{F}_{CD} = F \left[\frac{1.2\mathbf{i} + 0.6\mathbf{j} + 0.3\mathbf{k}}{\sqrt{1.2^2 + 0.6^2 + 0.3^2}} \right] = F(0.873\mathbf{i} + 0.436\mathbf{j} + 0.218\mathbf{k})$$

$$\sum M_z = 0: 25(9.81)(0.6) - 0.873F(1.2) = 0$$

$$\underline{F = 140.5 \text{ N}}$$

$$\sum M_y = 0: B_x(1.3) - 25(9.81)(0.65) + 0.873F(0.3) = 0$$

$$B_x = 94.3 \text{ N}$$

$$\sum F_x = 0: A_x + 94.3 + 0.873F - 25(9.81) = 0$$

$$A_x = 28.3 \text{ N}$$

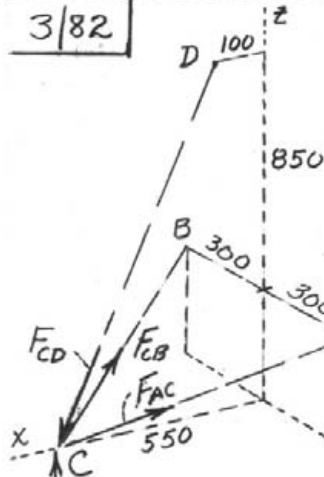
$$\sum M_x = 0: -B_y(1.3) + 0.218F(0.6) = 0$$

$$B_y = 14.15 \text{ N}$$

$$\sum F_y = 0: A_y + 14.15 + 0.436F = 0, A_y = -75.5 \text{ N}$$

$$A_n = \sqrt{75.5^2 + 28.3^2} = \underline{80.6 \text{ N}}, B_n = \sqrt{94.3^2 + 14.15^2} = \underline{95.4 \text{ N}}$$

3/82



$R = \frac{1}{3} mg$; From Table C2
for Mars $g = 3.73 \text{ m/s}^2$

$$R = \frac{1}{3} 600(3.73) = 746 \text{ N}$$

$$\bar{AC} = \sqrt{(550)^2 + (300)^2 + (350)^2} = 718 \text{ mm}$$

$$\bar{CD} = \sqrt{(450)^2 + (1200)^2} = 1282 \text{ mm}$$

$$\underline{F}_{AC} = \frac{F_{AC}}{718} (-550\underline{i} + 300\underline{j} + 350\underline{k})$$

$$\underline{F}_{CB} = \frac{F_{CB}}{718} (-550\underline{i} - 300\underline{j} + 350\underline{k})$$

$$\underline{F}_{CD} = \frac{F_{CD}}{1282} (450\underline{i} - 1200\underline{k})$$

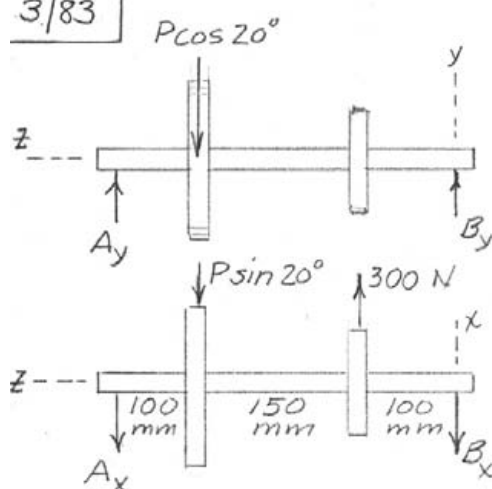
$\sum \underline{F} = \underline{0}$ with $F_{CB} = F_{AC}$ gives upon collecting terms

$$\left(-\frac{2F_{AC}}{718} 550 + \frac{F_{CD}}{1282} 450 \right) \underline{i} + \left(746 + \frac{2F_{AC}}{718} 350 - \frac{F_{CD}}{1282} 1200 \right) \underline{k} = \underline{0}$$

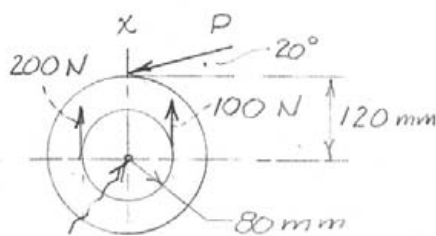
Equate coefficients to zero & solve simultaneously
to get $F_{CD} = 1046 \text{ N}$ compression

$$F_{AC} = F_{CB} = 240 \text{ N tension}$$

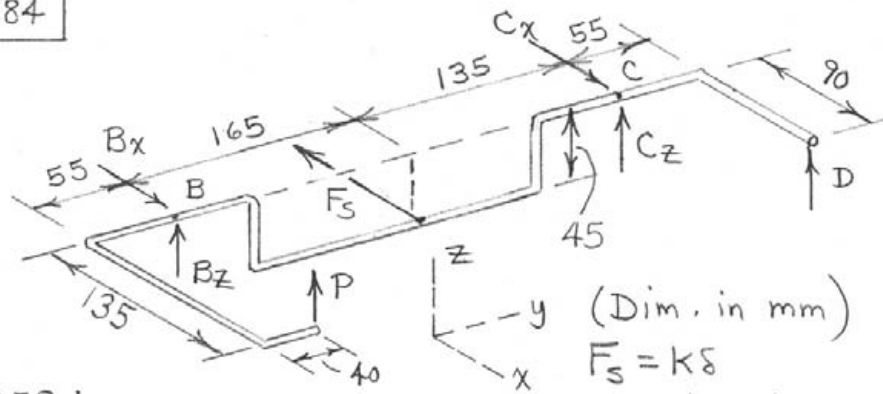
3/83



$$\begin{aligned}\Sigma M_z &= 0; \\ (200 - 100) 0.080 \\ - 0.120 P \cos 20^\circ &= 0 \\ P &= 70.9 \text{ N}\end{aligned}$$



$$\begin{aligned}y-z; \Sigma M_A &= 0; 0.35 B_y - 70.9 \cos 20^\circ (0.1) = 0, B_y = 19.05 \text{ N} \\ \Sigma F_y &= 0; A_y + 19.05 - 70.9 \cos 20^\circ = 0, A_y = 47.6 \text{ N} \\ x-z; \Sigma M_A &= 0; 300 (0.25) - 70.9 \sin 20^\circ (0.1) - 0.35 B_x = 0 \\ B_x &= 207 \text{ N} \\ \Sigma F_x &= 0; A_x + 70.9 \sin 20^\circ + 207 - 300 = 0, A_x = 68.4 \text{ N} \\ A &= \sqrt{68.4^2 + 47.6^2} = 83.3 \text{ N}, B = \sqrt{207^2 + 19.05^2} = 208 \text{ N}\end{aligned}$$



$$D = 0 :$$

$$\sum M_{BC} = 0 : -P(135) + 54(45) = 0, \quad P = \underline{18 \text{ N}_{\min}}$$

$$\sum M_{B_x} = 0 : -18(15) + C_z(300) = 0, \quad C_z = 0.9 \text{ N}$$

$$\sum M_{B_z} = 0 : 54(165) - C_x(300) = 0, \quad C_x = 29.7 \text{ N}$$

$$\sum F_x = 0 : 29.7 + B_x - 54 = 0, \quad B_x = 24.3 \text{ N}$$

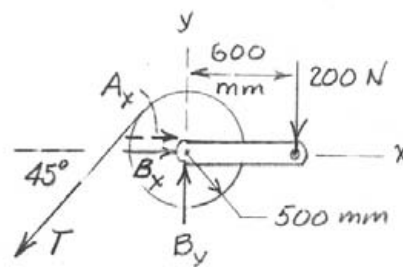
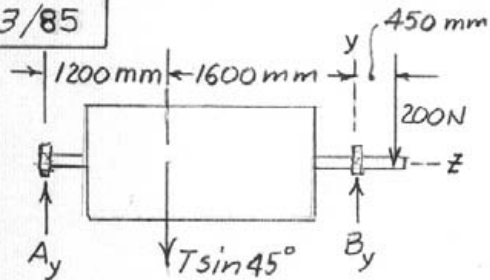
$$\sum F_z = 0 : 0.9 + B_z + 18 = 0, \quad B_z = -18.9 \text{ N}$$

$$B = \sqrt{B_x^2 + B_z^2} = \underline{30.8 \text{ N}}, \quad C = \sqrt{C_x^2 + C_z^2} = \underline{29.7 \text{ N}}$$

$$\text{If } P = P_{\min}/2 = 18/2 = 9 \text{ N}, \quad (D \neq 0) :$$

$$\sum M_{BC} = 0 : -9(135) + 54(45) - D(90) = 0, \quad D = \underline{13.5 \text{ N}}$$

3/85



$$\sum M_z = 0; 0.500T - 0.600(200) = 0, \quad T = 240 \text{ N}$$

$$\sum M_y = 0; 2.800A_y + 0.450(200) - 240(0.707)(1.600) = 0$$

$$A_y = 64.8 \text{ N}$$

$$\sum F_y = 0; 64.8 - 240(0.707) + B_y - 200 = 0, \quad B_y = 305 \text{ N}$$

$$\sum M_x = 0; 240(0.707)(1.600) - 2.800A_x = 0, \quad A_x = 97.0 \text{ N}$$

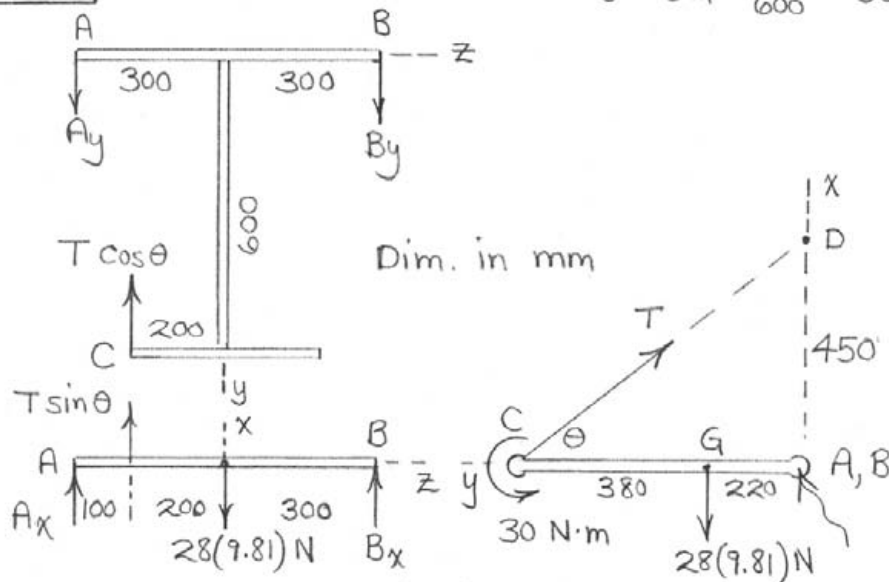
$$\sum F_x = 0; 97.0 + B_x - 240(0.707) = 0, \quad B_x = 72.7 \text{ N}$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(97.0)^2 + (64.8)^2} = 116.7 \text{ N}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(72.7)^2 + (305)^2} = 313 \text{ N}$$

3/86

$$\theta = \tan^{-1} \frac{450}{600} = 36.9^\circ$$



$$\begin{aligned} (x-y) \sum M_z = 0: & 28(9.81)(0.220) - T \sin 36.9^\circ (0.600) + 30 = 0, & T = 251 \text{ N} \\ (x-z) \sum M_B = 0: & 28(9.81)(0.300) - 251 \sin 36.9^\circ (0.500) - 0.600 A_x = 0, & A_x = 11.74 \text{ N} \\ \sum F_x = 0: & 11.74 + 251 \sin 36.9^\circ - 28(9.81) + B_x = 0, & B_x = 112.2 \text{ N} \\ (y-z) \sum M_B = 0: & 251 \cos 36.9^\circ (0.500) - 0.6 A_y = 0, & A_y = 167.5 \text{ N} \\ \sum F_y = 0: & 167.5 + B_y - 251 \cos 36.9^\circ = 0, & B_y = 33.5 \text{ N} \end{aligned}$$

$$A = \sqrt{11.74^2 + 167.5^2} = 167.9 \text{ N}$$

$$B = \sqrt{112.2^2 + 33.5^2} = 117.1 \text{ N}$$

Couple may be applied at any place on rigid body with the same external effect.

3/87

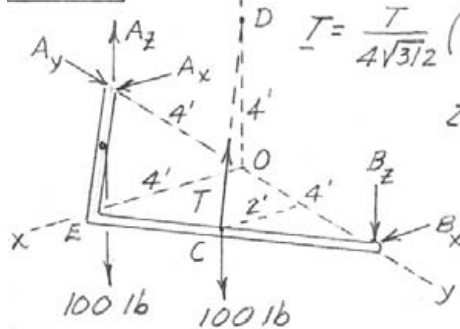
$$\overline{CD} = \sqrt{(4/\sqrt{2})^2 + 4^2} = 4\sqrt{3/2} \text{ ft}$$

$$\underline{T} = \frac{T}{4\sqrt{3/2}} (-2\underline{i} - 2\underline{j} + 4\underline{k}) = \frac{T}{\sqrt{6}} (-\underline{i} - \underline{j} + 2\underline{k})$$

$$\sum M_{AB} = 0; 200(2) - T_z(2) = 0$$

$$T_z = \frac{2T}{\sqrt{6}} = 200, T = 100\sqrt{6} \text{ lb}$$

$$\underline{T} = 245 \text{ lb}$$



$$(\sum M_B)_x = 0; 100(2) + 100(6) - 200(2) - 8A_z = 0, A_z = 50 \text{ lb}$$

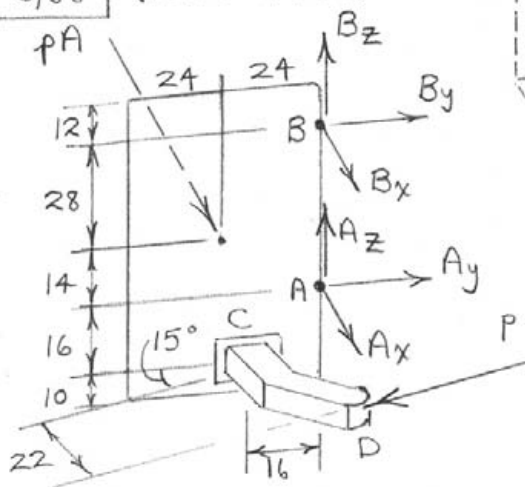
$$\sum F_y = 0; A_y - \frac{100\sqrt{6}}{\sqrt{6}} = 0, A_y = 100 \text{ lb}$$

$$(\sum M_B)_z = 0; 8A_x - \frac{100\sqrt{6}}{\sqrt{6}}(2) - \frac{100\sqrt{6}}{\sqrt{6}}(2) = 0, A_x = 50 \text{ lb}$$

$$A = \sqrt{50^2 + 100^2 + 50^2} = 122.5 \text{ lb}$$

3/88

(Dim. in mm)



z
y (parallel to rudder & horizontal)
x

$$pA = 4(10^{-5})(48)(80) = 0.1536 \text{ N}$$

$$\sum M_{AB} = 0 : -P(22 - 16 \sin 15^\circ) + 0.1536(24) = 0$$

$$P = 0.206 \text{ N}$$

$$\sum M_{Bx} = 0 : A_y(42) - 0.206 \cos 15^\circ(58) = 0$$

$$A_y = 0.275 \text{ N}$$

$$\sum M_{Ax} = 0 : -B_y(42) - 0.206 \cos 15^\circ(16) = 0$$

$$B_y = -0.0760 \text{ N}$$

3/89

From Prob. 2/105,

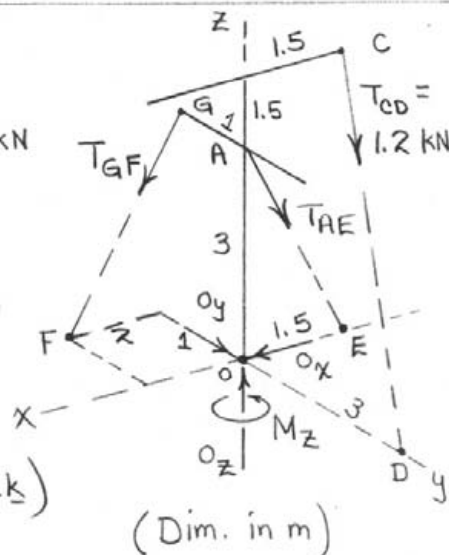
$$\underline{T}_{CD} = 0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k} \text{ kN}$$

$$\underline{T}_{AE} = T_{AE} \frac{-1.5\underline{i} - 3\underline{k}}{\sqrt{1.5^2 + 3^2}}$$

$$= T_{AE} (-0.447\underline{i} - 0.894\underline{k})$$

$$\underline{T}_{GF} = T_{GF} \frac{2\underline{i} - 3\underline{k}}{\sqrt{2^2 + 3^2}}$$

$$= T_{GF} (0.555\underline{i} - 0.832\underline{k})$$



$$\Sigma \underline{M}_O = \underline{0} : \underline{OC} \times \underline{T}_{CD} + \underline{OA} \times \underline{T}_{AE} + \underline{OG} \times \underline{T}_{GF} + M_z \underline{k} = \underline{0}$$

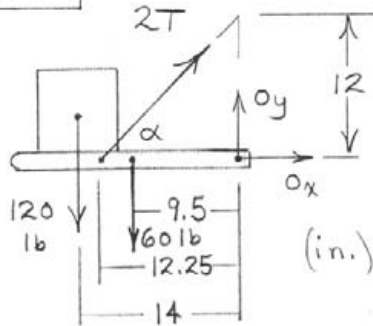
$$(-1.5\underline{i} + 4.5\underline{k}) \times (0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}) + 3\underline{k} \times$$

$$T_{AE} (-0.447\underline{i} - 0.894\underline{k}) + (-\underline{j} + 3\underline{k}) \times T_{GF} (0.555\underline{i} - 0.832\underline{k})$$

$$+ M_z \underline{k} = \underline{0} \Rightarrow \begin{cases} 0.832 T_{GF} - 2.89 = 0 \\ 1.664 T_{GF} - 1.342 T_{AE} = 0 \\ 0.555 T_{GF} - 0.962 + M_z = 0 \end{cases}$$

Solve to obtain $\underline{T}_{GF} = 3.47 \text{ kN}, \underline{T}_{AE} = 4.30 \text{ kN},$
 $M_z = -0.962 \text{ kN}\cdot\text{m}$

3/9

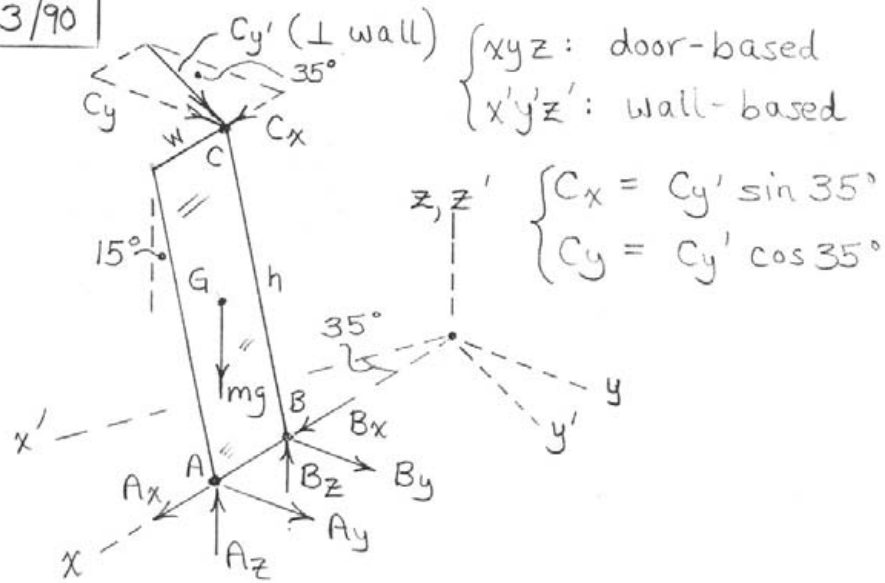


$$\alpha = \tan^{-1} \frac{12}{12.25} = 44.4^\circ$$

$$\sum M_o = 0 : 120(14) + 60(9.5) - 2T \sin 44.4^\circ (12.25) = 0$$

$$\underline{T = 131.2 \text{ lb}}$$

3/90



$$\begin{cases} C_x = C_{y'} \sin 35^\circ \\ C_y = C_{y'} \cos 35^\circ \end{cases}$$

$$\sum M_x = 0: mg \frac{h}{2} \sin 15^\circ - C_y h \cos 15^\circ = 0, C_y = 0.1340 mg$$

$$C_{y'} = \frac{C_y}{\cos 35^\circ} = 0.1636 mg, C_x = C_y \tan 35^\circ = 0.0938 mg$$

$$\sum M_{Bz} = 0: C_x h \sin 15^\circ + A_y w = 0, A_y = -0.0243 mg \frac{h}{w}$$

$$\sum M_{By} = 0: mg \frac{w}{2} + C_x h \cos 15^\circ - A_z w = 0$$

$$A_z = mg \left(\frac{1}{2} + 0.0906 \frac{h}{w} \right)$$

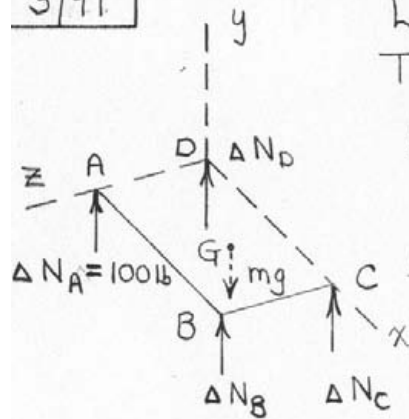
$$\sum M_{Az} = 0: -B_y w - C_y w + C_x h \sin 15^\circ = 0$$

$$B_y = mg \left(0.0243 \frac{h}{w} - 0.1340 \right)$$

$$\sum M_{Ay} = 0: B_z w - mg \frac{h}{2} + C_x h \cos 15^\circ = 0$$

$$B_z = mg \left(\frac{1}{2} - 0.0906 \frac{h}{w} \right)$$

3/91



Location of G does not change.

Thus,

$$\Delta N_D = -100 \text{ lb} \quad (\text{preserves total rear-axle loading})$$

$$\Delta N_B = -100 \text{ lb} \quad (\text{preserves total right-side loading})$$

$$\Delta N_C = 100 \text{ lb} \quad (\text{preserves total normal force; preserves total front-axle loading})$$

(Note: The results for ΔN_B & ΔN_C hold only if the track (distance between tire centers) at the front is equal to that at the rear.)

3/92

$$\sum F_z = 0; 2T \cos \beta - mg = 0 \quad \text{--- (1)}$$

$$\sum M_z = 0; 2T \sin \beta \cos \frac{\alpha}{2} \left(\frac{b}{2} \right) - M = 0 \dots (2)$$

$$\overline{CD} = 2 \frac{b}{2} \sin \frac{\alpha}{2} = b \sin \beta, \quad \beta = \frac{\alpha}{2} \quad \text{--- (3)}$$

Divide (2) by (1) & substitute (3) & get

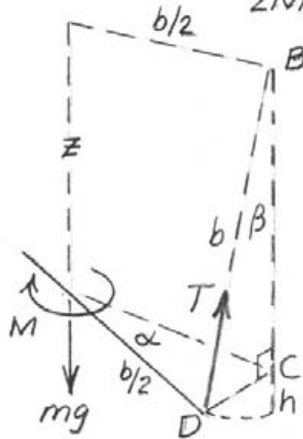
$$\frac{2T \frac{b}{2} \sin \beta \cos \beta}{2T \cos \beta} = \frac{M}{mg}, \quad \sin \beta = \frac{2M}{bmg}$$

$$\text{Thus } \cos \beta = \sqrt{1 - \left(\frac{2M}{bm g}\right)^2}$$

$$h = b(1 - \cos \beta)$$

$$\text{so } h = b \left(1 - \sqrt{1 - \left(\frac{2M}{bmg} \right)^2} \right)$$

For $h \rightarrow b$, $\cos \beta \rightarrow 0$, $\sin \beta \rightarrow \pi/2$ & $M \rightarrow \frac{bmg}{2}$



3/93

$\overline{AC} = \overline{AD} = \sqrt{4^2 + 5^2 + 10^2} = 11.87 \text{ m} ; |\underline{P}| = 20 \text{ kN}$
 $\underline{T}_1 = \frac{T_1}{11.87} (-4\underline{i} - 5\underline{j} - 10\underline{k}) ; \sin 20^\circ = 0.3420$
 $\underline{T}_2 = \frac{T_2}{11.87} (4\underline{i} - 5\underline{j} - 10\underline{k}) \quad \cos 20^\circ = 0.9397$

$\sum \underline{M}_x = 0 ; \underline{r}_{EC} \times \underline{T}_2 \cdot \underline{i} + \underline{r}_{FD} \times \underline{T}_1 \cdot \underline{i}$
 $+ \underline{r}_{BM} \times \underline{P} \cdot \underline{i} = 0$
 $-2\underline{j} \times \frac{T_2}{11.87} (4\underline{i} - 5\underline{j} - 10\underline{k}) \cdot \underline{i}$
 $-2\underline{j} \times \frac{T_1}{11.87} (-4\underline{i} - 5\underline{j} - 10\underline{k}) \cdot \underline{i} + (1.5\underline{j} + 5\underline{k}) \times 20 (0.342\underline{i}$
 $+ 0.9397\underline{j}) \cdot \underline{i} = 0$

Simplify & get $T_1 + T_2 = 55.79 \dots (1)$

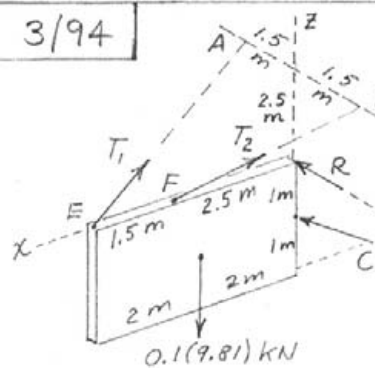
$\sum \underline{M}_z = 0 ; \underline{r}_{BA} \times (\underline{T}_1 + \underline{T}_2) \cdot \underline{k} + \underline{r}_{BM} \times \underline{P} \cdot \underline{k} = 0$

$(3\underline{j} + 10\underline{k}) \times \frac{1}{11.87} ([-4T_1 + 4T_2]\underline{i} + [-5T_1 - 5T_2]\underline{j} - [10T_1 + 10T_2]\underline{k}) \cdot \underline{k}$

Simplify to $T_1 - T_2 = 10.14 \dots (2)$

Solve (1) & (2) & get $T_1 = 33.0 \text{ kN}, T_2 = 22.8 \text{ kN}$

3/94



$$l_1 = \overline{AE} = \sqrt{4^2 + 1.5^2 + 2.5^2} = 4.95 \text{ m}$$

$$l_2 = \overline{BF} = \sqrt{2.5^2 + 1.5^2 + 2.5^2} = 3.84 \text{ m}$$

$$\underline{T}_1 = \frac{T_1}{l_1}(-4\underline{i} - 1.5\underline{j} + 2.5\underline{k})$$

$$\underline{T}_2 = \frac{T_2}{l_2}(-2.5\underline{i} + 1.5\underline{j} + 2.5\underline{k})$$

Note: \underline{T}_1 , \underline{T}_2 , \underline{R} , and weight all pass through x-axis, so \underline{C} must also. Thus $C_y = 0$

$$\sum M_{AB} = 0; C_x(3.5) - 9.81(2) = 0, C_x = 0.561 \text{ kN}$$

$$\sum M_z = 0; 4\underline{i} \times \frac{T_1}{l_1}(-1.5\underline{j}) + 2.5\underline{i} \times \frac{T_2}{l_2}(1.5\underline{j}) = 0, 8T_1/l_1 = 5T_2/l_2$$

$$\sum F_x = 0; -\frac{T_1}{l_1}(4) - \frac{T_2}{l_2}(2.5) + 0.561 = 0, 8T_1/l_1 + 5T_2/l_2 = 1.121 \text{ kN}$$

$$\text{solve \& get } T_1 = 1.121 l_1 / 16 = 0.347 \text{ kN}, T_2 = \frac{1.121}{10} l_2 = 0.431 \text{ kN}$$

$$\sum F_y = 0; \frac{1.121}{16} l_1 \frac{1.5}{l_1} - \frac{1.121}{10} l_2 \frac{1.5}{l_2} + R = 0, R = 0.0631 \text{ kN}$$

$$\sum F_z = 0; \frac{1.121}{16} l_1 \frac{2.5}{l_1} + \frac{1.121}{10} l_2 \frac{2.5}{l_2} + C_z - 0.981 = 0, C_z = 0.526 \text{ kN}$$

$$\text{Thus } C = \sqrt{(0.561)^2 + (0.526)^2} = 0.768 \text{ kN}$$

► 3/95

$\overline{AE} = 2400 \sin 30^\circ = 1200 \text{ mm}$, $\overline{AH} = 600 \text{ mm}$, $\overline{HE} = 1200 \cos 30^\circ = 600\sqrt{3} \text{ mm}$

$$\vec{T} = \frac{T}{\sqrt{2}} \left(-\frac{1}{2}\vec{i} + \frac{\sqrt{3}}{2}\vec{j} + \frac{1}{2}\vec{k} \right)$$

$$\sum M_z = 0; T_y \overline{AD} - W_y \overline{GF} = 0$$

$$\frac{T\sqrt{3}}{2\sqrt{2}} (1200) = 40(9.81) \frac{\sqrt{3}}{2} (600)$$

$$T = 20(9.81)\sqrt{2} = \underline{277 \text{ N}}$$

$$\sum M_x = 0; -B_y \overline{AB} + W_y \overline{FA} = 0$$

$$2400 B_y = 40(9.81) \frac{\sqrt{3}}{2} (1200)$$

$$B_y = 98.1\sqrt{3} = 169.9 \text{ N}$$

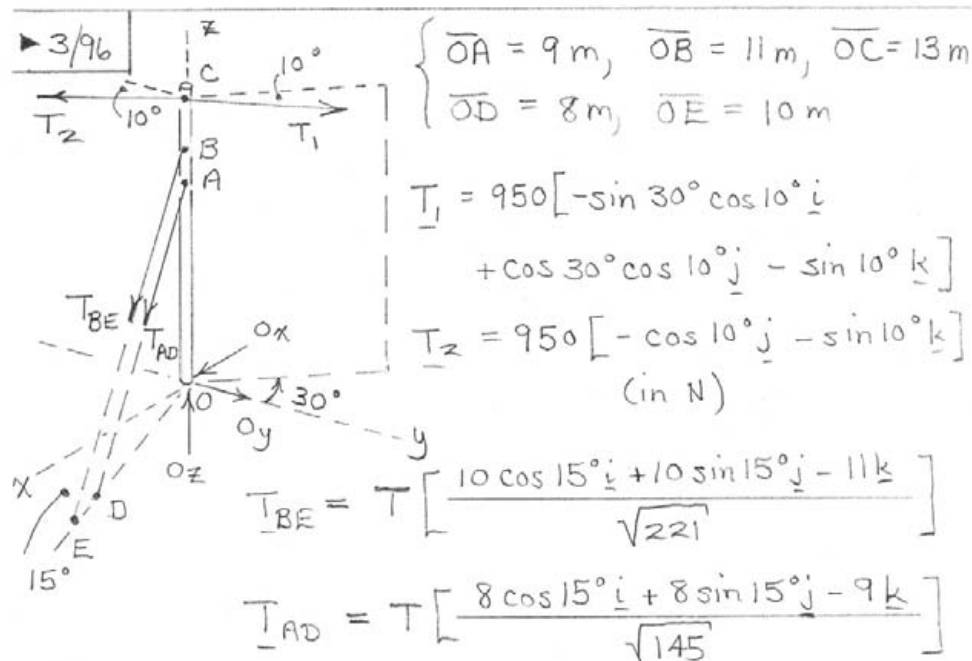
$$\sum M_y = 0; B_x \overline{AB} + W_z \overline{GF} - T_z \overline{AD} = 0$$

$$2400 B_x + 40(9.81) 0.5(600) - \frac{277}{2\sqrt{2}} (1200) = 0$$

$$B_x = 0$$

$$B = B_y = \underline{169.9 \text{ N}}$$

($B_x = 0$ can be obtained by inspection by noting $\sum M_{AE} = 0$ eliminates all terms except $B_x \overline{BE}$ so $B_x = 0$)



$$\sum \underline{M}_O = \underline{0}: \underline{r}_{OC} \times (\underline{T}_1 + \underline{T}_2) + \underline{r}_{OB} \times \underline{T}_{BE} + \underline{r}_{OA} \times \underline{T}_{AD} = \underline{0}$$

Set $\underline{r}_{OC} = 13\text{ k m}$, $\underline{r}_{OB} = 11\text{ k m}$, & $\underline{r}_{OA} = 9\text{ k m}$, carry out cross products, and set either the \underline{i} -component or the \underline{j} -component to zero to obtain $T = 471\text{ N}$.

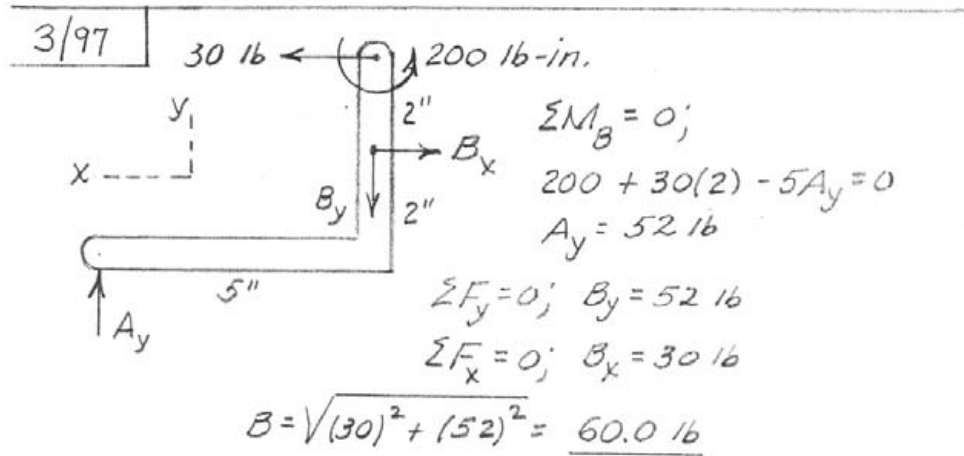
$$\sum F_x = 0: -950 \sin 30^\circ \cos 10^\circ + 471 \frac{10 \cos 15^\circ}{\sqrt{221}} + 471 \frac{8 \cos 15^\circ}{\sqrt{145}} + O_x = 0$$

$$O_x = -140.0\text{ N}$$

$$\sum F_y = 0: 950 \cos 30^\circ \cos 10^\circ - 950 \cos 10^\circ + 471 \frac{10 \sin 15^\circ}{\sqrt{221}} + 471 \frac{8 \sin 15^\circ}{\sqrt{145}} + O_y = 0$$

$$O_y = -37.5\text{ N}$$

$$\text{Then } O = \sqrt{O_x^2 + O_y^2} = 144.9\text{ N}$$



3/98

Isolate wheel of unicycle:

$$\alpha = \tan^{-1}\left(\frac{0.075}{9}\right) = 0.477^\circ$$

$$50(9.81) \text{ N}$$

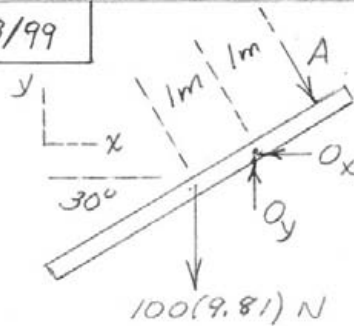
$$+\uparrow \sum F = 0: 2T \sin \alpha - 50(9.81) = 0$$

$$T = 29400 \text{ N}$$

$$\text{or } \underline{T = 29.4 \text{ kN}}$$



3/99



$$\sum M_O = 0; A(1) - (981 \cos 30^\circ)1 = 0$$

$$A = 850 \text{ N}$$

$$\sum F_x = 0; O_x - 850 \sin 30^\circ = 0$$

$$O_x = 425 \text{ N}$$

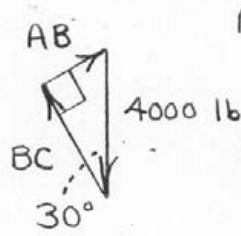
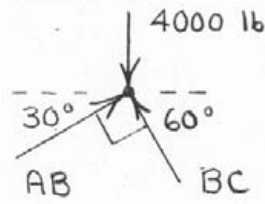
$$\sum F_y = 0; O_y - 850 \cos 30^\circ - 981 = 0$$

$$O_y = 1717 \text{ N}$$

$$O = \sqrt{425^2 + 1717^2} = 1769 \text{ N}$$

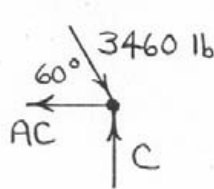
4/1

Joint B:



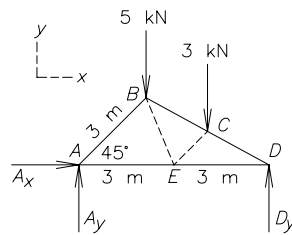
$$\begin{aligned} AB &= 4000 \left(\frac{1}{2} \right) \\ &= \underline{2000 \text{ lb C}} \\ BC &= 4000 \frac{\sqrt{3}}{2} \\ &= \underline{3460 \text{ lb C}} \end{aligned}$$

Joint C:



$$\rightarrow \sum F_x = 0: -AC + 3460 \cos 60^\circ = 0$$

$$\underline{AC = 1732 \text{ lb T}}$$



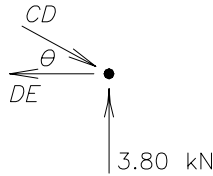
As a whole:

Note: $\overline{CE} = 1.5$ m by similar triangles

$$\Sigma M_A = 0: 5(3 \cos 45^\circ) + 3(3 + 1.5 \cos 45^\circ) - 6D_y = 0$$

$$D_y = 3.80 \text{ kN}$$

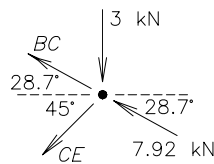
Joint D:



$$\theta = \tan^{-1} \frac{3 \sin 45^\circ}{6 - 3 \cos 45^\circ} = 28.7^\circ$$

$$\begin{cases} \Sigma F_y = 0: 3.80 - CD \sin 28.7^\circ = 0, CD = 7.92 \text{ kN } C \\ \Sigma F_x = 0: 7.92 \cos 28.7^\circ - DE = 0, DE = 6.94 \text{ kN } T \end{cases}$$

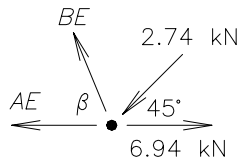
Joint C:



$$\begin{cases} \Sigma F_x = 0: -BC \cos 28.7^\circ - CE \cos 45^\circ - 7.92 \cos 28.7^\circ = 0 \\ \Sigma F_y = 0: BC \sin 28.7^\circ - CE \sin 45^\circ + 7.92 \sin 28.7^\circ - 3 = 0 \end{cases}$$

Solve simultaneously to obtain: $BC = -5.70 \text{ kN } (C)$
 $CE = -2.74 \text{ kN } (C)$

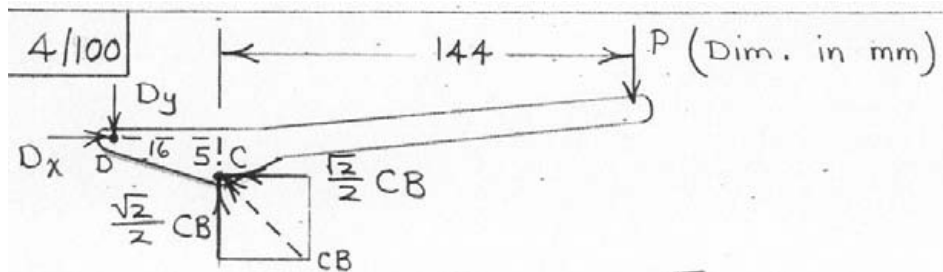
Joint E:



$$\beta = \frac{180^\circ - 45^\circ}{2} = 67.5^\circ$$

$$\Sigma F_y = 0: BE \sin 67.5^\circ - 2.74 \sin 45^\circ = 0$$

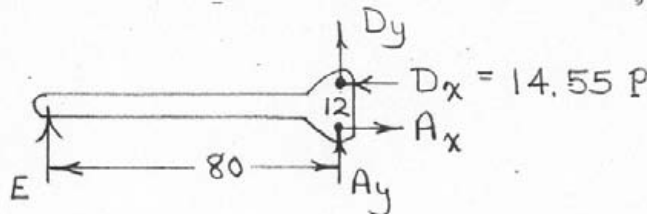
$$BE = 2.10 \text{ kN } T$$



$$\rightarrow \sum M_D = 0: P(160) + \frac{\sqrt{2}}{2} CB (5) - \frac{\sqrt{2}}{2} CB (16) = 0$$

$$CB = 20.6 P$$

$$\rightarrow \sum F = 0: Dx - 20.6 P \frac{\sqrt{2}}{2} = 0, Dx = 14.55 P$$



$$\rightarrow \sum M_A = 0: E(80) - 14.55 P (12) = 0$$

$$\underline{E = 2.18 P}$$

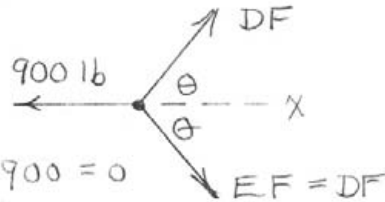
(Note: Mechanical advantage will increase as CB becomes more aligned with CD.)

4/101

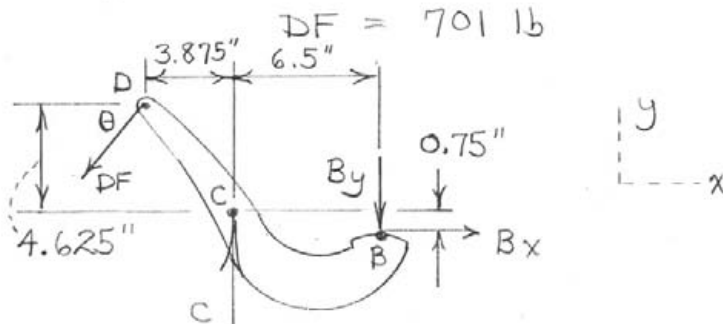
FBD of joint F :

$$\theta = \tan^{-1} \left(\frac{4.625}{3.875} \right) = 50.0^\circ$$

$$\Sigma F_x = 0: 2 DF \cos 50.0^\circ - 900 = 0$$



$$DF = 701 \text{ lb}$$



$$\Sigma F_x = 0: -701 \cos 50.0^\circ + B_x = 0, B_x = 450 \text{ lb}$$

$$\Sigma M_B = 0: -C(6.5) + 701 \cos 50.0^\circ(4.625 + 0.75) + 701 \sin 50.0^\circ(10.375) = 0, C = 1229 \text{ lb}$$

$$\Sigma F_y = 0: -701 \sin 50.0^\circ + 1229 - B_y = 0$$

$$B_y = 692 \text{ lb} = B_n = A_n$$

(No horizontal force component at C because of symmetry.)

4/102 Member DFH:

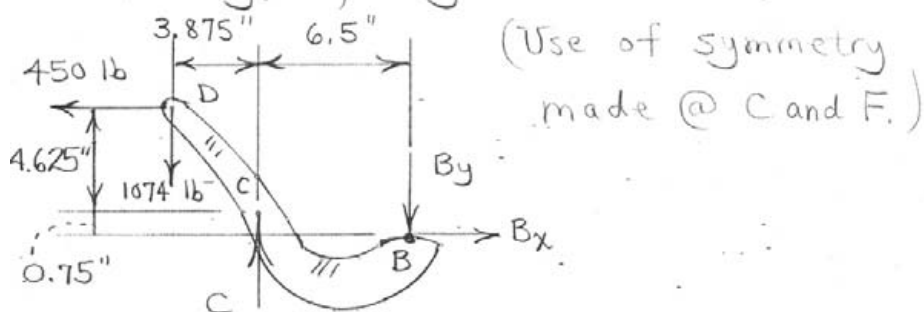
From $\sum F_x = 0$,

$$D_x = 450 \text{ lb}$$

$\sum M_D = 0$:

$$F(3.875) - 450(9.25) = 0, \quad F = 1074 \text{ lb}$$

From $\sum F_y = 0$, $D_y = 1074 \text{ lb}$



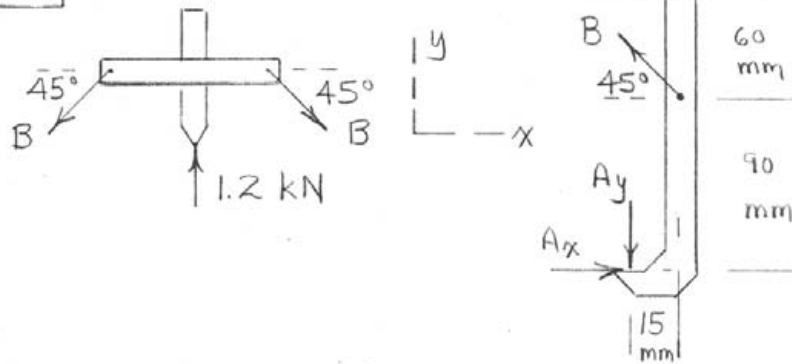
$$\sum M_B = 0: 450(5.375) + 1074(10.375) - C(6.5) = 0$$

$$C = 2090 \text{ lb}$$

$$\sum F_y = 0: -1074 + 2090 - B_y = 0$$

$$B_y = 1012 \text{ lb}$$

4/103



(Upper bar & screw)

$$\sum F_y = 0: -2B \sin 45^\circ + 1.2 = 0, \quad B = 0.849 \text{ kN}$$

(ABD)

$$\sum M_A = 0: 150D - 0.849 \cos 45^\circ (90) - 0.849 \sin 45^\circ (15) = 0$$

$$D = 0.420 \text{ kN}$$

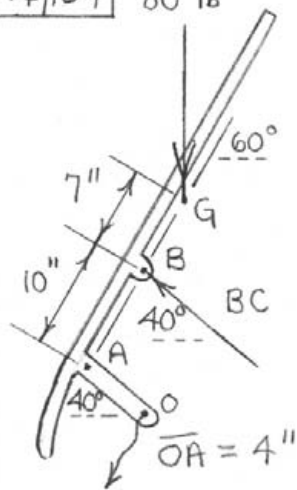
$$\sum F_x = 0: A_x - 0.849 \cos 45^\circ + 0.420 = 0, \quad A_x = 0.1800 \text{ kN}$$

$$\sum F_y = 0: -A_y + 0.849 \sin 45^\circ = 0, \quad A_y = 0.6 \text{ kN}$$

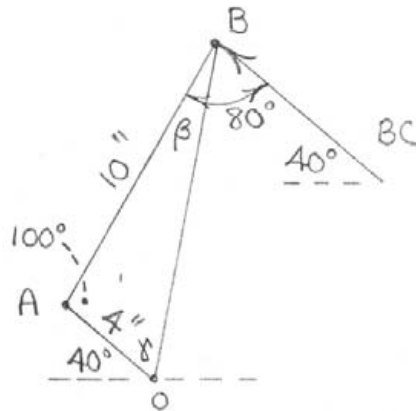
$$A = \sqrt{A_x^2 + A_y^2} = \underline{0.626 \text{ kN}}$$

4/104

80 lb



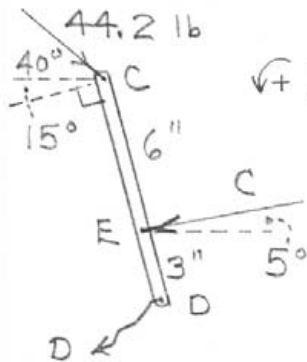
Blowup:



$$\overline{OB}^2 = 4^2 + 10^2 - 2(4)(10) \cos 100^\circ, \quad \overline{OB} = 11.40 \text{ in.}$$

$$\frac{\sin \beta}{4''} = \frac{\sin 100^\circ}{11.40''}, \quad \beta = 20.2^\circ$$

$$\begin{aligned} \curvearrowright \sum M_O = 0: & -80(17 \cos 60^\circ - 4 \cos 40^\circ) \\ & + BC(11.40 \sin(80^\circ - 20.2^\circ)) = 0, \quad BC = 44.2 \text{ lb} \end{aligned}$$

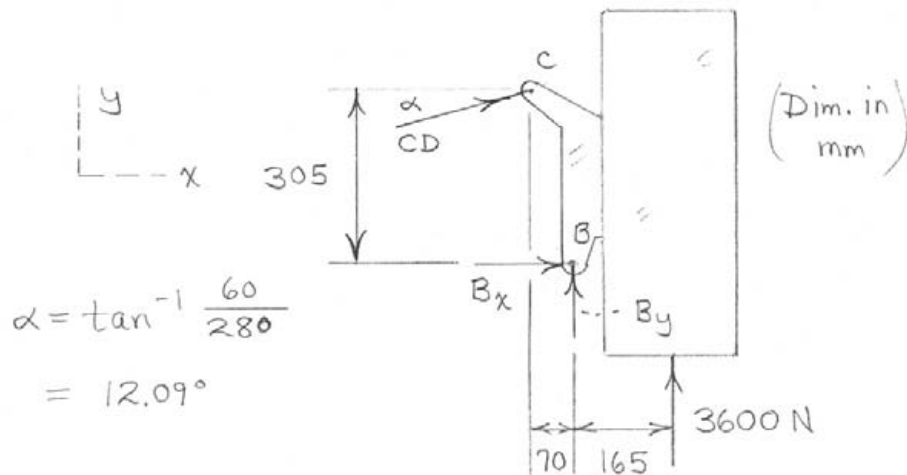


$$\begin{aligned} \curvearrowright \sum M_D = 0: & -44.2 \cos 55^\circ (9) \\ & + C \cos 10^\circ (3) = 0 \end{aligned}$$

$$\underline{C = 77.2 \text{ lb}}$$

4/105

Wheel unit:



$$\alpha = \tan^{-1} \frac{60}{280}$$

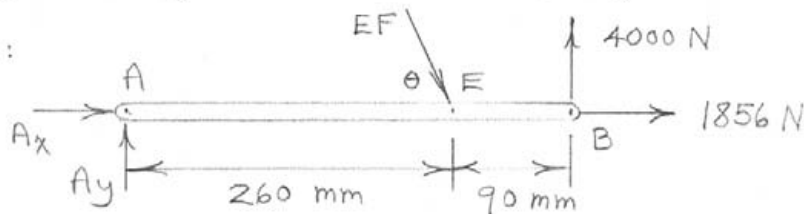
$$= 12.09^\circ$$

$$\sum M_B = 0: 3600(165) - CD \cos \alpha (305) - CD \sin \alpha (70) = 0, CD = 1898 \text{ N}$$

$$\sum F_x = 0: B_x + CD \cos \alpha = 0, B_x = -1856 \text{ N}$$

$$\sum F_y = 0: B_y + CD \sin \alpha + 3600 = 0, B_y = -4000 \text{ N}$$

AEB:



$$\theta = \tan^{-1} \left(\frac{435}{200} \right) = 65.3^\circ$$

$$\sum M_A = 0: -EF \sin \theta (260) + 4000(350) = 0$$

$$EF = 5920 \text{ N}$$

$$\sum F_x = 0: A_x + EF \cos \theta + 1856 = 0$$

$$A_x = -4330 \text{ N}$$

$$\sum F_y = 0: A_y - EF \sin \theta + 4000 = 0$$

$$A_y = 1384 \text{ N}$$

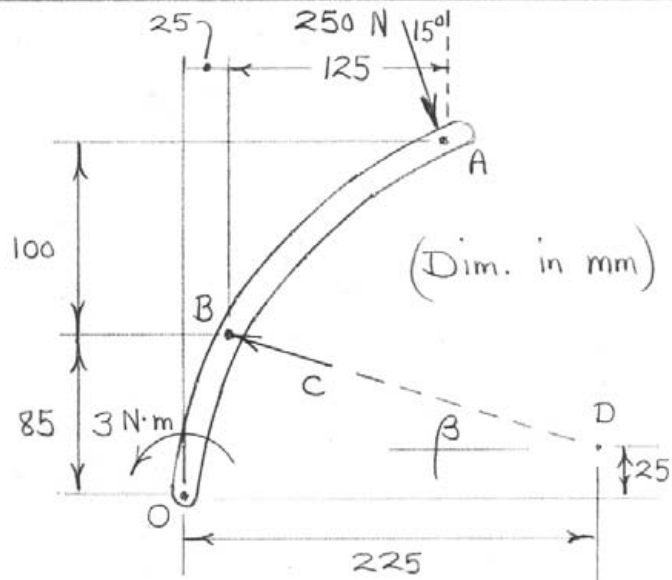
$$\text{So } A = \sqrt{(-4330)^2 + 1384^2} = 4550 \text{ N}$$

$$B = \sqrt{1856^2 + 4000^2} = 4410 \text{ N}$$

$$C = D = 1898 \text{ N}, E = F = 5920 \text{ N}$$

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$$\beta = \tan^{-1} \frac{60}{200} = 16.70^\circ$$



$$\sum M_O = 0: 3000 - 250 \cos 15^\circ (150) - 250 \sin 15^\circ (185) + C \cos \beta (85) + C \sin \beta (25) = 0$$

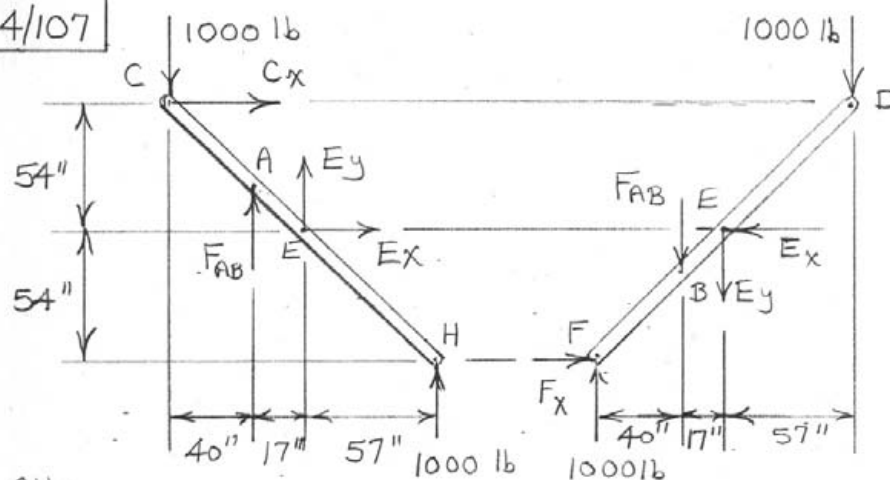
$$C = 510 \text{ N}$$

$$C = pA: 510 = p \left(\frac{\pi 45^2}{4} \right)$$

$$p = 0.321 \frac{\text{N}}{\text{mm}^2} \text{ or } 321\,000 \text{ Pa}$$

(gauge pressure)

4/107



CH:

$$\sum F_x = 0: C_x + E_x = 0 \quad (1)$$

$$\sum F_y = 0: F_{AB} + E_y + 1000 - 1000 = 0 \quad (2)$$

$$\sum M_C = 0: F_{AB}(40) + E_y(57) + E_x(54) + 1000(114) = 0 \quad (3)$$

DF:

$$\sum F_x = 0: F_x - E_x = 0 \quad (4)$$

$$\sum F_y = 0: (\text{Same as Eq. (2)})$$

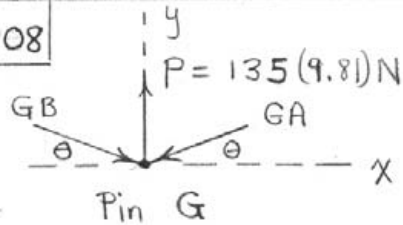
$$\sum M_F = 0: -F_{AB}(40) - E_y(57) + E_x(54) - 1000(114) = 0 \quad (5)$$

$$\text{Solution: } F_{AB} = 6710 \text{ lb}$$

$$C_x = F_x = E_x = 0, \quad E_y = -6710 \text{ lb}$$

(C_x , F_x evident from overall FBDs!)

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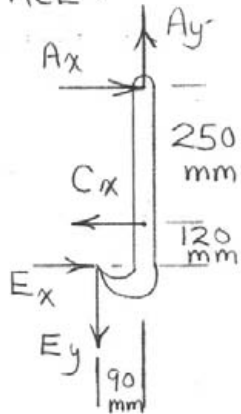


$$\theta = \cos^{-1} \frac{340}{350} = 13.73^\circ$$

$$\sum F_y = 0 : 135(9.81) - 2 G_A \sin 13.73^\circ = 0$$

$$G_A = G_B = 2790 \text{ N}$$

ACE:



$$A_x = 2790 \cos 13.73^\circ = 2710 \text{ N}$$

$$A_y = 2790 \sin 13.73^\circ = 662 \text{ N}$$

$$\sum F_y = 0 \Rightarrow E_y = 662 \text{ N}$$

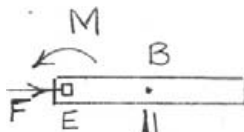
$$\sum M_C = 0 : 2710(250) - 662(90)$$

$$- E_x(120) = 0, \quad E_x = 5150 \text{ N}$$

$$E = \sqrt{E_x^2 + E_y^2} = 5190 \text{ N}$$

$$\text{or } \underline{5.19 \text{ kN}}$$

$$F_{AB} = 250 \text{ N}$$



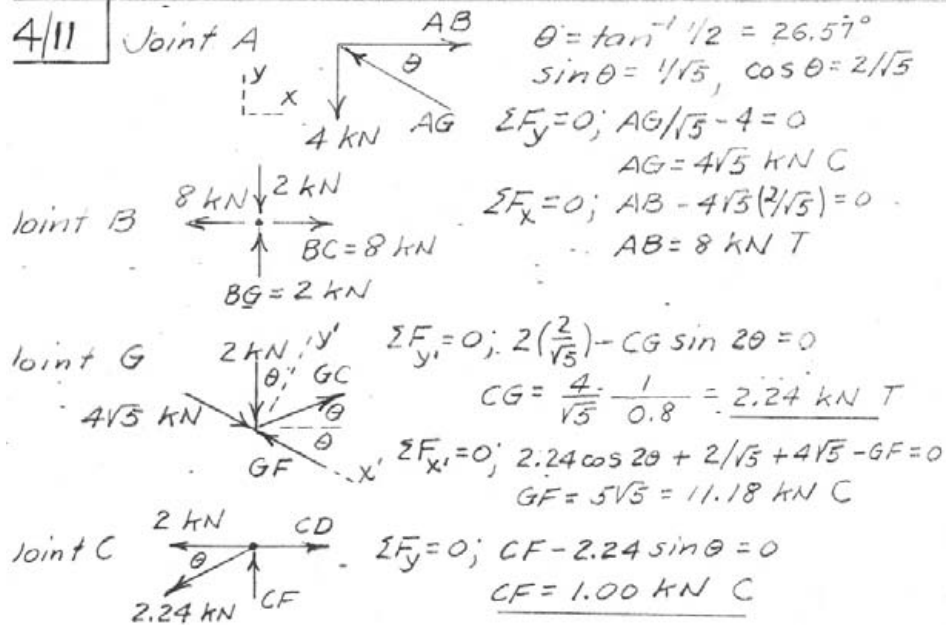
$$+\uparrow \Sigma F = 0 : C - 250 \cos 5^\circ = 0$$

C = 249 N

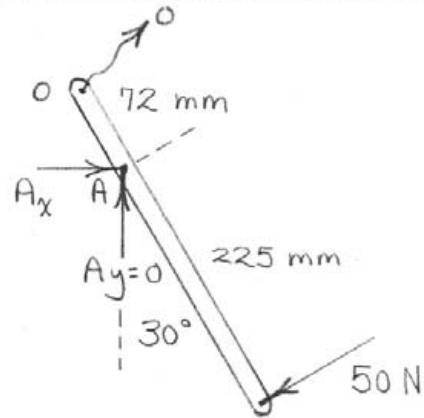
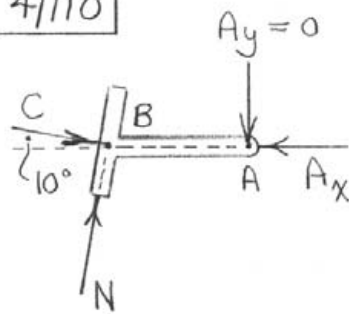
5. $F_{AB} = 250 \text{ N}$

(a factor $\frac{C}{P} = 4.97$)

4/11



4/110



$A_y = 0$ because AB is a three-force body.

Body OA:

$$\sum M_O = 0 : A_x (72 \cos 30^\circ) - 50(72 + 225) = 0$$

$$A_x = 238 \text{ N}$$

Body AB:

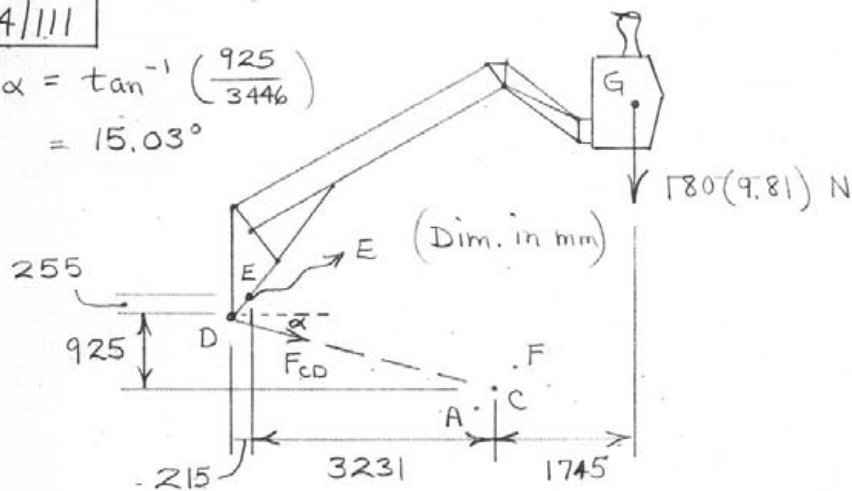
$$\sum F = 0 : C - 238 \cos 10^\circ = 0$$

$$C = 235 \text{ N}$$

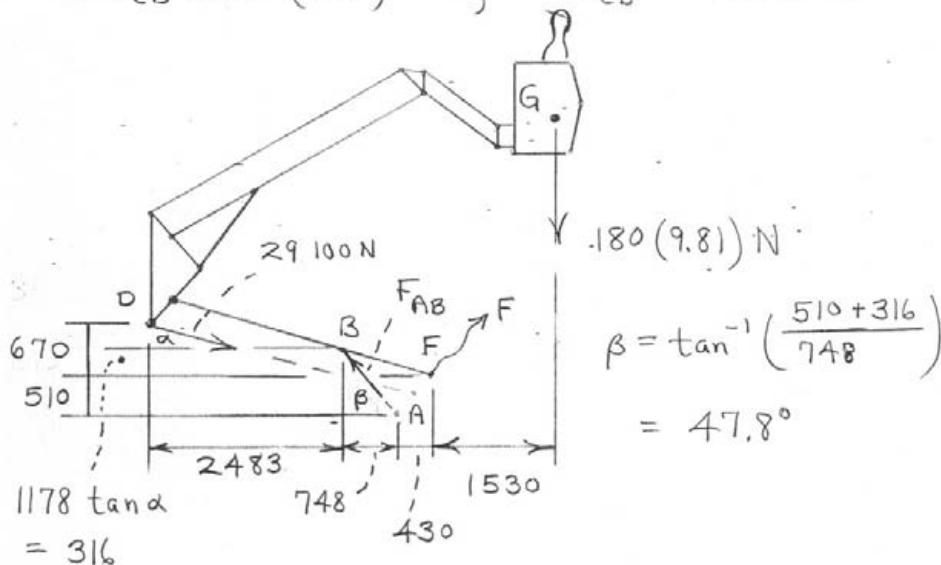
$$\left(\text{a factor } \frac{C}{P} = 4.69 \right)$$

4/111

$$\alpha = \tan^{-1} \left(\frac{925}{3446} \right) = 15.03^\circ$$



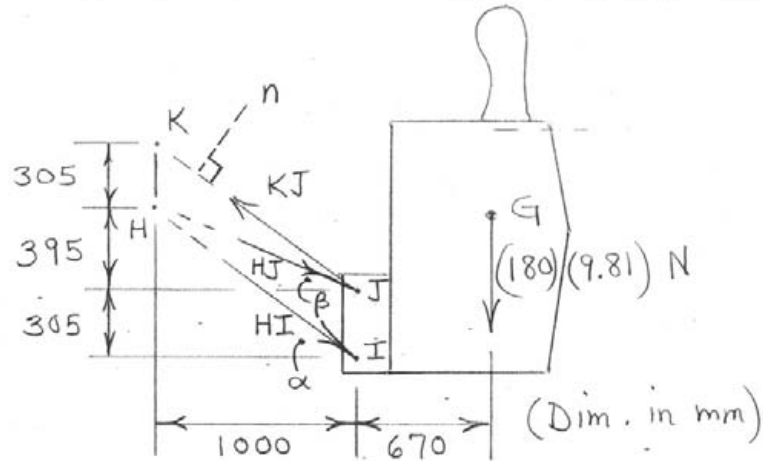
$$\begin{aligned} \sum M_E = 0: & -180(9.81)(4976) + F_{CD} \cos \alpha (255) \\ & + F_{CD} \sin \alpha (215) = 0, \quad F_{CD} = 29100 \text{ N} \end{aligned}$$



$$\beta = \tan^{-1} \left(\frac{510 + 316}{748} \right) = 47.8^\circ$$

$$\begin{aligned} \sum M_F = 0: & -29100 \cos \alpha (670) + 29100 \sin \alpha (3661) \\ & - F_{AB} \cos \beta (510) - F_{AB} \sin \beta (430) - 180(9.81)(1530) = 0 \\ & F_{AB} = 9200 \text{ N} \end{aligned}$$

4/112

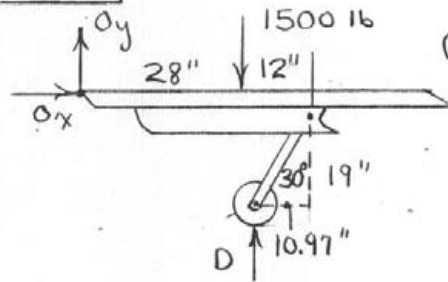


$$\alpha = \tan^{-1} \frac{700}{1000} = 35.0^\circ; \quad \beta = \tan^{-1} \frac{395}{1000} = 21.6^\circ$$

$$\sum F_n = 0: \quad HJ \sin(\alpha - \beta) - 180(9.81) \cos \alpha = 0$$

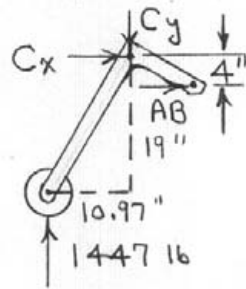
$$\underline{HJ = 6220 \text{ N}}$$

4/113 Entire ramp + mechanism :



$$\begin{aligned}\sum M_O &= 0: D(28+12-10.97) \\ &\quad - 1500(28) = 0 \\ D &= 1447 \text{ lb}\end{aligned}$$

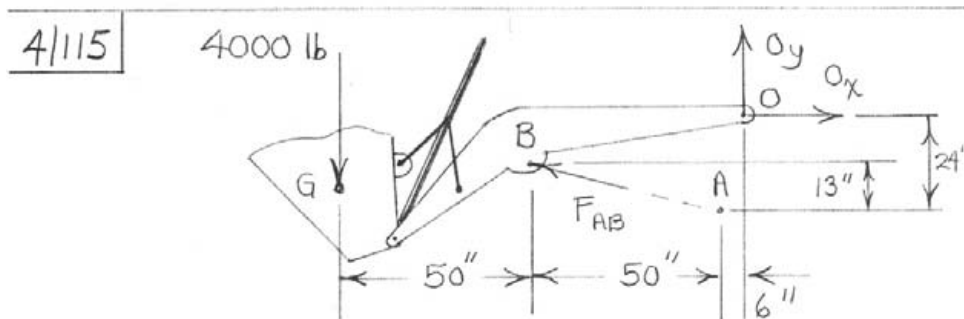
Crank BCD :



$$\begin{aligned}\sum M_C &= 0: -1447(10.97) \\ &\quad + AB(4) = 0, \quad AB = 3970 \text{ lb} \\ &\quad \text{(Cylinder is in compression)}\end{aligned}$$

4/114

$\frac{700}{\sin 60^\circ} = \frac{250}{\sin \theta}$, $\theta = \sin^{-1} 0.3093 = 18.02^\circ$
 $\cos \theta = 0.9510$, $\tan \theta = 0.3252$
 $\bar{AC} = 700(0.9510) + 250(0.5) = 791 \text{ mm}$
 $\sum M_C = 0; 24(250)(0.866) - 791 A_x = 0$
 $A_x = 6.57 \text{ kN}$
 $\bar{AE} = \bar{AC} \tan \theta = 791(0.3252) = 257 \text{ mm}$
 $\sum M_E = 0; A_y(257) - 24(250)(0.866 + 257) = 0$
 $A_y = 44.2 \text{ kN}$
 $A = \sqrt{6.57^2 + 44.2^2} = 44.7 \text{ kN}$



$$\angle OBA = \tan^{-1}\left(\frac{11}{56}\right) + \tan^{-1}\left(\frac{13}{50}\right) = 11.11^\circ + 14.57^\circ = 25.7^\circ$$

$$\overline{OB} = \sqrt{11^2 + 56^2} = 57.1 \text{ in.}$$

$$\sum M_O = 0: 4000(106) - F_{AB} \sin 25.7^\circ (57.1) = 0$$

$$F_{AB} = 17,140 \text{ lb}$$

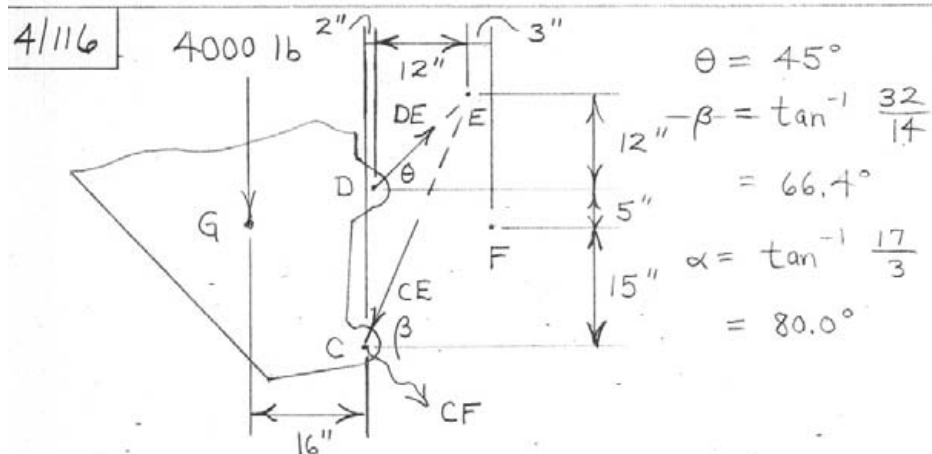
$$\sum F_x = 0: O_x - 17,140 \cos 14.57^\circ = 0$$

$$O_x = 16,590 \text{ lb}$$

$$\sum F_y = 0: O_y + 17,140 \sin 14.57^\circ - 4000 = 0$$

$$O_y = -313 \text{ lb}$$

$$O = \sqrt{16,590^2 + 313^2} = 16,590 \text{ lb}$$



$$\sum M_C = 0: 4000(16) - DE \cos 45^\circ(20) + DE \sin 45^\circ(2) = 0$$

$$DE = 5030 \text{ lb}$$

FBD of joint E:

Free Body Diagram of joint E showing forces DE , CE , and EF at angles θ , β , and α respectively.

$$\sum F_x = 0:$$

$$-DE \cos \theta + CE \cos \beta + EF \cos \alpha = 0$$

$$\sum F_y = 0:$$

$$-DE \sin \theta + CE \sin \beta - EF \sin \alpha = 0$$

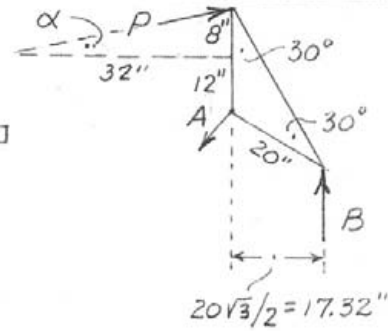
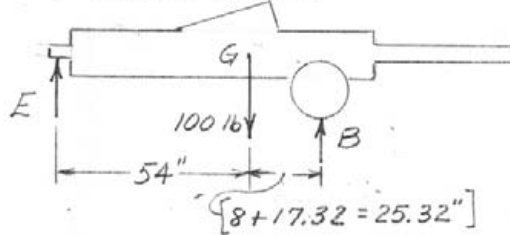
Solve simultaneously to obtain

$$\begin{cases} CE = 7440 \text{ lb} \\ EF = 3310 \text{ lb} \end{cases}$$

4/117

$$\tan \alpha = \frac{8}{32}, \alpha = 14.04^\circ$$

$$\cos \alpha = 0.9701$$

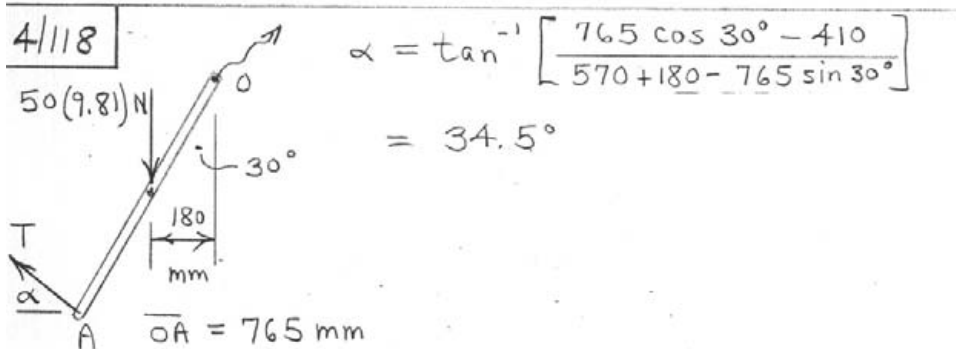


$$\sum M_E = 0; (54 + 25.32) B - 54(100) = 0$$

$$B = 68.1 \text{ lb}$$

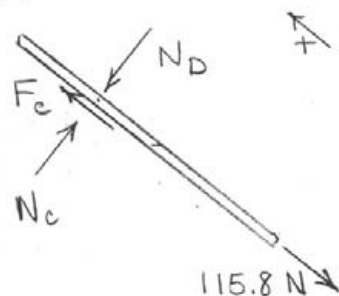
$$\sum M_A = 0; 0.9701 P(20) - 68.1(17.32) = 0$$

$$P = 60.8 \text{ lb}$$



$$\uparrow \sum M_O = 0: 50(9.81)(180) - T \cos \alpha (765 \cos 30^\circ) - T \sin \alpha (765 \sin 30^\circ) = 0, T = 115.8 \text{ N}$$

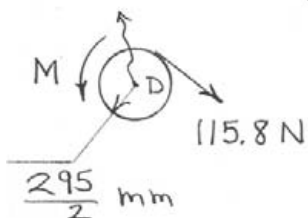
AB:



$$\uparrow \sum F = 0: F_c - 115.8 = 0$$

$$F_c = 115.8 \text{ N}$$

Pinion:

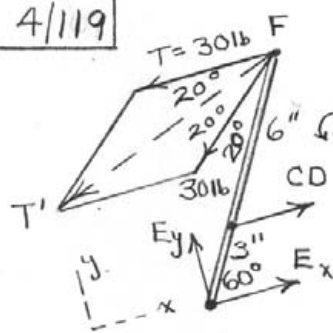


$$\uparrow \sum M_D = 0: M - 115.8 \left(\frac{295}{2} \right) = 0$$

$$M = 17080 \text{ N}\cdot\text{mm}$$

or 17.08 N·m

4/119



$$T'^2 = 30^2 + 30^2 - 2(30)(30)\cos 140^\circ$$

$$T' = 56.4 \text{ lb}$$

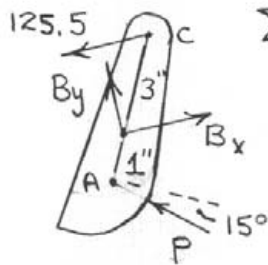
$$\sum M_E = 0: -CD(3 \sin 60^\circ) + 56.4(9 \sin 40^\circ) = 0, \quad CD = 125.5 \text{ lb}$$

$$\sum F_x = 0: E_x + 125.5 - 56.4 \cos 20^\circ = 0$$

$$E_x = -72.6 \text{ lb}$$

$$\sum F_y = 0: E_y - 56.4 \sin 20^\circ = 0, \quad E_y = 19.28 \text{ lb}$$

$$E = \sqrt{E_x^2 + E_y^2} = 75.1 \text{ lb}$$

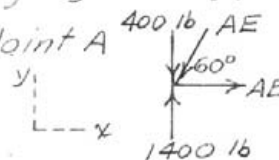


$$\sum M_B = 0: 125.5(3 \sin 60^\circ)$$

$$-P \cos 15^\circ (1) = 0, \quad P = 338 \text{ lb}$$

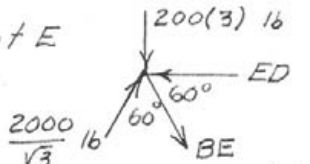
4/12 Total weight of truss = $7(400) = 2800$ lb

By symmetry, reactions at A & C are 1400 lb

Joint A  $\sum F_y = 0; AE \cos 30^\circ + 400 - 1400 = 0$
 $AE = 2000/\sqrt{3}$ lb C
 $\sum F_x = 0; AB - \frac{2000}{\sqrt{3}} \cos 60^\circ = 0$
 $AB = 1000/\sqrt{3}$ lb T

By symmetry $BC = 1000/\sqrt{3}$ lb T

$CD = 2000/\sqrt{3}$ lb C

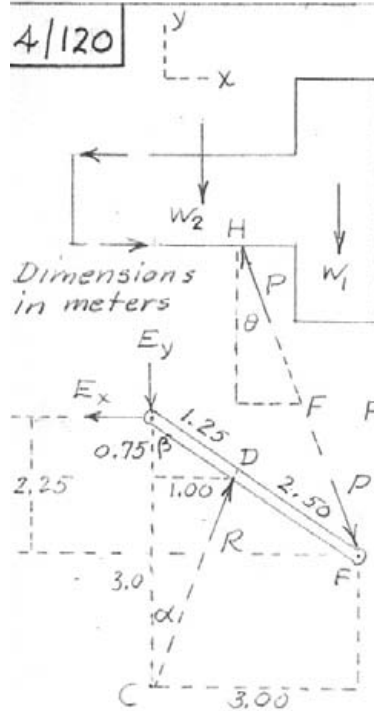
Joint E  $\sum F_y = 0; BE \sin 60^\circ - \frac{2000}{\sqrt{3}} \sin 60^\circ + 600 = 0$
 $BE = 800/\sqrt{3}$ lb T

& by symmetry $BD = 800/\sqrt{3}$ lb T

$\sum F_x = 0; ED - \frac{2000}{\sqrt{3}} \sin 30^\circ - \frac{800}{\sqrt{3}} \sin 30^\circ = 0$

$ED = 1400/\sqrt{3}$ lb C

4/120



$$\sin \theta = \frac{(3.00 - 1.75)}{3.25} = \frac{5}{13}, \cos \theta = \frac{12}{13}$$

$$\cos \beta = \frac{0.75}{1.25} = \frac{3}{5}, \sin \beta = \frac{4}{5}$$

$$\tan \alpha = 1/3, \sin \alpha = 1/\sqrt{10}, \cos \alpha = 3/\sqrt{10}$$

$$W_1 = 1.5(9.81) = 14.72 \text{ kN}$$

$$W_2 = 2(9.81) = 19.62 \text{ kN}$$

$$\sum F_y = 0; \frac{12}{13}P - 14.72 - 19.62 = 0$$

$$P = 37.2 \text{ kN}$$

$$\text{For EF; } \sum M_E = 0; 37.2\left(\frac{12}{13}\right)(3) - 37.2\left(\frac{5}{13}\right)(2.25)$$

$$- \frac{R}{\sqrt{10}}(3.0 + 0.75) = 0$$

$$R = 59.7 \text{ kN}$$

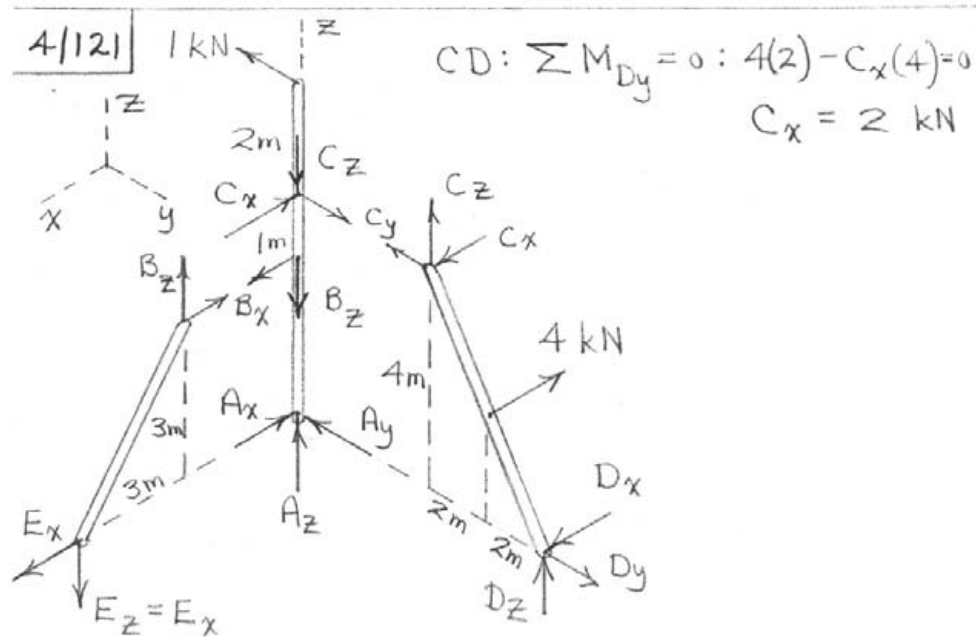
$$\sum F_x = 0; E_x - 59.7/\sqrt{10} - 37.2(5/13) = 0$$

$$E_x = 33.2 \text{ kN}$$

$$\sum F_y = 0; E_y + 37.2(12/13) - 59.7(3/\sqrt{10}) = 0$$

$$E_y = 22.3 \text{ kN}$$

$$R = \sqrt{33.2^2 + 22.3^2} = 40.0 \text{ kN}$$



ABC: $\sum M_{By} = 0: 2(1) - A_x(3) = 0, \quad A_x = 0.667 \text{ kN}$
 $\sum F_x = 0: B_x - 0.667 - 2 = 0, \quad B_x = 2.67 \text{ kN}$

EB: $B_z = E_z = B_x = E_x = 2.67 \text{ kN}$

ABC: $\sum M_{Ax} = 0: 1(6) - C_y(4) = 0, \quad C_y = 1.50 \text{ kN}$

CD: $\sum M_{Dx} = 0: C_z = C_y = 1.50 \text{ kN}$

ABC: $\sum F_z = 0: A_z - 2.67 - 1.50 = 0, \quad A_z = 4.17 \text{ kN}$

$\sum F_y = 0: A_y + 1 - 1.50 = 0, \quad A_y = 0.50 \text{ kN}$

$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \underline{4.25 \text{ kN}}$

$$\theta = \tan^{-1} \frac{5 \sin 50^\circ}{7 + 5 \cos 50^\circ} = 20.6^\circ$$

$$\theta = \tan^{-1} \frac{5 \sin 50^\circ}{7 + 5 \cos 50^\circ} = 20.6^\circ$$

$$d = 7 \sin 20.6^\circ = 2.46 \text{ ft}$$

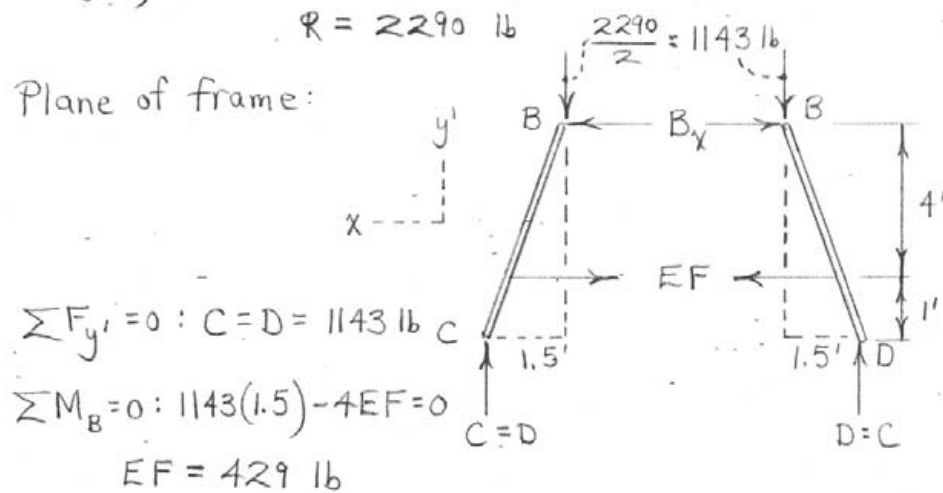
$$\beta + 20.6^\circ = 50^\circ, \quad \beta = 29.4^\circ$$

$$\sum M_x = 0 : (5 \cos 50^\circ)(1200) - 2.46T = 0, \quad \underline{T = 1569 \text{ lb}}$$

$$\sum F_{(C-D)-B} = 0 : R - 1200 \cos 40^\circ - 1569 \cos 29.4^\circ = 0$$

$$R = 2290 \text{ lb}$$

Plane of frame:



$$\sum F_{y'} = 0 : C = D = 1143 \text{ lb } C$$

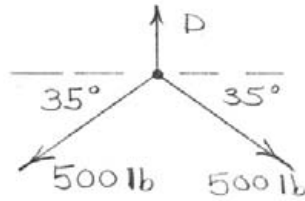
$$\sum M_B = 0: 1143(1.5) - 4EF = 0$$

$$EF = 429 \text{ lb}$$

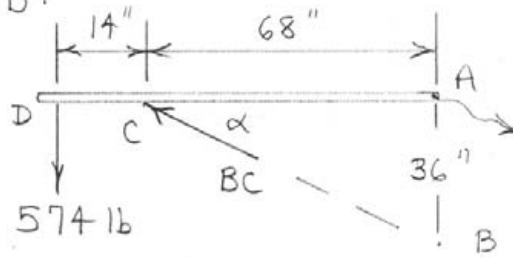
4/123 At D:

$$\uparrow \Sigma F = 0: D - 2(500) \sin 35^\circ = 0$$

$$D = 574 \text{ lb}$$



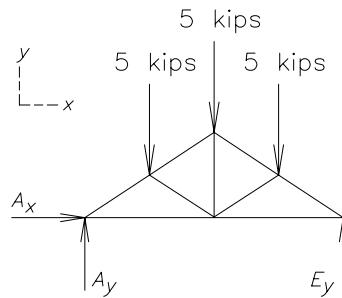
ACD:



$$\alpha = \tan^{-1} \frac{36}{68} = 27.9^\circ$$

$$\uparrow \Sigma M_A = 0: 574(82) - BC \sin 27.9^\circ (68) = 0$$

$$BC = \underline{1478 \text{ lb} = F}$$

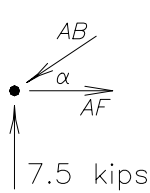


As a whole:

By symmetry, $A_y = E_y = 7.5$ kips

$$\Sigma F_x = 0: A_x = 0$$

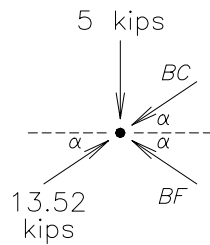
Joint A:



$$\alpha = \tan^{-1} \frac{10}{15} = 33.7^\circ$$

$$\begin{cases} \Sigma F_x = 0: -13.52 \cos 33.7^\circ + AF = 0, \underline{AF = 11.25 \text{ kips } T} \\ \Sigma F_y = 0: -AB \sin 33.7^\circ + 7.5 = 0, \underline{AB = 13.52 \text{ kips } C} \end{cases}$$

Joint B:

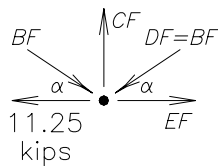


$$\begin{cases} \Sigma F_x = 0: (13.52 - BC - BF) \cos \alpha = 0 \\ \Sigma F_y = 0: (13.52 - BC + BF) \sin \alpha - 5 = 0 \end{cases}$$

Solve simultaneously to obtain:

$$\underline{BC = 9.01 \text{ kips } C}, \quad \underline{BF = 4.51 \text{ kips } C}$$

Joint F:



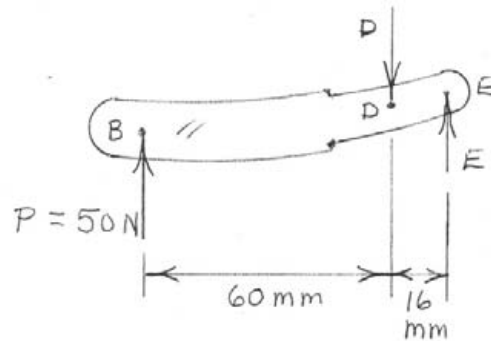
By symmetry, $\underline{EF = AF = 11.25 \text{ kips } T}$
& $\underline{DF = BF = 4.51 \text{ kips } C}$

$$\Sigma F_y = 0: CF - 2(4.51) \sin 33.7^\circ = 0, \quad \underline{CF = 5 \text{ kips } T}$$

By symmetry, $\underline{CD = 9.01 \text{ kips } C}, \underline{DE = 13.52 \text{ kips } C}, \underline{DF = 4.51 \text{ kips } C}$

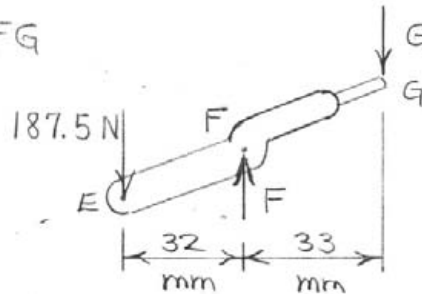
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Handle BDE



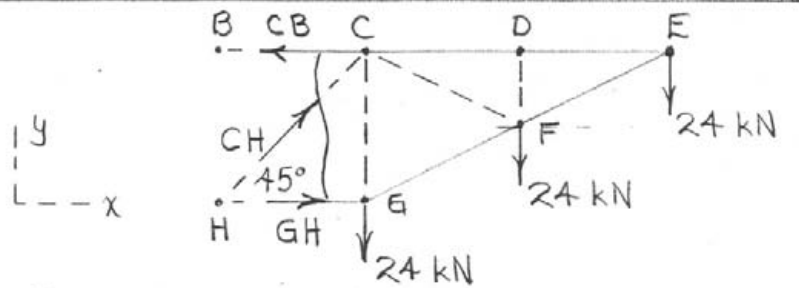
$$\uparrow \Sigma M_D = 0: -50(60) + E(16) = 0, \quad E = 187.5 \text{ N}$$

Jaw EFG



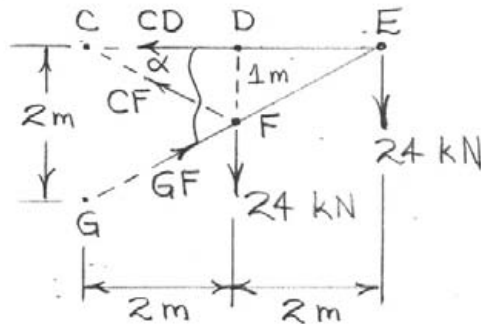
$$\uparrow \Sigma M_F = 0: 187.5(32) - G(33) = 0, \quad \underline{G = 181.8 \text{ N}}$$

4/126



$$\sum F_y = 0: CH \sin 45^\circ - 3(24) = 0, \quad \underline{CH = 101.8 \text{ kN C}}$$

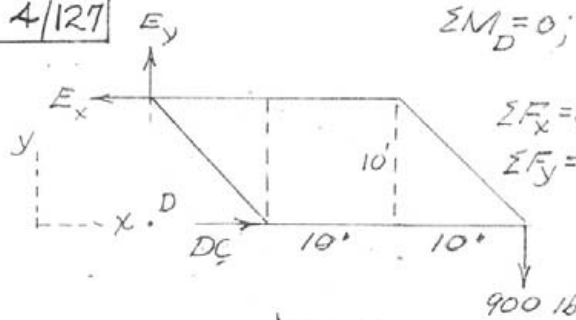
$$\alpha = \tan^{-1} \frac{1}{2} = 26.6^\circ$$



$$\sum M_E = 0: 24(2) - (CF \sin 26.6^\circ)(4) = 0$$

$$\underline{CF = 26.8 \text{ kN T}}$$

4/127

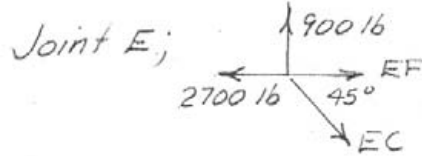


$$\sum M_D = 0; 900(30) - 10E_x = 0$$

$$E_x = 2700 \text{ lb}$$

$$\sum F_x = 0; DC = 2700 \text{ lb C}$$

$$\sum F_y = 0; E_y = 900 \text{ lb}$$



Joint E;

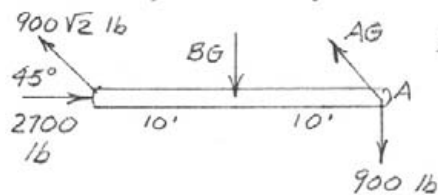
$$\sum F_y = 0; EC/\sqrt{2} - 900 = 0$$

$$EC = 900\sqrt{2} \text{ lb}$$

$$\sum F_x = 0; EF + \frac{900\sqrt{2}}{\sqrt{2}} - 2700 = 0$$

$$EF = 1800 \text{ lb T}$$

Joint F; $FG = EF$, $FC = 0$

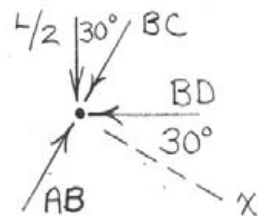


$$\sum M_A = 0; 10BG - \frac{900\sqrt{2}}{\sqrt{2}} 20 = 0$$

$$BG = 1800 \text{ lb C}$$

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Joint B :

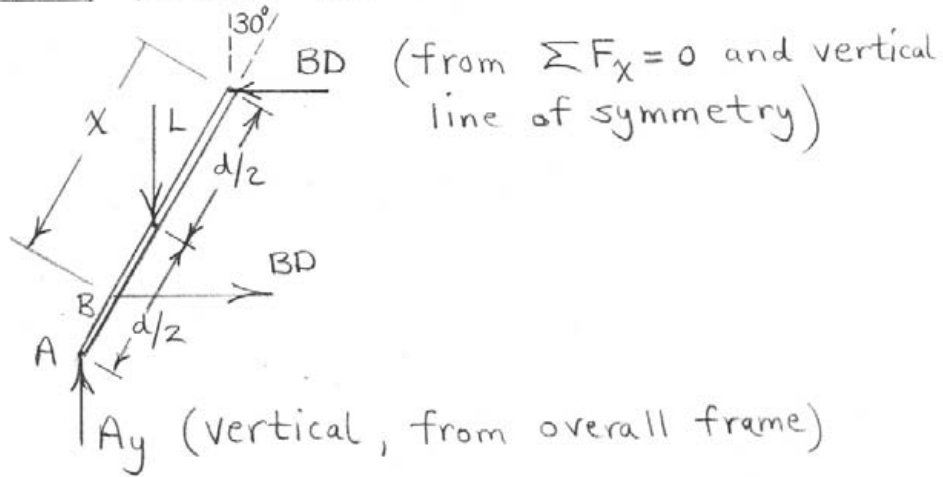


$$\sum F_x = 0: \frac{L}{2} \left(\frac{1}{2} \right) - BD \frac{\sqrt{3}}{2} = 0$$

$$BD = \frac{\sqrt{3}}{6} L \quad C, \text{ independent of } x.$$

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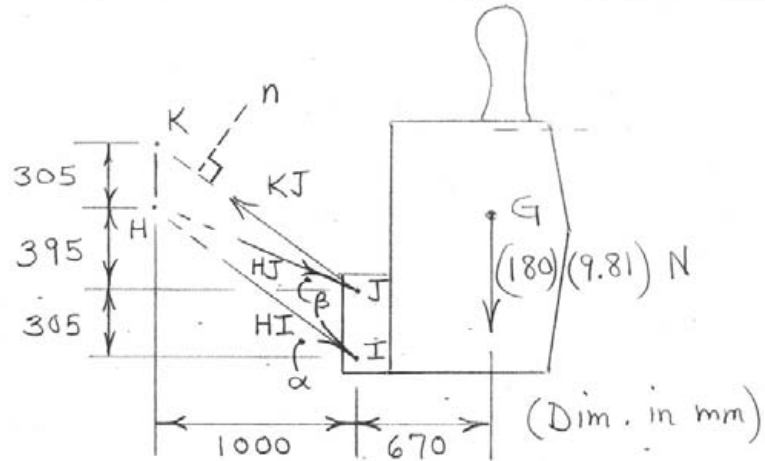
Member ABC:



$$\begin{aligned} \curvearrowright \sum M_A = 0: & \quad BD(d \cos 30^\circ) - BD(d-x) \cos 30^\circ \\ & \quad - L\left(\frac{d}{2} \sin 30^\circ\right) = 0 \\ BD = & \quad \frac{Ld}{2x} \tan 30^\circ = \frac{0.289Ld}{x} \end{aligned}$$

(x cannot be zero)

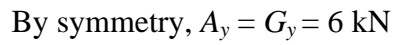
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$$\alpha = \tan^{-1} \frac{700}{1000} = 35.0^\circ; \quad \beta = \tan^{-1} \frac{395}{1000} = 21.6^\circ$$

$$\sum F_n = 0: \quad HJ \sin(\alpha - \beta) - 180(9.81) \cos \alpha = 0$$

$$\underline{HJ = 6220 \text{ N}}$$

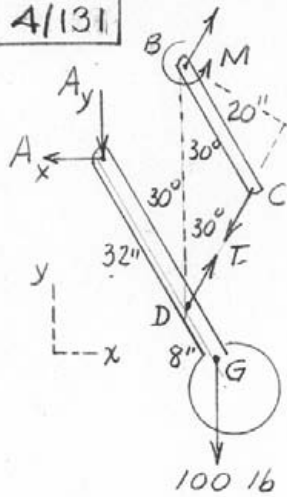


$$\overline{AC} = \frac{3}{\sqrt{2}} \text{ m}$$

$$\Sigma F_y = 0: 6 + CD \sin 18.43^\circ - 1.5 - 3 = 0, CD = -4.74 \text{ kN or } \underline{4.74 \text{ kN } C}$$

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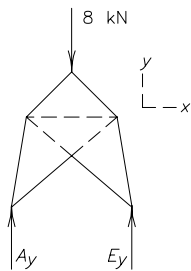


$$ADG; \sum M_A = 0; 100(40 \sin 30^\circ) - T(32 \cos 30^\circ) = 0$$

$$T = 72.2 \text{ lb}$$

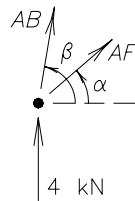
$$BC; \sum M_B = 0; 72.2(20 \cos 30^\circ) - M = 0$$

$$M = 1250 \text{ lb-in.}$$



By symmetry, $A_y = E_y = 4 \text{ kN}$

Joint A:

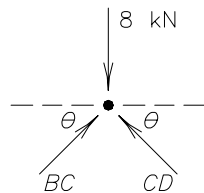


$$\alpha = \tan^{-1} \frac{3}{3.5} = 40.6^\circ, \beta = \tan^{-1} \frac{3}{0.5} = 80.5^\circ$$

$$\begin{cases} \Sigma F_x = 0: AB \cos 80.5^\circ + AF \cos 40.6^\circ + 4 = 0 \\ \Sigma F_y = 0: AB \sin 80.5^\circ + AF \sin 40.6^\circ = 0 \end{cases}$$

Solve to obtain: $AB = -4.73 \text{ kN}$ or 4.73 kN C

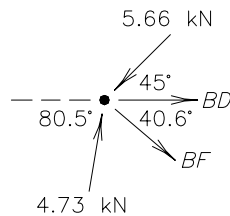
Joint C:



By symmetry, $BC = CD$

$$\Sigma F_y = 0: 2BC \sin 45^\circ - 8 = 0, BC = 5.66 \text{ kN C}$$

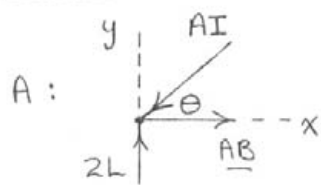
Joint B:



$$\Sigma F_y = 0: 4.73 \sin 80.5^\circ - BF \sin 40.6^\circ - 5.66 \sin 45^\circ = 0$$

$$\underline{BF = 1.024 \text{ kN T}}$$

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From whole truss, $A = F = 2L$ 

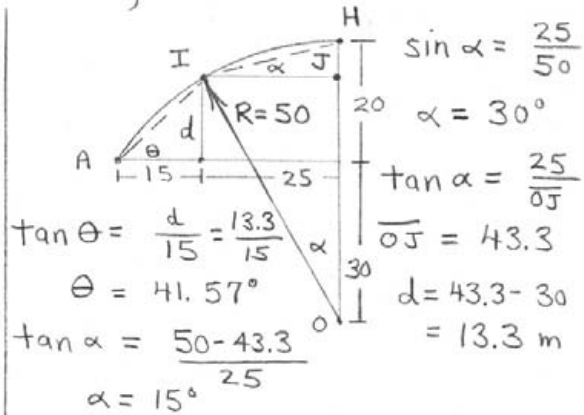
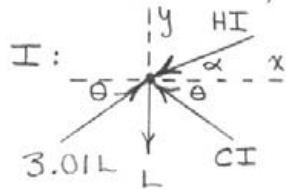
A:

$$y: 2L - AI \sin \theta$$

$$AI = 3.01L \text{ C}$$

$$x: -3.01 \cos \theta + AB = 0$$

$$AB = 2.26L \text{ T}$$

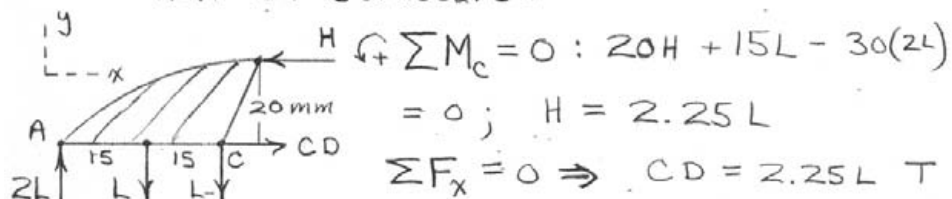
From joint B, $BI = L \text{ T}$ 

$$I: \begin{cases} \sum F_x = 0: (3.01L - CI) \cos \theta - HI \cos \alpha = 0 \\ \sum F_y = 0: (3.01L + CI) \sin \theta - HI \sin \alpha - L = 0 \end{cases}$$

Solve to obtain $\frac{CI}{HI} = \frac{-0.458L \text{ T}}{2.69L \text{ C}}$

4/134 From whole structure, $A = F = 2L$

Half of structure:

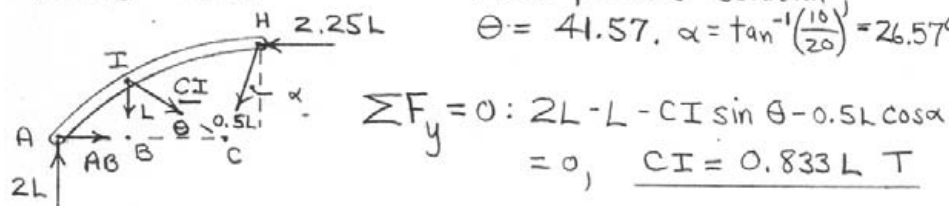


From joint B, $BI = L \text{ T}$

Member AIH:

From previous solution

$$\theta = 41.57, \alpha = \tan^{-1}\left(\frac{10}{20}\right) = 26.57^\circ$$

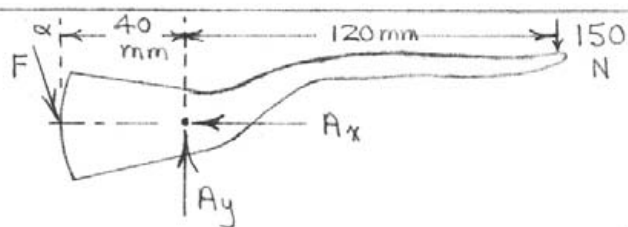
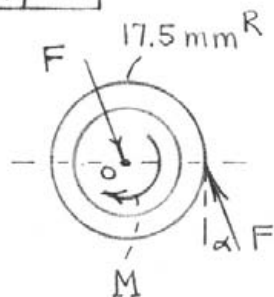


$$\sum F_x = 0: AB + 0.833L \cos \theta - 0.5L \sin \alpha - 2.25L = 0$$

$$AB = 1.850L \text{ T}$$

Problem not solvable without CH data.

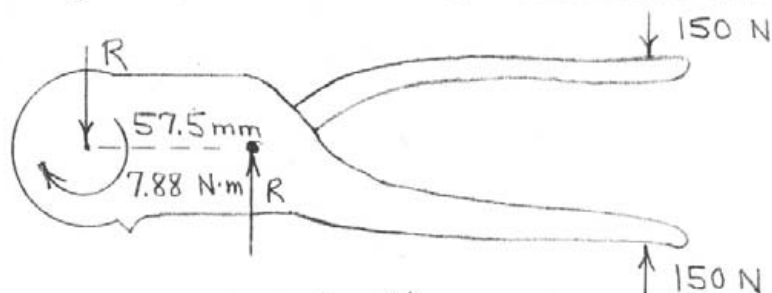
4/135



$$\sum M_A = 0: 40 F \cos \alpha = 120 (150)$$

$$F \cos \alpha = 450 \text{ N}$$

$$\sum M_o = 0: 450 (0.0175) = M = 7.88 \text{ N}\cdot\text{m}$$

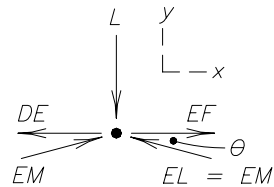


$$\sum M = 0: 57.5 (10^{-3}) R = 7.88$$

$$R = 137.0 \text{ N}$$

We can begin at joint E without finding the external reactions.

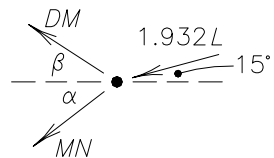
Joint E:



$$\theta = \tan^{-1} \frac{24 - 24 \sin 60^\circ}{24 \sin 30^\circ} = 15^\circ$$

$$\begin{cases} \Sigma F_y = 0: 2(EM \sin 15^\circ) - L = 0, \underline{EM = 1.932 L C} \\ \Sigma F_x = 0: 1.932L \cos 15^\circ - DE = 0, \underline{DE = 1.366 L C} \end{cases}$$

Joint M:



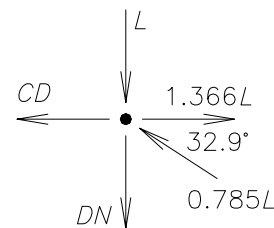
$$\alpha = \tan^{-1} \frac{24 \sin 60^\circ - 24 \sin 45^\circ}{24 \cos 45^\circ - 24 \cos 60^\circ} = 37.5^\circ$$

$$\beta = \tan^{-1} \frac{24 - 24 \sin 60^\circ}{24 \cos 45^\circ - 24 \cos 60^\circ} = 32.9^\circ$$

$$\begin{cases} \Sigma F_x = 0: -DM \cos 32.9^\circ - MN \cos 37.5^\circ - 1.932L \cos 15^\circ = 0 \\ \Sigma F_y = 0: DM \sin 32.9^\circ - MN \sin 37.5^\circ - 1.932 \sin 15^\circ = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{DM = 0.785 L C}$

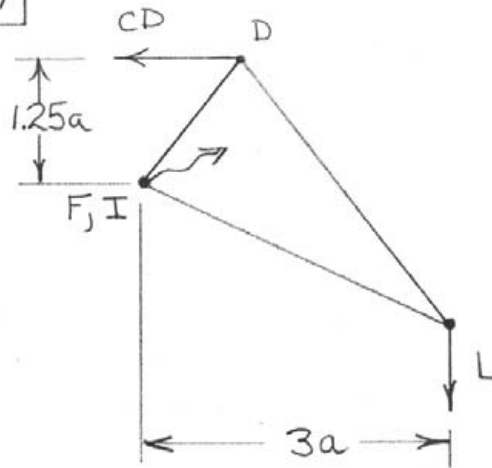
Joint D:



$$\Sigma F_y = 0: 0.785L \sin 32.9^\circ - L - DN = 0$$

$$\underline{DN = 0.574 L C}$$

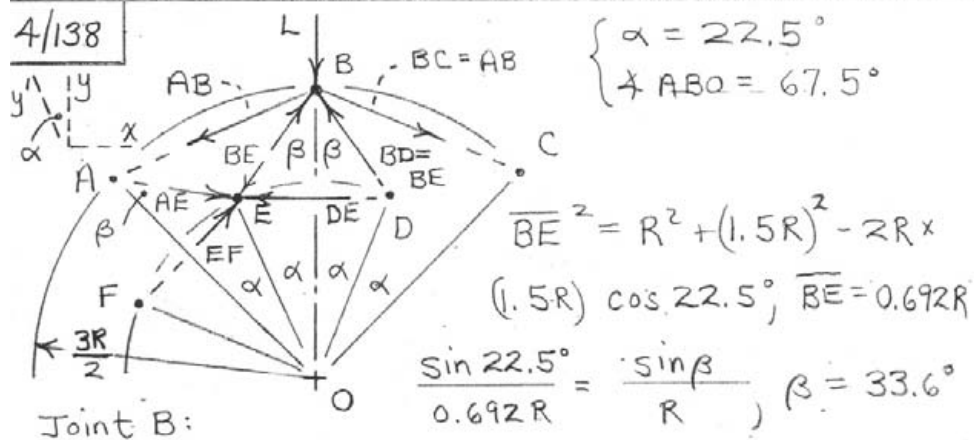
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$$\curvearrowright \sum M_F = 0: CD(1.25a) - L(3a) = 0$$

$$\underline{CD = 2.4L \text{ T}}$$

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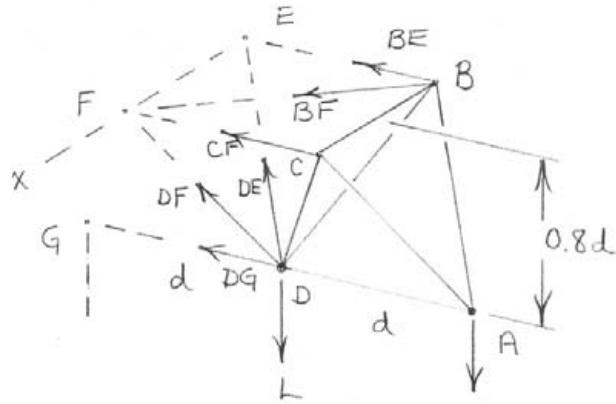
$$\sum F_y = 0 : 2(0.8L) \cos 33.6^\circ - 2AB \cos 67.5^\circ - L = 0$$

$$AB = 0.434L \text{ T}$$

For joint E, note that $EF = DE$ & $AE = BE$.

$$\sum F_y = 0 : -2(0.8L) \cos 56.1^\circ + 2DE \cos 67.5^\circ = 0$$

$$DE = 1.166L \text{ C}$$



$$\sum M_x = 0: -DG(0.8d) - L\left(\frac{d}{2}\right) - L\left(\frac{3d}{2}\right) = 0$$

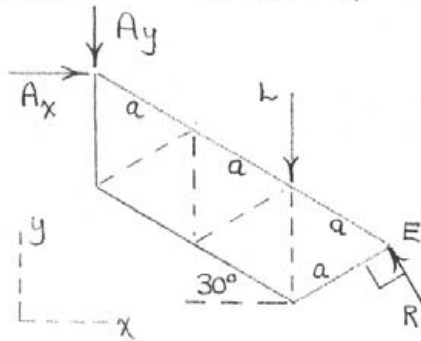
$$DG = -2.5L \text{ (or } 2.5L \text{ C)}$$

$$\text{Similarly, } AD = 0.625L \text{ C}$$

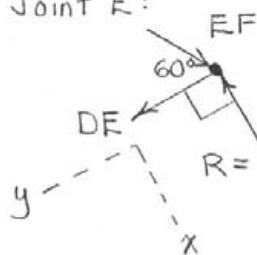
4/14

$$+2 \sum M_A = 0 : L(2a \cos 30^\circ) - R(3a \sin 30^\circ) = 0$$

$$R = \frac{2L}{\sqrt{3}}$$



Joint E:



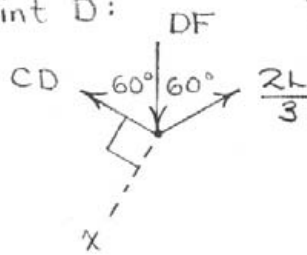
$$\sum F_x = 0 : EF \sin 60^\circ - \frac{2L}{\sqrt{3}} = 0$$

$$EF = \frac{4L}{3} \quad C$$

$$\sum F_y = 0 : DE - \frac{4L}{3} \cos 60^\circ = 0$$

$$DE = \frac{2L}{3} \quad T$$

Joint D:



$$\sum F_x = 0 : DF \cos 30^\circ - \frac{2L}{3} \cos 30^\circ = 0$$

$$DF = \frac{2L}{3} \quad C$$

► 4/140

Vector expressions for forces at A

(treated as tensions) with $F_{AE} = F_{AF} = F_1$,

$F_{BE} = F_{BF} = P$, $F_{BD} = F_{BC} = C$, are

$$\underline{F}_{AE} = \frac{F_1}{1.552} (-1.2\underline{i} - 0.4\underline{j} + 0.9\underline{k}), \underline{F}_{AF} = \frac{F_1}{1.552} (-1.2\underline{i} + 0.4\underline{j} + 0.9\underline{k})$$

$$\underline{F}_{AB} = \frac{F_{AB}}{1.432} (-0.3\underline{i} + 1.4\underline{k}), \underline{F} = 2.2\underline{k}. \text{ For joint}$$

$$A, \Sigma \underline{F} = 0 \text{ gives } \left[\frac{F_{AB}}{1.432} (-0.3) + \frac{2F_1}{1.552} (-1.2) \right] \underline{i}$$

$$+ \left[2.2 + \frac{F_{AB}}{1.432} (1.4) + \frac{2F_1}{1.552} (0.9) \right] \underline{k} = 0$$

Solve to get $F_{AB} = -2.681 \text{ kN}$, $F_1 = 0.363 \text{ kN}$

$$\text{On B: } \underline{F}_{BE} = \frac{P}{1.105} (-0.9\underline{i} - 0.4\underline{j} - 0.5\underline{k})$$

$$\underline{F}_{BF} = \frac{P}{1.105} (-0.9\underline{i} + 0.4\underline{j} - 0.5\underline{k}), \underline{F}_{BD} = \frac{C}{1.105} (-0.9\underline{i} - 0.4\underline{j} + 0.5\underline{k})$$

$$\underline{F}_{BC} = \frac{C}{1.105} (-0.9\underline{i} + 0.4\underline{j} + 0.5\underline{k})$$

For joint B, $\Sigma \underline{F} = 0$ gives

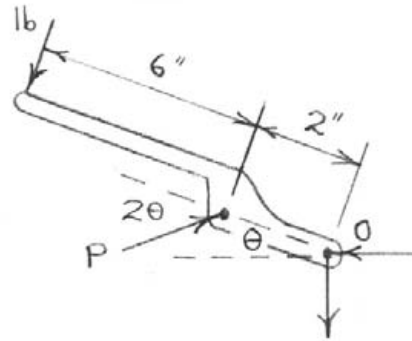
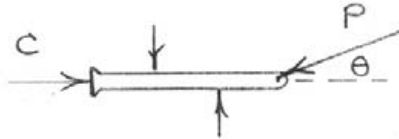
$$\left(\frac{-1.8P}{1.105} - \frac{1.8C}{1.105} + 0.3 \frac{-2.681}{1.432} \right) \underline{i} + \left(\frac{-P}{1.105} + \frac{C}{1.105} - 1.4 \frac{-2.681}{1.432} \right) \underline{k}$$

$$+ 0\underline{j} = 0. \text{ Solve to get } P = 1.620 \text{ kN}, \\ C = -1.275 \text{ kN}, \underline{F}_{BE} = P = 1.620 \text{ kN}$$

* 4/141

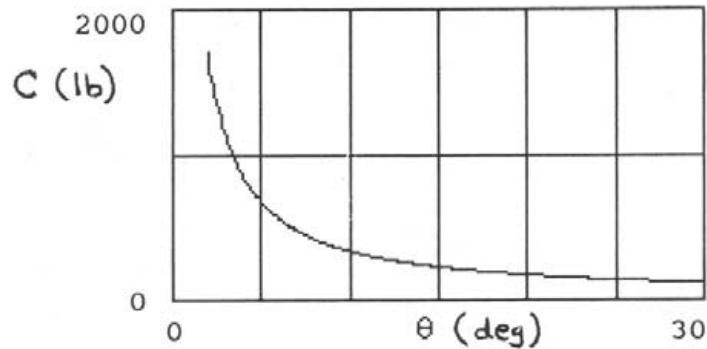
Shaft: $\Sigma F = 0$ gives

$$C = P \cos \theta$$

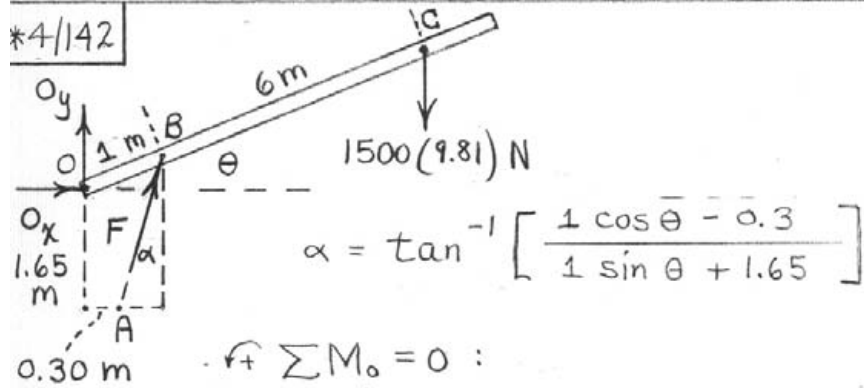


Lever: $\Sigma M_o = 0: 30(8) - P \sin 2\theta (2) = 0$

$$P = \frac{120}{\sin 2\theta} = \frac{60}{\sin \theta \cos \theta}, \quad C = \frac{60}{\sin \theta} \quad (1b)$$



*4/142

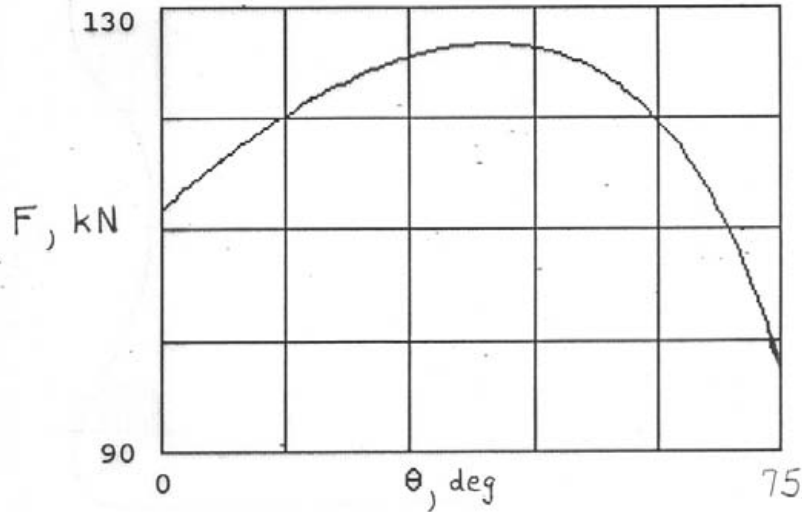


$$\alpha = \tan^{-1} \left[\frac{1 \cos \theta - 0.3}{1 \sin \theta + 1.65} \right]$$

$$\sum M_o = 0 :$$

$$(F \cos \alpha)(1 \cos \theta) - (F \sin \alpha)(1 \sin \theta) - 1500(9.81)(7 \cos \theta) = 0$$

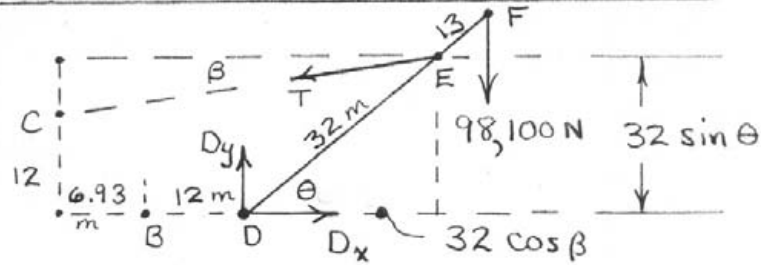
$$F = \frac{103.0 \cos \theta}{(\cos \alpha \cos \theta - \sin \alpha \sin \theta)} \quad (\text{in kN})$$



$$F_{\max} = 126.7 \text{ kN @ } \theta = 39.8^\circ$$

*4/143

Boom:

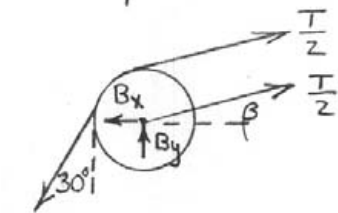


$$\beta = \tan^{-1} \left(\frac{32 \sin \theta - 12}{6.93 + 12 + 32 \cos \theta} \right) = \tan^{-1} \left(\frac{32 \sin \theta - 12}{32 \cos \theta + 18.93} \right) \quad (1)$$

$$\sum M_D = 0: (T \cos \beta)(32 \sin \theta) - (T \sin \beta)(32 \cos \theta) - (98,100)(45 \cos \theta) = 0$$

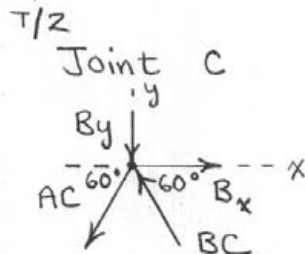
$$\text{So } T = \frac{137,953 \cos \theta}{\cos \beta \sin \theta - \sin \beta \cos \theta} \quad (\text{in N}) \quad (2)$$

Pulley at C:



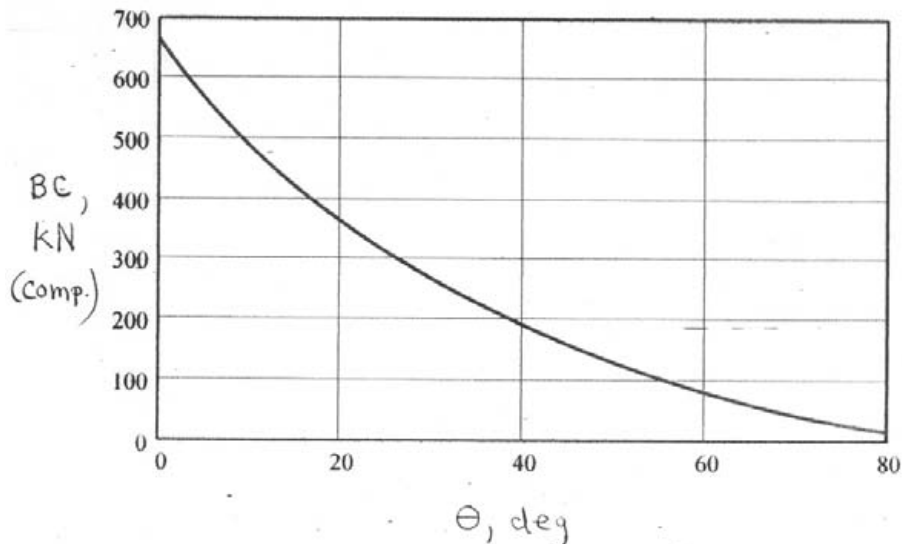
Equilibrium yields

$$\begin{cases} B_x = T \cos \beta - \frac{T}{2} \sin 30^\circ \\ B_y = -T \sin \beta + \frac{T}{2} \cos 30^\circ \end{cases} \quad (3)$$



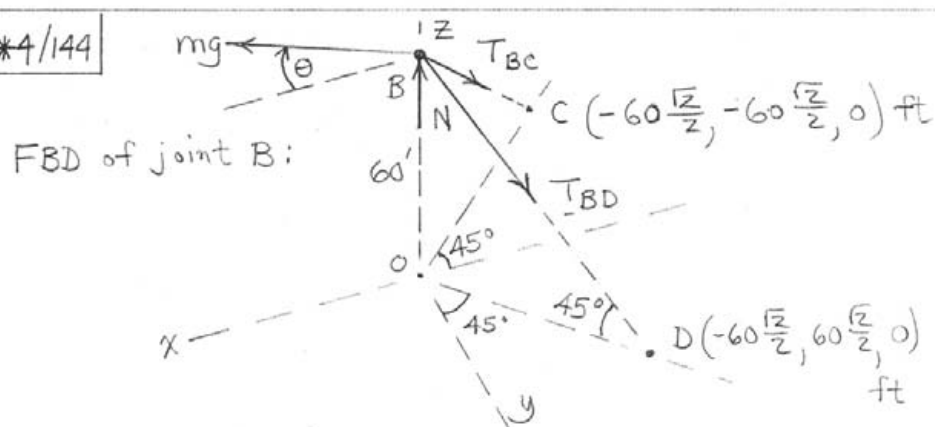
Equilibrium yields

$$\begin{aligned} BC &= B_x + 0.5774 B_y \quad (4) \\ (AC &= B_x - 0.5774 B_y) \end{aligned}$$



$$BC @ \theta = 40^\circ = 190.5 \text{ kN}$$

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$$\underline{T}_{BC} = T_{BC} \left[\frac{-60 \frac{\sqrt{2}}{2} \underline{i} - 60 \frac{\sqrt{2}}{2} \underline{j} - 60 \underline{k}}{\sqrt{2(60)^2 \frac{2}{4} + 60^2}} \right]$$

$$= T_{BC} [-0.5 \underline{i} - 0.5 \underline{j} - 0.707 \underline{k}]$$

$$\underline{T}_{BD} = T_{BD} [-0.5 \underline{i} + 0.5 \underline{j} - 0.707 \underline{k}]$$

$$\Sigma F_x = 0: -0.5 T_{BC} - 0.5 T_{BD} + mg \cos \theta = 0 \quad (1)$$

$$\Sigma F_y = 0: -0.5 T_{BC} + 0.5 T_{BD} - mg \sin \theta = 0 \quad (2)$$

Solve simultaneously:

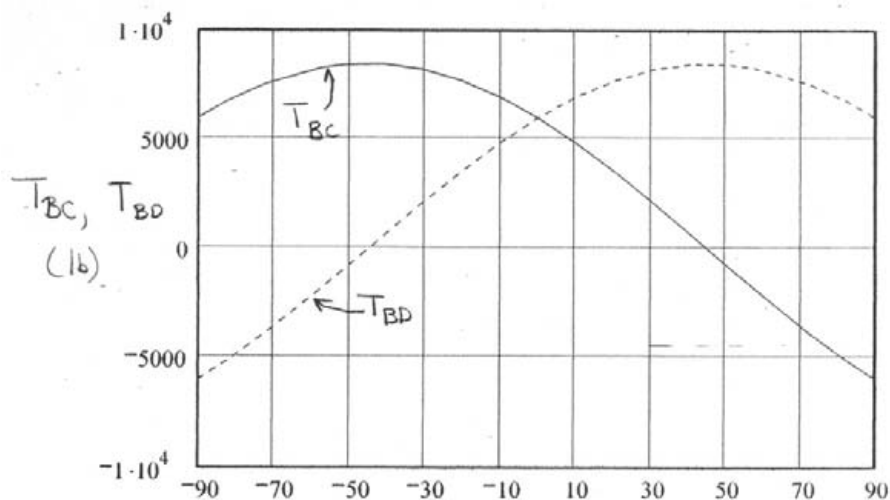
$$T_{BC} = mg (\cos \theta - \sin \theta)$$

$$T_{BD} = mg (\cos \theta + \sin \theta)$$

Maximum values

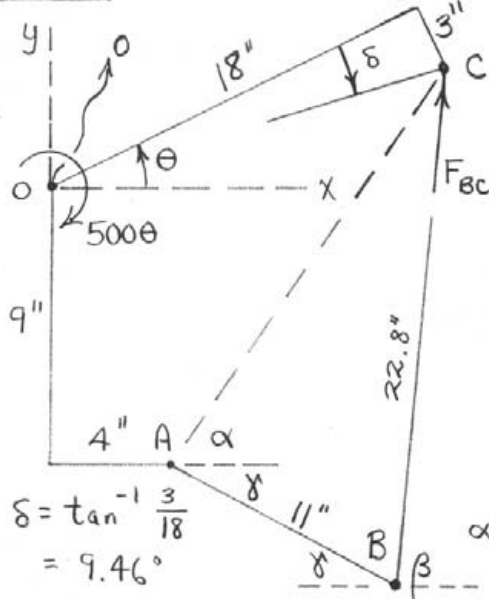
$$T_{BC \max} = 8490 \text{ lb @ } \theta = -45^\circ$$

$$T_{BD \max} = 8490 \text{ lb @ } \theta = 45^\circ$$



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$$\overline{OC} = \sqrt{18^2 + 3^2}$$



$$\delta = \tan^{-1} \frac{3}{18}$$

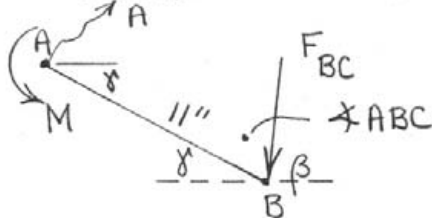
$$= 9.46^\circ$$

$$\gamma = \angle BAC - \alpha; \quad \beta = 180^\circ - \gamma - \angle ABC$$

From above FBD of door:

$$\sum M_O = 0: -500\theta + F_{BC} (\overline{OC} \sin(\beta - \theta + \delta)) = 0$$

Then from FBD of AB:



$$\sum M_A = 0: M - F_{BC} (11 \sin \angle ABC) = 0$$

Solve the above equations (in order) with θ

varied from 0° to 90° to obtain the following plots for M and F_{BC} .

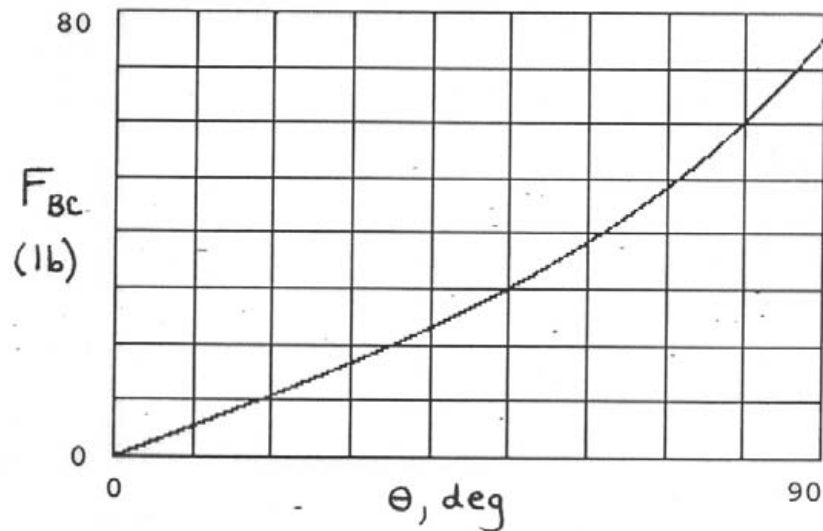
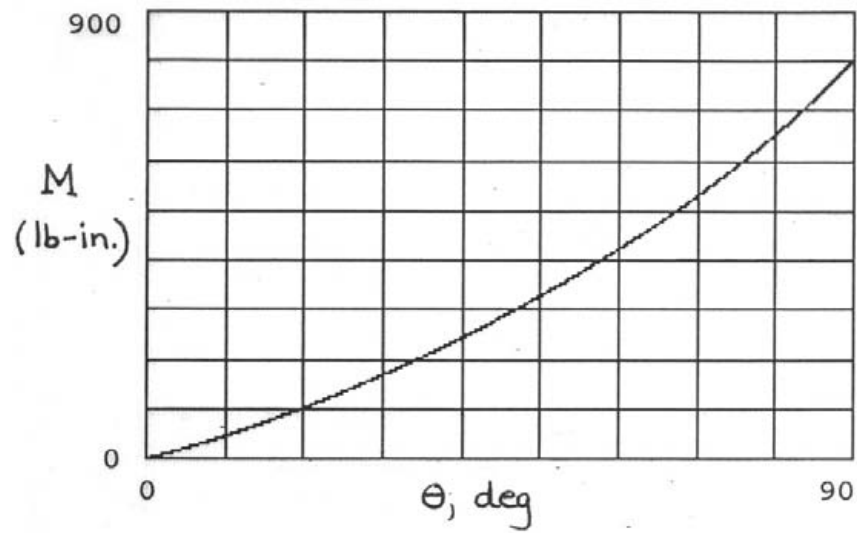
$$\overline{AB} = 11; \quad \overline{BC} = \sqrt{14^2 + 17^2}$$

$$\overline{AC} = \left\{ [18 \cos \theta + 3 \sin \theta - 4]^2 + [18 \sin \theta - 3 \cos \theta + 9]^2 \right\}^{1/2}$$

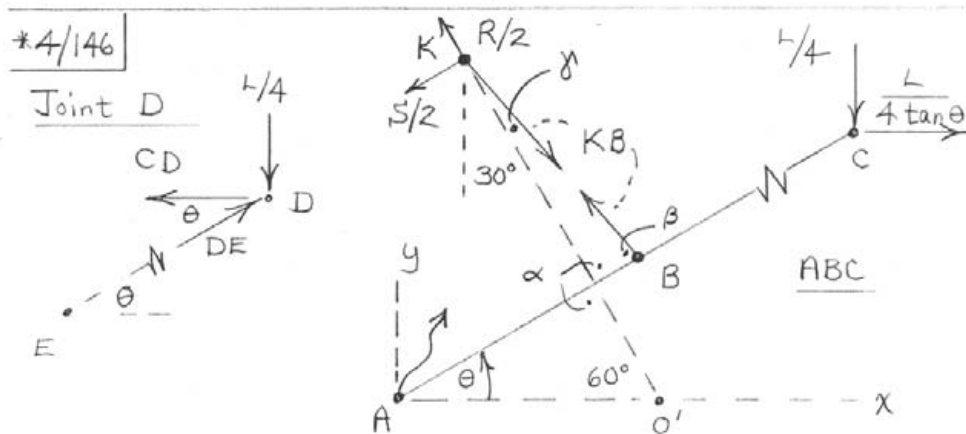
$$\angle BAC = \cos^{-1} \left[\frac{\overline{AB}^2 + \overline{AC}^2 - \overline{BC}^2}{2 \overline{AB} \overline{AC}} \right]$$

$$\angle ABC = \cos^{-1} \left[\frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2 \overline{AB} \overline{BC}} \right]$$

$$\alpha = \tan^{-1} \left[\frac{18 \sin \theta - 3 \cos \theta + 9}{18 \cos \theta + 3 \sin \theta - 4} \right]$$



At $\theta = 45^\circ$, $M = 285$ lb-in.



From FBD of joint D,

$$\begin{cases} \sum F_y = 0 : DE \sin \theta - \frac{L}{4} = 0, & DE = \frac{L}{4 \sin \theta} C \\ \sum F_x = 0 : \frac{L}{4 \sin \theta} \cos \theta - CD = 0, & CD = \frac{L}{4 \tan \theta} T \end{cases}$$

From above right diagrams:

$$\begin{aligned} \underline{r}_K &= \underline{r}_{B/A} + \underline{r}_{K/B} = \overline{AB} (\cos \theta \underline{i} + \sin \theta \underline{j}) \\ &\quad + \overline{BK} (-\sin (\gamma + 30^\circ) \underline{i} + \cos (\gamma + 30^\circ) \underline{j}) \end{aligned}$$

Also;

$$\underline{r}_K = \underline{r}_{O'/A} + \underline{r}_{K/O'} = \overline{AO'} \underline{i} + \overline{O'K} (-\cos 60^\circ \underline{i} + \sin 60^\circ \underline{j})$$

Equate the two \underline{r}_K expressions, then equate corresponding \underline{i} - and \underline{j} -coefficients, and then numerically solve for γ and $\overline{O'K}$ as θ varies from 5° to 90° .

$$\text{Then } \alpha = 180^\circ - 60^\circ - \theta$$

$$\text{and } \beta = 180^\circ - \gamma - \alpha$$

From FBD of ABC:

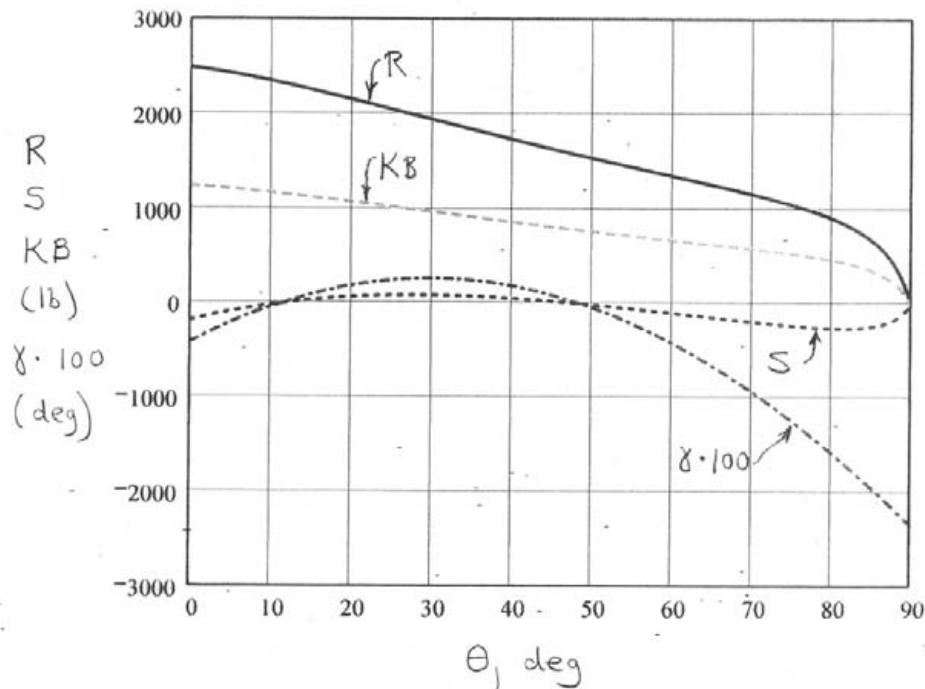
$$\sum M_A = 0: -\frac{L}{4}(\overline{AC} \cos \theta) - \frac{L}{4 \tan \theta}(\overline{AC} \sin \theta) + KB \cos(\beta - \theta)(\overline{AB} \sin \theta) + KB \sin(\beta - \theta)(\overline{AB} \cos \theta) = 0$$

Solve this for force KB. From the final FBD,

$$\frac{R}{2} - KB \cos \gamma = 0 \Rightarrow \text{Gives } R$$

$$\frac{S}{2} - KB \sin \gamma = 0 \Rightarrow \text{Gives } S$$

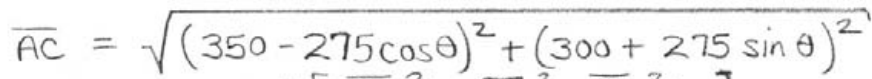
For $L = 800$ lb, $\overline{AO'} = 8.75$ ", $\overline{AB} = 8$ ", $\overline{BK} = 9$ ", and $\overline{AC} = 22.5$ ", the following plots are constructed:



$$R_{\max} = 2490 \text{ lb @ } \theta = 0$$

$$|S|_{\max} = 259 \text{ lb @ } \theta = 81.6^\circ$$

(Dim. in mm)

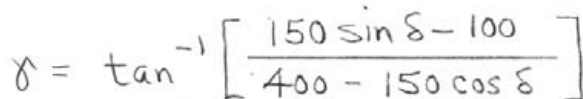


$$\alpha = \tan^{-1} \left[\frac{300 + 275 \sin \theta}{350 - 275 \cos \theta} \right]$$

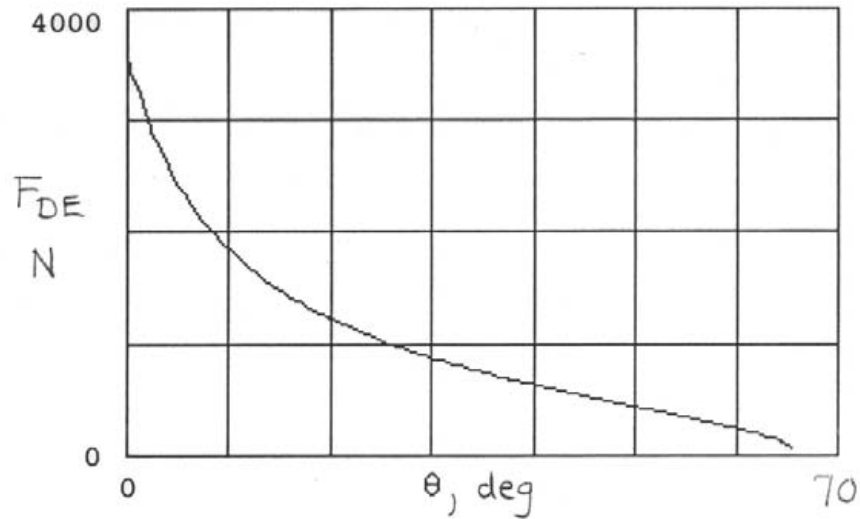
$$\delta = 180^\circ - \alpha - \angle ACB$$

$mg = 30(9.81) \text{ N}$
 125
 275
 θ
 F_{AB}
 $(90^\circ - \alpha + \angle CAB - \theta) =$
 $[90^\circ - (\alpha + \theta - \angle CAB)]$
 $[(90^\circ - \alpha) + \angle CAB]$

$$F_{AB} = \frac{mg \overline{OG} \cos \theta}{\overline{OA} \sin (\alpha + \theta - \angle CAB)}$$



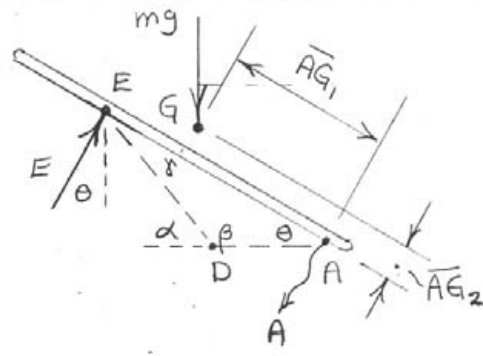
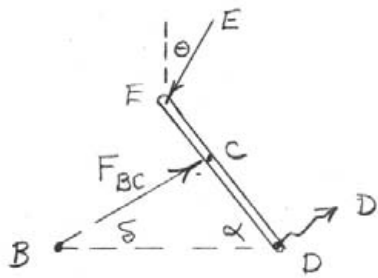
To find θ_{\max} , set the above expression for \overline{AC} to $\overline{AC} = 600 \text{ mm}$ and solve for $\theta_{\max} = 65.9^\circ$. Numerically evaluate the above expressions for $0 \leq \theta \leq \theta_{\max}$ to obtain the following plot:



$$(F_{DE})_{\max} = 3580 \text{ N @ } \theta = 0$$

$$(F_{DE})_{\min} = 0 \text{ @ } \theta_{\max} = 65.9^\circ \text{ (links AB and BC are collinear and serve as (an unstable!) prop for the door)}$$

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$$\frac{\sin \gamma}{AD} = \frac{\sin \theta}{DE}, \quad \gamma = \sin^{-1} \left(\frac{AD \sin \theta}{DE} \right)$$

$$\beta = 180 - (\theta + \gamma), \quad \alpha = 180 - \beta$$

$$AE = \sqrt{DE^2 + AD^2 - 2(DE)(AD) \cos \beta}$$

$$BC = \sqrt{BD^2 + CD^2 - 2(BD)(CD) \cos \alpha}$$

$$\frac{\sin \alpha}{BC} = \frac{\sin \delta}{CD}, \quad \delta = \sin^{-1} \left(\frac{CD \sin \alpha}{BC} \right)$$

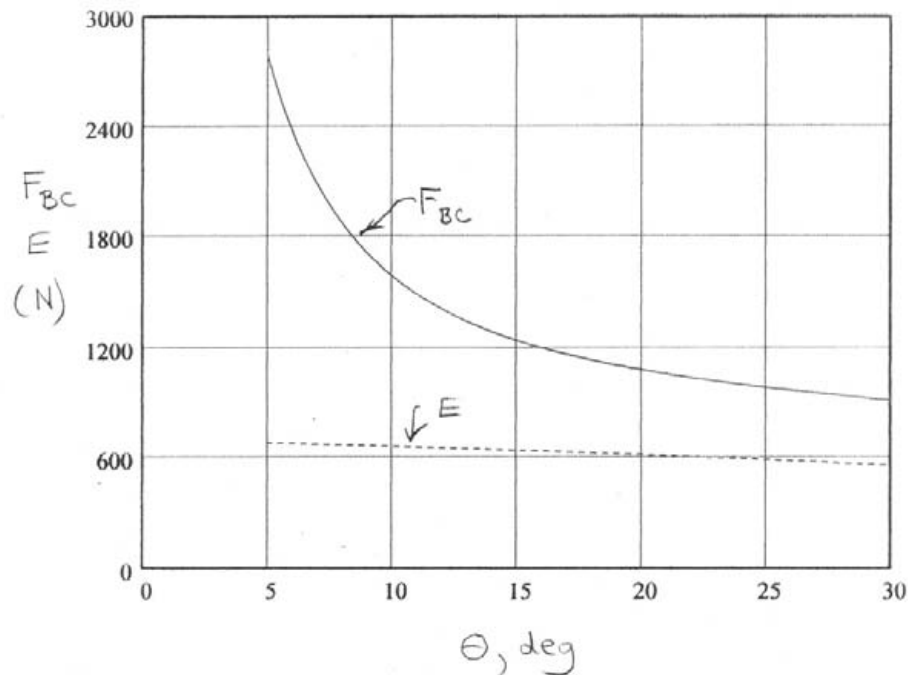
$$\text{Conveyor: } \sum M_A = 0: -E(AE) + mg(AG_1) \cos \theta - mg(AG_2) \sin \theta = 0 \quad (1)$$

$$DE: \sum M_D = 0:$$

$$E \cos \theta (\overline{DE} \cos \alpha) + E \sin \theta (\overline{DE} \sin \alpha)$$

$$- F_{BC} \sin \delta (\overline{BD}) = 0 \quad (2)$$

Solve Eqs. (1) & (2) for E and F_{BC} as functions of θ for the values $\overline{AD} = 1060 \text{ mm}$, $\overline{BD} = 1660 \text{ mm}$, $\overline{DE} = 1945 \text{ mm}$, $\overline{CD} = 1150 \text{ mm}$, $\overline{AG}_1 = 2130 \text{ mm}$, $\overline{AG}_2 = 500 \text{ mm}$, and $mg = 100(9.81) \text{ N}$ to obtain the following plot:



At $\theta = 5^\circ$, $(F_{BC})_{\max} = 2800 \text{ N}$

*4/149 $\theta = 0$: $C_y = \frac{pA}{2} = \frac{p\pi r^2}{2}$

$$= \frac{35(10^6)\pi(0.050)^2}{2}$$

$$= 137400 \text{ N}$$

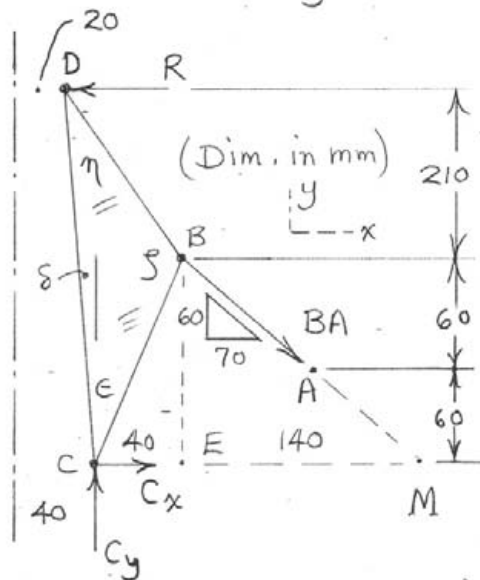
$$\delta = \tan^{-1} \frac{20}{330} = 3.47^\circ$$

$$\overline{CD} = \sqrt{20^2 + 330^2}$$

$$= 331 \text{ mm}$$

$$\text{Similarly, } \overline{BD} = 218 \text{ mm}$$

$$\overline{BC} = 126.5 \text{ mm}$$

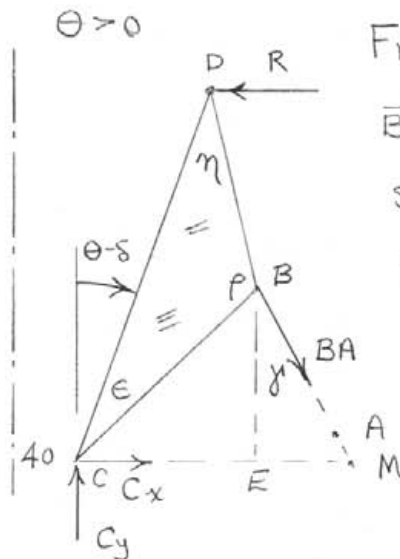


$$\sum M_M = 0: R(330) - 137400(180) = 0, \underline{R = 75000 \text{ N}}$$

$$\sum F_y = 0: -BA \sin(\tan^{-1} \frac{6}{7}) + 137400 = 0, \underline{BA = 211000 \text{ N}}$$

$$\sum F_x = 0: -75000 + 211000 \cos(\tan^{-1} \frac{6}{7}) + C_x = 0$$

$$\underline{C_x = -85400 \text{ N}}$$



Find ϵ , ρ , and η :

$$\overline{BD}^2 = \overline{CD}^2 + \overline{BC}^2 - 2\overline{CD}\overline{BC}\cos\epsilon$$

Solve for ϵ , then use

$$\frac{\sin \rho}{\overline{CD}} = \frac{\sin \epsilon}{\overline{BD}} \quad \text{for } \rho$$

$$\frac{\sin \rho}{\overline{CD}} = \frac{\sin \eta}{\overline{BC}} \quad \text{for } \eta$$

$$\overline{CE} = \overline{BC} \sin(\theta - \delta + \epsilon)$$

$$\overline{BE} = \overline{BC} \cos(\theta - \delta + \epsilon)$$

$$110 = \overline{AB} \sin \gamma + \overline{BC} \sin(\theta - \delta + \epsilon) \Rightarrow \text{Solve for } \gamma$$

$$\text{Then } \overline{EM} = \overline{BE} \tan \gamma \quad \text{and} \quad \overline{CM} = \overline{CE} + \overline{EM}$$

$$\sum M_M = 0: R(\overline{CD} \cos(\theta - \delta)) - C_y(\overline{CM}) = 0$$

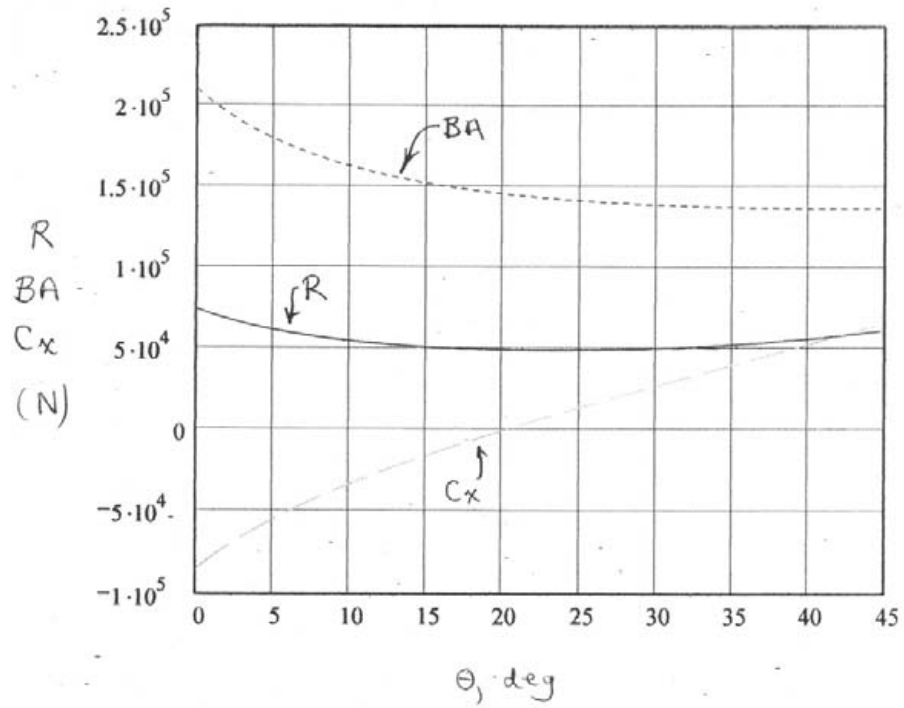
$$R = \frac{C_y \overline{CM}}{\overline{CD} \cos(\theta - \delta)}$$

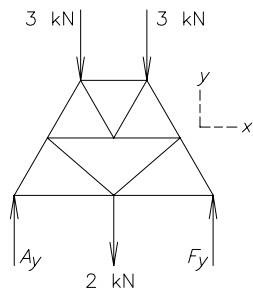
$$\sum F_y = 0: C_y - BA \cos \gamma = 0, \quad \underline{BA = \frac{C_y}{\cos \gamma}}$$

$$\sum F_x = 0: -R + BA \sin \gamma + C_x = 0, \quad \underline{C_x = R - BA \sin \gamma}$$

Solve all of the above in sequential order to obtain the plots below. Note that

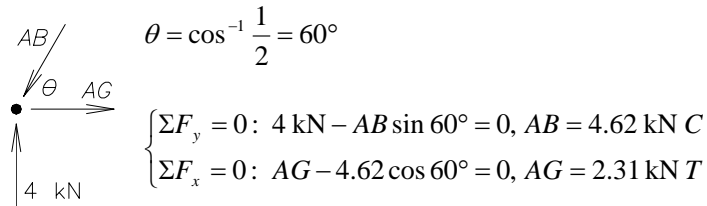
$$\underline{R_{\min} = 49400 \text{ N} @ \theta = 23.2^\circ}$$



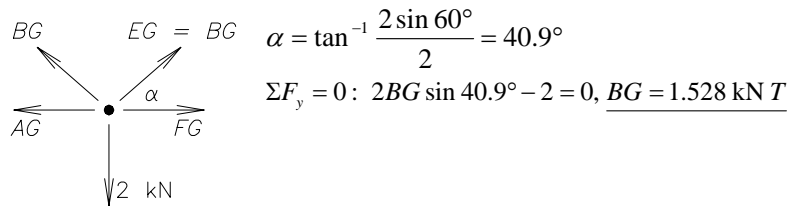


By symmetry, $A_y = F_y = 4 \text{ kN}$

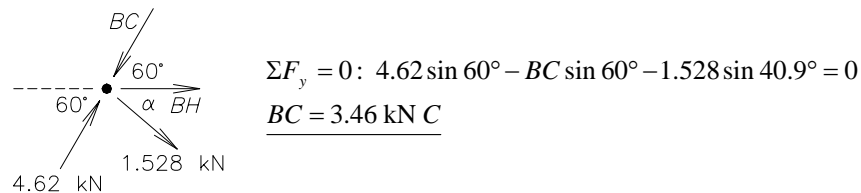
Joint A:



Joint G:



Joint B:

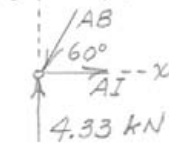


4/16 Truss as a whole

$$\sum M_F = 0; 2(a + \frac{a}{2}) + 4(2a + \frac{a}{2}) - A(3a) = 0$$

$$A = 13/3 = 4.33 \text{ kN}$$

Joint A



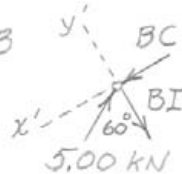
$$\sum F_y = 0; AB \sin 60^\circ - 4.33 = 0$$

$$AB = 5.00 \text{ kN C}$$

$$\sum F_x = 0; AI - 5 \cos 60^\circ = 0$$

$$AI = 2.50 \text{ kN T}$$

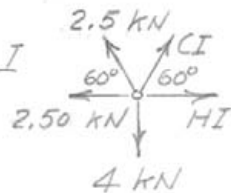
Joint B



$$\sum F_{y'} = 0; 5.00 \cos 60^\circ - BI = 0$$

$$BI = 2.50 \text{ kN T}$$

Joint I



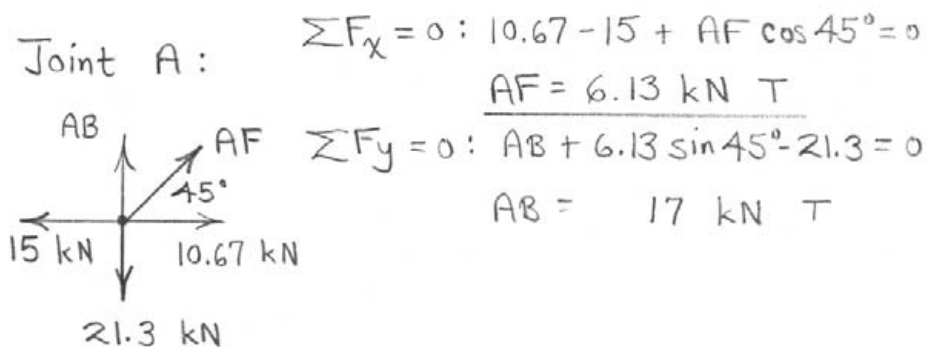
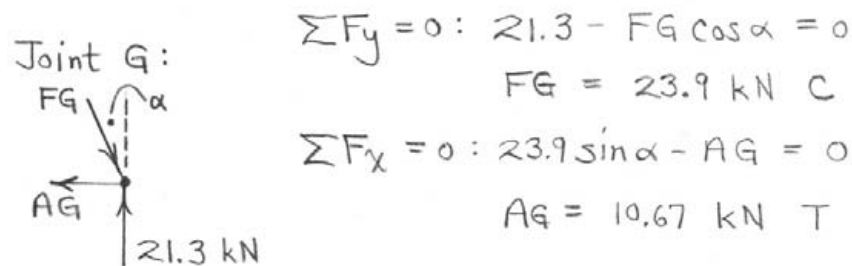
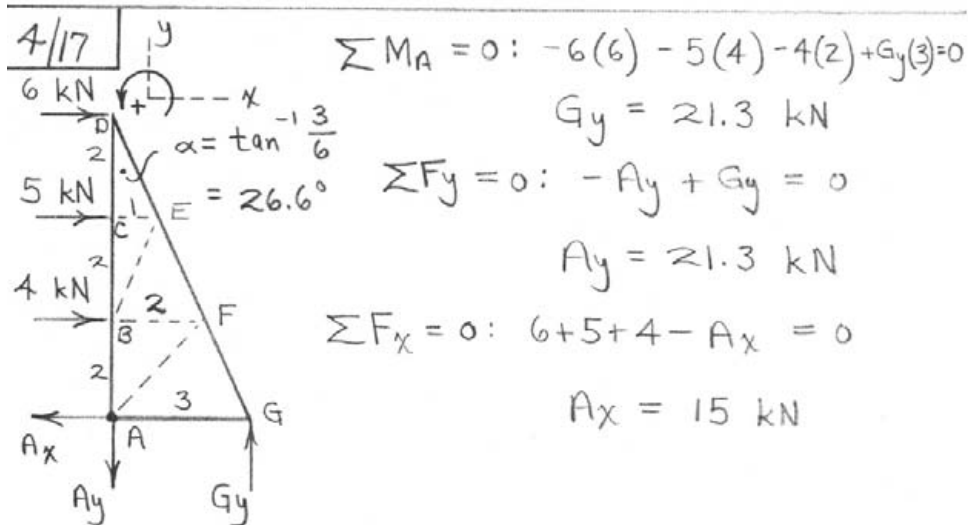
$$\sum F_y = 0; (CI + 2.5) \sin 60^\circ - 4 = 0$$

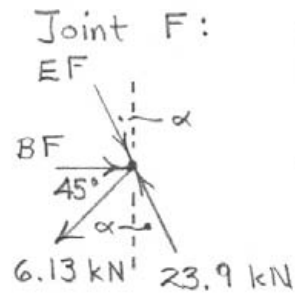
$$CI = 2.12 \text{ kN T}$$

$$\sum F_x = 0; HI + 2.12 \cos 60^\circ$$

$$- 2.50 - 2.5 \cos 60^\circ = 0$$

$$HI = 2.69 \text{ kN T}$$

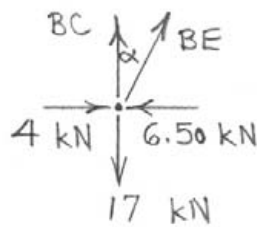




$$\sum F_y = 0: -EF \cos \alpha + 23.9 \cos \alpha - 6.13 \sin 45^\circ = 0, \quad EF = 19.01 \text{ kN C}$$

$$\sum F_x = 0: 19.01 \sin \alpha + BF - 6.13 \cos 45^\circ - 23.9 \sin \alpha = 0, \quad \underline{BF = 6.50 \text{ kN C}}$$

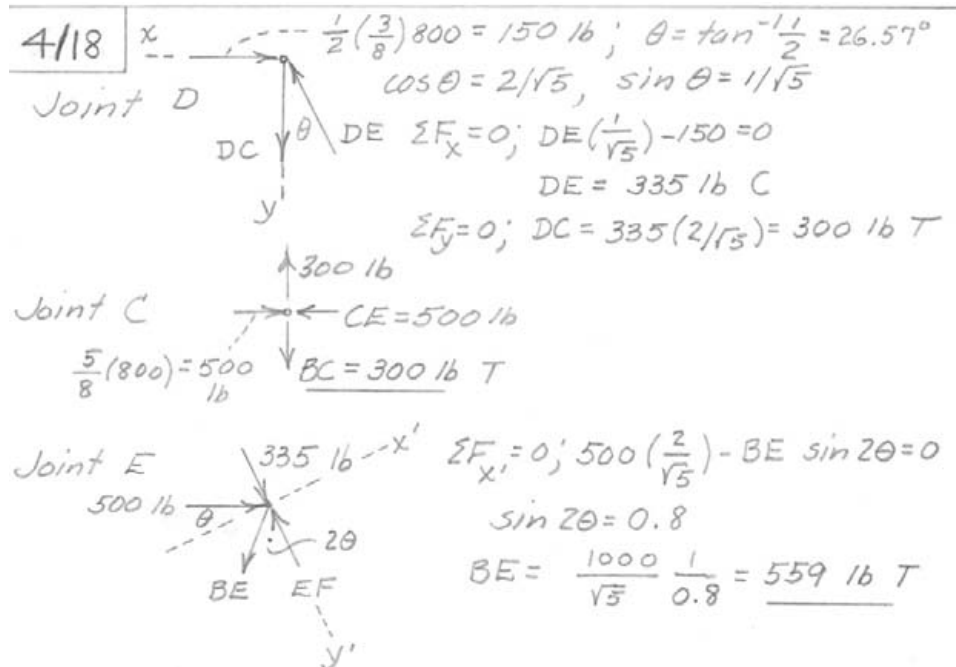
Joint B:



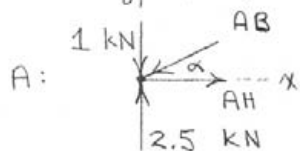
$$\sum F_x = 0: BE \sin \alpha + 4 - 6.50 = 0$$

$$\underline{BE = 5.59 \text{ kN T}}$$

From inspection of joint C, CE = 5 kN C



4/19 By symmetry, $A = E = 2.5 \text{ kN}$; $\alpha = \tan^{-1}(\frac{2}{4}) = 26.6^\circ$

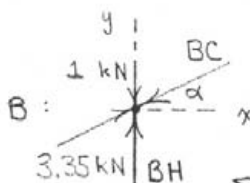


$$\sum F_y = 0: 2.5 - 1 - AB \sin \alpha = 0$$

$$AB = 3.35 \text{ kN C}$$

$$\sum F_x = 0: -3.35 \cos \alpha + AH = 0$$

$$AH = 3 \text{ kN T}$$

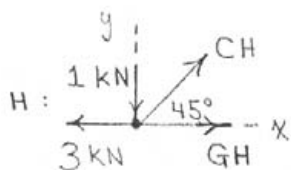


$$\sum F_x = 0: 3.35 \cos \alpha - BC \cos \alpha = 0$$

$$BC = 3.35 \text{ kN C}$$

$$\sum F_y = 0: -1 + (3.35 - 3.35) \sin \alpha + BH = 0$$

$$BH = 1 \text{ kN C}$$



$$\sum F_y = 0: -1 + CH \sin 45^\circ = 0$$

$$CH = 1.414 \text{ kN T}$$

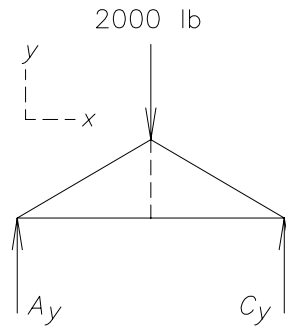
$$\sum F_x = 0: -3 + 1.41 \cos 45^\circ + GH = 0$$

$$GH = 2 \text{ kN T}$$

By inspection of joint G and $\sum F_y = 0$, $CG = 0$.

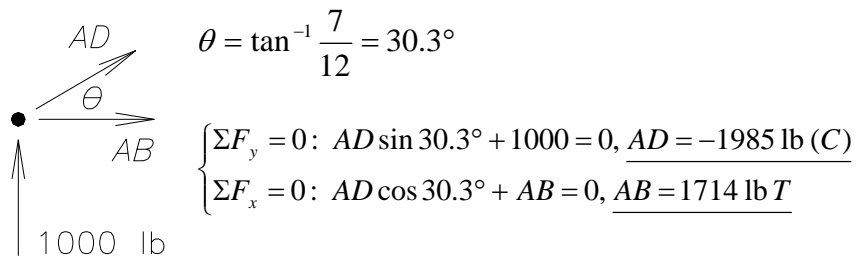
By symmetry,

$$\left\{ \begin{array}{l} DE = AB = 3.35 \text{ kN C} \\ CD = BC = 3.35 \text{ kN C} \\ EF = AH = 3 \text{ kN T} \\ DF = BH = 1 \text{ kN C} \\ CF = CH = 1.414 \text{ kN T} \\ FG = GH = 2 \text{ kN T} \end{array} \right.$$



By symmetry, $A_y = C_y = 1000 \text{ lb}$

Joint A:

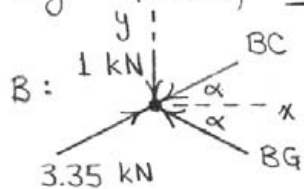


By symmetry: $CD = AC = 1985 \text{ lb C}$
 $BC = AB = 1714 \text{ lb T}$

By inspection of joint B, BD is a zero-force member.

4/20 By symmetry, $A = E = 2.5 \text{ kN}$; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6^\circ$

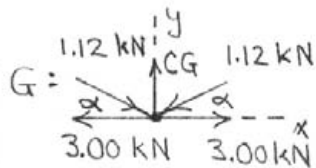
Joint A analysis same as Prob. 4/19: $\begin{cases} AB = 3.35 \text{ kN C} \\ AH = 3.00 \text{ kN T} \end{cases}$
 By inspection, $BH = 0$ and $GH = AH$.



$$\sum F_y = 0: -1 + 3.35 \sin \alpha + BG \sin \alpha - BC \sin \alpha = 0$$

$$\sum F_x = 0: 3.35 \cos \alpha - BC \cos \alpha - BG \cos \alpha = 0$$

$\Rightarrow BC = 2.24 \text{ kN C}, \quad BG = 1.118 \text{ kN C}$



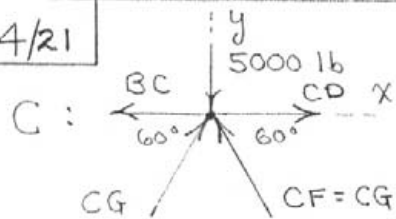
$$\sum F_y = 0: CG - 2(1.12) \sin \alpha = 0$$

$$CG = 1.00 \text{ kN T}$$

By symmetry,

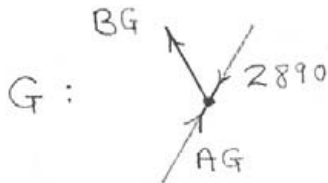
$$\left\{ \begin{array}{l} DE = AB = 3.35 \text{ kN C} \\ CD = BC = 2.24 \text{ kN C} \\ EF = AH = 3.00 \text{ kN T} \\ DF = BH = 0 \\ FG = GH = 3.00 \text{ kN T} \\ DG = BG = 1.118 \text{ kN C} \end{array} \right.$$

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Arguing symmetry,

$$\sum F_y = 0: 2CG \sin 60^\circ - 5000 = 0, \quad \underline{CG = 2890 \text{ lb C}}$$

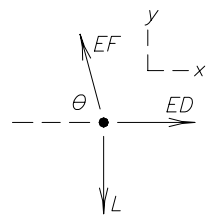


By inspection,

$$AG = \underline{2890 \text{ lb C}}$$

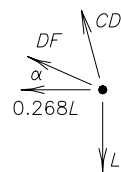
$$BG = \underline{0}$$

From joint B : $\underline{AB = 0, BC = 0.}$
 (Right truss symmetric to left one.)

Joint *E*:

$$\theta = \frac{180^\circ - 30^\circ}{2} = 75^\circ$$

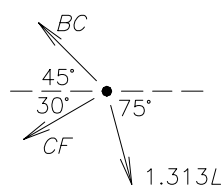
$$\begin{cases} \Sigma F_y = 0: EF \sin 75^\circ - L = 0, EF = 1.035L \text{ T} \\ \Sigma F_x = 0: -1.035L \cos 75^\circ + ED = 0, ED = 0.268L \text{ T} \end{cases}$$

Joint *D*:

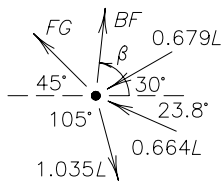
$$\overline{DF}^2 = R^2 + 4R^2 - 2(R)(2R) \cos 30^\circ, \overline{DF} = 1.239R$$

$$\frac{\sin 30^\circ}{1.239R} = \frac{\sin \alpha}{R}, \alpha = 23.8^\circ$$

$$\begin{cases} \Sigma F_x = 0: -0.268L - DF \cos 23.8^\circ - CD \cos 75^\circ = 0 \\ \Sigma F_y = 0: -L + DF \sin 23.8^\circ + CD \sin 75^\circ = 0 \end{cases}$$

Solve simultaneously to obtain: $CD = 1.313L \text{ T}$, $DF = 0.664L \text{ C}$ Joint *C*:

$$\begin{cases} \Sigma F_x = 0: -BC \cos 45^\circ - CF \cos 30^\circ + 1.313L \cos 75^\circ = 0 \\ \Sigma F_y = 0: BC \sin 45^\circ - CF \sin 30^\circ - 1.313L \sin 75^\circ = 0 \end{cases}$$

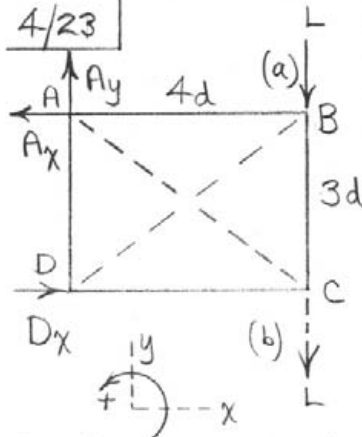
Solve simultaneously to obtain: $CF = -0.674L \text{ (C)}$ Joint *F*:

$$\beta = 30^\circ + 23.8^\circ + 30^\circ = 83.8^\circ$$

$$\begin{cases} \Sigma F_x = 0: -FG \cos 45^\circ + BF \cos \beta + EF \cos 75^\circ + CF \cos 30^\circ + DF \cos \alpha = 0 \\ \Sigma F_y = 0: FG \sin 45^\circ + BF \sin \beta - EF \sin 75^\circ + CF \sin 30^\circ - DF \sin \alpha = 0 \end{cases}$$

Solve simultaneously to obtain: $BF = 1.814L \text{ T}$

4/23



$$\sum F_y = 0: A_y - L = 0, A_y = L$$

$$\sum M_A = 0: D_x(3d) - L(4d) = 0$$

$$D_x = \frac{4L}{3}$$

$$\sum F_x = 0: A_x = D_x = \frac{4L}{3}$$

(Reactions hold for (a) & (b).)

(a) Assume that BD goes slack. From an inspection of joint B, $\underline{AB=0}$ and $\underline{BC=L\ C.}$ Similarly, from joint D, $\underline{AD=0}$ and $\underline{CD=\frac{4L}{3}\ C.}$

Joint A:

$$\begin{cases} \sum F_y = 0: L - \frac{3}{5}AC = 0, AC = \frac{5L}{3}\ T \\ \sum F_x = 0: -\frac{4L}{3} + \frac{5L}{3} \cdot \frac{4}{5} = 0 \checkmark \end{cases}$$

Because AC is in tension, assumption is valid.

(b) Assume that BD goes slack. From joint B, $\underline{AB=BC=0.}$ From joint D, $\underline{AD=0}$ & $\underline{CD=\frac{4L}{3}\ C.}$

Joint C:

$$\begin{cases} \sum F_y = 0: AC(\frac{3}{5}) - L = 0, AC = \frac{5L}{3}\ T \\ \sum F_x = 0: \frac{4L}{3} - \frac{5L}{3}(\frac{4}{5}) = 0 \checkmark \end{cases}$$

4/24 $m = \text{no. of two-force members}$
 $j = \text{no. of joints}$

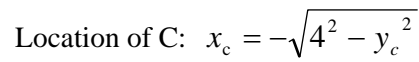
(a) $[m+3=13] > [2j=12]$ so redundant members.

Remove one member connecting B, C, D, and E.

(b) $[m+3=12] = [2j=12]$ so sufficient no. of members, but redundancy in external supports. Place A or F on roller.

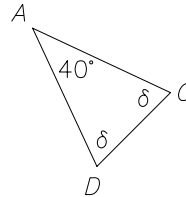
(c) $[m+3=9] > [2j=8]$ so redundant members. Supports are also redundant. Remove AE or BE. Supports are then OK.

(d) $[m+3=12] = [2j=12]$ so sufficient no. of members, but redundancy in external supports. Place B on roller or remove member CD.



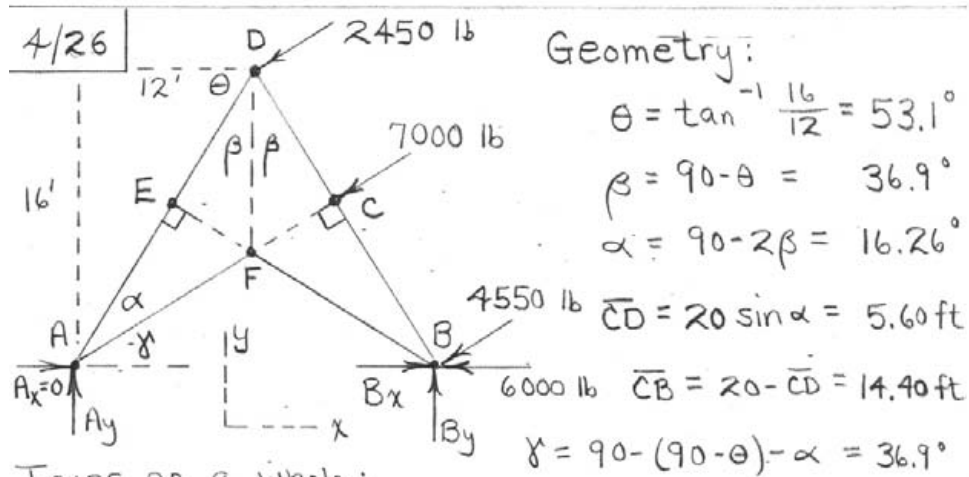
$$x_c = -(4 - AC \cos 25^\circ), y_c = 4 - AC \sin 25^\circ$$

$$\begin{cases} \Sigma F_y = 0: CB \sin 18.04^\circ - 500 = 0, CB = 1615 \text{ lb } C \\ \Sigma F_x = 0: 1615 \cos 18.04^\circ - AB = 0, AB = 1535 \text{ lb } T \end{cases}$$



$$\begin{cases} \Sigma F_y = 0: AC \sin 25^\circ - CD \sin 45^\circ - 1615 \sin 18.04^\circ = 0 \\ \Sigma F_x = 0: -AC \cos 25^\circ - CD \cos 45^\circ - 1615 \cos 18.04^\circ = 0 \end{cases}$$

By symmetry across a line from the origin to point A, $AD = AC = 779 \text{ lb } C$



Truss as a whole:

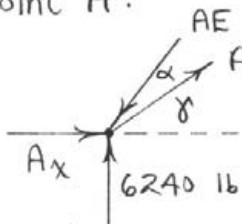
$$\sum M_B = 0: 7000(14.4) + 2450(20) - 24A_y = 0$$

$$A_y = 6240 \text{ lb}$$

$$\sum F_y = 0: B_y + 6240 - (2450 + 7000 + 4550) \sin 36.9^\circ = 0, \quad B_y = 2160 \text{ lb}$$

$$\sum F_x = 0: B_x - (2450 + 7000 + 4550) \cos 36.9^\circ - 6000 = 0, \quad B_x = 17,200 \text{ lb}$$

Joint A:



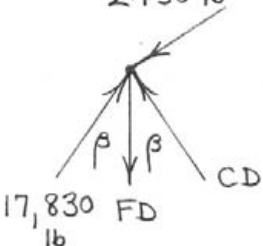
$$\Sigma F_x = 0 : -AE \cos 53.1^\circ + AF \cos 36.9^\circ = 0$$

$$\Sigma F_y = 0 : 6240 - AE \sin 53.1^\circ + AF \sin 36.9^\circ = 0$$

$$\Rightarrow AF = 13,380 \text{ lb T}, AE = 17,830 \text{ lb C}$$

From joint E, $ED = AE = 17,830 \text{ lb C}$

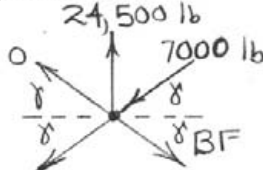
Joint D:



$$\Sigma F_x = 0 : 17,830 \sin 36.9^\circ - CD \sin 36.9^\circ - 2450 \cos 36.9^\circ = 0, CD = 14,570 \text{ lb C}$$

$$\Sigma F_y = 0 : (14,570 + 17,830) \cos 36.9^\circ - FD - 2450 \sin 36.9^\circ = 0, FD = 24,500 \text{ lb T}$$

Joint F:



$$\Sigma F_x = 0 : BF \cos 36.9^\circ - (13,380 + 7000) \cos 36.9^\circ = 0, BF = 20,400 \text{ lb T}$$

(Joint B checks.)

The maximum force occurs in member FD:

$$\underline{FD = 24,500 \text{ lb T}}$$

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$$\theta = \tan^{-1} \frac{1}{2} = 26.6^\circ$$

Joint A

$$\sum F_y = 0; AB \sin 26.6^\circ - 1.8 \cos 15^\circ = 0$$

$$AB = 3.89 \text{ kN C}$$

$$\sum F_x = 0; AE = 1.8 \sin 15^\circ + 3.89 \cos \theta$$

$$AE = 3.94 \text{ kN T}$$

Joint E gives $EB = 0$

Joint B

$$\sum F_y = 0 \text{ so } DB = 0$$

$$CB = 3.89 \text{ kN C}$$

without diagonals, FD would lengthen & CJ shorten, so FD is tension & CJ = 0

Joint H

$$\sum F_y = 0; HG \sin 26.6^\circ - 1.8 \cos 15^\circ = 0$$

$$HG = 3.89 \text{ kN C}$$

$$\sum F_x = 0; HI + 1.8 \sin 15^\circ - 3.89 \cos 26.6^\circ = 0$$

$$HI = 3.01 \text{ kN T}$$

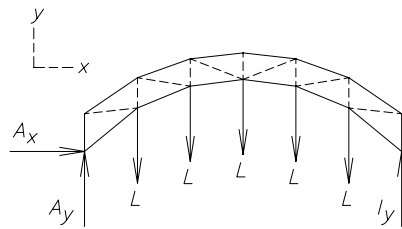
with $IG = GJ = JC = 0$, $JD = HI$

Joint D

$$\sum F_x = 0; FD \cos 45^\circ + 3.01 = 3.94$$

$$FD = 1.318 \text{ kN T}$$

$$\sum F_y = 0; CD = 1.318 \sin 45^\circ = 0.932 \text{ kN C}$$



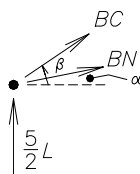
By symmetry, $A_y = I_y = 5/2L$, $A_x = 0$
By inspection of point A, $A_N = 0$

Coordinate origin is at the center of the two concentric arcs.

Location of N: $x^2 + y^2 = 16^2$
For $x_N = -8$ m: $y_N^2 = -8^2 + 16^2$
 $y_N = 13.86$ m N = (-8, 13.86) m

Location of A: $y_A^2 = -12^2 + 16^2$, $y_A = 10.58$ m, A = (-12, 10.58) m
Location of B: $y_B^2 = -12^2 + 18^2$, $y_B = 13.42$ m, B = (-12, 13.41) m
Location of C: $y_C^2 = -8^2 + 18^2$, $y_C = 16.12$ m, C = (-8, 16.12) m
Location of D: $y_D^2 = -4^2 + 18^2$, $y_D = 17.55$ m, C = (-4, 17.55) m
Location of M: $y_M^2 = -4^2 + 16^2$, $y_M = 15.49$ m, C = (-4, 15.49) m

Joint B:



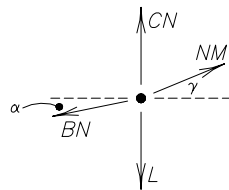
$$\alpha = \tan^{-1} \frac{13.86 - 13.42}{-8 + 12} = 6.28^\circ$$

$$\beta = \tan^{-1} \frac{16.12 - 13.42}{-8 + 12} = 34.1^\circ$$

$$\begin{cases} \Sigma F_x = 0: BC \cos 34.1^\circ + BN \cos 6.28^\circ = 0 \\ \Sigma F_y = 0: (5/2)L + BC \sin 34.1^\circ + BN \sin 6.28^\circ = 0 \end{cases}$$

Solve simultaneously to obtain: $BN = 4.44L$ T, $BC = -5.32L$ (C)

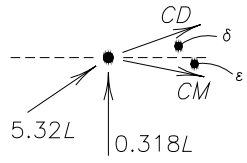
Joint N:



$$\gamma = \tan^{-1} \frac{15.49 - 13.86}{-4 + 8} = 22.2^\circ$$

$$\begin{cases} \Sigma F_x = 0: -4.44L \cos 6.28^\circ + NM \cos 22.2^\circ = 0, & NM = 4.76L \text{ T} \\ \Sigma F_y = 0: -4.44L \sin 6.28^\circ + 4.76 \sin 22.2^\circ + CN - L = 0, & CN = 0.318L \text{ C} \end{cases}$$

Joint C:



$$\delta = \tan^{-1} \frac{17.55 - 16.12}{-4 + 8} = 19.61^\circ$$

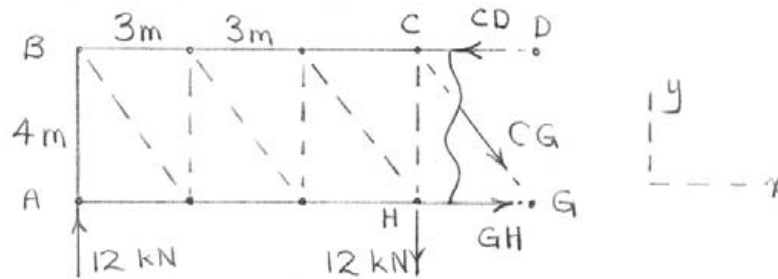
$$\varepsilon = \tan^{-1} \frac{16.12 - 15.49}{8 - 4} = 8.99^\circ$$

$$\begin{cases} \Sigma F_x = 0: 5.32L \cos 34.1^\circ + CD \cos 19.61^\circ + CM \cos 8.99^\circ = 0 \\ \Sigma F_y = 0: 5.32L \sin 34.1^\circ + 0.318L + CD \sin 19.61^\circ - CM \sin 8.99^\circ = 0 \end{cases}$$

Solve simultaneously to obtain: $CM = 3.41L \text{ T}$

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From truss as a whole, $A_y = F_y = 12 \text{ kN}$ and $A_x = 0$. Left section:

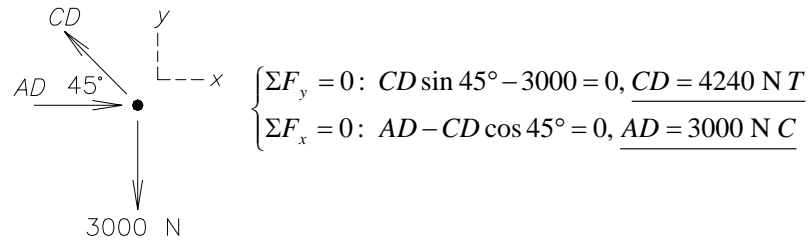


By inspection of $\sum F_y = 0$, $CG = 0$.

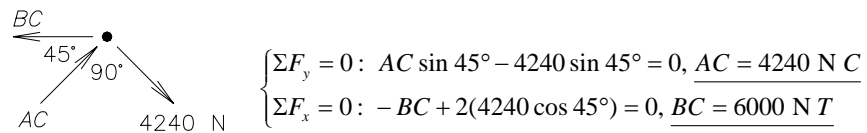
$\sum M_C = 0: GH(4) - 12(9) = 0$, $GH = 27 \text{ kN T}$

We can begin at joint D without finding the external reactions.

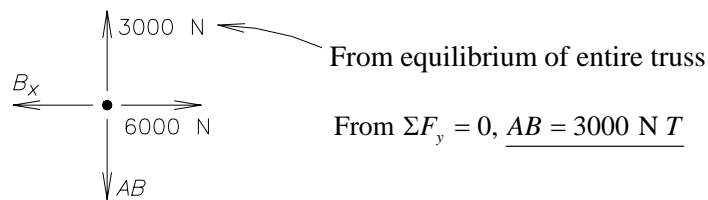
Joint D:

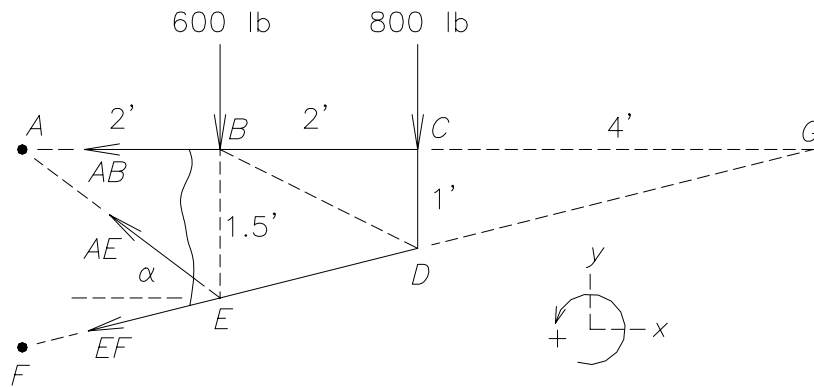


Joint C:



Joint B:

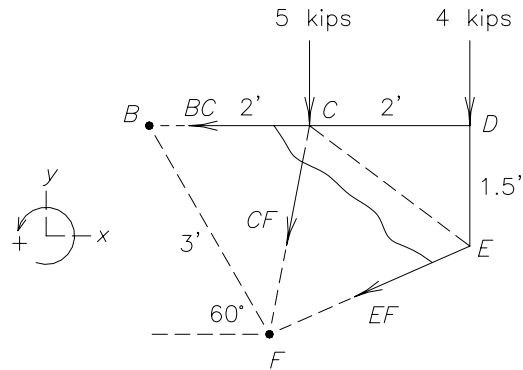




$$\alpha = \tan^{-1} \frac{1.5}{2} = 36.9^\circ$$

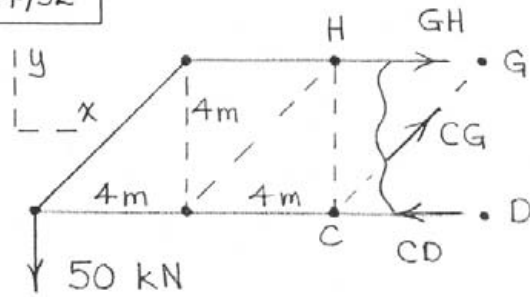
$$\Sigma M_G = 0: 800(4) + 600(6) - AE \cos 36.9^\circ(1.5) - AE \sin 36.9^\circ(6) = 0$$

$$\underline{AE = 1417 \text{ lb } T}$$



$$\Sigma M_F = 0: BC(3 \sin 60^\circ) - 5(0.5) - 4(2.5) = 0, \underline{BC = 4.81 \text{ kips } T}$$

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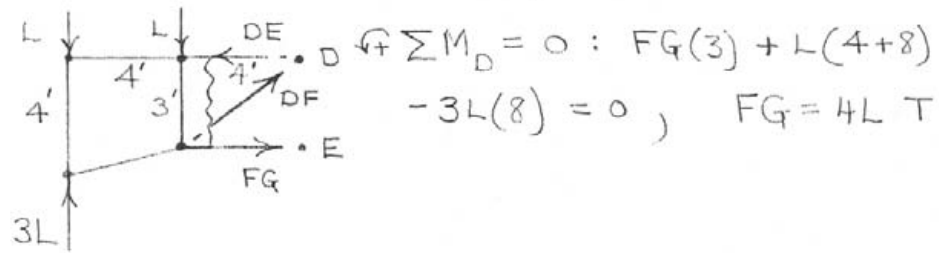


$$\sum F_y = 0 : CG \sin 45^\circ - 50 = 0, \quad \underline{CG = 70.7 \text{ kN T}}$$

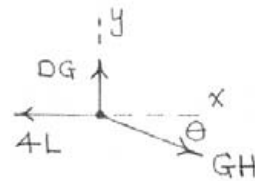
$$\sum M_C = 0 : GH(4) - 50(8) = 0, \quad \underline{GH = 100 \text{ kN T}}$$

All members except EF are statically determinate,
so above solution is unaffected by the
redundant support.

4/33 From entire truss, $A = B = 3L$.



Joint G:



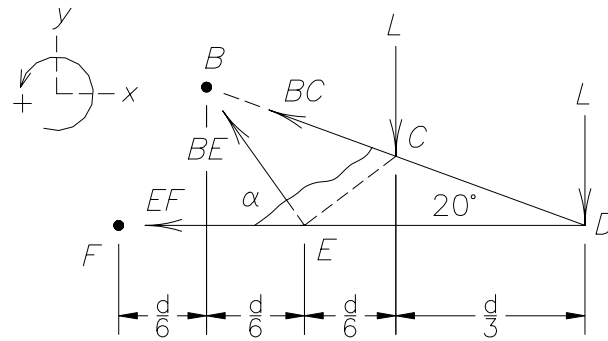
$$\theta = \tan^{-1}\left(\frac{1}{4}\right) = 14.04^\circ$$

$$\sum F_x = 0 : -4L + GH \cos \theta = 0$$

$$GH = 4.12L \text{ T}$$

$$\sum F_y = 0 : DG - 4.12 \sin \theta = 0$$

$$\underline{DG = L \text{ T}}$$

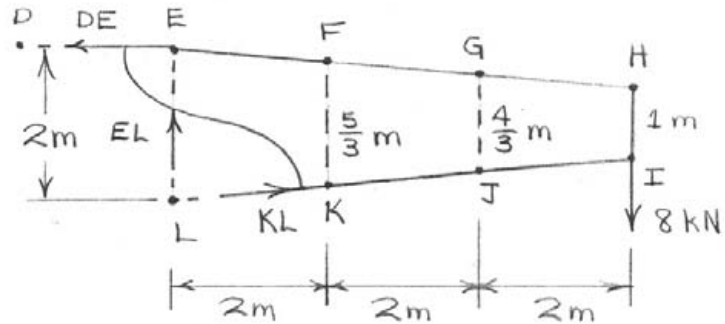


$$\alpha = \tan^{-1} \frac{\frac{4}{6}d \tan 20^\circ}{\frac{d}{6}} = 55.5^\circ$$

$$\Sigma M_D = 0: L\left(\frac{d}{3}\right) - BE\left(\frac{d}{2}\right)(\sin 55.5^\circ) = 0$$

$$\underline{BE = 0.809LT}$$

4/35



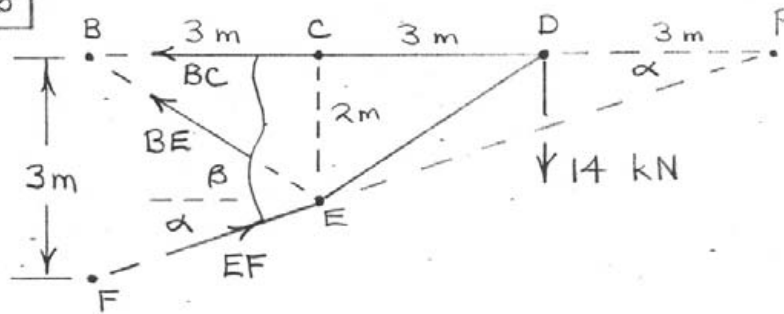
$$\sum M_L = 0: DE(2) - 8(6) = 0, \quad \underline{DE = 24 \text{ kN T}}$$

Joint D:

$$\sum F_x = 0: 24 - DL \cos 45^\circ = 0$$

$$\underline{DL = 33.9 \text{ kN C}}$$

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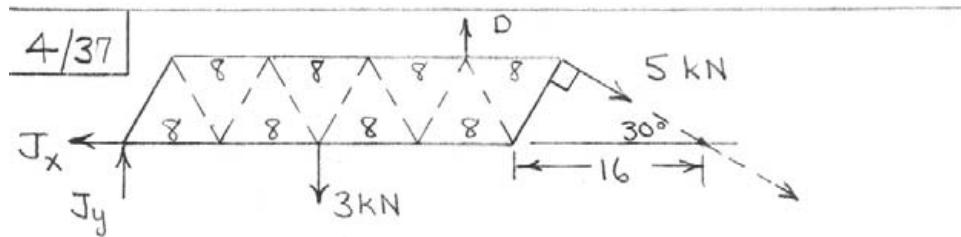


$$\alpha = \tan^{-1}\left(\frac{2}{6}\right) = 18.43^\circ, \quad \beta = \tan^{-1}\frac{2}{3} = 33.7^\circ$$

$$\curvearrowright \sum M_E = 0: BC(2) - 14(3) = 0, \quad \underline{BC = 21 \text{ kN T}}$$

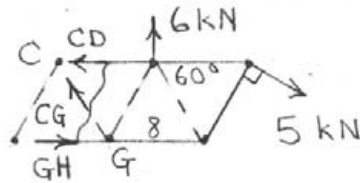
$$\curvearrowright \sum M_P = 0: -BE \sin \beta (9) + 14(3) = 0, \quad \underline{BE = 8.41 \text{ kN T}}$$

$$\curvearrowright \sum M_B = 0: EF \cos \alpha (3) - 14(6) = 0, \quad \underline{EF = 29.5 \text{ kN C}}$$



$$\sum M_J = 0: -3(16) + D(28) - (5 \sin 30^\circ)(48) = 0$$

$$D = 6 \text{ kN}$$



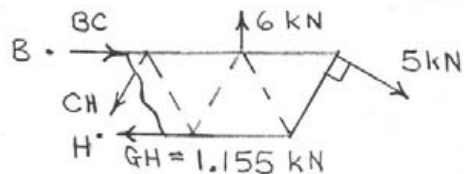
$$\sum F_y = 0: CG \sin 60^\circ + 6 - 5 \sin 30^\circ = 0$$

$$CG = -4.041 \text{ kN C}$$

$$\sum M_G = 0: CD(8 \sin 60^\circ) + 6(4) - (5 \sin 30^\circ)(24) = 0, \quad CD = 5.20 \text{ kN T}$$

$$\sum M_C = 0: GH(8 \sin 60^\circ) + 6(8) + (5 \sin 30^\circ)(16) = 0$$

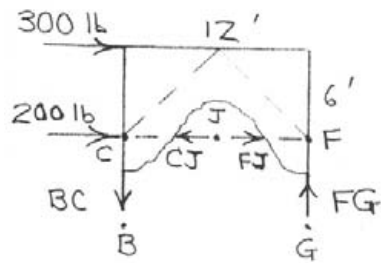
$$GH = -1.155 \text{ kN T}$$



$$\sum M_H = 0: 6(12) - (5 \sin 30^\circ)(32) - BC(8 \sin 60^\circ) = 0$$

$$BC = -1.155 \text{ kN T}$$

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$$\sum M_C = 0: -300(6) + FG(12) = 0$$

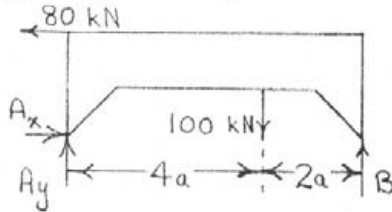
$$FG = 150 \text{ lb C}$$

$$\sum M_F = 0: -300(6) + BC(12) = 0$$

$$BC = 150 \text{ lb T}$$

4/39 $\sum M_A = 0: B(6a) - 100(4a) + 80(2a) = 0$

$B = 40 \text{ kN}$



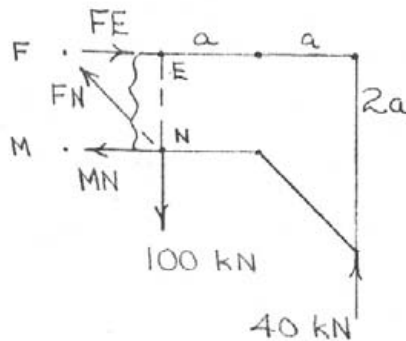
For section,

$\sum F_y = 0: \frac{FN}{\sqrt{2}} + 40 - 100 = 0$

$FN = 84.8 \text{ kN T}$

$\sum M_E = 0: 40(2a) - 84.8 \frac{a}{\sqrt{2}}$

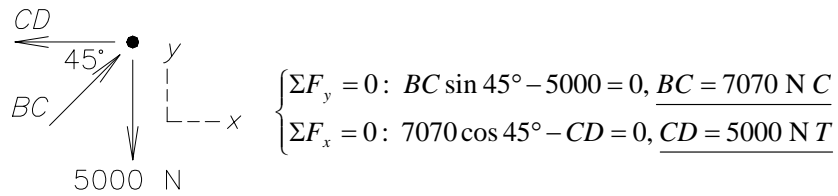
$-MN(a) = 0, \underline{MN = 20 \text{ kN T}}$



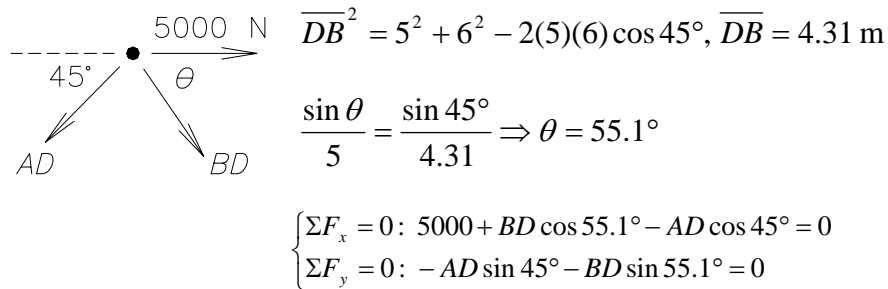
For section through GF & LM, $\sum F_y = 0$ gives
 $GM = 84.8 \text{ kN T}$

We can begin at joint C without finding the external reactions.

Joint C:

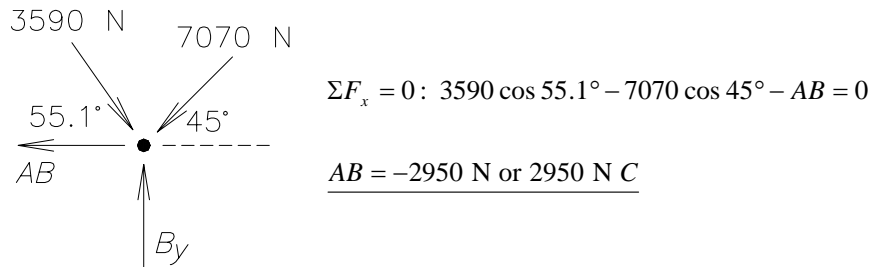


Joint D:



Solve simultaneously to obtain: $\underline{BD = -3590 \text{ N or } 3590 \text{ N C}}$
 $\underline{AD = 4170 \text{ N T}}$

Joint B:



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Diagram showing a cable structure with points O, P, A, B, C, F, G. A vertical line contains points O, P, A, G. A horizontal line contains points A, B, F. A cable segment BC is shown. A dashed line BF is also shown. Angles are given: α at O and B, 30° at A, and 15° at A. Distances are given: 5' from A to B, and 5' from B to F. Forces are given: 9 kips at A and 3 kips at G.

Calculation for α :

$$\alpha = \tan^{-1} \left[\frac{5 \tan 30^\circ + 2(5 \tan 15^\circ)}{5} \right]$$

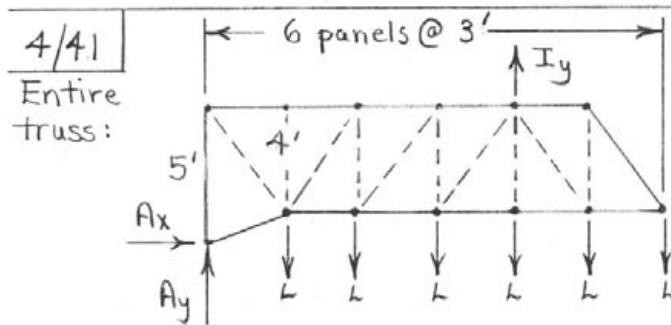
$$= 48.1^\circ$$

Calculation for \overline{AO} :

$$\begin{aligned} \overline{AO} &= \overline{AP} + \overline{PO} \\ &= \overline{AP} + \overline{PB} \tan \alpha \\ &= 5 \tan 30^\circ + 5 \tan \alpha \\ &= 8.45' \end{aligned}$$

$$\sum M_A = 0 : (BF \cos \alpha) \overline{AO} - 3(5) = 0$$

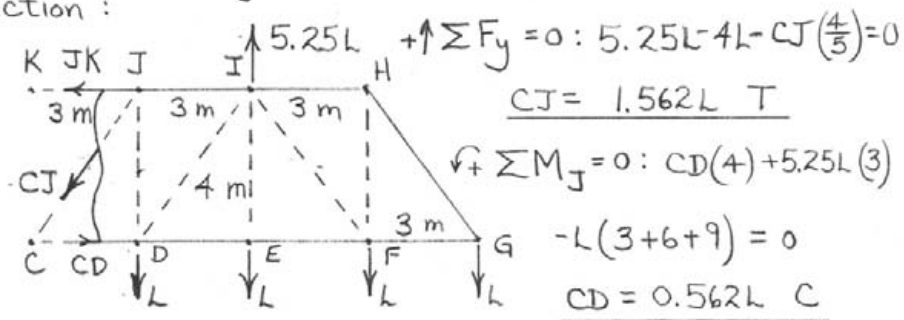
$$BF = 2.66 \text{ kips C}$$



$$\uparrow \sum M_A = 0: I_y (12) - L(3+6+9+12+15+18) = 0$$

$$I_y = 5.25L$$

Section:

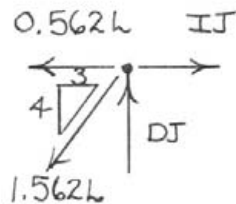


From $\sum F_x = 0$, $JK = 0.562L \text{ T}$.

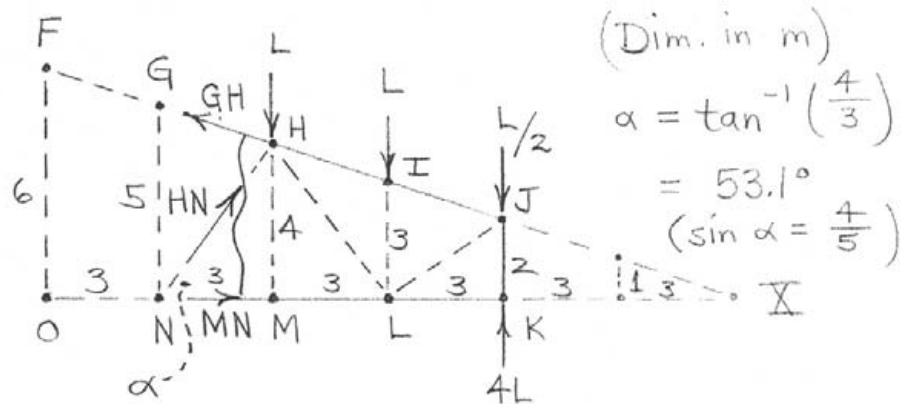
Joint J:

$$\uparrow \sum F_y = 0: DJ - 1.562L\left(\frac{4}{5}\right) = 0$$

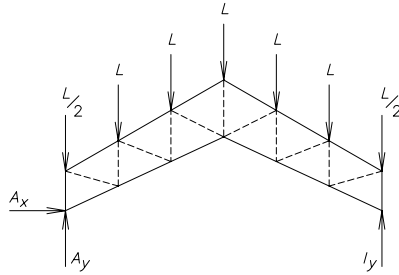
$$DJ = 1.250L \text{ C}$$



4/42 From the truss as a whole, the external reactions at A and K are $4L$ (up).

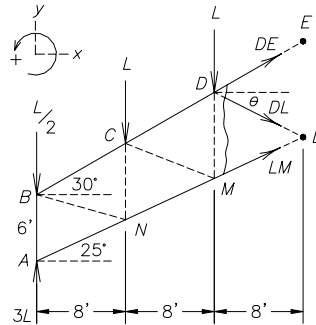


$$\begin{aligned} \sum M_X = 0 : & \left(\frac{L}{2} - 4L\right)6 + L(9) + L(12) \\ & - HN\left(\frac{4}{5}\right)(15) = 0, \quad \underline{HN = 0} \end{aligned}$$



By symmetry, $A_y = I_y = 3L$

$$\Sigma F_x = 0: A_x = 0$$



$$\theta = \tan^{-1} \frac{6 + 16 \tan 30^\circ - 24 \tan 25^\circ}{8} = 26.8^\circ$$

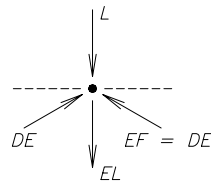
$$\Sigma M_L = 0: L(8) + L(16) + \frac{L}{2}(24) - 3L(24) - DE(24 \tan 30^\circ - 24 \tan 25^\circ + 6) \sin 60^\circ = 0$$

$$DE = -4.80L \text{ or } \underline{4.80L \text{ C}}$$

$$\begin{cases} \Sigma F_x = 0: -4.80L \cos 30^\circ + DL \cos 26.8^\circ + LM \cos 25^\circ = 0 \\ \Sigma F_y = 0: -4.80L \sin 30^\circ - DL \sin 26.8^\circ + LM \sin 25^\circ + 3L - 2L - \frac{L}{2} \end{cases}$$

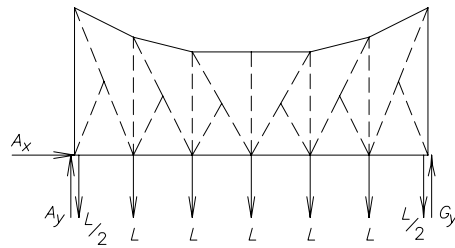
Solve simultaneously to obtain: $\underline{DL = 0.0446LT}$, $\underline{LM = 4.54LT}$

Joint E:



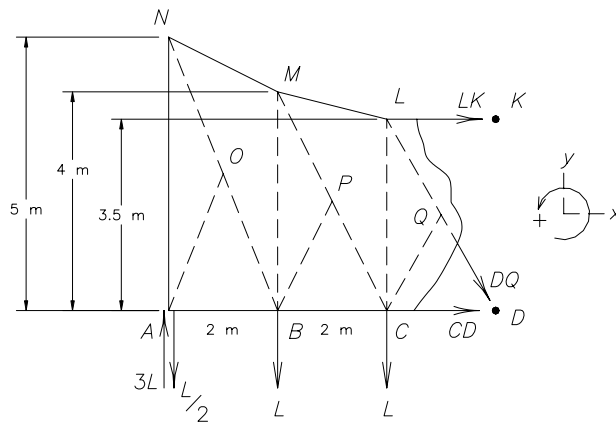
$$\Sigma F_y = 0: 2(4.80L \sin 30^\circ) - L - EL = 0$$

$$\underline{EL = 3.80LT}$$



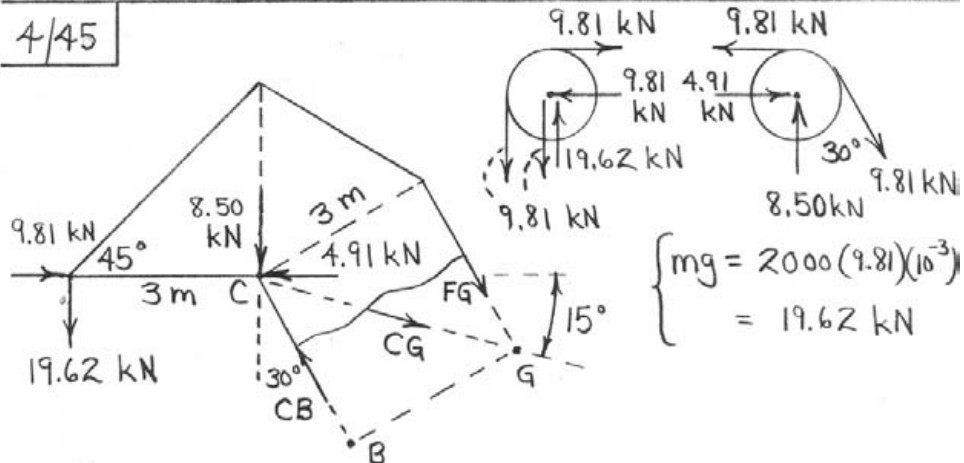
By symmetry, $A_y = G_y = 3L$

$$\Sigma F_x = 0: A_x = 0$$



$$\Sigma F_y = 0: 3L - 2L - \frac{L}{2} - DQ \sin\left(\tan^{-1} \frac{3.5}{2}\right) = 0, \underline{DQ = 0.576L T}$$

By inspection: CQ = 0



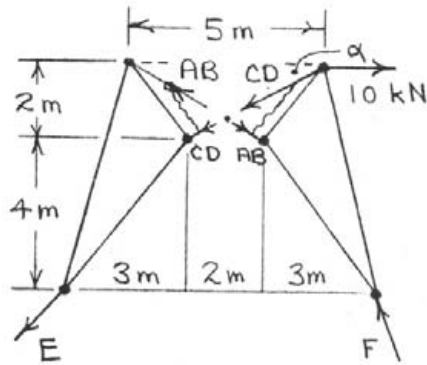
$$\uparrow + \sum M_C = 0: 19.62(3) - FG(3) = 0, \quad \underline{FG = 19.62 \text{ kN T}}$$

$$\begin{aligned} \uparrow + \sum M_G = 0: & 19.62(3 + 3\sqrt{2} \cos 15^\circ) - (9.81 - 4.91)(3\sqrt{2} \sin 15^\circ) \\ & + 8.50(3\sqrt{2} \cos 15^\circ) - CB(3) = 0 \\ & \underline{CB = 56.2 \text{ kN C}} \end{aligned}$$

$$\begin{aligned} \rightarrow \sum F_x = 0: & CG \cos 15^\circ + 19.62 \sin 30^\circ - 56.2 \sin 30^\circ \\ & + 9.81 - 4.91 = 0, \quad \underline{CG = 13.87 \text{ kN T}} \end{aligned}$$

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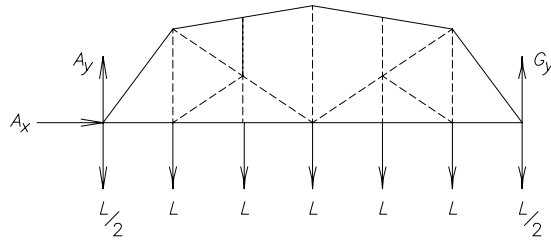
$$\alpha = \tan^{-1}\left(\frac{2}{3.5}\right) = 29.7^\circ$$



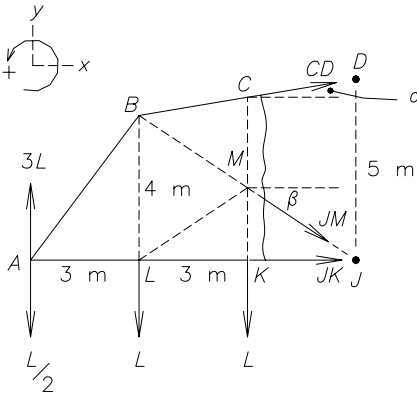
$$\begin{aligned} \text{I. } \sum M_E = 0: & \quad CD(4 \cos \alpha) \\ & \quad - CD(3 \sin \alpha) - AB(6 \cos \alpha) \\ & \quad - AB(1.5 \sin \alpha) = 0 \\ & \quad CD = 3.00 AB \end{aligned}$$

$$\begin{aligned} \text{II. } \sum M_F = 0: & \quad 10(6) + AB(4 \cos \alpha) - AB(3 \sin \alpha) \\ & \quad - CD(6 \cos \alpha) - CD(1.5 \sin \alpha) = 0 \\ & \quad 60 + 1.985 AB - 5.954 CD = 0 \end{aligned}$$

Solving simultaneously, $\underline{AB = 3.78 \text{ kN C.}}$



By symmetry, $A_y = G_y = 3L$ $\Sigma F_x = 0: A_x = 0$



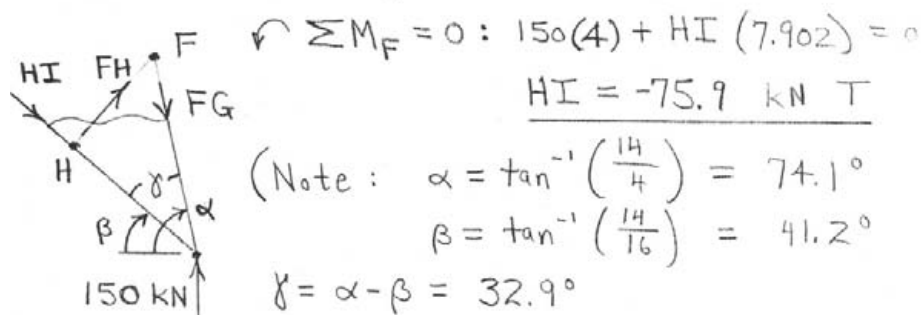
$$\alpha = \tan^{-1} \frac{1}{6} = 9.46^\circ, \beta = \tan^{-1} \frac{4}{6} = 33.7^\circ$$

$$\Sigma M_J = 0: \frac{L}{2}(9) + L(6) + L(3) - 3L(9) - CD(5)[\sin(90^\circ - 9.46^\circ)] = 0$$

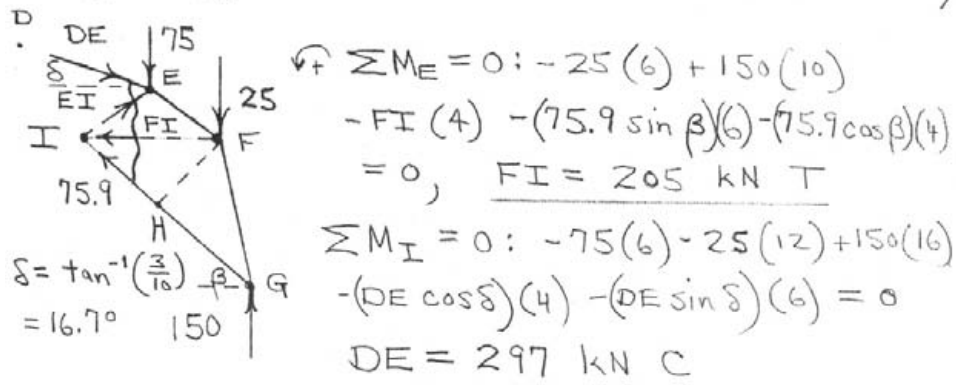
$$CD = -2.74L (C)$$

$$\Sigma F_y = 0: 3L + CD \sin 9.46^\circ - \frac{L}{2} - 2L - JM \sin 33.7^\circ = 0, \underline{JM = 0.0901L T}$$

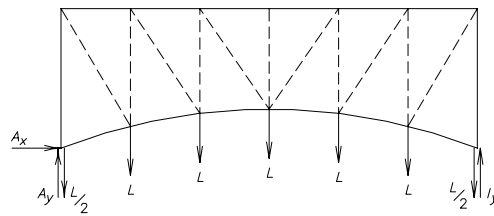
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By symmetry, $A = G = 150 \text{ kN}$ 

Then $d_\perp = FG \sin \gamma = \sqrt{14^2 + 4^2} \sin \gamma = 7.902 \text{ m}$

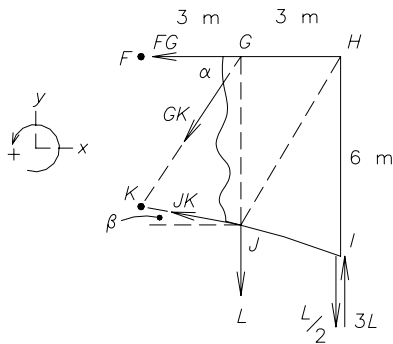


$\sum F_y = 0: -75 - 25 + 150 - 297 \sin \delta + 75.9 \sin \beta + EI \frac{4}{\sqrt{52}} = 0$
 $EI = -26.4 \text{ kN T}$



By symmetry, $A_y = H_y = 3L$

$$\Sigma F_x = 0: A_x = 0$$



Origin at center of arc

Location of I : $y_I^2 = 25^2 - 9^2$, $I = (9, 23.3)$ m

Location of J : $y_J^2 = 25^2 - 6^2$, $J = (6, 24.3)$ m

Location of G : $y_G = y_I + 6$, $G = (6, 29.3)$ m

Location of K : $y_K^2 = 25^2 - 3^2$, $K = (3, 24.8)$ m

$$\alpha = \tan^{-1} \frac{29.3 - 24.8}{3} = 56.3^\circ, \beta = \tan^{-1} \frac{24.8 - 24.3}{3} = 10.39^\circ$$

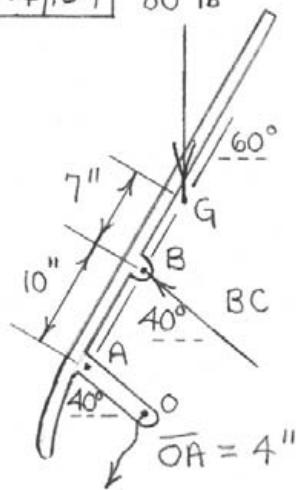
$$\Sigma M_K = 0: 3L(6) - \frac{L}{2}(6) + FG(y_G - y_K) - L(3) = 0, FG = -2.66L \text{ (C)}$$

$$\begin{cases} \Sigma F_x = 0: 2.66L - GK \cos 56.3^\circ - JK \cos 10.39^\circ = 0 \\ \Sigma F_y = 0: \frac{3}{2}L + JK \sin 10.39^\circ - GK \sin 56.3^\circ = 0 \end{cases}$$

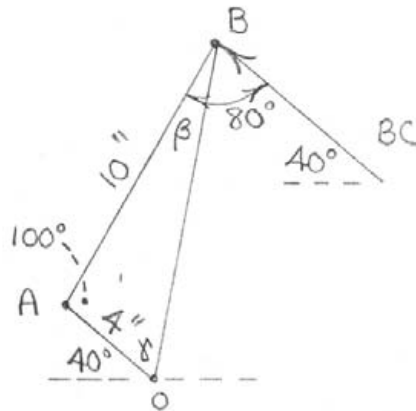
Solve simultaneously to obtain: $GK = 2.13L$

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80 lb



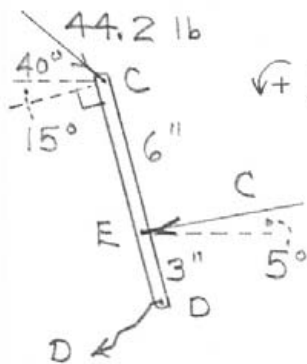
Blowup:



$$\overline{OB}^2 = 4^2 + 10^2 - 2(4)(10) \cos 100^\circ, \quad \overline{OB} = 11.40 \text{ in.}$$

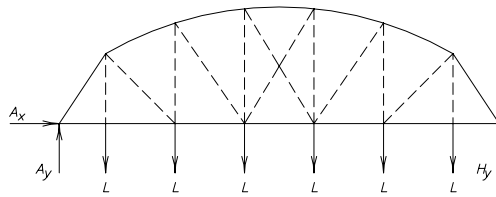
$$\frac{\sin \beta}{4''} = \frac{\sin 100^\circ}{11.40''}, \quad \beta = 20.2^\circ$$

$$\begin{aligned} \curvearrowright \sum M_O = 0: & -80(17 \cos 60^\circ - 4 \cos 40^\circ) \\ & + BC(11.40 \sin(80^\circ - 20.2^\circ)) = 0, \quad BC = 44.2 \text{ lb} \end{aligned}$$



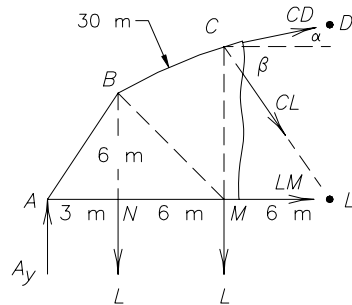
$$\begin{aligned} \curvearrowright \sum M_D = 0: & -44.2 \cos 55^\circ (9) \\ & + C \cos 10^\circ (3) = 0 \end{aligned}$$

$$\underline{C = 77.2 \text{ lb}}$$



By symmetry, $A_y = H_y = 3L$

$$\Sigma F_x = 0: A_x = 0$$



Origin at center of arc

$$\text{Location of } B: y_B^2 = 30^2 - (-15)^2, B = (-15, 26.0) \text{ m}$$

$$\text{Location of } A: y_A = y_B - 6, A = (-18, 20.0) \text{ m}$$

$$\text{Location of } C: y_C^2 = 30^2 - (-9)^2, C = (-9, 28.6) \text{ m}$$

$$\text{Location of } M: y_M = y_A, M = (-9, 20.0) \text{ m}$$

$$\text{Location of } D: y_D^2 = 30^2 - (-3)^2, D = (-3, 29.8) \text{ m}$$

$$\text{Location of } L: y_L = y_A, L = (-3, 20.0) \text{ m}$$

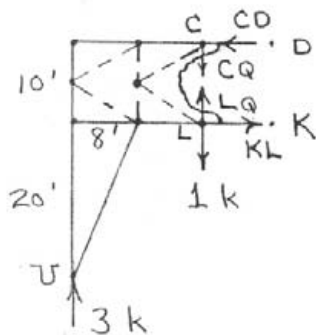
$$\alpha = \tan^{-1} \frac{29.8 - 28.6}{6} = 11.60^\circ, \beta = \tan^{-1} \frac{28.6 - 20.0}{6} = 55.2^\circ$$

$$\Sigma M_L = 0: -3L(15) + L(6) + L(12) - CD(29.8 - 20.0)\sin(90^\circ - 11.60^\circ) = 0$$

$$CD = -2.79L \text{ or } CD = 2.79L$$

$$\Sigma F_y = 0: 3L - L - L - 2.79L \sin 11.60^\circ - CL \sin 55.2^\circ = 0, \underline{CL = 0.534LT}$$

►4/51 From truss as a whole, $\begin{cases} U = 3 \text{ kips} \\ V = 4 \text{ kips} \end{cases}$



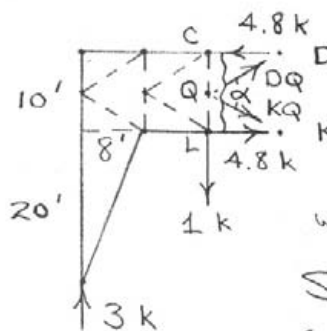
$$\sum M_C = 0: KL(10) - 3(16) = 0$$

$$KL = 4.8 \text{ kips T}$$

$$\sum M_L = 0: CD(10) - 3(16) = 0$$

$$CD = 4.8 \text{ kips C}$$

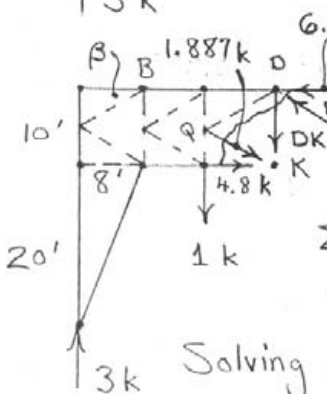
(From a similar right-hand section, $DE = 6.40 \text{ kips C}$.)



$$\sum M_D = 0: -3(24) + 1(8) + KQ(\sqrt{8^2 + 5^2} \sin \alpha) + 4.8(10) = 0,$$

$$\text{where } \alpha = 180 - 2 \tan^{-1}\left(\frac{8}{5}\right) = 64.0^\circ$$

$$\text{Solving, } KQ = 1.887 \text{ kips T}$$



$$\beta = \tan^{-1}\left(\frac{5}{8}\right) = 32.0^\circ$$

$$\sum F_x = 0: -6.40 + 1.887 \cos \beta + 4.8 - DR \cos \beta = 0, DR = 0$$

$$\sum M_B = 0: -3(8) - 1(8) + 4.8(10) - DK(16) = 0$$

$$\text{Solving, } \underline{DK = 1 \text{ kip T}}$$

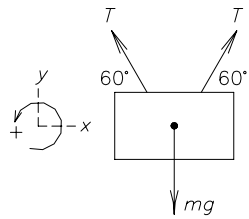
4/60 The truss as a whole is statically determinate with six supporting constraints. $j=6 \neq m=12$; $3j = m+6$, so there are sufficient members for stability. C, B, and D are fixed so E is fixed. A and F are also fixed, so the truss is a rigid unit. From an inspection of joint F, $AF=0$, $BF=0$, $EF=LT$. From an inspection of joint A, $AB=AD=AE=0$. We can now go to joint

E and solve for all unknowns there:

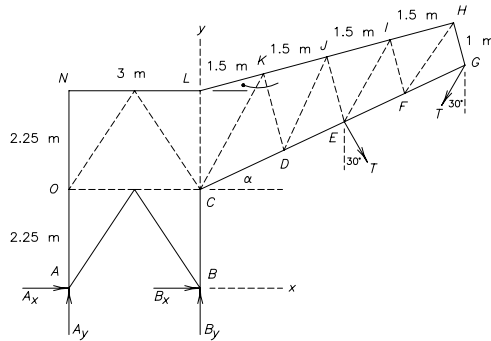
$$\begin{aligned}\underline{CE} &= CE[0.866\mathbf{j} - 0.5\mathbf{k}] \\ \underline{DE} &= DE[0.866\mathbf{j} + 0.5\mathbf{k}] \\ \underline{BE} &= BE\left[\frac{1.5\mathbf{i} + 0.866\mathbf{j} - 0.5\mathbf{k}}{\sqrt{1.5^2 + 0.866^2 + 0.5^2}}\right] \\ &= BE[0.832\mathbf{i} + 0.480\mathbf{j} - 0.277\mathbf{k}]\end{aligned}$$

$$\left. \begin{aligned}\sum F_x &= 0: 0.832BE + L = 0 \\ \sum F_y &= 0: 0.480BE + 0.866CE + 0.866DE - L = 0 \\ \sum F_z &= 0: -0.277BE - 0.5CE + 0.5DE = 0\end{aligned} \right\}$$

Solution: $\underline{BE = -1.202L (C)}, \quad \underline{CE = 1.244L T}$
 $\underline{DE = 0.577L T}$



$$\Sigma F_y = 0: 2T \sin 60^\circ - 1000(9.81) = 0, T = 5660 \text{ N}$$



$$\text{From } \Sigma F_x = 0, A_x + B_x = 0$$

$$C = (0, 2.25) \text{ m}$$

$$L = (0, 4.5) \text{ m}$$

$$H = (6\cos 15^\circ, 4.5 + 6\sin 15^\circ) = (5.80, 6.05) \text{ m}$$

$$G = (6\cos 15^\circ + 1\cos 75^\circ, 4.5 + 6\sin 15^\circ - 1\sin 75^\circ) = (6.05, 5.09) \text{ m}$$

$$\alpha = \tan^{-1} \frac{5.09 - 2.25}{6.05} = 25.1^\circ$$

$$FG = EF = DE = \frac{1.5}{\cos(25.1^\circ - 15^\circ)} = 1.524 \text{ m}$$

$$E = (6\cos 15^\circ + 1\cos 75^\circ - 2(1.524\cos 25.1^\circ), 4.5 + 6\sin 15^\circ - 1\sin 75^\circ - 2(1.524\sin 25.1^\circ)) = (3.30, 3.79) \text{ m}$$

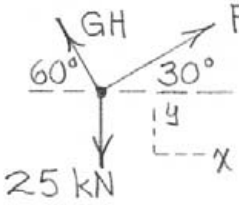
$$\Sigma M_B = 0: -A_y(3) - T \sin 60^\circ(3.30) - T \cos 60^\circ(3.79) - T \sin 60^\circ(6.05) + T \cos 60^\circ(5.09) = 0$$

$$A_y = -14\,070 \text{ N}$$

$$\Sigma F_y = 0: -14\,070 + B_y - 1000(9.81) = 0, B_y = 23\,900 \text{ N}$$

► 4/54

Joint G:



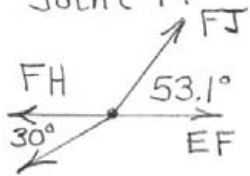
$$\begin{cases} \sum F_x = 0: -GH\left(\frac{1}{2}\right) + FG\left(\frac{\sqrt{3}}{2}\right) = 0 \\ \sum F_y = 0: GH\left(\frac{\sqrt{3}}{2}\right) + FG\left(\frac{1}{2}\right) - 25 = 0 \end{cases}$$

1st eq.: $GH = \sqrt{3} FG$

$$2^{\text{nd}} \text{ eq.: } \sqrt{3} FG \left(\frac{\sqrt{3}}{2} \right) + \frac{1}{2} FG = 25$$

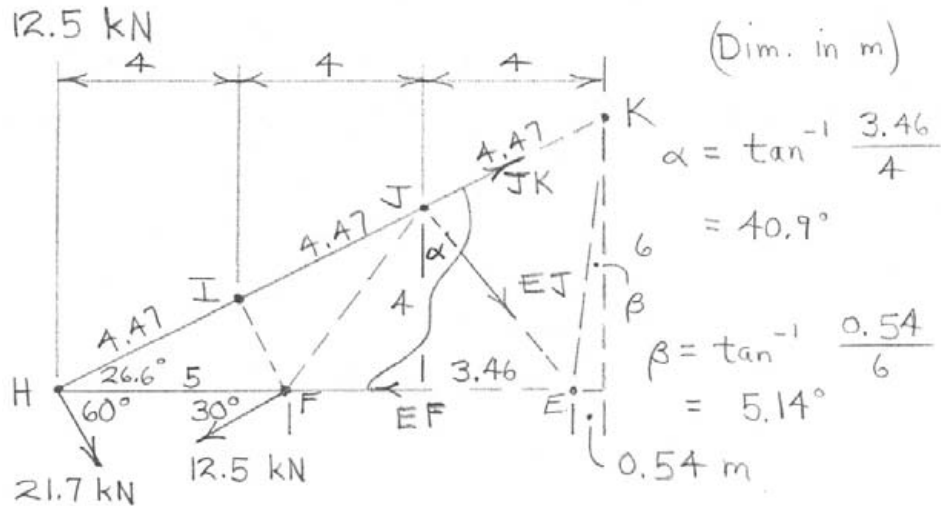
$$\Rightarrow FG = 12.5 \text{ kN T}, GH = 21.7 \text{ kN T}$$

Joint F:



$$\Sigma F_y = 0: FJ(\sin 53.1^\circ) - 12.5 \sin 30^\circ = 0$$

$$\underline{F_J = 7.81 \text{ kN T}}$$



$$\uparrow + \sum M_H = 0: -12.5\left(\frac{1}{2}\right)(5) - EJ[\cos 40.9^\circ(8) + \sin 40.9^\circ(4)] = 0, \quad EJ = -3.61 \text{ kN}$$

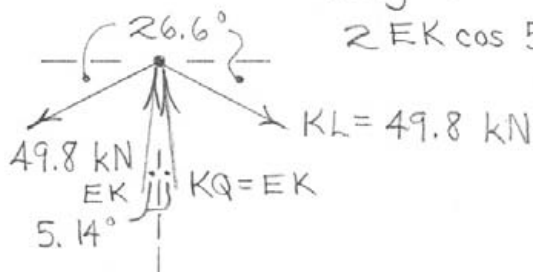
So $EJ = 3.61 \text{ kN C}$

$$\sum F_y = 0: JK \sin 26.6^\circ + 3.61 \cos 40.9^\circ - 12.5\left(\frac{1}{2}\right) - 21.7\left(\frac{\sqrt{3}}{2}\right) = 0, \quad JK = 49.8 \text{ kN T}$$

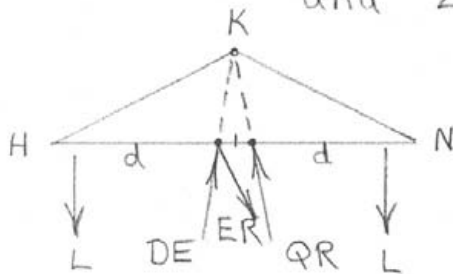
Joint K, using symmetry:

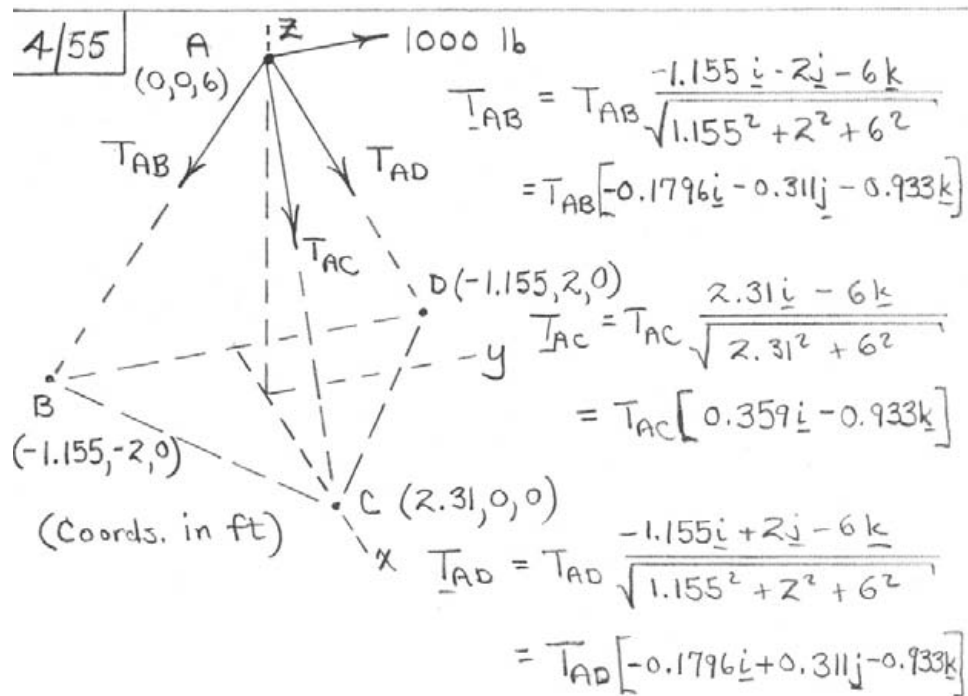
$$\sum F_y = 0: -2(49.8) \sin 26.6^\circ + 2EK \cos 5.14^\circ = 0,$$

$EK = 22.4 \text{ kN C}$



By symmetry of loads L and $\sum M_K = 0$, $ER = 0$





$$\sum F_x = 0: -0.1796 T_{AB} + 0.359 T_{AC} - 0.1796 T_{AD} = 0 \quad (1)$$

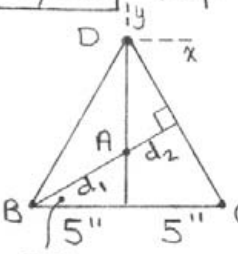
$$\sum F_y = 0: 1000 - 0.311 T_{AB} + 0.311 T_{AD} = 0 \quad (2)$$

$$\sum F_z = 0: -0.933 T_{AB} - 0.933 T_{AC} - 0.933 T_{AD} = 0 \quad (3)$$

Solve Eqs. (1)-(3):

$$\begin{cases} T_{AB} = 1607 \text{ lb} & (T) \\ T_{AC} = 0 \\ T_{AD} = -1607 \text{ lb} & (C) \end{cases}$$

4/56 Top view of base:



$$\cos 30^\circ = \frac{d_1 + d_2}{10}, \quad d_1 + d_2 = 8.66''$$

$$\cos 30^\circ = \frac{5}{d_1}, \quad d_1 = 5.77''$$

$$d_2 = 8.66 - 5.77 = 2.89''$$

For joint A, assuming symmetry:

$$\underline{F}_{BA} = P \left[\frac{5\underline{i} + 2.89\underline{j} + 16\underline{k}}{(5^2 + 2.89^2 + 16^2)^{1/2}} \right] = P(0.294\underline{i} + 0.1697\underline{j} + 0.941\underline{k})$$

$$\underline{F}_{CA} = P(-0.294\underline{i} + 0.170\underline{j} + 0.941\underline{k}), \quad \underline{F}_{DA} = P(-0.339\underline{j} + 0.941\underline{k})$$

$$\Sigma F_z = 0 \text{ at A: } 3(0.941 P) - 800 = 0, \quad P = 283 \text{ lb}$$

For joint C, assuming symmetry:

$$\underline{F}_{BC} = -Q\underline{i}, \quad \underline{F}_{CD} = Q(-\sin 30^\circ \underline{i} + \cos 30^\circ \underline{j})$$

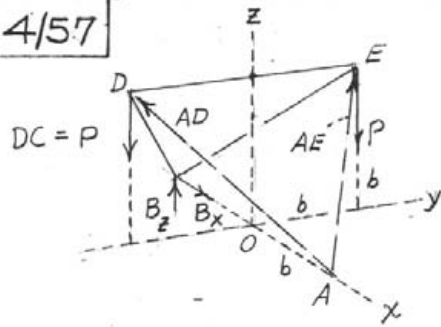
$$\text{Normal } \underline{N} = 267\underline{k} \text{ lb}$$

$$\Sigma \underline{F} = 0 \text{ at C: } \underline{N} + \underline{F}_{BC} + \underline{F}_{CD} + \underline{F}_{AC} = 0$$

$$267\underline{k} - Q\underline{i} + 283(0.294\underline{i} - 0.1697\underline{j} - 0.941\underline{k}) + Q(-0.5\underline{i} + 0.866\underline{j}) = 0$$

Solving, $Q = 55.6 \text{ lb} \Rightarrow \underline{BC} = \underline{BD} = \underline{CD} = 55.6 \text{ lb T}$

4/57



For truss as a whole

$\sum M_x = 0$ gives $DC = P$

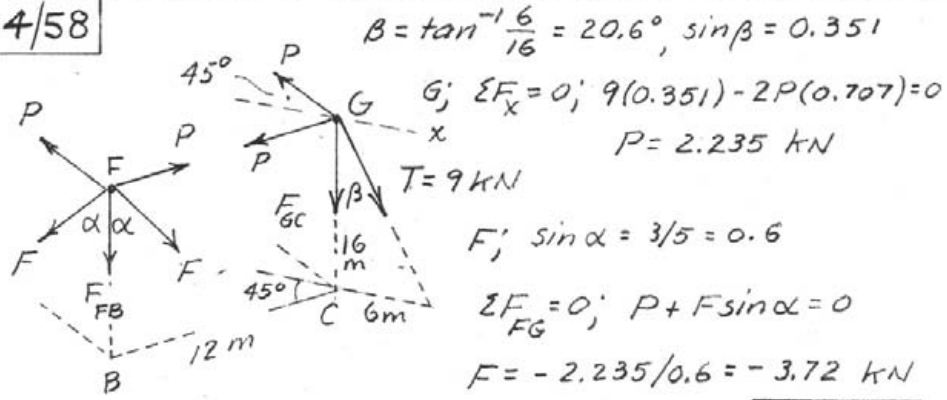
$\sum M_y = 0$ & $\sum F_z = 0$ gives $B_z = P$

For unit BDE

$\sum M_{DE} = 0$; $B_z b - B_x b = 0$

$$\underline{B_x = B_z = P}$$

4/58



4/59 From truss as a whole $\sum M = 0$ gives tension in vertical wire at C $T_C = \frac{1}{3}mg$

By symmetry & $\sum F_z = 0$; $T_A = T_B = \frac{1}{3}mg$

Joint A; $mg/3$

$F_{AB} = F_{AB} \underline{j}$
 $F_{AC} = \frac{F_{AC}}{5} (3\underline{i} + 4\underline{j})$
 $F_{AD} = \frac{F_{AD}}{\sqrt{53}} (\underline{i} + 4\underline{j} - 6\underline{k})$

$\underline{AD} = \sqrt{4^2 + 1^2 + 6^2} = \sqrt{53} \text{ m}$

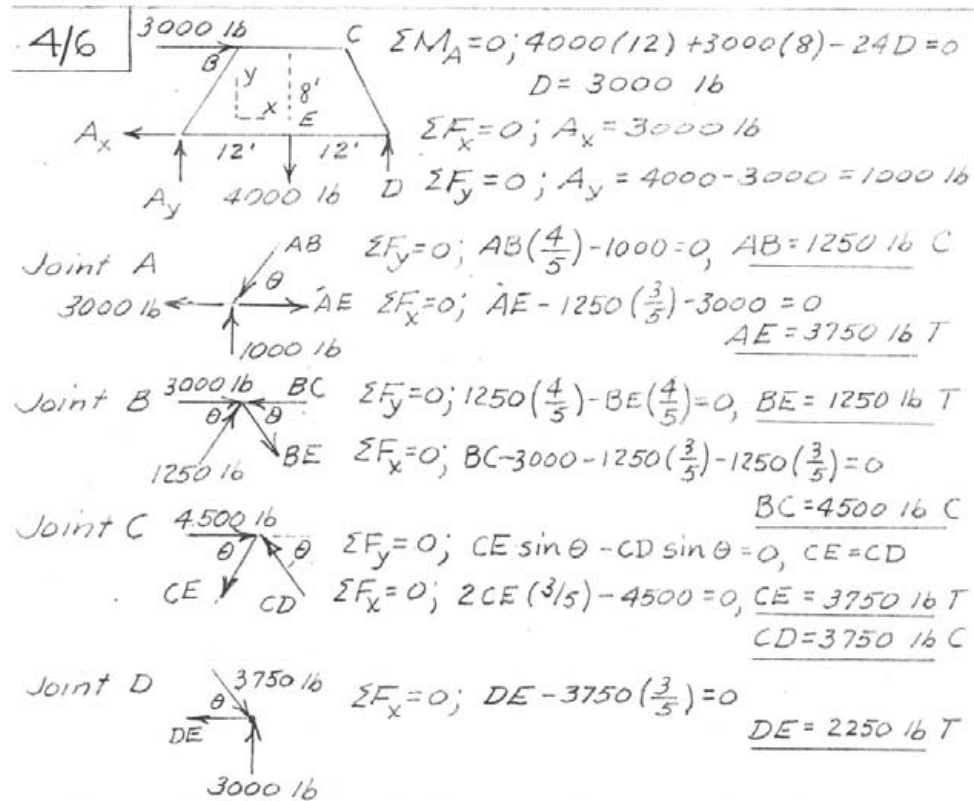
$\sum \underline{F} = \underline{0}; \frac{mg}{3} \underline{k} + F_{AB} + F_{AC} + F_{AD} = \underline{0}$

Substitute & collect terms to get

$$\left(\frac{3F_{AC}}{5} + \frac{F_{AD}}{\sqrt{53}} \right) \underline{i} + \left(F_{AB} + \frac{4F_{AC}}{5} + \frac{4F_{AD}}{\sqrt{53}} \right) \underline{j} + \left(\frac{mg}{3} - \frac{6F_{AD}}{\sqrt{53}} \right) \underline{k} = \underline{0}$$

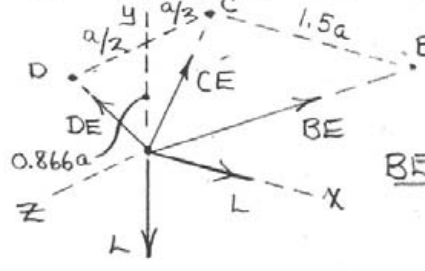
Equate coefficients of \underline{i} , \underline{j} , & \underline{k} -terms to zero & get

$F_{AD} = \frac{\sqrt{53}}{18} mg$, $F_{AC} = -\frac{5}{54} mg$, $F_{AB} = -\frac{4}{27} mg$



4/60 The truss as a whole is statically determinate with six supporting constraints. $j=6 \neq m=12$; $3j = m+6$, so there are sufficient members for stability. C, B, and D are fixed so E is fixed. A and F are also fixed, so the truss is a rigid unit. From an inspection of joint F, $AF=0$, $BF=0$, $EF=LT$. From an inspection of joint A, $AB=AD=AE=0$. We can now go to joint

E and solve for all unknowns there:



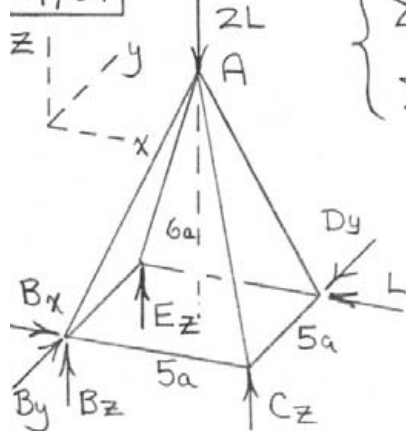
$$\begin{aligned} \underline{CE} &= CE[0.866\mathbf{j} - 0.5\mathbf{k}] \\ \underline{DE} &= DE[0.866\mathbf{j} + 0.5\mathbf{k}] \\ \underline{BE} &= BE\left[\frac{1.5\mathbf{i} + 0.866\mathbf{j} - 0.5\mathbf{k}}{\sqrt{1.5^2 + 0.866^2 + 0.5^2}}\right] \\ &= BE[0.832\mathbf{i} + 0.480\mathbf{j} - 0.277\mathbf{k}] \end{aligned}$$

$$\left. \begin{aligned} \sum F_x &= 0: 0.832BE + L = 0 \\ \sum F_y &= 0: 0.480BE + 0.866CE + 0.866DE - L = 0 \\ \sum F_z &= 0: -0.277BE - 0.5CE + 0.5DE = 0 \end{aligned} \right\}$$

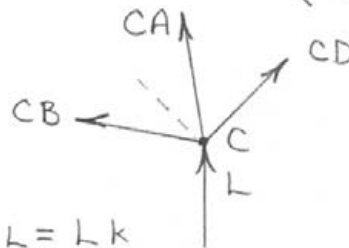
Solution: $\underline{BE = -1.202L (C)}, \quad \underline{CE = 1.244L T}$
 $\underline{DE = 0.577L T}$

4/61

$$\begin{cases} \sum M_{BE} = 0 \Rightarrow C_z = L \\ \sum M_{Bz} = 0 \Rightarrow D_y = L \end{cases}$$



Joint C: (tensions assumed)



$$\underline{CB} = -CB \underline{i}, \quad \underline{CD} = CD \underline{j}, \quad \underline{L} = L \underline{k}$$

$$\underline{CA} = CA \left(\frac{-2.5a \underline{i} + 2.5a \underline{j} + 6a \underline{k}}{\sqrt{(2.5^2 + 2.5^2 + 6^2)a^2}} \right) = CA(-0.359 \underline{i} + 0.359 \underline{j} + 0.862 \underline{k})$$

$$\sum \underline{F} = 0 \text{ yields: } \begin{cases} \underline{i}: -CB - 0.359 CA = 0 \\ \underline{j}: CD + 0.359 CA = 0 \\ \underline{k}: L + 0.862 CA = 0 \end{cases} \Rightarrow \begin{cases} CA = -0.1667L \\ CD = +0.417L \end{cases}$$

Joint D:

 $\sum \underline{F} = 0$ yields

$$\begin{cases} \underline{i}: -DE - L - 0.359 DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{j}: -0.417L - L - 0.359 DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{k}: 0.862 DA = 0 \end{cases}$$

$$DA = 0, \quad \underline{DB} = -2.00L$$

4/62

$j = \text{number of joints} = 7$

$m_i = \text{initial number of members} = 11$

$$[m_i + 6 = 17] < [3j = 21]$$

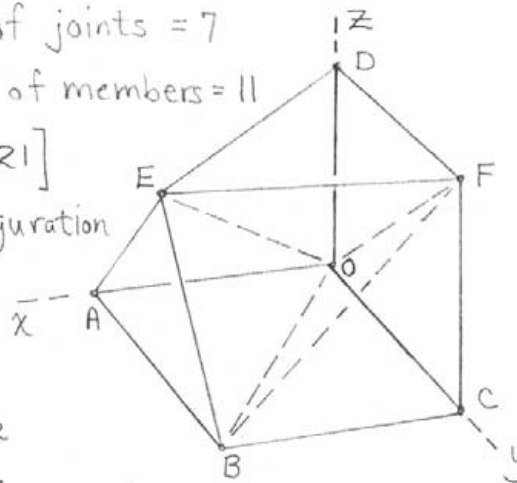
So the initial configuration

lacks $21 - 17 = 4$

members for internal

stability. A stable

configuration is achieved by



(1) Adding OB & OE to produce the rigid tetrahedron ABE O.

(2) Adding OF to produce the rigid tetrahedron ODEF.

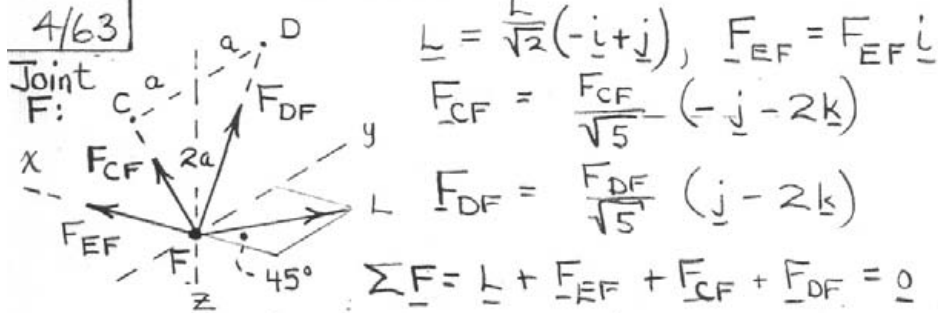
(3) Adding BF to produce the rigid tetrahedrons OBCF and OBEF.

With 4 new members, $m = 15$ and $m + 6 = 21$.

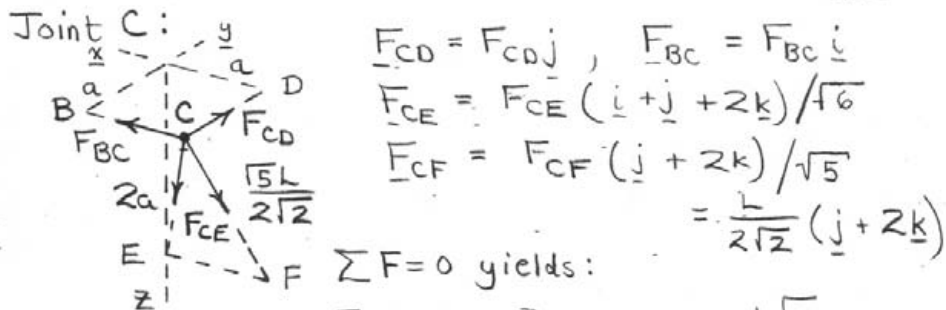
The number of joints remains $j = 7$; $3j = 21$.

So $m + 6 = 3j$; sufficient number of members now present.

4/63



$$\begin{aligned} \underline{i}: -\frac{L}{\sqrt{2}} + F_{EF} &= 0 & F_{EF} &= \frac{L}{\sqrt{2}} \\ \underline{j}: \frac{L}{\sqrt{2}} - \frac{F_{CF}}{\sqrt{5}} + \frac{F_{DF}}{\sqrt{5}} &= 0 \\ \underline{k}: -\frac{2}{\sqrt{5}}F_{CF} - \frac{2}{\sqrt{5}}F_{DF} &= 0 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} F_{CF} = \frac{\sqrt{5}L}{2\sqrt{2}} \\ F_{DF} = -\frac{\sqrt{5}L}{2\sqrt{2}} \end{array}$$



$$\begin{aligned} \underline{i}: 0 + F_{BC} + \frac{F_{CE}}{\sqrt{6}} &= 0 \\ \underline{j}: F_{CD} + \frac{F_{CE}}{\sqrt{6}} + \frac{L}{2\sqrt{2}} &= 0 \\ \underline{k}: \frac{2F_{CE}}{\sqrt{6}} + \frac{L}{\sqrt{2}} &= 0 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} F_{BC} = \frac{L\sqrt{2}}{4} \\ F_{CD} = 0 \\ F_{CE} = -\frac{L\sqrt{3}}{2} \end{array}$$

4/64

$T_{AB} = T_{AB} \frac{0.36\mathbf{i} + 0.8\mathbf{k}}{\sqrt{0.36^2 + 0.8^2}}$
 $= T_{AB} [0.410\mathbf{i} + 0.912\mathbf{k}]$
 $T_{AE} = T_{AE} \frac{-0.36\mathbf{i} - 0.3\mathbf{j} + 0.44\mathbf{k}}{\sqrt{0.36^2 + 0.3^2 + 0.44^2}}$
 $= T_{AE} [-0.560\mathbf{i} - 0.467\mathbf{j} + 0.684\mathbf{k}]$
 $T_{AF} = T_{AF} \frac{-0.36\mathbf{i} + 0.3\mathbf{j} + 0.44\mathbf{k}}{\sqrt{0.36^2 + 0.3^2 + 0.44^2}} = T_{AF} [-0.560\mathbf{i} + 0.467\mathbf{j} + 0.684\mathbf{k}]$

$$\left. \begin{aligned} \sum F_x = 0: & 0.410T_{AB} - 0.560T_{AE} - 0.560T_{AF} = 0 \\ \sum F_y = 0: & -0.467T_{AE} + 0.467T_{AF} = 0 \\ \sum F_z = 0: & 0.912T_{AB} + 0.684T_{AE} + 0.684T_{AF} - 5 = 0 \end{aligned} \right\}$$

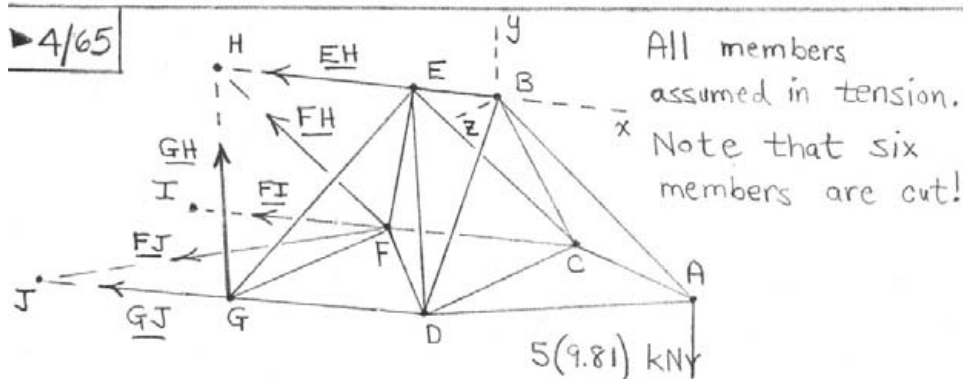
Solution: $T_{AB} = 3.54 \text{ kN}$, $T_{AE} = T_{AF} = 1.296 \text{ kN}$

$T_{BC} = T_{BC} \frac{-0.72\mathbf{i} + 0.3\mathbf{j} + 0.36\mathbf{k}}{\sqrt{0.72^2 + 0.3^2 + 0.36^2}}$
 $= T_{BC} [-0.838\mathbf{i} + 0.349\mathbf{j} + 0.419\mathbf{k}]$
 $T_{BD} = T_{BD} [-0.838\mathbf{i} - 0.349\mathbf{j} + 0.419\mathbf{k}]$
 $T_{BE} = T_{BE} [-0.838\mathbf{i} - 0.349\mathbf{j} - 0.419\mathbf{k}]$
 $T_{BF} = T_{BF} [-0.838\mathbf{i} + 0.349\mathbf{j} - 0.419\mathbf{k}]$
 Note: $T_{BD} = T_{BC}$; $T_{BE} = T_{BF}$ (Symmetry)

Set $\sum \mathbf{F} = 0$ to obtain $T_{BD} = T_{BC} = 1.491 \text{ kN}$

$T_{BE} = T_{BF} = -2.36 \text{ kN (C)}$

► 4/65



$$\underline{GJ} = -GJ\underline{i}, \quad \underline{FI} = -FI\underline{i}, \quad \underline{FJ} = \frac{FJ}{\sqrt{2}}(-\underline{i} + \underline{k})$$

$$\begin{aligned} \sum \underline{M}_H = 0: & -49.05(5)\underline{k} + (-2 \cos 30^\circ \underline{j} + 2 \sin 30^\circ \underline{k}) \\ & \times (-GJ)\underline{i} + (-2 \cos 30^\circ \underline{j} - 2 \sin 30^\circ \underline{k}) \times (-FI)\underline{i} \\ & + (\underline{i} - 2 \cos 30^\circ \underline{j} - \underline{k}) \times \frac{FJ}{\sqrt{2}}(-\underline{i} + \underline{k}) = 0. \end{aligned}$$

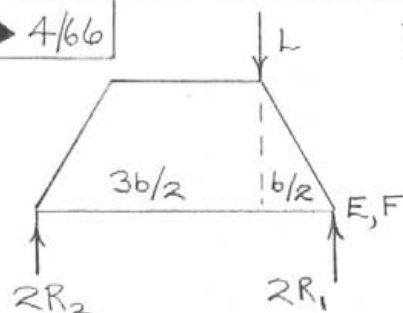
Equating unit vector coefficients to zero:

$$-1.225 FJ = 0 \Rightarrow \underline{FJ = 0}$$

$$\left. \begin{aligned} -GJ + FI &= 0 \\ -1.732 GJ - 1.732 FI &= 245 \end{aligned} \right\} \begin{aligned} FI &= GJ = \\ & -70.8 \text{ kN} \end{aligned}$$

$$\therefore \underline{\text{Force in GJ} = 70.8 \text{ kN C}}$$

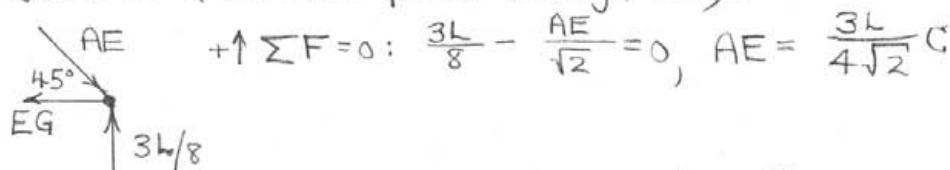
4/66



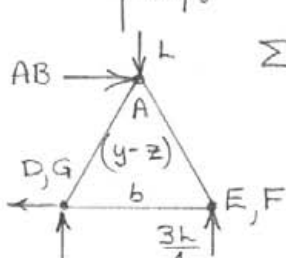
$$\sum M_{EF} = 0: 2R_2(2b) - L \frac{b}{2} = 0$$

$$R_2 = \frac{L}{8}, R_1 = \frac{3L}{8}$$

Joint E (Vertical plane through AE):

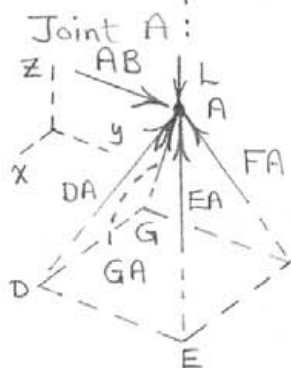


$$+\uparrow \sum F = 0: \frac{3L}{8} - \frac{AE}{\sqrt{2}} = 0, AE = \frac{3L}{4\sqrt{2}} C$$



$$\sum M_{DG} = 0: AB \frac{b}{\sqrt{2}} + L \frac{b}{2} - \frac{3L}{4} b = 0$$

$$AB = \frac{L\sqrt{2}}{4} \text{ (Comp)}$$



$$\underline{DA} = DA \left(-\frac{1}{2}\underline{i} + \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k} \right)$$

$$\underline{EA} = \frac{3L}{4\sqrt{2}} \left(-\frac{1}{2}\underline{i} - \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k} \right)$$

$$\underline{GA} = GA \left(\frac{1}{2}\underline{i} + \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k} \right)$$

$$\underline{FA} = \frac{3L}{4\sqrt{2}} \left(\frac{1}{2}\underline{i} - \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k} \right)$$

$$L = -L\underline{k}, AB = \frac{L\sqrt{2}}{4}\underline{j}$$

$$\sum \underline{F} = 0: L + AB + DA + EA + GA + FA = 0$$

Note that $DA = GA$ & $EA = FA$ by symmetry & obtain

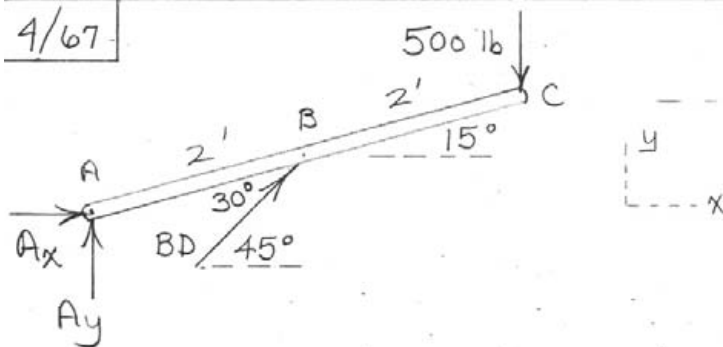
$$\underline{i}: -\frac{1}{2}DA - \frac{3L}{8\sqrt{2}} + \frac{1}{2}DA + \frac{3L}{8\sqrt{2}} = 0, 0 = 0 \checkmark$$

$$\underline{j}: \frac{L\sqrt{2}}{4} + \frac{1}{2}DA - \frac{3L}{8\sqrt{2}} + \frac{1}{2}DA - \frac{3L}{8\sqrt{2}} = 0, DA = GA$$

$$\underline{k}: -L + \frac{L}{8} + \frac{3L}{8} + \frac{L}{8} + \frac{3L}{8} = 0, 0 = 0 \checkmark$$

$$= \frac{L\sqrt{2}}{8} \text{ (Comp.)}$$

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$$\uparrow \Sigma M_A = 0: BD \sin 30^\circ (2) - 500 (4 \cos 15^\circ) = 0$$

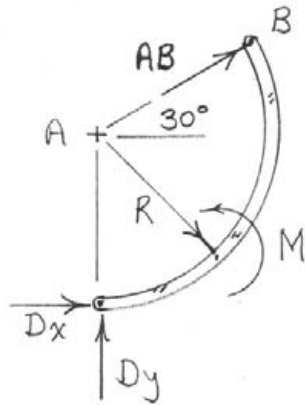
$$BD = 1932 \text{ lb}$$

Forces at B and D = 1932 lb

$$\Sigma F_x = 0: A_x + 1932 \cos 45^\circ = 0, A_x = -1366 \text{ lb}$$

$$\Sigma F_y = 0: A_y + 1932 \sin 45^\circ - 500 = 0, A_y = -866 \text{ lb}$$

$$A = \sqrt{1366^2 + 866^2} = 1617 \text{ lb}$$



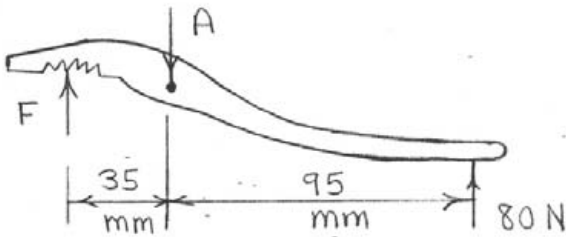
$$\sum M_D = 0 : -AB \cos 30^\circ (R) + M = 0$$

$$AB = \frac{M}{R \cos 30^\circ} = \frac{M}{R \sqrt{3}/2} = \frac{2\sqrt{3} M}{3R}$$

Load is a couple, so reactions form a couple:

$$\underline{A = D = \frac{2\sqrt{3} M}{3R}}$$

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$$\sum M_A = 0: 80(95) - F(35) = 0, \quad \underline{F = 217\text{ N}}$$

$$\sum F = 0: 217 - A + 80 = 0, \quad \underline{A = 297\text{ N}}$$

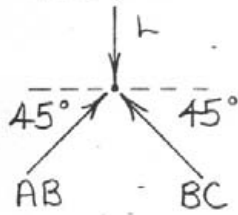
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From $\Sigma F_x = 0$, $AB = BC$

Joint B:

$$\Sigma F_y = 0: 2AB \frac{\sqrt{2}}{2} - L = 0$$

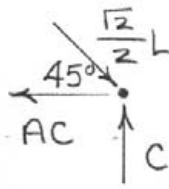
$$AB = \frac{\sqrt{2}}{2} L = BC$$



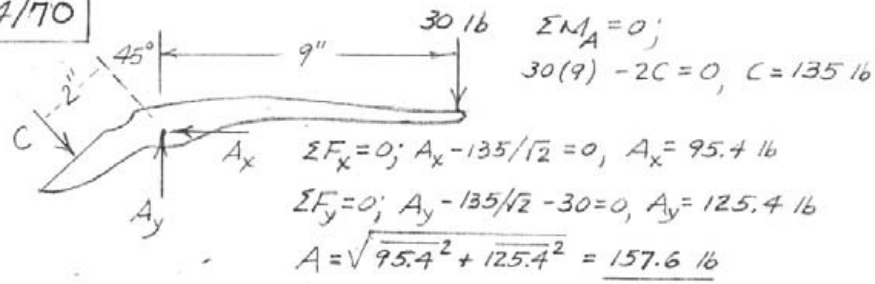
Joint C:

$$\Sigma F_x = 0: \frac{\sqrt{2}}{2} L \left(\frac{\sqrt{2}}{2} \right) - AC = 0$$

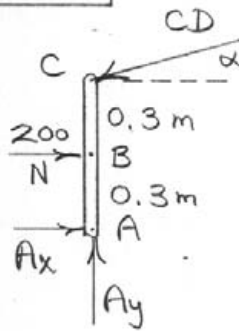
$$\underline{AC = \frac{L}{2} \text{ T}}$$



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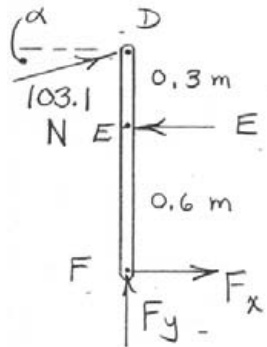


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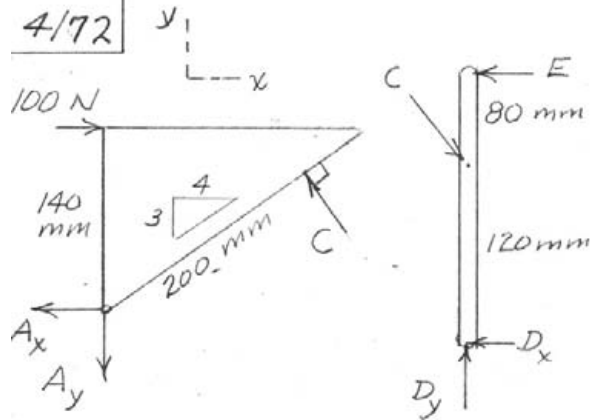
$$\alpha = \tan^{-1} \frac{0.3}{1.2} = 14.04^\circ$$

$$\begin{aligned} \sum M_A = 0: & \quad CD \cos \alpha (0.6) \\ & \quad - 200(0.3) = 0, \quad \underline{CD = 103.1 \text{ N}} \end{aligned}$$



$$\begin{aligned} \sum M_F = 0: & \quad -103.1 \cos \alpha (0.9) \\ & \quad + E(0.6) = 0, \quad \underline{E = 150 \text{ N}} \end{aligned}$$

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Triangle:

$$\sum M_A = 0; 200C - 140(100) = 0, \quad C = 70 \text{ N}$$

Bar:

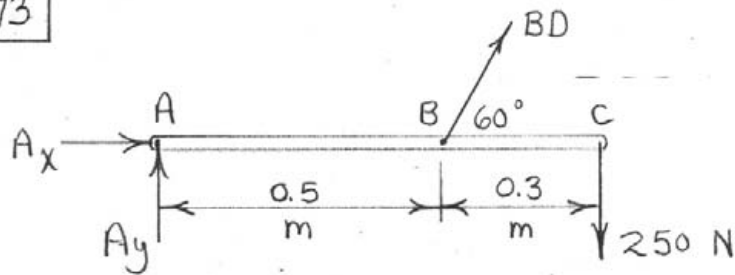
$$\sum M_D = 0; 200E - 70\left(\frac{3}{5}\right)120 = 0; \quad E = 25.2 \text{ N}$$

$$\sum F_x = 0; D_x + 25.2 - 70\left(\frac{3}{5}\right) = 0, \quad D_x = 16.8 \text{ N}$$

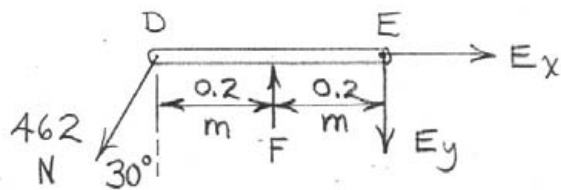
$$\sum F_y = 0; D_y - 70\left(\frac{4}{5}\right) = 0, \quad D_y = 56 \text{ N}$$

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{16.8^2 + 56^2} = \underline{58.5 \text{ N}}$$

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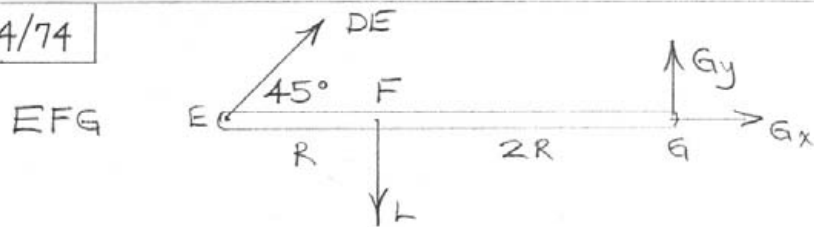


$$\begin{aligned} \curvearrow + \sum M_A = 0 : (BD \sin 60^\circ)(0.5) - 250(0.8) &= 0 \\ BD &= 462 \text{ N} \end{aligned}$$



$$\begin{aligned} \curvearrow + \sum M_E = 0 : 462 \cos 30^\circ (0.4) + F(0.2) &= 0 \\ F &= 800 \text{ N} \end{aligned}$$

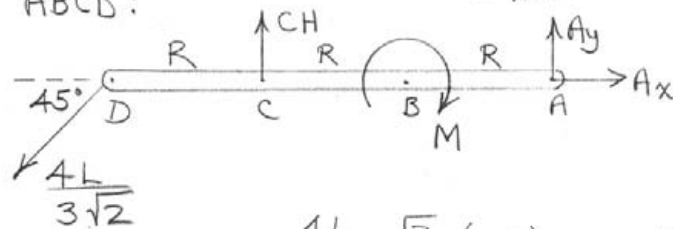
4/74



$$\sum M_G = 0: L(2R) - DE \frac{\sqrt{2}}{2} (3R) = 0$$

$$DE = \frac{4L}{3\sqrt{2}}$$

ABCD:

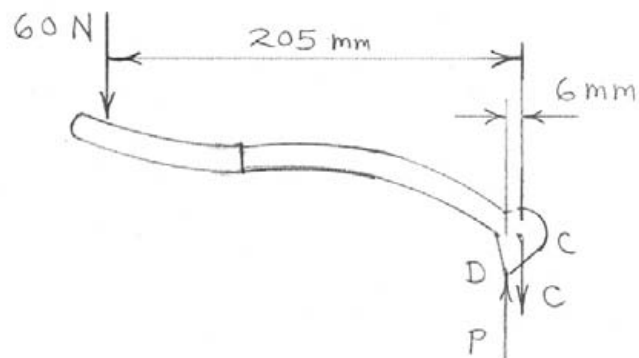


$$\sum M_A = 0: \frac{4L}{3\sqrt{2}} \frac{\sqrt{2}}{2} (3R) - CH(2R) - M = 0$$

$$CH = L - \frac{M}{2R} \text{ (tension if positive)}$$

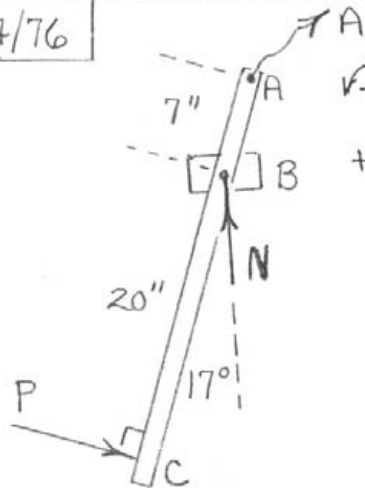
$$CH = 0 \text{ if } M = 2RL$$

4/75 Upper handle/cam unit:



$$\begin{aligned} \uparrow + \sum M_C = 0: & 60(205) - P(6) = 0 \\ & P = 2050 \text{ N} \end{aligned}$$

4/76



$$\sum M_A = 0:$$

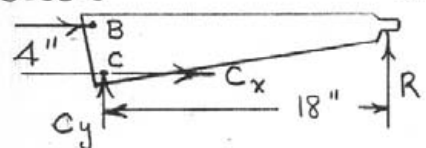
$$+P(27) - N \sin 17^\circ (7) = 0$$

$$\underline{N = 13.19P}$$

4/77 Piston force = $(500)(20) = 10,000 \text{ lb}$
 Force in link AB = $10,000/2 = 5000 \text{ lb}$

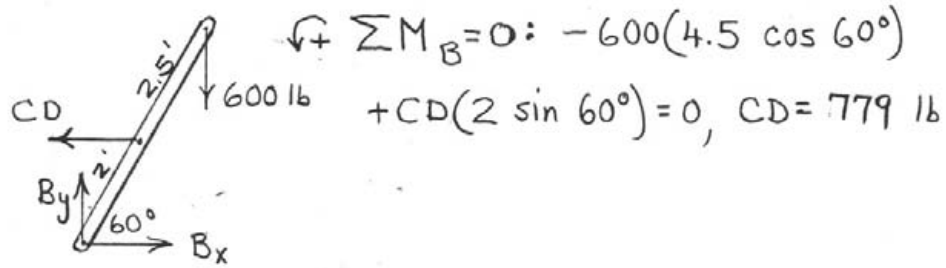
Lower jaw:

5000 lb

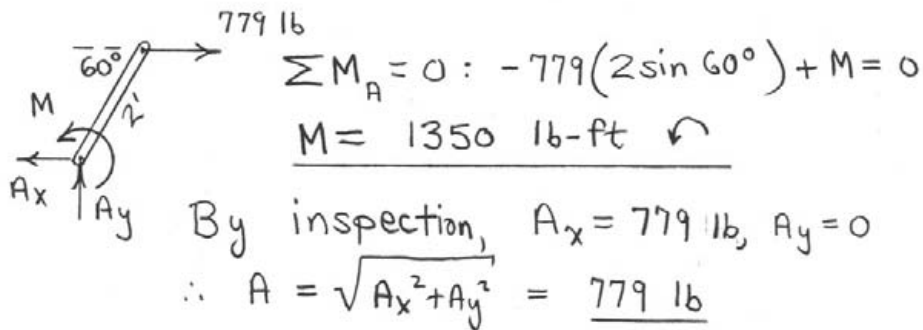


$\downarrow + \sum M_C = 0: R(18) - 5000(4)$
 $= 0, \quad R = 1111 \text{ lb}$

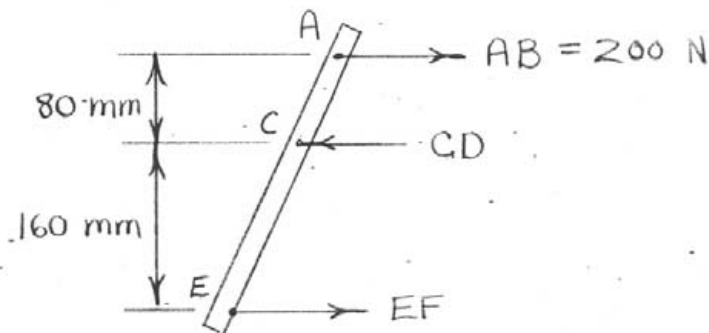
4/78 Member BE :



Member AC :



4/79



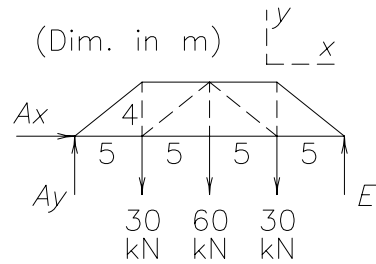
$$\curvearrowleft \sum M_C = 0 : -200(80) + EF(160) = 0$$

$$EF = 100 \text{ N } \quad \underline{\hspace{1cm}}$$

$$\rightarrow \sum F = 0 : 200 - CD + 100 = 0$$

$$CD = 300 \text{ N.}$$

So force supported by pin C is F = 300 N



As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

$A_y = E = 60 \text{ kN}$ by

$\Sigma F_y = 0$ and symmetry.

Joint A: $(\theta = \tan^{-1}(4/5) = 38.7^\circ)$

$$\begin{cases} \Sigma F_y = 0: 60 - AB \sin \theta = 0, \underline{AB = 96.0 \text{ kN } C} \\ \Sigma F_x = 0: AH - 96.0 \cos \theta, \underline{AH = 75 \text{ kN } T} \end{cases}$$

Joint B:

$$\begin{cases} \Sigma F_x = 0: BC + 96.0 \sin 51.3^\circ = 0, \underline{BC = -75 \text{ kN } (C)} \\ \Sigma F_y = 0: -BH + 96.0 \cos 51.3^\circ = 0, \underline{BH = 60 \text{ kN } T} \end{cases}$$

Joint H:

$$\begin{cases} \Sigma F_y = 0: -CH \sin \theta + 30 = 0, \underline{CH = 48.0 \text{ kN } C} \\ \Sigma F_x = 0: 48.0 \cos \theta + GH - 75 = 0, \underline{GH = 112.5 \text{ kN } T} \end{cases}$$

Joint G:

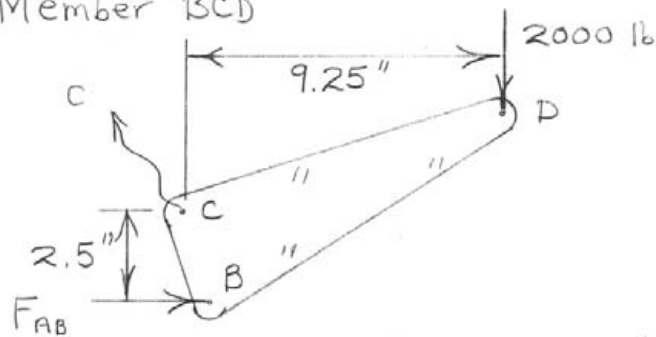
$$\Sigma F_y = 0 \Rightarrow CG = 60 \text{ kN } T$$

By symmetry:

$FG = 112.5 \text{ kN } T, CF = 48.0 \text{ kN } C$
 $CD = 75 \text{ kN } C, DF = 60 \text{ kN } T$
 $EF = 75 \text{ kN } T, DE = 96.0 \text{ kN } C$

4/80

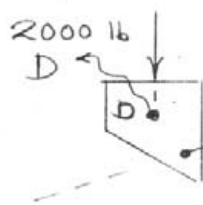
Member BCD



$$\sum M_C = 0: F_{AB}(2.5) - 2000(9.25) = 0$$

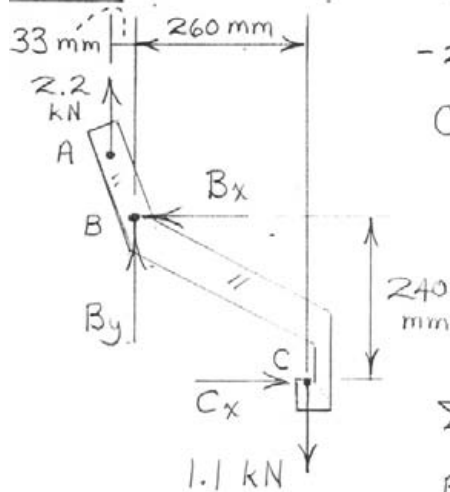
$$F_{AB} = 7400 \text{ lb}$$

The force in member EF is zero, as is disclosed by a FBD of the lifting pad:



For equilibrium, F_{EF} must be concurrent at D, but it cannot be, so it is zero!

4/81



$$\sum M_B = 0:$$

$$-2.2(33) + C_x(240) - 1.1(260) = 0$$

$$C_x = 1.494 \text{ kN}$$

$$\sum F_x = 0:$$

$$1.494 - B_x = 0$$

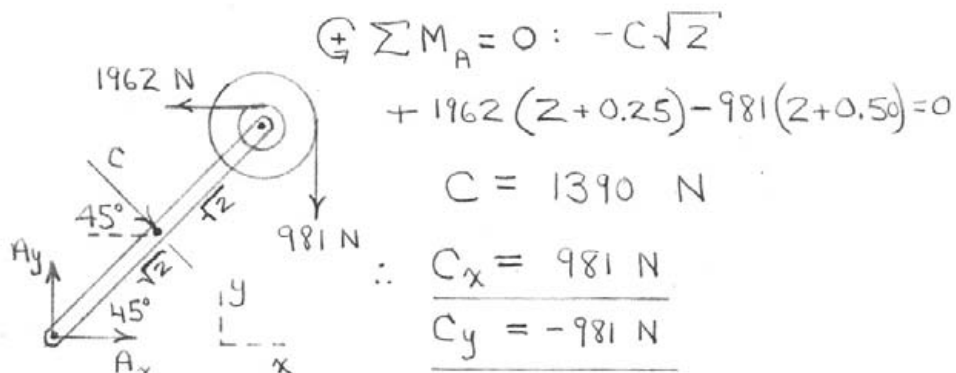
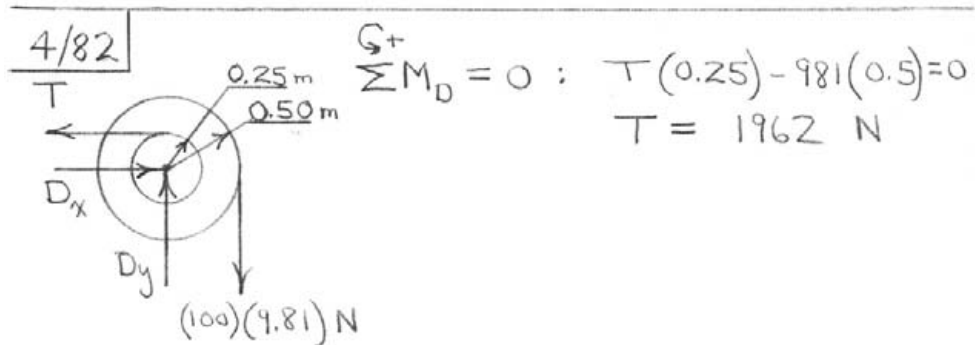
$$B_x = 1.494 \text{ kN}$$

$$\sum F_y = 0:$$

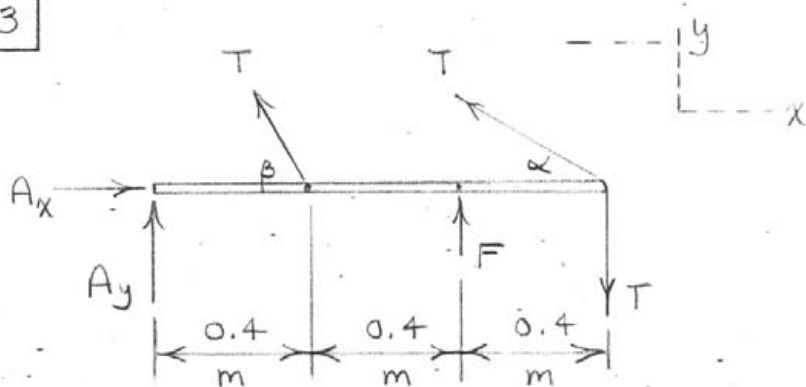
$$B_y + 2.2 - 1.1 = 0$$

$$B_y = -1.1 \text{ kN}$$

$$B = \sqrt{1.494^2 + 1.1^2} = \underline{1.855 \text{ kN}}$$



4/83



$$\begin{cases} T = 60(9.81) = 589 \text{ N} \\ \alpha = \tan^{-1}\left(\frac{0.5}{1.2}\right) = 22.6^\circ \\ \beta = \tan^{-1}\left(\frac{0.5}{0.4}\right) = 51.3^\circ \end{cases}$$

$$\sum M_A = 0: T \sin \beta (0.4) + T \sin \alpha (1.2)$$

$$- T (1.2) + F (0.8) = 0, \quad \underline{F = 314 \text{ N}}$$

(Contact at bottom roller)

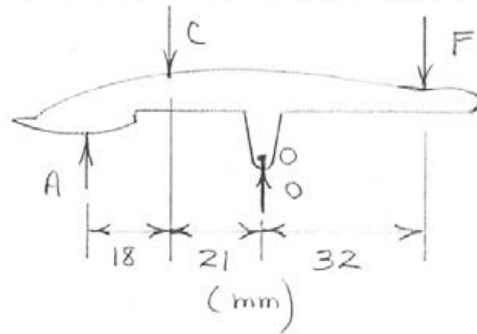
$$\sum F_x = 0: A_x - T \cos \beta - T \cos \alpha = 0, \quad A_x = 911 \text{ N}$$

$$\sum F_y = 0: A_y + T \sin \beta + T \sin \alpha - T + F = 0$$

$$A_y = -411 \text{ N}$$

$$A = \sqrt{A_x^2 + A_y^2} = \underline{999 \text{ N}}$$

4/84

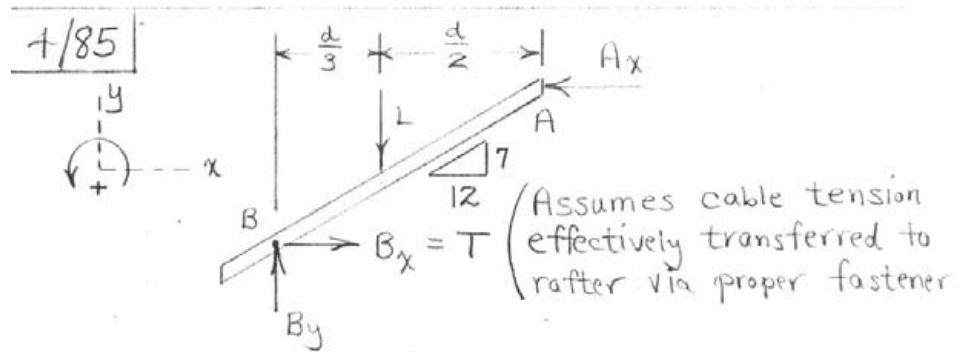


$F = 15 \text{ N}$ when $A = 0$:

$$\uparrow \Sigma M_o = 0: C(21) - 15(32) = 0, \quad C = 22.9 \text{ N}$$

When $F = 0$:

$$\uparrow \Sigma M_o = 0: -A(39) + 22.9(21) = 0, \quad \underline{A = 12.31 \text{ N}}$$

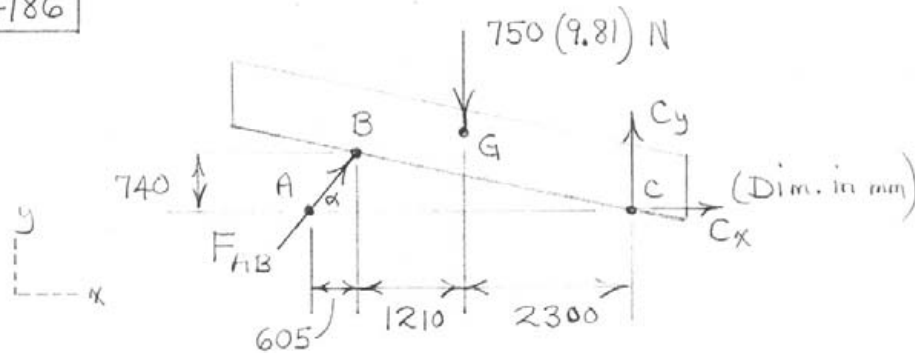


$$\sum F_x = 0 \Rightarrow T = A_x$$

$$\sum M_B = 0 : -L\left(\frac{d}{3}\right) + T\left(\frac{5d}{6} \cdot \frac{7}{12}\right) = 0$$

$$T = \frac{24}{35} L$$

4/86



$$\alpha = \tan^{-1}\left(\frac{740}{605}\right) = 50.7^\circ$$

$$\sum M_C = 0: 750(9.81)(2300) - F_{AB} \sin \alpha (4115) = 0$$

$$F_{AB} = 5310 \text{ N}$$

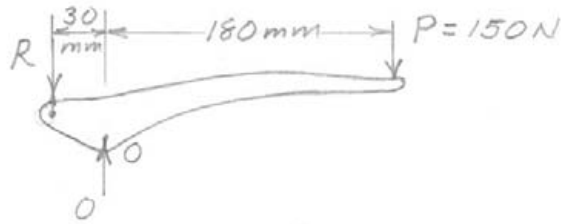
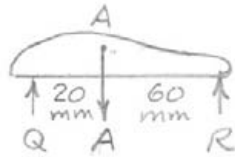
$$\sum F_x = 0: 5310 \cos 50.7^\circ + C_x = 0, \quad C_x = -3360 \text{ N}$$

$$\sum F_y = 0: 5310 \sin 50.7^\circ - 750(9.81) + C_y = 0$$

$$C_y = 3250 \text{ N}$$

$$C = \sqrt{3360^2 + 3250^2} = 4670 \text{ N}$$

4/87

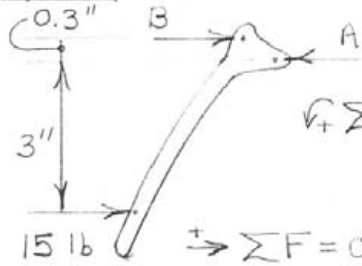


Handle: $\sum M_O = 0$; $30R - 180(150) = 0$, $R = 900 \text{ N}$

Jaw: $\sum M_A = 0$; $20Q - 60(900) = 0$, $Q = 2700 \text{ N}$
 or $Q = 2.7 \text{ kN}$

4/88

Left handle (with use of symmetry):

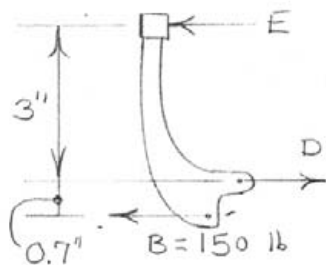


$$\curvearrowright \sum M_A = 0: 15(3) - B(0.3) = 0$$

$$\underline{B = 150 \text{ lb}}$$

$$\rightarrow \sum F = 0: 15 + 150 - A = 0, \quad \underline{A = 165 \text{ lb}}$$

Left jaw:



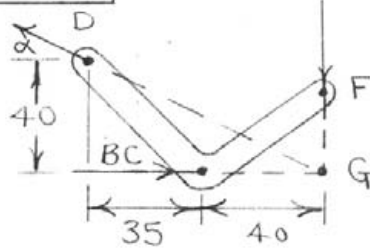
$$\curvearrowright \sum M_D = 0: E(3) - 150(0.7) = 0$$

$$\underline{E = 35 \text{ lb}}$$

$$\rightarrow \sum F = 0: -35 - 150 + D = 0$$

$$\underline{D = 185 \text{ lb}}$$

4/89



DCF is a three-force body; forces intersect at G.

$$\alpha = \tan^{-1}\left(\frac{40}{75}\right) = 28.1^\circ$$

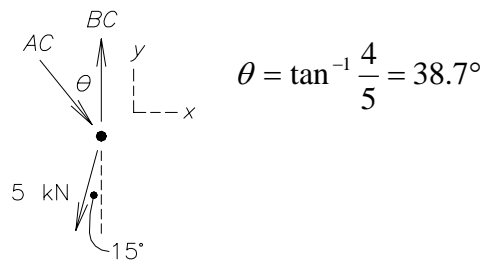
$$\sum F_y = 0: -200 + D \sin \alpha = 0$$

$$D = 425 \text{ N}$$

$$\sum F_x = 0: -D \cos \alpha + BC = 0, \quad BC = 375 \text{ N}$$

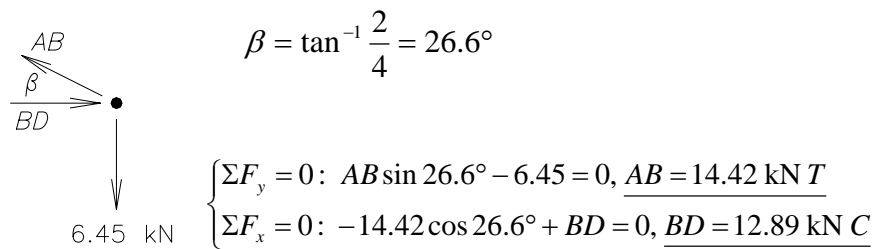
(BC in compression)

Joint C:



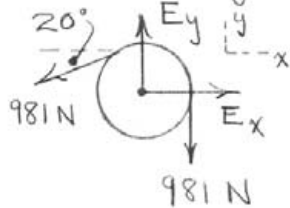
$$\begin{cases} \Sigma F_x = 0: -5 \sin 15^\circ + AC \sin 38.7^\circ = 0, \underline{AC = 2.07 \text{ kN } C} \\ \Sigma F_y = 0: BC - 5 \cos 15^\circ - 2.07 \cos 38.7^\circ = 0, \underline{BC = 6.45 \text{ kN } T} \end{cases}$$

Joint B:



4/90

Pulley :



$$\sum F_x = 0: E_x - 981 \cos 20^\circ = 0$$

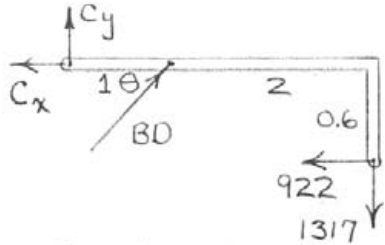
$$E_x = 922 \text{ N}$$

$$\sum F_y = 0: E_y - 981 - 981 \sin 20^\circ = 0$$

$$E_y = 1317 \text{ N}$$

Member CDE :

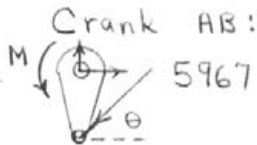
$$\theta = \tan^{-1}\left(\frac{1.15}{1}\right) = 48.99^\circ$$



$$\sum M_C = 0: (BD \sin \theta)(1)$$

$$- 922(0.6) - 1317(3) = 0$$

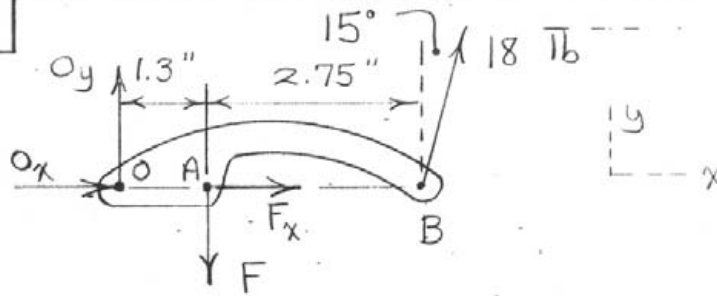
$$BD = 5967 \text{ N}$$



$$\sum M_A = 0: M - (5967 \cos \theta)(0.5) = 0$$

$$M = 1958 \text{ N}\cdot\text{m} \quad \curvearrowright$$

4/91

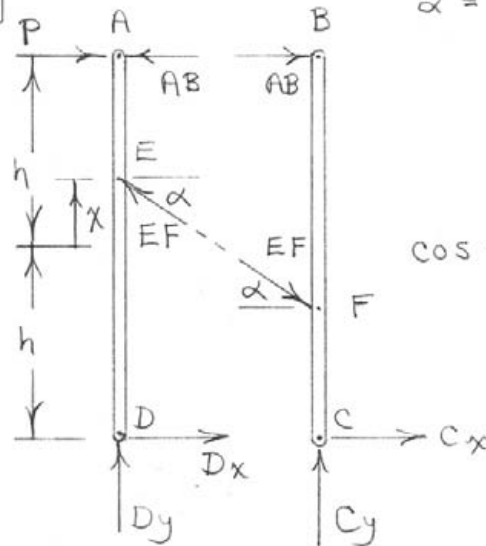


$$\sum M_O = 0 : -F(1.3) + 18 \cos 15^\circ (2.75 + 1.3) = 0$$

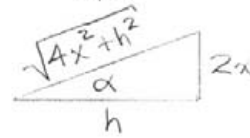
$$F = 54.2 \text{ lb}$$

(Note: Treatment of member OC as a three-force body would yield a constraint relationship between O_x and O_y .)

4/92



$$\alpha = \tan^{-1} \frac{x}{h/2} = \tan^{-1} \frac{2x}{h}$$



$$\cos \alpha = \frac{h}{\sqrt{4x^2 + h^2}}$$

Member AED, $\sum M_D = 0$:

$$AB(2h) + EF \cos \alpha (h+x) - P(2h) = 0 \quad (1)$$

Member BFC, $\sum M_C = 0$:

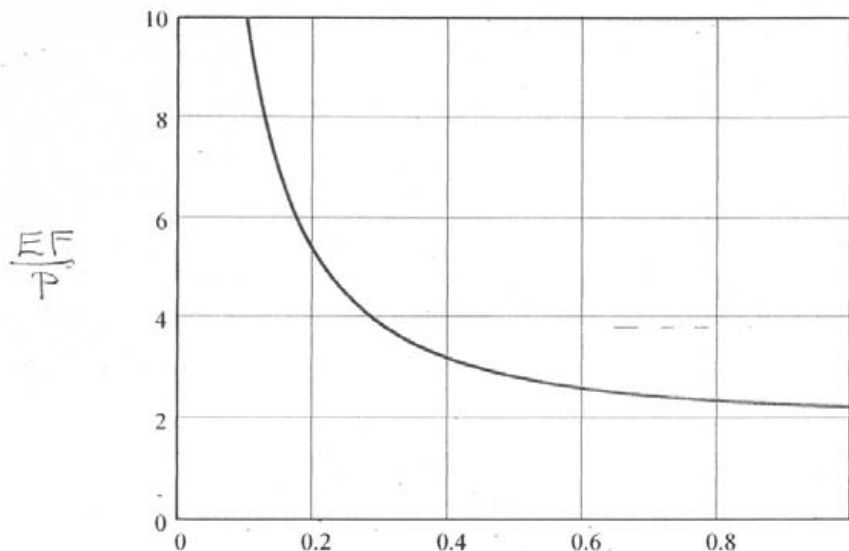
$$-AB(2h) - EF \cos \alpha (h-x) = 0 \quad (2)$$

Add (1) & (2), use the above expression for

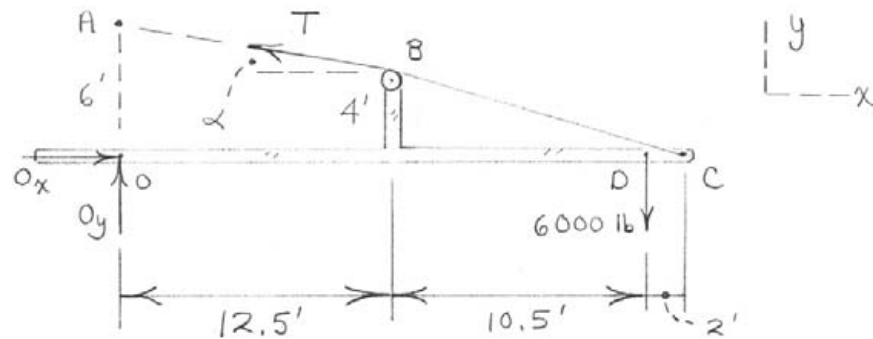
$\cos \alpha$, and solve for EF:

$$EF = P \sqrt{4 + \frac{1}{(x/h)^2}}$$

Undefined @ $x=0$,
but structure
collapses for $x=0$



4/93 FBD of OBC and portion of cable:



$$\alpha = \tan^{-1} \left(\frac{2}{12.5} \right) = 9.09^\circ$$

$$\curvearrowright \sum M_O = 0 : T \cos \alpha (4) + T \sin \alpha (12.5) - 6000 (23) = 0$$

$$T = 23,300 \text{ lb}$$

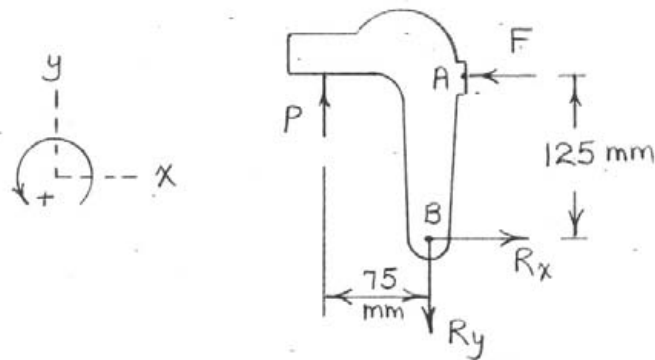
$$\sum F_x = 0 : O_x - 23,300 \cos \alpha = 0$$

$$O_x = 23,000 \text{ lb}$$

$$\sum F_y = 0 : O_y + 23,300 \sin \alpha - 6000 = 0$$

$$O_y = 2320 \text{ lb}$$

$$O = \sqrt{23,000^2 + 2320^2} = 23,100 \text{ lb}$$



For $P = 3 \text{ kN}$:

$$\sum M_B = 0: 125F - 3(75) = 0, \quad F = 1.8 \text{ kN}$$

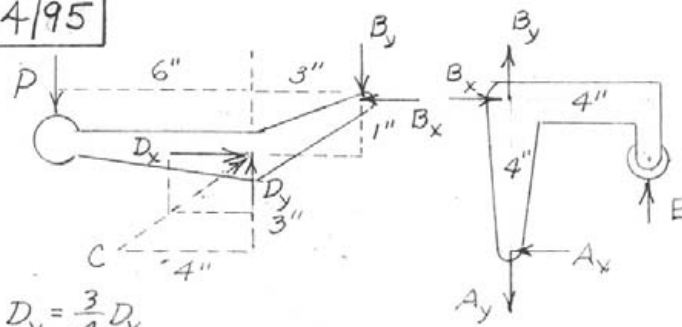
$$\text{For } F = 2(1.8) = 3.6 \text{ kN}, \quad P = 3(2) = 6 \text{ kN}$$

$$\sum F_x = 0: R_x - 3.6 = 0, \quad R_x = 3.6 \text{ kN}$$

$$\sum F_y = 0: -R_y + 6 = 0, \quad R_y = 6 \text{ kN}$$

$$R = \sqrt{3.6^2 + 6^2} = \underline{7.00 \text{ kN}}$$

4/95



$$D_y = \frac{3}{4} D_x$$

Lever BD; $\sum M_B = 0$; $9P + 1D_x - 3(\frac{3}{4}D_x) = 0$, $D_x = \frac{36}{5}P$

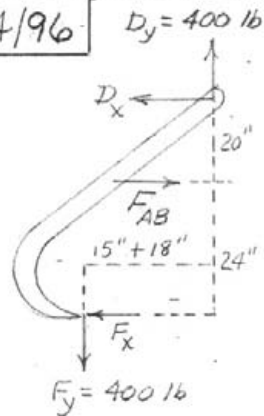
$$D_y = \frac{3}{4} D_x = \frac{27}{5}P$$

$$\sum F_x = 0; B_x = D_x = \frac{36}{5}P$$

$$\sum F_y = 0; B_y + P - \frac{27}{5}P = 0, B_y = \frac{22}{5}P$$

ABE; $\sum M_A = 0$; $4E - 4(\frac{36}{5}P) = 0$, $E = \frac{36}{5}P = 7.2P$

4/96



From tongs as a whole

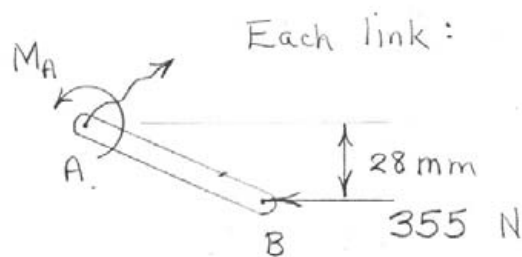
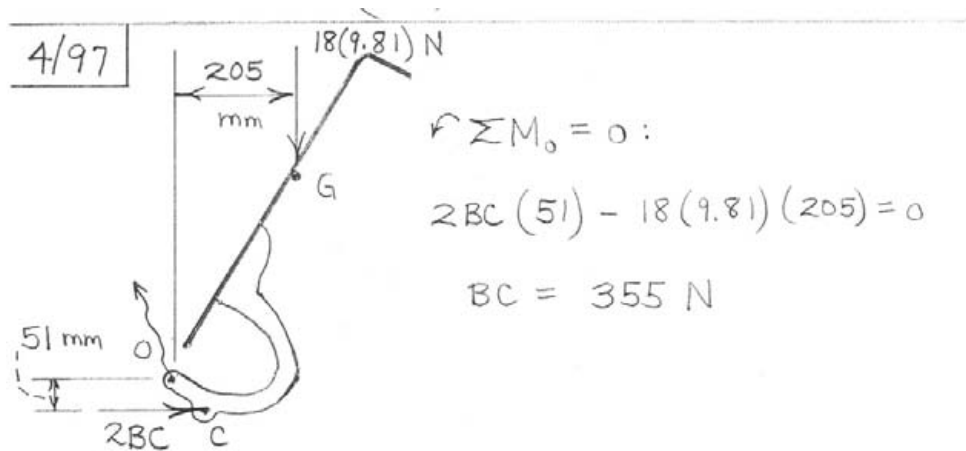
$$D_y = \frac{1}{2}(800) = 400 \text{ lb} = F_y$$

$$\text{From ED, } D_x = \frac{18}{12} D_y = \frac{18}{12}(400) = 600 \text{ lb}$$

From DF, $\sum M_F = 0$;

$$F_{AB}(24) - 600(44) - 400(18 + 15) = 0$$

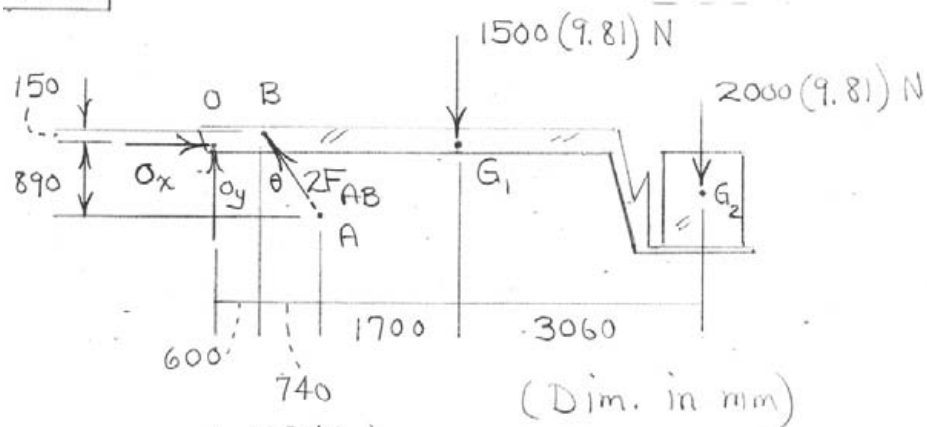
$$F_{AB} = \underline{1650 \text{ lb}} \text{ tension}$$



$$\sum M_A = 0: M_A - 355(28) = 0$$

$$M_A = 9940 \text{ N}\cdot\text{mm} \text{ or } \underline{9.94 \text{ N}\cdot\text{m CCW}}$$

4/98

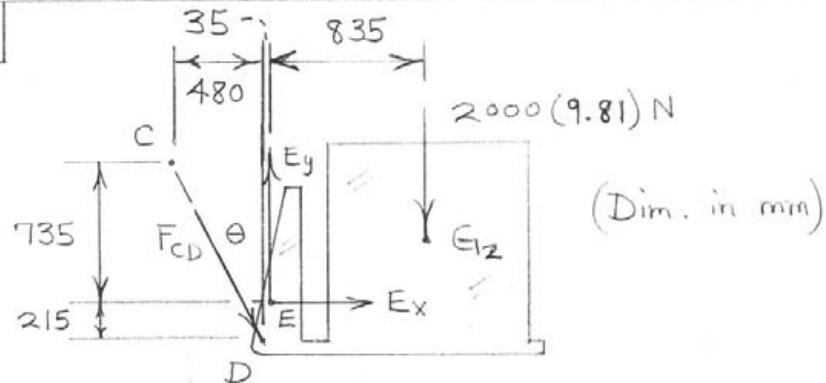


$$\theta = \tan^{-1} \left(\frac{740}{1040} \right) = 35.4^\circ$$

$$\sum M_O = 0: 2F_{AB} \cos \theta (600) + 2F_{AB} \sin \theta (150) - 1500(9.81)(3040) - 2000(9.81)(6100) = 0$$

$$F_{AB} = 142800 \text{ N or } \underline{142.8 \text{ kN}}$$

4/99



$$\theta = \tan^{-1} \left(\frac{480}{950} \right) = 26.8^\circ$$

$$\begin{aligned} \uparrow + \sum M_E = 0: & -2000(9.81)(835) + F_{CD} \cos \theta (35) \\ & + F_{CD} \sin \theta (215) = 0 \end{aligned}$$

$$F_{CD} = 127,800 \text{ N}$$

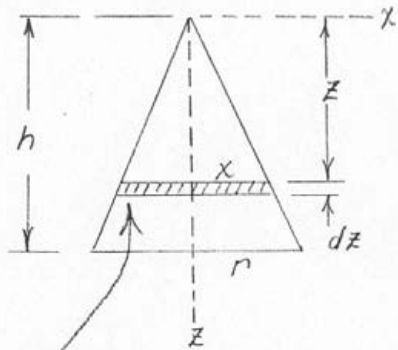
$$\text{or } \underline{127.8 \text{ kN}}$$

5/1

The horizontal coordinate to the centroid is $14 - \frac{1}{3}(14-2) = \underline{10}$

5/10

$$x = \frac{r}{h} z, \quad dV = \pi x^2 dz = \pi \frac{r^2}{h^2} z^2 dz$$



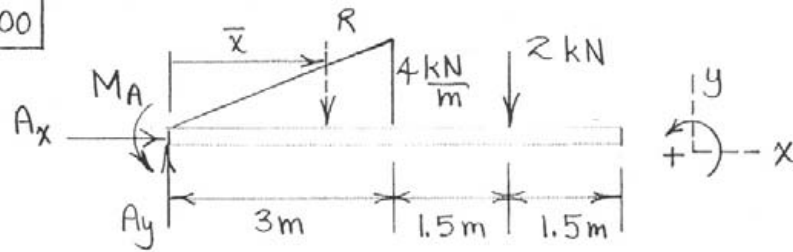
$$V = \pi \frac{r^2}{h^2} \int_0^h z^2 dz = \frac{\pi r^2 h}{3}$$

$$\int z dV = \pi \frac{r^2}{h^2} \int_0^h z^3 dz = \frac{\pi r^2 h^2}{4}$$

$$\bar{z} = \int z dV / V = \underline{3h/4}$$

(Disk-shaped element viewed edge-on.)

5/100



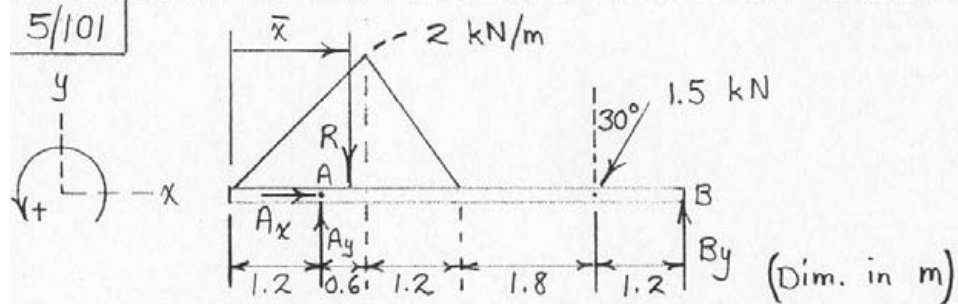
$$R = \frac{1}{2}(3)(4) = 6 \text{ kN} @ \bar{x} = \frac{2}{3}(3) = 2 \text{ m}$$

$$\sum M_A = 0: M_A - 6(2) - 2(4.5) = 0, \quad M_A = 21 \text{ kN}\cdot\text{m}$$

$$\sum F_y = 0: A_y - 6 - 2 = 0, \quad A_y = 8 \text{ kN}$$

$$\sum F_x = 0: A_x = 0$$

5/101



$$R = \frac{1}{2} (1.2 + 0.6 + 1.2) (2) = 3 \text{ kN}$$

$$\bar{x} = \frac{(1.2 + 0.6) + (1.2 + 0.6 + 1.2)}{3} = 1.6 \text{ m}$$

$$\sum F_x = 0 : A_x - 1.5 \sin 30^\circ = 0, \quad \underline{A_x = 750 \text{ N}}$$

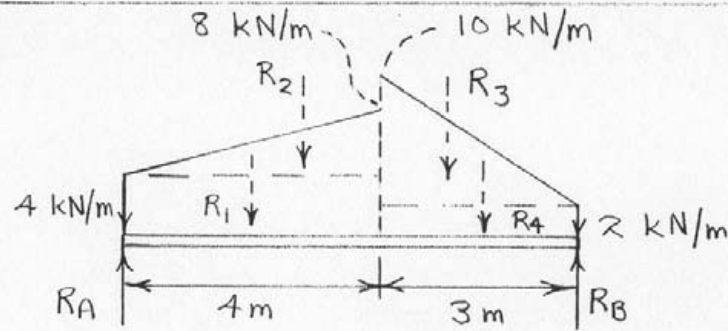
$$\sum M_A = 0 : -3 (1.6 - 1.2) - 1.5 \cos 30^\circ (3.6) - B_y (4.8) = 0$$

$$\underline{B_y = 1.224 \text{ kN}}$$

$$\sum F_y = 0 : A_y - 3 - 1.5 \cos 30^\circ + 1.224 = 0$$

$$\underline{A_y = 3.07 \text{ kN}}$$

5/102



$$R_1 = 4(4) = 16 \text{ kN}, \quad R_2 = \frac{1}{2}(4)(4) = 8 \text{ kN}$$

$$R_3 = \frac{1}{2}(8)(3) = 12 \text{ kN}, \quad R_4 = 2(3) = 6 \text{ kN}$$

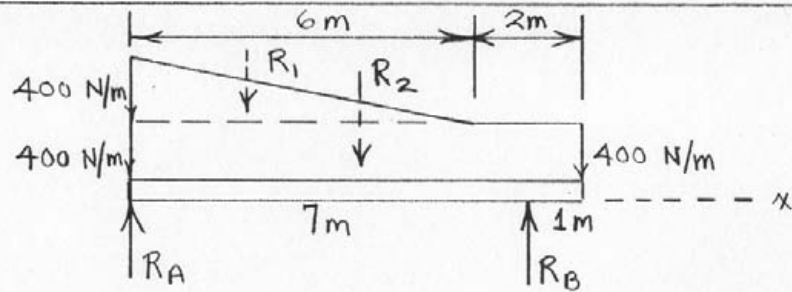
$$+\circlearrowleft \sum M_A = 0: 7R_B - 16(2) - 8\left(\frac{2}{3}4\right) - 12(4+1)$$

$$- 6(4+1.5) = 0, \quad \underline{R_B = 20.9 \text{ kN}}$$

$$+\uparrow \sum F = 0: R_A + 20.9 - (16 + 8 + 12 + 6) = 0$$

$$\underline{R_A = 21.1 \text{ kN}}$$

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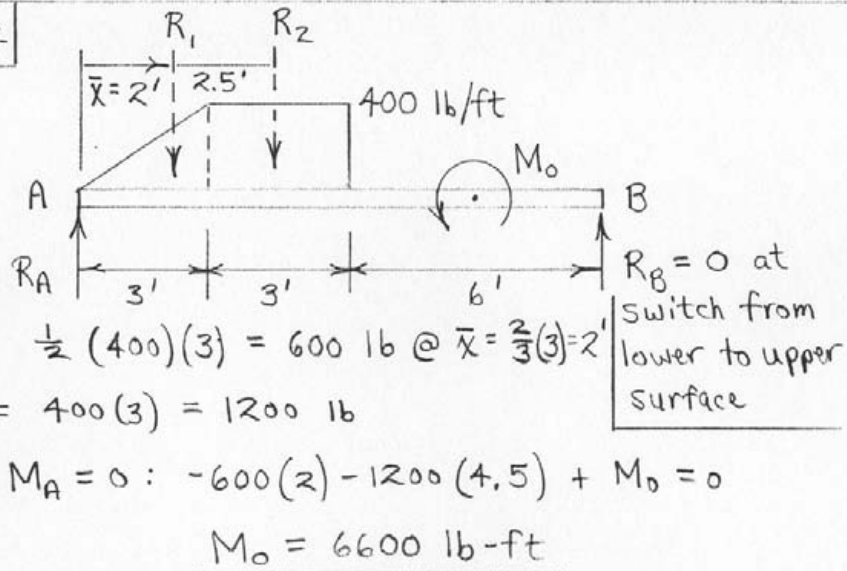
$$R_1 = \frac{1}{2}(400)(6) = 1200 \text{ N @ } \bar{x}_1 = \frac{1}{3}(6) = 2 \text{ m}$$

$$R_2 = 400(2) = 800 \text{ N @ } \bar{x}_2 = \frac{1}{2}(2) = 1 \text{ m}$$

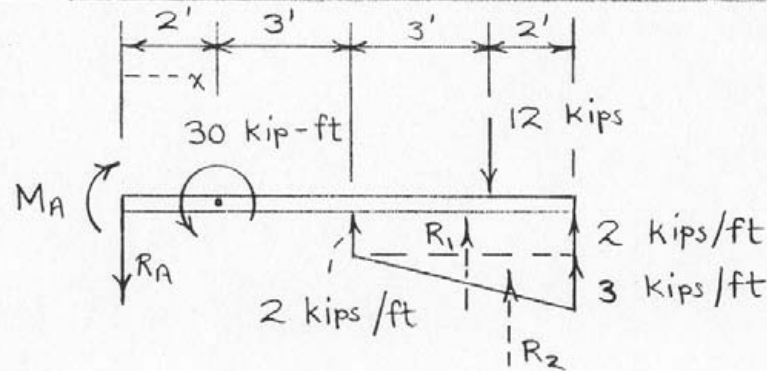
$$\curvearrowleft + \sum M_A = 0: R_B(7) - 1200(2) - 800(4) = 0, \underline{R_B = 2170 \text{ N}}$$

$$+\uparrow \sum F = 0: R_A - 1200 - 800 + 2170 = 0, \underline{R_A = 2230 \text{ N}}$$

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$$R_1 = 2(5) = 10 \text{ kips @ } \bar{x}_1 = 7.5'$$

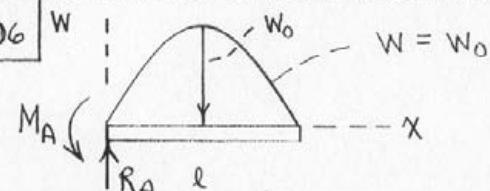
$$R_2 = \frac{1}{2}(3)(5) = 7.5 \text{ kips @ } \bar{x}_2 = 8.33'$$

$$\circlearrowleft \sum M_A = 0 : M_A - 30 + 12(8) - 10(7.5) - 7.5(8.33) = 0$$

$$\underline{M_A = 71.5 \text{ kip-ft}}$$

$$+\uparrow \sum F = 0 : -R_A + 10 + 7.5 - 12 = 0, \quad \underline{R_A = 5.5 \text{ kips}}$$

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$w = w_0 \sin \frac{\pi}{l} x$

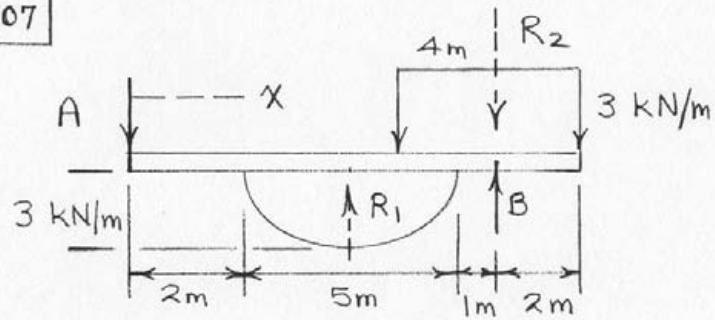
$$R = \int w dx = \int_0^l w_0 \sin \frac{\pi}{l} x = -w_0 \frac{l}{\pi} \cos \frac{\pi}{l} x \Big|_0^l = \frac{2w_0 l}{\pi}$$

$\bar{x} = \frac{l}{2}$, by inspection

$$\uparrow \sum M_A = 0: M_A - \frac{2w_0 l}{\pi} \left(\frac{l}{2} \right) = 0, \quad M_A = \frac{w_0 l^2}{\pi}$$

$$+\uparrow \sum F = 0: R_A - \frac{2w_0 l}{\pi} = 0, \quad R_A = \frac{2w_0 l}{\pi}$$

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$$R_1 = \frac{\pi(5)(3)}{4} = 11.78 \text{ kN} \quad @ \quad \bar{x}_1 = 4.5 \text{ m}$$

$$R_2 = 4(3) = 12 \text{ kN} \quad @ \quad \bar{x}_2 = 8 \text{ m}$$

$$\circlearrowleft \sum M_B = 0 : A(8) - 11.78(3.5) = 0, \quad \underline{A = 5.15 \text{ kN}}$$

$$+\uparrow \sum F = 0 : -5.15 + 11.78 - 12 + B = 0, \quad \underline{B = 5.37 \text{ kN}}$$

Contact at A is at the upper roller.

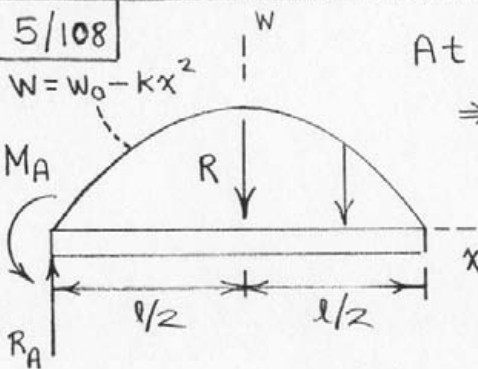
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$W = W_0 - kx^2$

At $x = \frac{l}{2}$, $W = 0 = W_0 - k\left(\frac{l}{2}\right)^2$
 $\Rightarrow k = \frac{4W_0}{l^2}$

$W = W_0\left(1 - \frac{4}{l^2}x^2\right)$

$\bar{x} = 0$, by inspection

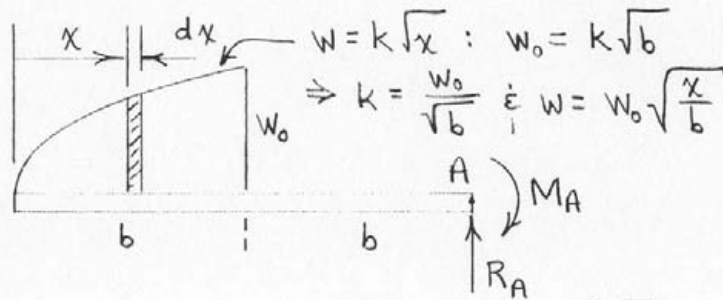


$R = \int W dx = 2 \int_0^{l/2} W_0\left(1 - \frac{4}{l^2}x^2\right) dx$
 $= 2W_0 \left[x - \frac{4}{3l^2}x^3 \right]_0^{l/2} = \frac{2}{3}W_0l$

$+\uparrow \Sigma F = 0: R_A - \frac{2}{3}W_0l = 0, \quad R_A = \frac{2}{3}W_0l$

$\curvearrowright + \Sigma M_A = 0: M_A - \frac{2}{3}W_0l\left(\frac{l}{2}\right) = 0, \quad M_A = \frac{1}{3}W_0l^2$

5/109



The force on dx is $dF = w dx = w_0 \sqrt{\frac{x}{b}} dx$

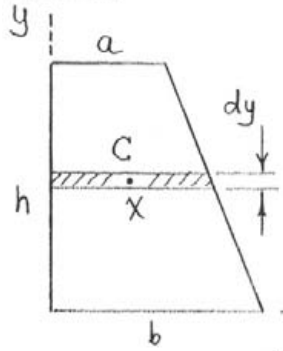
$$\text{Total load } F = \int dF = \int_0^b w_0 \sqrt{\frac{x}{b}} dx = \frac{2}{3} w_0 b$$

$$\text{So } \underline{R_A = \frac{2}{3} w_0 b}$$

The moment of dF about A is $w dx (2b - x)$

$$\begin{aligned} M_A &= \int_0^b w (2b - x) dx = \int_0^b w_0 \sqrt{\frac{x}{b}} (2b - x) dx \\ &= \frac{w_0}{\sqrt{b}} \int_0^b (2b\sqrt{x} - x^{3/2}) dx = \underline{\underline{\frac{14}{15} w_0 b^2}} \end{aligned}$$

5/11



$$x = \left(\frac{a-b}{h}\right)y + b$$

$$dA = x dy = \left[\left(\frac{a-b}{h}\right)y + b\right] dy$$

$$A = \left(\frac{a+b}{2}\right)h = \frac{h}{2}(a+b)$$

$$\int x_c dA = \int_0^h \frac{x}{2} \left[\left(\frac{a-b}{h}\right)y + b\right] dy$$

$$x = \frac{1}{2} \int_0^h \left[\left(\frac{a-b}{h}\right)y + b\right]^2 dy$$

$$\int x_c dA = \frac{1}{2} \int_0^h \left[\left(\frac{a-b}{h}\right)^2 y^2 + 2b\left(\frac{a-b}{h}\right)y + b^2\right] dy$$

$$= \frac{1}{2} \left[\left(\frac{a-b}{h}\right)^2 \frac{y^3}{3} + 2b\left(\frac{a-b}{h}\right) \frac{y^2}{2} + b^2 y \right]_0^h$$

$$= \frac{h}{6} (a^2 + b^2 + ab)$$

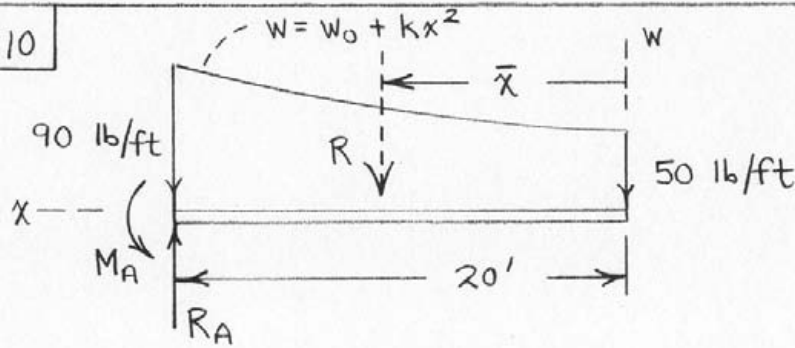
$$\int y_c dA = \int_0^h y \left[\left(\frac{a-b}{h}\right)y + b\right] dy = \int_0^h \left[\left(\frac{a-b}{h}\right)y^2 + by\right] dy$$

$$= \left[\left(\frac{a-b}{h}\right) \frac{y^3}{3} + b \frac{y^2}{2} \right]_0^h = \frac{h^2}{3} \left(a + \frac{b}{2}\right)$$

$$\bar{x} = \frac{\int x_c dA}{A} = \frac{\frac{h}{6} (a^2 + b^2 + ab)}{\frac{h}{2} (a+b)} = \frac{a^2 + b^2 + ab}{3(a+b)}$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{\frac{h^2}{3} \left(a + \frac{b}{2}\right)}{\frac{h}{2} (a+b)} = \frac{h(2a+b)}{3(a+b)}$$

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$$\text{At } x=0, w=50 = w_0$$

$$\text{At } x=20', w=90 = 50 + k(20)^2, k = \frac{1}{10} \text{ lb/ft}^3$$

$$\text{So } w = 50 + \frac{x^2}{10} \quad (\text{lb/ft})$$

$$R = \int w dx = \int_0^{20} \left(50 + \frac{x^2}{10}\right) dx = \left[50x + \frac{x^3}{30}\right]_0^{20} = 1267 \text{ lb}$$

$$\bar{x} = \frac{\int x w dx}{R} = \frac{\int_0^{20} x \left(50 + \frac{x^2}{10}\right) dx}{1267} = \frac{\left[\frac{50}{2} x^2 + \frac{x^4}{40}\right]_0^{20}}{1267}$$

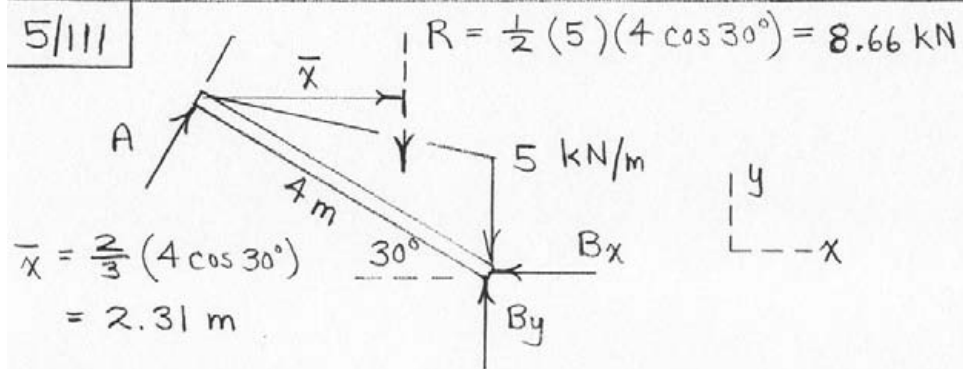
$$= 11.05 \text{ ft}$$

$$+\uparrow \sum F = 0: R_A - 1267 = 0,$$

$$R_A = 1267 \text{ lb}$$

$$\curvearrowright \sum M_A = 0: M_A - 1267(20 - 11.05) = 0, M_A = 11.33(10^3) \text{ lb-ft}$$

5/111



$$\sum F_x = 0 : A \sin 30^\circ - B_x = 0$$

$$\sum F_y = 0 : A \cos 30^\circ + B_y - 8.66 = 0$$

$$\circlearrowleft \sum M_B = 0 : -A(4) + 8.66\left(\frac{1}{3} 4 \cos 30^\circ\right) = 0$$

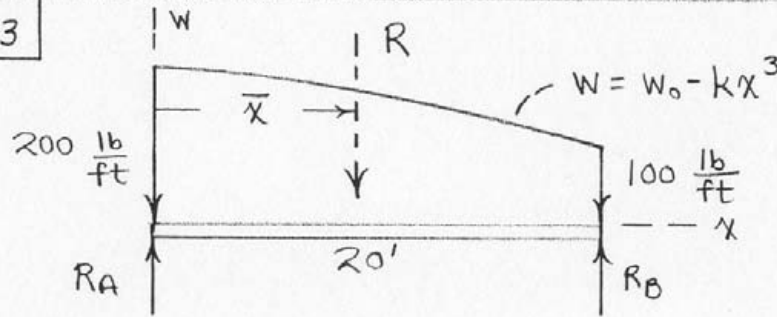
$$A = 2.5 \text{ kN}, \quad B_x = 1.25 \text{ kN}, \quad B_y = 6.50 \text{ kN}$$

$$B = \sqrt{B_x^2 + B_y^2} = 6.61 \text{ kN}$$

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$R_1 = 8(3) = 24 \text{ kN}$
 $w = w_0 - kx^2$
 $w_0 = 8 \text{ kN/m}$
 At $x = 2 \text{ m}$:
 $0 = 8 - k(2)^2, \quad k = 2 \text{ kN/m}^3$
 $R_2 = \int w dx = \int_0^2 (8 - 2x^2) dx = \left(8x - \frac{2}{3}x^3 \right)_0^2$
 $= 10.67 \text{ kN}$
 $\bar{x} = \frac{\int x w dx}{R} = \frac{\int_0^2 (8x - 2x^3) dx}{10.67} = \frac{\left(4x^2 - \frac{1}{2}x^4 \right)_0^2}{10.67}$
 $= 0.75 \text{ m}$
 Equilibrium considerations $\begin{matrix} y \\ \curvearrowleft \\ x \end{matrix}$
 $\sum F_x = 0 \Rightarrow A_x = 0$
 $\sum F_y = 0: A_y - 24 - 10.67 = 0, \quad A_y = 34.7 \text{ kN}$
 $\sum M_A = 0: M_A - 24(1.5) - 10.67(3.75) = 0$
 $M_A = 76 \text{ kN}\cdot\text{m}$

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$$\text{At } x=0, W = W_0 = 200 \text{ lb/ft}$$

$$\text{At } x=20', W = 200 - k(20)^3 = 100, \quad k = \frac{1}{80} \frac{\text{lb}}{\text{ft}^4}$$

$$\text{So } W = 200 - \frac{x^3}{80} \quad (\text{lb/ft})$$

$$R = \int W dx = \int_0^{20} \left(200 - \frac{x^3}{80}\right) dx = \left[200x - \frac{x^4}{320}\right]_0^{20} = 3500 \text{ lb}$$

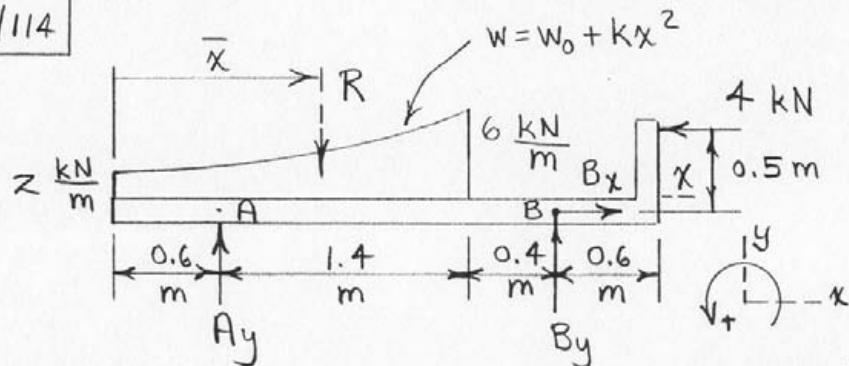
$$\bar{x} = \frac{\int x W dx}{R} = \frac{\int_0^{20} \left(200 - \frac{x^3}{80}\right) x dx}{3500} = \frac{\left[100x^2 - \frac{x^5}{400}\right]_0^{20}}{3500}$$

$$= 9.14'$$

$$\curvearrowleft \sum M_A = 0: 20 R_B - 3500(9.14) = 0, \quad \underline{R_B = 1600 \text{ lb}}$$

$$+\uparrow \sum F = 0: R_A + 1600 - 3500 = 0, \quad \underline{R_A = 1900 \text{ lb}}$$

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$$w = 2 + kx^2 : 6 = 2 + k(2)^2, \quad k = 1 \text{ kN/m}^3, \quad w = 2 + x^2$$

$$R = \int w dx = \int_0^2 (2 + x^2) dx = 2x + \frac{x^3}{3} \Big|_0^2 = 6.67 \text{ kN}$$

$$\bar{x} = \frac{\int x w dx}{R} = \frac{\int_0^2 (2x + x^3) dx}{6.67} = \frac{x^2 + \frac{x^4}{4}}{6.67} \Big|_0^2 = 1.2 \text{ m}$$

$$\sum F_x = 0: B_x - 4 = 0, \quad B_x = 4 \text{ kN}$$

$$\sum M_B = 0: 4(0.5) + 6.67(2.4 - 1.2) - A_y(1.8) = 0$$

$$A_y = 5.56 \text{ kN}$$

$$\sum F_y = 0: 5.56 + B_y - 6.67 = 0, \quad B_y = 1.11 \text{ kN}$$

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Diagram details: Beam length 5m. Point load 2 kN at left end. Reaction RA at 1m, RB at 5m. Distributed load w' from 2 kN/m to 4 kN/m. Segments: 1m, 1m, 3m.

$$w' = kx'^2$$

At $x' = 3\text{ m}$:

$$4 = k(3)^2$$

$$k = \frac{4}{9} \text{ N/m}^3$$

$$w' = \frac{4}{9} x'^2$$

$$R_2 = \int_0^3 w' dx' = \int_0^3 \frac{4}{9} x'^2 = \frac{4}{9} \frac{x'^3}{3} \Big|_0^3 = 4 \text{ kN}$$

$$\bar{x}'_2 = \int x' w' dx' / R_2 = \frac{1}{4} \int_0^3 \frac{4}{9} x'^3 = \frac{1}{4} \frac{4}{9} \frac{x'^4}{4} \Big|_0^3 = 2.25 \text{ m}$$

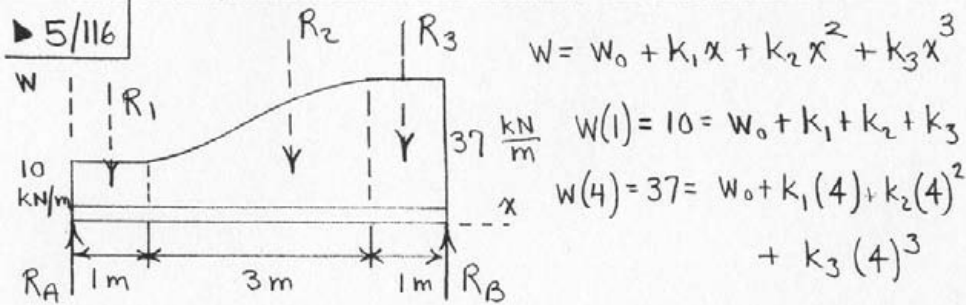
$$R_1 = 2(5) = 10 \text{ kN @ } \bar{x}_1 = 2.5 \text{ m} \quad \bar{x}_2 = 4.25 \text{ m}$$

$$\uparrow + \sum M_A = 0: -10(1.5) - 4(3.25) + 4R_B = 0$$

$$\underline{R_B = 7 \text{ kN}}$$

$$\uparrow + \sum F = 0: 7 + R_A - 4 - 10 = 0, \quad \underline{R_A = 7 \text{ kN}}$$

► 5/116



$$w = w_0 + k_1x + k_2x^2 + k_3x^3$$

$$w(1) = 10 = w_0 + k_1 + k_2 + k_3$$

$$w(4) = 37 = w_0 + k_1(4) + k_2(4)^2 + k_3(4)^3$$

$$\frac{dw}{dx} = k_1 + 2k_2x + 3k_3x^2 : \begin{cases} 0 = k_1 + 2k_2(1) + 3k_3(1)^2 \\ 0 = k_1 + 2k_2(4) + 3k_3(4)^2 \end{cases}$$

Solve simultaneously to get $w = 21 - 24x + 15x^2 - 2x^3$

$$R_2 = \int w dx = \int_1^4 (21 - 24x + 15x^2 - 2x^3) dx$$

$$= \left[21x - 12x^2 + 5x^3 - \frac{1}{2}x^4 \right]_1^4 = 70.5 \text{ kN}$$

$$\bar{x}_2 = \frac{1}{R_2} \int x w dx = \frac{1}{70.5} \int_1^4 (21 - 24x + 15x^2 - 2x^3) x dx$$

$$= \frac{1}{70.5} \left[\frac{21}{2}x^2 - 8x^3 + \frac{15}{4}x^4 - \frac{2}{5}x^5 \right]_1^4 = 2.84 \text{ m}$$

$$R_1 = 10(1) = 10 \text{ kN @ } \bar{x}_1 = 0.5 \text{ m}$$

$$R_3 = 37(1) = 37 \text{ kN @ } \bar{x}_3 = 4.5 \text{ m}$$

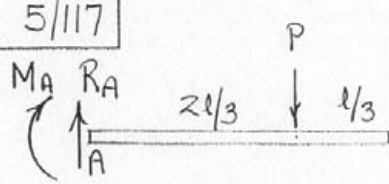
$$\sum M_A = 0 : 5R_B - 10(0.5) - 70.5(2.84) - 37(4.5) = 0$$

$$R_B = 74.4 \text{ kN}$$

$$\sum F = 0 : 74.4 - 10 - 70.5 - 37 + R_A = 0, \quad R_A = 43.1 \text{ kN}$$

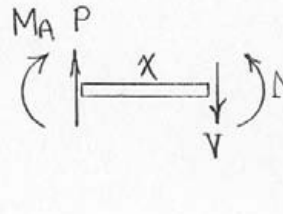
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$\uparrow \Sigma F = 0 \Rightarrow R_A = P$
 $\curvearrowright \Sigma M_A = 0: -M_A - P\left(\frac{2l}{3}\right) = 0$
 $M_A = -\frac{2Pl}{3}$



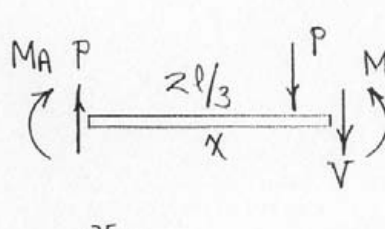
$$0 < x < \frac{2l}{3} :$$

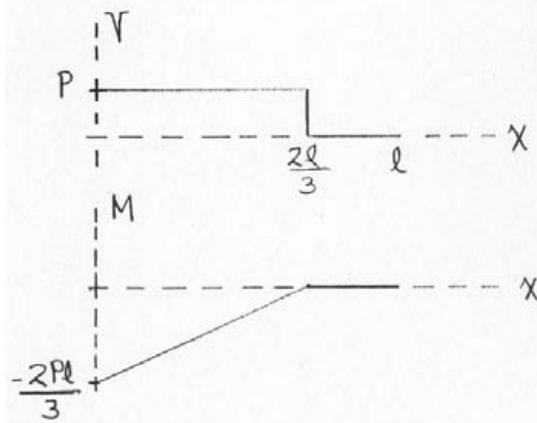
$\uparrow \Sigma F = 0 \Rightarrow V = P$
 $\curvearrowright \Sigma M = 0: \frac{2Pl}{3} - Px + M = 0$
 $M = P\left(x - \frac{2l}{3}\right)$



$$\frac{2l}{3} < x < l :$$

$\uparrow \Sigma F = 0: P - P - V = 0, V = 0$
 $\curvearrowright \Sigma M = 0: \frac{2Pl}{3} - P\left(\frac{2l}{3}\right) + M = 0$
 $M = 0$

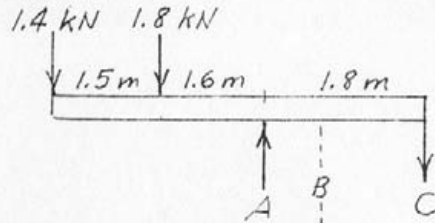




At $x = \frac{l}{2} :$

$V = P$
 $M = P\left(\frac{l}{2} - \frac{2l}{3}\right)$
 $= -\frac{Pl}{6}$

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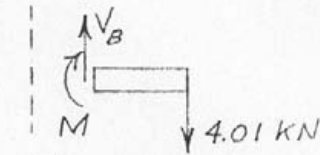
$$\sum M_A = 0;$$

$$1.4(3.1) + 1.8(1.6) - 1.8C = 0$$

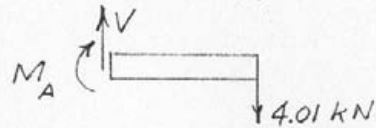
$$C = 4.01 \text{ kN}$$

$$\sum F = 0; A - 4.01 - 1.4 - 1.8 = 0$$

$$A = 7.21 \text{ kN}$$



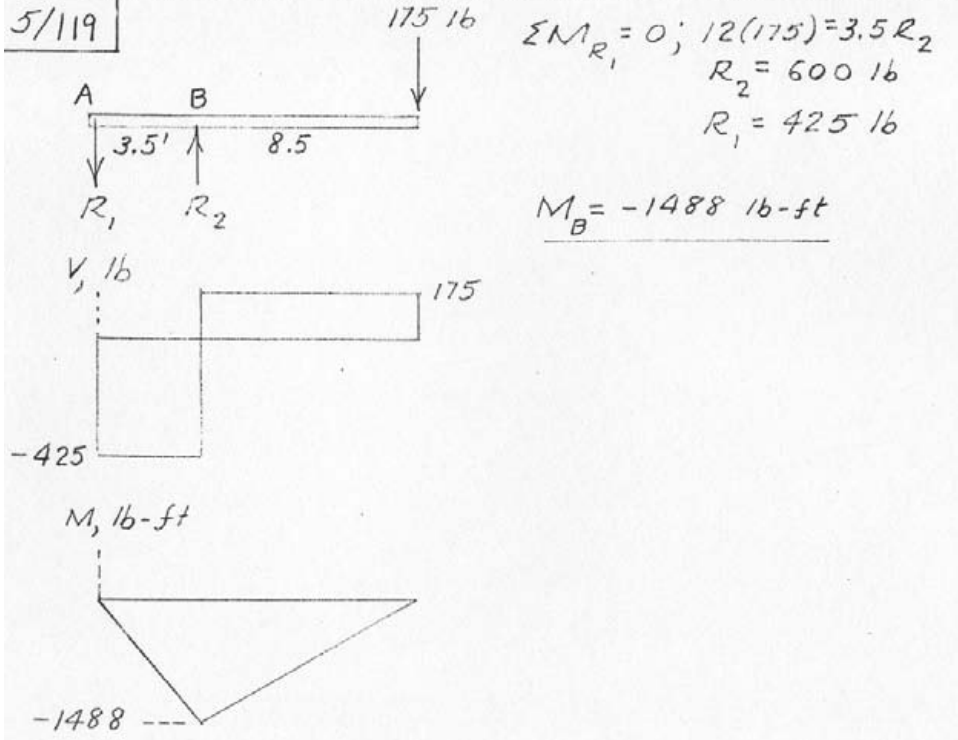
$$\sum F = 0; \underline{V_B = 4.01 \text{ kN}}$$



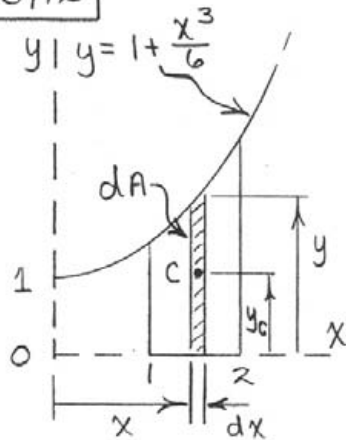
$$\sum M_A = 0; M_A + 4.01(1.8) = 0$$

$$\underline{M_A = -7.22 \text{ kN}\cdot\text{m}}$$

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$$dA = y dx = \left(1 + \frac{x^3}{6}\right) dx$$

$$A = \int dA = \int_1^2 \left(1 + \frac{x^3}{6}\right) dx$$

$$= x + \frac{x^4}{24} \Big|_1^2 = \frac{39}{24}$$

$$\int x_c dA = \int_1^2 x \left(1 + \frac{x^3}{6}\right) dx$$

$$= \frac{x^2}{2} + \frac{x^5}{30} \Big|_1^2 = \frac{38}{15}$$

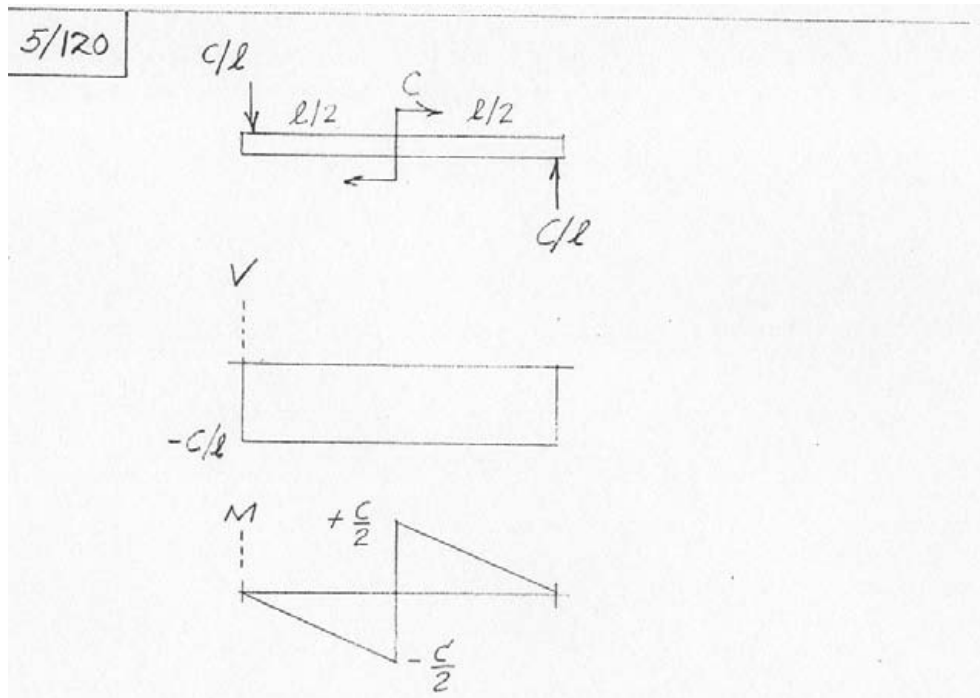
$$\int y_c dA = \int \frac{y}{2} y dx = \int \frac{y^2}{2} dx = \frac{1}{2} \int_1^2 \left(1 + \frac{x^3}{6}\right)^2 dx$$

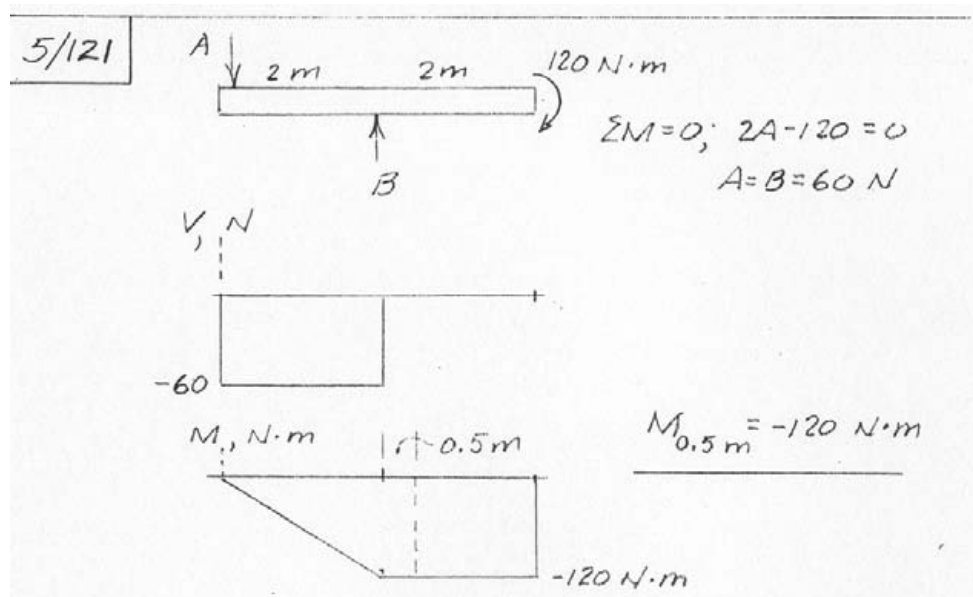
$$= \frac{1}{2} \int_1^2 \left(1 + \frac{x^3}{3} + \frac{x^6}{36}\right) dx = \frac{1}{2} \left(x + \frac{x^4}{12} + \frac{x^7}{252}\right) \Big|_1^2$$

$$= \frac{347}{252}$$

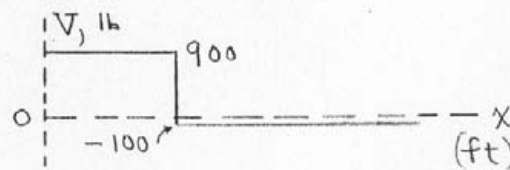
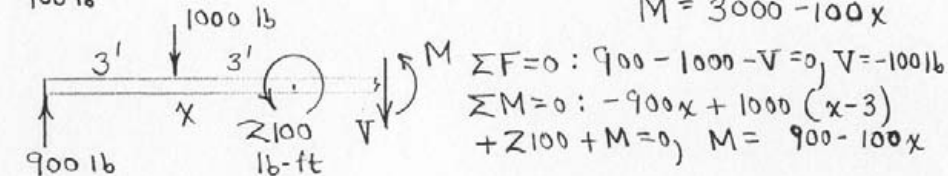
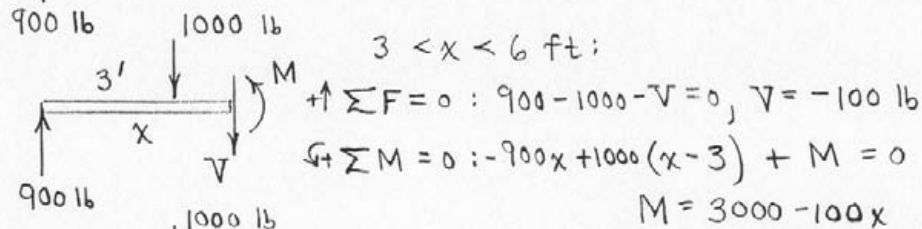
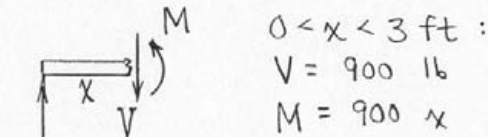
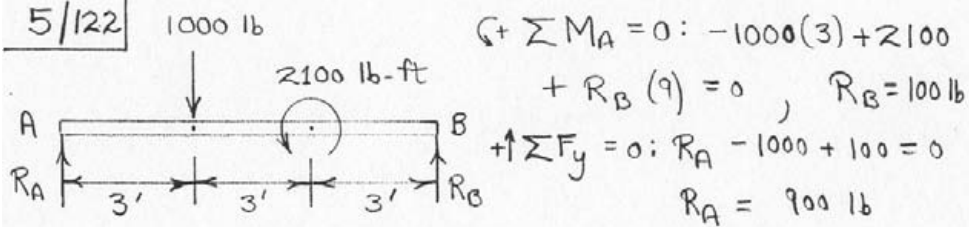
$$\text{So } \bar{x} = \frac{\int x_c dA}{\int dA} = \frac{38/15}{39/24} = \underline{1.559}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA} = \frac{347/252}{39/24} = \underline{0.847}$$



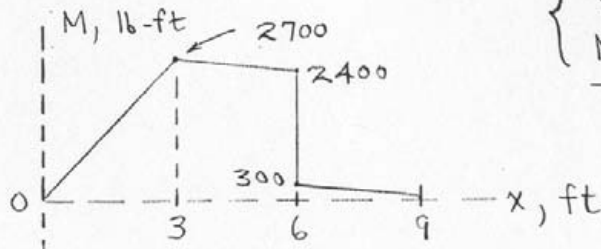


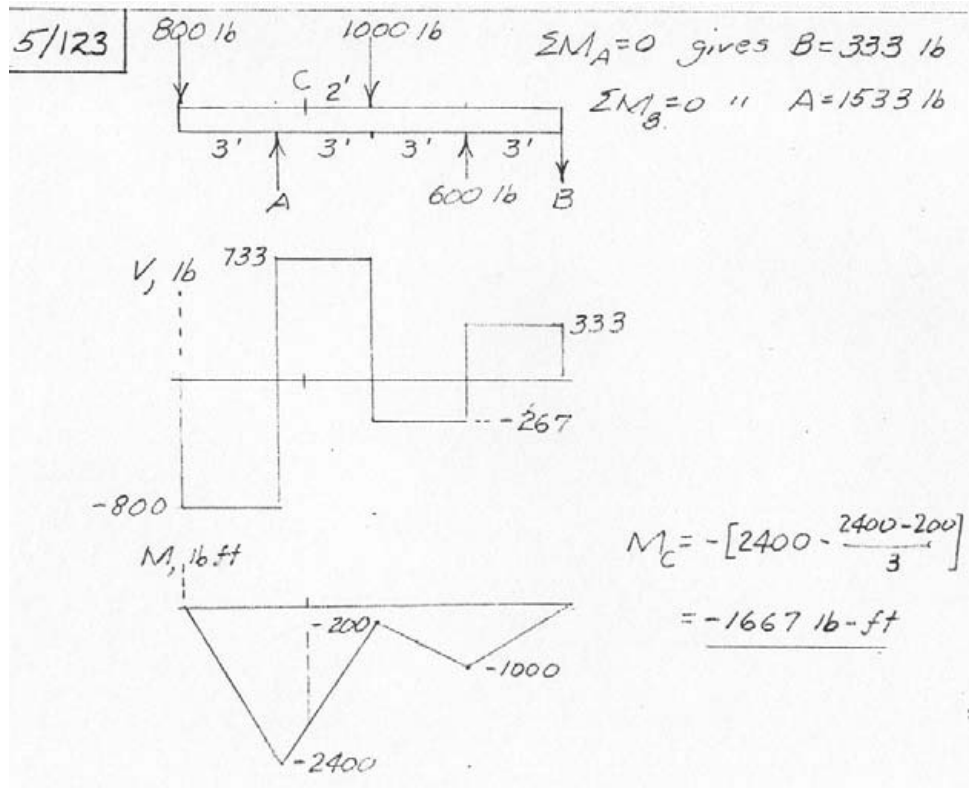
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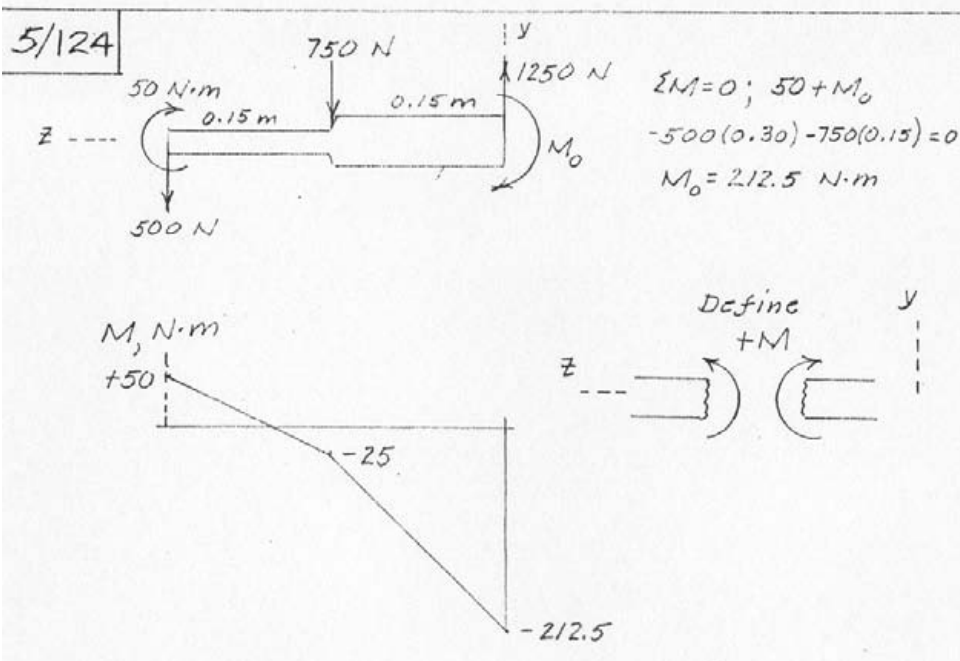


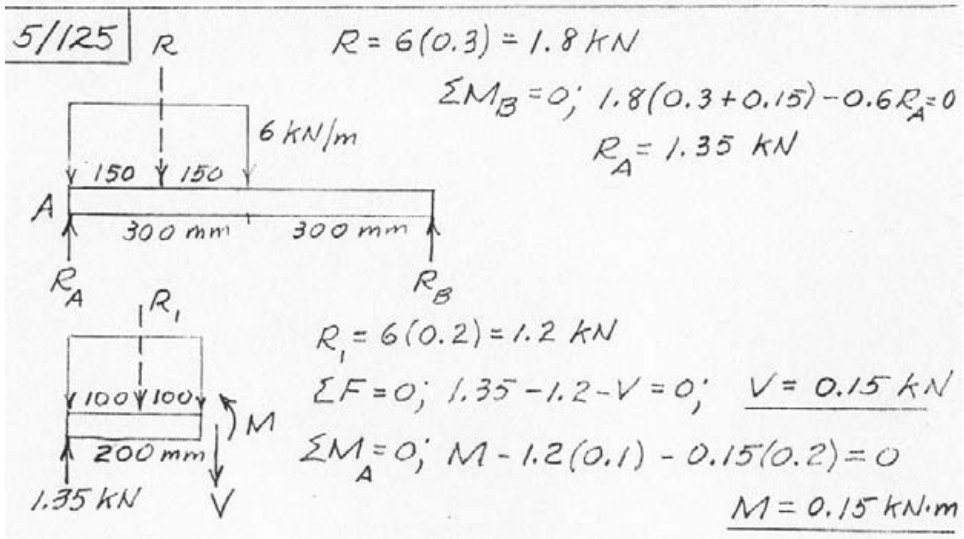
Values at middle:

$$\begin{cases} V = -100 \text{ lb} \\ M = 2550 \text{ lb-ft} \end{cases}$$



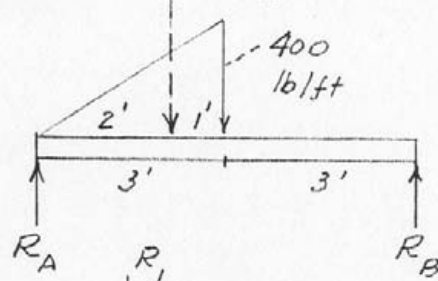






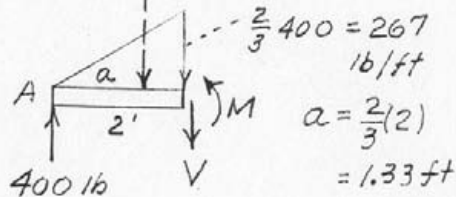
5/126

$$R = \frac{400}{2}(3) = 600 \text{ lb}$$



$$\sum M_B = 0; 600(4) - 6R_A = 0$$

$$R_A = 400 \text{ lb}$$



$$R_1 = \frac{267}{2} \cdot 2 = 267 \text{ lb}$$

$$\sum F = 0; 400 - V - 267 = 0$$

$$V = 133.3 \text{ lb}$$

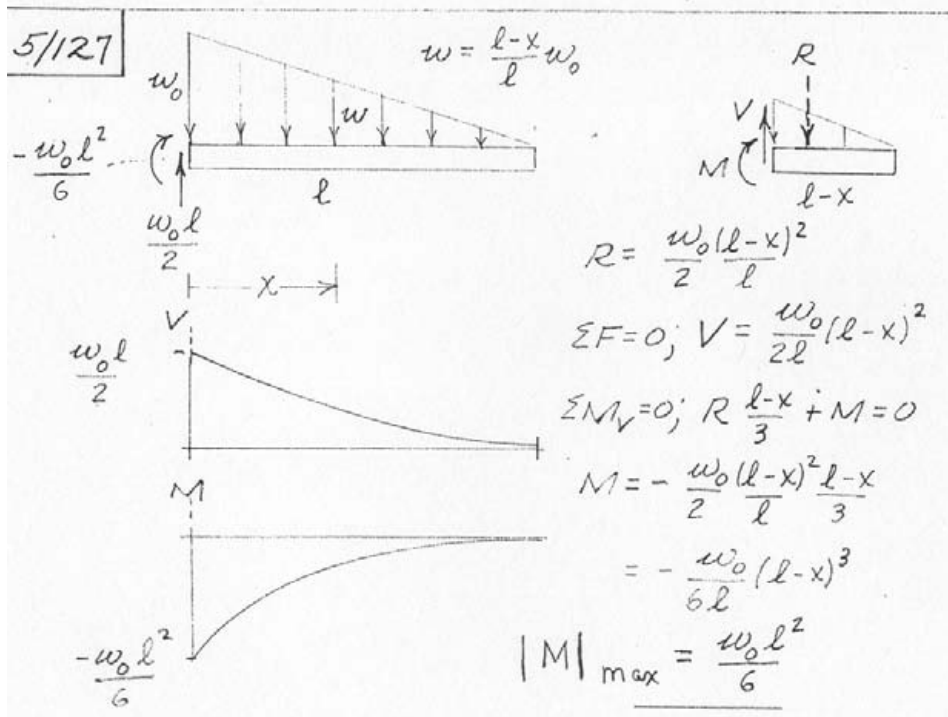
$$a = \frac{2}{3}(2)$$

$$= 1.33 \text{ ft}$$

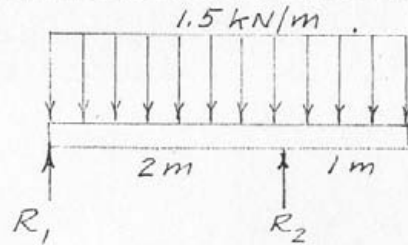
$$\sum M_A = 0;$$

$$-133.3(2) + M - 267(1.33) = 0$$

$$M = 622 \text{ lb-ft}$$

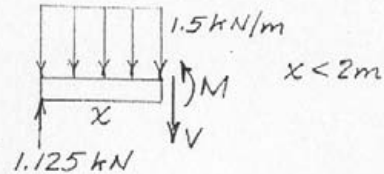
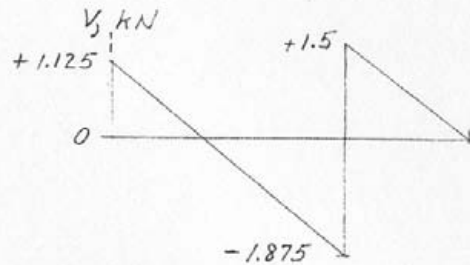


5/128



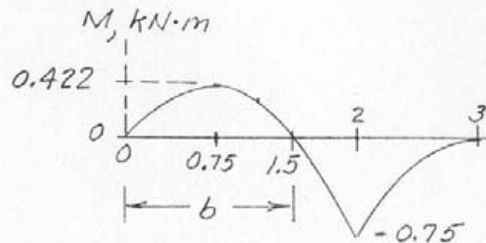
$$\begin{aligned}\sum M_{R_1} &= 0; \\ 1.5(3)(1.5) - 2R_2 &= 0 \\ R_2 &= 3.375 \text{ kN}\end{aligned}$$

$$\begin{aligned}\sum F &= 0; R_1 + 3.375 - 1.5(3) = 0 \\ R_1 &= 1.125 \text{ kN}\end{aligned}$$



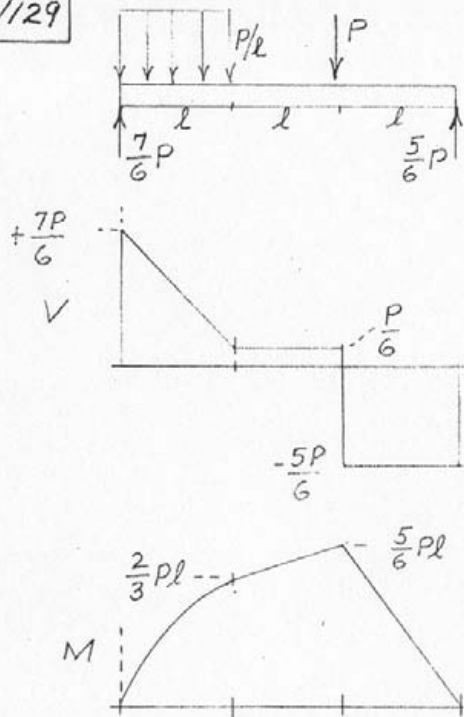
$$\begin{aligned}\sum F &= 0; V + 1.5x - 1.125 = 0 \\ V &= 1.125 - 1.5x\end{aligned}$$

$$\begin{aligned}\sum M_V &= 0; M + 1.5 \frac{x^2}{2} - 1.125x = 0 \\ M &= 1.125x - 0.75x^2\end{aligned}$$



$$b = 1.5 \text{ m}$$

5/129



Free body diagram for $x < l$:

$$\sum F = 0; -V - \frac{P}{l}x + \frac{7P}{6} = 0$$

$$V = P\left(\frac{7}{6} - \frac{x}{l}\right)$$

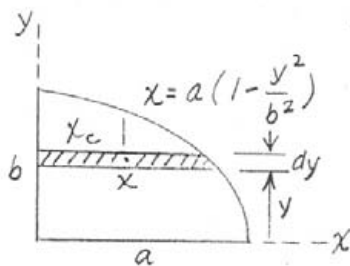
$$\sum M_V = 0; M + \frac{P}{l}x \frac{x}{2} - \frac{7P}{6}x = 0$$

$$M = \frac{Px}{2}\left(\frac{7}{3} - \frac{x}{l}\right)$$

$$M_{\max} = \frac{5}{6}Pl$$

5/13

$$dA = x dy, \quad A = \int_0^b a \left(1 - \frac{y^2}{b^2}\right) dy = a \left[y - \frac{y^3}{3b^2} \right]_0^b = \frac{2}{3} ab$$



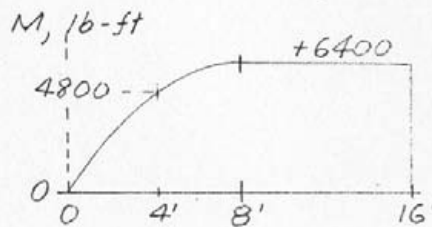
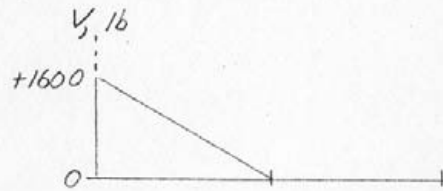
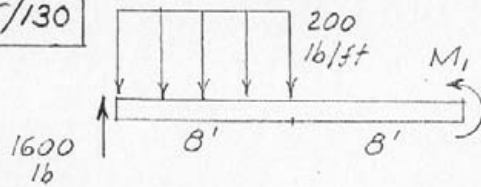
$$\begin{aligned} \bar{x}A &= \int x_c dA = \int_0^b \frac{x}{2} x dy \\ &= \frac{1}{2} \int_0^b a^2 \left(1 - \frac{2y^2}{b^2} + \frac{y^4}{b^4}\right) dy \\ &= \frac{a^2}{2} \left[y - \frac{2y^3}{3b^2} + \frac{y^5}{5b^4} \right]_0^b = \frac{4}{15} a^2 b \end{aligned}$$

$$\bar{x} = \frac{4a^2b/15}{2ab/3} = \frac{2}{5} a$$

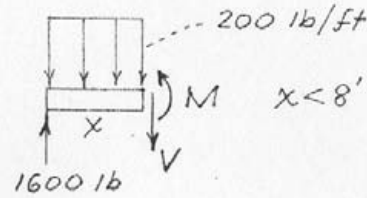
$$\bar{y}A = \int y dA = \int_0^b a \left(y - \frac{y^3}{b^2} \right) dy = a \left[\frac{y^2}{2} - \frac{y^4}{4b^2} \right]_0^b = \frac{1}{4} ab^2$$

$$\bar{y} = \frac{ab^2/4}{2ab/3} = \frac{3}{8} b$$

5/130



For zero moment
on left end,
 $M_1 = 200(8)4 = 6400 \text{ lb-ft}$



$$\sum F = 0; V + 200x - 1600 = 0$$

$$V = 1600 - 200x$$

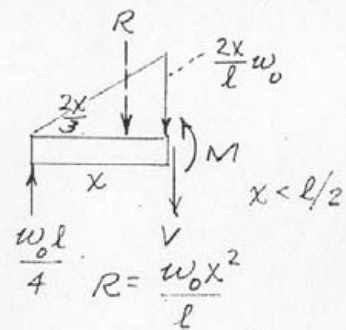
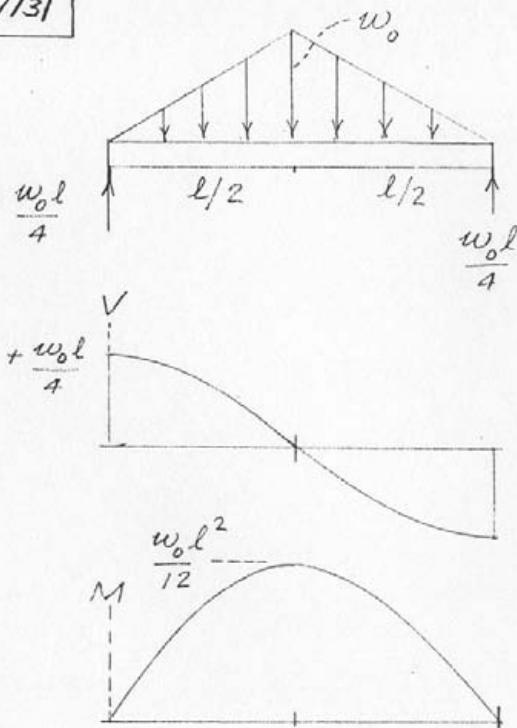
$$\sum M_V = 0; M + 200 \frac{x^2}{2} - 1600x = 0$$

$$M = 1600x - 100x^2$$

$$= 100x(16 - x)$$

At $x = 4$, $M = 100(12)$
 $= 4800 \text{ lb-ft}$

5/131



$$\sum F = 0; V + \frac{w_0 x^2}{l} - \frac{w_0 l}{4} = 0$$

$$V = w_0 l \left(\frac{1}{4} - \frac{x^2}{l^2} \right)$$

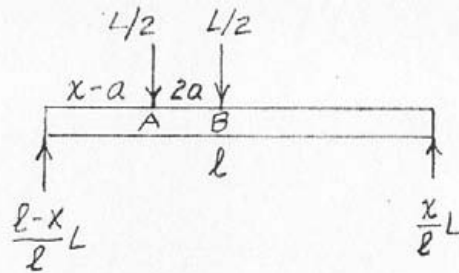
$$\sum M_V = 0; M + R \frac{x}{3} - \frac{w_0 l}{4} x = 0$$

$$M = w_0 x l \left(\frac{1}{4} - \frac{x^2}{3l^2} \right)$$

$$M_{\max} = \frac{w_0 l^2}{12}$$

$$\begin{aligned}
 5/132 \quad w &= -\frac{dV}{dx} = -\frac{d}{dx}(2200x - 40x^3) = \underline{-2200 + 120x^2} \\
 dM &= V dx; \quad \int_{-1600}^M dM = \int_1^3 (2200x - 40x^3) dx \\
 M - (-1600) &= 2200 \frac{3^2 - 1^2}{2} - 10(3^4 - 1^4) = 8000 \\
 M &= 8000 - 1600 = \underline{6400 \text{ lb-ft}}
 \end{aligned}$$

5/134



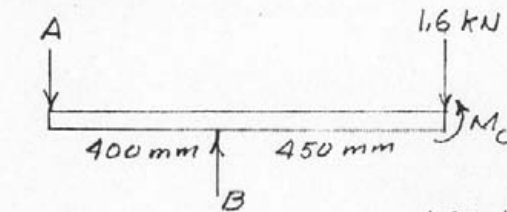
$$M_A = \frac{l-x}{l}L(x-a) = \frac{L}{l}(-x^2 + [a+l]x - al)$$

$$\frac{dM_A}{dx} = \frac{L}{l}(-2x + a + l) = 0 \text{ for max. } M_A; \quad x = \frac{a+l}{2}$$

$$\therefore M_{A \max} = \frac{L}{4l}(l-a)^2; \quad M_B_{x=\frac{a+l}{2}} = \frac{L}{4l}(l^2 - 2al - 3a^2) < M_A$$

(By symmetry, a second and equal maximum bending moment occurs at $x = \frac{l-a}{2}$.)

5/135



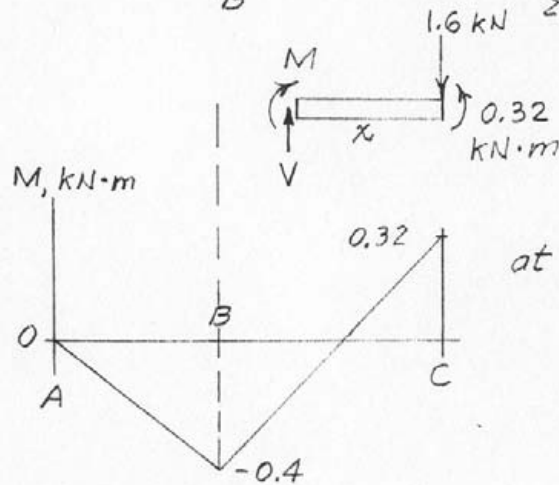
$$M_C = 1.6(0.200) = 0.32 \text{ kN}\cdot\text{m}$$

$$\sum M_A = 0; 0.4B + 0.32 - 0.85(1.6) = 0$$

$$B = 2.6 \text{ kN}$$

$$\sum F = 0; A + 1.6 - 2.6 = 0$$

$$A = 1.0 \text{ kN}$$



$$\sum M = 0$$

$$0.32 - 1.6x - M = 0$$

$$M = 0.32 - 1.6x$$

$$\text{at } B, x = 0.45 \text{ m}$$

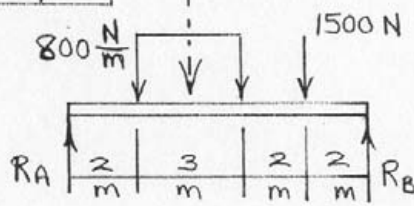
$$M_B = -0.40 \text{ kN}\cdot\text{m}$$

$$M = 0 \text{ when}$$

$$x = 0.32 / 1.6 = 0.2 \text{ m}$$

5/136

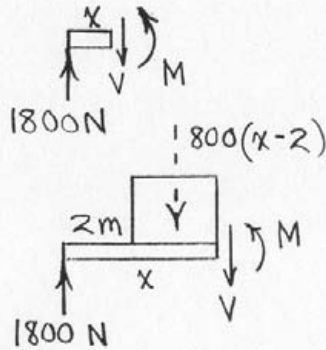
$$R = 800(3) = 2400 \text{ N}$$



$$\sum M_A = 0: R_B(9) - 1500(7) - 2400(3.5) = 0, R_B = 2100 \text{ N}$$

$$\sum F = 0: R_A - 2400 - 1500 + 2100 = 0$$

$$R_A = 1800 \text{ N}$$



$$0 < x < 2 \text{ m}:$$

$$\sum F = 0 \Rightarrow V = 1800 \text{ N}$$

$$\sum M = 0 \Rightarrow M = 1800x$$

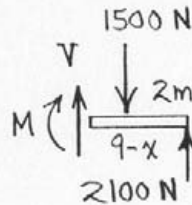
$$2 < x < 5 \text{ m}:$$

$$\sum F = 0: 1800 - 800(x-2) - V = 0$$

$$V = 3400 - 800x$$

$$\sum M = 0: M + 800(x-2) \frac{x-2}{2} + 1800x = 0$$

$$M = -400x^2 + 3400x - 1600$$



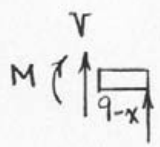
$$5 < x < 7:$$

$$\sum F = 0: 2100 - 1500 + V = 0$$

$$V = -600 \text{ N}$$

$$\sum M = 0: -M - 1500(7-x) + 2100(9-x) = 0$$

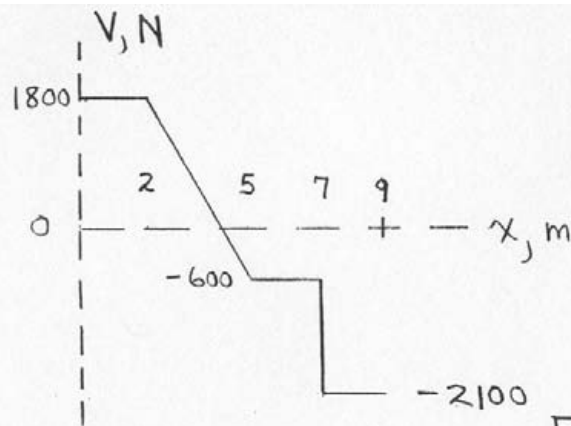
$$M = 8400 - 600x$$



$$7 < x < 9 \text{ m}:$$

$$\sum F = 0 \Rightarrow V = -2100 \text{ N}$$

$$\sum M = 0 \Rightarrow M = 18900 - 2100x$$



At $x = 6 \text{ m}$:

$$V = -600 \text{ N}$$

$$M = 8400 - 600(6) = 4800 \text{ N}\cdot\text{m}$$

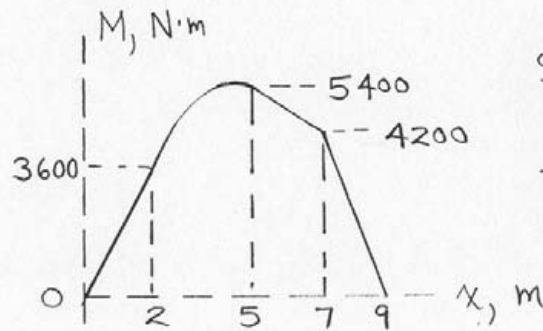
For M_{\max} ,

$$\frac{dM}{dx} = 0$$

$$\frac{d}{dx} (-400x^2 + 3400x - 1600)$$

$$= -800x + 3400 = 0$$

$$x = 4.25 \text{ m}$$



$$M_{\max} = -400(4.25)^2 + 3400(4.25) - 1600 = 5620 \text{ N}\cdot\text{m}$$

5/137

$$R = 800(3) = 2400 \text{ N}$$

$$\sum M_A = 0: R_B(9) - 2400(3.5)$$

$$- 4200 = 0, R_B = 1400 \text{ N}$$

$$\sum F = 0 \Rightarrow R_A = 1000 \text{ N}$$

$$0 < x < 2 \text{ m}:$$

$$\sum F = 0 \Rightarrow V = 1000 \text{ N}$$

$$\sum M = 0 \Rightarrow M = 1000x$$

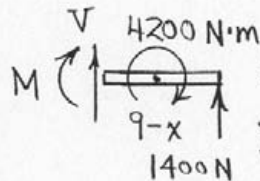
$$2 < x < 5 \text{ m}:$$

$$\sum F = 0: 1000 - 800(x-2) - V = 0$$

$$V = 2600 - 800x$$

$$\sum M = 0: M + 800(x-2)\frac{x-2}{2} - 1000x = 0$$

$$M = -400x^2 + 2600x - 1600$$

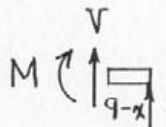


$$5 < x < 7 \text{ m}:$$

$$\sum F = 0: V + 1400 = 0, V = -1400 \text{ N}$$

$$\sum M = 0: -M - 4200 + 1400(9-x) = 0$$

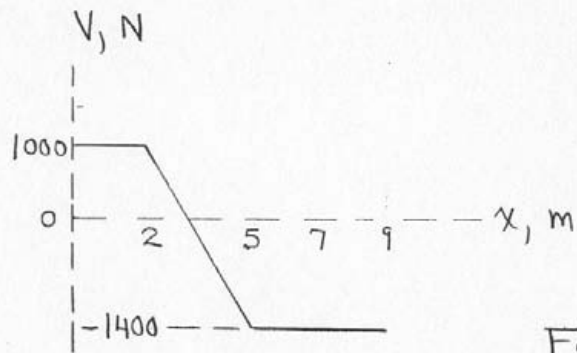
$$M = 8400 - 1400x$$



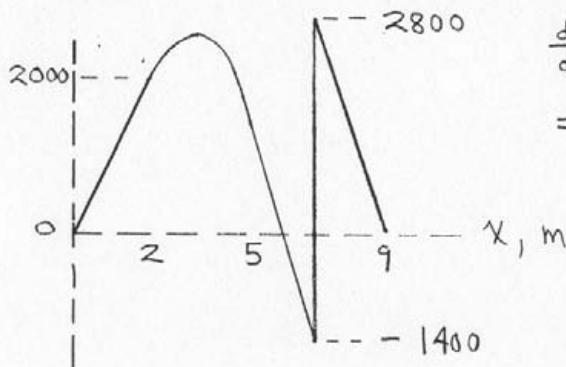
$$7 < x < 9 \text{ m}:$$

$$\sum F = 0 \Rightarrow V = -1400 \text{ N}$$

$$\sum M = 0 \Rightarrow M = 12600 - 1400x$$



$$\begin{aligned} \text{At } x &= 6 \text{ m,} \\ V &= -1400 \text{ N} \\ M &= 8400 - 1400(6) \\ &= 0 \end{aligned}$$

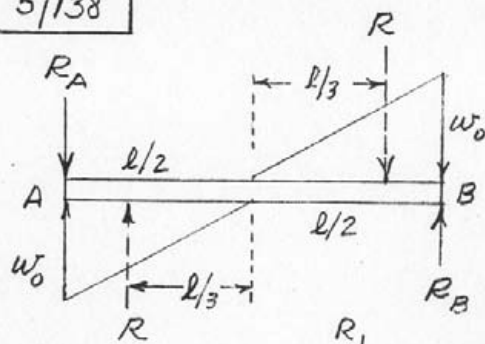


$$\begin{aligned} \text{For } M_{\max}, \frac{dM}{dx} &= 0 \\ \frac{d}{dx} (-400x^2 + 2600x - 1600) \\ &= -800x + 2600 = 0 \\ x &= 3.25 \text{ m} \end{aligned}$$

$$M_{x=3.25} = -400(3.25)^2 + 2600(3.25) - 1600 = 2625 \text{ N}\cdot\text{m}$$

$$\text{So } \underline{M_{\max} = 2800 \text{ N}\cdot\text{m}}$$

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$$R = \frac{w_0 l}{2} \cdot \frac{1}{2} = \frac{w_0 l}{4}$$

By inspection of the two couples, $R_A = R_B$

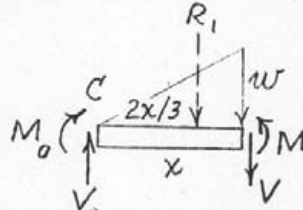
$$\sum M = 0; R_A l - \frac{w_0 l}{4} \cdot \frac{2l}{3} = 0$$

$$R_A = R_B = \frac{w_0 l}{6}$$

For complete right half of beam $\sum F = 0$ gives

$$V_0 = R - R_B = \frac{w_0 l}{12}$$

$$M_0 = 0$$



For section of length x , $w = \frac{x}{l/2} w_0$ & $R_1 = \frac{w}{2} x = \frac{w_0 x^2}{l}$

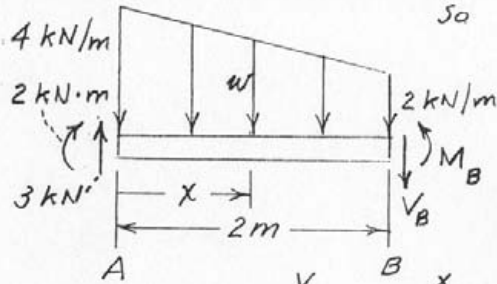
$$\sum F = 0; V + \frac{w_0 x^2}{l} - \frac{w_0 l}{12} = 0, \quad V = \frac{w_0 l}{12} \left(1 - \frac{x^2}{l^2} \right)$$

$$\sum M_C = 0; M - \frac{w_0 x^2}{l} \cdot \frac{2x}{3} - \frac{w_0 l}{12} \left(1 - \frac{x^2}{l^2} \right) x = 0, \quad M = \frac{w_0 l x}{3} \left(\frac{1}{4} - \frac{x^2}{l^2} \right)$$

5/139

$$w = 4 - kx \quad \text{where } k = \frac{4-2}{2} = 1 \text{ kN/m}^2$$

$$\text{So } w = 4 - x \text{ kN/m}$$



$$w = -\frac{dV}{dx}; \quad \int_3^V dV = -\int_0^x (4-x) dx, \quad V = 3 - \left[4x - \frac{x^2}{2} \right]_0^x$$

$$= 3 - 4x + \frac{x^2}{2}$$

$$\text{when } x = 2 \text{ m, } V = V_B = -3 \text{ kN}$$

$$V = \frac{dM}{dx}; \quad \int_2^{M_B} dM = \int_0^2 (3 - 4x + \frac{x^2}{2}) dx$$

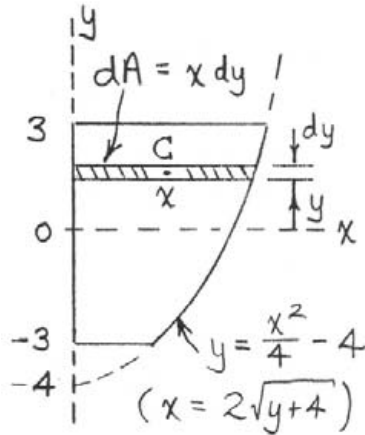
$$M_B - 2 = \left[3x - 2x^2 + \frac{x^3}{6} \right]_0^2 = -\frac{2}{3}$$

$$M_B = 2 - \frac{2}{3} = \frac{4}{3} \text{ kN}\cdot\text{m}$$

5/14

$$A = \int dA = \int x dy = \int_{-3}^3 2\sqrt{y+4} dy$$

$$= 2 \frac{2}{3} (y+4)^{3/2} \Big|_{-3}^3 = 23.4$$



$$\int x_c dA = \int \frac{x}{2} x dy = \frac{1}{2} \int x^2 dy$$

$$= \frac{1}{2} \int_{-3}^3 4(y+4) dy = 2 \left(\frac{y^2}{2} + 4y \right) \Big|_{-3}^3$$

$$= 48$$

$$\int y_c dA = \int y x dy = \int_{-3}^3 y 2\sqrt{y+4} dy$$

$$= 2 \frac{2}{15} (3y-8)(y+4)^{3/2} \Big|_{-3}^3 = 9.47$$

$$\bar{x} = \frac{\int x_c dA}{\int dA} = \frac{48}{23.4} = \underline{2.05}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA} = \frac{9.47}{23.4} = \underline{0.405}$$

5/140

$dR = w dx'$

From $w = w_0 + kx^2 = 100 + kx^2$;
 $400 = 100 + k(10)^2$
 $k = 3 \text{ lb/ft}^3$, $w = 100 + 3x^2$

$\uparrow \Sigma F = 0 : V - \int_x^{10} w dx' = 0$

$V = \int_x^{10} (100 + 3x'^2) dx' = 100x' + x'^3 \Big|_x^{10}$

$V = 2000 - 100x - x^3 \quad (\text{in lb if } x \text{ is in ft})$

$\curvearrowright \Sigma M = 0 : -M - \int_x^{10} (x' - x) w dx' = 0$

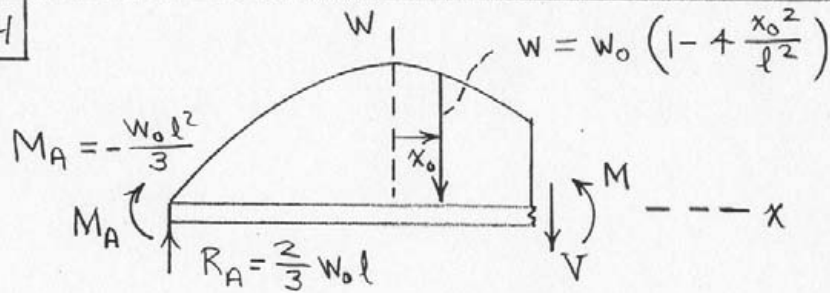
$M = - \int_x^{10} (-100x + 100x' + 3x'^3 - 3xx'^2) dx'$

$= - \left[-100xx' + 50x'^2 - xx'^3 + \frac{3}{4}x'^4 \right] \Big|_x^{10}$

$= -12,500 + 2000x - 50x^2 - \frac{1}{4}x^4$

$(\text{in lb-ft if } x \text{ is in ft})$

5/141



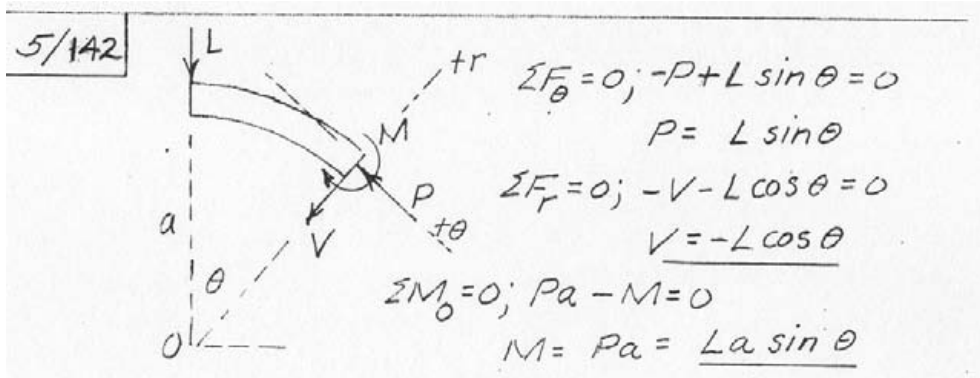
(R_A and M_A from Prob. 5/104)

$$\uparrow \Sigma F = 0: \frac{2}{3} w_0 l - \int_{-l/2}^x w_0 \left(1 - 4 \frac{x_0^2}{l^2}\right) dx_0 - V = 0$$

$$V = w_0 \left(\frac{l}{3} - x + \frac{4x^3}{3l^2}\right)$$

$$\circlearrowleft \Sigma M = 0: M + \int_{-l/2}^x w_0 \left(1 - 4 \frac{x_0^2}{l^2}\right) (x - x_0) dx_0$$

$$+ \frac{w_0 l^2}{3} - \frac{2}{3} w_0 l \left(\frac{l}{2} + x\right) = 0, \quad M = w_0 \left(-\frac{l^2}{16} + \frac{x l}{3} - \frac{x^2}{2} + \frac{x^4}{3l^2}\right)$$



►5/143

From areas under shear diagram:

$$|M_2| = \frac{1}{2} w x^2$$

$$|M_1| = \frac{w}{2} \left(\frac{L}{2} - x \right)^2 - \frac{1}{2} w x^2$$

$$= \frac{wL}{2} \left(\frac{L}{4} - x \right)$$

$|M|_{\min}$ occurs when

$|M_1| = |M_2|$. So

$$\frac{wL}{2} \left(\frac{L}{4} - x \right) = \frac{1}{2} w x^2$$

$$\text{or } x^2 + Lx - \frac{L^2}{4} = 0$$

$$\Rightarrow x = \frac{L}{2} (-1 \pm \sqrt{2})$$

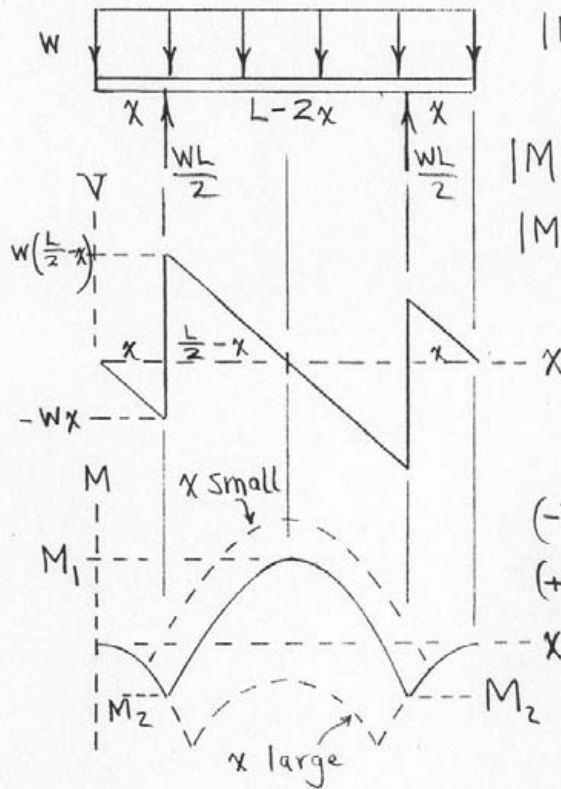
(-) sign: irrelevant

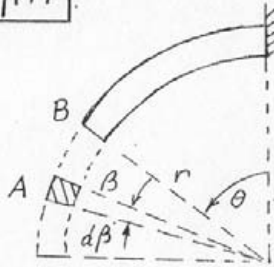
(+) sign: $x = 0.207L$

$$M_{\max} = |M_1|$$

$$= \frac{1}{2} w (0.207L)^2$$

$$= 0.0214 w L^2$$





On element A, $dF = wr d\beta$

Torsion about B due to dF is

$$dT = dF(r - r \cos \beta) = wr^2(1 - \cos \beta) d\beta$$

$$T = wr^2 \int_0^{\pi/2 - \theta} (1 - \cos \beta) d\beta$$

$$= wr^2 \left[\beta - \sin \beta \right]_0^{\pi/2 - \theta}$$

$$= wr^2 \left[\frac{\pi}{2} - \theta - \sin \left(\frac{\pi}{2} - \theta \right) \right]$$

$$= wr^2 \left[\frac{\pi}{2} - \theta - \cos \theta \right]$$

Bending moment about B due to dF is

$$dM = dF r \sin \beta = wr^2 \sin \beta d\beta$$

$$M = wr^2 \int_0^{\pi/2 - \theta} \sin \beta d\beta = -wr^2 \cos \beta \Big|_0^{\pi/2 - \theta}$$

$$= -wr^2 [\sin \theta - 1] = \underline{wr^2(1 - \sin \theta)}$$

5/145

Given: $\begin{cases} 2s = 100 \text{ ft}, & s = 50 \text{ ft} \\ \mu = 0.00624 \text{ lb/ft} \\ T = 10 \text{ lb} \end{cases}$

$$T^2 = \mu^2 s^2 + T_0^2 : 10^2 = (0.00624 \cdot 50)^2 + T_0^2$$

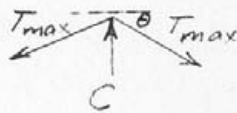
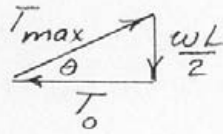
$$T_0 = 9.995 \text{ lb}$$

(Eq. 5/22) $T = T_0 + \mu y : 10 = 9.995 + 0.00624 h$

$$h = 0.780 \text{ ft or } \underline{h = 9.36 \text{ in.}}$$

5/146 $L = 4200 \text{ ft}$, $h = 470 \text{ ft}$, $w = \frac{21,300}{2} = 10650 \text{ lb/ft}$
for each cable

$$T_0 = \frac{wL^2}{8h} = \frac{10650 (4200)^2}{8(470)} = \underline{50.0(10^6) \text{ lb}}$$

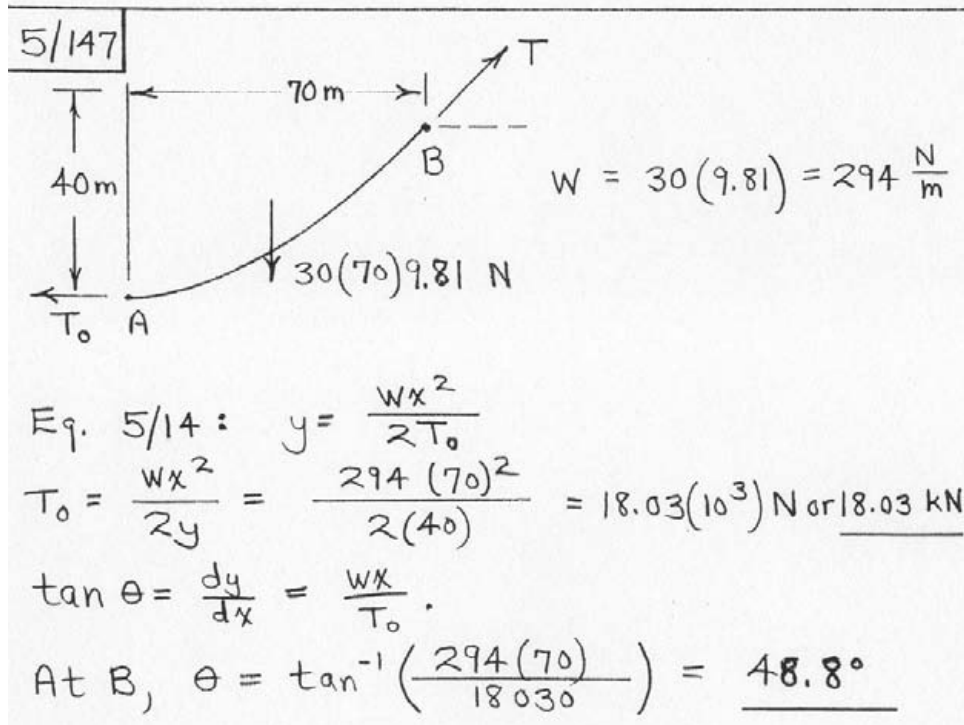


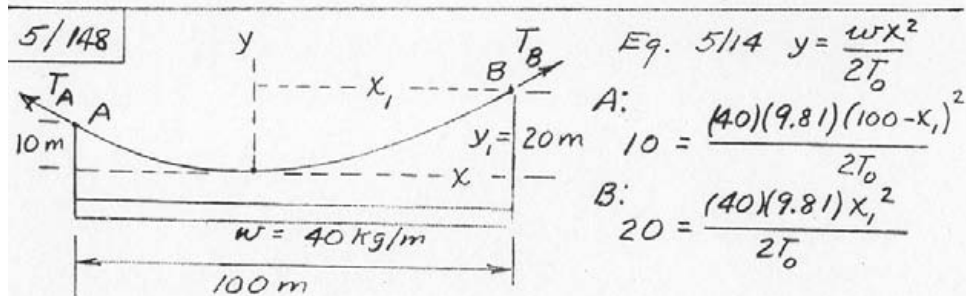
$$C = 2 T_{\max} \sin \theta$$

$$= 2 \left(\frac{wL}{2} \right) = wL$$

$$= 10650 (4200)$$

$$= \underline{44.7(10^6) \text{ lb}}$$





Combine & get $x_1^2 = 2(100-x_1)^2$, $x_1^2 - 400x_1 + 20000 = 0$

$x_1 = \frac{400}{2} \pm \frac{1}{2} \sqrt{(16-8)10^4} = 100(2 \pm \sqrt{2}) = 58.6 \text{ m (or 341 m)}$

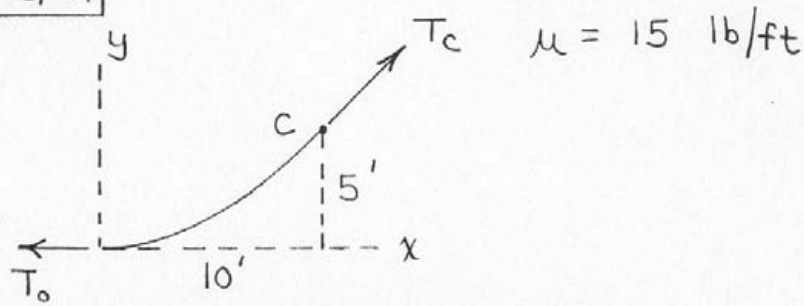
At B; $T_0 = \frac{40(9.81)(58.6)^2}{2(20)} = 33660 \text{ N or } T_0 = 33.7 \text{ kN}$

Also, $T^2 = T_0^2 + w^2x^2$ so

for B, $T_B = \sqrt{(33.7)^2 + [(40)(9.81)(10^{-3})]^2(58.6)^2} = 40.8 \text{ kN}$

for A, $T_A = \sqrt{(33.7)^2 + [(40)(9.81)(10^{-3})]^2(100-58.6)^2} = 37.4 \text{ kN}$

*5/149



Eq. 5/19 evaluated at point C:

$$5 = \frac{T_0}{\mu} \left[\cosh \frac{10}{T_0/\mu} - 1 \right]$$

Numerical solution: $\frac{T_0}{\mu} = 10.74 \text{ ft}$

$$\text{Then } T_0 = 10.74(15) = 161.1 \text{ lb}$$

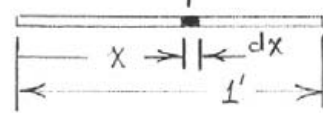
Eq. 5/22 evaluated at C:

$$T_c = 161.1 + 15(5) = \underline{236 \text{ lb}}$$

$$\text{Eq. 5/20 : } s = 10.74 \sinh \frac{10}{10.74} = 11.51 \text{ ft}$$

$$L = 2s = \underline{23.0 \text{ ft}}$$

5/15

 $dm = \rho dx$ (ρ = mass per unit length)


$$m = \int dm = \int \rho dx = \int_0^{l'} \rho_0 \left(1 - \frac{x}{2}\right) dx$$

$$= \rho_0 \left[x - \frac{x^2}{4} \right]_0^{l'} = \frac{3}{4} \rho_0 l'$$

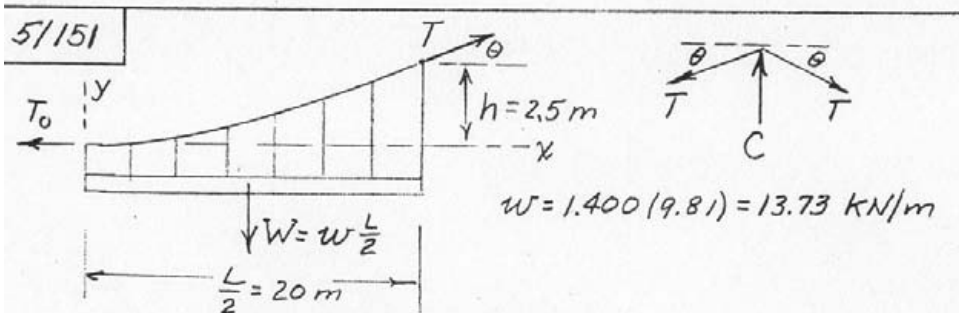
$$\int x dm = \int_0^{l'} x \rho_0 \left(1 - \frac{x}{2}\right) dx = \rho_0 \left[\frac{x^2}{2} - \frac{x^3}{6} \right]_0^{l'} = \frac{\rho_0 l'^3}{3}$$

$$\bar{x} = \frac{\int x dm}{\int dm} = \frac{\rho_0 l'^3 / 3}{3 \rho_0 l' / 4} = \frac{4}{9} l'$$

5/150 Eq. 5/22, $T_B = T_o + \mu y_B$, $T_A = T_o + \mu y_A$

so $T_B - T_A = \mu(y_B - y_A)$ or $T_B - T_A = \mu h$

Thus $h = \frac{1}{\mu}(T_B - T_A) = \frac{1}{0.12(9.81)}(230 - 110) = \underline{101.9 \text{ m}}$



From Eq. 5/15b, $T = \frac{wL}{2} \sqrt{1 + (L/4h)^2}$

$$= \frac{13.73(40)}{2} \sqrt{1 + \left[\frac{40}{4(2.5)} \right]^2} = 1133 \text{ kN}$$

$$T^2 = W^2 + T_0^2, \quad T_0 = \sqrt{(1133)^2 - [(13.73)(20)]^2} = 1099 \text{ kN}$$

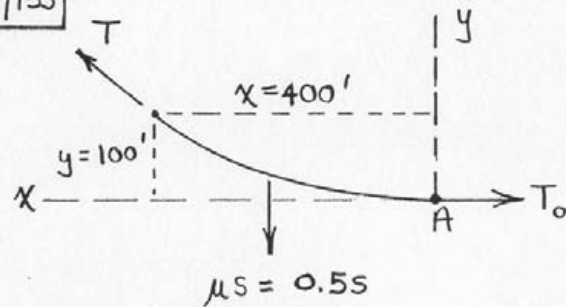
$$\frac{dy}{dx} = \tan \theta = \frac{w(L/2)}{T_0} = \frac{13.73(20)}{1099} = 0.250, \quad \theta = 14.04^\circ$$

$$\sum F_y = 0 \text{ at support}; \quad 2T \sin \theta - C = 0$$

$$C = 2(1133) \sin 14.04^\circ = \underline{549 \text{ kN}}$$

$$\begin{aligned}
 &5/152 \quad \text{Eq. 5/15} \quad T = w \sqrt{x^2 + (L^2/8h)^2} \\
 &\quad \text{so } \Delta T = \Delta w \sqrt{x^2 + (L^2/8h)^2} \\
 &\text{For each cable } 480,000 = \Delta w \sqrt{(600)^2 + \left(\frac{[2500]^2}{8[500]}\right)^2} \\
 &\quad = 1674 \Delta w \\
 &\quad \Delta w = 286.8 \text{ lb/ft per cable} \\
 &w' = 2\Delta w = 2(286.8) = \underline{574 \text{ lb/ft for both cables}}
 \end{aligned}$$

*5/153



Catenary Eq. 5/19 : $y = \frac{T_0}{\mu} \left(\cosh \frac{\mu x}{T_0} - 1 \right)$

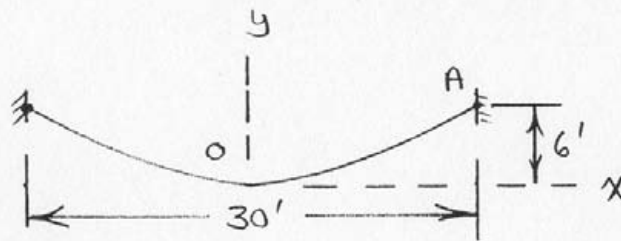
$$100 = \frac{T_0}{0.5} \left(\cosh \frac{0.5(400)}{T_0} - 1 \right)$$

Solve numerically to obtain $T_0 = 408 \text{ lb}$

Parabolic Eq. 5/14 : $y = \frac{wx^2}{2T_0} \approx \frac{\mu x^2}{2T_0}$

$$\text{So } T_0 \approx \frac{\mu x^2}{2y} = \frac{0.5(400)^2}{2(100)} = \underline{400 \text{ lb}}$$

*5/154



Eq. 5/19, from o to A: $6 = \frac{T_0}{\mu} \left[\cosh \frac{\mu}{T_0} (15) - 1 \right]$

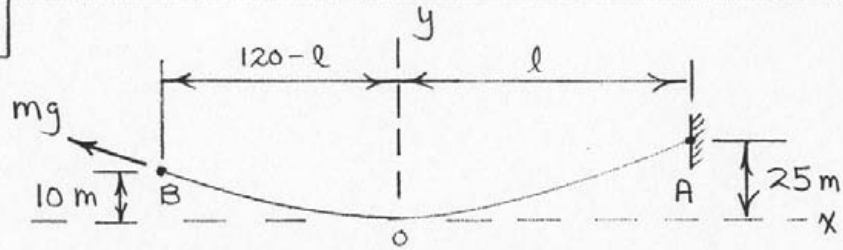
By numerical or graphical means, $\frac{T_0}{\mu} = 19.68 \text{ m}$

Eq. 5/20: $s = \frac{L}{2} = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$\therefore L = 2 \left(\frac{T_0}{\mu} \right) \sinh \frac{\mu}{T_0} 15$

$= 2(19.68) \sinh \frac{15}{19.68} = \underline{33.0 \text{ ft}}$

5/155



$$\text{Eq. 5/14: } y = \frac{wx^2}{2T_0}$$

$$\text{At A: } 25 = \frac{40l^2}{2T_0}, \quad \text{At B: } 10 = \frac{40(120-l)^2}{2T_0}$$

$$\text{Eliminate } T_0: \quad 0.6l^2 - 240l + 14400 = 0$$

$$l = 73.5 \text{ m (or } l = 326 \text{ m)}$$

$$T_0 = \frac{4}{5} l^2 = \frac{4}{5} (73.5)^2 = 4320 \text{ N}$$

$$\text{Section O-B: } (mg)^2 = T_0^2 + (wx)^2$$

$$(9.81 \text{ m})^2 = (4320)^2 + [40(120-73.5)]^2$$

$$m = 480 \text{ kg}$$

*5/156 Please refer to the diagram in the solution to Prob. 5/155. Eq. 5/19:

$$y = \frac{T_0}{\mu} \left[\cosh \frac{\mu x}{T_0} - 1 \right]$$

$$\left. \begin{aligned} \text{At B: } 10 &= \frac{T_0}{40} \left[\cosh \frac{40(120-l)}{T_0} - 1 \right] \\ \text{At A: } 25 &= \frac{T_0}{40} \left[\cosh \frac{40l}{T_0} - 1 \right] \end{aligned} \right\}$$

Numerical solution:
$$\begin{cases} T_0 = 4440 \text{ N} \\ l = 73.2 \text{ m} \end{cases}$$

Eq. 5/20: $s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$$\text{At B: } s_B = \frac{4440}{40} \sinh \frac{40(120-73.2)}{4440} = 48.2 \text{ m}$$

Equilibrium of section OB:

$$(mg)^2 = T_0^2 + (\mu s_B)^2: m^2(9.81)^2 = 4440^2 + (40 \cdot 48.2)^2$$

$$\underline{m = 494 \text{ kg}}$$

5/157 Eq. 5/19 is $y = \frac{T_0}{\mu} (\cosh \frac{\mu x}{T_0} - 1)$

$$\begin{aligned} \text{So } H - 2 &= \frac{300(10^3)}{22(9.81)} \left(\cosh \frac{22(9.81)(250)}{300(10^3)} - 1 \right) \\ &= 1390 (\cosh 0.1798 - 1) = 1390 (1.0162 - 1) \\ &= 22.5 \text{ m} \end{aligned}$$

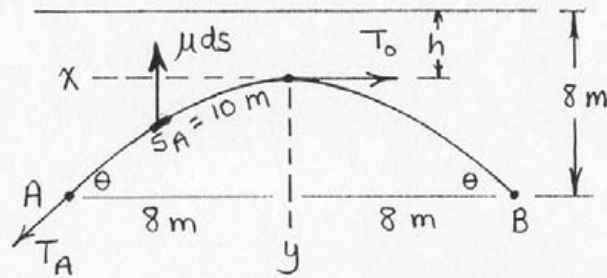
Thus $H = 24.5 \text{ m}$

$$\begin{aligned} \text{Eq. 5/20, } s &= \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0} = \frac{300(10^3)}{22(9.81)} \sinh \frac{22(9.81)250}{300(10^3)} \\ &= 1390 (0.1808) = \underline{251 \text{ m}} \end{aligned}$$

* 5/158

$$\mu = 560 - 100$$

$$= 460 \text{ N/m}$$



$$\text{Eq. 5/20: } s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0} : 10 = \frac{T_0}{460} \sinh \frac{460(8)}{T_0}$$

Numerical or graphical solution: $T_0 = 3110 \text{ N}$

$$\text{Eq. 5/21: } T = T_0 \cosh \frac{\mu x}{T_0} = 3110 \cosh \frac{460(8)}{3110}$$

$$T_A = 5550 \text{ N}$$

$$\text{Eq. 5/22: } T = T_0 + \mu y \text{ at A: } 5550 = 3110 + 460y$$

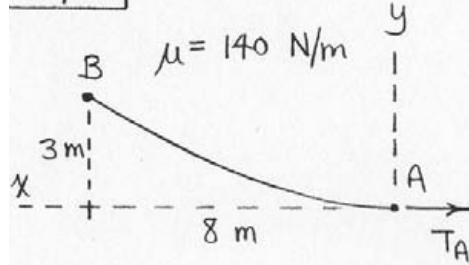
$$y = 5.31 \text{ m}$$

$$\text{Then } h = 8 - 5.31 = 2.69 \text{ m}$$

$$\text{From Eq. 5/19, } \frac{dy}{dx} = \tan \theta = \sinh \frac{\mu x}{T_0}$$

$$\tan \theta = \sinh \frac{460(8)}{3110} = 55.9^\circ$$

*5/159



$$\text{Eq. 5/19: } y = \frac{T_0}{\mu} \left[\cosh \frac{\mu x}{T_0} - 1 \right]$$

$$\text{At B: } 3 = \frac{T_A}{140} \left[\cosh \frac{140(8)}{T_A} - 1 \right]$$

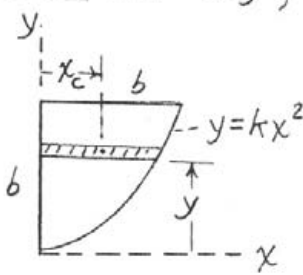
$$\text{Numerical solution: } \underline{T_A = 1559 \text{ N}}$$

$$\text{Eq. 5/20: } s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$$

$$\text{At B: } L = \frac{1559}{140} \sinh \frac{140(8)}{1559} = \underline{8.71 \text{ m}}$$

5/16

$dA = x dy$; $k = \frac{b}{b^2} = \frac{1}{b}$ so $x^2 = by$, $x_c = x/2$



$\bar{x} = \frac{\int x_c dA}{\int dA} = \frac{\int_0^b \frac{x}{2} x dy}{\int_0^b x dy}$

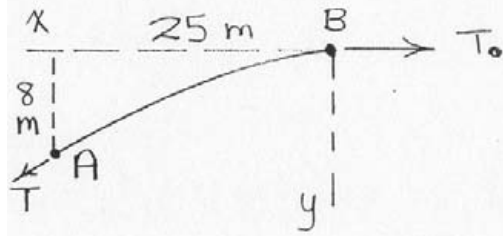
$= \frac{\frac{1}{2} \int_0^b by dy}{\int_0^b \sqrt{by} dy}$

$= \frac{\frac{1}{4} b^3 / \frac{2}{3} b^2}{\frac{2}{3} b^2} = \frac{3}{8} b$

$\bar{y} = \frac{\int y dA}{\int dA} = \frac{\int_0^b y \sqrt{by} dy}{\frac{2}{3} b^2} = \frac{\frac{2}{5} b^3 / \frac{2}{3} b^2}{\frac{2}{3} b^2} = \frac{3}{5} b$

*5/160

$$\mu = 30 \text{ N/m}$$



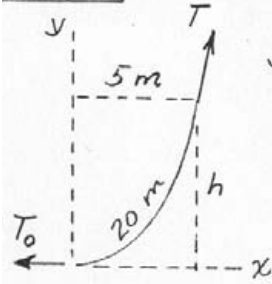
$$\text{Eq. 5/19: } y = \frac{T_0}{\mu} \left[\cosh \frac{\mu x}{T_0} - 1 \right]$$

$$\text{At A: } 8 = \frac{T_0}{\mu} \left[\cosh \frac{25\mu}{T_0} - 1 \right]$$

$$\text{Numerical solution: } \frac{T_0}{\mu} = 40.3 \text{ m}$$

$$T_0 = 40.3 (30) = \underline{1210 \text{ N}}$$

*5/161



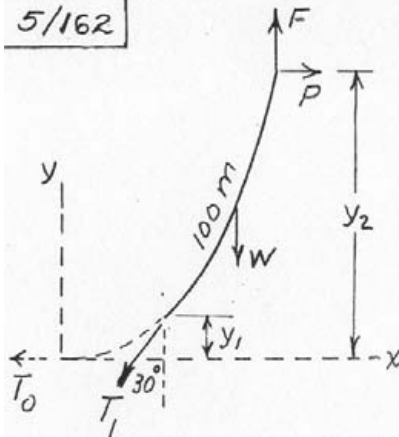
$$\text{Eq. 5/20, } 20 = \frac{T_0}{\mu} \sinh \frac{5\mu}{T_0}$$

Solve by computer or graphically
& get $T_0/\mu = 1.532 \text{ m}$

$$\text{Eq. 5/19, } y = 1.532 \left(\cosh \frac{5}{1.532} - 1 \right)$$

$$h = y = 1.532 (13.09 - 1) = \underline{18.53 \text{ m}}$$

5/162



$$W = 0.51(9.81)(100) = 500.3 \text{ N}$$

$$T_1 = M/r = 400 / \frac{0.5}{2} = 1600 \text{ N}$$

$$\sum F_x = 0; P - 1600 \sin 30^\circ = 0$$

$$P = 800 \text{ N } (= T_0)$$

$$\sum F_y = 0; F - 500.3 - 1600 \cos 30^\circ = 0$$

$$F = 1886 \text{ N}$$

$$T = \sqrt{F^2 + P^2} = \sqrt{(1886)^2 + (800)^2} = 2049 \text{ N}$$

$$\text{From Eq. 5/22, } T = T_0 + \mu y, \quad 2049 = 800 + 5.003 y_2$$

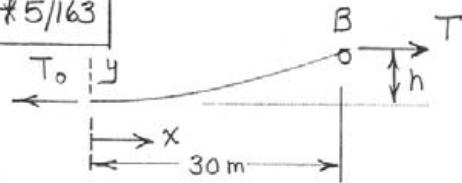
$$y_2 = 249.6 \text{ m}$$

$$\text{Also, } 1600 = 800 + 5.003 y_1$$

$$y_1 = 159.9 \text{ m}$$

$$H = y_2 - y_1 = \underline{89.7 \text{ m}}$$

#5/163

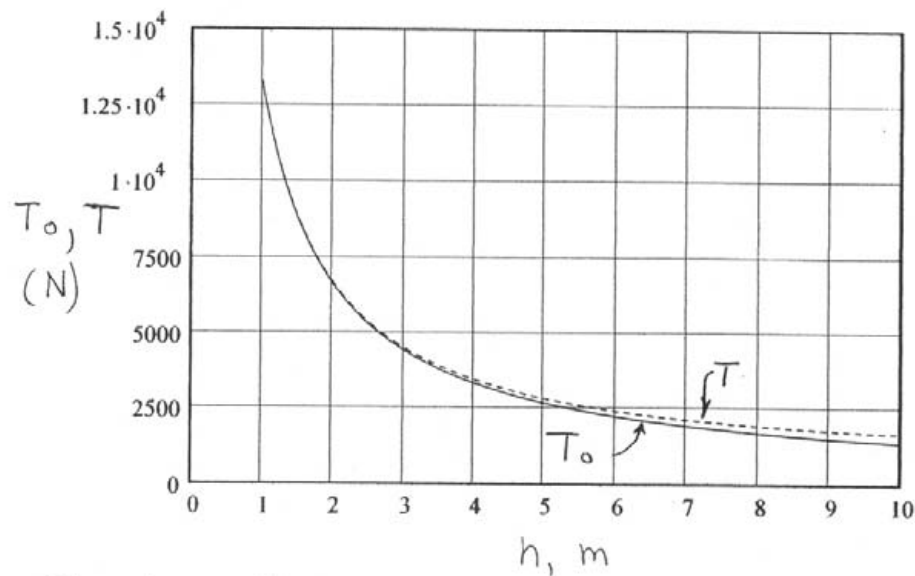


$$\mu = \rho g = 3(9.81) \\ = 29.4 \text{ N/m}$$

$$\text{Eq. 5/19 @ B: } h = \frac{T_0}{\mu} \left[\cosh \frac{\mu x}{T_0} - 1 \right] \quad (1)$$

$$\text{Eq. 5/22 @ B: } T = T_0 + \mu h \quad (2)$$

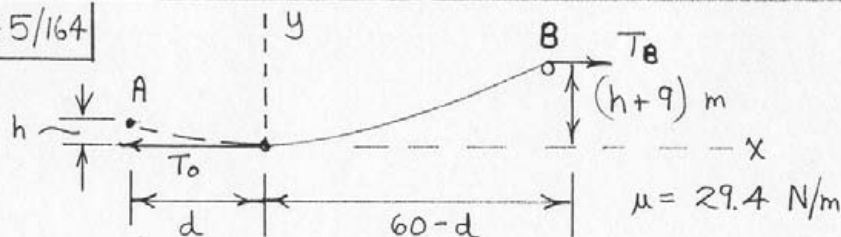
Numerically solve (1), then (2) over $1 \leq h \leq 10 \text{ m}$ to obtain



For $h = 2 \text{ m}$,

$$\underline{T_0 = 6630 \text{ N}}, \quad \underline{T = 6690 \text{ N}}$$

* 5/164

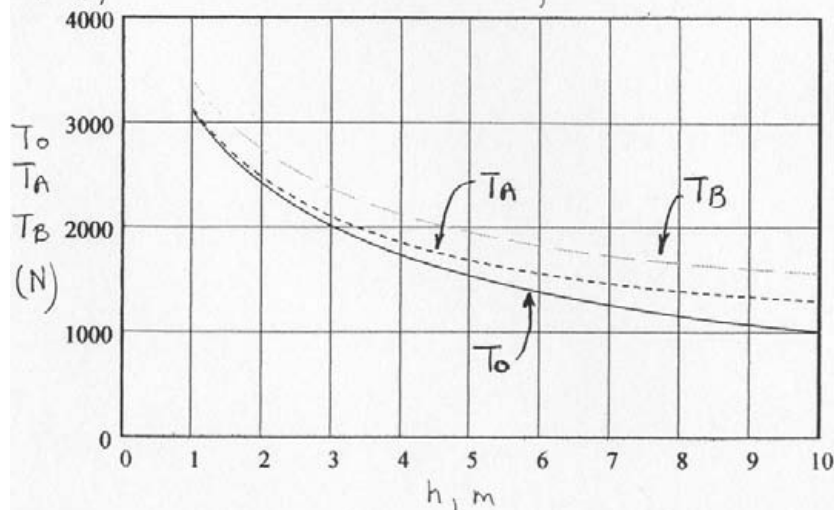


$$\text{Eq. 5/19 @ B: } h+9 = \frac{T_0}{\mu} \left[\cosh\left(\frac{\mu}{T_0}(60-d)\right) - 1 \right] \quad (1)$$

$$\text{Eq. 5/19 @ A: } h = \frac{T_0}{\mu} \left[\cosh\left(\frac{\mu}{T_0}d\right) - 1 \right] \quad (2)$$

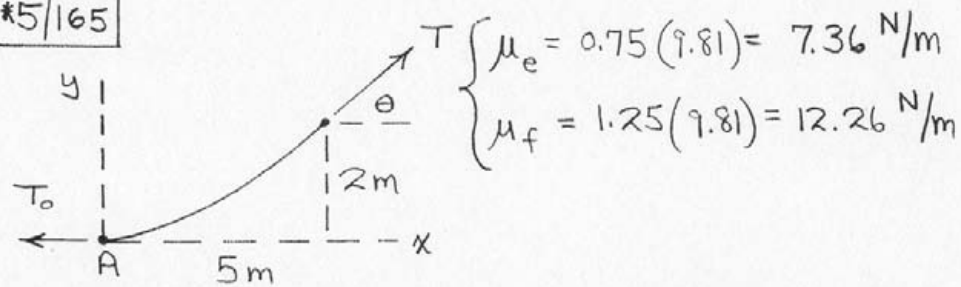
$$\text{Then Eq. 5/22: } \begin{cases} T_A = T_0 + \mu h \\ T_B = T_0 + \mu(h+9) \end{cases} \quad (3) \quad (4)$$

Numerically solve (1) and (2) (for T_0 & d), then (3) & (4) over $1 \leq h \leq 10$ m, to obtain



When $h = 2$ m, $T_0 = 2410$ N, $T_A = 2470$ N, $T_B = 2730$ N

*5/165



$$\begin{cases} \mu_e = 0.75(9.81) = 7.36 \text{ N/m} \\ \mu_f = 1.25(9.81) = 12.26 \text{ N/m} \end{cases}$$

$$\text{Eq. 5/19: } 2 = \frac{T_0}{\mu} \left[\cosh \frac{\mu}{T_0} 5 - 1 \right]$$

$$\text{Numerical solution: } \frac{T_0}{\mu} = 6.56 \text{ m}$$

$$\text{Slope } \frac{dy}{dx} = \sinh \frac{\mu x}{T_0} = \sinh \frac{5}{6.56} = 0.838$$

$$\theta = \tan^{-1} 0.838 = \underline{40.0^\circ} \quad (\text{empty or full})$$

$$\begin{aligned} T_0 &= 6.56 \mu = 6.56(7.36) = 48.3 \text{ N (empty)} \\ &= 6.56(12.26) = 80.4 \text{ N (full)} \end{aligned}$$

$$\begin{aligned} T &= T_0 + \mu y = 48.3 + 7.36(2) = \underline{63.0 \text{ N (empty)}} \\ &= 80.4 + 12.26(2) = \underline{105.0 \text{ N (full)}} \end{aligned}$$

*5/166 (a) Use $w = \mu = 1.2(9.81) = 11.77 \text{ N/m}$

Eq. 5/14: $y = \frac{wx^2}{2T_0}$ @ A: $2.4 = \frac{11.77(5)^2}{2T_0}$

$T_0 = 61.3 \text{ N}$

So $y_p = \frac{11.77x^2}{2(61.3)} = 0.096x^2$ (see plots below)

Eq. 5/16: $s_A = l_A \left[1 + \frac{2}{3} \left(\frac{h_A}{l_A} \right)^2 - \frac{2}{5} \left(\frac{h_A}{l_A} \right)^4 + \dots \right]$

$= 5 \left[1 + \frac{2}{3} \left(\frac{2.4}{5} \right)^2 - \frac{2}{5} \left(\frac{2.4}{5} \right)^4 + \dots \right] = 5.66 \text{ m}$

So the required length is $L_p = 2s_A = \underline{11.32 \text{ m}}$

(b) Eq. 5/9: $y = \frac{T_0}{\mu} \left(\cosh \frac{\mu x}{T_0} - 1 \right)$

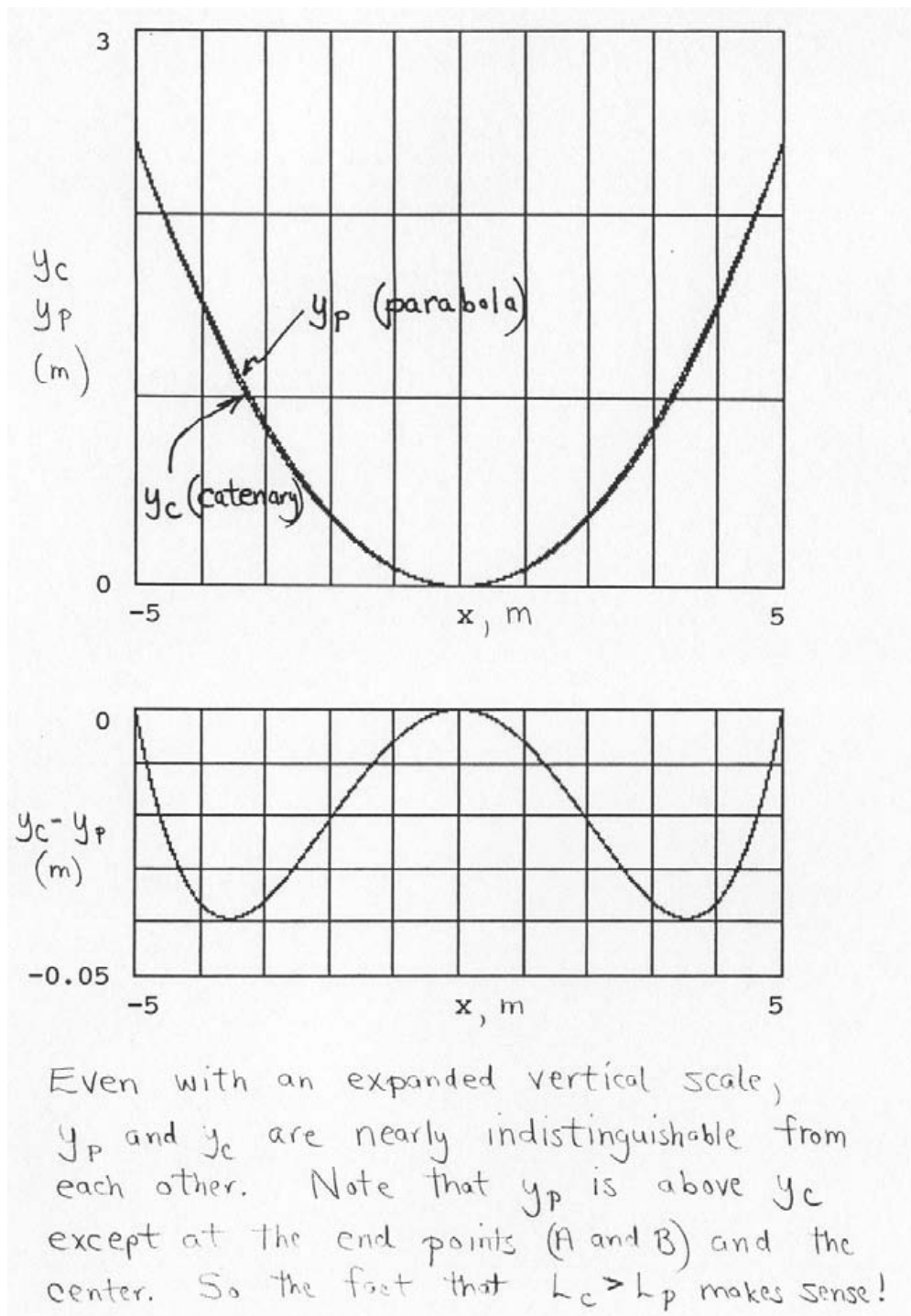
At A: $2.4 = \frac{T_0}{11.77} \left[\cosh \frac{11.77(5)}{T_0} - 1 \right]$

Numerical solution: $T_0 = 65.5 \text{ N}$

So $y_c = 5.57 \left[\cosh(0.1796x) - 1 \right]$ (see plots)

Eq. 5/20: $s_A = \frac{65.5}{11.77} \sinh \frac{11.77(5)}{65.6} = 5.70 \text{ m}$

The required length is $L_c = 2s_A = \underline{11.40 \text{ m}}$



5/167 From Eq. 5/19 with $x=100$ m, $y=32$ m

$$32 = \left(\frac{T_0}{\mu}\right) \left[\cosh\left(\frac{100\mu}{T_0}\right) - 1 \right]$$

Solve by computer or graphically & get $\frac{T_0}{\mu} = 161.3$ m

From Eq. 5/22, $60(10^3) = T_0 + 32\mu$

Solve simultaneously & get $\mu = 310$ N/m

Thus, $\rho = \frac{\mu}{g} - \rho_{\text{air}} = \frac{310}{9.81} - 18.2 = \underline{13.44 \text{ kg/m of ice}}$

$$\text{*5/168} \quad \mu = 0.5(9.81) = 4.90 \text{ N/m}$$

$$\text{Eq. 5/19 @ A : } y_A = \frac{T_0}{\mu} \left[\cosh \frac{\mu x_A}{T_0} - 1 \right] \quad (1)$$

$$\text{Eq. 5/19 @ B : } y_A + 9 = \frac{T_0}{\mu} \left[\cosh \left(\frac{\mu}{T_0} (x_A + 12) \right) - 1 \right] \quad (2)$$

$$\frac{dy}{dx} = \sinh \frac{\mu x}{T_0} \quad \text{@ A :}$$

$$\tan 15^\circ = \sinh \left[\frac{\mu}{T_0} (x_A) \right] \quad (3)$$

Solve Eqs. (1) - (3) numerically :

$$T_0 = 71.5 \text{ N}, \quad x_A = 3.86 \text{ m}, \quad y_A = 0.514 \text{ m}$$

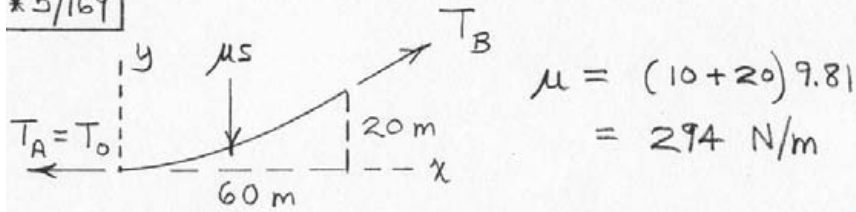
Then

$$\begin{aligned} T_B &= T_0 + \mu y_B = 71.5 + 4.90 (0.514 + 9) \\ &= \underline{118.2 \text{ N}} \end{aligned}$$

$$\tan \theta_B = \sinh \left[\frac{4.90}{71.5} (3.86 + 12) \right]$$

$$\underline{\theta_B = 52.8^\circ}$$

*5/169



Eq. 5/19: $y = \frac{T_0}{\mu} \left(\cosh \frac{\mu x}{T_0} - 1 \right)$

Numbers: $20 = \frac{T_A}{294} \left(\cosh \frac{294(60)}{T_A} - 1 \right)$

Numerical solution: $T_A = 27400 \text{ N}$
or 27.4 kN

Eq. 5/22: $T = T_0 + \mu y$

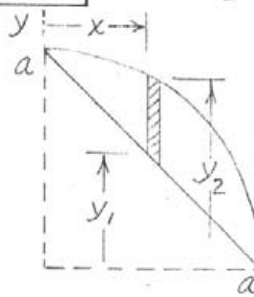
$T_B = 27400 + 294(20) = 33300 \text{ N}$
or 33.3 kN

Eq. 5/20: $s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$= \frac{27400}{294} \sinh \frac{294(60)}{27400} = \underline{64.2 \text{ m}}$

5/17

$$x^2 + y_2^2 = a^2; \quad x + y_1 = a$$



$$dA = (y_2 - y_1) dx = (\sqrt{a^2 - x^2} - a + x) dx$$

$$A = \int_0^a (\sqrt{a^2 - x^2} - a + x) dx$$

$$= \left[\frac{1}{2} (x\sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a}) - ax + \frac{x^2}{2} \right]_0^a$$

$$= \frac{a^2}{4} (\pi - 2)$$

$$\int x_c dA = \int_0^a (x\sqrt{a^2 - x^2} - ax + x^2) dx$$

$$= \left[-\frac{1}{3} \sqrt{(a^2 - x^2)^3} - \frac{ax^2}{2} + \frac{x^3}{3} \right]_0^a = \frac{a^3}{6}$$

$$\bar{x} = \int x_c dA / A = \frac{a^3}{6} / \frac{a^2}{4} (\pi - 2) = \frac{2a}{3(\pi - 2)} = \bar{y} \text{ by symm.}$$

*5/170 Eq. 5/19: $y = \frac{T_0}{\mu} \left(\cosh \frac{\mu x}{T_0} - 1 \right)$

Numbers and rearranging:

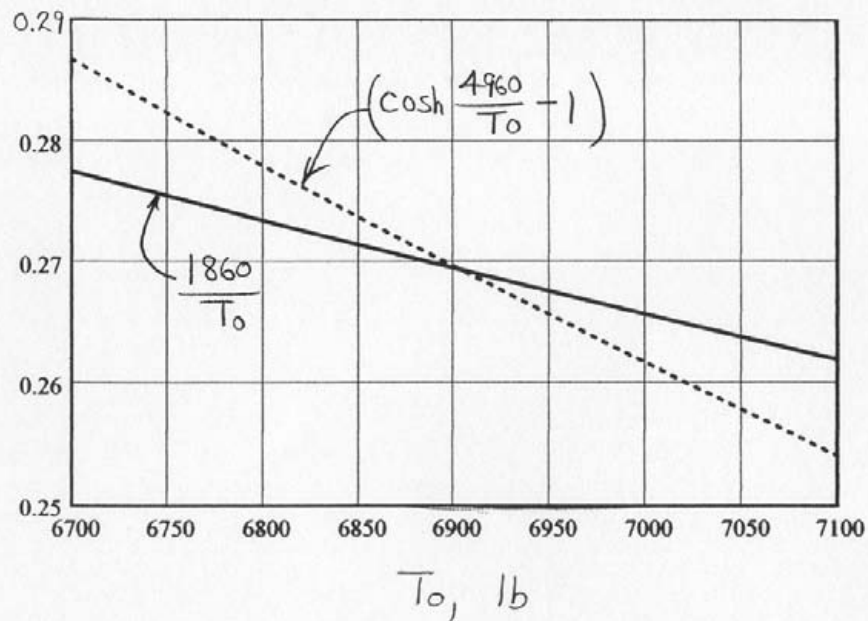
$$600 = \frac{T_0}{3.10} \left(\cosh \frac{3.10(1600)}{T_0} - 1 \right) \text{ or } \frac{1860}{T_0} = \cosh \frac{4960}{T_0} - 1$$

Numerical or graphical (see below) solution:

$T_0 = 6900 \text{ lb}$. Then from Eq. 5/22:

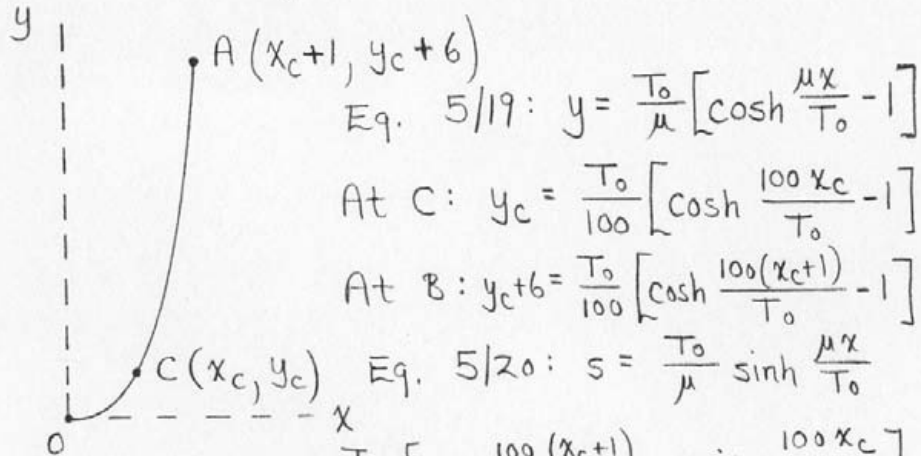
$$T = T_0 + \mu y = 6900 + 3.10(600) = \underline{8760 \text{ lb}}$$

$$\begin{aligned} \text{Eq. 5/20: } s &= \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0} \\ &= \frac{6900}{3.10} \sinh \frac{3.10(1600)}{6900} = \underline{1741 \text{ ft}} \end{aligned}$$



*5/171 Architect's plan : $(T_A)_{\text{arch}} = 6(100) = 600 \text{ N}$

Builder's arrangement :



$$\text{So } s_A - s_c = 6.1 = \frac{T_0}{100} \left[\sinh \frac{100(x_c + 1)}{T_0} - \sinh \frac{100 x_c}{T_0} \right]$$

Numerical solution of three equations :

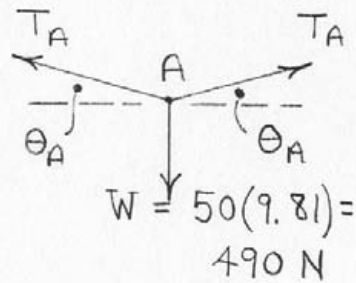
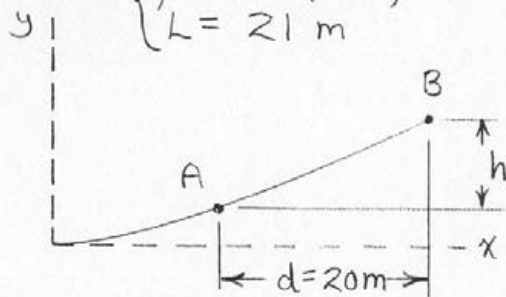
$$x_c = 1.071 \text{ m}, \quad y_c = 1.088 \text{ m}, \quad T_0 = 65.5 \text{ N}$$

$$\text{Eq. 5/22: } T = T_0 + \mu y, \text{ so } T_A = 65.5 + 100(1.088 + 6) = 774 \text{ N}$$

$$\text{Percent increase } n = \frac{774 - 600}{600} (100) = \underline{29.0\%} \quad (!)$$

*5/172

$$\begin{cases} \mu = 1.2 (9.81) = 11.77 \text{ N/m} \\ L = 21 \text{ m} \end{cases}$$



From FBD of junction ring at A,

$$\uparrow \Sigma F = 0: 2T_A \sin \theta_A - W = 0$$

$$\text{or } [T_0 + \mu y_A] \sin \left[\tan^{-1} \left(\sinh \frac{\mu x_A}{T_0} \right) \right] - \frac{W}{2} = 0 \quad (1)$$

$$\text{Eq. 5/19 @ A: } y_A = \frac{T_0}{\mu} \left[\cosh \frac{\mu x_A}{T_0} - 1 \right] \quad (2)$$

$$\text{Eq. 5/19 @ B: } y_A + h = \frac{T_0}{\mu} \left[\cosh \frac{\mu (x_A + d)}{T_0} - 1 \right] \quad (3)$$

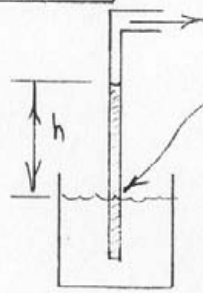
$$\text{Eq. 5/20: } s_B - s_A = \frac{T_0}{\mu} \left[\sinh \frac{\mu (x_A + d)}{T_0} - \sinh \frac{\mu x_A}{T_0} \right] = L \quad (4)$$

Solution of (1) - (4) with $W = 0$: $h = 5.57 \text{ m}$

With $W \neq 0$: $h = 6.30 \text{ m}$

$$\text{So } \delta = 6.30 - 5.57 = \underline{0.724 \text{ m}}$$

5/173

vacuum
source(Assume vacuum pump
reduces pressure to zero.)

Pressure here must be atmospheric:

$$\rho g h = P_{at}$$

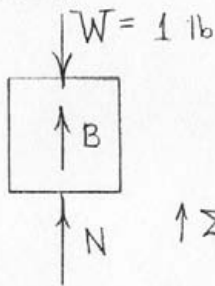
$$(1000)(9.81)(h) = 1.0133 (10^5)$$

$$\underline{h = 10.33 \text{ m}}$$

For mercury: $13\,570 (9.81)(h) = 1.0133 (10^5)$

$$\underline{h = 0.761 \text{ m}} \quad (29.97 \text{ in.})$$

5/174

The buoyancy force B is

$$B = \mu_w V = \mu_w \frac{W}{\mu_{ss}}$$

$$= \frac{62.4}{490} (1) = 0.1273 \text{ lb}$$

$$\uparrow \Sigma F = 0: N + 0.1273 - 1 = 0, N = 0.873 \text{ lb}$$

So the force which the weight exerts on the beaker bottom is 0.873 lb (down). The scale reading, however, increases by 1 lb as the weight is added. The difference is due to the increased water pressure on the beaker bottom as the water level rises.

5/175

Force on bottom = weight of water

$$= \rho g V = (1000 \frac{\text{kg}}{\text{m}^3})(9.81 \frac{\text{m}}{\text{s}^2})(0.3\text{m})(0.7\text{m})(0.4\text{m})$$

$$= \underline{824 \text{ N}} \quad (\text{down, at center of bottom})$$

$$\text{Force on front \& back} = P_{av} A_f = \frac{\rho g h}{2} A_f$$

$$= \frac{1000 (9.81) (0.4)}{2} (0.7)(0.4) = \underline{549 \text{ N}} \quad \left(\begin{array}{l} \text{outward, at} \\ \frac{2}{3} \text{ depth} \end{array} \right)$$

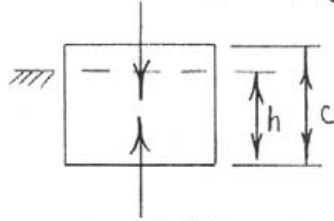
$$\text{Force on each end glass} = P_{av} A_e = \frac{\rho g h}{2} A_e$$

$$= \frac{1000 (9.81) (0.4)}{2} (0.3)(0.4) = \underline{235 \text{ N}} \quad \left(\begin{array}{l} \text{outward, at} \\ \frac{2}{3} \text{ depth} \end{array} \right)$$

(All side forces centered horizontally)

5/176

$$W = mg = \rho_1 V g = \rho_1 abc g$$



$$B = \rho_2 V_{\text{sub}} g = \rho_2 ab h g$$

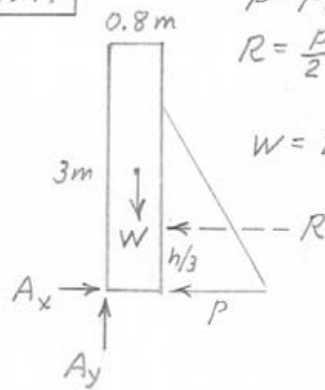
$$\uparrow \Sigma F = 0: \rho_2 ab h g - \rho_1 abc g = 0, \quad h = \frac{\rho_1}{\rho_2} c$$

$$r = \frac{h}{c} = \frac{\rho_1}{\rho_2}$$

$$\text{Oak in water: } r = \frac{800}{1000} = \underline{0.8}$$

$$\text{Steel in mercury: } r = \frac{7830}{13570} = \underline{0.577}$$

5/177



$$p = \rho g h = 1760(9.81)h$$

$$R = \frac{p}{2} h (1) = 1760(9.81) \frac{h^2}{2} = 8633h^2 \text{ N}$$

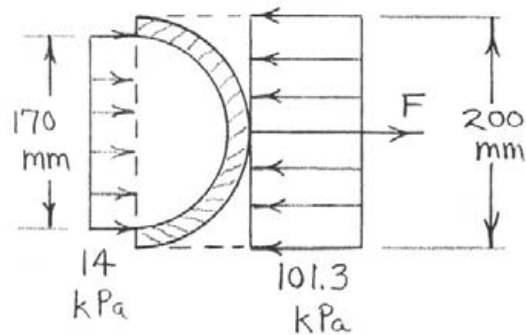
per meter of horizontal width

$$W = 2400(9.81)(3)(0.8) = 56.5(10^3) \text{ N}$$

$$\Sigma M_A = 0; R \frac{h}{3} - 0.4W = 0$$

$$8633h^2 \frac{h}{3} = 0.4(56.5)(10^3)$$

$$h^3 = 7.85, \quad \underline{h = 1.988 \text{ m}}$$



$$\rightarrow \sum F = 0: F + 14(10^3) \frac{\pi (0.170)^2}{4} - 101.3(10^3) \frac{\pi (0.200)^2}{4} = 0$$

$$F = 2860 \text{ N or } \underline{F = 2.86 \text{ kN}}$$

5/179

$$F = pA = \rho g h A = \rho g h \pi r^2$$

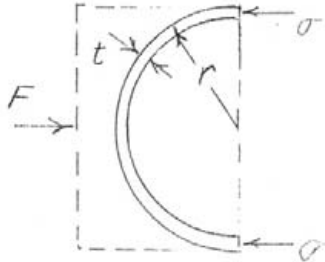
$$\sum F = 0; \rho g h \pi r^2 - \sigma (\pi r^2 - \pi [r-t]^2)$$

$$\sigma = \frac{\rho g h r}{2t} \frac{1}{1 - t/2r}$$

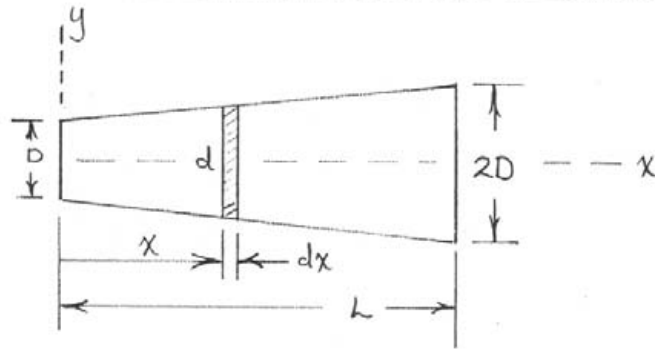
$$= \frac{1.03(10^3)(9.81) \frac{1.500}{2} 3(10^3)}{2(0.025)} \frac{1}{1 - \frac{25}{1500}}$$

$$= 454.7(10^6)(1.0169)$$

$$\text{or } \sigma = \underline{463 \text{ MPa}}$$



5/18



For constant density, $\bar{x}V = \int x dV$

Diameter $d = D \left(1 + \frac{x}{L}\right)$

$$\text{So } dV = \frac{\pi d^2}{4} dx = \frac{\pi D^2}{4} \left(1 + \frac{x}{L}\right)^2 dx$$

$$V = \frac{\pi D^2}{4} \int_0^L \left(1 + \frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4} \left[x + \frac{x^2}{L} + \frac{x^3}{3L^2} \right]_0^L$$

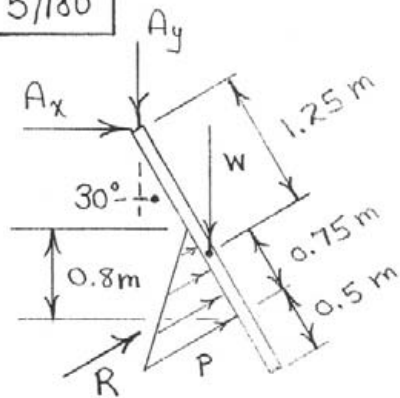
$$= \frac{7}{12} \pi D^2 L$$

$$\int x dV = \frac{\pi D^2}{4} \int_0^L x \left(1 + \frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4} \left[x^2 + \frac{2x^3}{3L} + \frac{x^4}{4L^2} \right]_0^L$$

$$= \frac{17}{48} \pi D^2 L^2$$

$$\bar{x} = \frac{\frac{17}{48} \pi D^2 L^2}{\frac{7}{12} \pi D^2 L} = \frac{17}{28} L$$

5/180

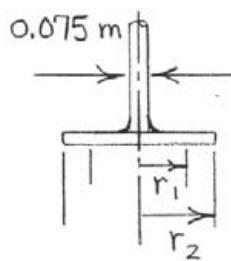


$$P = \rho g h = 1000(9.81)(0.8) \\ = 7850 \text{ Pa}$$

$$R = \frac{1}{2}(7850)\left(\frac{0.8}{\cos 30^\circ}\right) \\ = 3620 \text{ N/m}$$

$$\begin{aligned} \sum M_A = 0: & \quad w(1.25 \sin 30^\circ) - 3620\left(2.5 - 0.5 - \frac{1}{3} \frac{0.8}{\cos 30^\circ}\right) \\ & = 0; \quad \underline{w = 9810 \text{ N/m}} \end{aligned}$$

5/181



$$\begin{cases} r_1 = 0.2 \text{ m} \\ r_2 = 0.3 \text{ m} \end{cases}$$

$$p = \rho g h = 1030 (9.81) (0.6) = 6060 \text{ Pa}$$

$$R = pA = 6060 \pi (0.3^2 - \frac{0.075^2}{4}) = 1687 \text{ N}$$

Pressure supported by seal

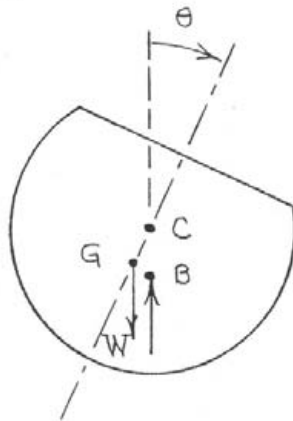
$$\sigma = \frac{R}{\pi(r_2^2 - r_1^2)} = \frac{1687}{\pi(0.3^2 - 0.2^2)}$$

$$= 10740 \text{ Pa or } \underline{10.74 \text{ kPa}}$$

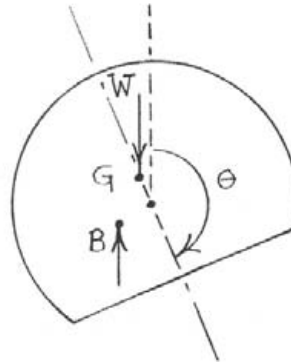
Force to lift plunger

$$P = R = 1687 \text{ N or } \underline{1.687 \text{ kN}}$$

5/182



CCW couple tends to
make $\theta = 0$



CW couple tends to
make $\theta = 180^\circ$

5/183

$$p = \rho g h = 1030 (9.81) (1000) = 10.104 (10^6) \text{ Pa}$$

$$R = pA = 10.104 (10^6) \pi (0.175)^2$$

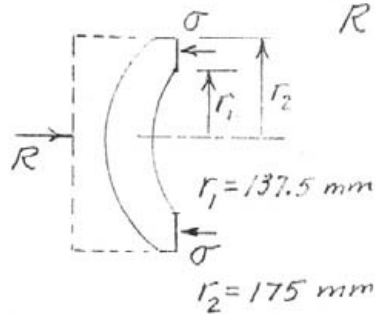
$$= 972 (10^3) \text{ N}$$

$$\sigma = R/A_o$$

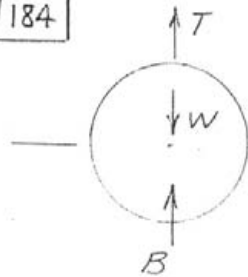
$$= \frac{972 (10^3)}{\pi ([0.175]^2 - [0.1375]^2)}$$

$$= 26.4 (10^6) \text{ Pa}$$

$$\text{or } \sigma = 26.4 \text{ MPa}$$



5/184



Let γ_c = wt. density of concrete
 $= 150 \text{ lb/ft}^3$

γ_w = wt. density of fresh
 water $= 62.4 \text{ lb/ft}^3$

L = length of cylinder $= 6 \text{ ft}$

r = radius of cylinder $= 2 \text{ ft}$

For equil., $T = W - B$

$$= \gamma_c \pi r^2 L - \gamma_w \frac{\pi r^2}{2} L$$

$$= \pi r^2 L \left(\gamma_c - \frac{1}{2} \gamma_w \right)$$

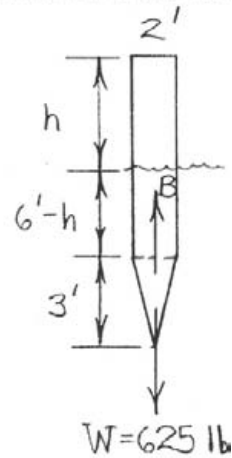
$$= \pi (2^2) (6) \left(150 - \frac{62.4}{2} \right) = \underline{8960 \text{ lb}}$$

5/185

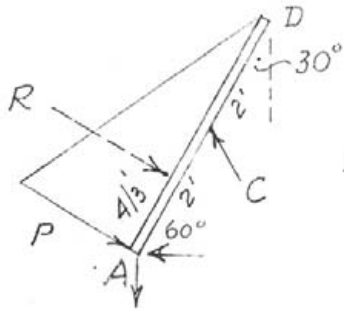
$$\begin{aligned}
 B &= \mu V = 64 \left[\pi \frac{z^2}{4} (6-h) + \frac{1}{3} \pi \frac{z^2}{4} (3) \right] \\
 &= 64 \pi \frac{z^2}{4} [6-h+1] \\
 &= 64 \pi [7-h]
 \end{aligned}$$

$$W = B : 625 = 64 \pi [7-h]$$

$$\underline{h = 3.89 \text{ ft}}$$



5/186



$$p = \rho g h = 62.4 (4 \cos 30^\circ) = 216 \text{ lb/ft}^2$$

$$R = \frac{p}{2} \text{ Area per panel}$$

$$= \frac{216}{2} (4)(2) = 865 \text{ lb}$$

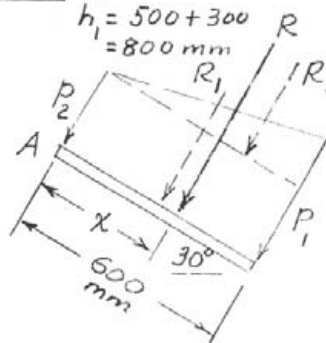
$$\sum M_A = 0; 2C \cos 30^\circ - 865 (4/3) = 0$$

$$C = \frac{865(2)}{3 \cos 30^\circ} = \underline{666 \text{ lb}}$$

5/187

$$h_2 = 500 \text{ mm}$$

$$h_1 = 500 + 300 = 800 \text{ mm}$$



$$p_2 = \rho g h_2 = 900(9.81)(0.500) = 4.415(10^3) \text{ Pa}$$

$$p_1 = \rho g h_1 = 900(9.81)(0.800) = 7.063(10^3) \text{ Pa}$$

$$R_1 = p_2 A = 4.415(10^3)(0.600)(0.400) = 1059 \text{ N}$$

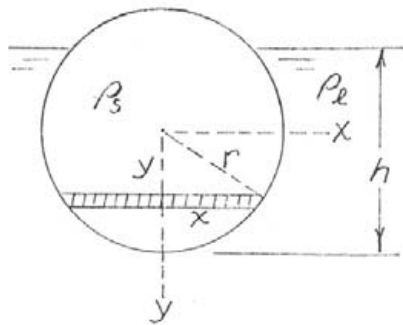
$$R_2 = \frac{p_1 - p_2}{2} A = \frac{(7.063 - 4.415)10^3}{2}(0.600)(0.400) = 318 \text{ N}$$

$$R = R_1 + R_2 = 1059 + 318 = 1377 \text{ N}$$

$$R x = \sum M_A, \quad 1377 x = 1060(300) + 318(400)$$

$$x = \frac{445000}{1377} = 323 \text{ mm}$$

5/188

Buoyancy $B = \rho_l g V$, $V = \text{submerged volume}$ 

$$W = mg = \rho_s \frac{4}{3} \pi r^3 g$$

$$dV = \pi x^2 dy = \pi (r^2 - y^2) dy$$

$$B = \rho_l g \pi \int_{-(h-r)}^r (r^2 - y^2) dy$$

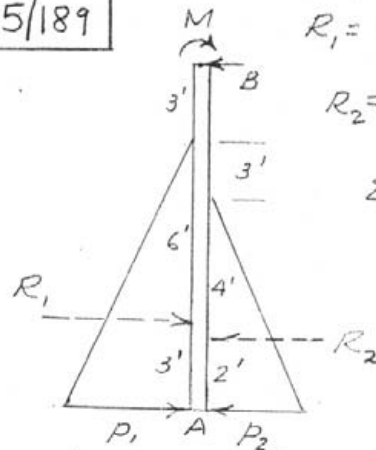
$$= \rho_l g \pi \left[r^2 y - \frac{y^3}{3} \right]_{-(h-r)}^r$$

$$= \frac{1}{3} \rho_l g \pi h^2 (3r - h)$$

Thus with $B = W$, $\rho_s g \frac{4}{3} \pi r^3 = \frac{1}{3} \rho_l g \pi h^2 (3r - h)$

Rearrange & get $\rho_s = \rho_l \left(\frac{h}{2r} \right)^2 \left(3 - \frac{h}{r} \right)$

5/189



Horiz. length of gate is 10 ft

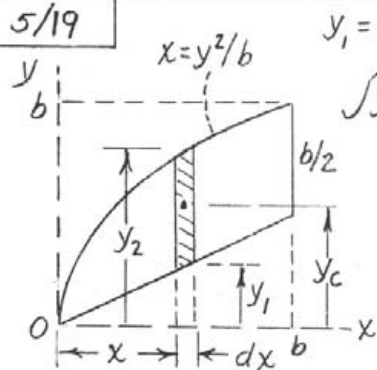
$$R_1 = \frac{p_1 A_1}{2} = \frac{62.4(9)}{2}(9)(10) = 25,270 \text{ lb}$$

$$R_2 = \frac{p_2 A_2}{2} = \frac{64(6)}{2}(6)(10) = 11,520 \text{ lb}$$

$$\sum M_B = 0; M + 11,520(10) - 25,270(9) = 0$$

$$M = 11.22(10^4) \text{ lb-ft}$$

5/19



$$y_1 = x/2, \quad y_2 = \sqrt{bx}$$

$$\int y_c dA = \int_0^b \frac{y_1 + y_2}{2} (y_2 - y_1) dx$$

$$= \frac{1}{2} \int_0^b (y_2^2 - y_1^2) dx$$

$$= \frac{1}{2} \int_0^b (bx - \frac{x^2}{4}) dx$$

$$= \frac{1}{2} \left[\frac{bx^2}{2} - \frac{x^3}{12} \right]_0^b = \frac{5}{24} b^3$$

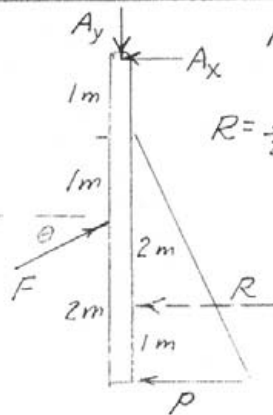
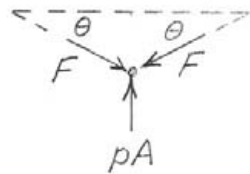
$$\int dA = \int_0^b (y_2 - y_1) dx = \int_0^b (\sqrt{bx} - x/2) dx = \left[\frac{2\sqrt{b}}{3} x^{3/2} - \frac{x^2}{4} \right]_0^b$$

$$= \frac{5}{12} b^2$$

$$\bar{y} = \frac{\int y_c dA}{\int dA} = \frac{\frac{5}{24} b^3}{\frac{5}{12} b^2} = \underline{\underline{\frac{b}{2}}}$$

5/190

$$\theta = \tan^{-1} \frac{0.5}{1} = 26.6^\circ$$



$$p = \rho g h = 1.0(9.81)(3) = 29.4 \text{ kPa}$$

$$R = \frac{p}{2} \text{ Area} = \frac{29.4}{2} (3)(2) = 88.3 \text{ kN}$$

Gate:

$$\sum M_A = 0; (F \cos 26.6^\circ) 2 - 88.3 (3) = 0, F = 148.1 \text{ kN}$$

Toggle:

$$\sum F = 0; pA - 2F \sin \theta = 0; \frac{\pi (0.150)^2}{4} p = 2(148.1)(10^3) \sin 26.6^\circ$$

$$p = 7.49(10^6) \text{ Pa or } \underline{p = 7.49 \text{ MPa}}$$

5/191 Submerged volume V is

$$V = 2(350)(40)(25) + 6\pi \frac{(30)^2}{4}(h-25) \\ = 70(10^4) + 0.4241(10^4)(h-25)$$

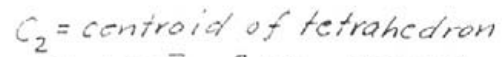
$$B = \gamma V = 64(70)10^4 + 64(0.4241)(10^4)(h-25) \\ = 4480(10^4) + 27.14(10^4)(h-25)$$

$$W = 26,000(2240) = 5824(10^4) \text{ lb}$$

$$B = W, \quad 27.14(10^4)(h-25) = (5824 - 4480)10^4 \\ h-25 = 49.5, \quad \underline{h = 74.5 \text{ ft}}$$

5/192 C_1 = centroid of triangular area

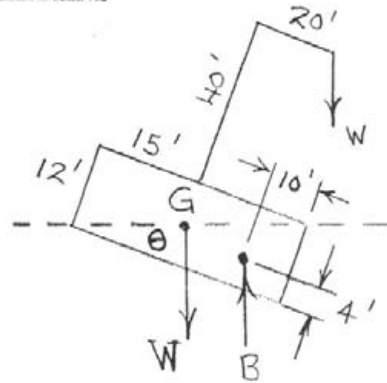
so $\bar{y} = \frac{2}{3}(3) = 2$ ft


$$\text{So } H = \bar{Y} = \frac{3}{4}(3) = \underline{2.25 \text{ ft}}$$

$$= (62.4)(2) \frac{1}{2} (3)(3)$$

$$= 562 \text{ lb}$$

5/193



$$\theta = \tan^{-1} \frac{12}{30} = 21.8^\circ$$

Moment arm of B about G

$$(15-10) \cos \theta - (6-4) \sin \theta = 3.90 \text{ ft}$$

Moment arm of W about G

$$(40+6) \sin \theta + 20 \cos \theta = 35.7 \text{ ft}$$

$$B = \rho g V = 64(12)(15)(80) = 922,000 \text{ lb}$$

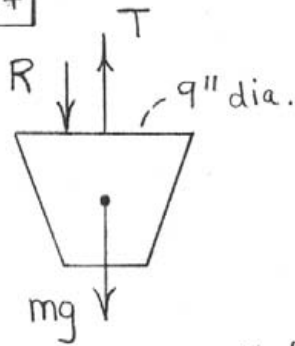
$$\sum M_G = 0: 35.7 W - 3.90 (922,000) = 0$$

$$W = 100,800 \text{ lb}$$

$$W = B - w = 922,000 - 100,800 = 821,000 \text{ lb}$$

$$\text{or } W = \frac{821,000}{2240} = 366 \text{ long tons}$$

5/194



$$p = \rho g h = 62.4(20) = 1248 \frac{\text{lb}}{\text{ft}^2}$$

$$R = pA = 1248 \left[\pi \left(\frac{4.5}{12} \right)^2 \right] = 551 \text{ lb}$$

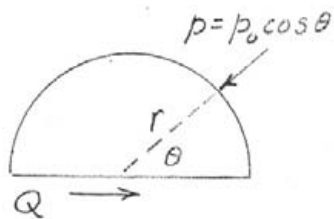
$$mg = \rho V g = \rho g \left[\frac{7\pi r^2 h}{24} \right] \quad (\text{from Prob. 5/35})$$

$$= 450 \left[\frac{7\pi \left(\frac{4.5}{12} \right)^2 \left(\frac{12}{12} \right)}{24} \right] = 58.0 \text{ lb}$$

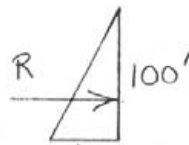
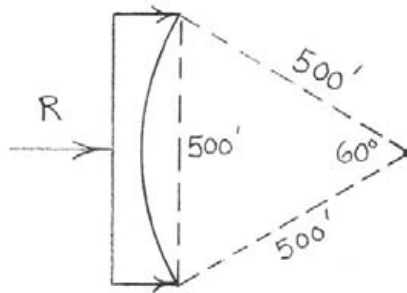
$$+\uparrow \Sigma F = 0 : T - 551 - 58.0 = 0, \quad \underline{T = 609 \text{ lb}}$$

5/195

$$\begin{aligned}
 Q &= \int_0^{\pi} (p_0 \cos \theta) (\cos \theta) r d\theta \\
 &= p_0 r \int_0^{\pi} \cos^2 \theta d\theta \\
 &= p_0 r \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi} \\
 &= p_0 r \left[\frac{\pi}{2} \right] \\
 Q &= \frac{1}{2} \pi r p_0
 \end{aligned}$$



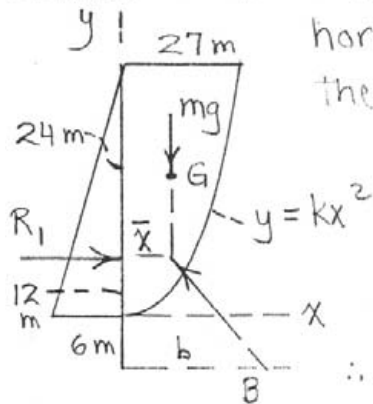
5/196



$$P_{100} = \mu h = 62.4(100) = 6240 \frac{\text{lb}}{\text{ft}^2}$$

$$R = P_{av} (\text{Area}) = \frac{6240}{2} (500)(100) = \underline{156.0 (10^6) \text{ lb}}$$

5/197 Take a vertical section of water of unit horizontal length. Let ρ be the water density in t/m^3 .



$$y = kx^2: 36 = k(27)^2, k = \frac{4}{81} \text{ m}^{-1}$$

$$\bar{x} = \frac{\int x dA}{\int dA}, dA = x dy = 2 \frac{4}{81} x^2 dx$$

$$\therefore \bar{x} = \frac{\int_0^{27} \frac{x}{2} \frac{8}{81} x^2 dx}{\int_0^{27} \frac{8}{81} x^2 dx} = 10.12 \text{ m}$$

$$A = \int dA = 648 \text{ m}^2, mg = 648 \rho g$$

$$R_1 = \frac{1}{2} 36 \rho g (36)(1) = 648 \rho g$$

Resultant of mg & R_1 passes through B, so

$$\sum M_B = 0. \text{ Thus } 648 \rho g (18) = 648 \rho g (b - 10.12)$$

$$b = 28.1 \text{ m}$$

5/198 | The gage pressure 12 m below the surface

is $p = \rho gh = (1000)(9.81)(12) = 117\,700 \text{ N/m}^2$.

(a) Cover area $A_{\text{cov}} = \pi \left(\frac{0.75}{2}\right)\left(\frac{0.5}{2}\right) = 0.295 \text{ m}^2$

Force on cover $= p A_{\text{cov}} = 34\,700 \text{ N}$

Seal area $A_s = A_{\text{cov}} - \pi \left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right) = 0.1325 \text{ m}^2$

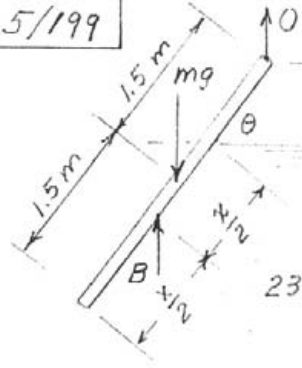
$\sigma A_s = p A_{\text{cov}}, \quad \sigma = \frac{34\,700}{0.1325} = 262\,000 \frac{\text{N}}{\text{m}^2}$

or $\sigma = 262 \text{ kPa}$

(b) $16 \Delta T = p A_{\text{hole}} = 117\,700 \left[\pi \left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right) \right]$

$\Delta T = 1192 \text{ N}$

5/199



Let A = cross-sectional area of plank

$$mg = 800(3)A(9.81) = 23.54(10^3)A \text{ N}$$

$$B = \rho_w g A x$$

$$= 1000(9.81)A \left(3 - \frac{1}{\sin \theta}\right)$$

$$\sum M_O = 0;$$

$$23.54(10^3)A(1.5 \cos \theta) - 9.81(10^3)A \left(3 - \frac{1}{\sin \theta}\right) \cdot \left[\frac{1}{2} \left(3 - \frac{1}{\sin \theta}\right) + \frac{1}{\sin \theta} \right] \cos \theta = 0$$

$$\left[\frac{1}{2} \left(3 - \frac{1}{\sin \theta}\right) + \frac{1}{\sin \theta} \right] \cos \theta = 0$$

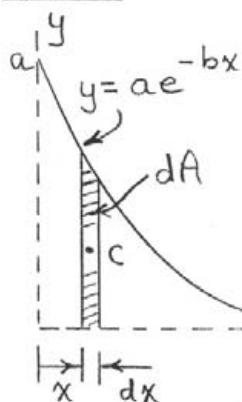
Simplify & set $23.54(1.5) = \frac{9.81}{2} \left(9 - \frac{1}{\sin^2 \theta}\right)$

& $\sin^2 \theta = 0.5556, \quad \sin \theta = 0.7454, \quad \theta = 48.2^\circ$

5/2 From Sample Problem 5/3 with $r=8$
and $\alpha = 120^\circ = \frac{2}{3}\pi$:

$$\bar{r} = \frac{2}{3}(8) \frac{\sin 120^\circ}{2\pi/3} = \underline{2.21}$$

5/20



$$dA = y dx = ae^{-bx} dx$$

$$A = \int dA = \int_0^{\infty} ae^{-bx} dx$$

$$= -\frac{a}{b} e^{-bx} \Big|_0^{\infty} = -\frac{a}{b} [0 - 1]$$

$$= a/b$$

$$\int x_c dA = a \int_0^{\infty} x e^{-bx} dx$$

$$= a \frac{e^{-bx}}{b^2} [-bx - 1]_0^{\infty}$$

$$= -\frac{a}{b^2} [bx e^{-bx} + e^{-bx}]_0^{\infty} = -\frac{a}{b^2} [0 + 0 - (0 + 1)]$$

$$= a/b^2$$

$$\int y_c dA = \int \frac{y}{2} y dx = \int \frac{y^2}{2} dx$$

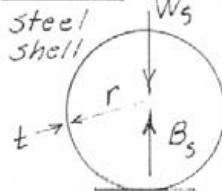
$$= \frac{1}{2} \int_0^{\infty} a^2 e^{-2bx} dx = \frac{a^2}{2} \frac{e^{-2bx}}{-2b} \Big|_0^{\infty}$$

$$= -\frac{a^2}{4b} [0 - 1] = \frac{a^2}{4b}$$

$$\bar{x} = \frac{\int x_c dA}{\int dA} = \frac{a/b^2}{a/b} = \frac{1}{b}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA} = \frac{a^2/4b}{a/b} = \frac{a}{4}$$

5/200

steel
shell

Lead

Ballast

For equilibrium $W_s + W_L = B_s + B_L$ ρ_s = density of steel = 7.83 Mg/m^3 ρ_w = " " salt water = 1.03 Mg/m^3 ρ_L = " " lead = 11.37 Mg/m^3 $r = 1.00 \text{ m}$, $t = 0.035 \text{ m}$ V_L = volume of lead, m^3 m = mass of lead = $\rho_L V_L$

$$\text{so } \rho_s g \frac{4}{3} \pi r^2 t + \rho_L g V_L = \rho_w g \frac{4}{3} \pi (r + \frac{t}{2})^3 + \rho_w g V_L$$

$$V_L g (\rho_L - \rho_w) = 4 \pi r^2 g \left[\frac{r}{3} (1 + \frac{t}{2r})^3 \rho_w - \rho_s t \right]$$

$$m (1 - \frac{\rho_w}{\rho_L}) = 4 \pi r^2 \left[\frac{r}{3} (1 + \frac{t}{2r})^3 \rho_w - \rho_s t \right]$$

$$m \left(1 - \frac{1.03}{11.37} \right) = 4 \pi (1)^2 \left[\frac{1}{3} (1 + \frac{0.035}{2})^3 1.03 - 7.83 (0.035) \right]$$

$$0.9094 m = 1.1008 ; \quad m = \underline{1.210 \text{ Mg}} \text{ (metric tons)}$$

5/201

The pressure at the bottom of the 3-m wall is $p = \rho gh = 2400(9.81)(3) = 70\,600 \text{ N/m}^2$

Each tie controls an area A given by

$$pA = T, \quad A = \frac{T}{p} = \frac{6500}{70\,600} = 0.0920 \text{ m}^2$$

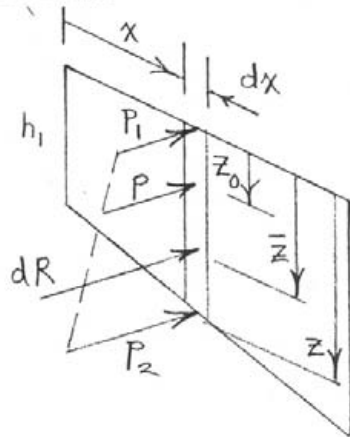
This square area has a side d given by

$$d^2 = A, \quad d = 0.303 \text{ m}$$

Using the pressure at the very bottom of the wall gives us a conservative design; a good figure for d would be $d = 0.300 \text{ m}$.

5/202

Method I: Direct integration



ρ = water density, $p = \rho g (b + z_0)$

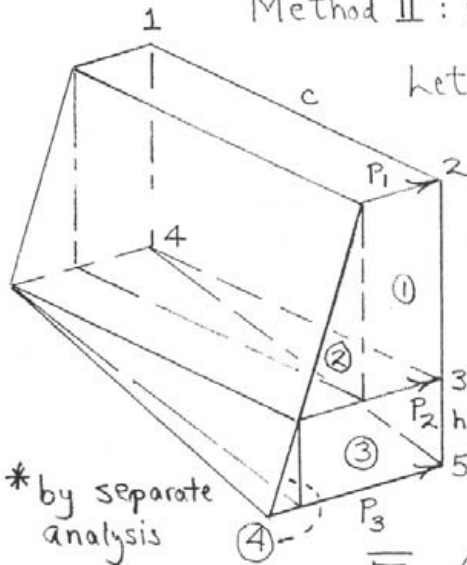
$$z = h_1 + kx = f(x)$$

(1) Calculate dR for elemental area $z dx$ & find \bar{z}

(2) Integrate & get $R = \int dR$

$$(3) R\bar{Z} = \int \bar{z} dR$$

Method II: Geometry of pressure-area volumes



Let $A_a = 1-2-3-4$, $A_b = 3-4-5$

$$R_1 = p_1 A_a, \quad \bar{z}_1 = h_1/2$$

$$R_2 = \frac{1}{2} (p_2 - p_1) A_a, \quad \bar{z}_2 = \frac{zh_1}{3}$$

$$R_3 = p_2 A_b, \quad \bar{z}_3 = h_1 + \frac{2}{3} (h_2 - h_1)$$

$$R_4 = \frac{1}{2} (p_3 - p_2) (h_2 - h_1) \frac{c}{3}$$

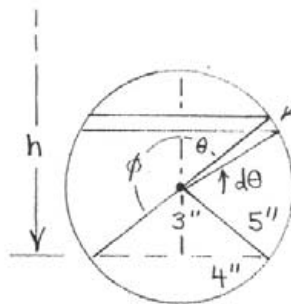
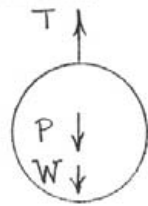
$$\bar{z}_4 = \frac{1}{2} (h_2 + h_1) *$$

$$R = R_1 + R_2 + R_3 + R_4$$

$$\bar{Z} = (R_1 \bar{z}_1 + R_2 \bar{z}_2 + R_3 \bar{z}_3 + R_4 \bar{z}_4) / R$$

* by separate analysis

5/203



$$\Sigma F = 0 : T - P - W = 0$$

$$\text{Pressure force } P = \int p dA \cos \theta$$

$$\rho_g = 62.4 / 1728 = 0.0361 \text{ lb/in.}^3$$

$$\phi = 180^\circ - \cos^{-1}(0.6) = 126.9^\circ$$

$$V = \frac{126.9}{180} \frac{4}{3} \pi (5^3) = 369 \text{ in.}^3, B = 369 \rho_g \text{ (lb)}$$

$$p = \rho_g (h - 3 - r \cos \theta) = \rho_g (h' - r \cos \theta), h' = h - 3$$

$$P = \int_0^\phi \rho_g (h' - r \cos \theta) (2\pi r \sin \theta) r d\theta \cos \theta$$

$$= 2\pi r^2 \rho_g \int_0^\phi (h' \sin \theta \cos \theta - r \sin \theta \cos^2 \theta) d\theta$$

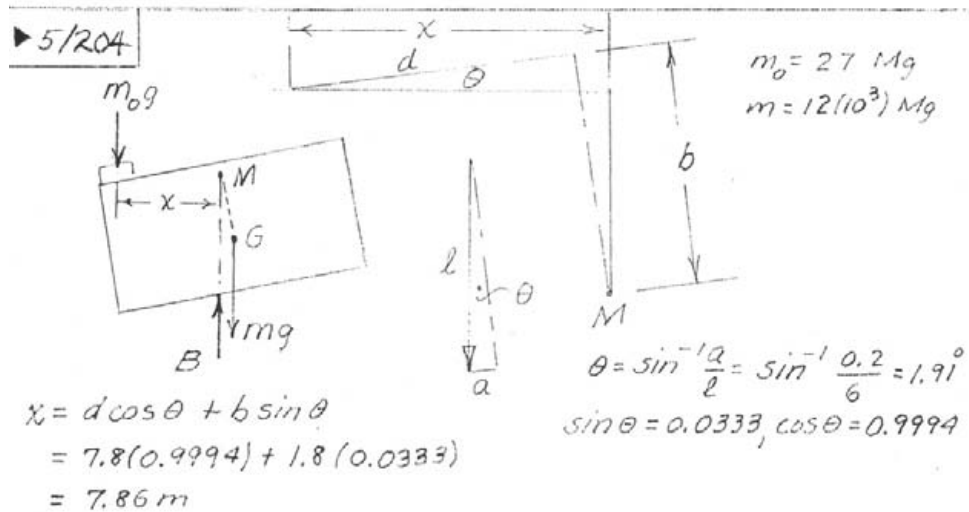
$$= 2\pi r^2 \rho_g \left[\int_{\theta=0}^{\theta=\phi} \frac{h'}{4} \sin 2\theta d(2\theta) - \int_0^\phi r (\sin \theta - \sin^3 \theta) d\theta \right]$$

$$= 2\pi r^2 \rho_g \left[-\frac{h'}{4} \cos 2\theta - r (-\cos \theta + \frac{\cos \theta}{3} (2 + \sin^2 \theta)) \right]_0^\phi$$

$$= 2\pi r^2 \rho_g [0.32 h' - 2.03] \text{ (lb)}$$

$$\text{When } T = W, P = 0 : h' = \frac{2.03}{0.32} = h - 3$$

$$h = 9.33 \text{ in.}$$

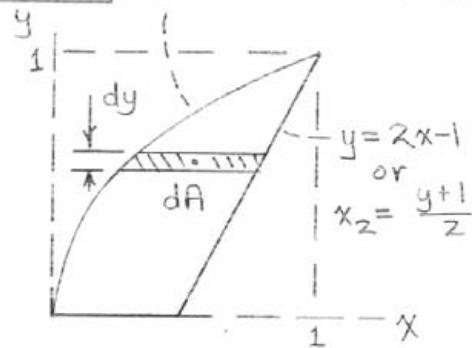


$$\sum M_M = 0; \quad mg \cdot \overline{GM} \sin \theta - m_0 g x = 0$$

$$\overline{GM} = \frac{m_0 x}{m \sin \theta} = \frac{27(7.86)}{12(10^3)(0.0333)} = 0.530 \text{ m}$$

5/205

$$y = x^{1/3} \text{ or } x_1 = y^3$$



$$dA = (x_2 - x_1) dy$$

$$= \left(\frac{y}{2} + \frac{1}{2} - y^3 \right) dy$$

$$A = \int dA = \int_0^1 \left(\frac{y}{2} + \frac{1}{2} - y^3 \right) dy = \left(\frac{y^2}{4} + \frac{y}{2} - \frac{y^4}{4} \right) \Big|_0^1 = \frac{1}{2}$$

$$\int y_c dA = \int_0^1 \left(\frac{y^2}{2} + \frac{y}{2} - y^4 \right) dy = \left(\frac{y^3}{6} + \frac{y^2}{4} - \frac{y^5}{5} \right) \Big|_0^1 = \frac{13}{60}$$

$$\int x_c dA = \int_0^1 \left(\frac{x_1 + x_2}{2} \right) (x_2 - x_1) dy = \frac{1}{2} \int_0^1 (x_2^2 - x_1^2) dy$$

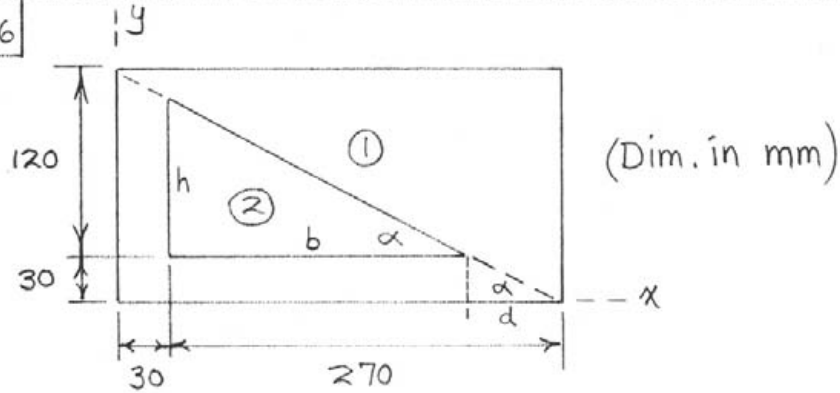
$$= \frac{1}{2} \int_0^1 \left(\frac{y^2}{4} + \frac{y}{2} + \frac{1}{4} - y^6 \right) dy = \frac{1}{2} \left(\frac{y^3}{12} + \frac{y^2}{4} + \frac{y}{4} - \frac{y^7}{7} \right) \Big|_0^1$$

$$= \frac{37}{168}$$

$$\bar{y} = \frac{\int y_c dA}{\int dA} = \frac{13/60}{1/2} = \frac{13}{30}$$

$$\bar{x} = \frac{\int x_c dA}{\int dA} = \frac{37/168}{1/2} = \frac{37}{84}$$

5/206



$$\tan \alpha = \frac{150}{300} = \frac{1}{2}; \quad \frac{30}{d} = \tan \alpha = \frac{1}{2}, \text{ so } d = 60 \text{ mm}$$

$$\text{Then } b = 300 - (30 + 60) = 210 \text{ mm}$$

$$\frac{h}{b} = \tan \alpha = \frac{1}{2}, \quad h = \frac{b}{2} = 105 \text{ mm}$$

$$A_1 = 150 (300) = 45\,000 \text{ mm}^2$$

$$\bar{x}_1 = 150 \text{ mm}, \quad \bar{y}_1 = 75 \text{ mm}$$

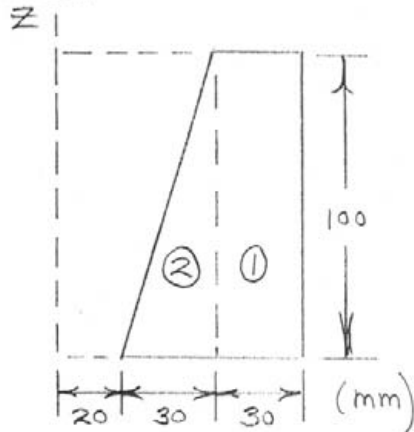
$$A_2 = -\frac{1}{2}bh = -\frac{1}{2}(210)(105) = -11\,025 \text{ mm}^2$$

$$\bar{x}_2 = 30 + \frac{1}{3}(210) = 100 \text{ mm}, \quad \bar{y}_2 = 30 + \frac{1}{3}(105) = 65 \text{ mm}$$

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{45\,000(150) - 11\,025(100)}{45\,000 - 11\,025} = \underline{166.2 \text{ mm}}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{45\,000(75) - 11\,025(65)}{45\,000 - 11\,025} = \underline{78.2 \text{ mm}}$$

5/207



$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi (65)(30 \cdot 100) = 1.225 (10^6) \text{ mm}^3$$

$$V_2 = 2\pi \bar{r}_2 A_2 = 2\pi (40) \frac{30 \cdot 100}{2} = 0.377 (10^6) \text{ mm}^3$$

$$V = V_1 + V_2 = 1.602 (10^6) \text{ mm}^3$$

$$m = \rho V = 7210 \frac{\text{kg}}{\text{m}^3} \left[1.602 (10^6) \text{ mm}^3 \cdot \left(\frac{1 \text{ m}}{1000 \text{ mm}} \right)^3 \right]$$

$$= \underline{11.55 \text{ kg}}$$

For circular arc, $\bar{y} = \frac{b \sin 30^\circ}{\pi/6} = 3b/\pi$

$$\bar{z} = b \quad (\text{See Samp. Prob. 5/1})$$

$$\bar{Y} = \frac{\sum \bar{y} L}{\sum L} = \frac{b(0) + 2b\left(\frac{b}{2} \cos 30^\circ\right) + \frac{\pi}{3}b\left(\frac{3b}{\pi}\right)}{b + 2b + \frac{\pi}{3}b}$$

$$= \underline{0.461b}$$

$$\bar{Z} = \frac{\sum \bar{z} L}{\sum L} = \frac{b\left(\frac{b}{2}\right) + 2b(b) + \frac{\pi}{3}b(b)}{b + 2b + \frac{\pi}{3}b}$$

$$= \underline{0.876b}$$

5/209

$$L = \pi a/2;$$

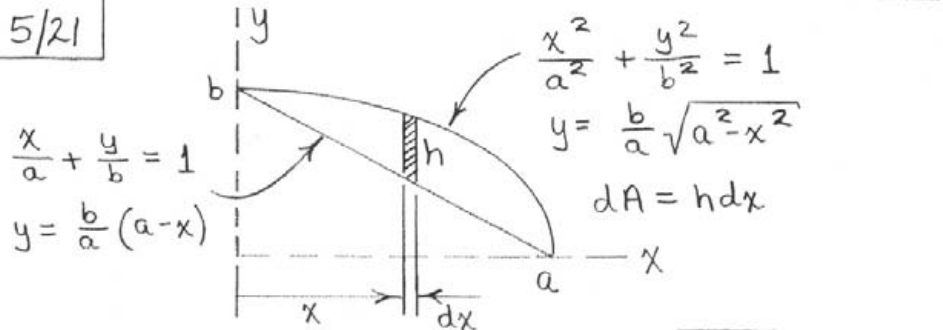
$$\bar{r} = 2a - \frac{2a}{\pi} = 2a\left(1 - \frac{1}{\pi}\right)$$



$$A = \frac{\pi}{2} \bar{r} L = \frac{\pi}{2} 2a\left(1 - \frac{1}{\pi}\right) \frac{\pi a}{2}$$

$$= \frac{\pi a^2}{2} (\pi - 1)$$

5/21



$$h = \frac{b}{a} \sqrt{a^2 - x^2} - \frac{b}{a}(a-x) = \frac{b}{a} [\sqrt{a^2 - x^2} - (a-x)]$$

$$A\bar{x} = \int x dA$$

$$A = \int_0^a \frac{b}{a} [\sqrt{a^2 - x^2} - (a-x)] dx$$

$$= \frac{b}{a} \left[\frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} - \left(ax - \frac{x^2}{2} \right) \right]_0^a$$

$$\int x dA = \int_0^a x \frac{b}{a} [\sqrt{a^2 - x^2} - (a-x)] dx = \frac{ab}{2} \left[\frac{\pi}{2} - 1 \right]$$

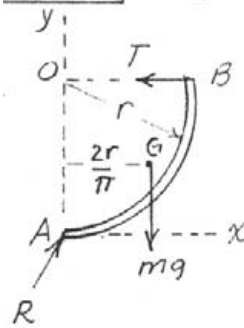
$$= \frac{b}{a} \left[-\frac{1}{3} \sqrt{(a^2 - x^2)^3} - \left(\frac{ax^2}{2} - \frac{x^3}{3} \right) \right]_0^a = \frac{ba^2}{6}$$

$$\bar{x} = \frac{\int x dA}{A} = \frac{ba^2/6}{\frac{ab}{2} (\frac{\pi}{2} - 1)} = \frac{a}{3(\frac{\pi}{2} - 1)}$$

$$\text{Similarly, } \bar{y} = \frac{b}{3(\frac{\pi}{2} - 1)}$$

5/210

$$\sum M_A = 0; \quad mg \frac{2r}{\pi} - Tr = 0, \quad T = \frac{2mg}{\pi}$$



$$\sum F_x = 0; \quad R_x = \frac{2mg}{\pi}$$

$$\sum F_y = 0; \quad R_y = mg$$

$$R = \sqrt{R_x^2 + R_y^2} = mg \sqrt{\frac{4}{\pi^2} + 1} = \underline{1.185 mg}$$

5/211

Diagram of a right triangular lamina with base $2R$ and height H . A horizontal disk of thickness dx is shown at height y from the base. The radius of the disk is x . The hypotenuse is labeled with the equation $y = \frac{R}{H}x$. The mass of the disk is labeled $dm(\text{disk})$.

$$dm = \rho dV = \rho \pi \left(\frac{2R-y}{2} \right)^2 dx$$

$$= \frac{\pi \rho}{4} (4R^2 - 4Ry + y^2) dx$$

$$= \frac{\pi \rho}{4} \left[4R^2 - 4R \frac{R}{H} x + \left(\frac{R}{H} x \right)^2 \right] dx$$

$$= \frac{\pi \rho R^2}{4} \left[4 - 4 \frac{x}{H} + \frac{x^2}{H^2} \right] dx$$

$$m = \int dm = \int_0^H \frac{\pi \rho R^2}{4} \left[4 - 4 \frac{x}{H} + \frac{x^2}{H^2} \right] dx$$

$$= \frac{7}{12} \pi \rho R^2 H$$

$$\int x_c dm = \int_0^H \frac{\pi \rho R^2}{4} \left[4x - 4 \frac{x^2}{H} + \frac{x^3}{H^2} \right] dx$$

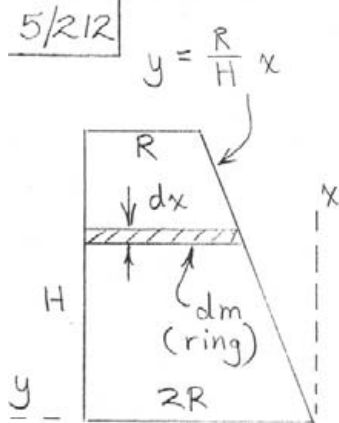
$$= \frac{11}{48} \pi \rho R^2 H^2$$

$$\text{Then } \bar{x} = \frac{\int x_c dm}{\int dm} = \frac{\frac{11}{48} \pi \rho R^2 H^2}{\frac{7}{12} \pi \rho R^2 H}$$

$$= \frac{11}{28} H = \bar{h}$$

(ρ = mass per unit volume)

5/212



$$dm = \rho dA = \rho \left[\frac{2R-y}{2} \cdot 2\pi k \right]$$

$$= \pi \rho \left[2R - \frac{R}{H}x \right] k dx$$

$$= \pi \rho R \left[2 - \frac{x}{H} \right] k dx$$

$$m = \int dm = \int_0^H \pi \rho R \left(2 - \frac{x}{H} \right) k dx$$

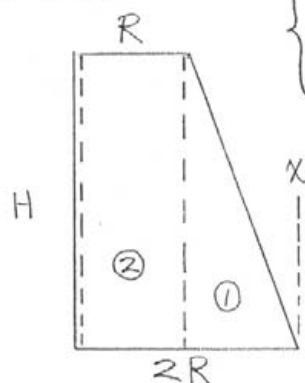
$$= \frac{3}{2} k \pi \rho R H$$

$$\int x_c dm = \int_0^H \pi \rho R k \left(2x - \frac{x^2}{H} \right) dx = \frac{2}{3} k \pi \rho R H^2$$

$$\bar{x} = \frac{\int x_c dm}{\int dm} = \frac{\frac{2}{3} k \pi \rho R H^2}{\frac{3}{2} k \pi \rho R H} = \frac{4}{9} H = \bar{h}$$

(ρ = mass per unit area, k is a parameter which accounts for the variation of the slant height of the differential elemental ring around the circumference of the body (the same for all elements.))

5/213



- { ① Solid body (Prob. 5/211)
 { ② Cylindrical hole

(Refer to the solution for Prob. 5/211)

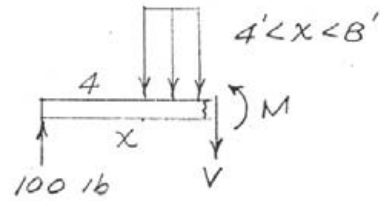
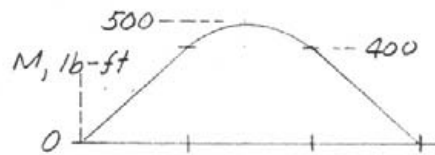
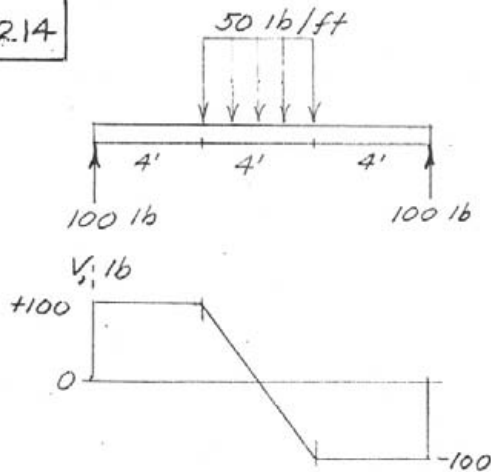
Body	m	\bar{x}	$\bar{x} m$
①	$\frac{7}{12} \pi \rho R^2 H$	$\frac{11}{28} H$	$\frac{77}{336} \pi \rho R^2 H^2$
②	$-\pi \rho \left(\frac{R}{2}\right)^2 H$	$\frac{H}{2}$	$-\frac{1}{8} \pi \rho R^2 H^2$

$$\Sigma m = \frac{1}{3} \pi \rho R^2 H$$

$$\Sigma \bar{x} m = \frac{35}{336} \pi \rho R^2 H^2$$

$$\bar{X} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{\frac{35}{336} \pi \rho R^2 H^2}{\frac{1}{3} \pi \rho R^2 H} = \frac{5}{16} H = \bar{h}$$

5/2.14



$$\sum F = 0; V + 50(x-4) - 100 = 0$$

$$V = 50(6-x)$$

$$\sum M_v = 0; M + 50 \frac{(x-4)^2}{2} - 100x = 0$$

$$M = 25(-x^2 + 12x - 16)$$

Set $\frac{dM}{dx} = 0$ to get

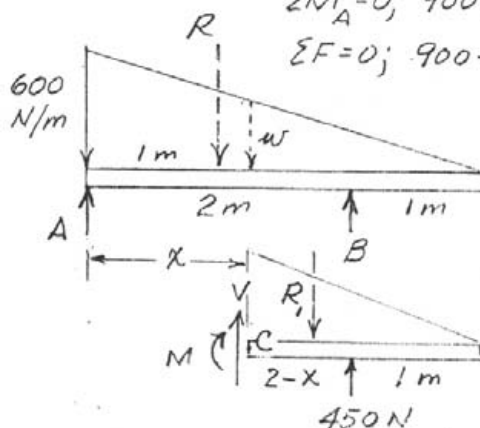
$$M_{\max} = 500 \text{ lb-ft}$$

$$@ x = 6 \text{ ft}$$

5/215 $R = \frac{600}{2}(3) = 900 \text{ N}$;

$\sum M_A = 0$; $900(1) - 2B = 0$, $B = 450 \text{ N}$

$\sum F = 0$; $900 - 450 - A = 0$, $A = 450 \text{ N}$



$R_1 = \frac{w}{2}(2-x+1) = \frac{3-x}{2}w$

where $w = \frac{3-x}{3} 600 \text{ N/m}$

So $R_1 = 100(3-x)^2 \text{ N}$

$\sum M_C = 0$;

$450(2-x) = 100(3-x)\frac{2(3-x)}{3} + M$

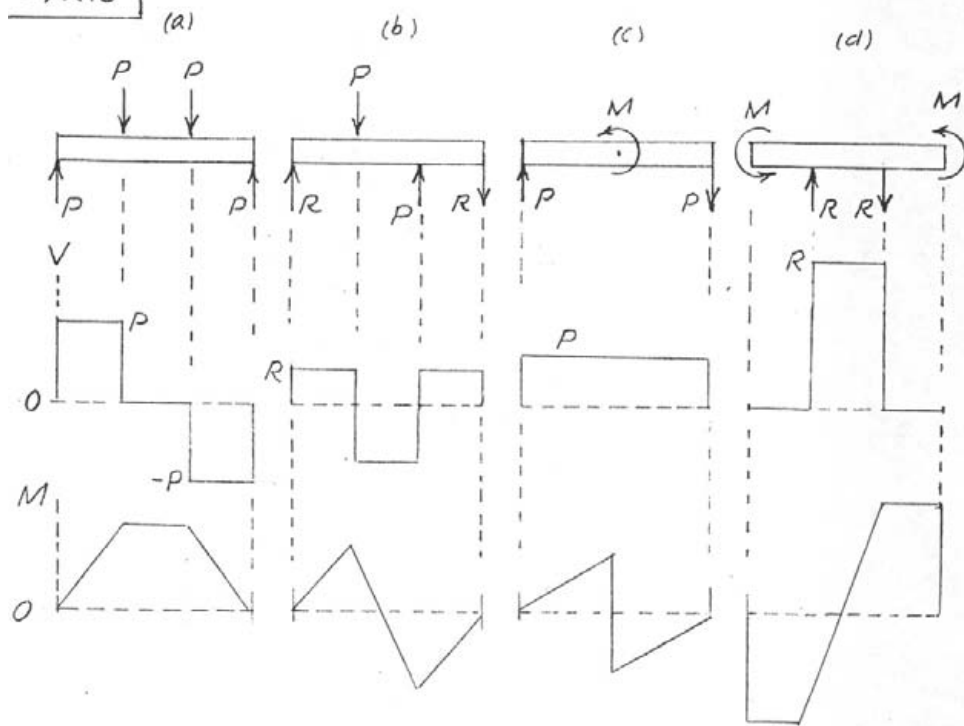
$M = 450(2-x) - \frac{100}{3}(3-x)^3 \quad 0 \leq x \leq 2 \text{ m}$

$\frac{dM}{dx} = -450 + 100(3-x)^2 = 0 \text{ for max or min}$

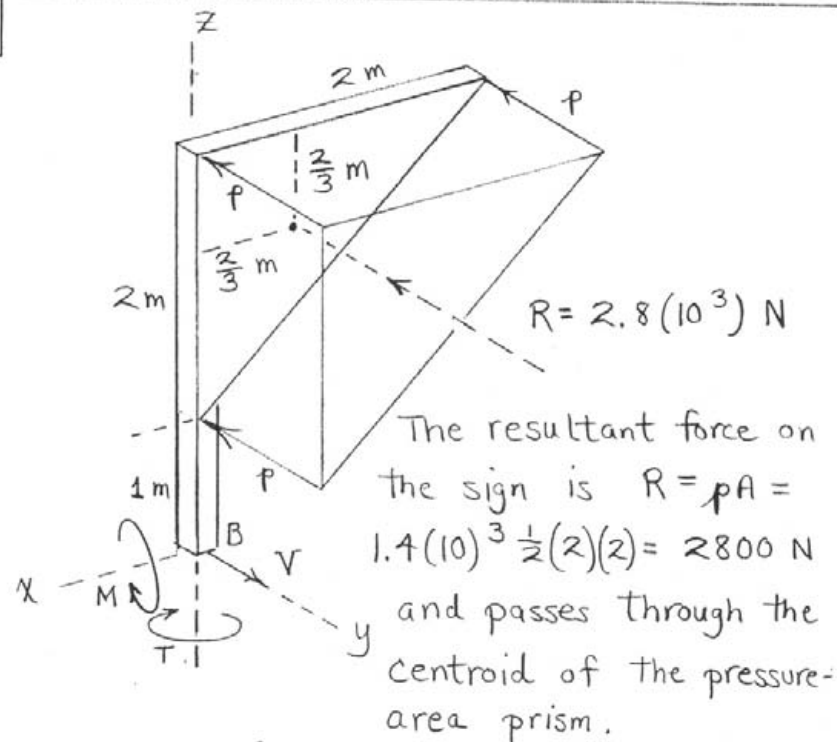
$3-x = \pm \sqrt{4.5}, \quad x = 3(1 \pm 1/\sqrt{2}) = \underline{0.879 \text{ m}}$
(or 5.12)

$M_{\max} = 450(2-0.879) - \frac{100}{3}(3-0.879)^3 = \underline{186.4 \text{ N}\cdot\text{m}}$

5/2/16



5/217



By inspection,
$$\begin{cases} V = 2.8 \text{ kN} \\ M = 2.8 \left(3 - \frac{2}{3}\right) = 6.53 \text{ kN}\cdot\text{m} \\ T = 2.8 \left(\frac{2}{3}\right) = 1.867 \text{ kN}\cdot\text{m} \end{cases}$$

5/218

$$\bar{X} = 0 = \frac{\pi D^2}{4} r \cos \theta + \frac{\pi d^2}{4} r \cos 30^\circ - \frac{\pi d^2}{4} r$$

$$D^2 \cos \theta = d^2 \left(1 - \frac{\sqrt{3}}{2}\right)$$

$$\bar{Y} = 0 = \frac{\pi D^2}{4} r \sin \theta - \frac{\pi d^2}{4} r - \frac{\pi d^2}{4} r \sin 30^\circ$$

$$D^2 \sin \theta = d^2 \left(1 + \frac{1}{2}\right)$$

Divide : $\frac{\sin \theta}{\cos \theta} = \frac{3/2}{1 - \sqrt{3}/2}, \quad \theta = 84.9^\circ$

$$D^2 = \frac{3d^2/2}{\sin 84.9^\circ}, \quad \underline{D = 1.227 d}$$

$$\underline{5/219} \quad \text{Hole: } V = -9h \text{ (in.}^3\text{)}, \quad \bar{z} = h/2$$

$$\text{Cylinder: } V = \pi 6^2(10) = 1131 \text{ in.}^3, \quad \bar{z} = 5 \text{ in.}$$

$$\bar{z} = \frac{\sum \bar{z}V}{\sum V} = \frac{-9h(\frac{h}{2}) + 1131(5)}{-9h + 1131}$$

$$\text{For max. } \bar{z}, \quad \frac{d\bar{z}}{dh} = 0$$

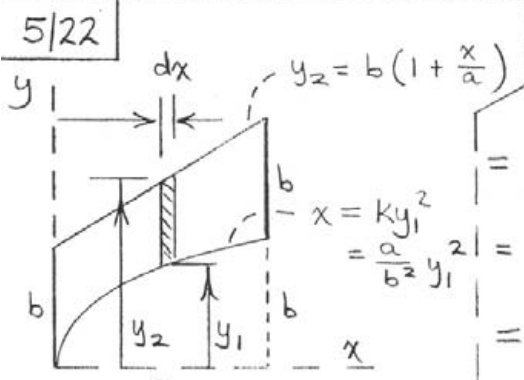
$$\frac{d\bar{z}}{dh} = \frac{(-9h + 1131)(-9) - (-\frac{9h^2}{2} + 5655)(-9)}{(-9h + 1131)^2} = 0$$

$$\Rightarrow 9h^2 - 1131h - 4.5h^2 + 5655 = 0$$

$$4.5h^2 - 1131h + 5655 = 0$$

$$h = \frac{1131 \pm \sqrt{1131^2 - 4(4.5)(5655)}}{9} = \underline{5.10 \text{ in. or } 246 \text{ in.}}$$

5/22



$$A = \int_0^a (y_2 - y_1) dx$$

$$= \int_0^a \left[b \left(1 + \frac{x}{a} \right) - b \left(\frac{x}{a} \right)^2 \right] dx$$

$$= b \left(x + \frac{x^2}{2a} \right) - \frac{b}{\sqrt{a}} \frac{2}{3} x^{3/2} \Big|_0^a$$

$$= \frac{5}{6} ab$$

$$\int y_c dA = \int_0^a \left(\frac{y_1 + y_2}{2} \right) (y_2 - y_1) dx = \frac{1}{2} \int_0^a (y_2^2 - y_1^2) dx$$

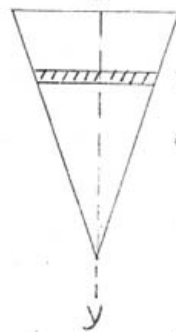
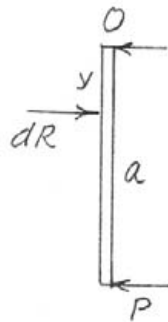
$$= \frac{1}{2} \int_0^a \left[b^2 \left(1 + \frac{x}{a} \right)^2 - \frac{b^2}{a} x \right] dx$$

$$= \frac{1}{2} \left[b^2 \left(x + \frac{x^2}{a} + \frac{x^3}{3a^2} \right) - \frac{b^2 x^2}{2a} \right] \Big|_0^a = \frac{11}{12} ab^2$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{11ab^2/12}{5ab/6} = \frac{11}{10} b$$

5/220 $dA = 2x dy = \frac{b}{a}(a-y) dy$, $p = \rho g (h+y)$

$$dR = \rho g (h+y) \frac{b}{a} (a-y) dy$$



$$\sum M_O = 0; Pa - \int y dR = 0$$

$$Pa = \rho g \frac{b}{a} \int_0^a y(h+y)(a-y) dy$$

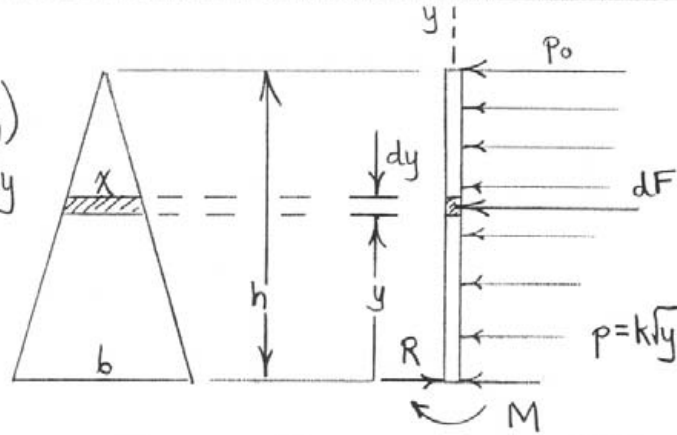
$$Pa = \rho g \frac{b}{a} \frac{a^3}{6} (h + \frac{a}{2})$$

$$P = \frac{\rho g a b}{6} (h + \frac{a}{2})$$

5/221

$$x = \frac{b}{h}(h-y)$$

$$dA = \frac{b}{h}(h-y) dy$$



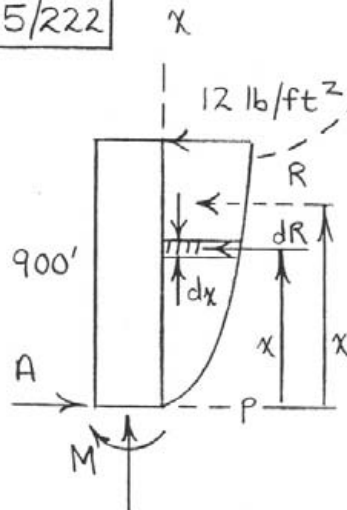
$$p = k\sqrt{y} : p_0 = k\sqrt{h}, k = \frac{p_0}{\sqrt{h}}, \text{ so } p = p_0\sqrt{\frac{y}{h}}$$

$$dF = p dA = p_0\sqrt{\frac{y}{h}} \frac{b}{h}(h-y) dy$$

$$dM = y dF = p_0 \frac{b}{h\sqrt{h}} (y^{3/2} h - y^{5/2}) dy$$

$$M = \int_0^h dM = \frac{p_0 b}{h\sqrt{h}} \left[\frac{y^{5/2}}{5/2} h - \frac{y^{7/2}}{7/2} \right]_0^h = \frac{4}{35} p_0 b h^2$$

5/222



$$p = k\sqrt{x} : 12 = k\sqrt{900}$$

$$k = 0.4 \frac{\text{lb}}{\text{ft}^{5/2}}, \quad p = 0.4\sqrt{x}$$

$$dR = 200 p dx = 200(0.4\sqrt{x})$$

$$= 80\sqrt{x} dx$$

$$R = \int dR = \int_0^{900'} 80\sqrt{x} dx$$

$$= 80 \frac{2}{3} x^{3/2} \Big|_0^{900'} = 1.440(10^6) \text{ lb}$$

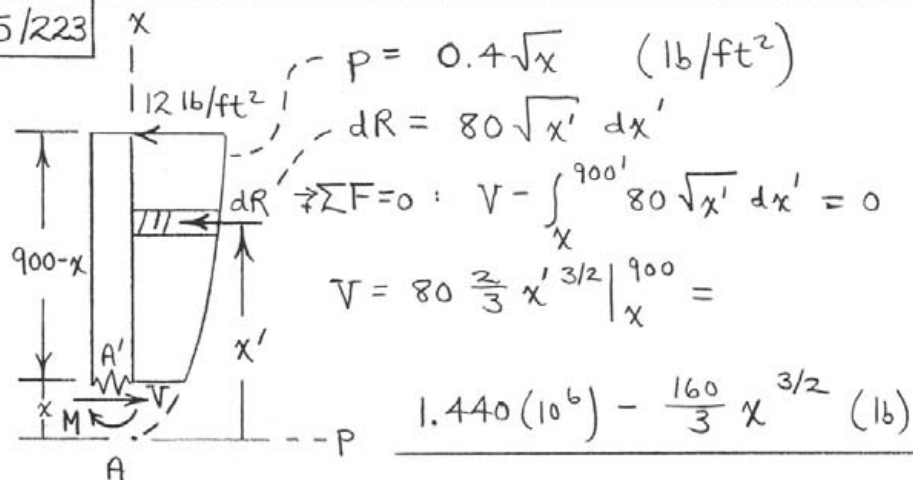
$$\int x dR = \int_0^{900} 80 x^{3/2} dx = 80 \frac{2}{5} x^{5/2} \Big|_0^{900'}$$

$$= 7.78(10^8) \text{ lb-ft}$$

Thus ,

$$\frac{A = 1.440(10^6) \text{ lb}}{M = 7.78(10^8) \text{ lb-ft}}$$

5/223

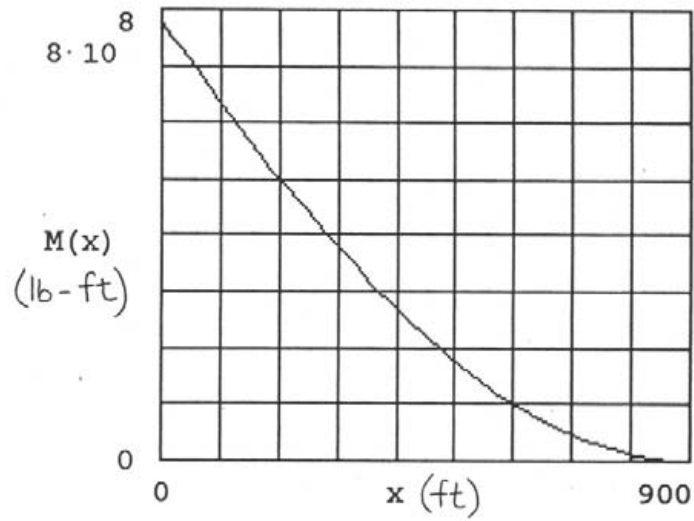
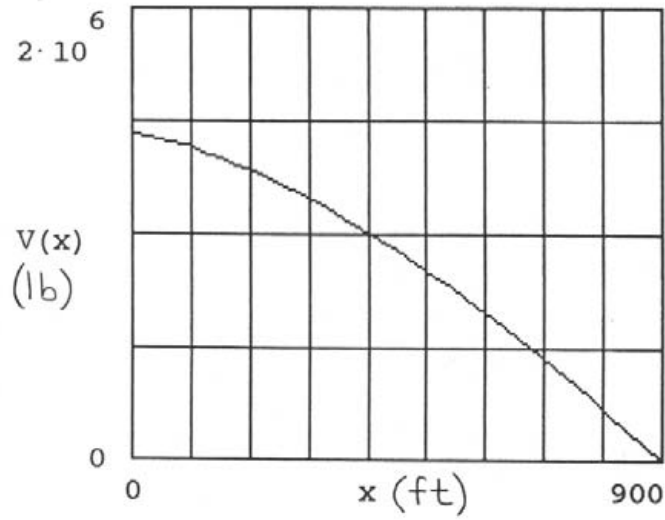


$$V \Big|_{x=450'} = 0.931(10^6) \text{ lb}$$

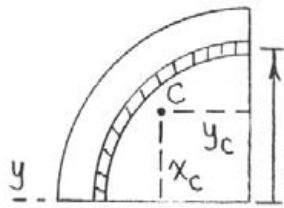
$$\sum M_{A'} = 0: M - \int_x^{900} 80\sqrt{x'} (x' - x) dx' = 0$$

$$\begin{aligned}
 M &= 80 \left\{ \frac{2}{5} x'^{5/2} - x \frac{2}{3} x'^{3/2} \right\} \Big|_x^{900} \\
 &= 7.78(10^8) - 1.440(10^6)x + \frac{64}{3} x^{5/2}
 \end{aligned}$$

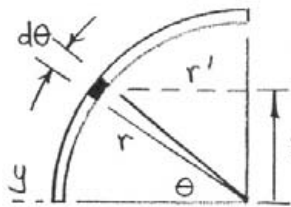
$$M \Big|_{x=450'} = 2.21(10^8) \text{ lb-ft}$$



► 5/224



$$(r' = r \cos \theta)$$



Element is quarter-circular ring

parallel to x-y plane with area

$$dA = \frac{\pi r'}{2} r d\theta = \frac{\pi}{2} r^2 \cos \theta d\theta$$

Centroid of element is at

$$x_c = y_c = \frac{2r'}{\pi} = \frac{2}{\pi} r \cos \theta \quad (\text{S.P. 5/1})$$

$$\int x_c dA = \int \left(\frac{2}{\pi} r \cos \theta \right) \frac{\pi}{2} r^2 \cos \theta d\theta$$

$$= r^3 \int_0^{\pi/2} \cos^2 \theta d\theta = \frac{\pi r^3}{4}$$

$$\int y_c dA = \int x_c dA = \frac{\pi r^3}{4}$$

$$\int z_c dA = \int (r \sin \theta) \left(\frac{\pi}{2} r^2 \cos \theta d\theta \right) = \frac{\pi r^3}{2} \int_0^{\pi/2} \sin \theta \cos \theta d\theta$$

$$= \pi r^3 / 4$$

$$A = \int \frac{\pi}{2} r^2 \cos \theta d\theta = \frac{\pi r^2}{2} \int_0^{\pi/2} \cos \theta d\theta = \pi r^2 / 2$$

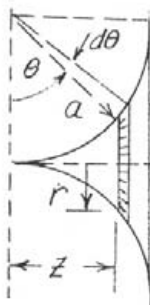
From $\bar{x} = \frac{\int x_c dA}{A}$, etc, we have

$$\bar{x} = \bar{y} = \bar{z} = \frac{\pi r^3 / 4}{\pi r^2 / 2} = \underline{r/2}$$

► 5/225

$$dA = 2\pi r a d\theta = 2\pi a^2(1 - \cos\theta) d\theta$$

$$\begin{aligned} \int \bar{z} dA &= \int_0^{\pi/2} (a \sin\theta) (2\pi a^2) (1 - \cos\theta) d\theta \\ &= 2\pi a^3 \int_0^{\pi/2} (\sin\theta - \sin\theta \cos\theta) d\theta \\ &= 2\pi a^3 \left(1 - \frac{1}{2}\right) = \pi a^3 \end{aligned}$$

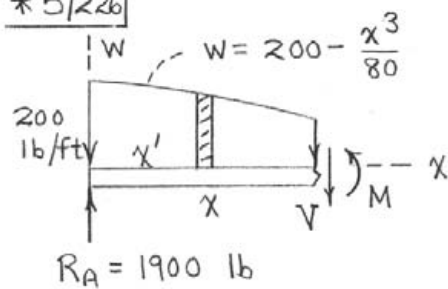


$$\int dA = 2\pi a^2 \int_0^{\pi/2} (1 - \cos\theta) d\theta = 2\pi a^2 \left(\frac{\pi}{2} - 1\right)$$

$$\bar{z} = \frac{\int \bar{z} dA}{A} = \frac{\pi a^3}{2\pi a^2 \left(\frac{\pi}{2} - 1\right)} = \frac{a}{\pi - 2}$$

*5/226

(w & R_A from Prob. 5/113)

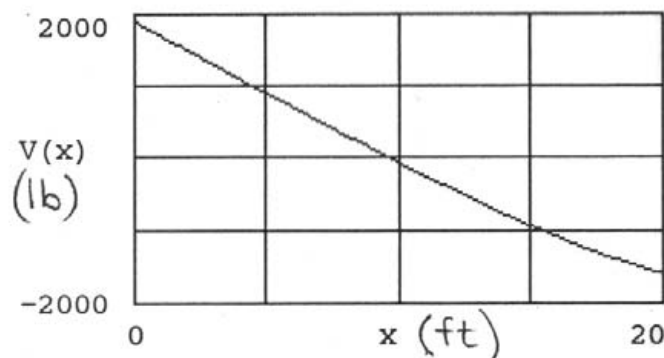


$$+\uparrow \sum F = 0 : 1900 - V - \int_0^x \left(200 - \frac{x'^3}{80} \right) dx' = 0$$

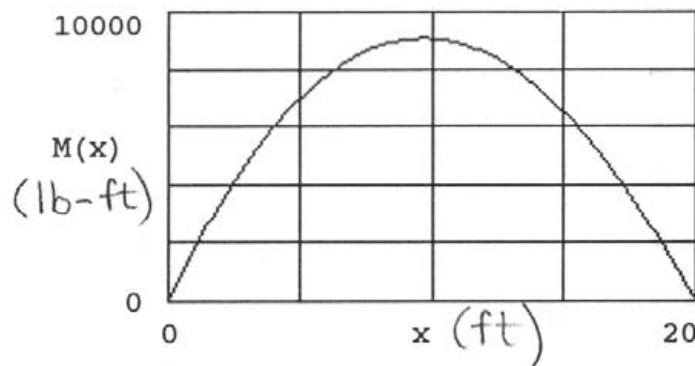
$$V = 1900 - 200x + \frac{x^4}{320} \quad (1b)$$

$$+\circlearrowleft \sum M = 0 : M + \int_0^x \left(200 - \frac{x'^3}{80} \right) (x - x') dx' - 1900x = 0$$

$$M = 1900x - 100x^2 + \frac{x^5}{1600} \quad (1b-ft)$$

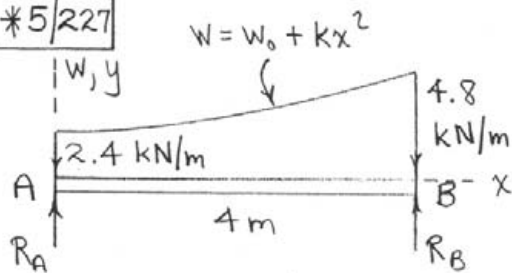


$$V_{\max} = 1900 \text{ lb @ } x = 0$$



$$M_{\max} = 9080 \text{ lb-ft @ } x = 9.63'$$

*5/227



$$W_0 = 2.4 \text{ kN/m}$$

$$4.8 = 2.4 + k(4)^2$$

$$k = 0.15 \text{ kN/m}^3$$

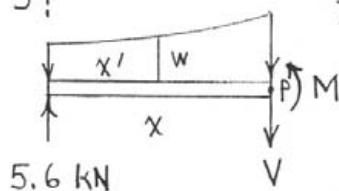
$$\text{So } W = 2.4 + 0.15x^2$$

$$\sum M_A = 0: \int_0^4 (2.4 + 0.15x^2)x dx - 4R_B = 0, R_B = 7.2 \text{ kN}$$

$$\sum F_y = 0: R_A + 7.2 - \int_0^4 (2.4 + 0.15x^2) dx = 0, R_A = 5.6 \text{ kN}$$

$$\sum F_y = 0: 5.6 - \int_0^x (2.4 + 0.15x^2) dx - V = 0$$

$$V = 5.6 - 2.4x - 0.05x^3$$



$$\sum M_P = 0: M + \int_0^x (2.4 + 0.15x'^2)(x - x') dx' - 5.6x = 0$$

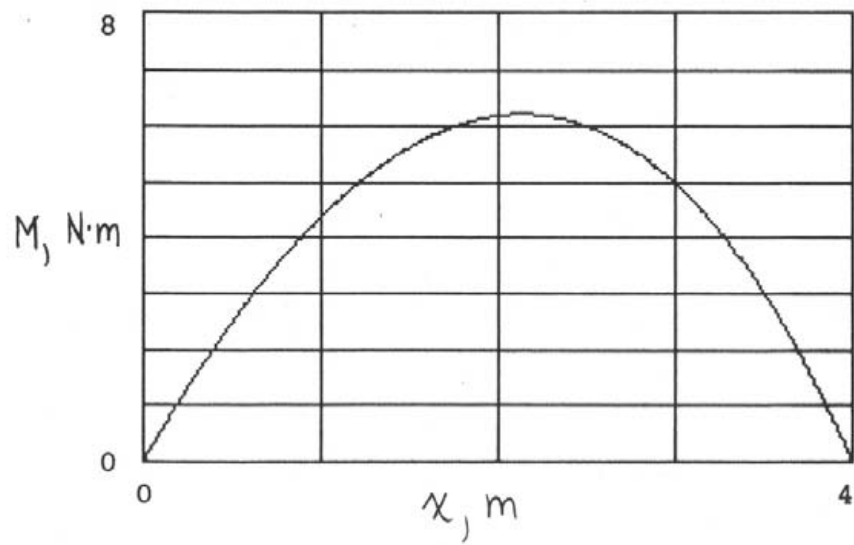
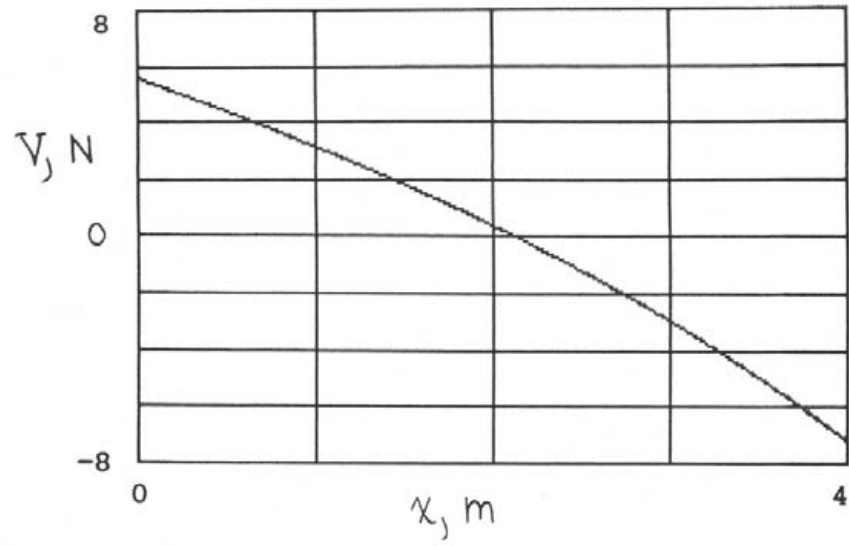
$$M = 5.6x - \left[2.4xx' + \frac{0.15}{3}x'^3x - \frac{2.4}{2}x'^2 - \frac{0.15}{4}x'^4 \right]_0^x$$

$$= 5.6x - 1.2x^2 - 0.0125x^4$$

$$\text{For a maximum, } \frac{dM}{dx} = 5.6 - 2.4x - 0.05x^3 = 0$$

Solve numerically to obtain

$$M_{\max} = 6.23 \text{ kN}\cdot\text{m} @ x = 2.13 \text{ m}$$

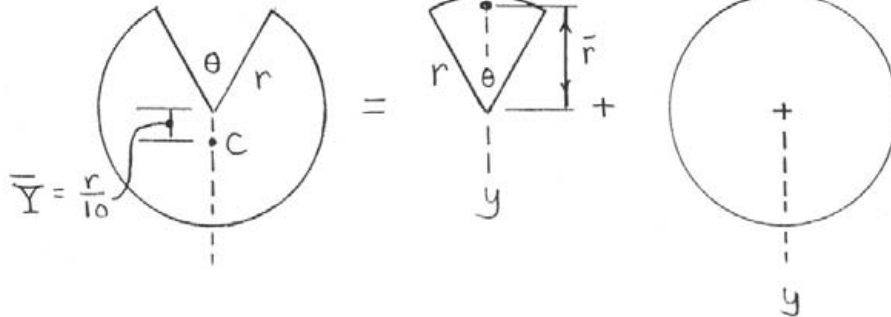


*5/228

$$\text{Arc} = (2\pi - \theta)r$$

Negative arc
 $r\theta$

Positive arc
 2π



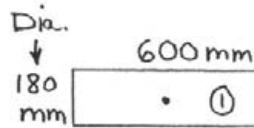
From Sample Problem 5/1, $\bar{r} = \frac{r \sin \frac{\theta}{2}}{\theta/2}$

$$\bar{Y} = \frac{\sum L \bar{y}}{\sum L} : \quad \frac{r}{10} = \frac{(-r\theta)(-r \frac{\sin \frac{\theta}{2}}{\theta/2}) + 2\pi r(0)}{(2\pi - \theta)r}$$

Simplify to $10 \sin \frac{\theta}{2} = \pi - \frac{\theta}{2}$ and solve numerically to obtain $\theta = 33.1^\circ$

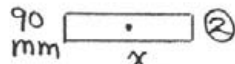
*5/229

$$V_1 = \frac{\pi 180^2}{4} (600) = 15\,270(10^3) \text{ mm}^3$$



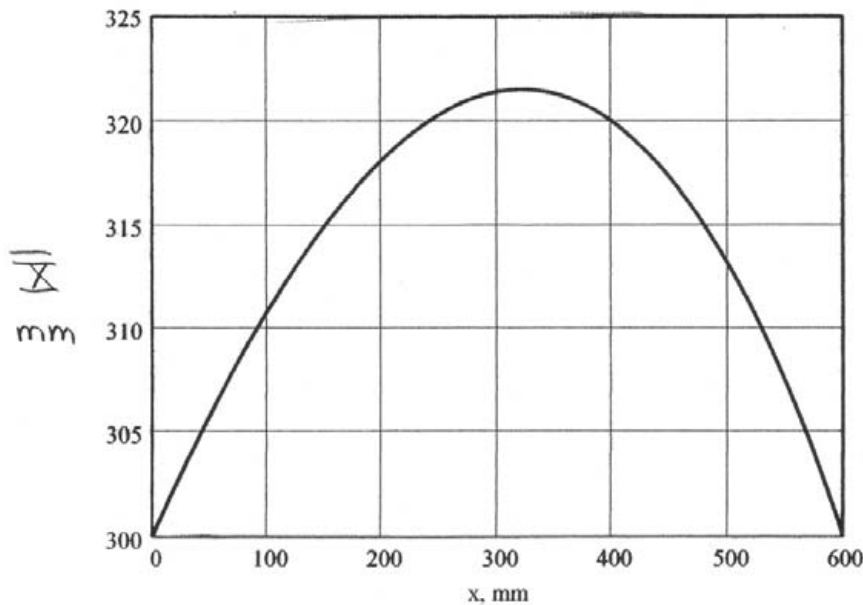
$$\bar{x}_1 = 300 \text{ mm}$$

$$V_2 = -\frac{\pi 90^2}{4} x = -6.36(10^3)x$$



$$\bar{x}_2 = -\frac{x}{2}$$

$$\begin{aligned} \bar{X} &= \frac{\sum V \bar{x}}{\sum V} = \frac{15\,270(10^3)(300) - 6.36(10^3)x\left(\frac{x}{2}\right)}{15\,270(10^3) - 6.36(10^3)x} \\ &= \frac{4580(10^3) - 3.18x^2}{15\,270 - 6.36x} \end{aligned}$$



$$\bar{X}_{\max} = 322 \text{ mm @ } x = 322 \text{ mm}$$

5/23

From Sample Problem 5/3,

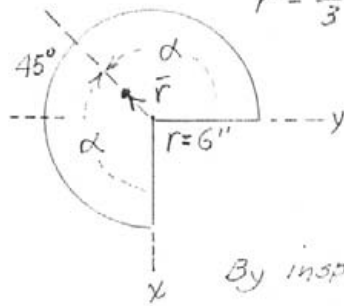
$$\bar{r} = \frac{2}{3} \frac{r \sin \alpha}{\alpha} \text{ where } \alpha = 135^\circ \text{ or } \frac{3}{4}\pi \text{ rad}$$

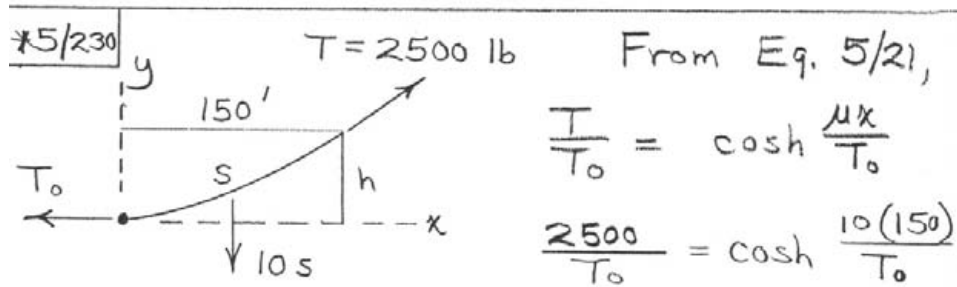
$$\bar{r} = \frac{2}{3}(6) \frac{\sin 135^\circ}{3\pi/4}$$

$$= 4 \frac{4}{3\pi} \frac{1}{\sqrt{2}} = \frac{16}{3\pi\sqrt{2}} \text{ in.}$$

$$\bar{x} = \bar{y} = -\bar{r}/\sqrt{2} = -\frac{8}{3\pi} \text{ in.}$$

By inspection $\bar{z} = 5 \text{ in.}$



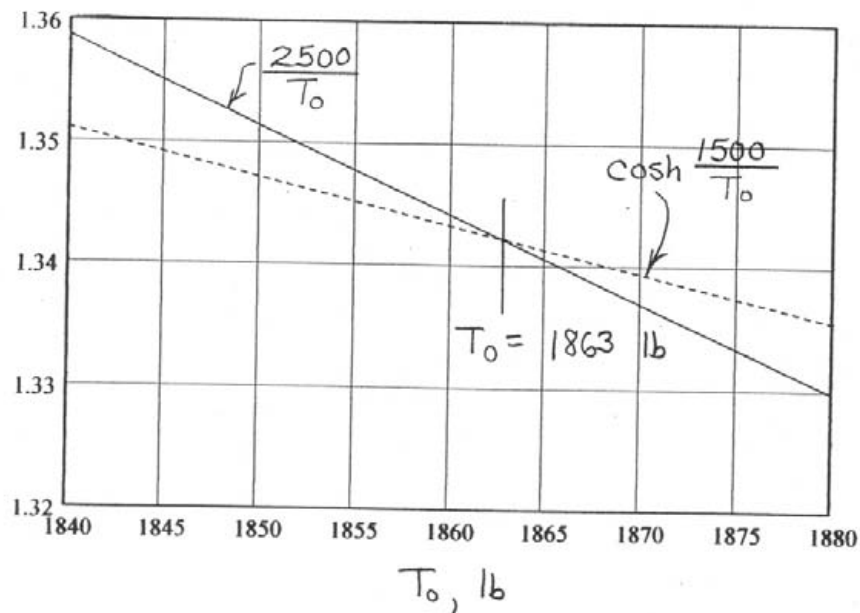


Numerical or graphical (see below) solution:

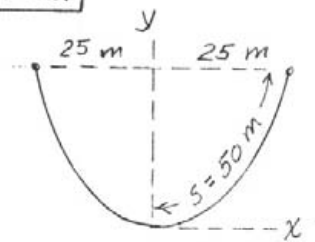
$T_0 = 1863$ lb. Then Eq. 5/22:

$$T = T_0 + \mu h : 2500 = 1863 + 10h, \quad h = 63.7 \text{ ft}$$

$$\begin{aligned} 5/22: S = 2s &= 2 \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0} \\ &= 2 \frac{1863}{10} \sinh \frac{10(150)}{1863} = 333 \text{ ft} \end{aligned}$$



*5/231



$$\text{Eq. 5/20} \quad S = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$$

$$50 = \frac{T_0}{\mu} \sinh \frac{25\mu}{T_0}$$

$$\frac{50\mu}{T_0} - \sinh \frac{25\mu}{T_0} = R = 0$$

Write and run program for $R = f\left(\frac{\mu}{T_0}\right)$ & find μ/T_0 for $R=0$. Result is $\mu/T_0 = 0.0871$

$$\text{From Eq. 5/19, } y = \frac{T_0}{\mu} \left(\cosh \frac{\mu x}{T_0} - 1 \right)$$

$$= \frac{1}{0.0871} (\cosh 0.0871[25] - 1)$$

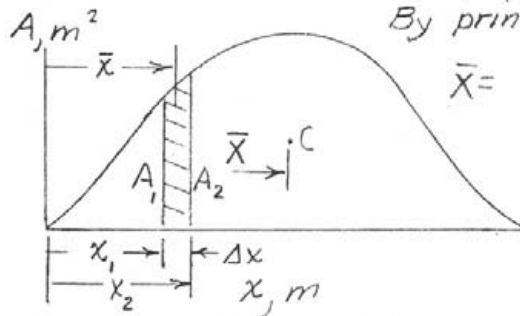
$$h = y = 3.468/0.0871 = \underline{39.8 \text{ m}}$$

Result depends only on the geometry of the catenary.

*5/232

$V =$ submerged volume

By principle of moments



$$\bar{X} = \frac{\sum \bar{x} \Delta V}{\Delta V}$$

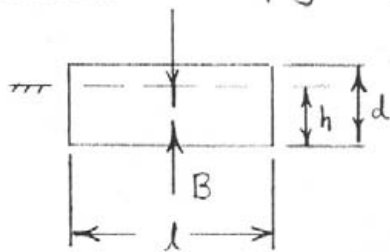
$$\bar{x} = \frac{1}{2}(x_1 + x_2)$$

$$\Delta V = \frac{1}{2}(A_1 + A_2)(x_2 - x_1)$$

$$\begin{aligned} \text{Thus } \bar{X} &= \frac{\sum \frac{1}{2}(x_1 + x_2) \frac{1}{2}(A_1 + A_2)(x_2 - x_1)}{\sum \frac{1}{2}(A_1 + A_2)(x_2 - x_1)} \\ &= \frac{\sum \frac{1}{2}(A_1 + A_2)(x_2^2 - x_1^2)}{\sum (A_1 + A_2)(x_2 - x_1)} \end{aligned}$$

Write & carry out program for quotient of sums
& get $\bar{X} = 24 \text{ m}$

*5/233 $W = \rho_1 V g = \rho_1 \pi \frac{d^2}{4} l g$

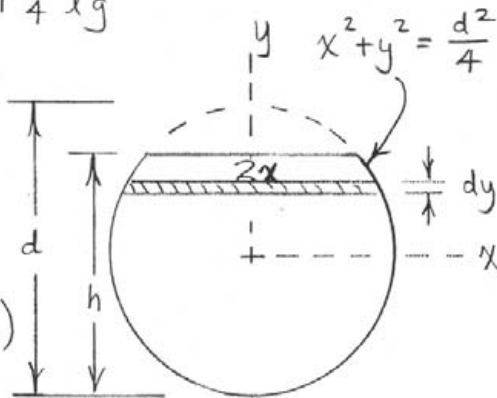


$B = \rho_2 V_{\text{sub}} g$ (Need V_{sub})

$A = \int dA = \int_{-d/2}^{h-d/2} 2x dy$

$$= 2 \int_{-d/2}^{h-d/2} \sqrt{\frac{d^2}{4} - y^2} dy = 2 \cdot \frac{1}{2} \left[y \sqrt{\frac{d^2}{4} - y^2} + \frac{d^2}{4} \sin^{-1} \frac{2y}{d} \right]_{-d/2}^{h-d/2}$$

$$= \left(h - \frac{d}{2}\right) \sqrt{hd - h^2} + \frac{d^2}{4} \sin^{-1} \left(\frac{2h-d}{d}\right) + \frac{\pi d^2}{8}$$



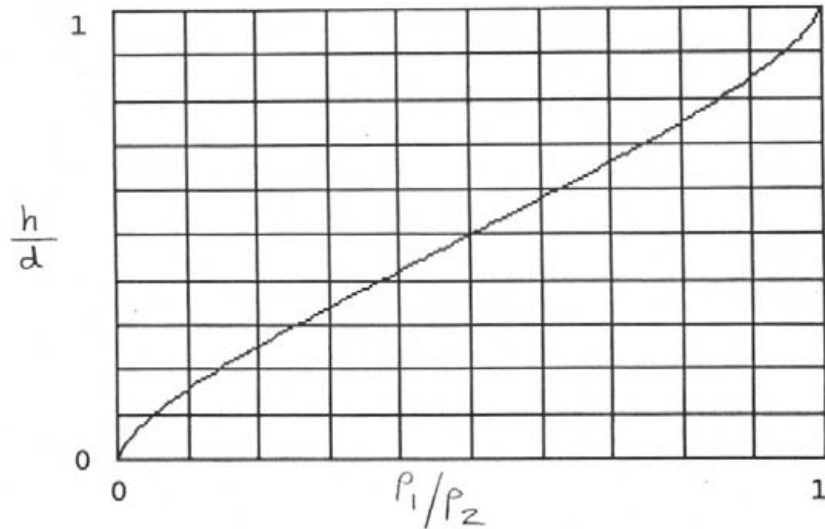
So $B = \rho_2 g A l$, A given just above

$\uparrow \sum F = 0 : \rho_2 g A l - \rho_1 \pi \frac{d^2}{4} l g = 0$

or $\left(h - \frac{d}{2}\right) \sqrt{hd - h^2} + \frac{d^2}{4} \sin^{-1} \left(\frac{2h-d}{d}\right) + \frac{\pi d^2}{8} = \frac{\rho_1}{\rho_2} \pi \frac{d^2}{4}$

Strategy : Set $d=1$ and numerically solve the above equation for h for values of

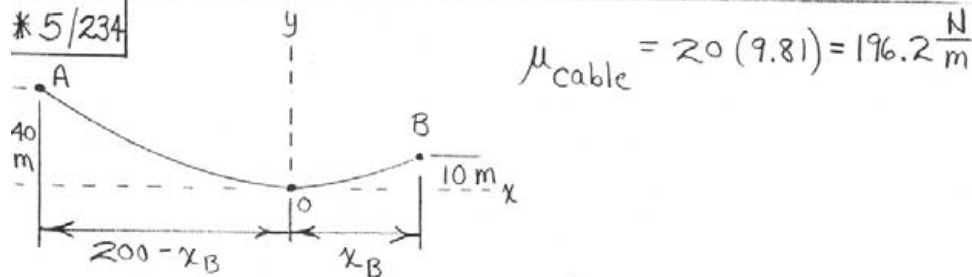
$\frac{\rho_1}{\rho_2}$ between 0 and 1:



For pine wood and salt water, $\rho_1 = 480 \frac{\text{kg}}{\text{m}^3}$
 and $\rho_2 = 1030 \frac{\text{kg}}{\text{m}^3}$. So $\frac{\rho_1}{\rho_2} = \frac{480}{1030} = 0.466$

Numerical solution : $\frac{h}{d} = \underline{0.473} = r$

* 5/234



$$\begin{aligned} \text{Eq. 5/19: } y &= \frac{T_0}{\mu} \left[\cosh \frac{x}{T_0/\mu} - 1 \right] \\ \text{At B: } 10 &= \frac{T_0}{\mu} \left[\cosh \frac{x_B}{T_0/\mu} - 1 \right] \\ \text{At A: } 40 &= \frac{T_0}{\mu} \left[\cosh \frac{200 - x_B}{T_0/\mu} - 1 \right] \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Eq. 5/19: } y &= \frac{T_0}{\mu} \left[\cosh \frac{x}{T_0/\mu} - 1 \right] \\ \text{At B: } 10 &= \frac{T_0}{\mu} \left[\cosh \frac{x_B}{T_0/\mu} - 1 \right] \\ \text{At A: } 40 &= \frac{T_0}{\mu} \left[\cosh \frac{200 - x_B}{T_0/\mu} - 1 \right] \right\}$$

Simultaneous numerical solution: $\begin{cases} x_B = 67.1 \text{ m} \\ T_0/\mu = 227 \text{ m} \end{cases}$

$$T_A = T_0 + \mu y_A : 75\,000 = 227\mu + \mu(40)$$

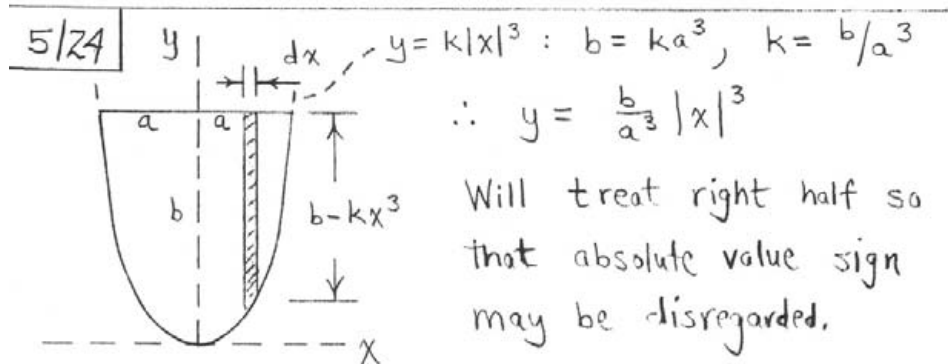
$$\mu = 281 \text{ N/m}$$

$$\mu = \mu_{\text{cable}} + \mu_{\text{ice}} : 281 = 196.2 + \mu_{\text{ice}}$$

$$\mu_{\text{ice}} = 84.7 \text{ N/m}$$

$$\rho = \frac{\mu}{g} = \frac{84.7}{9.81} = 8.63 \text{ kg/m}$$

The configuration does not depend on μ .



$$A = 2 \int_0^a \left(b - \frac{b}{a^3} x^3 \right) dx = 2 \left[bx - \frac{b}{4a^3} x^4 \right]_0^a = \frac{3}{2} ba$$

$$\int y_c dA = 2 \int_0^a \frac{b + \frac{b}{a^3} x^3}{2} \left(b - \frac{b}{a^3} x^3 \right) dx$$

$$= \int_0^a \left(b^2 - \frac{b^2}{a^6} x^6 \right) dx = b^2 \left[x - \frac{x^7}{7a^6} \right]_0^a = \frac{6ab^2}{7}$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{6ab^2/7}{3ab/2} = \frac{4}{7} b$$

$$\bar{x} = 0, \text{ by inspection.}$$

5/25

$$A = \pi \frac{ab}{4} - \frac{1}{2} a \frac{b}{2} = \frac{ab}{4} (\pi - 1)$$

$$y_2 = b \left(1 - \frac{x^2}{a^2}\right)$$

$$dA = (y_2 - y_1) dx$$

$$= \left[b \sqrt{1 - \frac{x^2}{a^2}} - \left(-\frac{b}{2a}x + \frac{b}{2}\right) \right] dx$$

$$= \frac{b}{a} \left[\sqrt{a^2 - x^2} + \frac{x}{2} - \frac{a}{2} \right] dx$$

$$y_1 = -\frac{b}{2a}x + \frac{b}{2}$$

$$\int x_c dA = \int_0^a x \frac{b}{a} \left[\sqrt{a^2 - x^2} + \frac{x}{2} - \frac{a}{2} \right] dx$$

$$= \frac{b}{a} \left[-\frac{1}{3} \sqrt{(a^2 - x^2)^3} + \frac{x^3}{6} - \frac{ax^2}{4} \right]_0^a = \frac{1}{4} ba^2$$

$$\int y_c dA = \int_0^a \left(\frac{y_2 + y_1}{2} \right) (y_2 - y_1) dx = \frac{1}{2} \int_0^a (y_2^2 - y_1^2) dx$$

$$= \frac{1}{2} \int_0^a \left[b^2 \left(1 - \frac{x^2}{a^2}\right) - \left(-\frac{b}{2a}x + \frac{b}{2}\right)^2 \right] dx$$

$$= \frac{1}{2} \int_0^a \left[b^2 \left(1 - \frac{x^2}{a^2}\right) - \frac{b^2}{4a^2}x^2 + \frac{b^2}{2a}x - \frac{b^2}{4} \right] dx$$

$$= \frac{1}{2} \left[b^2 \left(x - \frac{x^3}{3a^2}\right) - \frac{b^2}{4a^2} \frac{x^3}{3} + \frac{b^2}{2a} \frac{x^2}{2} - \frac{b^2}{4}x \right]_0^a$$

$$= \frac{7}{24} ab^2$$

$$\bar{x} = \frac{\int x_c dA}{A} = \frac{\frac{1}{4} ba^2}{\frac{ab}{4} (\pi - 1)} = \frac{a}{\pi - 1}$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{\frac{7}{24} ab^2}{\frac{ab}{4} (\pi - 1)} = \frac{7b}{6(\pi - 1)}$$

5/26 $\bar{x} = \frac{\int x dA}{\int dA}$

$dA = (y_2 - y_1) dx = \left(\sqrt{a^2 - x^2} - \frac{a-x}{2} \right) dx$

$A = \int_0^a \left(\sqrt{a^2 - x^2} - \frac{a-x}{2} \right) dx$

$= \left[\frac{1}{2} \left(x \sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a} - \frac{1}{2} (ax - \frac{x^2}{2}) \right) \right]_0^a = \frac{a^2}{4} (\pi - 1)$

Or, by inspection,

$A = \frac{1}{4} \pi a^2 - \frac{1}{2} a \frac{a}{2} = \frac{a^2}{4} (\pi - 1)$

$\int x dA = \int_0^a \left[x \sqrt{a^2 - x^2} - \frac{1}{2} (ax - x^2) \right] dx$

$= \left[-\frac{1}{3} \sqrt{(a^2 - x^2)^3} - \frac{1}{2} \left(\frac{ax^2}{2} - \frac{x^3}{3} \right) \right]_0^a = \frac{a^3}{4}$

So $\bar{x} = \frac{\frac{a^3}{4}}{\frac{a^2}{4} (\pi - 1)} = \frac{a}{\pi - 1}$

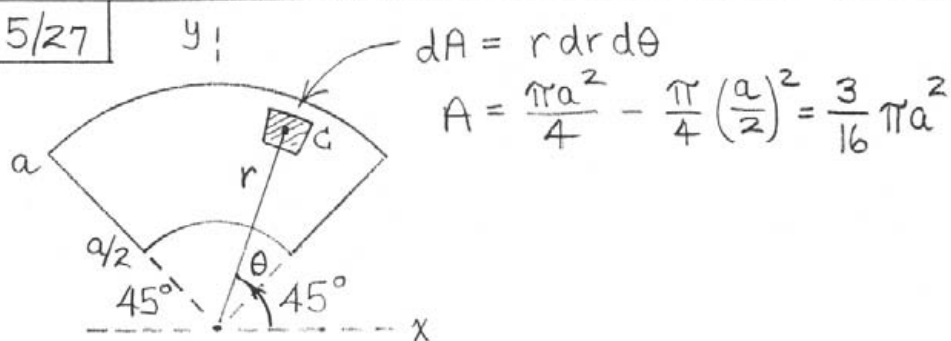
Now, $\bar{y} = \frac{\int y_c dA}{A}$, where $y_c = \frac{y_1 + y_2}{2}$

$\int y_c dA = \int \frac{y_1 + y_2}{2} (y_2 - y_1) dx = \frac{1}{2} \int_0^a (y_2^2 - y_1^2) dx$

$= \frac{1}{2} \int_0^a \left(a^2 - x^2 - \frac{a^2 - 2ax + x^2}{4} \right) dx = \frac{1}{8} \int_0^a (3a^2 + 2ax - 5x^2) dx$

$= \frac{7}{24} a^3$. Then $\bar{y} = \frac{7a^3/24}{\frac{a^2}{4} (\pi - 1)} = \frac{7a}{6(\pi - 1)}$

5/27



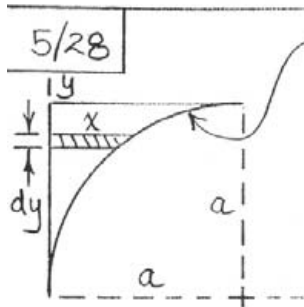
$$\int y_c dA = \iint (r \sin \theta) r dr d\theta$$

$$= \int_{\pi/4}^{3\pi/4} \int_{a/2}^a \sin \theta r^2 dr d\theta = \int_{\pi/4}^{3\pi/4} \sin \theta \frac{7}{24} a^3 d\theta$$

$$= \frac{7}{24} a^3 (-\cos \theta) \Big|_{\pi/4}^{3\pi/4} = \frac{7\sqrt{2}}{24} a^3$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{\frac{7\sqrt{2}}{24} a^3}{\frac{3}{16} \pi a^2} = \frac{14\sqrt{2}}{9\pi} a$$

5/28



$$(x-a)^2 + y^2 = a^2$$

$$dA = x dy = (a - \sqrt{a^2 - y^2}) dy$$

$$\int x_c dA = \int \frac{x}{2} x dy$$

$$= \frac{1}{2} \int_0^a (a - \sqrt{a^2 - y^2})^2 dy$$

$$\int x_c dA = \int_0^a (a^2 - a\sqrt{a^2 - y^2} - \frac{y^2}{2}) dy$$

$$= \left[a^2 y - \frac{a}{2} (y \sqrt{a^2 - y^2} + a^2 \sin^{-1} \frac{y}{a}) - \frac{y^3}{6} \right]_0^a$$

$$= \left(\frac{5}{6} - \frac{\pi}{4} \right) a^3$$

$$\int y_c dA = \int y x dy = \int_0^a (ay - y \sqrt{a^2 - y^2}) dy$$

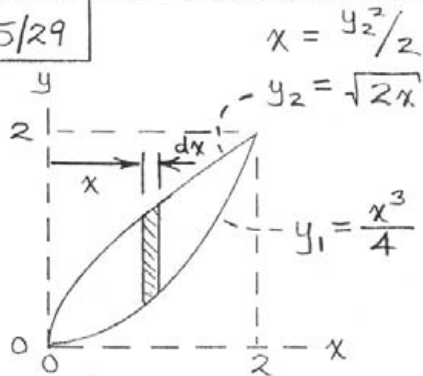
$$= \left[\frac{ay^2}{2} + \frac{1}{3} \sqrt{(a^2 - y^2)^3} \right]_0^a = \frac{a^3}{6}$$

$$A = a^2 - \frac{1}{4} \pi a^2 = a^2 \left(1 - \frac{\pi}{4} \right)$$

$$\bar{x} = \frac{\int x_c dA}{A} = \frac{\left(\frac{5}{6} - \frac{\pi}{4} \right) a^3}{\left(1 - \frac{\pi}{4} \right) a^2} = \frac{10 - 3\pi}{3(4 - \pi)} a = \underline{0.223a}$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{a^3/6}{\left(1 - \frac{\pi}{4} \right) a^2} = \frac{2a}{3(4 - \pi)} = \underline{0.777a}$$

5/29



$$A = \int dA = \int_0^2 (y_2 - y_1) dx = \int_0^2 \left(\sqrt{2x} - \frac{x^3}{4} \right) dx$$

$$= \left(\frac{2\sqrt{2}}{3} x^{3/2} - \frac{x^4}{16} \right) \Big|_0^2 = 5/3$$

$$\int x_c dA = \int_0^2 x \left(\sqrt{2x} - \frac{x^3}{4} \right) dx$$

$$= \left(\frac{2\sqrt{2}}{5} x^{5/2} - \frac{x^5}{20} \right) \Big|_0^2 = 8/5$$

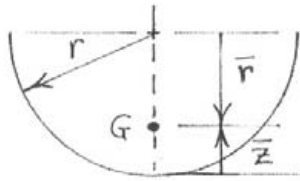
$$\int y_c dA = \int_0^2 \left(\frac{y_1 + y_2}{2} \right) (y_2 - y_1) dx = \int_0^2 \frac{1}{2} (y_2^2 - y_1^2) dx$$

$$= \frac{1}{2} \int_0^2 \left(2x - \frac{x^6}{16} \right) dx = \frac{1}{2} \left[x^2 - \frac{x^7}{7(16)} \right] \Big|_0^2 = 10/7$$

$$\bar{x} = \int x_c dA / A = \frac{8/5}{5/3} = \frac{24}{25}$$

$$\bar{y} = \int y_c dA / A = \frac{10/7}{5/3} = \frac{6}{7}$$

5/3



$$\bar{r} = \frac{2r}{\pi} = \frac{2(120)}{\pi} = 76.4 \text{ mm}$$

$$\bar{z} = r - \bar{r} = 120 - 76.4 = \underline{43.6 \text{ mm}}$$

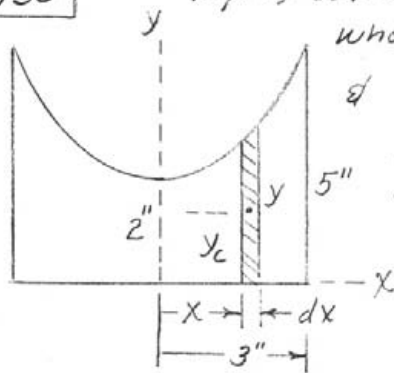
$$\bar{x} = -120 \text{ mm}, \text{ by inspection}$$

5/30

Eq. of curve: $y = 2 + kx^2$ when $x = 3''$, $y = 5''$ so $k = 1/3$

$$\therefore y = 2 + x^2/3$$

By symmetry use only half of area.



$$A = \int_0^3 y dx = \int_0^3 \left(2 + \frac{x^2}{3}\right) dx$$

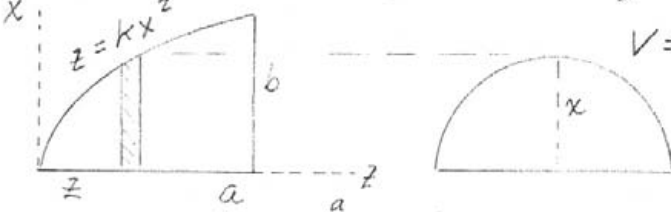
$$= \left[2x + \frac{x^3}{9}\right]_0^3 = 9 \text{ in.}^2$$

$$\bar{y}A = \int y_c dA = \int \frac{y}{2} dA = \frac{1}{2} \int_0^3 y^2 dx = \frac{1}{2} \int_0^3 \left(4 + \frac{4}{3}x^2 + \frac{x^4}{9}\right) dx$$

$$= \frac{1}{2} \left[4x + \frac{4x^3}{9} + \frac{x^5}{45}\right]_0^3 = \frac{1}{2} \left[12 + 12 + \frac{27}{5}\right] = 14.70 \text{ in.}^3$$

$$\text{Thus } \bar{h} = \bar{y} = \frac{14.70}{9} = \underline{1.633 \text{ in.}}$$

5/31 $a = kb^2$ so $z = \frac{a}{b^2}x^2$ $dV = \frac{\pi x^2}{2} dz = \frac{\pi}{2} \frac{b^2 z}{a} dz$



$$V = \frac{\pi b^2}{2a} \int_0^a z dz$$

$$= \frac{\pi b^2}{2a} \frac{a^2}{2}$$

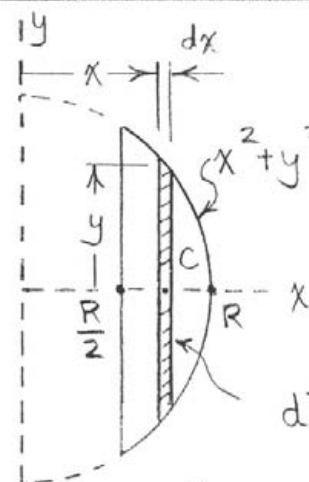
$$= \frac{\pi}{4} ab^2$$

$$\int z_c dV = \int z dV = \int_0^a z \frac{\pi}{2} \frac{b^2 z}{a} dz$$

$$= \frac{\pi b^2}{2a} \int_0^a z^2 dz = \frac{\pi b^2}{2a} \frac{a^3}{3} = \frac{\pi b^2 a^2}{6}$$

$$\bar{z} = \int z_c dV / V = \frac{\pi b^2 a^2}{6} / \frac{\pi}{4} ab^2 = \underline{2a/3}$$

5/32



(Note: Shaded element is a circular slice viewed edge-on.)

$$dV = \pi y^2 dx = \pi (R^2 - x^2) dx$$

$$V = \int dV = \int_{R/2}^R \pi (R^2 - x^2) dx = \pi \left[R^2 x - \frac{x^3}{3} \right]_{R/2}^R$$

$$= \pi \left[R^3 - \frac{R^3}{3} - \left(\frac{R^3}{2} - \frac{R^3}{24} \right) \right] = \frac{5}{24} \pi R^3$$

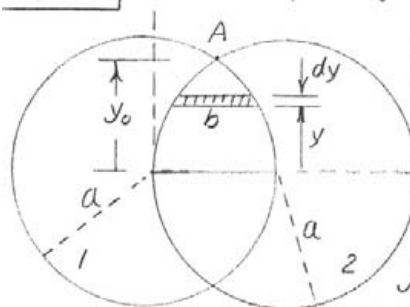
$$\int x_C dV = \int_{R/2}^R x \pi (R^2 - x^2) dx = \pi \left[\frac{R^2 x^2}{2} - \frac{x^4}{4} \right]_{R/2}^R$$

$$= \pi \left[\frac{R^4}{2} - \frac{R^4}{4} - \left(\frac{R^4}{8} - \frac{R^4}{64} \right) \right] = \frac{9}{64} \pi R^4$$

$$\bar{x} = \frac{\int x_C dV}{\int dV} = \frac{\frac{9}{64} \pi R^4}{\frac{5}{24} \pi R^3} = \underline{\underline{\frac{27}{40} R}}$$

5/33

$$(1), x^2 + y^2 = a^2; (2), (x-a)^2 + y^2 = a^2$$



$$(1) x_1 = +\sqrt{a^2 - y^2}$$

$$(2) x_2 = a - \sqrt{a^2 - y^2} \quad (\text{Note sign})$$

$$dA = b dy = (x_1 - x_2) dy$$

$$= (2\sqrt{a^2 - y^2} - a) dy$$

$$\int y dA = \int_0^{y_0} (2y\sqrt{a^2 - y^2} - ay) dy$$

For pt A, $x_1 = x_2$ or $\sqrt{a^2 - y^2} = a - \sqrt{a^2 - y^2}$, $2\sqrt{a^2 - y^2} = a$

$$y = y_0 = \sqrt{3}a/2$$

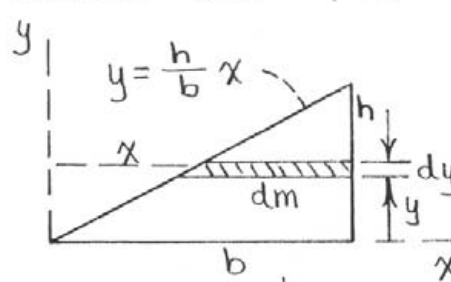
$$\text{so } \int y dA = \left[-\frac{2}{3}\sqrt{a^2 - y^2}^3 - \frac{ay^2}{2} \right]_0^{y_0} = 5a^3/24 = 0.208a^3$$

$$\int dA = \int_0^{y_0} (2\sqrt{a^2 - y^2} - a) dy = \left(y\sqrt{a^2 - y^2} + a^2 \sin^{-1} \frac{y}{a} - ay \right)_0^{y_0}$$

$$= a^2 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) = 0.614a^2$$

$$\bar{y} = \int y dA / \int dA = \frac{0.208a^3}{0.614a^2} = 0.339a$$

5/34



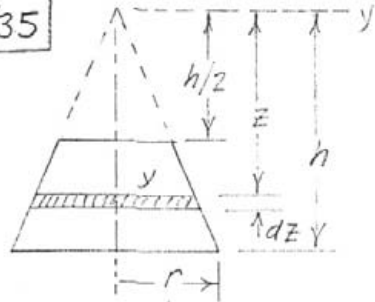
$$\begin{aligned}
 dm &= \rho dV = \rho dA t = t \rho (b-x) dy \\
 &= \left[t_0 \left(\frac{y}{h} + 1 \right) \right] \rho (b-x) dy \\
 &= t_0 \rho \left(\frac{y}{h} + 1 \right) \left(b - \frac{b}{h} y \right) dy \\
 &= t_0 \rho b \left(1 - \frac{y^2}{h^2} \right) dy
 \end{aligned}$$

$$\begin{aligned}
 m &= \int dm = \int_0^h t_0 \rho b \left(1 - \frac{y^2}{h^2} \right) dy = t_0 \rho b \left[y - \frac{y^3}{3h^2} \right]_0^h \\
 &= \frac{2}{3} \rho t_0 b h
 \end{aligned}$$

$$\begin{aligned}
 \int y_c dm &= \int t_0 \rho b \left(1 - \frac{y^2}{h^2} \right) y dy = t_0 \rho b \left[\frac{y^2}{2} - \frac{y^4}{4h^2} \right]_0^h \\
 &= \frac{1}{4} \rho t_0 b h^2
 \end{aligned}$$

$$\bar{y} = \frac{\int y_c dm}{m} = \frac{\frac{1}{4} \rho t_0 b h^2}{\frac{2}{3} \rho t_0 b h} = \underline{\underline{\frac{3}{8} h}}$$

5/35



$$dV = \pi y^2 dz \quad \text{where } y = \frac{r}{h} z$$

$$= \pi \frac{r^2}{h^2} z^2 dz$$

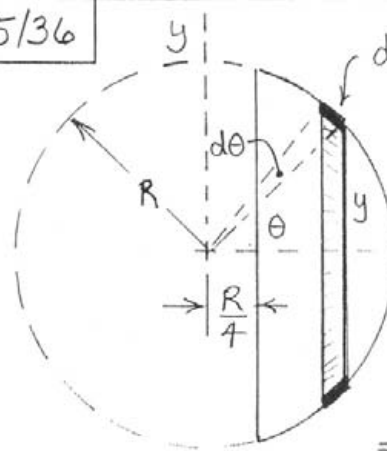
$$V = \pi \frac{r^2}{h^2} \int_{h/2}^h z^2 dz = \frac{7\pi r^2 h}{24}$$

$$\int \bar{z}_c dV = \int_{h/2}^h z \pi \frac{r^2}{h^2} z^2 dz = \frac{15}{64} \pi r^2 h^2$$

$$\bar{z} = \int \bar{z}_c dV / V = \frac{15}{64} \pi r^2 h^2 / \frac{7}{24} \pi r^2 h = \frac{45}{56} h$$

$$\bar{h} = h - \bar{z} = \frac{11}{56} h$$

5/36

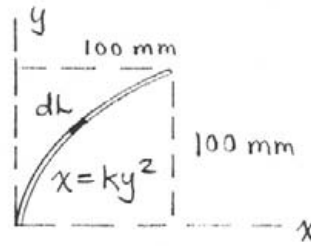


$$\begin{aligned}
 dm &= \rho dA = \rho (R d\theta) (2\pi y) \\
 &= 2\pi \rho R (R \sin \theta) d\theta \\
 &= 2\pi \rho R^2 \sin \theta d\theta \\
 m &= \int dm \\
 m &= \int_0^{\cos^{-1} \frac{1}{4}} 2\pi \rho R^2 \sin \theta d\theta \\
 &= 2\pi \rho R^2 (-\cos \theta) \Big|_0^{\cos^{-1} \frac{1}{4}} \\
 &= 1.5 \pi \rho R^2
 \end{aligned}$$

$$\begin{aligned}
 \int x_c dm &= \int (R \cos \theta) (2\pi \rho R^2 \sin \theta d\theta) \\
 &= 2\pi \rho R^3 \int \cos \theta \sin \theta d\theta = \pi \rho R^3 \int \sin 2\theta d\theta \\
 &= \pi \rho R^3 \left(-\frac{1}{2} \cos 2\theta \right) \Big|_0^{\cos^{-1} \frac{1}{4}} = \frac{15}{16} \pi \rho R^3 \\
 \bar{x} &= \frac{\int x_c dm}{\int dm} = \frac{\frac{15}{16} \pi \rho R^3}{1.5 \pi \rho R^2} = \frac{5}{8} R
 \end{aligned}$$

5/37

$$x = ky^2 = \frac{y^2}{100}, \quad \frac{dx}{dy} = \frac{y}{50}$$

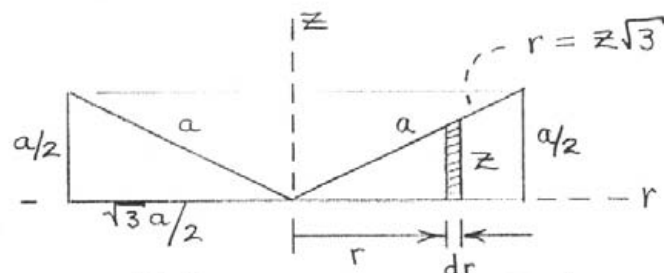


$$\begin{aligned} L &= \int dl = \int_0^{100} \sqrt{1 + (dx/dy)^2} dy = \int_0^{100} \sqrt{1 + \frac{y^2}{50^2}} dy \\ &= \frac{1}{50} \int_0^{100} \sqrt{50^2 + y^2} dy = \frac{1}{50 \cdot 2} \left[y \sqrt{50^2 + y^2} + 50^2 \ln(y + \sqrt{50^2 + y^2}) \right]_0^{100} \\ &= 147.9 \text{ mm} \end{aligned}$$

$$\begin{aligned} \int y_c dL &= \frac{1}{50} \int_0^{100} y \sqrt{50^2 + y^2} dy = \frac{1}{50} \frac{1}{3} (50^2 + y^2)^{3/2} \Big|_0^{100} \\ &= 8480 \text{ mm}^2 \end{aligned}$$

$$\bar{y} = \frac{\int y_c dL}{L} = \frac{8480}{147.9} = \underline{57.4 \text{ mm}}$$

5/38



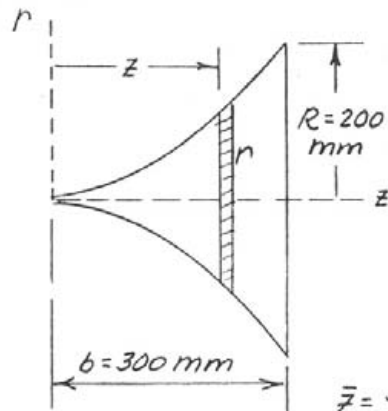
$$V = \int dV = \int_0^{\sqrt{3}a/2} 2\pi r z dr = \frac{2\pi}{\sqrt{3}} \int_0^{\sqrt{3}a/2} r^2 dr$$

$$= \frac{2\pi}{\sqrt{3}} \left(\frac{\sqrt{3}a/2}{3} \right)^3 = \frac{\pi a^3}{4}$$

$$\int z_c dV = \int_0^{\sqrt{3}a/2} \frac{r}{2\sqrt{3}} \frac{2\pi}{\sqrt{3}} r^2 dr = \frac{\pi}{3} \left[\frac{r^4}{4} \right]_0^{\sqrt{3}a/2} = \frac{3\pi a^4}{64}$$

$$\bar{z} = \frac{\int z_c dV}{V} = \frac{3\pi a^4/64}{\pi a^3/4} = \frac{3a}{16}$$

5/39 $r = kz^3$, $R = kb^3$, so $r = \frac{R}{b^3} z^3$



$$dV = \pi r^2 dz = \pi \frac{R^2}{b^6} z^6 dz$$

$$V = \frac{\pi R^2}{b^6} \int_0^b z^6 dz = \frac{\pi R^2}{7} b$$

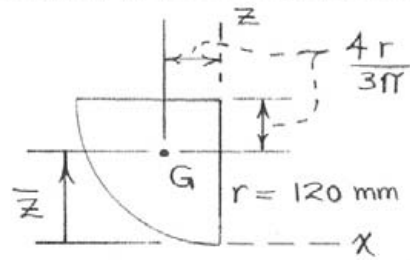
$$\int z_c dV = \int z dV = \frac{\pi R^2}{b^6} \int_0^b z^7 dz$$

$$= \frac{\pi R^2}{8} b^2$$

$$\bar{z} = \frac{\int z_c dV}{V} = \frac{\pi R^2 b^2 / 8}{\pi R^2 b / 7}$$

$$= 7b/8 = \frac{7(300)}{8} = \underline{263 \text{ mm}}$$

5/4



$$\bar{x} = -\frac{4r}{3\pi} = -\frac{4(120)}{3\pi} = \underline{-50.9 \text{ mm}}$$

$$\bar{y} = \underline{120 \text{ mm}}, \text{ by inspection}$$

$$\bar{z} = r - \frac{4r}{3\pi} = 120 - 50.9 = \underline{69.1 \text{ mm}}$$

►5/40

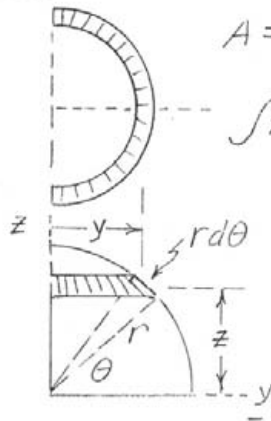
$$dA = \pi y r d\theta = \pi r^2 \cos \theta d\theta$$

$$A = \pi r^2 \int_0^{\pi/2} \cos \theta d\theta = \pi r^2 \sin \theta \Big|_0^{\pi/2} = \pi r^2$$

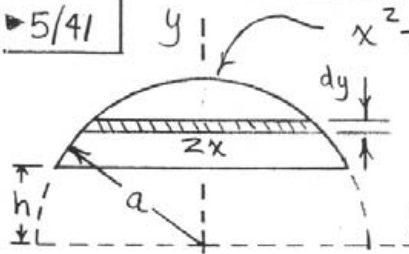
$$\int z_c dA = \int z dA = \int r \sin \theta \pi r^2 \cos \theta d\theta$$

$$= \pi r^3 \int_0^{\pi/2} \frac{\sin 2\theta}{2} d\theta = \frac{\pi r^3}{2}$$

$$\bar{z} = \int z_c dA / A = \frac{\pi r^3 / 2}{\pi r^2} = \frac{r}{2}$$



5/4/



$$x^2 + y^2 = a^2, \quad x = +\sqrt{a^2 - y^2}$$

$$dA = 2x dy = 2\sqrt{a^2 - y^2} dy$$

$$A = \int dA = \int_h^a 2\sqrt{a^2 - y^2} dy$$

$$= 2\left(\frac{1}{2}\right) \left[y\sqrt{a^2 - y^2} + a^2 \sin^{-1} \frac{y}{a} \right]_h^a$$

$$= a^2 \left(\frac{\pi}{2} - \sin^{-1} \frac{h}{a} \right) - h\sqrt{a^2 - h^2}$$

$$\int y dA = \int_h^a y 2\sqrt{a^2 - y^2} dy = 2\left(-\frac{1}{3}\right) (a^2 - y^2)^{3/2} \Big|_h^a$$

$$= \frac{2}{3} (a^2 - h^2)^{3/2}$$

$$\bar{y} = \frac{\int y dA}{\int dA} = \frac{\frac{2}{3} (a^2 - h^2)^{3/2}}{a^2 \left(\frac{\pi}{2} - \sin^{-1} \frac{h}{a} \right) - h\sqrt{a^2 - h^2}}$$

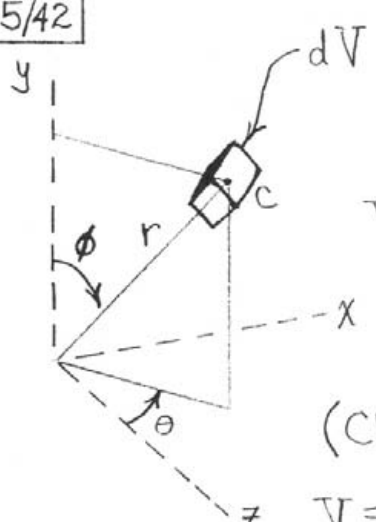
Special cases

$$h = 0 : \bar{y} = \frac{\frac{2}{3} a^3}{a^2 \frac{\pi}{2}} = \frac{4a}{3\pi} \quad (\text{the correct result})$$

$$h = \frac{a}{4} : \bar{y} = \frac{\frac{2}{3} (a^2 - (\frac{a}{4})^2)^{3/2}}{a^2 \left(\frac{\pi}{2} - \sin^{-1} \frac{1}{4} \right) - \frac{a}{4} \sqrt{a^2 - (\frac{a}{4})^2}} = \underline{0.562a}$$

$$h = \frac{a}{2} : \bar{y} = \frac{\frac{2}{3} (a^2 - (\frac{a}{2})^2)^{3/2}}{a^2 \left(\frac{\pi}{2} - \sin^{-1} \frac{1}{2} \right) - \frac{a}{2} \sqrt{a^2 - (\frac{a}{2})^2}} = \underline{0.705a}$$

► 5/42



$$dV = dr (r d\phi) (r \sin \phi d\theta)$$

$$= r^2 \sin \phi d\phi d\theta dr$$

$$V = \int dV = \int_{R/2}^R \int_0^\pi \int_0^\pi r^2 \sin \phi d\phi d\theta dr$$

$$= \frac{7}{12} \pi R^3$$

(Check V by

$$V = \frac{\frac{4}{3} \pi R^3}{2} - \frac{\frac{4}{3} \pi (\frac{R}{2})^3}{2} = \frac{7}{12} \pi R^3)$$

$$\int x_c dV = \int (r \sin \phi \sin \theta) (r^2 \sin \phi d\phi d\theta dr)$$

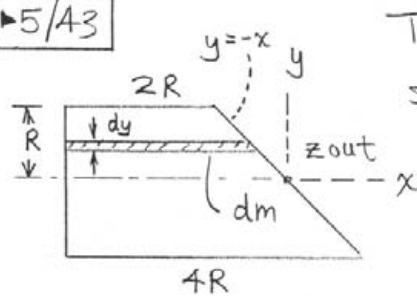
$$= \int_{R/2}^R \int_0^\pi \int_0^\pi r^3 \sin^2 \phi \sin \theta d\phi d\theta dr = \frac{15}{64} \pi R^4$$

$$\bar{x} = \frac{\int x_c dV}{V} = \frac{\frac{15}{64} \pi R^4}{\frac{7}{12} \pi R^3} = \frac{45}{112} R$$

(Compare to $\bar{x} = \frac{3}{8} R$ for no hole.)

Note: A hemispherical shell of radius r and thickness dr would be a better element.

5/43



Temporarily use coordinates shown. Element is a thin rectangular plate.

$$\begin{aligned} dm &= \rho dV = \rho (3R+x)(2z) dy \\ &= 2\rho [3R-y] [\sqrt{R^2-y^2}] dy \\ &= 6\rho R \sqrt{R^2-y^2} dy - 2\rho y \sqrt{R^2-y^2} dy \end{aligned}$$

$$m = \int dm = \int_{-R}^R 6\rho R \sqrt{R^2-y^2} dy - \int_{-R}^R 2\rho y \sqrt{R^2-y^2} dy$$

$$\begin{aligned} m &= 6\rho R \left[\frac{1}{2} (y \sqrt{R^2-y^2} + R^2 \sin^{-1} \frac{y}{R}) \right]_{-R}^R - 2\rho \left[-\frac{1}{3} \sqrt{R^2-y^2}^3 \right]_{-R}^R \\ &= 3\pi \rho R^3 \end{aligned}$$

$$\int x_c dm = \int_{-R}^R \left(\frac{x-3R}{2} \right) (6\rho R \sqrt{R^2-y^2} dy - 2\rho y \sqrt{R^2-y^2} dy)$$

$$\begin{aligned} I_1 &= -3\rho R \int_{-R}^R y \sqrt{R^2-y^2} dy = -3\rho R \left[-\frac{1}{3} \sqrt{R^2-y^2}^3 \right]_{-R}^R \\ &= 0 \end{aligned}$$

$$I_2 = -9\rho R^2 \int_{-R}^R \sqrt{R^2-y^2} dy$$

$$= -9\rho R^2 \cdot \frac{1}{2} \left(y \sqrt{R^2-y^2} + R^2 \sin^{-1} \left(\frac{y}{R} \right) \right)_{-R}^R$$

$$= -\frac{9}{2} \rho R^2 \left(R^2 \frac{\pi}{2} - (-R^2 \frac{\pi}{2}) \right) = -\frac{9}{2} \pi \rho R^4$$

$$\begin{aligned}
 I_3 &= \rho \int_{-R}^R y^2 \sqrt{R^2 - y^2} dy \\
 &= \rho \left(-\frac{y}{4} \sqrt{(R^2 - y^2)^3} + \frac{R^2}{8} \left(y \sqrt{R^2 - y^2} + R^2 \sin^{-1} \frac{y}{R} \right) \right) \Bigg|_{-R}^R \\
 &= \rho \left(\frac{R^2}{8} \cdot \left(R^2 \left(\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right) \right) \right) = \frac{1}{8} \pi \rho R^4
 \end{aligned}$$

$$I_4 = -3\rho R \int_{-R}^R y \sqrt{R^2 - y^2} dy = 0$$

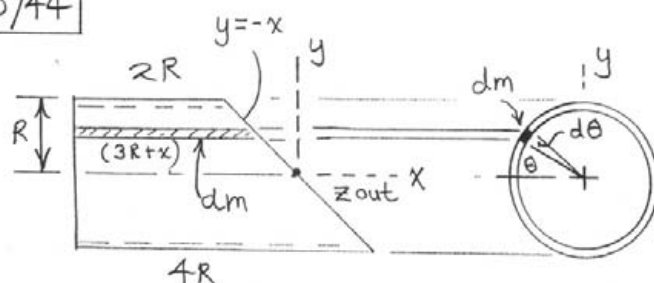
$$\begin{aligned}
 \text{So total is } \int x_c dm &= \left(-\frac{9}{2} + \frac{1}{8} \right) \pi \rho R^4 \\
 &= -\frac{35}{8} \pi \rho R^4
 \end{aligned}$$

$$\begin{aligned}
 \text{Then } \bar{x} &= \frac{\int x_c dm}{\int dm} \\
 &= \frac{-\frac{35}{8} \pi \rho R^4}{3\pi \rho R^3} = -\frac{35}{24} R
 \end{aligned}$$

Relative to left (flat) end, then,

$$\bar{x} = 3R - \frac{35}{24}R = \underline{\underline{\frac{37}{24}R}} \quad \underline{\underline{(1.542R)}}$$

5/44



Temporarily use coordinates shown. Element is a "stick". ρ is mass per unit area here.

$$\begin{aligned} dm &= \rho(3R+x)Rd\theta = \rho R(3R-y)d\theta \\ &= \rho R(3R-R\sin\theta)d\theta = \rho R^2(3-\sin\theta)d\theta \end{aligned}$$

$$\begin{aligned} m &= \int dm = \int_0^{2\pi} \rho R^2(3-\sin\theta)d\theta \\ &= \rho R^2 \left[3\theta + \cos\theta \right]_0^{2\pi} = 6\pi \rho R^2 \end{aligned}$$

$$\begin{aligned} x_c &= -3R + \frac{3R+x}{2} = \frac{x-3R}{2} = -\frac{y+3R}{2} \\ &= -R \frac{\sin\theta+3}{2} \end{aligned}$$

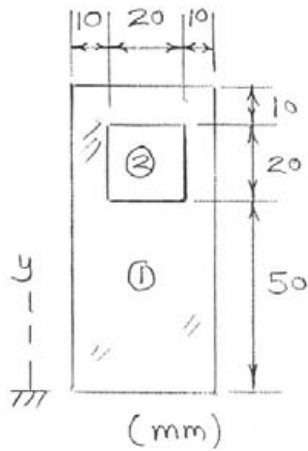
$$\begin{aligned} \int x_c dm &= \int_0^{2\pi} \left[-\frac{R}{2}(\sin\theta+3) \right] \rho R^2(3-\sin\theta)d\theta \\ &= -\frac{\rho R^3}{2} \int_0^{2\pi} (9-\sin^2\theta)d\theta \\ &= -\frac{\rho R^3}{2} \left[9\theta - \left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \right]_0^{2\pi} = -\rho R^3 \left(\frac{17\pi}{2} \right) \end{aligned}$$

$$\text{So } \bar{x} = \frac{\int x_c dm}{\int dm} = \frac{-\rho R^3 (17\pi/2)}{6\pi \rho R^2} = 1.417R$$

Relative to left (flat) end,

$$\bar{x} = 3R - 1.417R = \underline{1.583R}$$

5/45

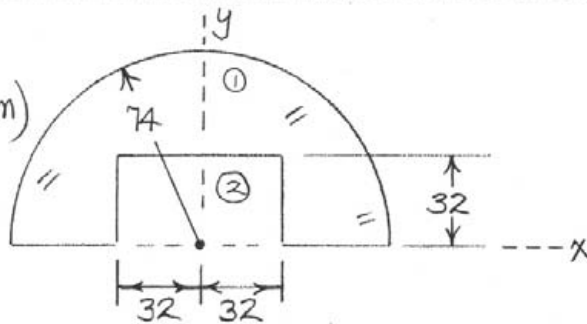


Component	$A \text{ (mm}^2\text{)}$	$\bar{y} \text{ (mm)}$	$\bar{y}A \text{ (mm}^3\text{)}$
1	$80(40) = 3200$	40	128 000
2	$-20(20) = -400$	60	-24 000
	$\Sigma A = 2800$		$\Sigma \bar{y}A = 104 000$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{104 000}{2800} = \underline{37.1 \text{ mm}}$$

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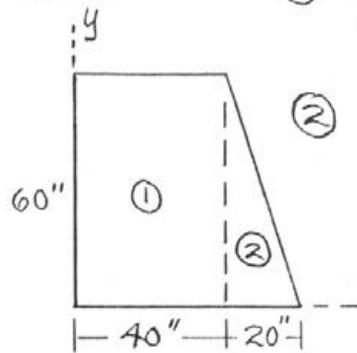
(Dim. in mm)



$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{\pi \frac{74^2}{2} \left(\frac{4(74)}{3\pi} \right) - 64(32) \left(\frac{32}{2} \right)}{\pi \frac{74^2}{2} - 64(32)}$$

$$= \underline{36.2 \text{ mm}}$$

5/47



$$\textcircled{1} A_1 = 40(60) = 2400 \text{ in.}^2$$

$$\bar{x}_1 = 20 \text{ in.}, \bar{y}_1 = 30 \text{ in.}$$

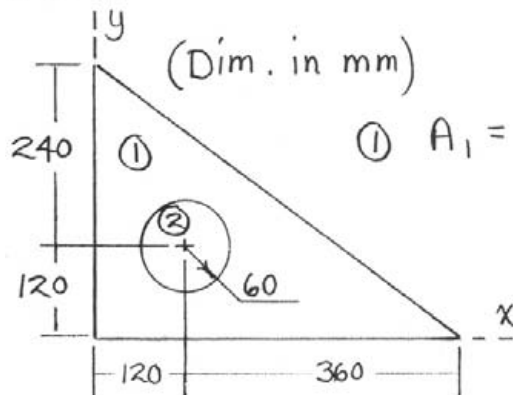
$$\textcircled{2} A_2 = \frac{1}{2}(20)(60) = 600 \text{ in.}^2$$

$$\bar{x}_2 = 40 + \frac{20}{3} = 46.7 \text{ in.}$$

$$\bar{y}_2 = \frac{60}{3} = 20 \text{ in.}$$

$$\bar{X} = \frac{\sum A \bar{x}}{\sum A} = \frac{2400(20) + 600(46.7)}{2400 + 600} = 25.3 \text{ in.}$$

$$\bar{Y} = \frac{\sum A \bar{y}}{\sum A} = \frac{2400(30) + 600(20)}{2400 + 600} = 28.0 \text{ in.}$$



$$\textcircled{1} \quad A_1 = \frac{1}{2}(480)(360) \\ = 86\,400 \text{ mm}^2$$

$$\bar{x}_1 = \frac{1}{3}(480) = 160 \text{ mm}$$

$$\bar{y}_1 = \frac{1}{3}(360) = 120 \text{ mm}$$

$$\textcircled{2} \quad A_2 = -\pi(60)^2 = -11\,310 \text{ mm}^2$$

$$\bar{x}_2 = \bar{y}_2 = 120 \text{ mm}$$

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{86\,400(160) - 11\,310(120)}{86\,400 - 11\,310} = \underline{166.0 \text{ mm}}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{86\,400(120) - 11\,310(120)}{86\,400 - 11\,310} = \underline{120 \text{ mm}}$$

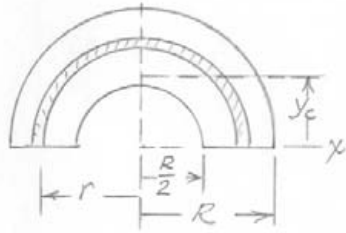
(Because the circular hole is located at the \bar{y} for the triangle, its presence does not alter the \bar{y} result.)

5/49 Triangle: $A = \frac{1}{2}(2h)h = h^2$
 $\bar{y} = h/3$
 Semi-circular hole: $A = -\frac{1}{2}\pi(h/2)^2 = -\pi h^2/8$
 $\bar{y} = 4(h/2)/3\pi = \frac{2h}{3\pi}$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{h^2(h/3) - (\pi h^2/8)(2h/3\pi)}{h^2 - \pi h^2/8} = \frac{h}{4(1 - \pi/8)} = 0.412h$$

5/5

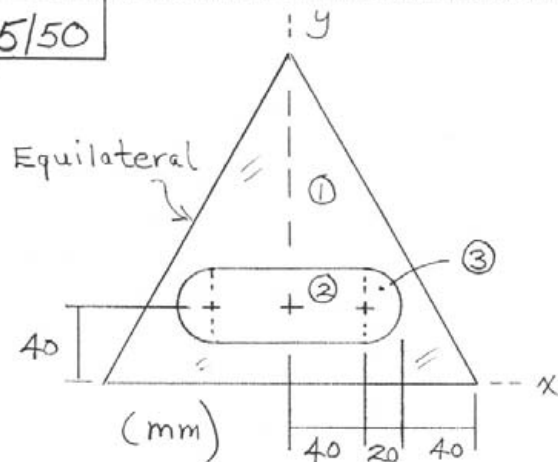
$$y_c = \frac{2r}{\pi}; dA = \pi r dr, A = \pi \int_{R/2}^R r dr = 3\pi R^2/8$$



$$\begin{aligned} \int y_c dA &= \int_{R/2}^R \frac{2r}{\pi} \pi r dr \\ &= \frac{2r^3}{3} \Big|_{R/2}^R = \frac{7}{12} R^3 \end{aligned}$$

$$\bar{y} A = \int y_c dA; \quad \bar{y} = \frac{7}{12} R^3 / \frac{3\pi R^2}{8} = \frac{14R}{9\pi}$$

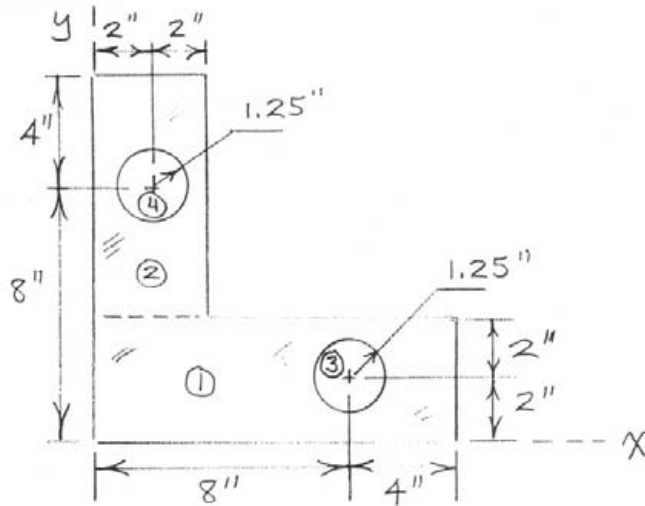
5/50



Component	$A \text{ (mm}^2\text{)}$	$\bar{y} \text{ (mm)}$	$\bar{y} A \text{ (mm}^3\text{)}$
Triangle 1	17 320	57.7	10^6
Rectangle 2	- 3200	40	- 128 000
2 semicircles 3	- 1257	40	- 50,300
$\Sigma A = 12 860$		$\Sigma \bar{y} A = 822 000$	

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{822 000}{12 860} = 63.9 \text{ mm}$$

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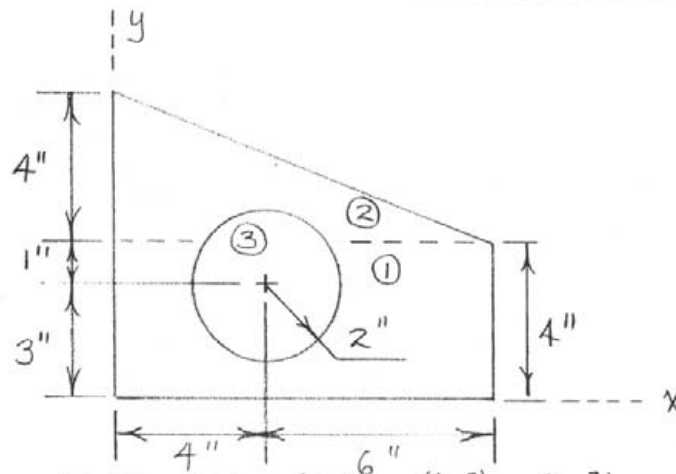
Comp.	$A \text{ (in.}^2\text{)}$	$\bar{x} \text{ (in.)}$	$\bar{x} A \text{ (in.}^3\text{)}$
①	48	6	288
②	32	2	64
③	- 4.91	8	- 39.3
④	- 4.91	2	- 9.82

$$\Sigma A = 70.2$$

$$\Sigma \bar{x} A = 303$$

$$\bar{X} = \bar{Y} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{303}{70.2} = \underline{4.32 \text{ in.}}$$

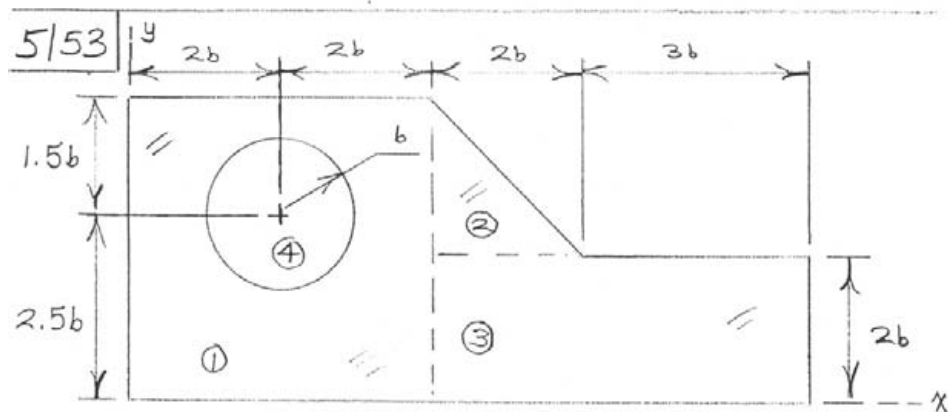
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Part	(in. ²) A	(in.) \bar{x}	(in.) \bar{y}	(in. ³) $A\bar{x}$	(in. ³) $A\bar{y}$
1	40	5	2	200	80
2	20	$10/3$	$4 + 4/3$	66.7	106.7
3	$-\pi 2^2$	4	3	-50.3	-37.7
Totals	47.4			216	149.0

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{216}{47.4} = 4.56 \text{ in.}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{149.0}{47.4} = 3.14 \text{ in.}$$



Comp.	A	\bar{x}	\bar{y}	$\bar{x}A$	$\bar{y}A$
1	$16b^2$	$2b$	$2b$	$32b^3$	$32b^3$
2	$2b^2$	$(4b + \frac{2b}{3})$	$(2.5b + \frac{2b}{3})$	$9.33b^3$	$6.33b^3$
3	$10b^2$	$(4b + \frac{5b}{2})$	b	$65b^3$	$10b^3$
4	$-\pi b^2$	$2b$	$2.5b$	$-2\pi b^3$	$-2.5\pi b^3$

$$\Sigma A = 24.9b^2$$

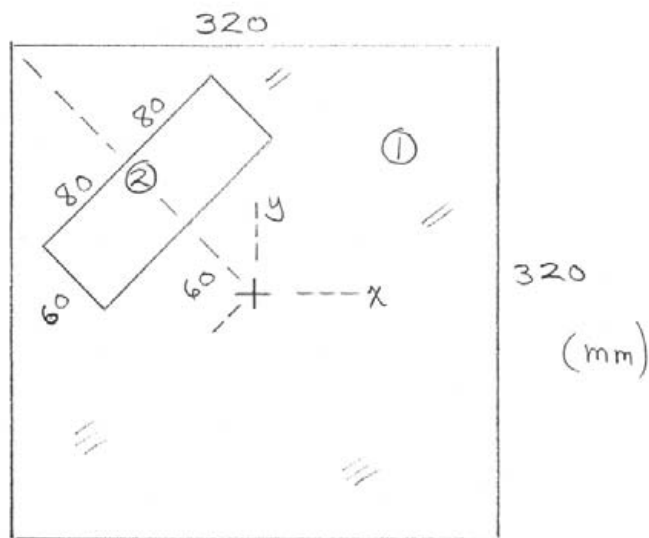
$$\Sigma \bar{x}A = 100.1b^3$$

$$\Sigma \bar{y}A = 40.5b^3$$

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{100.1b^3}{24.9b^2} = 4.02b$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{40.5b^3}{24.9b^2} = 1.628b$$

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Comp.	A mm^2	\bar{x} mm	\bar{y} mm	$\bar{x}A$ mm^3	$\bar{y}A$ mm^3
1	$(320)^2$	0	0	0	0
2	$-160(60)$	$-90\frac{\sqrt{2}}{2}$	$90\frac{\sqrt{2}}{2}$	611 000	-611 000

$$\Sigma A = 92\,800$$

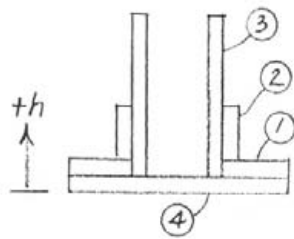
$$\Sigma \bar{x}A = 611\,000$$

$$\Sigma \bar{y}A = -611\,000$$

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{611\,000}{92\,800} = \underline{6.58 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{-611\,000}{92\,800} = \underline{-6.58 \text{ mm}}$$

5/55



Part	Size mm	A mm ²	\bar{h} mm	$A\bar{h}$ mm ³
①	10x40	800	15	12000
②	10x40	800	40	32000
③	10x120	2400	70	168000
④	10x160	1600	5	8000
Σ		5600		220000

$$\bar{H} = \frac{\Sigma A\bar{h}}{\Sigma A} = \frac{220000}{5600} = 39.3 \text{ mm}$$

$$5/56 \quad \bar{X} = \frac{\sum \bar{x} L}{\sum L}, \quad \bar{Y} = \frac{\sum \bar{y} L}{\sum L}$$

$$L_1 = \pi r = 150\pi \text{ mm}$$

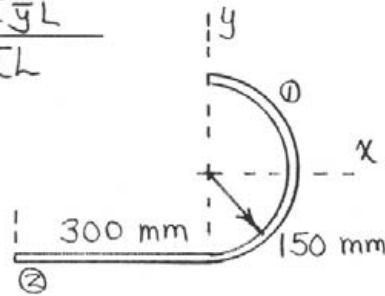
$$\bar{x}_1 = \frac{2r}{\pi} = \frac{300}{\pi} \text{ mm}$$

(from Sample Prob. 5/1)

$$\bar{y}_1 = 0$$

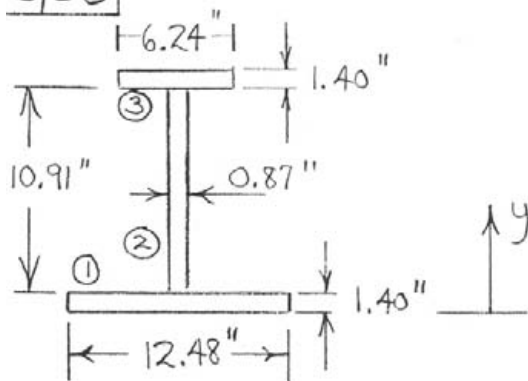
$$L_2 = 300 \text{ mm}, \quad \bar{x}_2 = -150 \text{ mm}, \quad \bar{y}_2 = -150 \text{ mm}$$

$$\text{So } \begin{cases} \bar{X} = \frac{\frac{300}{\pi}(150\pi) + 300(-150)}{150\pi + 300} = 0 \\ \bar{Y} = \frac{\frac{300}{\pi}(0) + 300(-150)}{150\pi + 300} = -58.3 \text{ mm} \end{cases}$$



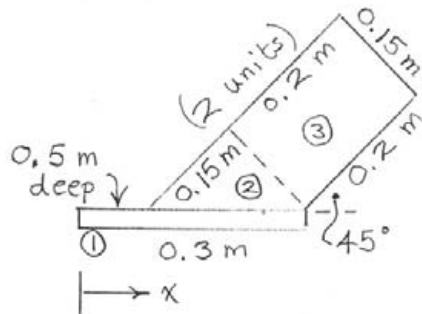
$$\frac{5/57}{\bar{z}} = \frac{\sum m \bar{z}}{\sum m} = \frac{2(0) + 1.5(90) + 1(180)}{2 + 1.5 + 1} = \underline{70 \text{ mm}}$$

5/58



Comp.	A (in. ²)	\bar{y} (in.)	$A\bar{y}$ (in. ³)
①	$(12.48)(1.40)$	$\frac{1.40}{2}$	12.23
②	$(10.91)(0.87)$	$1.4 + \frac{10.91}{2}$	65.1
③	$(6.24)(1.40)$	$1.4 + 10.91 + \frac{1.4}{2}$	113.7
$\Sigma A = 35.7$			$\Sigma A\bar{y} = 191.0$
$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 5.35 \text{ in.}$			

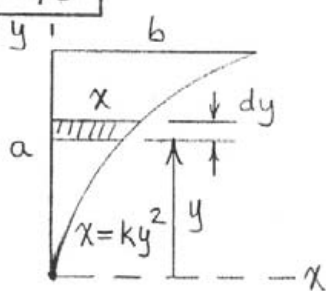
5/59 $\rho = \text{mass per unit area}$



Comp.	m	\bar{x}	$\bar{x} m$
①	$(0.5)(0.3) \rho$	0.15	0.0225ρ
②	$2 \frac{(0.15)(0.15)}{2} \rho$	$0.3 - \left[\frac{2}{3}(0.15) + \frac{1}{3}(0.15) \right] \frac{\sqrt{2}}{2}$	0.00436ρ
③	$2 (0.2)(0.15) \rho$	$0.3 + \left(\frac{0.2 - 0.15}{2} \right) \frac{\sqrt{2}}{2}$	0.01906ρ
$\Sigma m = 0.232 \rho$		$\Sigma \bar{x} m = 0.0459 \rho$	

$$\bar{X} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{0.0459 \rho}{0.232 \rho} = \underline{0.1975 \text{ m}}$$

5/6



$$b = ka^2, \quad k = \frac{b}{a^2}, \quad x = \frac{b}{a^2} y^2$$

$$A = \int x dy = \int_0^a \frac{b}{a^2} y^2 dy$$

$$= \frac{b}{a^2} \frac{a^3}{3} = \frac{1}{3} ab$$

$$\int x_c dA = \int \frac{x}{2} x dy = \int \frac{b^2 y^4}{2a^4} dy = \frac{ab^2}{10}$$

$$\bar{x} = \int x_c dA / A = \frac{ab^2/10}{ab/3} = \underline{\underline{\frac{3}{10} b}}$$

$$\int y_c dA = \int y x dy = \int_0^a y \frac{b}{a^2} y^2 dy = \frac{ba^2}{4}$$

$$\bar{y} = \int y_c dA / A = \frac{ba^2/4}{ba/3} = \underline{\underline{\frac{3}{4} a}}$$

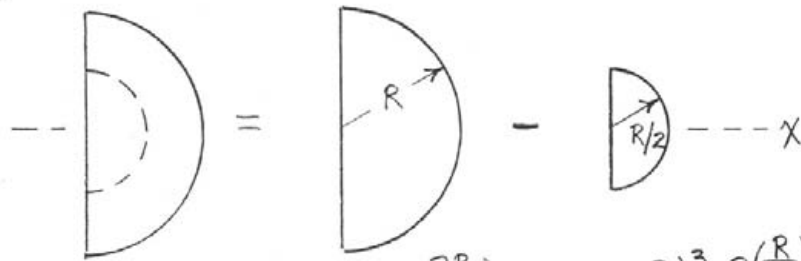
5/60

$A = \frac{\pi a^2}{4}$
 $A = -\frac{1}{2}a\left(\frac{a}{2}\right) = -\frac{a^2}{4}$

$\bar{X} = \frac{\sum \bar{x} A}{\sum A} = \frac{\frac{\pi a^2}{4} \frac{4a}{3\pi} - \frac{a^2}{4} \frac{a}{3}}{\frac{\pi a^2}{4} - \frac{a^2}{4}} = \frac{a}{\pi - 1}$

$\bar{Y} = \frac{\sum \bar{y} A}{\sum A} = \frac{\frac{\pi a^2}{4} \frac{4a}{3\pi} - \frac{a^2}{4} \frac{a}{6}}{\frac{\pi a^2}{4} - \frac{a^2}{4}} = \frac{7}{6} \frac{a}{\pi - 1}$

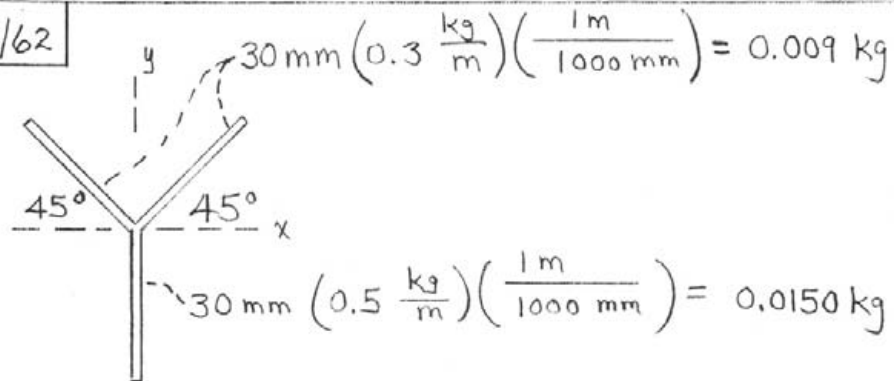
5/61



$$\bar{X} = \frac{\sum V \bar{x}}{\sum V} = \frac{\frac{2}{3} \pi R^3 \left(\frac{3R}{8} \right) - \frac{2}{3} \pi \left(\frac{R}{2} \right)^3 \frac{3 \left(\frac{R}{2} \right)}{8}}{\frac{2}{3} \pi R^3 - \frac{2}{3} \pi \left(\frac{R}{2} \right)^3}$$

$$= \frac{45}{112} R$$

5/62

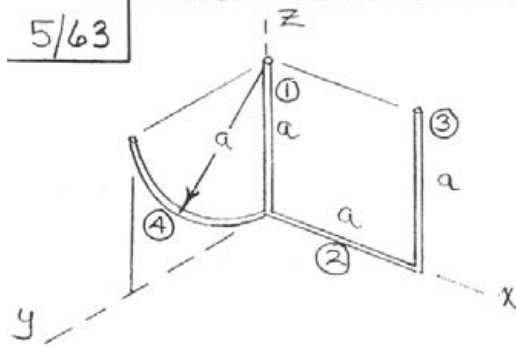


$$\bar{Y} = \frac{\sum m \bar{y}}{\sum m} = \frac{2(0.009)(15 \sin 45^\circ) - 0.0150(15)}{2(0.009) + 0.0150}$$

$$= \underline{-1.033 \text{ mm}}$$

$$\bar{X} = 0, \text{ by inspection.}$$

5/63


 $(\rho = \text{mass/length})$

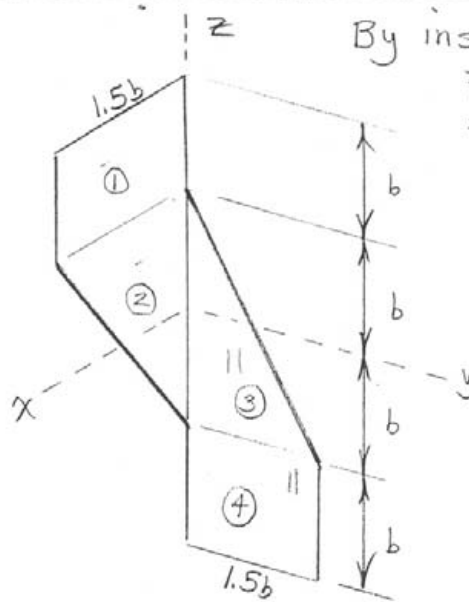
Comp.	m	\bar{x}	\bar{y}	\bar{z}	$m\bar{x}$	$m\bar{y}$	$m\bar{z}$
1	ρa	0	0	$a/2$	0	0	$\rho a^2/2$
2	ρa	$a/2$	0	0	$\rho a^2/2$	0	0
3	ρa	a	0	$a/2$	ρa^2	0	$\rho a^2/2$
4	$\rho \pi a/2$	0	$-2a/\pi$	$a(1-\frac{2}{\pi})$	0	$-\rho a^2$	$\rho a^2(\frac{\pi}{2}-1)$
Totals	$\rho a(3+\frac{\pi}{2})$				$\frac{3}{2}\rho a^2$	$-\rho a^2$	$\frac{\pi}{2}\rho a^2$

$$\bar{X} = \frac{\sum m\bar{x}}{\sum m} = \frac{\frac{3}{2}\rho a^2}{\rho a(3+\frac{\pi}{2})} = \frac{3a}{6+\pi}$$

$$\bar{Y} = \frac{\sum m\bar{y}}{\sum m} = \frac{-\rho a^2}{\rho a(3+\pi/2)} = -\frac{2a}{6+\pi}$$

$$\bar{Z} = \frac{\sum m\bar{z}}{\sum m} = \frac{\frac{\pi}{2}\rho a^2}{\rho a(3+\pi/2)} = \frac{\pi a}{6+\pi}$$

5/64



By inspection,

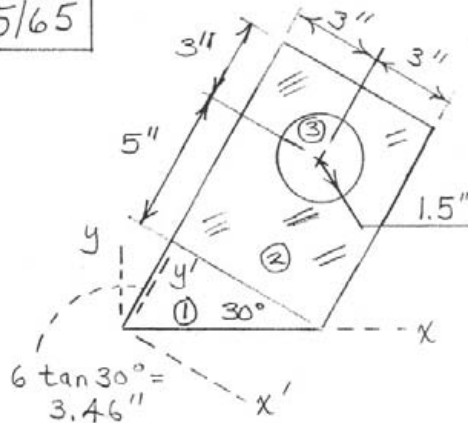
$$\bar{z} = 0$$

$$\bar{x} = \bar{y}$$

Let $\rho =$
area density

Comp.	m	\bar{x}	$m\bar{x}$
1	$1.5b^2\rho$	$1.5b/2$	$1.125b^3\rho$
2	$1.5b^2\rho$	$0.5b$	$0.75b^3\rho$
3	$1.5b^2\rho$	0	0
4	$1.5b^2\rho$	0	0
	$\Sigma A = 6b^2$		$\Sigma \bar{x}A = 1.875b^3$
$\bar{x} = \frac{\Sigma \bar{x}m}{\Sigma m} = \frac{1.875b^3\rho}{6b^2\rho} = 0.312b = \bar{y}$			

5/65



Perform calculations
in $x'-y'$ coordinates,
then convert to $x-y$.

Comp.	A	\bar{x}'	\bar{y}'	$\bar{x}'A$	$\bar{y}'A$
1	$\frac{3.46(6)}{2}$	$\frac{6}{3}$	$3.46\left(\frac{2}{3}\right)$	20.8	24
2	48	3	7.46	144	358
3	$-\pi(1.5)^2$	3	8.46	-21.2	-59.8
$\Sigma A = 51.3$				143.6	322

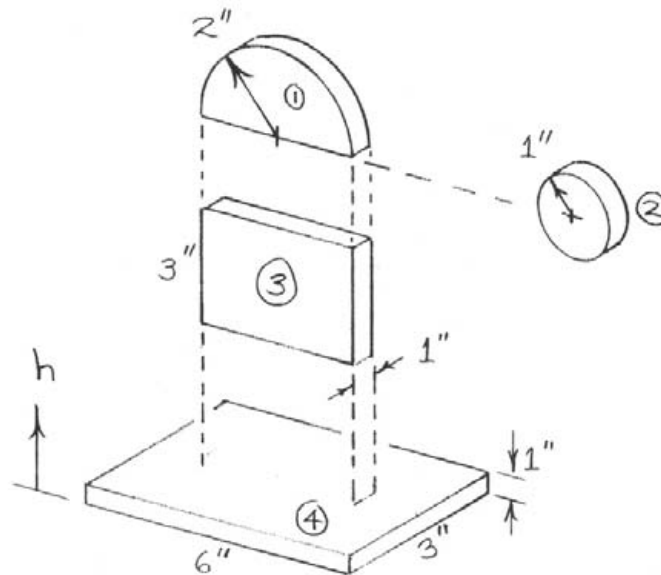
$$\bar{X}' = \frac{\Sigma \bar{x}'A}{\Sigma A} = \frac{143.6}{51.3} = 2.80 \text{ in.}$$

$$\bar{Y}' = \frac{\Sigma \bar{y}'A}{\Sigma A} = \frac{322}{51.3} = 6.28 \text{ in.}$$

Then

$$\bar{X} = 2.80 \cos 30^\circ + 6.28 \sin 30^\circ = \underline{5.56 \text{ in.}}$$

$$\bar{Y} = -2.80 \sin 30^\circ + 6.28 \cos 30^\circ = \underline{4.04 \text{ in.}}$$



Part	$V, \text{in.}^3$	$\bar{h}, \text{in.}$	$V\bar{h}, \text{in.}^4$
①	6.28	4.85	30.5
②	-3.14	4	-12.57
③	12	2.5	30
④	18	0.5	9
Totals	33.1		56.9

$$\bar{H} = \frac{\sum V\bar{h}}{\sum V} = \frac{56.9}{33.1} = \underline{1.717 \text{ in.}}$$

Comp.	V	\bar{x}	\bar{y}	\bar{z}	$V\bar{x}$	$V\bar{y}$	$V\bar{z}$
Rect. Sol.	$8.08(10^6)$	0	0	0	0	0	0
Cyl.	$2.26(10^6)$	185	0	0	$418(10^6)$	0	0
Rod	$17.67(10^3)$	0	175	0	0	$3.09(10^6)$	0
Sphere	$524(10^3)$	0	275	0	0	$144.0(10^6)$	0
Totals	$10.88(10^6)$				$418(10^6)$	$147.1(10^6)$	0

$$\bar{X} = \frac{\sum V\bar{x}}{\sum V} = \frac{418(10^6)}{10.88(10^6)} = \underline{38.5 \text{ mm}}$$

$$\bar{Y} = \frac{\sum V\bar{y}}{\sum V} = \frac{147.1(10^6)}{10.88(10^6)} = \underline{13.52 \text{ mm}}$$

$$\bar{Z} = \underline{0}$$

5/68



Length = 35 mm



Length = 25 mm

$$\begin{cases} V_1 = \frac{\pi}{2} (30)^2 (35) = 49\,500 \text{ mm}^3 \\ \bar{x}_1 = -\frac{4(30)}{3\pi} = -12.73 \text{ mm} \\ \bar{z}_1 = 17.5 \text{ mm} \end{cases}$$

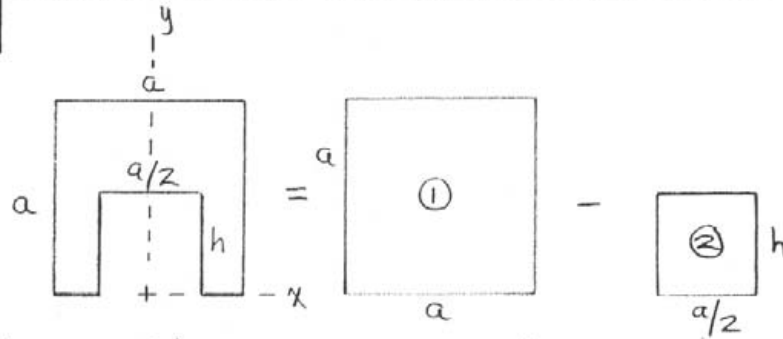
$$V_2 = -\frac{\pi}{2} (20)^2 (25) = -15\,710 \text{ mm}^3$$

$$\bar{x}_2 = -\frac{4(20)}{3\pi} = -8.49 \text{ mm}$$

$$\bar{z}_2 = \frac{1}{2}(10 + 35) = 22.5 \text{ mm}$$

$$\begin{aligned} \bar{\bar{x}} &= \frac{\sum V \bar{x}}{\sum V} = \frac{49\,500(-12.73) - 15\,710(-8.49)}{49\,500 - 15\,710} \\ &= \underline{-14.71 \text{ mm}} \end{aligned}$$

$$\begin{aligned} \bar{\bar{z}} &= \frac{\sum V \bar{z}}{\sum V} = \frac{49\,500(17.5) - 15\,710(22.5)}{49\,500 - 15\,710} \\ &= \underline{15.17 \text{ mm}} \end{aligned}$$



$$A_1 = a^2, \quad \bar{y}_1 = a/2; \quad A_2 = -ah/2, \quad \bar{y}_2 = h/2$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{a^2(\frac{a}{2}) - \frac{ah}{2}(\frac{h}{2})}{a^2 - \frac{ah}{2}} = \frac{1}{2} \frac{a^2 - h^2/2}{a - h/2}$$

$$\frac{d\bar{Y}}{dh} = \frac{1}{2} \frac{(a - h/2)(-h) - (a^2 - h^2/2)(-\frac{1}{2})}{(a - h/2)^2}$$

$$= \frac{1}{2} \frac{\frac{h^2}{4} - ah + \frac{a^2}{2}}{(a - \frac{h}{2})^2} = 0 \quad \text{for max } \bar{Y}.$$

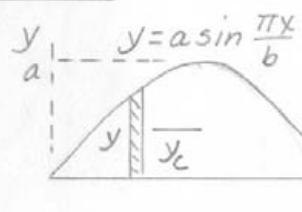
$$\text{So } \frac{1}{4}(h^2 - 4ah + 2a^2) = 0, \quad h = a(2 \pm \sqrt{2})$$

The plus sign is rejected because h must be less than a .

$$\text{Hence } h = a(2 - \sqrt{2}) = \underline{0.586a}$$

(Note that $\bar{Y} = 0.586a$ for $h = 0.586a$)

5/7



$$dA = y dx = a \sin \frac{\pi x}{b} dx$$

$$A = \int_0^b a \sin \frac{\pi x}{b} dx = -\frac{ab}{\pi} \cos \frac{\pi x}{b} \Big|_0^b = \frac{2ab}{\pi}$$

$$\int y_c dA = \int \frac{y}{2} dA = \int_0^b \frac{1}{2} a^2 \sin^2 \frac{\pi x}{b} dx = \frac{a^2 b}{2 \pi} \left(\frac{\pi x}{2b} - \frac{\sin \frac{2\pi x}{b}}{4} \right) \Big|_0^b = \frac{a^2 b}{4}$$

$$\bar{y} = \int y_c dA / A = \frac{a^2 b / 4}{2ab / \pi} = \frac{\pi a}{8}$$

$$\boxed{5/70} \text{ Cube: } \begin{cases} V_1 = 350^3 = 42\,875\,000 \text{ mm}^3 \\ \bar{z}_1 = 175 \text{ mm} \end{cases}$$

$$\text{Hole: } \begin{cases} V_2 = -\pi (100)^2 h \\ \bar{z}_2 = \frac{h}{2} \end{cases}$$

$$\bar{z} = \frac{\sum V \bar{z}}{\sum V} = \frac{42\,875\,000 (175) - \pi (100)^2 h \frac{h}{2}}{42\,875\,000 - \pi (100)^2 h}$$

For The maximum \bar{z} , set $\frac{d\bar{z}}{dh} = 0$:

$$\frac{(42\,875\,000 - \pi 100^2 h)(-\pi 100^2 h) - (42\,875\,000 (175) - \pi 100^2 \frac{h^2}{2}) \times (-\pi 100^2)}{(denominator)} = 0$$

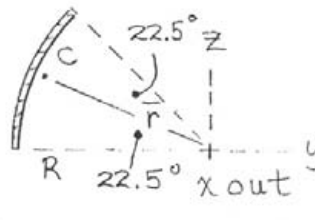
Set numerator equal to zero to obtain

$$\underline{h = 187.9 \text{ mm}}$$

5/71

Detail of "hole" (2): [1 = cyl. shell]

$$\bar{r} = \frac{R \sin 22.5^\circ}{\pi/8} = 0.974R$$



$$\begin{cases} \bar{x} = -\frac{L}{8} - \frac{1}{2} \frac{3L}{8} = -\frac{5}{16} L \\ \bar{y} = -\bar{r} \cos 22.5^\circ = -0.900R \\ \bar{z} = \bar{r} \sin 22.5^\circ = 0.373R \end{cases}$$

Comp	m	\bar{x}	\bar{y}	\bar{z}
①	$2\pi RL$	$-\frac{L}{2}$	0	0
②	$-\frac{3}{32}\pi RL$	$-\frac{5L}{16}$	$-0.900R$	$0.373R$

$$\left. \begin{array}{l} \text{①} \\ \text{②} \end{array} \right\} \Sigma m = 1.906\pi RL$$

	$\bar{x}m$	$\bar{y}m$	$\bar{z}m$
①	$-\pi RL^2$	0	0
②	$+0.0293\pi RL^2$	$+0.0844\pi R^2L$	$-0.0350\pi R^2L$

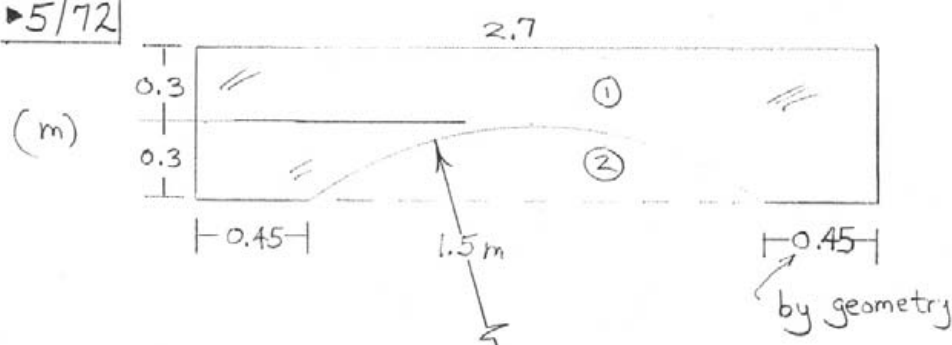
$$\Sigma \bar{x}m = -0.971\pi RL^2 \quad \Sigma \bar{y}m = 0.0844\pi R^2L \quad \Sigma \bar{z}m = -0.0350\pi R^2L$$

$$\bar{\bar{X}} = \frac{\Sigma \bar{x}m}{\Sigma m} = \frac{-0.971\pi RL^2}{1.906\pi RL} = -0.509L$$

$$\bar{\bar{Y}} = \frac{\Sigma \bar{y}m}{\Sigma m} = \frac{0.0844\pi R^2L}{1.906\pi RL} = 0.0443R$$

$$\bar{\bar{Z}} = \frac{\Sigma \bar{z}m}{\Sigma m} = \frac{-0.0350\pi R^2L}{1.906\pi RL} = -0.01834R$$

► 5/72



From Prob. 5/41, relative to the base of the current area, we have for the circular portion with $a = 1.5$ m & $h = 1.2$ m:

$$\bar{y} = \frac{\frac{2}{3} (1.5^2 - 1.2^2)^{3/2}}{1.5^2 \left[\frac{\pi}{2} - \sin^{-1} \left(\frac{1.2}{1.5} \right) \right] - 1.2 \sqrt{1.5^2 - 1.2^2}} - 1.2$$

$$= 0.1211 \text{ m}$$

Comp.	A	\bar{y}	$\bar{y} A$
①	$(2.7)(0.6)$	0.3	0.486
②	$- 0.368^*$	0.1211	$- 0.0445$
	$\Sigma A = 1.252$		$\Sigma \bar{y} A = 0.441$

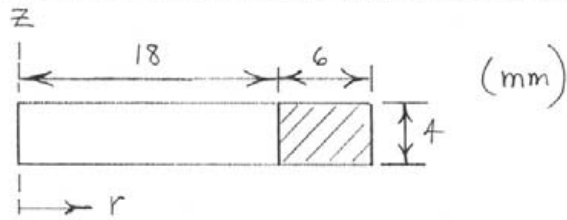
$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.441}{1.252} = 0.353 \text{ m}$$

* From Prob. 5/41:

$$A = 1.5^2 \left(\frac{\pi}{2} - \sin^{-1} \frac{1.2}{1.5} \right) - 1.2 \sqrt{1.5^2 - 1.2^2}$$

$$= 0.368 \text{ m}^2$$

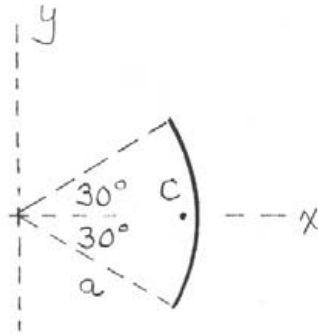
5/73



$$A = 2\pi \bar{r} L = 2\pi \left(18 + \frac{6}{2}\right) (6 + 6 + 4 + 4)$$
$$= \underline{2640 \text{ mm}^2}$$

$$V = 2\pi \bar{r} A = 2\pi \left(18 + \frac{6}{2}\right) (6 \cdot 4)$$
$$= \underline{3170 \text{ mm}^3}$$

5/74

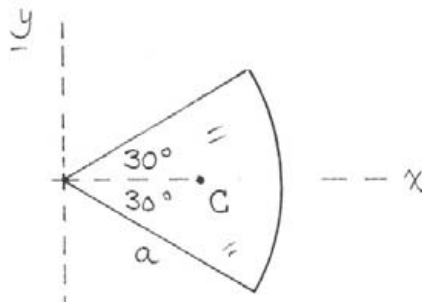


From Table D/3,

$$\bar{x} = \frac{a \sin \pi/6}{\pi/6} = \frac{3a}{\pi}$$

$$A = 2\pi \bar{x} L = 2\pi \left(\frac{3a}{\pi} \right) \left(\frac{\pi}{3} a \right) \\ = \underline{2\pi a^2}$$

5/75



From Table D/3:

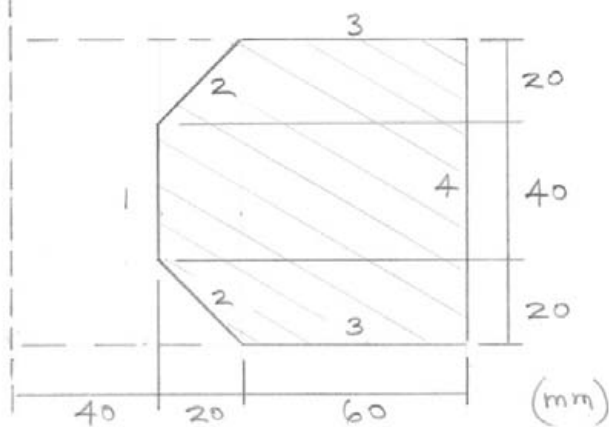
$$\bar{x} = \frac{2}{3} a \frac{\sin \pi/6}{\pi/6} = \frac{2a}{3} \frac{1}{2} \frac{6}{\pi} = \frac{2a}{\pi}$$

$$\begin{aligned} V &= \pi \bar{x} A = \pi \left(\frac{2a}{\pi} \right) \left(\frac{\pi a^2}{6} \right) \\ &= \frac{\pi a^3}{3} \end{aligned}$$

5/76	$V = \theta \bar{r} A = \pi \left(8 + \frac{2}{3} 12 \right) \frac{1}{2} (12)(12) = 3620 \text{ mm}^3$
------	---

5/77

z



$$A_1 = \pi \bar{r}_1 L_1 = \pi (40)(40) = 1600\pi \text{ mm}^2$$

$$A_2 = 2(\pi \bar{r}_2 L_2) = 2\pi (50)(20\sqrt{2}) = 2830\pi \text{ mm}^2$$

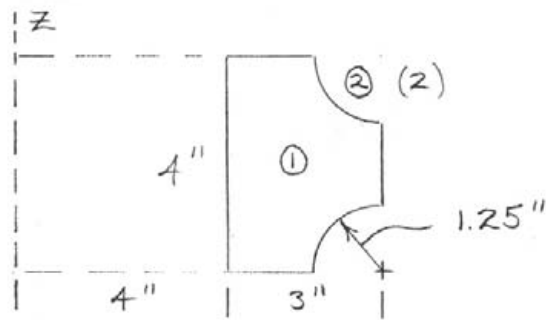
$$A_3 = 2(\pi \bar{r}_3 L_3) = 2\pi (90)(60) = 10800\pi \text{ mm}^2$$

$$A_4 = \pi \bar{r}_4 L_4 = \pi (120)(80) = 9600\pi \text{ mm}^2$$

$$\text{End faces: } 2\{80(60) + 40(20) + 20(20)\} = 12000 \text{ mm}^2$$

$$\text{Total: } \underline{90000 \text{ mm}^2}$$

5/78



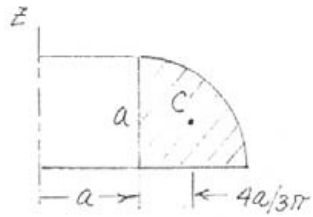
$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi \left(4 + \frac{3}{2}\right) (4 \cdot 3) = 415 \text{ in.}^3$$

$$V_2 = -2(2\pi \bar{r}_2 A_2) = -4\pi \left(7 - \frac{4(1.25)}{3\pi}\right) \left(\frac{\pi (1.25)^2}{4}\right)$$

$$= -99.8 \text{ in.}^3$$

$$V = V_1 + V_2 = \underline{315 \text{ in.}^3}$$

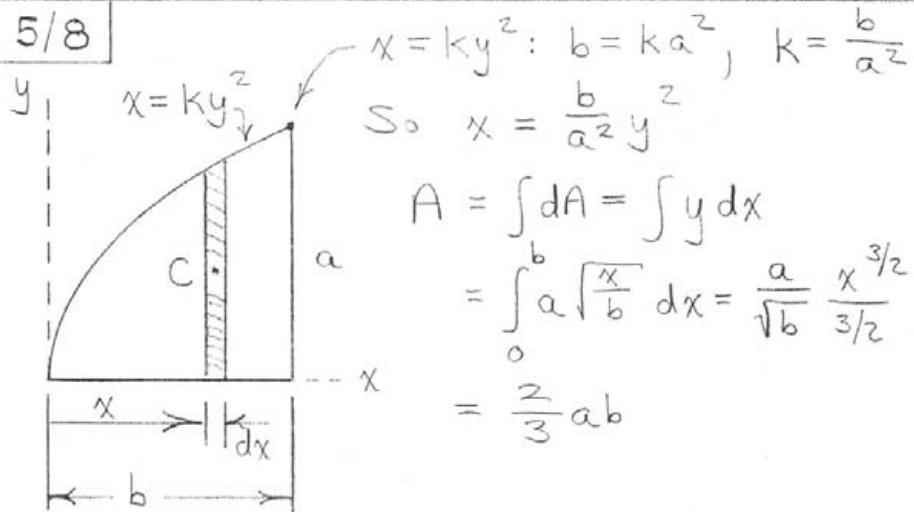
5/79



$$V = \bar{r} \theta A = \left(a + \frac{4a}{3\pi}\right) \frac{\pi}{2} \frac{\pi a^2}{4}$$

$$= \frac{\pi a^3}{24} (3\pi + 4)$$

5/8



$$x = ky^2: b = ka^2, k = \frac{b}{a^2}$$

$$\text{So } x = \frac{b}{a^2} y^2$$

$$A = \int dA = \int y dx$$

$$= \int_0^b a \sqrt{\frac{x}{b}} dx = \frac{a}{\sqrt{b}} \frac{x^{3/2}}{3/2} \Big|_0^b$$

$$= \frac{2}{3} ab$$

$$\int x_c dA = \int_0^b \frac{a}{\sqrt{b}} x^{3/2} dx = \frac{a}{\sqrt{b}} \frac{x^{5/2}}{5/2} \Big|_0^b = \frac{2}{5} ab^2$$

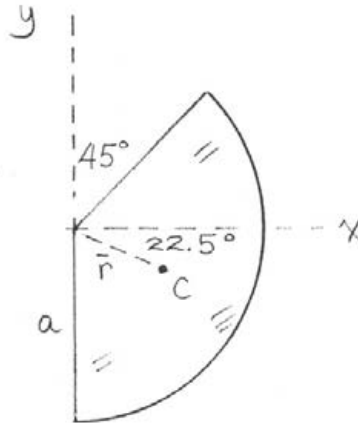
$$\int y_c dA = \int \frac{y}{2} y dx = \frac{1}{2} \int y^2 dx$$

$$= \frac{1}{2} \int_0^b \frac{a^2}{b} x dx = \frac{1}{2} \frac{a^2}{b} \frac{x^2}{2} \Big|_0^b = \frac{1}{4} a^2 b$$

$$\bar{x} = \frac{\int x_c dA}{A} = \frac{\frac{2}{5} ab^2}{\frac{2}{3} ab} = \frac{3}{5} b$$

$$\bar{y} = \frac{\int y_c dA}{A} = \frac{\frac{1}{4} a^2 b}{\frac{2}{3} ab} = \frac{3}{8} a$$

5/80



From Table D/3, with $\alpha = 67.5^\circ (0.375\pi)$

$$\bar{r} = \frac{2}{3} \frac{a \sin(0.375\pi)}{0.375\pi} = 0.523a$$

$$\begin{aligned} V &= 2\pi \bar{x} A = 2\pi (0.523a \cos 22.5^\circ) \left(\pi a^2 \frac{0.75\pi}{2\pi} \right) \\ &= \underline{3.58a^3} \end{aligned}$$

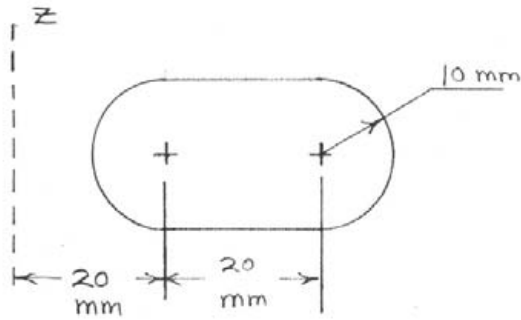
5/81

$$A = 2\pi rL + \pi dh$$

$$= 2\pi(8.2)(34) + \pi(8)(18) = 2204 \text{ ft}^2$$

$$\text{No. of gal. for 2 coats} = \frac{2204}{500} \times 2 = \underline{8.82 \text{ gal}}$$

5/82



$$A = 2\pi r L = 2\pi (30) [20 + 20 + \pi(10) + \pi(10)]$$

$$= \underline{19\,380 \text{ mm}^2}$$

$$V = 2\pi r A = 2\pi (30) [20(20) + \pi(10^2)]$$

$$= \underline{134\,600 \text{ mm}^3}$$

For steel,

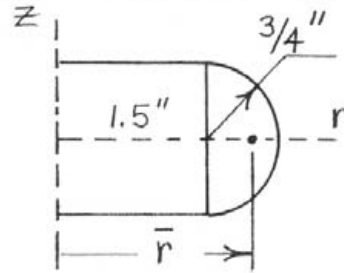
$$m = \rho V = 7830 \frac{\text{kg}}{\text{m}^3} \left(\frac{1 \text{ m}}{1000 \text{ mm}} \right)^3 (134\,600 \text{ mm}^3)$$

$$= \underline{1.054 \text{ kg}}$$

5/83

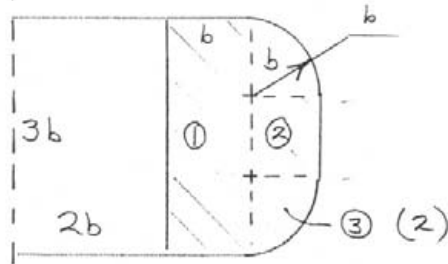
$$\begin{aligned}
 V &= 2\pi \bar{r}_{\text{area}} A \\
 &= 2\pi \left(1.5 + \frac{4(0.75)}{3\pi}\right) \frac{\pi(0.75)^2}{2} \\
 &= \underline{10.09 \text{ in.}^3}
 \end{aligned}$$

$$\begin{aligned}
 A &= 2\pi \bar{r}_{\text{arc}} L \\
 &= 2\pi \left(1.5 + \frac{2(0.75)}{\pi}\right) (\pi \times 0.75) \\
 &= \underline{29.3 \text{ in.}^2}
 \end{aligned}
 \quad \left\{ \begin{array}{l} \bar{r}_{\text{area}} = \frac{4r}{3\pi} + 1.5 \\ \bar{r}_{\text{arc}} = \frac{2r}{\pi} + 1.5 \end{array} \right.$$



5/84

Z:



$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi \left(2b + \frac{b}{2}\right) (3b^2) = 15\pi b^3$$

$$V_2 = 2\pi \bar{r}_2 A_2 = 2\pi \left(3b + \frac{b}{2}\right) (b^2) = 7\pi b^3$$

$$V_3 = 2(2\pi \bar{r}_3 A_3) = 4\pi \left(3b + \frac{4b}{3\pi}\right) \left(\frac{\pi b^2}{4}\right) = \pi b^3 \left(3 + \frac{4}{3\pi}\right)$$

$$V = V_1 + V_2 + V_3 = \pi b^3 \left(\frac{70}{3} + 3\pi\right) = 102.9 b^3$$

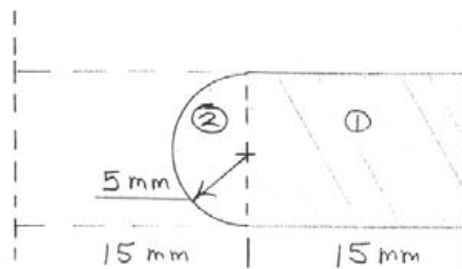
$$A_{1\text{inner}} = 2\pi \bar{r}_{1i} L_{1i} = 2\pi (2b) (3b) = 12\pi b^2$$

$$A_{1\text{top \& bott.}} = 2 \cdot 2\pi \bar{r}_{1t} L_{1t} = 4\pi \left(\frac{5b}{2}\right) (b) = 10\pi b^2$$

$$A_2 = 2\pi \bar{r}_2 L_2 = 2\pi (4b) (b) = 8\pi b^2$$

$$A_3 = 2 \cdot 2\pi \bar{r}_3 L_3 = 4\pi \left(3b + \frac{2b}{\pi}\right) \left(\frac{\pi b}{2}\right) = 2\pi^2 b^2 \left(3 + \frac{2}{\pi}\right)$$

$$A = A_1 + A_2 + A_3 = \pi b^2 (34 + 6\pi) = 166.0 b^2$$



$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi \left(15 + \frac{15}{2}\right) (15 \cdot 10) = 21\,200 \text{ mm}^3$$

$$V_2 = 2\pi \bar{r}_2 A_2 = 2\pi \left(15 - \frac{4.5}{3\pi}\right) \left(\frac{\pi 5^2}{2}\right) = 3\,180 \text{ mm}^3$$

$$V = V_1 + V_2 = \underline{24\,400 \text{ mm}^3}$$

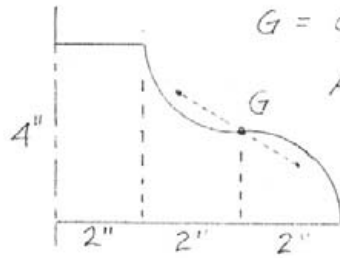
$$A_{1, \text{outer}} = 2\pi \bar{r}_{1o} L_{1o} = 2\pi (30) (10) = 1885 \text{ mm}^2$$

$$A_{1, \text{top \& bottom}} = 2 \cdot 2\pi \bar{r}_{1t} L_{1t} = 4\pi \left(15 + \frac{15}{2}\right) (15) = 4240 \text{ mm}^2$$

$$A_2 = 2\pi \bar{r}_2 L_2 = 2\pi \left(15 - \frac{2.5}{\pi}\right) (\pi 5) = 1166 \text{ mm}^2$$

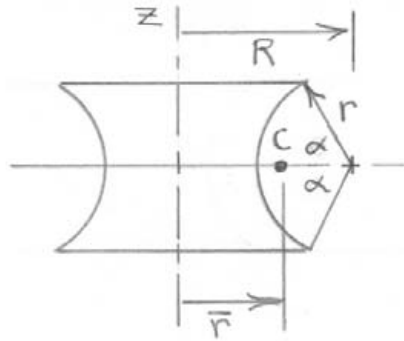
$$A = A_1 + A_2 = \underline{7290 \text{ mm}^2}$$

5/86



$G = \text{centroid by symmetry}$

$$\begin{aligned} A &= 2\pi \bar{r} L = 2\pi (4) \pi (2) \\ &= 16\pi^2 = \underline{157.9 \text{ in}^2} \end{aligned}$$



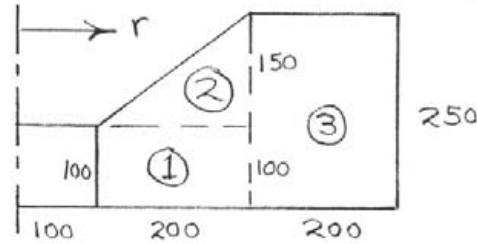
$$\bar{r} = R - r \frac{\sin \alpha}{\alpha} \quad \text{from Sample Problem 5/1}$$

$$L = 2r\alpha$$

$$A = 2\pi \bar{r} L = 2\pi \left(R - r \frac{\sin \alpha}{\alpha} \right) (2r\alpha)$$

$$= \underline{4\pi r (R\alpha - r \sin \alpha)}$$

5/88



$$V = 2\pi \bar{r} A = 2\pi \sum_{i=1}^3 \bar{r}_i A_i$$

$$A_1 = 200(100) = 20(10^3) \text{ mm}^2, \quad r_1 = 100 + \frac{200}{2} = 200 \text{ mm}$$

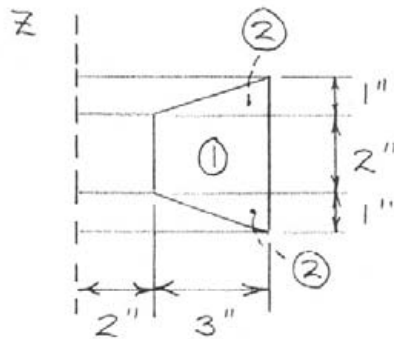
$$A_2 = \frac{200(150)}{2} = 15(10^3) \text{ mm}^2, \quad r_2 = 100 + \frac{2}{3}200 = 233 \text{ mm}$$

$$A_3 = 200(250) = 50(10^3) \text{ mm}^2, \quad r_3 = 300 + \frac{200}{2} = 400 \text{ mm}$$

$$V = 2\pi \left[200(20 \times 10^3) + 233(15 \times 10^3) + 400(50 \times 10^3) \right]$$

$$= 172.8(10^6) \text{ mm}^3 \quad \text{or} \quad \underline{V = 0.1728 \text{ m}^3}$$

5/8.9



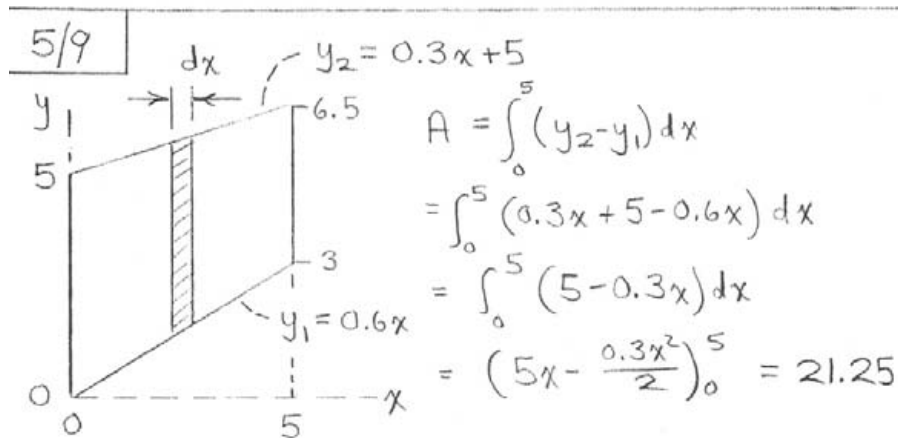
$$V_1 = \pi \bar{r}_1 A_1 = \pi \left(2 + \frac{3}{2} \right) (3 \cdot 2) = 66.0 \text{ in.}^3$$

$$V_2 = 2 \left(\pi \bar{r}_2 A_2 \right) = 2 \pi \left(5 - \frac{3}{3} \right) \left(\frac{1}{2} (3) (1) \right) = 37.7 \text{ in.}^3$$

$$V = V_1 + V_2 = 103.7 \text{ in.}^3$$

$$W = \mu V = \left(168 \frac{\text{lb}}{\text{ft}^3} \right) (103.7 \text{ in.}^3) \left(\frac{1 \text{ ft}}{12 \text{ in.}} \right)^3$$

$$= \underline{10.08 \text{ lb}}$$



(Note: Trapezoidal area formula could be used)

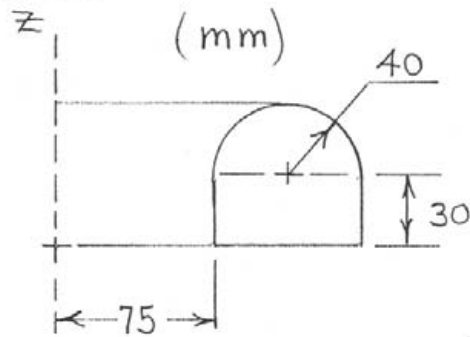
$$\int x_c dA = \int_0^5 x(5 - 0.3x) dx = \left(\frac{5}{2}x^2 - 0.1x^3 \right)_0^5 = 50$$

$$\begin{aligned} \int y_c dA &= \int_0^5 \left(\frac{y_1 + y_2}{2} \right) (y_2 - y_1) dx = \frac{1}{2} \int_0^5 (y_2^2 - y_1^2) dx \\ &= \frac{1}{2} \int_0^5 [(0.3x + 5)^2 - (0.6x)^2] dx = \frac{1}{2} \int_0^5 (25 + 3x - 0.27x^2) dx \\ &= \frac{1}{2} \left[25x + \frac{3x^2}{2} - \frac{0.27x^3}{3} \right]_0^5 = 75.6 \end{aligned}$$

$$\bar{x} = \int x_c dA / A = 50 / 21.25 = \underline{2.35}$$

$$\bar{y} = \int y_c dA / A = 75.6 / 21.25 = \underline{3.56}$$

5/90



$$A_1 = \pi \bar{r} L = \pi (75 + 40) (2 \cdot 30 + 80 + \pi 40)$$

$$= 96\,000 \text{ mm}^2$$

$$\text{End areas } A_2 = 2 \left(\frac{\pi}{2} \cdot 40^2 + 80(30) \right)$$

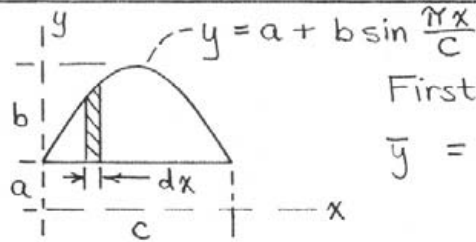
$$= 9\,830 \text{ mm}^2$$

$$\text{Total area } A = A_1 + A_2 = \underline{105\,800 \text{ mm}^2}$$

$$V = \pi \bar{r} A = \pi (75 + 40) (30 \cdot 80 + \pi 40^2 / 2)$$

$$= \underline{1.775 (10^6) \text{ mm}^3}$$

5/91

First, find \bar{y} by

$$\bar{y} = \int y_c dA / \int dA$$

$$A = \int dA = \int_0^c b \sin \frac{\pi x}{c} dx = -\frac{bc}{\pi} \cos \frac{\pi x}{c} \Big|_0^c = \frac{2bc}{\pi}$$

$$\int y_c dA = \int_0^c \left[a + \frac{b}{2} \sin \frac{\pi x}{c} \right] \left[b \sin \frac{\pi x}{c} dx \right]$$

$$= \int_0^c ab \sin \frac{\pi x}{c} dx + \int_0^c \frac{b^2}{2} \sin^2 \frac{\pi x}{c} dx$$

$$= -ab \frac{c}{\pi} \cos \frac{\pi x}{c} \Big|_0^c + \frac{b^2}{2} \left[\frac{x}{2} - \frac{1}{4} \sin \frac{2\pi x}{c} \right]_0^c$$

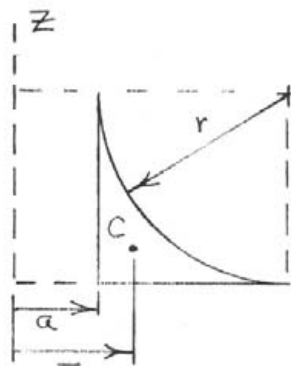
$$= \frac{2abc}{\pi} + \frac{b^2c}{4} = bc \left[\frac{2a}{\pi} + \frac{b}{4} \right]$$

$$\therefore \bar{y} = \frac{bc \left[\frac{2a}{\pi} + \frac{b}{4} \right]}{2bc/\pi} = a + \frac{b}{8} \pi$$

$$V = 2\pi \bar{y} A = 2\pi \left(a + \frac{b}{8} \pi \right) (2bc/\pi)$$

$$= \underline{4bc \left(a + \frac{b\pi}{8} \right)}$$

5/92

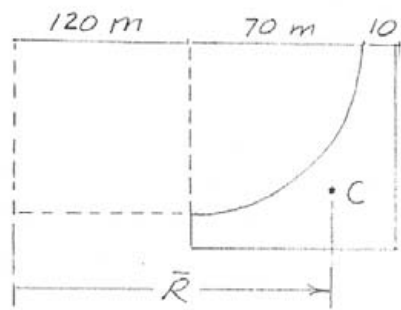


(Refer to Sample Problem 5/3)

$$\bar{r} = \frac{r^2(a + r/2) - \frac{\pi r^2}{4}(a + r - \frac{4r}{3\pi})}{r^2 - \pi r^2/4} = a + \frac{10-3\pi}{3(4-\pi)} r$$

$$\begin{aligned} V &= \theta \bar{r} A = \frac{\pi}{2} \left[a + \frac{10-3\pi}{3(4-\pi)} r \right] \left(1 - \frac{\pi}{4} \right) r^2 \\ &= \frac{\pi r^2}{8} \left[(4-\pi) a + \frac{10-3\pi}{3} r \right] \end{aligned}$$

5/93



$$\text{Square: } A = 80^2 = 6400 \text{ m}^2$$

$$\frac{1}{4} \text{ Circle: } A = \frac{1}{4} \pi (70^2) = 3848 \text{ m}^2$$

$$\text{Net area} = 2552 \text{ m}^2$$

$$\bar{r}_{\text{square}} = 120 + 40 = 160 \text{ m}$$

$$\bar{r}_{\frac{1}{4} \text{ circle}} = 120 + \frac{4(70)}{3\pi} = 149.7 \text{ m}$$

$$\bar{R} = \frac{\sum A \bar{r}}{\sum A} = \frac{6400(160) - 3848(149.7)}{2552}$$

$$= 175.5 \text{ m}$$

$$V = \theta \bar{R} A = \frac{\pi}{3} (175.5)(2552) = 469000 \text{ m}^3$$

$$m = \rho V = 2.40 (469000) = \underline{1.126 \times 10^6 \text{ Mg}}$$

5/94 From the solution to Prob. 5/11 ,

$$\bar{r} = 8 - \frac{2}{3} \frac{2(1.5) + 2}{1.5 + 2} = 7.05 \text{ m}$$

$$A = \frac{2 + 1.5}{2} (2) = 3.5 \text{ m}^2$$

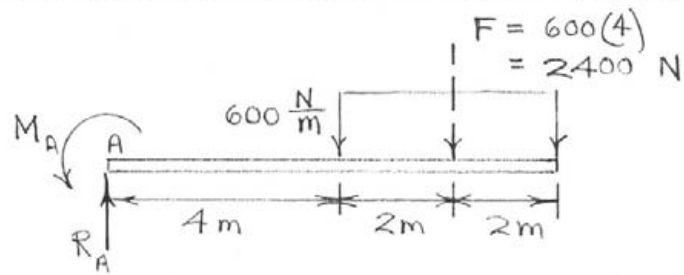
$$\theta = \pi/3$$

$$\text{So } V = \theta \bar{r} A = \frac{\pi}{3} (7.05) (3.5) = 25.8 \text{ m}^3$$

$$W = \rho_g V = 2400 (9.81) (25.8) = 608 (10^3) \text{ N}$$

$$\text{or } \underline{W = 608 \text{ kN}}$$

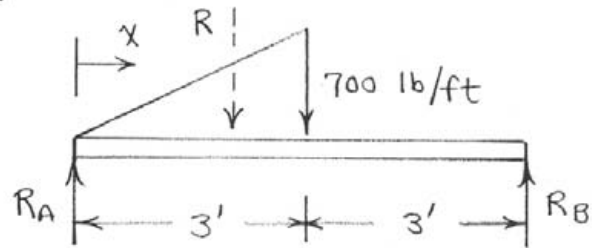
5/95



$$+\uparrow \sum F = 0 : R_A - 2400 = 0 , \quad \underline{R_A = 2.4 \text{ kN}}$$

$$+\circlearrowleft \sum M_A = 0 : M_A - 2400(4) = 0 , \quad \underline{M_A = 14.4 \text{ kN}\cdot\text{m}}$$

5/96

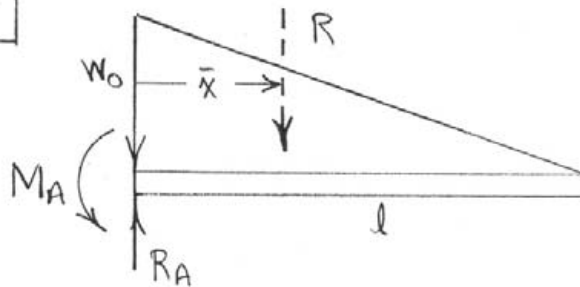


$$R = \frac{1}{2} (700)(6) = 1050 \text{ lb} \quad @ \quad \bar{x} = \frac{2}{3}(6) = 4'$$

$$\circlearrowleft \sum M_A = 0: R_B (6) - 1050 (4) = 0, \quad \underline{R_B = 700 \text{ lb}}$$

$$+\uparrow \sum F = 0: R_A + 700 - 1050 = 0, \quad \underline{R_A = 350 \text{ lb}}$$

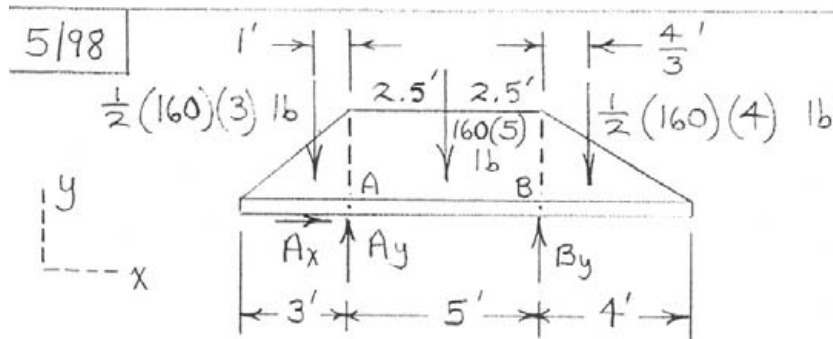
5/97



$$R = \frac{1}{2} w_0 l \quad @ \quad \bar{x} = \frac{1}{3} l$$

$$\curvearrowleft + \sum M_A = 0 : M_A - \frac{1}{2} w_0 l \left(\frac{l}{3} \right) = 0, \quad \underline{M_A = \frac{1}{6} w_0 l^2}$$

$$+\uparrow \sum F = 0 : R_A - \frac{1}{2} w_0 l = 0, \quad \underline{R_A = \frac{1}{2} w_0 l}$$



$$\sum M_A = 0 : 240(1) - 800(2.5) - 320(6.33) + B_y(5) = 0$$

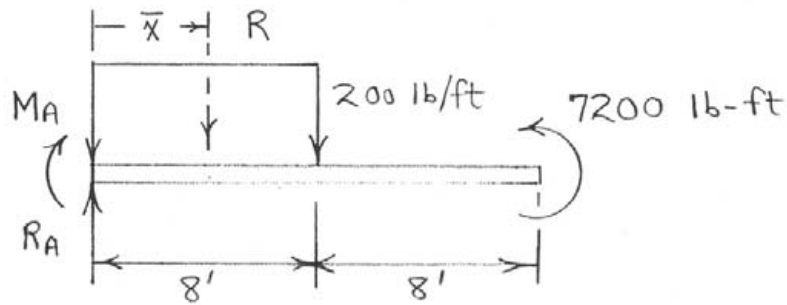
$$B_y = 757 \text{ lb}$$

$$\sum F_y = 0 : A_y + 757 - 240 - 800 - 320 = 0$$

$$A_y = 603 \text{ lb}$$

$$\sum F_x = 0 \Rightarrow A_x = 0$$

5/99



$$R = 200(8) = 1600 \text{ lb} \quad @ \quad \bar{x} = 4'$$

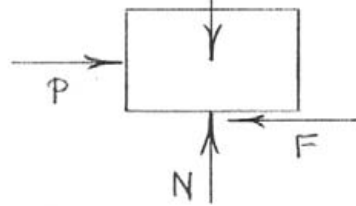
$$\downarrow + \sum M_A = 0 : -M_A - 1600(4) + 7200 = 0$$

$$\underline{M_A = 800 \text{ lb-ft}}$$

$$+\uparrow \sum F = 0 : R_A - 1600 = 0 , \quad \underline{R_A = 1600 \text{ lb}}$$

6/1

$$90(9.81) = 883 \text{ N}$$



$$\sum F_y = 0: N - 883 = 0, N = 883 \text{ N}$$

$$F_{\max} = \mu_s N = 0.5(883) = 441 \text{ N}$$

$$\sum F_x = 0 \text{ yields } F = P \text{ for equilibrium}$$

$$(a) P = 300 \text{ N}, \underline{F = 300 \text{ N}} < F_{\max}, \text{ OK}$$

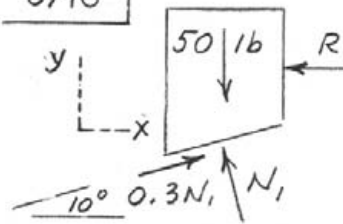
$$(b) P = 400 \text{ N}, \underline{F = 400 \text{ N}} < F_{\max}, \text{ OK}$$

$$(c) P = 500 \text{ N}, F = 500 \text{ N} > F_{\max}, \text{ motion}$$

$$\text{So } F = \mu_k N = 0.4(883) = \underline{353 \text{ N}}$$

(all to the left)

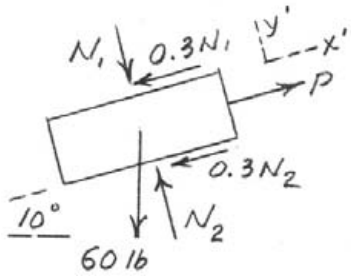
6/10

50-lb block: $\Sigma F_y = 0$;

$$N_1 \cos 10^\circ + 0.3 N_1 \sin 10^\circ - 50 = 0$$

$$N_1 (0.985 + 0.0521) = 50$$

$$N_1 = 50 / 1.0369 = 48.2 \text{ lb}$$



60-lb block;

$$\Sigma F_{y'} = 0; N_2 - 48.2 - 60 \cos 10^\circ = 0$$

$$N_2 = 107.3 \text{ lb}$$

$$\Sigma F_{x'} = 0; P - 0.3(48.2 + 107.3) - 60 \sin 10^\circ = 0$$

$$P = 46.7 + 10.4$$

$$\underline{P = 57.1 \text{ lb}}$$

$$\frac{6}{100} \quad T_2 = T_1 e^{\mu \beta}, \quad \beta = 2 \text{ turns} + 60^\circ$$

$$= 2(360^\circ) + 60^\circ = 780^\circ$$

$$\text{or } 13.61 \text{ rad}$$

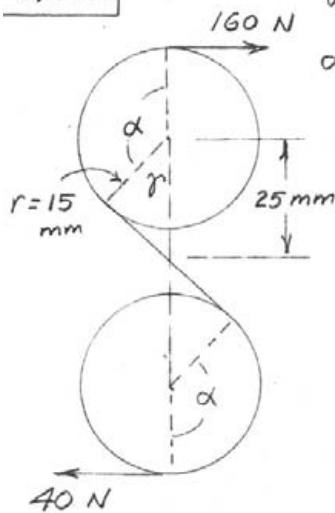
$$T = \left(\frac{2}{16}\right) e^{0.7(13.61)} = \underline{1720 \text{ lb}}$$

6/101 $\frac{4}{mg} = e^{\mu/\beta}$, $\frac{mg}{1.6} = e^{\mu/\beta}$

Thus $\frac{4}{mg} = \frac{mg}{1.6}$, $m^2 g^2 = 4(1.6)$

$m = \frac{\sqrt{4(1.6)}}{9.81} = 0.258 \text{ Mg}$ or $m = 258 \text{ kg}$

6/102



$$\gamma = \cos^{-1} \frac{15}{25} = 53.13^\circ$$

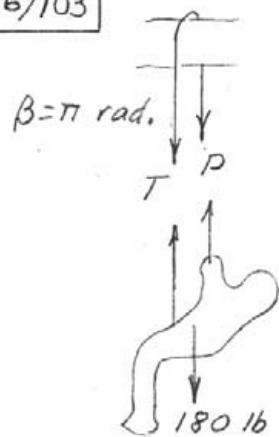
$$\alpha = 180 - 53.13 = 126.87^\circ$$

$$\beta = 2\alpha = \frac{2(126.87)}{180} \pi = 4.429 \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu\beta}, \quad \frac{160}{40} = e^{4.429\mu}$$

$$4.429\mu = 1.3863, \quad \mu = 0.313$$

6/103



$$T = P e^{\mu \beta}$$

$$= P e^{0.6\pi} = 6.59 P$$

$$\text{Man; } \sum F = 0; T + P = 180$$

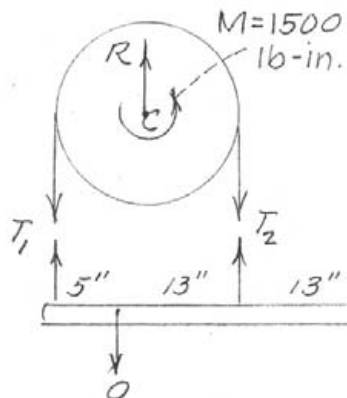
Combine & get

$$P(1 + 6.59) = 180 \text{ lb}$$

$$P = \frac{180}{7.59} = \underline{23.7 \text{ lb}}$$

6/104

$$T_2/T_1 = e^{\mu\beta} = e^{0.20\pi} = 1.874 \quad \text{--- (a)}$$



$$\sum M_C = 0; (T_2 - T_1) R = 1500$$

$$T_2 - T_1 = 166.7 \quad \text{--- (b)}$$

$$\text{Lever: } \sum M_O = 0; 26P + 5T_1 - 13T_2 = 0 \quad \text{--- (c)}$$

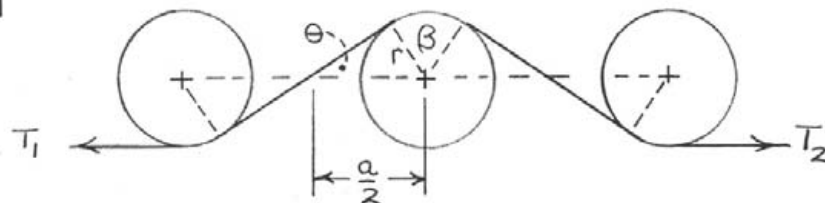
Combine (a) & (b) & get

$$T_2 = 1.874(T_2 - 166.7), T_2 = 357 \text{ lb.}$$

$$T_1 = 357 - 166.7 = 190.7 \text{ lb}$$

Substitute in (c) & get

$$26P + 5(190.7) - 13(357) = 0, \quad \underline{P = 142.0 \text{ lb}}$$

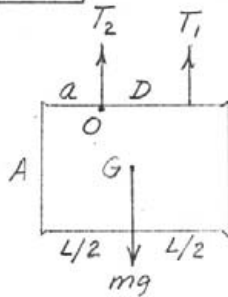


$$\theta = \sin^{-1} \frac{r}{a/2}, \quad \beta = 2\theta = 2 \sin^{-1} \frac{2r}{a}$$

$$\frac{T_2}{T_1} = e^{\mu\beta} : 1.15 = e^{0.1\beta}, \quad \beta = 1.398 \text{ rad or } 80.1^\circ$$

$$\text{Thus } 80.1^\circ = 2 \sin^{-1} \frac{2(20)}{a}, \quad \underline{a = 62.2 \text{ mm}}$$

6/106

Slipping impends for rope when $T_2 = T_1 e^{\mu\beta}$ (1)

Equil. of drum

$$\sum F = 0; T_1 + T_2 = mg \quad \text{--- (2)}$$


$$\sum M_O = 0; mg\left(\frac{L}{2} - a\right) = T_1 D \quad \text{--- (3)}$$

$$(2) \& (3) \quad T_1\left(\frac{L}{2} - a - D\right) = T_2\left(a - \frac{L}{2}\right)$$

Combine with (1) & get

$$\frac{\frac{L}{2} - a - D}{a - \frac{L}{2}} = e^{\mu\beta}, \quad a = \frac{L}{2} - \frac{D}{1 + e^{\mu\pi}} \quad \text{where } \beta = \pi \text{ rad.}$$

6/107



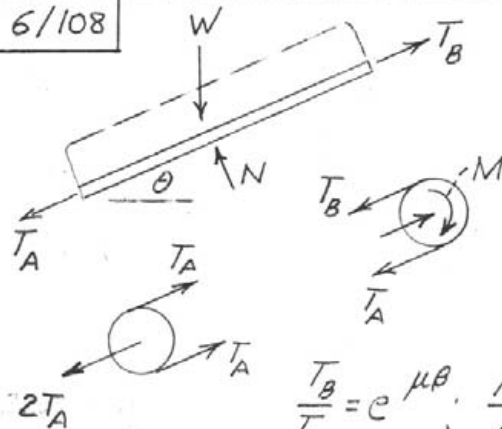
$$L = 75(9.81) \text{ N}, \quad T_2 = T_1 e^{\mu \beta}$$

$$75(9.81) \text{ N} \quad 75(9.81) - 10 = 10 e^{\mu(3 + \frac{1}{2})2\pi}$$

$$72.6 = e^{21.99\mu}$$

$$21.99\mu = 4.285, \quad \underline{\mu = 0.1948}$$

6/108



$$\theta = \tan^{-1} \frac{20}{50} = 21.8^\circ$$

$$\sin \theta = 0.3714$$

$$W \sin \theta = 30(150)(0.3714) = 1671 \text{ lb}$$

$$T_B = 1000 + \frac{1671}{2} = 1836 \text{ lb}$$

$$T_A = 1000 - \frac{1671}{2} = 164 \text{ lb}$$

$$\frac{T_B}{T_A} = e^{\mu \theta}, \quad \frac{1836}{164} = e^{\pi \mu} = 11.17$$

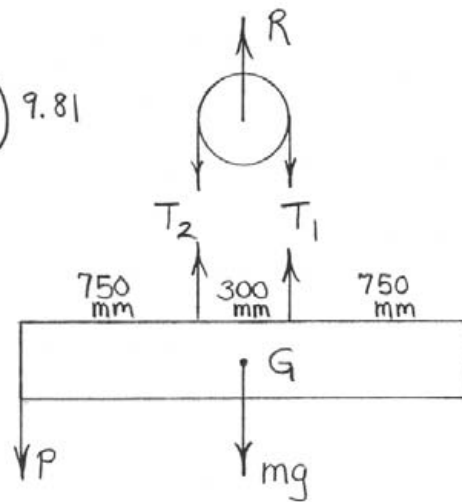
$$\pi \mu = \ln 11.17 = 2.413$$

$$\mu = \frac{2.413}{\pi} = 0.768$$

6/109

$$mg = 74(0.75 + 0.30 + 0.75) 9.81$$

$$= 1307 \text{ N}$$



$$\text{Beam } +\uparrow \Sigma F = 0 : T_1 + T_2 - P - 1307 = 0 \quad (1)$$

$$G + \Sigma M_G = 0 : P(0.75 + \frac{0.30}{2}) + \frac{0.30}{2} T_1 - \frac{0.30}{2} T_2 = 0$$

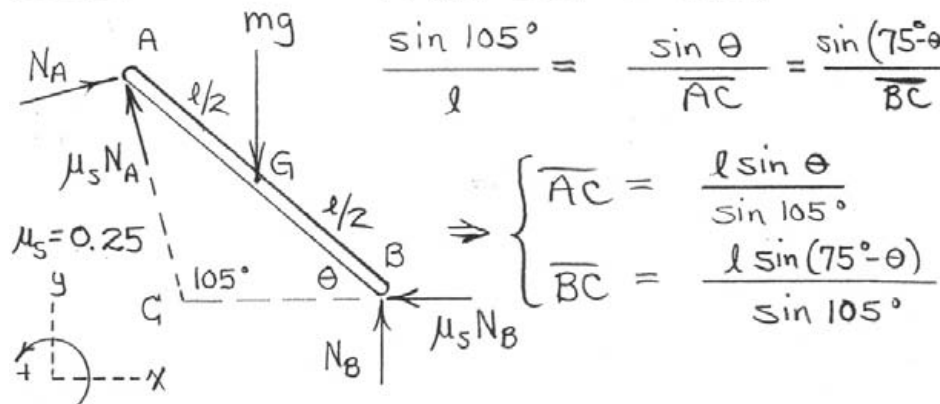
$$\text{Drum : } T_2 = T_1 e^{\mu \beta} = T_1 e^{0.5\pi} \quad (2)$$

$$\text{Solve Eqs. (1), (2), and (3) : } \begin{cases} T_1 = 252 \text{ N} \\ T_2 = 1215 \text{ N} \\ P = 160.3 \text{ N} \end{cases}$$

6/11

From law of sines:

$$\frac{\sin 105^\circ}{l} = \frac{\sin \theta}{AC} = \frac{\sin(75^\circ - \theta)}{BC}$$



$$\Rightarrow \begin{cases} AC = \frac{l \sin \theta}{\sin 105^\circ} \\ BC = \frac{l \sin(75^\circ - \theta)}{\sin 105^\circ} \end{cases}$$

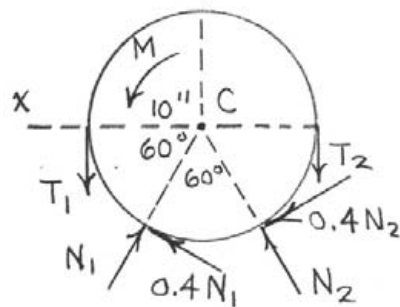
$$\sum F_x = 0: N_A \cos 15^\circ - 0.25 N_A \sin 15^\circ - 0.25 N_B = 0$$

$$\sum F_y = 0: N_A \sin 15^\circ + 0.25 N_A \cos 15^\circ + N_B - mg = 0$$

$$\sum M_C = 0: -N_A \left(\frac{l \sin \theta}{\sin 105^\circ} \right) + N_B \left(\frac{l \sin(75^\circ - \theta)}{\sin 105^\circ} \right) - mg \left[\frac{l \sin(75^\circ - \theta)}{\sin 105^\circ} - \frac{l}{2} \cos \theta \right] = 0$$

$$\text{Solution: } \begin{cases} N_A = 0.244 mg \\ N_B = 0.878 mg \\ \theta = 59.9^\circ \end{cases}$$

6/110



Lever: $\sum M_O = 0: \frac{4T_2}{\sqrt{2}} = 25(15 + \frac{4}{\sqrt{2}})$

$$T_2 = 157.6 \text{ lb}$$

Band: $\frac{T_2}{T_1} = e^{\mu\beta}$

$$\frac{157.6}{T_1} = e^{0.3\pi}, \quad T_1 = 61.4 \text{ lb}$$

Pipe: $\sum F_y = 0:$

$$N_1 \left(\frac{\sqrt{3}}{2} + 0.4 \left(\frac{1}{2} \right) \right) + N_2 \left(\frac{\sqrt{3}}{2} - 0.4 \left(\frac{1}{2} \right) \right) - 157.6 - 61.4 = 0 \quad (a)$$

$$\sum F_x = 0: N_1 \left(\frac{1}{2} - 0.4 \frac{\sqrt{3}}{2} \right) - N_2 \left(\frac{1}{2} + 0.4 \frac{\sqrt{3}}{2} \right) = 0 \quad (b)$$

Solve (a) & (b): $N_1 = 184.5 \text{ lb}, \quad N_2 = 33.5 \text{ lb}$

$$\sum M_C = 0: M - 157.6(10) + 61.4(10)$$

$$- 0.4(184.5 + 33.5)(10) = 0, \quad \underline{M = 1834 \text{ lb-in.}}$$

6/111

 T_2

Equilibrium of A gives



$$T_1 = mg \sin \theta$$

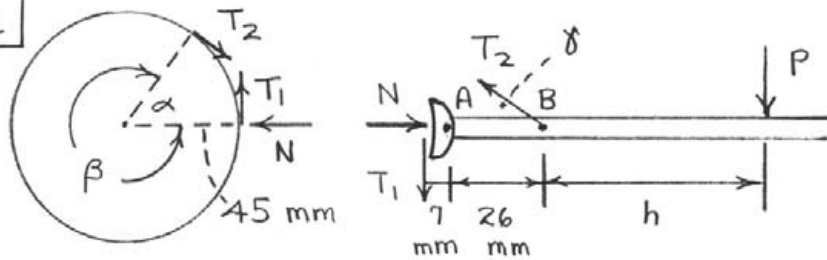
$$T_2 = mg \cos \theta$$

$$\frac{T_2}{T_1} = e^{\mu\beta} \cdot \frac{mg \cos \theta}{mg \sin \theta} = e^{3\pi\mu/2}$$

$$\text{For } \theta = 20^\circ, \cot 20^\circ = e^{3\pi\mu/2} = 2.747$$

$$\text{or } \mu = \frac{2}{3\pi} \ln 2.747 = \underline{0.214}$$

6/112



$$\cos \alpha = \frac{45}{45+7+26}, \quad \alpha = 54.8^\circ, \quad \gamma = 180 - 90 - \alpha = 35.2^\circ$$

$$\beta = 360 - \alpha = 305^\circ \text{ or } 5.33 \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu \beta} = e^{0.25(5.33)} = 3.79 \quad (a)$$

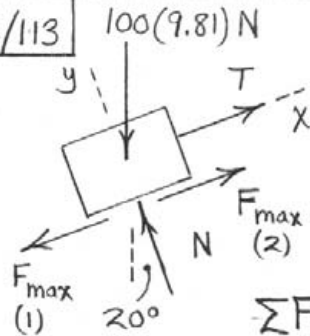
$$\text{Bar: } \sum M_P = 0: T_1(h+7+26) - T_2 \sin \gamma (h) = 0$$

$$\text{or } \frac{T_2}{T_1} = \frac{h+33}{0.577 h} \quad (b)$$

$$\text{From (a) \& (b), } h = 27.8 \text{ mm}$$

(For actual wrench, $h \approx 100 \text{ mm}$)

6/113



$$\Sigma F_y = 0: N - 100(9.81) \cos 20^\circ = 0$$

$$N = 922 \text{ N (throughout)}$$

(a) No cord friction: $T = mg$

(1) Motion impends up incline:

$$\Sigma F_x = 0: -0.3(922) - 981 \sin 20^\circ + T = 0$$

$$(F_{\max} = 0.3N)$$

$$T = 612 \text{ N}, \quad m = \frac{612}{9.81} = 62.4 \text{ kg}$$

(2) Motion impends down the incline:

$$\Sigma F_x = 0: 0.3(922) - 981 \sin 20^\circ + T = 0, \quad T = 59.0 \text{ N}$$

$$m = \frac{59.0}{9.81} = 6.01 \text{ kg}; \quad \text{so } 6.01 \text{ kg} \leq m \leq 62.4 \text{ kg}$$

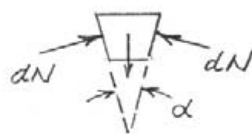
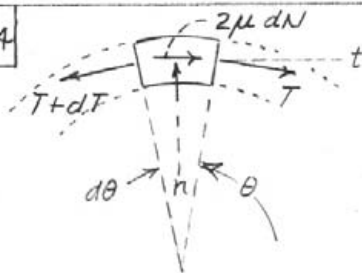
(b) Cord friction: $T \neq mg$, $\beta = 110^\circ \left(\frac{\pi}{180^\circ} \right) = 1.920 \text{ rad}$

$$(1) \text{ Motion impends up: } \frac{mg}{612} = e^{0.3(1.920)}, \quad m = 111.0 \text{ kg}$$

$$(2) \text{ Motion impends down: } \frac{59.0}{mg} = e^{0.3(1.920)}, \quad m = 3.38 \text{ kg}$$

$$\text{So } \underline{3.38 \text{ kg} \leq m \leq 111.0 \text{ kg}}$$

6/114



Cross section of belt

$$\sum F_n = 0; T \sin \frac{d\theta}{2} + (T + dT) \sin \frac{d\theta}{2} = 2 dN \sin \frac{\alpha}{2}$$

$$\text{or } T d\theta = 2 dN \sin \frac{\alpha}{2}$$

$$\sum F_t = 0; T \cos \frac{d\theta}{2} + 2\mu dN = (T + dT) \cos \frac{d\theta}{2}$$

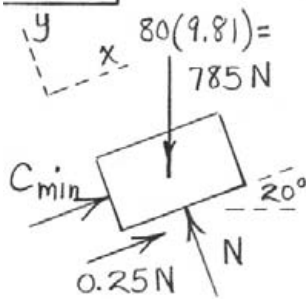
$$\text{or } 2\mu dN = dT$$

$$\text{combine \& get } \frac{dT}{T} = \frac{\mu}{\sin \frac{\alpha}{2}} d\theta$$

$$\int_{T_1}^{T_2} \frac{dT}{T} = \frac{\mu}{\sin \frac{\alpha}{2}} \int_0^{\beta} d\theta, \ln \frac{T_2}{T_1} = \frac{\mu\beta}{\sin \frac{\alpha}{2}}, \frac{T_2}{T_1} = e^{\mu\beta/\sin \frac{\alpha}{2}}$$

$$n = 1/\sin 17.5^\circ = \underline{3.33}$$

6/115

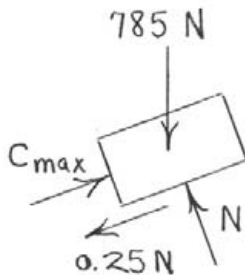


$$\Sigma F_y = 0 : N - 785 \cos 20^\circ = 0$$

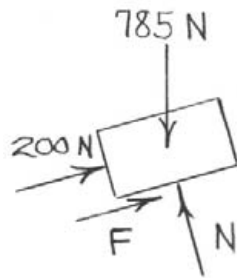
$$N = 737 \text{ N Throughout}$$

(a) C_{\min}

$$\Sigma F_x = 0 : C_{\min} + 0.25 (737) - 785 \sin 20^\circ = 0, \quad \underline{C_{\min} = 84.0 \text{ N}}$$

(b) C_{\max}

$$\Sigma F_x = 0 : C_{\max} - 0.25 (737) - 785 \sin 20^\circ = 0, \quad \underline{C_{\max} = 453 \text{ N}}$$

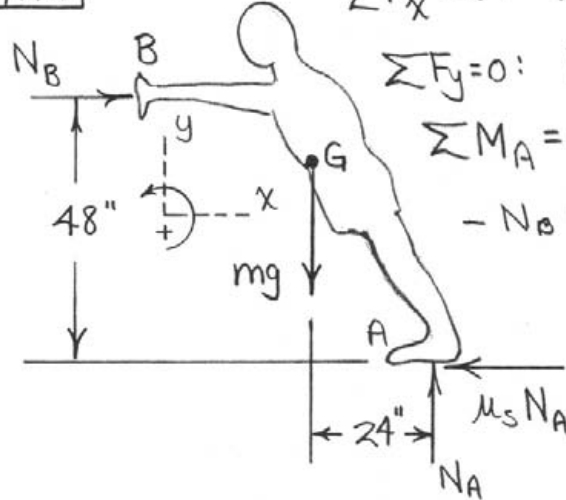
(c) $C = 200 \text{ N}$

$$\Sigma F_x = 0 : 200 + F - 785 \sin 20^\circ = 0$$

$$\underline{F = 68.4 \text{ N up the incline}}$$

$$(F < \mu_s N = 0.25 (737) = 184.4 \text{ N, so no motion occurs})$$

6/116



$$\sum F_x = 0: N_B - \mu_s N_A = 0 \quad (1)$$

$$\sum F_y = 0: N_A - mg = 0 \quad (2)$$

$$\sum M_A = 0:$$

$$-N_B (48) + mg (24) = 0 \quad (3)$$

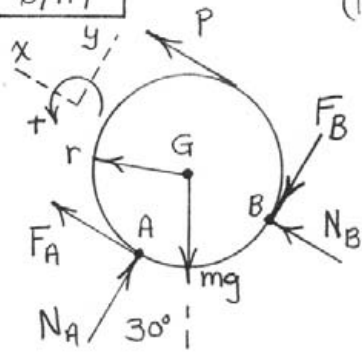
Solve Eqs. (1)-(3) to obtain

$$N_A = mg$$

$$N_B = \frac{1}{2} mg$$

$$\mu_s = \frac{1}{2}$$

6/117



(1) Assume no slippage until contact at B is lost:

$$F_B = N_B = 0$$

$$\begin{cases} \Sigma F_x = 0: P + F_A + mg \sin 30^\circ = 0 \\ \Sigma F_y = 0: N_A - mg \cos 30^\circ = 0 \\ \Sigma M_G = 0: Pr - F_A r = 0 \end{cases}$$

Solution: $P = F_A = \frac{mg}{4}$, $N_A = 0.866mg$

$$(F_A)_{\max} = \mu_s N_A = 0.25(0.866mg) = 0.217mg < F_A;$$

Assumption invalid

(2) Assume rotational slippage impends:

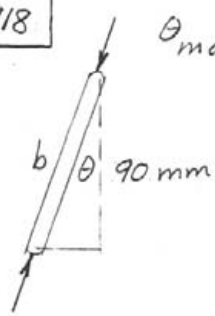
$$F_A = \mu_s N_A = 0.25 N_A, \quad F_B = \mu_s N_B = 0.25 N_B$$

$$\begin{cases} \Sigma F_x = 0: P + 0.25 N_A - mg \sin 30^\circ + N_B = 0 \\ \Sigma F_y = 0: N_A - mg \cos 30^\circ - 0.25 N_B = 0 \\ \Sigma M_G = 0: Pr - 0.25 N_A r - 0.25 N_B r = 0 \end{cases}$$

Solution: $P = 0.232mg$, $N_A = 0.878mg$, $N_B = 0.0487mg$

So rotational slippage occurs first, at $P = 0.232mg$

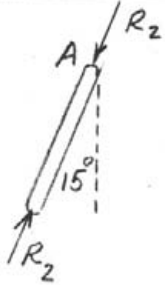
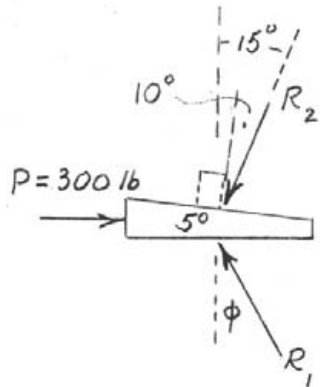
6/118



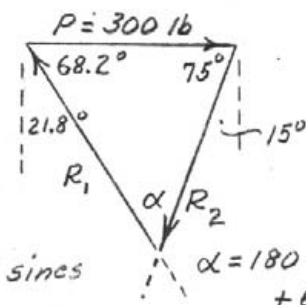
$$\theta_{\max} = \phi = \tan^{-1} \mu = \tan^{-1} 0.40 = 21.80^\circ$$

$$b = 90 / \cos 21.80^\circ = \underline{96.9 \text{ mm}}$$

6/119



Wedge $\phi = \tan^{-1} 0.40$
 $= 21.80^\circ$



Law of sines

$$\frac{R_2}{\sin 68.2} = \frac{300}{\sin 36.8^\circ}$$

$$R_2 = 300 \frac{0.9285}{0.5990} = 465 \text{ lb}$$

Friction force at A is $F = R_2 \sin 15^\circ = 465 (0.2588)$
 $= 120.3 \text{ lb}$

$10^\circ < (\tan^{-1} 0.4 = 21.8^\circ)$ so toggle
 does not slip & $F \neq \mu N$.

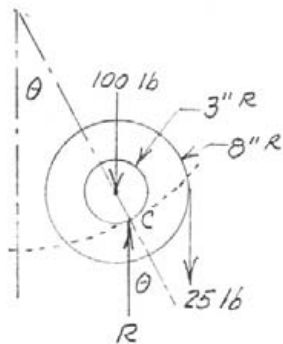
6/12

$$\sum M_C = 0; 100(3 \sin \theta) - 25(8 - 3 \sin \theta) = 0$$

$$\sin \theta = 8/15, \quad \theta = 32.2^\circ$$

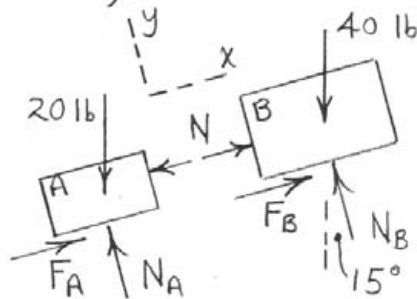
$$\mu_{\min} = \tan \theta = \tan 32.2^\circ$$

$$= 0.630$$



6/120 From $\theta_{\max} = \tan^{-1} \mu_s$, we have

$$\left. \begin{aligned} (\theta_{\max})_A &= \tan^{-1} 0.30 = 16.70^\circ \\ (\theta_{\max})_B &= \tan^{-1} 0.20 = 11.31^\circ \\ (\theta_{\max})_C &= \tan^{-1} 0.35 = 19.29^\circ \end{aligned} \right\} \begin{array}{l} \text{So C remains stationary} \\ \text{By themselves, B} \\ \text{would slide, A would} \\ \text{not.} \end{array}$$



From $\Sigma F_y = 0$:

$$N_A = 19.32 \text{ lb}$$

$$N_B = 38.6 \text{ lb}$$

Assume that slipping impends for B:

$$\Sigma F_x = 0: N + 0.2(38.6) - 40 \sin 15^\circ = 0, \quad N = 2.63 \text{ lb}$$

$$\Sigma F_x = 0 \text{ for A: } F_A - 20 \sin 15^\circ - 2.63 = 0$$

$$F_A = 7.80 \text{ lb}; \quad (F_A)_{\max} = 0.30(19.32) = 5.80 \text{ lb}$$

Because $(F_A)_{\max} < F$, both A and B slip.

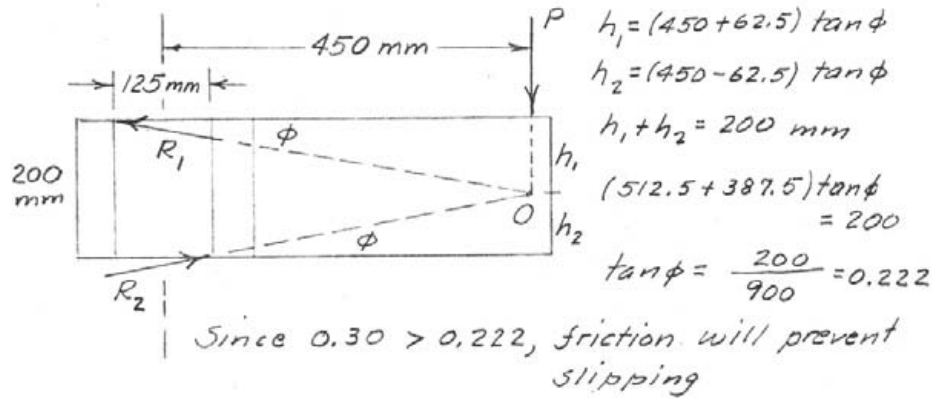
$$\boxed{6/121} \quad \text{Friction angle } \phi = \tan^{-1} \mu = \tan^{-1} 0.15 = 8.53^\circ$$

$$\tan \alpha = \frac{h}{2\pi r}; \quad \text{Critical when } \alpha = \phi$$

$$\therefore \text{Lead } h = 2\pi r \tan \phi = 2\pi \frac{3/8}{2} \tan 8.53^\circ$$
$$= 0.1767 \text{ in. per revolution}$$

$$N = 1/L = 1/0.1767 = \underline{5.66 \text{ Threads per inch}}$$

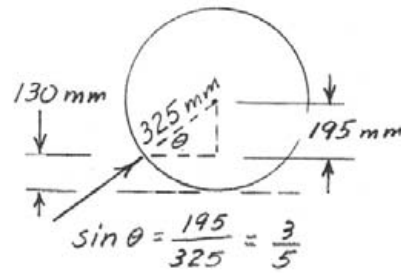
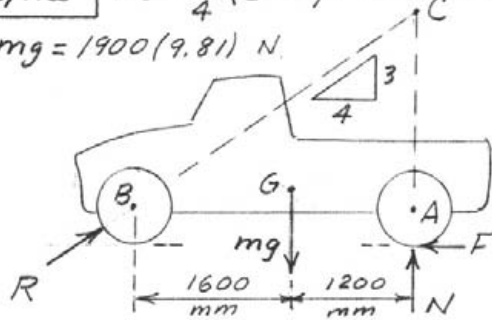
6/122

Forces concurrent at O gives minimum ϕ 

6/123

$$\bar{AC} = \frac{3}{4} (2800) = 2100 \text{ mm}$$

$$mg = 1900(9.81) \text{ N}$$



$$\sum M_C = 0; F(2100 + 325) - 1200 mg = 0, F = 0.495 mg$$

$$\sum M_B = 0; 1600 mg + 0.495 mg (325) - 2800 N = 0, N = 0.629 mg$$

$$\text{Thus } \mu_{\min} = F/N = 0.495/0.629 = \underline{0.787}$$

$$M = Fr = 0.495(1900)(9.81)(0.325) = 2998 \text{ N}\cdot\text{m}$$

$$\text{or } \underline{M = 3.00 \text{ kN}\cdot\text{m}}$$

$$\begin{aligned} \phi &= \tan^{-1} \mu_s = \tan^{-1}(0.2) = 11.31^\circ \\ \alpha &= \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{2.5}{2\pi(5)} = 4.55^\circ \end{aligned}$$

(a) Tighten: $M = Pr \tan(\phi + \alpha)$

$$F(100) = 600 \left(\frac{10}{2} \right) \tan(11.31^\circ + 4.55^\circ)$$

$$\underline{F = 8.52 \text{ N}}$$

(b) Loosen: $M = Pr \tan(\phi - \alpha)$

$$F(100) = 600 \left(\frac{10}{2} \right) \tan(11.31^\circ - 4.55^\circ)$$

$$\underline{F = 3.56 \text{ N}}$$

$$6/125 \text{ Helix angle } \alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{8}{2\pi \frac{25}{2}} = 5.82^\circ$$

$$\phi = \tan^{-1} 0.25 = 14.04^\circ$$

$$\text{Screw: (a) } M_s = Wr \tan(\alpha + \phi) = 4\left(\frac{25}{2}\right) \tan 19.86^\circ \\ = 18.05 \text{ N}\cdot\text{m}$$

$$(b) M_s = Wr \tan(\phi - \alpha) = 4\left(\frac{25}{2}\right) \tan 8.22^\circ \\ = 7.22 \text{ N}\cdot\text{m}$$

$$\text{Bearing } M_b = \frac{1}{2} \mu P(R_o + R_i) = \frac{1}{2} (0.25)(4) \frac{20+4}{2} \\ \text{(worn)} \\ = 6.00 \text{ N}\cdot\text{m}$$

$$\text{Total moment (a) } M = 18.05 + 6.00 = \underline{24.1 \text{ N}\cdot\text{m}}$$

$$(b) M = 7.22 + 6.00 = \underline{13.22 \text{ N}\cdot\text{m}}$$

6/126

$$\text{Helix angle } \alpha = \tan^{-1} \frac{24}{40\pi} = 10.81^\circ$$

$$\text{Friction angle } \phi = \tan^{-1} \mu = \tan^{-1} 0.15 = 8.53^\circ$$

$\alpha > \phi$ so screw is not self-locking.

$$\alpha + \phi = 19.34^\circ ; \quad \alpha - \phi = 2.28^\circ$$

$$(a) M = P r \tan(\alpha - \phi) : 60 = P (0.020) \tan 2.28^\circ$$

$$P = 75\,300 \text{ N or } \underline{75.3 \text{ kN}}$$

$$(b) M = P r \tan(\alpha + \phi) : 60 = P (0.020) \tan 19.34^\circ$$

$$P = 8550 \text{ N or } \underline{8.55 \text{ kN}}$$

6/127

$$\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{13}{2\pi(78)} = 3.04^\circ$$

$$\phi = \tan^{-1} \mu = \tan^{-1} 0.25 = 14.04^\circ$$

(a) To raise, $M = Wr \tan(\alpha + \phi)$

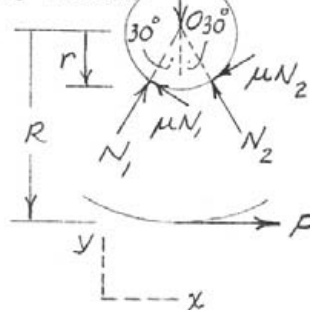
$$\begin{aligned} &= \frac{2.2(10^3) 9.81}{2} \frac{(0.078)}{2} \tan(3.04^\circ + 14.04^\circ) \\ &= \underline{129.3 \text{ N}\cdot\text{m}} \end{aligned}$$

(b) To lower, $M = Wr \tan(\phi - \alpha)$

$$\begin{aligned} &= \frac{2.2(10^3) 9.81}{2} \frac{(0.078)}{2} \tan(14.04^\circ - 3.04^\circ) \\ &= \underline{81.8 \text{ N}\cdot\text{m}} \end{aligned}$$

6/128

$mg = 3(9.81)$ kN
 $r = 25$ mm
 $R = 750$ mm



$$\sum M_O = 0; PR = \mu r(N_1 + N_2) \quad (a)$$

$$\sum F_x = 0; P + (N_1 - N_2)\frac{1}{2} = \mu(N_1 + N_2)\frac{\sqrt{3}}{2} \quad (b)$$

$$\sum F_y = 0; mg = (N_1 + N_2)\frac{\sqrt{3}}{2} + \mu(N_1 - N_2)\frac{1}{2} \quad (c)$$

Combine (a) & (c) to give

$$\frac{1}{2}(N_1 - N_2) = \frac{mg}{\mu} - \frac{\sqrt{3}}{2} \frac{PR}{\mu^2 r} \quad (d)$$

Combine (a) & (b) to give

$$\frac{1}{2}(N_1 - N_2) = -P + \frac{PR\sqrt{3}}{r} \quad (e)$$

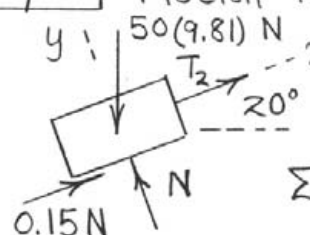
Combine (d) & (e) & get

$$P = \frac{mg/\mu}{\frac{R\sqrt{3}}{r} \left(1 + \frac{1}{\mu^2}\right) - 1} = \frac{3(9.81)/0.30}{\frac{750\sqrt{3}}{25} \left(1 + \frac{1}{0.3^2}\right) - 1} = 0.313 \text{ kN}$$

or 313 N

6/129

Motion impending down plane:



$$\sum F_y = 0: N = 50(9.81) \cos 20^\circ$$

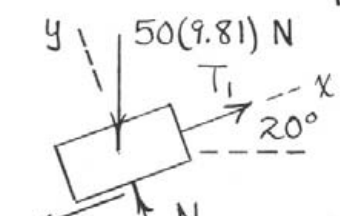
$$N = 461 \text{ N (throughout)}$$

$$\sum F_x = 0: T_2 - 50(9.81) \sin 20^\circ + 0.15(461) = 0$$

$$T_2 = T_1 e^{\mu\beta}: 98.6 = \frac{mg}{2} e^{0.25(110 \cdot \frac{\pi}{180})}$$

$$m = 12.44 \text{ kg}$$

Motion impending up plane:



$$\sum F_x = 0: T_1 - 50(9.81) \sin 20^\circ - 0.15(461) = 0$$

$$T_1 = 237 \text{ N}$$

$$T_2 = T_1 e^{\mu\beta}: \frac{mg}{2} = 237 e^{0.25(110 \cdot \frac{\pi}{180})}$$

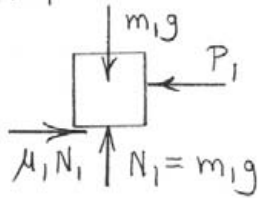
$$m = 78.0 \text{ kg}$$

So range is $12.44 \leq m \leq 78.0 \text{ kg}$

6/13

For impending slip between
person and board,

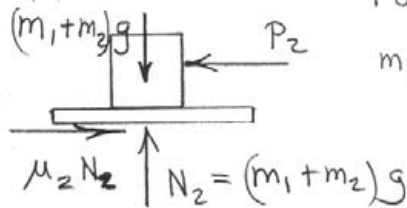
m_1 alone :



$$\rightarrow \sum F = 0: \mu_1 m_1 g - P_1 = 0$$

$$P_1 = \mu_1 m_1 g$$

m_1 plus m_2 :



For impending slip beneath
 m_2 , $\rightarrow \sum F = 0$:

$$-P_2 + \mu_2 (m_1 + m_2)g = 0$$

$$P_2 = \mu_2 (m_1 + m_2)g$$

For first slipping to occur beneath m_2 :

$$P_2 < P_1$$

$$\mu_2 (m_1 + m_2)g < \mu_1 m_1 g$$

$$\mu_2 < \frac{m_1}{m_1 + m_2} \mu_1$$

$$\text{Numbers: } \mu_2 < \frac{80}{80+10} (0.60) = 0.533$$

6/130

Assume equilibrium

$$\sum M_A = 0; 100(7) \frac{4}{5} - N(10) = 0, N = 56 \text{ lb}$$

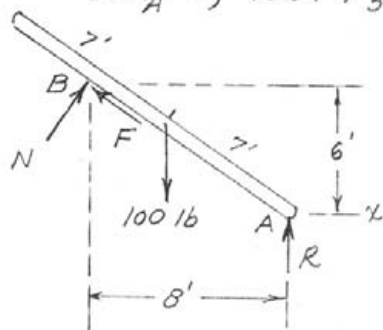
$$\sum F_x = 0; \frac{4}{5}F - \frac{3}{5}(56) = 0$$

$$F = 42 \text{ lb}$$

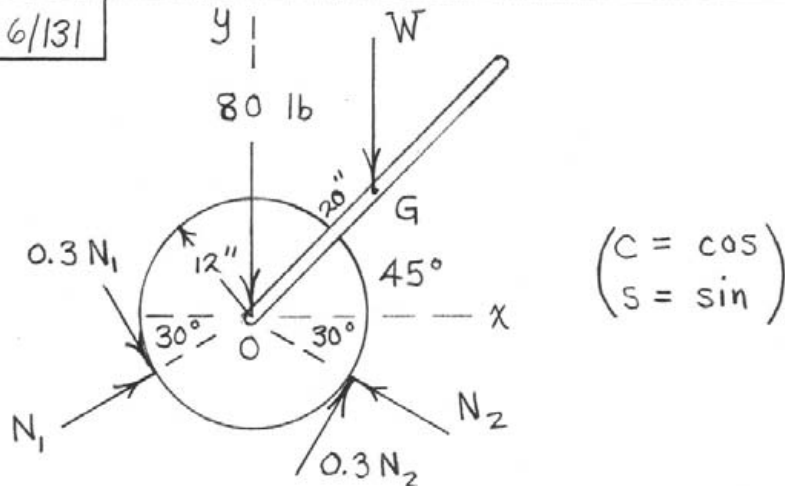
$$\mu_{s \min} = \frac{F}{N} = \frac{42}{56} = 0.75 < 0.80$$

so assumption is OK.

$$\boxed{F = 42 \text{ lb}}$$



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$$\Sigma F_x = 0: N_1 \cos 30^\circ + 0.3 N_1 \sin 30^\circ - N_2 \cos 30^\circ + 0.3 N_2 \sin 30^\circ = 0$$

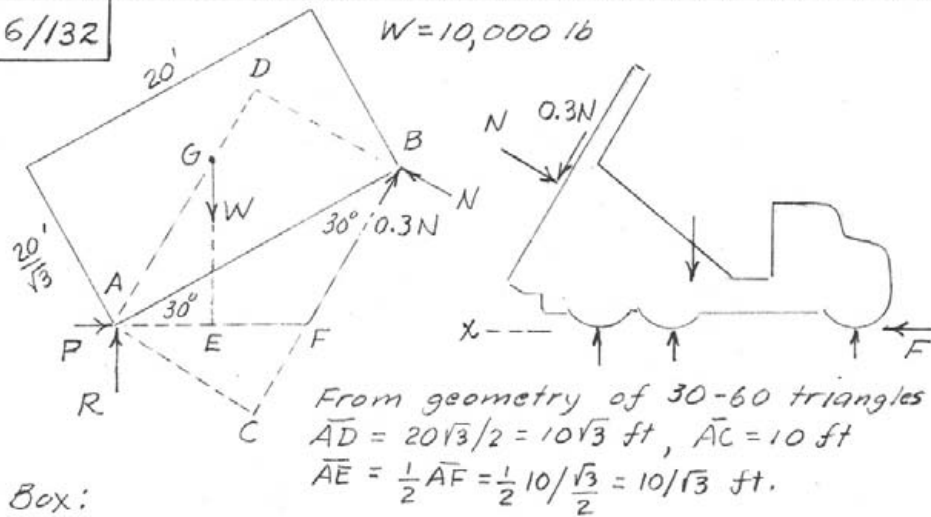
$$\Sigma F_y = 0: N_1 \sin 30^\circ - 0.3 N_1 \cos 30^\circ + N_2 \sin 30^\circ + 0.3 N_2 \cos 30^\circ - 80 - W = 0$$

$$\Sigma M_O = 0: (0.3 N_1 + 0.3 N_2)(12) - W(20 \cos 45^\circ) = 0$$

Solve to obtain

$$\begin{cases} N_1 = 113.9 \text{ lb} \\ N_2 = 161.6 \text{ lb} \\ \underline{W = 70.1 \text{ lb}} \end{cases}$$

6/132



Box:

$$\sum M_A = 0; 10,000(10/\sqrt{3}) - 10\sqrt{3}N - 10(0.3N) = 0$$

$$N = 2840 \text{ lb}$$

Truck:

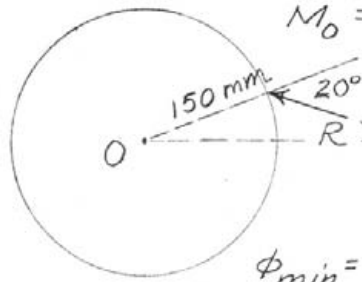
$$\sum F_x = 0; F = N\left(\frac{\sqrt{3}}{2}\right) - 0.3N\left(\frac{1}{2}\right) = 0.7160(2840) = \underline{2030 \text{ lb}}$$

6/133

$$M_0 = PL = 150(0.600) = 90 \text{ N}\cdot\text{m}$$

$$M_0 = (R \sin 20^\circ)(0.150) = 90$$

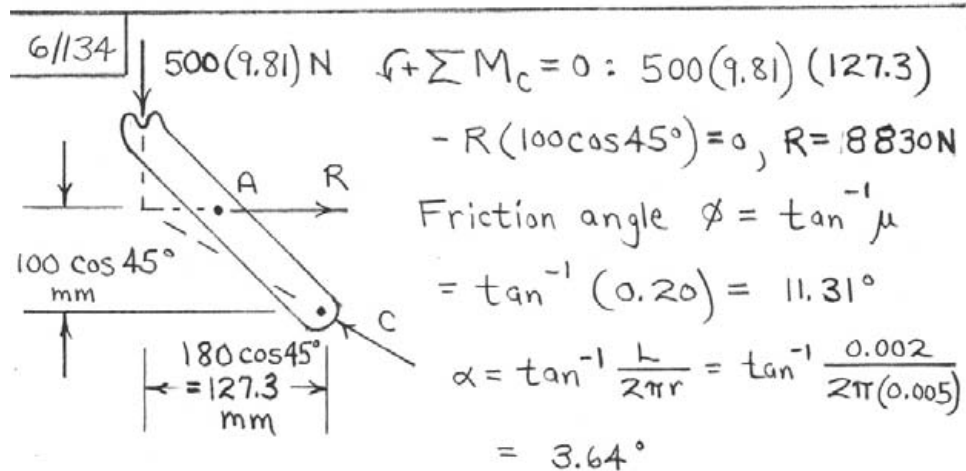
$$R = \frac{90}{0.3420(0.150)} = 1754 \text{ N}$$



Force on pin at A is

$$R = 1.754 \text{ kN}$$

$$\phi_{\min} = 20^\circ, \mu_s(\min) = \tan \phi_{\min} = \tan 20^\circ = 0.364$$



Raise load: $M = Rr \tan(\phi + \alpha)$

$$P(0.150) = 8830(0.005) \tan(11.31^\circ + 3.64^\circ)$$

$$P = 78.6 \text{ N}$$

Lower load: $M = Rr \tan(\phi - \alpha)$

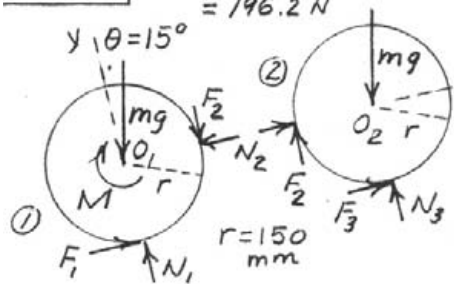
$$P(0.150) = 8830(0.005) \tan(11.31^\circ - 3.64^\circ)$$

$$P = 39.6 \text{ N}$$

6/135

$$mg = 20(9.81) = 196.2 \text{ N}$$

$$\mu_s = 0.60, \mu_k = 0.50$$



Assume slipping
occurs only at
contact between cylinders

$$\text{so } F_2 = \mu_k N_2 = \mu N_2$$

$$\left. \begin{aligned} \sum M_{O_1} &= 0; M = (F_1 - F_2)r \\ \sum M_{O_2} &= 0; F_2 = F_3 \end{aligned} \right\}$$

$$\left. \begin{aligned} \textcircled{1} \sum F_x &= 0; F_1 - N_2 - mg \sin \theta = 0 \\ \sum F_y &= 0; N_1 - F_2 - mg \cos \theta = 0 \\ \textcircled{2} \sum F_x &= 0; F_3 + N_2 - mg \sin \theta = 0 \\ \sum F_y &= 0; N_3 + F_2 - mg \cos \theta = 0 \end{aligned} \right\} \begin{array}{l} 7 \text{ eqs. in } 7 \text{ unknowns} \\ \text{Solve, substitute} \\ \text{values \& get} \end{array}$$

$$F_2 = \frac{\mu}{1+\mu} mg \sin \theta = F_3 = 16.93 \text{ N}$$

$$F_1 = \frac{2+\mu}{1+\mu} mg \sin \theta = 84.6 \text{ N}$$

$$N_1 = 206.4 \text{ N}$$

$$N_2 = 33.9 \text{ N}$$

$$N_3 = 172.6 \text{ N}$$

$$F_1 < \mu_s N_1 \quad \&$$

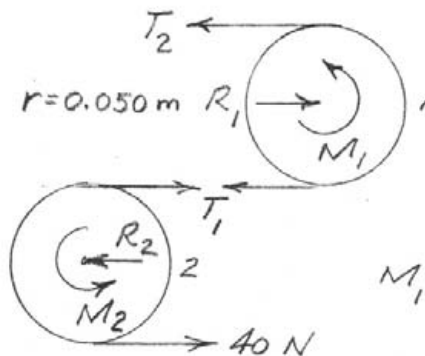
$$F_3 < \mu_s N_3 \quad \text{so}$$

assumption valid &

$$M = (84.6 - 16.93)(0.150)$$

$$= \underline{10.16 \text{ N}\cdot\text{m}}$$

6/136



$$\frac{T_1}{40} = e^{0.2(\pi)}, \quad \frac{T_1}{T_2} = e^{0.3(\pi)}$$

$$= 1.874 \quad = 2.566$$

$$T_1 = 40(1.874) = 74.98 \text{ N}$$

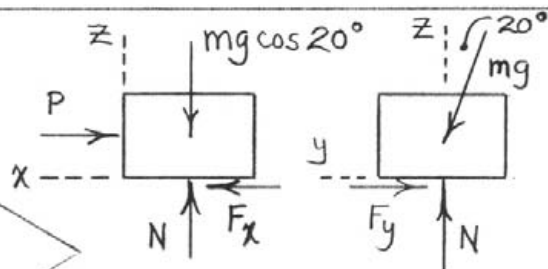
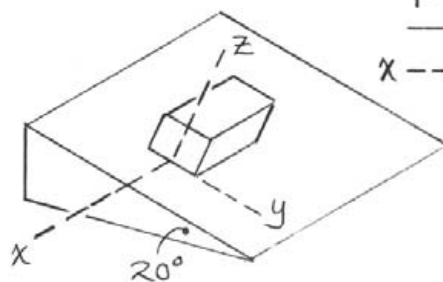
$$T_2 = 74.98 / 2.566 = 29.22 \text{ N}$$

$$M_1 = (T_1 - T_2)r = (74.98 - 29.22)(0.050)$$

$$= \underline{2.29 \text{ N}\cdot\text{m}}$$

$$M_2 = (T_1 - 40)r = (74.98 - 40)(0.050)$$

$$= \underline{1.75 \text{ N}\cdot\text{m}}$$



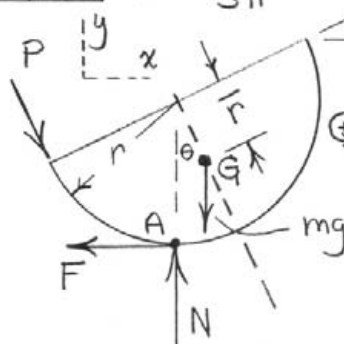
$$(x-z) \begin{cases} \sum F_z = 0 : N - 8(9.81) \cos 20^\circ = 0, & N = 73.7 \text{ N} \\ \sum F_x = 0 : F_x - P = 0, & F_x = P \end{cases}$$

$$(y-z) \begin{cases} \sum F_y = 0 : -F_y + 8(9.81) \sin 20^\circ = 0, & F_y = 26.8 \text{ N} \end{cases}$$

$$F = \sqrt{F_x^2 + F_y^2} = \mu_s N : \sqrt{P^2 + 26.8^2} = 0.5(73.7)$$

$$\underline{P = 25.3 \text{ N}}$$

*6/138 $\bar{r} = \frac{4r}{3\pi}$



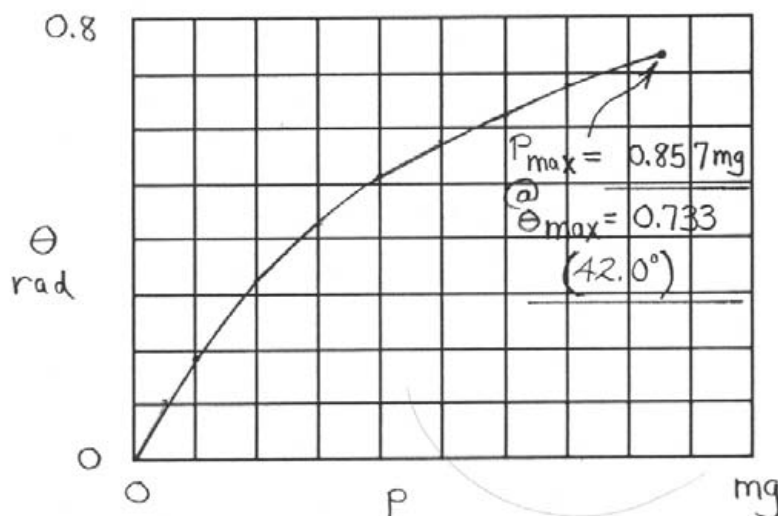
$$\sum F_x = 0: P \sin \theta - F = 0 \quad (1)$$

$$\sum F_y = 0: N - mg - P \cos \theta = 0 \quad (2)$$

$$\sum M_A = 0: Pr - N\bar{r} \sin \theta$$

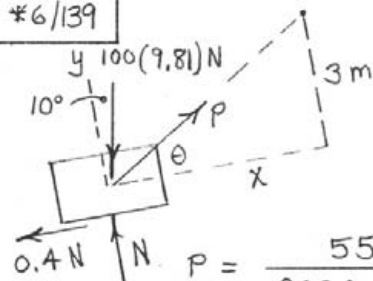
$$- F(r - \bar{r} \cos \theta) = 0 \quad (3)$$

Vary P and numerically solve Eqs. (1), (2), and (3) for θ , F , and N . Plot for θ :



(Note: Could vary θ & solve for P ; then plot!)

*6/139



$$\Sigma F_x = 0:$$

$$P \cos \theta - 0.4N - 981 \sin 10^\circ = 0$$

$$\Sigma F_y = 0:$$

$$N + P \sin \theta - 981 \cos 10^\circ = 0$$

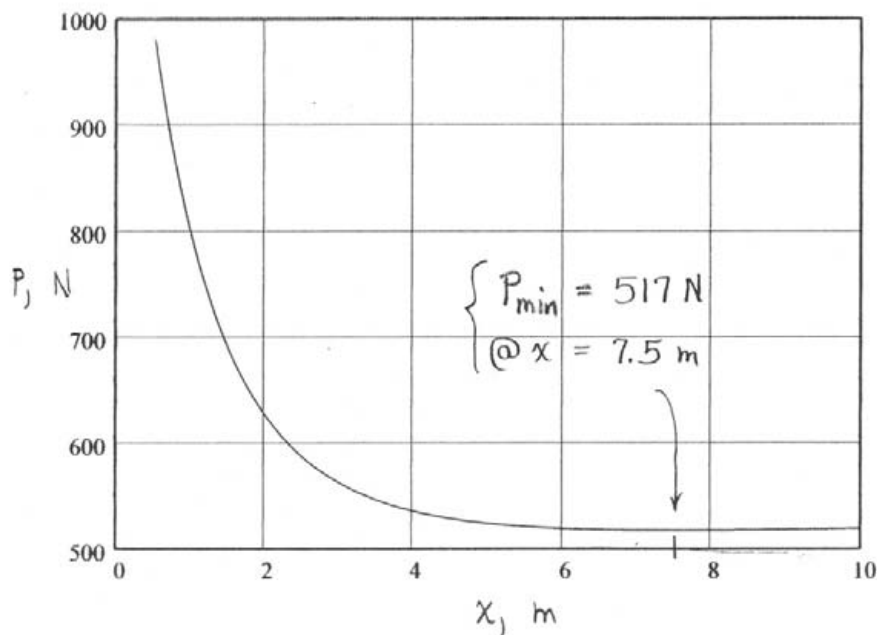
$$P = \frac{557}{\cos \theta + 0.4 \sin \theta}, \quad N = \frac{-170.3 \sin \theta + 966 \cos \theta}{\cos \theta + 0.4 \sin \theta}$$

$$\frac{dP}{d\theta} = \frac{-557(-\sin \theta + 0.4 \cos \theta)}{(\cos \theta + 0.4 \sin \theta)^2} = 0$$

$$\Rightarrow -\sin \theta + 0.4 \cos \theta = 0 \text{ or } \tan \theta = 0.4, \quad \theta = 21.8^\circ$$

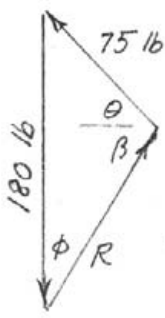
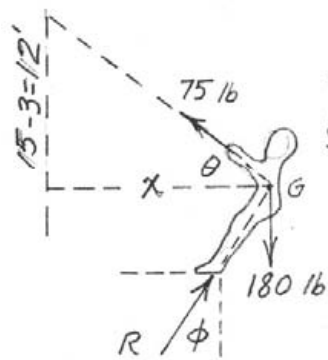
$$\text{From } \tan \theta = \frac{3}{x}, \quad x = \frac{3}{\tan \theta} = \frac{3}{0.4} = \underline{7.5 \text{ m}}$$

$$\text{At } \theta = 21.8^\circ, \quad \underline{P = 517 \text{ N}}$$



(Note: $N \rightarrow 0$ @ $x = 0.529 \text{ m}$)

6/14



$$\phi = \tan^{-1} 0.40 = 21.8^\circ$$

$$\beta = 90 - 21.8 = 68.2^\circ$$

Law of sines

$$\frac{180}{\sin(\theta + \beta)} = \frac{75}{\sin 21.8^\circ}$$

$$\theta + \beta = \sin^{-1} \frac{180 \sin 21.8}{75}$$

$$= \sin^{-1} 0.891 = 63.0^\circ$$

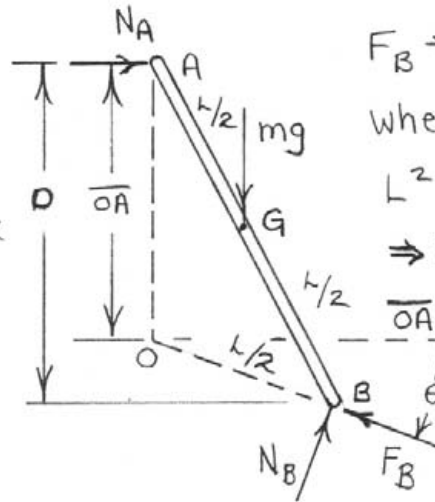
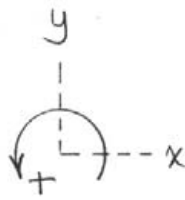
or 117.0°

63.0° sol. not possible,

$$\text{so } \theta = 117.0 - 68.2 = 48.8^\circ$$

$$\frac{12}{x} = \tan 48.8^\circ, \quad x = 12 / 1.14 = \underline{10.52 \text{ ft}}$$

*6/140



$$F_B \Rightarrow \mu_s N_B = 0.4 N_B$$

When slipping impends,

$$L^2 = D^2 + \left(\frac{L}{2} \cos \theta\right)^2$$

$$\Rightarrow D = L \sqrt{1 - \frac{\cos^2 \theta}{4}}$$

$$\overline{OA} = L \sqrt{1 - \frac{\cos^2 \theta}{4}} - \frac{L}{2} \sin \theta$$

$$= \frac{L}{2} \left[\sqrt{4 - \cos^2 \theta} - \sin \theta \right]$$

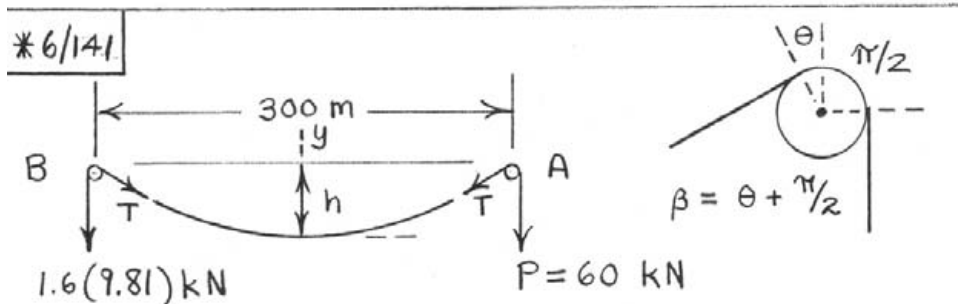
$$\Sigma F_x = 0 : N_A + N_B \sin \theta - 0.4 N_B \cos \theta = 0 \quad (1)$$

$$\Sigma F_y = 0 : N_B \cos \theta + 0.4 N_B \sin \theta - mg = 0 \quad (2)$$

$$\Sigma M_O = 0 : N_B \left(\frac{L}{2}\right) - N_A \frac{L}{2} \left[\sqrt{4 - \cos^2 \theta} - \sin \theta \right] - mg \frac{L}{4} \cos \theta = 0 \quad (3)$$

Numerical solution :

$$\begin{cases} N_A = 0.287 mg \\ N_B = 0.966 mg \\ \theta = 5.80^\circ \end{cases}$$



During slipping, Eq. 6/7 @ A: $60 = T e^{\mu_k \beta}$

$$\text{Eq. 6/7 @ B: } T = 1.6(9.81) e^{\mu_k \beta}$$

Eliminate $\mu_k \beta$ & get $T = 30.7 \text{ kN}$

$$\text{Then } e^{\mu_k \beta} = 60/30.7, \quad \mu_k \beta = 0.670$$

$$\text{Eq. 5/21: } T = T_0 \cosh \frac{\rho g x}{T_0} : 30.7 = T_0 \cosh \frac{12(9.81)10^{-3} \cdot 150}{T_0}$$

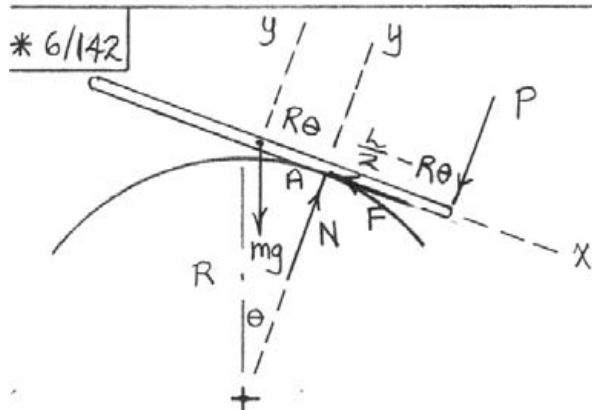
$$\text{or } 30.7 = T_0 \cosh \frac{17.66}{T_0}. \text{ Solve numerically: } T_0 = 23.8 \text{ kN}$$

$$\frac{dy}{dx} = \sinh \frac{\rho g x}{T_0} : \tan \theta = \sinh \frac{17.66}{23.8} = 0.810$$

$$\theta = 0.681 \text{ rad}, \quad \beta = \theta + \pi/2 = 2.25 \text{ rad}$$

$$\mu_k = \frac{0.670}{2.25} = \underline{0.298}$$

$$\begin{aligned} \text{Eq. 5/22: } T &= T_0 + \rho g h, \quad h = \frac{T - T_0}{\rho g} \\ &= \frac{30.7 - 23.8}{12(9.81)(10^{-3})} = \underline{58.1 \text{ m}} \end{aligned}$$



$$\Sigma F_x = 0 : mg \sin \theta - \mu_s N = 0 \quad (1)$$

$$\Sigma F_y = 0 : N - mg \cos \theta - P = 0 \quad (2)$$

$$\Sigma M_A = 0 : P \left(\frac{L}{2} - R\theta \right) - mg R\theta \cos \theta = 0 \quad (3)$$

$$(1) : N = \frac{mg \sin \theta}{\mu_s}$$

$$(2) : \frac{mg \sin \theta}{\mu_s} - mg \cos \theta = P$$

$$(3) : mg \left(\frac{\sin \theta}{\mu_s} - \cos \theta \right) \left(\frac{L}{2} - R\theta \right) - mg R\theta \cos \theta = 0$$

$$\text{Simplify : } \tan \theta = \mu_s \frac{1}{1 - \frac{2R\theta}{L}} = 0.15 \frac{1}{1 - 2(0.6)\theta}$$

$$\text{or } \tan \theta - \frac{0.15}{1 - 1.2\theta} = 0. \text{ Numerical solution: } \theta = 11.04^\circ$$

*6/143 From Prob. 6/142, the equilibrium

$$\text{equations are } mg \sin \theta - \mu_s N = 0 \quad (1)$$

$$N - mg \cos \theta - P = 0 \quad (2)$$

$$P \left(\frac{L}{2} - R\theta \right) - mg R\theta \cos \theta = 0 \quad (3)$$

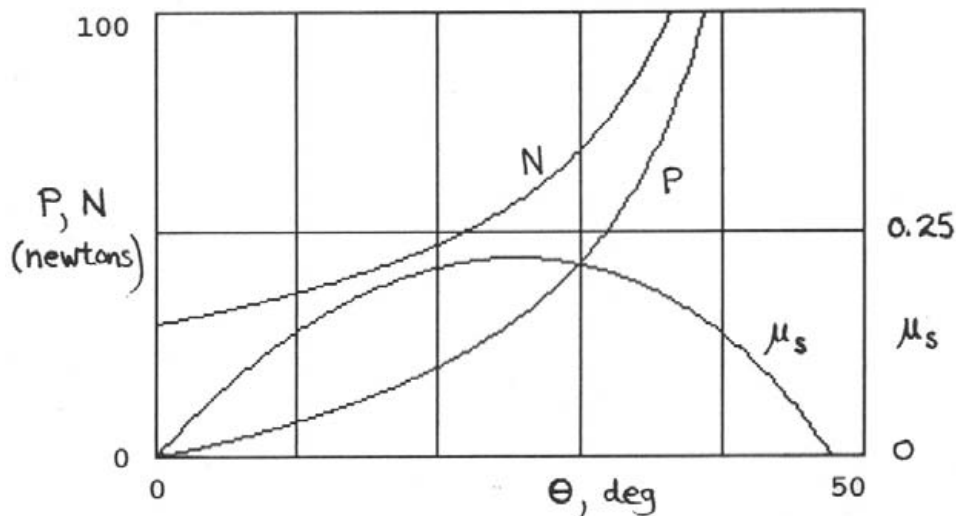
$$(2) \div (3): (N - mg \cos \theta) \left(\frac{L}{2} - R\theta \right) - mg R\theta \cos \theta = 0$$

$$N = mg \cos \theta \frac{1}{1 - \frac{2R\theta}{L}} = 3(9.81) \frac{\cos \theta}{1 - \frac{2(1.2)\theta}{2}} = \frac{29.4 \cos \theta}{1 - 1.2\theta}$$

$$(3): P = mg \frac{2R\theta}{L - 2R\theta} \cos \theta = \frac{35.3\theta \cos \theta}{1 - 1.2\theta}$$

$$(1) \div (2): \mu_s = \left(1 - \frac{2R\theta}{L} \right) \tan \theta = \frac{(1 - 1.2\theta) \tan \theta}{1}$$

$$\mu_{s \max} = 0.222 \text{ @ } \theta = 25.5^\circ$$



*6/144

$$\mu = 0.4$$

From geometry,

upper wheel :

$$\theta + \frac{\pi}{4} + \frac{\pi}{2} + \beta_2 = 2\pi$$

$$\beta_2 = \frac{5\pi}{4} - \theta$$

lower wheel :

$$\theta + \frac{\pi}{4} + \beta_1 + \pi = 2\pi$$

$$\beta_1 = \frac{3\pi}{4} - \theta$$

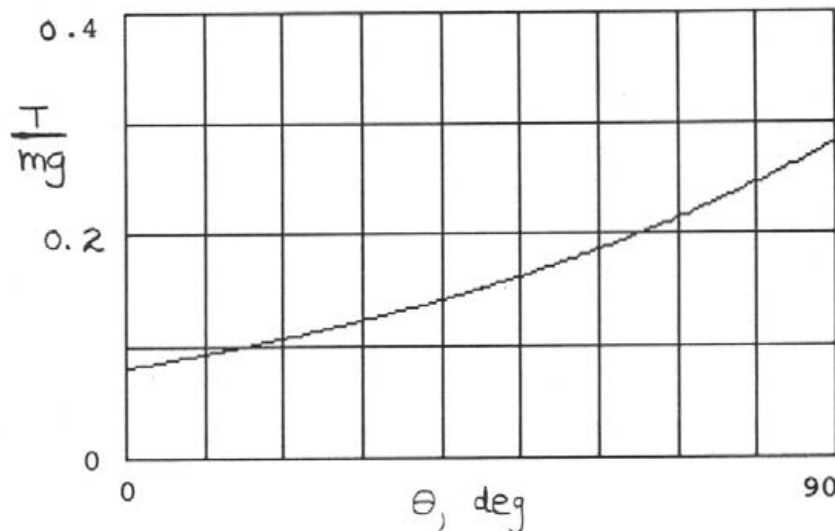
$$\text{Forces: } \begin{cases} mg = T_0 e^{\mu(\frac{3\pi}{4} - \theta)} \\ T_0 = T e^{\mu(\frac{5\pi}{4} - \theta)} \end{cases}$$

$$\text{So } mg = T e^{2\mu(\pi - \theta)} ; \quad \frac{T}{mg} = e^{-0.8(\pi - \theta)}$$

Device as a whole :

$$\begin{aligned} \sum M_O = 0 : 3rV - Tr - mg(r + 2r\sqrt{2}\cos\theta) &= 0 \\ V &= \frac{1}{3}[T + mg(1 + 2\sqrt{2}\cos\theta)] \end{aligned}$$

$$\text{For } \theta = 60^\circ, \quad \frac{T}{mg} = 0.1872, \quad V = 0.867mg$$



*6/145 Refer to Article 6/8. $dN = T d\theta$

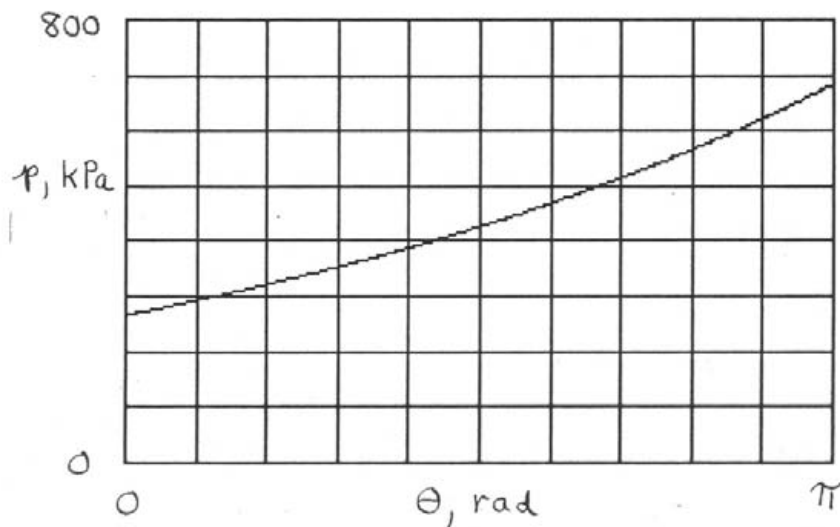
But $dN = p b r d\theta$ and $T = T_1 e^{\mu \theta}$ so

$$p b r d\theta = T_1 e^{\mu \theta} d\theta \text{ . Hence } p = \frac{T_1}{b r} e^{\mu \theta}$$

$$\text{Numbers : } p = \frac{1000 e^{0.3\theta}}{(0.050)(0.075)} \text{ Pa or } p = 267 e^{0.3\theta} \text{ kPa}$$

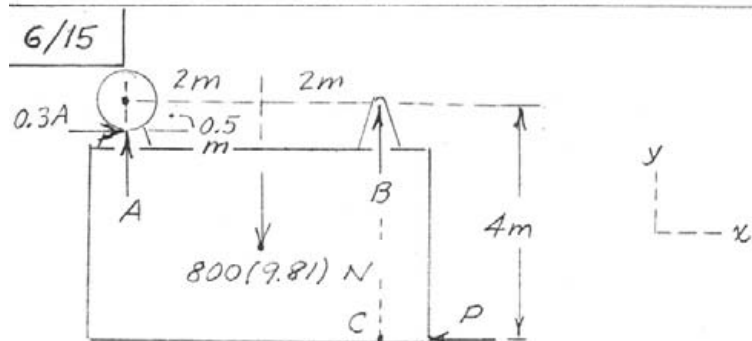
$$T_2 = T_1 e^{\mu \pi} = 1000 e^{0.3\pi} = \underline{2570 \text{ N}}$$

$$M = (T_2 - T_1) r = (2570 - 1000)(0.075) = \underline{117.5 \text{ N}\cdot\text{m}}$$



$$\underline{p_1 = 267 \text{ kPa}}$$

$$\underline{p_2 = 684 \text{ kPa}}$$

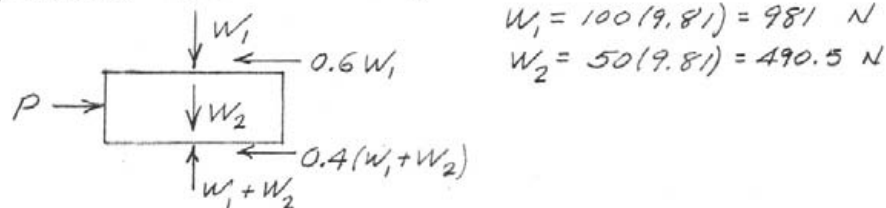


$$\sum M_C = 0; 800(9.81)2 - 4A - 0.3A(4 - 0.5) = 0, A = 3108 \text{ N}$$

$$\sum F_x = 0; P - 0.3(3108) = 0, \underline{P = 932 \text{ N}}$$

6/16 There are two possibilities

(a) Middle block moves; bottom one does not

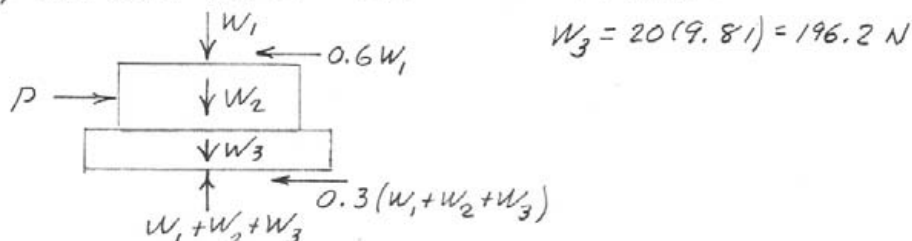


$$W_1 = 100(9.81) = 981 \text{ N}$$

$$W_2 = 50(9.81) = 490.5 \text{ N}$$

$$\Sigma F = 0; P = 0.6(981) + 0.4(981 + 490.5) = 1177 \text{ N}$$

(b) Bottom block moves with middle block

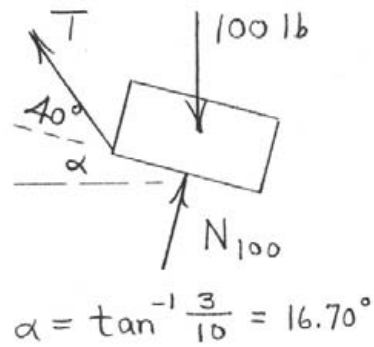
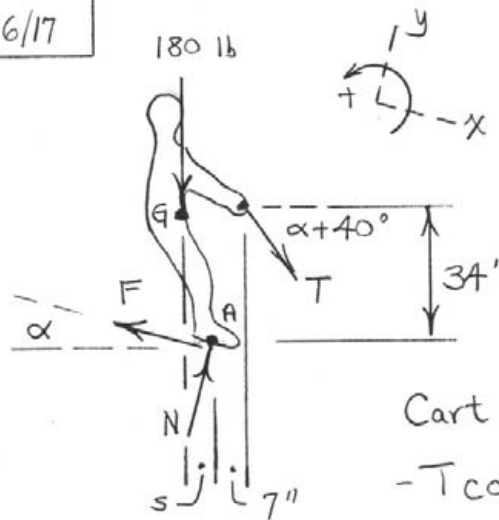


$$W_3 = 20(9.81) = 196.2 \text{ N}$$

$$\Sigma F = 0; P = 0.6(981) + 0.3(981 + 490.5 + 196.2) = 1089 \text{ N}$$

$$1088 < 1177 \text{ so case (b) occurs } \underline{P = 1089 \text{ N}}$$

6/17

Cart: $\sum F_x = 0$:

$$-T \cos 40^\circ + 100 \sin \alpha = 0$$

$$T = 37.5 \text{ lb}$$

Man:

$$\sum F_x = 0 : -F + 180 \sin \alpha + 37.5 \cos 40^\circ = 0$$

$$F = 80.5 \text{ lb}$$

$$\sum F_y = 0 : N - 180 \cos \alpha - 37.5 \sin 40^\circ = 0$$

$$N = 196.5 \text{ lb}$$

$$\mu_s = \frac{F}{N} = \frac{80.5}{196.5} = 0.409$$

$$\sum M_A = 0 : 180s - 37.5 \cos(\alpha + 40^\circ)(34)$$

$$- 37.5 \sin(\alpha + 40^\circ)(7) = 0$$

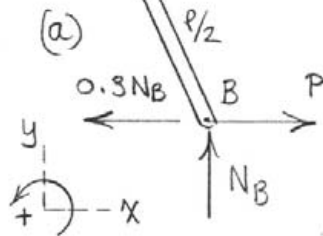
$$s = 5.11 \text{ in.}$$

6/18

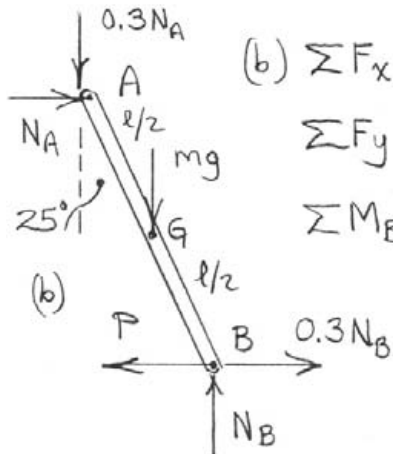
$$(a) \sum F_x = 0: P - 0.3N_B + N_A = 0$$

$$\sum F_y = 0: N_B - mg + 0.3N_A = 0$$

$$\sum M_B = 0: mg\left(\frac{l}{2} \sin 25^\circ\right) - N_A(l \cos 25^\circ) - 0.3N_A(l \sin 25^\circ) = 0$$



$$\text{Solution: } \begin{cases} N_A = 0.205 mg \\ N_B = 0.939 mg \\ P = 0.0771 mg \end{cases}$$



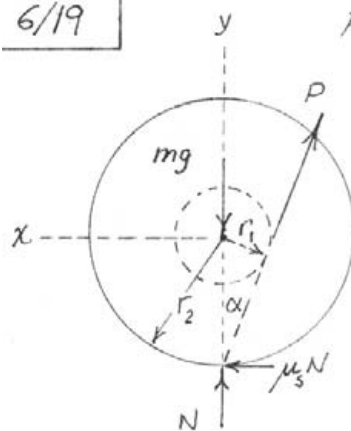
$$(b) \sum F_x = 0: -P + 0.3N_B + N_A = 0$$

$$\sum F_y = 0: N_B - mg - 0.3N_A = 0$$

$$\sum M_B = 0: mg\left(\frac{l}{2} \sin 25^\circ\right) - N_A(l \cos 25^\circ) + 0.3N_A(l \sin 25^\circ) = 0$$

$$\text{Solution: } \begin{cases} N_A = 0.271 mg \\ N_B = 1.081 mg \\ P = 0.595 mg \end{cases}$$

6/19



Forces must be concurrent

for equilibrium so $\alpha = \sin^{-1} \frac{r_1}{r_2}$

$$\sum F_y = 0; P \cos \alpha + N - mg = 0$$

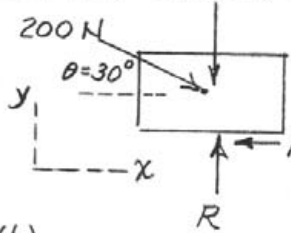
$$\sum F_x = 0; P \sin \alpha - \mu_s N = 0$$

Eliminate N & get

$$P = \frac{\mu_s mg}{\sin \alpha + \mu_s \cos \alpha} = \frac{\mu_s mg r_2}{r_1 + \mu_s \sqrt{r_2^2 - r_1^2}}$$

6/2

(a) $\Sigma F_x = 0; 200 \cos 30^\circ - \mu_s R = 0$
 $\Sigma F_y = 0; R - 200 \sin 30^\circ - 50(9.81) = 0$
 $R = 590.5 \text{ N}$
 So $\mu_s = \frac{200 \cos 30^\circ}{590.5} = 0.29$



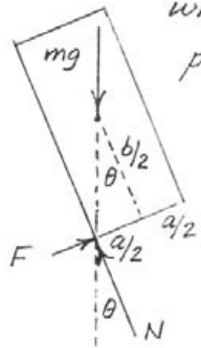
(b)
 For $\theta = 45^\circ$, $\Sigma F_x = 0$ gives $F = 200 \cos 45^\circ = 141.4 \text{ N}$
 which is $< \mu_s R_b$

6/20

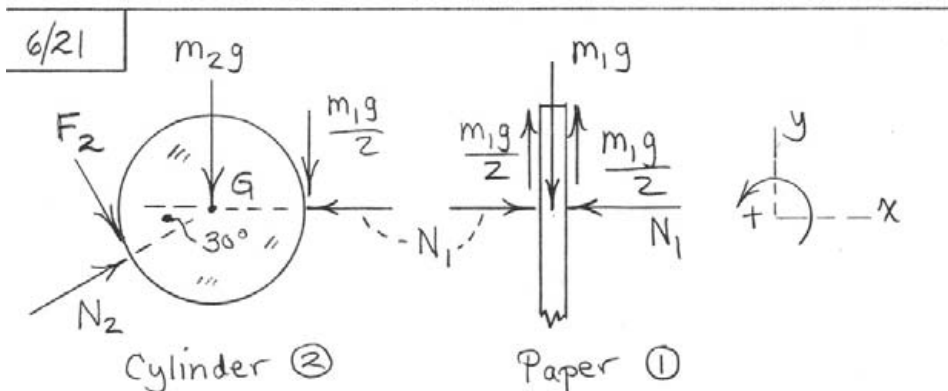
Block slips if $F = \mu N$ or $mg \sin \theta = \mu mg \cos \theta$

when angle reaches $\theta = \tan^{-1} \mu$

provided $\frac{a}{2} > \frac{b}{2} \tan \theta$ or $a > \mu b$



Tips first if $a < \mu b$



Cylinder:

$$\begin{cases} \sum F_x = 0: N_2 \cos 30^\circ + F_2 \sin 30^\circ - N_1 = 0 & (1) \\ \sum F_y = 0: N_2 \sin 30^\circ - F_2 \cos 30^\circ - m_2 g - \frac{m_1 g}{2} = 0 & (2) \\ \sum M_G = 0: F_2 r - \frac{m_1 g}{2} r = 0 & (3) \end{cases}$$

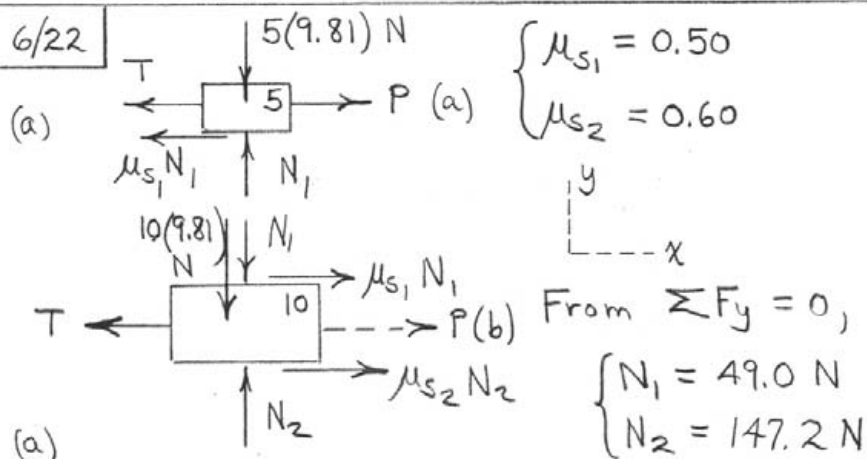
Solve simultaneously to obtain

$$\mu = \frac{m_1 g / 2}{N_1} = \frac{m_1}{3.73 m_1 + 3.46 m_2} > \frac{F_2}{N_2}$$

where we have assumed and then verified that slipping occurs first on right side of the cylinder.

For $m_1 \gg m_2$, $\mu = 0.268$

6/22



$$\sum F_x = 0: \begin{cases} P - T - 0.50(49.0) = 0 \\ -T + 0.50(49.0) + 0.60(147.2) = 0 \end{cases}$$

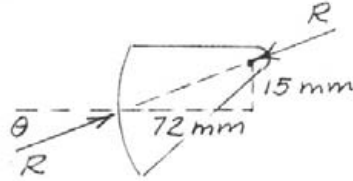
$$\underline{T = 112.8 \text{ N}}, \quad \underline{P = 137.3 \text{ N}}$$

(b) Now P is applied to 10-kg block & we reverse all friction forces above:

$$\sum F_x = 0: \begin{cases} -T + 0.50(49.0) = 0 \\ -T - 0.50(49.0) - 0.60(147.2) + P = 0 \end{cases}$$

$$\underline{T = 24.5 \text{ N}}, \quad \underline{P = 137.3 \text{ N}}$$

6/23



Req'd. minimum
coeff. of friction
is

$$\mu_s = \tan \phi = \tan \theta = \frac{15}{72} = 0.208$$

$$\Sigma F = 0 \text{ for rope; } 2R \sin \theta = 600$$

$$R = \frac{300}{15/\sqrt{15^2 + 72^2}} = 1471 \text{ N}$$

$$6/24 \quad \Sigma F_y = 0 : T - W \cos 10^\circ = 0$$

$$T = W \cos 10^\circ$$

100-lb block:

$$\Sigma F_y = 0 : N - 100 \cos 20^\circ = 0$$

$$N = 94.0 \text{ lb (throughout)}$$

(a) Motion impends down incline:

$$\Sigma F_x = 0 : 2T - 100 \sin 20^\circ + F_{\max} = 0$$

$$\text{With } F_{\max} = \mu_s N = 0.3(94.0)$$

$$= 28.2 \text{ lb and } T = W \cos 10^\circ,$$

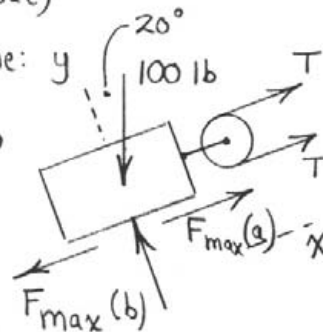
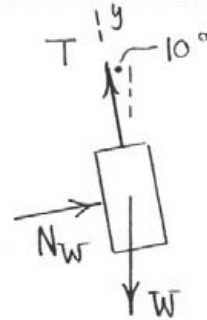
$$W = 3.05 \text{ lb}$$

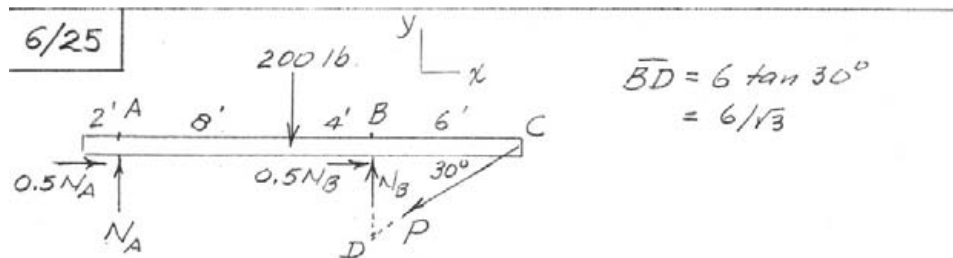
(b) Motion impends up incline:

$$\Sigma F_x = 0 : 2T - 100 \sin 20^\circ - F_{\max} = 0$$

$$\text{Similarly, } W = 31.7 \text{ lb}$$

Hence the allowable range is $\underline{3.05} \leq \underline{W} \leq \underline{31.7 \text{ lb}}$





$$\sum M_C = 0; 10(200) - 18N_A - 6N_B = 0$$

$$\sum M_D = 0; 4(200) - 0.5(N_A + N_B)\frac{6}{\sqrt{3}} - 12N_A = 0$$

solve simultaneously & get $N_A = 26.08 \text{ lb}$
 $N_B = 255.1 \text{ lb}$

$$\sum F_x = 0; 0.866P - 0.5(26.08 + 255.1) = 0$$

$$P = \underline{162.3 \text{ lb}}$$

6/26

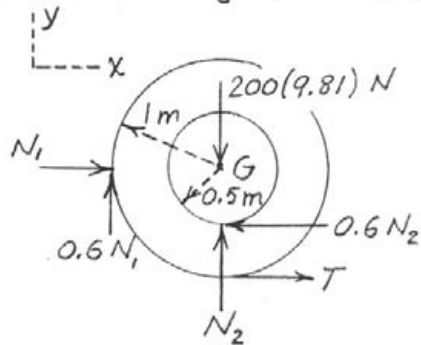
$$\Sigma M_G = 0; 0.6 N_1(1) + 0.6 N_2(0.5) - T(1) = 0$$

$$\Sigma F_x = 0; T - 0.6 N_2 + N_1 = 0$$

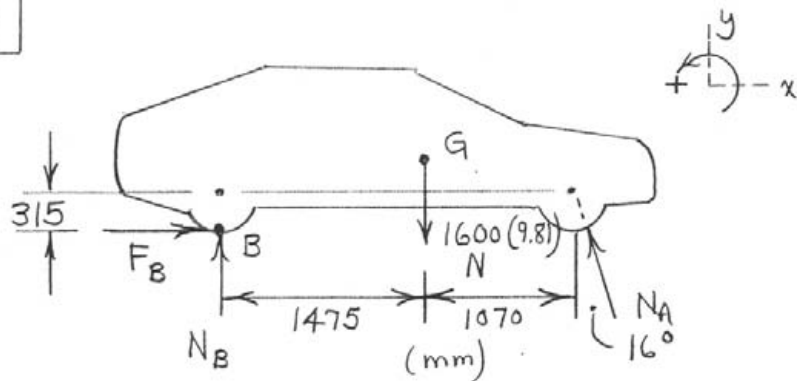
$$\Sigma F_y = 0; N_2 + 0.6 N_1 - 1962 = 0$$

Combine & get

$$N_1 = 331 \text{ N}, N_2 = 1764 \text{ N}, \underline{T = 727 \text{ N}}$$



6/27

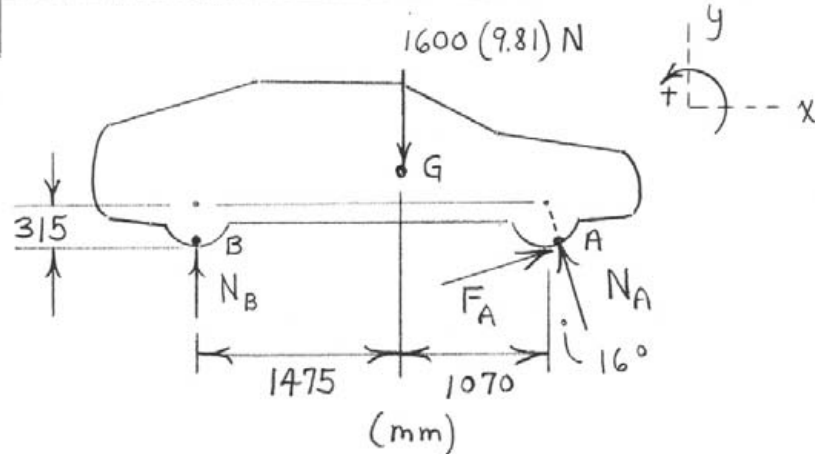


$$\begin{cases} \sum F_x = 0 : F_B - N_A \sin 16^\circ = 0 & (1) \\ \sum F_y = 0 : N_B + N_A \cos 16^\circ - 1600(9.81) = 0 & (2) \\ \sum M_B = 0 : -1600(9.81)(1475) + N_A \cos 16^\circ (2545) + N_A \sin 16^\circ (315) = 0 & (3) \end{cases}$$

$$\text{Solution : } \begin{cases} N_A = 9140 \text{ N} \\ N_B = 6910 \text{ N} \\ F_B = 2520 \text{ N} \end{cases}$$

$$\mu_s = \frac{F_B}{N_B} = \frac{2520}{6910} = \underline{0.365}$$

6/28



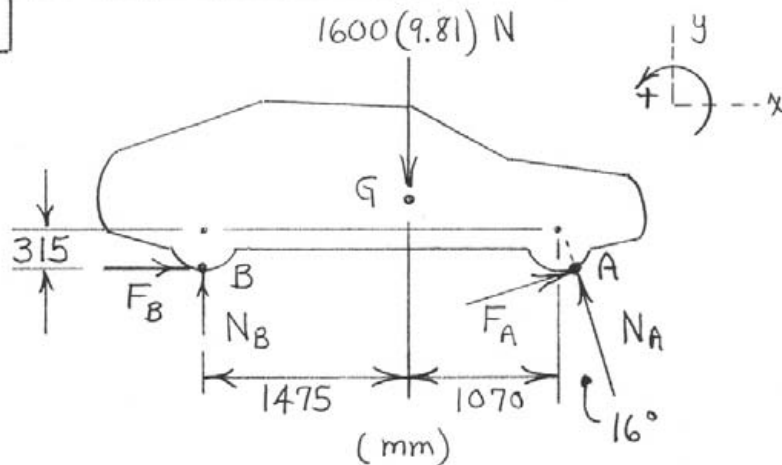
$$\begin{cases} \sum F_x = 0 : F_A \cos 16^\circ - N_A \sin 16^\circ = 0 & (1) \\ \sum F_y = 0 : N_B + F_A \sin 16^\circ + N_A \cos 16^\circ - 1600(9.81) = 0 & (2) \\ \sum M_A = 0 : -N_B (2545 + 315 \sin 16^\circ) + 1600(9.81) (1070 + 315 \sin 16^\circ) = 0 & (3) \end{cases}$$

$$\text{Solution : } \begin{cases} N_A = 8460 \text{ N} \\ N_B = 6900 \text{ N} \\ F_A = 2420 \text{ N} \end{cases}$$

$$\mu_s = \frac{F_A}{N_A} = \frac{2420}{8460} = 0.287$$

Note that the value of μ_s can be determined from Eq. (1) alone.

6/29



$$\sum F_x = 0: F_B + F_A \cos 16^\circ - N_A \sin 16^\circ = 0 \quad (1)$$

$$\sum F_y = 0: N_B + F_A \sin 16^\circ + N_A \cos 16^\circ - 1600(9.81) = 0 \quad (2)$$

$$\sum M_A = 0: -N_B(2545 + 315 \sin 16^\circ) + F_B(315 - 315 \cos 16^\circ) + 1600(9.81)(1070 + 315 \sin 16^\circ) = 0 \quad (3)$$

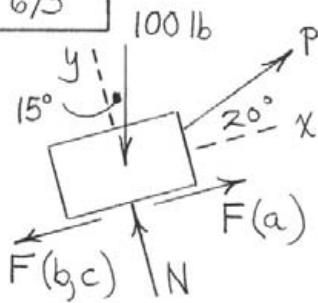
Also, set

$$\begin{cases} F_A = \mu_s N_A & (4) \\ F_B = \mu_s N_B & (5) \end{cases}$$

Solution:

$$\begin{cases} F_A = 1378 \text{ N} & F_B = 1087 \text{ N} \\ N_A = 8750 \text{ N} & N_B = 6900 \text{ N} \\ \mu_s = 0.1575 \end{cases}$$

6/3

(a) $P = 0$

$$\sum F_y = 0: N - 100 \cos 15^\circ = 0$$

$$N = 96.6 \text{ lb}$$

Assume equilibrium:

$$\sum F_x = 0: F - 100 \sin 15^\circ = 0$$

$$F = 25.9 \text{ lb}$$

$F_{\max} = \mu_s N = 0.25(96.6) = 24.1 < F$; assumption invalid and $F = F_k = \mu_k N = 0.2(96.6) = \underline{19.32 \text{ lb}}$ up the incline.

(b) $P = 40 \text{ lb}$

$$\sum F_y = 0: N - 100 \cos 15^\circ + 40 \sin 20^\circ = 0, \quad N = 82.9 \text{ lb}$$

$$\sum F_x = 0: 40 \cos 20^\circ - 100 \sin 15^\circ - F = 0, \quad F = \underline{11.71 \text{ lb}}$$

$$F_{\max} = \mu_s N = 0.25(82.9) = 20.7 \text{ lb}; \text{ assumption OK}$$

(c) $P = 60 \text{ lb}$

$$\sum F_y = 0: N - 100 \cos 15^\circ + 60 \sin 20^\circ = 0, \quad N = 76.1 \text{ lb}$$

$$\sum F_x = 0: 60 \cos 20^\circ - 100 \sin 15^\circ - F = 0, \quad F = 30.5 \text{ lb}$$

$$F_{\max} = \mu_s N = 0.25(76.1) = 19.02 \text{ lb} < F; \text{ assumption invalid}$$

$$F = \mu_k N = 0.2(76.1) = \underline{15.21 \text{ lb}} \text{ down the incline}$$

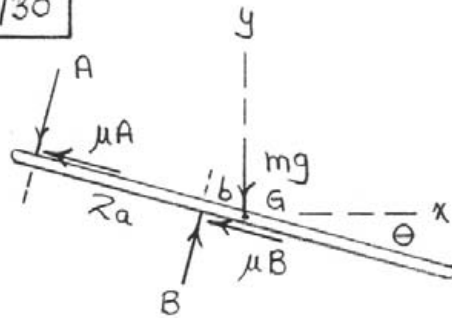
(d) To initiate motion, set $F = \mu_s N = 0.25N$ down the incline:

$$\sum F_y = 0: N - 100 \cos 15^\circ + P \sin 20^\circ = 0$$

$$\sum F_x = 0: P \cos 20^\circ - 100 \sin 15^\circ - 0.25N = 0$$

$$\text{Solve to obtain } \begin{cases} N = 79.9 \text{ lb} \\ P = \underline{48.8 \text{ lb}} \end{cases}$$

6/30



$$\sum M_G = 0 : A(2a+b) - Bb = 0, \quad \frac{B}{A} = \frac{2a+b}{b}$$

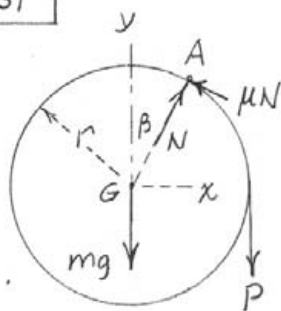
$$\sum F_x = 0 : (B-A)\sin\theta - \mu(A+B)\cos\theta = 0$$

$$\tan\theta = \mu \frac{A+B}{B-A} = \mu \frac{1 + B/A}{\frac{B}{A} - 1}$$

$$\text{Substitute } \frac{B}{A} : \tan\theta = \mu \frac{1 + (2a+b)/b}{(2a+b)/b - 1}$$

$$\text{or } \underline{\theta = \tan^{-1}\left(\mu \frac{a+b}{a}\right)}$$

6/31



$$\Sigma F_x = 0; N \sin \beta - \mu N \cos \beta = 0$$

$$\tan \beta = \mu$$

$$\beta = \tan^{-1} \mu$$

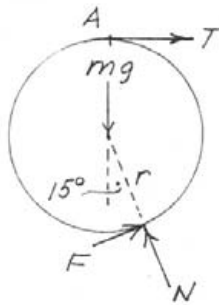
$$\Sigma M_A = 0;$$

$$P(r - r \sin \beta) - mgr \sin \beta = 0$$

$$P = mg \frac{\sin \beta}{1 - \sin \beta}$$

$$\text{But } \sin \beta = \frac{\mu}{\sqrt{1 + \mu^2}} \text{ so } P = mg \frac{\mu}{\sqrt{1 + \mu^2} - \mu}$$

6/32

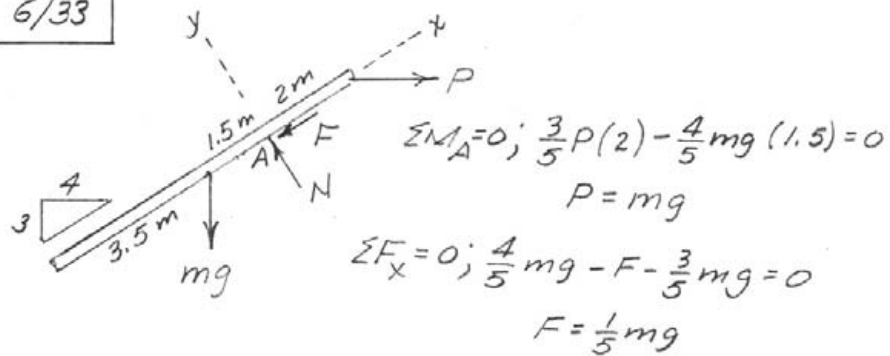
Assume equilibrium, $\sum M_A = 0$ 

$$F(r + r \cos 15^\circ) = N r \sin 15^\circ$$

$$\frac{F}{N} = \frac{0.2588}{1 + 0.9659} = 0.1317$$

But $F_{\max} = 0.20 \text{ N}$ ($> 0.1317 \text{ N}$)

so $F = 0.1317 \text{ N}$ can be supported
& paper rolls without slipping



$$\Sigma M_A = 0; \frac{3}{5}P(2) - \frac{4}{5}mg(1.5) = 0$$

$$P = mg$$

$$\Sigma F_x = 0; \frac{4}{5}mg - F - \frac{3}{5}mg = 0$$

$$F = \frac{1}{5}mg$$

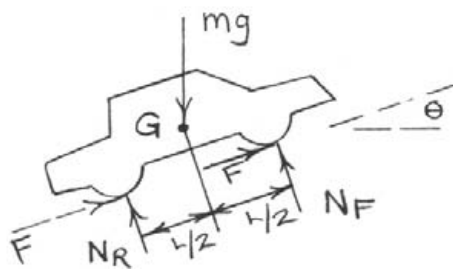
$$\Sigma F_y = 0; N - \frac{3}{5}mg - \frac{4}{5}mg = 0, N = \frac{7}{5}mg$$

$\mu_s(\min) = F/N = \frac{1/5}{7/5} = \frac{1}{7} = 0.14 < 0.30$ so
friction is sufficient to maintain equil.

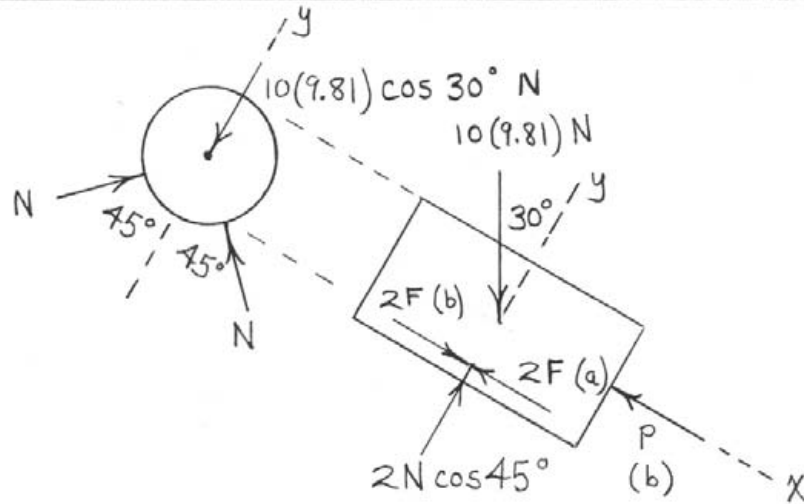
Ans. is yes. $F = \frac{1}{5}60(9.81) = \underline{117.7 \text{ N}}$

6/34

Consider the FBD below and the equilibrium equation $\Sigma M_G = 0$. The presence of the propulsive friction forces F , whether applied at the front or at the rear, increases the rear normal forces and decreases the front ones. Increased normals mean increased available propulsive friction forces.



Thus the rear-wheel drive car would climb the steeper grade.



$$\Sigma F_y = 0: 2N \cos 45^\circ - 10(9.81) \cos 30^\circ = 0, N = 60.1 \text{ N}$$

(a) $P = 0$

$$\Sigma F_x = 0: -2F + 10(9.81) \sin 30^\circ = 0, \underline{F = 24.5 \text{ N}}$$

$$\text{Check: } F_{\max} = \mu_s N = 0.5(60.1) = 30.0 \text{ N} > F = 24.5 \text{ N}$$

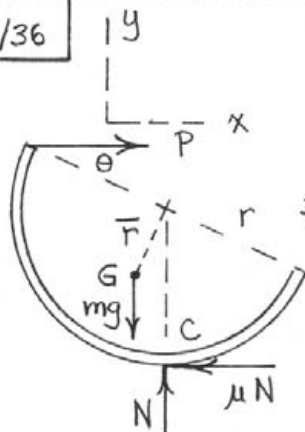
So we indeed have static equilibrium.

(b) $P \neq 0$

$$\Sigma F_x = 0: -P + 10(9.81) \sin 30^\circ + 2(0.5 \cdot 60.1) = 0$$

$$\underline{P = 109.1 \text{ N}}$$

6/36



$$\Sigma F_y = 0 \Rightarrow N = mg$$

$$\Sigma F_x = 0 \Rightarrow P = \mu_s N$$

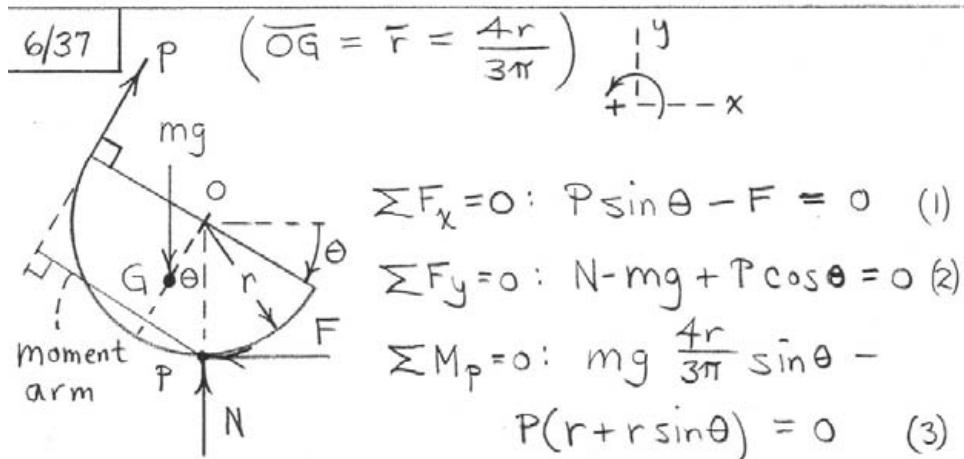
$$\Sigma M_C = 0 : P(r + r \sin \theta)$$

$$- mg \bar{r} \sin \theta = 0, \quad \bar{r} = \frac{2r}{\pi}$$

$$\therefore \mu_s mg(r + r \sin \theta) = mg \frac{2r}{\pi} \sin \theta$$

$$\sin \theta \left[\frac{2}{\pi} - \mu_s \right] = \mu_s, \quad \theta = \sin^{-1} \left(\frac{\pi \mu_s}{2 - \pi \mu_s} \right)$$

$$\theta = 90^\circ \text{ when } \frac{\pi \mu_s}{2 - \pi \mu_s} = 1 \text{ or } \underline{\mu_{90^\circ} = \frac{1}{\pi} = 0.318}$$



When slipping impends, $F = \mu_s N \quad (4)$

$$(4) \rightarrow (1): P \sin \theta - \mu_s N = 0 \quad (5)$$

$$(2): N = mg - P \cos \theta \quad (6)$$

$$(6) \rightarrow (5): P \sin \theta - \mu_s (mg - P \cos \theta) = 0$$

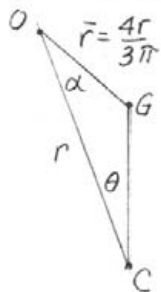
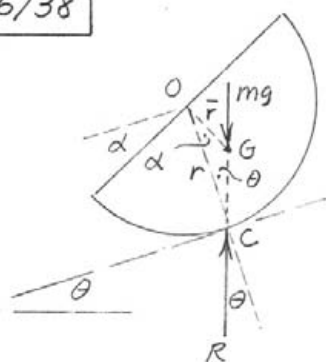
$$\Rightarrow P = \frac{\mu_s mg}{\sin \theta + \mu_s \cos \theta} \quad (7)$$

(7) \rightarrow (3) & simplification yields

$$\mu_s = \frac{4 \sin^2 \theta}{3\pi(1 + \sin \theta) - 4 \sin \theta \cos \theta}$$

For $\theta = 40^\circ$, $\mu_s = 0.1223$

(7) then gives $P = 0.1661 mg$



$$\frac{r}{\sin(\pi - \theta - \alpha)} = \frac{\bar{r}}{\sin \theta}$$

$$\frac{r}{\sin(\theta + \alpha)} = \frac{4r/3\pi}{\sin \theta}$$

Max. angle occurs when

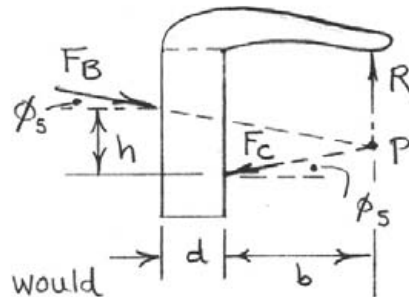
$$\theta = \phi = \tan^{-1} \mu = \tan^{-1} 0.30 = \underline{16.70^\circ}$$

$$\text{Thus } \sin(\theta + \alpha) = \frac{3\pi}{4} \sin \theta = \frac{3\pi}{4} (0.2873) = 0.6770$$

$$\text{so } \theta + \alpha = 42.6^\circ \text{ \& } \alpha = 42.6 - 16.7 = \underline{25.9^\circ}$$

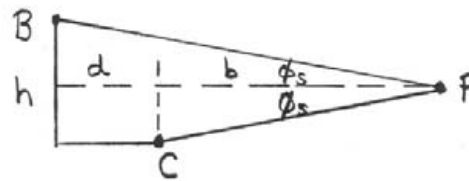
6/39

For equilibrium the three forces must be concurrent at P. For b larger than that shown, friction angles ϕ would



be less than $\phi_s = \tan^{-1} \mu_s$ and no slippage is possible.

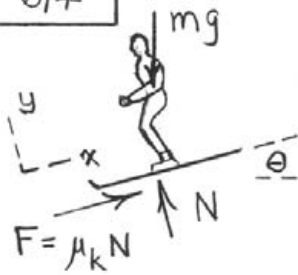
From geometry,



$$\frac{h - b \tan \phi_s}{b + d} = \tan \phi_s$$

$$h - b \mu_s = (b + d) \mu_s, \quad b = \frac{1}{2} \left(\frac{h}{\mu_s} - d \right)$$

6/4



$$\Sigma F_x = 0: 0.08N - mg \sin \theta = 0$$

$$N = \frac{mg \sin \theta}{0.08}$$

$$\Sigma F_y = 0: N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

$$\text{So } \frac{mg \sin \theta}{0.08} = mg \cos \theta, \tan \theta = 0.08, \underline{\theta = 4.57^\circ}$$

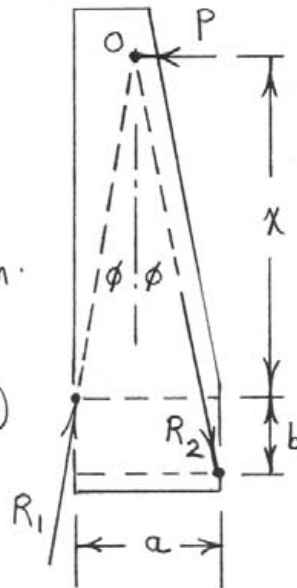
6/40

For equilibrium, the forces must be concurrent at O and $\phi = \tan^{-1} \mu_s$ for impending motion with $x = x_{\min}$.

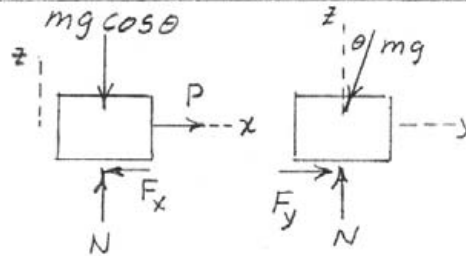
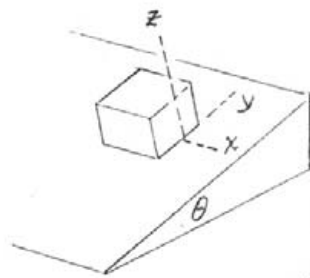
$$a = x \tan \phi + (x+b) \tan \phi$$

$$= x \mu_s + (x+b) \mu_s = \mu_s (2x+b)$$

$$x = \frac{a - b\mu_s}{2\mu_s}$$



6/41



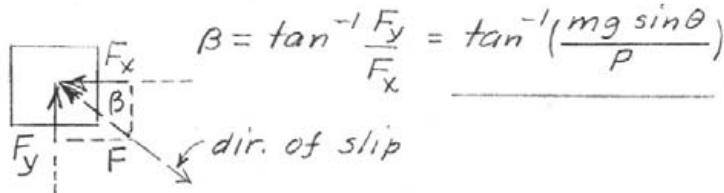
$$\sum F_y = 0; F_y = mg \sin \theta$$

$$\sum F_x = 0; F_x = P; \sum F_z = 0; N = mg \cos \theta$$

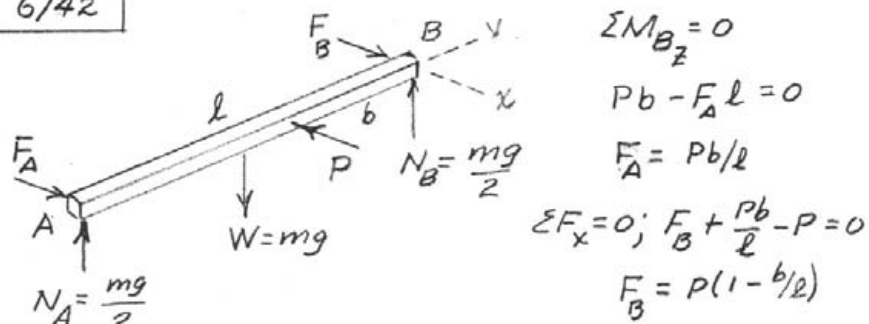
$$F = \mu_s N = \sqrt{F_x^2 + F_y^2} \text{ so } (\mu_s mg \cos \theta)^2 = P^2 + (mg \sin \theta)^2$$

$$\text{so } P = mg \sqrt{\mu_s^2 \cos^2 \theta - \sin^2 \theta}$$

where $\mu_s > \tan \theta$ otherwise block slips without P



6/42

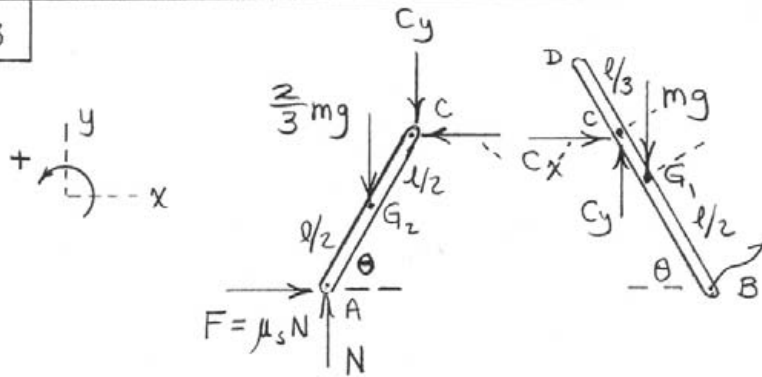


For $b < l/2$, $F_B > F_A$ so slips first at B

Hence $F_B = \mu_s N_B = \mu_s mg/2$

Thus $\mu_s mg/2 = P(1 - b/l)$ so $P = \frac{\mu_s mg}{2(1 - b/l)}$

& $F_A = \frac{\mu_s mg}{2(1 - b/l)} \frac{b}{l} = \frac{\mu_s mg}{2(\frac{l}{b} - 1)}$



Body BD:

$$\sum M_B = 0: -C_x \left(\frac{2l}{3} \sin \theta \right) - C_y \left(\frac{2l}{3} \cos \theta \right) + mg \frac{l}{2} \cos \theta = 0 \quad (1)$$

Body AC:

$$\sum F_x = 0: \mu_s N - C_x = 0 \quad (2)$$

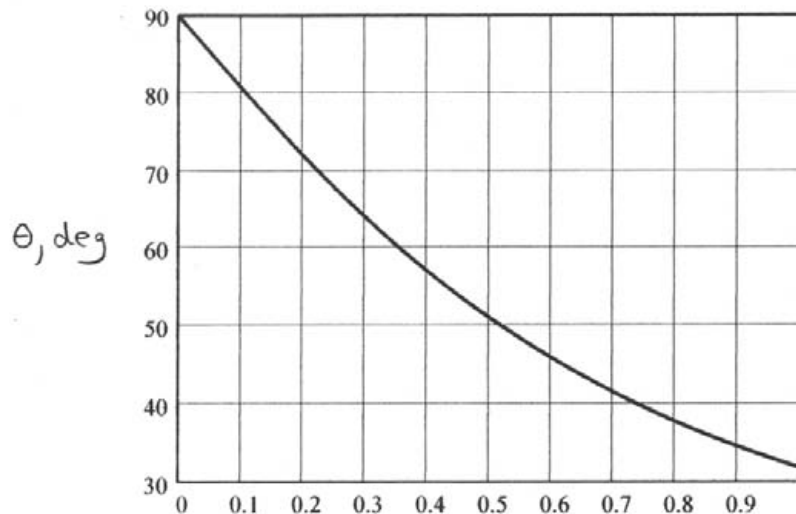
$$\sum F_y = 0: N - \frac{2}{3}mg - C_y = 0 \quad (3)$$

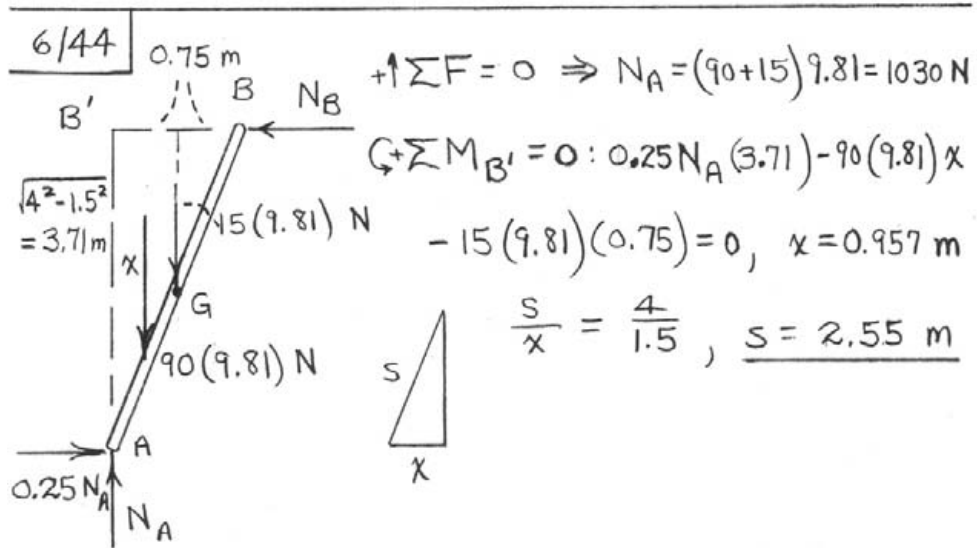
$$\sum M_C = 0: \mu_s N \left(\frac{2l}{3} \sin \theta \right) - N \left(\frac{2l}{3} \cos \theta \right) + \frac{2}{3}mg \left(\frac{l}{3} \cos \theta \right) = 0 \quad (4)$$

Solve Eqs. (1)-(4) for C_x , C_y , N , and θ as functions of μ_s . Solution for θ is

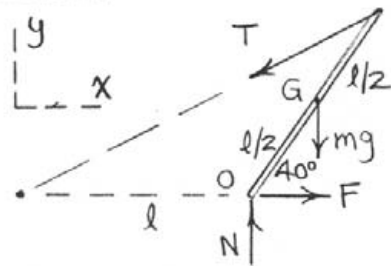
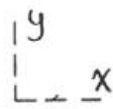
$$\theta = \tan^{-1} \frac{13}{21\mu_s} \text{ (plotted below)}$$

$$\begin{cases} \theta \rightarrow 90^\circ \text{ as } \mu_s \rightarrow 0 \\ \theta = 51.1^\circ \text{ for } \mu_s = 0.50 \\ \theta = 31.8^\circ \text{ for } \mu_s = 1 \end{cases}$$





6/45



$$\vec{T} = T \left\{ \frac{(-l - l \cos 40^\circ) \hat{i} - (l \sin 40^\circ) \hat{j}}{\sqrt{(l + l \cos 40^\circ)^2 + (l \sin 40^\circ)^2}} \right\}$$

$$= T \{-0.940 \hat{i} - 0.342 \hat{j}\}$$

$$\sum M_O = 0: 0.342 T(l) - mg \frac{l}{2} \cos 40^\circ = 0$$

$$T = 1.120 mg$$

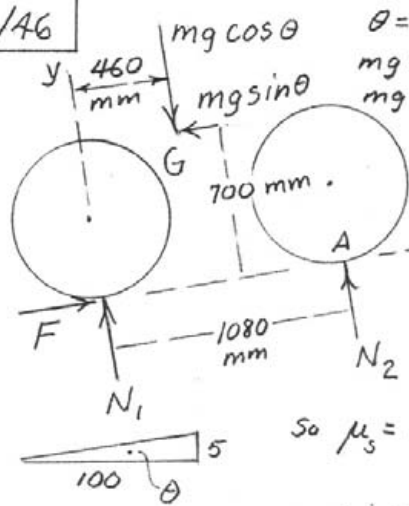
$$\sum F_x = 0: F - 1.120 mg (0.940) = 0, F = 1.052 mg$$

$$\sum F_y = 0: N - 1.120 mg (0.342) - mg = 0$$

$$N = 1.383 mg$$

$$\mu_s = \frac{F}{N} = \frac{1.052 mg}{1.383 mg} = \underline{0.761}$$

6/46



$$\theta = \tan^{-1} 5/100 = 2.86^\circ$$

$$mg \cos \theta = 82(9.81)(0.9998) = 803.4 \text{ N}$$

$$mg \sin \theta = 82(9.81)(0.0499) = 40.2 \text{ N}$$

$$\Sigma M_A = 0;$$

$$803.4(620) + 40.2(700)$$

$$-1080 N_1 = 0$$

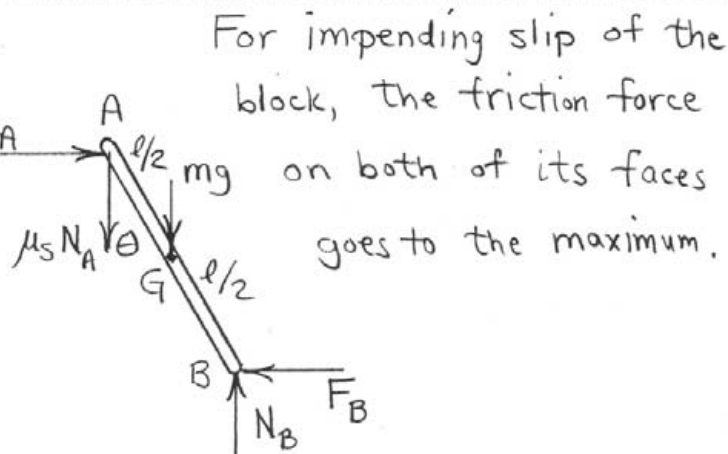
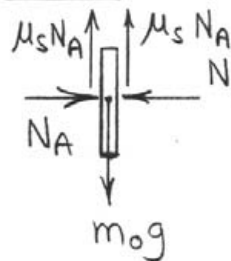
$$N_1 = 487.3 \text{ N}$$

$$\Sigma F_x = 0; F = 40.2 \text{ N}$$

$$\text{So } \mu_s = F/N_1 = 40.2/487.3 = \underline{0.082}$$

If μ_s were doubled, friction force remains 40.2 N

6/47



For impending slip of the block, the friction force on both of its faces goes to the maximum.

$$(\text{Block}) \uparrow \sum F = 0: 2\mu_s N_A - m_o g = 0, N_A = \frac{m_o g}{2\mu_s} \quad (1)$$

$$(\text{Bar}) \curvearrowright \sum M_B = 0: mg \left(\frac{l}{2} \sin \theta \right) + \mu_s N_A (l \sin \theta) - N_A (l \cos \theta) = 0 \quad (2)$$

With Eq. (1), Eq. (2) becomes

$$mg \left(\frac{l}{2} \sin \theta \right) + \mu_s \frac{m_o g}{2\mu_s} l \sin \theta - \frac{m_o g}{2\mu_s} l \cos \theta = 0$$

$$\text{Solving for } \theta: \theta = \tan^{-1} \left[\frac{1}{\mu_s \left(1 + \frac{m}{m_o} \right)} \right] = \theta_{\min}$$

$$(\text{Bar}) \rightarrow \sum F = 0: N_A - F_B = 0$$

$$F_B = N_A = \frac{m_o g}{2\mu_s}$$

$$\uparrow \sum F = 0: N_B - mg - \mu_s N_A = 0$$

$$N_B = mg + \mu_s \frac{m_o g}{2\mu_s} = \left(m + \frac{m_o}{2} \right) g$$

$$(\mu_s)_B = \frac{F_B}{N_B} = \frac{m_o g / (2\mu_s)}{\left(m + \frac{m_o}{2} \right) g} = \frac{1}{\mu_s \left(1 + 2 \frac{m}{m_o} \right)}$$

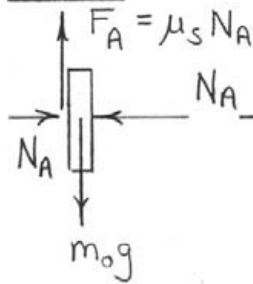
Numbers ($\mu_s = 0.5$ throughout)

$$(a) \frac{m}{m_o} = 0.1: \theta_{\min} = 61.2^\circ, (\mu_s)_B = 1.667$$

($\mu_s > 1$ perhaps unusual, but quite possible)

$$(b) \frac{m}{m_o} = 1: \theta_{\min} = 45^\circ, (\mu_s)_B = 0.667$$

6/48

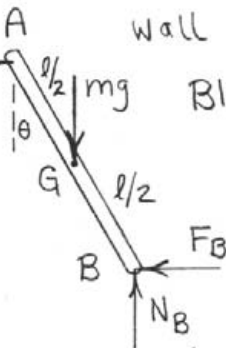


The friction force F_A between the block m_0 and the vertical wall is set to the maximum.

Block: $\uparrow \Sigma F = 0$:

$$\mu_s N_A = m_0 g$$

$$N_A = \frac{m_0 g}{\mu_s} \quad (1)$$



Bar: $\curvearrowright \Sigma M_B = 0$: $mg \left(\frac{l}{2} \sin \theta \right) - N_A (l \cos \theta) = 0$

$$N_A = \frac{mg}{2} \tan \theta \quad (2)$$

Combine (1) & (2): $\frac{m_0 g}{\mu_s} = \frac{mg}{2} \tan \theta$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{2}{\mu_s} \frac{m_0}{m} \right) = \theta_{\min}$$

Numbers: $\theta_{\min} = \tan^{-1} \left(\frac{2}{0.5} \cdot \frac{1}{10} \right) = 21.8^\circ$

Check to see that no slippage occurs at B.

(Bar) $\rightarrow \Sigma F = 0$: $N_A - F_B = 0$

So $F_B = N_A = \frac{mg}{2} \tan \theta$ (from Eq. (2))

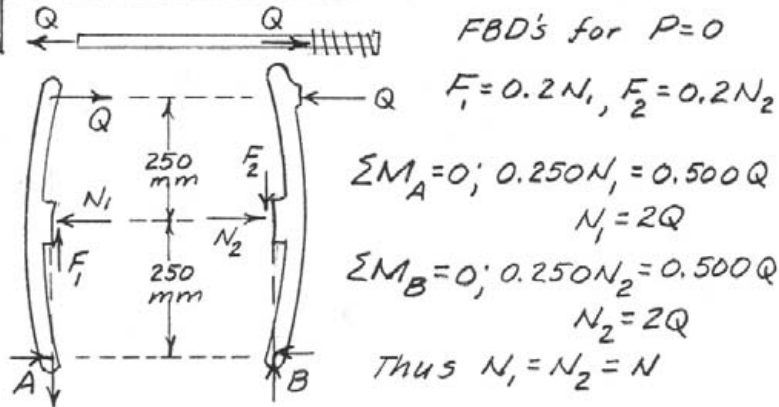
$\uparrow \Sigma F = 0$: $N_B - mg = 0$, $N_B = mg$

$$(F_B)_{\max} = \mu_s N_B = \mu_s mg$$

Numbers: $\left\{ \begin{aligned} F_B &= \frac{mg}{2} \tan 21.8^\circ = 0.2mg \\ (F_B)_{\max} &= 0.5mg \end{aligned} \right.$

$F_B < (F_B)_{\max}$, so no slippage at B.

6/49



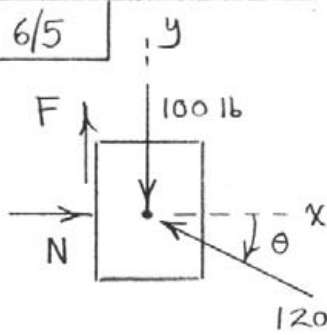
Flywheel: $\Sigma M = 0; M = F_1 r + F_2 r$

$$100 = 0.2N(0.200) + 0.2N(0.200)$$

$$N = 1250 \text{ N}$$

$$\text{So } Q = N/2 = 625 \text{ N} ; Q = k\delta, k = \frac{625}{0.030} = 20.8(10^3) \frac{\text{N}}{\text{m}}$$

6/5



$$\sum F_x = 0 : N - 120 \cos \theta = 0$$

$$\sum F_y = 0 : F - 100 + 120 \sin \theta = 0$$

$$(a) \theta = 15^\circ$$

$$N = 115.9 \text{ lb}, F = 68.9 \text{ lb}$$

$$F_{\max} = \mu_s N = 0.50 (115.9)$$

$$= 58.0 \text{ lb}$$

$F > F_{\max}$, so assumption of equilibrium is invalid and $F = F_k = \mu_k N = 0.40 (115.9)$

$$= \underline{46.4 \text{ lb (up)}}$$

$$(b) \theta = 30^\circ$$

$$N = 103.9 \text{ lb}, F = 40 \text{ lb}$$

$$F_{\max} = \mu_s N = 0.50 (103.9) = 52.0 \text{ lb}$$

$F < F_{\max}$, so equilibrium assumption OK

$$\nabla \underline{F = 40 \text{ lb (up)}}$$

► 6/50 Given : $\begin{cases} r_1 = \overline{OA} = 5/16'' \\ r_2 = \overline{OB} = 7/16'' \\ \alpha = 5^\circ \end{cases}$

$\phi > \beta$ so slipping would occur

first at A. Hence $\mu_{\min} = \tan \phi$.

Law of cosines: $b^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos \alpha$

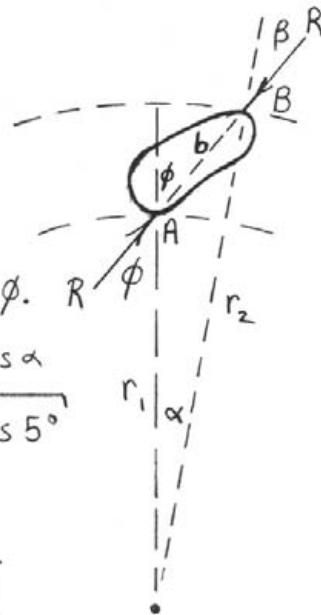
$$b = \sqrt{\left(\frac{5}{16}\right)^2 + \left(\frac{7}{16}\right)^2 - 2\left(\frac{5}{16}\right)\left(\frac{7}{16}\right) \cos 5^\circ}$$

$$= 0.1291 \text{ in.}$$

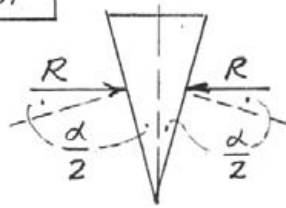
Law of sines : $\frac{b}{\sin \alpha} = \frac{r_2}{\sin(\pi - \phi)}$

$$\frac{0.1291}{\sin 5^\circ} = \frac{7/16}{\sin(\pi - \phi)} = \frac{7/16}{\sin \phi}$$

$$\phi = 17.18^\circ, \mu_{\min} = \tan \phi = \tan 17.18^\circ = \underline{0.309}$$



6/51



Critical α occurs when

$$\frac{\alpha}{2} = \phi = \tan^{-1} \mu$$

$$\text{so } \alpha = 2 \tan^{-1} 0.20$$

$$= 2(11.31^\circ) = \underline{22.6^\circ}$$

6/52 Self-locking condition is

$$\alpha = \phi$$

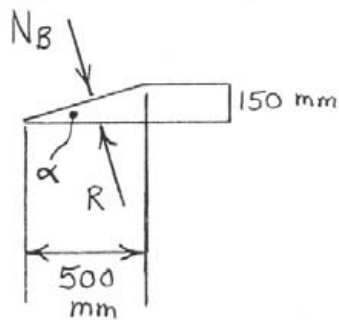
$$\tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \mu_s$$

$$\mu_s = \frac{L}{2\pi r} = \frac{5}{2\pi(3)}$$

$$= \underline{0.265}$$

6/53

FBD of portable ramp:



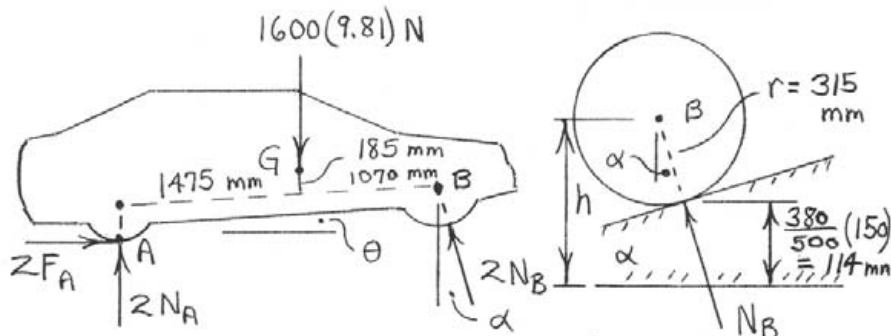
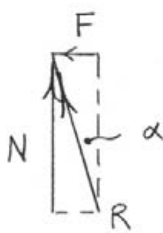
$$\alpha = \tan^{-1} \left(\frac{150}{500} \right) = 16.70^\circ$$

$$\tan \alpha = \frac{F}{N} = \mu_s$$

$$\text{So } \mu_s = \tan \alpha$$

$$= \tan 16.70^\circ$$

$$\mu_s = 0.3$$



$$\text{So } \theta = \sin^{-1} \frac{416 - 315}{2545} = 2.27^\circ$$

$$\begin{cases} h = 114 + 315 \cos \alpha \\ = 416 \text{ mm} \end{cases}$$

$$\begin{aligned} \sum M_A = 0: & -1600(9.81)(1475 \cos \theta - 185 \sin \theta) \\ & + 2N_B \cos(\alpha - \theta)(315 \sin \theta + 2545) + 2N_B \sin(\alpha - \theta) 315 \cos \theta = 0 \end{aligned}$$

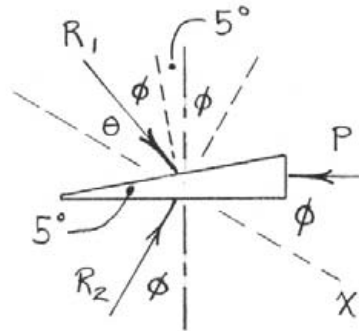
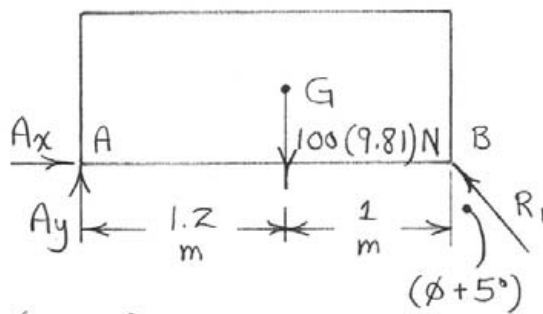
$$N_B = 4500 \text{ N}$$

$$\sum F = 0: 2F_A - 2N_B \sin \alpha = 0$$

$$F_A = 1294 \text{ N}$$

6/54

$$\phi = \tan^{-1}(0.60) = 31.0^\circ$$



(Door)

$$\sum M_A = 0: R_1 \cos(31.0^\circ + 5^\circ)(2.2) - 100(9.81)(1.2) = 0$$

$$R_1 = 661 \text{ N}$$

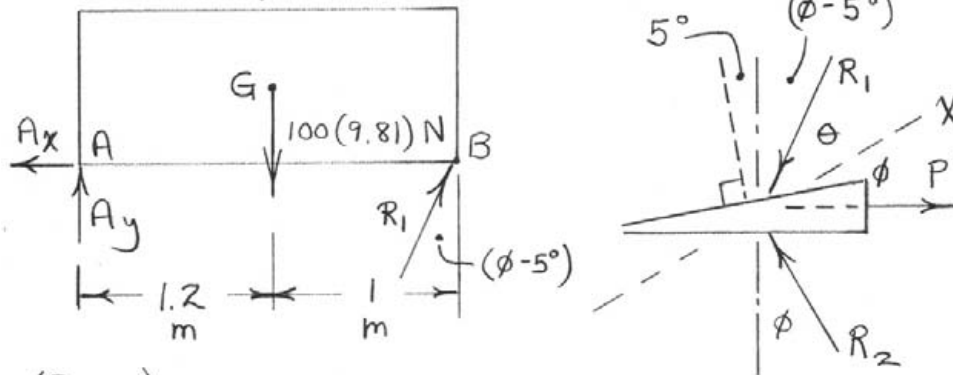
(Wedge) (Note $\theta = 90^\circ - 2\phi - 5^\circ = 23.1^\circ$)

$$\sum F_x = 0: 661 \cos 23.1^\circ - P \cos 31.0^\circ = 0$$

$$P = 709 \text{ N}$$

$$6/55 \quad \phi = \tan^{-1}(0.60) = 31.0^\circ$$

Assume A pinned:



(Door)

$$\sum M_A = 0: R_1 \cos(31.0^\circ - 5^\circ)(2.2) - 100(9.81)(1.2) = 0$$

$$R_1 = 595 \text{ N} \quad (3)$$

(Wedge) (Note $\theta = 90^\circ - 2\phi + 5^\circ = 33.1^\circ$) (4)

$$\sum F_x = 0: P' \cos 31.0^\circ - 595 \cos 33.1^\circ = 0$$

$$P' = 582 \text{ N} \quad (5)$$

$$\text{(Door)} \quad \sum F_x = 0: -A_x + 595 \sin(\phi - 5^\circ) = 0, A_x = 261 \text{ N}$$

$$\sum F_y = 0: A_y - 100(9.81) + 595 \cos(\phi - 5^\circ) = 0, A_y = 446 \text{ N}$$

$$\frac{A_x}{A_y} = \frac{261}{446} = 0.584 < (\mu_s = 0.6); \text{ assumption OK,}$$

A will not slip.

6/56

$$\text{Helix angle } \alpha = \tan^{-1} \frac{h}{\pi d} = \tan^{-1} \frac{1/12}{\pi (3/8)} = 4.05^\circ$$

$$\text{Friction angle } \phi = \tan^{-1} \mu_s = \tan^{-1} (0.20) = 11.31^\circ$$

$$\begin{aligned} \text{Tighten: } M &= Pr \tan (\alpha + \phi) = 80 \frac{3/8}{2} \tan (4.05^\circ + 11.31^\circ) \\ &= \underline{4.12 \text{ lb-in.}} \end{aligned}$$

$$\begin{aligned} \text{Loosen: } M' &= Pr \tan (\phi - \alpha) = 80 \frac{3/8}{2} \tan (11.31^\circ - 4.05^\circ) \\ &= \underline{1.912 \text{ lb-in.}} \quad \left(\begin{array}{l} \text{in direction opposite} \\ \text{to that of } M \end{array} \right) \end{aligned}$$

Note: $\alpha < \phi$, so screw is self-locking (a good feature for a clamp!)

$$6/57 \quad M = Wr \tan(\alpha + \phi)$$

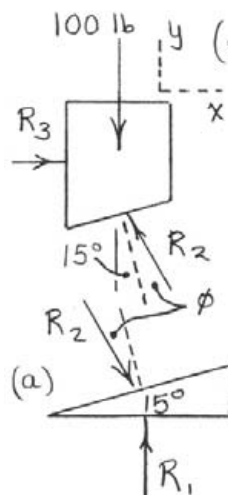
where $W = 450 \text{ N}$, $r = 0.025 \text{ m}$,

$$\alpha = \tan^{-1} \frac{\text{Lead}}{2\pi r} = \tan^{-1} \frac{0.020}{2\pi (0.025/2)} = \tan^{-1} 0.255$$
$$= 14.29^\circ$$

$$\phi = \tan^{-1} \mu = \tan^{-1} 0.20 = 11.31^\circ; \quad \phi + \alpha = 25.60^\circ$$

$$\text{So } M = 450 \left(\frac{0.025}{2} \right) \tan 25.60^\circ = \underline{2.69 \text{ N}\cdot\text{m}}$$

6/58 Friction angle $\phi = \tan^{-1}(0.2) = 11.31^\circ$



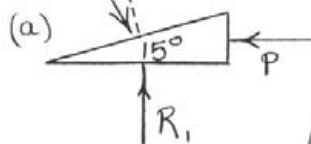
(a) Rollers under wedge

$$\sum F_y = 0: -100 + R_2 \cos(15^\circ + 11.31^\circ) = 0$$

$$R_2 = 111.6 \text{ lb}$$

$$\sum F_x = 0: R_2 \sin(15^\circ + 11.31^\circ) - P = 0$$

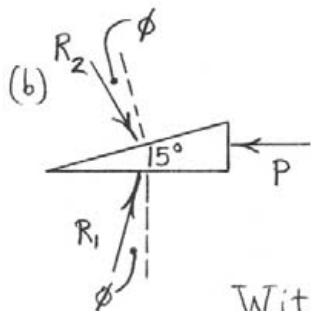
$$P = 49.4 \text{ lb}$$



(b) Rollers removed

Value of R_2 from 100-lb

body is unchanged.



$$\sum F_x = 0: R_2 \sin(15^\circ + 11.31^\circ) - P + R_1 \sin(11.31^\circ) = 0$$

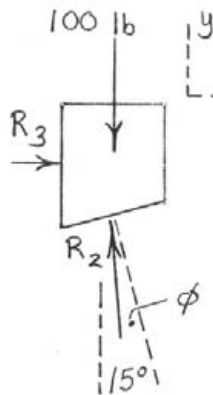
With R_1 determined from overall equilibrium as $R_1 = \frac{100}{\cos 11.31^\circ} = 102.0 \text{ lb}$, we solve for P as

$$P = 69.4 \text{ lb}$$

6/59

Friction angle $\phi = \tan^{-1}(0.2) = 11.31^\circ$

(a) Rollers under wedge

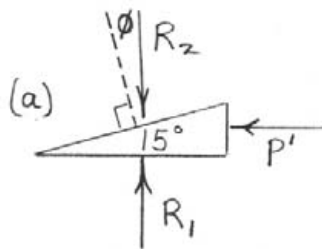


$$\sum F_y = 0: -100 + R_2 \cos(15^\circ - 11.31^\circ) = 0$$

$$R_2 = 100.2 \text{ lb}$$

$$\sum F_x = 0: R_2 \sin(15^\circ - 11.31^\circ) - P' = 0$$

$$P' = 6.45 \text{ lb (to the left)}$$



(b) Rollers removed

$$R_2 = 100.2 \text{ lb, from (a).}$$

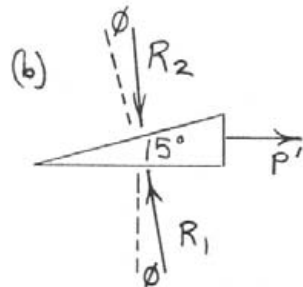
From overall equilibrium,

$$R_1 = \frac{100}{\cos 11.31^\circ} = 102.0 \text{ lb}$$

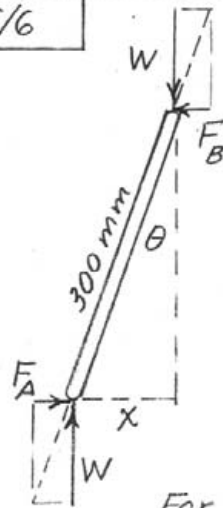
$$\sum F_x = 0: R_2 \sin(15^\circ - 11.31^\circ) + P' - R_1 \sin(11.31^\circ) = 0$$

$$- R_1 \sin(11.31^\circ) = 0$$

$$P' = 13.55 \text{ lb (to the right)}$$



6/6



For $x = 75 \text{ mm}$, $\theta = \sin^{-1} \frac{75}{300} = 14.5^\circ$

Friction angle $\phi = \tan^{-1} \mu_s$

for A is $\phi_A = \tan^{-1} 0.40 = 21.8^\circ$

" B " $\phi_B = \tan^{-1} 0.30 = 16.7^\circ$

Since $\theta < \phi_A$ & ϕ_B , bar does not slip & $F_A = F_B = W \tan \theta$

$$= 50(9.81) \tan 14.5^\circ$$

$$= \underline{126.6 \text{ N}}$$

For increased x bar slips first at B

with $\theta = \phi_B = 16.7^\circ$. Thus $x_{\max} = 300 \sin 16.7^\circ$

$$= \underline{86.2 \text{ mm}}$$

$$\boxed{6/60} \quad \text{Helix angle: } \alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{1/5}{2\pi \left(\frac{1.150}{2}\right)} = 3.17^\circ$$

$$\text{Friction angle } \phi = \tan^{-1} \mu = \tan^{-1}(0.25) = 14.04^\circ$$

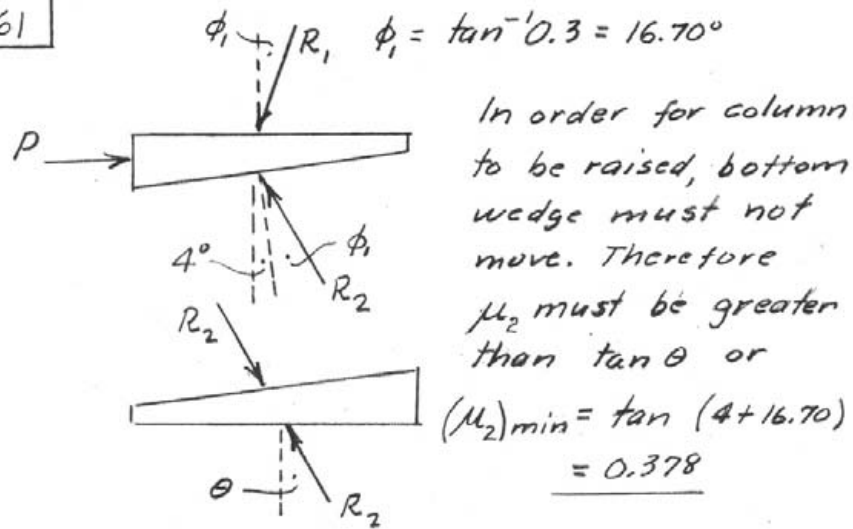
$$(a) \text{ To tighten, } M_a = 2Tr \tan(\alpha + \phi)$$

$$M_a = 2(10,000) \frac{1.150}{2} \tan(3.17^\circ + 14.04^\circ) = \underline{3560 \text{ lb-in.}}$$

$$(b) \text{ To loosen, } M_b = 2Tr \tan(\phi - \alpha)$$

$$M_b = 2(10,000) \frac{1.150}{2} \tan(14.04^\circ - 3.17^\circ) = \underline{2210 \text{ lb-in.}}$$

6/61



6/62

For the screw,

$$\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{6}{20\pi} = 5.45^\circ$$

$$\phi = \tan^{-1}(0.25) = 14.04^\circ$$

$$\text{Eq. 6/3: } M = Wr \tan(\alpha + \phi)$$

$$24 = P \frac{20/2}{1000} \tan(5.45^\circ + 14.04^\circ)$$

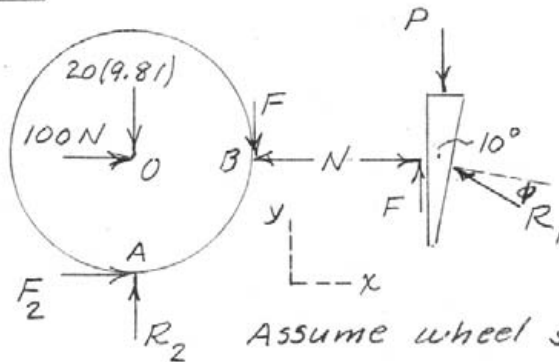
$$P = 6780 \text{ N (to remove collar)}$$

$$\text{Collar: } \mu p A = P : 0.30 p (0.050\pi \cdot 0.060) = 6780$$

$$p = 2.40(10^6) \text{ Pa or } \underline{2400 \text{ kPa}}$$

6/63

$$\phi = \tan^{-1} 0.30 = 16.70^\circ$$



Assume wheel slips at B

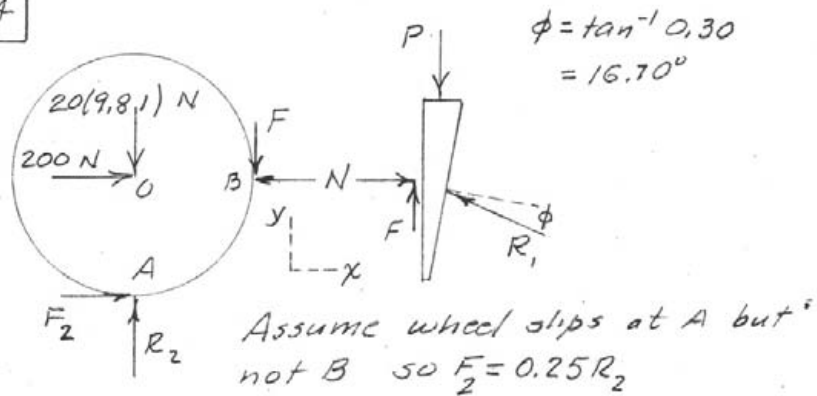
$$\begin{aligned} \text{wheel: } \Sigma M_O = 0; \quad F_2 = F = 0.3N \quad & \left. \begin{aligned} 0.7N &= 100 \\ N &= 142.9 \text{ N} \end{aligned} \right\} \\ \Sigma F_x = 0; \quad N &= 100 + F_2 \quad & F_2 = F = 0.3(142.9) \\ & & = 42.9 \end{aligned}$$

$$\Sigma F_y = 0; \quad R_2 = 196.2 + 42.9 = 239.1 \text{ N}$$

$$(F_2 = 42.9) < (0.25 \times 239.1 = 59.8) \quad \text{so assumption OK}$$

$$\text{Wedge: } \Sigma F_x = 0; \quad R_1 \cos 26.70^\circ = 142.9, \quad R_1 = 159.9 \text{ N}$$

$$\Sigma F_y = 0; \quad P = 42.9 + 159.9 \sin 26.70^\circ = \underline{114.7 \text{ N}}$$



Wheel: $\sum M_O = 0$; $F = F_2 = 0.25 R_2$

$\sum F_y = 0$; $R_2 = 196.2 + F = 196.2 + 0.25 R_2$ N

Solve & get $R_2 = 262$ N & $F = 0.25(262) = 65.4$

$\sum F_x = 0$; $N = 200 + 65.4 = 265.4$ N

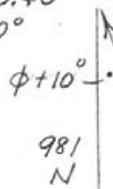
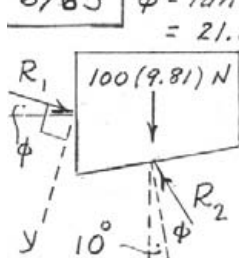
$(F = 65.4) < (0.3 \times 265.4 = 79.6)$ so assumption OK

Wedge: $\sum F_x = 0$; $R_1 \cos 26.70^\circ = 265.4$, $R_1 = 297$ N

$\sum F_y = 0$; $P = 65.4 + 297 \sin 26.70^\circ = \underline{198.8 \text{ N}}$

6/65

$$\phi = \tan^{-1} 0.40 = 21.80^\circ$$



$$\text{Block: } \Sigma F_y = 0;$$

$$981 / \cos 21.80^\circ$$

$$= R_2 \cos 53.60^\circ$$

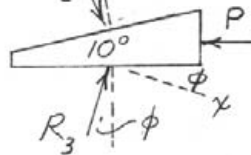
$$R_2 = 1535 \text{ N}$$

$$\text{Wedge: } \Sigma F_x = 0;$$

$$1535 \cos 36.40^\circ$$

$$= P \cos 21.80^\circ$$

$$P = 1331 \text{ N}$$



$$2\phi + 10^\circ = 53.60^\circ$$

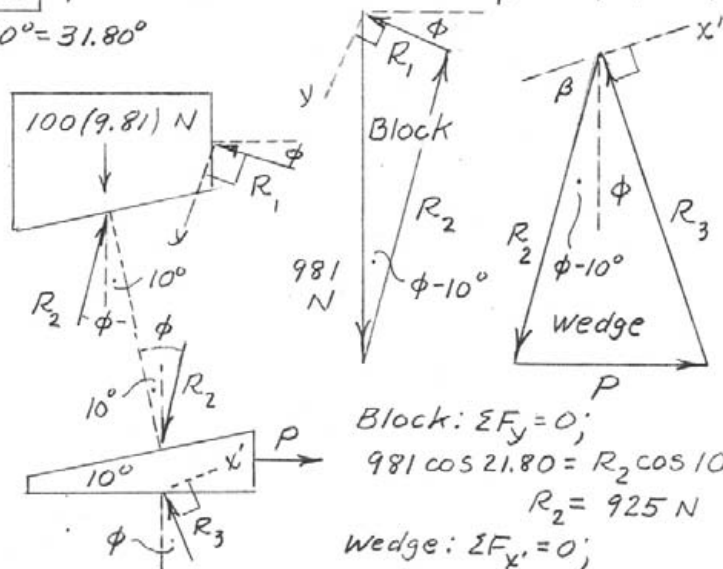
$$90^\circ - (2\phi + 10^\circ) = 36.40^\circ$$

$$\text{Screw: } \alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{10}{2\pi (15)} = 6.06^\circ; \phi = \tan^{-1} 0.25 = 14.04^\circ$$

$$\phi + \alpha = 20.09^\circ$$

$$M = Pr \tan(\phi + \alpha) = 1331 (0.015) \tan 20.09^\circ = \underline{7.30 \text{ N}\cdot\text{m}}$$

6/66 $\phi = \tan^{-1} 0.40 = 21.80^\circ$ $\beta = 90^\circ - (2\phi - 10^\circ) = 56.40^\circ$
 $\phi + 10^\circ = 31.80^\circ$



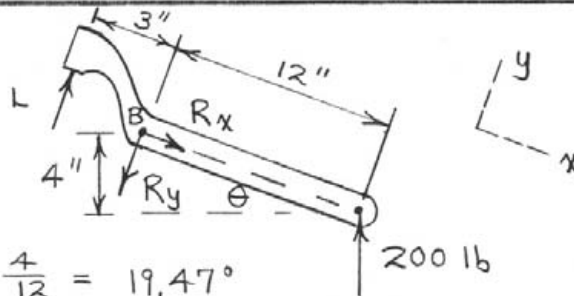
Block: $\Sigma F_y = 0$;
 $981 \cos 21.80^\circ = R_2 \cos 10^\circ$
 $R_2 = 925 \text{ N}$

Wedge: $\Sigma F_{x'} = 0$;
 $925 \cos 56.40^\circ = P \cos 21.80^\circ$
 $P = 551 \text{ N}$

For screw,
 $\alpha = 6.06^\circ$ & $\phi = 14.04^\circ$ (Prob. 6/65)

$M' = Pr \tan(\phi + \alpha) = 551(0.015) \tan 20.09^\circ = \underline{3.02 \text{ N}\cdot\text{m}}$
 Note (+) sign, $(\phi + \alpha)$

6/67



$$\theta = \sin^{-1} \frac{4}{12} = 19.47^\circ$$

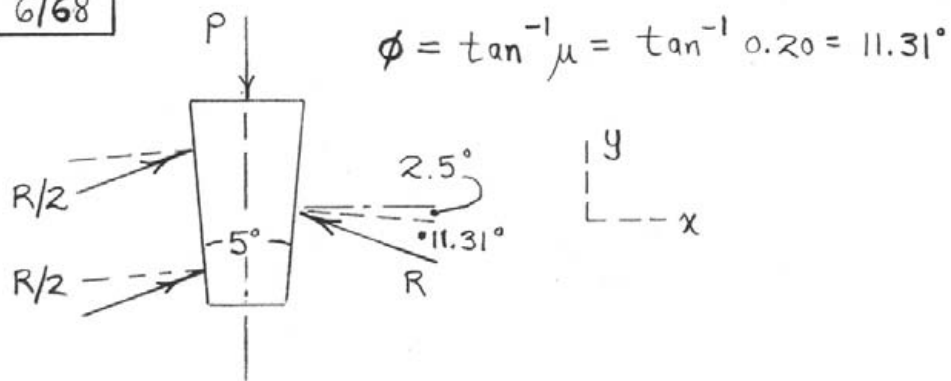
$$\sum M_B = 0: 200 \cos 19.47^\circ (12) - 3L = 0, \quad L = 754 \text{ lb}$$

$$\text{Screw: } \begin{cases} \text{helix angle } \alpha = \tan^{-1} \frac{1/12}{\pi(1/2)} = 3.04^\circ \\ \text{friction angle } \phi = \tan^{-1} (0.20) = 11.31^\circ \end{cases}$$

$$\begin{aligned} \text{Tighten screw: } M &= Lr \tan(\phi + \alpha) \\ &= 754 (0.25) \tan(11.31^\circ + 3.04^\circ) = 48.2 \text{ lb-in.} \end{aligned}$$

$$\begin{aligned} \text{Loosen screw: } M' &= Lr \tan(\phi - \alpha) \\ &= 754 (0.25) \tan(11.31^\circ - 3.04^\circ) = 27.4 \text{ lb-in.} \end{aligned}$$

6/68



$$\sum F_x = 0 \text{ for shaft: } R \cos(11.31^\circ + 2.5^\circ) = T = 200$$

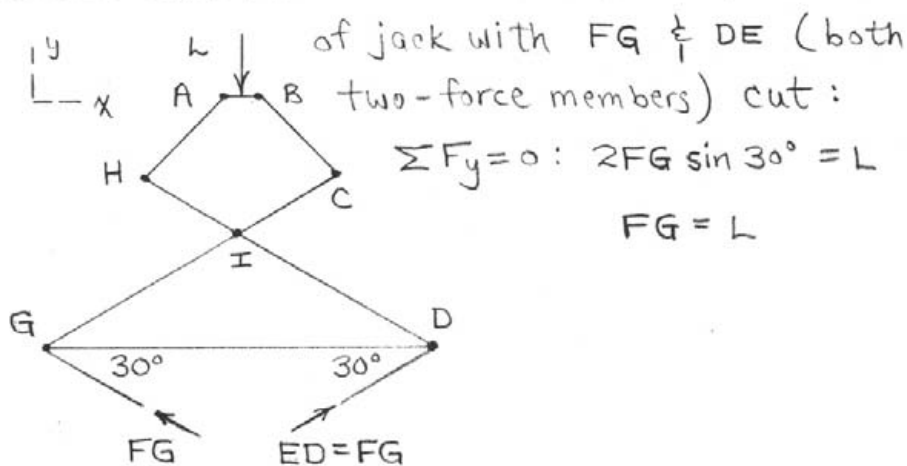
$$R = 206 \text{ lb}$$

$$\sum F_y = 0 \text{ for wedge: } P = 2R \sin(11.31^\circ + 2.5^\circ)$$

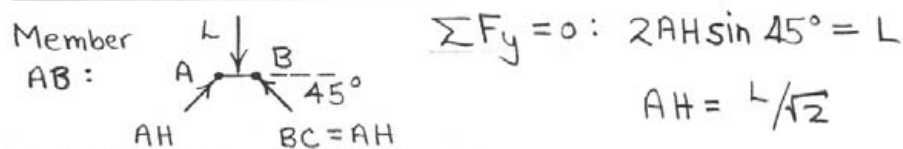
$$\underline{P = 98.3 \text{ lb}}$$

6/69

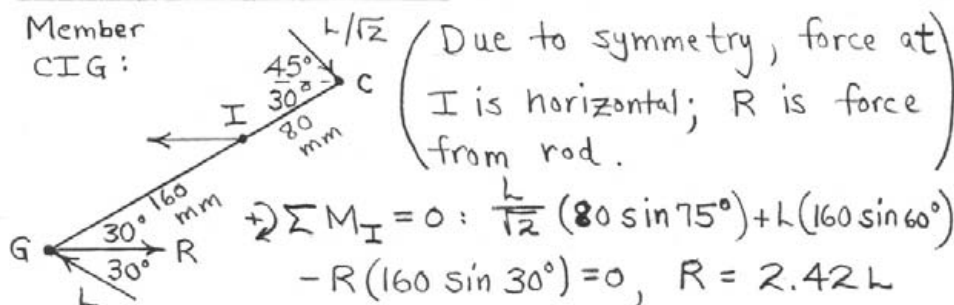
Calculate force in member FG. FBD



Member AB:



Member CIG:



Friction angle $\phi = \tan^{-1} \mu = \tan^{-1} (0.20) = 11.31^\circ$

$\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{0.003}{2\pi (0.010/2)} = 5.45^\circ$

To raise load: $M = Rr \tan(\alpha + \phi)$

$= [2.42(1000)(9.81)] \left[\frac{0.010}{2} \right] \tan(11.31^\circ + 5.45^\circ)$

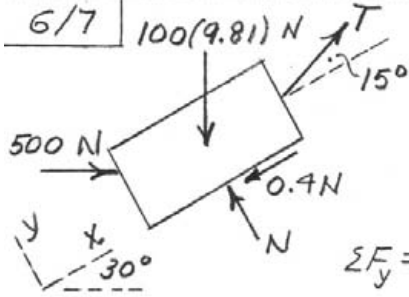
$= \underline{35.7 \text{ N}\cdot\text{m}}$

To lower load: $M' = Rr \tan(\phi - \alpha)$

$= [2.42(1000)(9.81)] \left[\frac{0.010}{2} \right] \tan(11.31^\circ - 5.45^\circ)$

$= \underline{12.15 \text{ N}\cdot\text{m}}$

6/7



$\sum F_x = 0;$
 $T \cos 15^\circ - 981 \sin 30^\circ - 0.4N + 500 \cos 30^\circ = 0$
 $0.966T - 0.4N = 57.5$

$\sum F_y = 0;$
 $N - 500 \sin 30^\circ - 981 \cos 30^\circ + T \sin 15^\circ = 0$
 $0.259T + N = 1099.6$

Solve simultaneously & get $N = 979 \text{ N}$

$T = 465 \text{ N}$

► 6/70 For equil. of screw (refer to prob. illust.)

$$\Sigma F = 0; W = \Sigma R, \cos(\alpha + \gamma) = \cos(\alpha + \gamma) \Sigma R,$$

$$\Sigma M = 0; M = \Sigma R, r \sin(\alpha + \gamma) = r \sin(\alpha + \gamma) \Sigma R,$$

$$\text{combine \& set } M = Wr \tan(\alpha + \gamma) = Wr \frac{\tan \alpha + \tan \gamma}{1 - \tan \alpha \tan \gamma}$$

$$\text{But } \tan \gamma = \frac{R \sin \phi}{R \cos \phi \cos \beta/2} = \mu / \cos \beta/2$$

$$\& \tan \beta/2 = \frac{L}{2h} \cos \alpha, \tan \frac{\theta}{2} = \frac{L}{2h}, \text{ so } \tan \frac{\beta}{2} = \tan \frac{\theta}{2} \cos \alpha$$

$$\& \cos \frac{\beta}{2} = 1 / \sqrt{1 + \tan^2 \frac{\theta}{2} \cos^2 \alpha}. \text{ Thus } \tan \gamma = \mu \sqrt{1 + \tan^2 \frac{\theta}{2} \cos^2 \alpha}$$

Hence

$$M = Wr \frac{\tan \alpha + \mu \sqrt{1 + \tan^2 \frac{\theta}{2} \cos^2 \alpha}}{1 - \mu \tan \alpha \sqrt{1 + \tan^2 \frac{\theta}{2} \cos^2 \alpha}}$$

$$\text{where } \tan \alpha = \frac{L}{2\pi r}$$

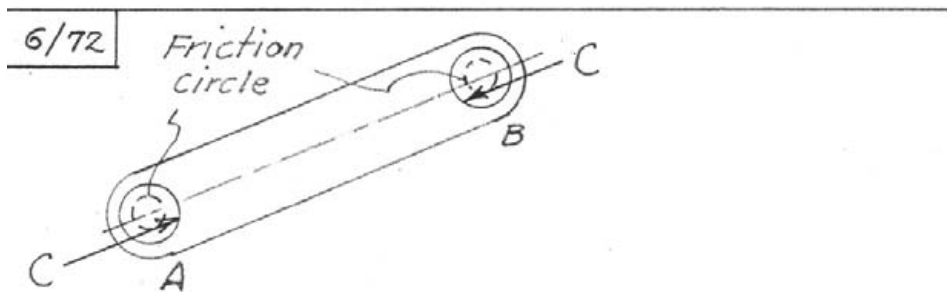
$$\boxed{6/71} \quad M = Rr \sin \phi, \quad \sin \phi = \frac{M}{Rr} = \frac{3}{2(40)(9.81)(0.040/2)}$$

$$\phi = 11.02^\circ$$

$$\mu = \tan \phi = \underline{0.1947}$$

$$r_f = r \sin \phi = \frac{0.040}{2} \sin 11.02^\circ = 0.00382 \text{ m}$$

$$\text{or } \underline{r_f = 3.82 \text{ mm}}$$



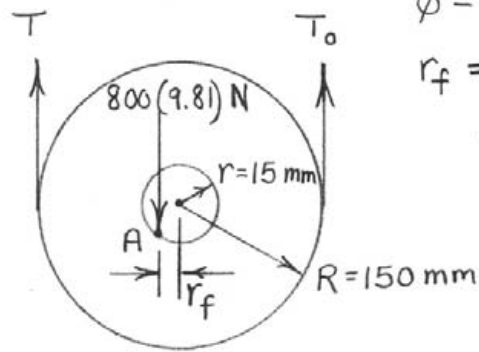
6/73

$$M = \frac{2}{3} \mu PR$$

$$A \text{ on } B: M = \frac{2}{3} (0.40)(80)\left(\frac{9}{2}\right) = \underline{96 \text{ lb-in.}}$$

$$B \text{ on } C: 96.0 = \frac{2}{3} \mu (80)\left(\frac{12}{2}\right), \underline{\mu = 0.3}$$

6/74



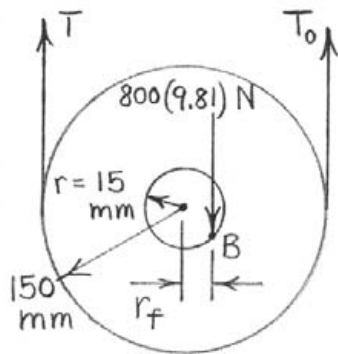
$$\phi = \tan^{-1} 0.25 = 14.04^\circ$$

$$r_f = r \sin \phi = 15 \sin 14.04^\circ = 3.64 \text{ mm}$$

$$+\uparrow \sum F = 0 : T + T_o - 800(9.81) = 0 \quad (1)$$

$$+\circlearrowleft \sum M_A = 0 : T(150 - 3.64) - T_o(150 + 3.64) = 0 \quad (2)$$

$$\text{Solve (1) \& (2) : } \begin{cases} T = 4020 \text{ N} \\ T_o = 3830 \text{ N} \end{cases}$$



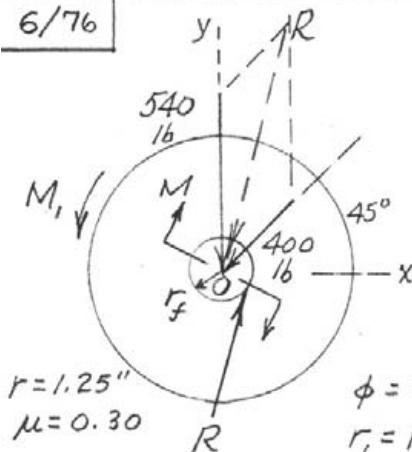
From the solution to
Prob. 6/74, $r_f = 3.64 \text{ mm}$

$$\uparrow \sum F = 0: T + T_0 - 800(9.81) = 0 \quad (1)$$

$$\curvearrowright \sum M_B = 0: T(150 + 3.64) - T_0(150 - 3.64) = 0 \quad (2)$$

$$\text{Solve (1) \& (2): } \begin{cases} T = 3830 \text{ N} \\ T_0 = 4020 \text{ N} \end{cases}$$

6/76



Replace 400-lb force by
force and couple M_1

$$M_1 = 400(22) = 8800 \text{ lb-in.}$$

$$R_x = 400 \cos 45^\circ = 282.8 \text{ lb}$$

$$R_y = 540 + 400 \sin 45^\circ = 822.8 \text{ lb}$$

$$R = \sqrt{(282.8)^2 + (822.8)^2} = 870.1 \text{ lb}$$

$$\phi = \tan^{-1} 0.30 = 16.70^\circ$$

$$r_f = r \sin \phi = 1.25 \sin 16.70^\circ = 0.359 \text{ in.}$$

$$\sum M_O = 0; M - 8800 - 870.1(0.359) = 0$$

$$M = 8800 + 313 = 9113 \text{ lb-in.}$$

$$\text{or } M = 9113/12 = \underline{759 \text{ lb-ft}}$$

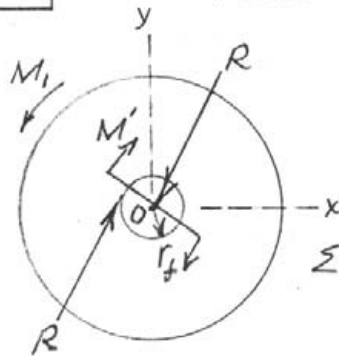
6/77

From Prob. 6/76, $R = 870.0 \text{ lb}$

$$\phi = 16.70^\circ$$

$$r_f = r \sin \phi = 0.359 \text{ in.}$$

$$M_1 = 8800 \text{ lb-in.}$$



$$\sum M_O = 0; M' + 870.0(0.359) - 8800 = 0$$

$$M' = 8800 - 313$$

$$= 8487 \text{ lb-in.}$$

$$\text{or } M' = \underline{707 \text{ lb-ft}}$$

$$\boxed{6/78} \quad \mu = 0.80 - kr : 0.50 = 0.80 - k(3), k = \frac{1}{10} \text{ in.}^{-1}$$

$$\text{So } \mu = 0.80 - \frac{r}{10} \quad (r \text{ in in.})$$

$$\text{Downward force } R = pA, \quad p = \frac{6+10}{\pi(3)^2} = \frac{16}{9\pi} \text{ lb/in.}^2$$

$$M_z = \int \mu p dA \times r = \int_0^{2\pi} \int_0^3 \left(0.80 - \frac{r}{10}\right) \frac{16r}{9\pi} r dr d\theta$$

$$= 2\pi \frac{16}{9\pi} \int_0^3 \left(0.80 - \frac{r}{10}\right) r^2 dr = \frac{32}{9} \left[\frac{0.80r^3}{3} - \frac{r^4}{40} \right]_0^3$$

$$= \underline{18.40 \text{ lb-in.}}$$

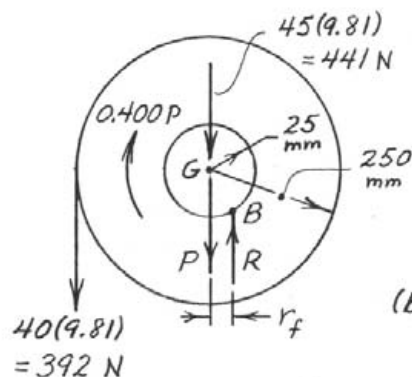
6/79

FBD of shaft and attached drum, with force P replacedby a force-couple system at G :

(a) $r_f = 0$:

$$\zeta + \sum M_G = 0: 392(0.250) - 0.400P = 0,$$

$$\underline{P = 245 \text{ N}}$$



(b) $\phi = \tan^{-1} 0.2 = 11.31^\circ$

$$r_f = r \sin \phi = 25 \sin 11.31^\circ = 4.90 \text{ mm}$$

$$\zeta + \sum M_B = 0: 392(0.250 + 0.00490) + 441(0.00490) + P(0.00490) - 0.400P = 0$$

$$\underline{P = 259 \text{ N}}$$

(This solution assumes that the bearing reaction can be represented by a single force R as shown above.)

6/8 $W_A = 200 \text{ lb}$

From $\uparrow \Sigma F_y = 0$,
 $N_{AB} = 200 \text{ lb}$, $N_B = 300 \text{ lb}$

$$\begin{cases} F_{AB_{\max}} = \mu_s N_{AB} = 0.5(200) = 100 \text{ lb} \\ F_{B_{\max}} = \mu_s N_B = 0.25(300) = 75 \text{ lb} \end{cases}$$

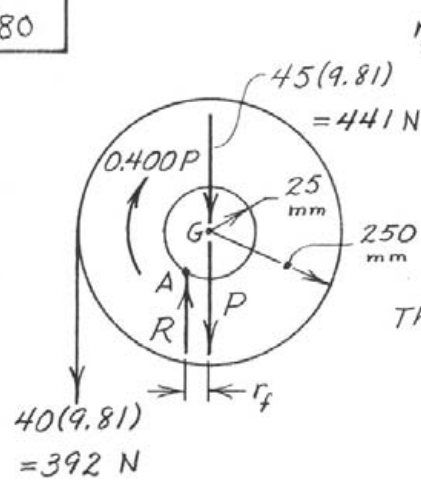
$W_B = 100 \text{ lb}$

(a) $P = 60 \text{ lb}$: Both blocks remain stationary, as
 $\rightarrow \Sigma F_x = 0$ yields $F_{AB} = 60 \text{ lb}$
 and $F_B = 60 \text{ lb}$, both less than the maximum

(b) $P = 80 \text{ lb}$: No relative motion between A and B, but sliding between B and the supporting surface; motion is to the right.

(c) $P = 120 \text{ lb}$: A slides to the right relative to B; $F_{AB} = \mu_k N_{AB} = 0.3(200) = 60 \text{ lb} < F_{B_{\max}}$,
 so B remains stationary.

6/80



$$r_f = 4.90 \text{ mm (from Prob. 6/79)}$$

$$\sum M_A = 0: 392(0.250 - 0.00490)$$

$$- 0.400P$$

$$- (441 + P)(0.00490) = 0$$

$$\underline{P = 232 \text{ N}}$$

The values are NOT symmetric:

$$\begin{cases} P_{\text{down}} = 232.18 \text{ N} \\ P_{\text{n.f.}} = 245.25 \text{ N} \\ P_{\text{up}} = 258.64 \text{ N} \end{cases}$$

$$6/81 \quad dM = (\mu p dA) r \quad \text{where } p = k/r^2$$

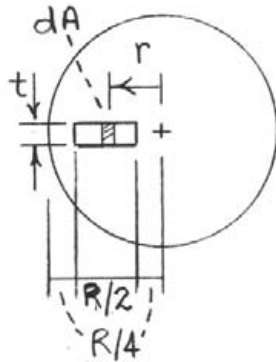
$$M = \int_0^{2\pi} \int_{r_i}^{r_o} \mu p r (r dr d\theta) = 2\pi \mu k \int_{r_i}^{r_o} dr = 2\pi \mu k (r_o - r_i)$$

$$\text{Also } L = \int p dA = \int_0^{2\pi} \int_{r_i}^{r_o} \frac{k}{r^2} r dr d\theta = 2\pi k \ln r \Big|_{r_i}^{r_o}$$

$$\text{or } L = 2\pi k \ln \frac{r_o}{r_i}, \quad 2\pi k = \frac{L}{\ln r_o/r_i}$$

$$\text{Thus } M = \mu L \frac{r_o - r_i}{\ln (r_o/r_i)}$$

6/82



(t small)

$$P = pA = p\left(t \frac{R}{2}\right)$$

$$M = \int \mu p r dA = \mu p \int_{R/4}^{3R/4} r t dr$$

$$= \mu p t \frac{r^2}{2} \Big|_{R/4}^{3R/4}$$

$$= \frac{\mu p t}{2} \left[\left(\frac{3R}{4}\right)^2 - \left(\frac{R}{4}\right)^2 \right]$$

$$= \frac{1}{4} \mu p t R^2$$

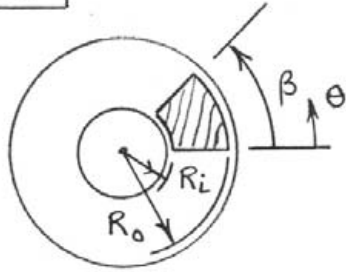
$$= \frac{1}{4} \mu \left(\frac{P}{t R/2} \right) t R^2 = \underline{\underline{\frac{1}{2} \mu P R}}$$

6/83 For constant pressure Eq. 6/5a gives

$$M = \frac{2}{3} \mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} = \frac{2}{3} (0.35)(1) \frac{(150)^3 - (75)^3}{(150)^2 - (75)^2} = 40.8 \text{ N}\cdot\text{m}$$

For wheel $\sum M = 0$; $F(0.3) - 40.8 = 0$, $F = 136.1 \text{ N}$

6/84



$$P = pA = p \int_0^\beta \int_{R_i}^{R_o} r dr d\theta$$

$$= \frac{p}{2} \int_0^\beta (R_o^2 - R_i^2) d\theta$$

$$= \frac{p}{2} (R_o^2 - R_i^2) \beta$$

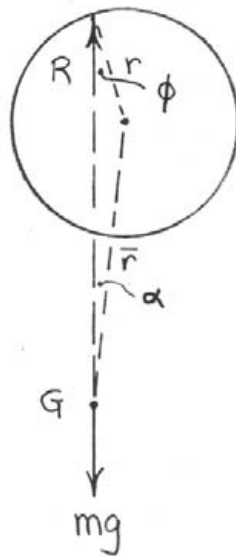
$$M = 2 \int \mu p r dA = 2 \mu p \int_0^\beta \int_{R_i}^{R_o} r^2 dr d\theta$$

$$= \frac{2 \mu p}{3} (R_o^3 - R_i^3) \beta$$

$$= \frac{2 \mu}{3} \frac{2 P}{(R_o^2 - R_i^2) \beta} (R_o^3 - R_i^3) \beta$$

$$= \frac{4 \mu p}{3} \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$$

Same form as Eq. 6/5a except for factor of 2 for 2 pads. No β dependence. Pressure variation with θ would not change the moment M .



$$r \sin \phi = \bar{r} \sin \alpha$$

$$\mu = \tan \phi = \frac{\bar{r} \sin \alpha}{\sqrt{r^2 - \bar{r}^2 \sin^2 \alpha}}$$

or

$$\mu = \frac{1}{\sqrt{\left(\frac{d/2}{\bar{r} \sin \alpha}\right)^2 - 1}}$$

$$6/86 \quad p = p_0 \left(1 - \frac{r}{2a}\right); \quad dA = 2\pi r dr$$

$$L = \int p dA = \int_0^a p_0 \left(1 - \frac{r}{2a}\right) 2\pi r dr = 2\pi p_0 \left[\frac{r^2}{2} - \frac{r^3}{6a} \right]_0^a$$

$$= \frac{2}{3} \pi p_0 a^2 \quad \text{so} \quad p_0 = \frac{3L}{2\pi a^2}$$

$$M = \int \mu p r dA = \int_0^a \mu p_0 \left(r - \frac{r^2}{2a}\right) 2\pi r dr = 2\pi \mu p_0 \left[\frac{r^3}{3} - \frac{r^4}{8a} \right]_0^a$$

$$= \frac{5}{12} \pi \mu p_0 a^3 = \frac{5}{8} \mu L a$$

$$6/87 \quad L = \frac{1}{4} (960 - 4 \times 40) = 200 \text{ lb}$$

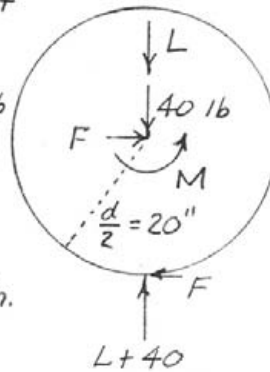
$$P = 4F$$

$$F = \frac{1}{4} 16 = 4 \text{ lb}$$

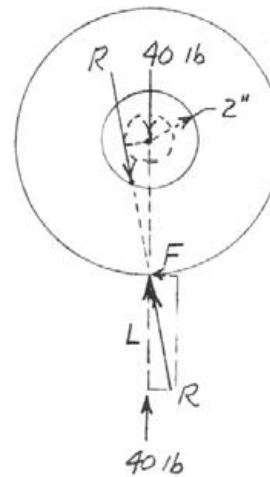
$$M = F \frac{d}{2}$$

$$= 4(20)$$

$$= 80 \text{ lb-in.}$$



\equiv

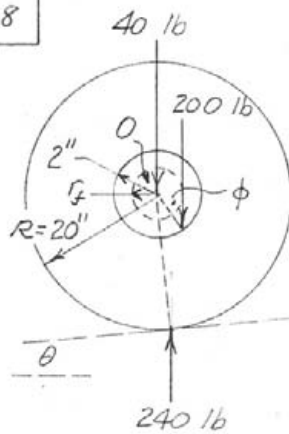


$$M = Rr \sin \phi \approx Lr \sin \phi$$

$$80 \approx 200(2) \sin \phi$$

$$\phi = \sin^{-1} 0.2 = 11.54^\circ, \quad \mu = \tan \phi = \underline{0.204}$$

6/88



$$r_f = r \sin \phi = 2 \sin (\tan^{-1} 0.2)$$

$$= 2 \sin^{-1} 11.31^\circ = 2 (0.1961)$$

$$= 0.3922 \text{ in.}$$

$$\sum M_O = 0;$$

$$200 r_f - 240 R \sin \theta = 0$$

$$\sin \theta = \frac{200 \times 0.3922}{240 \times 20} = 0.01634$$

$$\theta = 0.936^\circ$$

$$\frac{6}{89} \quad \alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{11}{2\pi \frac{120}{2}} = 1.671^\circ$$

$$\phi = \tan^{-1} 0.15 = 8.53^\circ$$

Screw: (a) Raise : $M_s = Wr \tan(\alpha + \phi)$

$$= \frac{1}{2} (10+3)(9.81) \frac{120}{2} \tan(1.671^\circ + 8.53^\circ) = 689 \text{ N}\cdot\text{m}$$

(b) Lower : $M_s = Wr \tan(\phi - \alpha)$

$$= \frac{1}{2} (10+3)(9.81) \frac{120}{2} \tan(8.53^\circ - 1.671^\circ) = 460 \text{ N}\cdot\text{m}$$

Collar bearing : $M_c = \frac{2}{3} \mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$

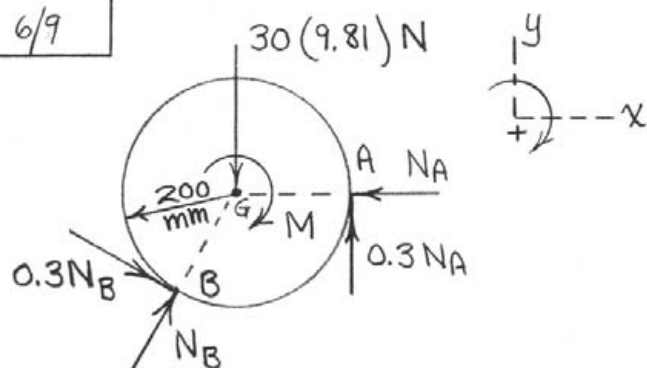
$$= \frac{2}{3} (0.15) \left(\frac{10+3}{2} + 0.9 \right) (9.81) \frac{(250/2)^3 - (125/2)^3}{(250/2)^2 - (125/2)^2}$$

$$= 1059 \text{ N}\cdot\text{m}$$

Total moment per screw

$$\begin{cases} (a) \quad M = 689 + 1059 = \underline{1747 \text{ N}\cdot\text{m}} \\ (b) \quad M = 460 + 1059 = \underline{1519 \text{ N}\cdot\text{m}} \end{cases}$$

6/9



$$\left\{ \begin{array}{l} \sum M_G = 0: M - 0.3(N_A + N_B) \cdot 0.2 = 0 \end{array} \right. \quad (1)$$

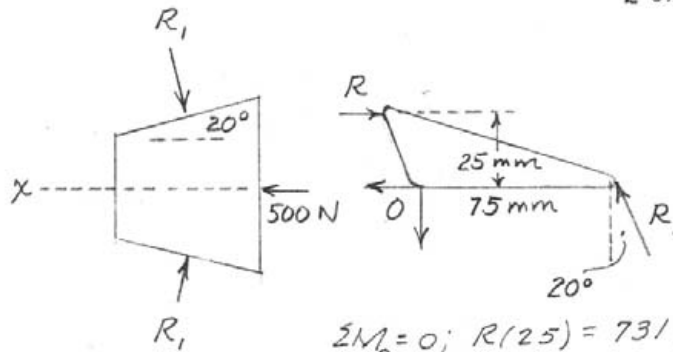
$$\left\{ \begin{array}{l} \sum F_x = 0: N_B \sin 30^\circ + 0.3 N_B \cos 30^\circ - N_A = 0 \end{array} \right. \quad (2)$$

$$\left\{ \begin{array}{l} \sum F_y = 0: N_B \cos 30^\circ - 0.3 N_B \sin 30^\circ - 30(9.81) \\ \quad + 0.3 N_A = 0 \end{array} \right. \quad (3)$$

Solution of Eqs. (1)-(3):

$$\begin{cases} N_B = 312 \text{ N} \\ N_A = 237 \text{ N} \\ M = 32.9 \text{ N}\cdot\text{m} \end{cases}$$

$$6/90 \quad \Sigma F_x = 0; 2R_1 \sin 20^\circ = 500, R_1 = \frac{500}{2 \sin 20^\circ} = 731 \text{ N}$$



$$\Sigma M_O = 0; R(25) = 731(75 \cos 20^\circ)$$

$$R = 2060 \text{ N}$$

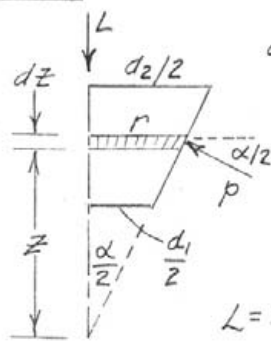
For 5 pairs of surfaces with uniform pressure

$$M = 5\left(\frac{2}{3}\right) \mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} = \frac{10}{3}(0.15)(4120) \frac{0.15^3 - 0.05^3}{0.15^2 - 0.05^2}$$

$$\underline{M = 335 \text{ N}\cdot\text{m}}$$

► 6/91

$$dL = p(2\pi r) ds \sin \frac{\alpha}{2} \text{ where } ds = dz / \cos \frac{\alpha}{2}$$



$$dL = 2\pi p \sin \frac{\alpha}{2} (z \tan \frac{\alpha}{2}) \frac{dz}{\cos \frac{\alpha}{2}} \left[dz \sqrt{\frac{1}{\cos^2 \frac{\alpha}{2}}} \right]$$

$$= 2\pi p \tan^2 \frac{\alpha}{2} z dz$$

$$L = 2\pi p \tan^2 \frac{\alpha}{2} \int_{z_1}^{z_2} z dz \text{ where } z_1 = \frac{d_1/2}{\tan \alpha/2}$$

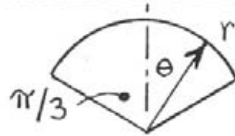
$$z_2 = \frac{d_2/2}{\tan \alpha/2}$$

$$L = \frac{\pi p (d_2^2 - d_1^2)}{4}$$

$$M = \int r \mu p dA = \mu p \int_{z_1}^{z_2} (z \tan \frac{\alpha}{2}) (2\pi z \tan \frac{\alpha}{2}) \frac{dz}{\cos \alpha/2}$$

$$= 2\pi \mu p \frac{\tan^2 \frac{\alpha}{2}}{\cos \alpha/2} \int_{z_1}^{z_2} z^2 dz = 2\pi \mu p \frac{1}{24 \sin \frac{\alpha}{2}} (d_2^3 - d_1^3)$$

$$M = \frac{\mu L}{3 \sin \frac{\alpha}{2}} \frac{d_2^3 - d_1^3}{d_2^2 - d_1^2}$$



$$dM = \mu p dA (r \sin \theta) = \mu p (r d\theta) (2\pi r \sin \theta) r \sin \theta$$

$$= 2\pi \mu r^3 p_0 \cos \theta \sin^2 \theta d\theta$$

$$M = 2\pi \mu r^3 p_0 \int_0^{\pi/3} \cos \theta \sin^2 \theta d\theta$$

$$= 2\pi \mu r^3 p_0 \left[\frac{\sin^3 \theta}{3} \right]_0^{\pi/3} = \frac{\sqrt{3}}{4} \pi \mu r^3 p_0$$

$$\text{But } L = \int p \cos \theta dA = \int_0^{\pi/3} p_0 \cos^2 \theta (r d\theta) (2\pi r \sin \theta)$$

$$= 2\pi r^2 p_0 \int_0^{\pi/3} \cos^2 \theta \sin \theta d\theta = 2\pi r^2 p_0 \left(-\frac{\cos^3 \theta}{3} \right)_0^{\pi/3}$$

$$= \frac{7}{12} \pi r^2 p_0$$

Substitute $\pi r^2 p_0 = \frac{12}{7} L$ to obtain

$$M = \frac{\sqrt{3}}{4} \mu r \left(\frac{12L}{7} \right) = \underline{\underline{\frac{3\sqrt{3}}{7} \mu r L}}$$

6/93

$$\frac{T_2}{T_1} = e^{\mu\beta} : \quad \frac{mg}{mg/10} = e^{\mu(3\pi)}, \quad \underline{\mu = 0.244}$$

6/94 Use $\frac{T_2}{T_1} = e^{\mu\beta}$, where $\beta = \frac{\pi}{2}$

(a) $\frac{P}{W} = e^{0.4(\pi/2)}$, $P = 1.874 W$

(b) $\frac{W}{P} = e^{0.4(\pi/2)}$, $P = 0.533 W$

$$\begin{array}{|l} 6/95 \\ \hline \end{array} \quad \frac{T_2}{T_1} = e^{\mu\beta}, \quad \frac{T}{200} = e^{0.30(5\pi/2)} \quad \text{where } \beta = \frac{5}{4}(2\pi) \text{ rad}$$

$$\frac{T}{200} = e^{2.356} = 10.55, \quad T = 10.55(200) = 2110 \text{ N}$$

or $T = 2.11 \text{ kN}$

6/96

$$T_2 = T_1 e^{\mu\beta}$$

$$18\,000 = 240 e^{0.30\beta}, \quad e^{0.30\beta} = 75.0$$

$$0.30\beta = 4.32, \quad \beta = 14.39 \text{ rad}$$

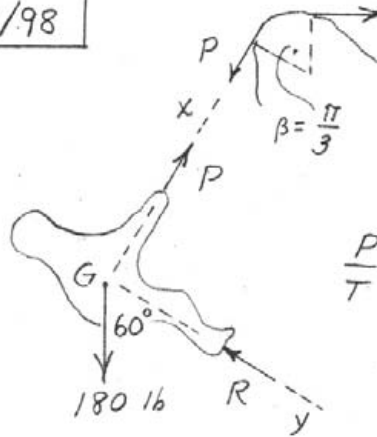
$$\text{No. of turns } n = \frac{\beta}{2\pi} = \frac{14.39}{2\pi} = \underline{2.29 \text{ turns}}$$

$$6/97 \quad T_2 = T_1 e^{\mu\beta}, \quad T_2 = 50 \text{ lb}, T_1 = 20 \text{ lb}, \beta = \pi$$

$$\text{So } \frac{50}{20} = e^{\pi\mu}, \quad e^{\pi\mu} = 2.5, \quad \pi\mu = \ln 2.5$$

$$\pi\mu = 0.9163, \quad \underline{\mu = 0.292}$$

6/98



Equil. of climber, $\Sigma F_x = 0$

gives $P = 180 \sin 60^\circ$

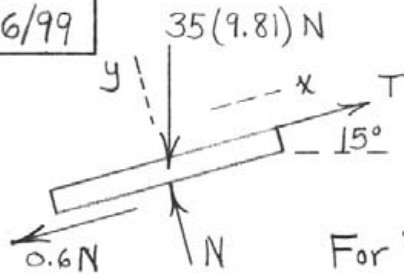
$$= 155.9 \text{ lb}$$

$$\frac{P}{T} = e^{\mu\beta}, \quad \frac{155.9}{75} = e^{\pi\mu/3}$$

$$\frac{\pi}{3}\mu = \ln 2.078 = 0.732$$

$$\mu = 3(0.732)/\pi = \underline{0.699}$$

6/99



$$\Sigma F_y = 0 :$$

$$N - 35(9.81) \cos 15^\circ = 0$$

$$N = 332 \text{ N}$$

For impending motion up incline:

$$-0.6(332) - 35(9.81) \sin 15^\circ + T = 0, \quad T = 288 \text{ N}$$

$$\text{Cord friction : } \frac{P}{288} = e^{0.4 \left[15 \frac{\pi}{180} \right]}$$

$$\underline{P = 320 \text{ N}}$$

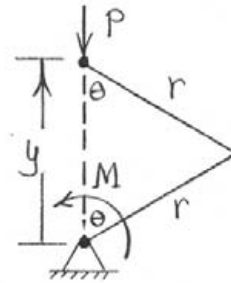
7/1

$$\delta U = 0: -M\delta\theta - P\delta y = 0$$

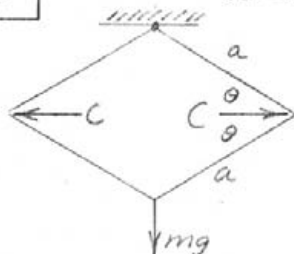
$$y = 2r\cos\theta, \quad \delta y = -2r\sin\theta\delta\theta$$

$$\text{So } M\delta\theta = P(2r\sin\theta\delta\theta)$$

$$\underline{M = 2Pr\sin\theta}$$



7/10

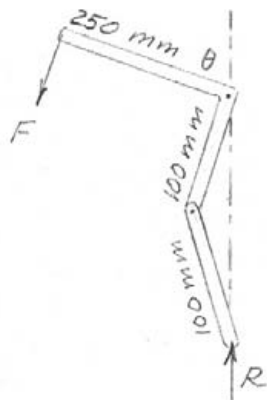


$$\delta U = 0; 2C \delta(a \cos \theta) + mg \delta(2a \sin \theta) = 0$$

$$-2Ca \sin \theta \delta \theta + 2mga \cos \theta \delta \theta = 0$$

$$C = mg \cot \theta$$

7/11



$$\delta U = 0$$

$$F(250 \delta\theta) - R \delta(2[100 \sin\theta]) = 0$$

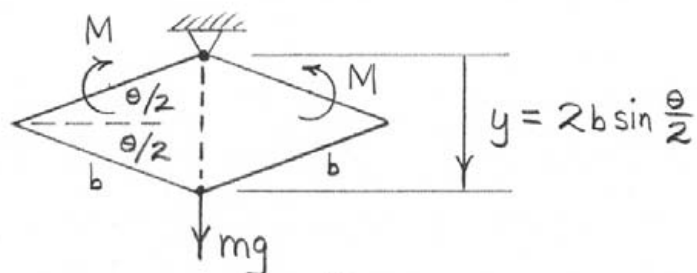
$$250F \delta\theta = 200R \cos\theta \delta\theta$$

$$\underline{F = 0.8R \cos\theta}$$

$$\boxed{7/12} \quad e = \frac{\text{output work}}{\text{input work}}$$

To raise, $0.75 = \frac{250(1/4)}{P(1)}, \quad \underline{P = 83.3 \text{ lb}}$

To lower, $0.75 = \frac{P'(1)}{250(1/4)}, \quad \underline{P' = 46.9 \text{ lb}}$

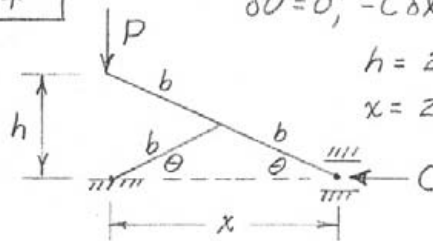


$$\delta U = 0 : mg \delta y - 2M \delta\left(\frac{\theta}{2}\right) = 0$$

$$\delta y = b \cos \frac{\theta}{2} \delta \theta, \text{ so } mgb \cos \frac{\theta}{2} \delta \theta = 2\left(\frac{1}{2}\right) M \delta \theta$$

$$\underline{M = mgb \cos \frac{\theta}{2}}$$

7/14



$$\delta U = 0; -C\delta x - P\delta h = 0$$

$$h = 2b \sin \theta, \delta h = 2b \cos \theta \delta \theta$$

$$x = 2b \cos \theta, \delta x = -2b \sin \theta \delta \theta$$

$$\text{Thus } -C(-2b \sin \theta \delta \theta) - P(2b \cos \theta \delta \theta) = 0$$

$$C \sin \theta = P \cos \theta, C = P \cot \theta$$

$$\text{But } \cot \theta = \sqrt{4b^2 - h^2}/h = \sqrt{(2b/h)^2 - 1}, \text{ so } \underline{C = P\sqrt{(2b/h)^2 - 1}}$$

7/15

$$\delta U = 0: 160 F \delta \theta - 0.4(160 F \delta \theta) - 100(9.81) \left(150 \delta \frac{\theta}{25}\right) = 0$$

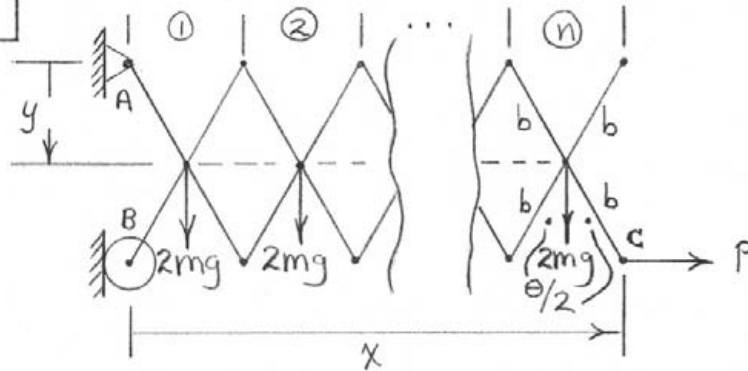
$$0.6(160)F = 981(6), \quad \underline{F = 61.3 \text{ N}}$$

7/16 Let $\delta\theta$ = virtual angle of input rotation

Then $\delta\theta/40$ = " " " output "

$$c = \frac{\text{output work}}{\text{input work}} = \frac{1180(\delta\theta/40)}{30\delta\theta} = \frac{1180}{30(40)} = \underline{0.983}$$

7/17



$$y = b \cos \frac{\theta}{2}, \quad \delta y = -\frac{b}{2} \sin \frac{\theta}{2} \delta \theta$$

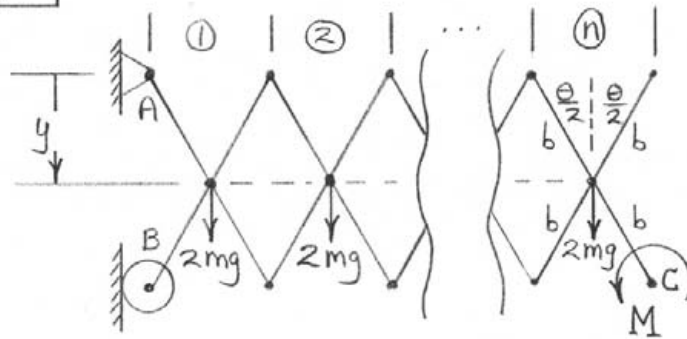
$$x = n(2b \sin \frac{\theta}{2}), \quad \delta x = nb \cos \frac{\theta}{2} \delta \theta$$

$$\delta U = 0: P \delta x + n(2mg) \delta y = 0$$

$$P(nb \cos \frac{\theta}{2} \delta \theta) = -2nmg(-\frac{b}{2} \sin \frac{\theta}{2} \delta \theta)$$

$$P = mg \tan \frac{\theta}{2}$$

P does not depend on the number n of sections present.



$$y = b \cos \frac{\theta}{2}, \quad \delta y = -\frac{b}{2} \sin \frac{\theta}{2} \delta \theta$$

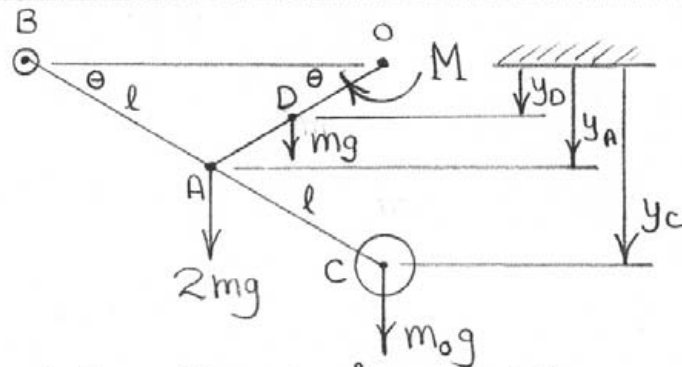
$$\delta U = 0 : M \delta \left(\frac{\theta}{2} \right) + n(2mg) \delta y = 0$$

$$\frac{M}{2} \delta \theta = -2nmg \left(-\frac{b}{2} \sin \frac{\theta}{2} \delta \theta \right)$$

$$\underline{M = 2nmg b \sin \frac{\theta}{2}}$$

M does depend on the number n of sections present.

7/19



$$y_D = \frac{l}{2} \sin \theta, \quad \delta y_D = \frac{l}{2} \cos \theta \delta \theta$$

$$y_A = l \sin \theta, \quad \delta y_A = l \cos \theta \delta \theta$$

$$y_C = 2l \sin \theta, \quad \delta y_C = 2l \cos \theta \delta \theta$$

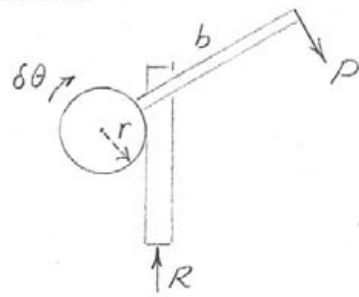
$$\delta U = 0: -M \delta \theta + mg \delta y_D + 2mg \delta y_A + m_0 g \delta y_C = 0$$

$$-M \delta \theta + mg \left(\frac{l}{2} \cos \theta \delta \theta \right) + 2mg (l \cos \theta \delta \theta) + m_0 g (2l \cos \theta \delta \theta) = 0$$

$$\Rightarrow M = \left(\frac{5}{2} m + 2m_0 \right) g l \cos \theta$$

$$\text{For } \theta = 30^\circ: \underline{M = \left(\frac{5}{4} m + m_0 \right) g l \sqrt{3}}$$

7/2



$$\delta U = 0; Pb\delta\theta - Rr\delta\theta = 0$$

$$\underline{R = P\frac{b}{r}}$$

$$\frac{7}{20} \quad e = \frac{\text{output work}}{\text{input work}}$$

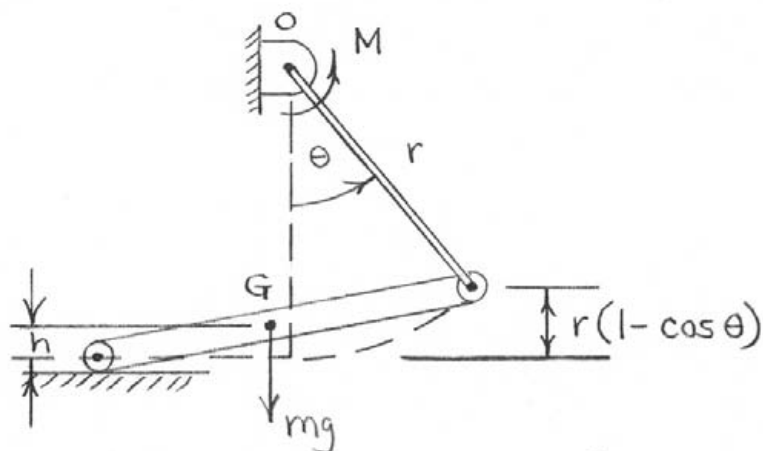
Let $\delta\theta$ = virtual crank angle, radians

δh = " movement of lifting pad, inches

where $\frac{\delta\theta}{\delta h} = \frac{12(2\pi)}{1}$, $\delta\theta = 24\pi \delta h$

To raise, $e = \frac{L \delta h}{Fr \delta\theta} = \frac{2700 \delta h}{10(6) 24\pi \delta h} = \underline{0.597}$

7/21



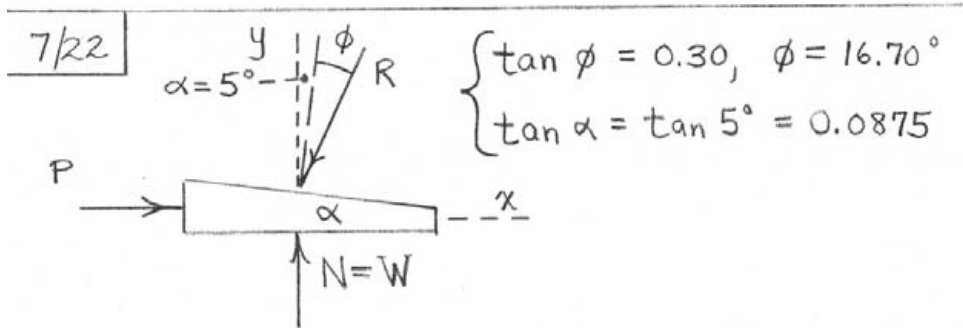
$$\delta U = 0: M \delta \theta - mg \delta h = 0, \quad h = \frac{r}{2}(1 - \cos \theta)$$

$$\delta h = \frac{r}{2} \sin \theta \delta \theta$$

$$\therefore M \delta \theta - mg \left(\frac{r}{2} \sin \theta \delta \theta \right) = 0$$

$$\left(M - \frac{mgr}{2} \sin \theta \right) \delta \theta = 0, \quad \theta = \sin^{-1} \left(\frac{2M}{mgr} \right)$$

$$M_{\max} = \frac{mgr}{2} @ \theta = 90^\circ. \quad \text{No equilibrium for } M > \frac{mgr}{2}$$



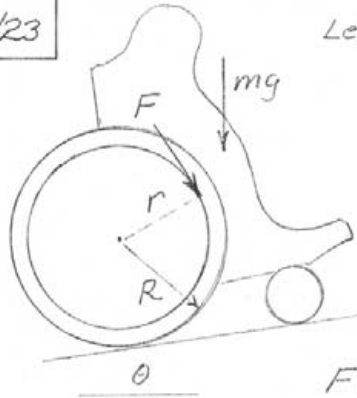
$$\left. \begin{aligned} \Sigma F_x = 0: P &= R \sin(\phi + \alpha) \\ \Sigma F_y = 0: W &= R \cos(\phi + \alpha) \end{aligned} \right\} \tan(\phi + \alpha) = \frac{P}{W}$$

Work input: $P \delta x$ } where $\delta y = \delta x \tan \alpha$
 Work output: $W \delta y$ }

$$e = \frac{W \delta y}{P \delta x} = \frac{\tan \alpha}{\tan(\phi + \alpha)} = \frac{\tan 5^\circ}{\tan(16.70^\circ + 5^\circ)}$$

or $e = 0.220$

7/23



Let β = angle through which wheel turns

s = corresponding displacement along incline.

$$s = R\beta \text{ so } \delta s = R\delta\beta$$

$$\delta U = 0$$

$$Fr\delta\beta - mg\delta s \sin\theta = 0$$

$$Fr\delta\beta = mg R \sin\theta \delta\beta$$

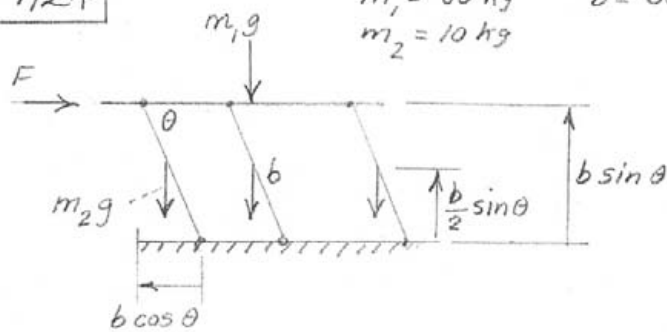
$$F = mg \frac{R}{r} \sin\theta$$

7/24

$$m_1 = 80 \text{ kg}$$

$$m_2 = 10 \text{ kg}$$

$$b = 600 \text{ mm}$$



$$\delta U = 0; -F \delta(b \cos \theta) - m_1 g \delta(b \sin \theta) - 3m_2 g \delta\left(\frac{b}{2} \sin \theta\right) = 0$$

$$F b \sin \theta \delta \theta = m_1 g b \cos \theta \delta \theta + \frac{3}{2} m_2 g b \cos \theta \delta \theta$$

$$F = g \cot \theta \left(m_1 + \frac{3}{2} m_2\right)$$

$$= 9.81 \left(80 + \frac{3}{2} 10\right) \cot \theta = \underline{932 \cot \theta \text{ N}}$$

Solution by force and moment equilibrium would require dismemberment with four FBD's and eventual elimination of unwanted forces and dimensions

7/25

$$\delta U = 0; \quad M \delta \theta - mg \delta h = 0$$

$$h = 2b \sin \theta, \quad \delta h = 2b \cos \theta \delta \theta$$

$$\text{So } M \delta \theta = mg (2b \cos \theta) \delta \theta$$

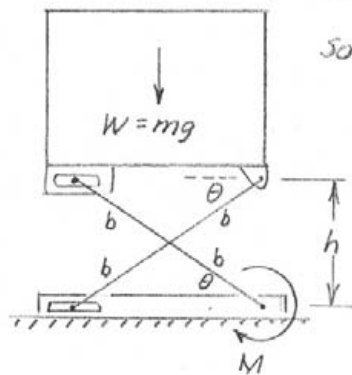
$$M = 2mg b \cos \theta$$

$$\text{But since } \sin \theta = \frac{h}{2b},$$

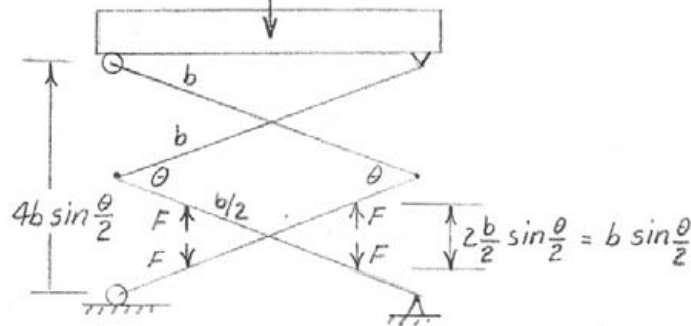
$$\cos \theta = \sqrt{1 - \left(\frac{h}{2b}\right)^2}$$

Thus

$$M = 2mg b \sqrt{1 - \left(\frac{h}{2b}\right)^2}$$



7/26

 mg $F = pA$ 

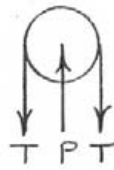
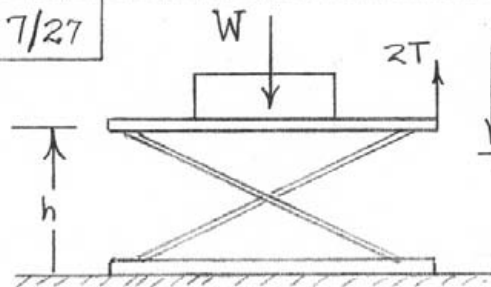
Work done by each cylinder is $F \delta(b \sin \frac{\theta}{2})$
 $= \frac{Fb}{2} \cos \frac{\theta}{2} \delta \theta$

$$\delta U = 0; -mg \delta(4b \sin \frac{\theta}{2}) + 2 \frac{Fb}{2} \cos \frac{\theta}{2} \delta \theta = 0$$

$$Fb \cos \frac{\theta}{2} = 2mgb \cos \frac{\theta}{2}, F = pA = 2mg$$

so $p = 2mg/A$ independent of b & θ

7/27

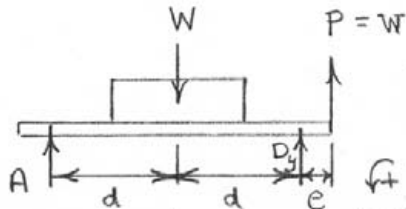


$$\delta U = 0: 2T\delta h$$

$$-W\delta h = 0$$

$$T = W/2$$

Pulley: $P = 2T$, so $P = W$
(independent of h)



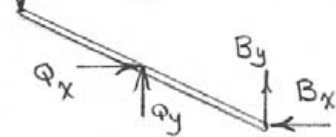
D_y and A form a clockwise couple

$$D_y = -A \quad (\text{Note } D_x = 0)$$

$$\sum M = 0: W(d+e) - A(2d) = 0$$

$$A = \frac{W}{2} \left(1 + \frac{e}{d} \right) = \frac{W}{2} \left(1 + \frac{2e}{\sqrt{l^2 - h^2}} \right)$$

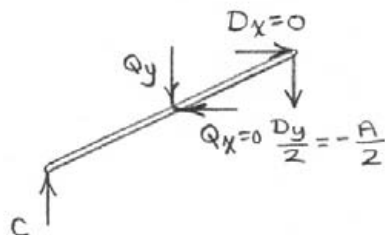
(Note: $d = \sqrt{l^2 - h^2}/2$)



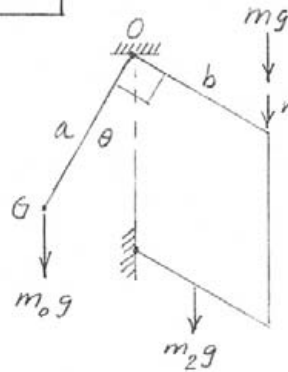
$$CD: \sum M_C = 0:$$

$$Q_y d - \frac{A}{2} (2d) = 0$$

$$Q = Q_y = A = \frac{W}{2} \left(1 + \frac{2e}{\sqrt{l^2 - h^2}} \right)$$



(From drawing, $d > e$,
so $Q \approx \frac{W}{2}$)



$$\delta U = 0; m_0 g \delta(a \cos \theta)$$

$$+ (m + m_1) g \delta(b \sin \theta)$$

$$m_1 g + m_2 g \delta\left(\frac{b}{2} \sin \theta\right) = 0$$

$$-m_0 a \sin \theta \delta \theta + (m + m_1) b \cos \theta \delta \theta$$

$$+ \frac{m_2 b}{2} \cos \theta \delta \theta = 0$$

$$m_0 a \tan \theta = m b + \left(m_1 + \frac{m_2}{2}\right) b \quad \text{--- (1)}$$

when $\theta = \theta_0$, $m g = 0$ so

$$m_0 a \tan \theta_0 = \left(m_1 + \frac{m_2}{2}\right) b \quad \text{--- (2)}$$

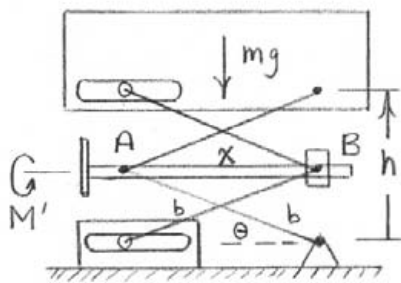
Eliminate m_1 & m_2 from Eqs. (1) & (2) & get

$$m_0 a \tan \theta = m b + m_0 a \tan \theta_0$$

$$m = \frac{a}{b} m_0 (\tan \theta - \tan \theta_0)$$

7/29 M' = necessary moment without friction

M' = necessary moment without friction



Let β = angle through which screw turns

$$\delta U = 0: M' \delta \beta - mg \delta h = 0$$

$$\frac{L}{2\pi} = \frac{-\delta(x)}{\delta\beta}, \quad \delta\beta = \frac{2\pi}{L}(-\delta x)$$

$$x = 2b \cos \theta, \quad \delta x = -2b \sin \theta \delta \theta$$

$$\delta\beta = \frac{4\pi b}{\lambda} \sin\theta \delta\theta$$

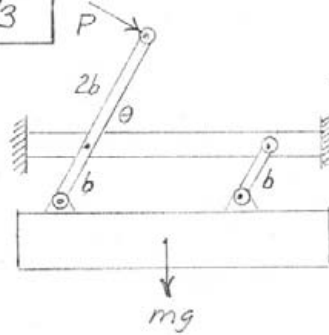
$$h = 4b \sin \theta, \quad \delta h = 4b \cos \theta \delta \theta$$

Thus $M' \frac{4\pi b}{L} \sin \theta \delta \theta - mg (4b \cos \theta \delta \theta) = 0$

$$M' = \frac{mgL}{r} \cot \theta$$

$$M = M_f + \frac{mgL}{r} \cot \theta$$

7/3



$$\delta U = 0;$$

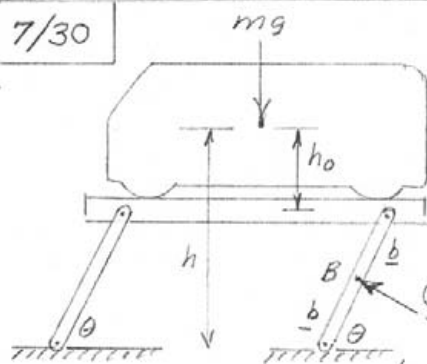
$$-P(2b \delta \theta) + mg \delta(b \sin \theta) = 0$$

$$2Pb \delta \theta = mg b \cos \theta \delta \theta$$

$$\cos \theta = \frac{2P}{mg}$$

$$\theta = \cos^{-1} \frac{2P}{mg}$$

7/30



$$\text{Let } \overline{AB} = l$$

$$\delta U = 0; C \delta l - mg \delta h = 0$$

$$l^2 = (b \sin \theta)^2 + (L - b \cos \theta)^2$$

$$\begin{aligned} 2l \delta l &= 2b^2 \sin \theta \cos \theta \delta \theta \\ &\quad + 2(L - b \cos \theta)(b \sin \theta \delta \theta) \\ &= 2Lb \sin \theta \delta \theta \end{aligned}$$

$$\delta l = \frac{Lb}{l} \sin \theta \delta \theta$$

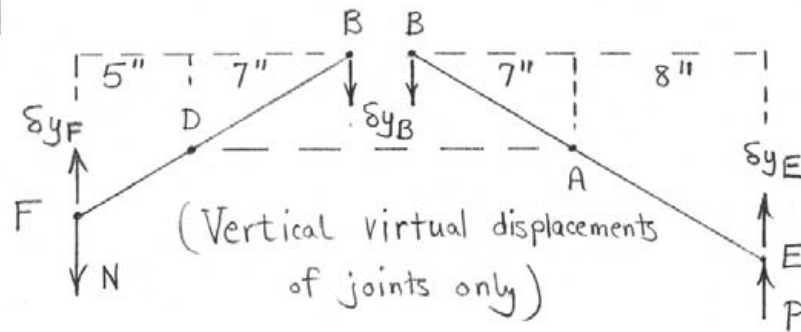
$$h = 2b \sin \theta + h_0, \quad \delta h = 2b \cos \theta \delta \theta + 0$$

$$\text{Thus } C \frac{Lb}{l} \sin \theta \delta \theta - mg (2b \cos \theta \delta \theta) = 0$$

$$C = 2mg \frac{l}{L} \cot \theta = \frac{2mg}{L} \sqrt{(b \sin \theta)^2 + (L - b \cos \theta)^2} \cot \theta$$

$$C = 2mg \sqrt{1 + \left(\frac{b}{L}\right)^2 - 2\frac{b}{L} \cos \theta} \cot \theta$$

7/31



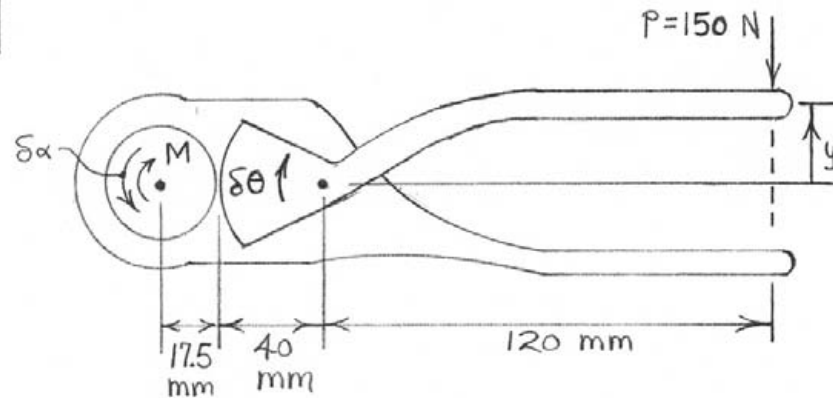
$$\delta y_B = \frac{7}{8} \delta y_E, \quad \delta y_F = \frac{5}{7} \delta y_B$$

$$\text{So } \delta y_F = \frac{5}{7} \cdot \frac{7}{8} \delta y_E = \frac{5}{8} \delta y_E$$

$$\delta U = 0: P \delta y_E - N \delta y_F = 0$$

$$P \delta y_E = N \left(\frac{5}{8} \delta y_E \right)$$

$$N = \frac{8}{5} P = \underline{1.6 P}$$



$\delta\alpha$ = rotation of socket on bolt head

$\delta\theta$ = rotation of upper handle (lower handle and frame taken as fixed)

$$17.5 \delta\alpha = 40 \delta\theta, \quad \delta y = -120 \delta\theta$$

$$\delta U = 0 : -M \delta\alpha + P(-\delta y) = 0$$

$$M \left(\frac{40}{17.5} \delta\theta \right) = 150 (120 \delta\theta)$$

$$M = 7880 \text{ N}\cdot\text{mm} \text{ or } \underline{M = 7.88 \text{ N}\cdot\text{m}}$$

7/33

$$\delta U = 0; \quad -P\delta x - W\delta h = 0$$

$$W = 250(9.81) \text{ N} \quad x^2 + y^2 = l^2, \quad x\delta x = -y\delta y$$

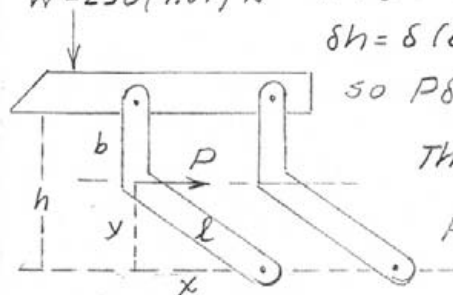
$$\delta h = \delta(b + y) = \delta y$$

$$\text{so } P\delta x = -W\delta y = -W\left(-\frac{x}{y}\right)\delta x$$

$$\text{Thus } P = W\frac{x}{y}$$

$$P = 250(9.81) \frac{500}{350} = 3500 \text{ N}$$

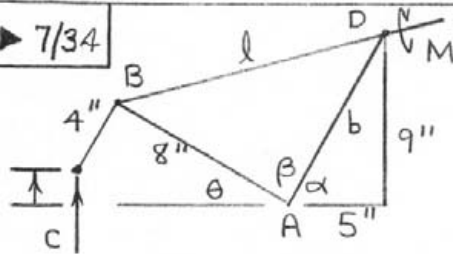
$$\text{or } \underline{P = 3.5 \text{ kN}}$$



$$h = 650 \text{ mm}, \quad b = 300 \text{ mm}$$

$$y = 350 \text{ mm}, \quad x = 500 \text{ mm}$$

► 7/34



$$\delta U = 0 : C \delta h + M \delta(2\pi \delta) = 0$$

where $\delta \delta$ = screw rotation in rev.

$$\text{With } L = \text{lead} = \frac{1}{6} \text{ in.}$$

$$\delta l = L \delta \delta = \frac{1}{6} \delta \delta \text{ in.}$$

$$b = \sqrt{5^2 + 9^2} = 10.30 \text{ in.}$$

$$h = 8 \sin \theta - 4 \cos \theta, \quad \delta h = (8 \cos \theta + 4 \sin \theta) \delta \theta$$

$$l^2 = 8^2 + b^2 - 2(8)b \cos \beta, \quad 2l \delta l = 16b \sin \beta \delta \beta$$

$$\text{Because } \alpha = \text{constant}, \quad \delta \beta = -\delta \theta : l \delta l = -8b \sin \beta \delta \theta$$

$$\text{Thus } C(8 \cos \theta + 4 \sin \theta) \delta \theta + M 2\pi 6 \frac{-8b}{l} \sin \beta \delta \theta = 0$$

$$C = 2\pi M \frac{12b}{l} \frac{\sin \beta}{2 \cos \theta + \sin \theta}$$

$$\text{For } \theta = 30^\circ, \quad \beta = 180^\circ - 30^\circ - \tan^{-1} \frac{9}{5} = 89.1^\circ$$

$$l = \sqrt{8^2 + 5^2 + 9^2 - 16(10.30)(0.0165)} = 12.93 \text{ in.}$$

$$C = 2\pi \frac{12(10.30) \sin 89.1^\circ}{12.93 (2 \cos 30^\circ + \sin 30^\circ)} M = 26.9 M$$

$$(M \text{ in lb-in.}, C \text{ in lb})$$

7/35

$$V = 6x^3 - 9x^2 - 7$$

$$\frac{dV}{dx} = 18x^2 - 18x = 0 \text{ for equil. } \underline{x=0 \text{ or } x=1}$$

$$\frac{d^2V}{dx^2} = 36x - 18 = -18 \text{ for } \underline{x=0 \text{ so unstable}}$$

$$= +18 \text{ " } \underline{x=1 \text{ so stable}}$$

7/36 $\delta =$ initial spring compression

$$V = V_g + V_e = mg \frac{L}{2} \cos \theta + \frac{1}{2} k (\delta + L \sin \theta)^2 + \frac{1}{2} k (\delta - L \sin \theta)^2$$

$$= \frac{1}{2} mgL \cos \theta + k (\delta^2 + L^2 \sin^2 \theta)$$

$$\frac{dV}{d\theta} = -\frac{1}{2} mgL \sin \theta + 2kL^2 \sin \theta \cos \theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{1}{2} mgL \cos \theta + 2kL^2 \cos 2\theta$$

For equilibrium, $\frac{dV}{d\theta} = 0$, so

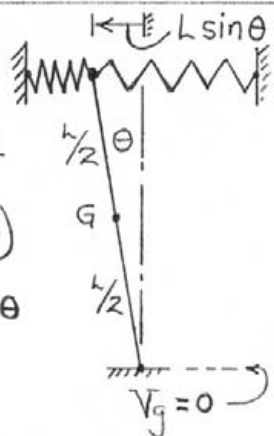
$$(-\frac{1}{2} mg + 2kL \cos \theta) \sin \theta = 0$$

$$\Rightarrow \theta = 0 \text{ or } \cos \theta = \frac{mg}{4kL}$$

$$\text{For } \theta = 0, \frac{d^2V}{d\theta^2} = -\frac{1}{2} mgL + 2kL^2$$

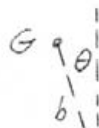
$$> 0 \text{ (Stable) if } 2kL^2 > \frac{1}{2} mgL$$

$$\text{So } k_{\min} = \frac{mg}{4L}$$



7/37

$$V = V_g = mgb \cos \theta$$



$$\frac{dV}{d\theta} = -mgb \sin \theta = 0 \text{ for equil.}$$

$$\theta = 0^\circ \text{ or } \theta = 180^\circ$$

$$V_g = 0 \quad \frac{d^2V}{d\theta^2} = -mgb \cos \theta$$

$$\theta = 0, \quad \frac{d^2V}{d\theta^2} = -mgb \text{ so } \underline{\text{unstable}}$$

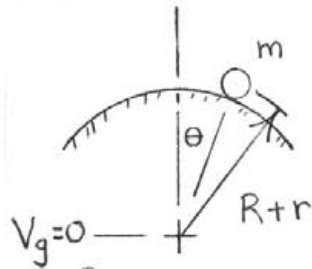
$$\theta = 180^\circ, \quad \frac{d^2V}{d\theta^2} = -mgb(-1) = +mgb \quad \underline{\text{stable}}$$

7/38

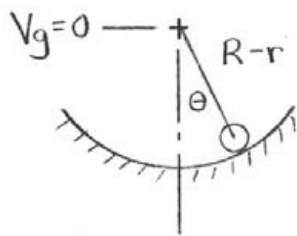
$$V = V_g = (R+r) \cos \theta$$

$$\frac{dV}{d\theta} = -(R+r) \sin \theta = 0 \quad \left(\begin{array}{l} \text{for} \\ \text{equil.} \end{array} \right)$$

$$\theta = 0, \pi \quad (\text{reject})$$



$$\frac{d^2V}{d\theta^2} = -(R+r) \cos \theta < 0 \quad @ \quad \theta = 0 : \underline{\text{unstable}}$$



$$V = V_g = -(R-r) \cos \theta$$

$$\frac{dV}{d\theta} = (R-r) \sin \theta = 0 \quad \left(\begin{array}{l} \text{for} \\ \text{equil.} \end{array} \right)$$

$$\theta = 0, \pi \quad (\text{reject})$$

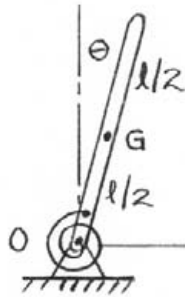
$$\frac{d^2V}{d\theta^2} = (R-r) \cos \theta > 0 \quad @ \quad \theta = 0 : \underline{\text{stable}}$$

7/39

$$V = V_e + V_g = \frac{1}{2} K \theta^2 + mg \frac{l}{2} \cos \theta$$

$$\frac{dV}{d\theta} = K\theta - mg \frac{l}{2} \sin \theta$$

$$\frac{d^2V}{d\theta^2} = K - mg \frac{l}{2} \cos \theta$$

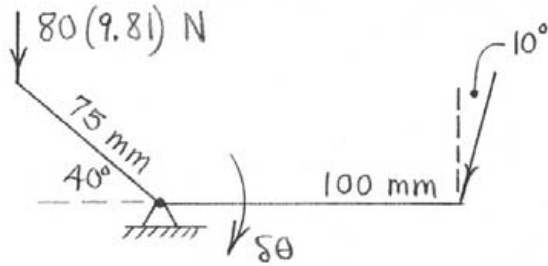


$$V_g = 0 \quad \frac{d^2V}{d\theta^2} > 0 \quad @ \quad \theta = 0 :$$

$$K - mg \frac{l}{2} > 0$$

$$\text{or} \quad \underline{l < \frac{2K}{mg}}$$

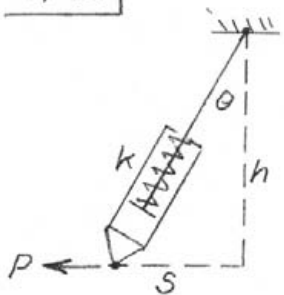
7/4



For a virtual displacement $\delta\theta$ of the lever,

$$\delta U = 0 : P \cos 10^\circ (100 \delta\theta) - 80(9.81) [75 \delta\theta \cos 40^\circ] = 0$$

$$\underline{P = 458 \text{ N}}$$



$x = \text{spring compression}$

$$= \frac{h}{\cos \theta} - h = h(\sec \theta - 1)$$

$$\delta U = \delta V_e ; P \delta s = \delta \left(\frac{1}{2} k x^2 \right)$$

$$= k x \delta x$$

$s = h \tan \theta$ so that

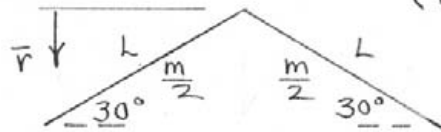
$$P \delta (h \tan \theta) = k h^2 (\sec \theta - 1) \sec \theta \tan \theta \delta \theta$$

$$P h \sec^2 \theta \delta \theta = k h^2 (\sec \theta - 1) \sec^2 \theta \sin \theta \delta \theta$$

$$P = k h (\sec \theta - 1) \sin \theta$$

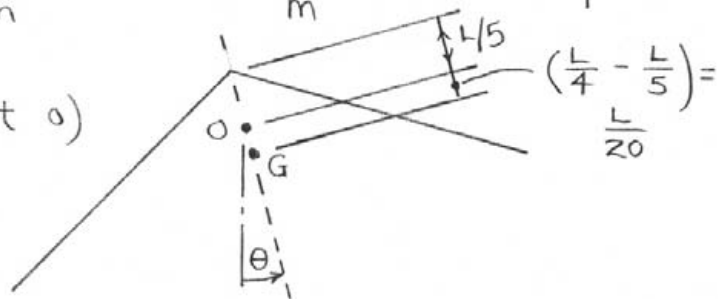
or $P = k h \tan \theta (1 - \cos \theta)$

7/41

(Total mass m)

$$\bar{r} = \frac{\sum m\bar{r}}{\sum m} = \frac{2\left[\frac{m}{2} \frac{L}{2} \sin 30^\circ\right]}{m} = \frac{L}{4}$$

(Datum at O)



$$\left(\frac{L}{4} - \frac{L}{5}\right) = \frac{L}{20}$$

$$V = V_g = -mg \frac{L}{20} \cos \theta$$

$$\frac{dV}{d\theta} = \frac{mgL}{20} \sin \theta \quad (\text{Zero @ } \theta = 0)$$

$$\frac{d^2V}{d\theta^2} = \frac{mgL}{20} \cos \theta > 0 \quad @ \quad \theta = 0 \quad \underline{\text{so stable.}}$$

7/42 Take $V_g = 0$ through AO & $V_e = 0$ when $\theta = 0$

$$\text{So } V_g = -mgh = -60(9.81)(0.7 \sin \theta) = -412.0 \sin \theta$$

$$V_e = \frac{1}{2} k x^2 = \frac{1}{2} (160) [2(1.4) \sin \frac{\theta}{2}]^2 = 627.2 \sin^2 \frac{\theta}{2}$$

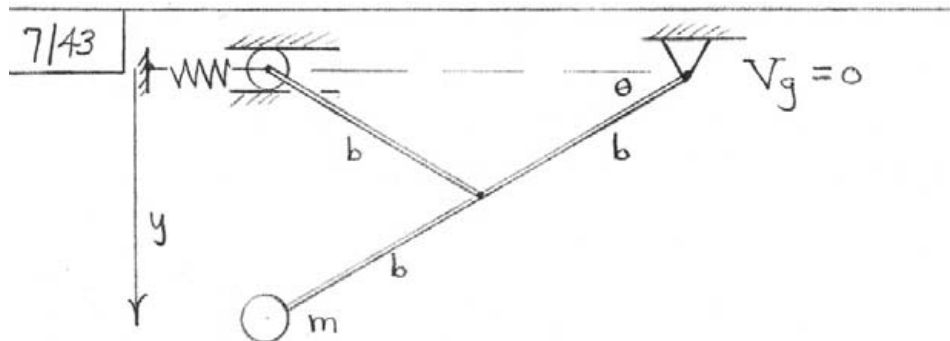
$$V = V_e + V_g = 627.2 \sin^2 \frac{\theta}{2} - 412.0 \sin \theta$$

$$\frac{dV}{d\theta} = \frac{2}{2} (627.2) \sin \frac{\theta}{2} \cos \frac{\theta}{2} - 412.0 \cos \theta$$

$$= 313.6 \sin \theta - 412.0 \cos \theta = 0 \text{ for equil.}$$

$$\text{or } \frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{412.0}{313.6} = 1.314$$

$$\underline{\theta = 52.7^\circ}$$



$$\text{Spring stretch} = 2b - 2b \cos \theta = 2b(1 - \cos \theta)$$

$$V_e = \frac{1}{2} k [2b(1 - \cos \theta)]^2 = 2kb^2(1 - \cos \theta)^2$$

$$V_g = -mg(2b \sin \theta) = -2mgb \sin \theta$$

$$V = 2kb^2(1 - \cos \theta)^2 - 2mgb \sin \theta$$

$$\frac{dV}{d\theta} = 4kb^2(1 - \cos \theta) \sin \theta - 2mgb \cos \theta = 0$$

(for equilibrium)

$$2kb(1 - \cos \theta) \sin \theta = mg \cos \theta, \quad k = \frac{mg}{2b} \frac{\cot \theta}{1 - \cos \theta}$$

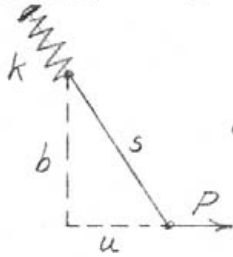
7/44

$$\delta V_e = kx \delta x \quad \text{where } x = s - b = \sqrt{b^2 + u^2} - b$$

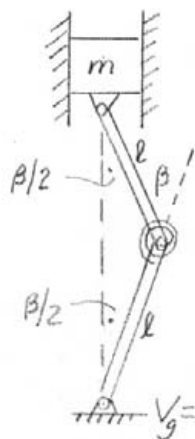
$$\delta x = \frac{u \delta u}{\sqrt{b^2 + u^2}}$$

$$\delta U' = \delta V_e; \quad P \delta u = k (\sqrt{b^2 + u^2} - b) \frac{u \delta u}{\sqrt{b^2 + u^2}}$$

$$P = \left(1 - \frac{b}{\sqrt{b^2 + u^2}}\right) ku$$



7/45



$$V_g = 2mg l \cos \frac{\beta}{2}, \quad V_e = \frac{1}{2} K \beta^2$$

$$V = V_g + V_e = 2mg l \cos \frac{\beta}{2} + \frac{1}{2} K \beta^2$$

$$\frac{dV}{d\beta} = -mg l \sin \frac{\beta}{2} + K\beta, \quad \frac{dV}{d\beta} = 0 \text{ for } \beta = 0$$

$$\frac{d^2V}{d\beta^2} = -\frac{1}{2} mg l \cos \frac{\beta}{2} + K$$

$$= -\frac{1}{2} mg l + K \text{ for } \beta = 0$$

$$= (+) \text{ stable if } K > \frac{1}{2} mg l$$

$$V_g = 0 \quad \text{Thus } \underline{K_{\min} = \frac{1}{2} mg l}$$

7/46

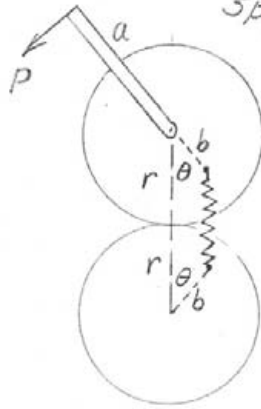
$$\delta U' = \delta V_e; \delta U' = Pa \delta \theta; \delta V_e = kx \delta x$$

$$\text{Spring stretch } x = (2r - 2b \cos \theta) - (2r - 2b) \\ = 2b(1 - \cos \theta)$$

$$\delta x = 2b \sin \theta \delta \theta$$

$$\text{Thus } Pa \delta \theta = 4b^2 k (1 - \cos \theta) \sin \theta \delta \theta$$

$$P = \frac{4kb^2}{a} \sin \theta (1 - \cos \theta)$$



7/47

$$V_e = \frac{1}{2} k x^2 \text{ where } x = 2a \sin \frac{\theta}{2}$$

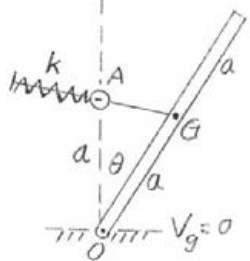
$$V_g = m g a \cos \theta$$

$$V = V_e + V_g = 2ka^2 \sin^2 \frac{\theta}{2} + m g a \cos \theta$$

$$\frac{dV}{d\theta} = 2ka^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} - m g a \sin \theta$$

$$= ka^2 \sin \theta - m g a \sin \theta = (ka - m g) a \sin \theta$$

$$\frac{dV}{d\theta} = 0 \text{ for equil. gives } \underline{k = m g / a \text{ for all } \theta}$$



7/48

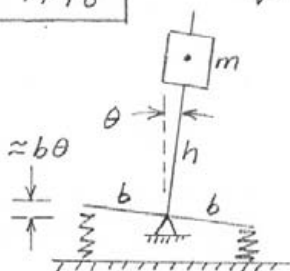
$$V = V_e + V_g = \frac{1}{2}k(\Delta + b\theta)^2 + \frac{1}{2}k(\Delta - b\theta)^2 + mgh \cos \theta$$

for θ small

$$V = k(\Delta^2 + b^2\theta^2) + mgh \cos \theta$$

$$\frac{dV}{d\theta} = 2kb^2\theta - mgh \sin \theta$$

$$\frac{d^2V}{d\theta^2} = 2kb^2 - mgh \cos \theta$$

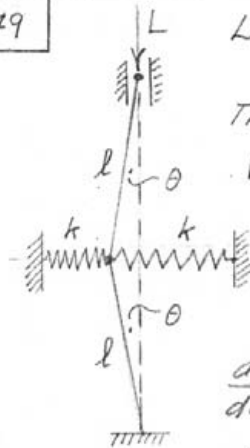


Let preset of
springs be Δ
when $\theta = 0$

For $\theta \rightarrow 0$, $\frac{d^2V}{d\theta^2}$ is (+) if $2kb^2 > mgh$

Thus $\theta = 0$ is stable if $h < \frac{2kb^2}{mg}$

7/49



Let Δ = initial compression in each spring

Then for small angles

$$V = V_e + V_g = 2 \left[\frac{1}{2} k (\Delta - l \sin \theta)^2 + \frac{1}{2} k (\Delta + l \sin \theta)^2 \right] + 2Ll \cos \theta$$

$$V = 2k(\Delta^2 + l^2 \sin^2 \theta) + 2Ll \cos \theta$$

$$\frac{dV}{d\theta} = 4kl^2 \sin \theta \cos \theta - 2Ll \sin \theta$$

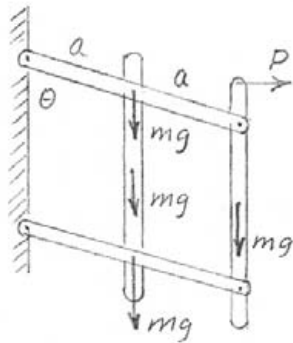
$$\frac{d^2V}{d\theta^2} = 4kl^2 \cos 2\theta - 2Ll \cos \theta$$

$$\frac{dV}{d\theta} = 0 \text{ for equil. } \theta = 0 \text{ \& } \theta = \cos^{-1} \frac{L}{2kl}$$

$$\text{For } \theta = 0, \frac{d^2V}{d\theta^2} = 4kl^2(1) - 2Ll(1) = (+) \text{ stable if}$$

$$k > \frac{L}{2l}$$

7/5

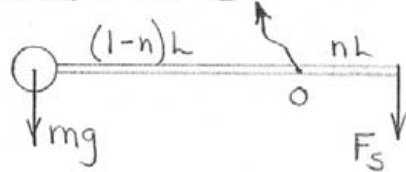


$$\delta U = 0; \quad P\delta(2a \sin \theta) + mg\delta(2a \cos \theta) + 3mg\delta(a \cos \theta) = 0$$

$$2Pa \cos \theta \delta \theta = 5mga \sin \theta \delta \theta$$

$$P = \frac{5}{2} mg \tan \theta$$

7/50 FBD @ $\theta = 0$:



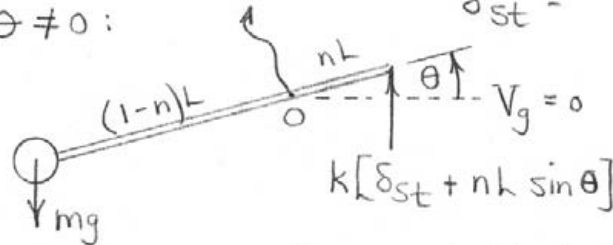
$$\uparrow \Sigma M_o = 0 :$$

$$mg(1-n)L - F_s(nL) = 0$$

$$F_s = \frac{mg(1-n)}{n} = k\delta_{st}$$

$$\delta_{st} = \frac{mg(1-n)}{nk}$$

$\theta \neq 0$:



$$V = V_g + V_e = -mg[(1-n)L \sin \theta] + \frac{1}{2}k \left[\frac{mg(1-n)}{kn} + nL \sin \theta \right]^2$$

$$= \frac{m^2 g^2 (1-n)^2}{2kn^2} + \frac{n^2 L^2 k \sin^2 \theta}{2}$$

$$\frac{dV}{d\theta} = n^2 L^2 k \sin \theta \cos \theta = 0 \quad \text{at } \theta = 0$$

$$\frac{d^2 V}{d\theta^2} = n^2 L^2 k [\cos^2 \theta - \sin^2 \theta] > 0 \quad \text{at } \theta = 0$$

for all $0 < n < 1$

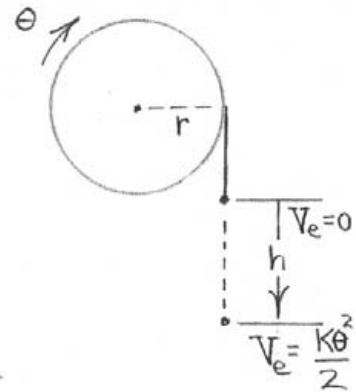
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$$\delta U' = \delta V: 0 - mgr \delta \theta + \delta \left(\frac{1}{2} K \theta^2 \right)$$

$$mgr = K \theta$$

$$\text{With } h = r\theta : mgr = K \left(\frac{h}{r} \right)$$

$$h = \frac{mgr^2}{K}$$



7/52

$$\text{Length } AB = 2(20) \cos \frac{\theta}{2} \text{ (in.)}$$

$$\text{Unstretched length} = 40 - 4 = 36 \text{ in.}$$

Spring stretch for arbitrary θ

$$\text{is } x = 40 \cos \frac{\theta}{2} - 36 \text{ in.}$$

$$V_e = 2 \frac{1}{2} k (40 \cos \frac{\theta}{2} - 36)^2$$

$$= k [1600 \cos^2 \frac{\theta}{2} - 2880 \cos \frac{\theta}{2} + 1296] \text{ in.-lb}$$

$$V_g = -3(20 \cos \theta) = -60 \cos \theta \text{ in.-lb}$$

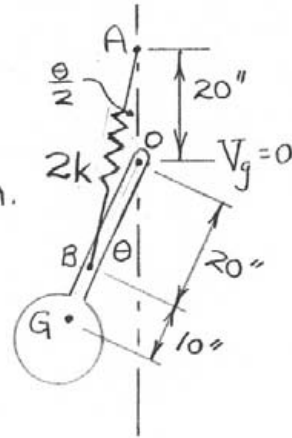
$$V = V_e + V_g = k [1600 \cos^2 \frac{\theta}{2} - 2880 \cos \frac{\theta}{2} + 1296] - 60 \cos \theta \text{ in.-lb}$$

$$\frac{dV}{d\theta} = k [-1600 \sin \frac{\theta}{2} \cos \frac{\theta}{2} + 1440 \sin \frac{\theta}{2}] + 60 \sin \theta$$

$$= k [-800 \sin \theta + 1440 \sin \frac{\theta}{2}] + 60 \sin \theta$$

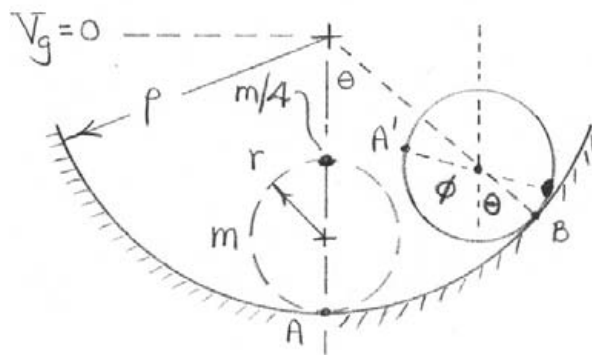
$$\frac{d^2V}{d\theta^2} = k [-800 \cos \theta + 720 \cos \frac{\theta}{2}] + 60 \cos \theta$$

$$\left(\frac{d^2V}{d\theta^2} \right)_{\theta=0} = k [-800 + 720] + 60 > 0 \text{ (Stable) if } k \text{ does not exceed } \frac{60}{80} = \underline{0.75 \text{ lb/in.} = k_{\max}}$$



7/53

$$\text{Arc } \widehat{AB} = \text{Arc } \widehat{A'B} : p\theta = r(\theta + \phi)$$



$$(p-r)\theta = r\phi$$

$$\phi = \frac{p-r}{r} \theta$$

$$V = V_g = -mg(p-r)\cos\theta - \frac{m}{4}g[(p-r)\cos\theta - r\cos\phi]$$

$$= \frac{mg}{4}[-5(p-r)\cos\theta + r\cos(\frac{p-r}{r}\theta)]$$

$$\frac{dV}{d\theta} = \frac{mg}{4}[5(p-r)\sin\theta - (p-r)\sin(\frac{p-r}{r}\theta)]$$

= 0 for equilibrium, $\theta = 0$ is desired solution

$$\frac{d^2V}{d\theta^2} = \frac{mg}{4}[5(p-r)\cos\theta - \frac{(p-r)^2}{r}\cos(\frac{p-r}{r}\theta)]$$

$$\text{At } \theta = 0: 5(p-r) - \frac{(p-r)^2}{r} > 0 \text{ for stability}$$

$$p^2 - 7pr + 6r^2 < 0$$

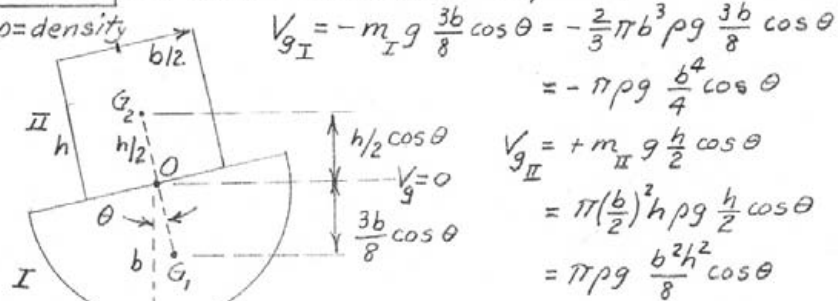
$$(p-6r)(p-r) < 0 \quad (p=r \text{ not possible})$$

$$\Rightarrow (p-6r) < 0 \quad \& \quad (p-r) > 0 \quad (\text{or the reverse which is impossible})$$

$$\text{So } \underline{p < 6r}$$

7/54 For solid hemisphere $\overline{OG}_1 = \frac{3b}{8}$

$\rho = \text{density}$



$$V_{gI} = -m_I g \frac{3b}{8} \cos \theta = -\frac{2}{3} \pi b^3 \rho g \frac{3b}{8} \cos \theta$$

$$= -\pi \rho g \frac{b^4}{4} \cos \theta$$

$$V_{gII} = +m_{II} g \frac{h}{2} \cos \theta$$

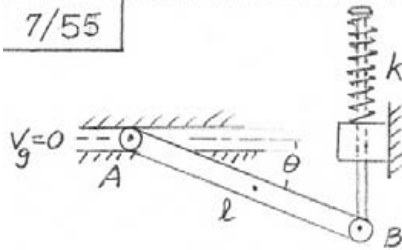
$$= \pi \left(\frac{b}{2}\right)^2 h \rho g \frac{h}{2} \cos \theta$$

$$= \pi \rho g \frac{b^2 h^2}{8} \cos \theta$$

$$V = V_{II} + V_I = \pi \rho g \frac{b^2}{4} \left(\frac{h^2}{2} - b^2 \right) \cos \theta$$

$$\frac{d^2 V}{d\theta^2} = -\pi \rho g \frac{b^2}{4} \left(\frac{h^2}{2} - b^2 \right) \cos \theta = +\pi \rho g \frac{b^2}{4} \left(b^2 - \frac{h^2}{2} \right) \cos \theta$$

$$= (+) \text{ stable if } \frac{h^2}{2} < b^2 \text{ or } \underline{h < b\sqrt{2}}$$



Spring compressed $l \sin \theta$
 so $V_e = \frac{1}{2} k (l \sin \theta)^2$

$$V_g = -mg \frac{l}{2} \sin \theta$$

$$V = V_e + V_g = \frac{k}{2} l^2 \sin^2 \theta - mg \frac{l}{2} \sin \theta$$

$$\begin{aligned} \frac{dV}{d\theta} &= k l^2 \sin \theta \cos \theta - mg \frac{l}{2} \cos \theta = \frac{k l^2}{2} \sin 2\theta - mg \frac{l}{2} \cos \theta \\ &= l \cos \theta (k l \sin \theta - mg/2) = 0 \text{ for equil.} \end{aligned}$$

$$(1) \cos \theta = 0, \theta = \pi/2$$

$$(2) \sin \theta = \frac{mg}{2kl}$$

$$\frac{d^2V}{d\theta^2} = k l^2 \cos 2\theta + mg \frac{l}{2} \sin \theta$$

$$\begin{aligned} \left(\frac{d^2V}{d\theta^2} \right)_{(1)} &= k l^2 \cos \pi + mg \frac{l}{2} (1) = k l^2 \left(-1 + \left[\frac{mg}{2kl} \right] \right) \\ &= (+) \text{ stable if } k < \frac{mg}{2l} \end{aligned}$$

$$\begin{aligned} \left(\frac{d^2V}{d\theta^2} \right)_{(2)} &= k l^2 \left(1 - 2 \left[\frac{mg}{2kl} \right]^2 \right) + \frac{mg l}{2} \frac{mg}{2kl} = k l^2 \left[1 - \left(\frac{mg}{2kl} \right)^2 \right] \\ &= (+) \text{ stable if } k > \frac{mg}{2l} \end{aligned}$$

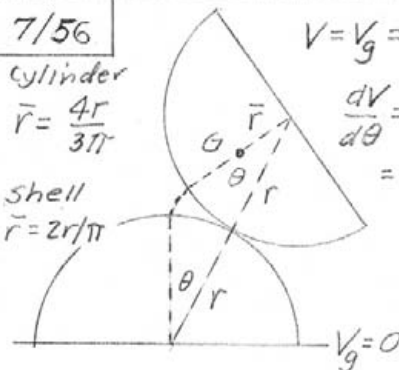
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cylinder

$$\bar{r} = \frac{4r}{3\pi}$$

shell

$$\bar{r} = 2r/\pi$$



$$V = V_g = mg(2r \cos \theta - \bar{r} \cos 2\theta)$$

$$\frac{dV}{d\theta} = mg(-2r \sin \theta + 2\bar{r} \sin 2\theta)$$

$$= 2mg(-r \sin \theta + \bar{r} \sin 2\theta)$$

$$\frac{d^2V}{d\theta^2} = 2mg(-r \cos \theta + 2\bar{r} \cos 2\theta)$$

$$\text{For } \theta = 0, \frac{d^2V}{d\theta^2} = 2mrg \left(-1 + \frac{2\bar{r}}{r}\right)$$

$$\text{For cylinder } \bar{r}/r = \frac{4}{3\pi}, \frac{d^2V}{d\theta^2} = 2mrg \left(-1 + \frac{8}{3\pi}\right) = (-)$$

unstable

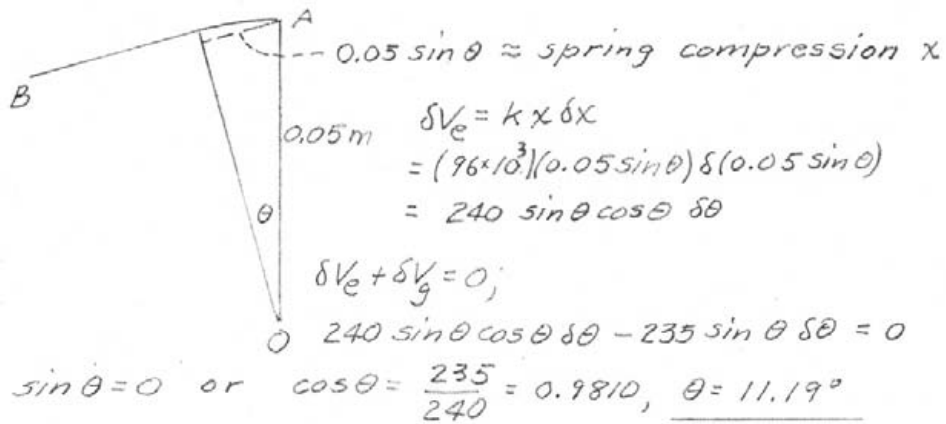
$$\text{For shell } \bar{r}/r = \frac{2}{\pi}, \frac{d^2V}{d\theta^2} = 2mrg \left(-1 + \frac{4}{\pi}\right) = (+)$$

stable

7/57

$$\delta V_g = \delta(0.3 \text{ mg} \cos \theta) = -0.3(80)(9.81) \sin \theta \delta \theta$$

$$= -235 \sin \theta \delta \theta$$



$$\delta V_e = k \delta x$$

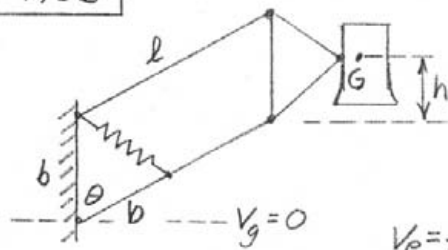
$$= (96 \times 10^3) (0.05 \sin \theta) \delta(0.05 \sin \theta)$$

$$= 240 \sin \theta \cos \theta \delta \theta$$

$$\delta V_e + \delta V_g = 0;$$

$$240 \sin \theta \cos \theta \delta \theta - 235 \sin \theta \delta \theta = 0$$

$$\sin \theta = 0 \text{ or } \cos \theta = \frac{235}{240} = 0.9810, \quad \theta = 11.19^\circ$$



$$V_g = mg(l \cos \theta + h)$$

$$\text{spring length} = 2b \sin \frac{\theta}{2}$$

$$\text{stretch } x = 2b \sin \frac{\theta}{2} - \frac{b}{2}$$

$$= \frac{b}{2} (4 \sin^2 \frac{\theta}{2} - 1)$$

$$V_e = \frac{1}{2} k x^2 = \frac{k b^2}{8} (4 \sin^2 \frac{\theta}{2} - 1)^2$$

$$V = V_e + V_g; \quad \frac{dV}{d\theta} = -mg l \sin \theta + \frac{k b^2}{4} (4 \sin^2 \frac{\theta}{2} - 1) 2 \cos \frac{\theta}{2}$$

$$= (k b^2 - mg l) \sin \theta - \frac{k b^2}{2} \cos \frac{\theta}{2} = 0 \text{ for equil.}$$

$$\text{Thus } \left[2(k b^2 - mg l) \sin \frac{\theta}{2} - \frac{k b^2}{2} \right] \cos \frac{\theta}{2} = 0$$

$$\& \text{ []} = 0 \text{ gives } k = \frac{mg l}{b^2} \frac{1}{1 - \frac{1}{4} \csc \frac{\theta}{2}}$$

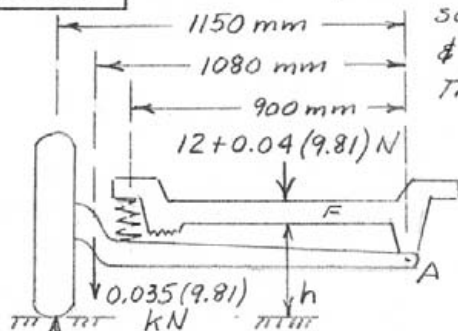
$$\frac{d^2 V}{d\theta^2} = (k b^2 - mg l) \cos \theta + \frac{k b^2}{4} \sin \frac{\theta}{2}; \text{ substitute } \sin \frac{\theta}{2} = \frac{k b^2}{4(k b^2 - mg l)}$$

$$\& \text{ get } \frac{d^2 V}{d\theta^2} = + (\text{stable}) \text{ within range of } \theta = 29^\circ, k = \infty \text{ to } \theta = 180^\circ, k = \frac{4mg l}{3b^2}$$

7/59

Let x = compression of spring (most easily seen by considering A fixed & wheels moving up)

Thus $x = \frac{900}{1150} (0.35 - h)$ meters



With F & hence A fixed,

$$\begin{aligned}\delta U' &= -2(6.54)\delta h \\ &\quad + 2(0.035)9.81 \frac{1080}{1150} \delta h \\ &= (-13.08 + 0.645)\delta h \\ &= -12.43\delta h\end{aligned}$$

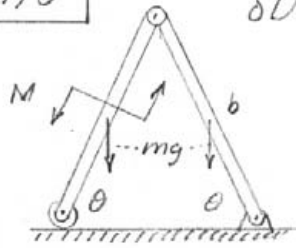
$$\begin{aligned}P &= 6 + 0.035(9.81) + 0.02(9.81) \\ &= 6.54 \text{ kN}\end{aligned}$$

$$\begin{aligned}\delta V_e &= 2(kx\delta x) = 2(120) \frac{900}{1150} (0.35 - h) \frac{900}{1150} (-\delta h) \\ &= -147.0(0.35 - h)\delta h\end{aligned}$$

$$\delta U' = \delta V_e; -12.43\delta h = -147.0(0.35 - h)\delta h$$

$$h = 0.35 - \frac{12.43}{147.0} = 0.265 \text{ m or } h = 265 \text{ mm}$$

7/6



$$\delta U = 0; \quad M \delta \theta - 2mg \delta \left(\frac{b}{2} \sin \theta \right) = 0$$

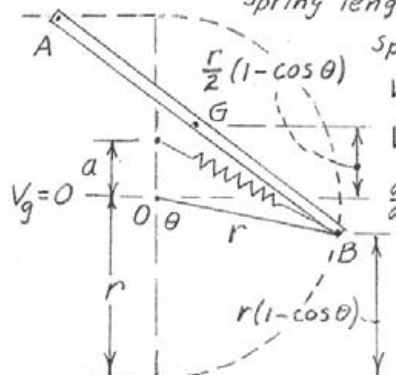
$$M \delta \theta = mg b \cos \theta \delta \theta$$

$$\theta = \cos^{-1} \frac{M}{mg b}$$

► 7/60

For $\frac{1}{2}$ of door $V_g = \frac{mgr}{2} (1 - \cos \theta)$

Spring length $l = \sqrt{r^2 + a^2 + 2ra \cos \theta}$



Spring stretch is $l - (r - a)$

$$V_e = \frac{1}{2} k (l - [r - a])^2$$

$$V = V_g + V_e = \frac{mgr}{4} (1 - \cos \theta) + \frac{k}{2} (l - [r - a])^2$$

$$\frac{dV}{d\theta} = \frac{mgr}{4} \sin \theta + k (l - [r - a]) \frac{-2ra \sin \theta}{2l}$$

$$= \left(\frac{mgr}{4} - kar \right) \sin \theta + \frac{kar(r-a) \sin \theta}{\sqrt{r^2 + a^2 + 2ra \cos \theta}}$$

$$\frac{d^2V}{d\theta^2} = \left(\frac{mgr}{4} - kar \right) \cos \theta + kar(r-a) \frac{l \cos \theta - \sin \theta (-ra \sin \theta)/l}{l^2}$$

$$\text{For } \theta = 0, \frac{d^2V}{d\theta^2} = \frac{mgr}{4} - kar + kar(r-a) \left(\frac{1}{r+a} + 0 \right)$$

$= 0$ for neutral equil. (insensitive response)

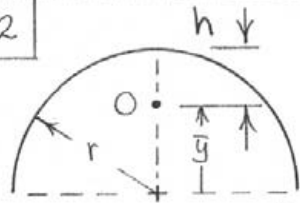
$$\text{Thus } \frac{mgr}{4} = kar \left(1 - \frac{r-a}{r+a} \right), \quad k = \frac{mg(r+a)}{8a^2}$$

7/61

For system $\delta U = 0$

$$P_1 \delta x - P_2 \delta x / 4 = 0, \quad P_2 = 4P_1 = 4(100) \\ = \underline{400 \text{ N}}$$

7/62

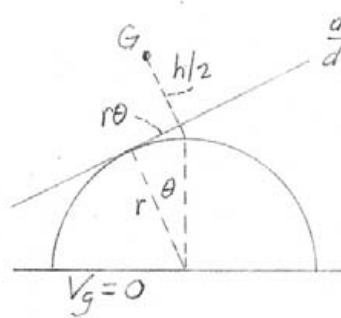


The pivot O must be at or above the mass center G of the shell, which is located at $\bar{y} = \frac{2r}{\pi}$. So

$$h_{\max} = r - \bar{y} = r\left(1 - \frac{2}{\pi}\right) = \underline{0.363r}$$

7/63

$$V = V_g = mg(r \cos \theta + r \theta \sin \theta + \frac{h}{2} \cos \theta)$$



$$\frac{dV}{d\theta} = mg(-r \sin \theta + r \sin \theta + r \theta \cos \theta - \frac{h}{2} \sin \theta)$$

$$= mg(r \theta \cos \theta - \frac{h}{2} \sin \theta)$$

$$\frac{d^2V}{d\theta^2} = mg(r \cos \theta - r \theta \sin \theta - \frac{h}{2} \cos \theta)$$

For $\theta = 0$ position, $\frac{d^2V}{d\theta^2} = mg(r - 0 - \frac{h}{2})$
 $= (+) \text{ stable if } \underline{h < 2r}$

7/6A Force & moment equilibrium (A)

(a), (b), (d)

Virtual work (B)

(c), (e), (f)

(c) $\delta U' = \delta V_g$

(e) $\delta U' = \delta V_g + \delta V_e$

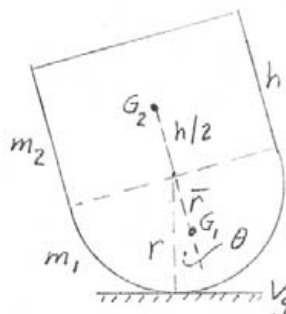
(f) $d^2V/d\theta^2$ must be (+)

7/65

Let ρ = mass per unit area of shell

$$m_1 = 2\pi r^2 \rho, \quad m_2 = 2\pi r h \rho$$

$$\bar{r} = r/2 \text{ for hemispherical shell}$$



$$V = V_{g_1} + V_{g_2}$$

$$= 2\pi r^2 \rho g (r - \bar{r} \cos \theta) + 2\pi r h \rho g (r + \frac{h}{2} \cos \theta)$$

$$= 2\pi r \rho g [(r^2 + hr) - \frac{1}{2}(r^2 - h^2) \cos \theta]$$

$$\frac{dV}{d\theta} = 2\pi r \rho g [0 + \frac{1}{2}(r^2 - h^2) \sin \theta]$$

$$\frac{d^2V}{d\theta^2} = \pi r \rho g (r^2 - h^2) \cos \theta$$

For equil. $\frac{dV}{d\theta} = 0$ gives $\theta = 0$ & $h = r$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = (+)$ if $h < r$

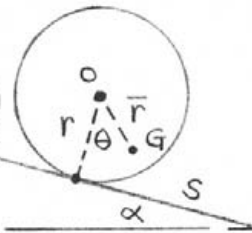
For $h = r$, neutral equilibrium

7/66

$$V_g = W(s \sin \alpha + r \cos \alpha - \bar{r} \cos(\theta - \alpha))$$

$$\text{But } s = s_0 + r\theta, \frac{ds}{d\theta} = r$$

$$V_g = 0$$



$$\text{So } \frac{dV_g}{d\theta} = W(r \sin \alpha + \bar{r} \sin(\theta - \alpha))$$

$$\text{For } \alpha = 10^\circ, r = 100 \text{ mm}, \bar{r} = 60 \text{ mm}:$$

$$\frac{dV_g}{d\theta} = W[0.1 \sin 10^\circ + 0.060 \sin(\theta - 10^\circ)]$$

$$= 0 \text{ for equilibrium}$$

$$-0.1 \sin 10^\circ = 0.060 \sin(\theta - 10^\circ)$$

$$\theta - 10^\circ = \sin^{-1}\left(\frac{-0.1}{0.06} \sin 10^\circ\right) = -16.82^\circ \text{ or } 196.8^\circ$$

$$\Rightarrow \underline{\theta = -6.82^\circ} \text{ or } \underline{\theta = 207^\circ}$$

$$\frac{d^2V_g}{d\theta^2} = W[0 + \bar{r} \cos(\theta - \alpha)]$$

$$\theta = -6.82^\circ: \frac{d^2V_g}{d\theta^2} = W\bar{r} \cos(-16.82^\circ) > 0 \text{ Stable}$$

$$\theta = 207^\circ: \frac{d^2V_g}{d\theta^2} = W\bar{r} \cos(196.8^\circ) < 0 \text{ Unstable}$$

$$7/67 \quad V = V_e = \frac{1}{2} k (2a \sin \frac{\theta}{2})^2 = 2ka^2 \sin^2 \frac{\theta}{2}$$

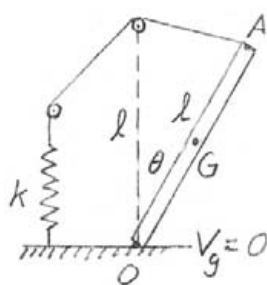
$$\delta U = \delta V_e; \quad p \delta (2a \sin \theta) = \delta (2ka^2 \sin^2 \frac{\theta}{2})$$

$$2Pa \cos \theta \delta \theta = 2ka^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \delta \theta$$

$$2Pa \cos \theta = ka^2 \sin \theta$$

$$\tan \theta = \frac{2P}{ka}, \quad \underline{\theta = \tan^{-1} \frac{2P}{ka}}$$

7/68 $V = V_e + V_g = \frac{1}{2}k(2l \sin \frac{\theta}{2})^2 + mg \frac{l}{2} \cos \theta$



$$\begin{aligned} \frac{dV}{d\theta} &= 2kl^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} - mg \frac{l}{2} \sin \theta \\ &= kl^2 \sin \theta - mg \frac{l}{2} \sin \theta \\ &= l(kl - \frac{1}{2}mg) \sin \theta = 0 \text{ for equil.} \\ \theta &= 0 \text{ and } k = \frac{mg}{2l} \text{ independent of } \theta \end{aligned}$$

$$\frac{d^2V}{d\theta^2} = l(kl - \frac{1}{2}mg) \cos \theta$$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = l(kl - \frac{1}{2}mg) = (+)$ stable if $k > \frac{mg}{2l}$
 $(-)$ unstable if $k < \frac{mg}{2l}$

For $k = \frac{mg}{2l}$, $V = \frac{mg l}{2}$ const., neutral equil

7/69

Total length of door is $2.5 + 0.6\pi/2 = 3.44$ m
 Unit mass is $135/3.44 = 39.22$ kg/m

Take $V_g = 0$ through A

Let potential energy of cylindrical portion be $-V_0$
 which remains constant

$$\text{So } V_g = 0 - V_0 - 39.22(9.81)x(0.6 + \frac{x}{2})$$

$$= -V_0 - 384.7x(0.6 + \frac{x}{2}) \quad \text{J}$$

$$V_e = 2(\frac{1}{2}K\theta^2) = \frac{10}{2\pi}\theta^2 = \frac{5}{\pi}(\frac{x}{0.080})^2 = 248.7x^2 \quad \text{J}$$

$$V = V_e + V_g = 248.7x^2 - V_0 - 384.7x(0.6 + \frac{x}{2})$$

$$\frac{dV}{dx} = 497.4x - 230.8 - 384.7x = 112.7x - 230.8$$

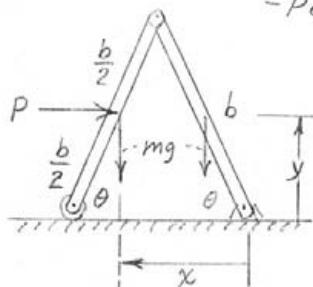
$$= 0 \text{ for equilibrium, so } x = \frac{230.8}{112.7} = \underline{2.05 \text{ m}}$$

$$\frac{d^2V}{dx^2} = 112.7 \quad (+) \text{ so } \underline{\text{stable}}$$

7/7

$$\delta U = 0; -P \delta x - 2mg \delta y = 0$$

$$-P \delta \left(\frac{3b}{2} \cos \theta \right) - 2mg \delta \left(\frac{b}{2} \sin \theta \right) = 0$$



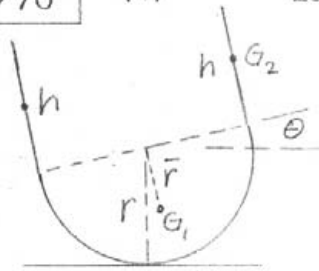
$$\frac{3Pb}{2} \sin \theta \delta \theta - mg b \cos \theta \delta \theta = 0$$

$$\tan \theta = \frac{2mg}{3P}$$

$$\theta = \tan^{-1} \frac{2mg}{3P}$$

7/70

(a)

(a) $V_g = 0$ Let ρ = mass per unit periphery of shell

$$\bar{r} = 2r/\pi$$

$$V_g = \rho \left[\pi r \left(r - \frac{2r}{\pi} \cos \theta \right) + h \left(r + r \sin \theta + \frac{h}{2} \cos \theta \right) + h \left(r - r \sin \theta + \frac{h}{2} \cos \theta \right) \right]$$

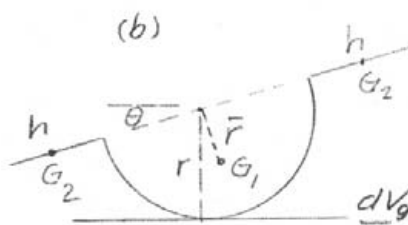
$$= \rho \left[\pi r^2 - 2r^2 \cos \theta + 2hr + h^2 \cos \theta \right]$$

$$\frac{dV_g}{d\theta} = \rho [2r^2 - h^2] \sin \theta, \quad \frac{d^2V_g}{d\theta^2} = \rho [2r^2 - h^2] \cos \theta$$

Equil. at $\theta = 0$ stable if $h < r\sqrt{2}$
 unstable if $h > r\sqrt{2}$

Neutral equil. if $h = r\sqrt{2}$ for any θ

(b)



$$V_g = \rho \left[\pi r \left(r - \frac{2r}{\pi} \cos \theta \right) + h \left(r + \left[r + \frac{h}{2} \sin \theta \right] \right) + h \left(r - \left[r + \frac{h}{2} \sin \theta \right] \right) \right]$$

$$= \rho \left[\pi r^2 - 2r^2 \cos \theta + 2hr \right]$$

$$\frac{dV_g}{d\theta} = 2\rho r^2 \sin \theta, \quad \theta = 0 \text{ for stable equil. independent of } \theta$$

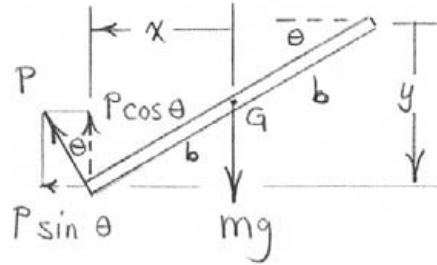
7/71

$$y = 2b \sin \theta$$

$$\delta y = 2b \cos \theta \delta \theta$$

$$x = b \cos \theta$$

$$\delta x = -b \sin \theta \delta \theta$$



$$\delta U = 0 : mg \delta \left(\frac{y}{2} \right) - P \cos \theta (\delta y) + P \sin \theta (\delta x) = 0$$

$$mg b \cos \theta \delta \theta - P \cos \theta (2b \cos \theta \delta \theta) + P \sin \theta (-b \sin \theta \delta \theta) = 0$$

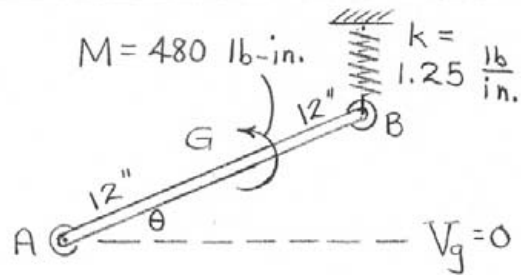
$$mg \cos \theta = P (\sin^2 \theta + 2 \cos^2 \theta) = P (1 + \cos^2 \theta)$$

$$P = \frac{mg \cos \theta}{1 + \cos^2 \theta}$$

* 7/73

$$\delta U' = \delta V_g$$

$$\delta U' = 480 \delta \theta$$



$$V = V_g + V_e = 20 (12 \sin \theta) + \frac{1}{2} (1.25) (24 \sin \theta)^2$$

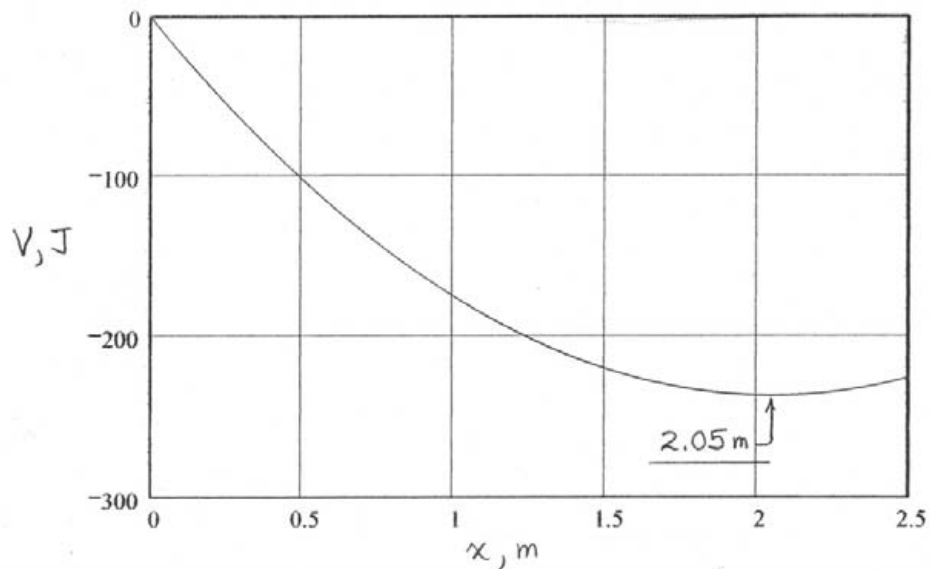
$$\delta V = 240 \cos \theta \delta \theta + 360 (2 \sin \theta \cos \theta) \delta \theta$$

$$\text{So } 480 \delta \theta = 240 \cos \theta \delta \theta + 360 \sin 2\theta \delta \theta$$

$$\text{or } 4 = 2 \cos \theta + 3 \sin 2\theta$$

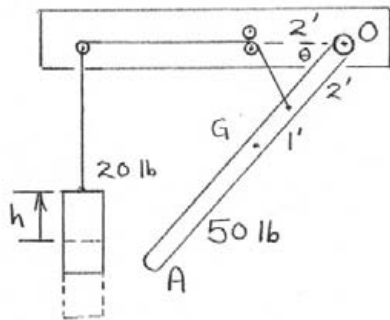
Numerically solve to obtain $\theta = 23.0^\circ$

* 7/74 From Prob. 7/69,
 $V = 248.7x^2 - V_0 - 384.7x(0.6 + \frac{x}{2})$ in Joules
 $= 56.33x^2 - 230.8x$ with $V_0 = 0$
 where V_0 is the potential energy of the cylindrical
 section. Compute V and get



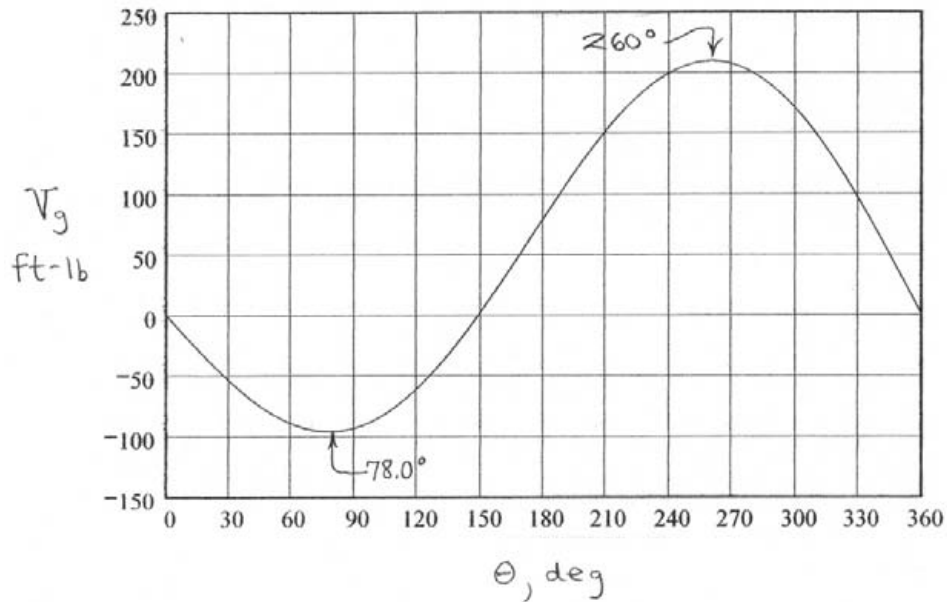
*7/75

$$h = 2 \left(2 \sin \frac{\theta}{2} \right) = 4 \sin \frac{\theta}{2}$$



$$V_g = 20(4 \sin \frac{\theta}{2}) - 50(3 \sin \theta)$$

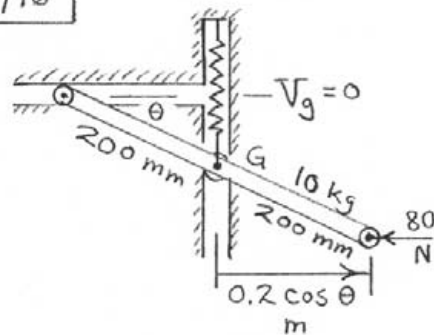
$$= 80 \sin \frac{\theta}{2} - 150 \sin \theta \text{ ft-lb}$$



$$\theta = 78.0^\circ \text{ stable}$$

$$\theta = 260^\circ \text{ unstable}$$

*7/76



$$\delta U = \delta V_g + \delta V_e$$

$$\delta U = -80 \delta (0.2 \cos \theta) = 16 \sin \theta \delta \theta$$

$$\begin{aligned} \delta V_g &= \delta (-mgh) = -\delta [10(9.81)(0.2 \sin \theta)] \\ &= -19.62 \cos \theta \delta \theta \end{aligned}$$

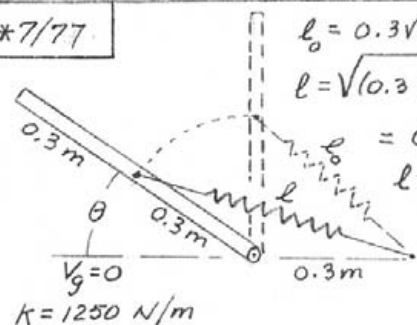
$$\begin{aligned} \delta V_e &= \delta \left(\frac{1}{2} k x^2 \right) = k x \delta x \\ &= 1500 (0.2 \sin \theta) \delta (0.2 \sin \theta) \\ &= 60 \sin \theta \cos \theta \delta \theta = 30 \sin 2\theta \delta \theta \end{aligned}$$

$$\text{Thus } 16 \sin \theta \delta \theta = -19.62 \cos \theta \delta \theta + 30 \sin 2\theta \delta \theta$$

$$\text{or } (16 \sin \theta + 19.62 \cos \theta - 30 \sin 2\theta) \delta \theta = 0$$

$$\text{Numerical solution : } \underline{\theta = 27.9^\circ}$$

*7/77



$$l_0 = 0.3\sqrt{2} = 0.4243 \text{ m}$$

$$l = \sqrt{(0.3 + 0.3\cos\theta)^2 + (0.3\sin\theta)^2}$$

$$= 0.3\sqrt{2}\sqrt{1+\cos\theta} = 0.4243\sqrt{1+\cos\theta}$$

$$l - l_0 = 0.4243(\sqrt{1+\cos\theta} - 1)$$

$$V_e = \frac{1}{2}k(l - l_0)^2$$

$$= \frac{1250}{2} \times 0.18 [2 + \cos\theta - 2\sqrt{1+\cos\theta}]$$

$$= 112.5 [2 + \cos\theta - 2\sqrt{1+\cos\theta}]$$

$$V_g = mgh = 10(9.81)(0.3\sin\theta)$$

$$= 29.43\sin\theta$$

$$V = V_e + V_g$$

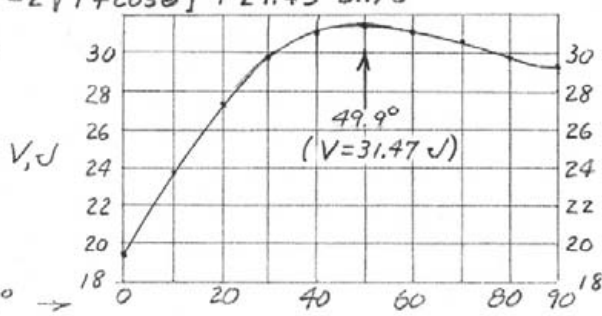
$$V = 112.5 [2 + \cos\theta - 2\sqrt{1+\cos\theta}] + 29.43\sin\theta$$

Compute V as a function of θ

and get \rightarrow

Unstable equilibrium occurs at $\theta = 49.9^\circ$

Stable equilibrium at $\theta = \pm 90^\circ$



* 7/78

$$y = 2(150) \cos \theta$$

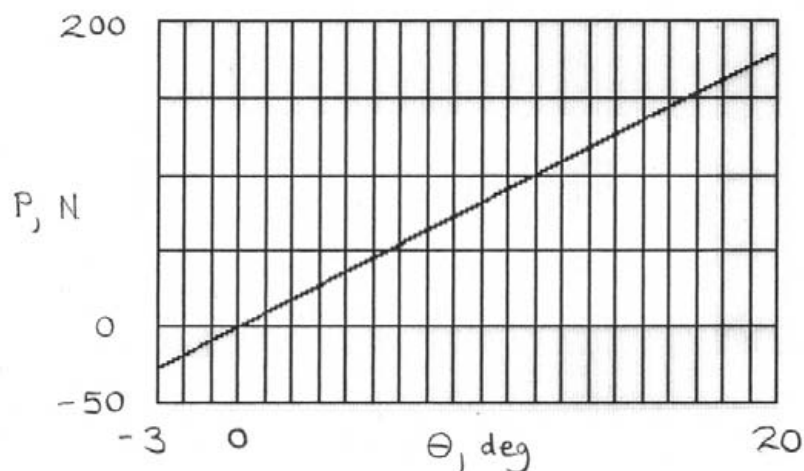
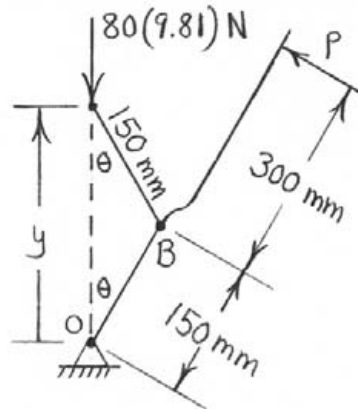
$$\delta y = -300 \sin \theta \delta \theta$$

$$\delta U = 0 :$$

$$-P(450 \delta \theta) - 80(9.81) \delta y = 0$$

$$P = \frac{80(9.81)(300 \sin \theta)}{450}$$

$$= 523 \sin \theta \text{ (in newtons)}$$



$$\left(\begin{array}{l} \text{At } \theta = -3^\circ, \quad P = -27.4 \text{ N} \\ \text{At } \theta = 20^\circ, \quad P = 178.9 \text{ N} \end{array} \right)$$

* 7/79

$$V_g = 20(9.81)(-0.2 \cos \theta)$$

$$= -39.24 \cos \theta \text{ J}$$

$$\overline{AB} = 2(0.4) \sin\left(\frac{180-\theta}{2}\right)$$

$$= 0.8 \cos \frac{\theta}{2}$$

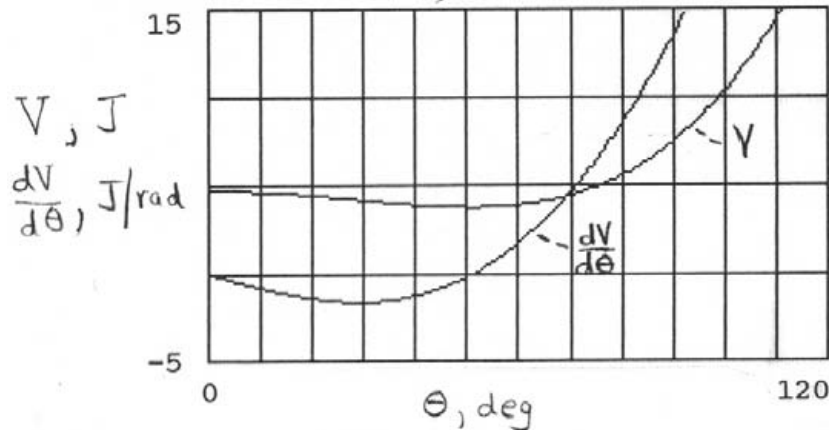
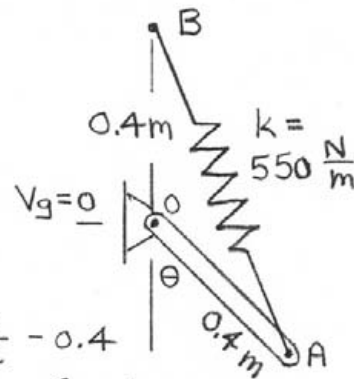
$$\text{Stretch of spring} = 0.8 \cos \frac{\theta}{2} - 0.4$$

$$= 0.4(2 \cos \frac{\theta}{2} - 1)$$

$$V_e = \frac{1}{2} 550 [0.4(2 \cos \frac{\theta}{2} - 1)]^2 = 44(2 \cos \frac{\theta}{2} - 1)^2$$

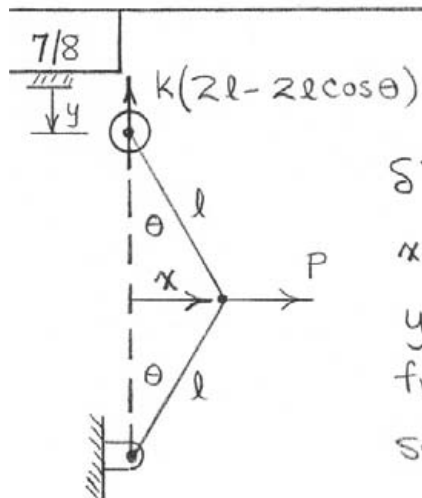
$$V = V_e + V_g = 44(2 \cos \frac{\theta}{2} - 1)^2 - 39.24 \cos \theta$$

$$dV/d\theta = 88(2 \cos \frac{\theta}{2} - 1)(-\sin \frac{\theta}{2}) + 39.24 \sin \theta$$



$\theta = 0$, unstable

$\theta = 51.1^\circ$, stable



$$\delta U = 0: P \delta x - k(2l - 2l \cos \theta) \delta y = 0$$

$$x = l \sin \theta, \quad \delta x = l \cos \theta \delta \theta$$

$y = 2l - 2l \cos \theta$ (measured from wheel position when spring is unstretched)

$$\delta y = 2l \sin \theta \delta \theta$$

$$\text{So } P(l \cos \theta \delta \theta) - k(2l - 2l \cos \theta)(2l \sin \theta \delta \theta) = 0$$

$$\Rightarrow P = \frac{4kl(\sin \theta - \sin \theta \cos \theta)}{\cos \theta}$$

$$\text{or } P = \underline{4kl(\tan \theta - \sin \theta)}$$

* 7/80

$$\text{Spring stretch} = 2(16 \sin \frac{\theta}{2}) - 8$$

$$= 8\left(4 \sin \frac{\theta}{2} - 1\right) \text{ in.}$$

$$V_e = \frac{1}{2} (12)(8)^2 (4 \sin \frac{\theta}{2} - 1)^2$$

$$= 384 \left(4 \sin \frac{\theta}{2} - 1 \right)^2 \text{ in.-lb}$$

$$V_g = 10(8 \cos \theta) + 10(16 + 8 \cos \theta) \quad k = 12 \text{ lb/in.} \quad V_g = 0$$

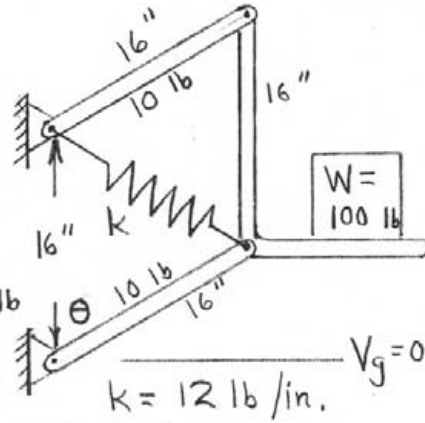
$$+100(16 \cos \theta) = 1760 \cos \theta + 160 \text{ in.} - 16$$

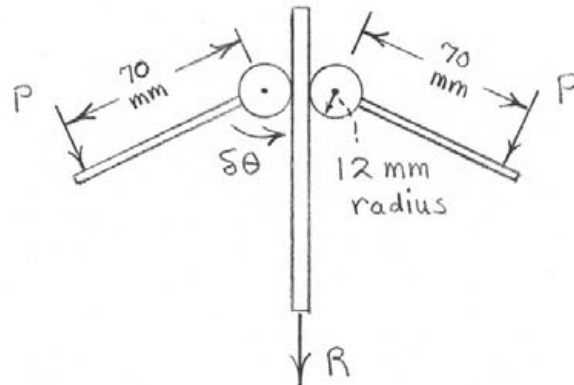
$$V = V_e + V_g = 384 \left(4 \sin \frac{\theta}{2} - 1 \right)^2 + 1760 \cos \theta + 160 \quad \text{in. - lb}$$

$$\frac{dV}{d\theta} = 768 \left(4 \sin \frac{\theta}{2} - 1 \right) 2 \cos \frac{\theta}{2} - 1760 \sin \theta$$

$= 0$ for equilibrium

Numerical solution : $\theta = 71.7^\circ$



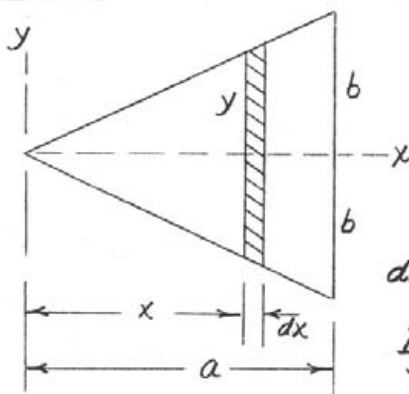


$$\delta U = 0: 2P(70)\delta\theta - R(12)\delta\theta = 0$$

$$\underline{R = 11.67 P}$$

A/1

Using the results of Sample Prob. A/1



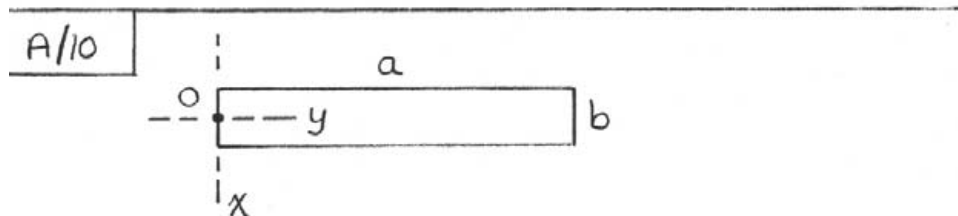
$$dI_x = \frac{1}{12} (2y)^3 dx = \frac{2}{3} y^3 dx$$

But $y = \frac{b}{a} x$ so

$$I_x = \frac{2b^3}{3a^3} \int_0^a x^3 dx = \frac{1}{6} ab^3$$

$$dI_y = x^2 (2y dx) = 2 \frac{b}{a} x^3 dx$$

$$I_y = \frac{2b}{a} \int_0^a x^3 dx = \frac{2b}{a} \frac{a^4}{4} = \frac{1}{2} ba^3$$



$$I_x = \frac{1}{3} A a^2 = \frac{1}{3} a^3 b, \quad I_y = \frac{1}{12} a b^3$$

$$I_o = I_x + I_y = \frac{1}{3} a b \left(a^2 + \frac{b^2}{4} \right)$$

$$n = \% \text{ error} = \frac{I_x - I_o}{I_o} (100) = \frac{-\frac{1}{12} a b^3}{\frac{1}{3} a b \left(a^2 + \frac{b^2}{4} \right)} 100$$

$$= -\frac{1}{4} \frac{b^2}{a^2 + \frac{b^2}{4}} 100$$

$$\text{For } \frac{b}{a} = \frac{1}{10}, \quad n = -25 \frac{1}{10^2 + \frac{1}{4}} = \underline{\underline{-0.249 \%}}$$

A/11

$$A \cong \frac{90-30-15}{360} 2\pi (300)(10) = 2360 \text{ mm}^2$$

$$I_o \cong Ar^2 = 2360(300)^2 = \underline{212(10^6) \text{ mm}^4}$$

A/12

$$I_x = \int y^2 dA, \quad I_y = \int x^2 dA$$

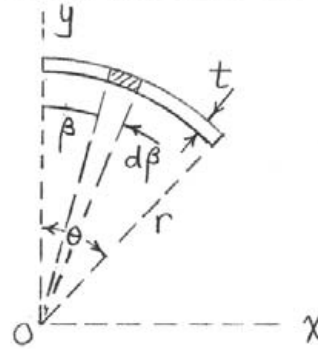
where $y = r \cos \beta$, $x = r \sin \beta$,

and $dA = r t d\beta$

$$I_x = \int_0^\theta r^2 \cos^2 \beta \, r t d\beta$$
$$= r^3 t \left[\frac{\beta}{2} + \frac{\sin 2\beta}{4} \right]_0^\theta$$

But $A = r \theta t$, so $I_x = \frac{Ar^2}{4} \left(2 + \frac{\sin 2\theta}{\theta} \right)$

$$I_y = \int_0^\theta r^2 \sin^2 \beta \, r t d\beta = r^3 t \left[\frac{\beta}{2} - \frac{\sin 2\beta}{4} \right]_0^\theta$$
$$= \frac{Ar^2}{4} \left(2 - \frac{\sin 2\theta}{\theta} \right)$$



$$I_o = I_x + I_y = Ar^2$$

For $\theta = 45^\circ$, $r = 300 \text{ mm}$, and $t = 10 \text{ mm}$

from Prob. A/11, we have

$$A = 45 \left(\frac{\pi}{180} \right) (300)(10) = 2360 \text{ mm}^2$$

$$\text{So } I_o = 2360 (300)^2 = \underline{212(10^6) \text{ mm}^4}$$

A/13

For complete ring,

$$I_o = Ar^2 = 2\pi r t r^2 = 2\pi r^3 t$$

and $I_o = I_x + I_y$, $I_x = I_y$

So for complete ring, $I_x = \frac{I_o}{2} = \pi r^3 t$

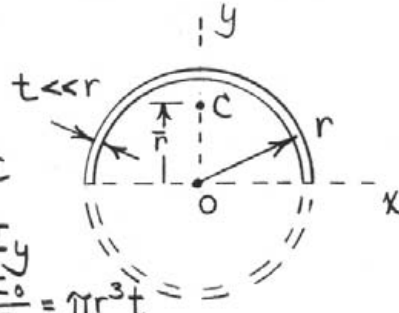
For half-ring, $I_x = \frac{1}{2} \pi r^3 t$ and $I_y = I_x$

by symmetry so $I_y = \frac{1}{2} \pi r^3 t$

For half-ring, $I_o = \frac{1}{2} (2\pi r^3 t) = \pi r^3 t$

$$I_c = I_o - A\bar{r}^2 = \pi r^3 t - \pi r t \left(\frac{2r}{\pi} \right)^2$$

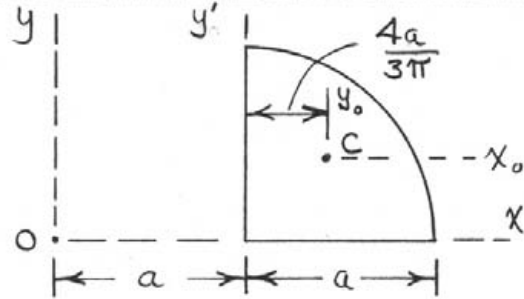
$$= \pi r^3 t \left(1 - \frac{4}{\pi^2} \right)$$



A/14

From Table D/3

$$\underline{I_x = I_{y'} = \frac{\pi a^4}{16}}$$



$$\text{So } I_{x_0} = I_{y_0} =$$

$$\frac{\pi a^4}{16} - \frac{\pi a^2}{4} \left(\frac{4a}{3\pi} \right)^2 = a^4 \left[\frac{\pi}{16} - \frac{4}{9\pi} \right]$$

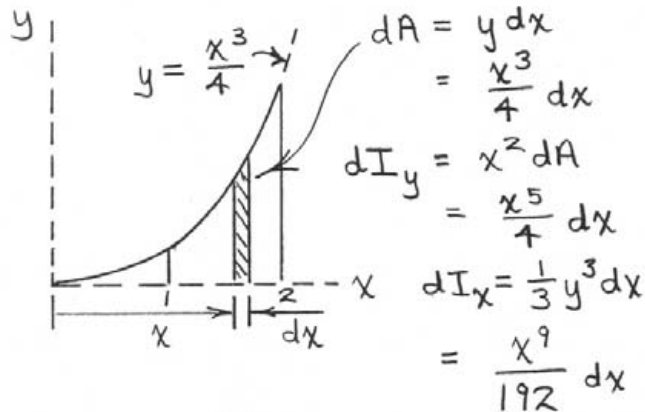
$$\begin{aligned} \text{Then } I_y &= I_{y_0} + A d^2 = a^4 \left[\frac{\pi}{16} - \frac{4}{9\pi} \right] + \frac{\pi a^2}{4} \left[a + \frac{4a}{3\pi} \right]^2 \\ &= a^4 \left[\frac{5\pi}{16} + \frac{2}{3} \right] \end{aligned}$$

$$I_0 = I_x + I_y = a^4 \left[\frac{3\pi}{8} + \frac{2}{3} \right]$$

$$k_0 = \sqrt{I_0/A} = \sqrt{a^4 \left[\frac{3\pi}{8} + \frac{2}{3} \right] / \frac{\pi a^2}{4}} = a \sqrt{\frac{8}{3\pi} + \frac{2}{3}}$$

$$\text{Decimal forms: } \begin{cases} I_x = 0.1963 a^4, & I_y = 1.648 a^4 \\ k_0 = 1.533 a \end{cases}$$

A/15



$$A = \int dA = \int_1^2 \frac{x^3}{4} dx = \left. \frac{1}{4} \frac{x^4}{4} \right|_1^2 = \frac{15}{16}$$

$$I_y = \int dI_y = \int_1^2 \frac{x^5}{4} dx = \left. \frac{1}{4} \frac{x^6}{6} \right|_1^2 = \frac{63}{24}$$

$$I_x = \int dI_x = \int_1^2 \frac{x^9}{192} dx = \left. \frac{1}{192} \frac{x^{10}}{10} \right|_1^2 = \frac{1023}{1920}$$

$$k_y = \sqrt{I_y/A} = \sqrt{\frac{63/24}{15/16}} = \sqrt{14/5} = 1.673$$

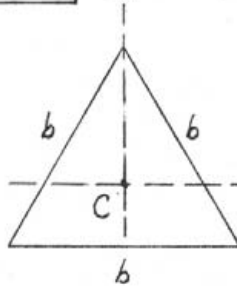
$$k_x = \sqrt{I_x/A} = \sqrt{\frac{1023/1920}{15/16}} = 0.754$$

$$k_z = \sqrt{k_x^2 + k_y^2} = \sqrt{1.673^2 + 0.754^2} = 1.835$$

A/16

 y_0

From Sample Problem A/2



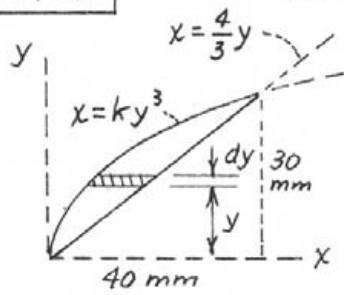
$$\bar{I}_x = \frac{1}{36} b \left(b \frac{\sqrt{3}}{2} \right)^3 = \frac{b^4}{96} \sqrt{3}$$

$$\bar{I}_y = 2 \left(\frac{1}{12} b \frac{\sqrt{3}}{2} \left[\frac{b}{2} \right]^3 \right) = \frac{b^4}{96} \sqrt{3}$$

$$\bar{J} = \bar{I}_x + \bar{I}_y = \frac{b^4}{48} \sqrt{3}$$

$$\bar{r} = \sqrt{\bar{J}/A} = \sqrt{\frac{b^4 \sqrt{3}}{48} / \left(\frac{b^2 \sqrt{3}}{4} \right)} = \frac{b}{2\sqrt{3}}$$

A/17

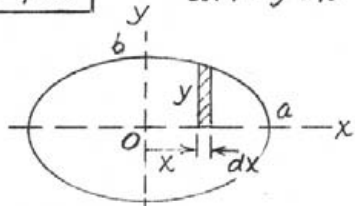
For $x = 40 \text{ mm}$ & $y = 30 \text{ mm}$, $k = \frac{40}{27(10^3)}$ 

$$dI_x = y^2 dA = y^2 \left(\frac{4}{3}y - \frac{4}{2700}y^3 \right) dy$$

$$I_x = \int_0^{30} \left(\frac{4}{3}y^3 - \frac{4}{2700}y^5 \right) dy$$

$$= \left[\frac{y^4}{3} - \frac{y^6}{4050} \right]_0^{30} = \underline{9(10^4) \text{ mm}^4}$$

A/18



$$dA = y dx = \frac{b}{a} \sqrt{a^2 - x^2} dx$$

$$A = \frac{4b}{a} \int_0^a \sqrt{a^2 - x^2} dx = \pi ab$$

$$dI_y = x^2 y dx = \frac{b}{a} x^2 \sqrt{a^2 - x^2} dx$$

$$I_y = \frac{4b}{a} \int_0^a x^2 \sqrt{a^2 - x^2} dx$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$= \frac{4b}{a} \left[-\frac{x}{4} \sqrt{(a^2 - x^2)^3} + \frac{a^2}{8} \left(x \sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a} \right) \right]_0^a$$

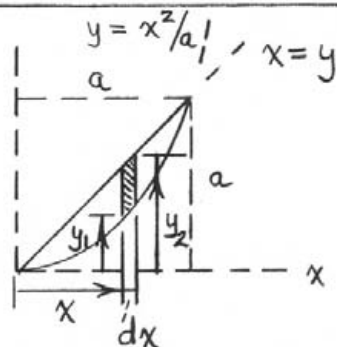
$$= \frac{b}{a} \left[0 + \frac{a^2}{2} \left(0 + a^2 \frac{\pi}{2} \right) \right] = \frac{\pi a^3 b}{4}$$

Similarly $I_x = \frac{\pi ab^3}{4}$

$$\text{So } I_0 = I_x + I_y = \frac{\pi ab}{4} (a^2 + b^2)$$

$$\therefore k_0 = \sqrt{I_0/A} = \frac{1}{2} \sqrt{a^2 + b^2}$$

A/19



$$dA = (y_2 - y_1) dx$$

$$= \left(x - \frac{x^2}{a}\right) dx$$

$$I_x = \int \frac{1}{3} y_2^3 dx - \int \frac{1}{3} y_1^3 dx = \frac{1}{3} \int_0^a \left(x^3 - \frac{x^6}{a^3}\right) dx$$

$$= \frac{1}{3} \left[\frac{x^4}{4} - \frac{x^7}{7a^3} \right]_0^a = \frac{a^4}{3} \left(\frac{1}{4} - \frac{1}{7} \right) = \underline{\underline{a^4/28}}$$

$$I_y = \int x^2 dA = \int x^2 \left(x - \frac{x^2}{a}\right) dx = \int_0^a \left(x^3 - \frac{x^4}{a}\right) dx$$

$$= \left[\frac{x^4}{4} - \frac{x^5}{5a} \right]_0^a = \frac{a^4}{4} - \frac{a^4}{5} = \underline{\underline{a^4/20}}$$

A/2

From Sample Prob. A/1,

$$I_{x_o} = \frac{1}{12} b h^3$$

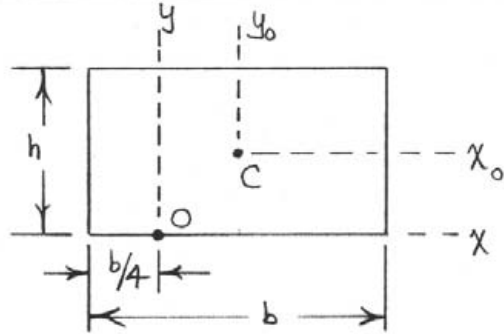
$$I_{y_o} = \frac{1}{12} h b^3$$

So

$$I_x = \frac{1}{12} b h^3 + b h \left(\frac{h}{2} \right)^2 = \frac{1}{3} b h^3$$

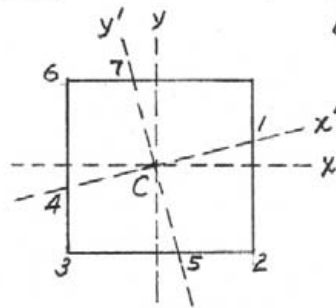
$$I_y = \frac{1}{12} h b^3 + b h \left(\frac{b}{4} \right)^2 = \frac{7}{48} h b^3$$

$$I_o = I_x + I_y = \frac{b h}{3} \left(h^2 + \frac{7}{16} b^2 \right)$$



A/20

$$I_C = I_x + I_y = I_{x'} + I_{y'}$$



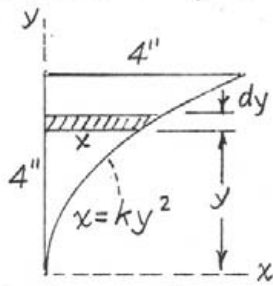
By symmetry of the figures
1-2-3-4 and 5-3-6-7

$I_{x'} = I_{y'}$. Thus since $I_x = I_y$,

$$2I_x = I_C = 2I_{x'} \text{ so } \underline{I_{x'} = I_x}$$

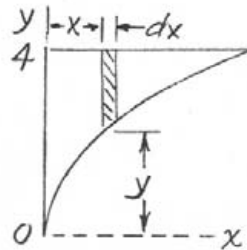
A/21

$$dI_x = y^2 dA = y^2 x dy = y^2 (y^2/4) dy = (y^4/4) dy$$



$$I_x = \frac{1}{4} \int_0^4 y^4 dy = \frac{1}{4} \frac{y^5}{5} \Big|_0^4 = \frac{256}{5} = \underline{51.2 \text{ in.}^4}$$

$$k = 1/4, x = y^2/4$$



$$dI_x = \frac{1}{3} (dx) (4^3 - y^3)$$

$$= \frac{1}{3} (4^3 - [2\sqrt{x}]^3) dx$$

$$I_x = \frac{1}{3} \int_0^4 (4^3 - 8x^{3/2}) dx$$

$$= \frac{1}{3} \left[4^3 x - 8 \frac{2}{5} x^{5/2} \right]_0^4$$

$$= \frac{256}{3} \left(1 - \frac{2}{5} \right) = \frac{256}{5} = \underline{51.2 \text{ in.}^4}$$

$$\underline{A/22} \quad (a) \quad k_o^2 = k_c^2 + \bar{OC}^2 \quad \text{where} \quad k_c^2 = I_c/A = \frac{40(10^4)}{1600} \text{ mm}^2$$

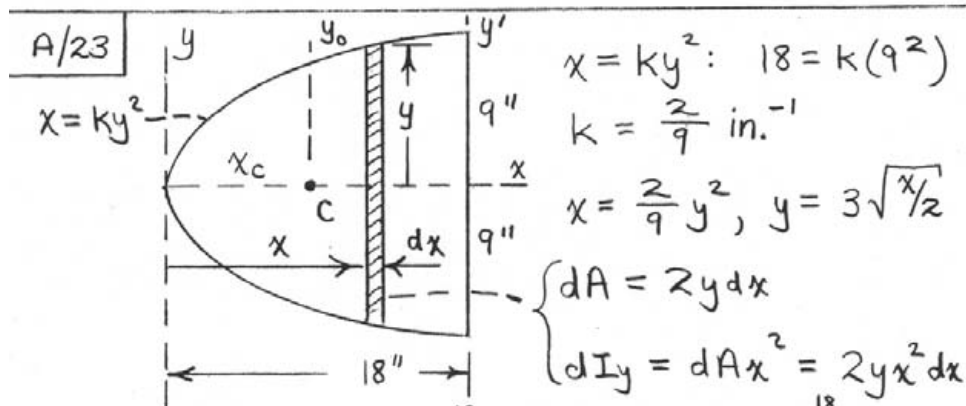
$$= 250 \text{ mm}^2$$

$$k_o^2 = 250 + (30\sqrt{2})^2 = 2050 \text{ mm}^2$$

$$k_o = \sqrt{2050} = \underline{45.3 \text{ mm}}$$

$$(b) \quad I_{x_o} = k_{x_o}^2 A \quad \& \quad I_{x_o} + I_{y_o} = I_c \quad \& \quad I_{y_o} = I_{x_o} \quad \text{so} \quad I_{x_o} = \frac{1}{2} I_c$$

$$\text{so } k_{x_o}^2 = I_c/2A, \quad k_{x_o} = \sqrt{\frac{40(10^4)}{2(1600)}} = \underline{11.18 \text{ mm}}$$



$$A = \int dA = \int 2y dx = \int_0^{18} 2(3\sqrt{x/2}) dx = \frac{6}{\sqrt{2}} \int_0^{18} x^{1/2} dx$$

$$= \frac{6}{\sqrt{2}} \frac{x^{3/2}}{3/2} \Big|_0^{18} = \frac{4}{\sqrt{2}} 18^{3/2} = 216 \text{ in.}^2$$

$$I_y = \int dI_y = \int 2(3\sqrt{x/2}) x^2 dx = \frac{6}{\sqrt{2}} \int_0^{18} x^{5/2} dx$$

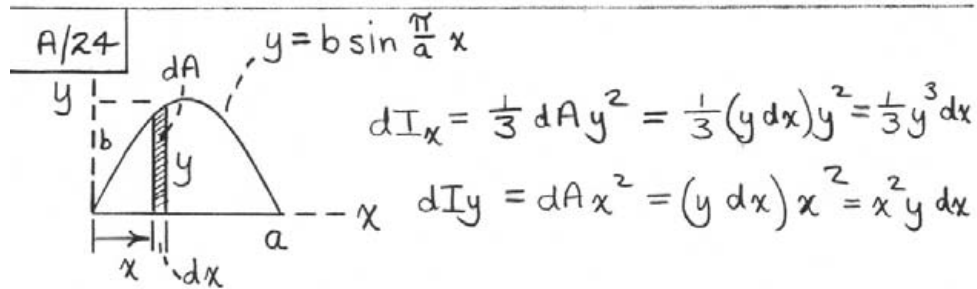
$$= \frac{6}{\sqrt{2}} \frac{x^{7/2}}{7/2} \Big|_0^{18} = 30,000 \text{ in.}^4$$

$$\int x dA = \int 2(3\sqrt{x/2}) x dx = \frac{6}{\sqrt{2}} \int_0^{18} x^{3/2} dx = 2330 \text{ in.}^3$$

$$x_c = \int x dA / A = 2330 / 216 = 10.80 \text{ in.}$$

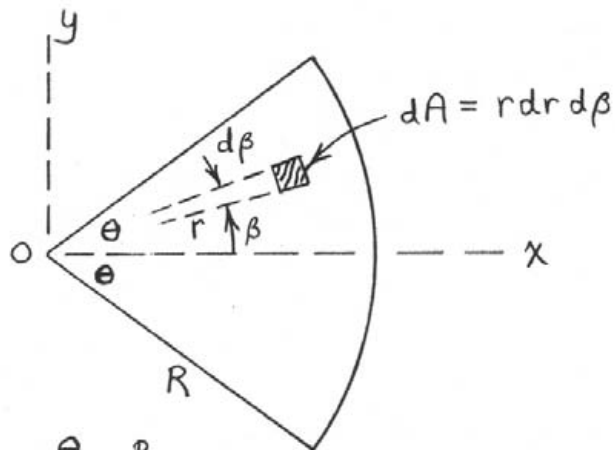
$$I_{y_0} = I_y - A x_c^2 = 30,000 - 216 (10.80)^2 = 4800 \text{ in.}^4$$

$$I_{y'} = I_{y_0} + A (18 - x_c)^2 = 4800 + 216 (18 - 10.8)^2 = 16,000 \text{ in.}^4$$



$$\begin{aligned}
 I_x &= \int dI_x = \int \frac{1}{3} y^3 dx = \frac{1}{3} \int_0^a \left(b \sin \frac{\pi}{a} x \right)^3 dx \\
 &= \frac{b^3}{3} \left[-\frac{a}{\pi} \cos \frac{\pi}{a} x + \frac{a}{3\pi} \cos^3 \frac{\pi}{a} x \right]_0^a \\
 &= \frac{4}{9} \frac{ab^3}{\pi}
 \end{aligned}$$

$$\begin{aligned}
 I_y &= \int dI_y = \int_0^a x^2 b \sin \frac{\pi}{a} x dx \\
 &= b \left[\frac{2x}{(\pi/a)^2} \sin \frac{\pi}{a} x + \frac{2}{(\pi/a)^3} \cos \frac{\pi}{a} x - \frac{x^2}{\pi/a} \cos \frac{\pi}{a} x \right]_0^a \\
 &= \frac{ba^3}{\pi} \left(1 - \frac{4}{\pi^2} \right)
 \end{aligned}$$



$$\begin{aligned}
 I_x &= \int y^2 dA = \int_{-\theta}^{\theta} \int_0^R (r \sin \beta)^2 r dr d\beta \\
 &= \frac{R^4}{4} \left(\frac{\beta}{2} - \frac{\sin 2\beta}{4} \right)_{-\theta}^{\theta} = \frac{R^4}{4} \left(\theta - \frac{1}{2} \sin 2\theta \right)
 \end{aligned}$$

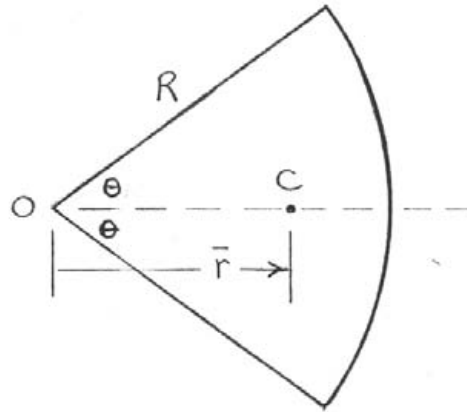
$$\begin{aligned}
 I_y &= \int x^2 dA = \int_{-\theta}^{\theta} \int_0^R (r \cos \beta)^2 r dr d\beta \\
 &= \frac{R^4}{4} \left(\frac{\beta}{2} + \frac{\sin 2\beta}{4} \right)_{-\theta}^{\theta} = \frac{R^4}{4} \left(\theta + \frac{1}{2} \sin 2\theta \right)
 \end{aligned}$$

A/26

$$I_c = I_o - A\bar{r}^2$$

$$\bar{r} = \frac{2}{3} \frac{R \sin \theta}{\theta}$$

$$A = \frac{2\theta}{2\pi} \pi R^2 = \theta R^2$$



From Prob. A/25,

$$I_o = I_x + I_y = \frac{R^4}{4} \left(\theta - \frac{\sin \theta}{2} + \theta + \frac{\sin \theta}{2} \right) = \frac{1}{2} R^4 \theta$$

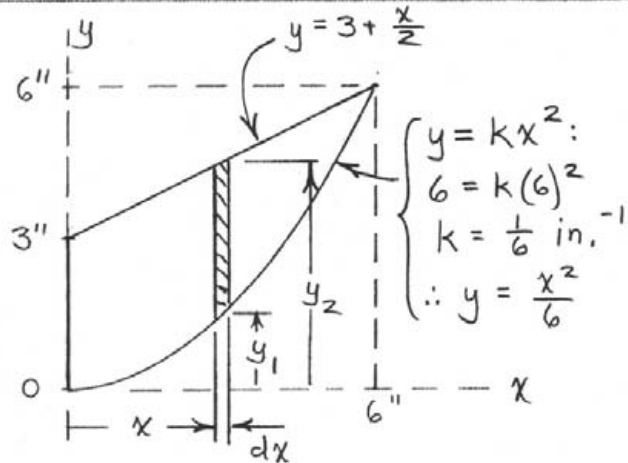
$$\text{So } I_c = \frac{1}{2} R^4 \theta - \theta R^2 \left(\frac{2}{3} \frac{R \sin \theta}{\theta} \right)^2$$

$$= \underline{R^4 \theta \left(\frac{1}{2} - \frac{4}{9} \frac{\sin^2 \theta}{\theta^2} \right)}$$

A/27

$$y_2 = 3 + \frac{x}{2}$$

$$y_1 = \frac{x^2}{6}$$



From Sample Problem A/1:

$$dI_x = \frac{1}{3} (y_2 dx) y_2^2 - \frac{1}{3} (y_1 dx) y_1^2$$

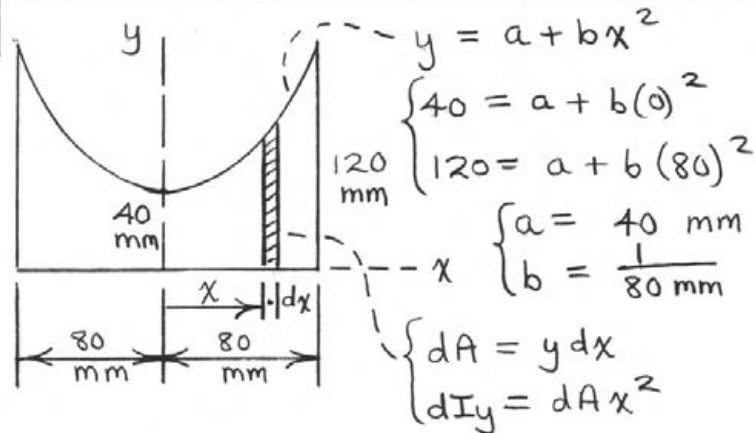
$$= \frac{1}{3} (y_2^3 - y_1^3) dx$$

$$I_x = \frac{1}{3} \int_0^6 \left[\left(3 + \frac{x}{2} \right)^3 - \left(\frac{x^2}{6} \right)^3 \right] dx$$

$$= \frac{1}{3} \left[27x + \frac{27}{4} x^2 + \frac{3}{4} x^3 + \frac{1}{32} x^4 - \frac{1}{1512} x^7 \right]_0^6$$

$$= \underline{140.8 \text{ in.}^4}$$

A/28



$$A = \int dA = \int y dx = 2 \int_0^{80} \left(40 + \frac{1}{80} x^2 \right) dx$$

$$= 2 \left[40x + \frac{x^3}{240} \right]_0^{80} = 10670 \text{ mm}^2$$

$$I_y = \int dI_y = \int x^2 y dx = 2 \int_0^{80} \left[40x^2 + \frac{x^4}{80} \right] dx$$

$$= 2 \left[\frac{40}{3} x^3 + \frac{x^5}{400} \right]_0^{80} = 30.0 (10^6) \text{ mm}^4$$

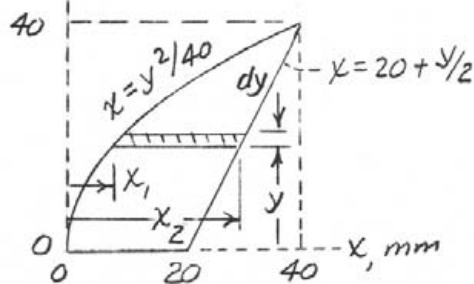
$$k_y = \sqrt{I_y/A} = \sqrt{\frac{30.0 (10^6)}{10670}} = \underline{53.1 \text{ mm}}$$

A/29

$$dI_y = \frac{1}{3} dy (x_2^3 - x_1^3) = \frac{1}{3} \left[\left(20 + \frac{y}{2} \right)^3 - \left(\frac{y^2}{40} \right)^3 \right] dy$$

y, mm

40



$$\int_0^{40} \left(20 + \frac{y}{2} \right)^3 dy$$

$$= \frac{1}{2} \left(20 + \frac{y}{2} \right)^4 \Big|_0^{40} = 120 (10^4) \text{ mm}^4$$

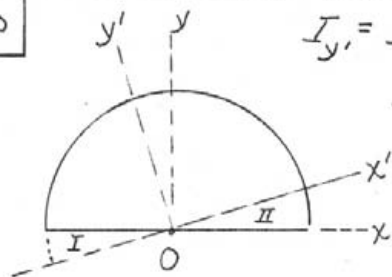
$$\int_0^{40} \frac{y^6}{40^3} dy = \frac{1}{40^3} \frac{y^7}{7} \Big|_0^{40}$$

$$= 36.57 (10^4) \text{ mm}^4$$

$$I_y = \frac{1}{3} (120 - 36.6) 10^4 = \underline{27.8 (10^4) \text{ mm}^4}$$

A/3	$I_x \approx Ad^2 = 300(15)^2 = 67.5(10^3) \text{ mm}^4$ $J_o = I_x + I_y = 67.5(10^3) + 35(10^3) = 102.5(10^3) \text{ mm}^4$ $r_o = \sqrt{J_o/A} = \sqrt{\frac{102.5(10^3)}{300}} = \underline{18.48 \text{ mm}}$
-----	--

A/30



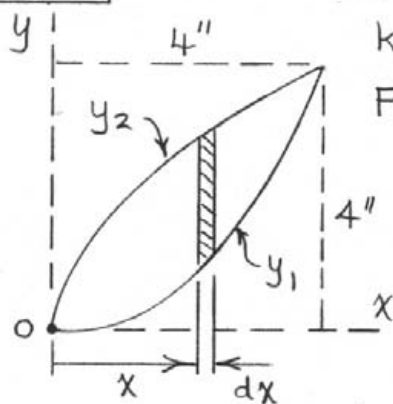
$$I_{y'} = I_{\text{half circle}} - I_I + I_{II}$$

But $I_I = I_{II}$ by symmetry

Similarly for $I_{x'}$

$$\text{So } \underline{I_{x'} = I_{y'} = I_x = I_y}$$

A/31



$$\text{From } y_1 = k_1 x^3 : 4 = k_1 4^3 \\ k_1 = \frac{1}{16} \text{ in.}^{-2} \quad \& \quad y_1 = \frac{x^3}{16}$$

$$\text{From } y_2 = k_2 \sqrt{x} : 4 = k_2 \sqrt{4} \\ k_2 = 2 \text{ in.}^{1/2} \quad \& \quad y_2 = 2\sqrt{x}$$

$$dA = (y_2 - y_1) dx \\ = \left(2\sqrt{x} - \frac{x^3}{16} \right) dx$$

$$I_y = \int x^2 dA = \int_0^4 x^2 \left(2x^{1/2} - \frac{x^3}{16} \right) dx \\ = \left[\frac{4}{7} x^{7/2} - \frac{x^6}{96} \right]_0^4 = \underline{30.5 \text{ in.}^4}$$

$$I_x = \int \left[\frac{1}{3} y_2^3 dx - \frac{1}{3} y_1^3 dx \right] = \frac{1}{3} \int_0^4 \left[(2\sqrt{x})^3 - \left(\frac{x^3}{16} \right)^3 \right] dx \\ = \frac{1}{3} \left[8 \cdot \frac{2}{5} x^{5/2} - \frac{x^{10}}{10 \cdot 16^3} \right]_0^4 = \underline{25.6 \text{ in.}^4}$$

$$I_o = I_x + I_y = \underline{56.1 \text{ in.}^4}$$

A/32

$$I_x + I_y = I_o, \quad I_x = I_y$$
$$\text{So } I_x = \frac{1}{2} I_o = \frac{1}{2} [16(10^6)]$$
$$= \underline{8(10^6) \text{ mm}^4}$$

$$\text{Similarly, } I_{x'} + I_{y'} = I_o$$
$$\text{and } I_{x'} = I_{y'}$$

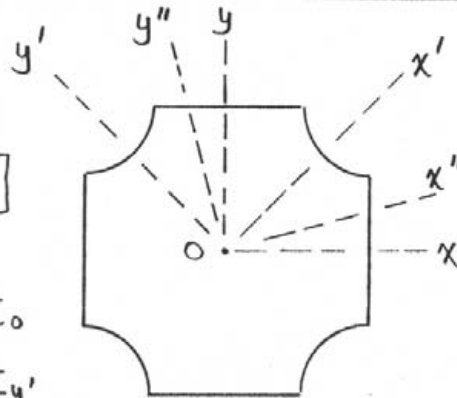
$$\text{So } I_{x'} = \frac{1}{2} I_o = I_x = \underline{8(10^6) \text{ mm}^4}$$

By symmetry it is seen that $I_{x''} = I_{y''}$

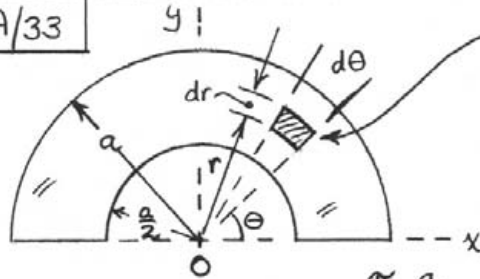
and because $I_{x''} + I_{y''} = I_o$, it follows that

The moment of inertia about every axis

Through O has the same value, $8(10^6) \text{ mm}^4$.



A/33



$$dA = r dr d\theta$$

$$A = \frac{1}{2} \left[\pi a^2 - \pi \left(\frac{a}{2} \right)^2 \right]$$

$$= \frac{3}{8} \pi a^2$$

$$I_x = \int y^2 dA = \int_0^{\pi} \int_{a/2}^a (r \sin \theta)^2 r dr d\theta$$

$$= \int_0^{\pi} \frac{15}{64} a^4 \sin^2 \theta d\theta = \frac{15}{128} \pi a^4$$

$$I_y = \int x^2 dA = 2 \int_0^{\pi/2} \int_{a/2}^a (r \cos \theta)^2 r dr d\theta$$

$$= 2 \int_{a/2}^a \frac{15}{64} a^4 \cos^2 \theta d\theta = \frac{15}{128} \pi a^4$$

$$\underline{k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{15}{128} \pi a^4}{\frac{3}{8} \pi a^2}} = \frac{\sqrt{5}}{4} a = k_y}$$

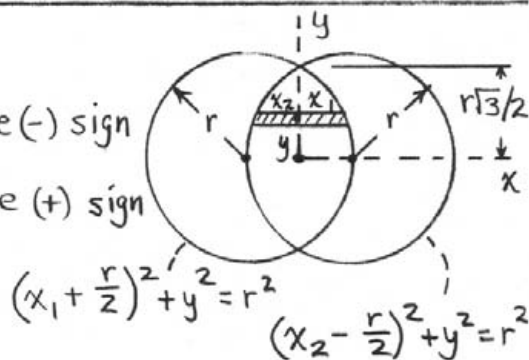
$$k_z^2 = k_x^2 + k_y^2 = 2 \left(\frac{5}{16} a^2 \right)$$

$$\underline{k_z = \frac{\sqrt{10}}{4} a}$$

► A/34

$$x_2 = \frac{r}{2} \pm \sqrt{r^2 - y^2} \quad \text{use (-) sign}$$

$$x_1 = -\frac{r}{2} \pm \sqrt{r^2 - y^2} \quad \text{use (+) sign}$$



$$(x_1 - x_2) = -\frac{r}{2} + \sqrt{r^2 - y^2} - \frac{r}{2} + \sqrt{r^2 - y^2} = 2\sqrt{r^2 - y^2} - r$$

$$dA = (2\sqrt{r^2 - y^2} - r) dy$$

$$I_x = \int y^2 dA = 2 \int_0^{r\sqrt{3}/2} y^2 (2\sqrt{r^2 - y^2} - r) dy$$

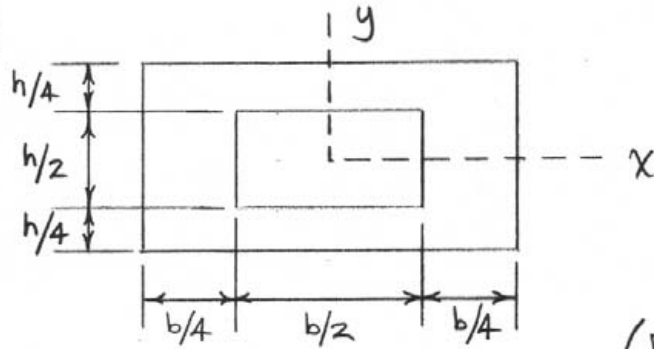
$$= 4 \left\{ -\frac{y}{4} \sqrt{(r^2 - y^2)^3} + \frac{r^2}{8} \left(y \sqrt{r^2 - y^2} + r^2 \sin^{-1} \frac{y}{r} \right) \right\} - \frac{2r^3}{3} y \Big|_0^{r\sqrt{3}/2}$$

$$= 4 \left\{ -\frac{r\sqrt{3}}{8} \frac{r^3}{8} + \frac{r^2}{8} \left(\frac{r\sqrt{3}}{2} \frac{r}{2} + r^2 \frac{\pi}{3} \right) \right\} - \frac{2\sqrt{3}}{8} r^4 - 0$$

$$= \frac{r^4}{2} \left\{ -\frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4} + \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right\} = \frac{r^4}{2} \left\{ \frac{\pi}{3} - \frac{3\sqrt{3}}{8} \right\}$$

$$= 0.1988 r^4$$

A/35



For solid plate : $I_z = \frac{1}{12} bh(b^2 + h^2)$ (From Sample Problem A/1)

For plate with hole :

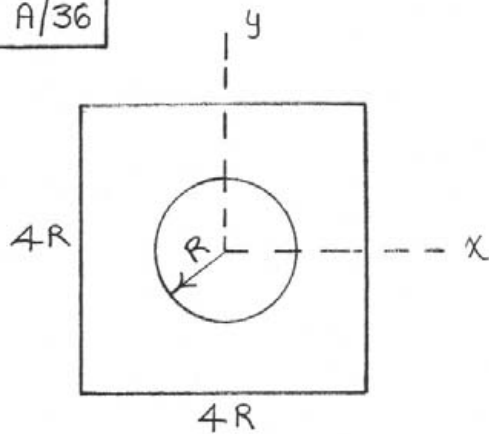
$$(I_z)_h = \frac{1}{12} bh(b^2 + h^2) - \frac{1}{12} \frac{b}{2} \frac{h}{2} \left[\left(\frac{b}{2} \right)^2 + \left(\frac{h}{2} \right)^2 \right]$$

$$= \frac{15}{16} \left[\frac{1}{12} bh(b^2 + h^2) \right]$$

The percent reduction $n = \frac{I_z - (I_z)_h}{I_z} (100)$

$$n = \left(\frac{1 - \frac{15}{16}}{1} \right) 100 = \underline{6.25\%}$$

A/36

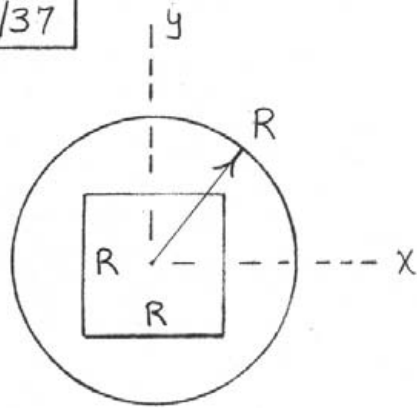


Without hole, $I_x = \frac{1}{12} (4R)(4R)^3 = \frac{64}{3} R^4$
 $(21.3 R^4)$

With hole, $I_x = \frac{64}{3} R^4 - \frac{1}{4} (\pi R^2) R^2$
 $= \underline{20.6 R^4}$

(a 3.68% reduction)

A/37



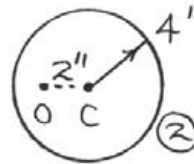
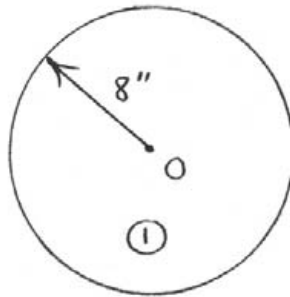
Without square hole:

$$I_z = 2I_x = 2 \left(\frac{1}{4} \pi R^2 \cdot R^2 \right) = \underline{1.571 R^4}$$

With hole:

$$I_z = 1.571 R^4 - 2 \left(\frac{1}{12} R \cdot R^3 \right) = \underline{1.404 R^4}$$

(a reduction of 10.61%)



$$\text{Area } A = A_1 - A_2 = \pi(8^2 - 4^2) = 48\pi \text{ in.}^2$$

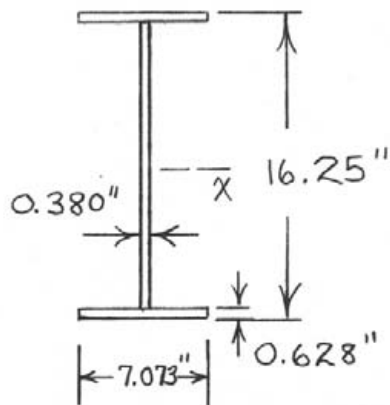
$$\textcircled{1} \quad I_{O_1} = \frac{1}{2}(\pi 8^2) 8^2 = 2048\pi \text{ in.}^4$$

$$\textcircled{2} \quad I_{O_2} = +\frac{1}{2}(\pi 4^2) 4^2 + \pi(4)^2(2^2) = +192\pi \text{ in.}^4$$

$$\text{So } I_O = I_{O_1} - I_{O_2} = 1856\pi \text{ in.}^4$$

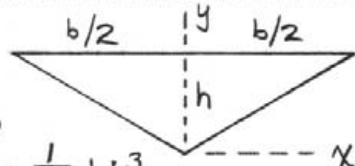
$$k_O = \sqrt{I_O/A} = \sqrt{\frac{1856\pi}{48\pi}} = \underline{6.22 \text{ in.}}$$

A/39



$$\begin{aligned}\bar{I}_x &= \frac{1}{12} (0.380) [16.25 - 2(0.628)]^3 \\ &\quad + 2 \left\{ \frac{1}{12} (7.073)(0.628)^3 + 7.073(0.628) \left[\frac{16.25}{2} - \frac{0.628}{2} \right]^2 \right\} \\ &= \underline{649 \text{ in.}^4}\end{aligned}$$

A/4



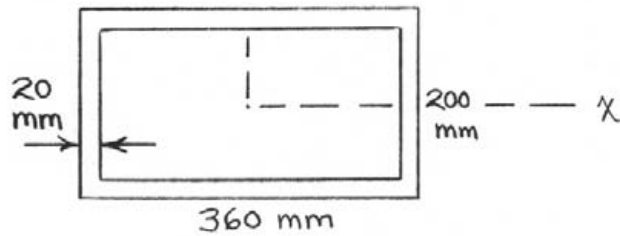
From Sample Problem A/2,

$$I_x = \frac{1}{4} b h^3, \quad I_y = 2 \left\{ \frac{1}{12} h \left(\frac{b}{2} \right)^3 \right\} = \frac{1}{48} h b^3$$

$$I_x = I_y \quad \text{if} \quad \frac{1}{4} b h^3 = \frac{1}{48} h b^3, \quad \underline{\frac{b}{h} = 2\sqrt{3}}$$

A/40

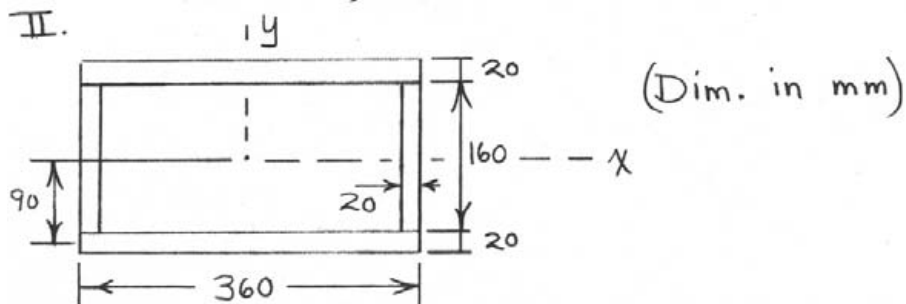
I.



$$I_x = \frac{1}{12} (360)(200)^3 - \frac{1}{12} (320)(160)^3$$

$$= \underline{130.8 (10^6) \text{ mm}^4}$$

II.



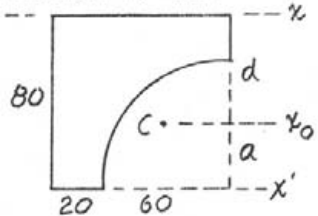
$$I_x = 2 \left[\frac{1}{12} (360)(20^3) + 360(20)(90)^2 \right]$$

$$+ 2 \left[\frac{1}{12} (20)(160)^3 \right] = \underline{130.8 (10^6) \text{ mm}^4}$$

A/41

Square: $I_x = \frac{1}{3} b^4 = \frac{1}{3} (80)^4 = 13.65 (10^6) \text{ mm}^4$

Dimen. in mm



Quarter-circle: $a = \frac{4r}{3\pi} = \frac{4(60)}{3\pi}$

$$= 25.46 \text{ mm}$$

$$d = 80 - 25.46 = 54.54 \text{ mm}$$

$$I_x = I_{x_0} + Ad^2 = I_{x_1} - Aa^2 + Ad^2$$

$$= \frac{1}{4} \frac{\pi r^4}{4} - \frac{\pi r^2}{4} (d^2 - a^2) = -\frac{\pi r^2}{4} \left(\frac{r^2}{4} + d^2 - a^2 \right)$$

$$= -\frac{\pi (60)^2}{4} \left[\frac{60^2}{4} + (54.54)^2 - (25.46)^2 \right]$$

$$= -9.120 (10^6) \text{ mm}^4$$

Total $I_x = (13.65 - 9.120) (10^6) = \underline{4.53 (10^6) \text{ mm}^4}$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A} = \frac{0 + (130)(20)(65+10)}{250(20)} = 39.0 \text{ mm}$$

The diagram shows an L-shaped cross-section composed of three rectangular parts labeled ①, ②, and ③. Part ① is a horizontal rectangle with a width of 120 mm and a height of 20 mm. Part ② is a vertical rectangle with a width of 20 mm and a height of 29.0 mm, attached to the right side of part ①. Part ③ is another vertical rectangle with a width of 20 mm and a height of 101.0 mm, attached to the bottom of part ②. A dashed horizontal line represents the x-axis, with the origin \$x_o\$ at the center of the horizontal leg. The distance from the top edge of the horizontal leg to the x-axis is \$h_1 = 29.0\$ mm. The distance from the x-axis to the bottom edge of the vertical leg is \$h_2 = 101.0\$ mm. The total height of the vertical leg is \$h_1 + h_2 = 130\$ mm. The parallel axis theorem is used to calculate the moment of inertia for each part about the x-axis.

$h_1 = 29.0 \text{ mm}$
 $h_2 = 101.0 \text{ mm}$
 $h_1 + h_2 = 130 \text{ mm}$

① $\bar{I}_x = \frac{1}{12}(120)(20^3) + (120)(20)(39.0)^2$
 $= 3.73(10^6) \text{ mm}^4$

② $\bar{I}_x = \frac{1}{3}(20)(29.0)^3 = 0.163(10^6) \text{ mm}^4$

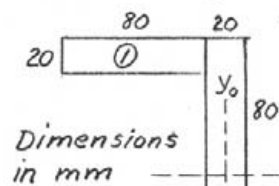
③ $\bar{I}_x = \frac{1}{3}(20)(101.0)^3 = 6.87(10^6) \text{ mm}^4$

$\bar{I}_x = (3.73 + 0.163 + 6.87)10^6$
 $= 10.76(10^6) \text{ mm}^4$

A/43

$$\textcircled{1} I_{x_o} = \frac{1}{12} (80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6) \text{ mm}^4$$

$$I_{y_o} = \frac{1}{12} (20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6) \text{ mm}^4$$



$$\textcircled{2} I_{x_o} = \frac{1}{12} (20)(160)^3 = 6.83(10^6) \text{ mm}^4$$

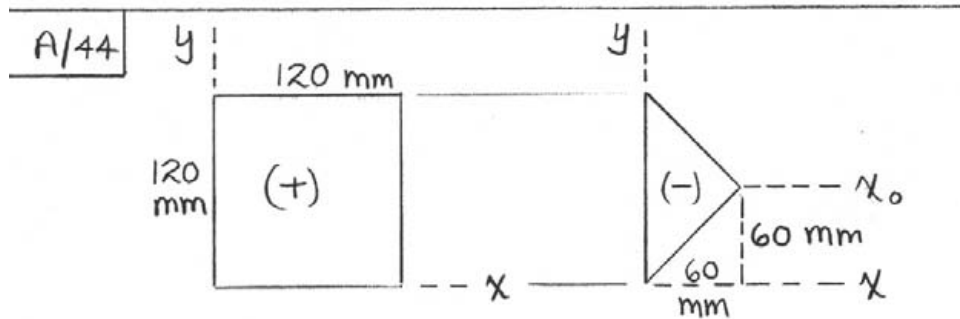
$$I_{y_o} = \frac{1}{12} (160)(20)^3 = 0.1067(10^6) \text{ mm}^4$$

$$\text{Total } \bar{I}_x = [2(7.89) + 6.83](10^6)$$

$$= 22.6(10^6) \text{ mm}^4$$

$$\bar{I}_y = [2(4.85) + 0.1067](10^6)$$

$$= 9.81(10^6) \text{ mm}^4$$



Positive area :

$$I_x = I_y = \frac{1}{3} A h^2 = \frac{1}{3} (120)^2 (120)^2 = 69.1 (10^6) \text{ mm}^4$$

$$I_z = I_x + I_y = 2 (69.1) (10^6) = 138.2 (10^6) \text{ mm}^4$$

Negative area :

$$I_x = I_{x_0} + A \bar{y}^2 = -2 \left(\frac{b h^3}{12} \right) - A (60)^2$$

$$= -2 \left(\frac{60 \times 60^3}{12} \right) - \frac{1}{2} (60) (120) (60)^2 = -15.12 (10^6) \text{ mm}^4$$

$$I_y = -\frac{b h^3}{12} = -\frac{120 (60)^3}{12} = -2.16 (10^6) \text{ mm}^4$$

$$I_z = I_x + I_y = -15.12 (10^6) - 2.16 (10^6) = -17.28 (10^6) \text{ mm}^4$$

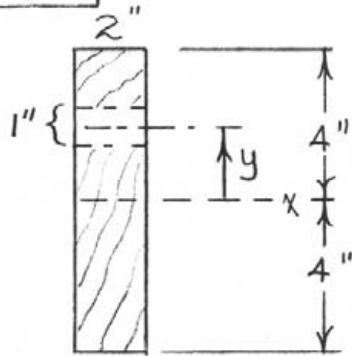
For composite area :

$$I_x = [69.1 - 15.12] 10^6 = \underline{54 (10^6) \text{ mm}^4}$$

$$I_y = [69.1 - 2.16] 10^6 = \underline{67.0 (10^6) \text{ mm}^4}$$

$$I_z = I_x + I_y = [54 + 67.0] 10^6 = \underline{121.0 (10^6) \text{ mm}^4}$$

A/45



Without hole,

$$I_x = \frac{1}{12} b h^3 = \frac{1}{12} (2) (8)^3$$

$$= 85.3 \text{ in.}^4$$

With hole, $I'_x = I_x - (\bar{I}_{\text{hole}} + A y^2)$

$$I'_x = 85.3 - \left[\frac{1}{12} (2) (1)^3 + (2) (1) y^2 \right]$$

$$= 85.2 - 2y^2 \quad (y \text{ in in., } I'_x \text{ in in.}^4)$$

Percent reduction $n = \frac{I_x - I'_x}{I_x} (100\%)$

$$n = \frac{85.3 - (85.2 - 2y^2)}{85.3} (100\%) = 0.1953 + 2.34y^2$$

(in percent)

For $y = 2 \text{ in.}$,

$$n = 0.1953 + 2.34(2)^2 = \underline{9.57\%}$$

A/46

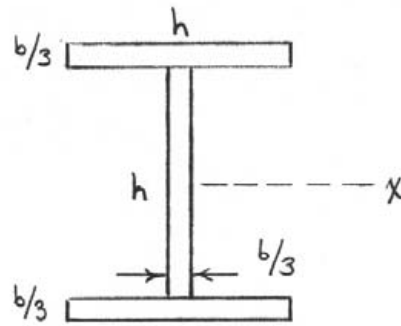
For area (a),

$$I_x = \frac{1}{12} b h^3$$

For area (b),

$$I_x = \frac{1}{12} \frac{b}{3} h^3 + 2 \left[\frac{1}{12} h \left(\frac{b}{3} \right)^3 + h \frac{b}{3} \left(\frac{h}{2} + \frac{b}{6} \right)^2 \right]$$

$$= \frac{hb}{9} \left(\frac{7}{4} h^2 + \frac{2}{9} b^2 + hb \right)$$



If $h = 200 \text{ mm}$ and $b = 60 \text{ mm}$, we have

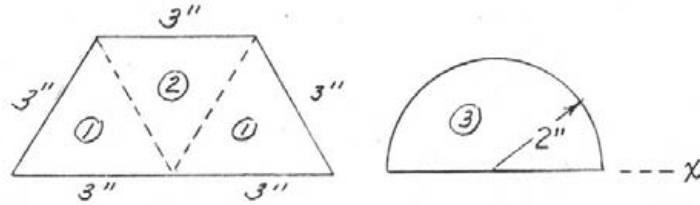
$$(a) I_x = \frac{1}{12} (60) (200)^3 = 40 (10^6) \text{ mm}^4$$

$$(b) I_x = \frac{200(60)}{9} \left(\frac{7}{4} (200)^2 + \frac{2}{9} (60)^2 + 200(60) \right)$$

$$= 110.4 (10^6) \text{ mm}^4$$

$$\text{Percent increase } n = \frac{110.4 - 40}{40} (100\%) = \underline{\underline{176.0\%}}$$

A/47



$$\textcircled{1} \quad I_x = 2 \frac{1}{12} (3) (3\sqrt{3}/2)^3 = \frac{81}{16} \sqrt{3} \text{ in.}^4$$

$$\textcircled{2} \quad I_x = \frac{1}{4} (3) (3\sqrt{3}/2)^3 = \frac{243}{32} \sqrt{3} \text{ in.}^4$$

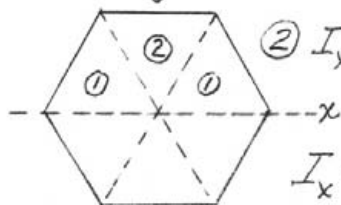
$$\textcircled{3} \quad I_x = -\frac{1}{2} \left(\frac{1}{4} \pi 2^4 \right) = -2\pi = -6.28 \text{ in.}^4$$

$$\text{Total } I_x = \frac{81}{16} \sqrt{3} + \frac{243}{32} \sqrt{3} - 6.28 = \underline{15.64 \text{ in.}^4}$$

A/48

From Sample Problem A2

$$\textcircled{1} I_x = \frac{1}{12} b (b\sqrt{3}/2)^3 = \frac{\sqrt{3}}{32} b^4$$

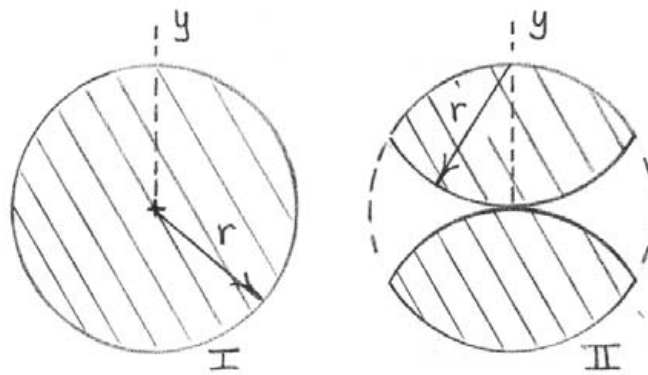


$$\textcircled{2} I_x = \frac{1}{4} b h^3 = \frac{1}{4} b (b\sqrt{3}/2)^3 = \frac{3\sqrt{3}}{32} b^4$$

$$I_x = 4I_{\textcircled{1}} + 2I_{\textcircled{2}} = \frac{\sqrt{3}}{8} b^4 + \frac{3\sqrt{3}}{16} b^4$$

$$= \underline{\underline{\frac{5\sqrt{3}}{16} b^4}}$$

A/49

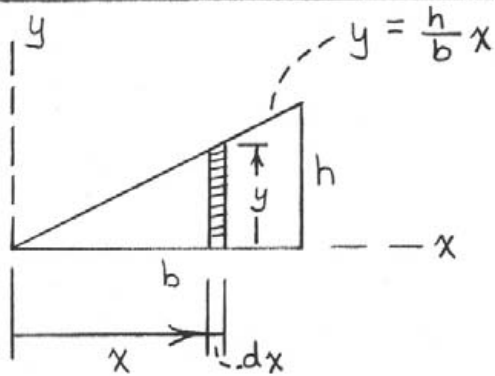


$$I_I = \frac{1}{4} \pi r^4; \quad I_{II} = 2(0.1988 r^4) = 0.398 r^4$$

$$I_y = I_I - I_{II} = 0.785 r^4 - 0.398 r^4 = \underline{0.388 r^4}$$

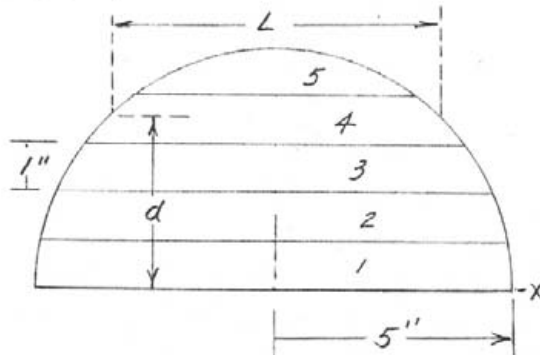
(Refer to Prob. A/34 for I_{II} .)

A/5



$$\begin{aligned} I_y &= \int x^2 dA = \int x^2 y dx = \int x^2 \left(\frac{h}{b} x \right) dx \\ &= \frac{h}{b} \int_0^b x^3 dx = \frac{h}{4b} x^4 \Big|_0^b = \underline{\underline{\frac{hb^3}{4}}} \end{aligned}$$

A/50

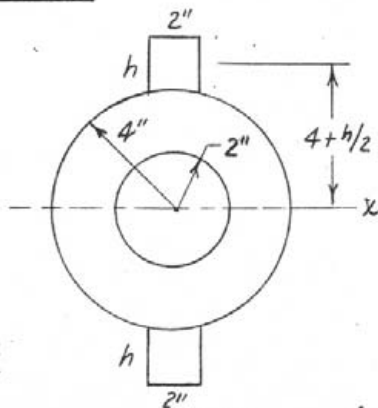


	L in.	A in. ²	d in.	Ad^2 in. ⁴
1	9.9	9.9	0.5	2.5
2	9.5	9.5	1.5	21.5
3	8.7	8.7	2.5	54.1
4	7.1	7.1	3.5	87.5
5	4.4	4.4	4.5	88.3

$$I_x \approx \sum Ad^2 = 253.8 \text{ in.}^4$$

$$I_{x \text{ exact}} = \frac{1}{2} \left(\frac{\pi r^4}{4} \right) = \frac{\pi}{8} 5^4 = 245.4 \text{ in.}^4; \text{ Error} = \frac{253.8 - 245.4}{245.4} \cdot 100 = +3\%$$

A/52



Circular section

$$I_x = \frac{\pi r_2^4}{4} - \frac{\pi r_1^4}{4}$$

$$= \frac{\pi}{4} (4^4 - 2^4) = 188.50 \text{ in.}^4$$

Rectangular strips

$$I_x = \bar{I}_x + Ad^2$$

$$I_x = 2 \left[\frac{1}{12} (2)(h)^3 + 2h \left(4 + \frac{h}{2} \right)^2 \right]$$

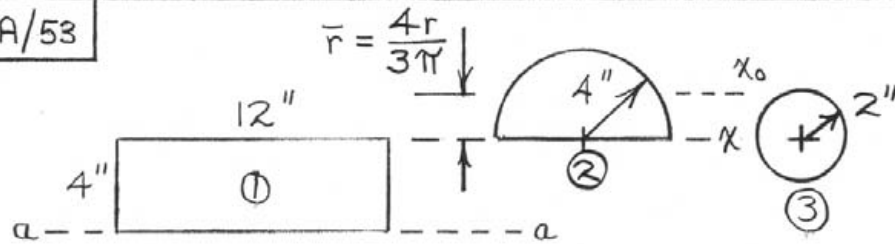
$$= \frac{h^3}{3} + 64h + 16h^2 + h^3$$

$$= \frac{4h^3}{3} + 16h^2 + 64h$$

To double stiffness $(I_x)_{\square} = [I_x]_o$

$$\text{so } \frac{4h^3}{3} + 16h^2 + 64h = 188.50$$

solve for h & get $h = 1.900 \text{ in.}$



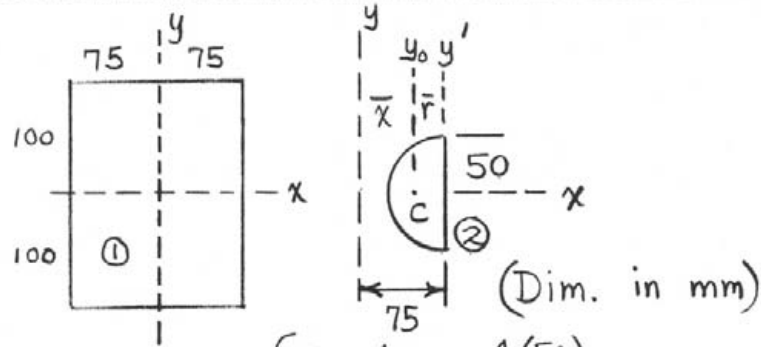
Part 1: $I_{a-a} = \frac{1}{3} (12) 4^3 = 256 \text{ in.}^4$

Part 2: $I_{a-a} = I_{x_0} + A \left(4 + \frac{4 \cdot 4}{3\pi} \right)^2$
 where $I_{x_0} = I_x - A \bar{r}^2 = \frac{1}{8} \pi 4^4 - \pi \frac{4^2}{2} \left(\frac{4 \cdot 4}{3\pi} \right)^2$
 $= 28.1 \text{ in.}^4$

So $I_{a-a} = 28.1 + \frac{\pi 4^2}{2} (32.5) = 844 \text{ in.}^4$

Part 3: $I_{a-a} = I_x + A (4)^2 = \frac{1}{4} \pi 2^4 + \pi 2^2 (4)^2$
 $= 214 \text{ in.}^4$

Combined: $I_{a-a} = 256 + 844 - 214 = 886 \text{ in.}^4$



$$\begin{cases} \bar{r} = \frac{4r}{3\pi} = \frac{4(50)}{3\pi} = 21.2 \text{ mm} \\ \bar{x} = 75 - 21.2 = 53.8 \text{ mm} \end{cases}$$

Part I: $I_x = \frac{1}{12} (150)(200)^3 = 100(10^6) \text{ mm}^4$

$$I_y = \frac{1}{12} (200)(150)^3 = 56.2(10^6) \text{ mm}^4$$

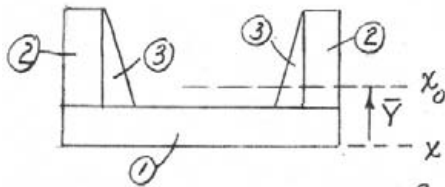
Parts II: $I_x = \frac{1}{4} \pi (50)^4 = 4.91(10^6) \text{ mm}^4$ (for both together)

$$\begin{aligned} I_y &= I_{y_0} + A\bar{x}^2 = I_{y_1} - A\bar{r}^2 + A\bar{x}^2 = I_{y_1} + A(\bar{x}^2 - \bar{r}^2) \\ &= \frac{1}{2} \left(\frac{1}{4} \pi 50^4 \right) + \frac{\pi (50)^2}{2} (53.8^2 - 21.2^2) \\ &= 12.04(10^6) \text{ mm}^4 \text{ for each, } 24.1(10^6) \text{ mm}^4 \text{ for both} \end{aligned}$$

Combined: $I_x = 100(10^6) - 4.91(10^6) = 95.1(10^6) \text{ mm}^4$

$$I_y = 56.2(10^6) - 24.1(10^6) = \underline{32.2(10^6) \text{ mm}^4}$$

Part	A in. ²	\bar{Y} in.	$\bar{Y}A$ in. ³	\bar{I}_x in. ⁴	d in.	Ad^2 in. ⁴
1	8.40	0.35	2.94	0.343	0.696	4.07
2	4.29	2.35	10.08	3.894	1.304	7.30
3	0.33	1.80	0.59	0.200	0.754	0.19
Totals	13.02	13.61	4.437	11.56		



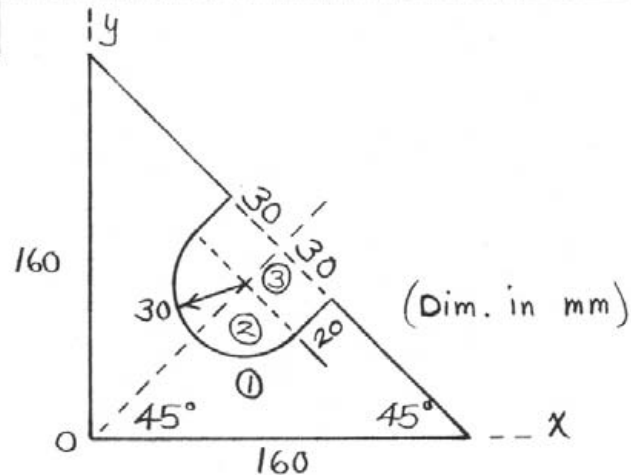
$$\textcircled{1} A_1 = 0.70 \times 12 = 8.40 \text{ in.}^2$$

$$\textcircled{2} A_2 = 0.65 \times 3.30 \times 2 = 4.29 \text{ in.}^2$$

$$\textcircled{3} A_3 = 2 \left(\frac{1}{2} \right) (0.10) (3.30) = 0.33 \text{ in.}^2$$

$$\bar{Y} = \frac{13.61}{13.02} = 1.046 \text{ in.}$$

$$\begin{aligned} \bar{I}_x = I_{x_0} &= \sum \bar{I}_x + \sum Ad^2 \\ &= 4.437 + 11.56 \\ &= \underline{16.00 \text{ in.}^4} \end{aligned}$$



$$\text{Triangle ①: } I_o = I_x + I_y = 2I_x = 2 \left[\frac{160(160)^3}{12} \right] \\ = 109.2 (10^6) \text{ mm}^4$$

Semicircular area ②:

$$I_o = -\frac{\pi(30)^4}{4} + \frac{\pi(30)^2}{2} \left(\frac{4 \cdot 30}{3\pi} \right)^2 - \frac{\pi(30)^2}{2} \left(\frac{160}{\sqrt{2}} - 20 - \frac{4 \cdot 30}{3\pi} \right)^2 \\ = -9.55 (10^6) \text{ mm}^4$$

Rectangular area ③:

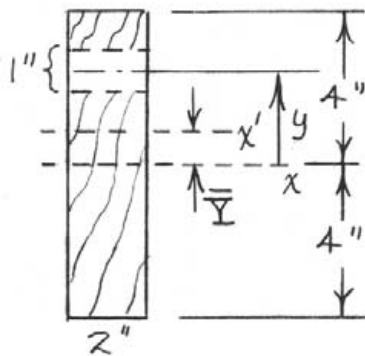
$$I_o = -\frac{60(20)}{12} (60^2 + 20^2) - 60(20) \left(\frac{160}{\sqrt{2}} - 10 \right)^2 \\ = -13.16 (10^6) \text{ mm}^4$$

$$\text{Total } I_o = \underline{86.5 (10^6) \text{ mm}^4}$$

►A/57

Without hole, $x' = x$ and $I_x = I_{x'} =$

$$\frac{1}{12}bh^3 = \frac{1}{12}(2)(8)^3 = 85.3 \text{ in.}^4$$



Centroid location :

$$\bar{Y} = \frac{\sum \bar{y} A}{\sum A} = \frac{8(2)(0) - 2(1)y}{8(2) - 2(1)} = -0.1429y \quad (\text{or } -\frac{1}{7}y)$$

$$\begin{aligned} I_{x'}' &= \frac{1}{3}(2)(4+\bar{Y})(4+\bar{Y})^2 + \frac{1}{3}(2)(4-\bar{Y})(4-\bar{Y})^2 \\ &\quad - \left[\frac{1}{12}(2)(1)^3 + 2(1)(y-\bar{Y})^2 \right] \\ &= 85.2 + 16\bar{Y}^2 - 2(y-\bar{Y})^2 \end{aligned}$$

$$\text{With } \bar{Y} = -\frac{1}{7}y : I_{x'}' = 85.2 + 16\left(-\frac{1}{7}y\right)^2 - 2\left(y + \frac{1}{7}y\right)^2$$

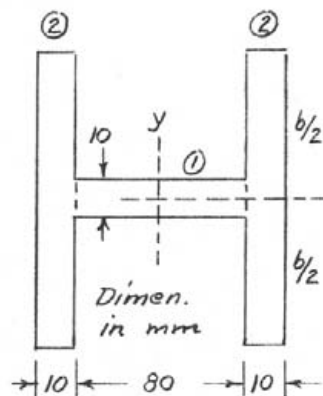
$$\text{or } I_{x'}' = 85.2 - \frac{112}{49}y^2 = 85.2 - 2.29y^2$$

$$\text{Percent reduction } n = \frac{I_{x'} - I_{x'}'}{I_{x'}} (100\%)$$

$$= \frac{(100\%)85.3 - (85.2 - 2.29y^2)}{85.3} = 0.1953 + 2.68y^2 \quad (\text{in percent})$$

$$\text{For } y = 2 \text{ in.}, \quad n = 10.91\%$$

► A/58



$$\textcircled{1} I_x = \frac{1}{12} (80)(10)^3 = 0.00667(10^6) \text{ mm}^4$$

$$I_y = \frac{1}{12} (10)(80)^3 = 0.427(10^6) \text{ mm}^4$$

$$\textcircled{2} I_x = 2 \left[\frac{1}{12} (10) b^3 \right] = 1.667 b^3$$

$$I_y = 2 \left[\frac{1}{12} b (10)^3 \right] + (10b)(45)^2$$

$$= 0.0407(10^6) b$$

$$\text{Total } I_x = \text{Total } I_y$$

$$(0.00667)(10^6) + 1.667 b^3$$

$$= (0.427 + 0.0407)(10^6)$$

$$\text{or } b^3 - 0.0244(10^6) b - 0.252(10^6) = 0$$

Solve by cubic formula; $\left[\frac{0.252(10^6)}{2} \right]^2 < \left[\frac{0.0244(10^6)}{3} \right]^3$ so 3 real roots

$$\cos u = \frac{q}{p\sqrt{p}} \text{ where } q = \frac{252(10^3)}{2} = 126(10^3), p = \frac{24.4(10^3)}{3} = 8.13(10^3)$$

$$\cos u = 126(10^3) / [8.13(10^3) 90.2] = 0.1718, u = 80.11^\circ$$

$$b_1 = 2\sqrt{p} \cos \frac{u}{3} = 2(90.2)(0.8933) = 161.1 \text{ mm} \text{ or } \underline{b = 161.1 \text{ mm}}$$

$$b_2 = 2\sqrt{p} \cos \left(\frac{u}{3} + 120^\circ \right) = (-1), b_3 = 2\sqrt{p} \cos \left(\frac{u}{3} + 240^\circ \right) = (-1)$$

$$\boxed{A/59} \quad A = (30)(60) = 1800 \text{ mm}^2 \text{ for each area.}$$

$$\bar{I}_{xy} = 0 \text{ for each area, so } I_{xy} = 0 + A d_x d_y.$$

$$(a) \quad I_{xy} = 50(40)(1800) = 360(10^4) \text{ mm}^4$$

$$(b) \quad I_{xy} = 50(-40)(1800) = -360(10^4) \text{ mm}^4$$

$$(c) \quad I_{xy} = (-50)(10)(1800) = \underline{-90(10^4) \text{ mm}^4}$$

$A/6$

$$I_P = I_C + A(3)^2, \quad I_{P'} = I_C + A(2)^2$$
$$I_P - I_{P'} = 50 = A(3^2 - 2^2), \quad \underline{A = 10 \text{ in.}^2}$$

A/60

$$\begin{aligned} I_{xy} &= -3(3) [(-3.5)(3.5) + (3.5)(-3.5)] \\ &= \underline{220 \text{ in.}^4} \end{aligned}$$

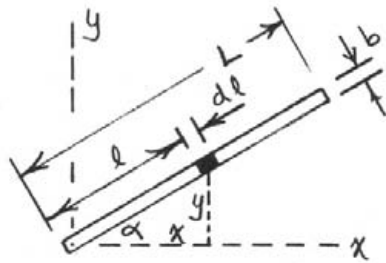
A/61

$$\begin{aligned} I_x &= \frac{1}{12} (400) (200)^3 - 3 \left[\frac{1}{4} \pi (30^4) + \pi (30)^2 (50)^2 \right] \\ &= \underline{2.44 (10^8) \text{ mm}^4} \end{aligned}$$

$$\begin{aligned} I_y &= \frac{1}{12} (200) (400)^3 - 3 \left[\frac{1}{4} \pi (30^4) + \pi (30)^2 (100)^2 \right] \\ &= \underline{9.80 (10^8) \text{ mm}^4} \end{aligned}$$

$$\begin{aligned} I_{xy} &= -\pi (30)^2 [(100)(50) + (-100)(50) + (-100)(-50)] \\ &= \underline{-14.14 (10^6) \text{ mm}^4} \end{aligned}$$

A/62



$$\begin{aligned}
 I_{xy} &= \int xy \, dA = \int_0^L (l \cos \alpha)(l \sin \alpha) b \, dl \\
 &= b \sin \alpha \cos \alpha \int_0^L l^2 \, dl \\
 &= \frac{bL^3}{3} \sin \alpha \cos \alpha \quad \text{or} \quad \underline{\underline{\frac{1}{6} bL^3 \sin 2\alpha}}
 \end{aligned}$$

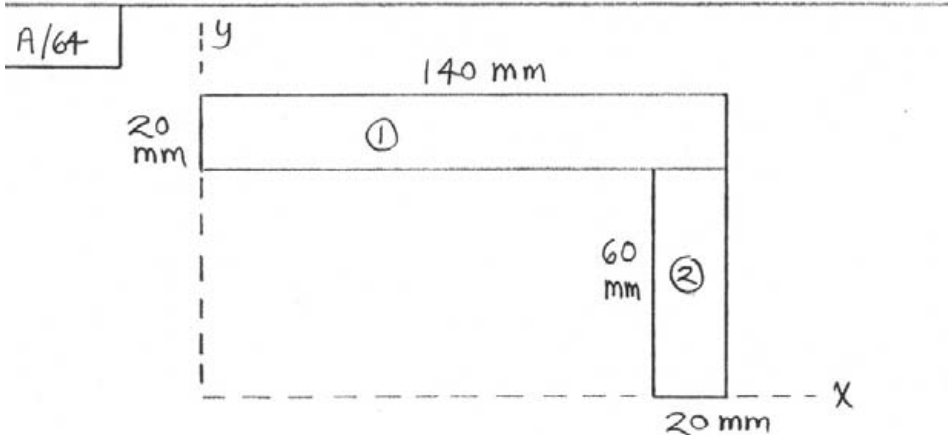
A/63

$$(a) \ I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)(40)(80)(50) \\ = \underline{9.60(10^6) \text{ mm}^4}$$

$$(b) \ I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(40)(\pi \cdot 25^2) \\ = \underline{-4.71(10^6) \text{ mm}^4}$$

$$(c) \ I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(-40)(80)(50) \\ = \underline{9.60(10^6) \text{ mm}^4}$$

$$(d) \ I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)\left(-40 - \frac{4(25)}{3\pi}\right) \\ \times (\pi \cdot 25^2)/2 \\ = \underline{-2.98(10^6) \text{ mm}^4}$$

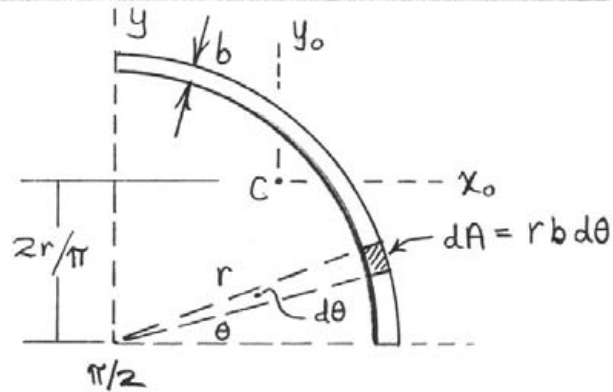


$$I_{xy_1} = 20(140)(70)(70) = 13.72(10^6) \text{ mm}^4$$

$$I_{xy_2} = 60(20)(130)(30) = 4.68(10^6) \text{ mm}^4$$

$$\text{Total: } I_{xy} = 18.40(10^6) \text{ mm}^4$$

A/65

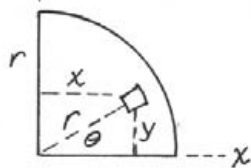


$$\begin{aligned}
 I_{xy} &= \int xy \, dA = \int (r \cos \theta)(r \sin \theta) r b \, d\theta \\
 &= r^3 b \left. \frac{\sin^2 \theta}{2} \right|_0^{\pi/2} = \underline{\frac{1}{2} r^3 b}
 \end{aligned}$$

$$\begin{aligned}
 I_{x_0 y_0} &= I_{xy} - d_x d_y A \\
 &= \frac{1}{2} r^3 b - \left(\frac{2r}{\pi} \right) \left(\frac{2r}{\pi} \right) r b \frac{\pi}{2} = \underline{\left(\frac{1}{2} - \frac{2}{\pi} \right) r^3 b}
 \end{aligned}$$

A/66

$$I_{xy} = \int xy dA = \int_0^{\pi/2} \int_0^r (r \cos \theta)(r \sin \theta) r dr d\theta$$



$$= \int_0^{\pi/2} \frac{\sin 2\theta}{2} \frac{r^4}{4} d\theta = \frac{r^4}{16} (-\cos 2\theta) \Big|_0^{\pi/2}$$

$$= \frac{r^4}{16} (1 - [-1]) = \frac{r^4}{8}$$

$$\bar{I}_{xy} = I_{xy} - d_x d_y A = \frac{r^4}{8} - \frac{4r}{3\pi} \left(\frac{4r}{3\pi} \right) \frac{\pi r^2}{4} = \frac{r^4}{8} \left(1 - \frac{32}{9\pi} \right)$$

$$= \underline{\underline{-0.01647 r^4}}$$

A/67

(1) By direct integration

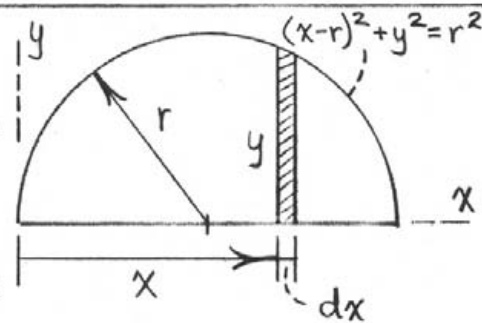
For elemental strip,

$$dI_{xy} = x \frac{y}{2} dA = \frac{xy}{2} y dx$$

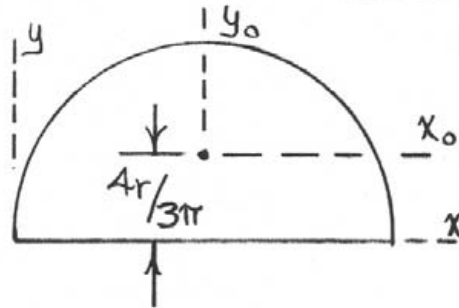
$$= \frac{x}{2} [r^2 - (x-r)^2] dx$$

$$I_{xy} = \frac{1}{2} \int_0^{2r} (xr^2 - x^3 + 2rx^2 - r^2x) dx$$

$$= \frac{1}{2} \left[\frac{x^2 r^2}{2} - \frac{x^4}{4} + \frac{2rx^3}{3} - \frac{r^2 x^2}{2} \right]_0^{2r} = \frac{2}{3} r^4$$

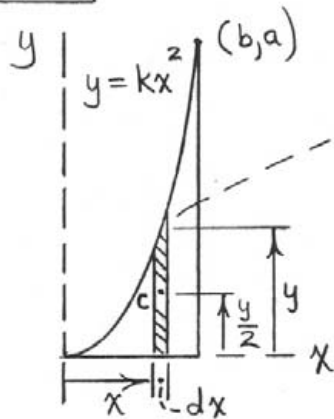


(2) By axis transfer



$$I_{xy} = I_{x_0 y_0} + A d_x d_y = 0 + \frac{\pi r^2}{2} (r) \left(\frac{4r}{3\pi} \right) = \frac{2}{3} r^4$$

A/68



$$y = kx^2: a = kb^2, k = a/b^2$$

$$\Rightarrow y = \frac{a}{b^2} x^2$$

$$dA = y dx = \frac{a}{b^2} x^2 dx$$

$$d\bar{I}_{xy} = 0$$

$$dI_{xy} = d\bar{I}_{xy} + dA(x)(\frac{y}{2})$$

$$= 0 + y dx (x)(\frac{y}{2})$$

$$= \frac{1}{2} \frac{a^2}{b^4} x^5 dx$$

$$I_{xy} = \int dI_{xy} = \int_0^b \frac{1}{2} \frac{a^2}{b^4} x^5 dx$$

$$= \frac{1}{2} \frac{a^2}{b^4} \frac{x^6}{6} \Big|_0^b = \underline{\underline{\frac{1}{12} a^2 b^2}}$$

A/69

$$x = \left(\frac{a-b}{h}\right)y + b$$

$$dI_{xy} = x dy \left(\frac{x}{2}\right) = x^2 \frac{y}{2} dy$$

$$= \left[\left(\frac{a-b}{h}\right)y + b\right]^2 \frac{y}{2} dy$$

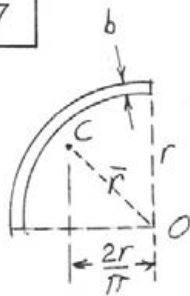
$$= \frac{1}{2} \left[\left(\frac{a-b}{h}\right)^2 y^3 + 2\left(\frac{a-b}{h}\right)by^2 + b^2 y \right] dy$$

$$I_{xy} = \int dI_{xy} = \frac{1}{2} \int_0^h \left[\left(\frac{a-b}{h}\right)^2 y^3 + 2\left(\frac{a-b}{h}\right)by^2 + b^2 y \right] dy$$

$$= \frac{1}{2} \left[\left(\frac{a-b}{h}\right)^2 \frac{y^4}{4} + 2\left(\frac{a-b}{h}\right)b \frac{y^3}{3} + b^2 \frac{y^2}{2} \right]_0^h$$

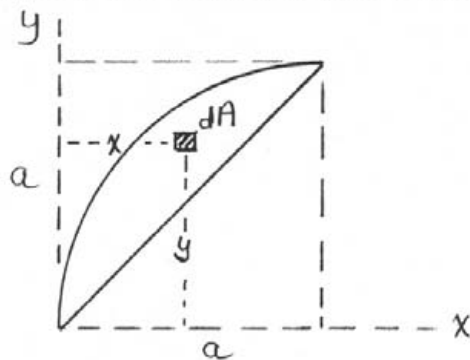
$$= \frac{h^2}{4} \left(\frac{a^2}{2} + \frac{ab}{3} + \frac{b^2}{6} \right) \text{ or } \frac{h^2}{24} (3a^2 + 2ab + b^2)$$

A/7



$$\begin{aligned} I_C &= I_O - A\bar{r}^2 \\ &= Ar^2 - A\left(\frac{2r}{\pi}\sqrt{2}\right)^2 \\ &= Ar^2\left(1 - \frac{8}{\pi^2}\right) = \frac{\pi br^3}{2}\left(1 - \frac{8}{\pi^2}\right) \\ &= br^3\left(\frac{\pi}{2} - \frac{4}{\pi}\right) \\ &= \underline{0.298 br^3} \end{aligned}$$

$$\begin{aligned}
 I_{xy} &= \int xy \, dA \\
 &= \iint xy \, dx \, dy \\
 &= \int_0^a y \left[\int_{x_1}^{x_2} x \, dx \right] dy
 \end{aligned}$$



where $x_2 = y$ and $(x_1 - a)^2 + y^2 = a^2$

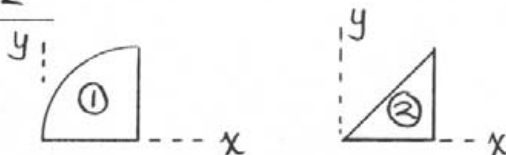
$$x_1 = a - \sqrt{a^2 - y^2}$$

(note minus sign)

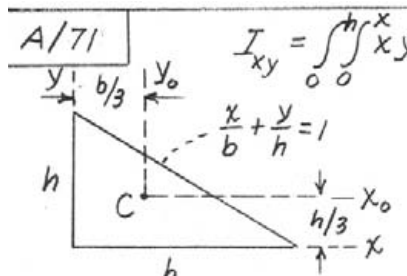
$$\int_{x_1}^{x_2} x \, dx = \frac{x^2}{2} \Big|_{a - \sqrt{a^2 - y^2}}^y = y^2 - a^2 + a\sqrt{a^2 - y^2}$$

$$\begin{aligned}
 \text{So } I_{xy} &= \int_0^a y (y^2 - a^2 + a\sqrt{a^2 - y^2}) \, dy \\
 &= \left[\frac{y^4}{4} - \frac{a^2 y^2}{2} - \frac{a}{3} \sqrt{(a^2 - y^2)^3} \right]_0^a \\
 &= \frac{a^4}{12}
 \end{aligned}$$

Alternatively



$$I_{xy} = (I_{xy})_{\text{①}} - (I_{xy})_{\text{②}}$$

$\frac{A}{71}$


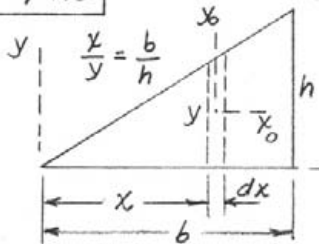
$$I_{xy} = \int_0^h \int_0^{b(1-y/h)} xy \, dx \, dy = \int_0^h y \left(\frac{x^2}{2} \right) \Big|_0^{b(1-y/h)} dy$$

$$= \int_0^h \frac{b^2}{2} \left(y - \frac{2y^2}{h} + \frac{y^3}{h^2} \right) dy$$

$$= \frac{b^2}{2} \left[\frac{h^2}{2} - \frac{2h^2}{3} + \frac{h^2}{4} \right] = \frac{b^2 h^2}{24}$$

$$I_{x_o y_o} = \bar{I}_{xy} = I_{xy} - d_x d_y A = \frac{b^2 h^2}{24} - \frac{bh}{2} \left(\frac{h}{3} \right) \left(\frac{b}{3} \right) = -\frac{b^2 h^2}{72}$$

A/72



First: $I_{xy} = \int_0^b \int_0^y xy \, dy \, dx$

$$= \int_0^b \left[\frac{x}{2} y^2 \right]_0^{\frac{hx}{b}} dx = \int_0^b \frac{h^2}{2b^2} x^3 dx$$

$$= \frac{h^2}{2b^2} \frac{b^4}{4} = \frac{b^2 h^2}{8}$$

Second: $dI_{xy} = dI_{x_0 y_0} + d_x d_y (dA)$

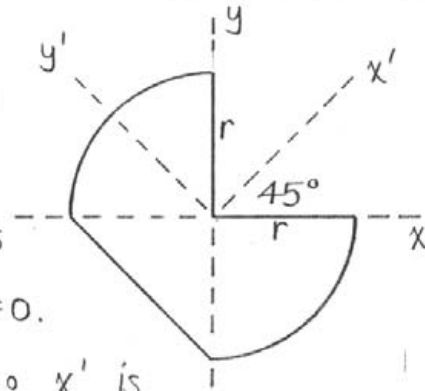
$$= 0 + \frac{y}{2} x (y dx) = \frac{h^2}{2b^2} x^3 dx$$

$$I_{xy} = \frac{h^2}{2b^2} \int_0^b x^3 dx = \frac{h^2}{2b^2} \frac{b^4}{4} = \frac{b^2 h^2}{8}$$

A/73

By inspection, x' is an axis of symmetry, so x' and y' are principal axes of inertia where $I_{x'y'} = 0$.

By inspection $I_{x'} > I_{y'}$ so x' is an axis of maximum inertia.



For each quarter-circular area,

$$I_x = I_y = \frac{1}{16}\pi r^4, \quad I_{xy} = -\frac{1}{8}r^4 \quad (\text{from Prob. A/66})$$

For triangular area

$$I_x = I_y = \frac{1}{12}r^4, \quad I_{xy} = \frac{1}{24}r^4 \quad (\text{from Prob. A/71})$$

with $h = b = r$

For Composite area

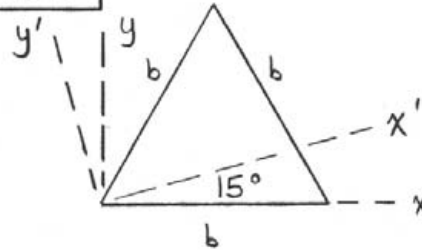
$$I_x = I_y = \left(\frac{1}{12} + 2\frac{\pi}{16}\right)r^4 = 0.476r^4$$

$$I_{xy} = \left(-\frac{1}{8}(2) + \frac{1}{24}\right)r^4 = -0.208r^4$$

Eq. A/11:

$$\begin{aligned} I_{\max} &= 0.476r^4 + \frac{1}{2}\sqrt{0 + 4(-0.208)^2r^8} \\ &= \underline{0.684r^4} \end{aligned}$$

A/74



$$I_x = \frac{1}{12} b \left(\frac{b\sqrt{3}}{2} \right)^3$$

$$= \frac{\sqrt{3}}{32} b^4$$

$$\bar{I}_y = \bar{I}_x = \frac{\sqrt{3}}{32} b^4 - \frac{1}{2} b^2 \frac{\sqrt{3}}{2} x$$

$$\left(\frac{1}{3} b \frac{\sqrt{3}}{2} \right)^2 = \frac{\sqrt{3}}{96} b^4$$

$$I_y = \bar{I}_y + A d_y^2 = \frac{\sqrt{3}}{96} b^4 + b \left(\frac{b\sqrt{3}}{4} \right) \left(\frac{b}{2} \right)^2 = \frac{7\sqrt{3}}{96} b^4$$

$$I_{xy} = \bar{I}_{xy} + A d_x d_y = 0 + b \left(\frac{b\sqrt{3}}{4} \right) \left(\frac{b}{2} \right) \left(\frac{1}{3} \frac{\sqrt{3}}{2} b \right)$$

$$= \frac{3}{48} b^4$$

Eqs. A/9 & A/9c, with $\theta = 15^\circ$:

$$I_{x'} = \frac{5\sqrt{3}}{96} b^4 - \frac{\sqrt{3}}{48} b^4 \cos 30^\circ - \frac{3}{48} b^4 \sin 30^\circ$$

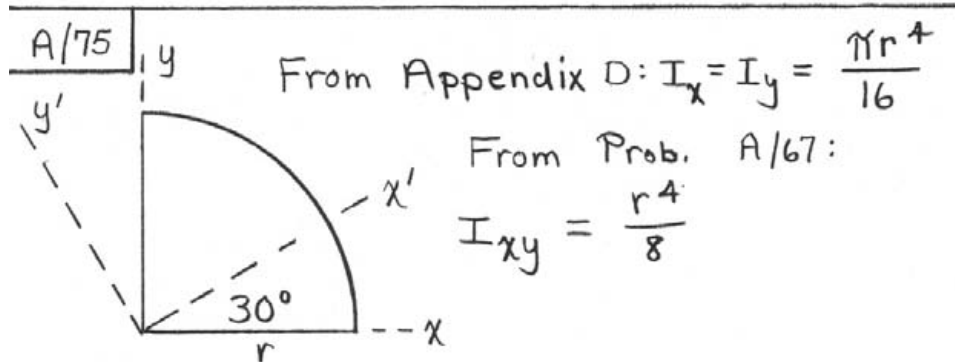
$$= 0.0277 b^4$$

$$I_{y'} = \frac{5\sqrt{3}}{96} b^4 + \frac{\sqrt{3}}{48} b^4 \cos 30^\circ + \frac{3}{48} b^4 \sin 30^\circ$$

$$= 0.1527 b^4$$

$$I_{x'y'} = -\frac{\sqrt{3}}{48} b^4 \sin 30^\circ + \frac{3}{48} b^4 \cos 30^\circ$$

$$= 0.0361 b^4$$



Eq. A/9: $I_{x'} = \frac{\pi r^4}{16} + 0 - \frac{r^4}{8} \sin 60^\circ = \frac{r^4}{16} [\pi - \sqrt{3}]$

$I_{y'} = \frac{\pi r^4}{16} - 0 + \frac{r^4}{8} \sin 60^\circ = \frac{r^4}{16} [\pi + \sqrt{3}]$

Eq. A/9a:

$I_{x'y'} = 0 + \frac{r^4}{8} \cos 60^\circ = \frac{r^4}{16}$

$$\boxed{A/76} \quad I_x = \frac{1}{3} b (b^3) = \frac{1}{3} b^4; \quad I_y = \frac{1}{3} b^4$$

$$I_{xy} = 0 + \frac{b}{2} \frac{b}{2} b^2 = \frac{1}{4} b^4$$

With $\theta = 30^\circ$, Eqs. A/9 & A/9a give

$$I_{x'} = \frac{b^4}{3} + 0 - \frac{1}{4} b^4 \sin 60^\circ = \left(\frac{1}{3} - \frac{\sqrt{3}}{8} \right) b^4 = \underline{0.1168 b^4}$$

$$I_{y'} = \frac{b^4}{3} + 0 + \frac{1}{4} b^4 \sin 60^\circ = \left(\frac{1}{3} + \frac{\sqrt{3}}{8} \right) b^4 = \underline{0.5498 b^4}$$

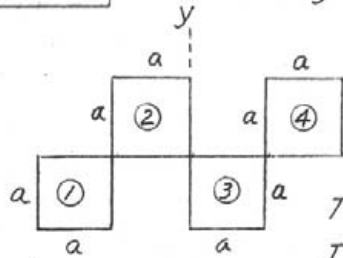
$$I_{x'y'} = 0 + \frac{b^4}{4} \frac{1}{2} = \frac{b^4}{8} = \underline{0.1250 b^4}$$

A/77 ① $I_x = a^4/3, I_y = a^4/2 + a^2(3a/2)^2 = 7a^4/3; I_{xy} = +3a^4/4$

② $I_x = a^4/3, I_y = a^4/3, I_{xy} = -a^4/4$

③ same as ②

④ same as ①



Thus for composite area

$$I_x = 4a^4/3, I_y = 16a^4/3, I_{xy} = +a^4$$

From Eq. A/11,

$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = \left(\frac{10}{3} + \sqrt{5}\right)a^4 = 5.57a^4$$

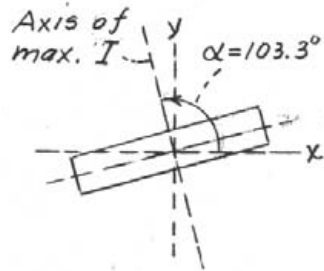
$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = \left(\frac{10}{3} - \sqrt{5}\right)a^4 = 1.097a^4$$

From Eq. A/10

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2a^4}{(16/3 - 4/3)a^4} = +\frac{1}{2}$$

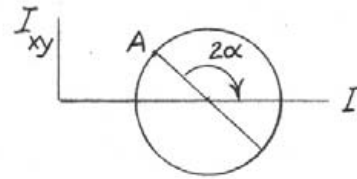
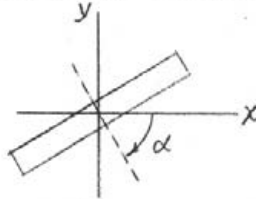
$$2\alpha = 26.6^\circ \text{ or } 206.6^\circ$$

$$\alpha = 13.3^\circ \text{ or } 103.3^\circ$$

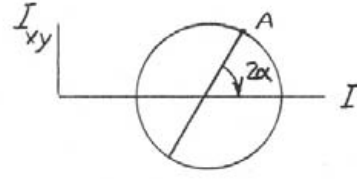
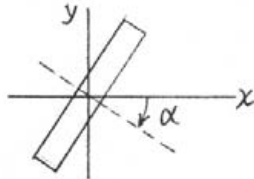


A/78

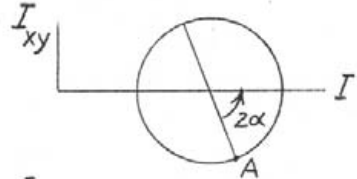
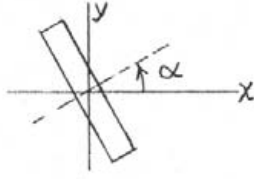
(a) $I_x < I_y, I_{xy}(+)$



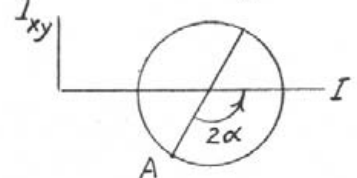
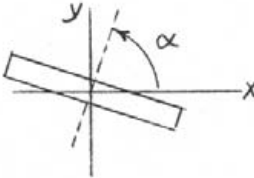
(b) $I_x > I_y, I_{xy}(+)$



(c) $I_x > I_y, I_{xy}(-)$

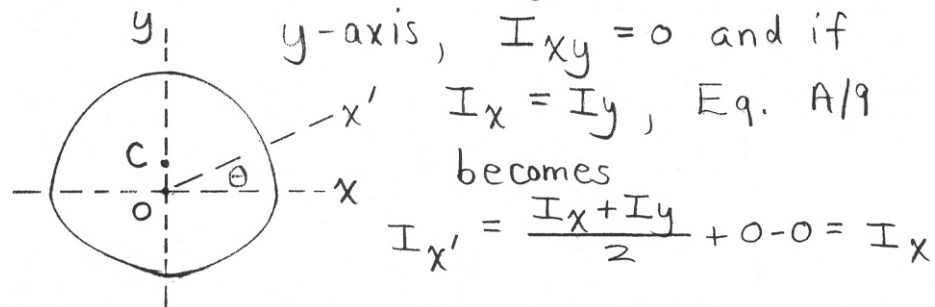


(d) $I_x < I_y, I_{xy}(-)$



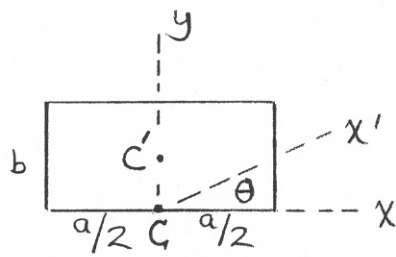
A/79

For an area symmetrical about the

y-axis, $I_{xy} = 0$ and if $I_x = I_y$, Eq. A/9

becomes

$$I_{x'} = \frac{I_x + I_y}{2} + 0 - 0 = I_x$$

So $I_{x'}$ is independent of θ .

The rectangular area

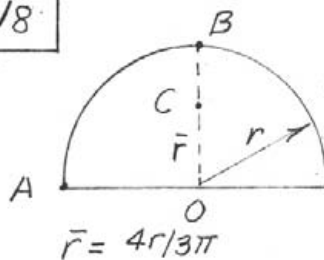
meets the condition

of y -axis symmetry, so

$$I_{xy} = 0. \quad I_x = I_y$$

$$\text{if } \frac{1}{3} ab^3 = \frac{1}{12} ba^3 \Rightarrow \underline{a = 2b}.$$

A/8



For complete circle

$$I_A = I_O + Ar^2 = \frac{1}{2}Ar^2 + Ar^2 = \frac{3}{2}Ar^2$$

For half circle

$$I_A = \frac{1}{2} \left(\frac{3}{2} \pi r^4 \right) = \frac{3}{4} \pi r^4$$

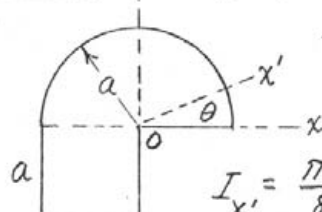
For half circle, $I_O = \frac{1}{4} \pi r^4$

$$I_B = I_C + A(r - \bar{r})^2 = I_O - A\bar{r}^2 + A(r - \bar{r})^2$$

$$= I_O + A(r^2 - 2r\bar{r})$$

$$= \frac{1}{4} \pi r^4 + \frac{\pi r^4}{2} \left(1 - \frac{8}{3\pi} \right) = r^4 \left(\frac{3\pi}{4} - \frac{4}{3} \right)$$

A/80



$$I_x = \frac{1}{8}\pi a^4 + \frac{1}{3}a^4, \quad I_y = \frac{1}{8}\pi a^4 + \frac{1}{3}a^4 = I_x$$

$$I_{xy} = 0 + a^2\left(-\frac{a}{2}\right)\left(-\frac{a}{2}\right) = \frac{a^4}{4}$$

From Eq. A/9

$$I_{x'} = \frac{\pi a^4}{8} + \frac{a^4}{3} + 0 - \frac{a^4}{4} \sin 2\theta$$

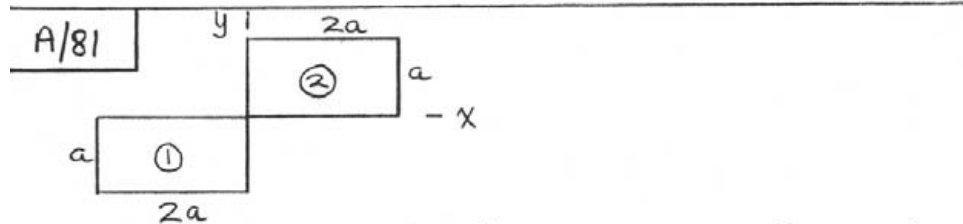
$$= a^4 \left(\frac{\pi}{8} + \frac{1}{3} - \frac{1}{4} \sin 2\theta \right)$$

$$\frac{dI_{x'}}{d\theta} = a^4 \left(-\frac{1}{2} \cos 2\theta \right) = 0 \text{ for max. or min.}$$

$$\text{so } 2\theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2} \text{ so } \theta = \frac{\pi}{4} \text{ or } \frac{3\pi}{4}$$

$$\theta = \frac{\pi}{4}, \quad I_{x'} = a^4 \left(\frac{\pi}{8} + \frac{1}{3} - \frac{1}{4} [1] \right) = \underline{0.476 a^4 \text{ (min)}}$$

$$I_{y'} = a^4 \left(\frac{\pi}{8} + \frac{1}{3} + \frac{1}{4} [1] \right) = \underline{0.976 a^4 \text{ (max)}}$$



$$\textcircled{1} I_x = \frac{1}{3} (2a)(a^3) = \frac{2}{3} a^4, \quad I_y = \frac{1}{3} (a)(2a)^3 = \frac{8}{3} a^4$$

$$I_{xy} = (2a^2)(a)\left(\frac{a}{2}\right) = a^4$$

$$\textcircled{2} I_x = \frac{2}{3} a^4, \quad I_y = \frac{8}{3} a^4, \quad I_{xy} = a^4$$

$$\text{Eq. A/11: } I_{\min} = \frac{I_x + I_y}{2} - \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$= \frac{10}{3} a^4 - \frac{1}{2} \sqrt{(-4a^4)^2 + 16a^8} = 0.505 a^4$$

$$I_{\max} = \frac{I_x + I_y}{2} + \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$= \frac{10}{3} a^4 + \frac{1}{2} \sqrt{(-4a^4)^2 + 16a^8} = 6.16 a^4$$

$$\text{Eq. A/10: } \tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4a^4}{\left(\frac{16}{3} - \frac{4}{3}\right)a^4}$$

$$2\alpha = 45^\circ \text{ or } 225^\circ$$

$$\alpha = 22.5^\circ \text{ for } I_{\min}$$

$$\text{or } \alpha = 112.5^\circ \text{ for } I_{\max}$$

A/82 From figure, I_{xy} is (-)

Add Eqs. A/11 & get $I_{max} + I_{min} = I_x + I_y$

$$\text{so } I_x + I_y = (12 + 2)10^6 = 14(10^6) \text{ mm}^4$$

From the 1st of Eqs. A/11,

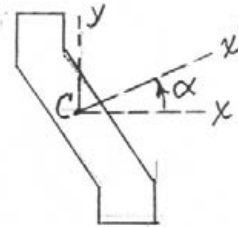
$$\begin{aligned}(I_x - I_y)^2 &= [2I_{max} - (I_x + I_y)]^2 - 4I_{xy}^2 \\ &= [2(12) - 14]^2 10^{12} - 4(-4)^2 10^{12} = 36(10^{12}) \text{ mm}^8\end{aligned}$$

$$\left. \begin{aligned}I_x - I_y &= 6(10^6) \text{ mm}^4 \\ I_x + I_y &= 14(10^6) \text{ mm}^4\end{aligned} \right\} \text{ add \& get } \begin{aligned}I_x &= 10(10^6) \text{ mm}^4 \\ \& \ I_y &= 4(10^6) \text{ mm}^4\end{aligned}$$

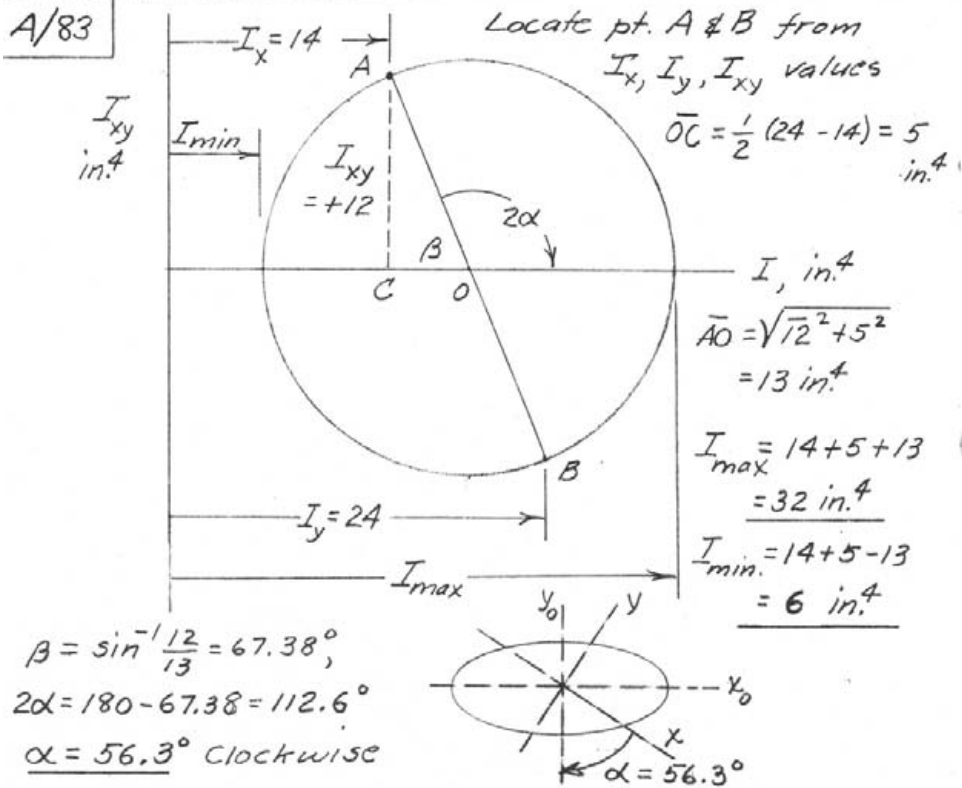
From Eq. A/10,

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(-4)(10^6)}{-6(10^6)} = 4/3$$

$$2\alpha = 53.13^\circ, \quad \alpha = 26.6^\circ$$



A/83



A/84

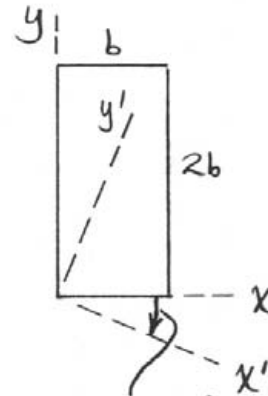
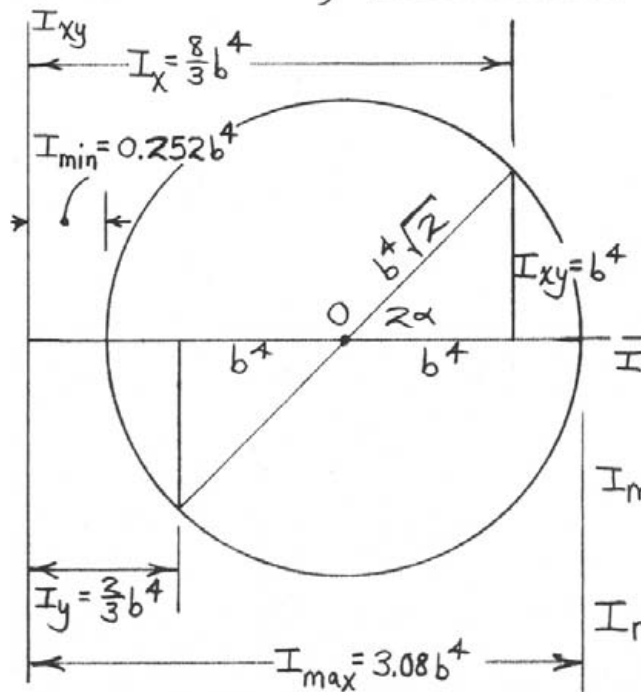
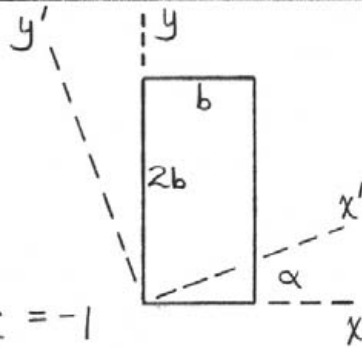
$$I_x = \frac{1}{3} 2b^2 (2b)^2 = \frac{8}{3} b^4$$

$$I_y = \frac{1}{3} 2b^2 (b)^2 = \frac{2}{3} b^4$$

$$I_{xy} = 2b^2 \left(\frac{b}{2}\right)(b) = b^4$$

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2b^4}{\frac{2}{3}b^4 - \frac{8}{3}b^4} = -1$$

$$\Rightarrow 2\alpha = -45^\circ, \quad \alpha = -22.5^\circ$$



$$\alpha = -22.5^\circ$$

$$I_{\max} = \frac{5}{3}b^4 + b^4\sqrt{2} = 3.08b^4$$

$$I_{\min} = \frac{5}{3}b^4 - b^4\sqrt{2} = 0.252b^4$$

A/85

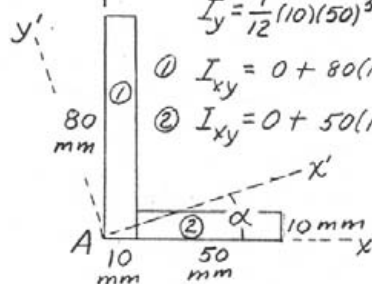
$$\textcircled{1} I_x = \frac{1}{3}(10)(80)^3 = 1.707(10^6) \text{ mm}^4, I_y = \frac{1}{3}(80)(10)^3 = 0.0267(10^6) \text{ mm}^4$$

$$\textcircled{2} I_x = \frac{1}{3}(50)(10)^3 = 0.0167(10^6) \text{ mm}^4,$$

$$I_y = \frac{1}{12}(10)(50)^3 + 10(50)(35)^2 = 0.7167(10^6) \text{ mm}^4$$

$$\textcircled{1} I_{xy} = 0 + 80(10)(40)(5) = 0.1600(10^6) \text{ mm}^4$$

$$\textcircled{2} I_{xy} = 0 + 50(10)(35)(5) = 0.0875(10^6) \text{ mm}^4$$



$$\text{Totals: } I_x = 1.723(10^6) \text{ mm}^4$$

$$I_y = 0.743(10^6) \text{ mm}^4$$

$$I_{xy} = 0.248(10^6) \text{ mm}^4$$

From Eqs. A/11

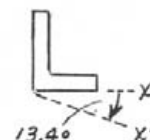
$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}, I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$I_{max} = \left[\frac{1.723 + 0.743}{2} + \frac{1}{2} \sqrt{(1.723 - 0.743)^2 + 4(0.248)^2} \right] 10^6 = 1.782(10^6) \text{ mm}^4$$

$$I_{min} = \left[\frac{1.723 + 0.743}{2} - \frac{1}{2} \sqrt{(1.723 - 0.743)^2 + 4(0.248)^2} \right] 10^6 = 0.684(10^6) \text{ mm}^4$$

From Eq. A/10

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(0.248)}{0.743 - 1.723} = -0.5051 \quad \left| \begin{array}{l} 2\alpha = -26.8^\circ \\ \alpha = -13.40^\circ \end{array} \right|$$



*A/86

$$I_x = \frac{1}{3}(100 \cdot 200)(100)^2 + \frac{1}{3}(100 \cdot 200)(200)^2 = 3.33(10^8) \text{ mm}^4$$

$$I_y = \frac{1}{3}(100 \cdot 300)(300)^2 + \frac{1}{3}(100 \cdot 100)(100)^2 = 9.33(10^8) \text{ mm}^4 \quad (\text{Dim. in mm})$$

$$I_{xy} = (100 \cdot 200)(50)(100) + (100 \cdot 200)(200)(50) = 3(10^8) \text{ mm}^4$$

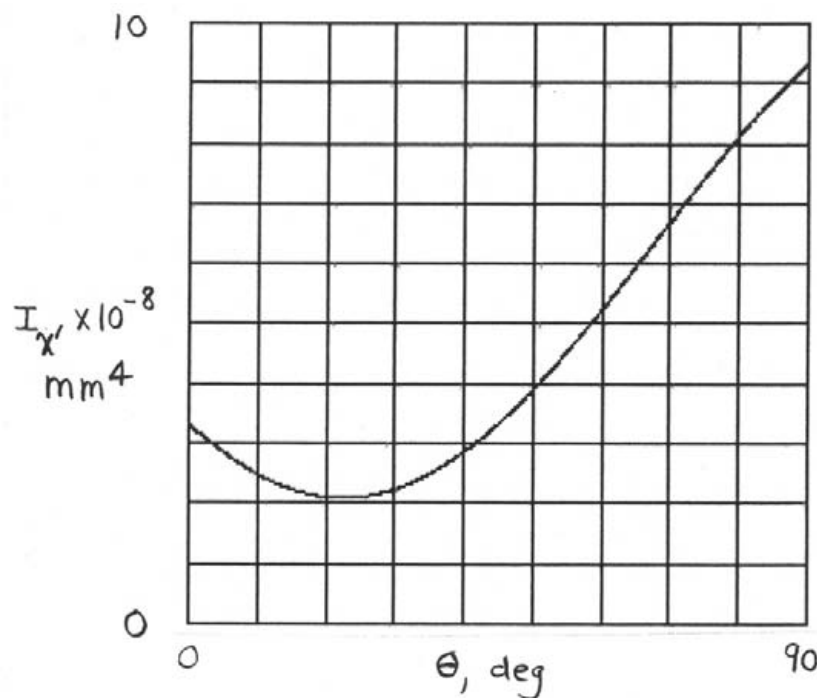
$$\text{Eq. A/9: } I_{x'} = \left\{ \frac{3.33 + 9.33}{2} + \frac{3.33 - 9.33}{2} \cos 2\theta - 3 \sin 2\theta \right\} 10^8$$

$$= \{ 6.33 - 3 \cos 2\theta - 3 \sin 2\theta \} 10^8 \text{ mm}^4$$

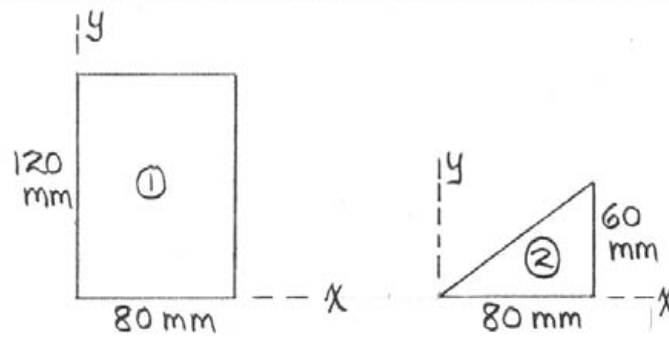
$$\frac{dI_{x'}}{d\theta} = 6 \{ \sin 2\theta - \cos 2\theta \} 10^8 \text{ mm}^4/\text{rad}$$

$$\text{Set } \frac{dI_{x'}}{d\theta} = 0 \text{ to find } (I_{x'})_{\min} = 2.09(10^8) \text{ mm}^4$$

$$\text{@ } \theta = 22.5^\circ$$



*A/87



$$\textcircled{1} I_x = \frac{1}{3}(80)(120)^3 = 46.1(10^6) \text{ mm}^4$$

$$I_y = \frac{1}{3}(120)(80)^3 = 20.5(10^6) \text{ mm}^4$$

$$I_{xy} = 80(120)(40)(60) = 23.0(10^6) \text{ mm}^4$$

$$\textcircled{2} I_x = -\frac{1}{12}(80)(60)^3 = -1.440(10^6) \text{ mm}^4$$

$$I_y = -\frac{1}{4}(60)(80)^3 = -7.68(10^6) \text{ mm}^4$$

$$I_{xy} = -\frac{b^2h^2}{8} = -\frac{80^2 60^2}{8} = -2.88(10^6) \text{ mm}^4$$

(See Prob. A/72)

So for the composite body:

$$\begin{cases} I_x = (46.1 - 1.440)10^6 = 44.6(10^6) \text{ mm}^4 \\ I_y = (20.5 - 7.68)10^6 = 12.80(10^6) \text{ mm}^4 \\ I_{xy} = (23.0 - 2.88)10^6 = 20.2(10^6) \text{ mm}^4 \end{cases}$$

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\theta - I_{xy} \sin 2\theta$$

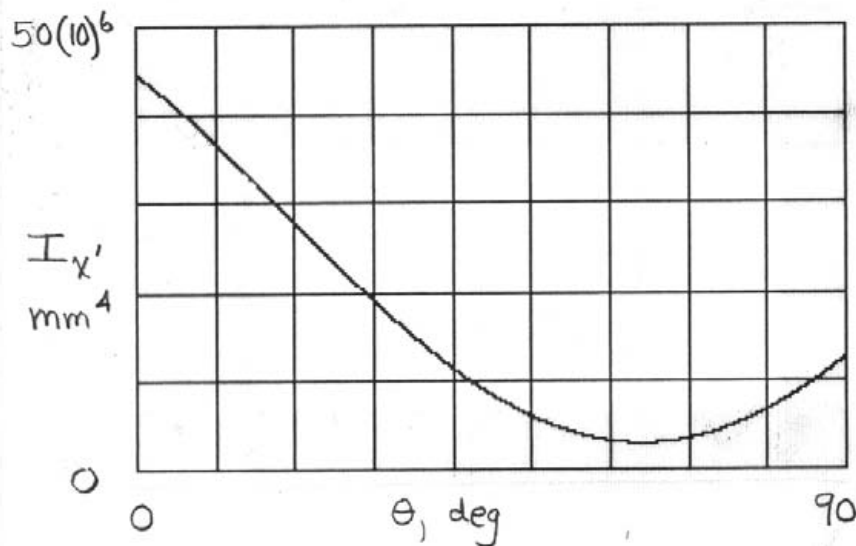
(See plot below)

$$I_{\min} = \frac{I_x + I_y}{2} - \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$= \left\{ \frac{44.6 + 12.80}{2} - \frac{1}{2} \sqrt{(44.6 - 12.80)^2 + 4(20.2)^2} \right\} 10^6$$

$$= 3.03 (10^6) \text{ mm}^4$$

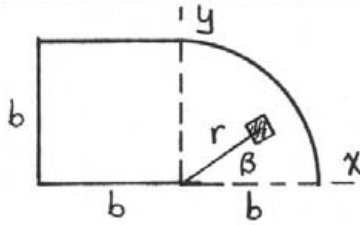
$$\tan 2\theta_{cr} = \frac{2I_{xy}}{I_y - I_x} = \frac{2(20.2)}{12.80 - 44.6} \quad \theta_{cr} = 64.1^\circ$$



* A/88

$$I_x = \frac{1}{3} b^4 + \frac{1}{16} \pi b^4 = 0.530 b^4$$

$$I_y = \frac{1}{3} b^4 + \frac{1}{16} \pi b^4 = 0.530 b^4$$



$$\begin{aligned} \text{Quarter circle: } I_{xy} &= \int_0^{\pi/2} \int_0^b (r \cos \beta)(r \sin \beta) r dr d\beta \\ &= \frac{r^4}{4} \Big|_0^b \times \left(-\frac{1}{4} \cos 2\beta \right) \Big|_0^{\pi/2} = \frac{b^4}{4} \left(\frac{1}{4} + \frac{1}{4} \right) = \frac{b^4}{8} \end{aligned}$$

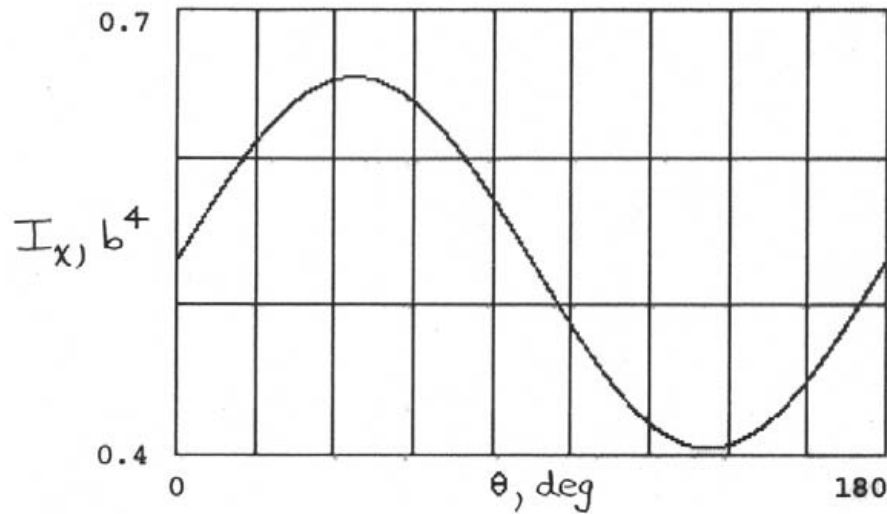
$$\text{Square: } I_{xy} = b^2 \left(-\frac{b}{2} \right) \left(\frac{b}{2} \right) = -\frac{b^4}{4} = -0.25 b^4$$

$$\text{Combined: } I_{xy} = \frac{b^4}{8} - \frac{b^4}{4} = -\frac{b^4}{8} = -0.125 b^4$$

$$\begin{aligned} \text{Eq. A/9: } I_{x'} &= \frac{2(0.530 b^4)}{2} + 0 - (-0.125 b^4) \sin 2\theta \\ &= (0.530 + 0.125 \sin 2\theta) b^4 \end{aligned}$$

For critical angle $\theta = \alpha$, Eq. A/10 gives

$$\tan 2\alpha = \frac{2(0.530 b^4)}{0}, \quad 2\alpha = \frac{\pi}{2}, \quad \alpha = \frac{\pi}{4}$$



$$I_{\max} = 0.655 b^4 \quad @ \quad \theta = 45^\circ$$

$$I_{\min} = 0.405 b^4 \quad @ \quad \theta = 135^\circ$$

Eqs. A/11:

$$\begin{aligned} I_{\max} &= 0.530 b^4 + \frac{1}{2} \sqrt{0^2 + 4(-0.125 b^4)^2} \\ &= 0.655 b^4 \quad \checkmark \end{aligned}$$

$$\begin{aligned} I_{\min} &= 0.530 b^4 - \frac{1}{2} \sqrt{0^2 + 4(-0.125 b^4)^2} \\ &= 0.405 b^4 \quad \checkmark \end{aligned}$$

* A/89

Complete rectangle:

$$I_x = \frac{1}{12}(8 \cdot 12)8^2 = 512 \text{ in.}^4$$

$$I_y = \frac{1}{12}(8 \cdot 12)12^2 = 1152 \text{ in.}^4$$

$$I_{xy} = 0$$

Each dashed triangle:

$$I_x = \frac{1}{4}(4)(4^3) = 64 \text{ in.}^4$$

(see Sample Problem A/2)

$$I_y = \frac{1}{36}4(4)^3 + \frac{1}{2}(4)(4)\left(2 + \frac{8}{3}\right)^2 = 181.3 \text{ in.}^4$$

(see Sample Problem A/2 and Eq. A/6)

$$I_{xy} = -\frac{4^4}{72} + \frac{1}{2}4^2\left(2 + \frac{8}{3}\right)\left(\frac{8}{3}\right) = 96 \text{ in.}^4$$

(see Prob. A/69 and Eq. A/8)

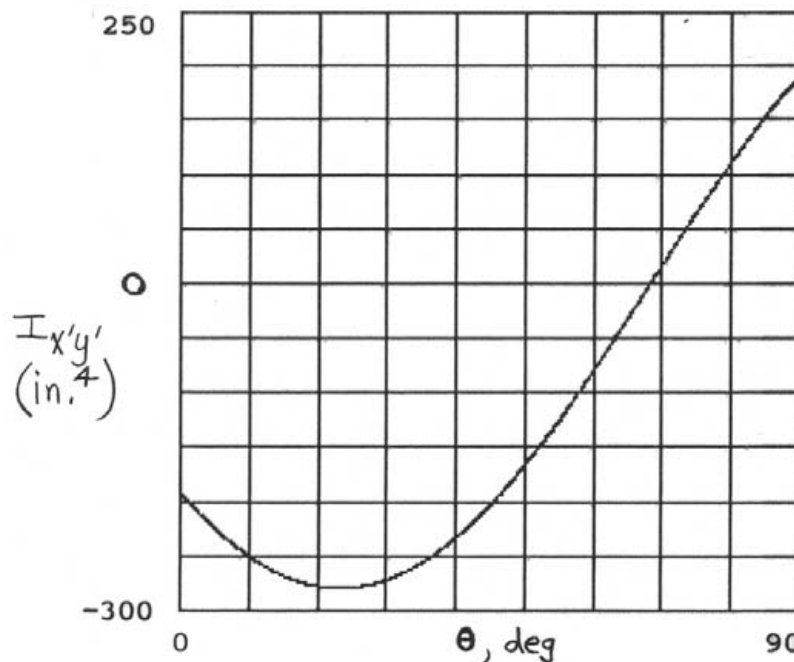
$$\text{Totals: } I_x = 384 \text{ in.}^4$$

$$I_y = 789 \text{ in.}^4$$

$$I_{xy} = -2(96) = -192 \text{ in.}^4$$

$$\text{Eq. A/9a: } I_{x'y'} = \frac{384 - 789}{2} \sin 2\theta - 192 \cos 2\theta$$

$$= -203 \sin 2\theta - 192 \cos 2\theta$$



$$I_{x'y'} = 0 \text{ @ } \theta = 68.5^\circ$$

A/9

From Sample Problem A/2,

$$I_x = \frac{1}{12}bh^3, \quad I_y = \frac{1}{12}b^3h$$

$$I_A = I_x + I_y$$

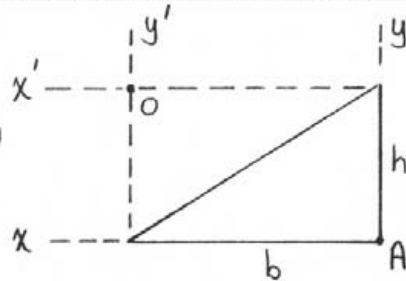
$$= \frac{1}{12}bh(h^2 + b^2)$$

$$k_A = \sqrt{I_A/A} = \sqrt{\frac{1}{12}bh(h^2 + b^2) / \frac{bh}{2}} = \sqrt{(h^2 + b^2)/6}$$

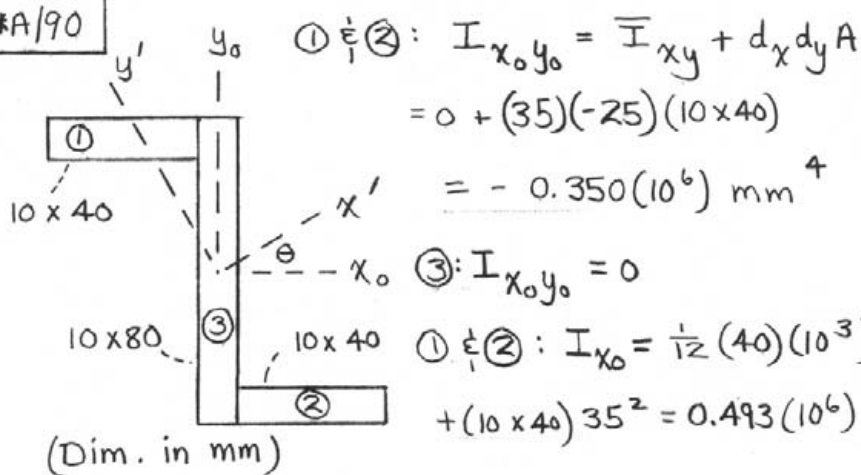
$$I_{x'} = \frac{1}{4}bh^3, \quad I_{y'} = \frac{1}{4}b^3h$$

$$I_o = I_{x'} + I_{y'} = \frac{1}{4}bh(h^2 + b^2)$$

$$k_o = \sqrt{I_o/A} = \sqrt{\frac{1}{4}bh(h^2 + b^2) / \frac{bh}{2}} = \sqrt{(h^2 + b^2)/2}$$



*A/90



$$I_{y_0} = \frac{1}{12} (10)(40)^3 + (10 \times 40) 25^2 = 0.303(10^6) \text{ mm}^4$$

$$\textcircled{3}: I_{x_0} = \frac{1}{12} (10)(80)^3 = 0.427(10^6) \text{ mm}^4$$

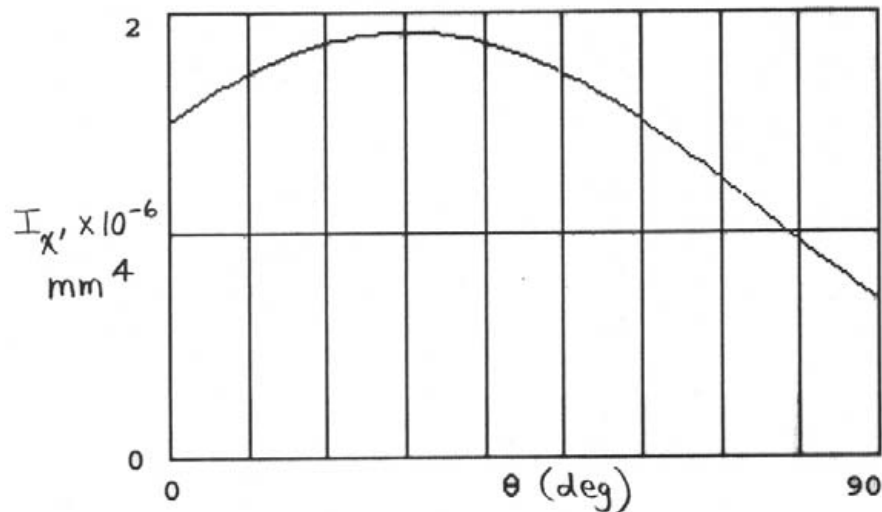
$$I_{y_0} = \frac{1}{12} (80)(10)^3 = 0.00667(10^6) \text{ mm}^4$$

$$\text{Totals: } \begin{cases} I_{x_0} = 1.413(10^6) \text{ mm}^4 \\ I_{y_0} = 0.613(10^6) \text{ mm}^4 \\ I_{x_0 y_0} = -0.700(10^6) \text{ mm}^4 \end{cases}$$

From Eq. A/9:

$$I_{x'} = \left[\frac{1.413 + 0.613}{2} + \frac{1.413 - 0.613}{2} \cos 2\theta + 0.700 \sin 2\theta \right] 10^6$$

$$= [1.013 + 0.4 \cos 2\theta + 0.7 \sin 2\theta] 10^6$$



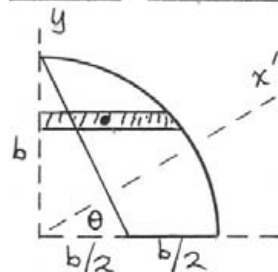
$$\begin{aligned}
 \text{Eq. (A/11): } I_{\max} &= \frac{I_x + I_y}{2} + \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2} \\
 &= \frac{1.413 + 0.613}{2} 10^6 + \frac{1}{2} \sqrt{(1.413 - 0.613)^2 (10^6)^2 + 4(-0.7 \times 10^6)^2} \\
 &= \underline{1.820 (10^6) \text{ mm}^4}
 \end{aligned}$$

$$\begin{aligned}
 \text{Eq. (A/10): } \tan 2\alpha &= \frac{2I_{xy}}{I_y - I_x} \\
 &= \frac{2(-0.7)}{0.613 - 1.413}
 \end{aligned}$$

$$\Rightarrow \alpha = \underline{30.1^\circ}, 120.1^\circ$$

(Values from Eqs. A/10 & A/11 agree with plot.)

*A/91 Quarter circle $I_x = I_y = \frac{1}{16} \pi r^4$



For horizontal strip:

$$dI_{xy} = 0 + (x dy) y \frac{x}{2} = \frac{b^2 y - y^3}{2} dy$$

$$I_{xy} = \frac{1}{2} \int_0^b (b^2 y - y^3) dy = \frac{b^4}{8}$$

Triangle $I_x = \frac{1}{12} \frac{b}{2} b^3 = \frac{b^4}{24}$

$$I_y = \frac{1}{12} b \left(\frac{b}{2}\right)^3 = \frac{b^4}{96}, \quad I_{xy} = \frac{1}{24} \left(\frac{b}{2}\right)^2 b^2 = \frac{b^4}{96}$$

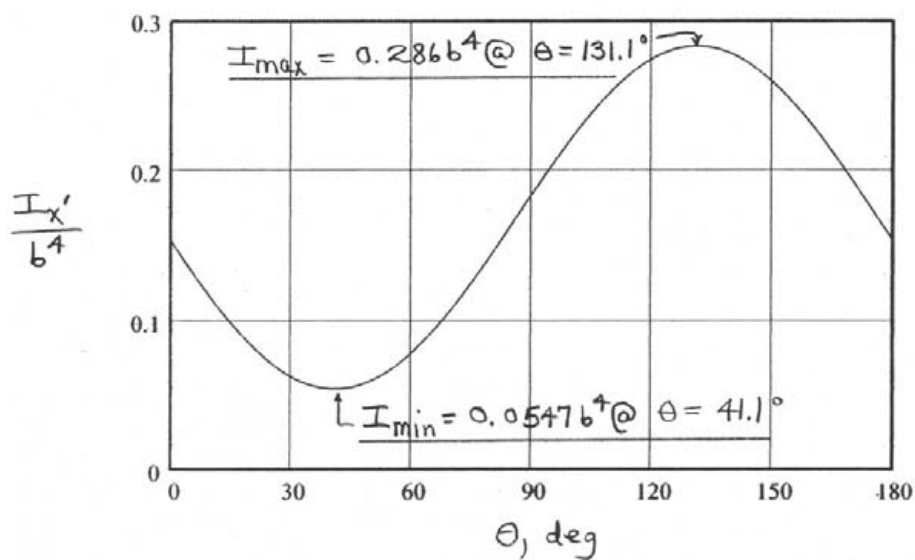
(Prob. A/71)

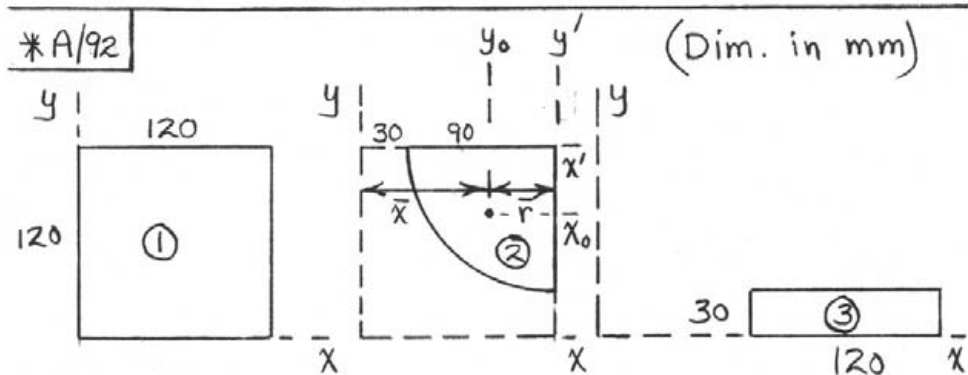
Composite

$$\begin{cases} I_x = \left(\frac{\pi}{16} - \frac{1}{24}\right) b^4 = 0.1547 b^4 \\ I_y = \left(\frac{\pi}{16} - \frac{1}{96}\right) b^4 = 0.1859 b^4 \\ I_{xy} = \left(\frac{1}{8} - \frac{1}{96}\right) b^4 = 0.1146 b^4 \end{cases}$$

From Eq. A/9:

$$\begin{aligned} I_{x'} &= \left[\frac{0.1547 + 0.1859}{2} + \frac{0.1547 - 0.1859}{2} \cos 2\theta - 0.1146 \sin 2\theta \right] b^4 \\ &= [0.1703 - 0.01562 \cos 2\theta - 0.1146 \sin 2\theta] b^4 \end{aligned}$$





$$\textcircled{1} I_x = I_y = \frac{1}{3} 120^4 = 69.1(10^6) \text{ mm}^4, A = 120^2 = 1.44(10^4) \text{ mm}^2$$

$$I_{xy} = 120^2(60)(60) = 51.8 \text{ mm}^4$$

$$\textcircled{2} \bar{r} = \frac{4r}{3\pi} = \frac{4(90)}{3\pi} = 38.2 \text{ mm}, \bar{x} = 120 - \bar{r} = 81.8 \text{ mm}$$

$$A = \frac{1}{4} \pi (90^2) = 0.636(10^4) \text{ mm}^2$$

$$I_{x'} = I_{y'} = \frac{1}{4} \left(\frac{1}{4} \pi \times 90^4 \right) = 12.88(10^6) \text{ mm}^4$$

$$I_x = I_y = I_{y_0} + A\bar{x}^2 = I_{y'} - A\bar{r}^2 + A\bar{x}^2$$

$$= [12.88 + 0.00636(81.8^2 - 38.2^2)] 10^6 = 46.2(10^6) \text{ mm}^4$$

$$I_{xy} = I_{x_0 y_0} + A\bar{x}\bar{y} = I_{x' y'} - A\bar{r}\bar{r} + A\bar{x}\bar{y}$$

$$\text{where } I_{x' y'} = \frac{r^4}{8} = \frac{90^4}{8} = 8.20(10^6) \text{ mm}^4 \left\{ \begin{array}{l} \text{See Sol. to} \\ \text{Prob. A/65} \end{array} \right.$$

$$\therefore I_{xy} = [8.20 + 0.00636(81.8^2 - 38.2^2)] 10^6 = 41.5(10^6) \text{ mm}^4$$

$$\textcircled{3} I_x = \frac{1}{3} (120) (30)^3 = 1.08 (10^6) \text{ mm}^4$$

$$I_y = \frac{1}{12} (30) (120)^3 + 30(120)(180^2) = 121.0 (10^6) \text{ mm}^4$$

$$I_{xy} = 30(120)(180)(15) = 9.72 (10^6) \text{ mm}^4$$

$$\text{Combined: } \begin{cases} I_x = (69.1 - 46.2 + 1.08) 10^6 = 24.0 (10^6) \text{ mm}^4 \\ I_y = (69.1 - 46.2 + 121.0) 10^6 = 143.9 (10^6) \text{ mm}^4 \\ I_{xy} = (51.8 - 41.5 + 9.72) 10^6 = 20.1 (10^6) \text{ mm}^4 \end{cases}$$

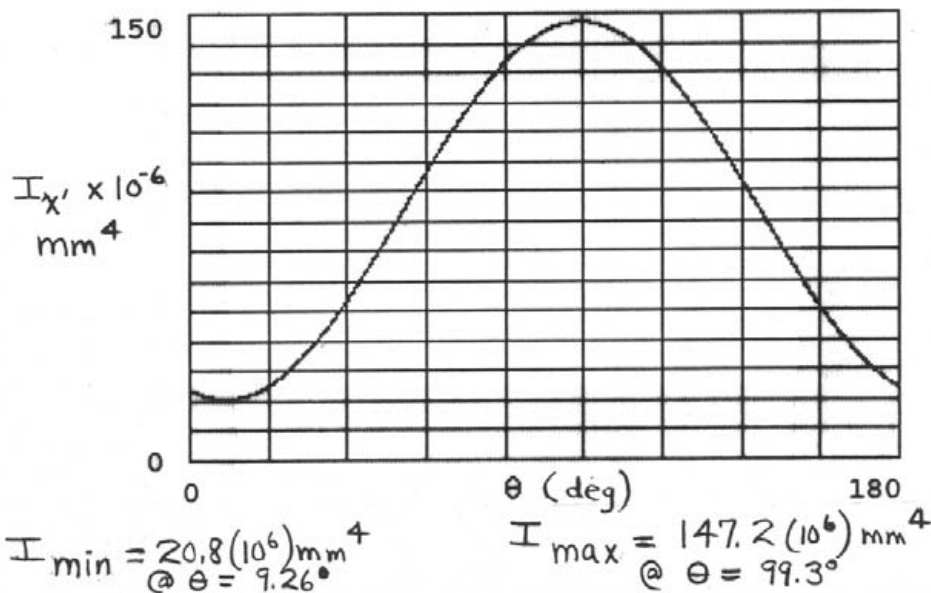
$$A/9: I_{x'} = \left\{ \frac{24.0 + 143.9}{2} + \frac{24.0 - 143.9}{2} \cos 2\theta - 20.1 \sin 2\theta \right\} \times 10^6$$

$$I_{x'} (10^{-6}) = 84.0 - 59.9 \cos 2\theta - 20.1 \sin 2\theta \text{ mm}^4$$

$$A/10: \tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(20.1)}{143.9 - 24.0} = 0.335$$

$$2\alpha = 18.51^\circ, \alpha = 9.26^\circ \text{ (minimum I)}$$

$$\alpha = 9.26 + 90 = 99.3^\circ \text{ (maximum I)}$$



$$I_{\min} = 20.8 (10^6) \text{ mm}^4 \text{ @ } \theta = 9.26^\circ$$

$$I_{\max} = 147.2 (10^6) \text{ mm}^4 \text{ @ } \theta = 99.3^\circ$$

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Appendix A: Area Moments of Inertia.

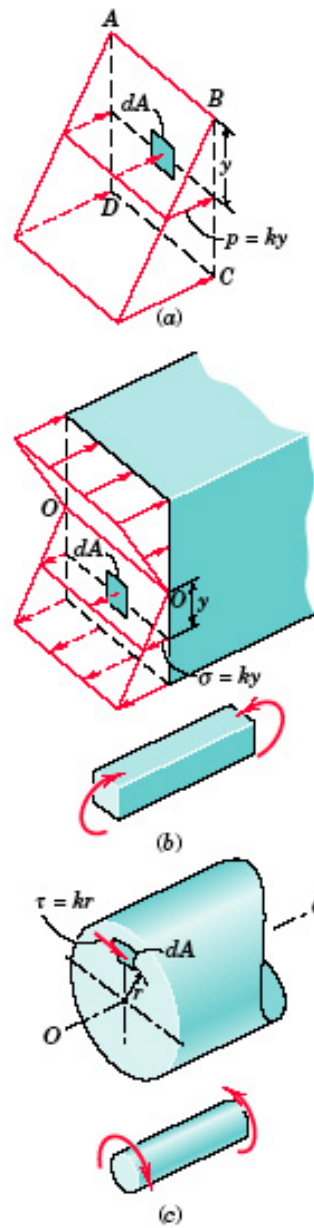


Figure A/1

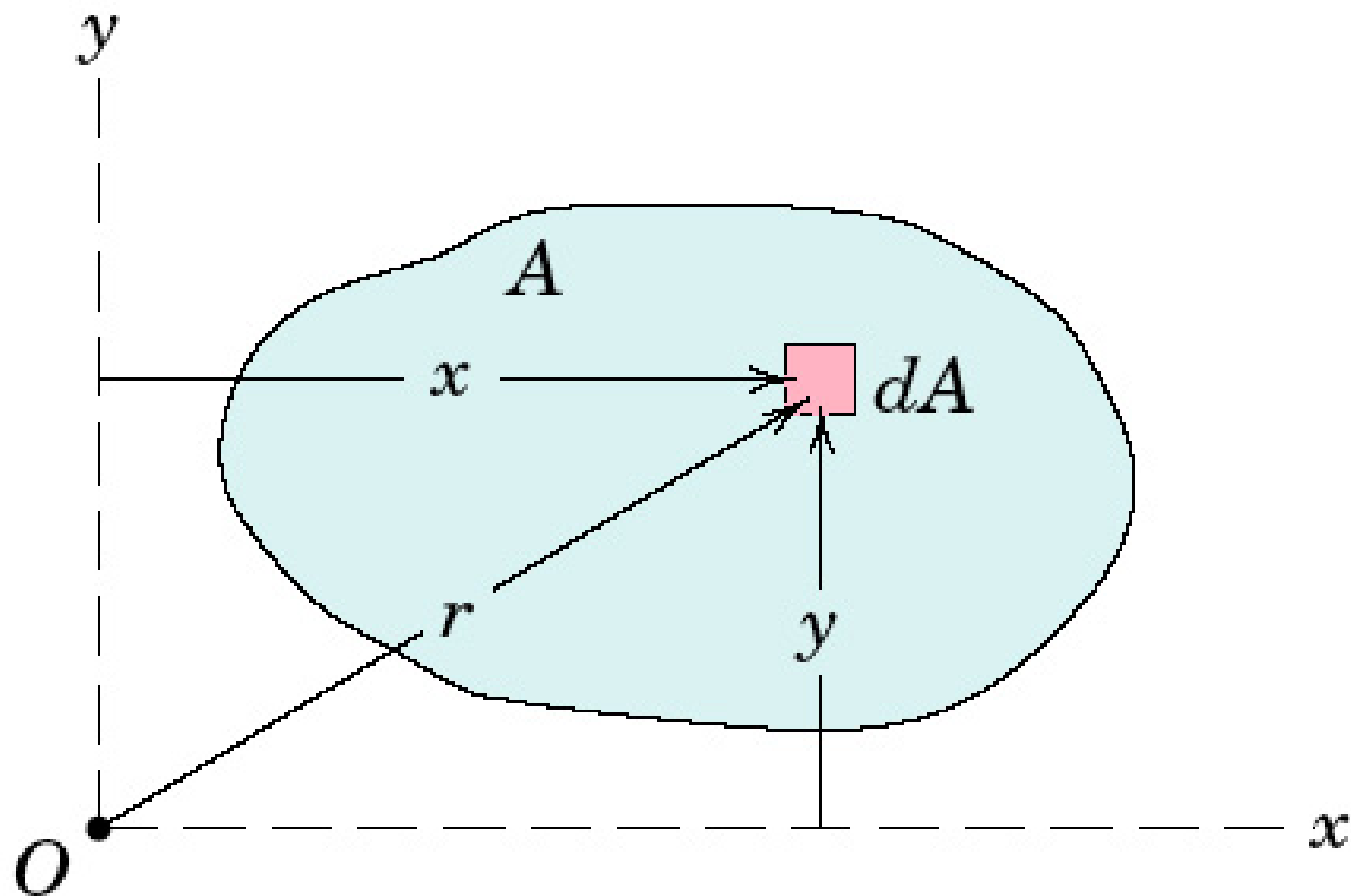


Figure A/2

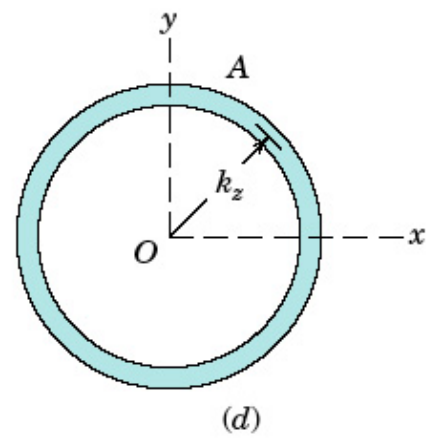
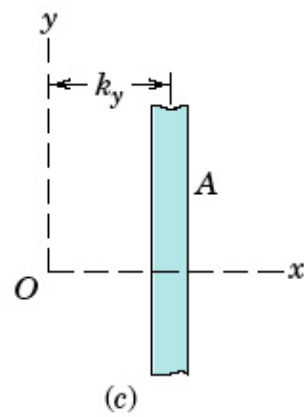
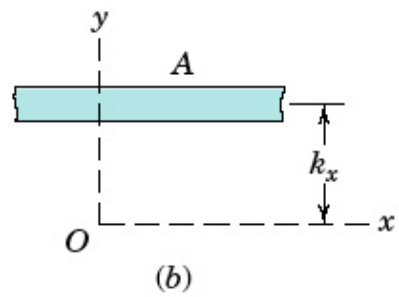
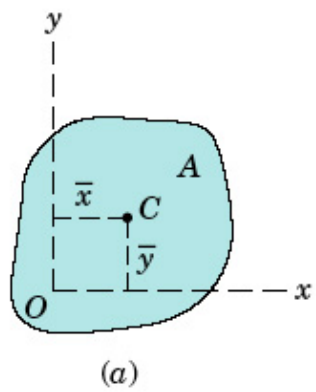


Figure A/3

A_03

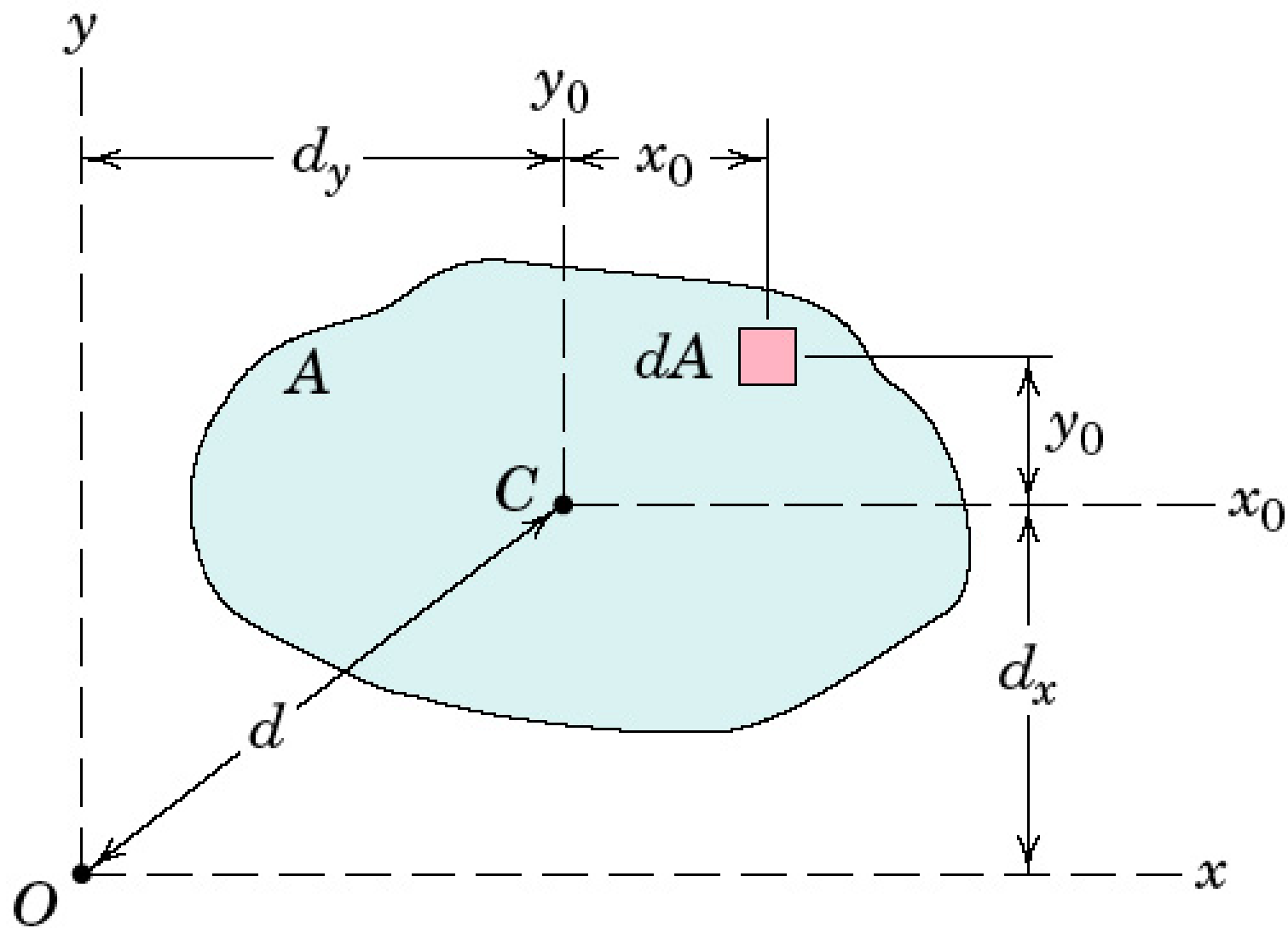


Figure A/4

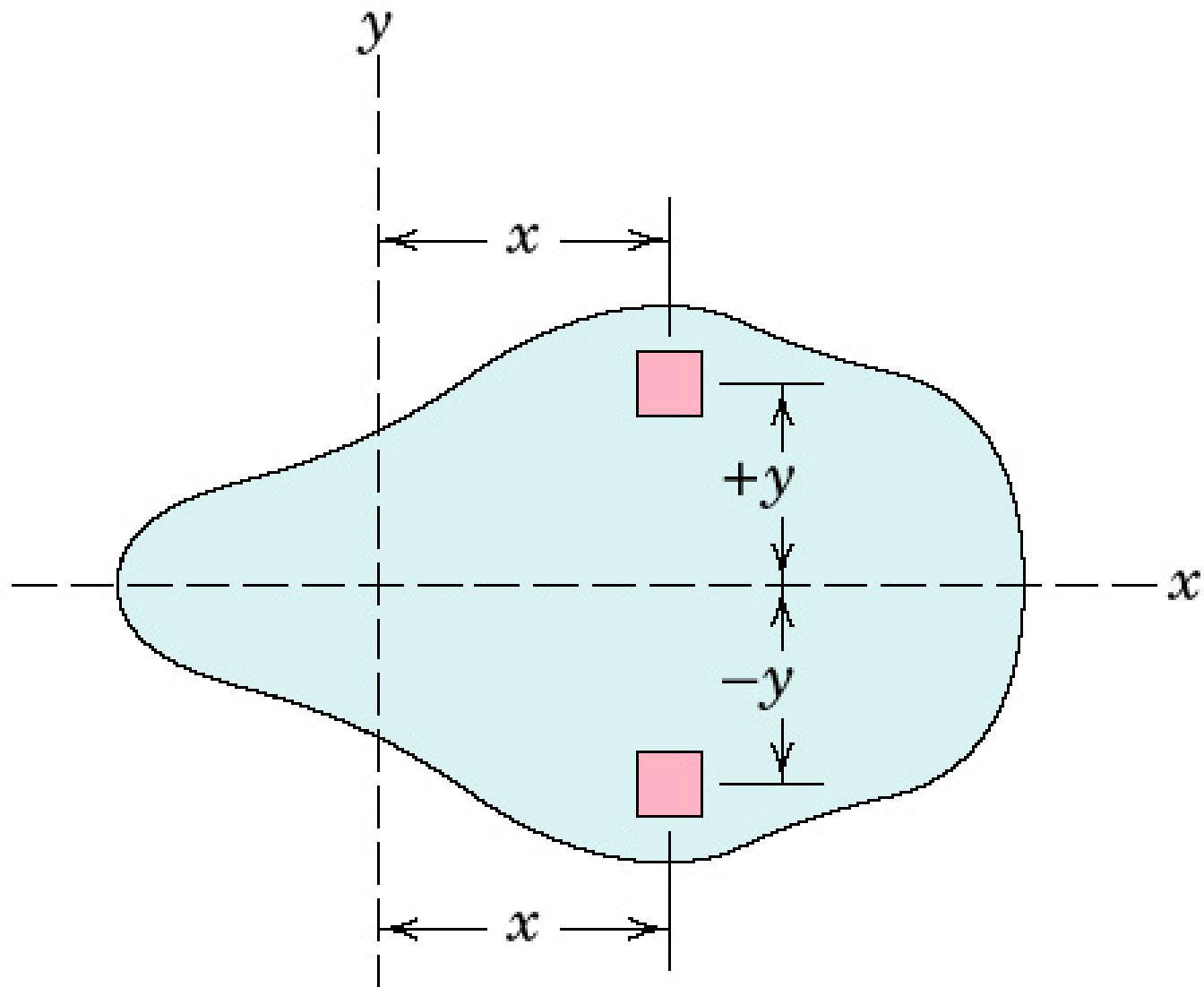


Figure A/5

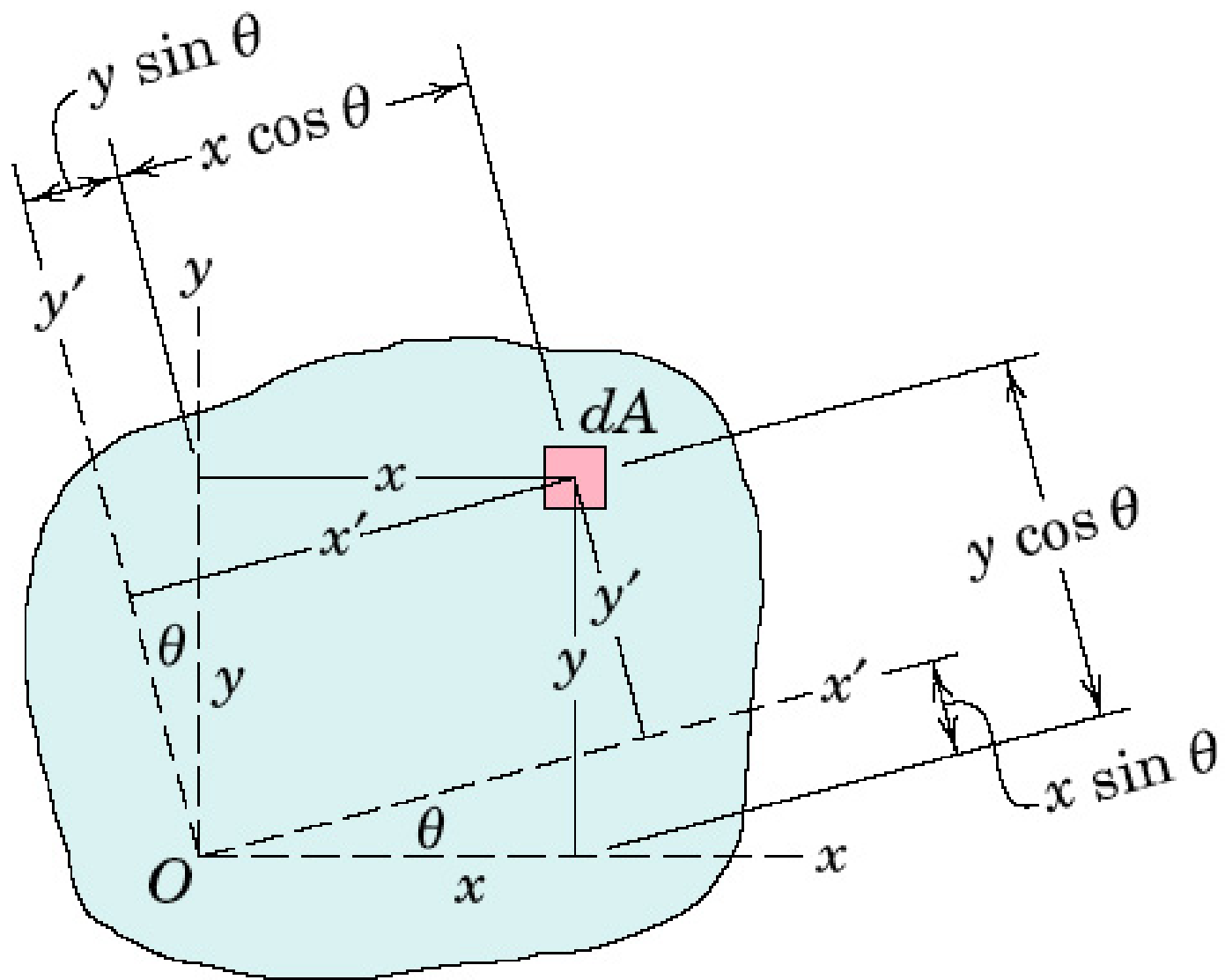
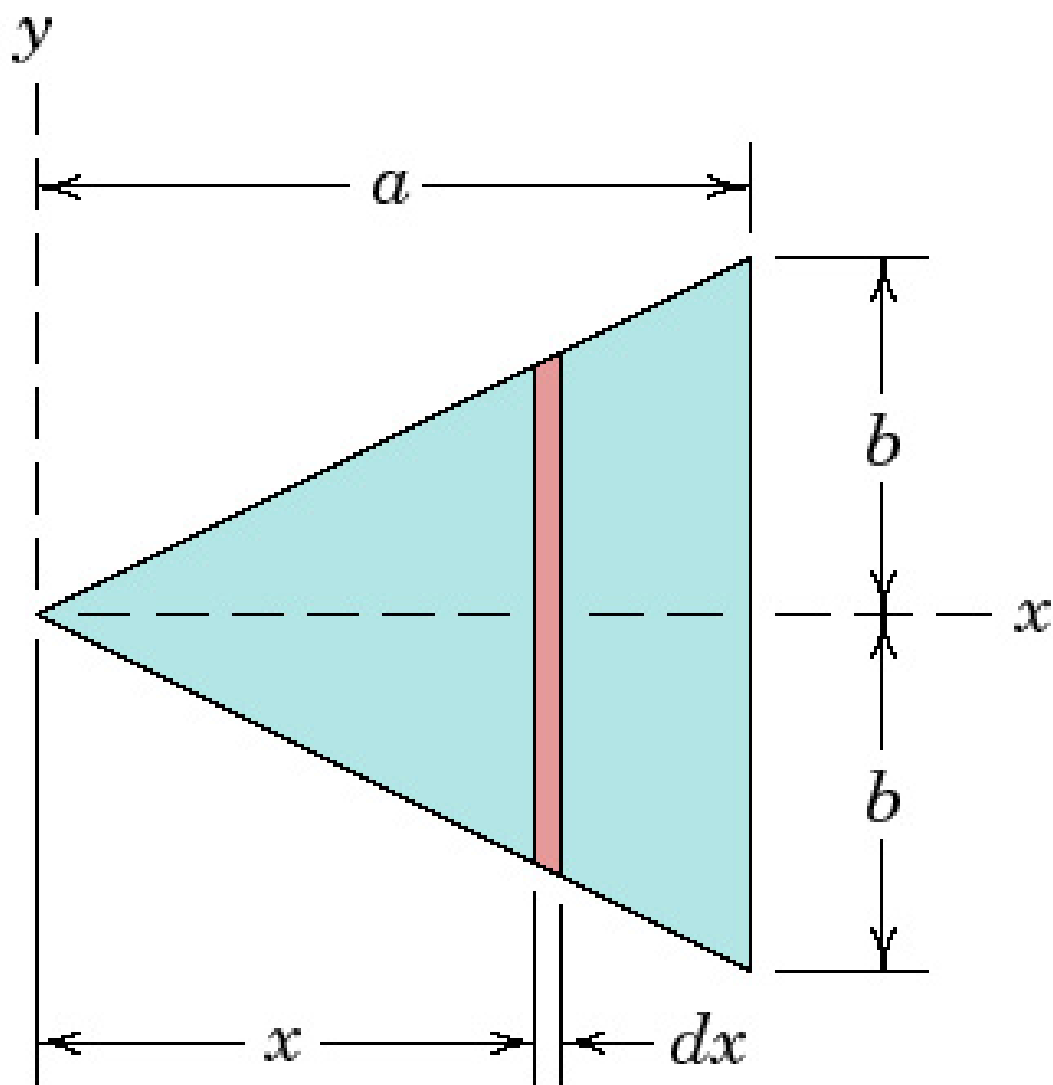
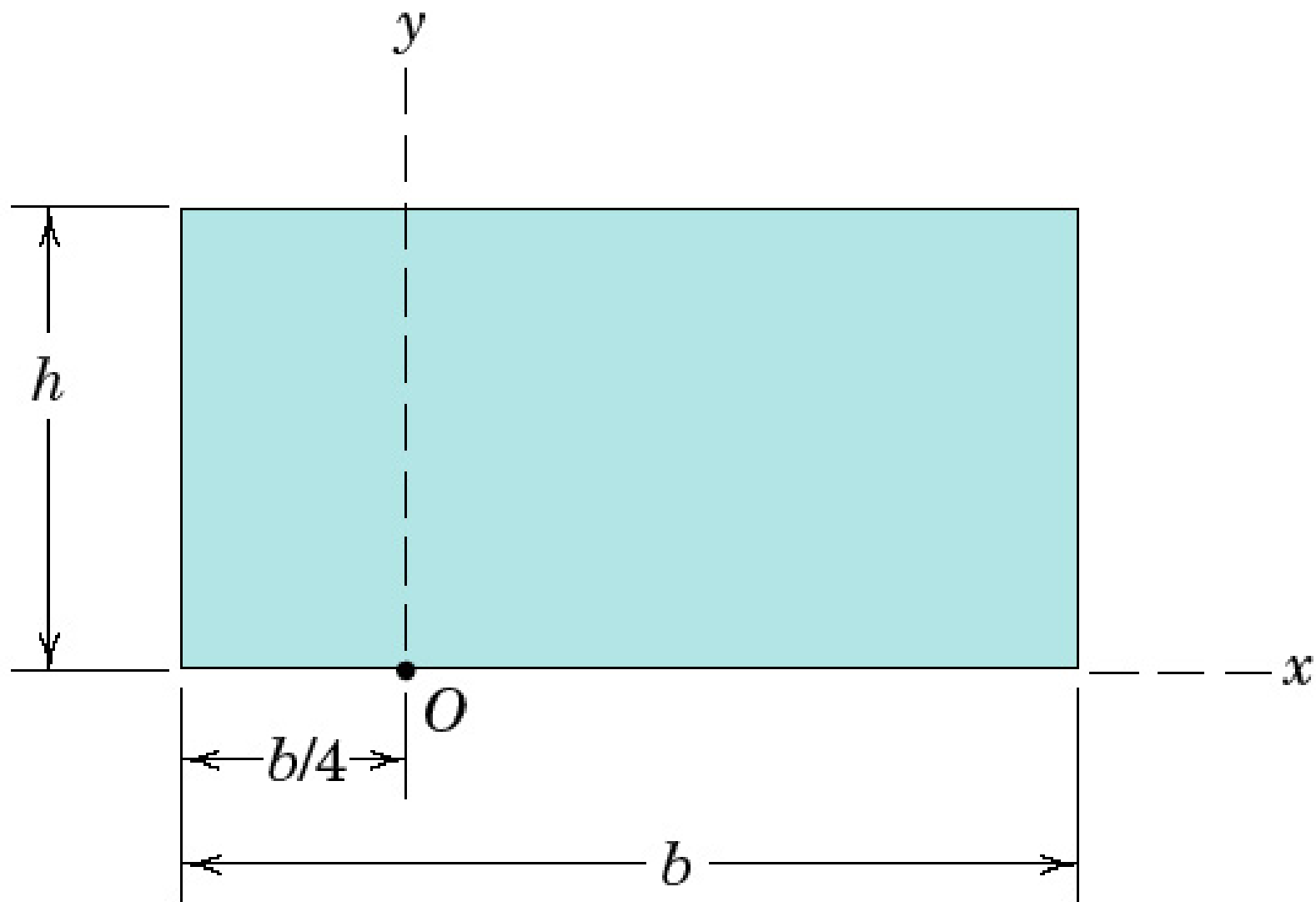


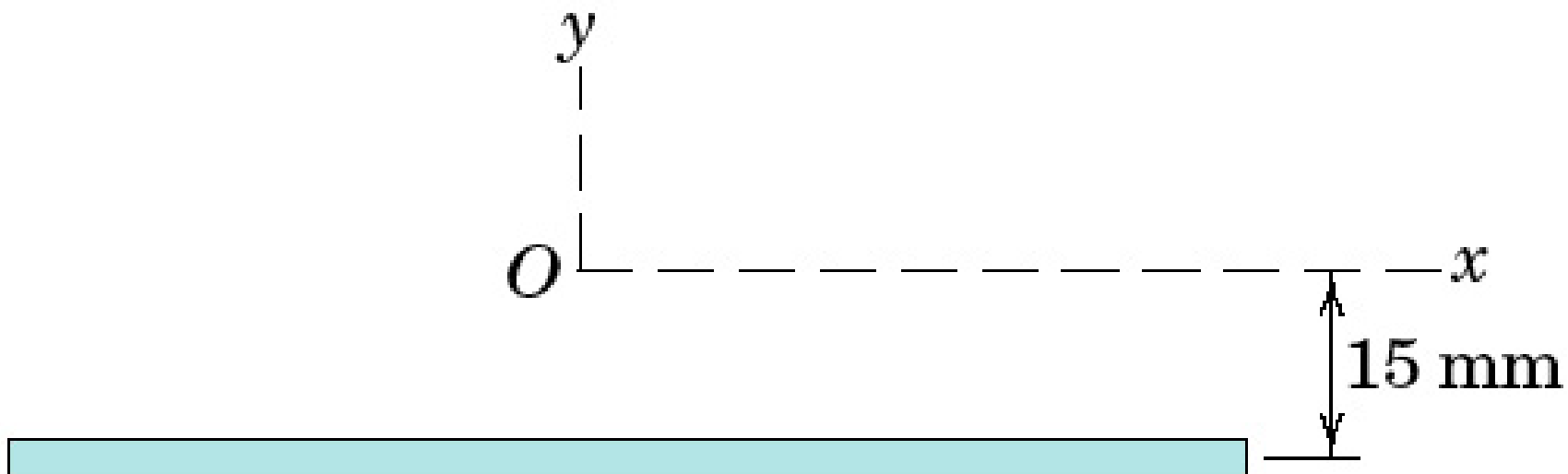
Figure A/6



Problem A/1

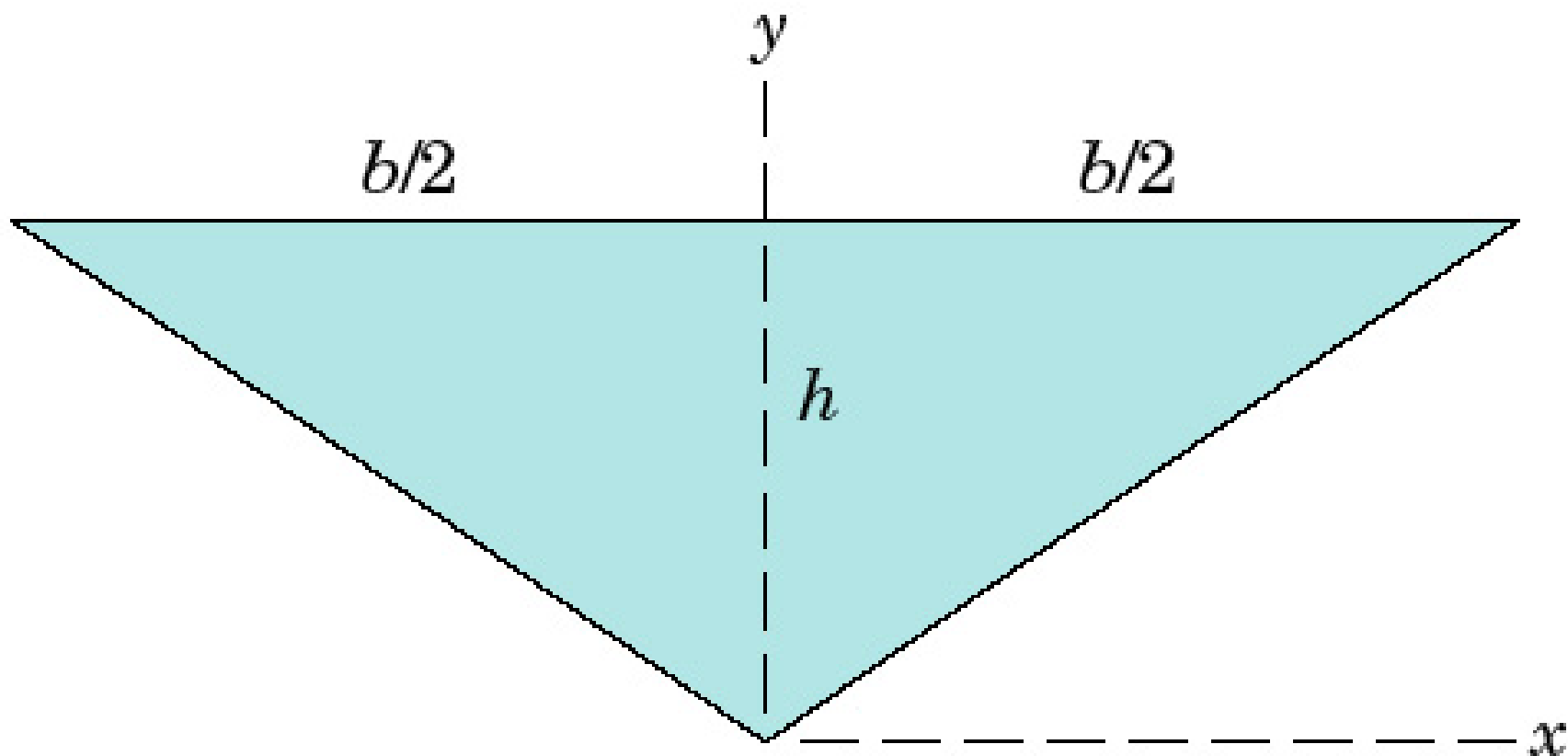


Problem A/2



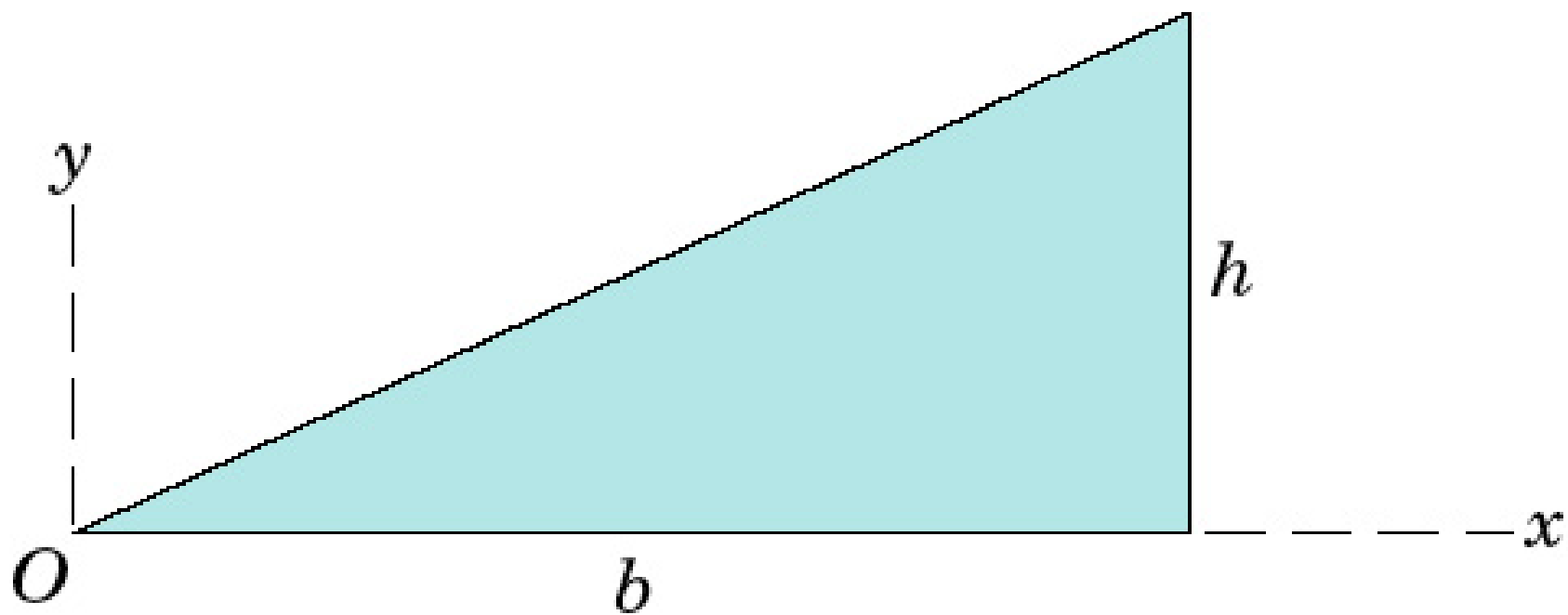
Problem A/3

P_A_03



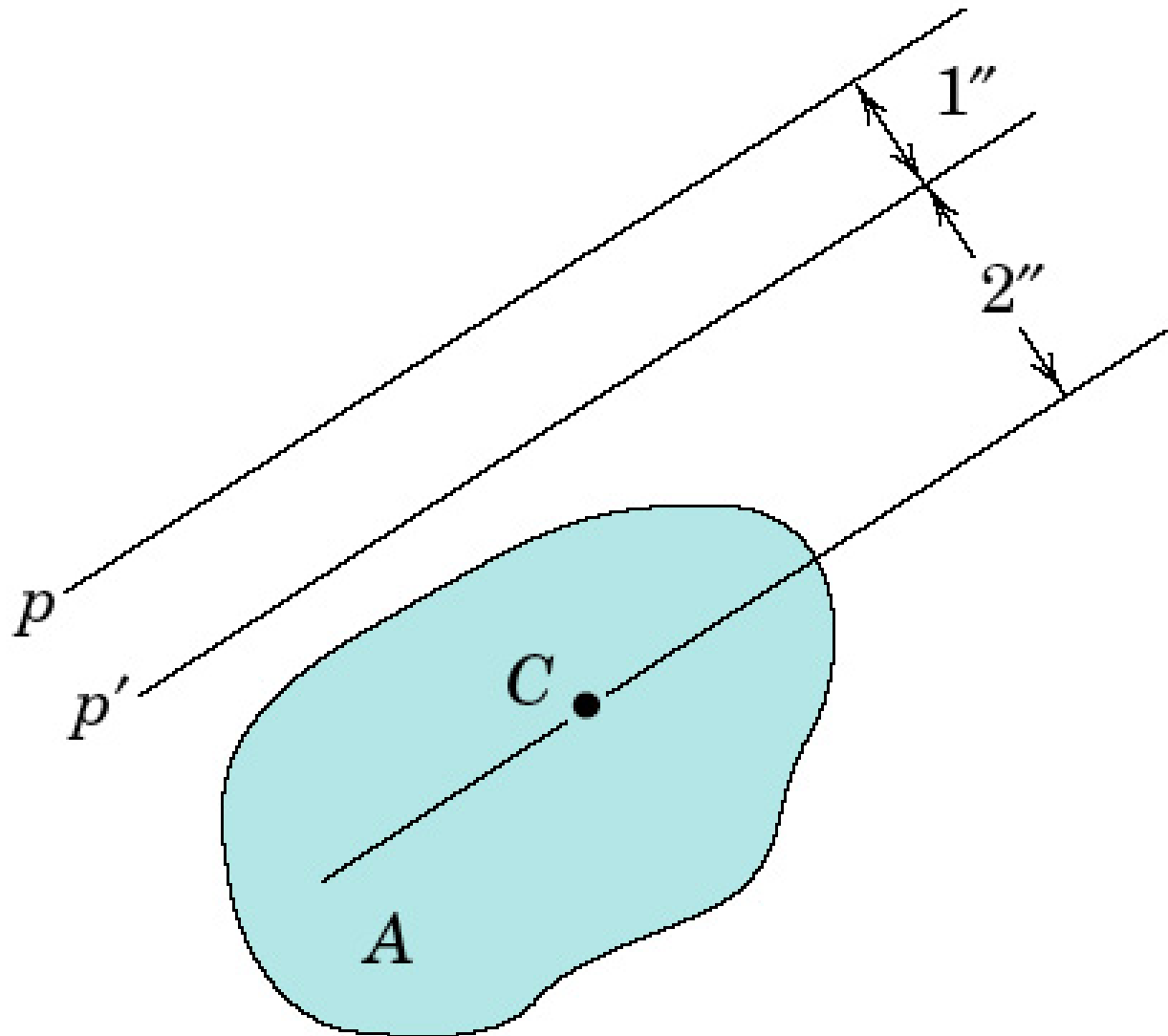
Problem A/4

P_A_04

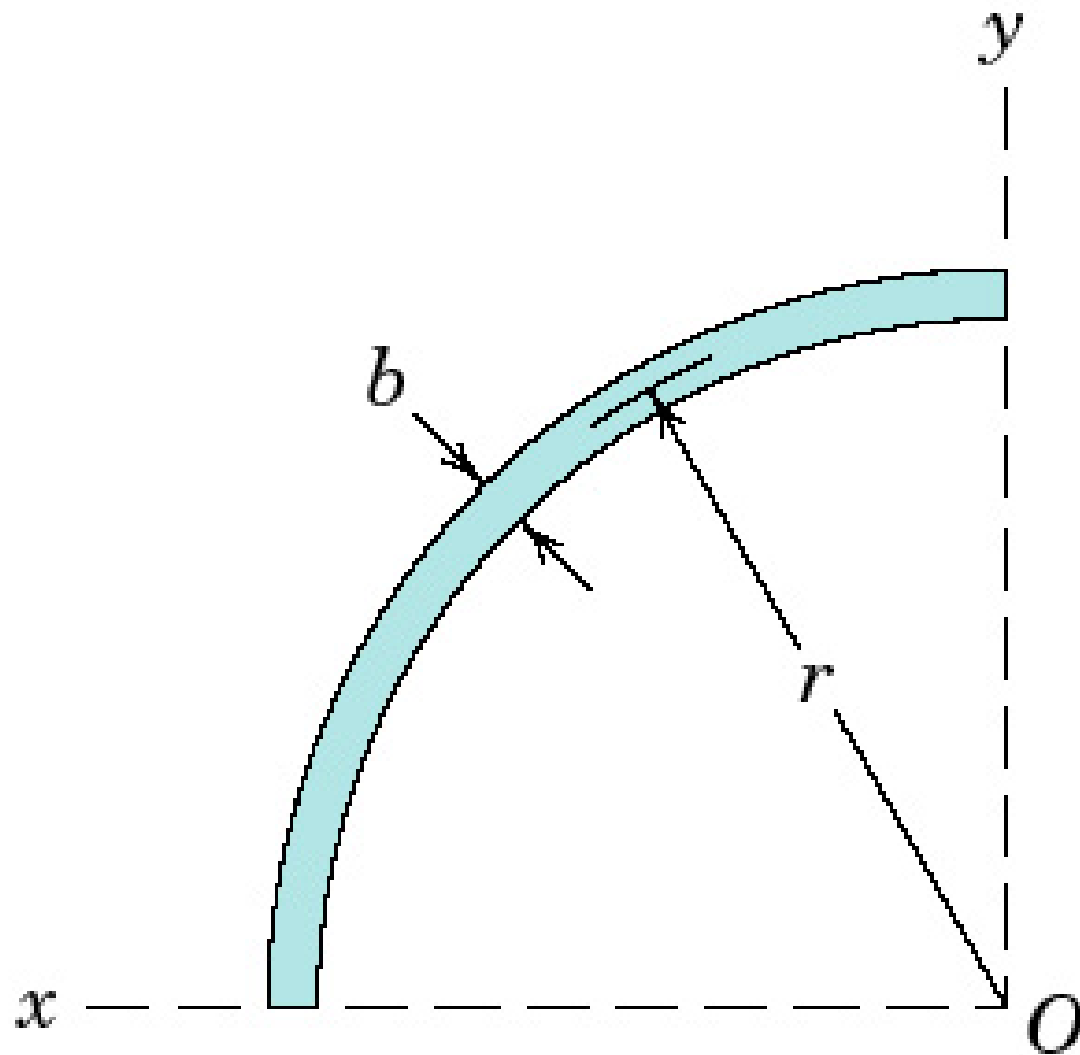


Problem A/5

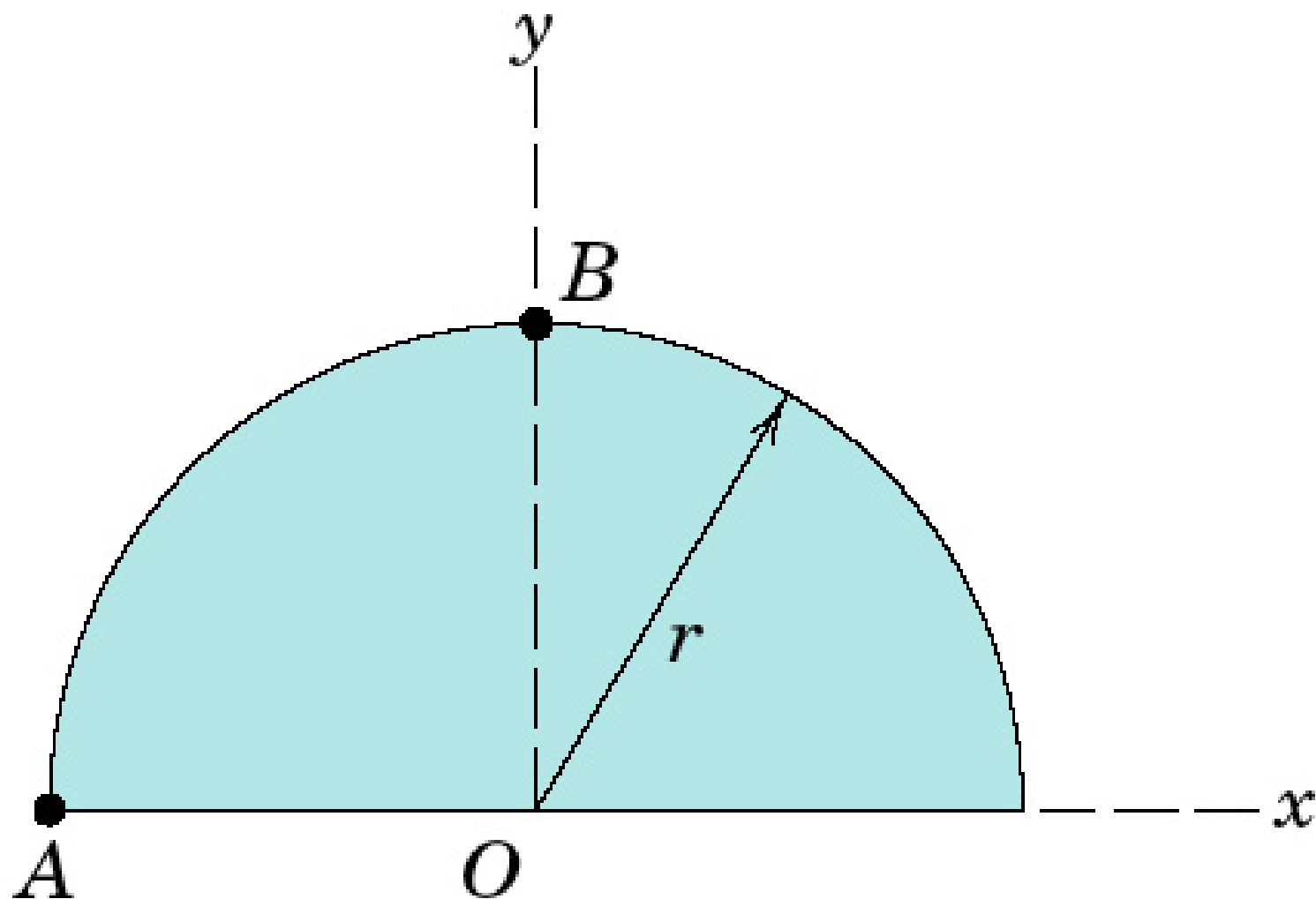
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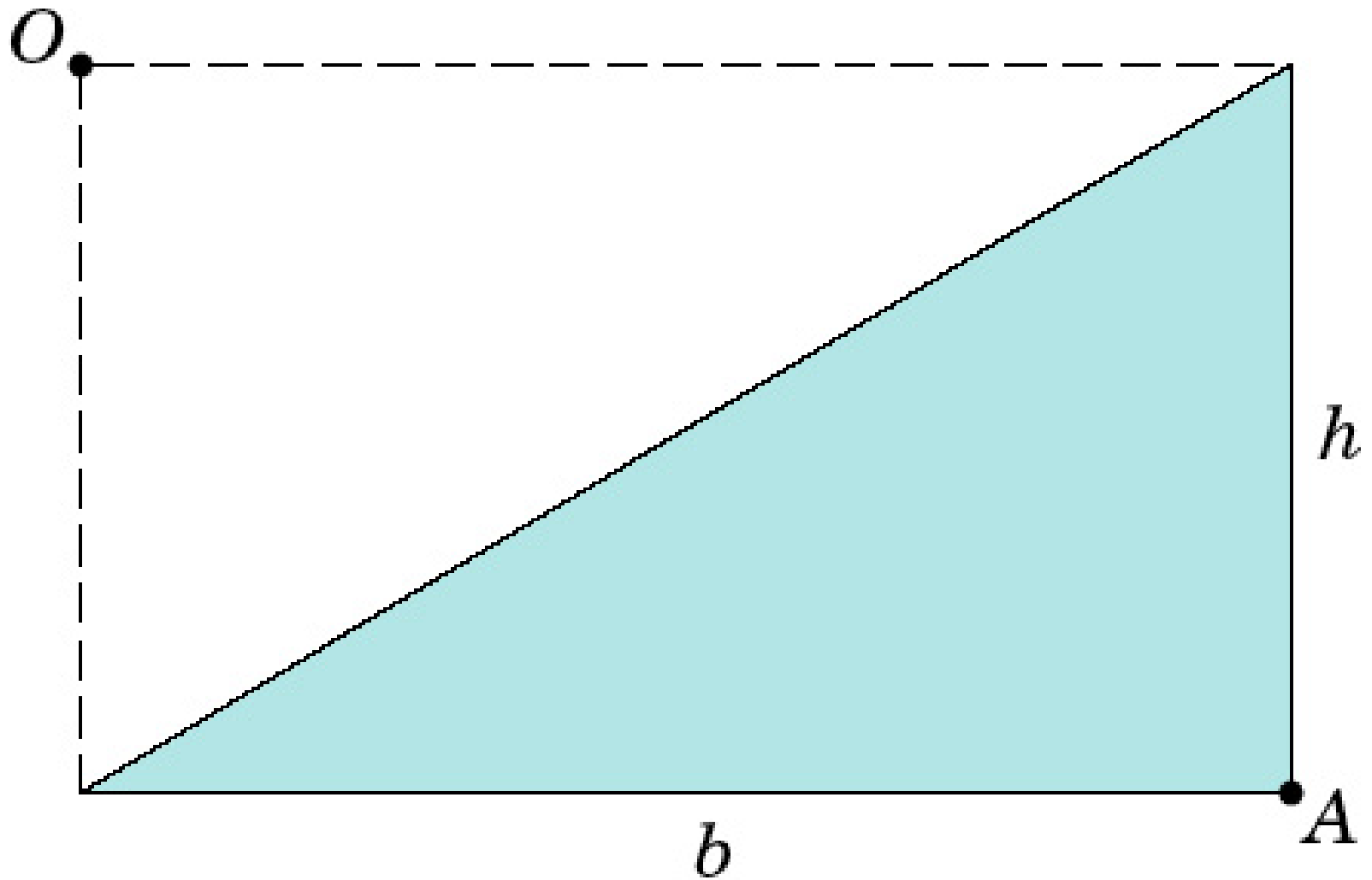
Problem A/6



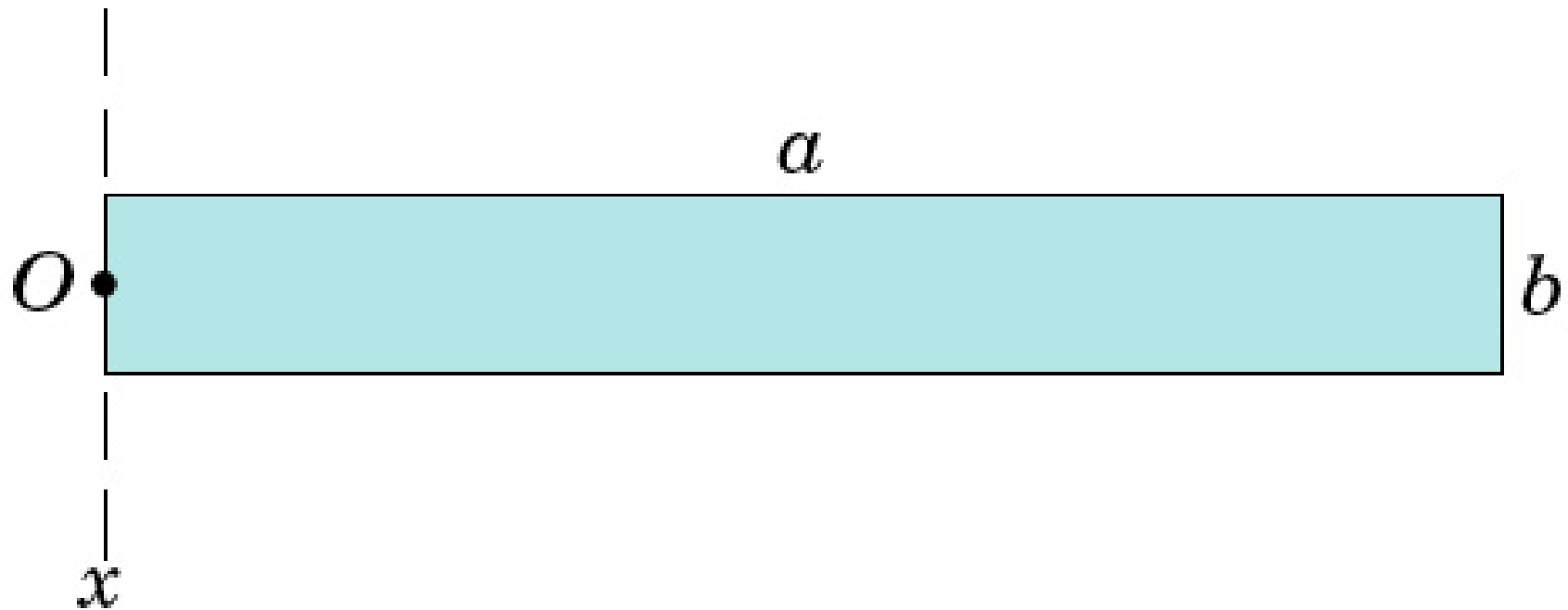
Problem A/7



Problem A/8

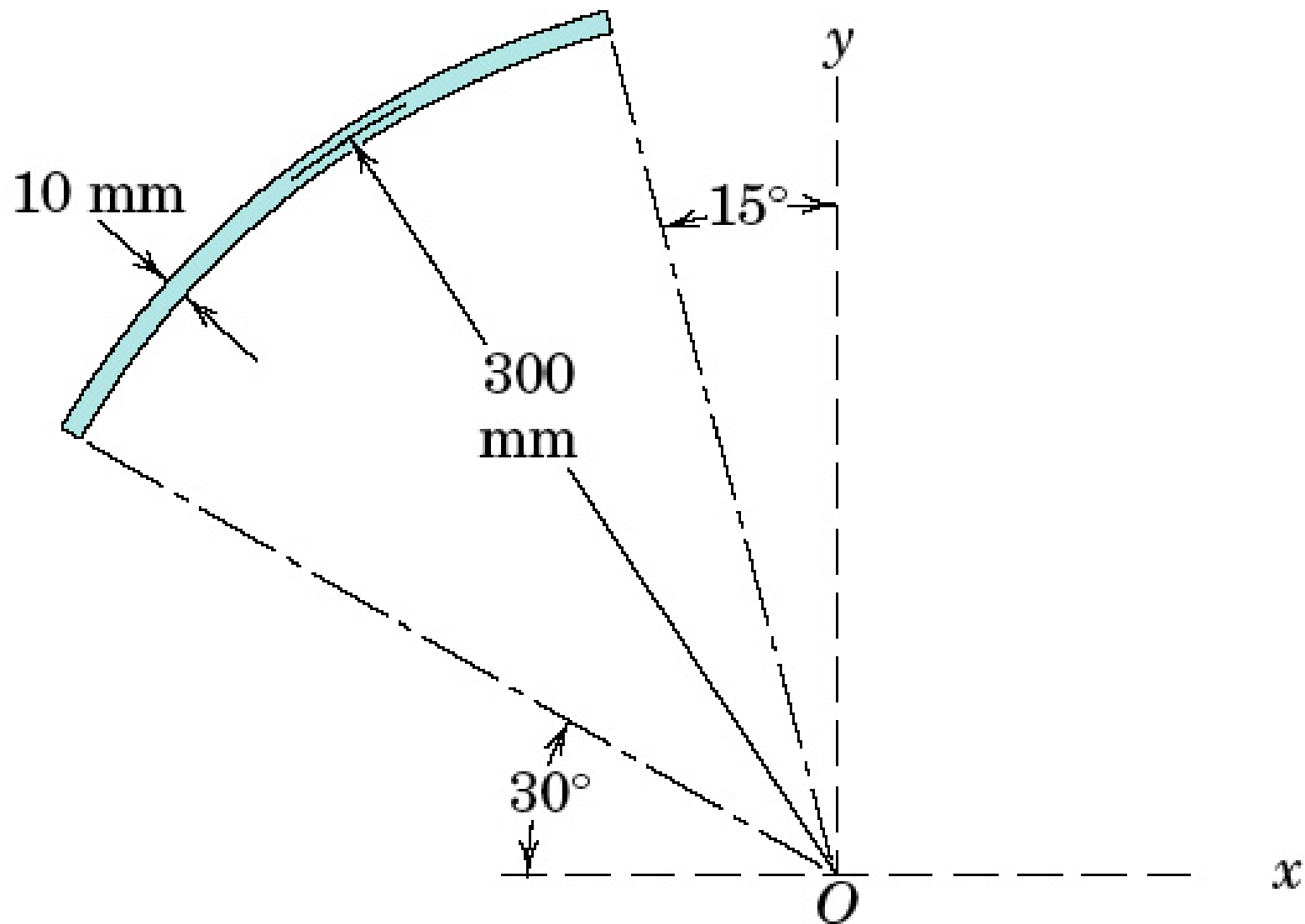


Problem A/9

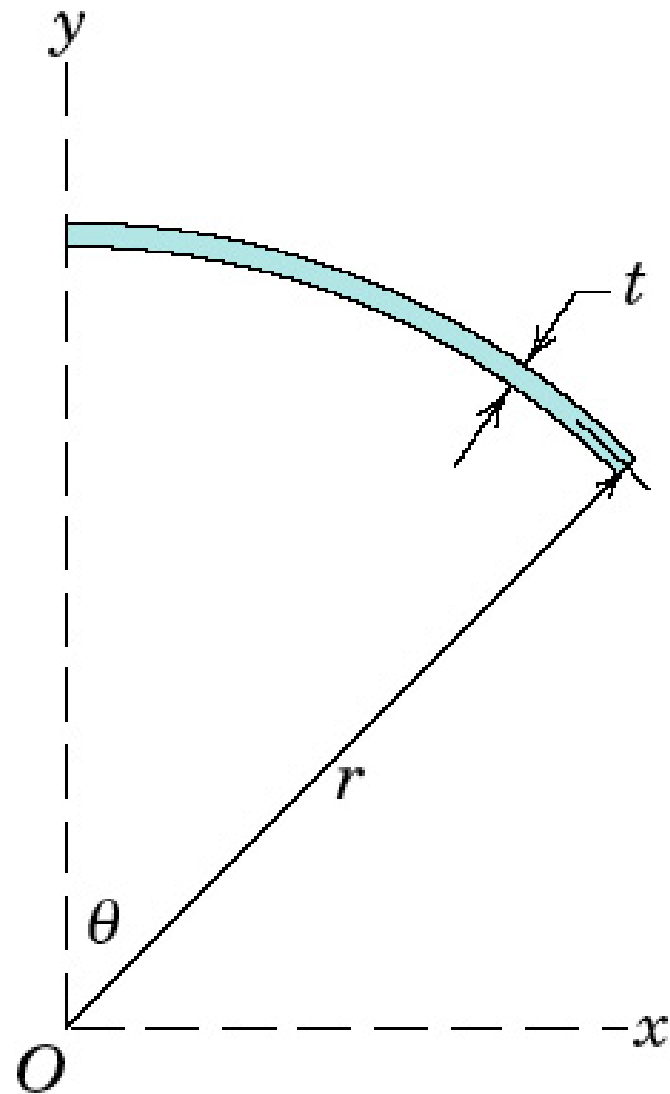


Problem A/10

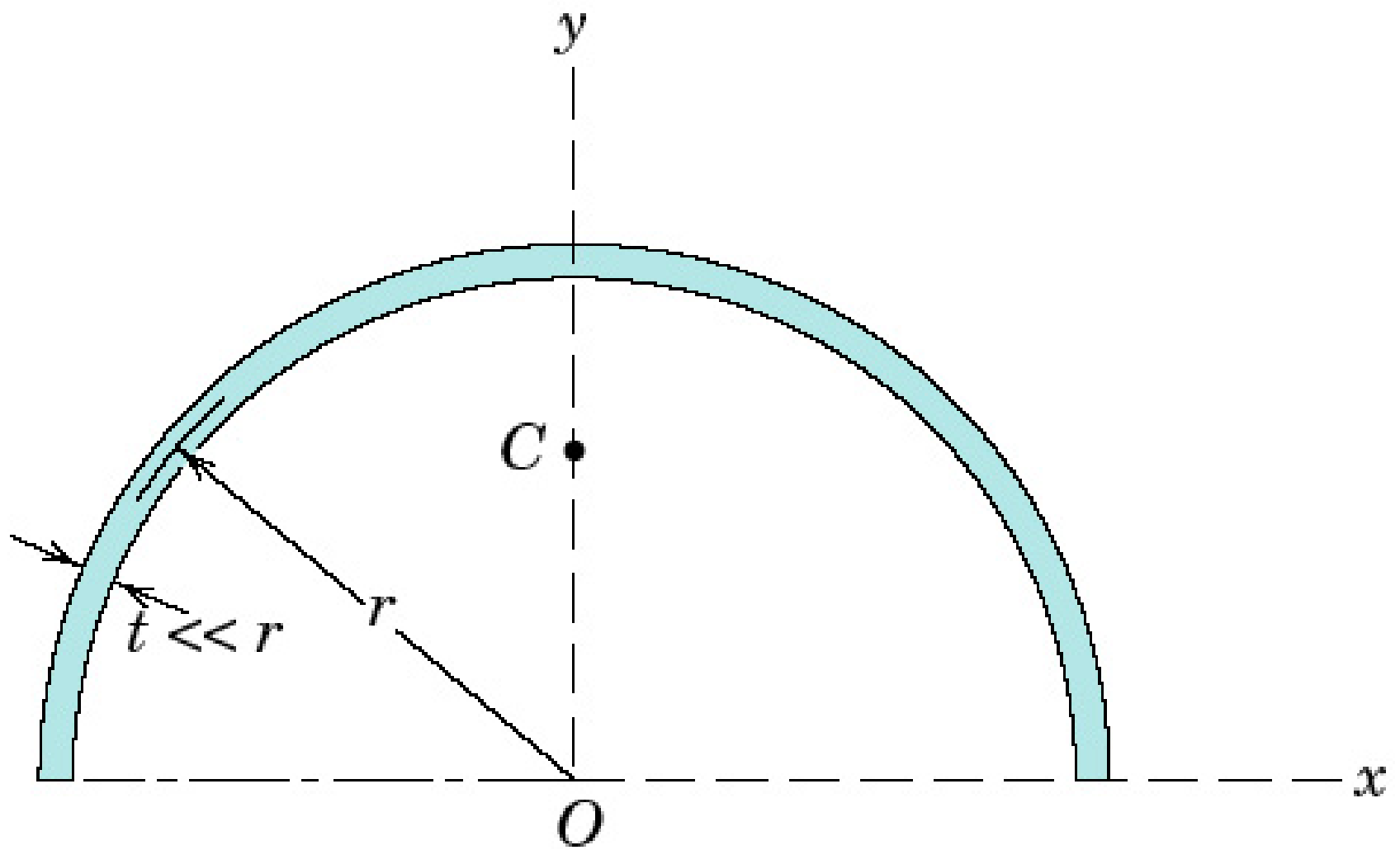
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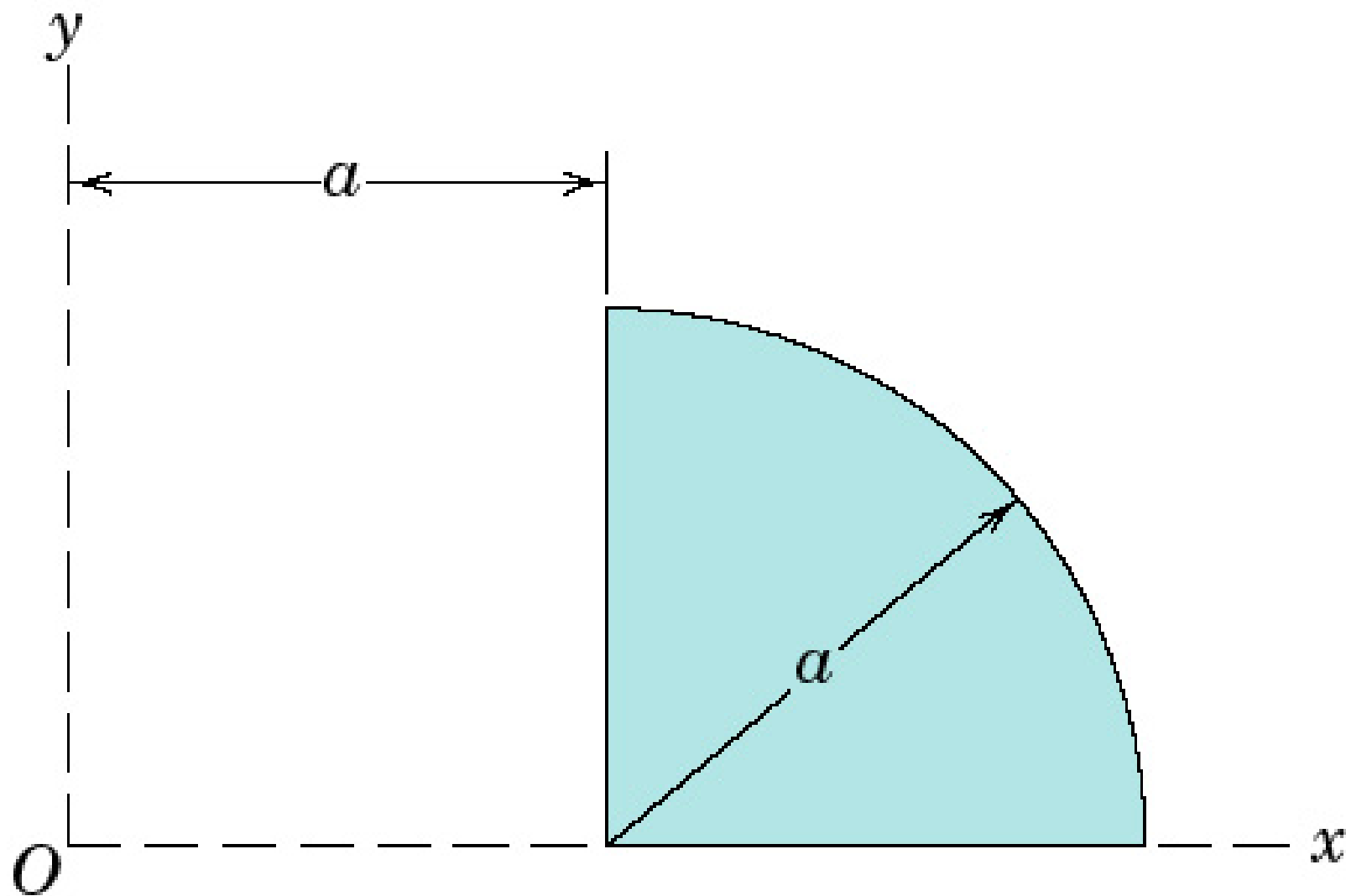
Problem A/11



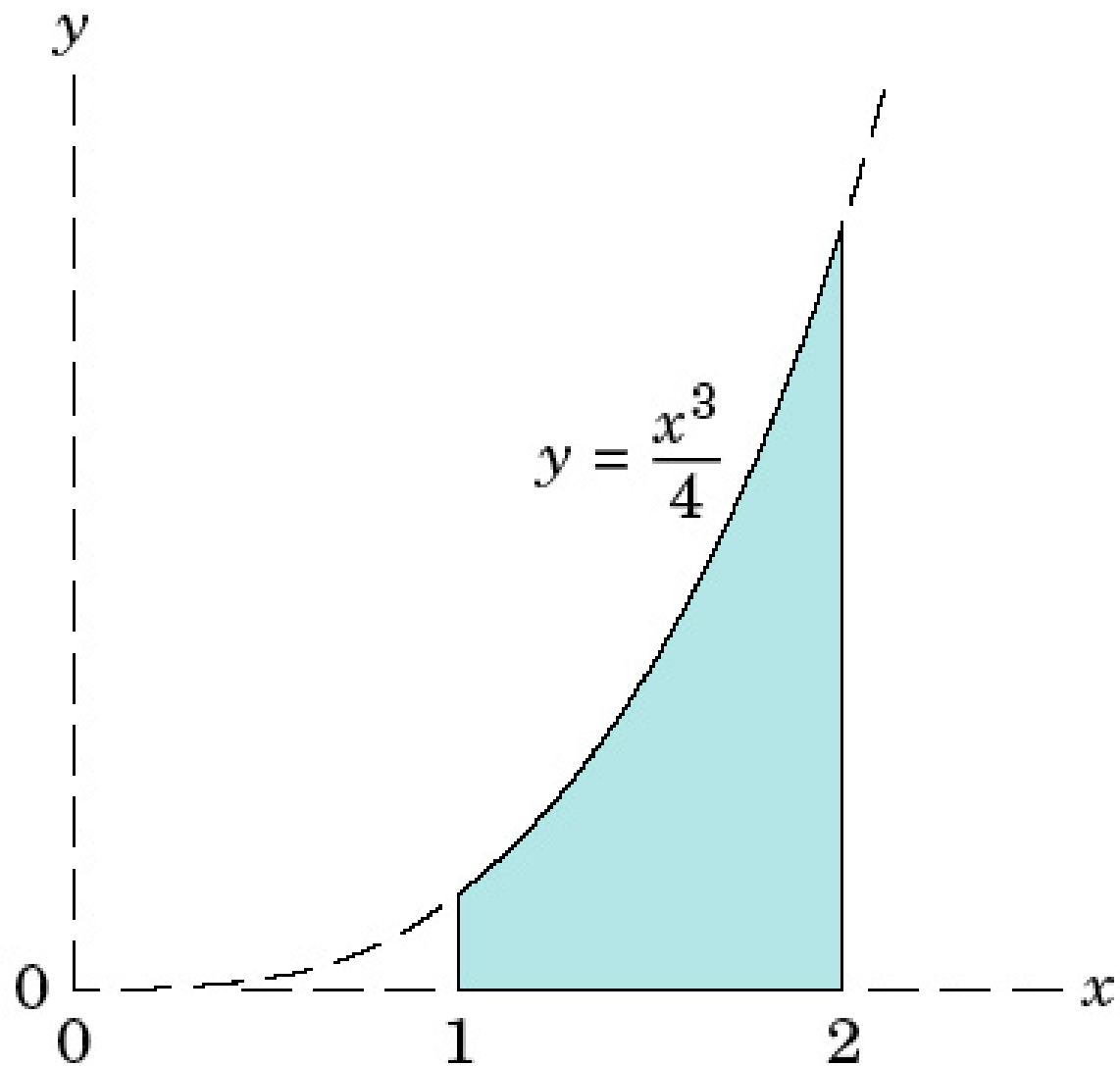
Problem A/12



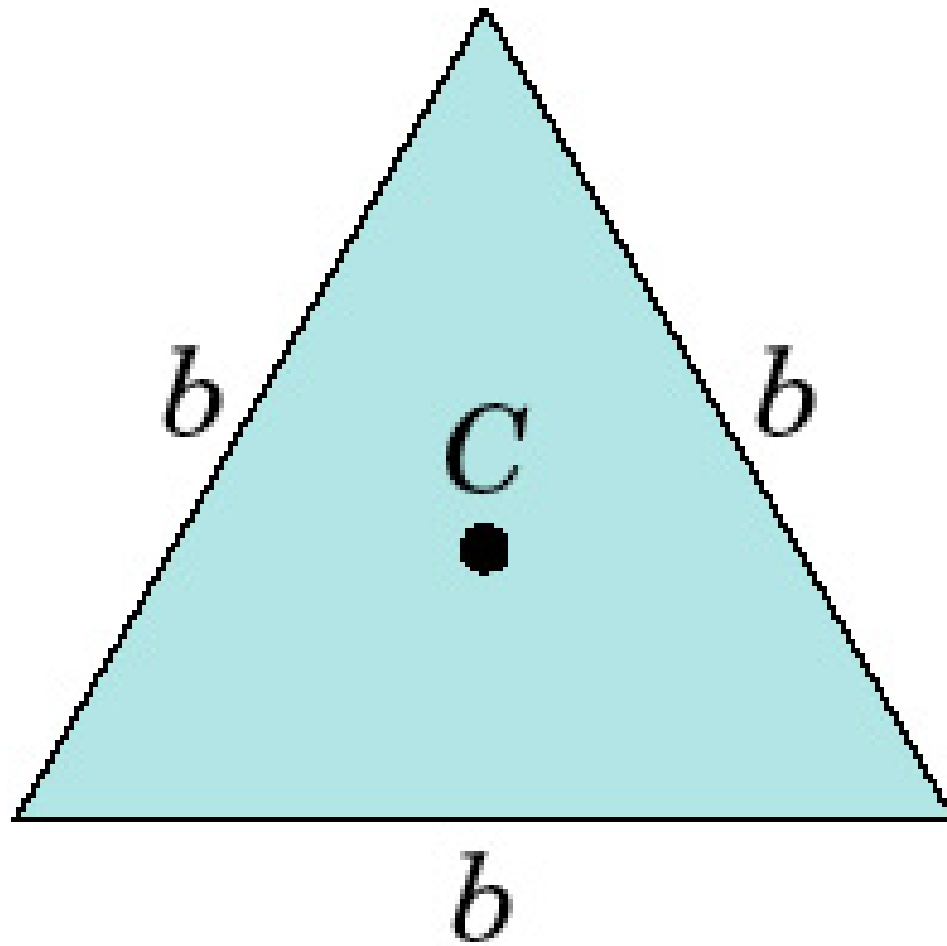
Problem A/13



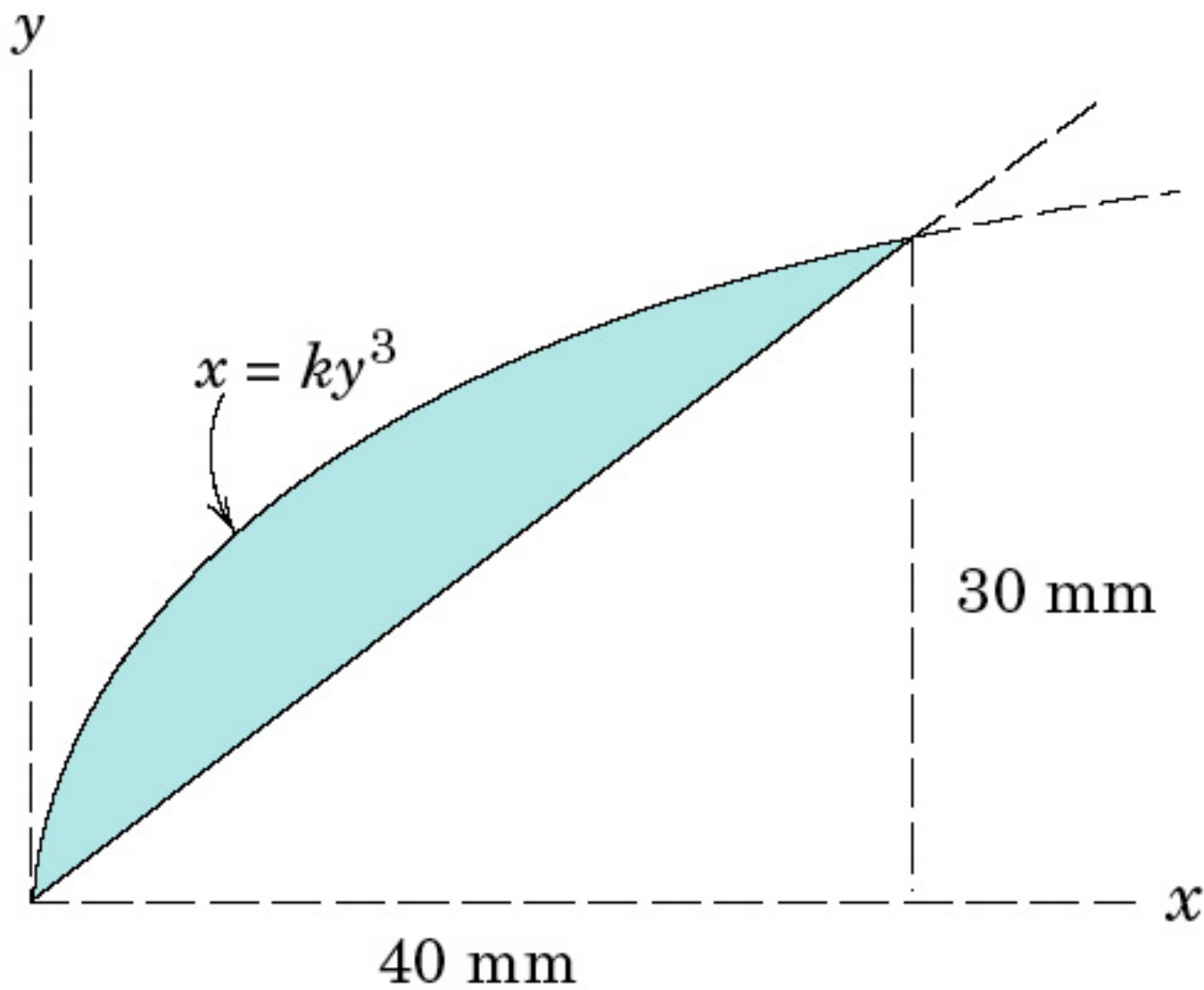
Problem A/14



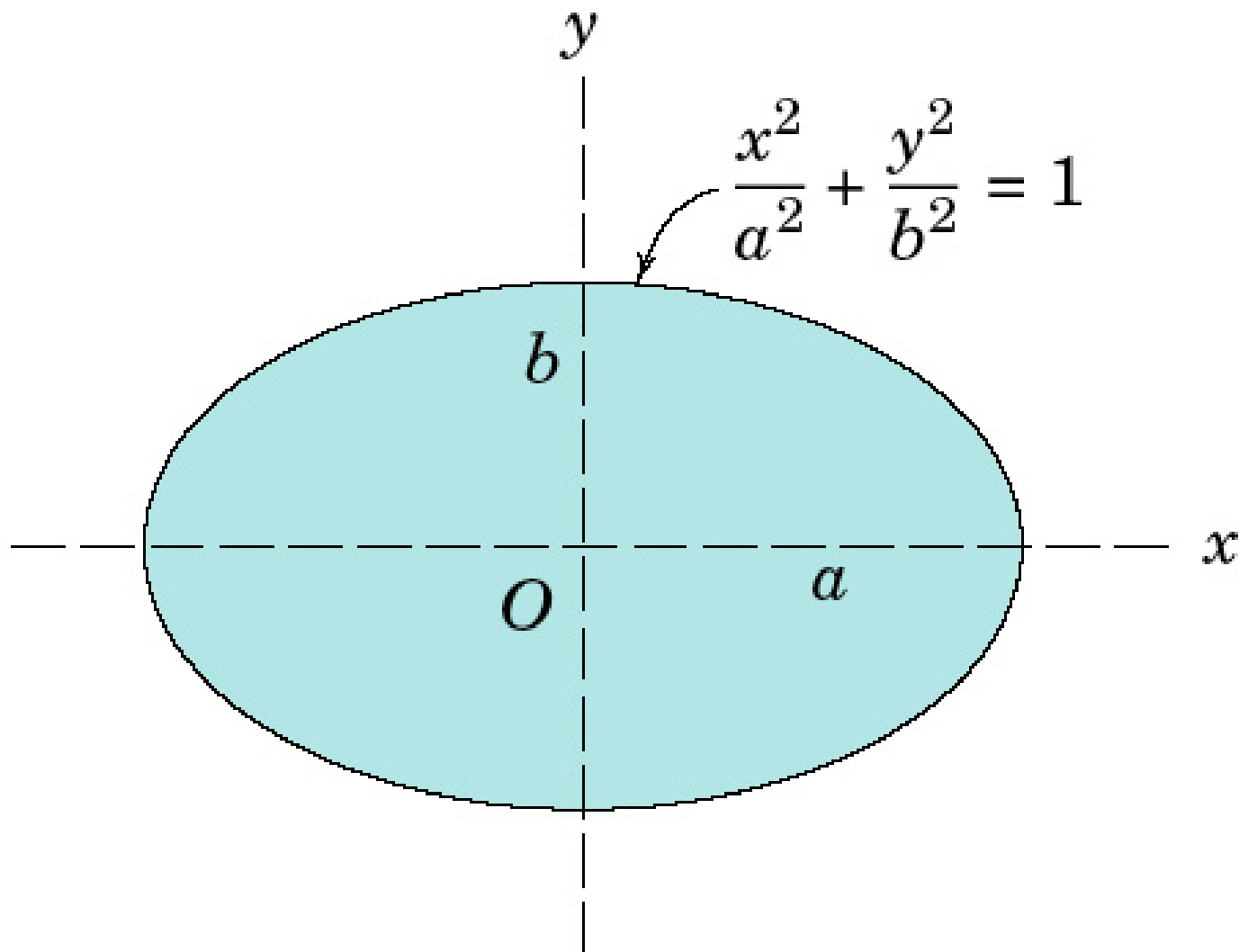
Problem A/15



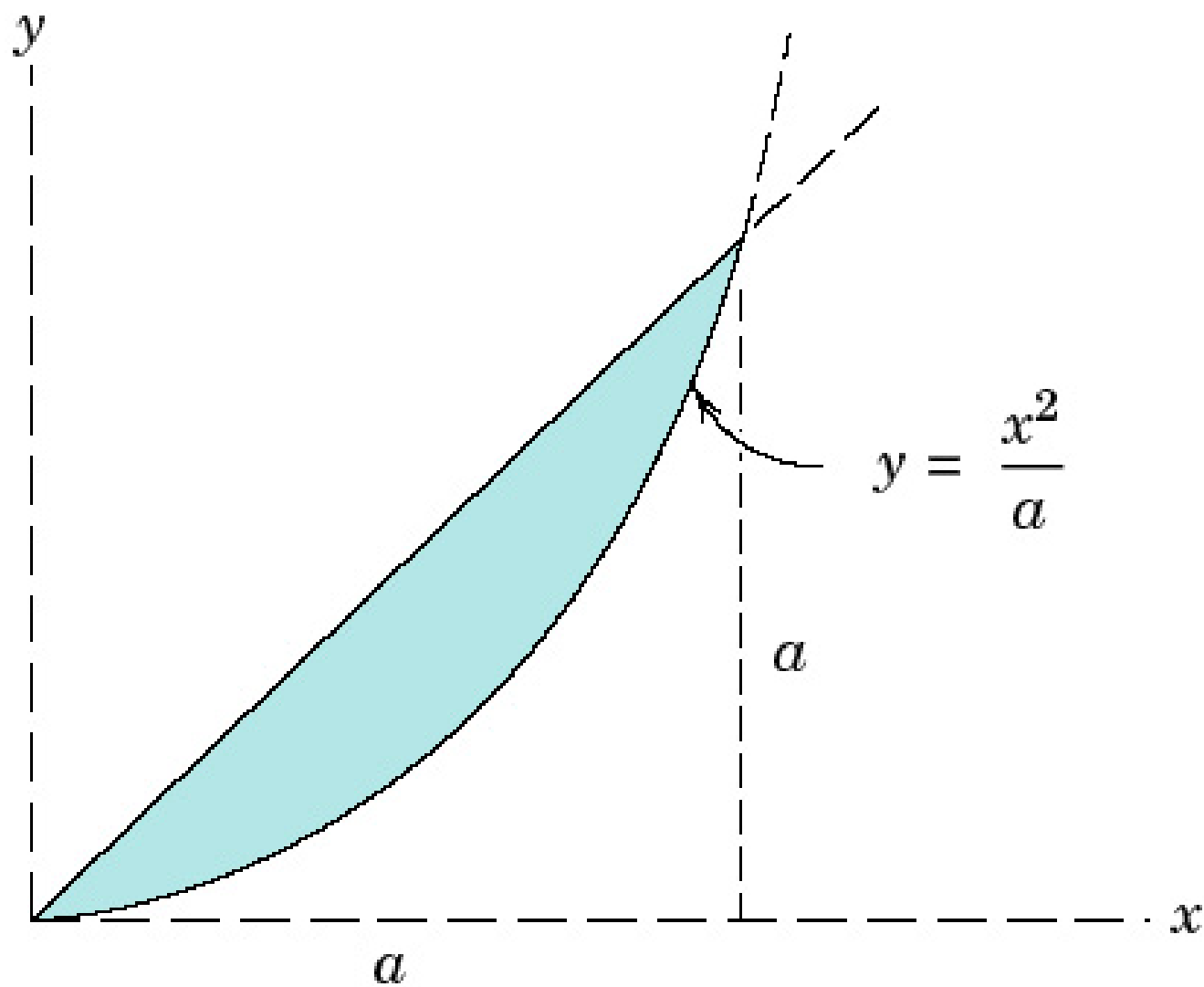
Problem A/16



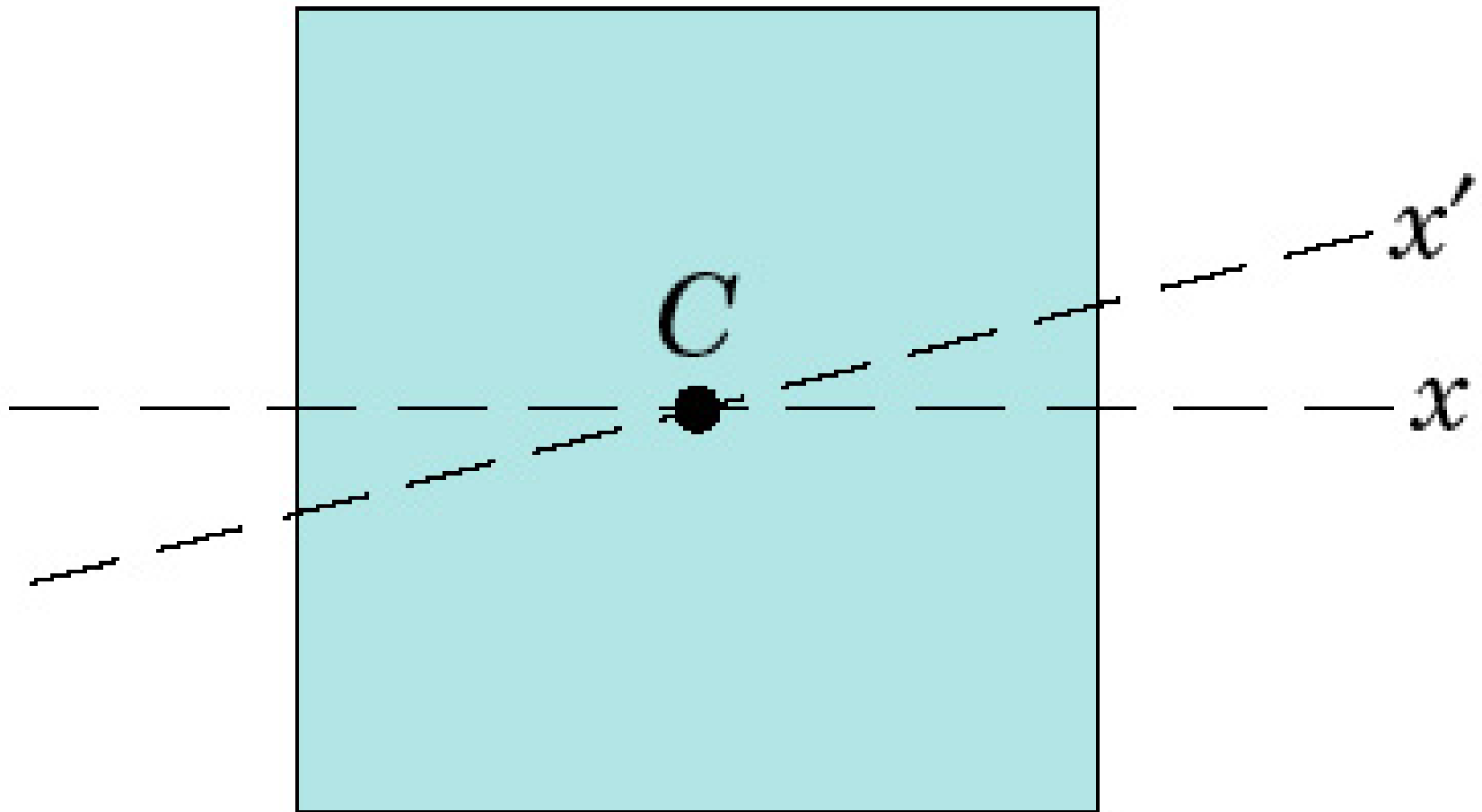
Problem A/17



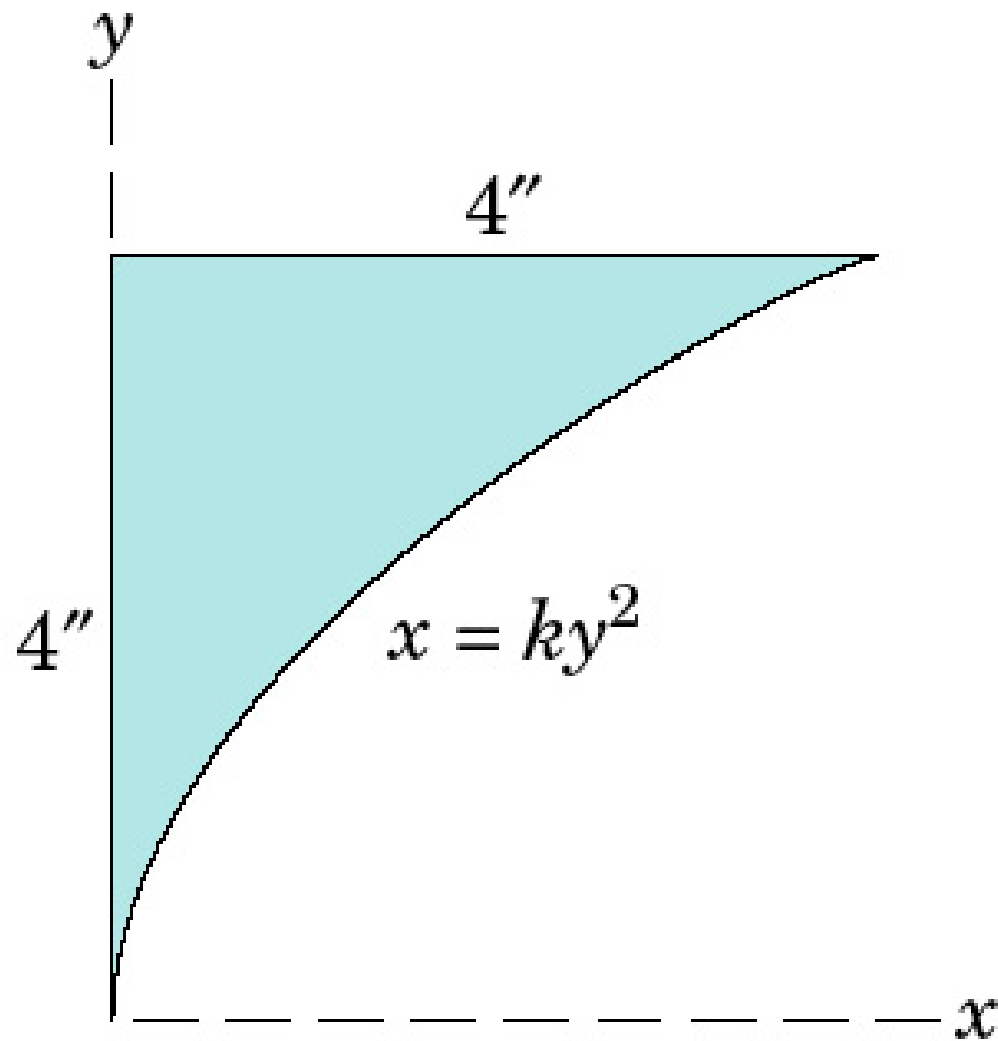
Problem A/18



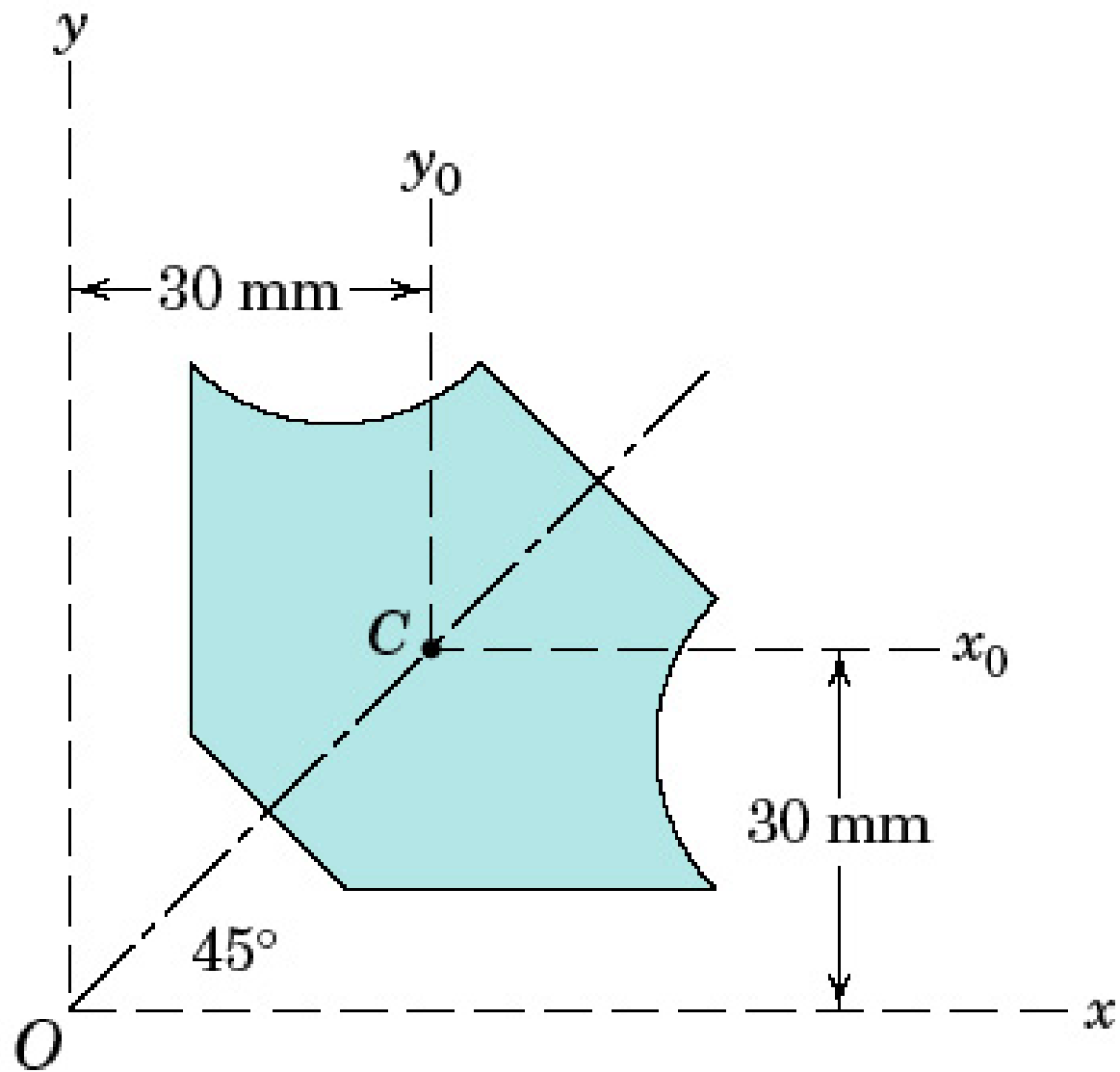
Problem A/19



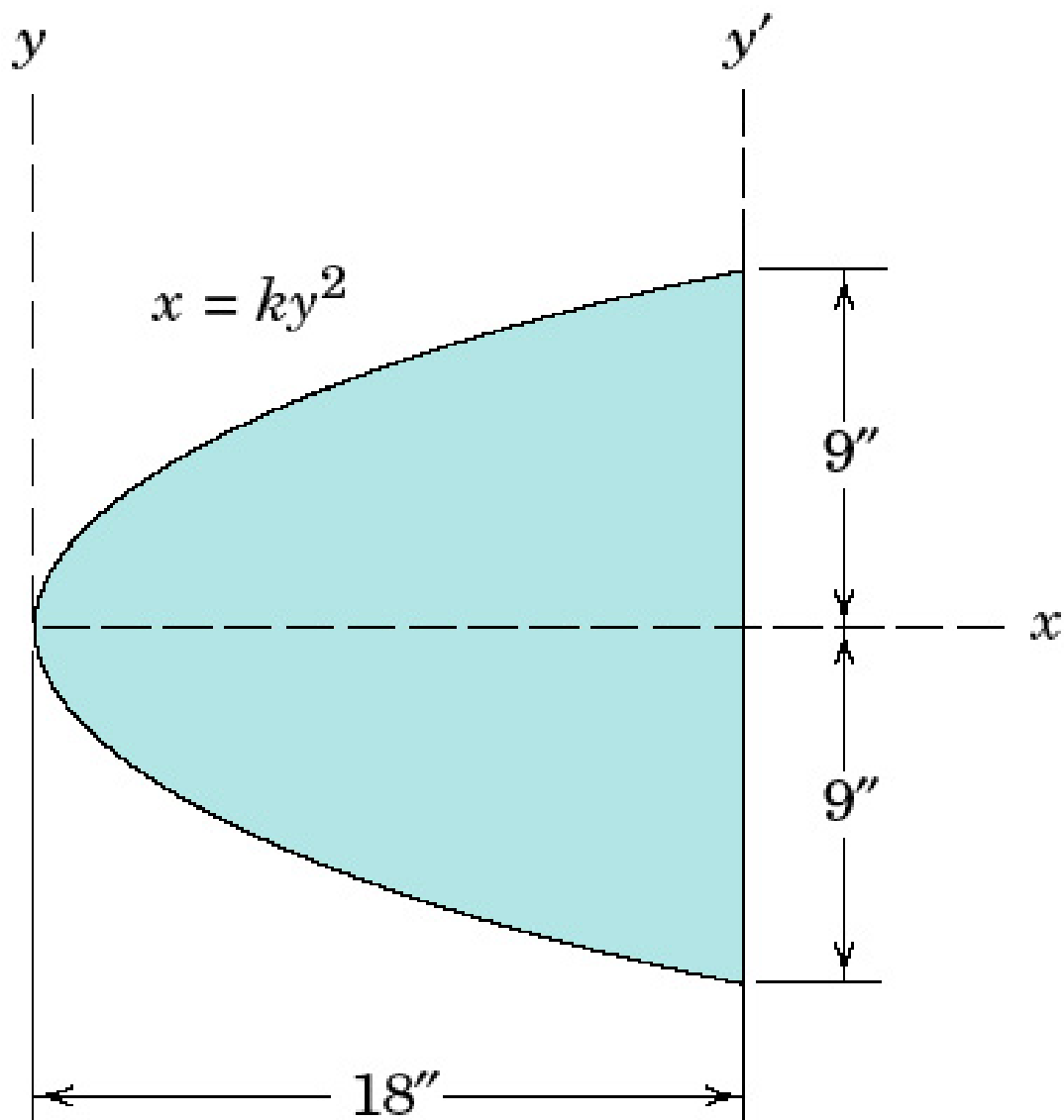
Problem A/20



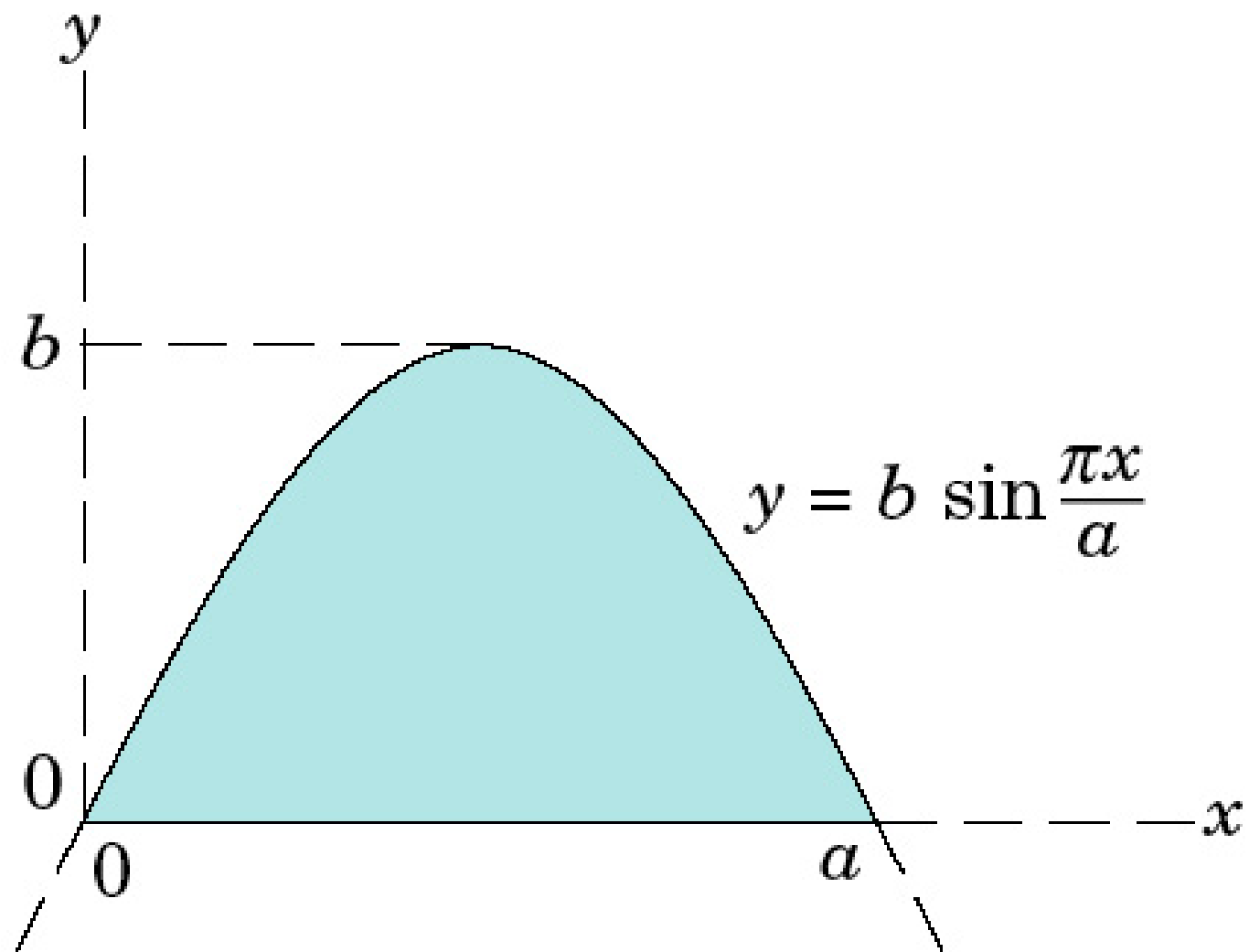
Problem A/21



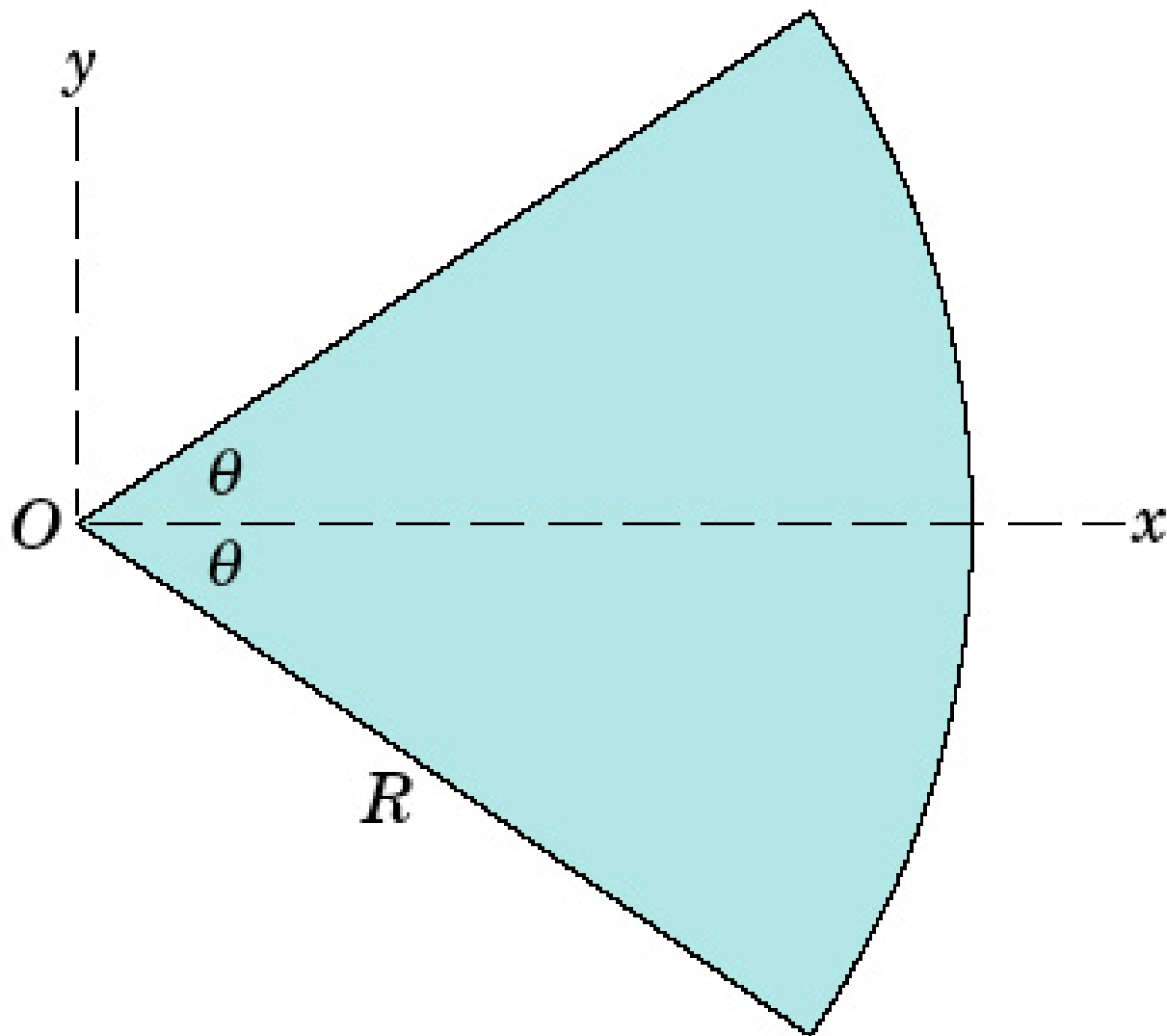
Problem A/22



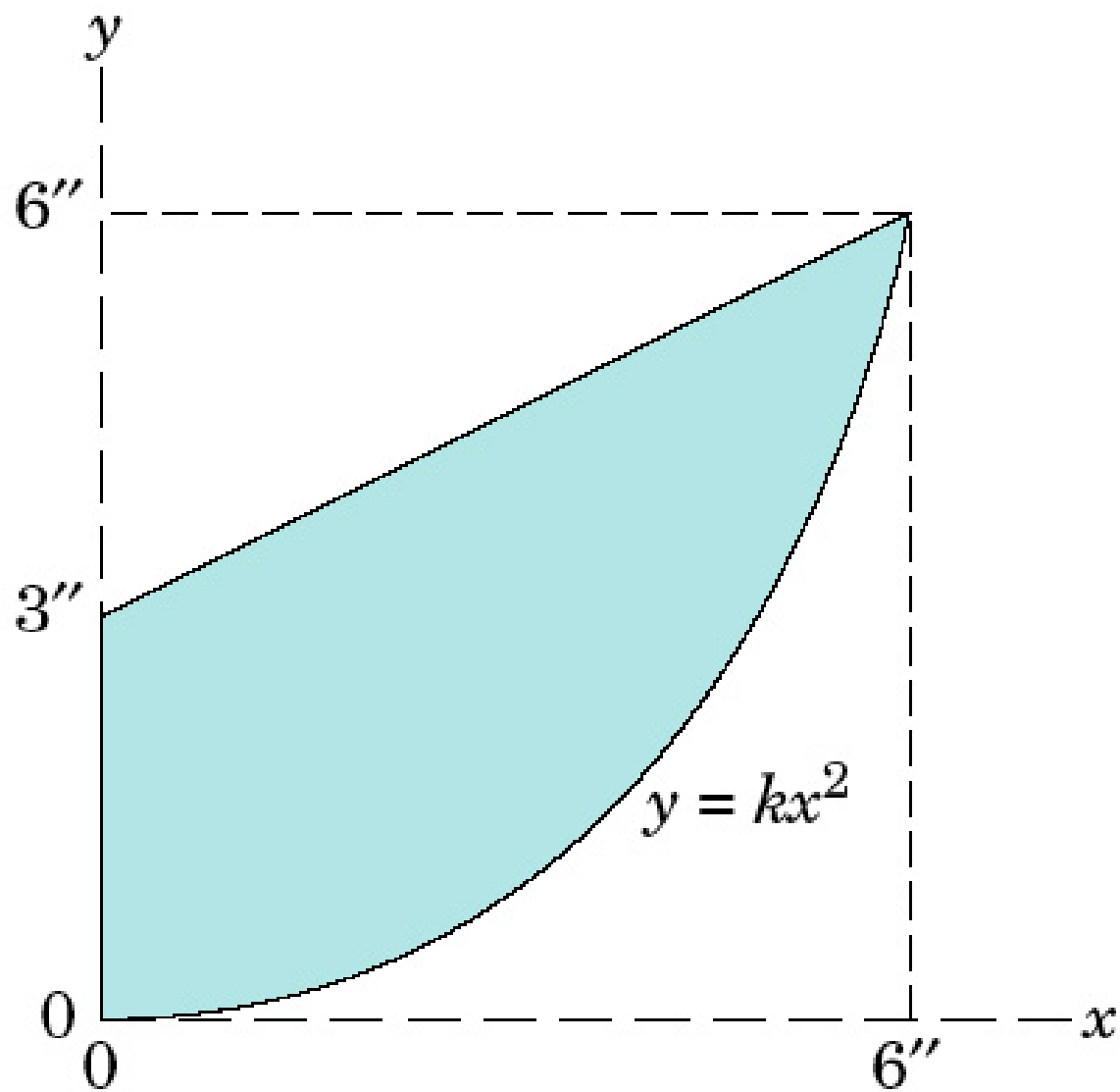
Problem A/23



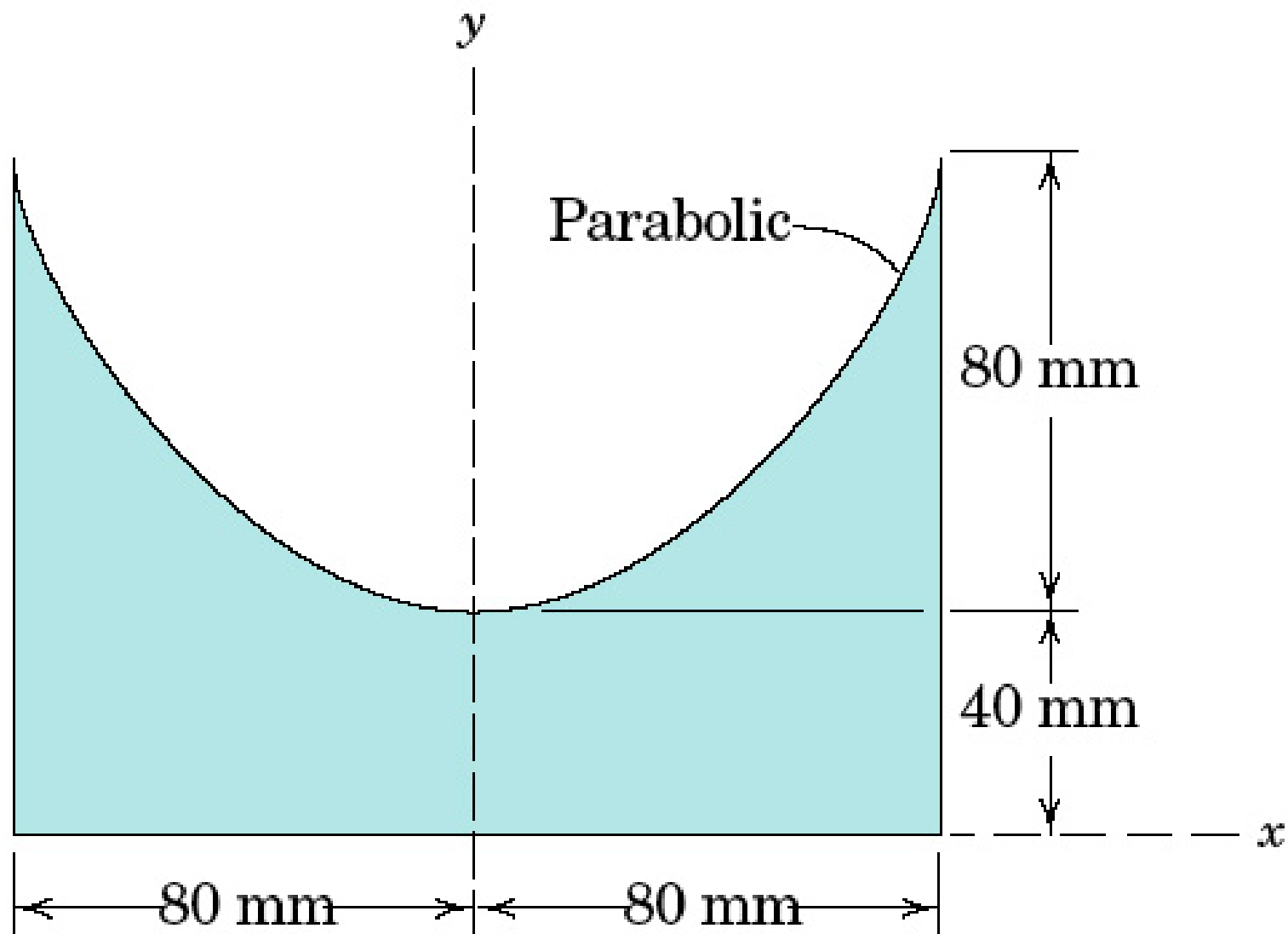
Problem A/24



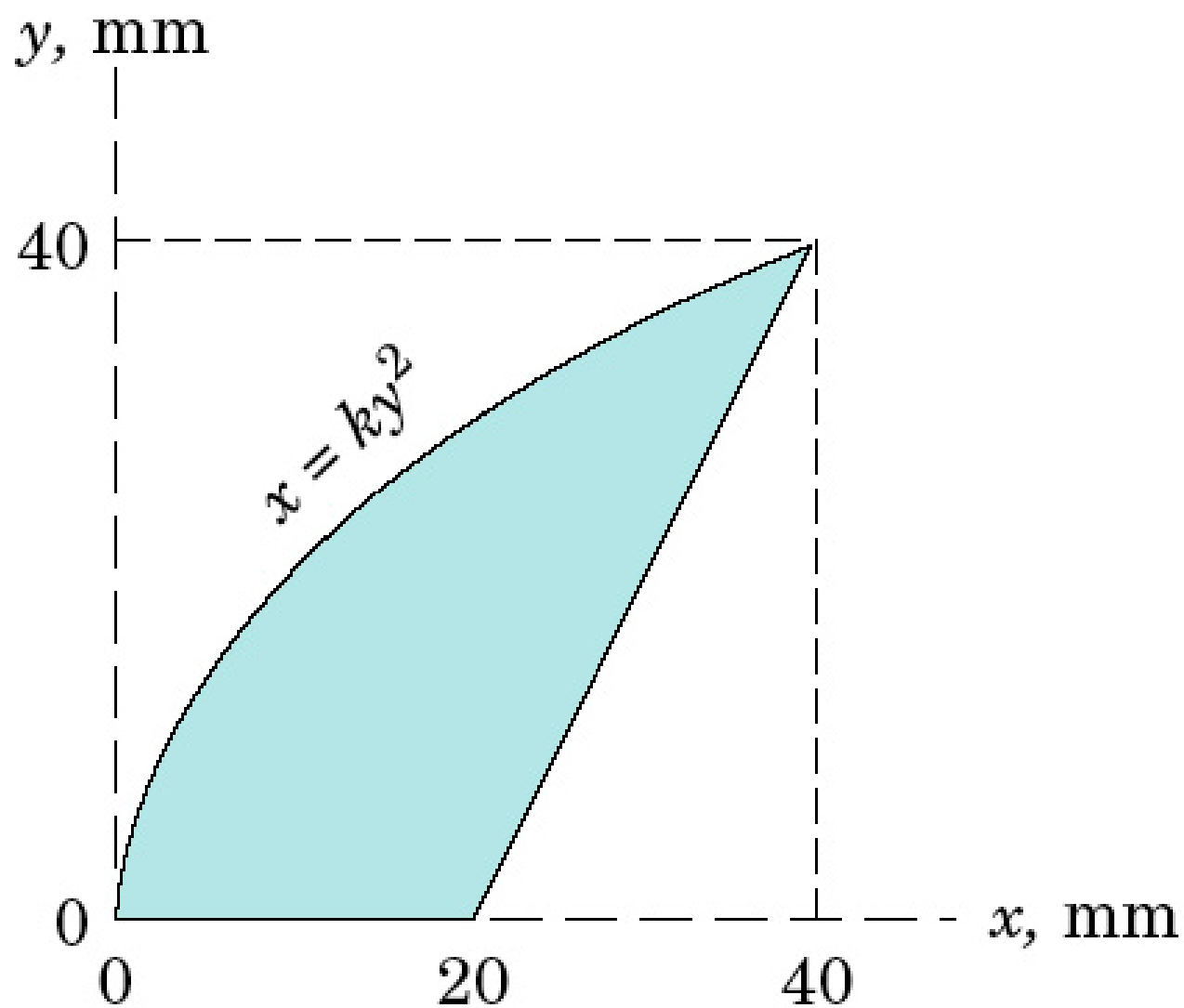
Problem A/25



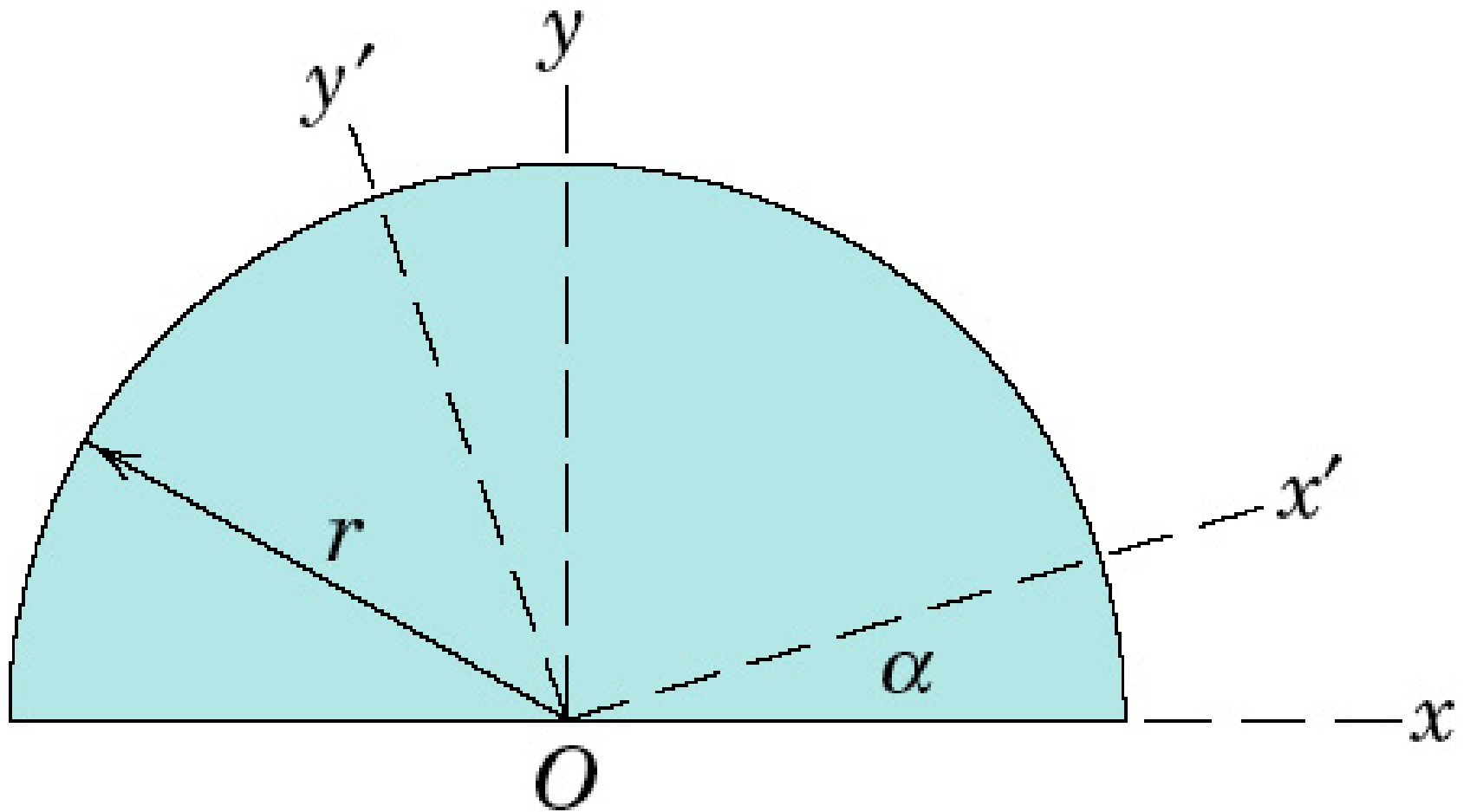
Problem A/27



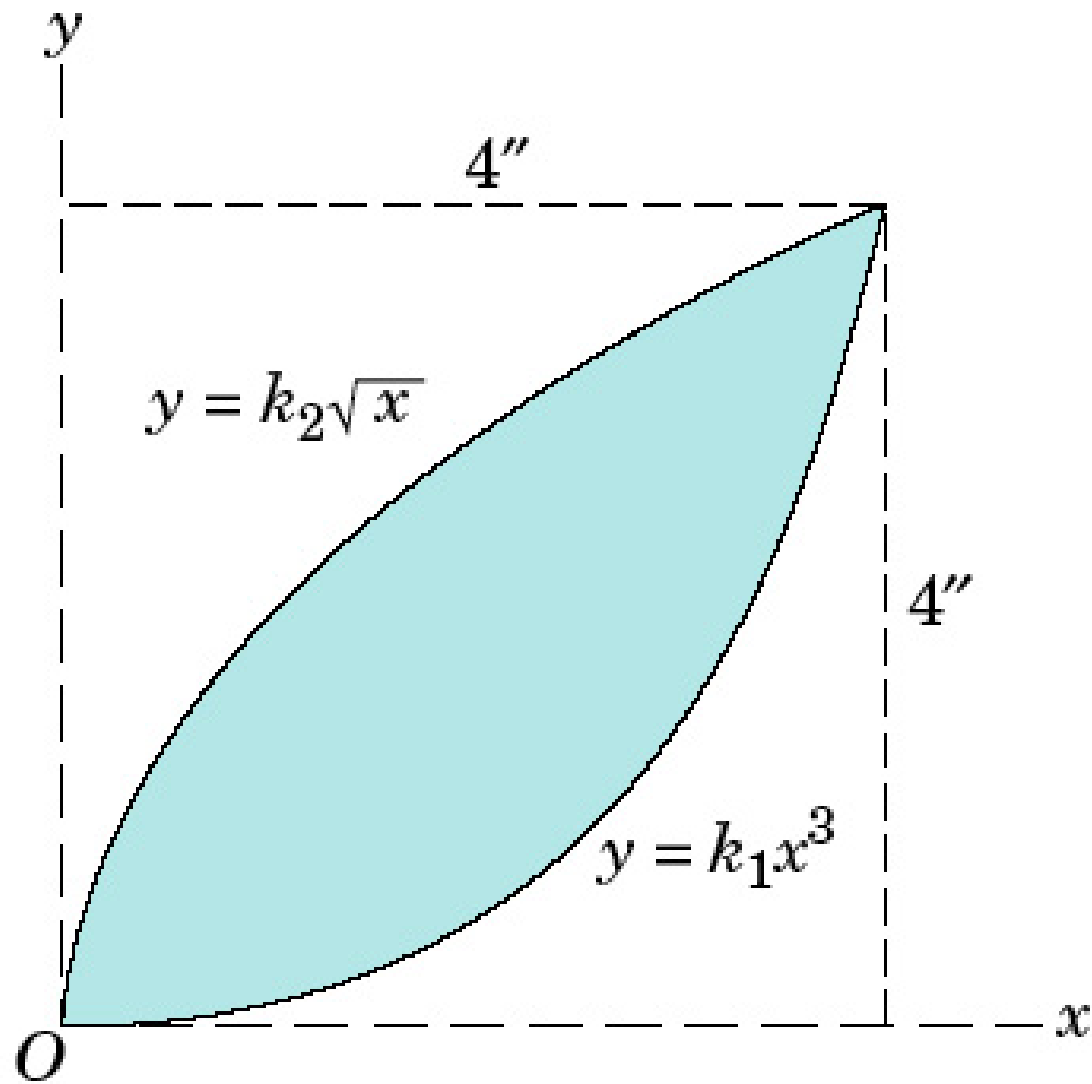
Problem A/28



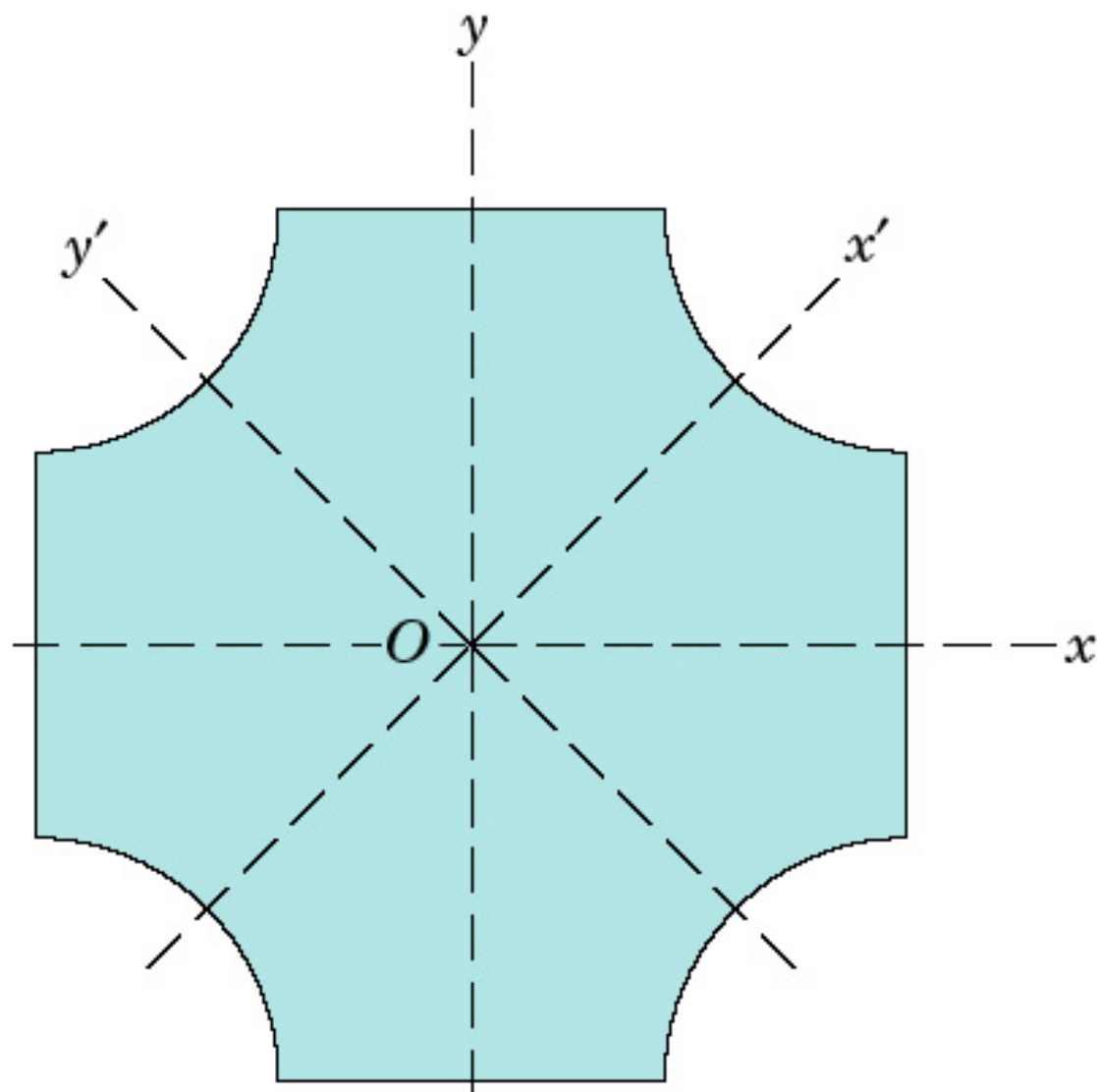
Problem A/29



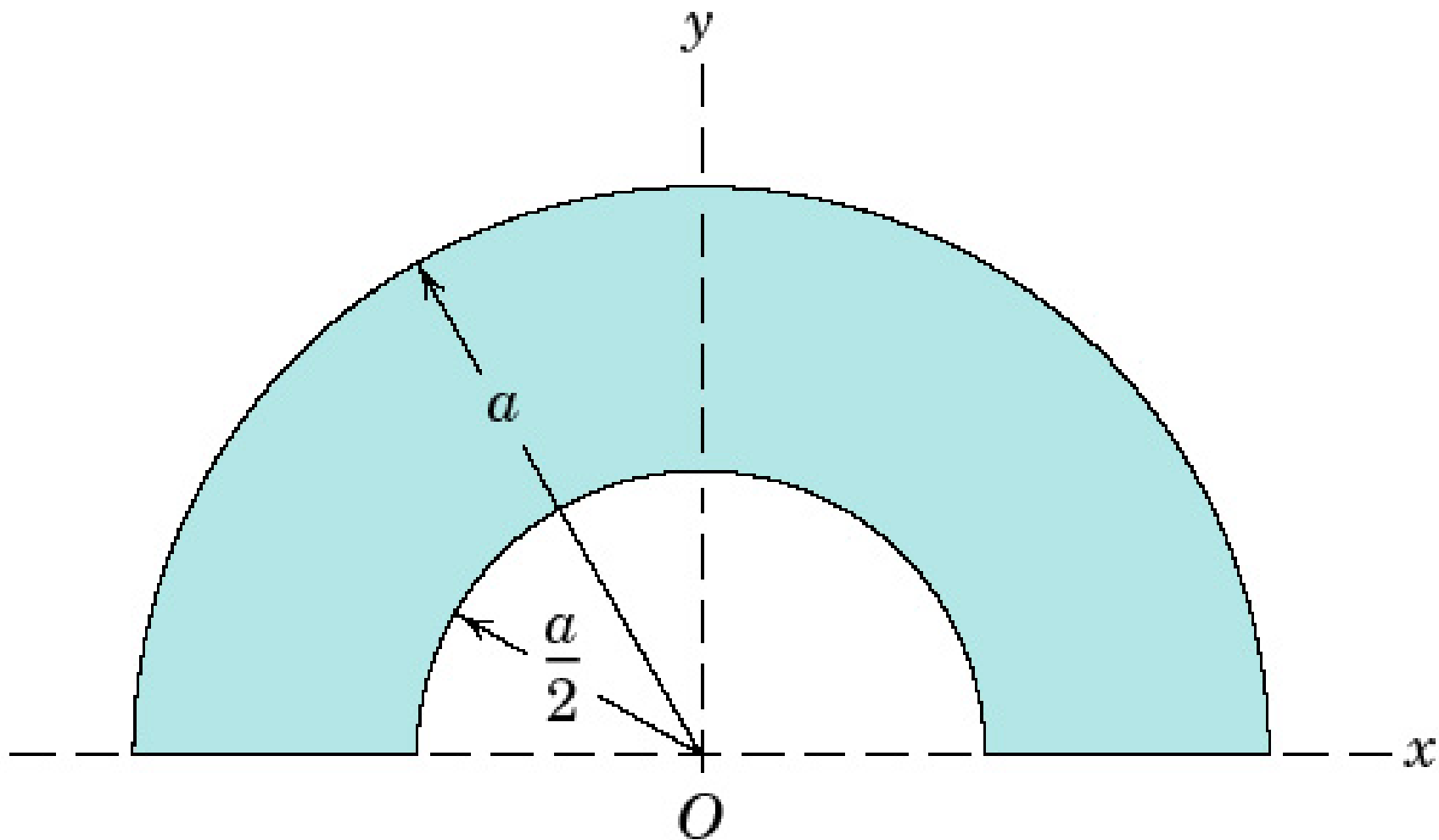
Problem A/30



Problem A/31

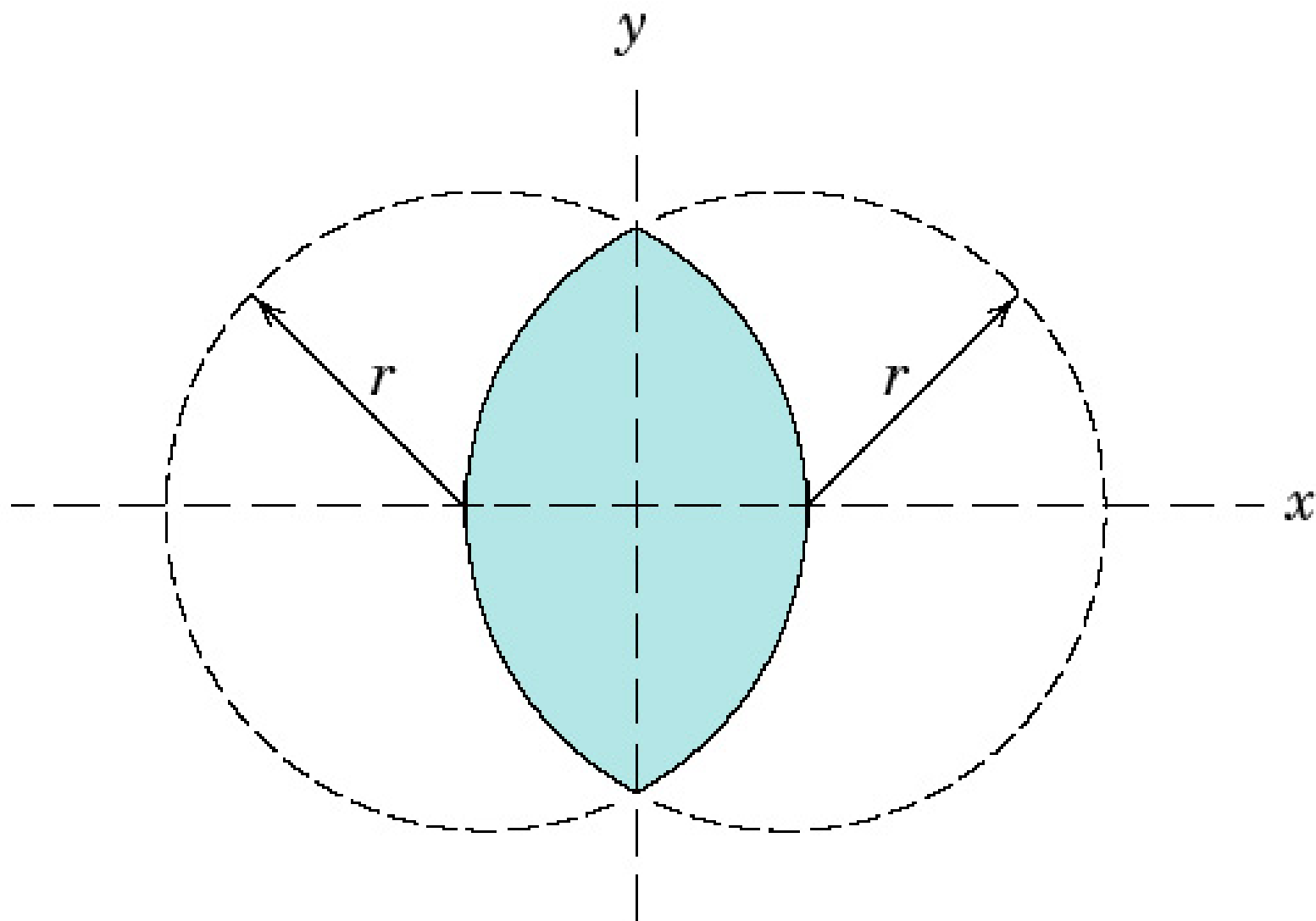


Problem A/32

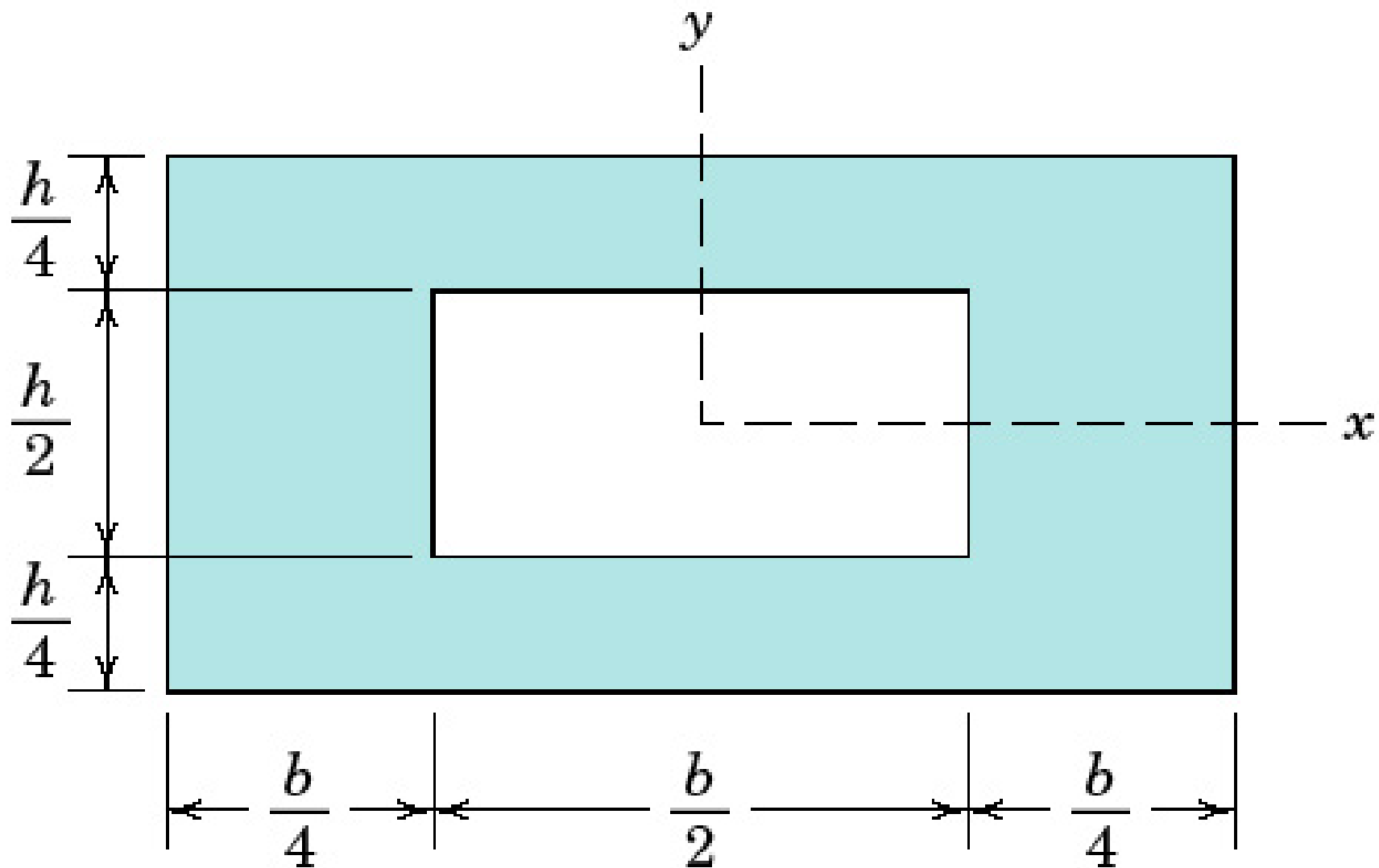


Problem A/33

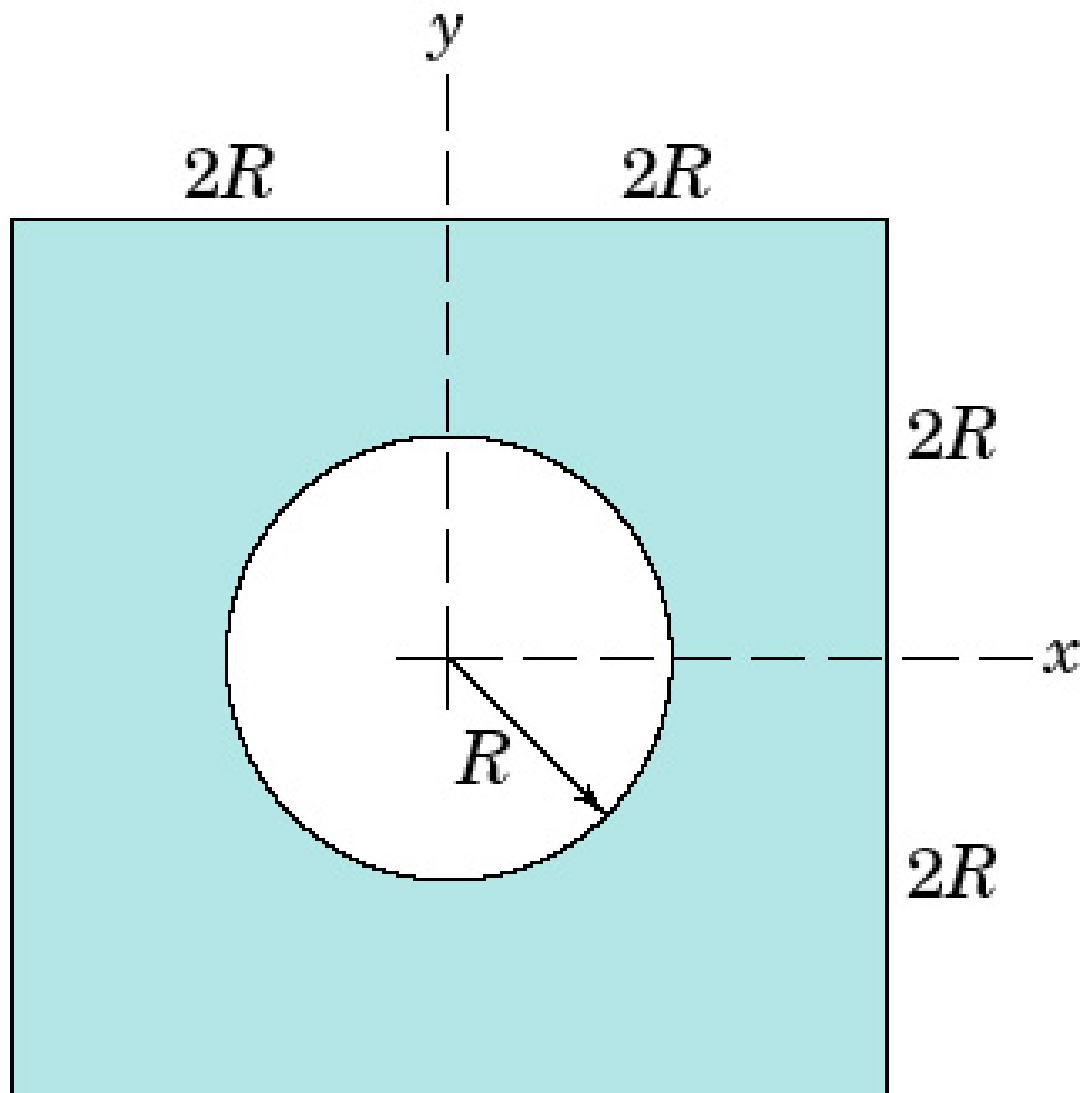
P_A_33



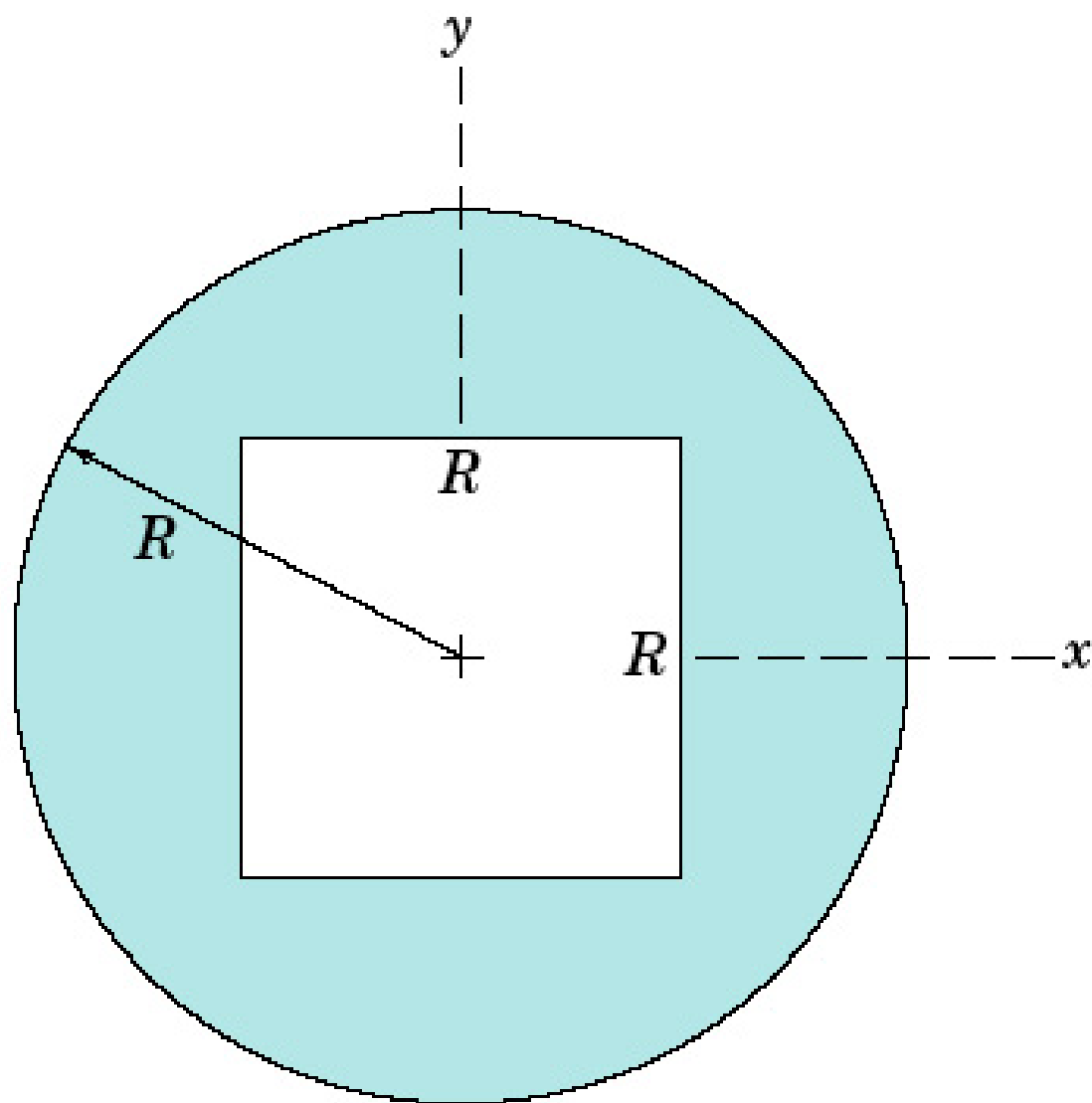
Problem A/34



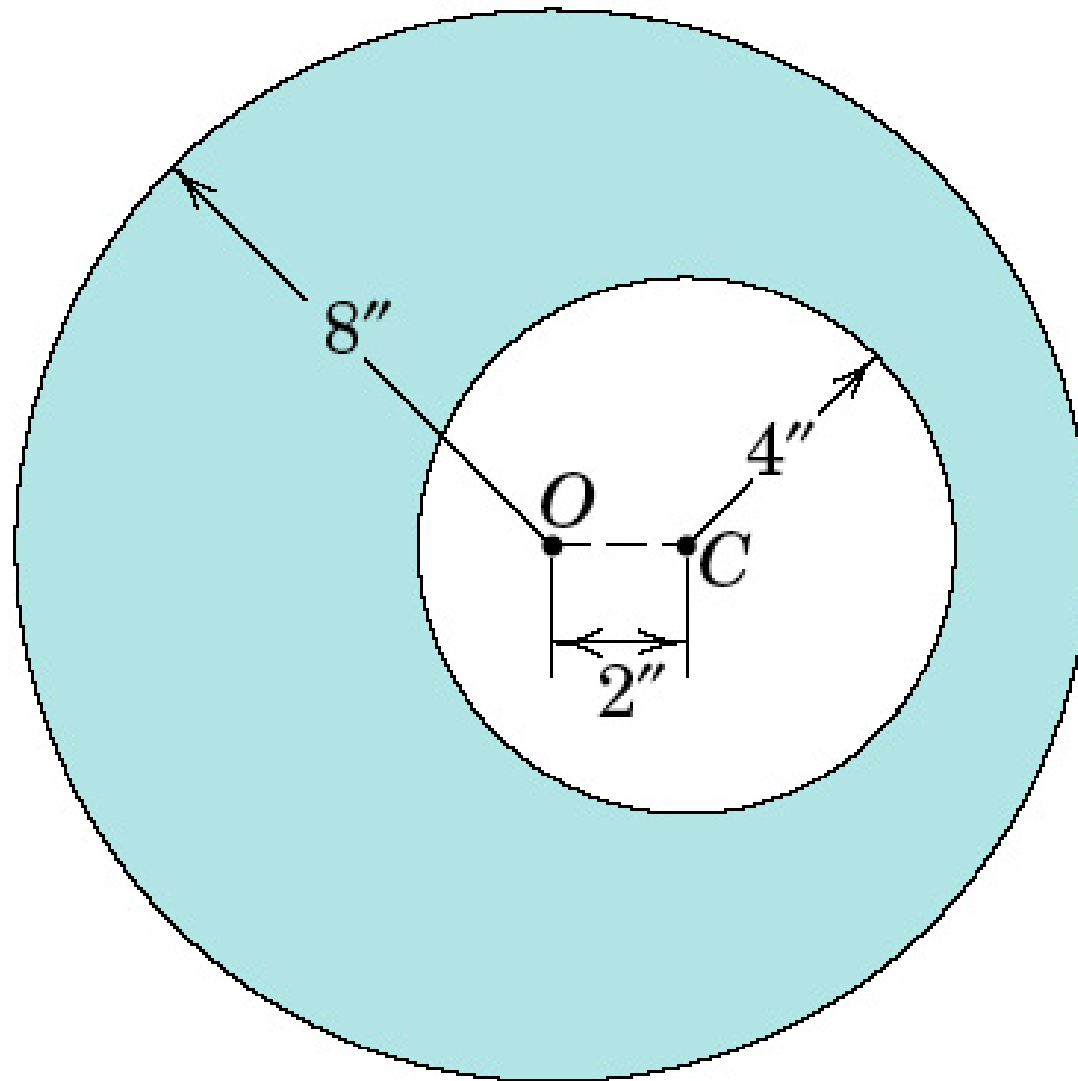
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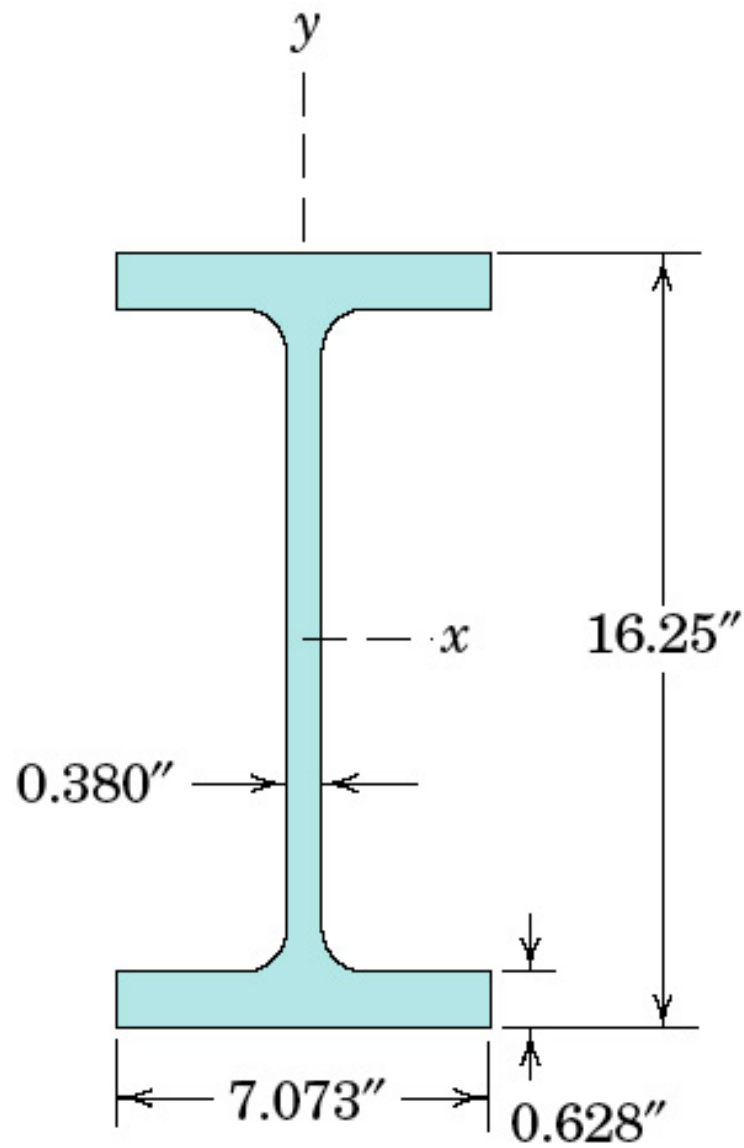
Problem A/36



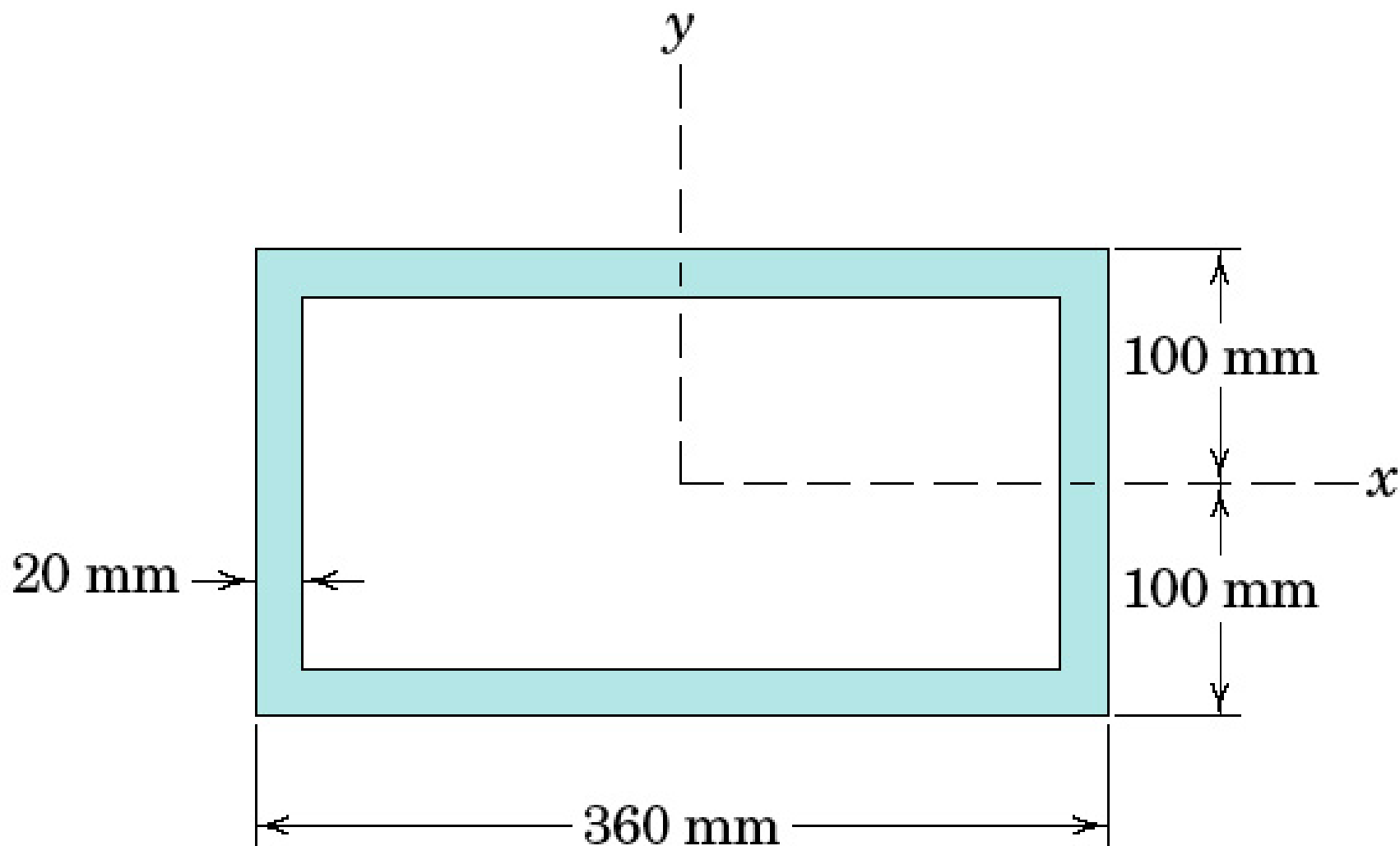
Problem A/37



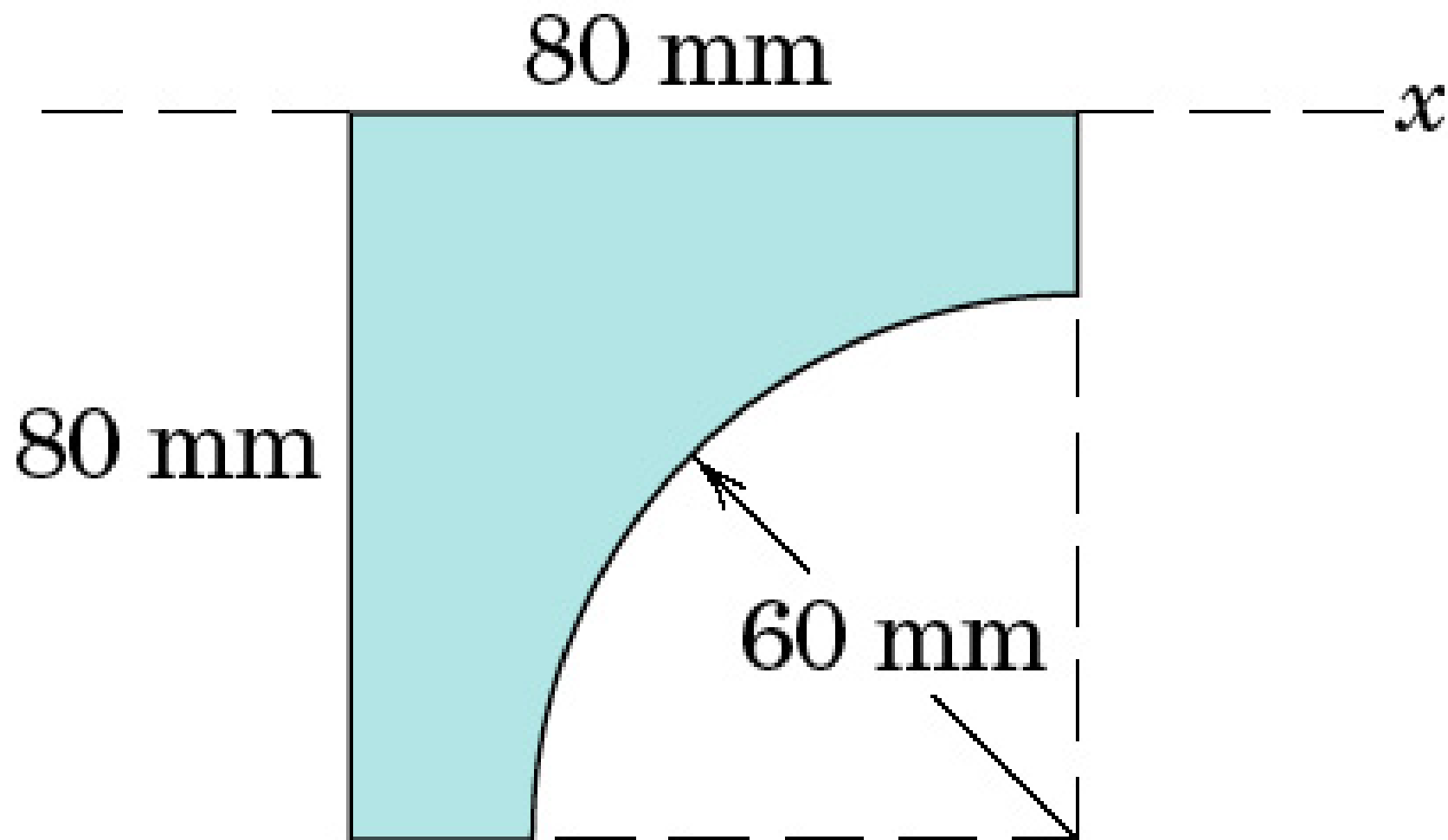
Problem A/38



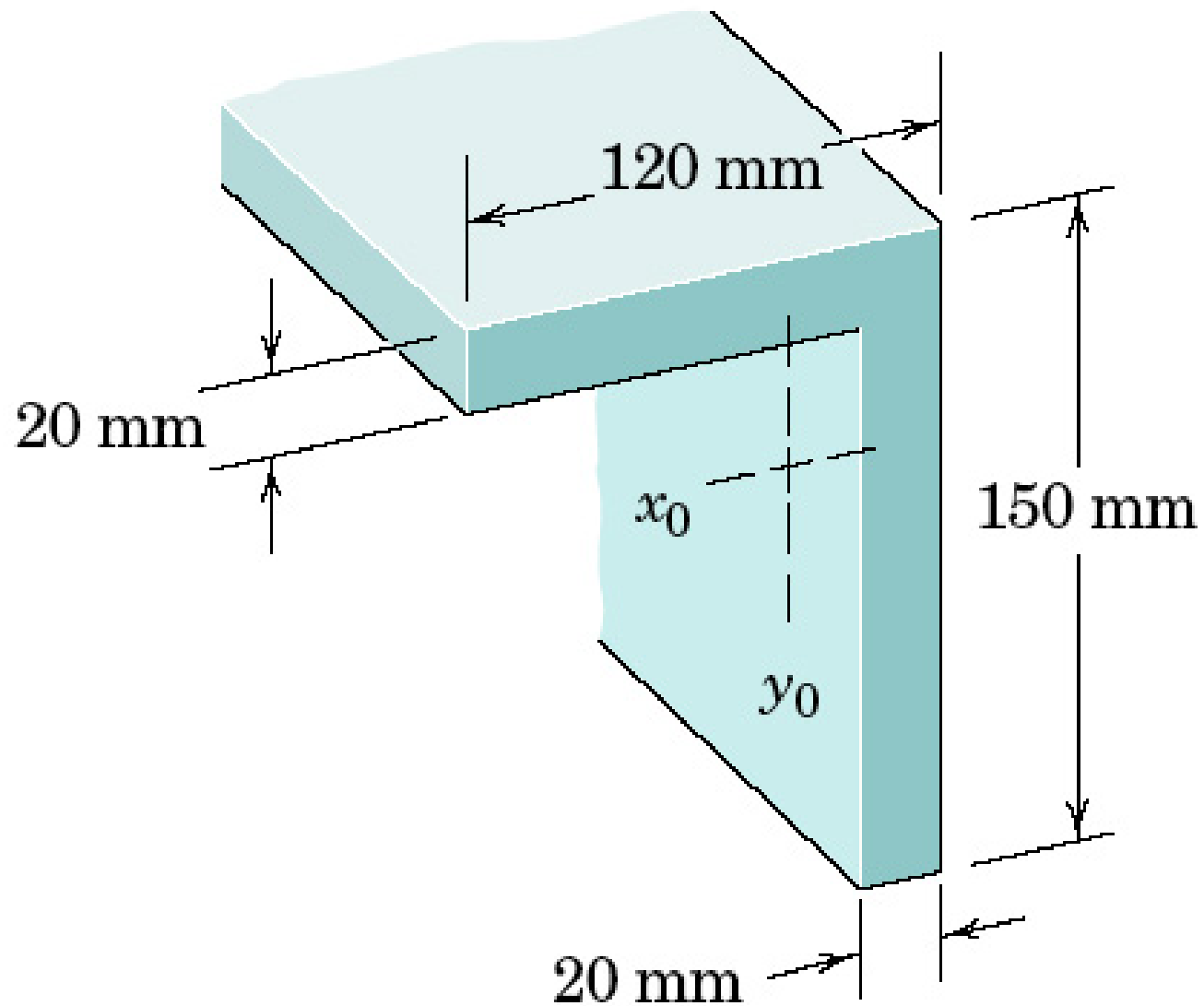
Problem A/39



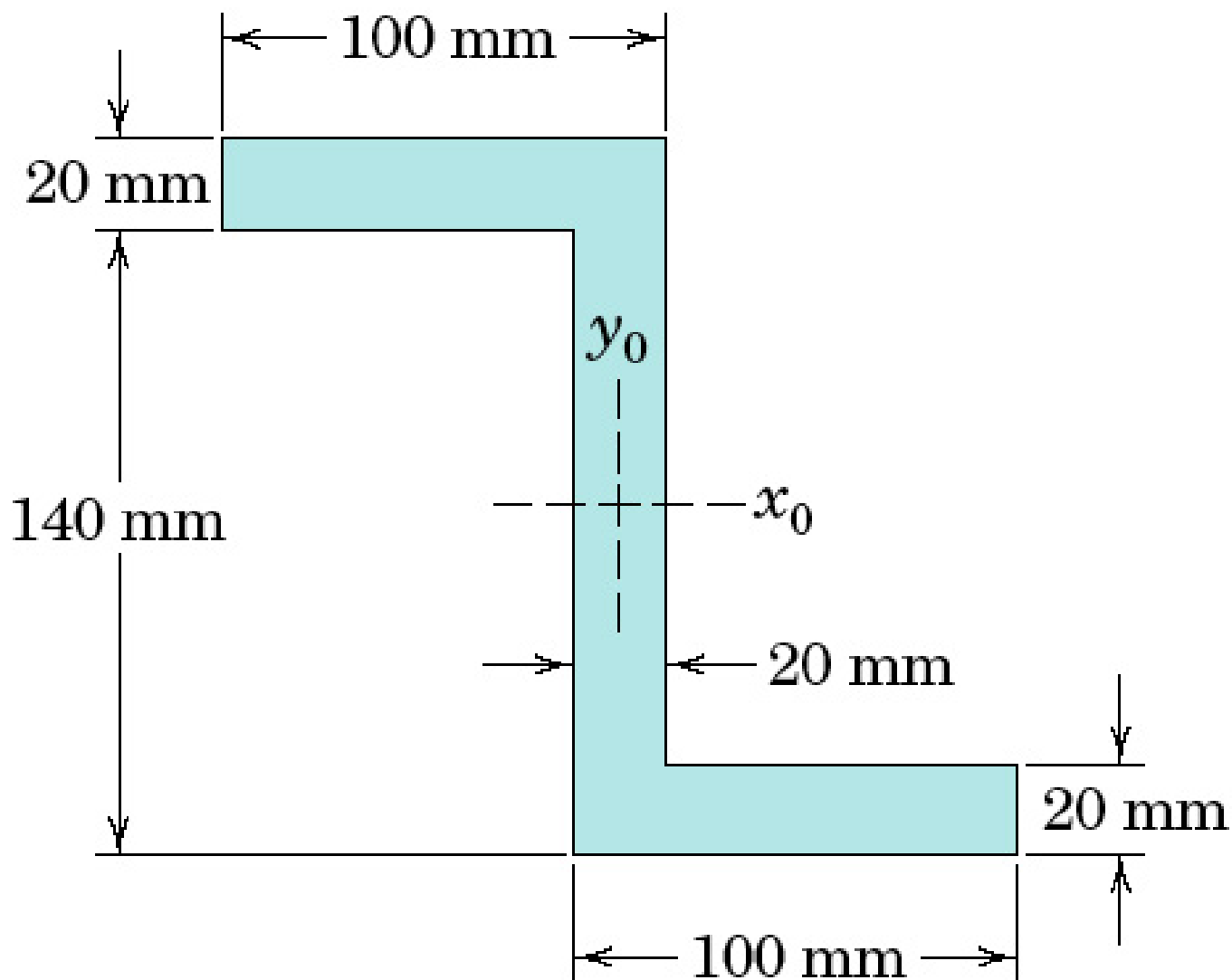
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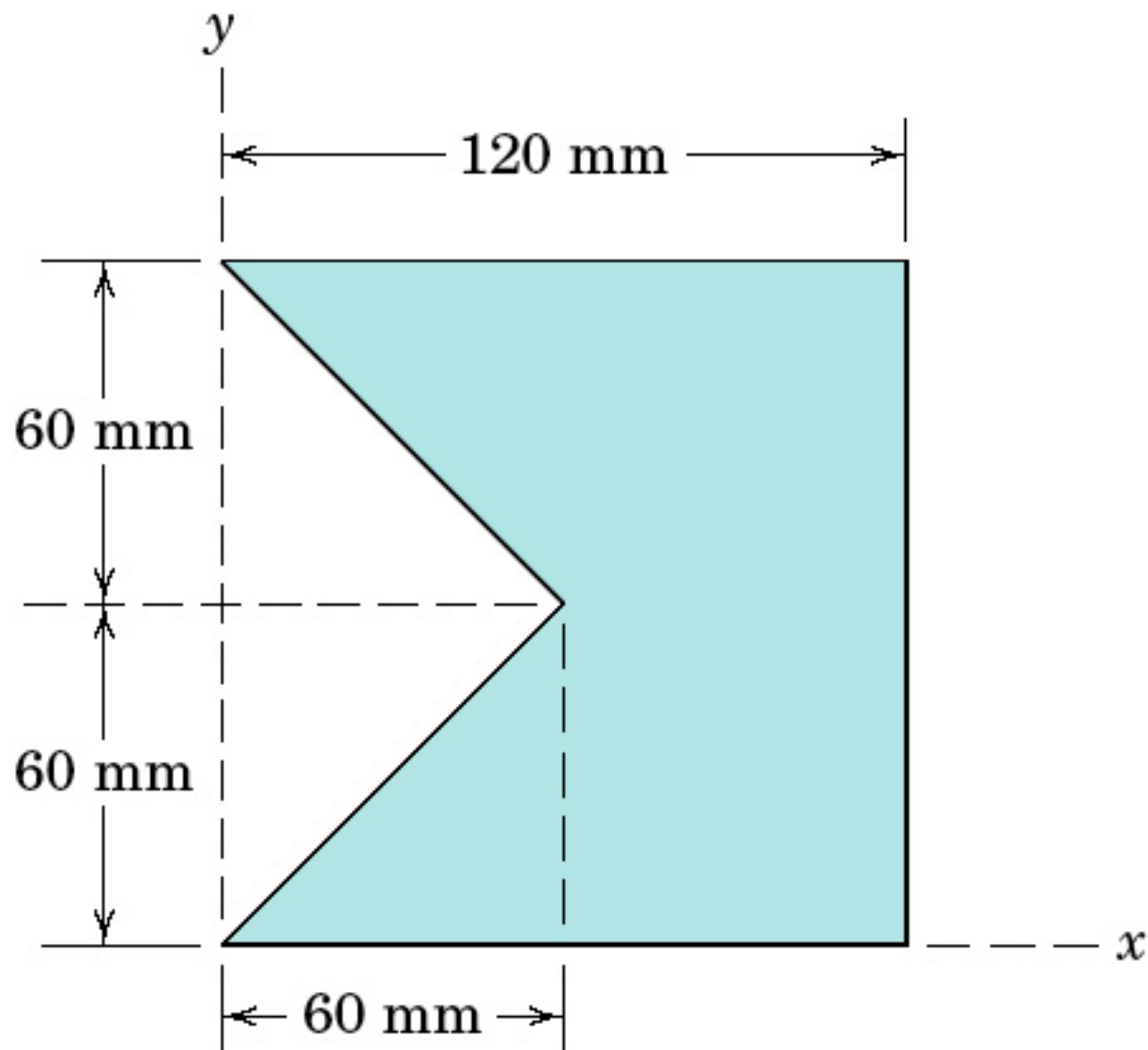
Problem A/41



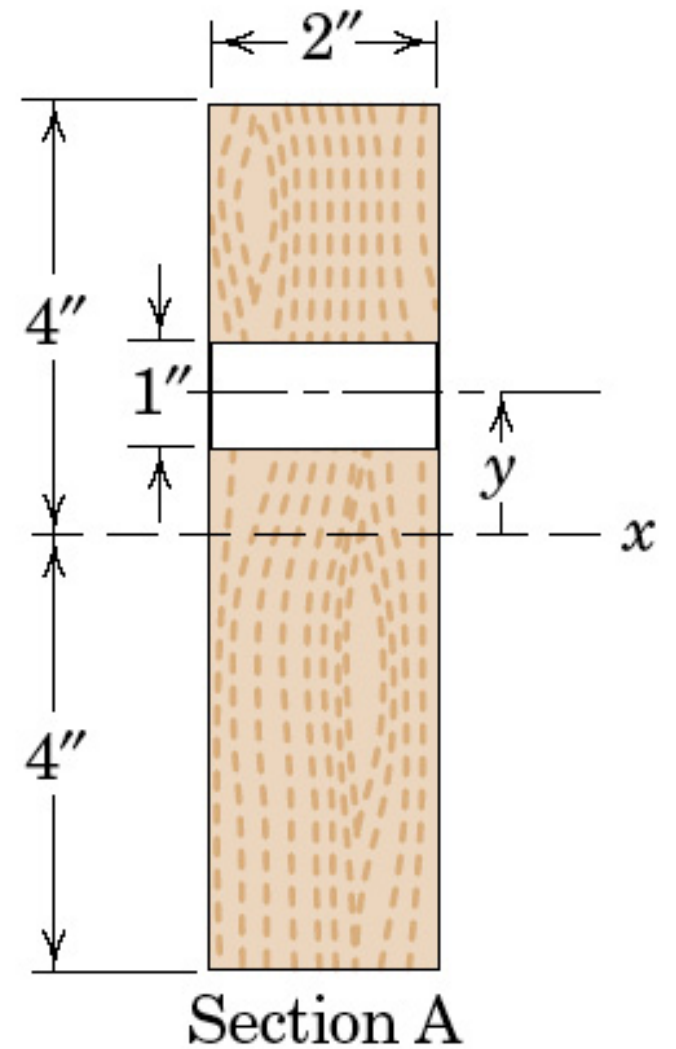
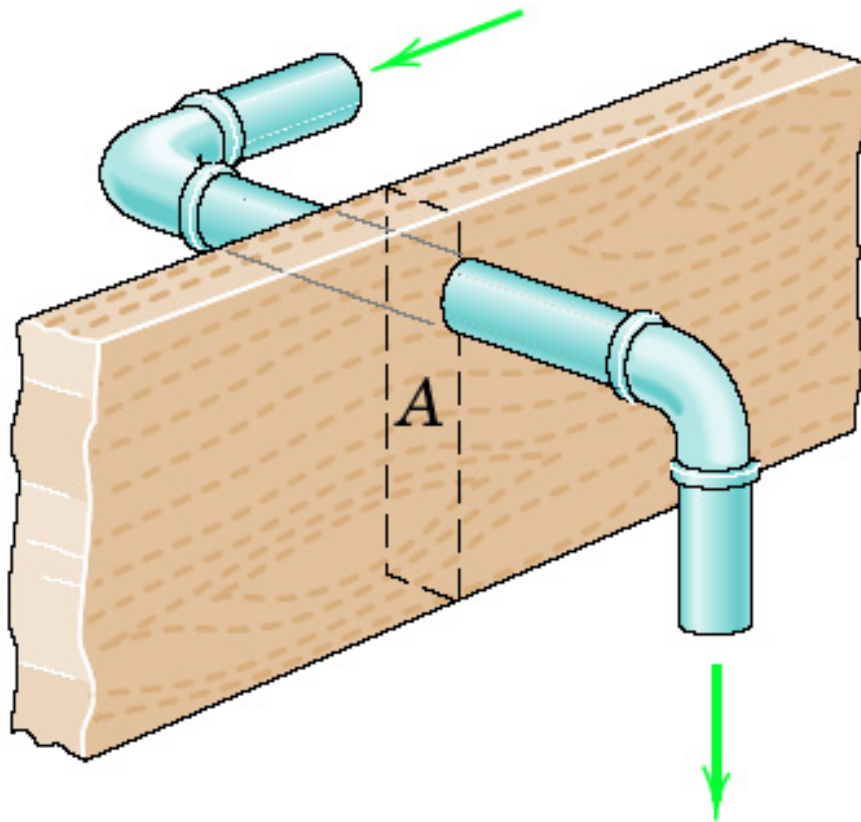
Problem A/42



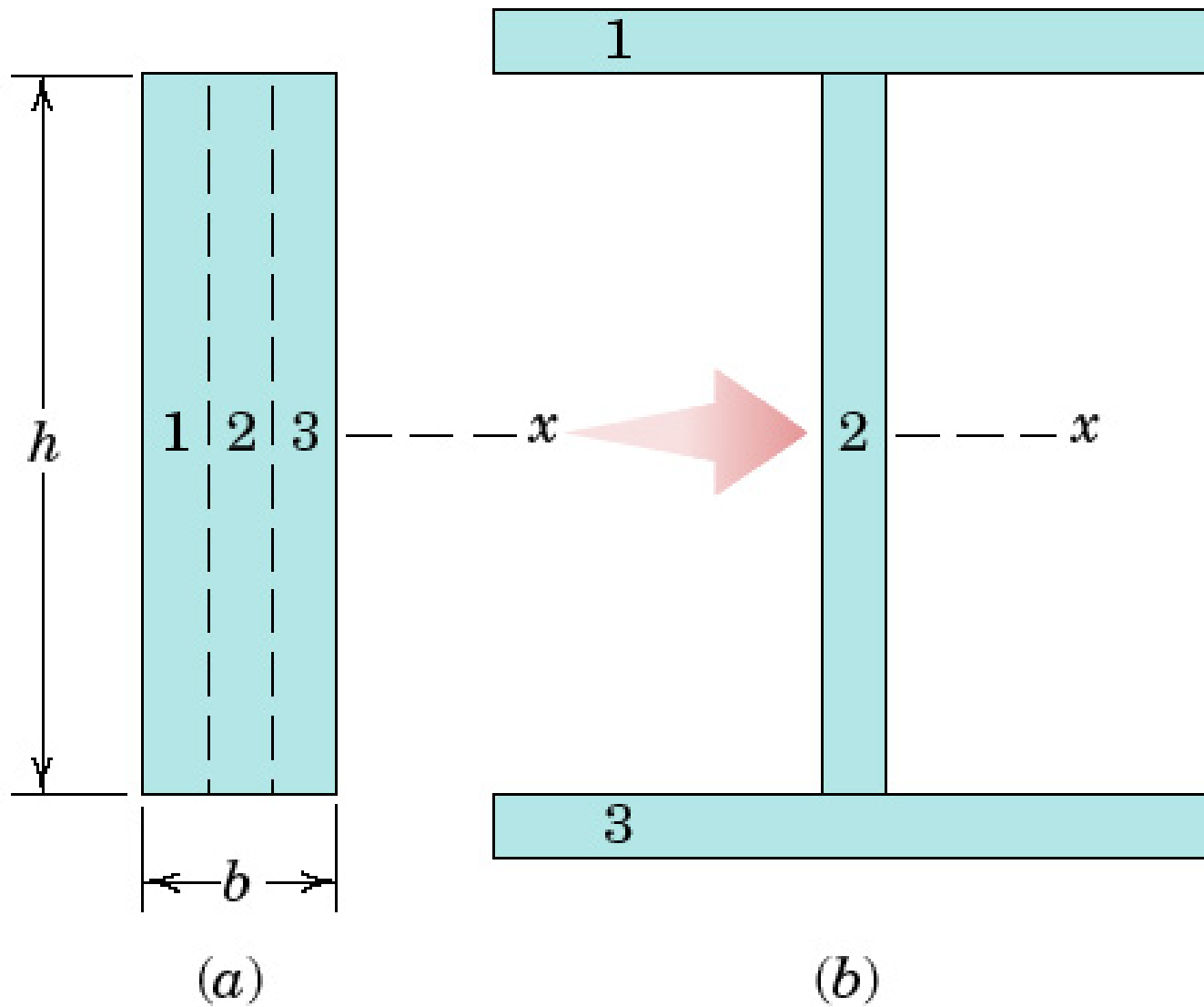
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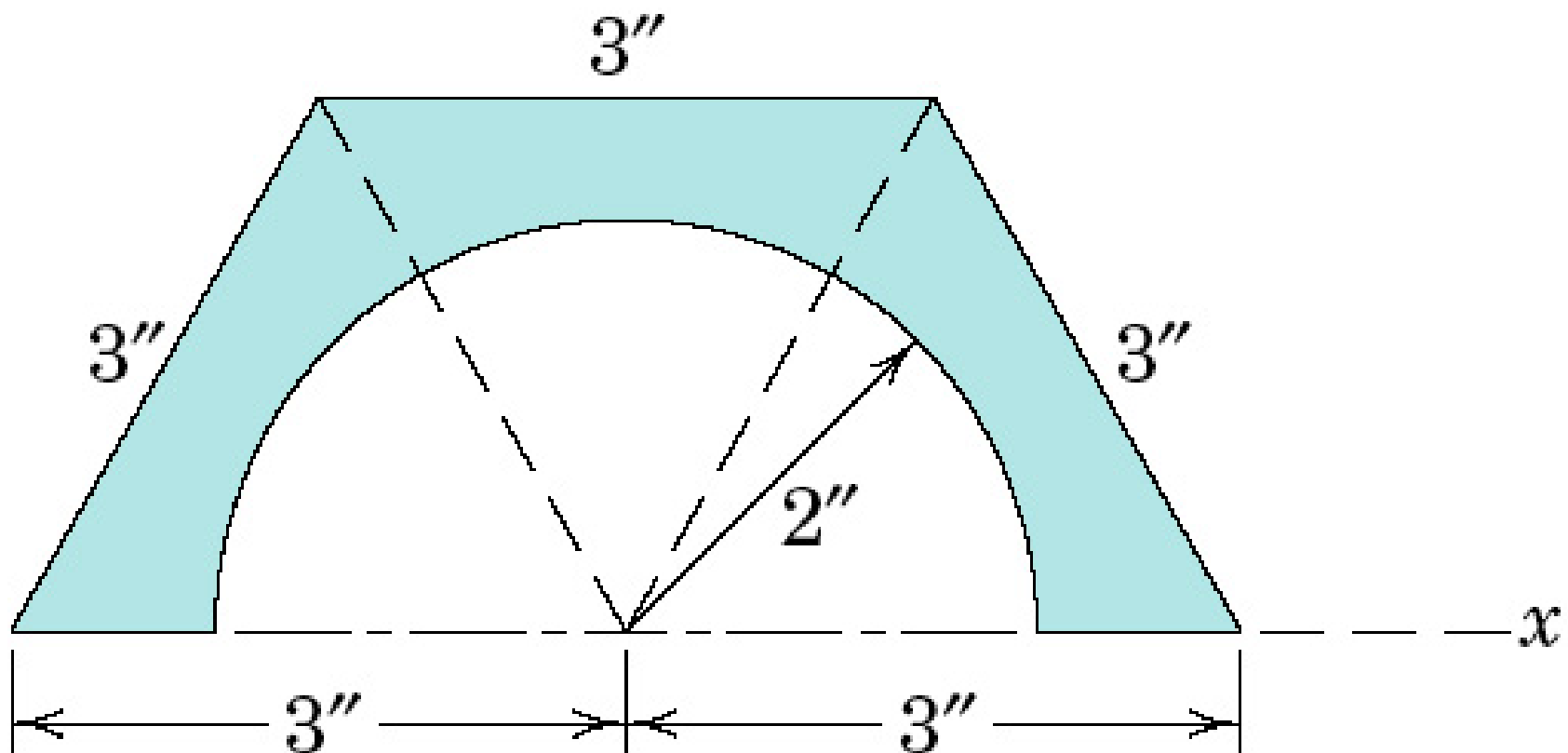
Problem A/44



Problem A/45

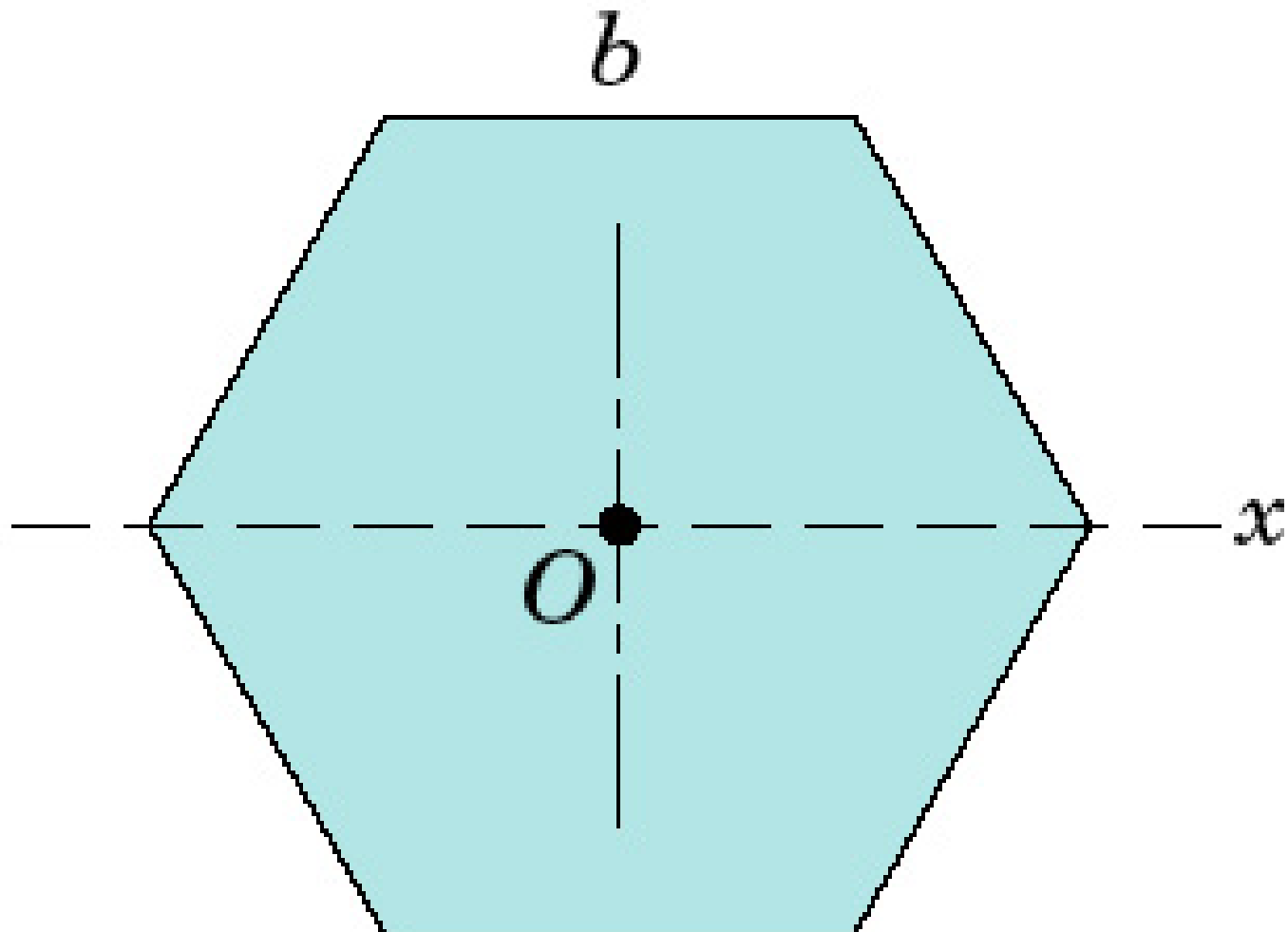


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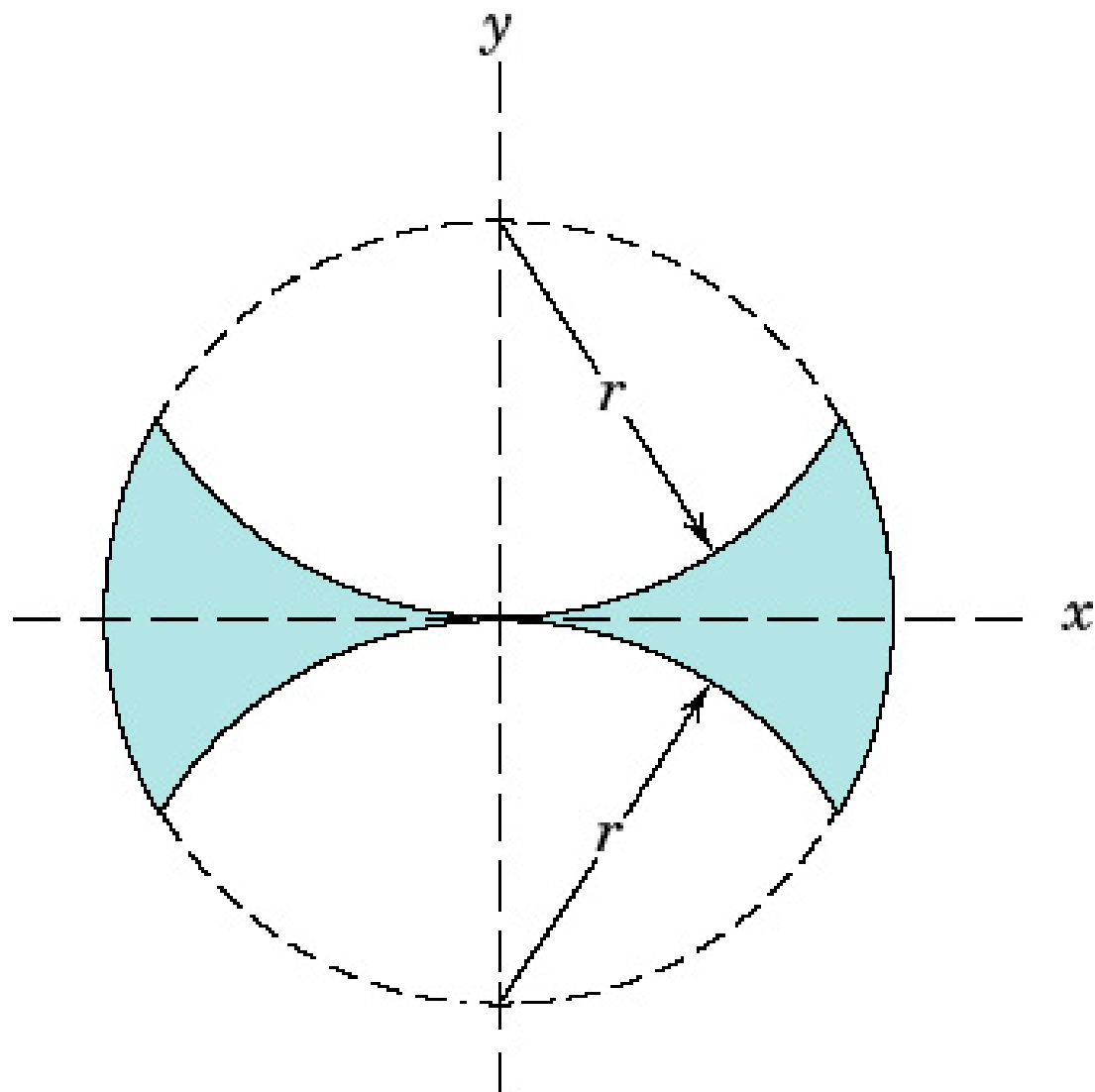


Problem A/47

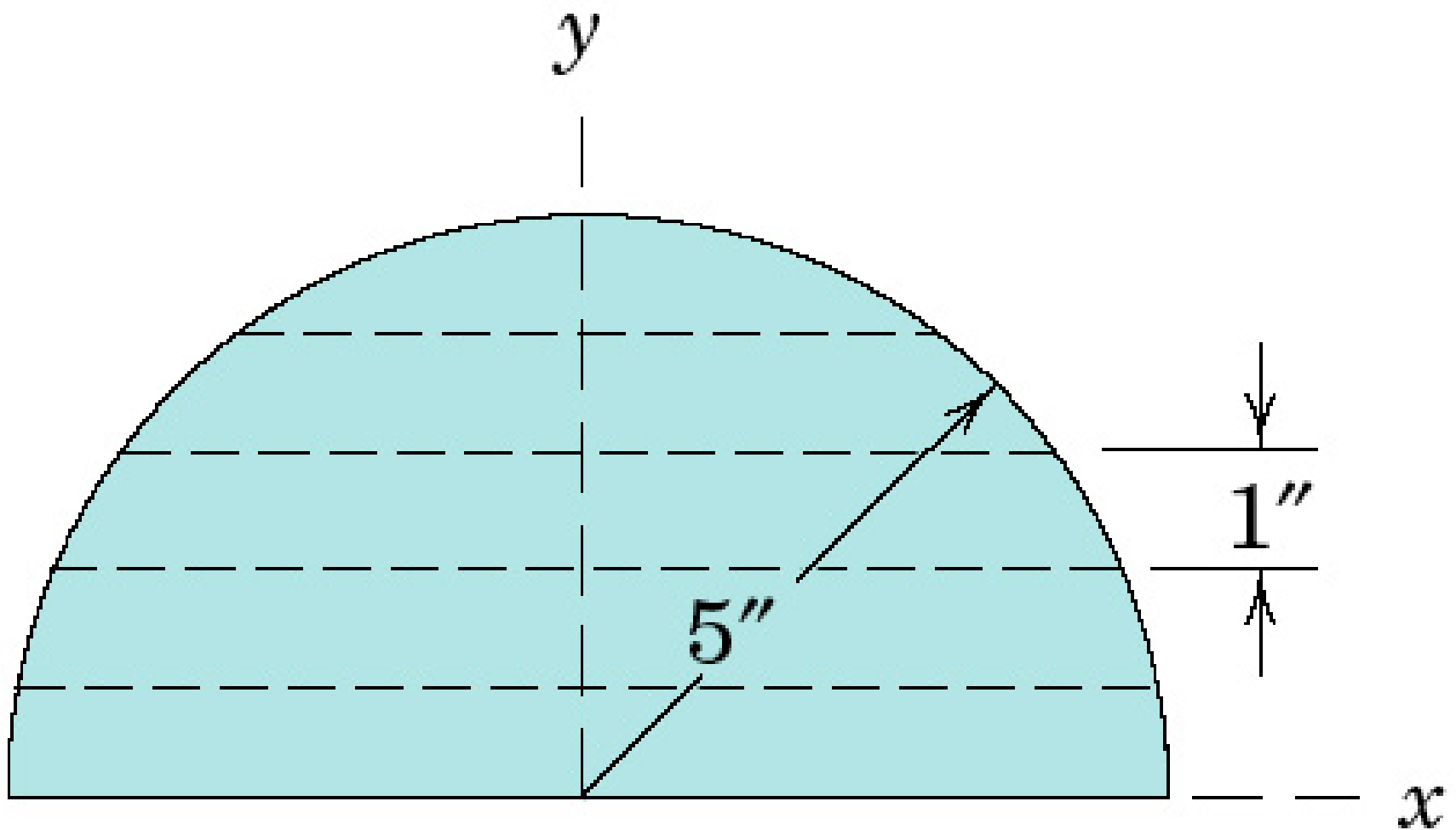
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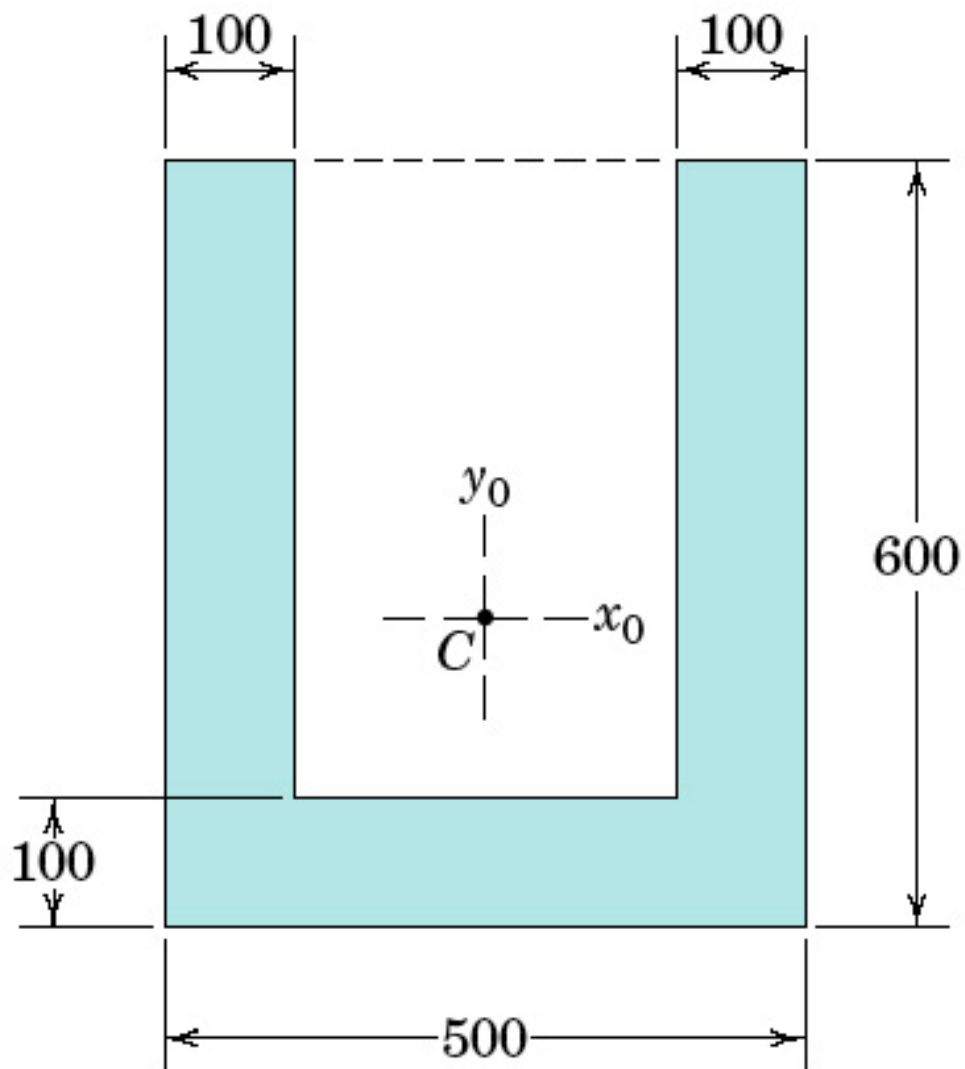
Problem A/48



Problem A/49

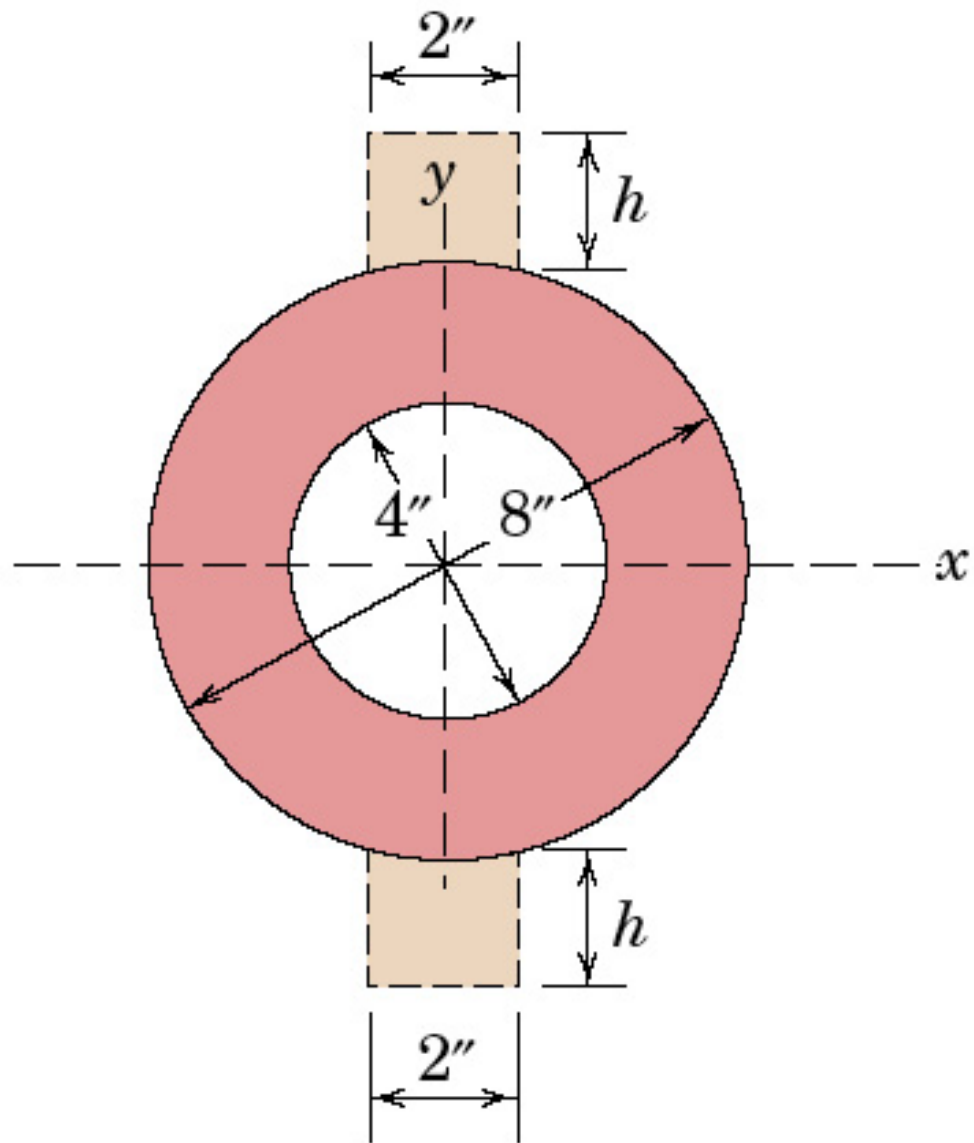


Problem A/50

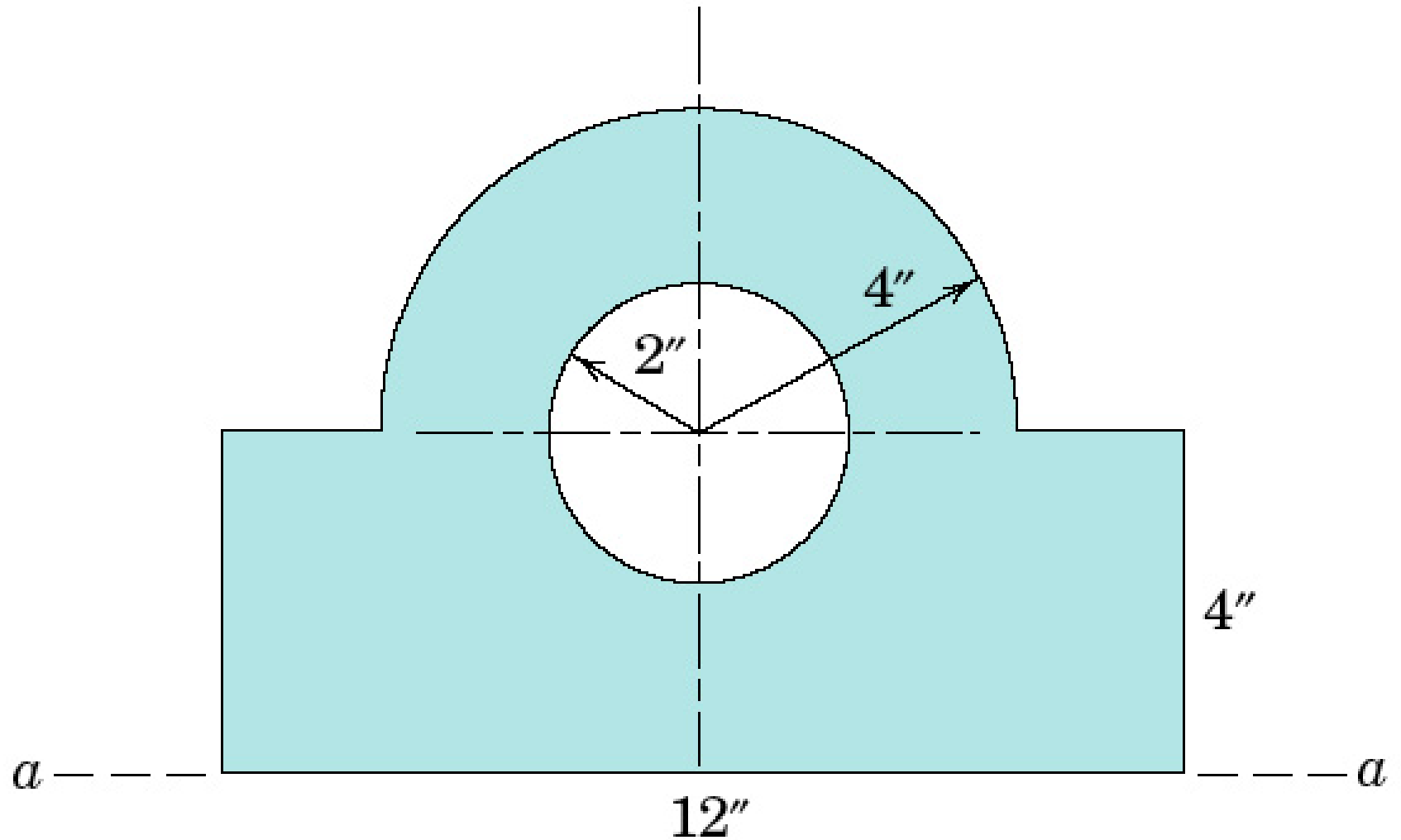


Dimensions in millimeters

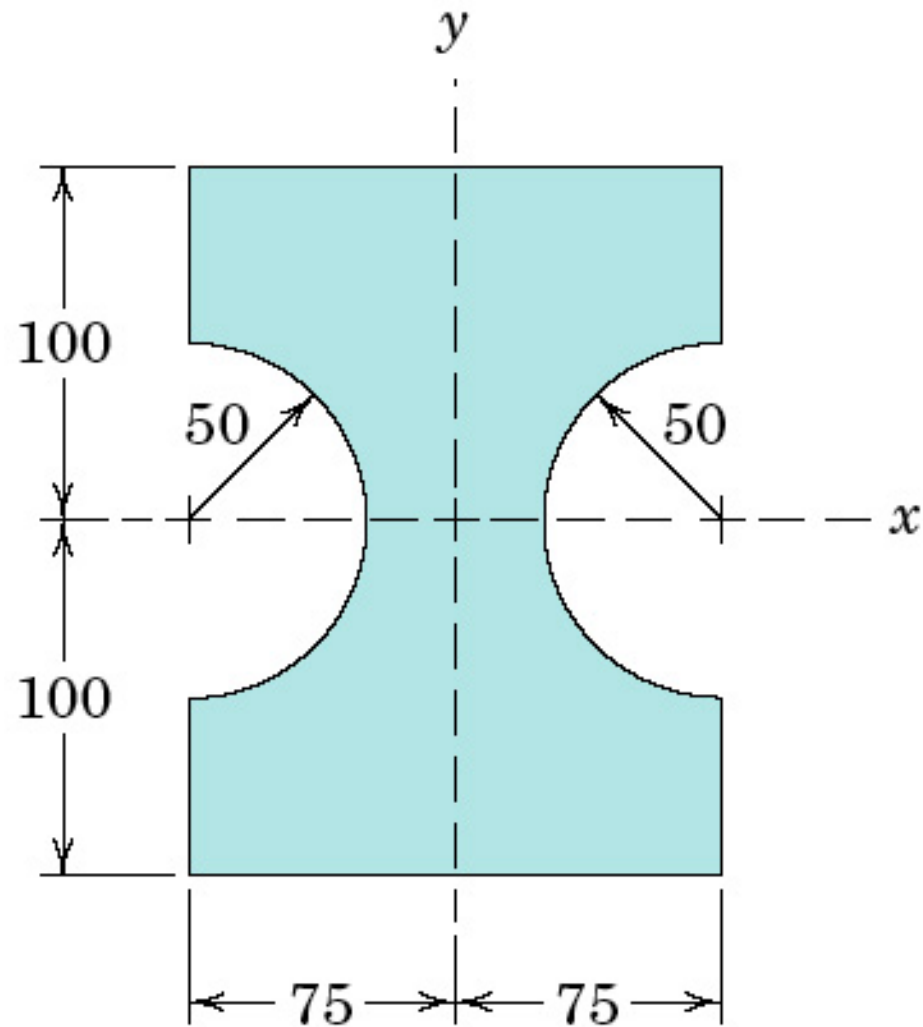
Problem A/51



Problem A/52

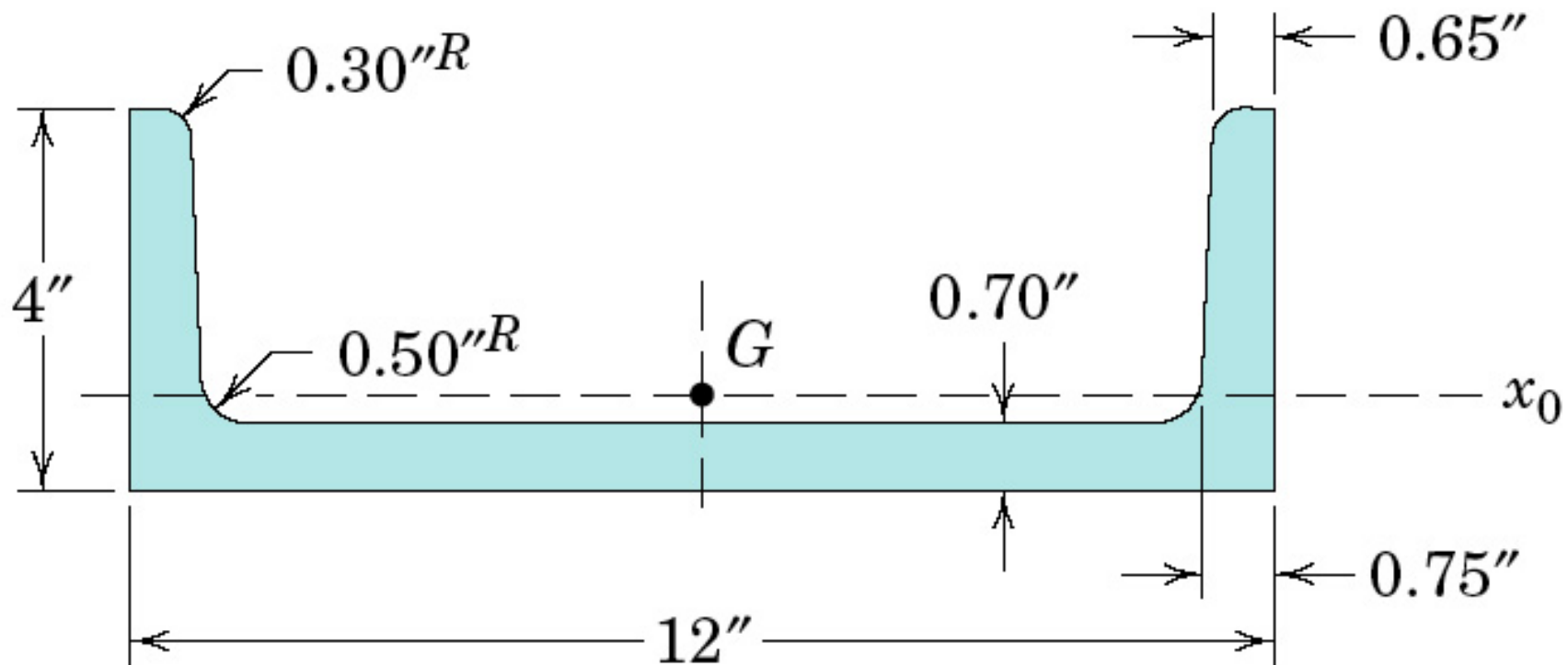


Problem A/53



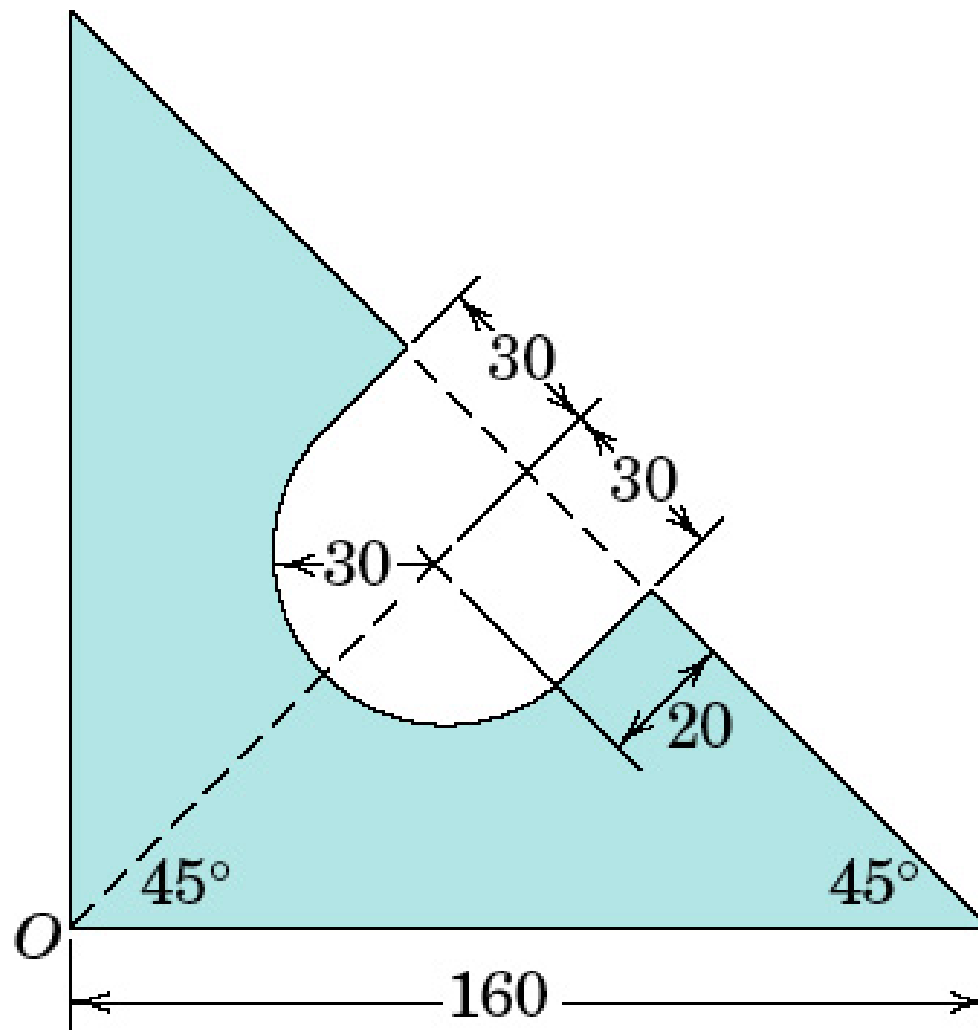
Dimensions in millimeters

Problem A/54



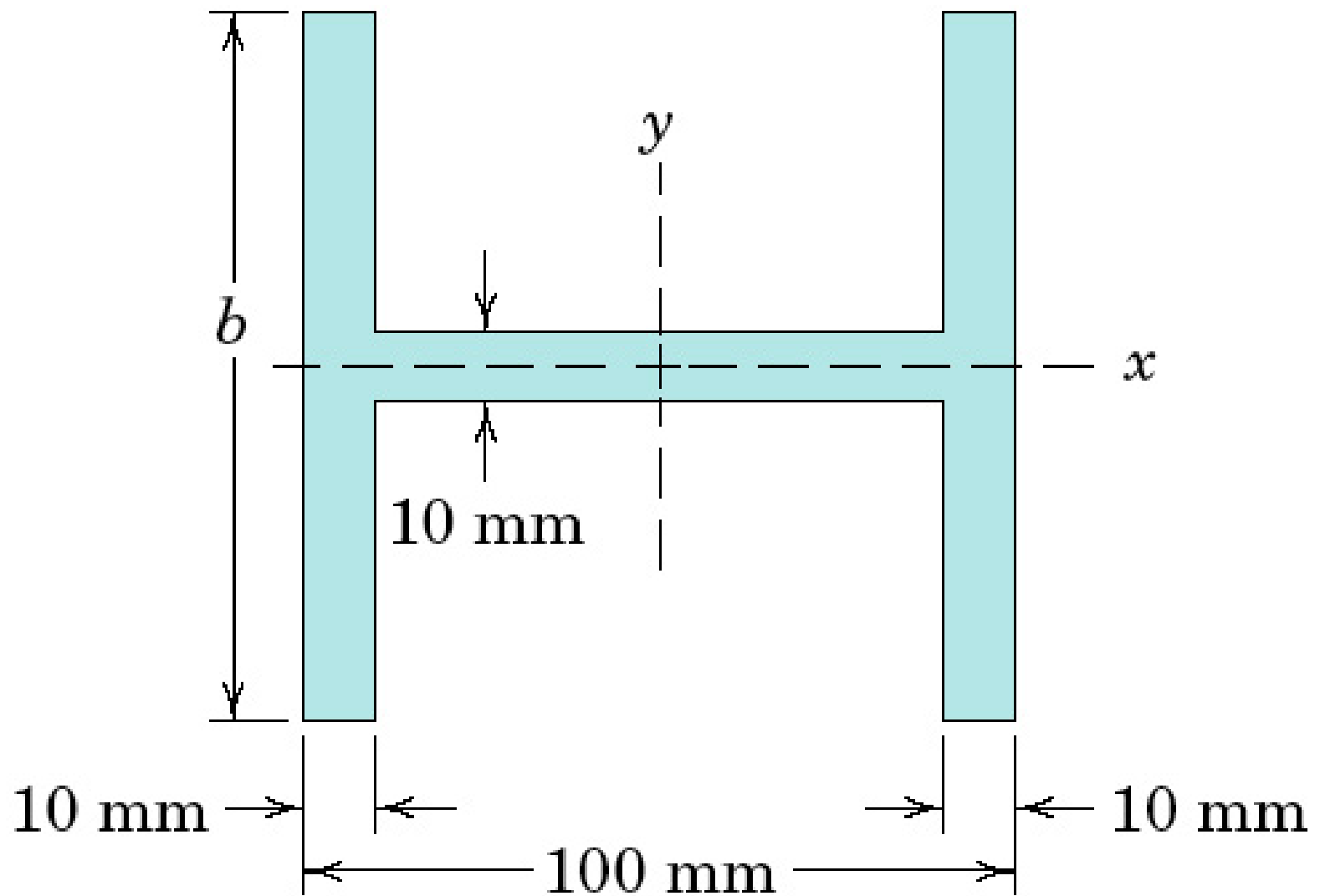
Problem A/55

P_A_55

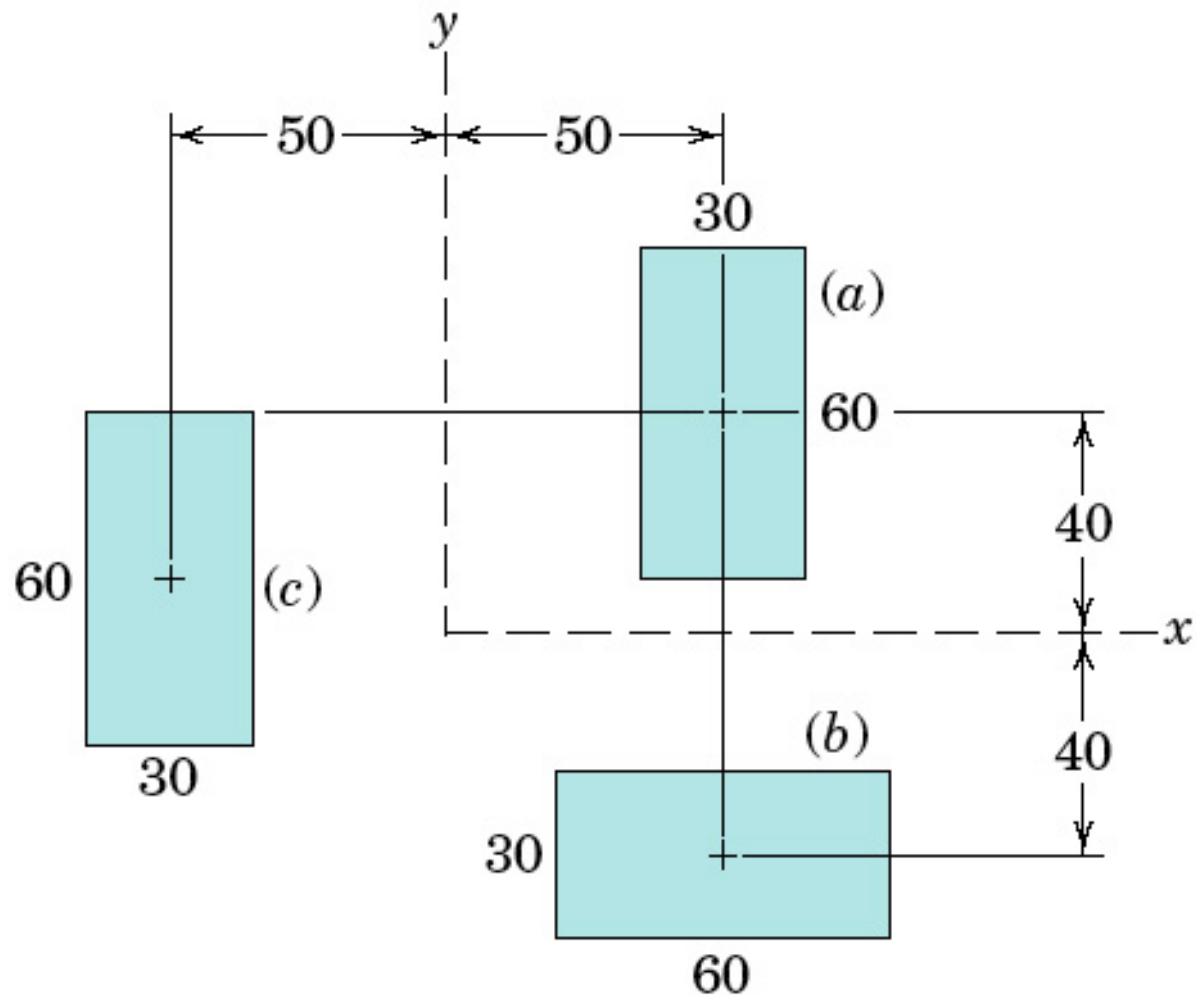


Dimensions in millimeters

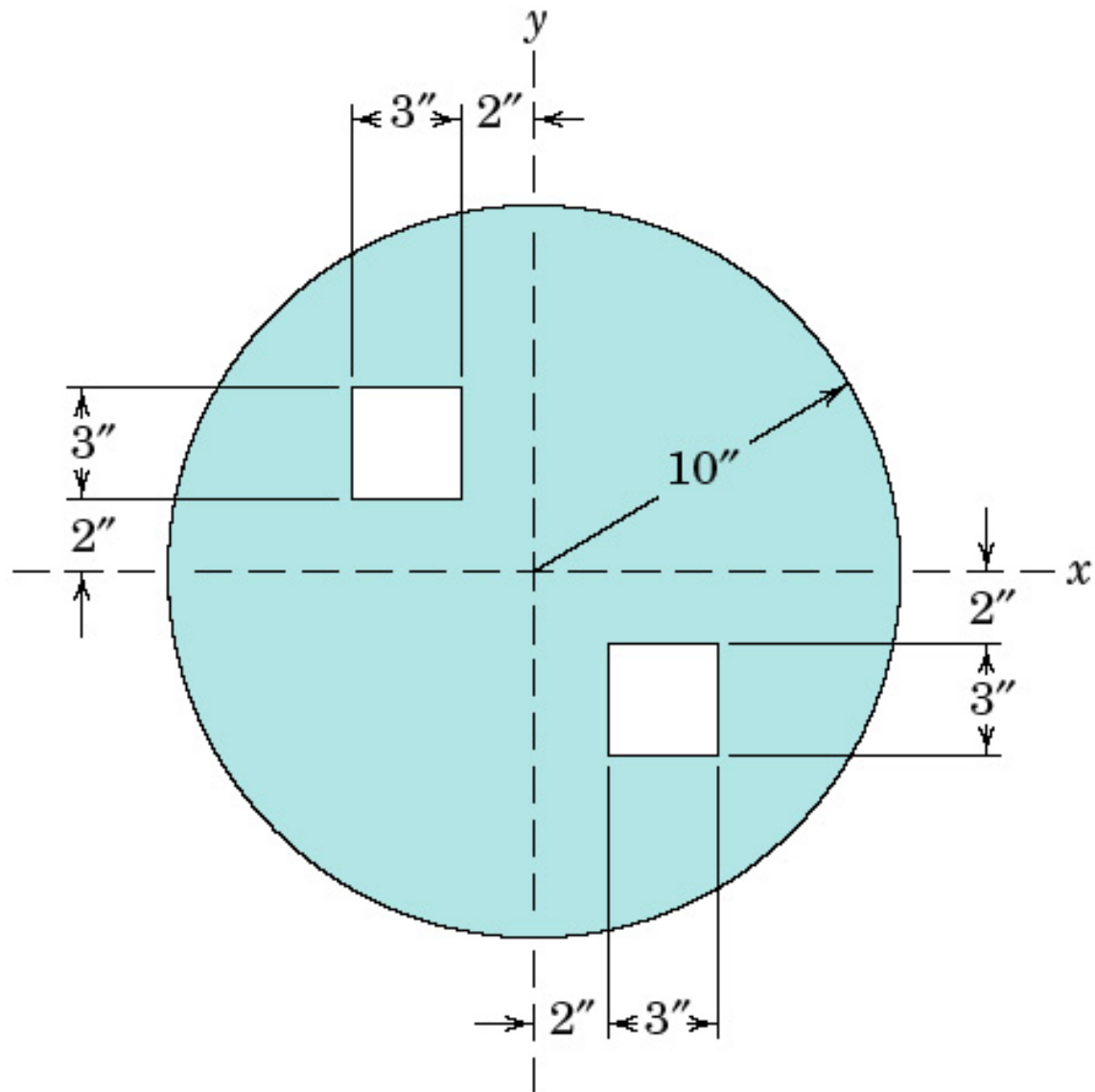
Problem A/56



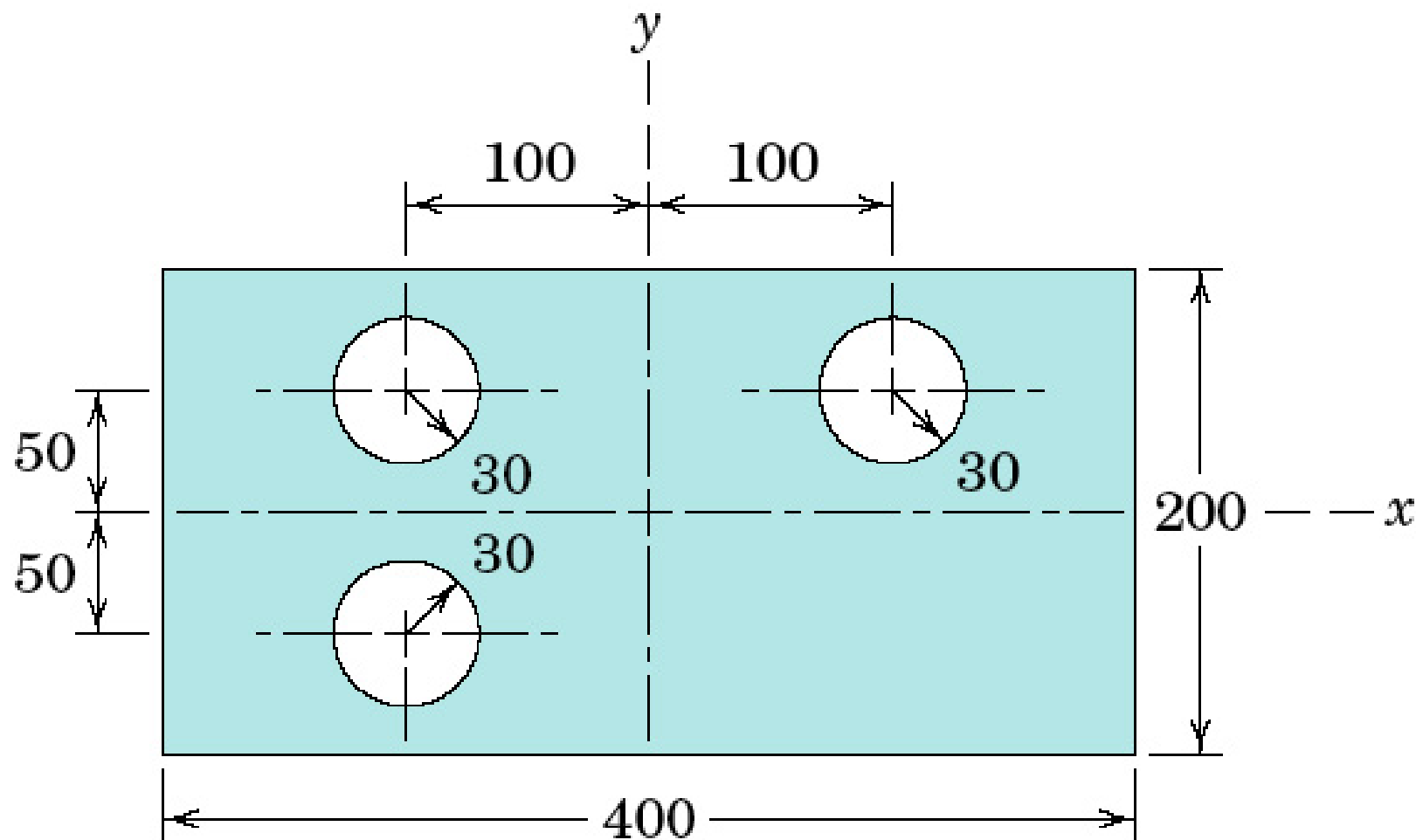
Problem A/58



Problem A/59

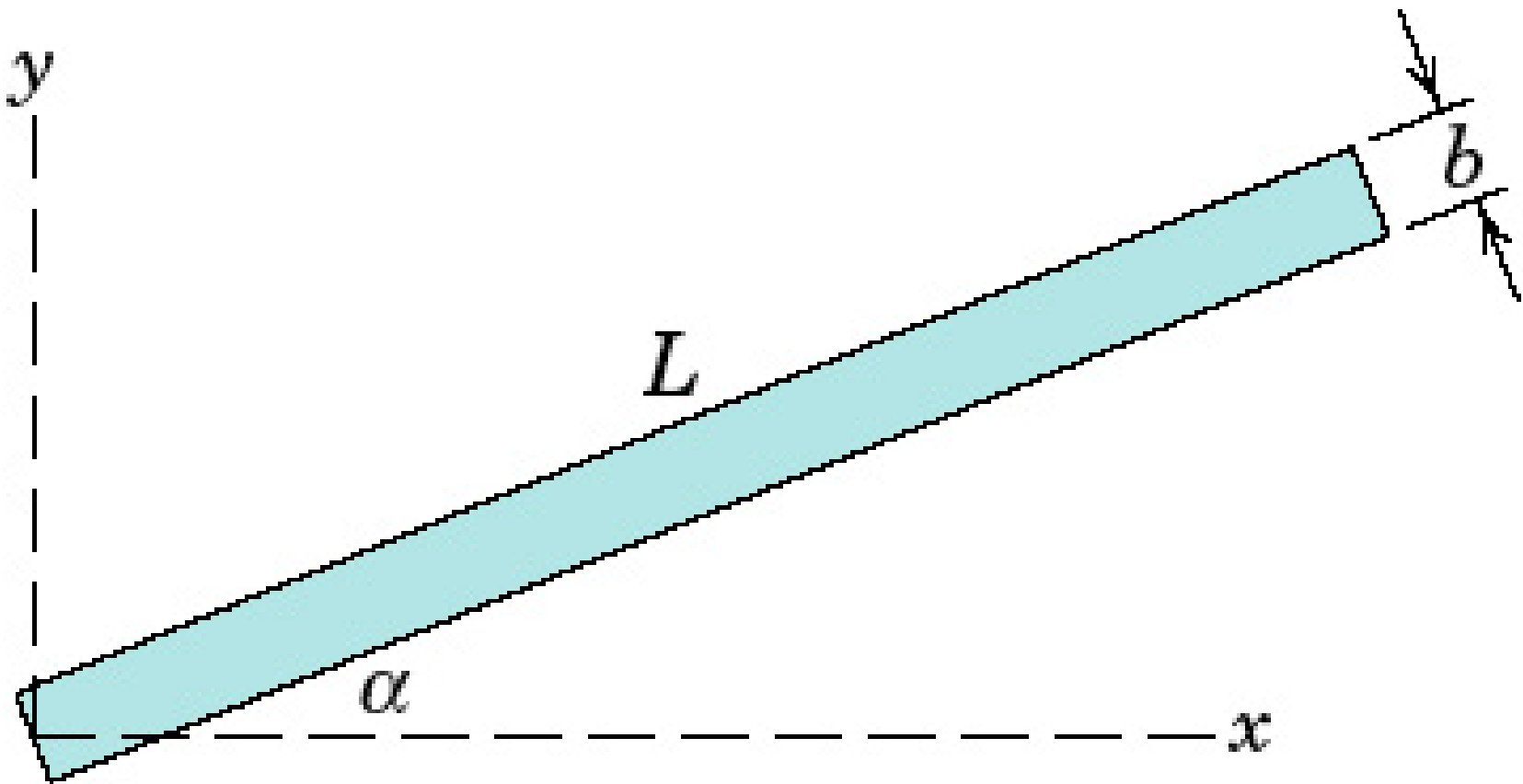


Problem A/60



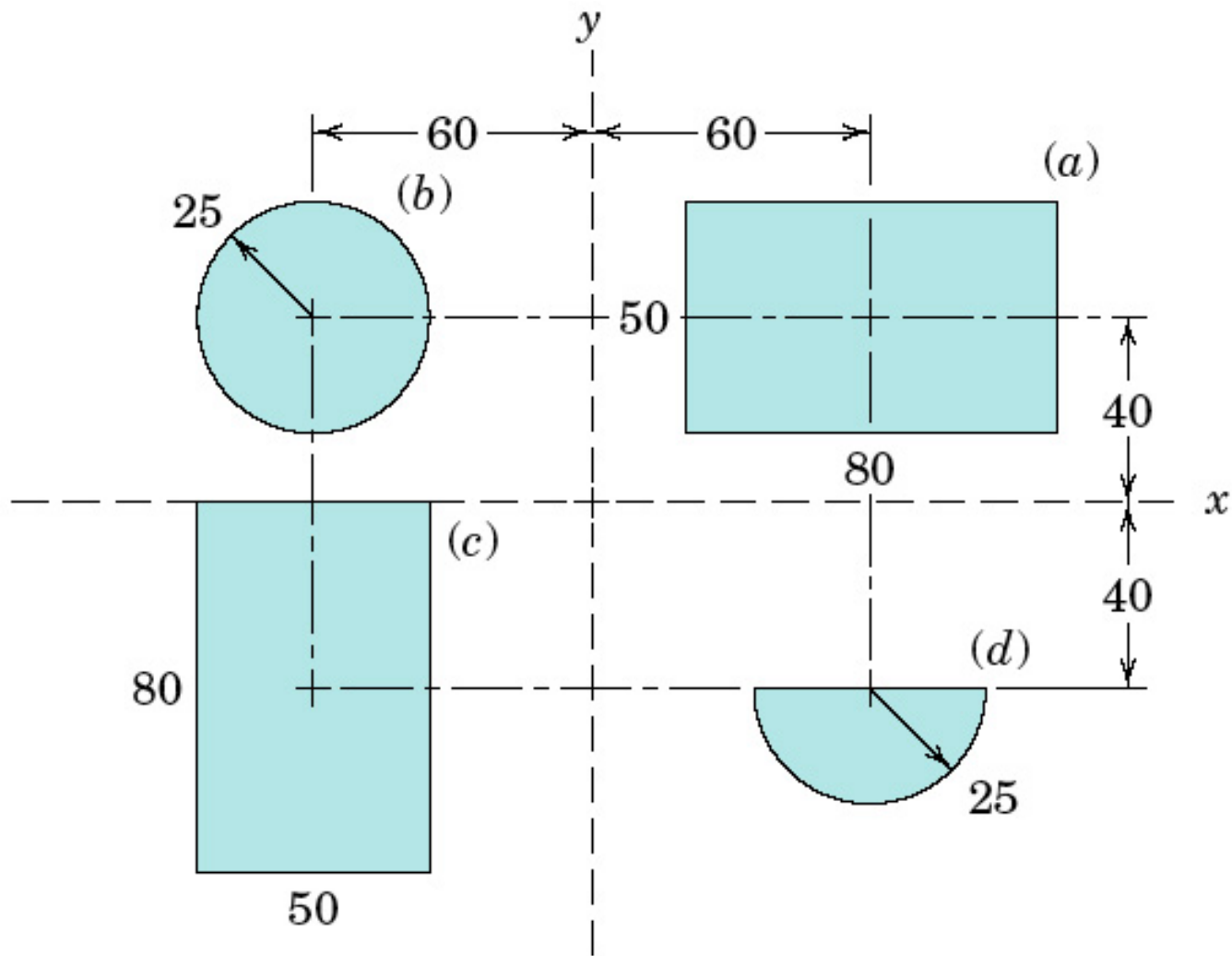
Dimensions in millimeters

Problem A/61



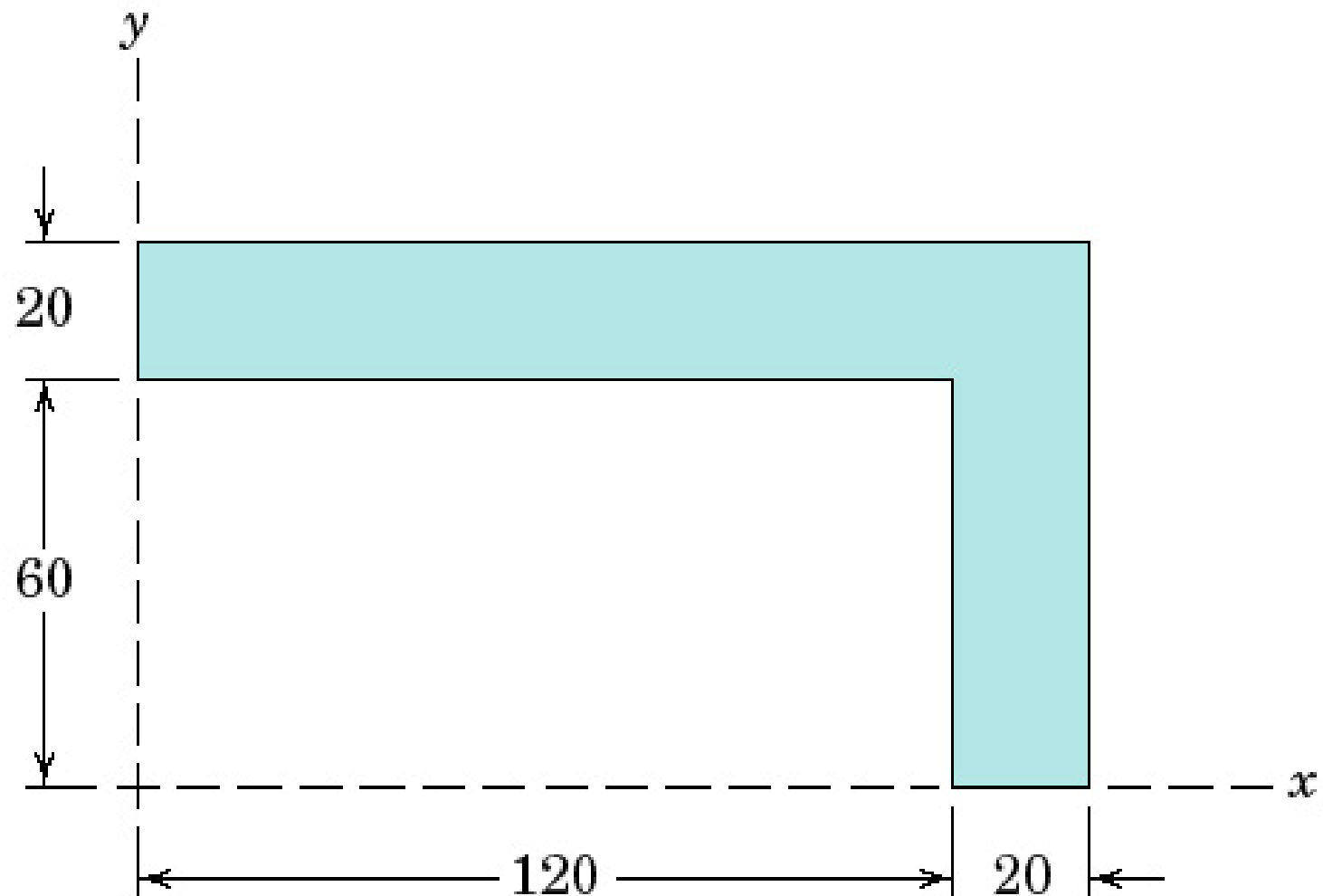
Problem A/62

P_A_62



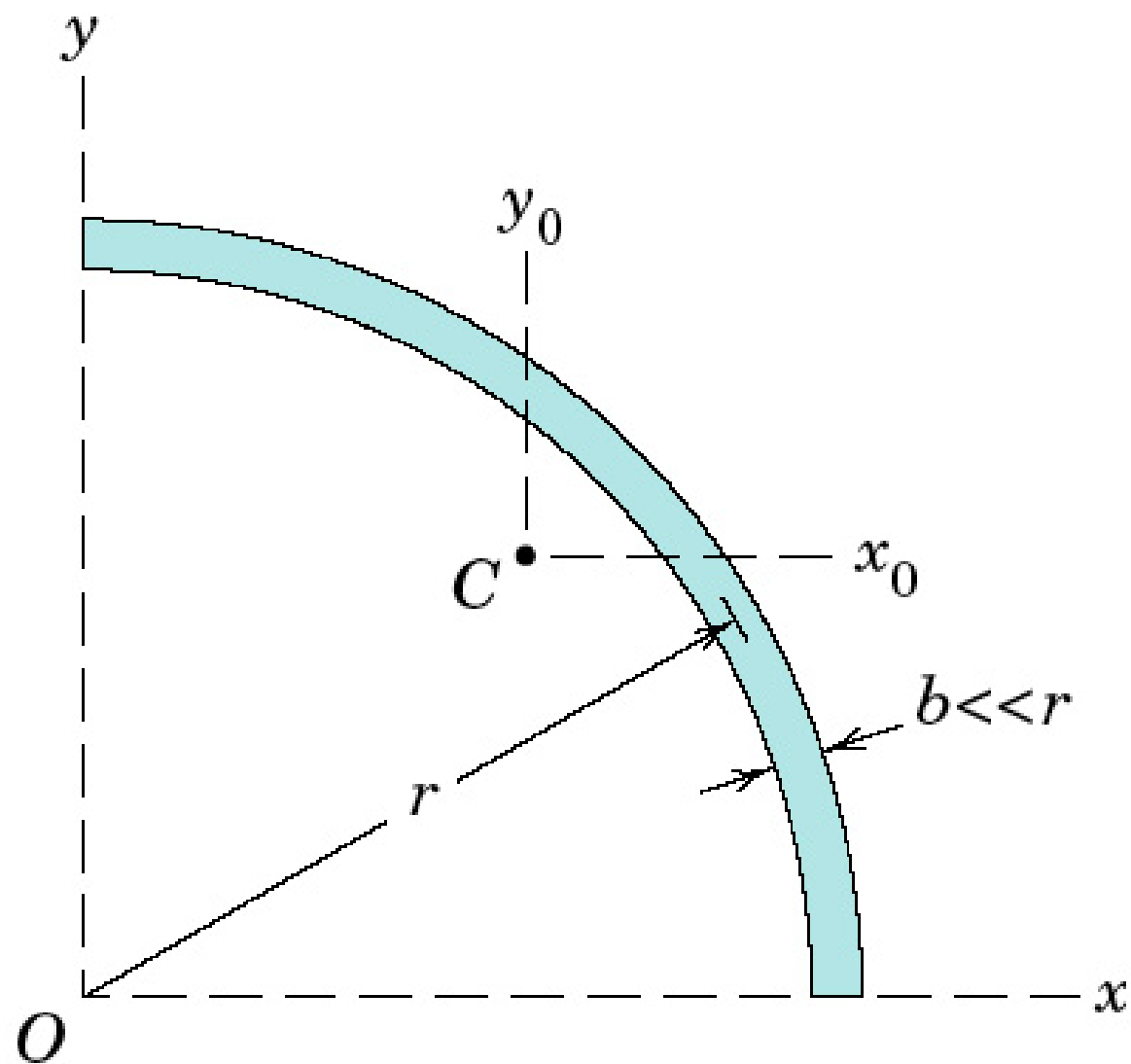
Dimensions in millimeters

Problem A/63

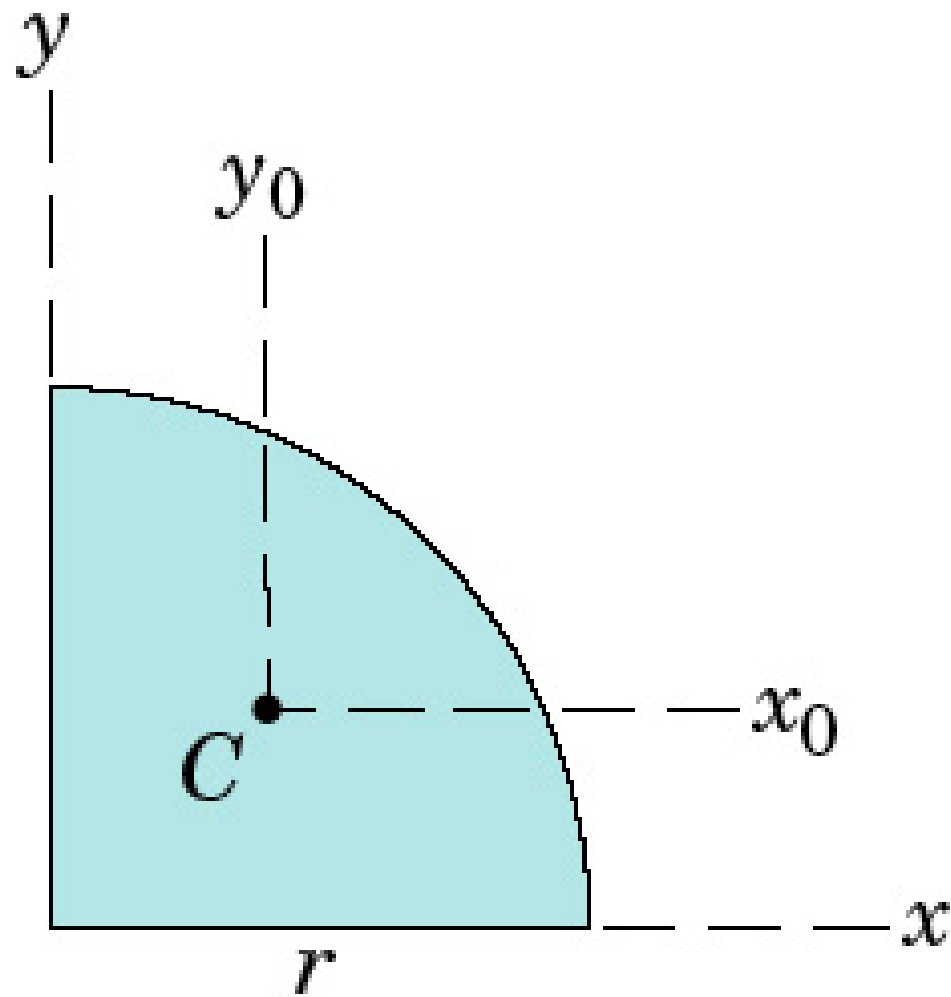


Dimensions in millimeters

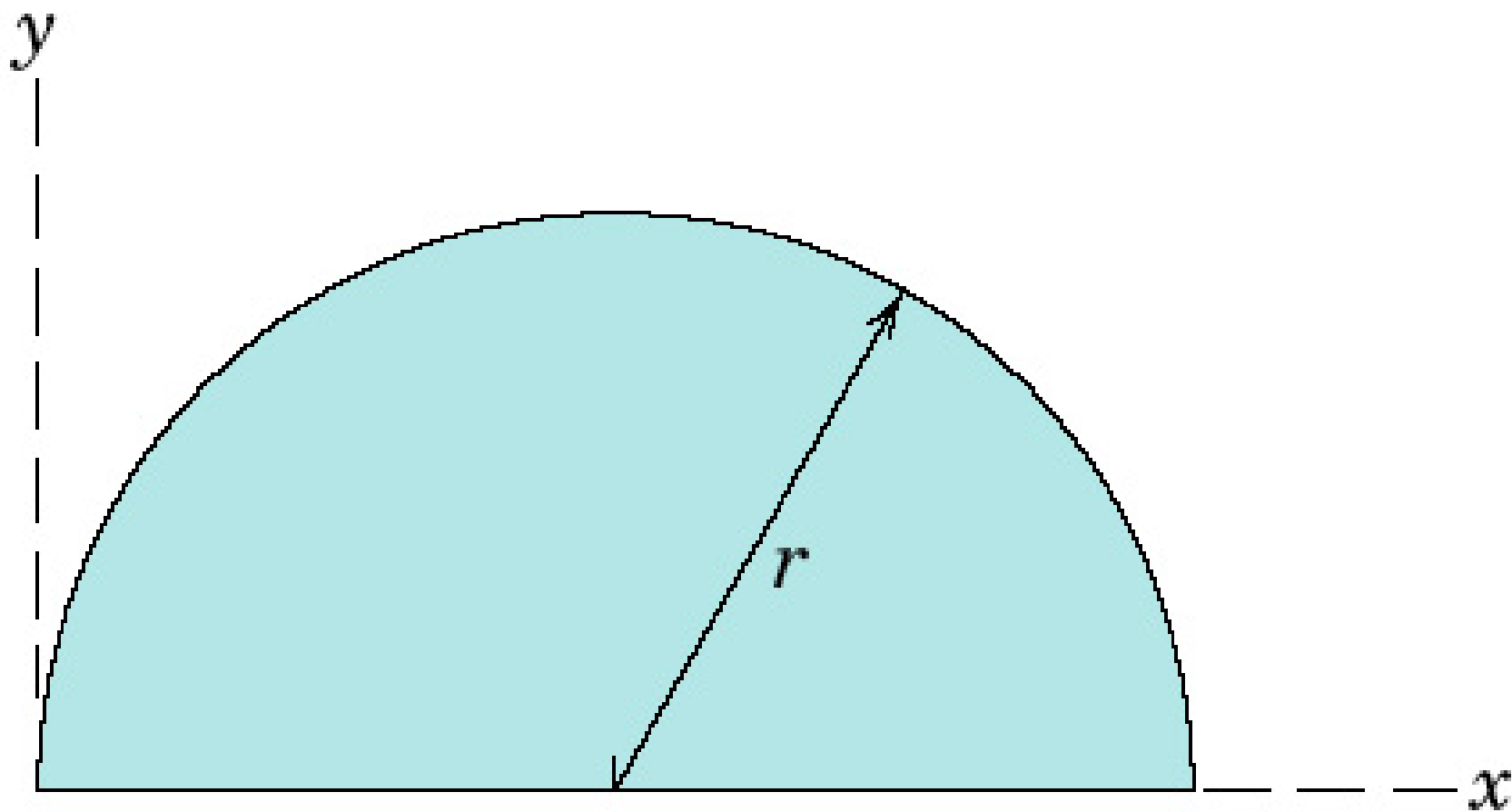
Problem A/64



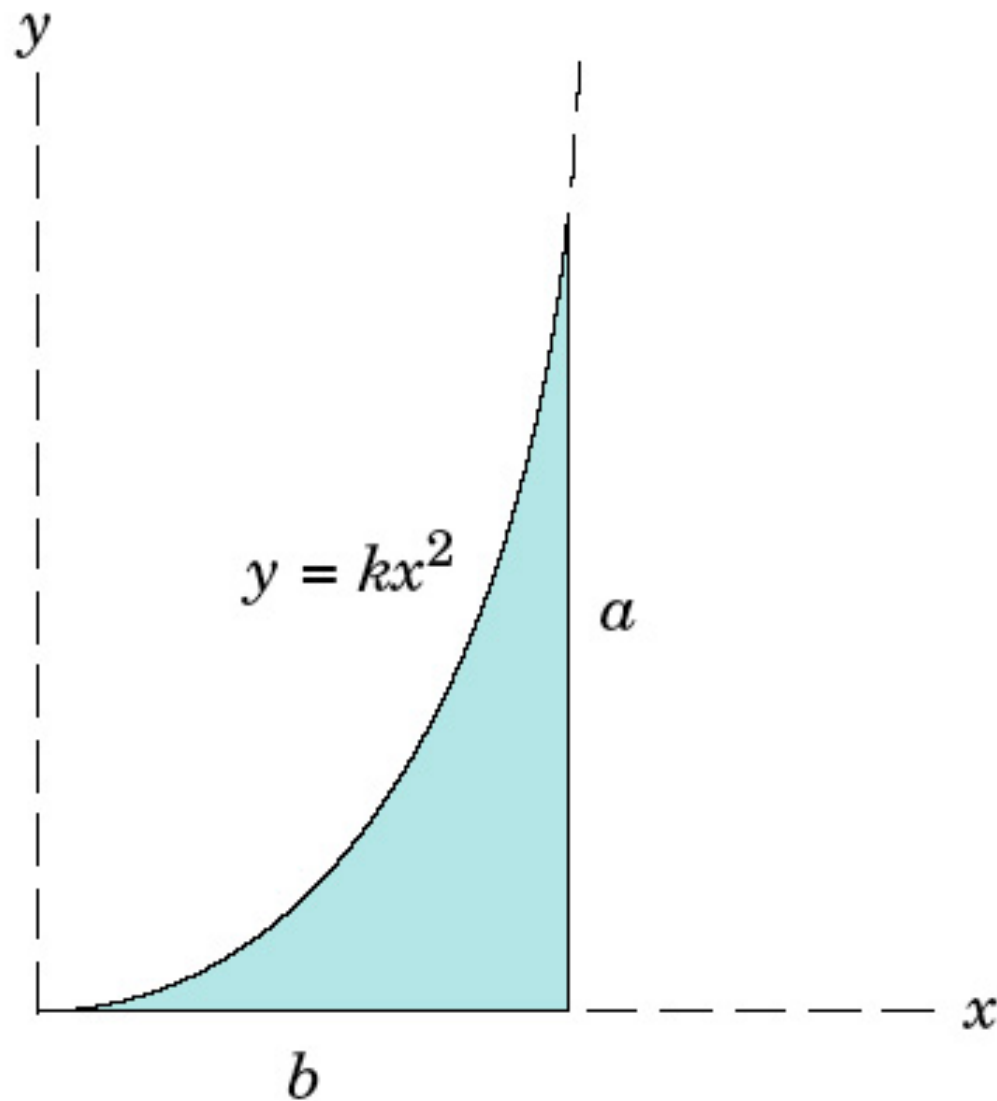
Problem A/65



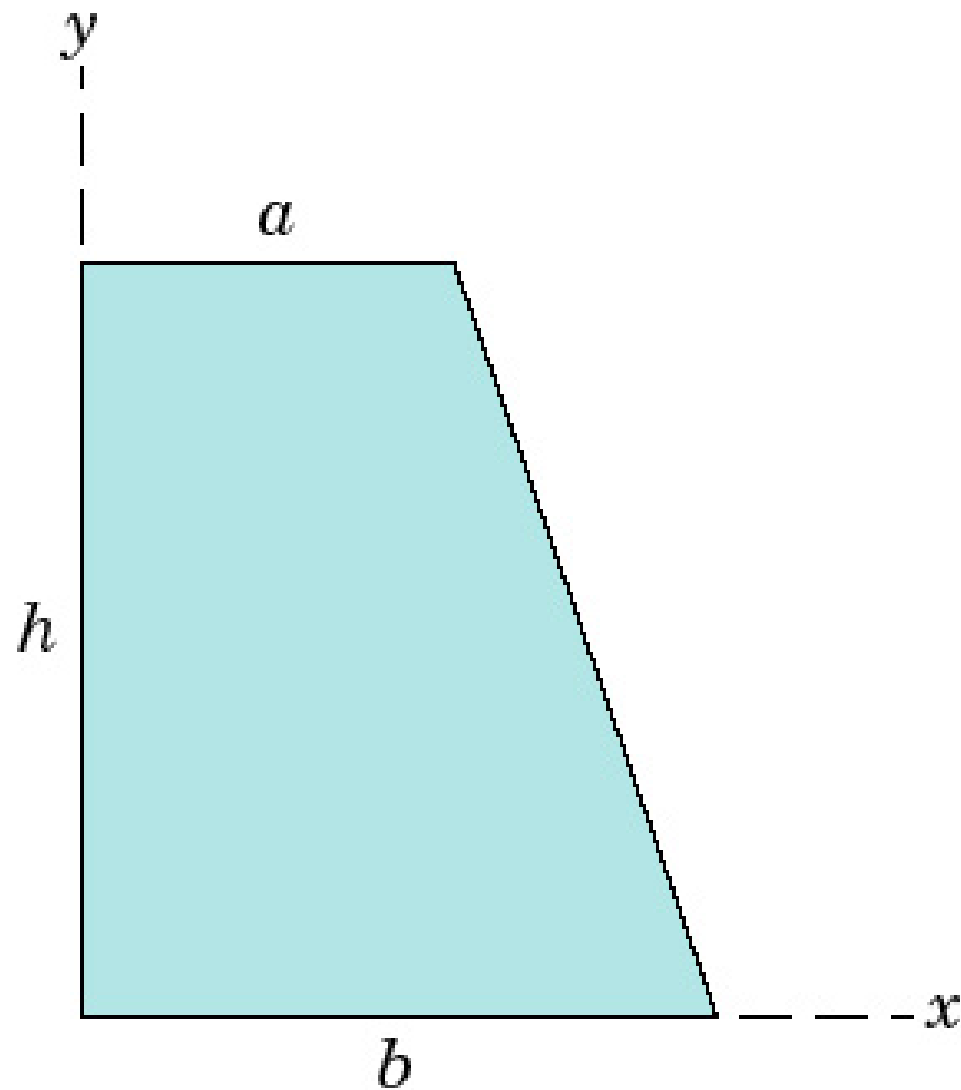
Problem A/66



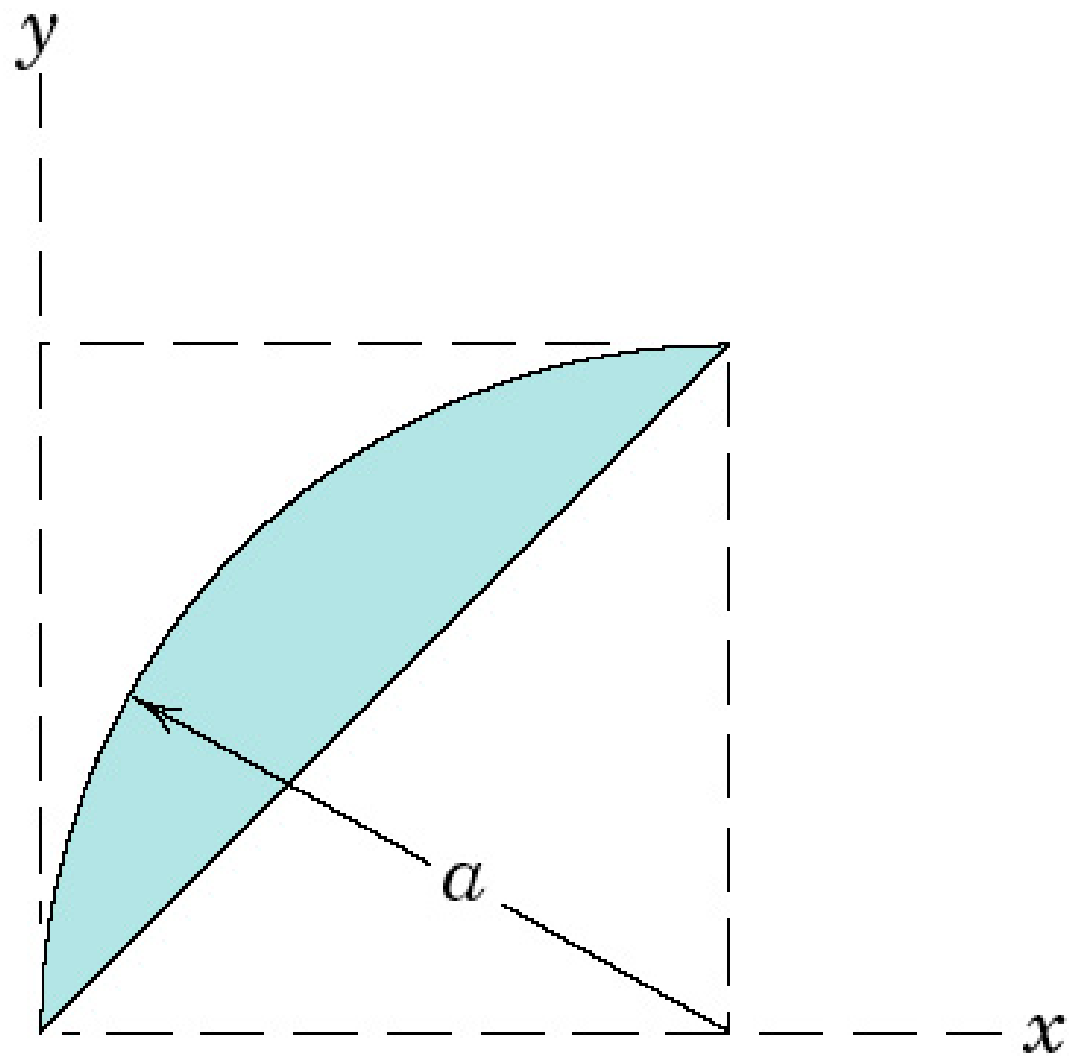
Problem A/67



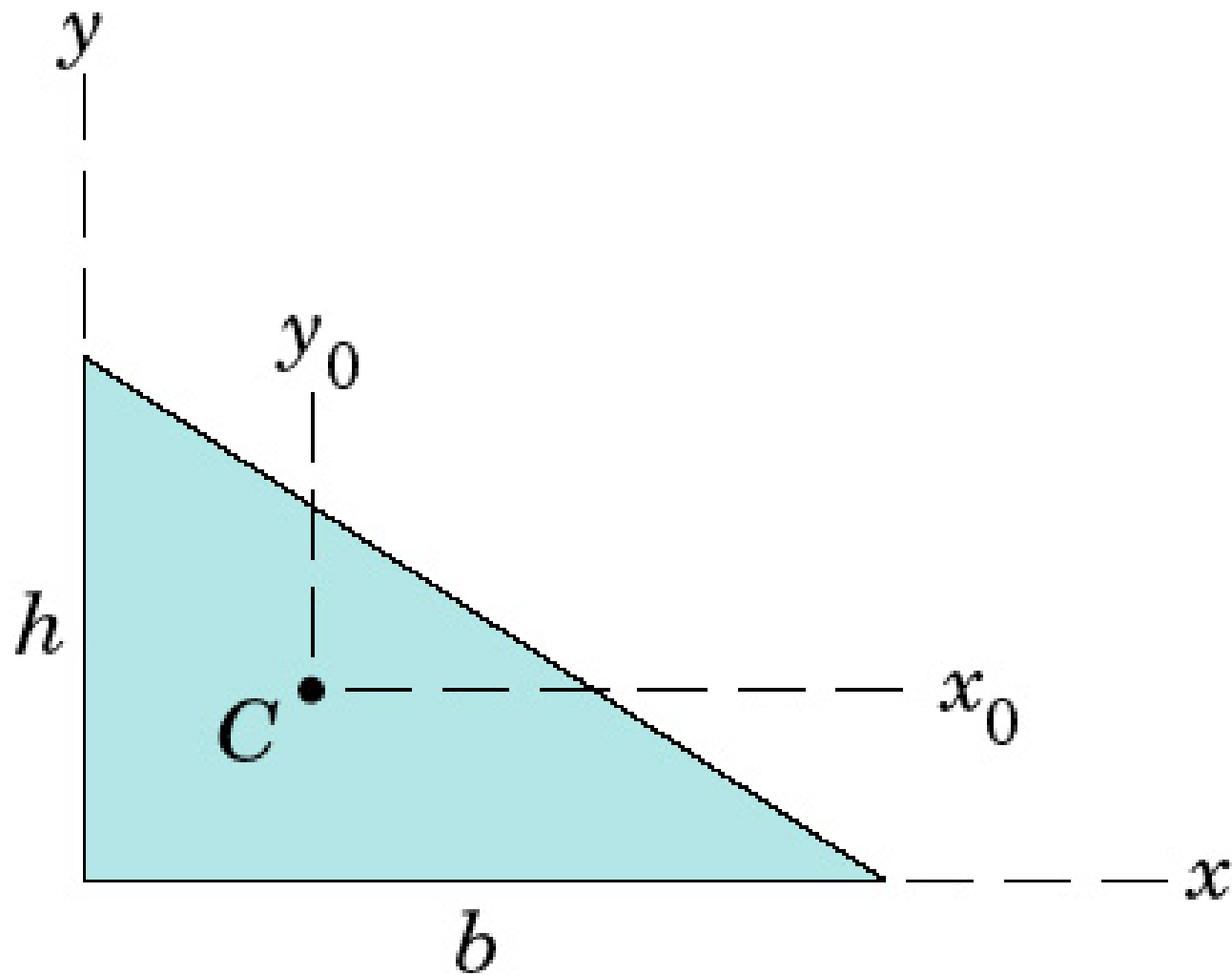
Problem A/68



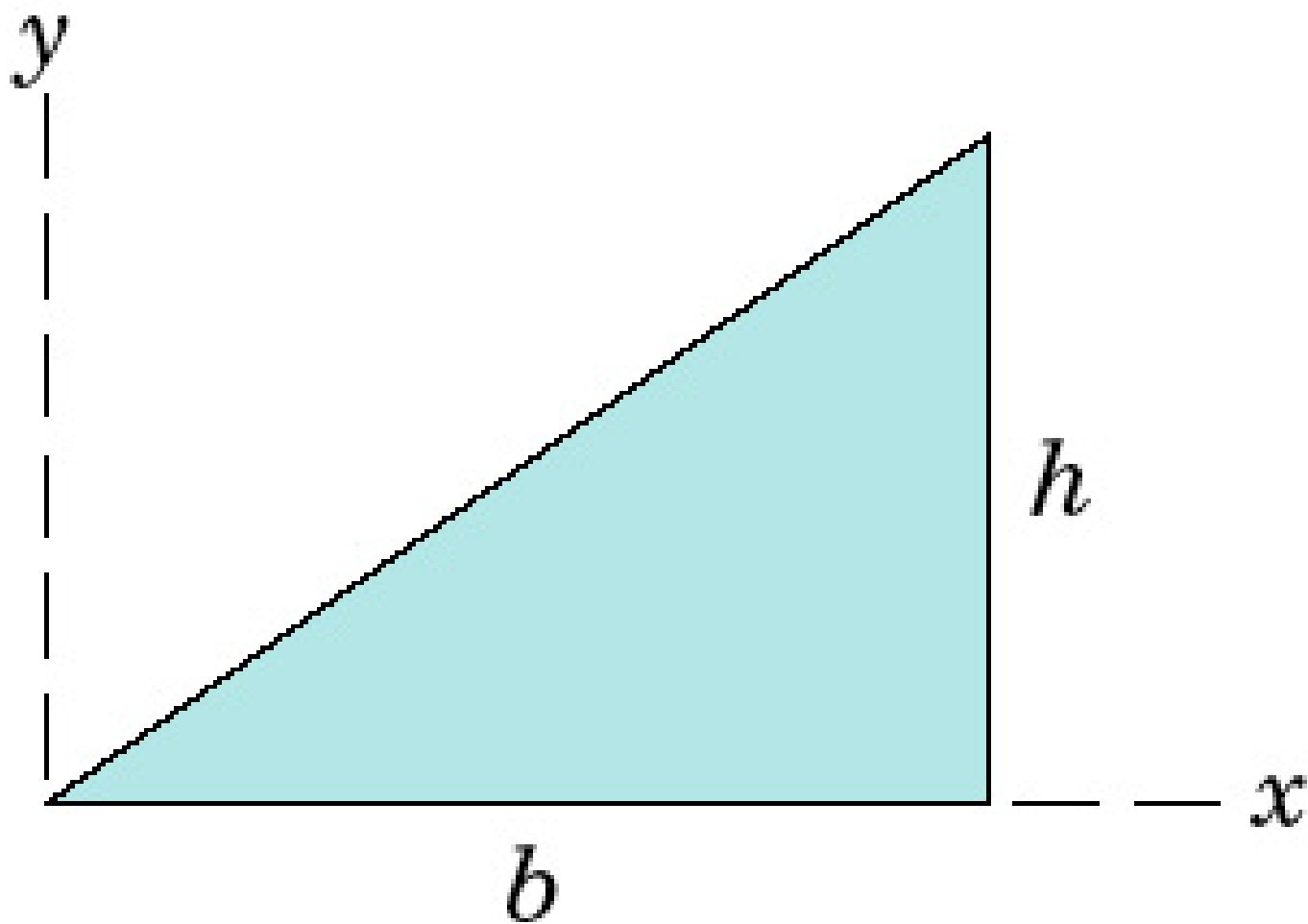
Problem A/69



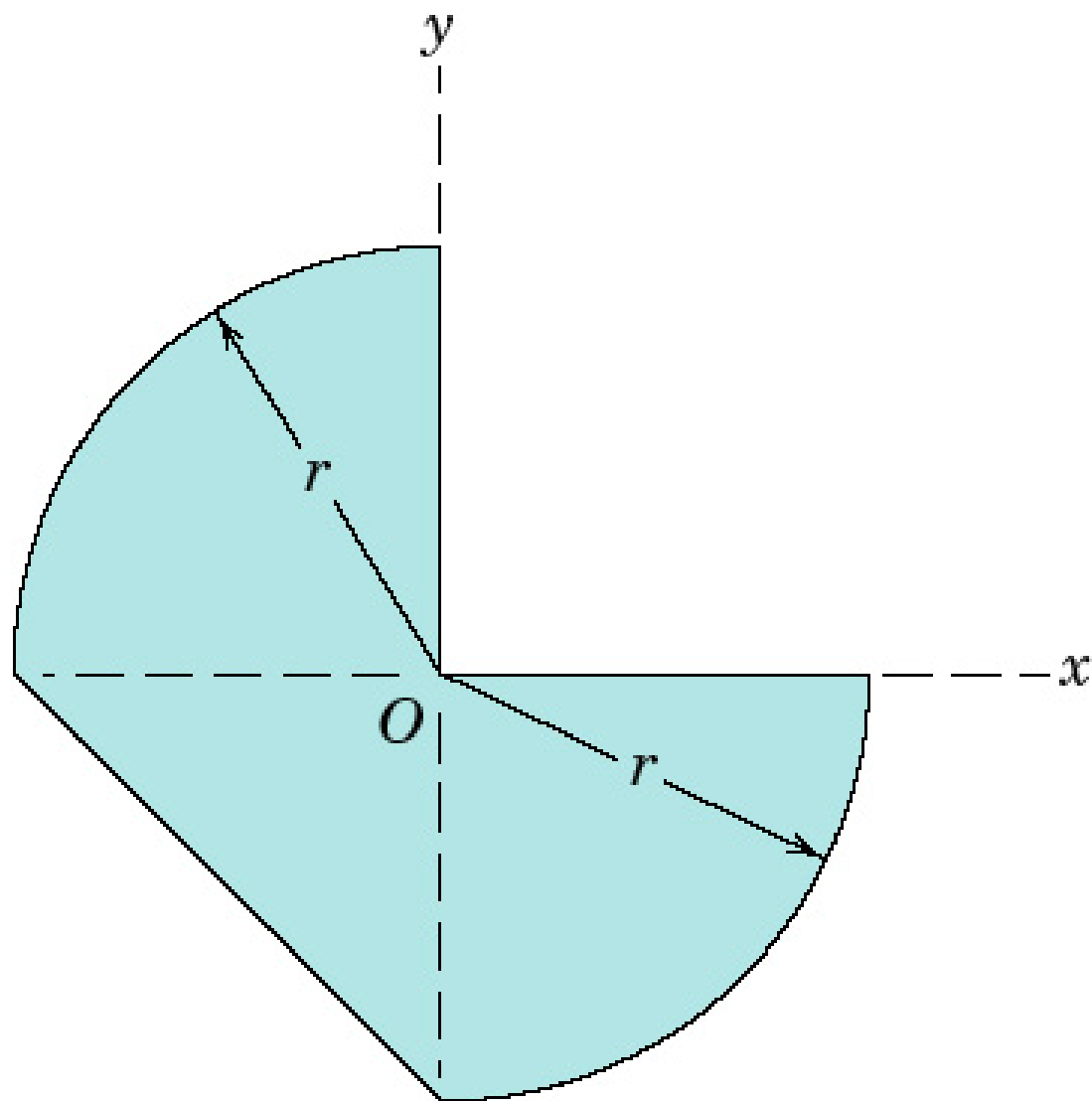
Problem A/70



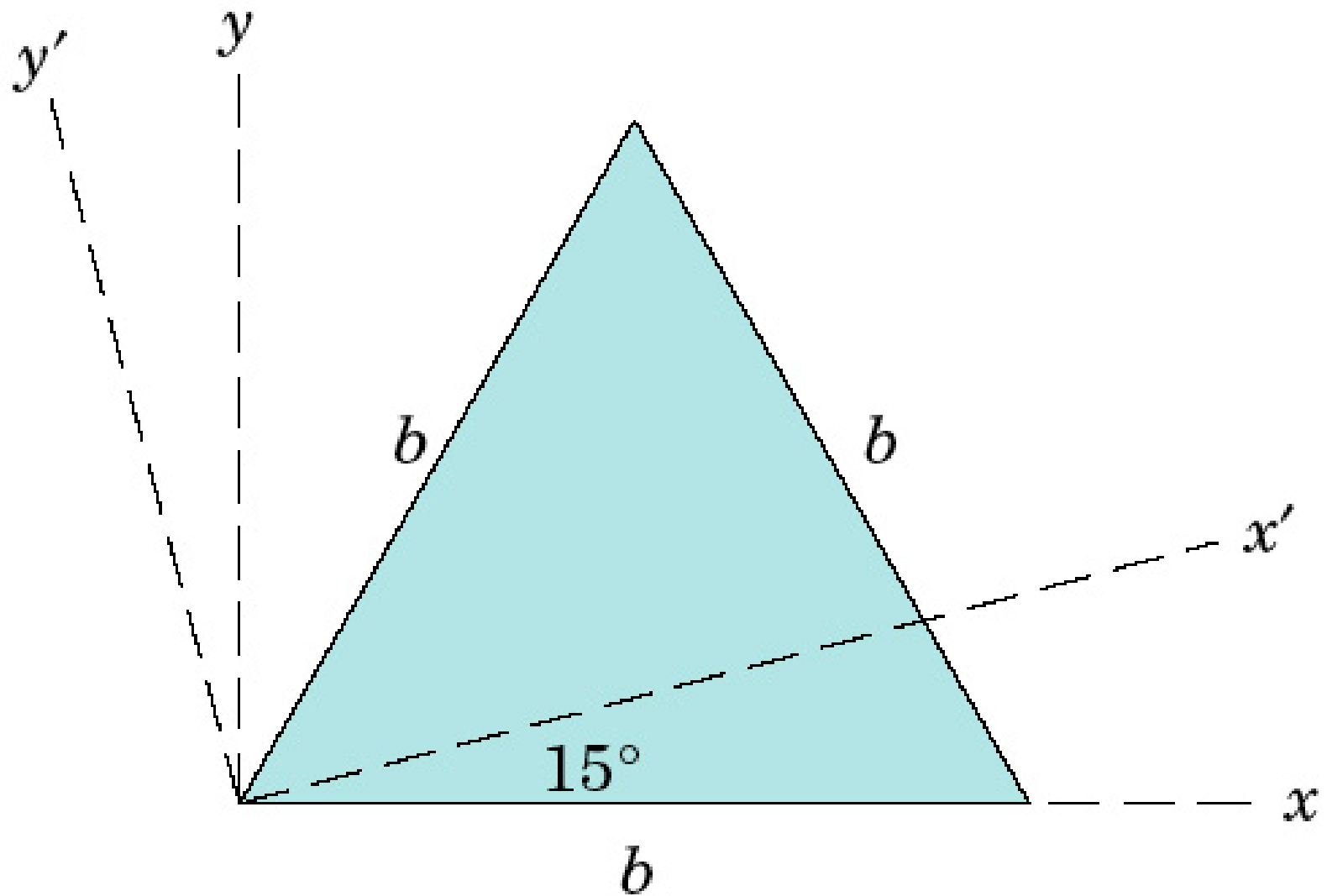
Problem A/71



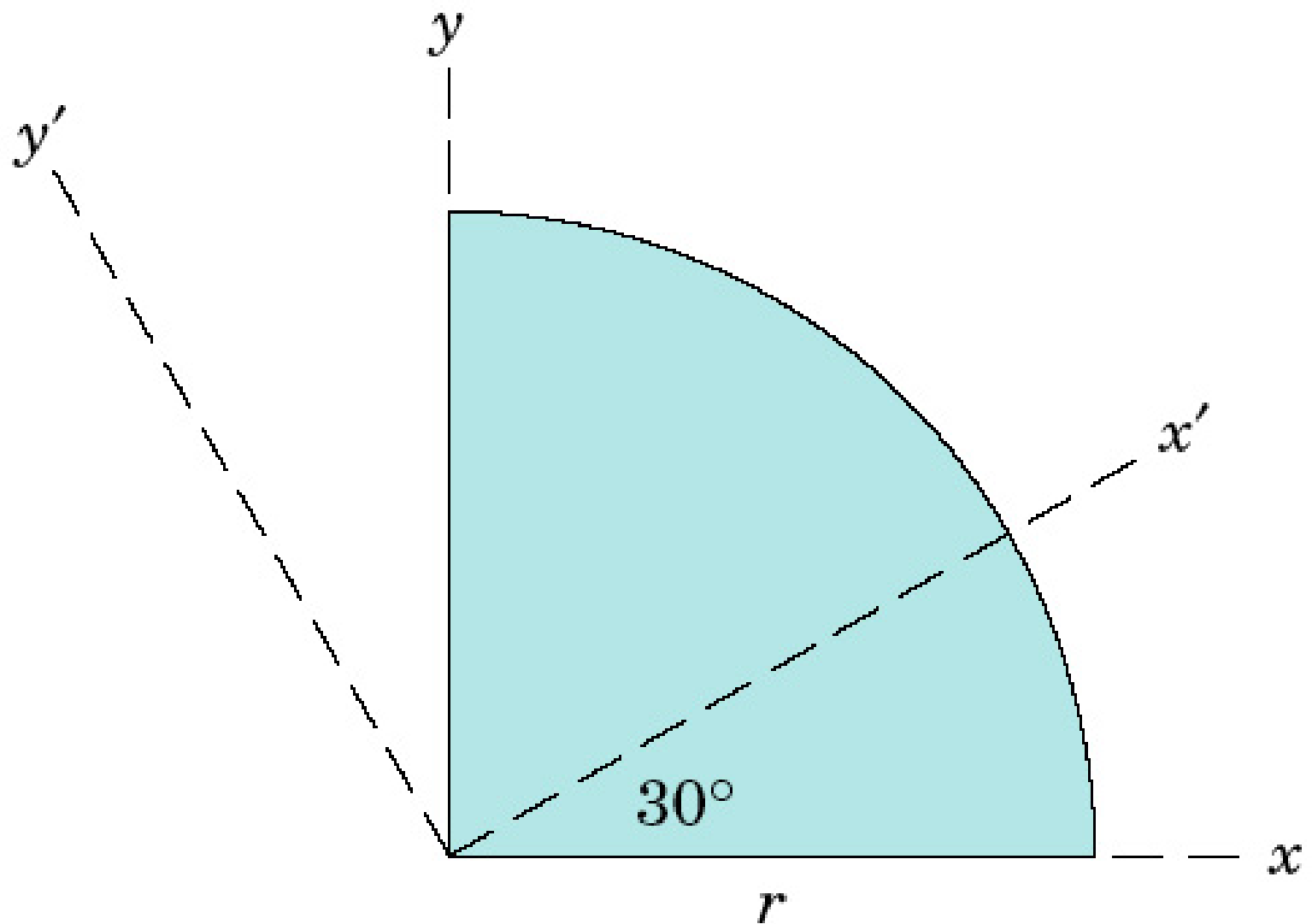
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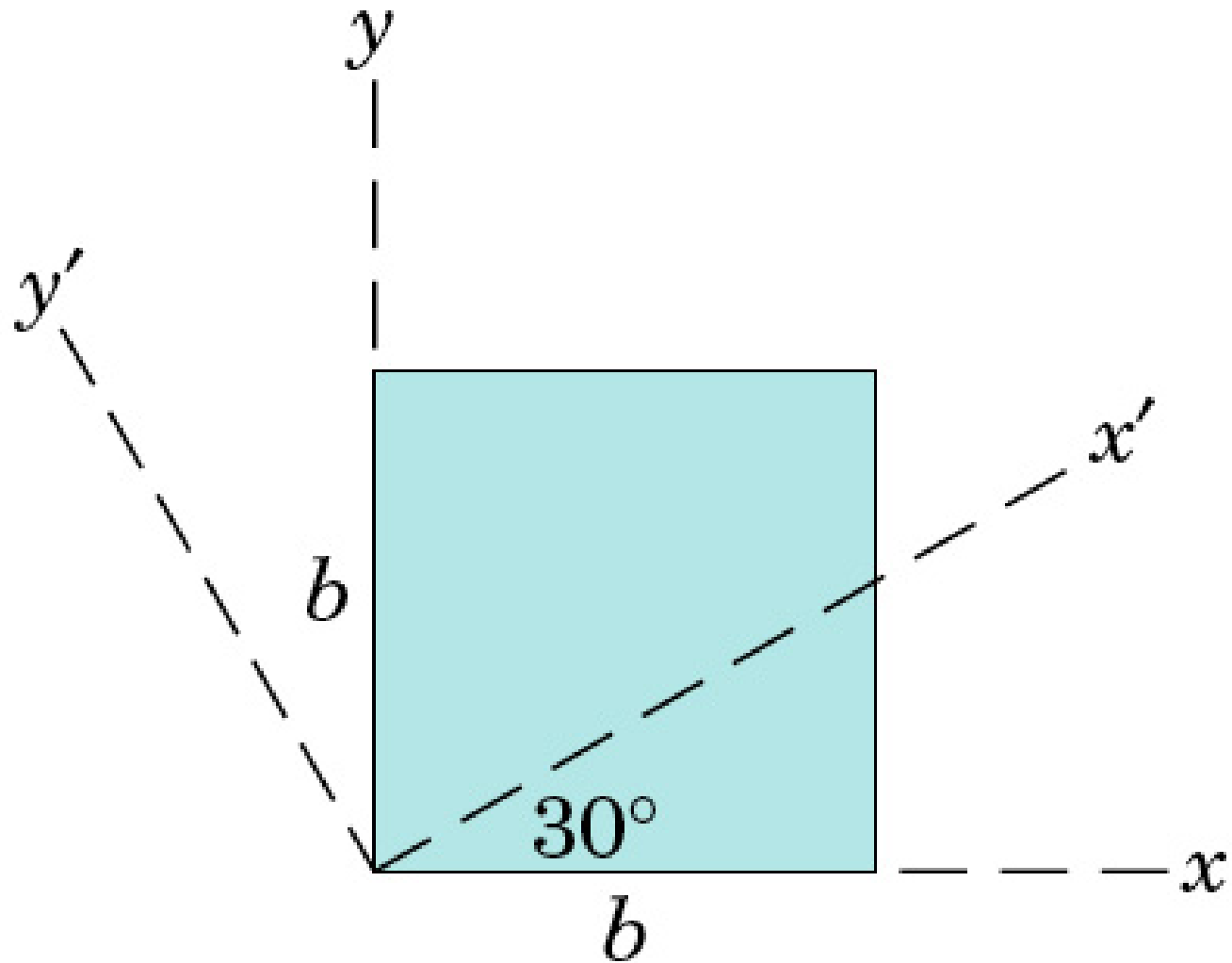
Problem A/73



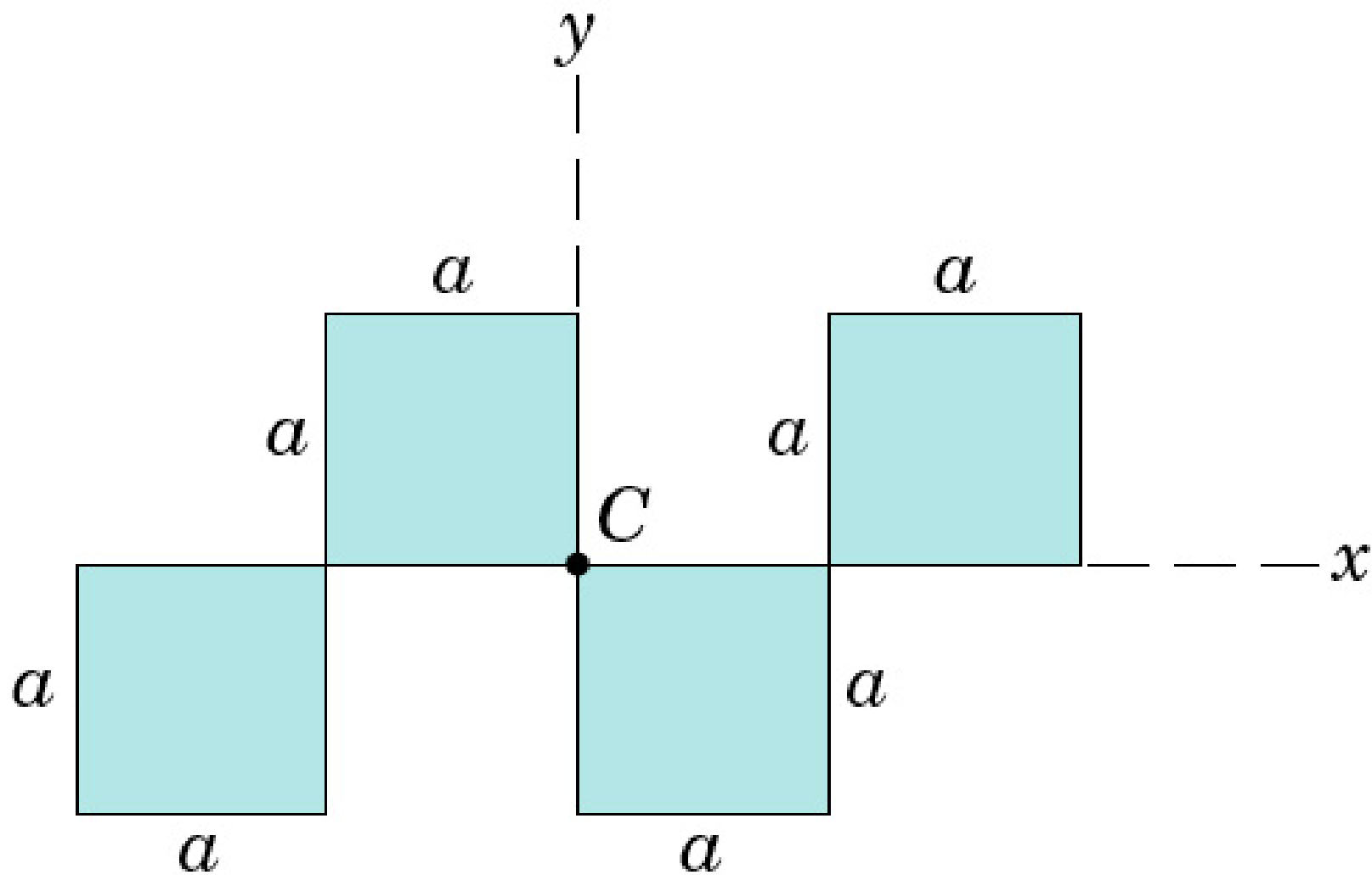
Problem A/74



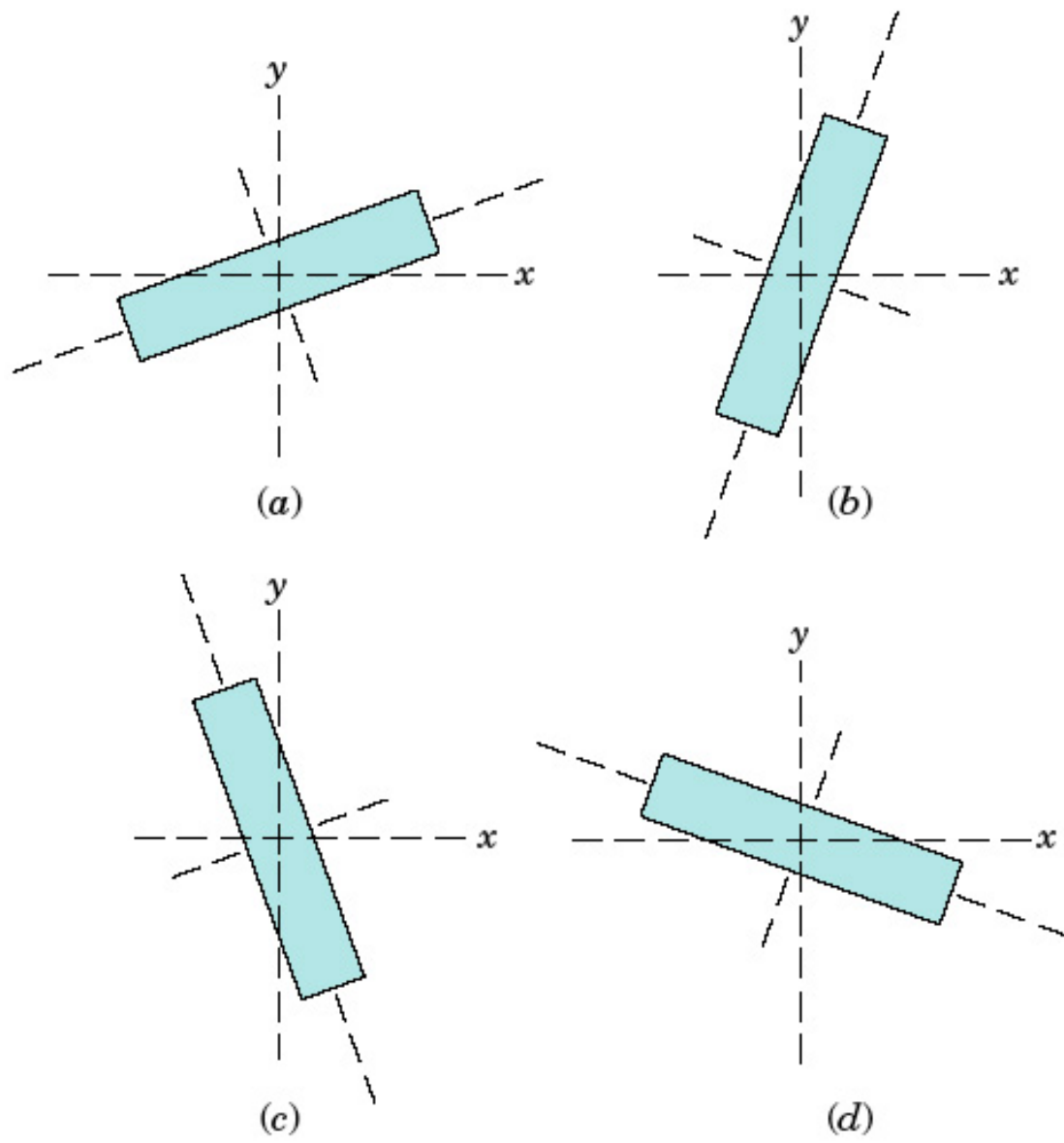
Problem A/75



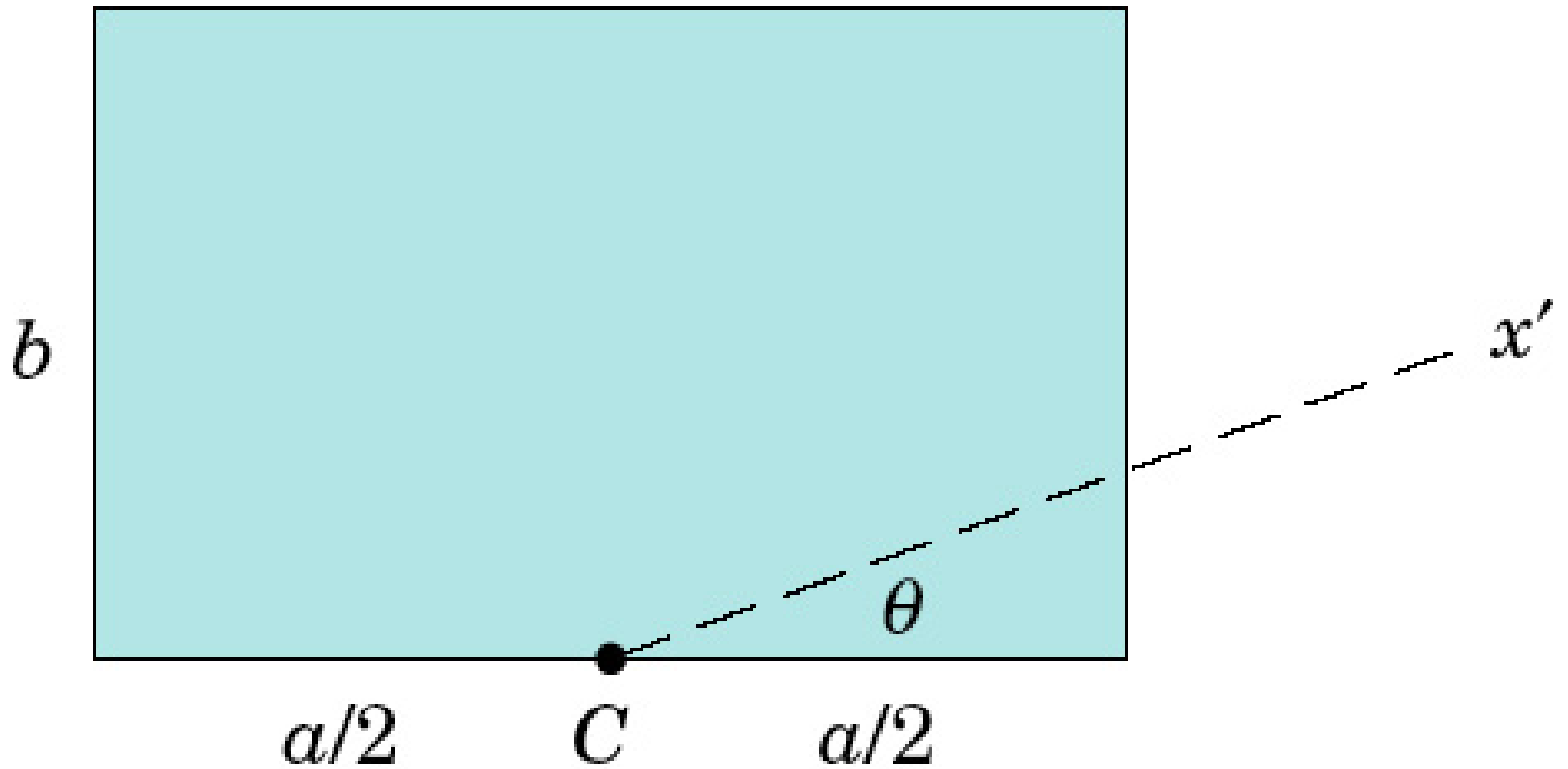
Problem A/76



Problem A/77

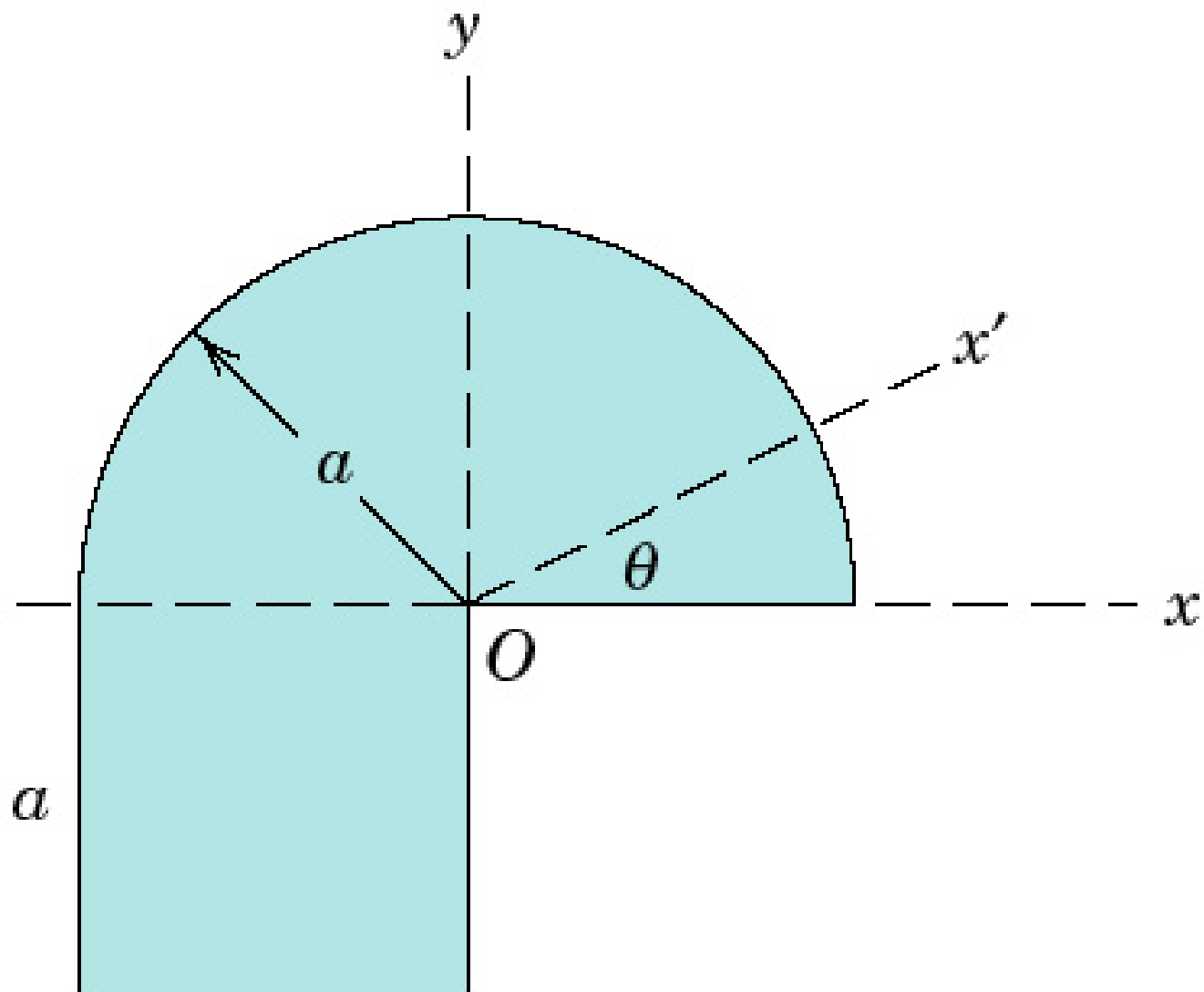


Problem A/78

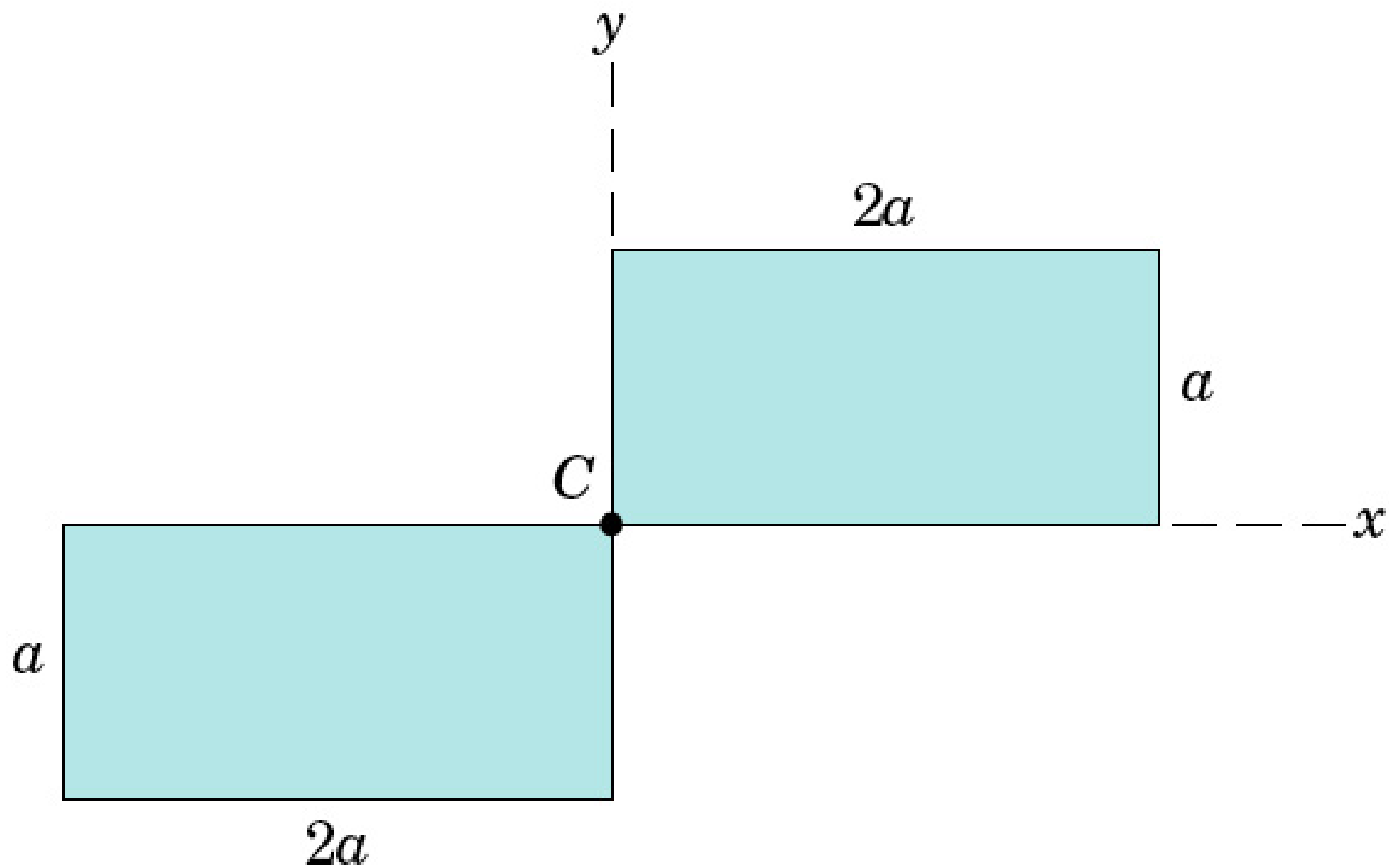


Problem A/79

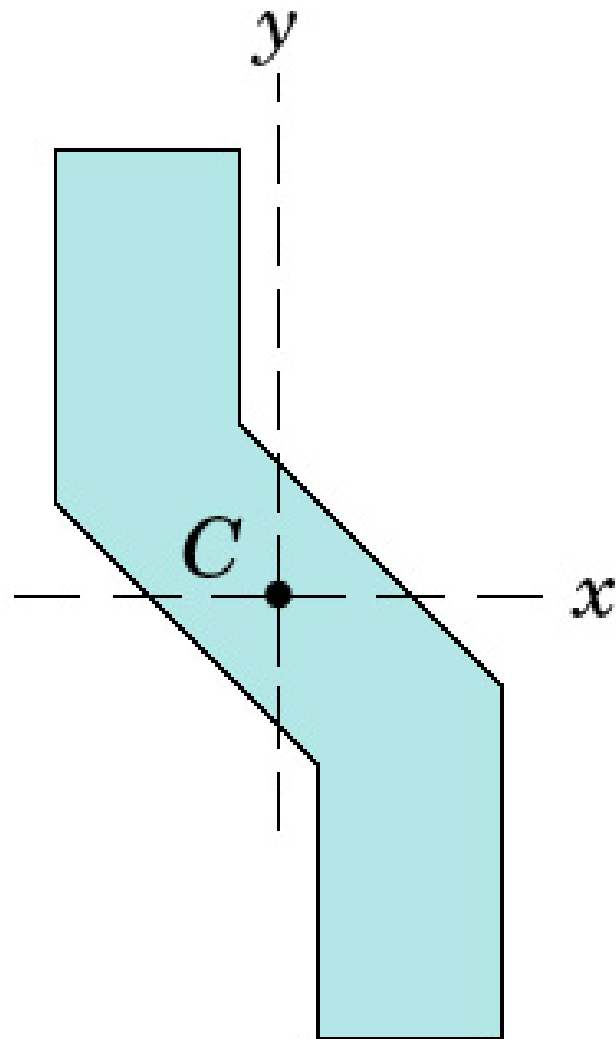
P_A_79



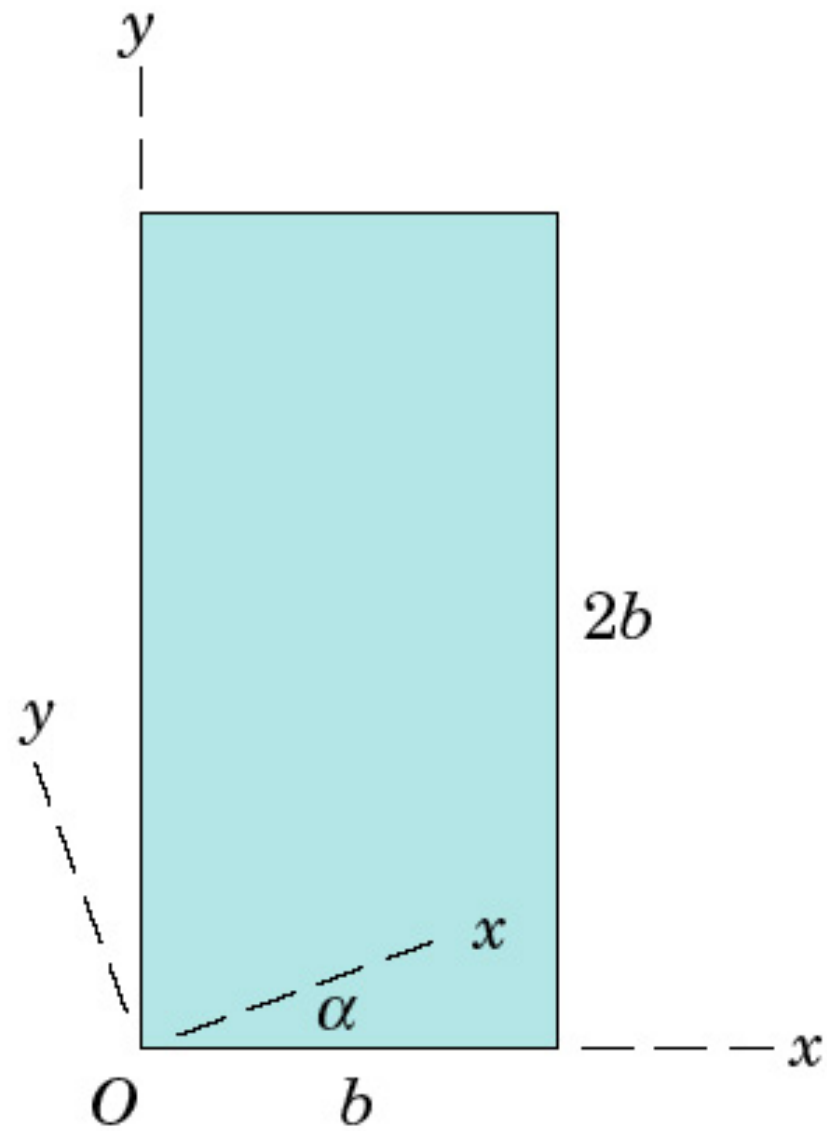
Problem A/80



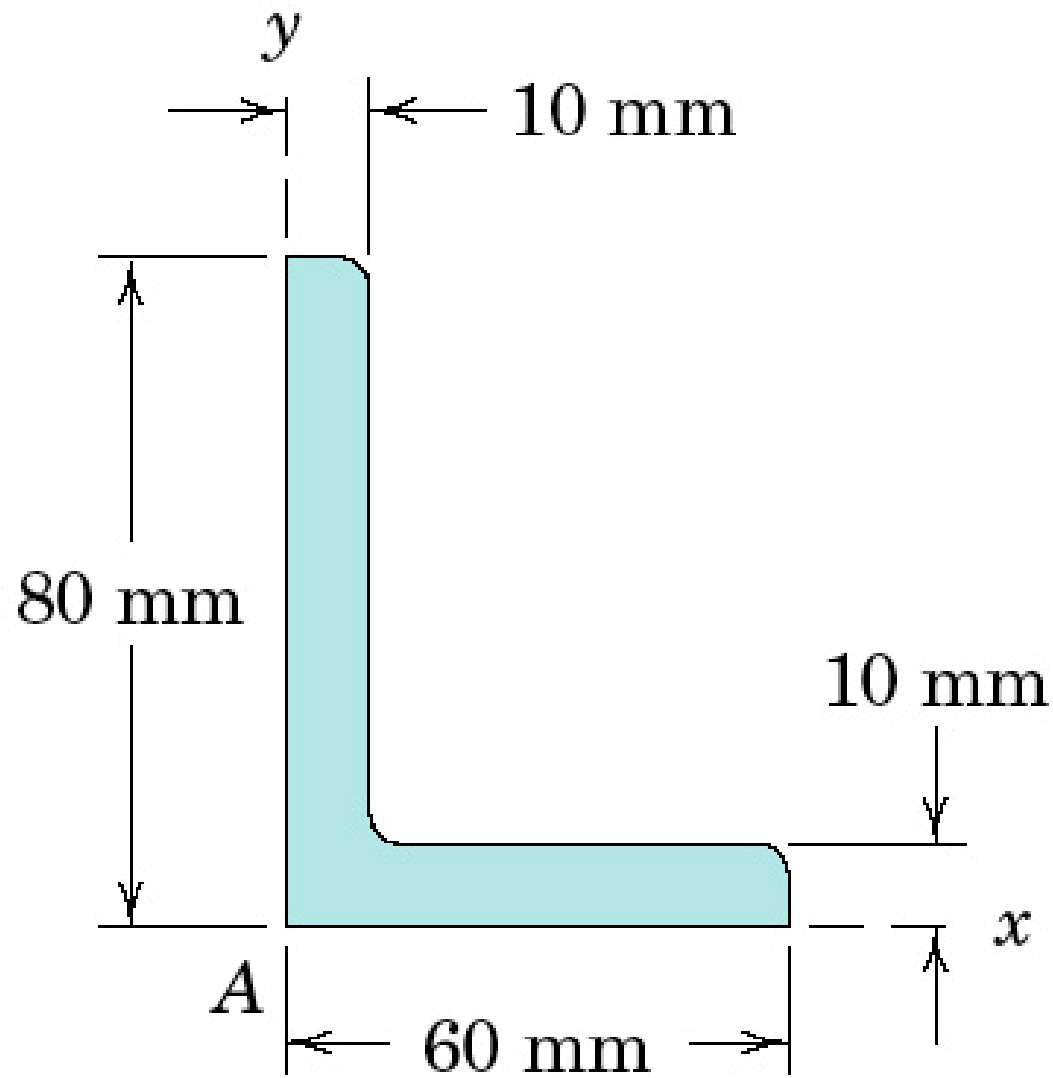
Problem A/81



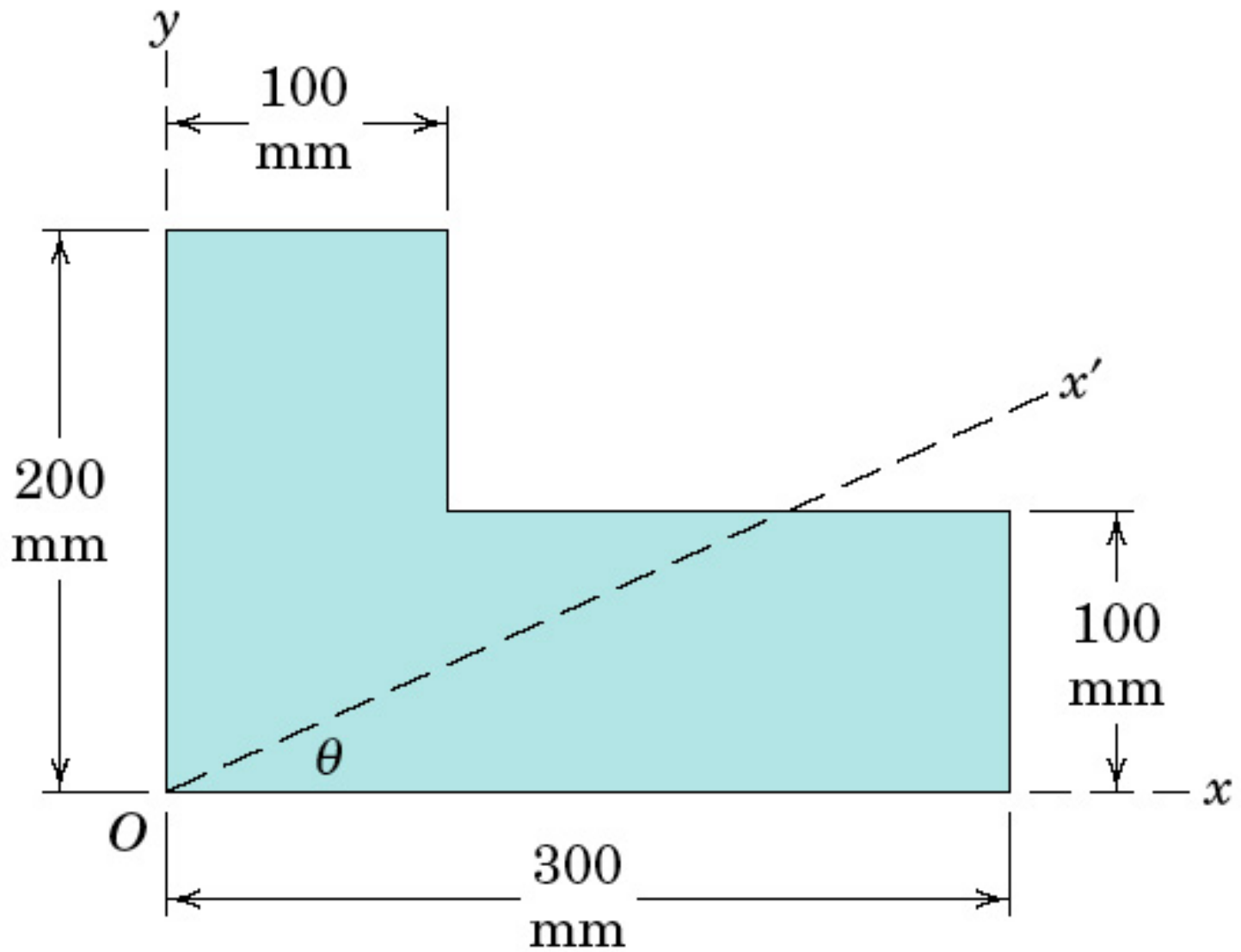
Problem A/82



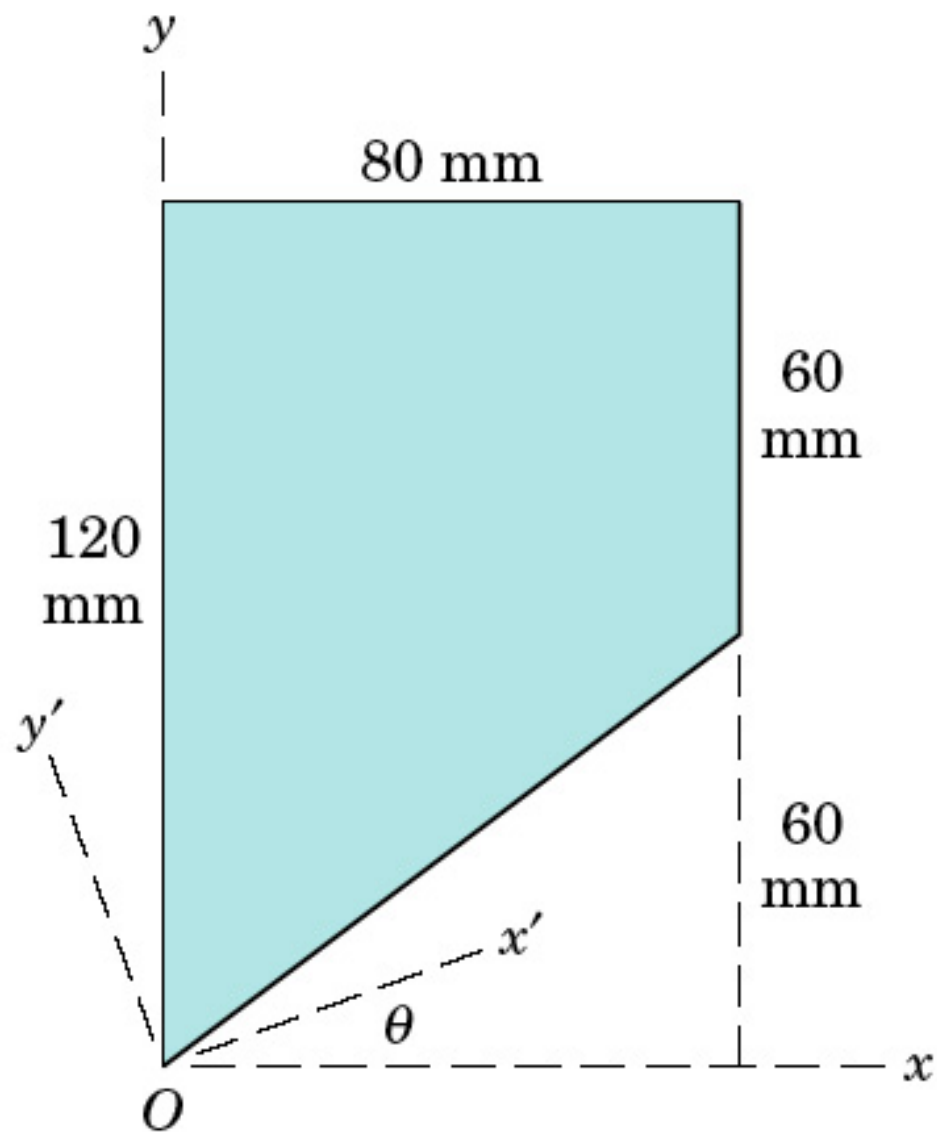
Problem A/84



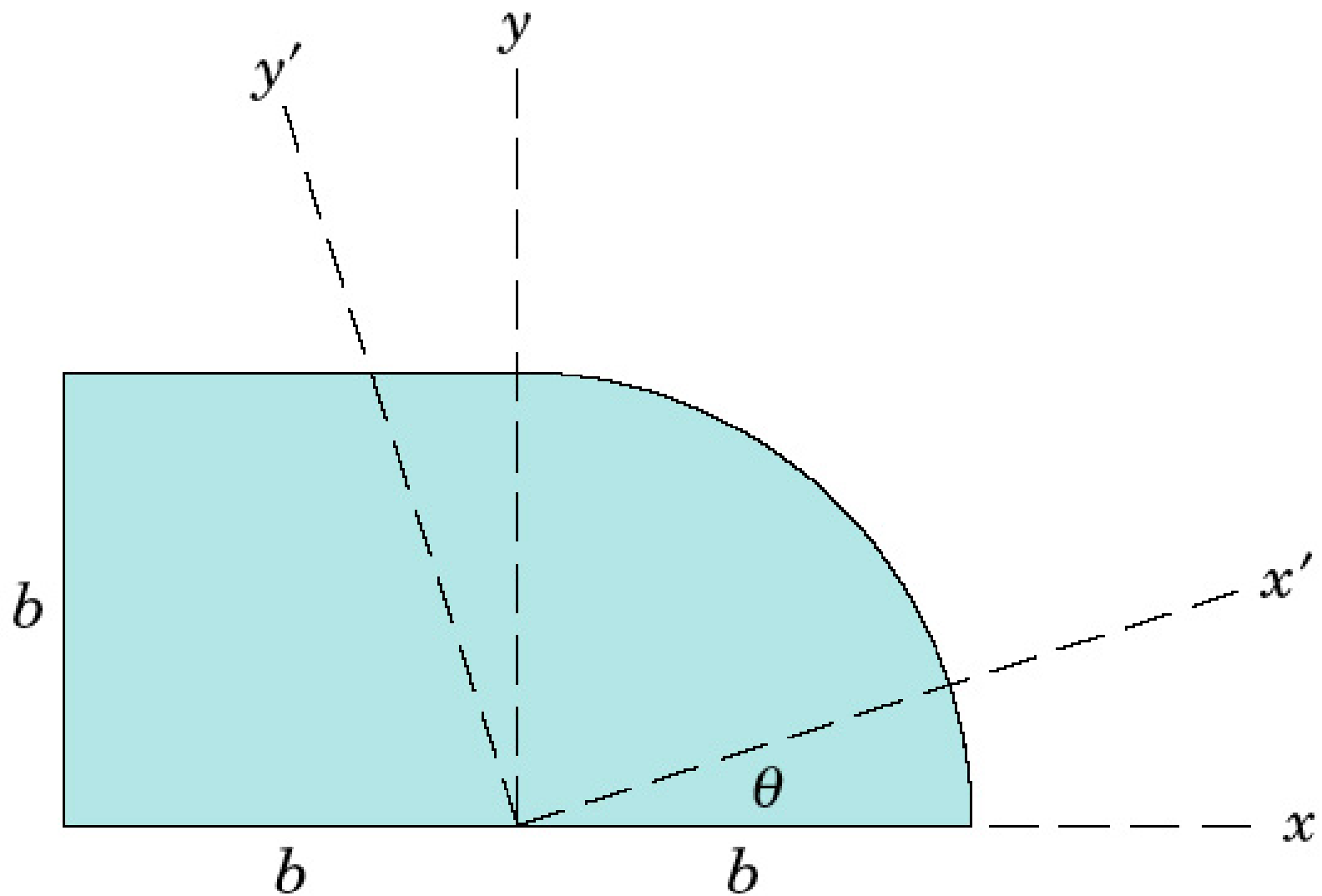
Problem A/85



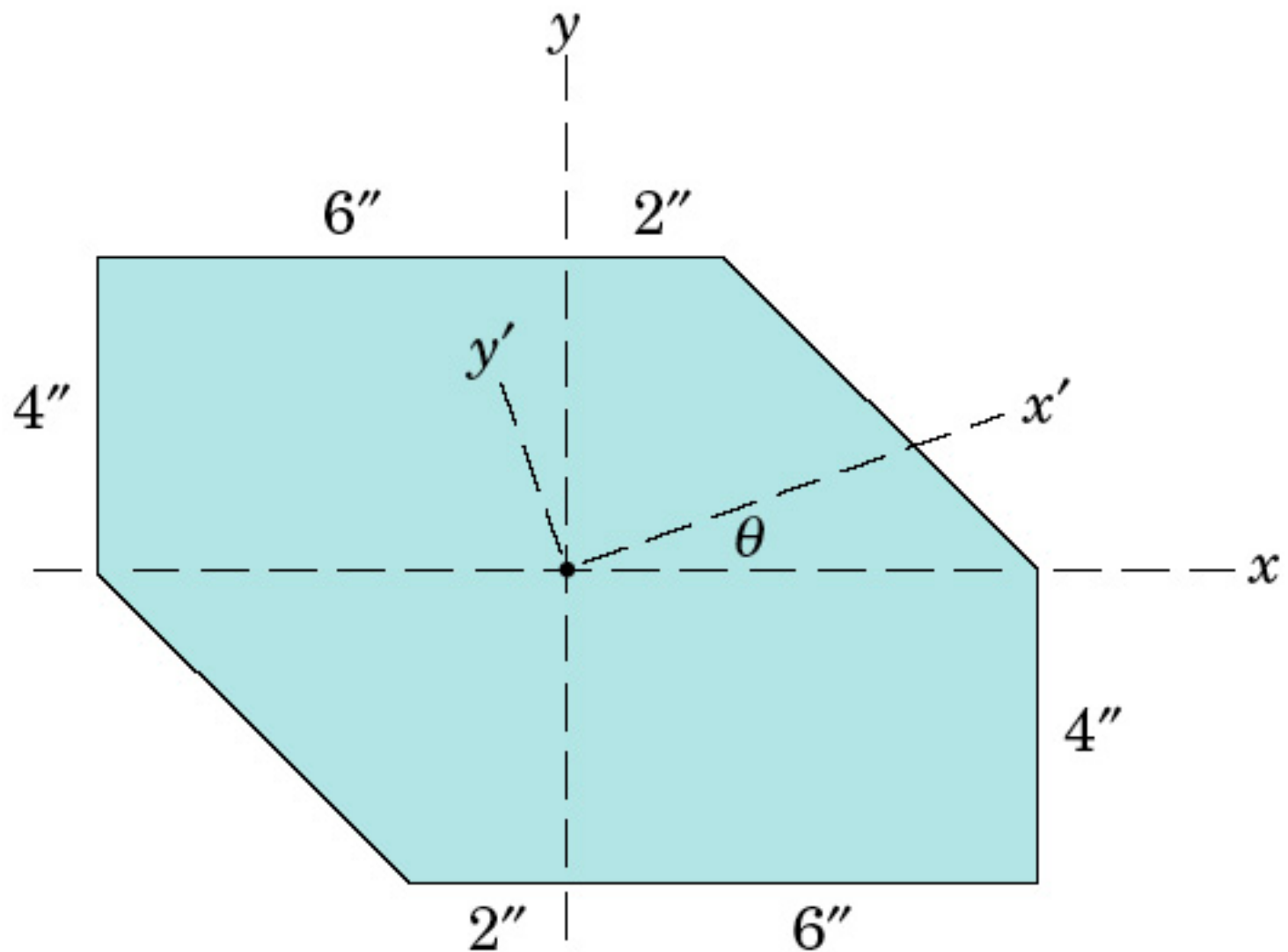
Problem A/86



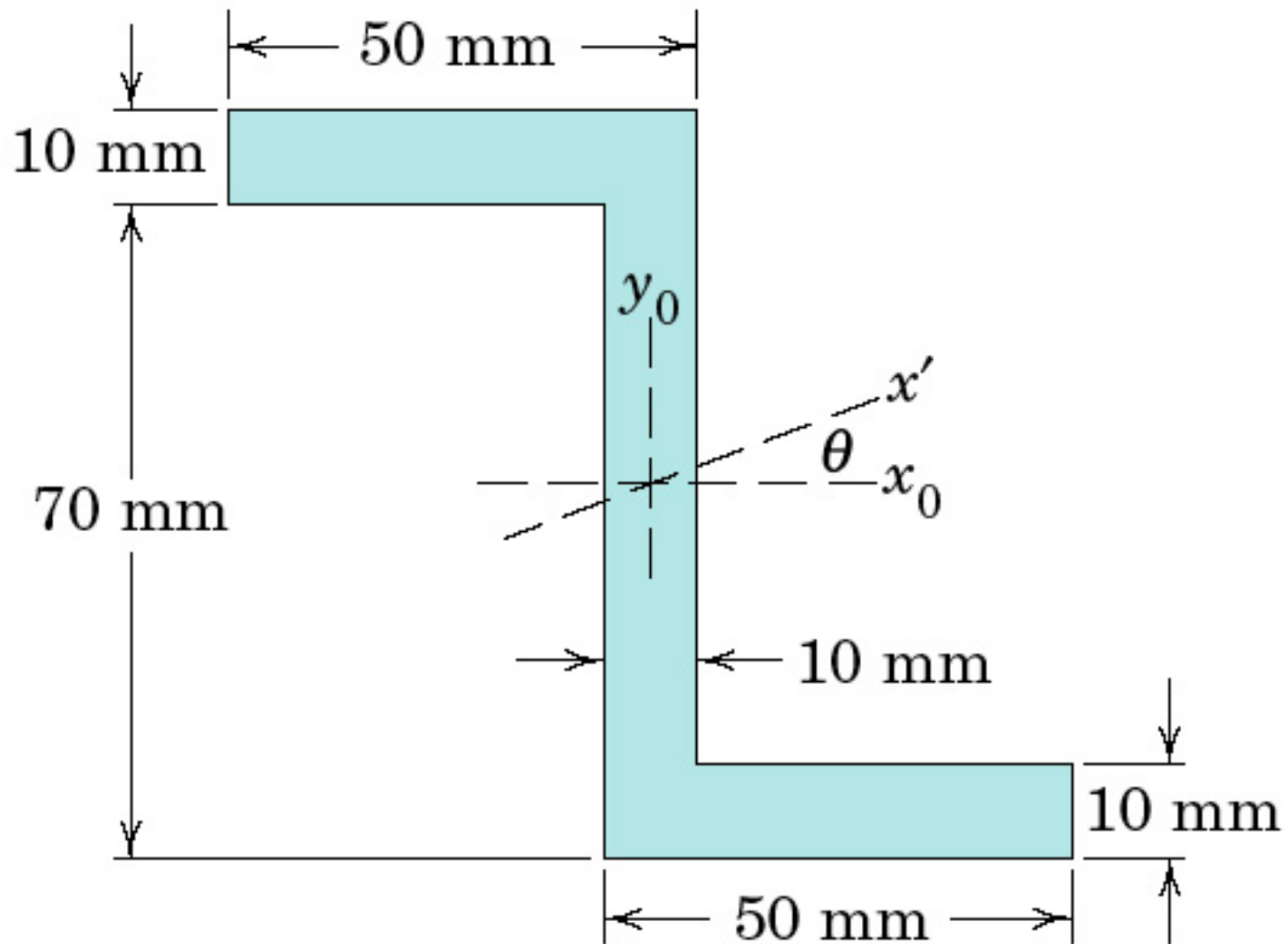
Problem A/87



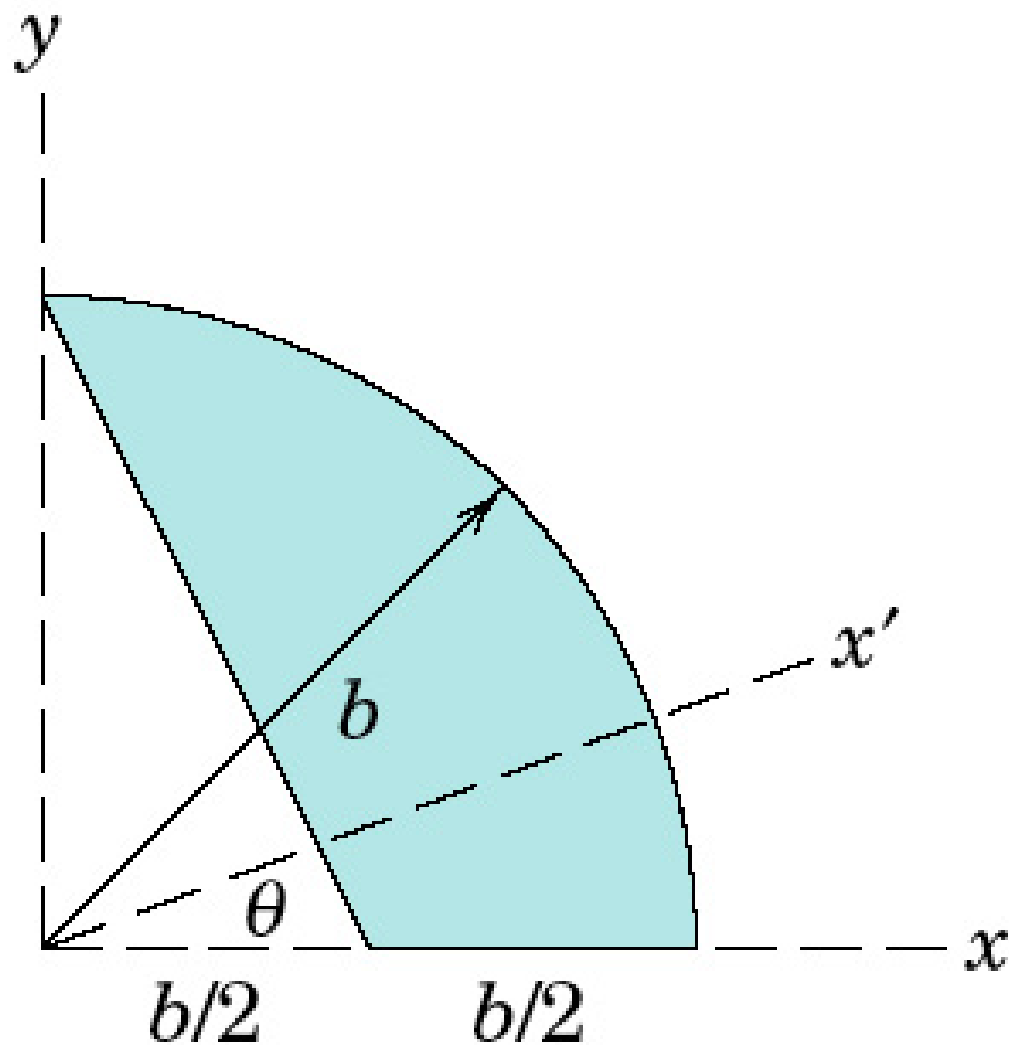
Problem A/88



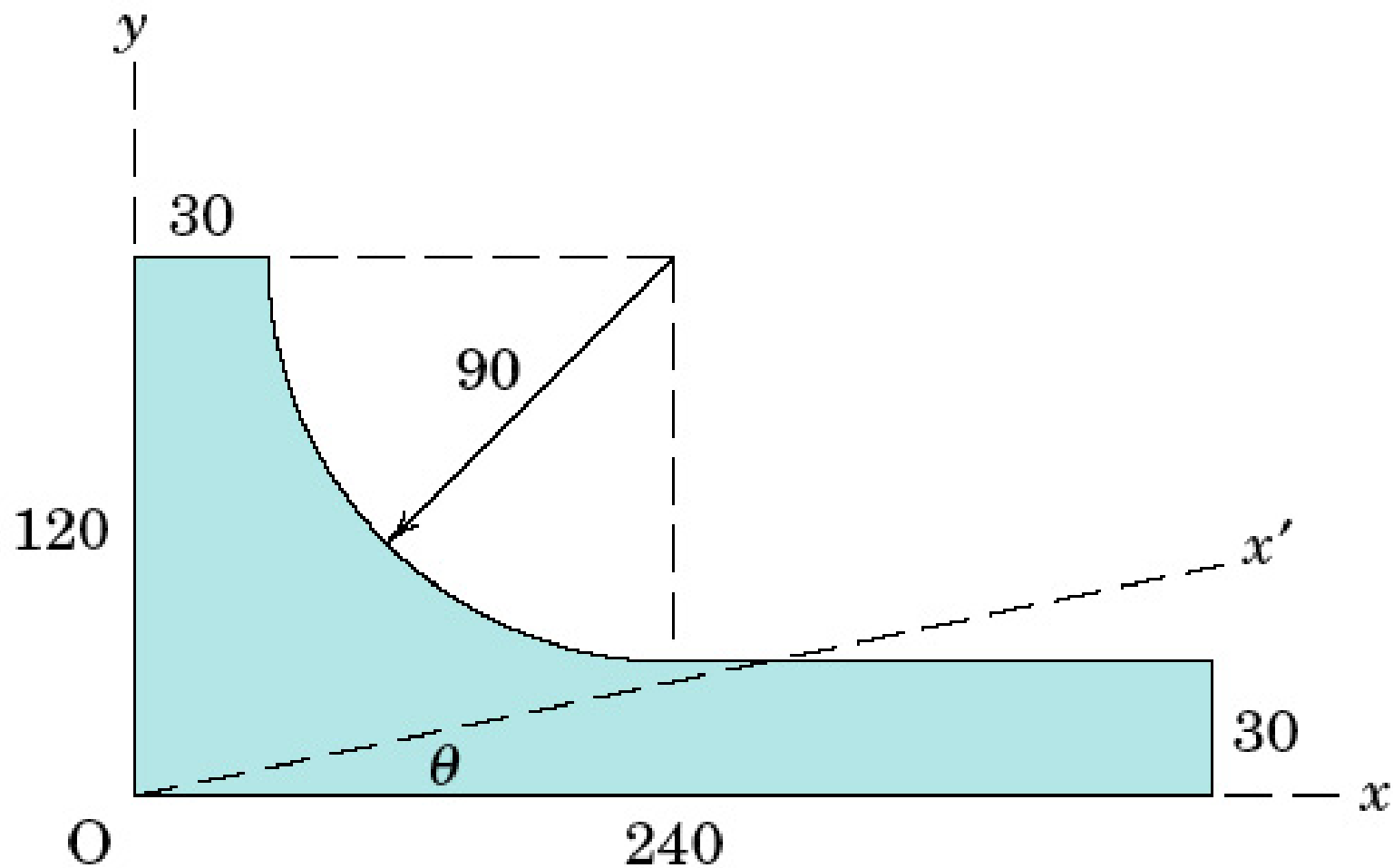
Problem A/89



Problem A/90



Problem A/91



Dimensions in millimeters

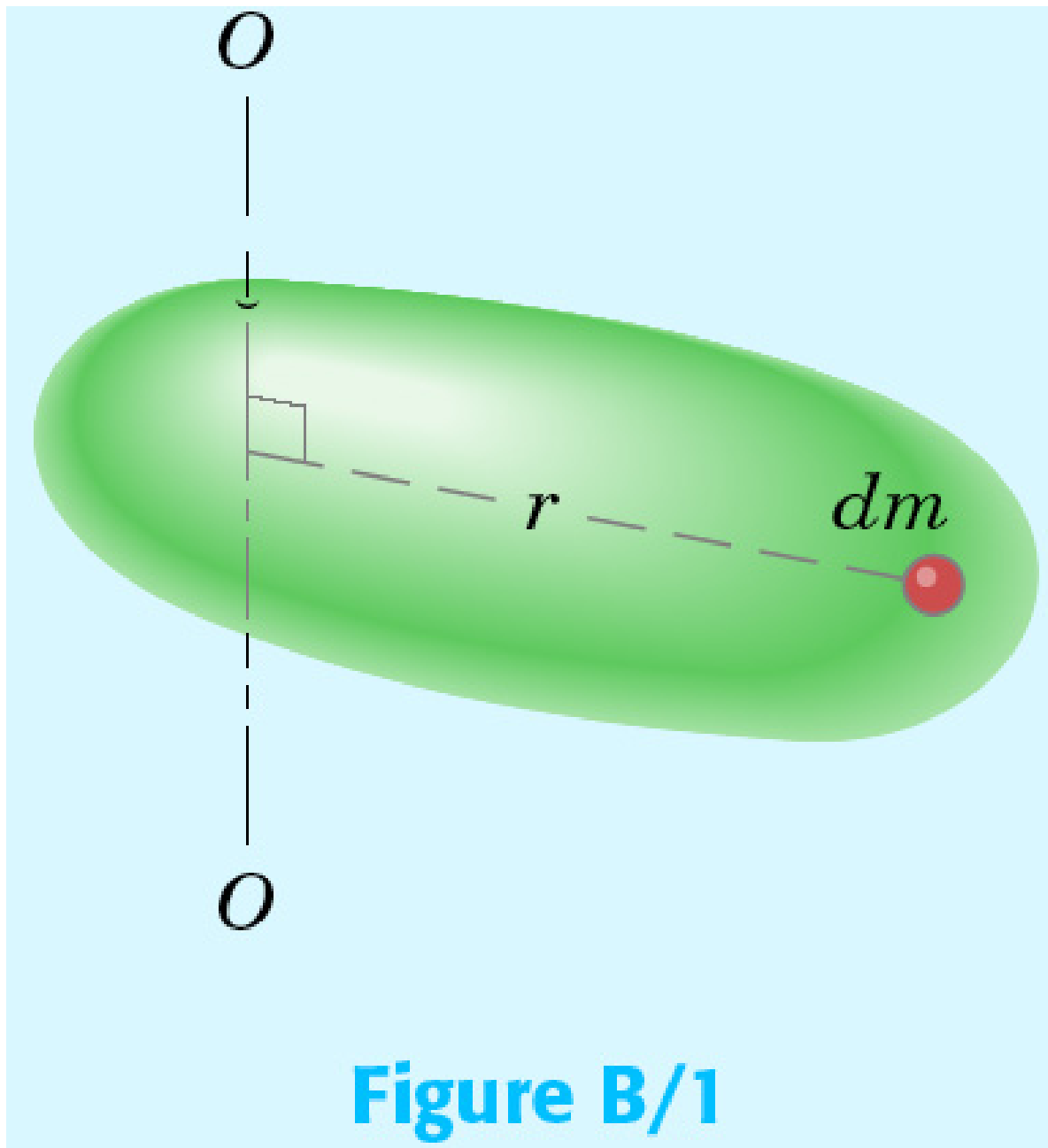
Problem A/92

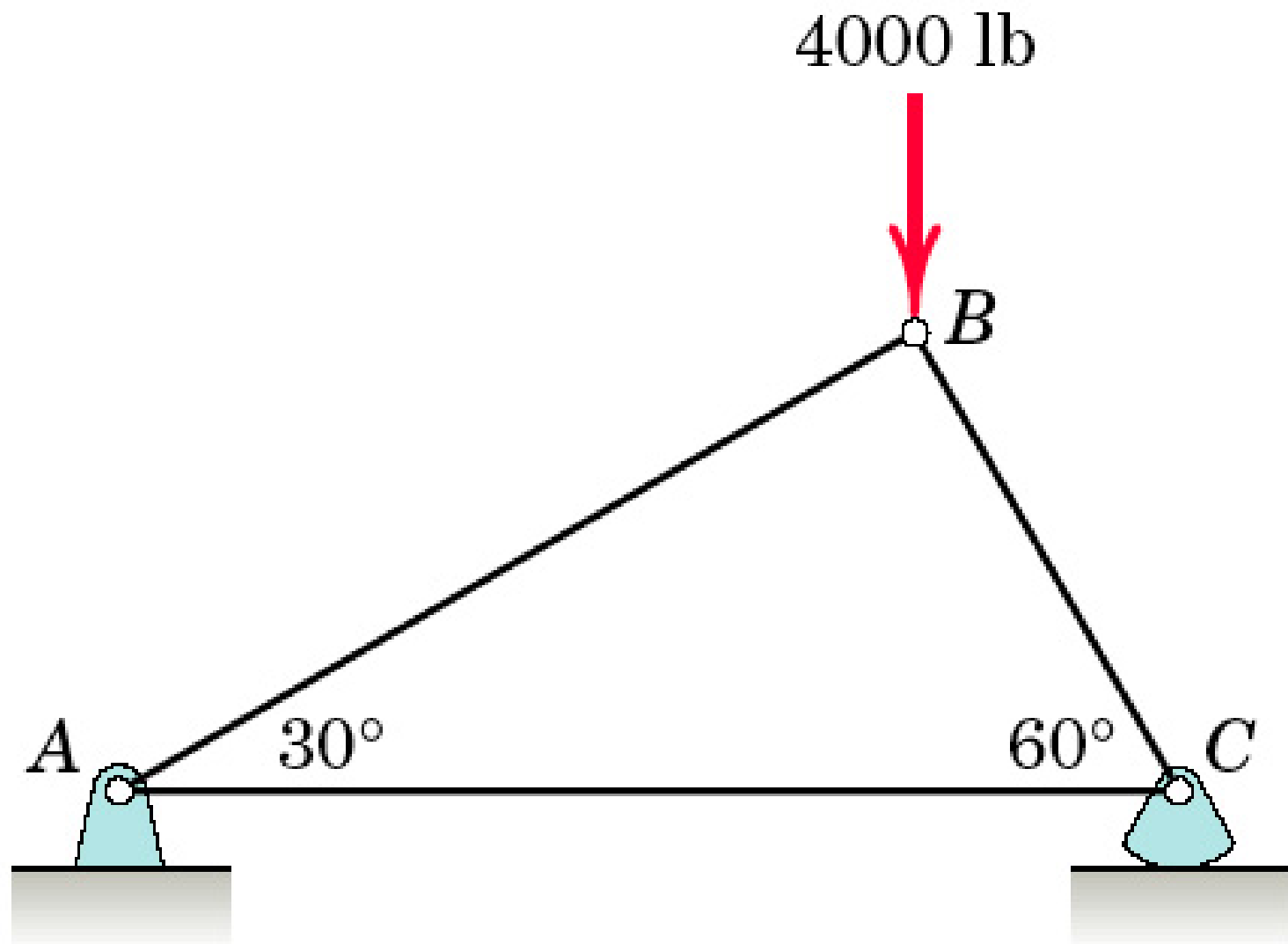
Text Illustrations

To Accompany

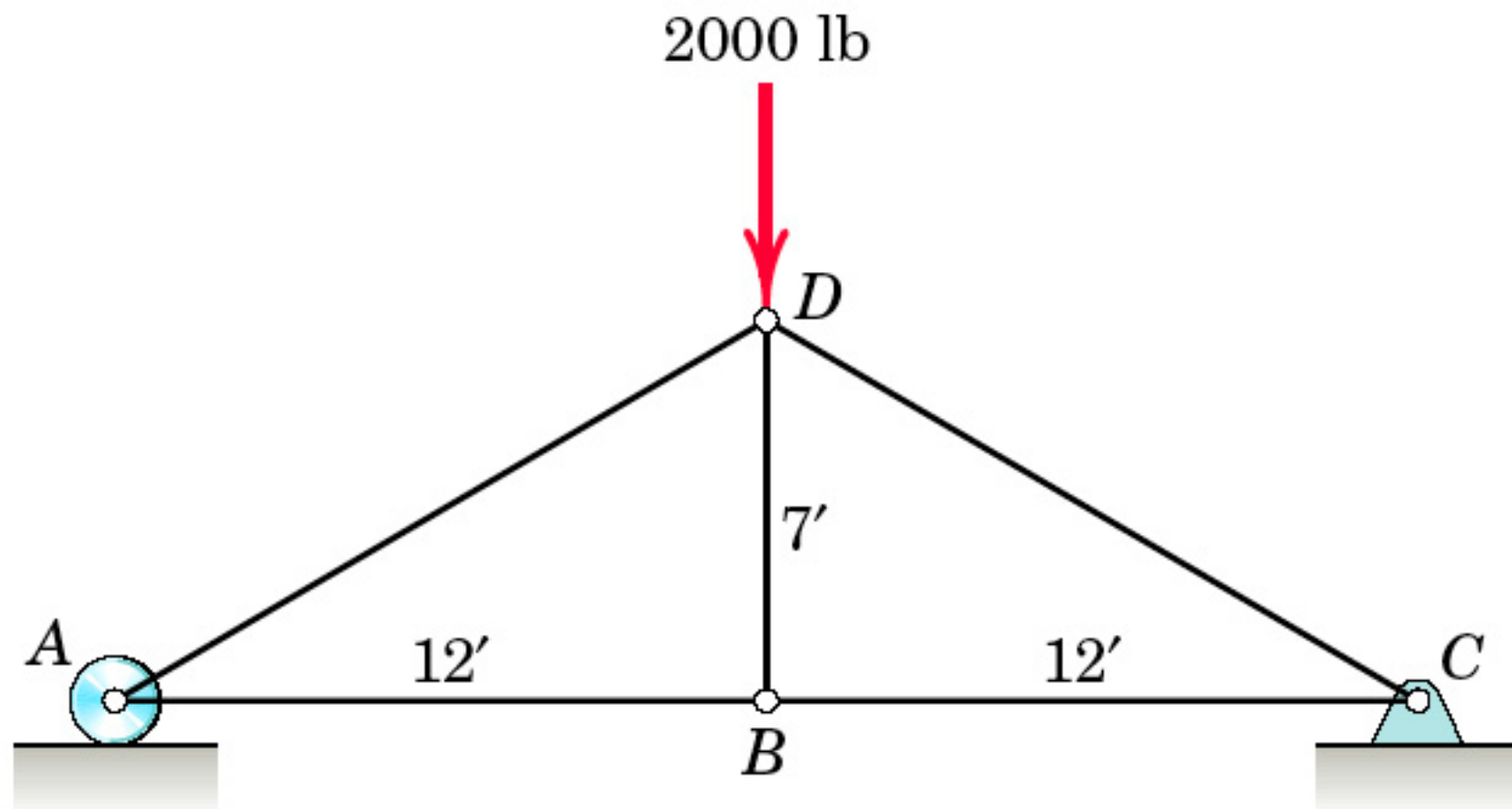
Meriam/Kraige – Statics 6e

Appendix B: Summary of Mass Moments of Inertia.



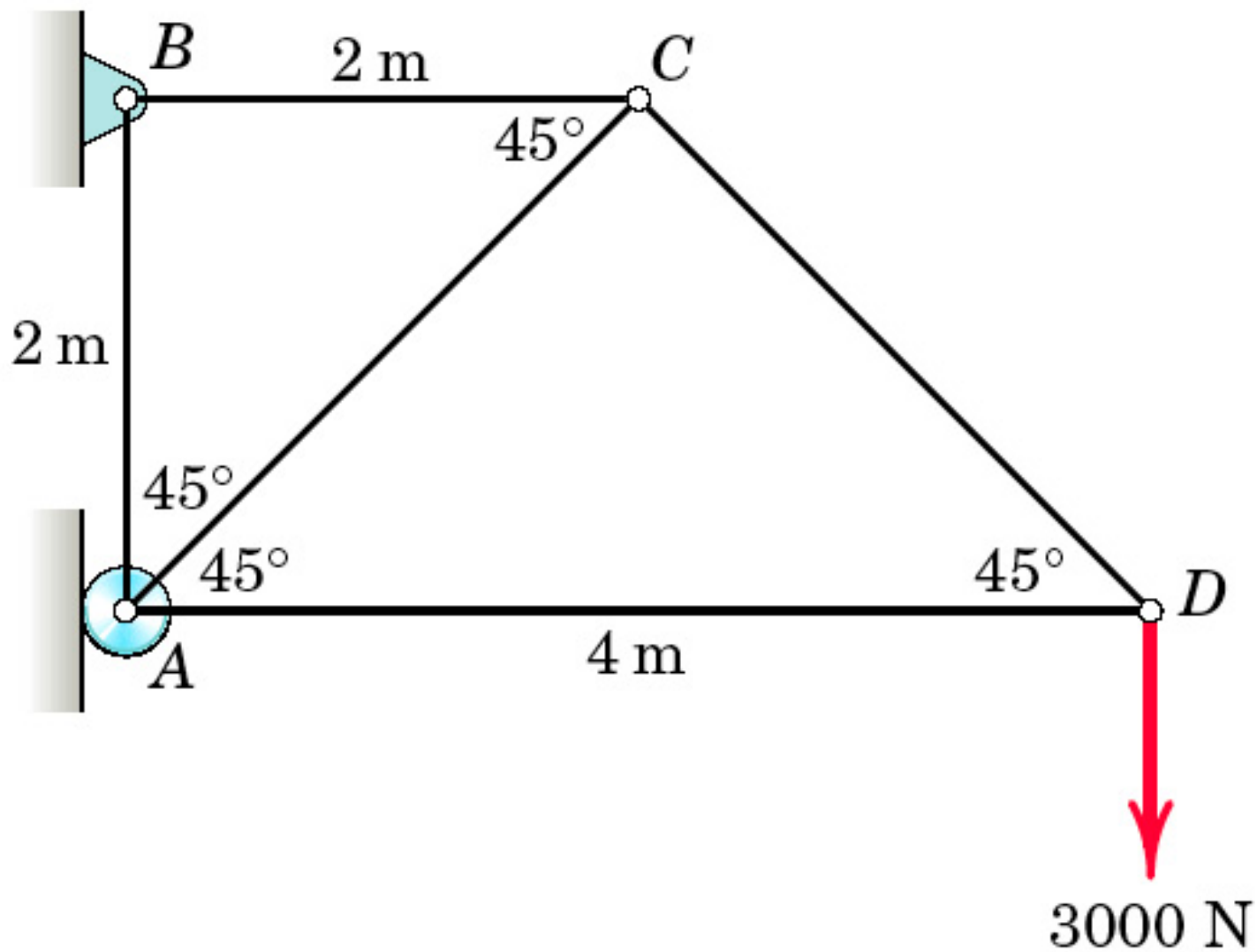


Problem 4/1

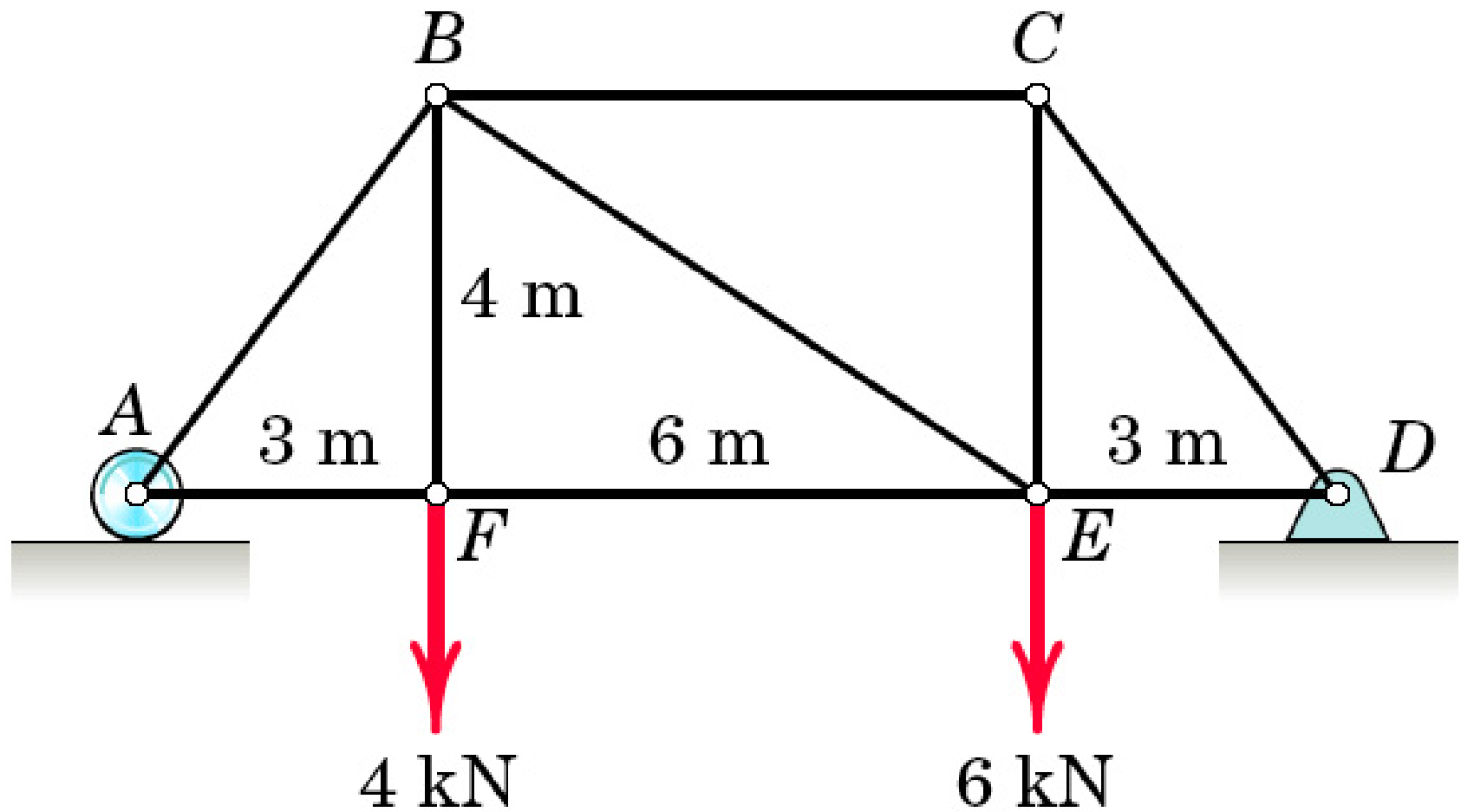


Problem 4/2

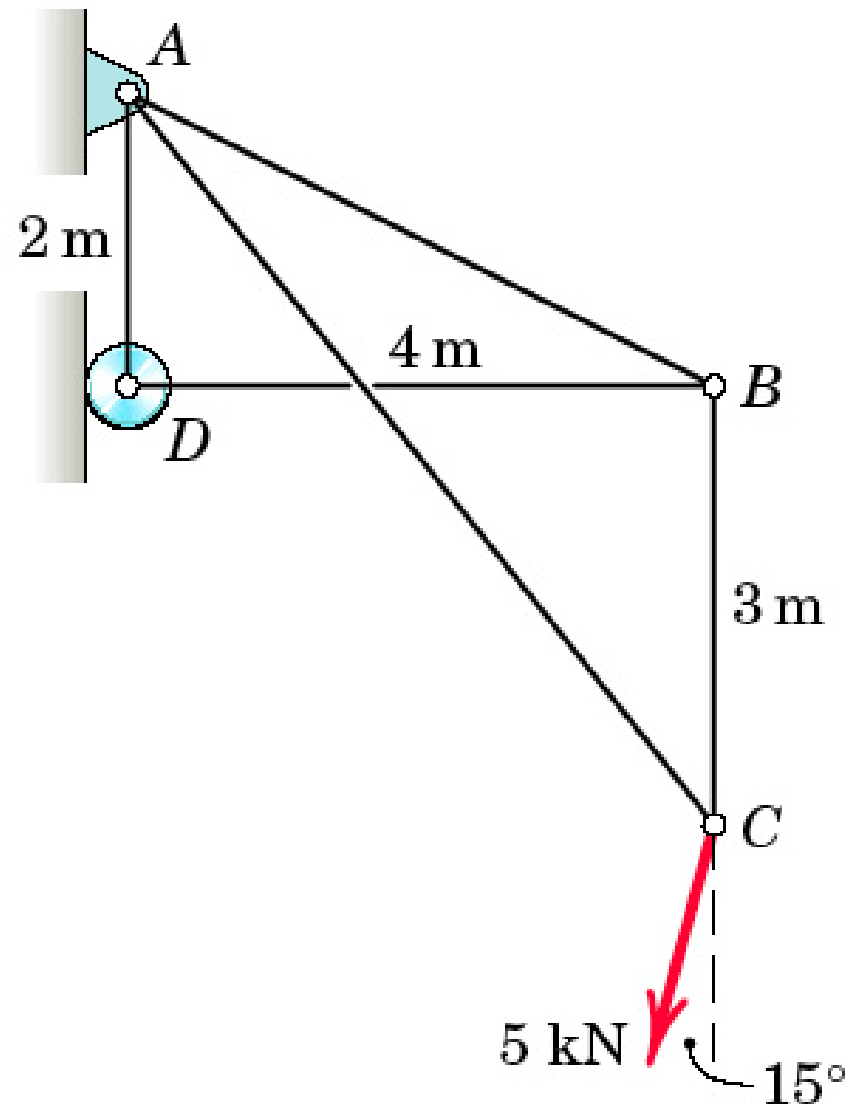
P_04_02



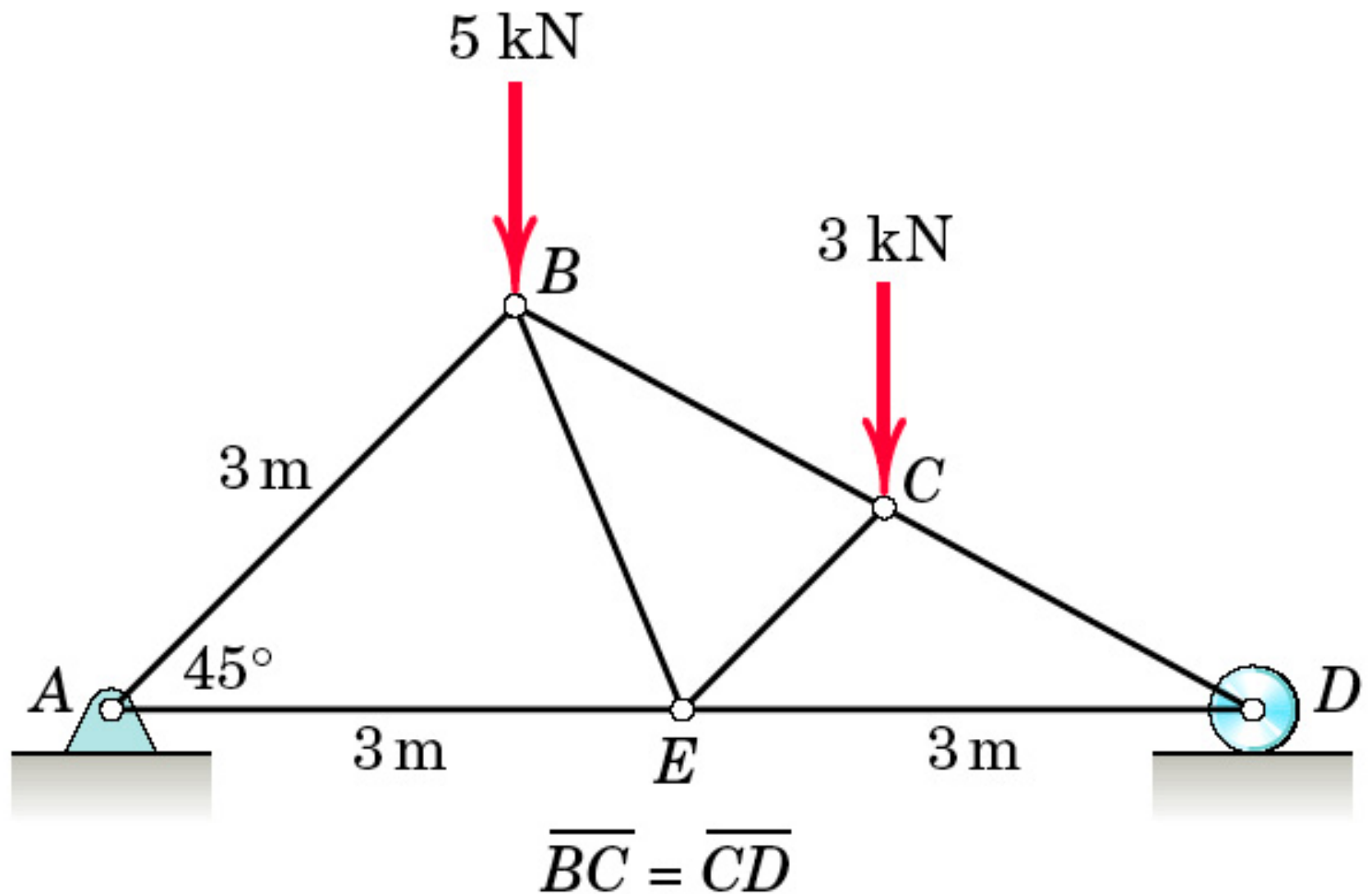
Problem 4/3



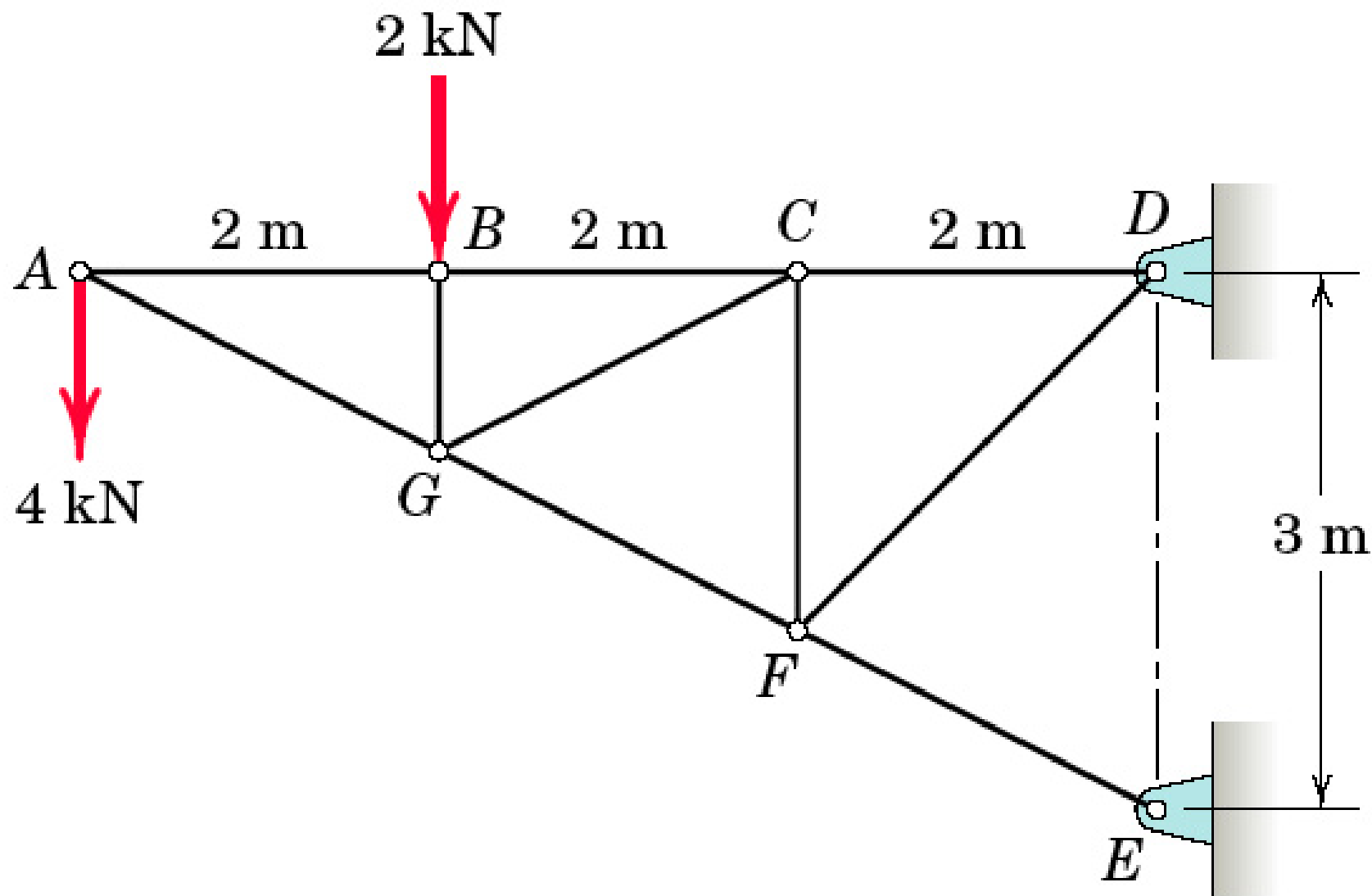
Problem 4/5



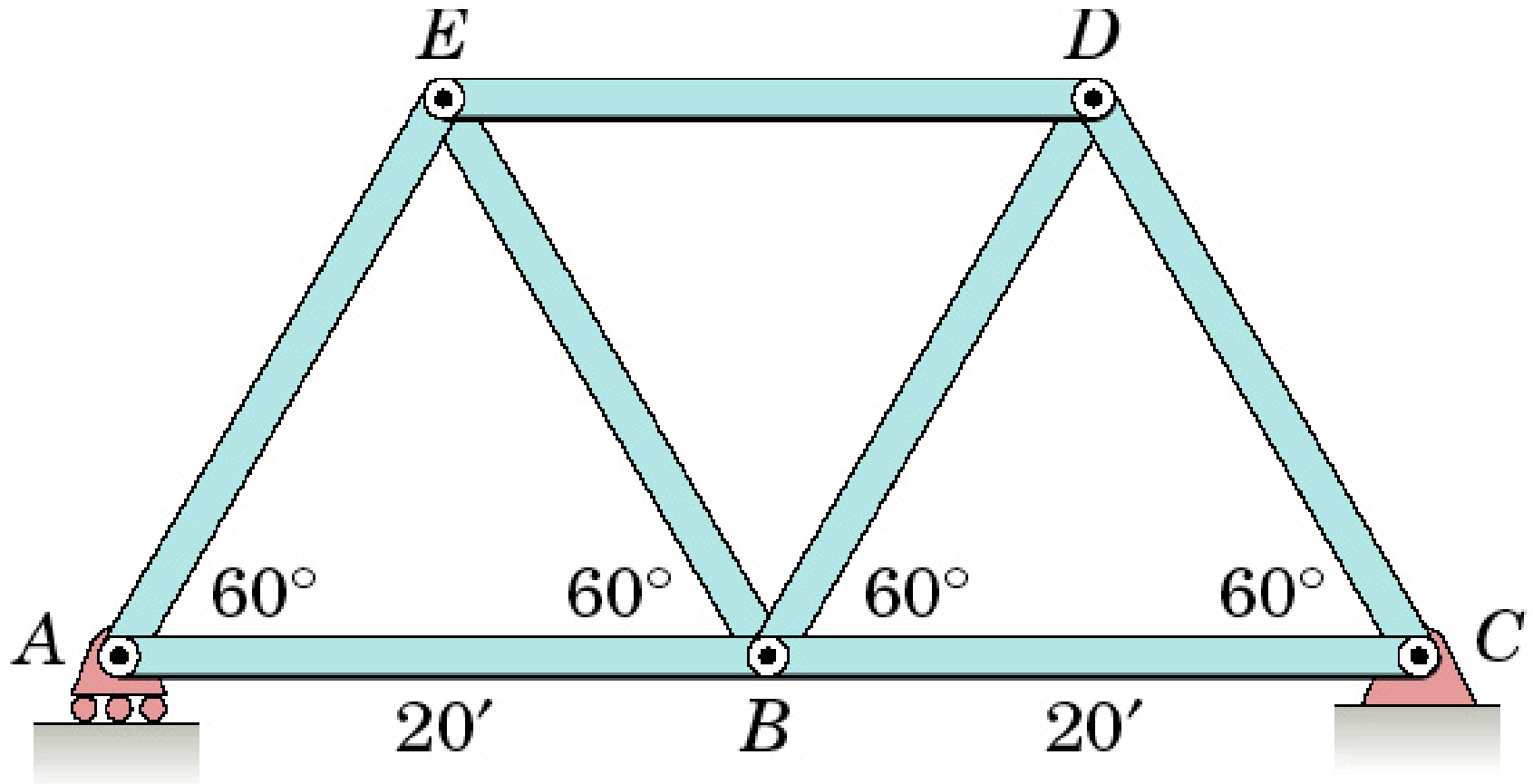
Problem 4/9



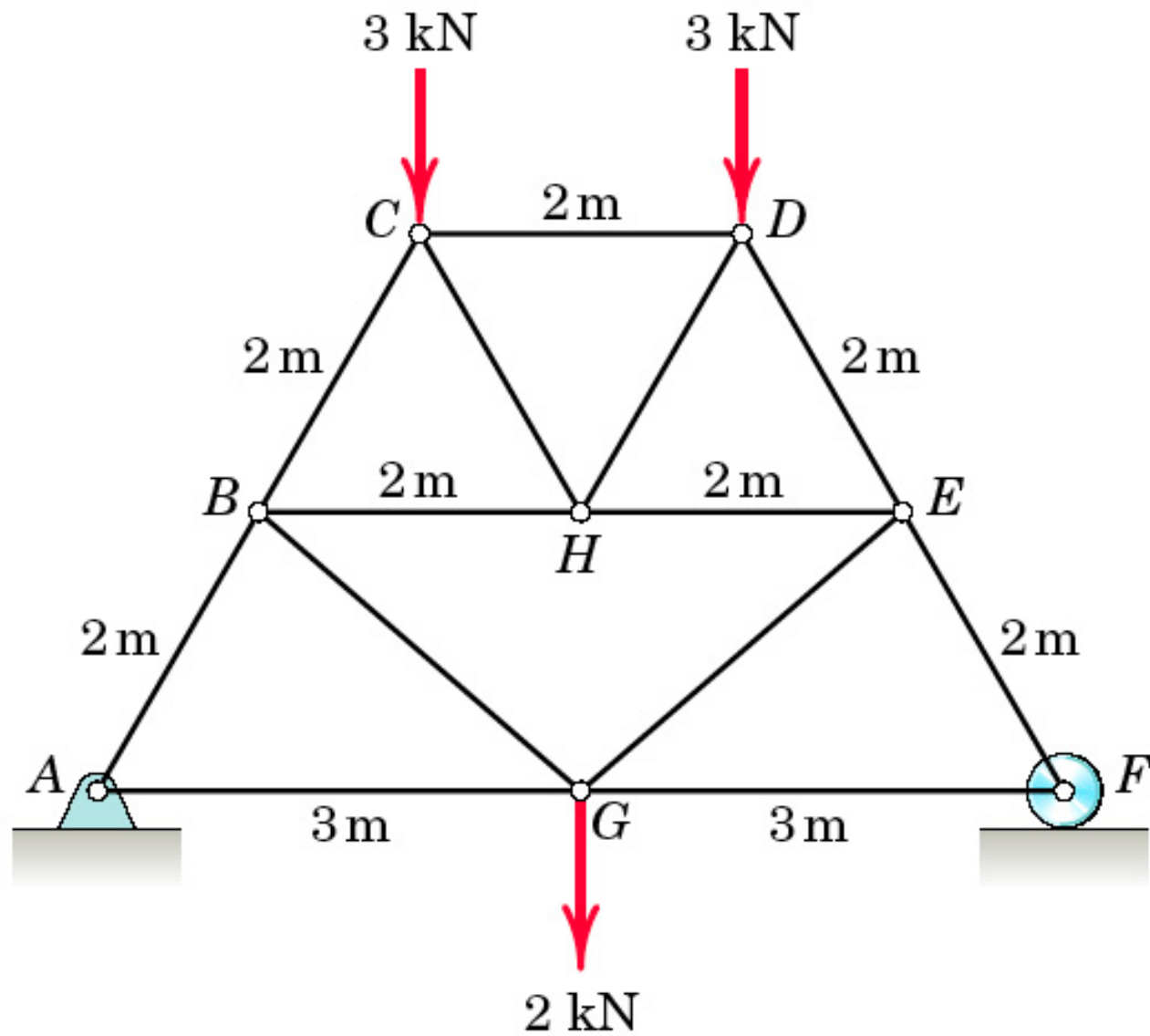
Problem 4/10



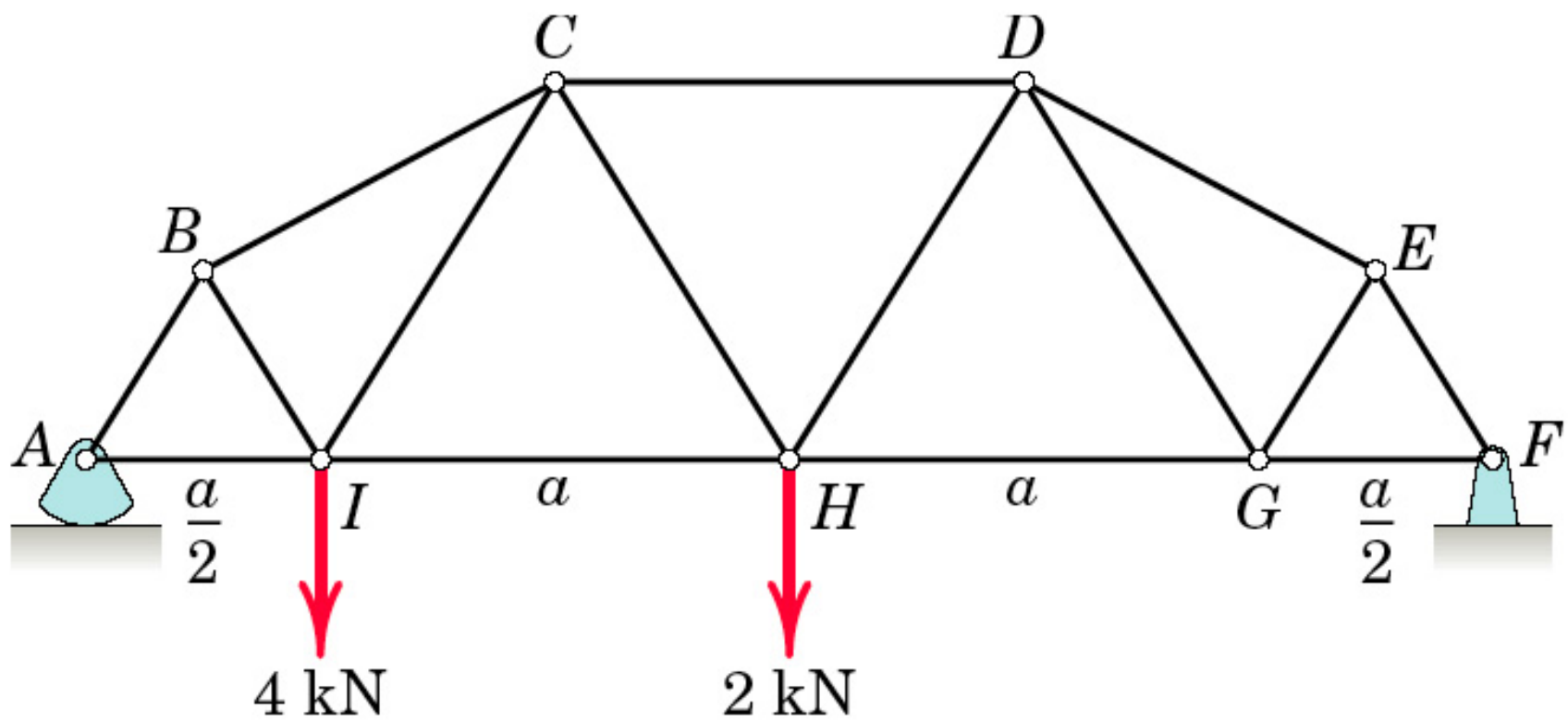
Problem 4/11



Problem 4/12



Problem 4/15



Problem 4/16

P_04_16

Text Illustrations

To Accompany

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Appendix D: Useful Tables.

TABLE D/1 PHYSICAL PROPERTIES*Density (kg/m³) and specific weight (lb/ft³)*

	kg/m ³	lb/ft ³		kg/m ³	lb/ft ³
Air*	1.2062	0.07530	Lead	11 370	710
Aluminum	2 690	168	Mercury	13 570	847
Concrete (av.)	2 400	150	Oil (av.)	900	56
Copper	8 910	556	Steel	7 830	489
Earth (wet, av.)	1 760	110	Titanium	3 080	192
(dry, av.)	1 280	80	Water (fresh)	1 000	62.4
Glass	2 590	162	(salt)	1 030	64
Gold	19 300	1205	Wood (soft pine)	480	30
Ice	900	56	(hard oak)	800	50
Iron (cast)	7 210	450			

* At 20°C (68°F) and atmospheric pressure

Coefficients of friction

(The coefficients in the following table represent typical values under normal working conditions. Actual coefficients for a given situation will depend on the exact nature of the contacting surfaces. A variation of 25 to 100 percent or more from these values could be expected in an actual application, depending on prevailing conditions of cleanliness, surface finish, pressure, lubrication, and velocity.)

CONTACTING SURFACE	TYPICAL VALUES OF COEFFICIENT OF FRICTION	
	STATIC, μ_s	KINETIC, μ_k
Steel on steel (dry)	0.6	0.4
Steel on steel (greasy)	0.1	0.05
Teflon on steel	0.04	0.04
Steel on babbitt (dry)	0.4	0.3
Steel on babbitt (greasy)	0.1	0.07
Brass on steel (dry)	0.5	0.4
Brake lining on cast iron	0.4	0.3
Rubber tires on smooth pavement (dry)	0.9	0.8
Wire rope on iron pulley (dry)	0.2	0.15
Hemp rope on metal	0.3	0.2
Metal on ice		0.02

TABLE D/2 SOLAR SYSTEM CONSTANTS

Universal gravitational constant	$G = 6.673(10^{-11}) \text{ m}^3/(\text{kg} \cdot \text{s}^2)$ $= 3.439(10^{-8}) \text{ ft}^4/(\text{lbf} \cdot \text{s}^4)$
Mass of Earth	$m_e = 5.976(10^{24}) \text{ kg}$ $= 4.095(10^{23}) \text{ lbf} \cdot \text{s}^2/\text{ft}$
Period of Earth's rotation (1 sidereal day)	$= 23 \text{ h } 56 \text{ min } 4 \text{ s}$ $= 23.9344 \text{ h}$
Angular velocity of Earth	$\omega = 0.7292(10^{-4}) \text{ rad/s}$
Mean angular velocity of Earth-Sun line	$\omega' = 0.1991(10^{-6}) \text{ rad/s}$
Mean velocity of Earth's center about Sun	$= 107\,200 \text{ km/h}$ $= 66,610 \text{ mi/h}$

BODY	MEAN DISTANCE TO SUN km (mi)	ECCENTRICITY OF ORBIT e	PERIOD OF ORBIT solar days	MEAN DIAMETER km (mi)	MASS RELATIVE TO EARTH	SURFACE GRAVITATIONAL ACCELERATION m/s^2 (ft/s^2)	ESCAPE VELOCITY km/s (mi/s)
Sun	—	—	—	1 392 000 (865 000)	333 000	274 (898)	616 (383)
Moon	384 398* (238 854)*	0.055	27.32	3 476 (2 160)	0.0123	1.62 (5.32)	2.37 (1.47)
Mercury	57.3×10^6 (35.6×10^6)	0.206	87.97	5 000 (3 100)	0.054	3.47 (11.4)	4.17 (2.59)
Venus	108×10^6 (67.2×10^6)	0.0068	224.70	12 400 (7 700)	0.815	8.44 (27.7)	10.24 (6.36)
Earth	149.6×10^6 (92.96×10^6)	0.0167	365.26	12 742† (7 918)†	1.000	9.821‡ (32.22)‡	11.18 (6.95)
Mars	227.9×10^6 (141.6×10^6)	0.093	686.98	6 788 (4 218)	0.107	3.73 (12.3)	5.03 (3.13)

* Mean distance to Earth (center-to-center)

† Diameter of sphere of equal volume, based on a spheroidal Earth with a polar diameter of 12 714 km (7900 mi) and an equatorial diameter of 12 756 km (7926 mi)

‡ For nonrotating spherical Earth, equivalent to absolute value at sea level and latitude 37.5°

TABLE D/3 PROPERTIES OF PLANE FIGURES


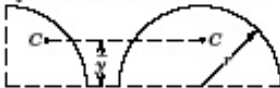
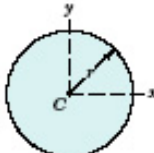
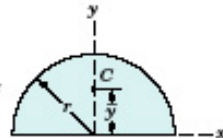
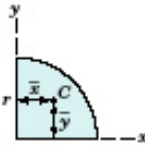
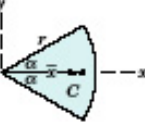
FIGURE	CENTROID	AREA MOMENTS OF INERTIA
<p>Arc Segment</p> 	$\bar{r} = \frac{r \sin \alpha}{\alpha}$	—
<p>Quarter and Semicircular Ares</p> 	$\bar{y} = \frac{2r}{\pi}$	—
<p>Circular Area</p> 	—	$I_x = I_y = \frac{\pi r^4}{4}$ $I_z = \frac{\pi r^4}{2}$
<p>Semicircular Area</p> 	$\bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{8}$ $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) r^4$ $I_z = \frac{\pi r^4}{4}$
<p>Quarter-Circular Area</p> 	$\bar{x} = \bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi} \right) r^4$ $I_z = \frac{\pi r^4}{8}$
<p>Area of Circular Sector</p> 	$\bar{x} = \frac{2}{3} \frac{r \sin \alpha}{\alpha}$	$I_x = \frac{r^4}{4} \left(\alpha - \frac{1}{2} \sin 2\alpha \right)$ $I_y = \frac{r^4}{4} \left(\alpha + \frac{1}{2} \sin 2\alpha \right)$ $I_z = \frac{1}{2} r^4 \alpha$

TABLE D/3 PROPERTIES OF PLANE FIGURES *Continued*

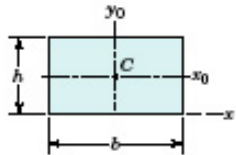
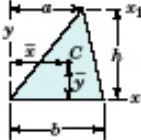
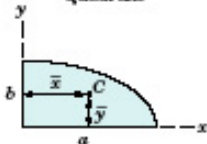
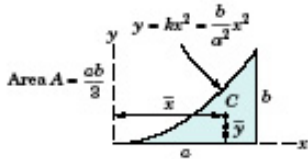
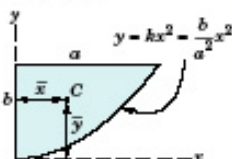
FIGURE	CENTROID	AREA MOMENTS OF INERTIA
<p>Rectangular Area</p> 	—	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh}{12}(b^2 + h^2)$
<p>Triangular Area</p> 	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$
<p>Area of Elliptical Quadrant</p> 	$\bar{x} = \frac{4a}{3\pi}$ $\bar{y} = \frac{4b}{3\pi}$	$I_x = \frac{\pi ab^3}{16}, \bar{I}_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)ab^3$ $I_y = \frac{\pi a^3b}{16}, \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^3b$ $I_z = \frac{\pi ab}{16}(a^2 + b^2)$
<p>Subparabolic Area</p> 	$\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3b}{10}$	$I_x = \frac{ab^3}{21}$ $I_y = \frac{a^3b}{5}$ $I_z = ab\left(\frac{a^2}{5} + \frac{b^2}{21}\right)$
<p>Parabolic Area</p> 	$\bar{x} = \frac{3a}{8}$ $\bar{y} = \frac{3b}{5}$	$I_x = \frac{2ab^3}{7}$ $I_y = \frac{2a^3b}{15}$ $I_z = 2ab\left(\frac{a^2}{15} + \frac{b^2}{7}\right)$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS *Continued*

(m = mass of body shown)

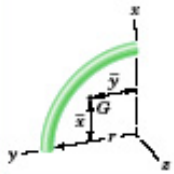
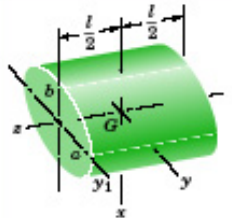
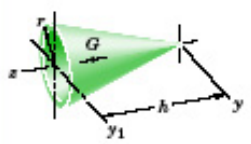
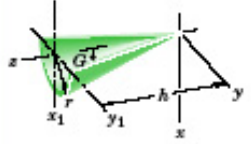
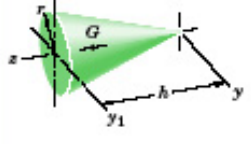
BODY	MASS CENTER	MASS MOMENTS OF INERTIA
 <p>Quarter-Circular Rod</p>	$\bar{x} = \bar{y}$ $= \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2}mr^2$ $I_z = mr^2$
 <p>Elliptical Cylinder</p>	—	$I_{xx} = \frac{1}{4}ma^2 + \frac{1}{12}ml^2$ $I_{yy} = \frac{1}{4}mb^2 + \frac{1}{12}ml^2$ $I_z = \frac{1}{4}m(a^2 + b^2)$ $I_{yy_1} = \frac{1}{4}mb^2 + \frac{1}{3}ml^2$
 <p>Conical Shell</p>	$\bar{z} = \frac{2h}{3}$	$I_{yy} = \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{yy_1} = \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_z = \frac{1}{2}mr^2$ $I_{yy} = \frac{1}{4}mr^2 + \frac{1}{18}mh^2$
 <p>Half Conical Shell</p>	$\bar{x} = \frac{4r}{3\pi}$ $\bar{z} = \frac{2h}{3}$	$I_{xx} = I_{yy}$ $= \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{xx_1} = I_{yy_1}$ $= \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_z = \frac{1}{2}mr^2$ $\bar{I}_z = \left(\frac{1}{2} - \frac{16}{9\pi^2} \right) mr^2$
 <p>Right Circular Cone</p>	$\bar{z} = \frac{3h}{4}$	$I_{yy} = \frac{3}{20}mr^2 + \frac{3}{8}mh^2$ $I_{yy_1} = \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_z = \frac{3}{10}mr^2$ $\bar{I}_{yy} = \frac{3}{20}mr^2 + \frac{3}{80}mh^2$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS *Continued*

(m = mass of body shown)

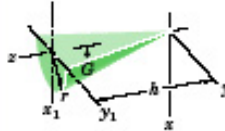
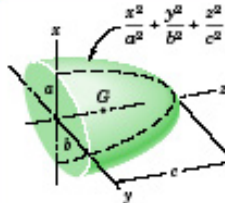
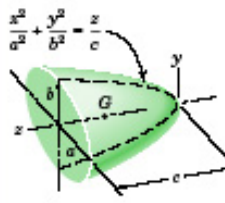
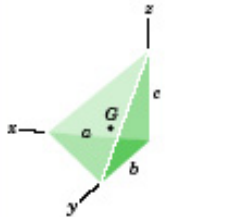
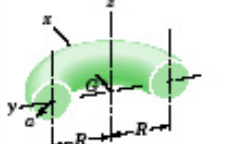
BODY	MASS CENTER	MASS MOMENTS OF INERTIA
 <p>Half Cone</p>	$\bar{x} = \frac{r}{\pi}$ $\bar{z} = \frac{3h}{4}$	$I_{xx} = I_{yy}$ $= \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{xx_1} = I_{yy_1}$ $= \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{zz} = \left(\frac{3}{10} - \frac{1}{\pi^2}\right)mr^2$
 <p>Semiellipsoid</p>	$\bar{z} = \frac{3c}{8}$	$I_{xx} = \frac{1}{8}m(b^2 + c^2)$ $I_{yy} = \frac{1}{8}m(a^2 + c^2)$ $I_{zz} = \frac{1}{8}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{8}m(b^2 + \frac{19}{64}c^2)$ $\bar{I}_{yy} = \frac{1}{8}m(a^2 + \frac{19}{64}c^2)$
 <p>Elliptic Paraboloid</p>	$\bar{z} = \frac{2c}{3}$	$I_{xx} = \frac{1}{8}mb^2 + \frac{1}{2}mc^2$ $I_{yy} = \frac{1}{8}ma^2 + \frac{1}{2}mc^2$ $I_{zz} = \frac{1}{8}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{8}m(b^2 + \frac{1}{3}c^2)$ $\bar{I}_{yy} = \frac{1}{8}m(a^2 + \frac{1}{3}c^2)$
 <p>Rectangular Tetrahedron</p>	$\bar{x} = \frac{a}{4}$ $\bar{y} = \frac{b}{4}$ $\bar{z} = \frac{c}{4}$	$I_{xx} = \frac{1}{10}m(b^2 + c^2)$ $I_{yy} = \frac{1}{10}m(a^2 + c^2)$ $I_{zz} = \frac{1}{10}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{3}{80}m(b^2 + c^2)$ $\bar{I}_{yy} = \frac{3}{80}m(a^2 + c^2)$ $\bar{I}_{zz} = \frac{3}{80}m(a^2 + b^2)$
 <p>Half Torus</p>	$\bar{x} = \frac{a^2 + 4R^2}{2\pi R}$	$I_{xx} = I_{yy} = \frac{1}{2}mR^2 + \frac{5}{8}ma^2$ $I_{zz} = mR^2 + \frac{3}{4}ma^2$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS

(m = mass of body shown)

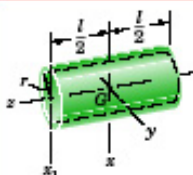
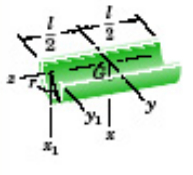
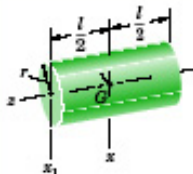
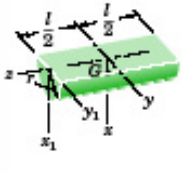
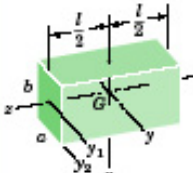
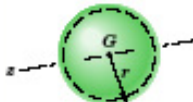

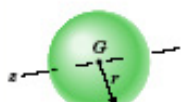
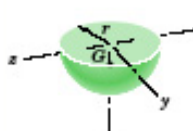
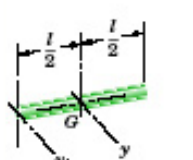
BODY	MASS CENTER	MASS MOMENTS OF INERTIA
 <p>Circular Cylindrical Shell</p>	—	$I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$ $I_{x,y_1} = \frac{1}{2}mr^2 + \frac{1}{3}ml^2$ $I_z = mr^2$
 <p>Half Cylindrical Shell</p>	$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy}$ $= \frac{1}{2}mr^2 + \frac{1}{12}ml^2$ $I_{x,y_1} = I_{y,y_1}$ $= \frac{1}{2}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = mr^2$ $\bar{I}_{zz} = \left(1 - \frac{4}{\pi^2}\right)mr^2$
 <p>Circular Cylinder</p>	—	$I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x,y_1} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_z = \frac{1}{2}mr^2$
 <p>Semicylinder</p>	$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy}$ $= \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x,y_1} = I_{y,y_1}$ $= \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
 <p>Rectangular Parallelepiped</p>	—	$I_{xx} = \frac{1}{12}m(a^2 + l^2)$ $I_{yy} = \frac{1}{12}m(b^2 + l^2)$ $I_z = \frac{1}{12}m(a^2 + b^2)$ $I_{y,y_1} = \frac{1}{12}mb^2 + \frac{1}{3}ml^2$ $I_{y_1,y_1} = \frac{1}{3}m(b^2 + l^2)$

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS *Continued*

(m = mass of body shown)

BODY	MASS CENTER	MASS MOMENTS OF INERTIA
 <p>Spherical Shell</p>	—	$I_{zz} = \frac{2}{3}mr^2$
 <p>Hemispherical Shell</p>	$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}mr^2$ $I_{yy} = I_{zz} = \frac{8}{15}mr^2$
 <p>Sphere</p>	—	$I_{zz} = \frac{2}{5}mr^2$
 <p>Hemisphere</p>	$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$ $I_{yy} = I_{zz} = \frac{83}{320}mr^2$
 <p>Uniform Slender Rod</p>	—	$I_{yy} = \frac{1}{12}ml^2$ $I_{y,y_1} = \frac{1}{3}ml^2$

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 1 : Introduction to Statics.

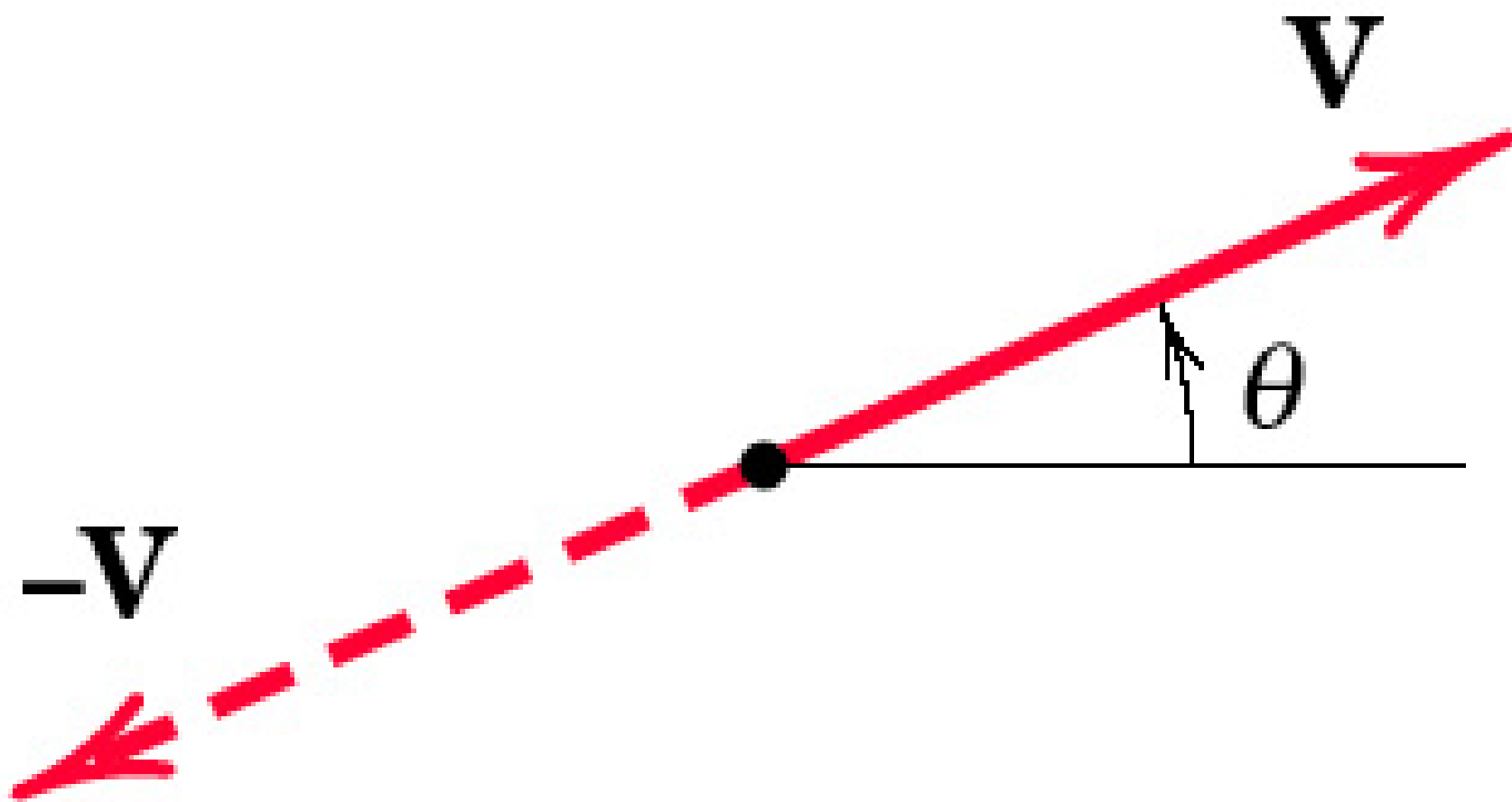


Figure 1 / 1

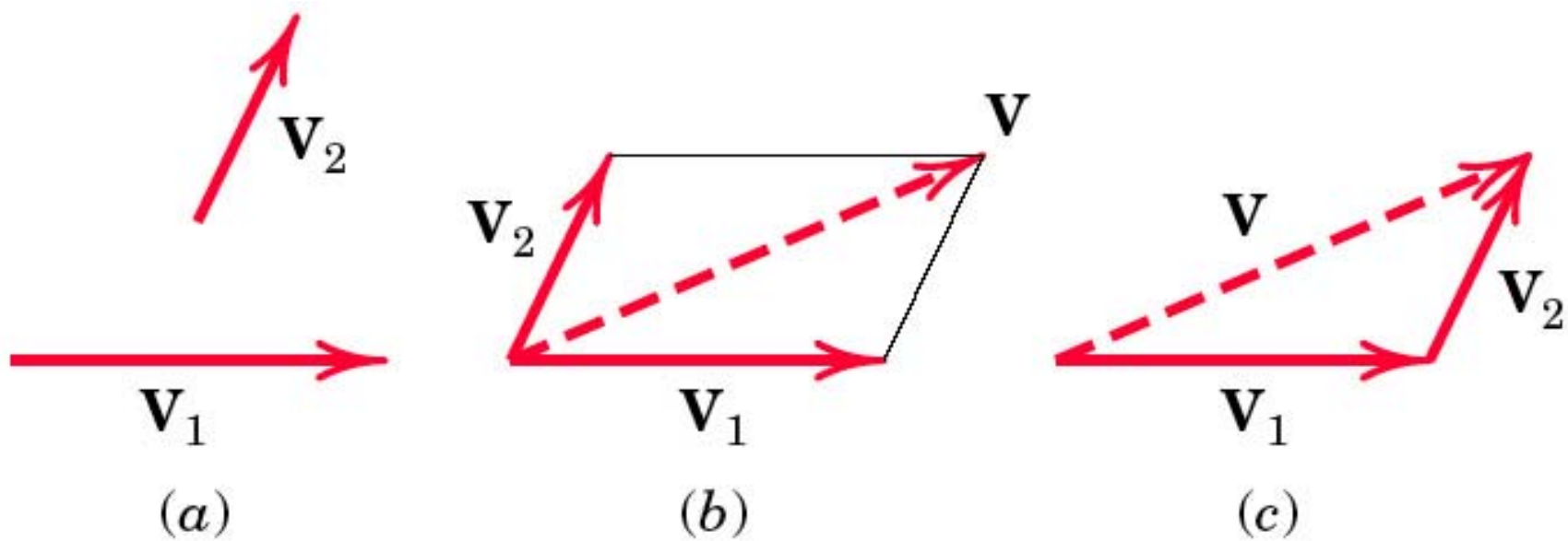


Figure 1/2

01_02

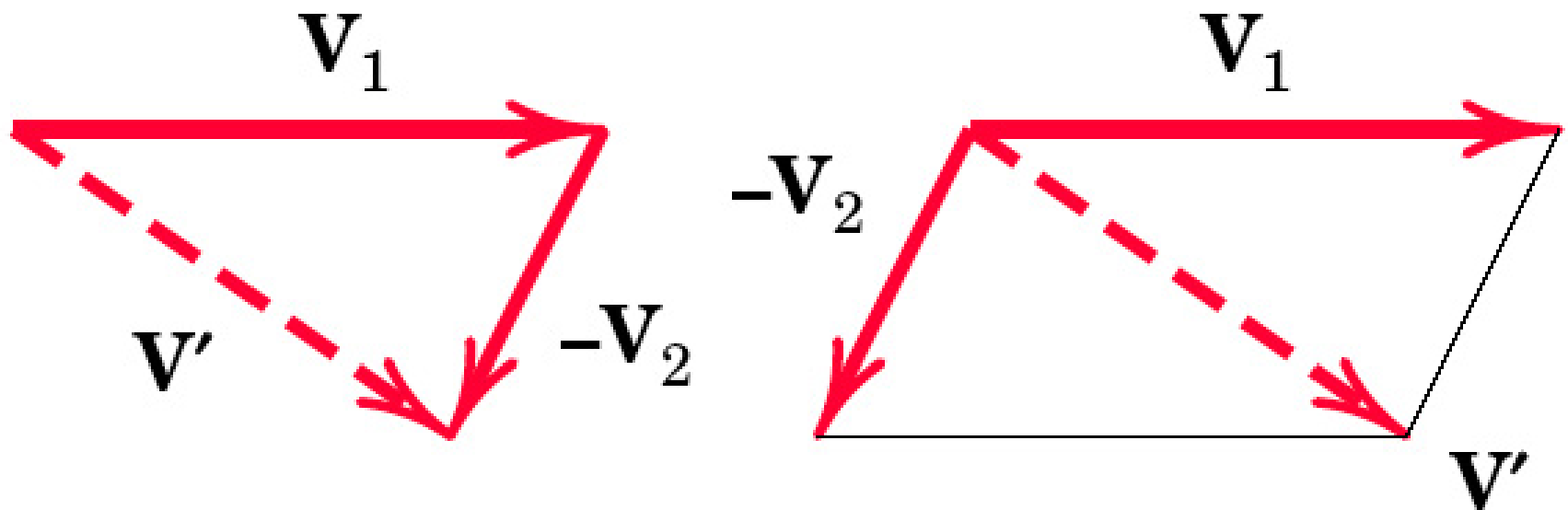


Figure 1/3

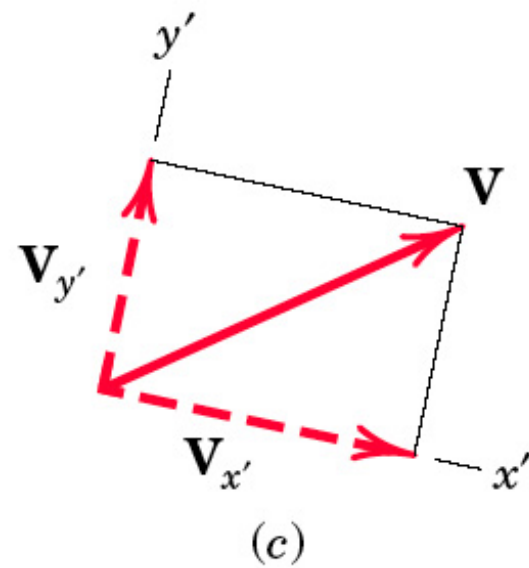
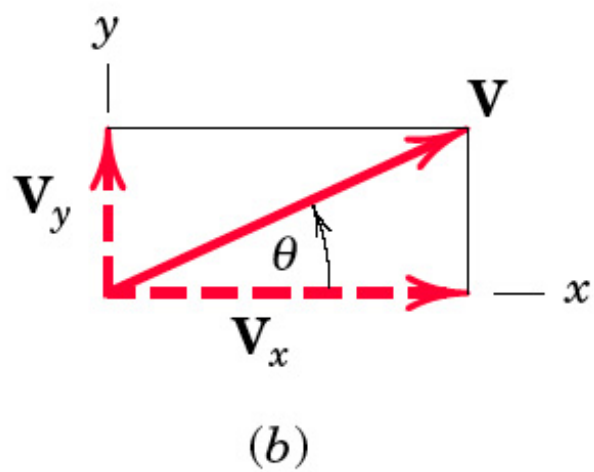
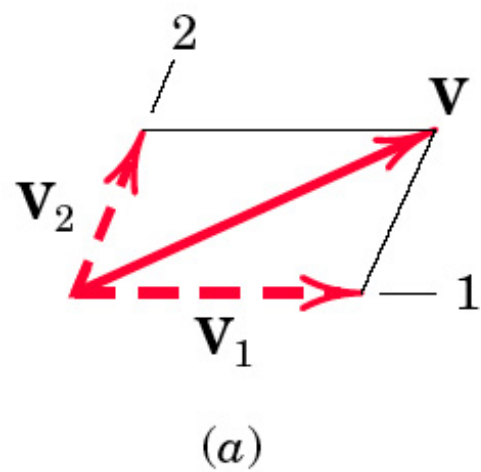


Figure 1/4

01_04

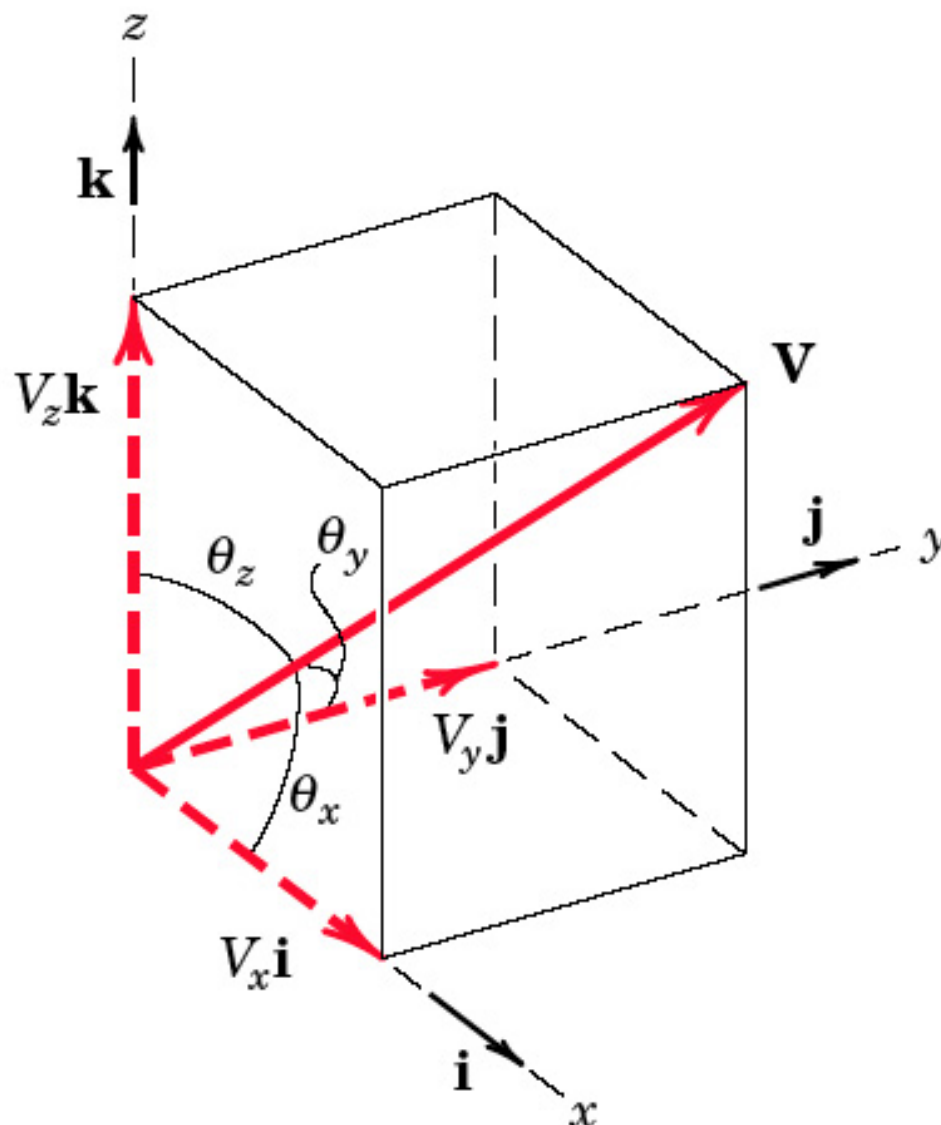


Figure 1/5

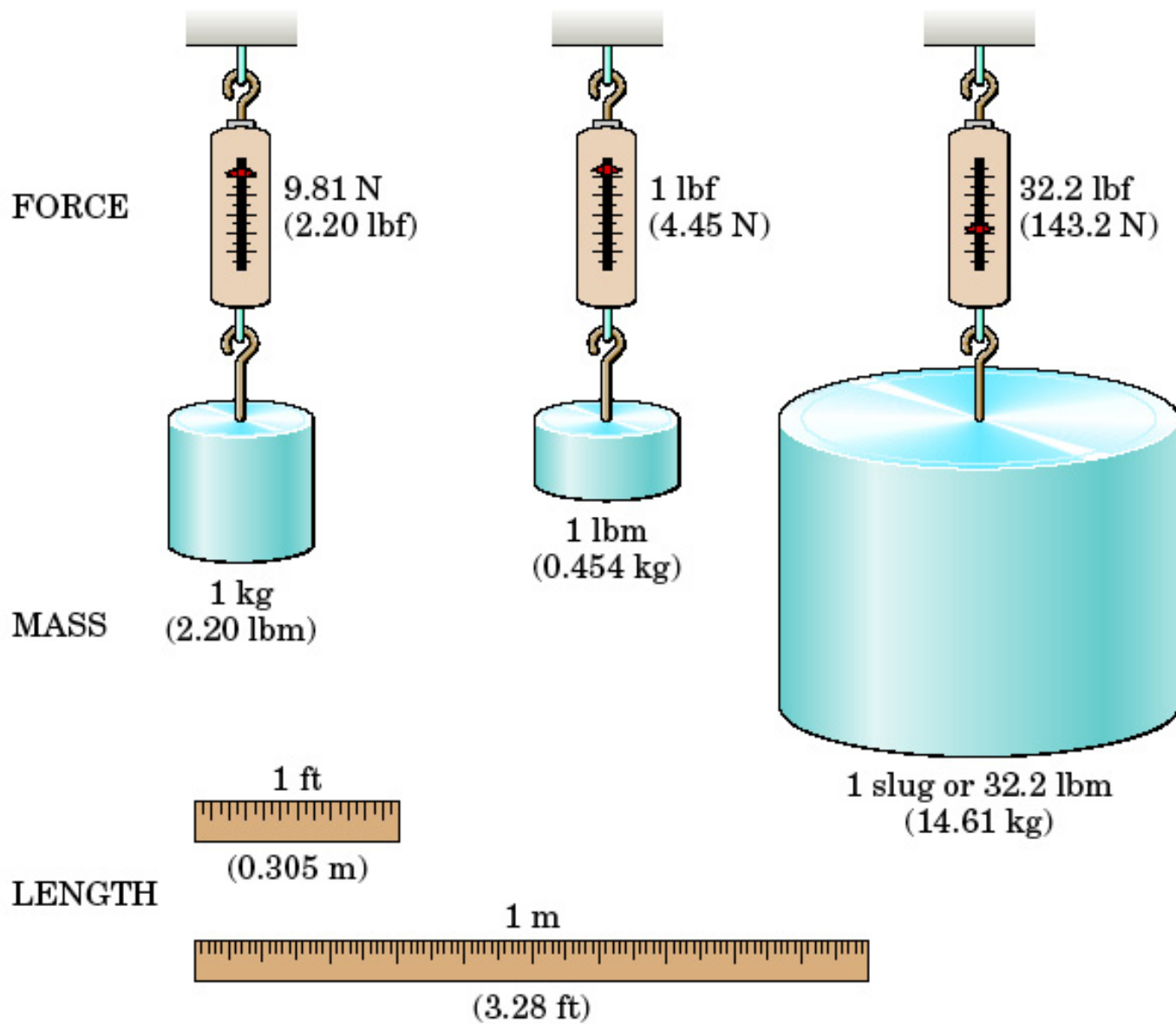


Figure 1/6

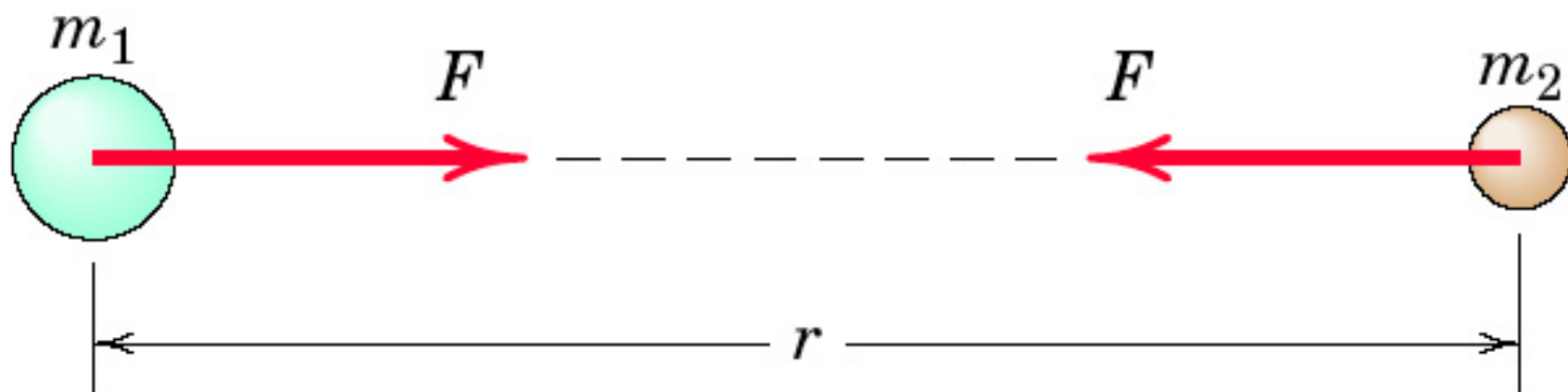


Figure 1/7

01_07

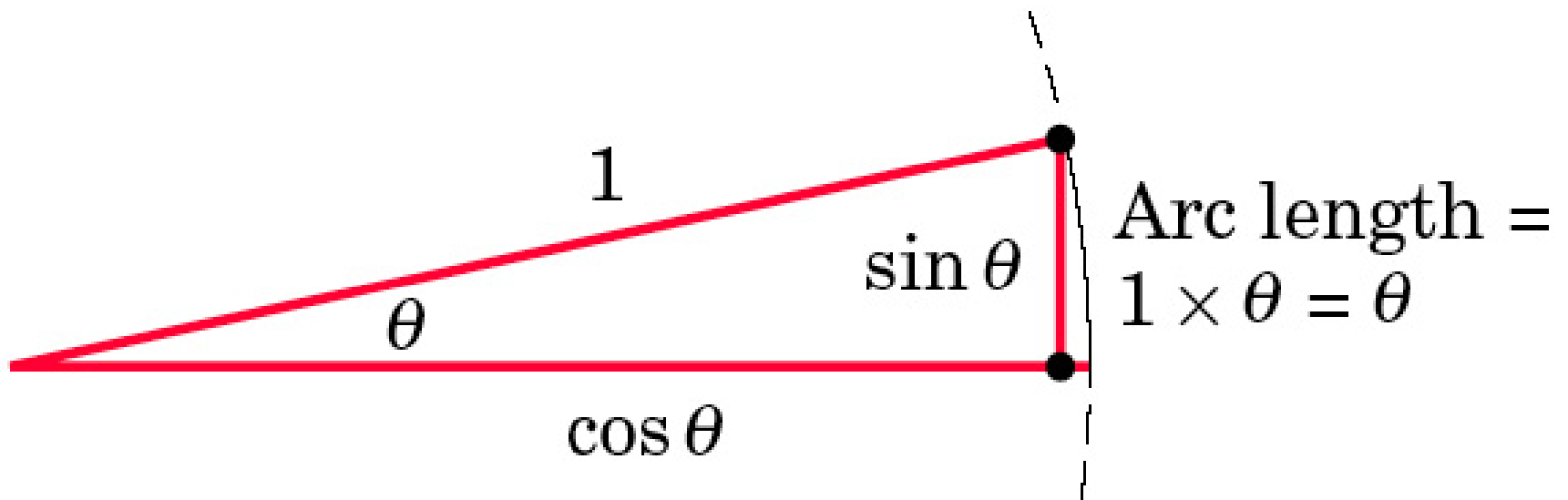
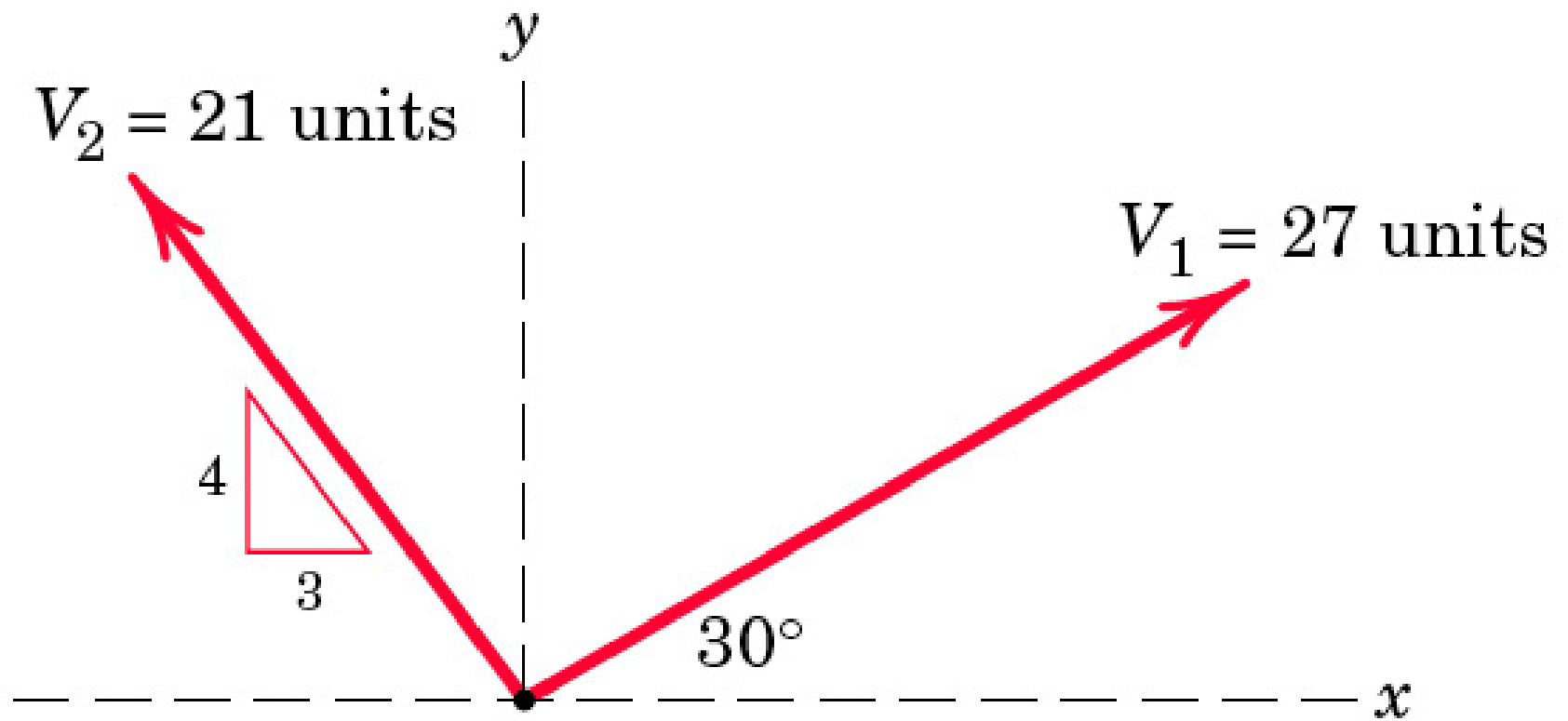


Figure 1/8

01_08



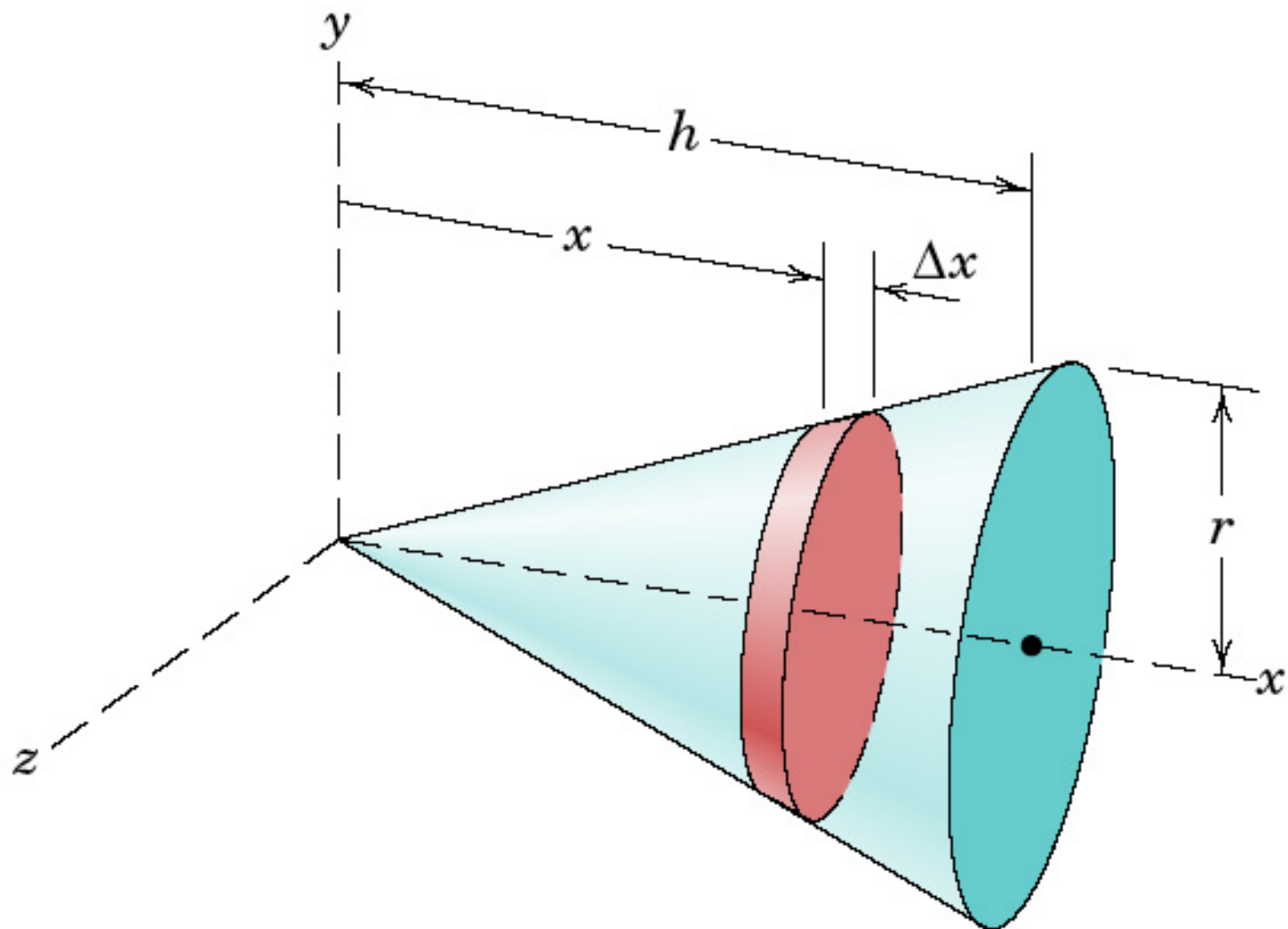
Problem 1/2

P_01_02



Problem 1/9

P_01_09



Problem 1/10

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 2 : Force Systems.

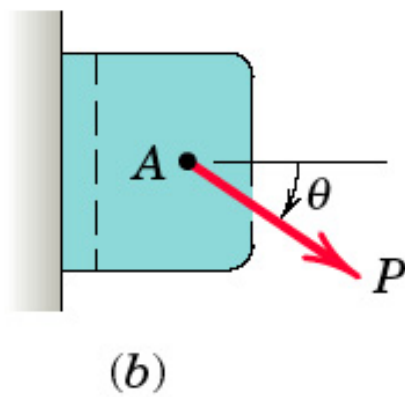
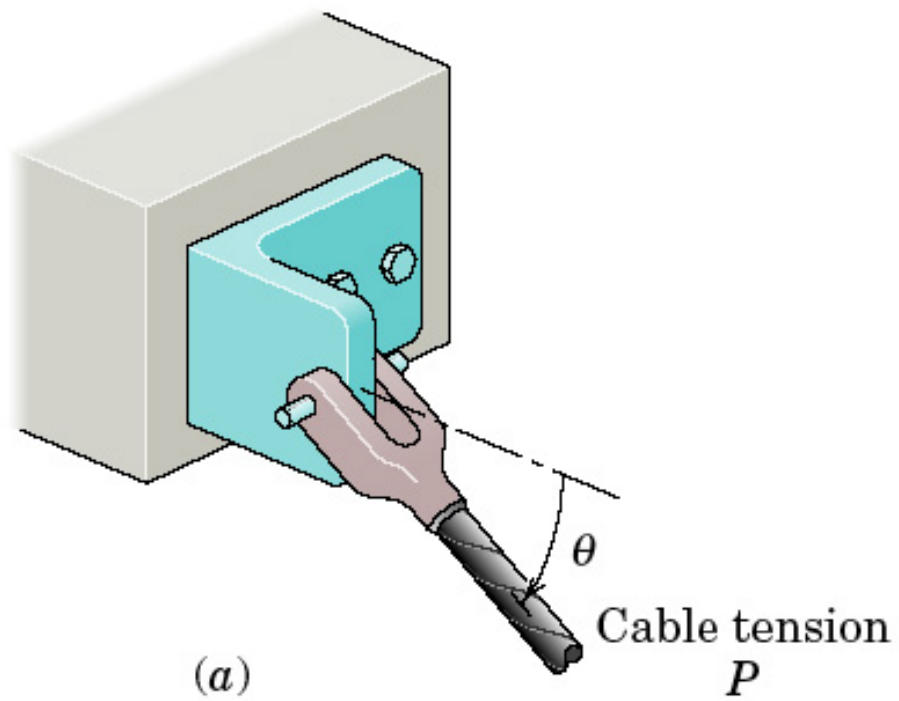


Figure 2/1

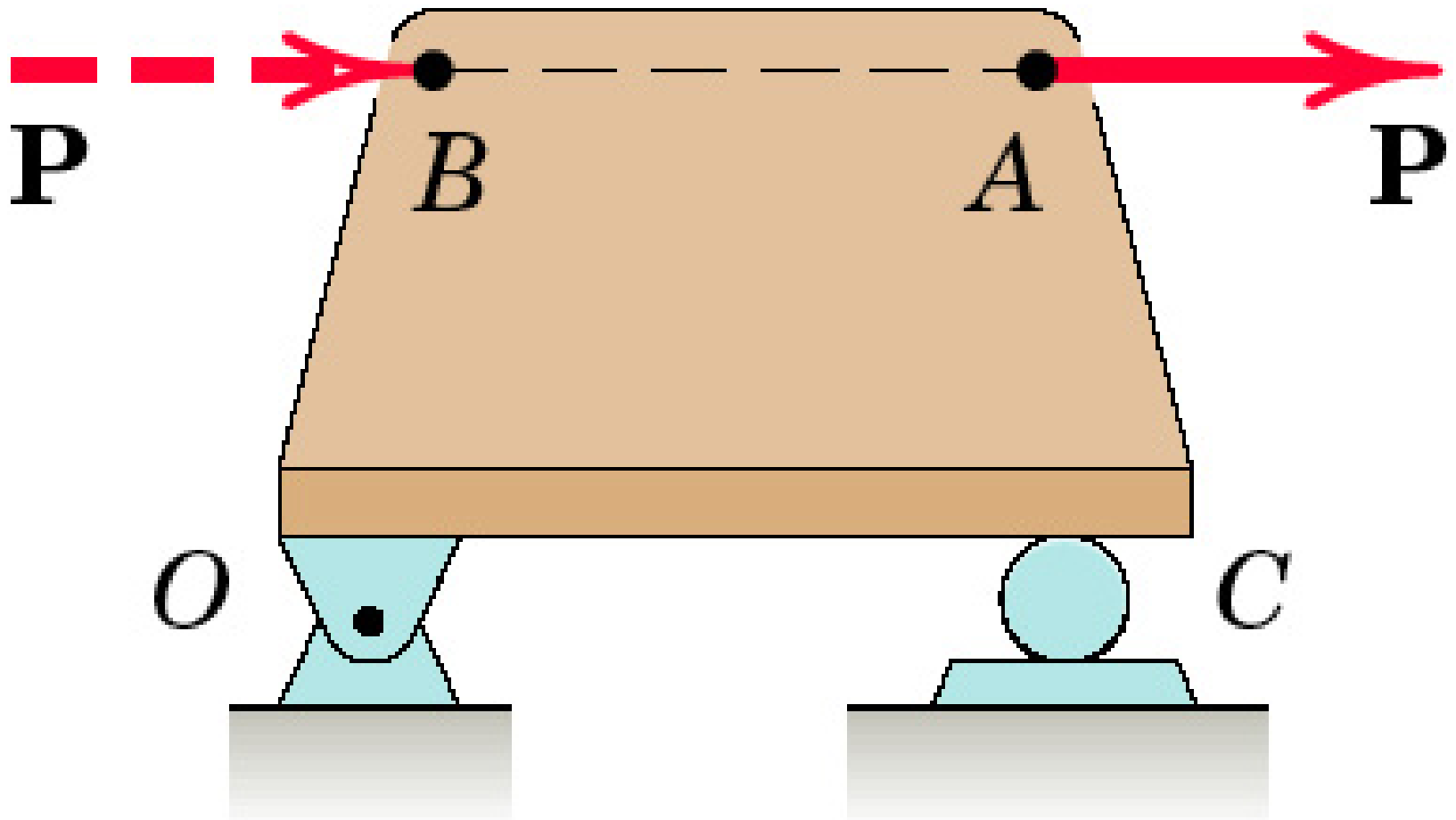
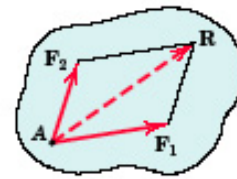
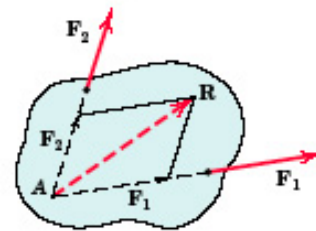


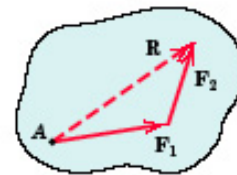
Figure 2/2



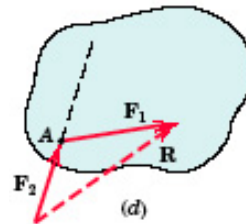
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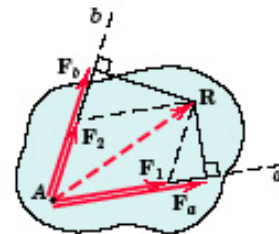
(b)



(c)



(d)



(e)

Figure 2/3

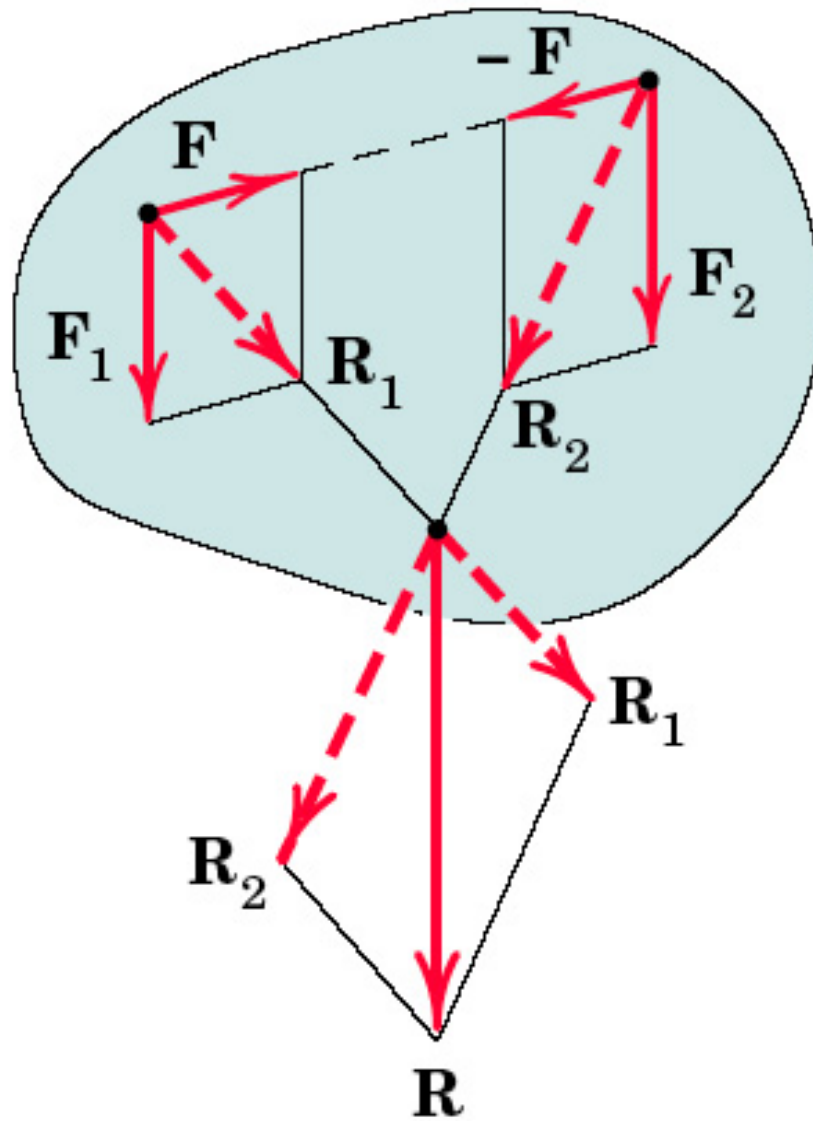


Figure 2/4

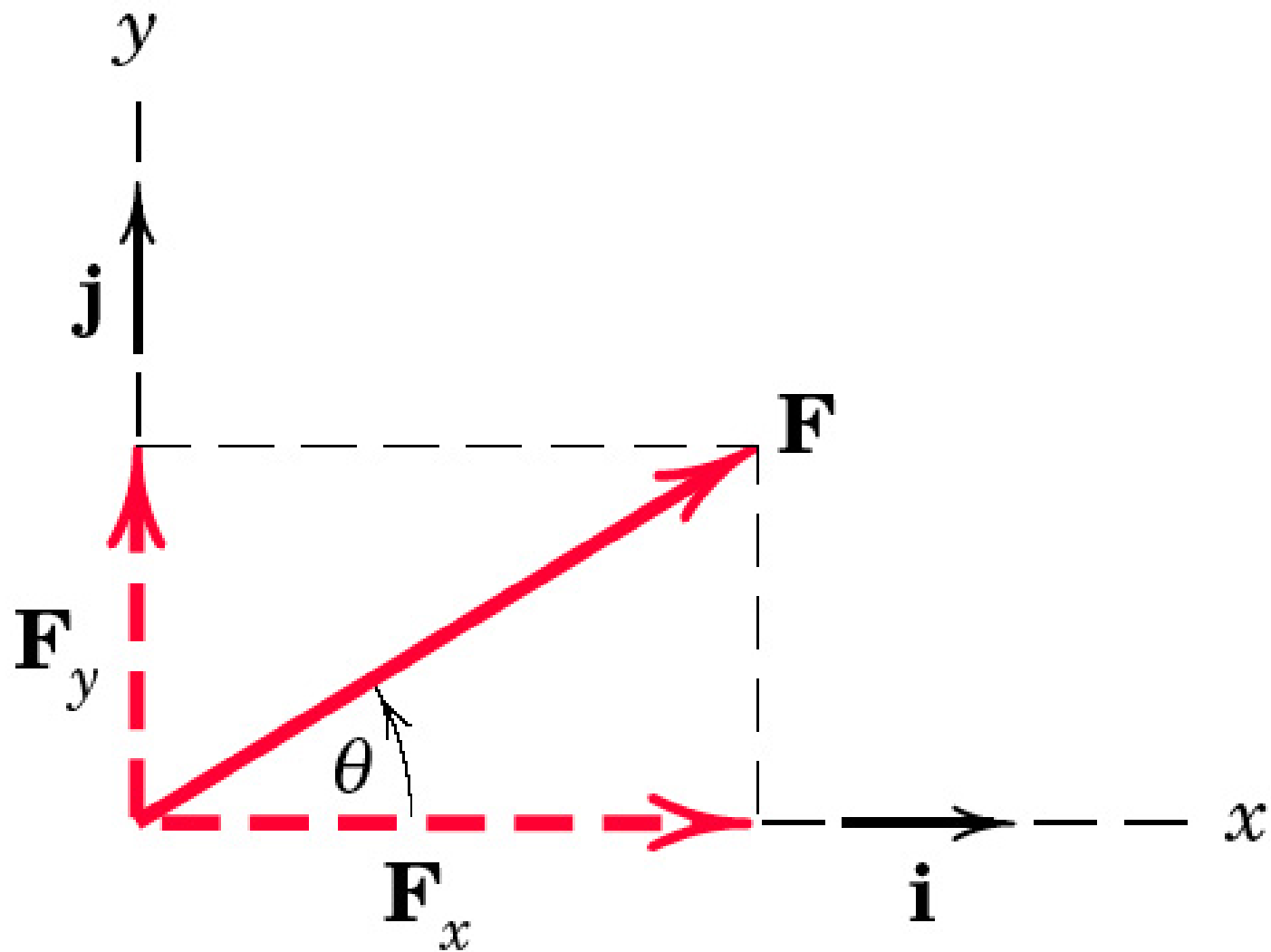


Figure 2/5

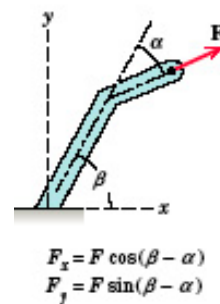
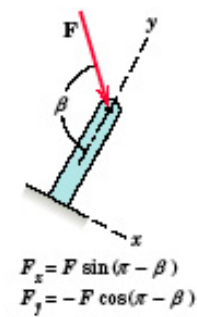
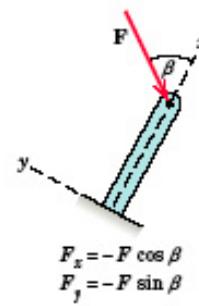
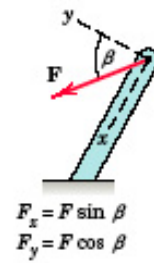


Figure 2/6

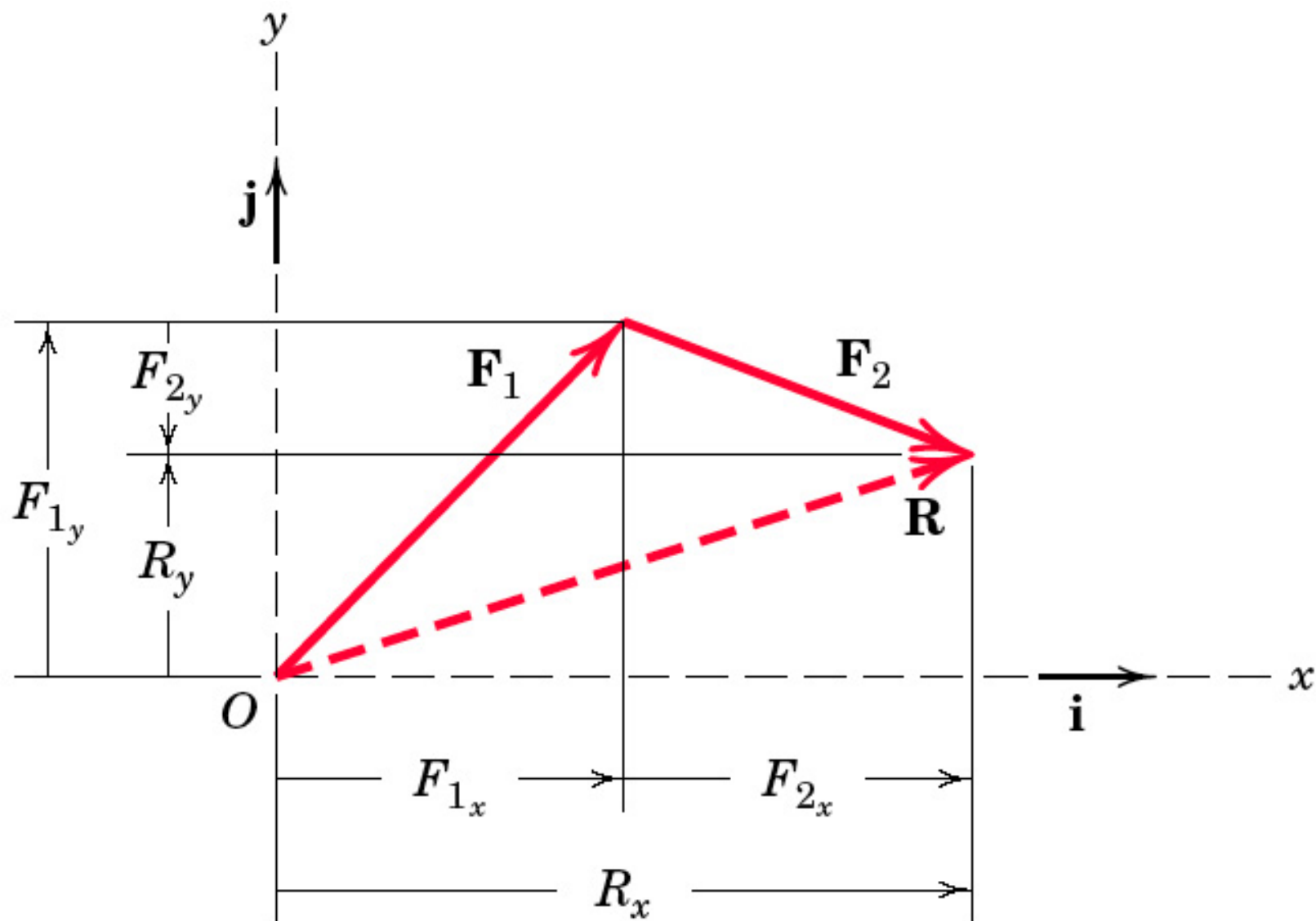


Figure 2/7

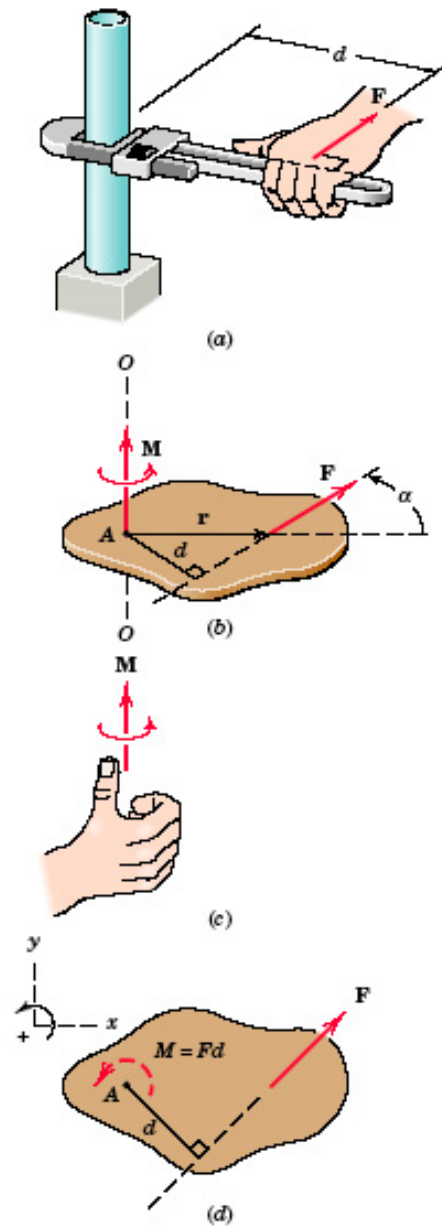


Figure 2/8

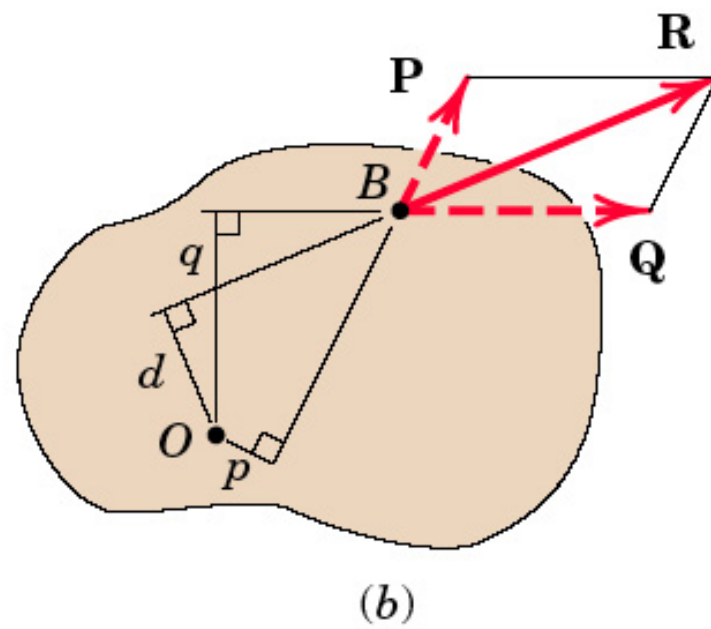
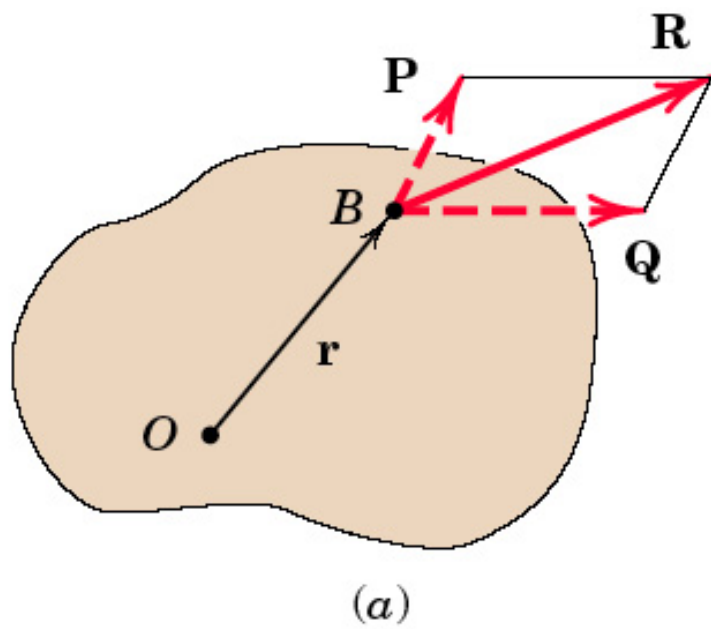
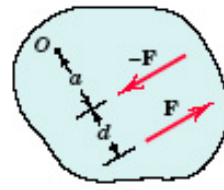
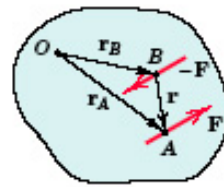


Figure 2/9

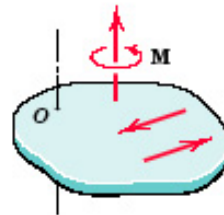
02_09



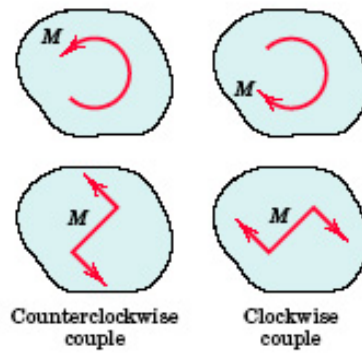
(a)



(b)



(c)



(d)

Figure 2/10

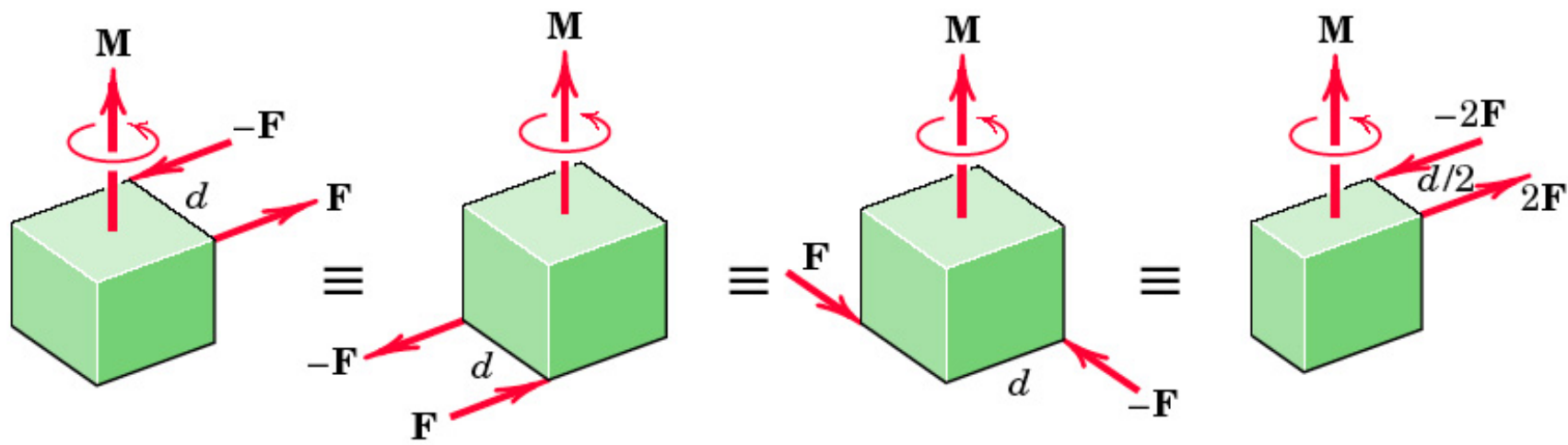


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02_11

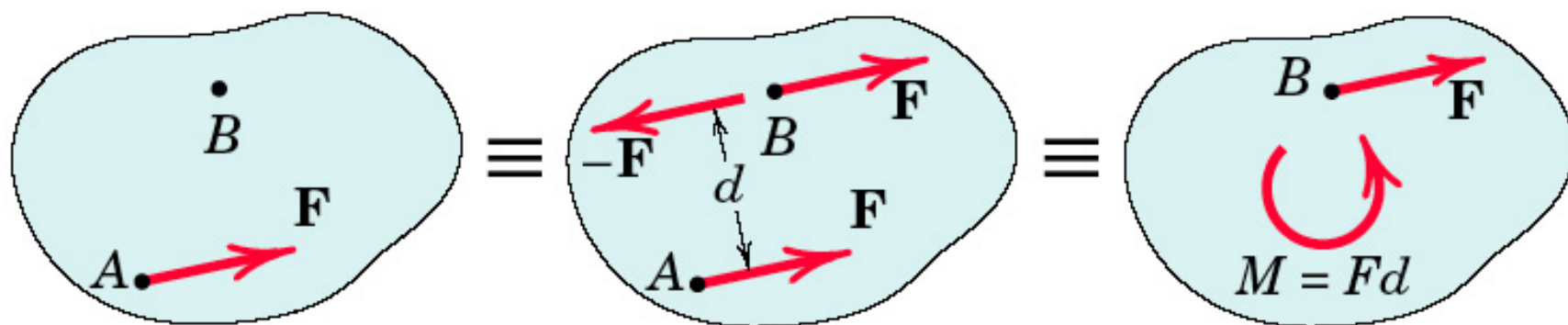
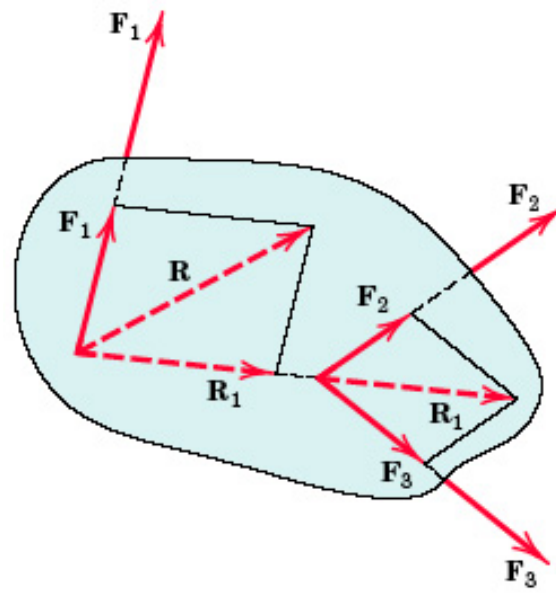
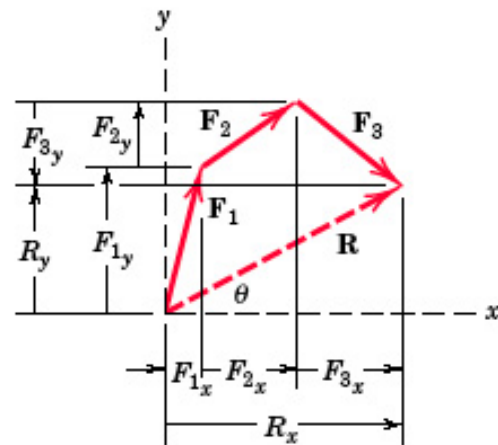


Figure 2/12



(a)



(b)

Figure 2/13

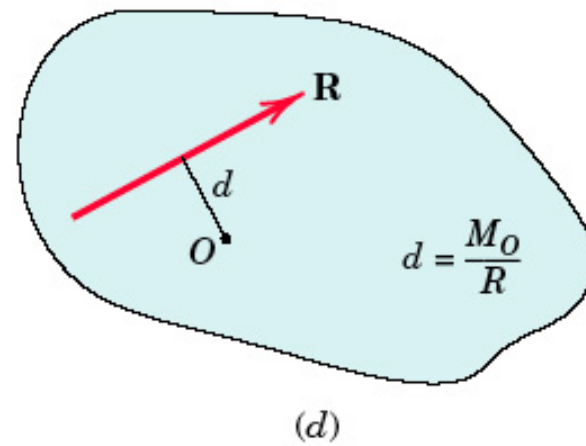
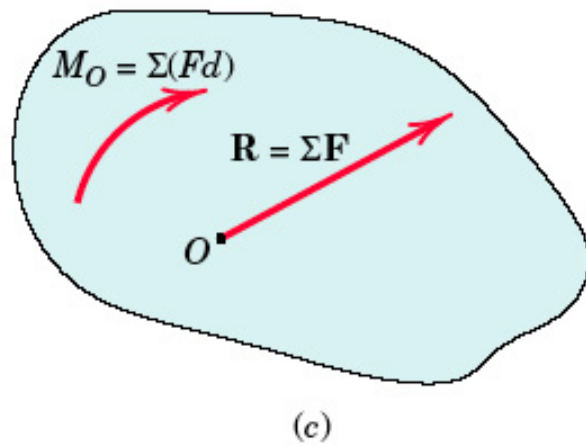
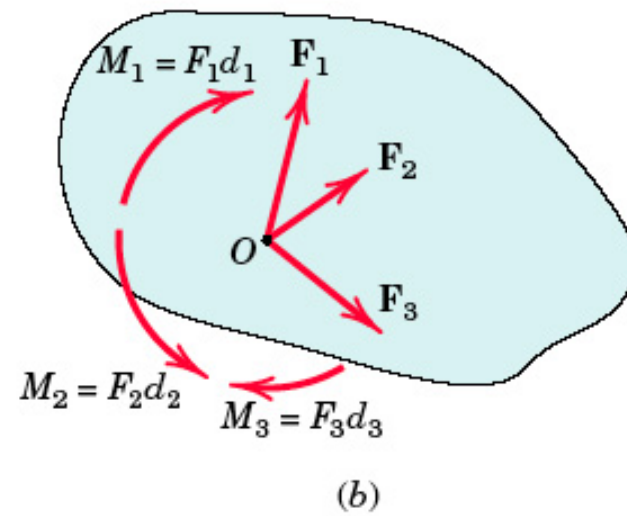
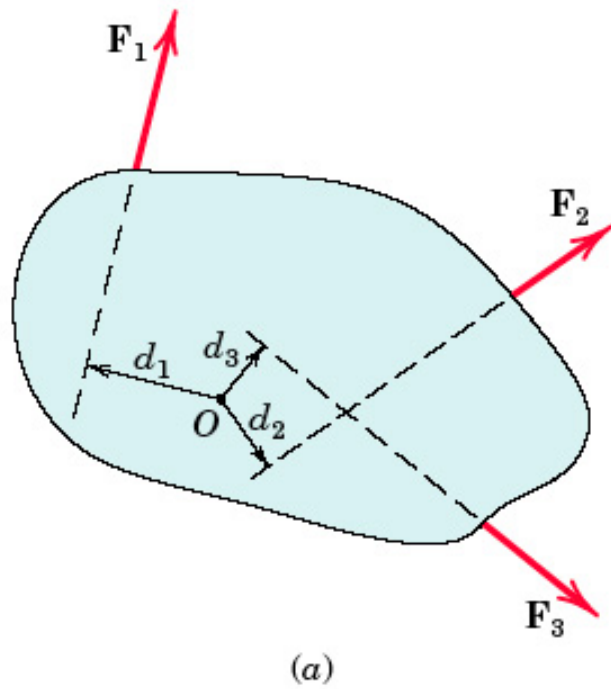


Figure 2/14

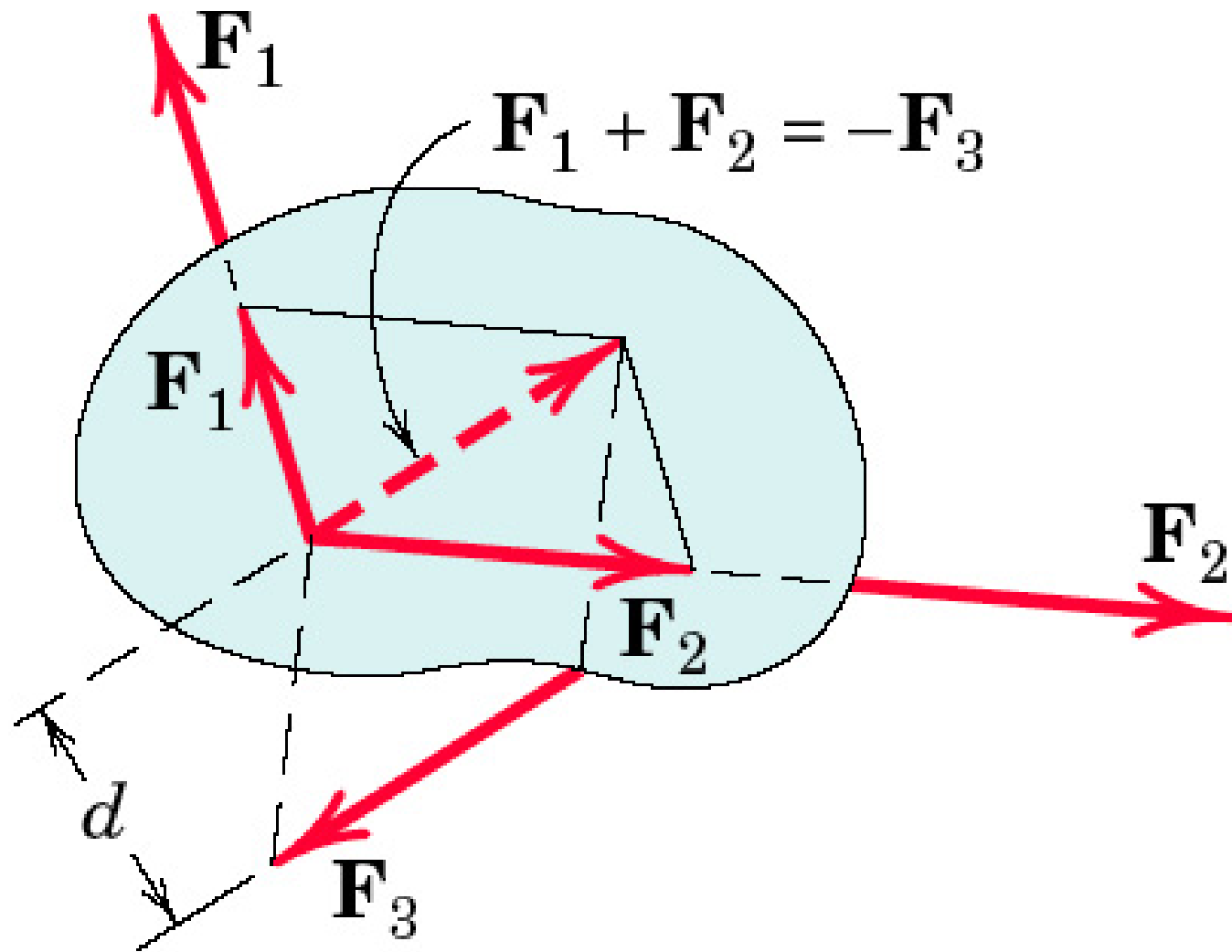


Figure 2/15

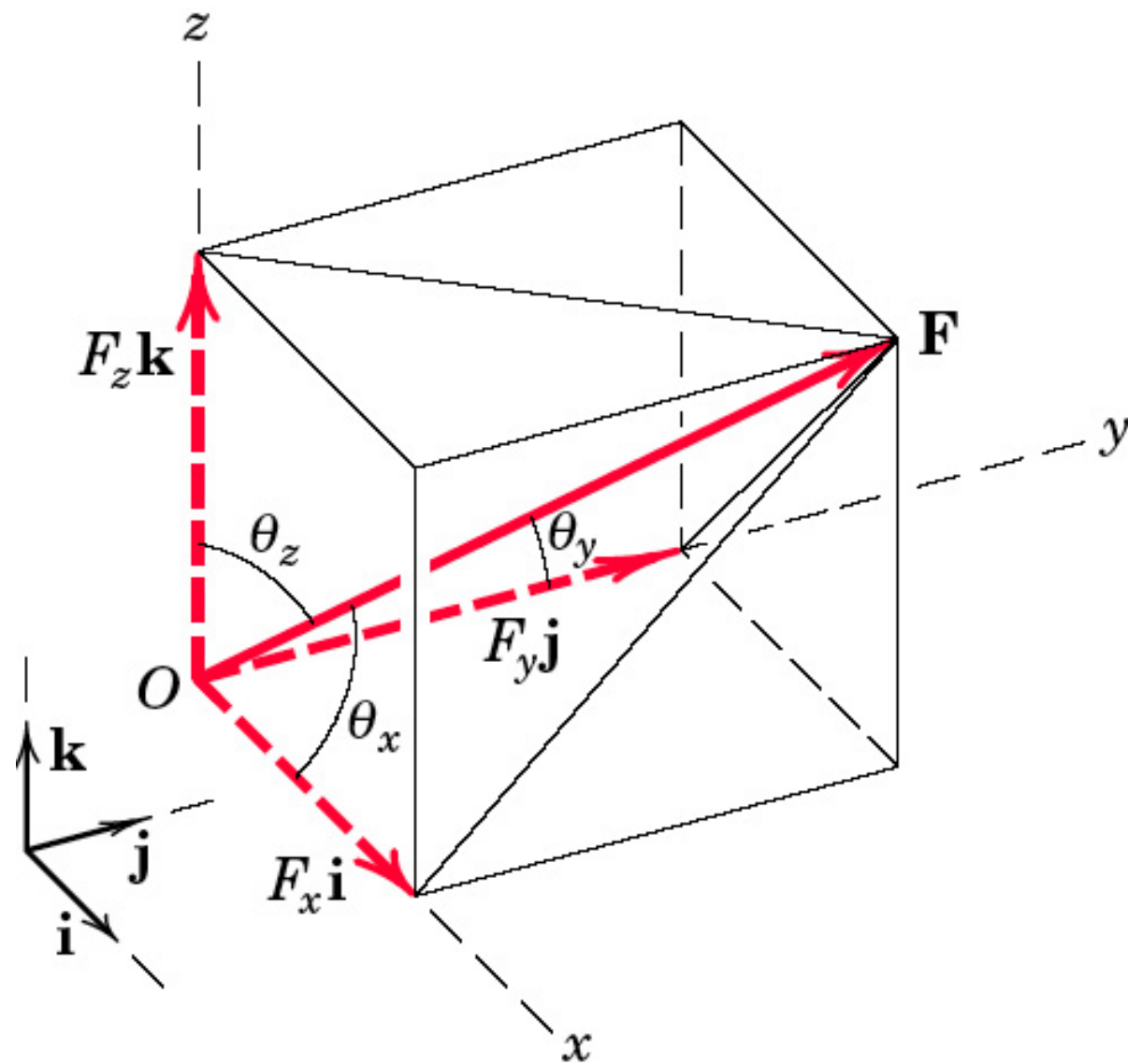


Figure 2/16

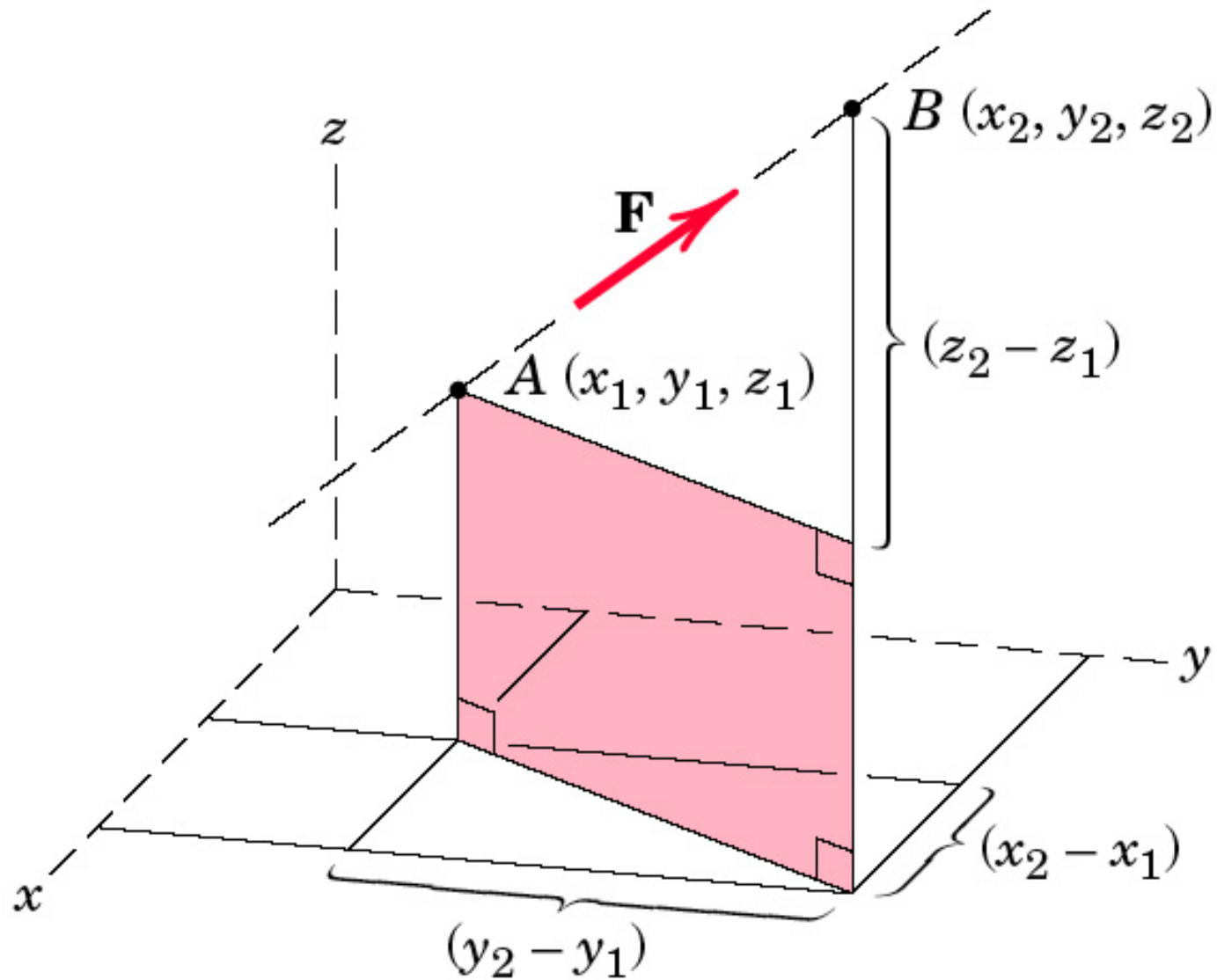


Figure 2/17

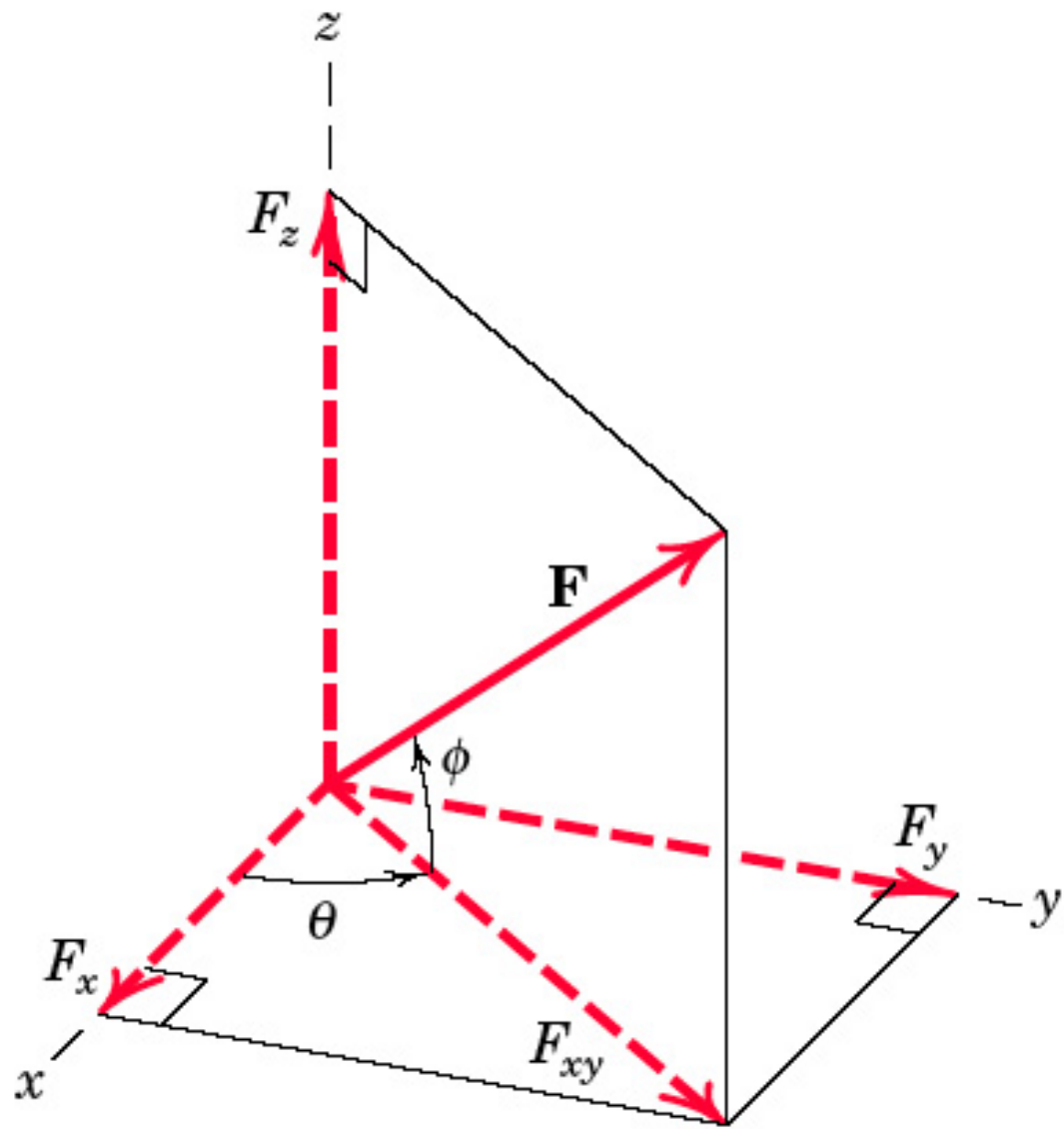
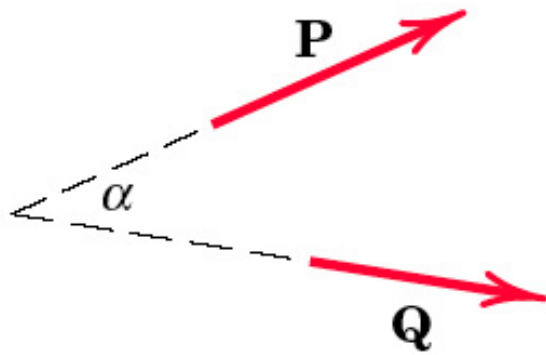
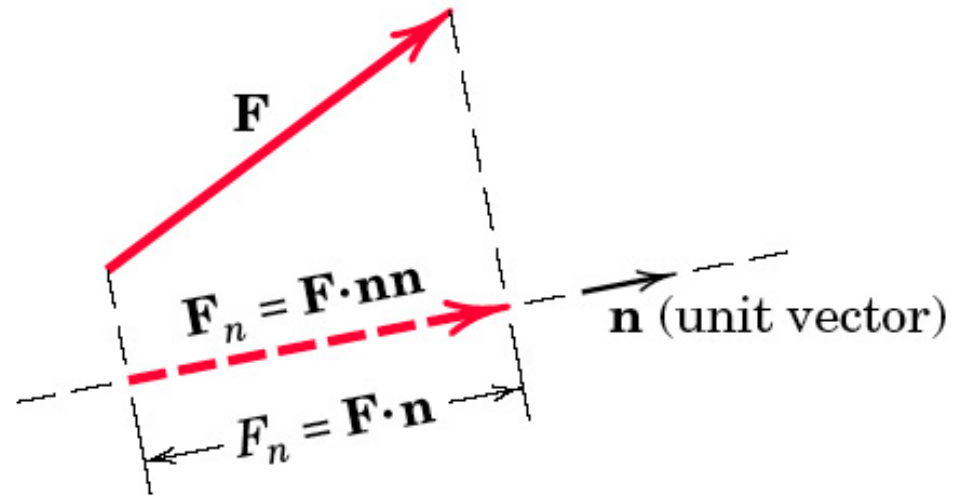


Figure 2/18



(a)



(b)

Figure 2/19

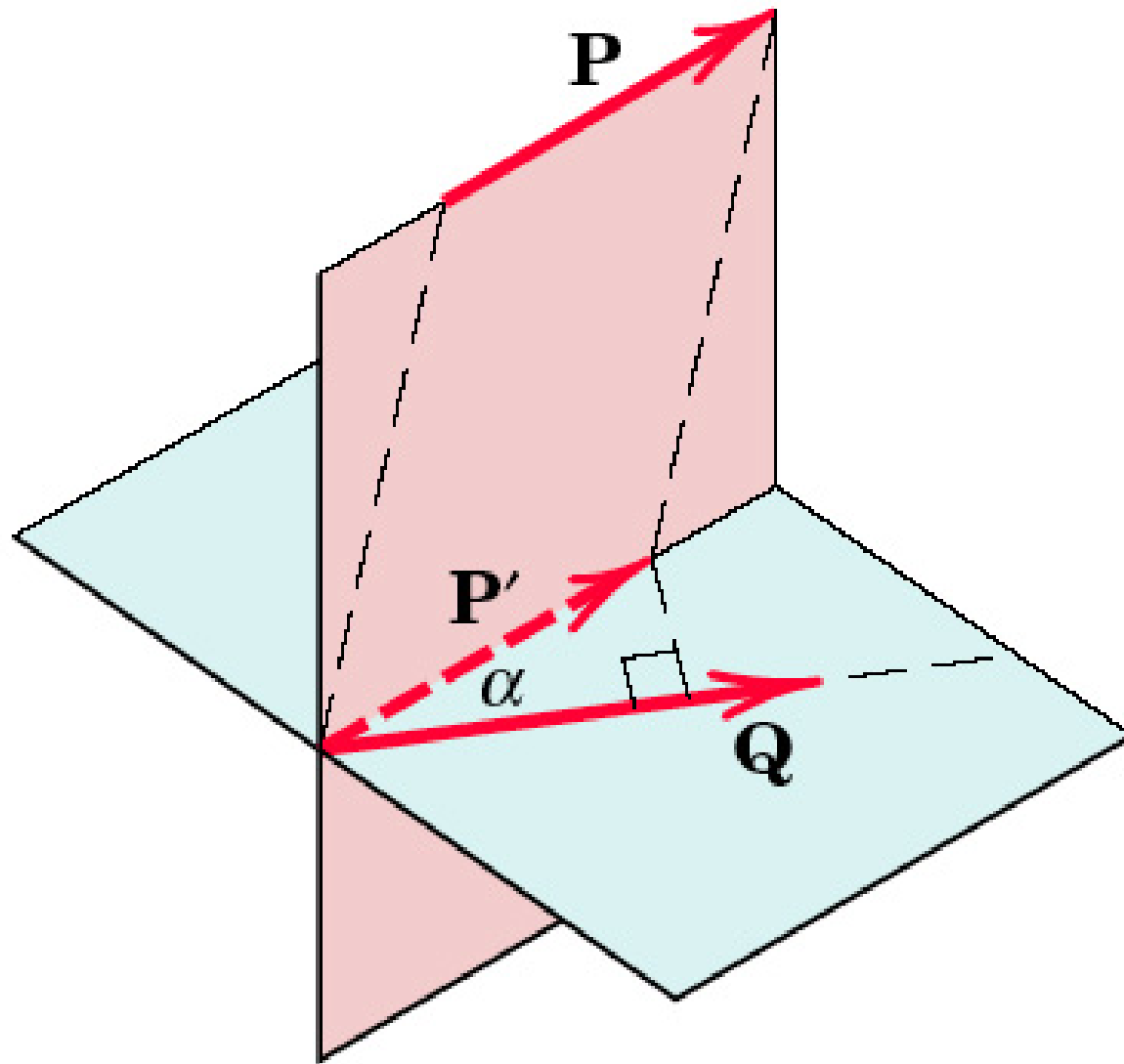


Figure 2/20

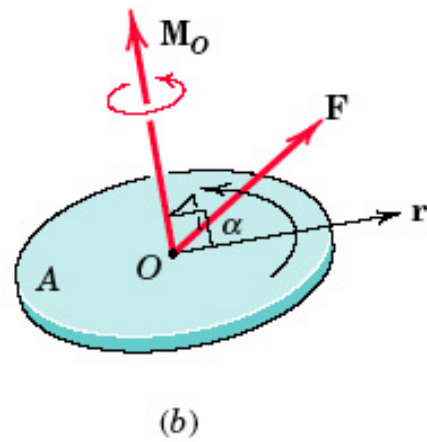
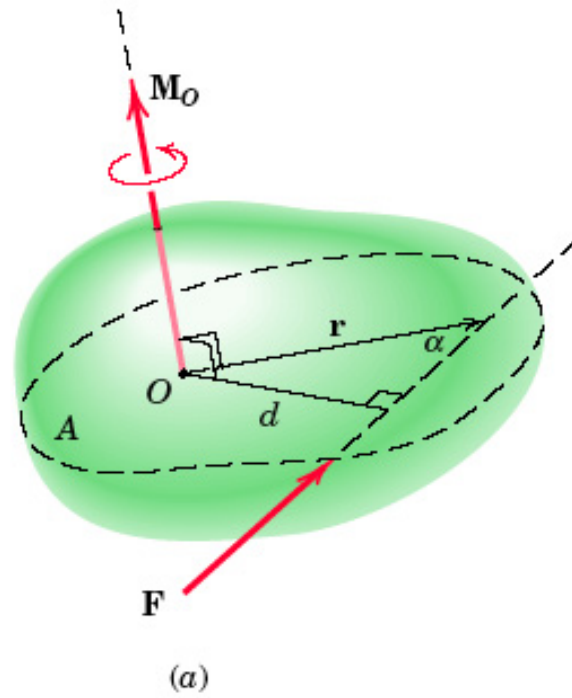


Figure 2/21

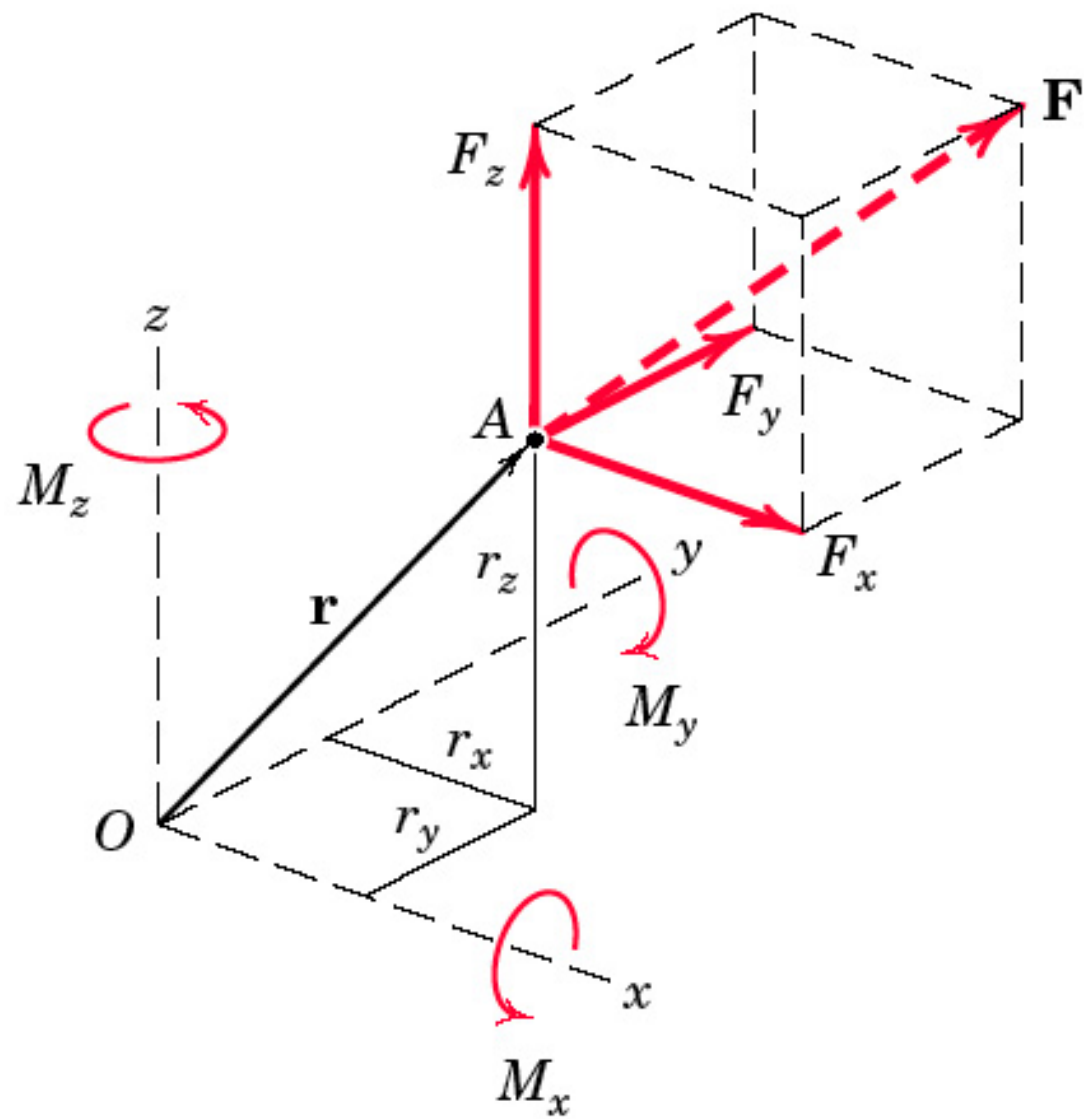


Figure 2/22

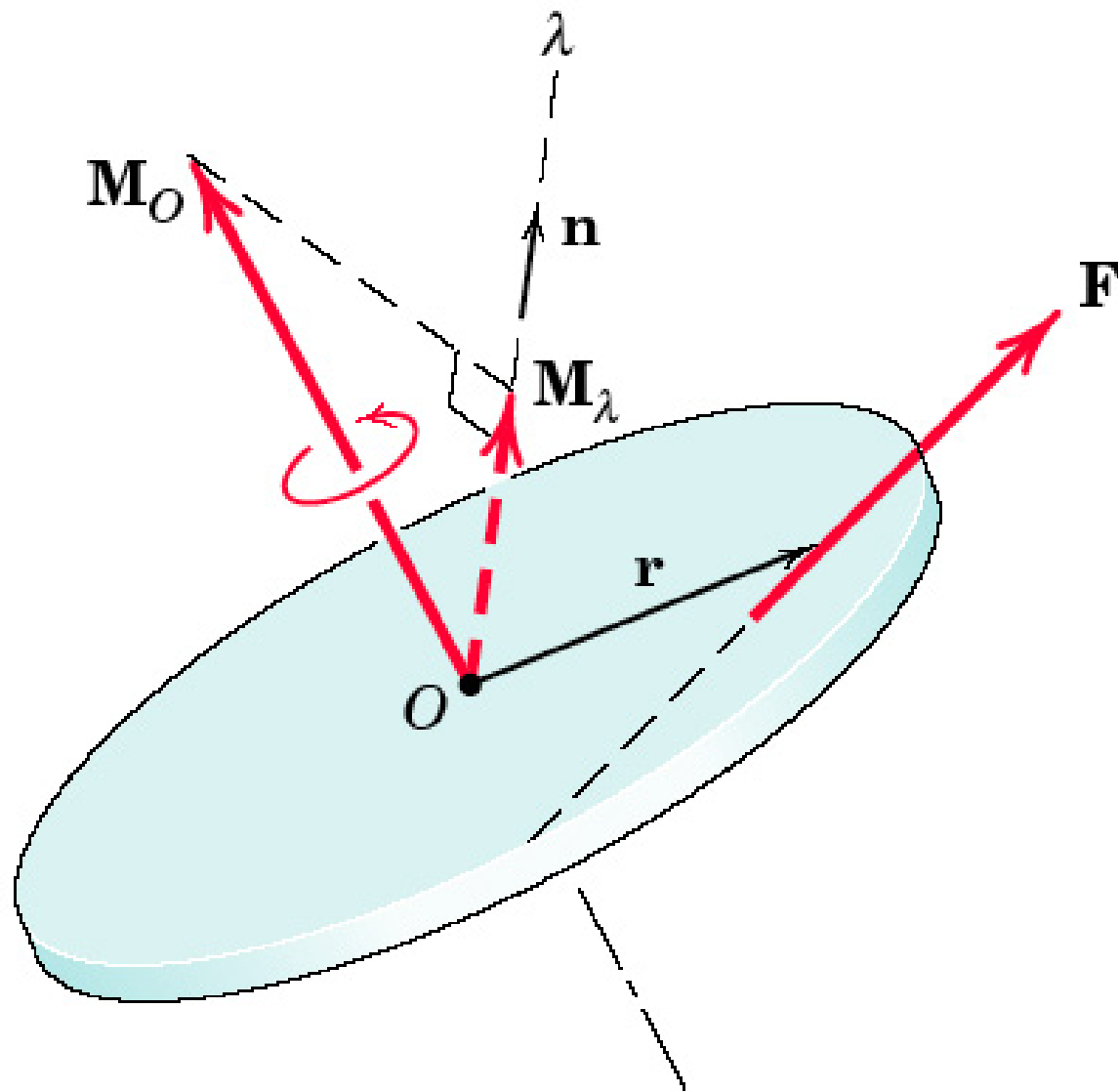


Figure 2/23

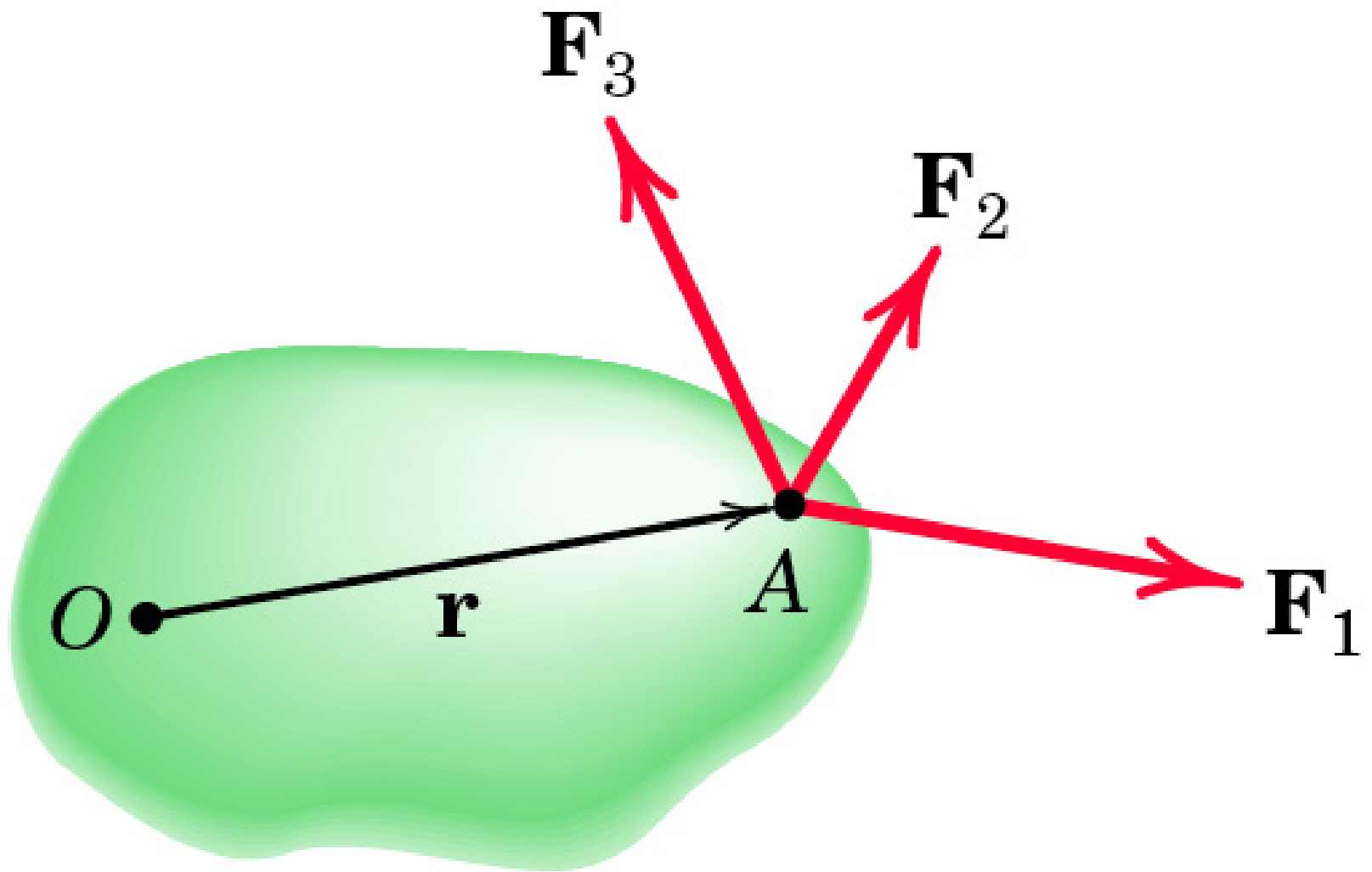


Figure 2/24

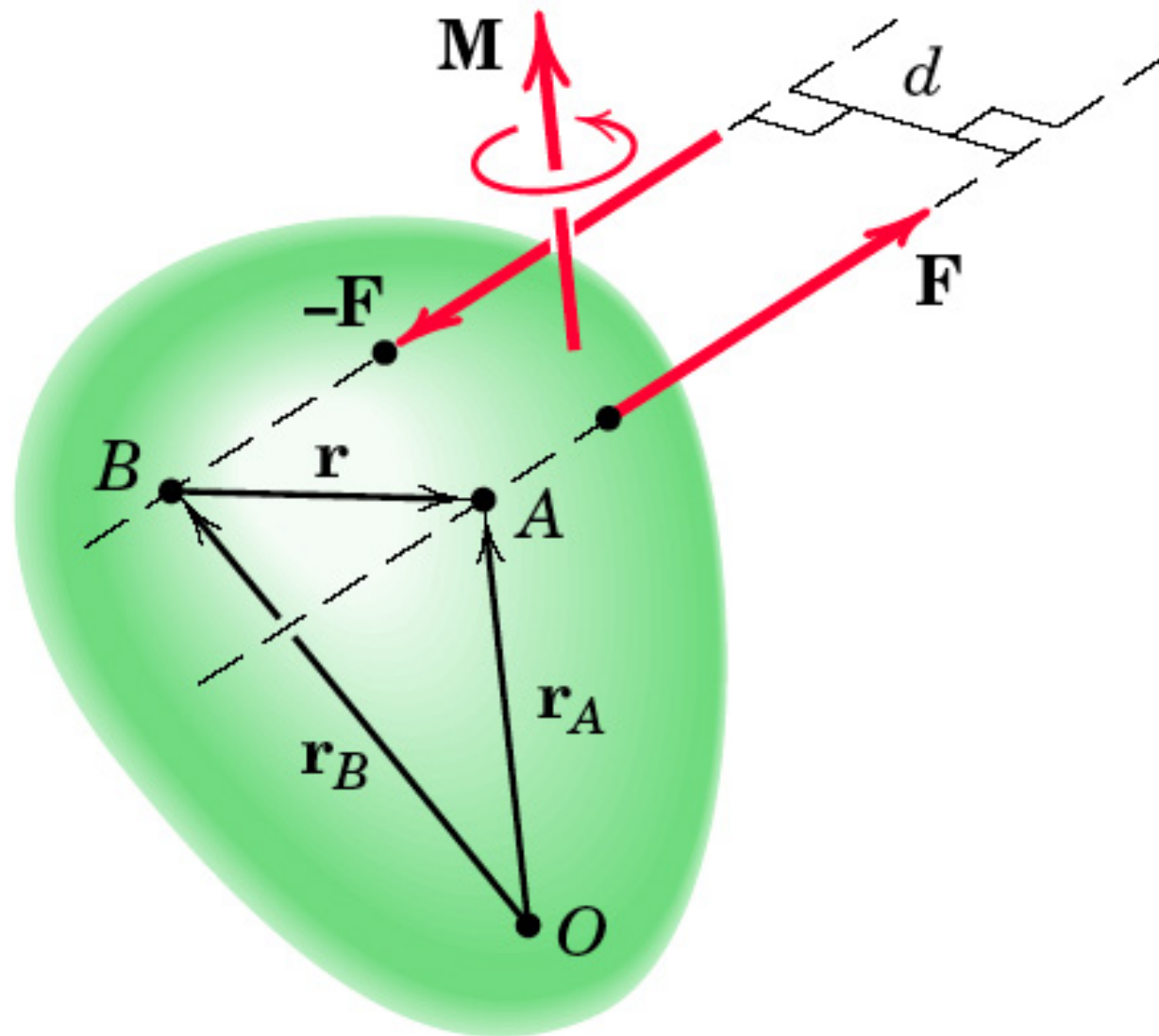


Figure 2/25

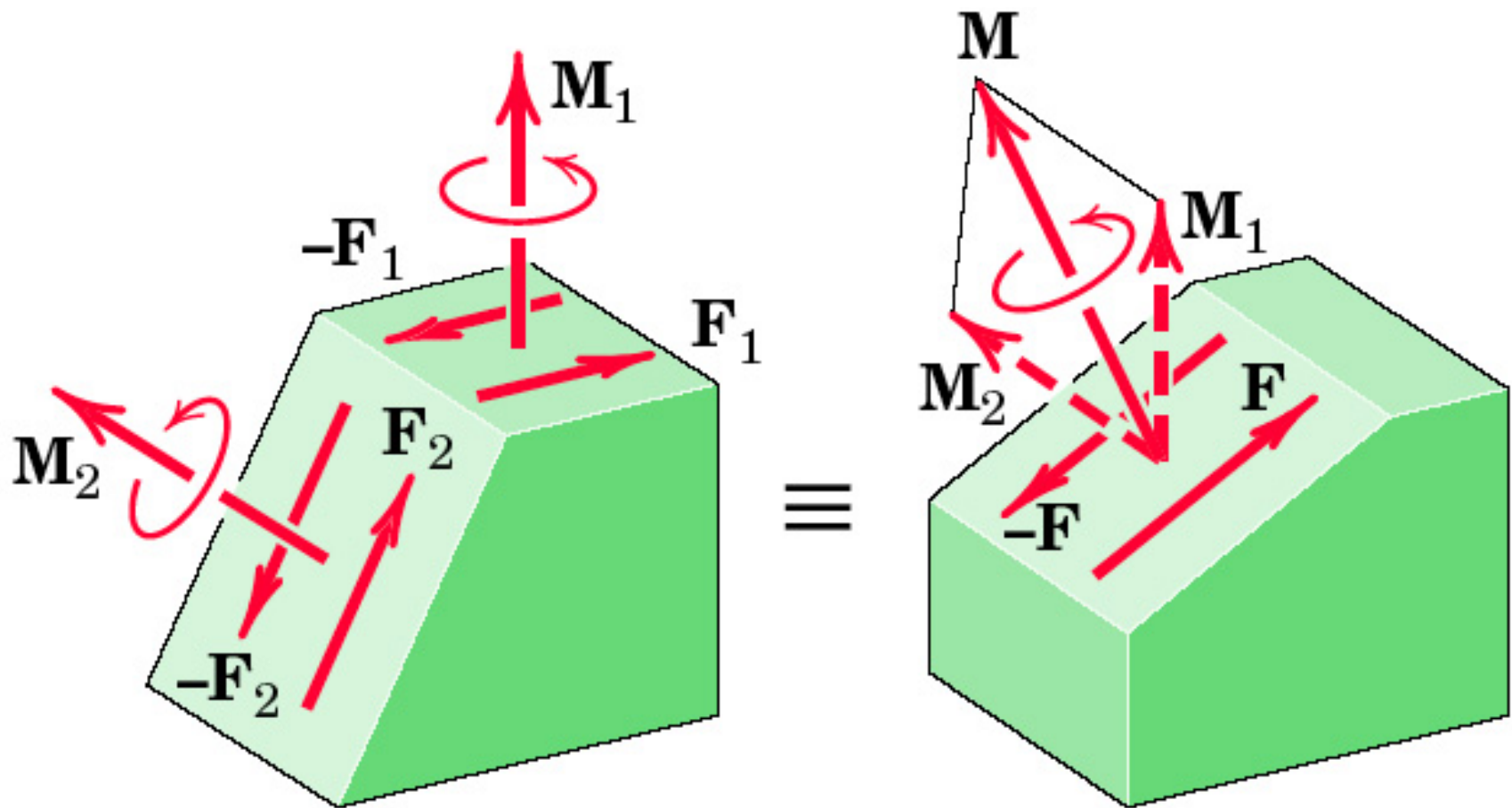


Figure 2/26

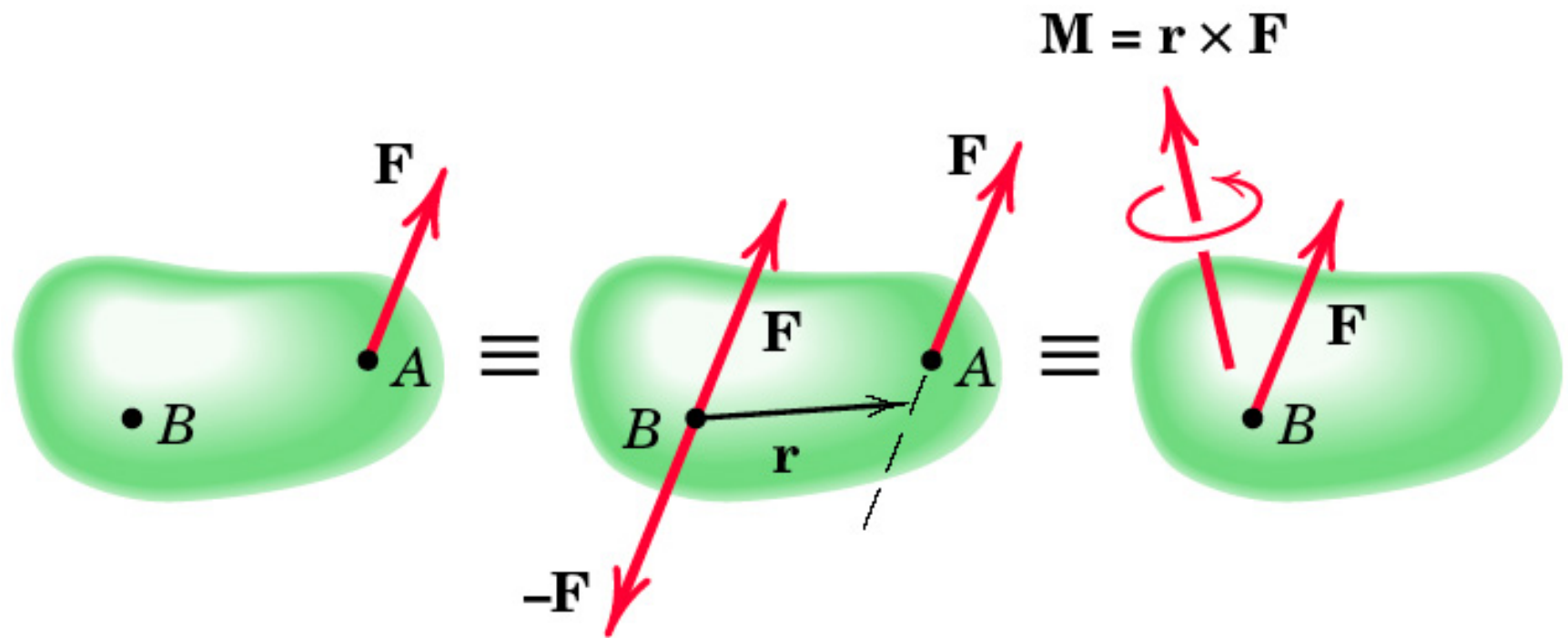


Figure 2/27

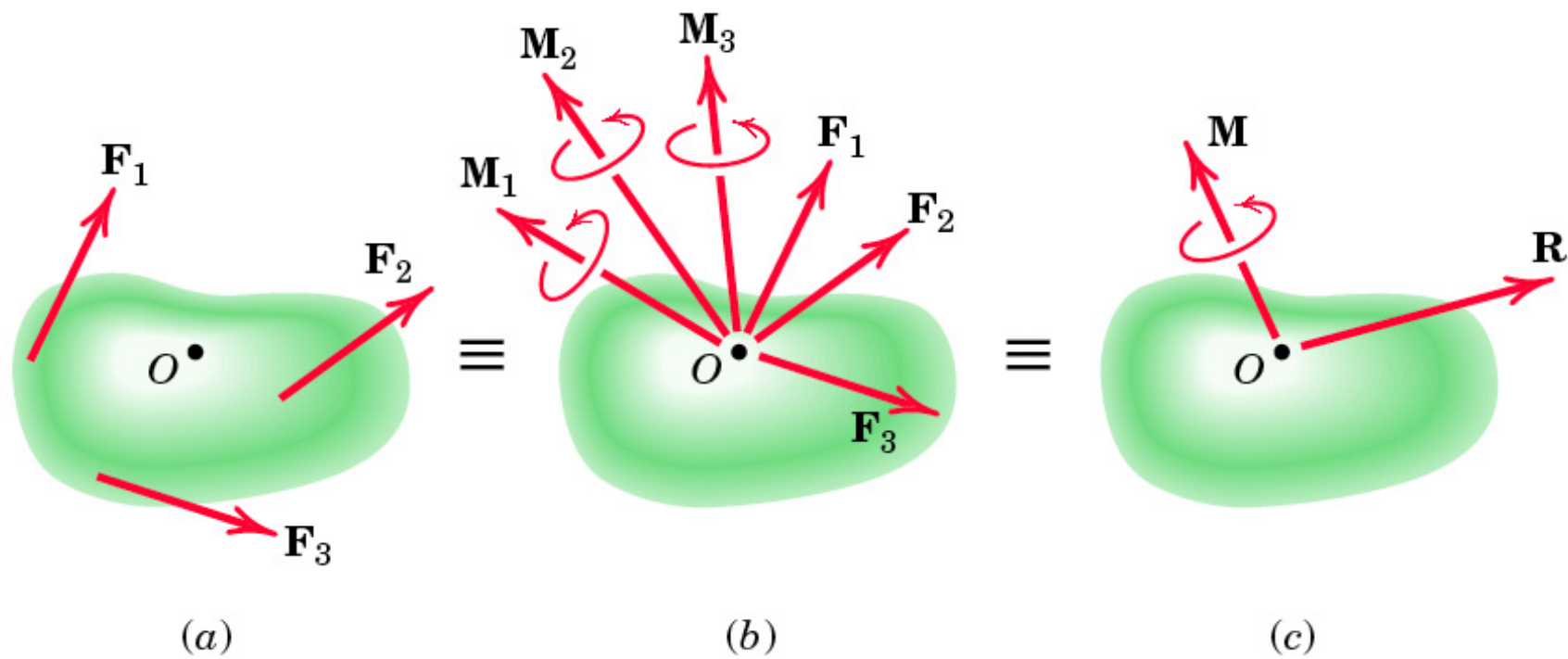
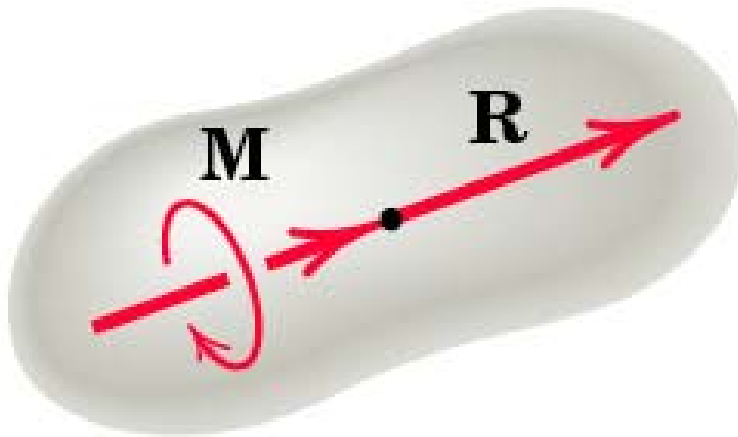
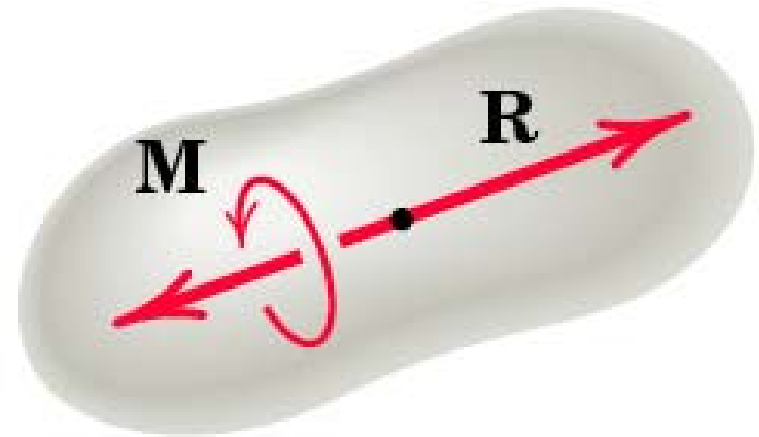


Figure 2/28

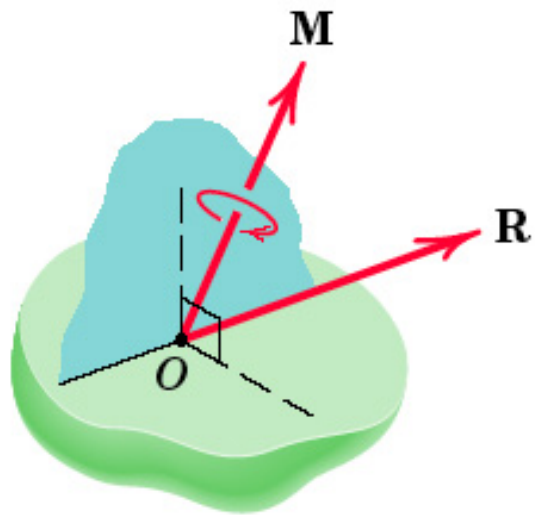


Positive wrench

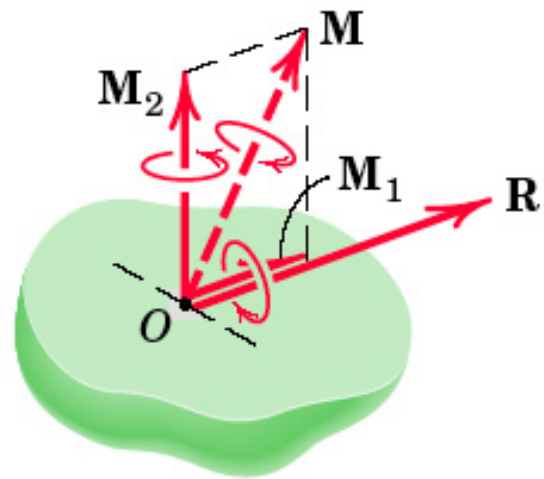


Negative wrench

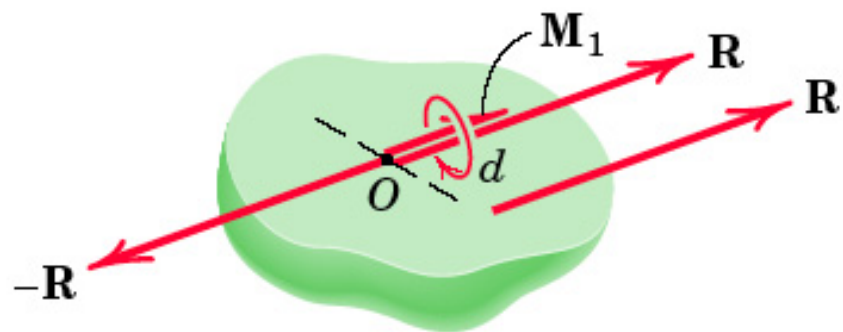
Figure 2/29



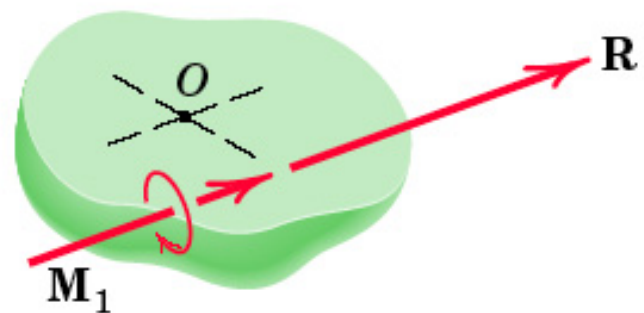
(a)



(b)

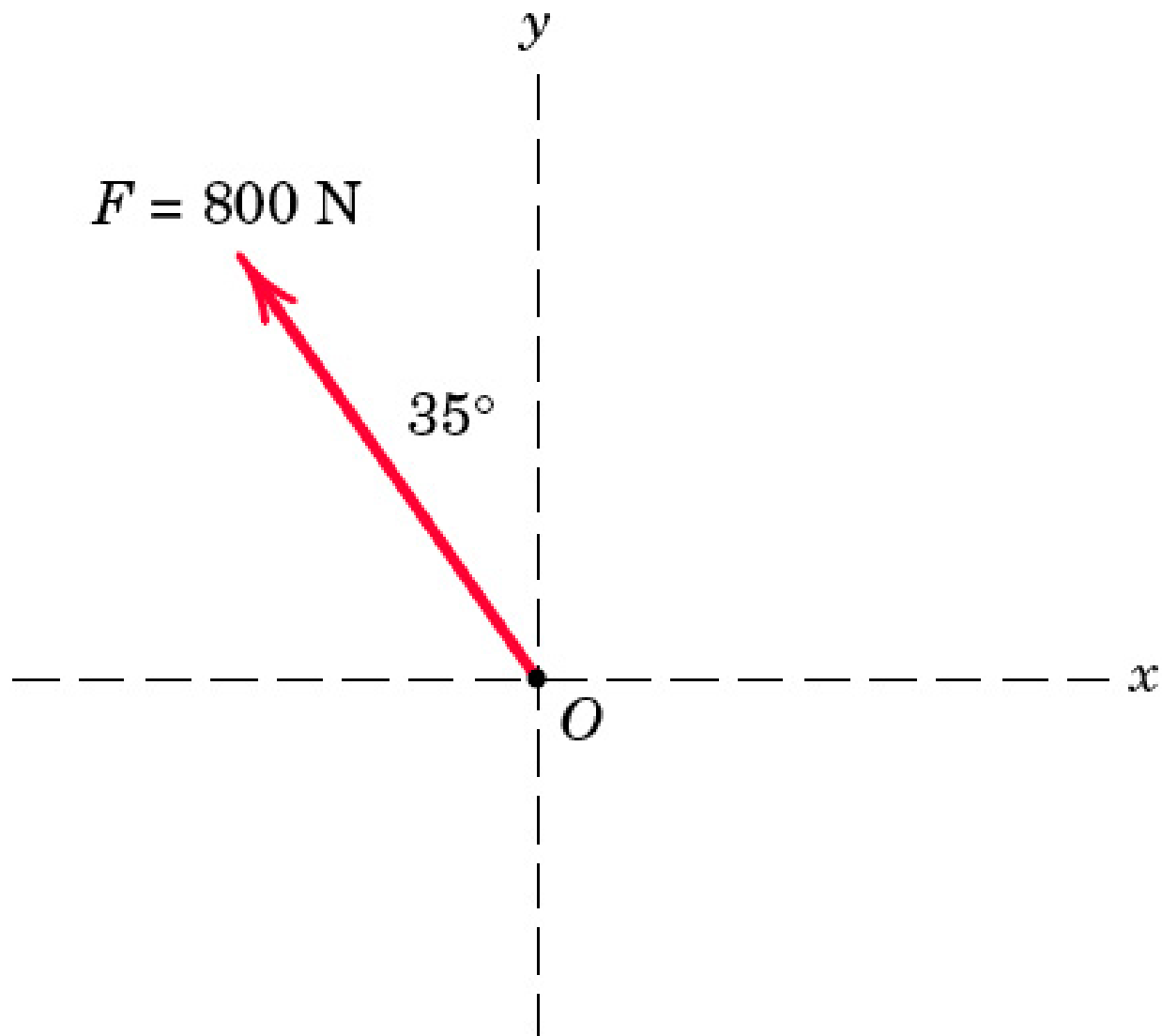


(c)

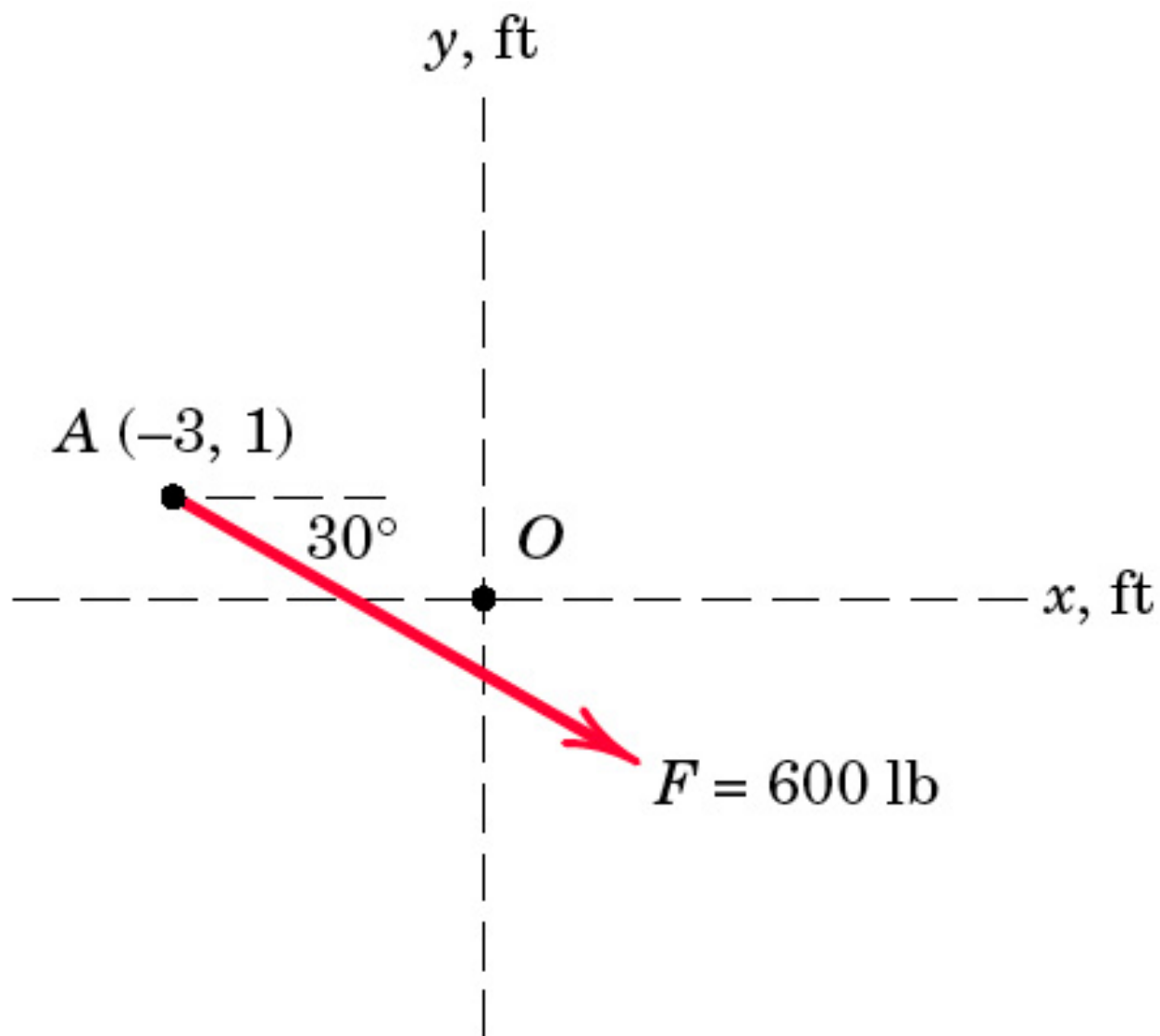


(d)

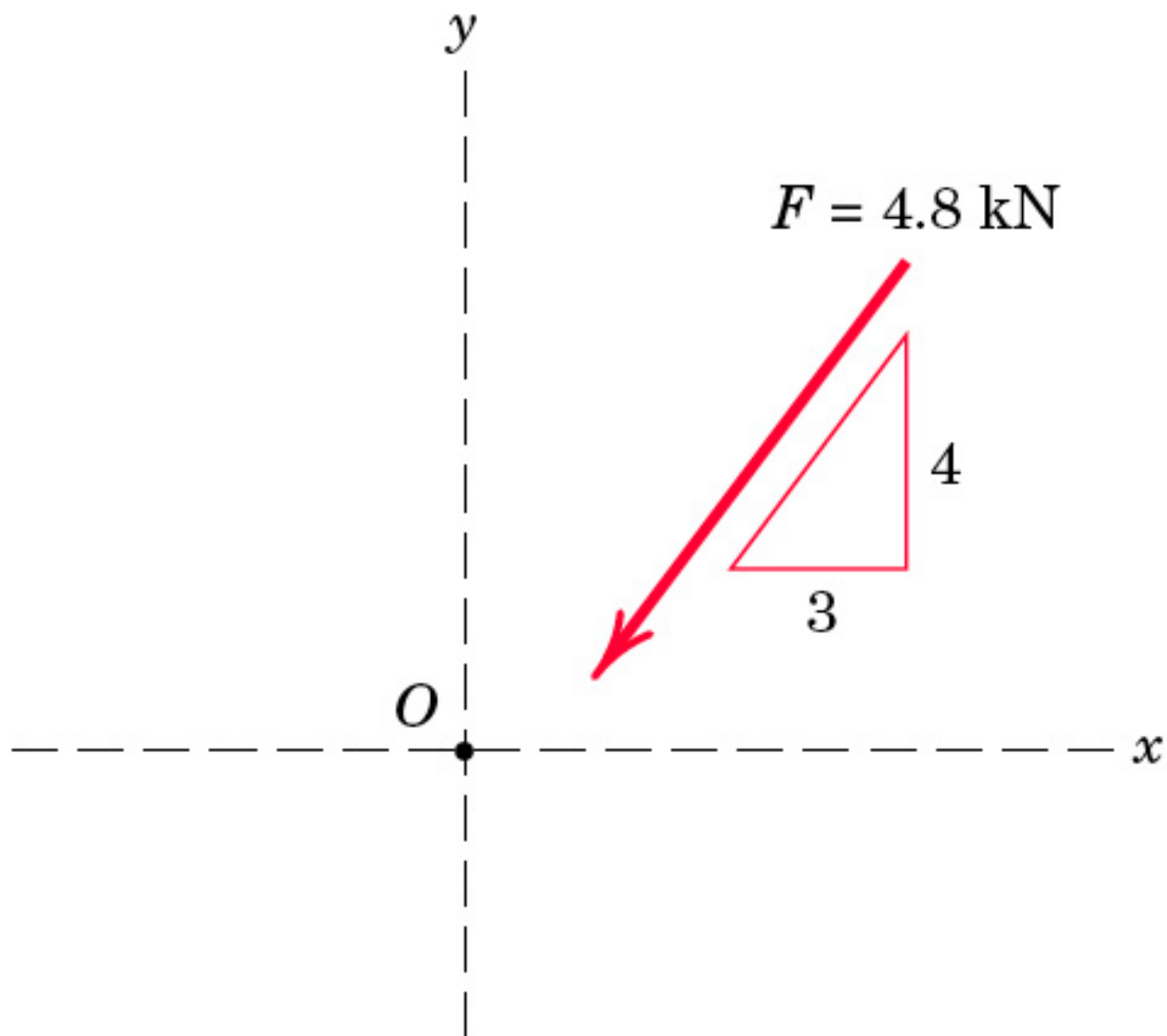
Figure 2/30



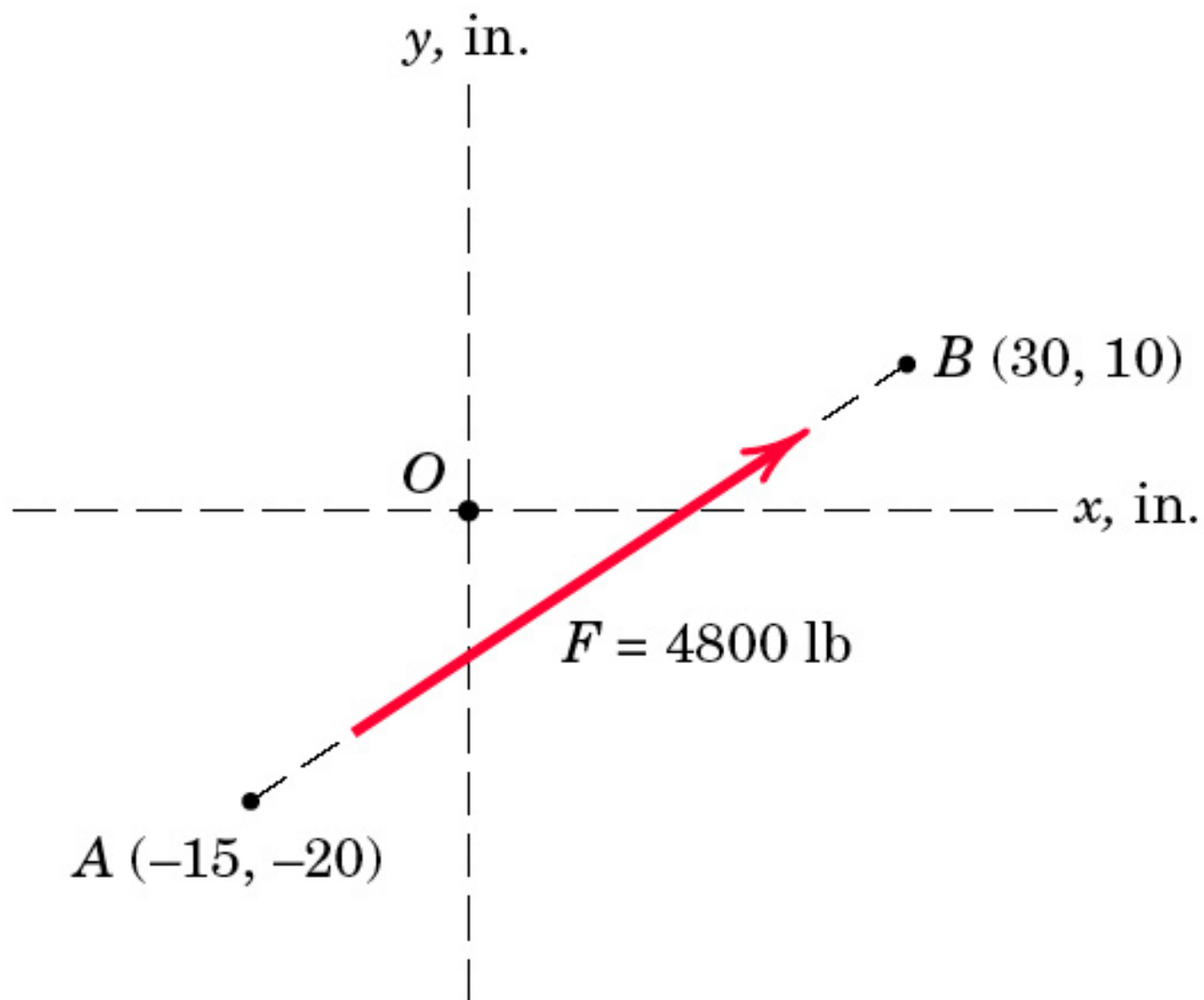
Problem 2/1



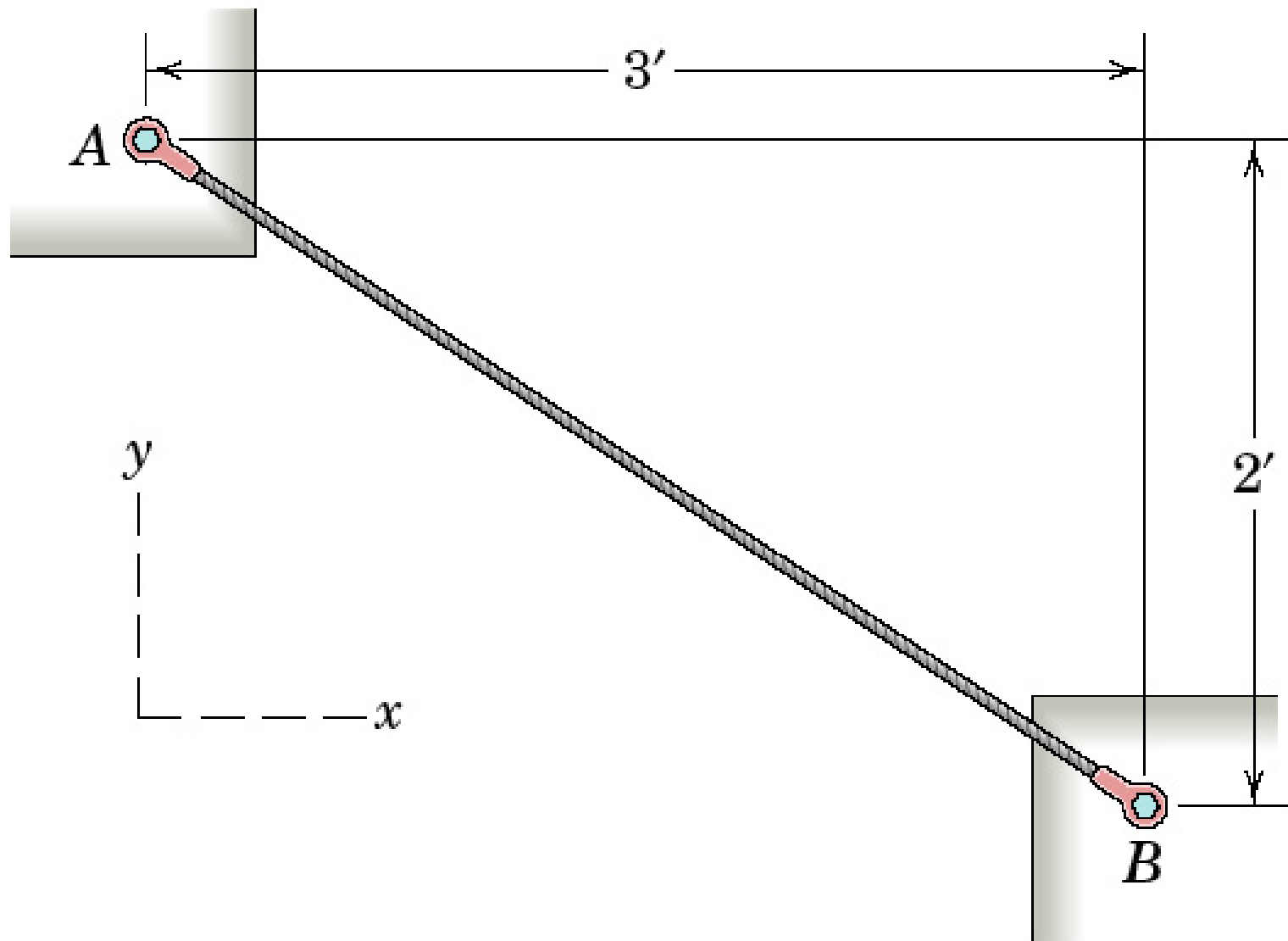
Problem 2/2



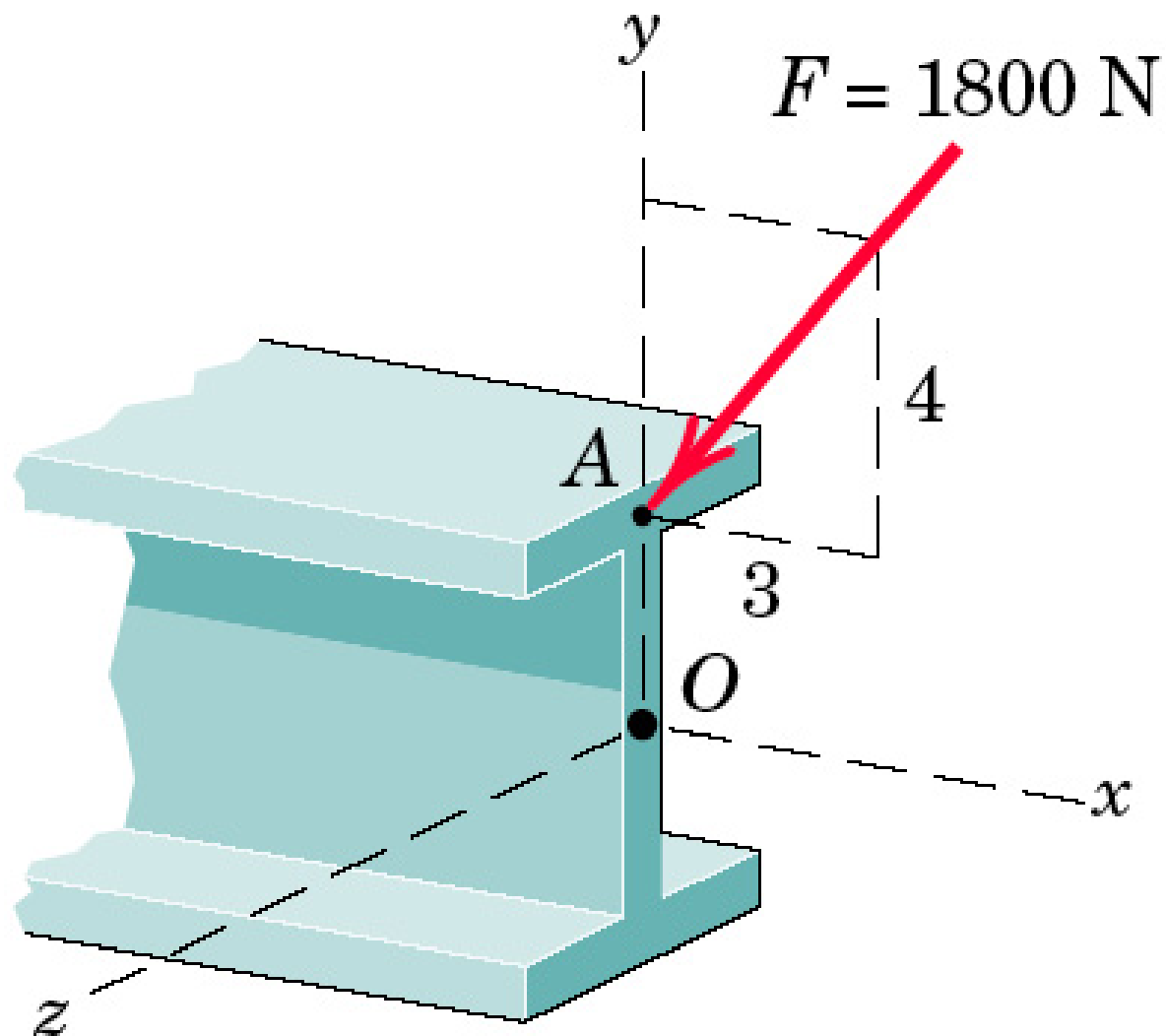
Problem 2/3



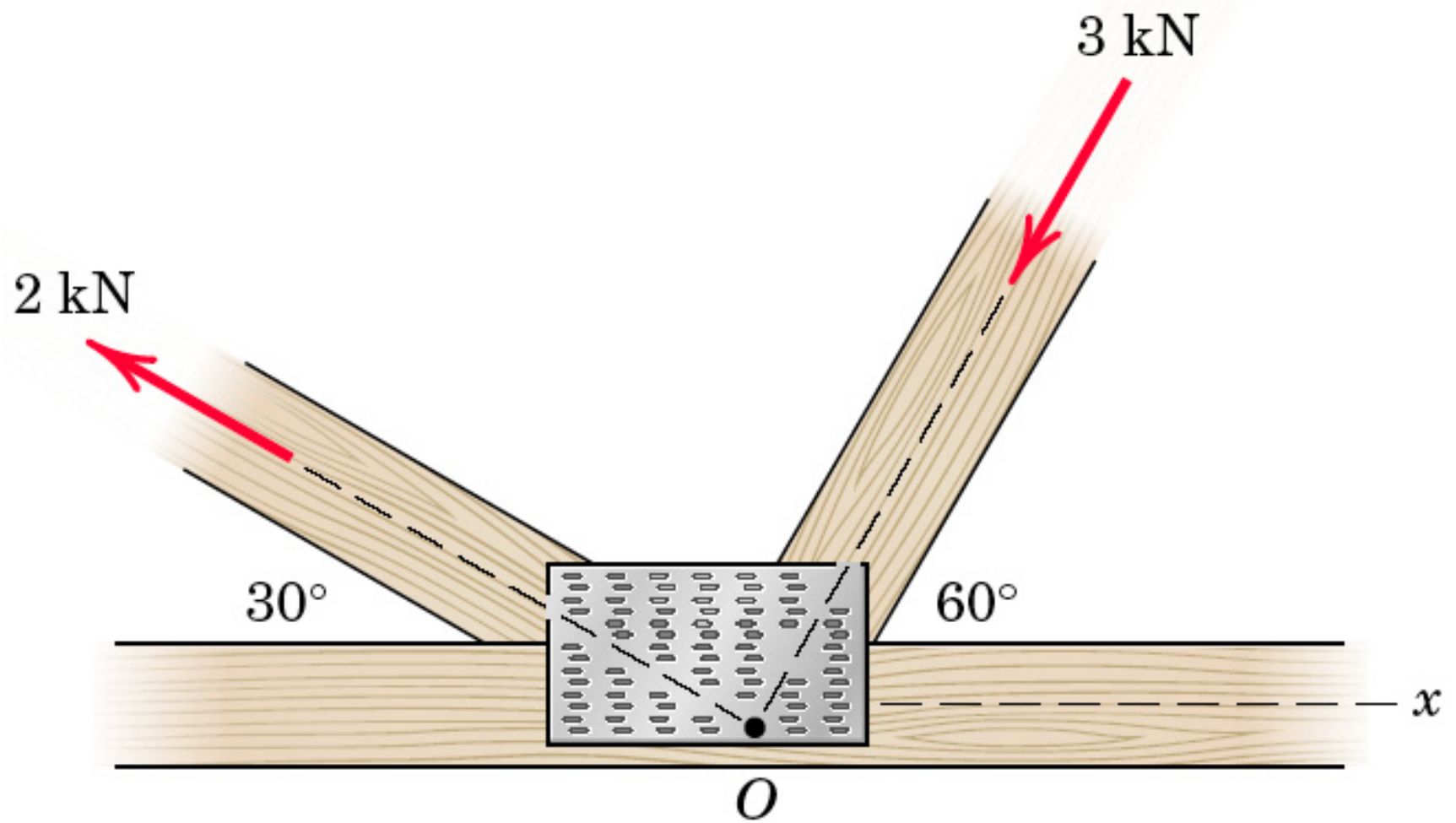
Problem 2/4



Problem 2/5

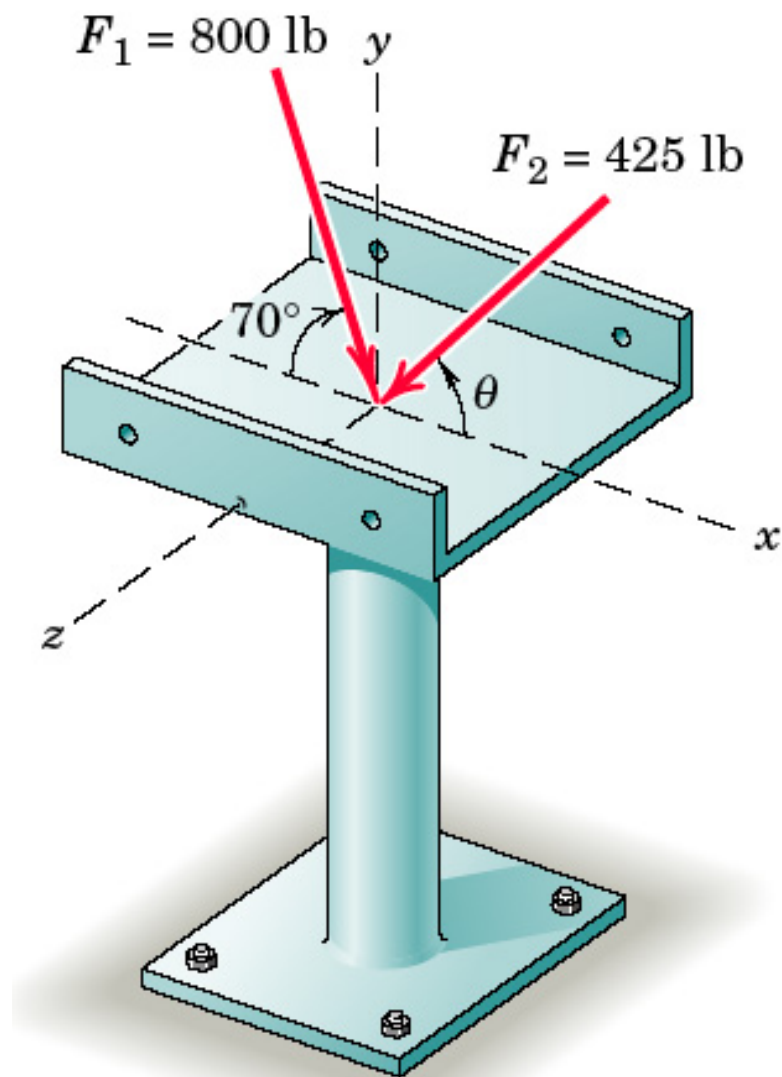


Problem 2/6

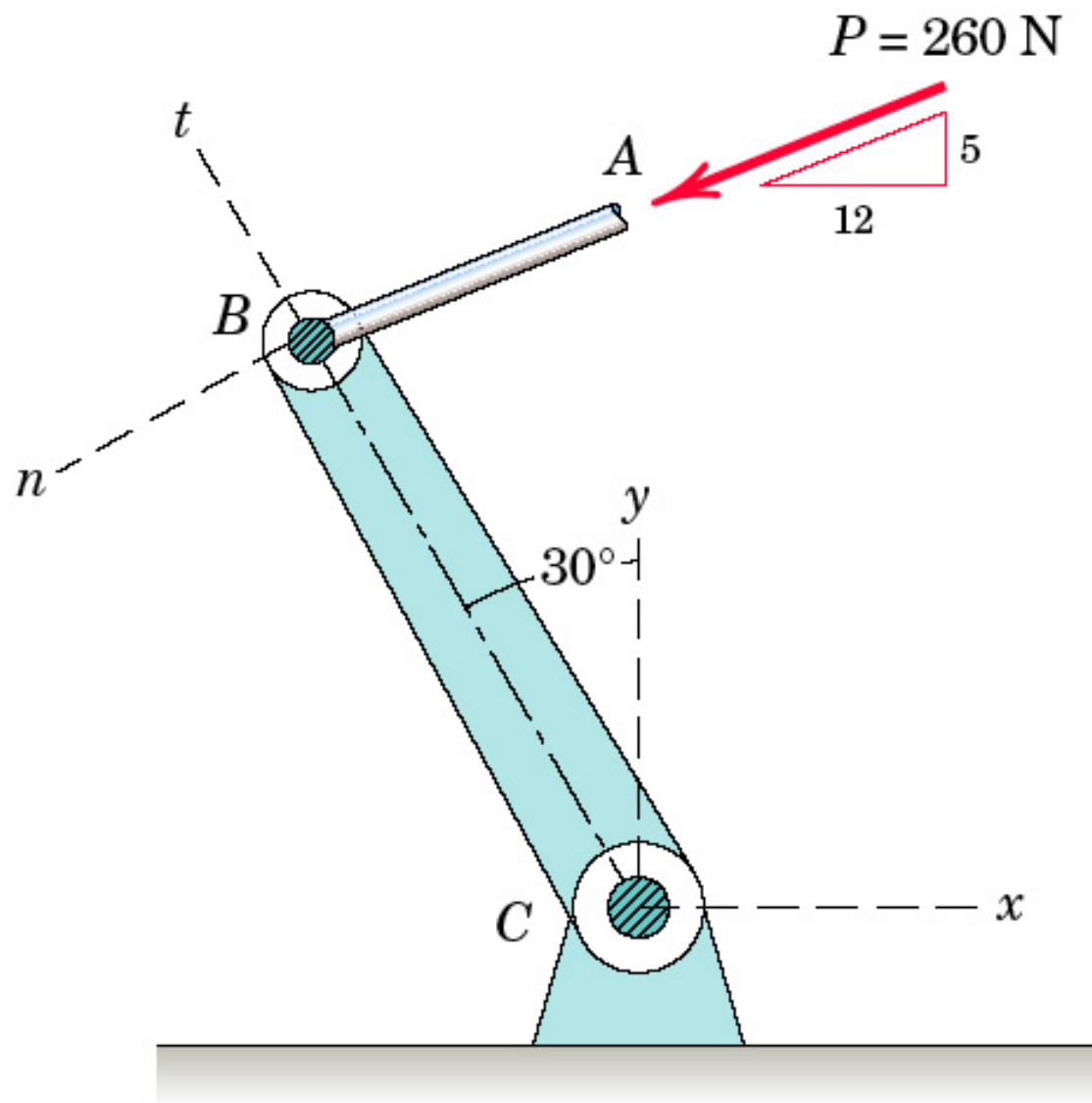


Problem 2/7

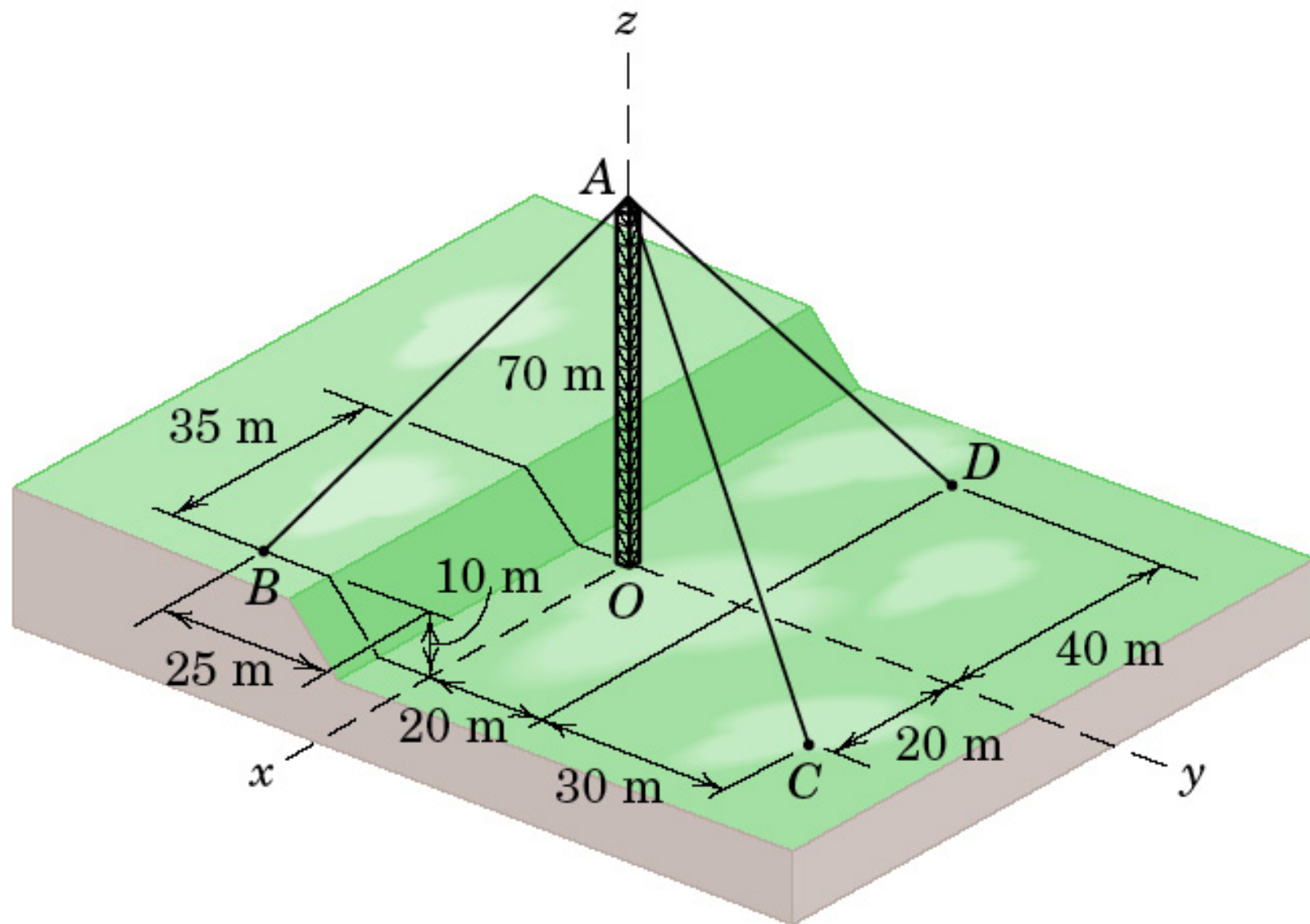
P_02_07



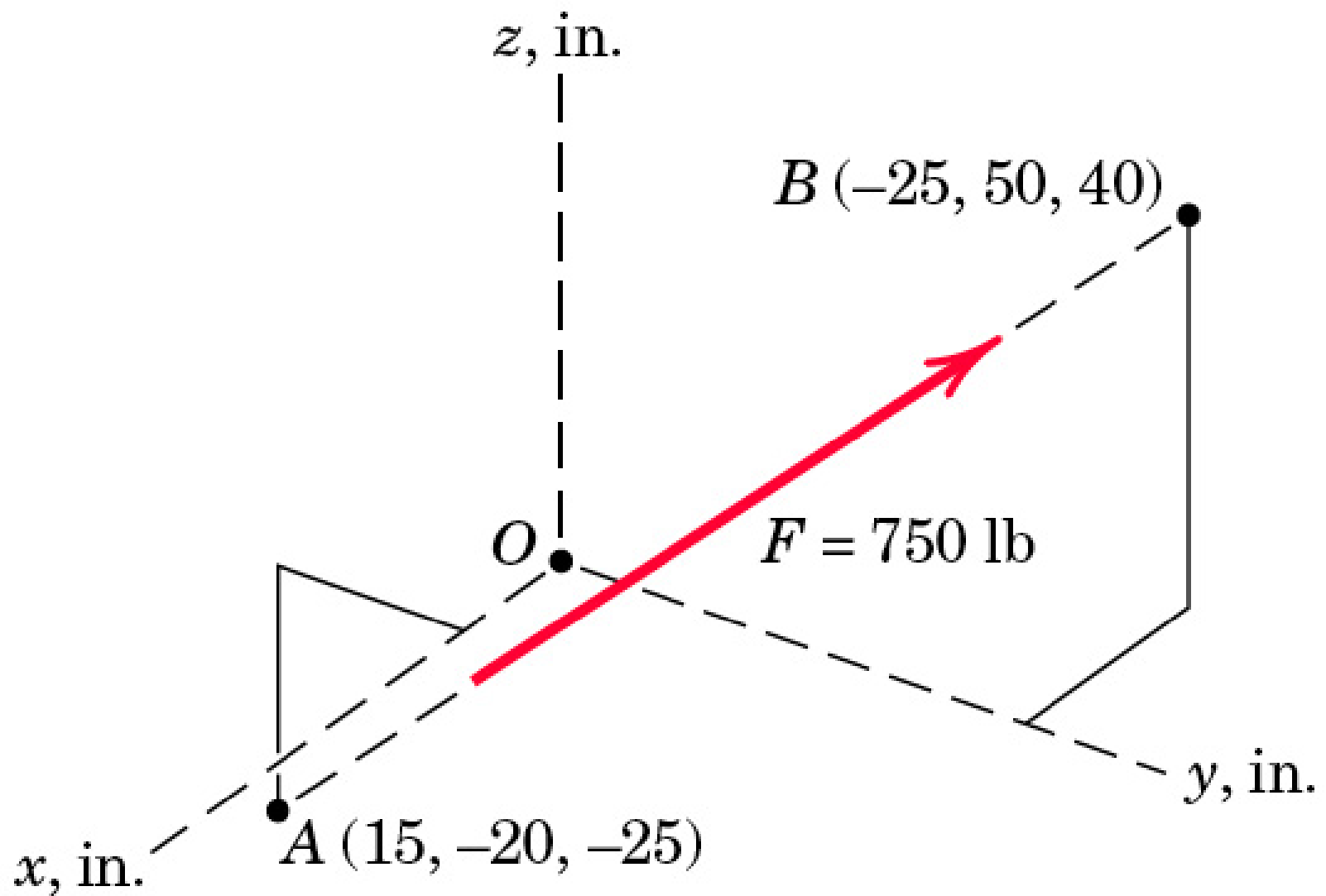
Problem 2/8



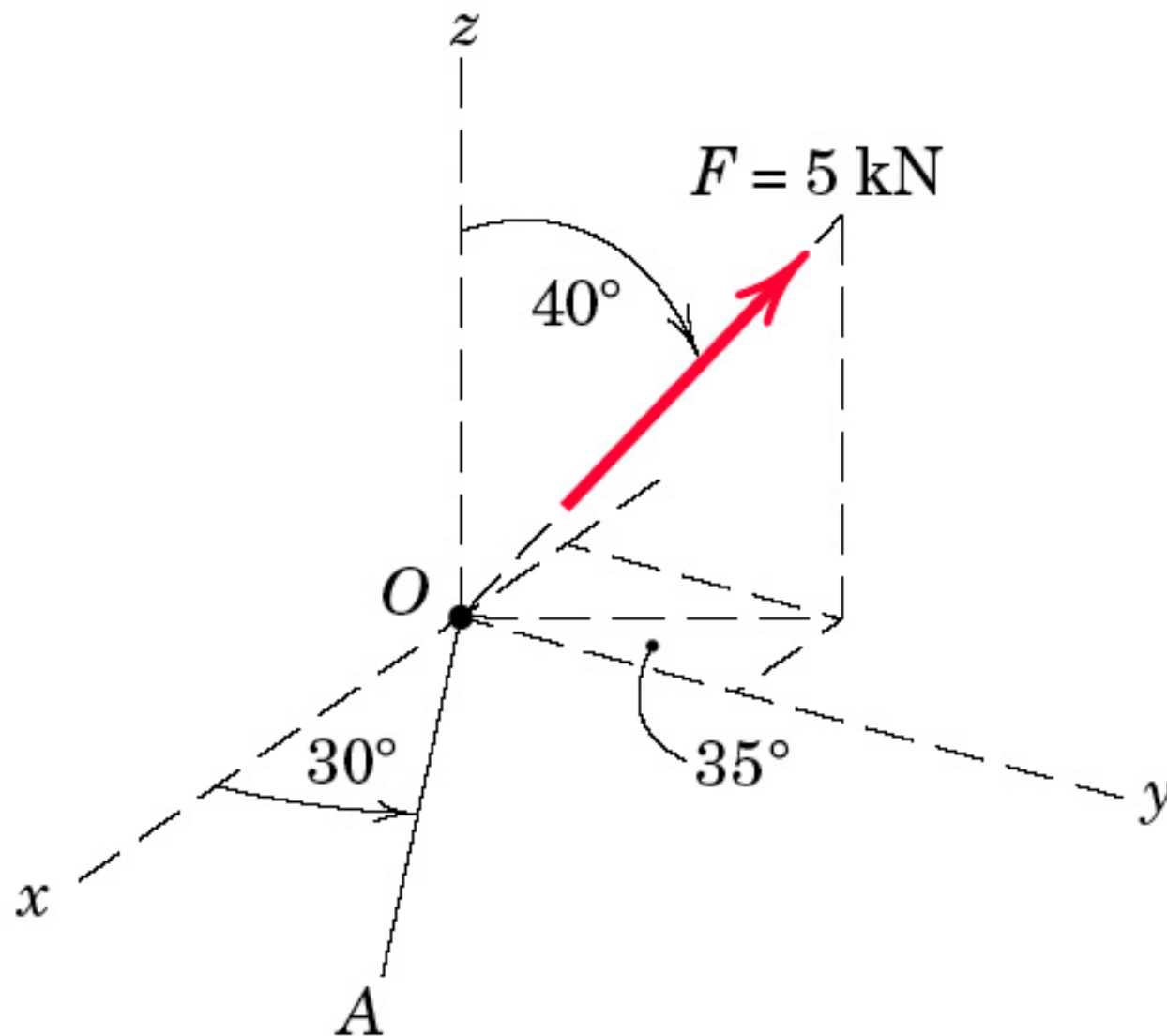
Problem 2/9



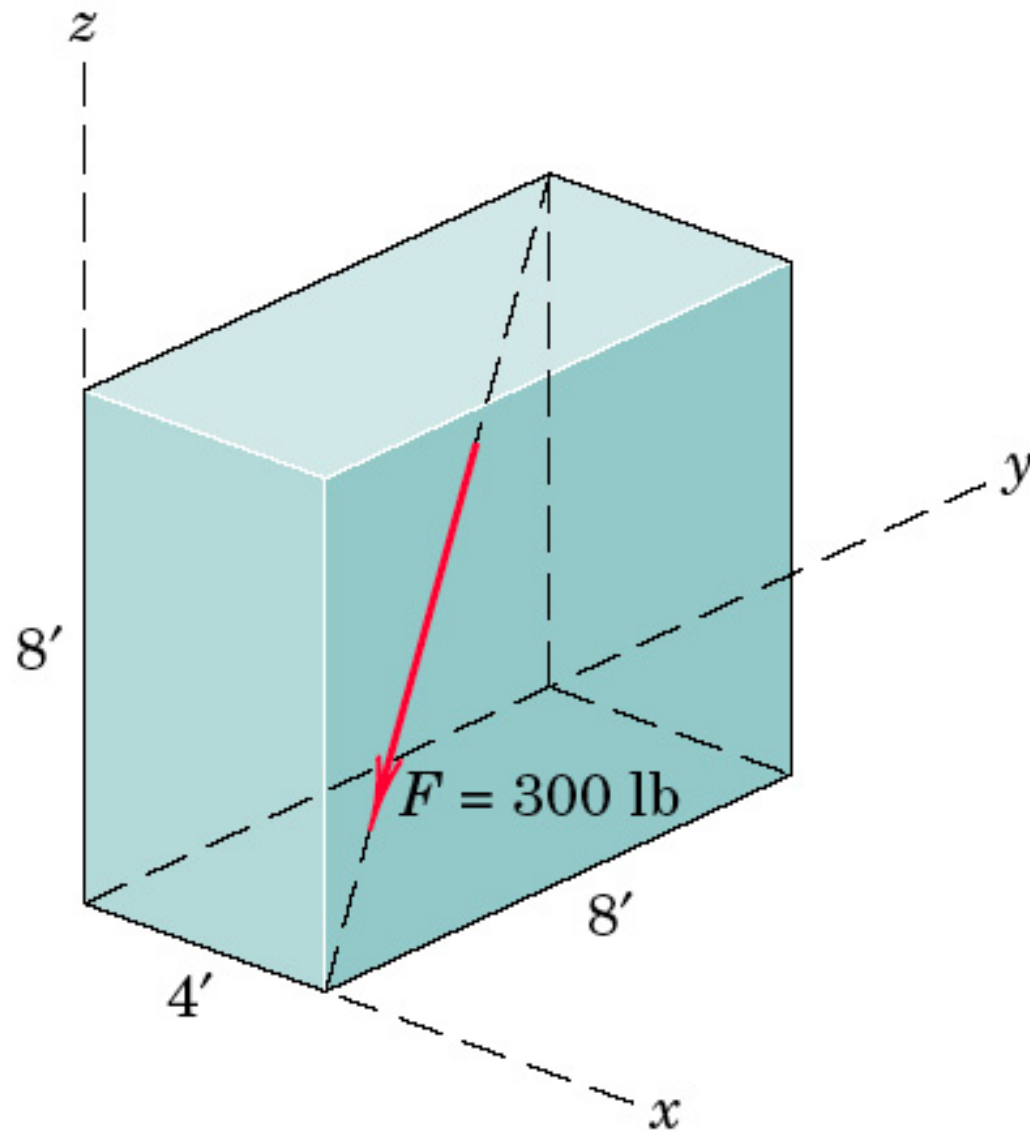
Problem 2/100



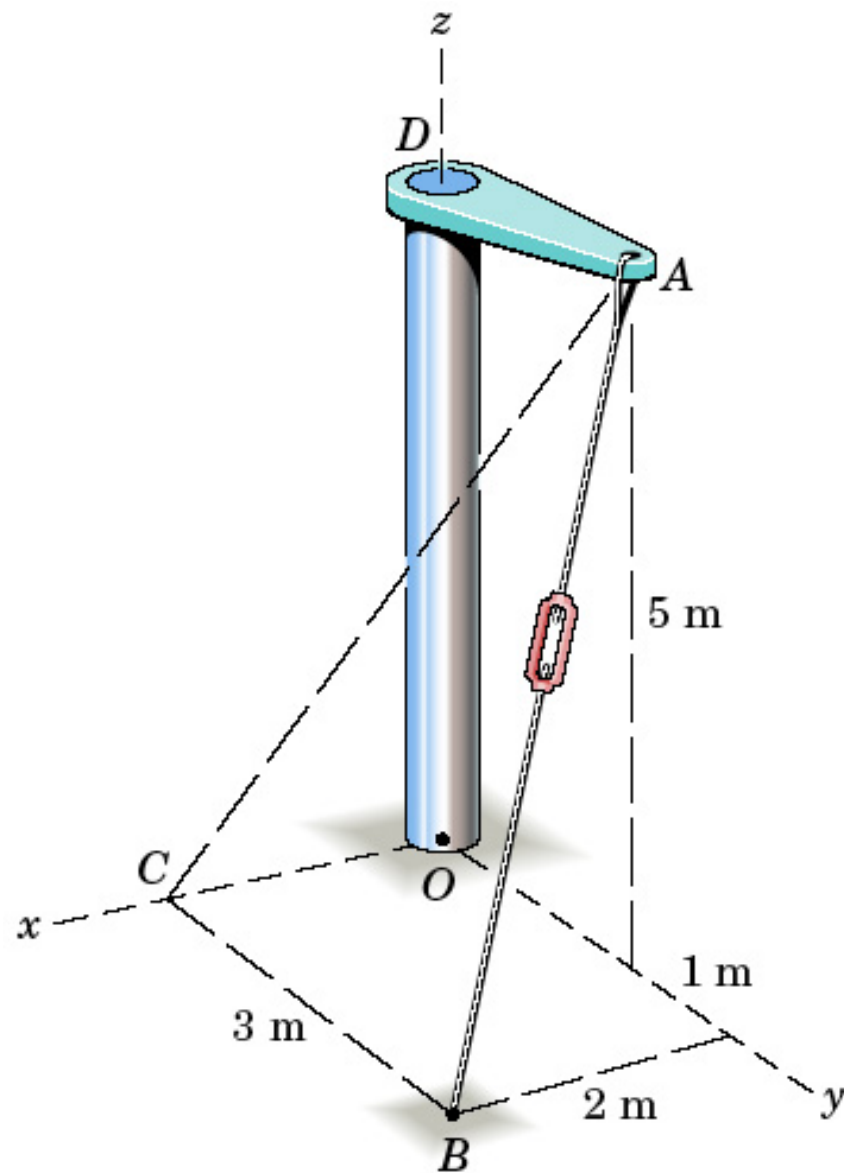
Problem 2/101



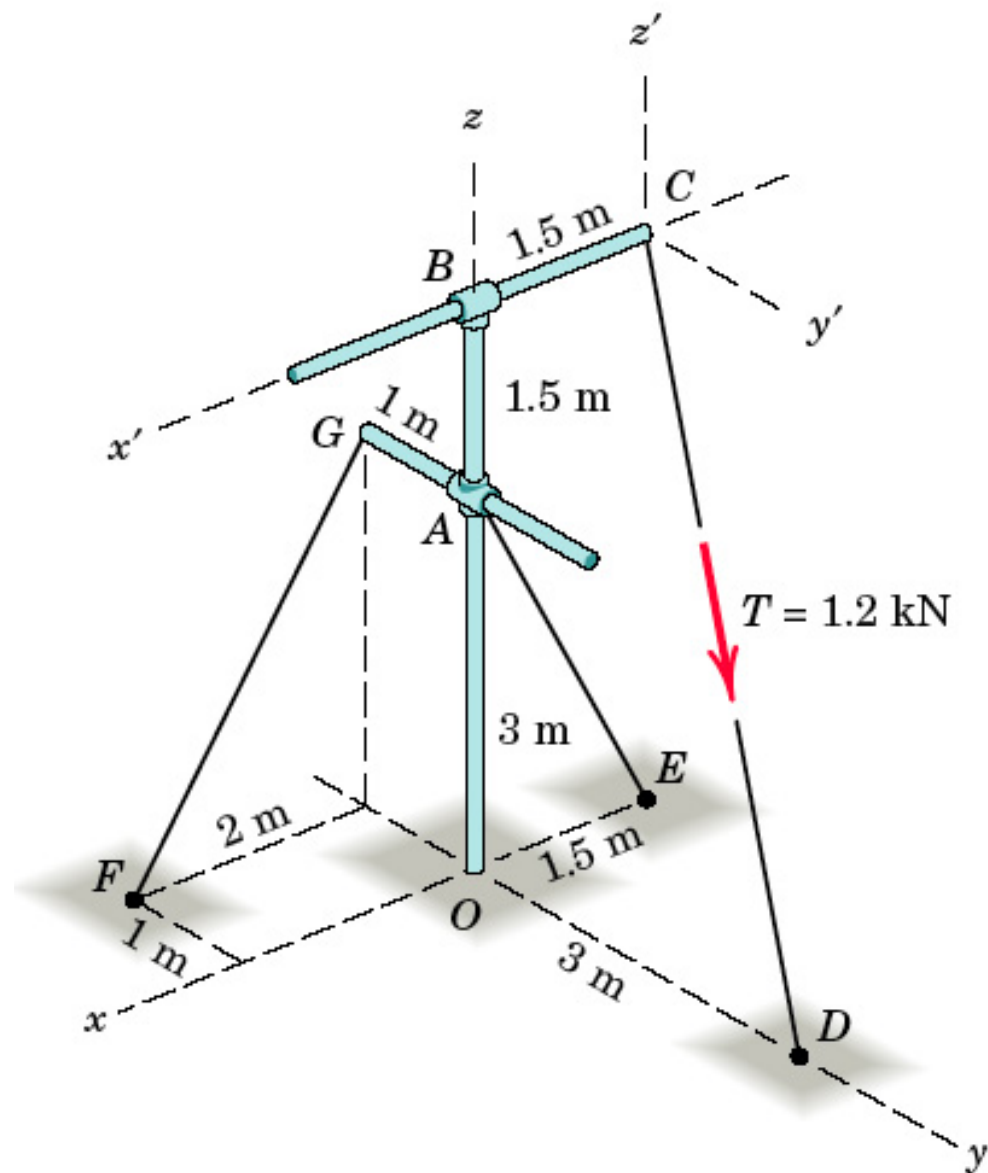
Problem 2/102



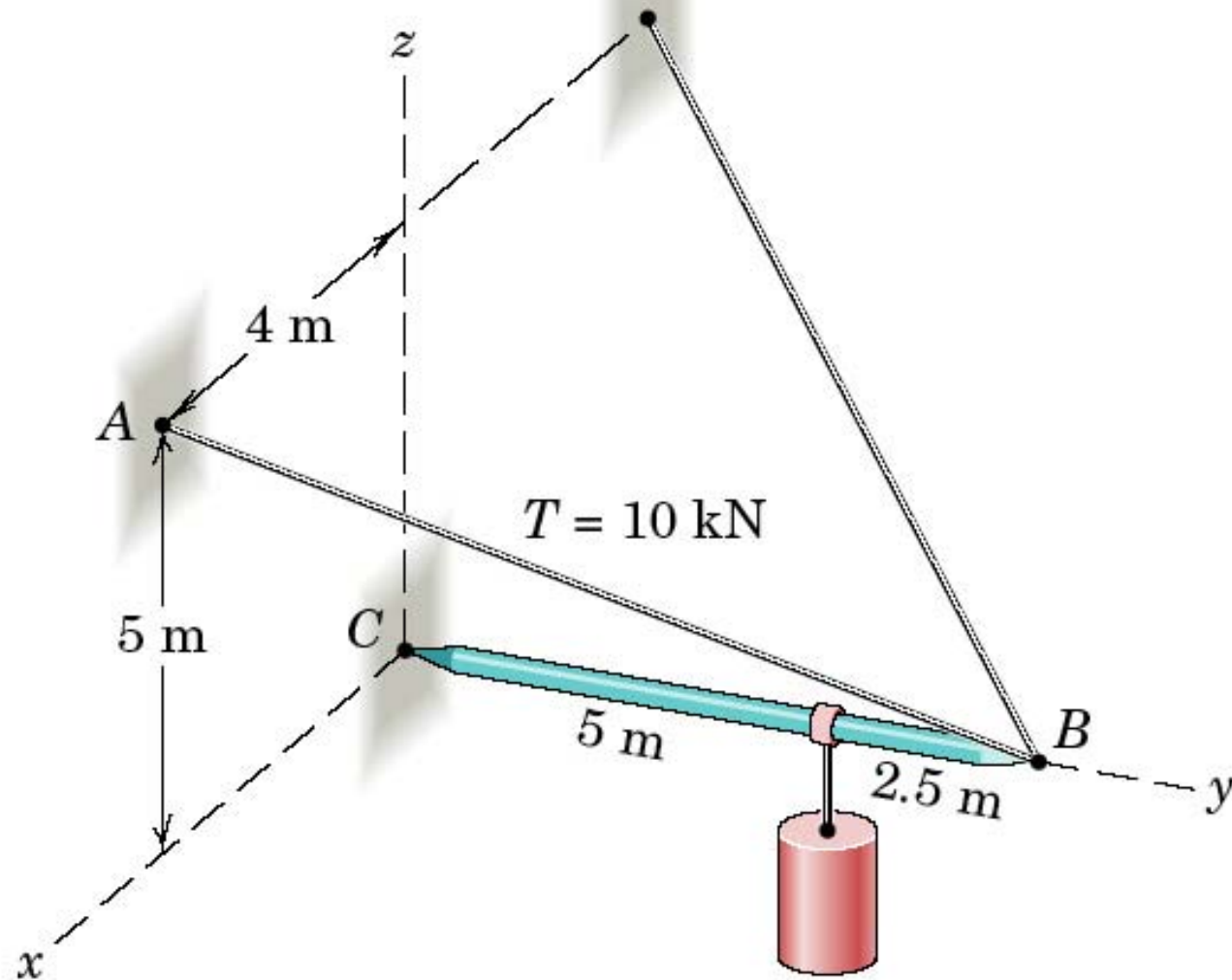
Problem 2/103



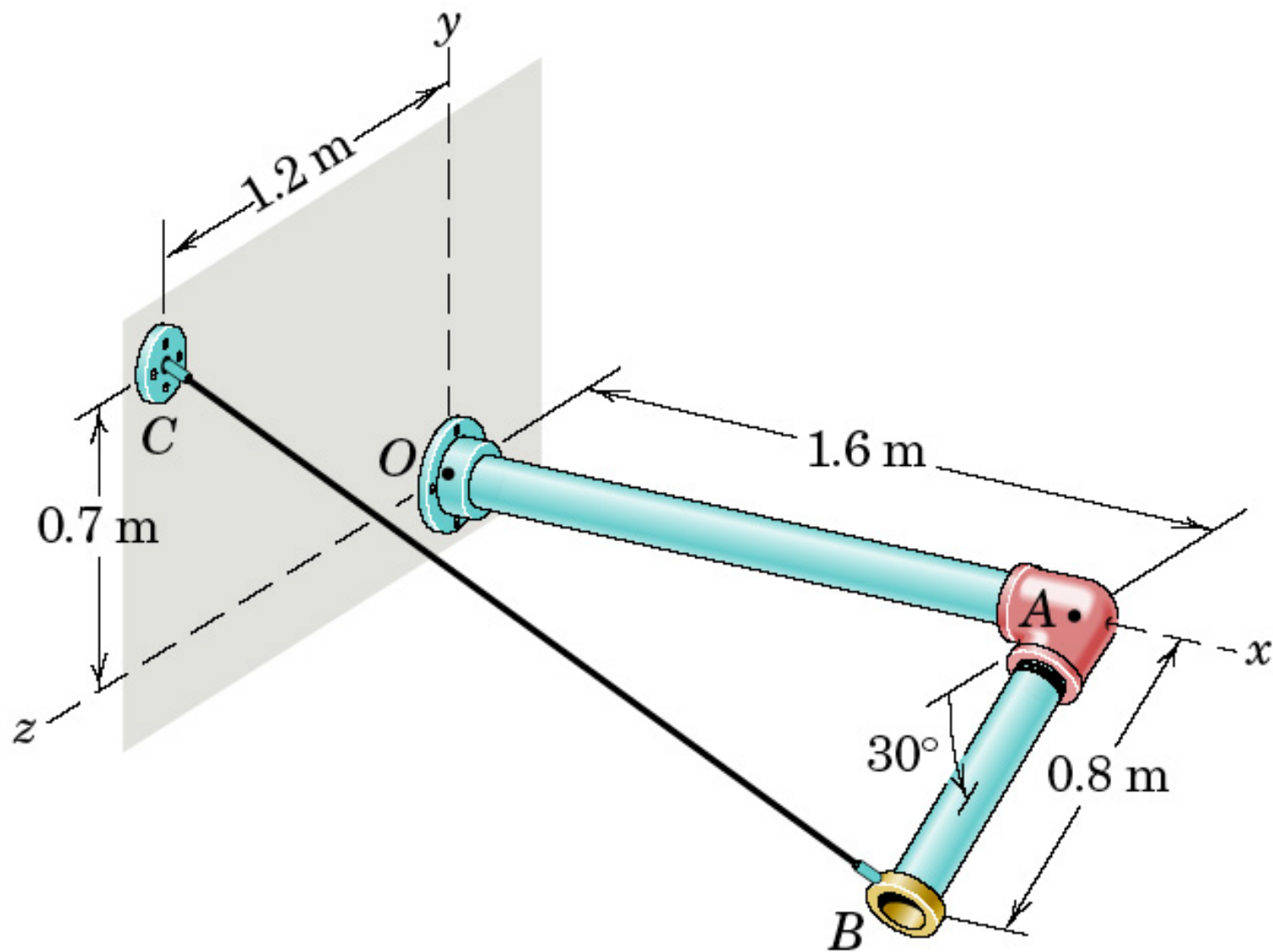
Problem 2/104



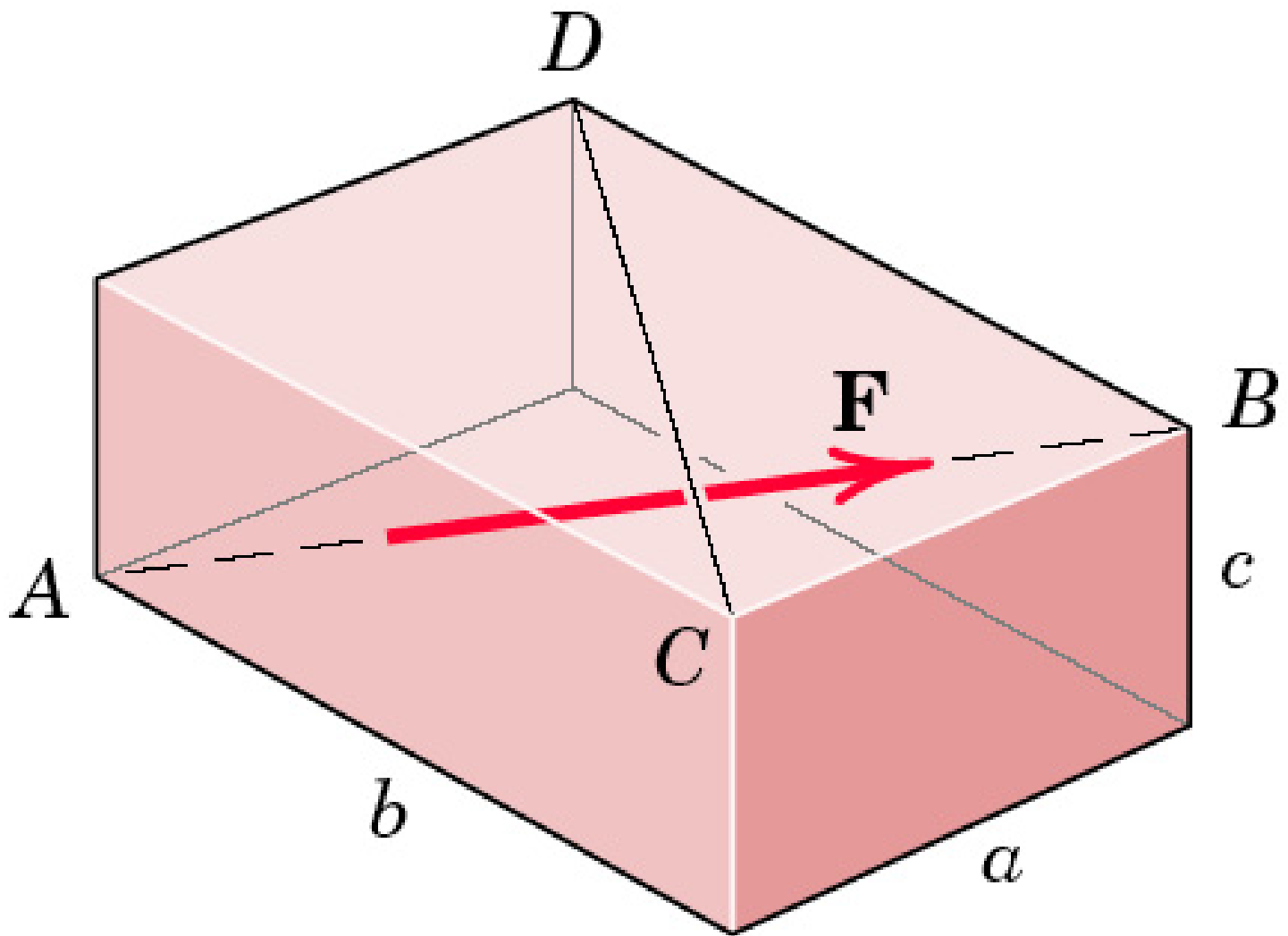
Problem 2/105



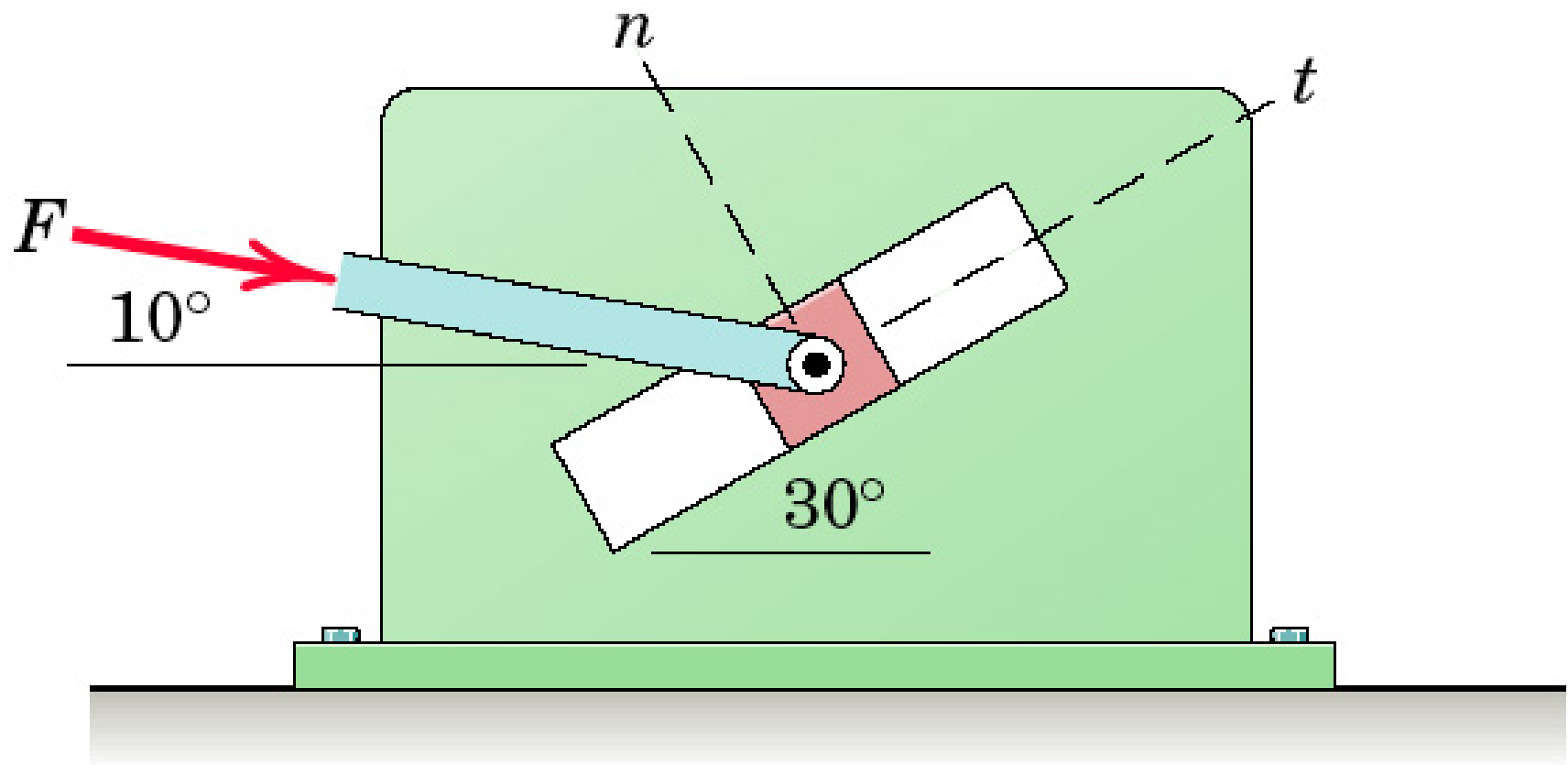
Problem 2/107



Problem 2/108

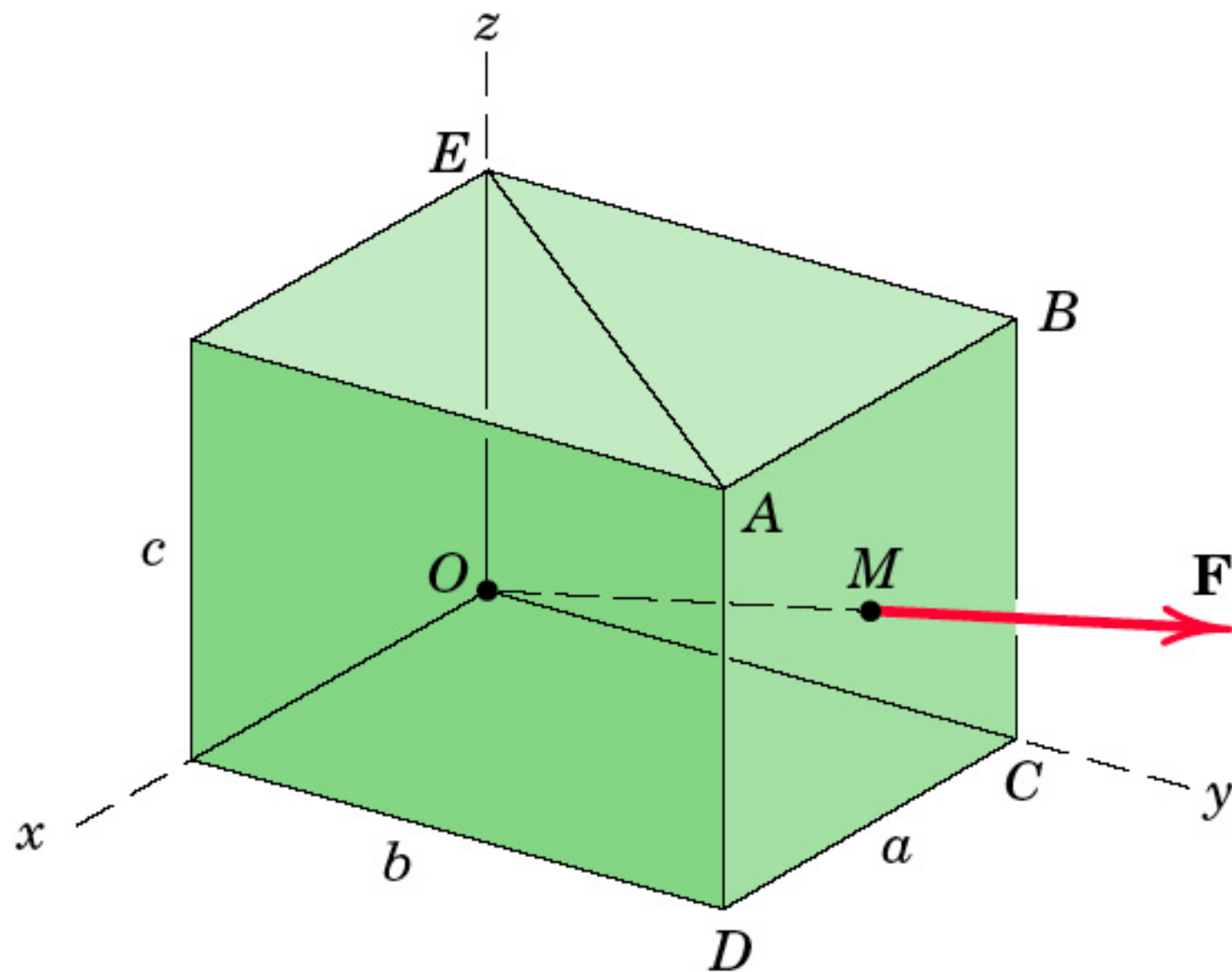


Problem 2/109

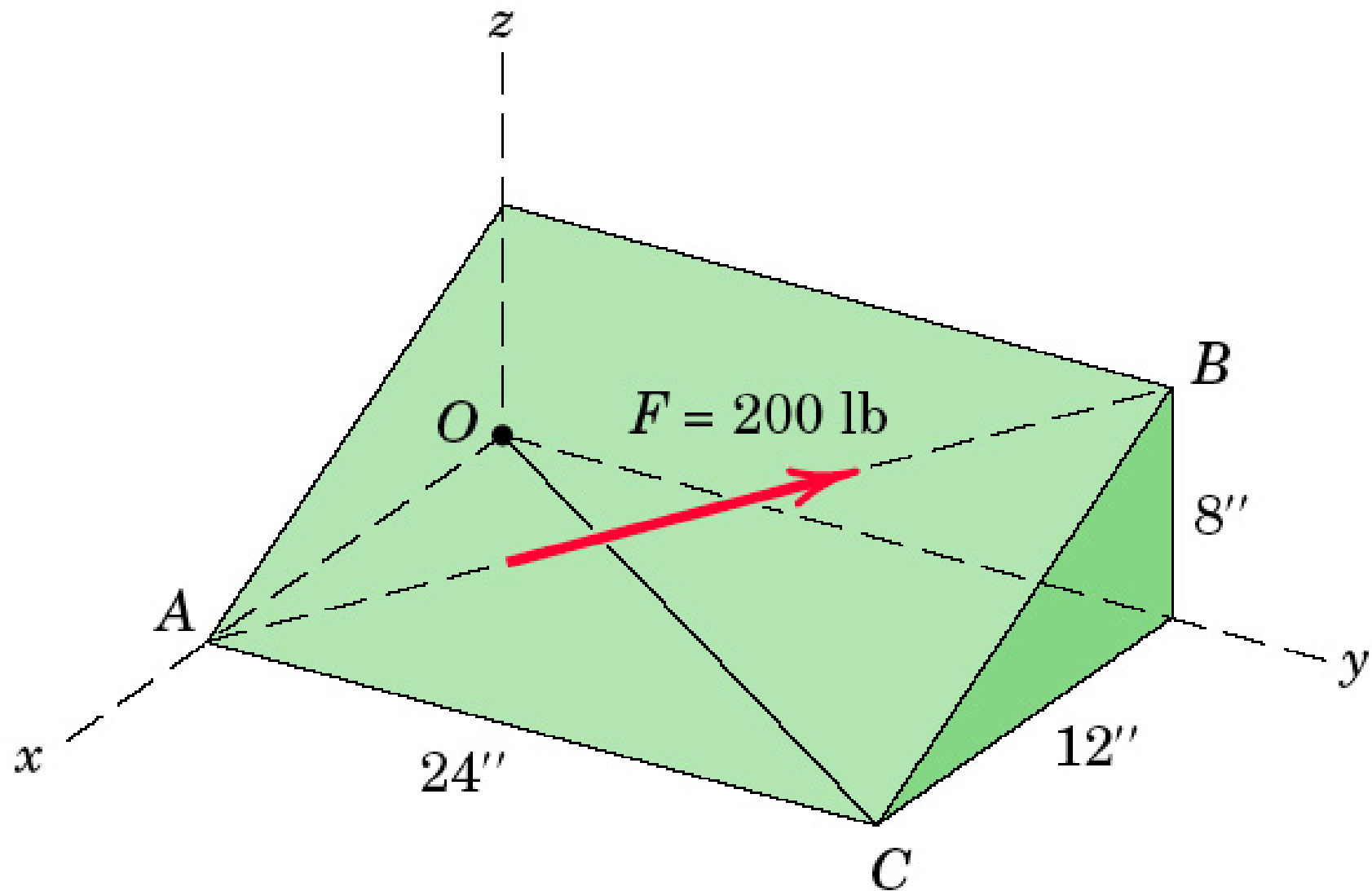


Problem 2/11

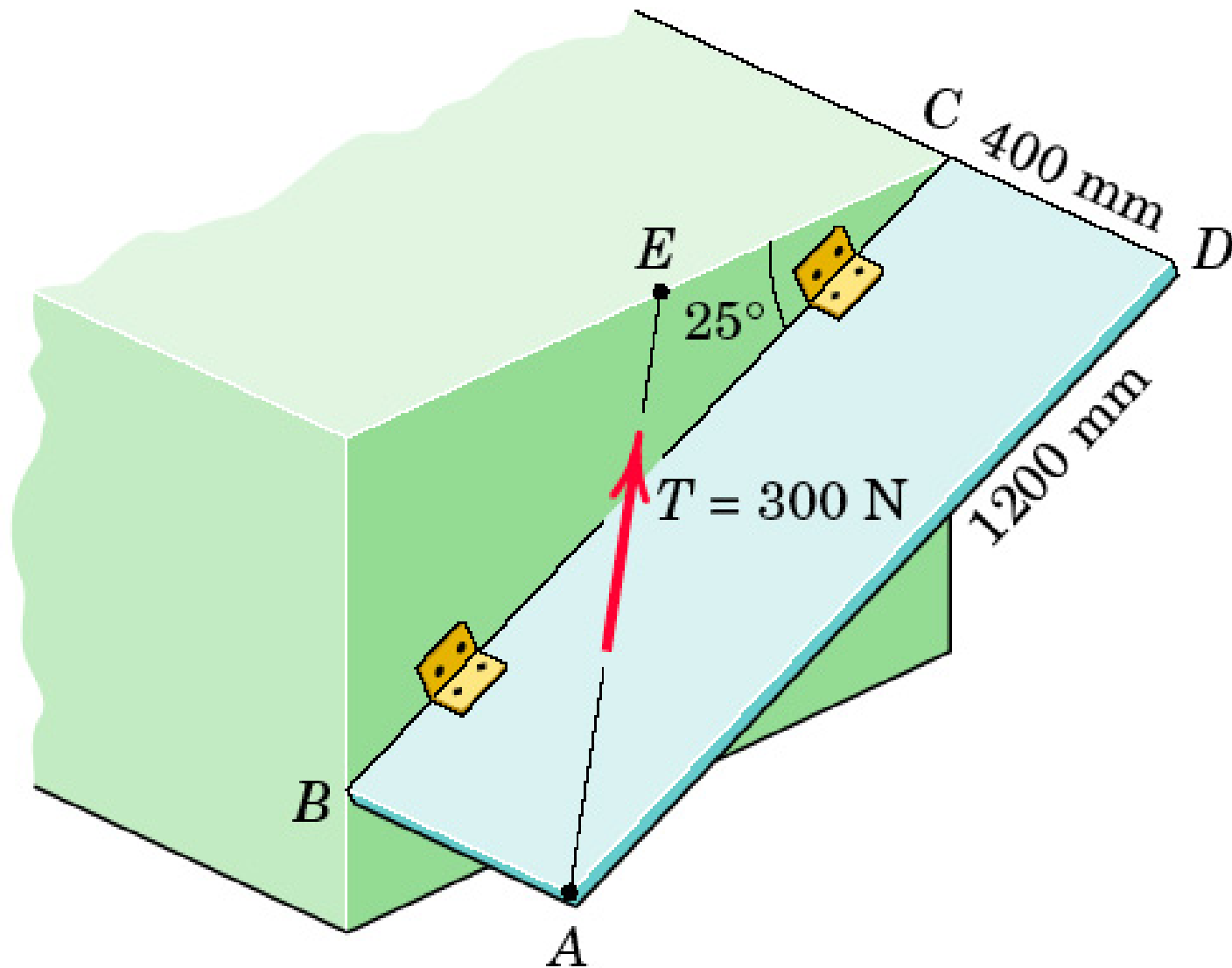
P_02_11



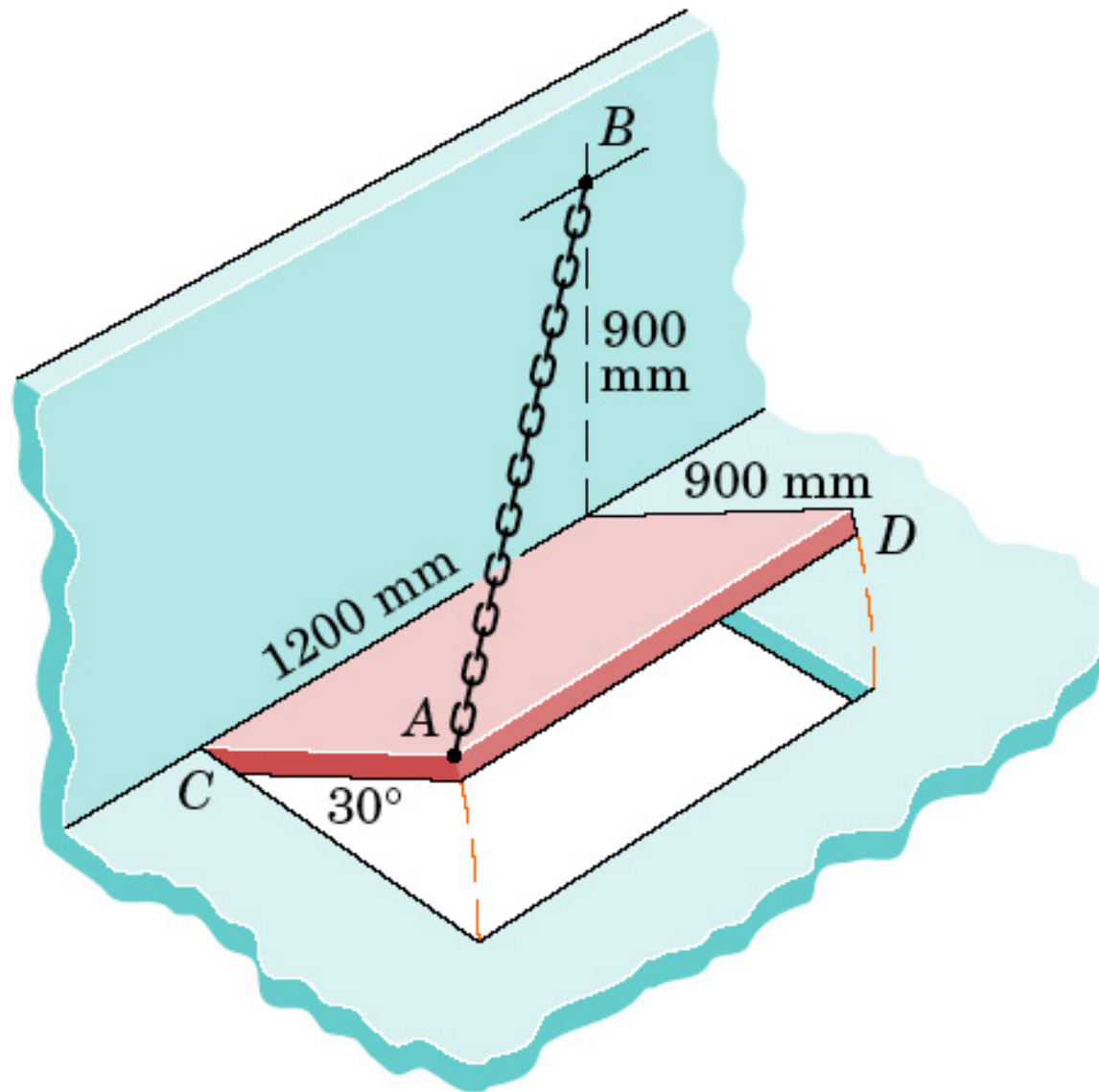
Problem 2/110



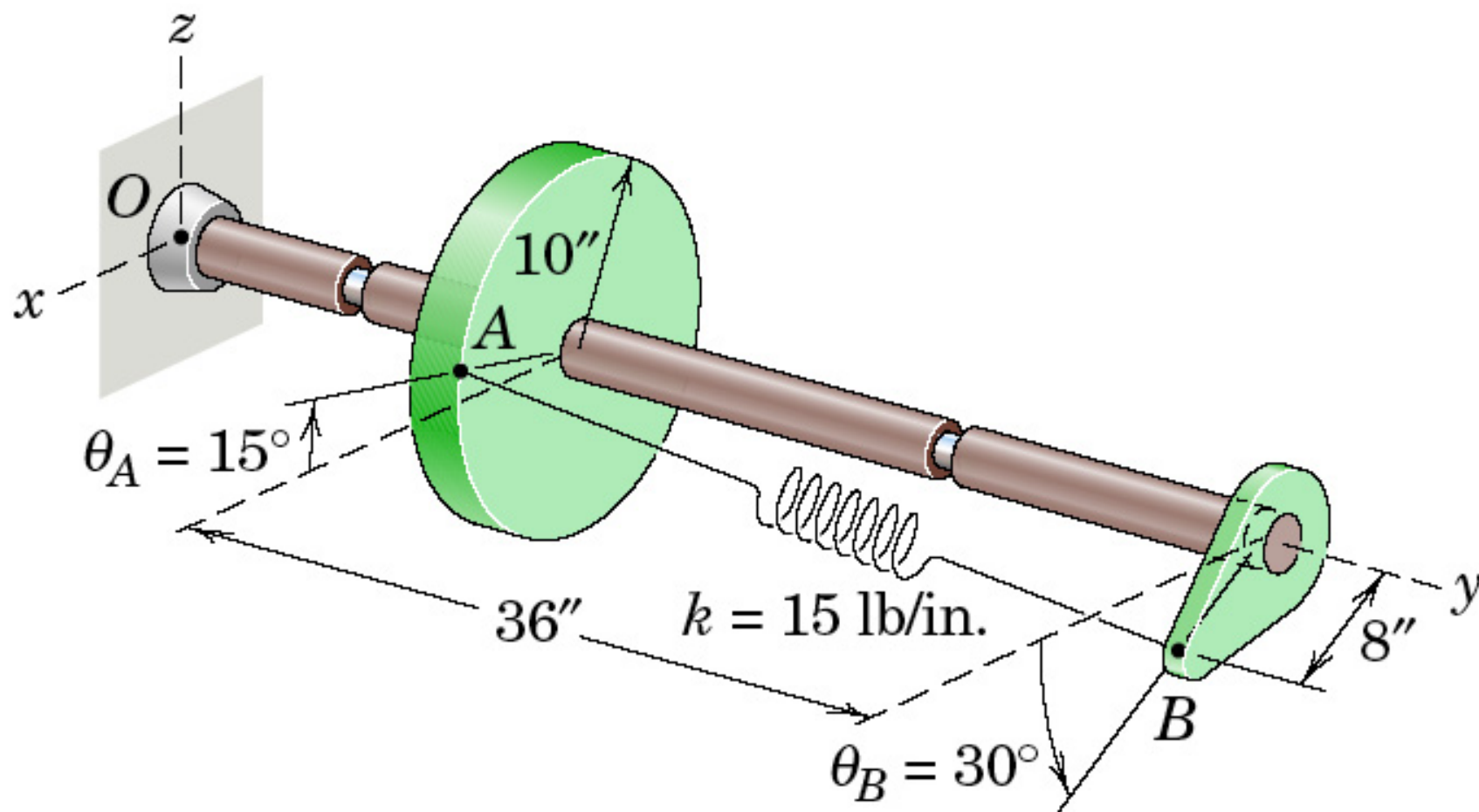
Problem 2/111



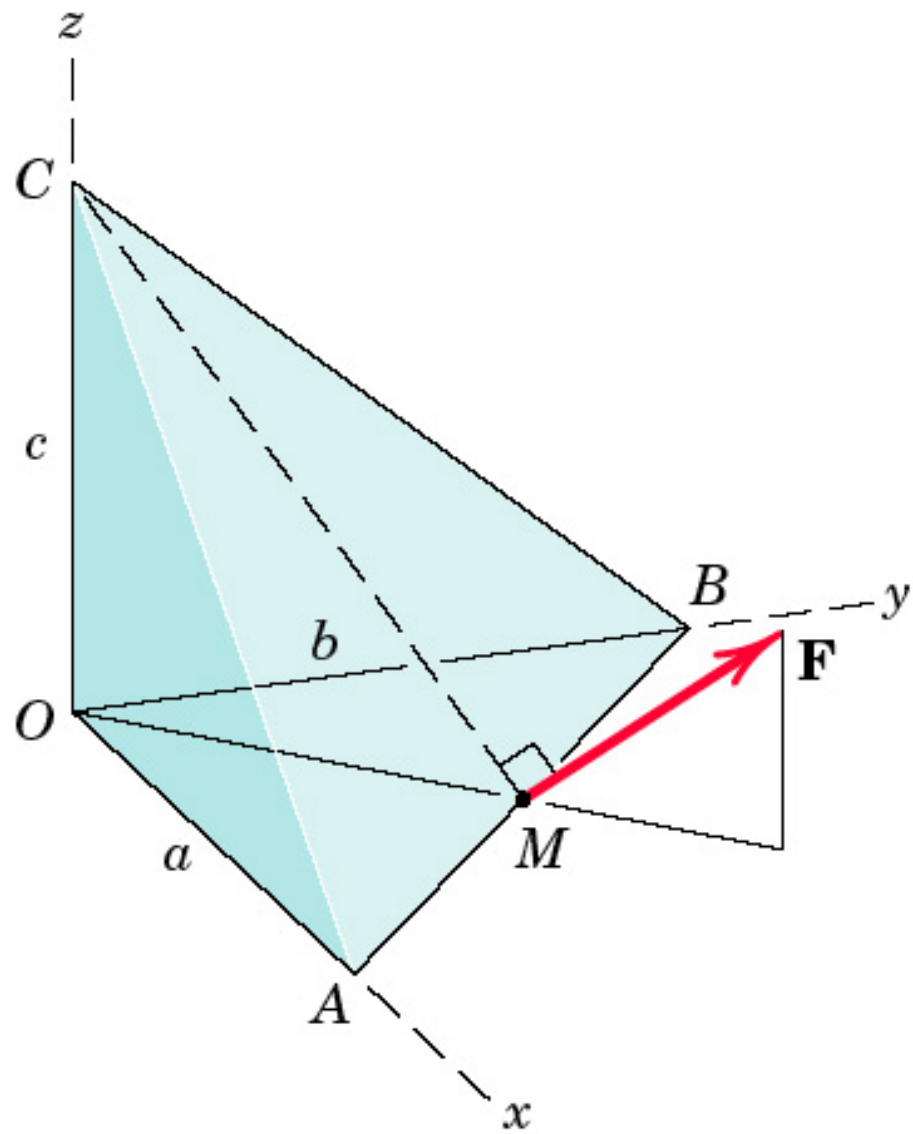
Problem 2/112



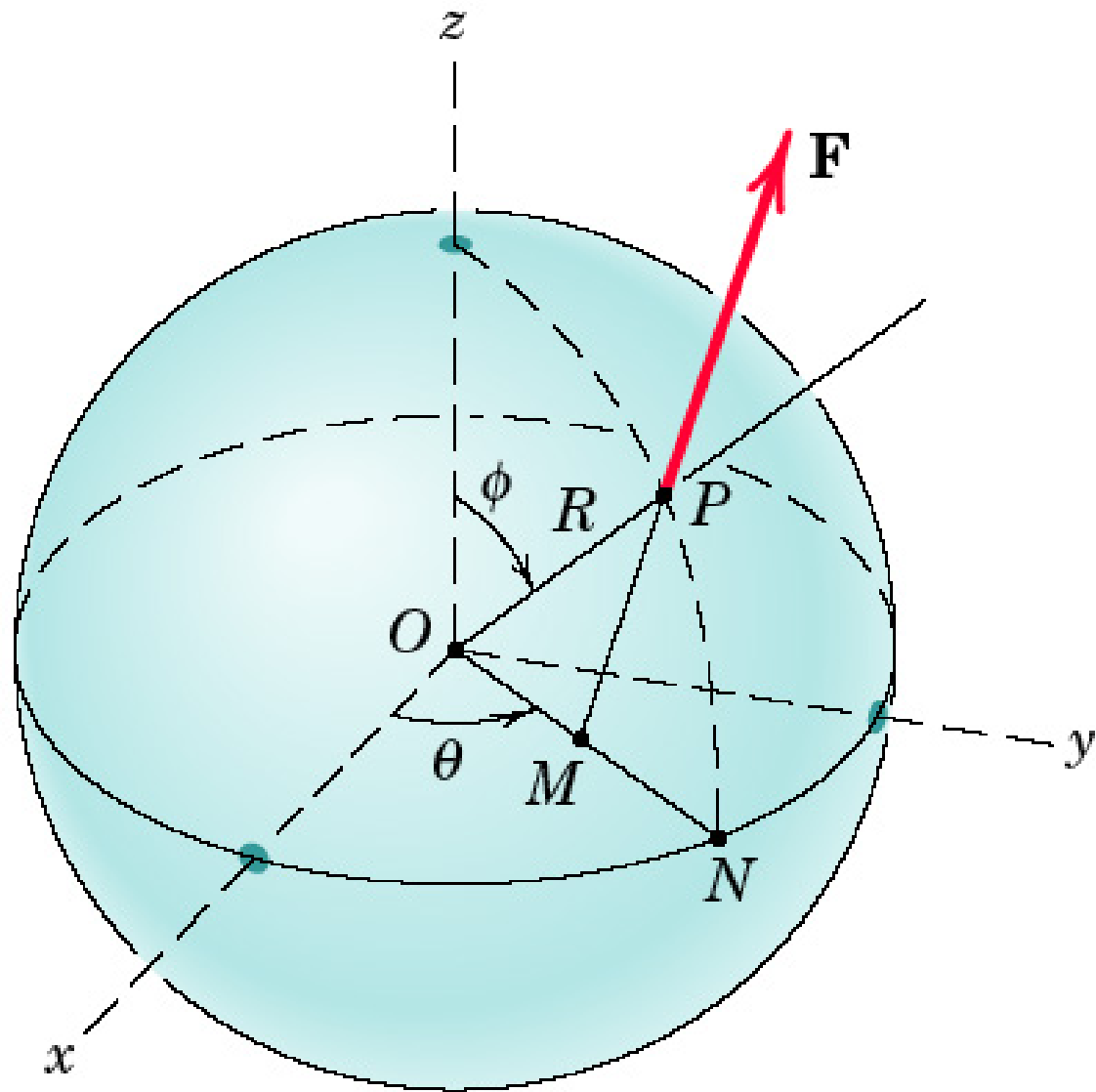
Problem 2/113



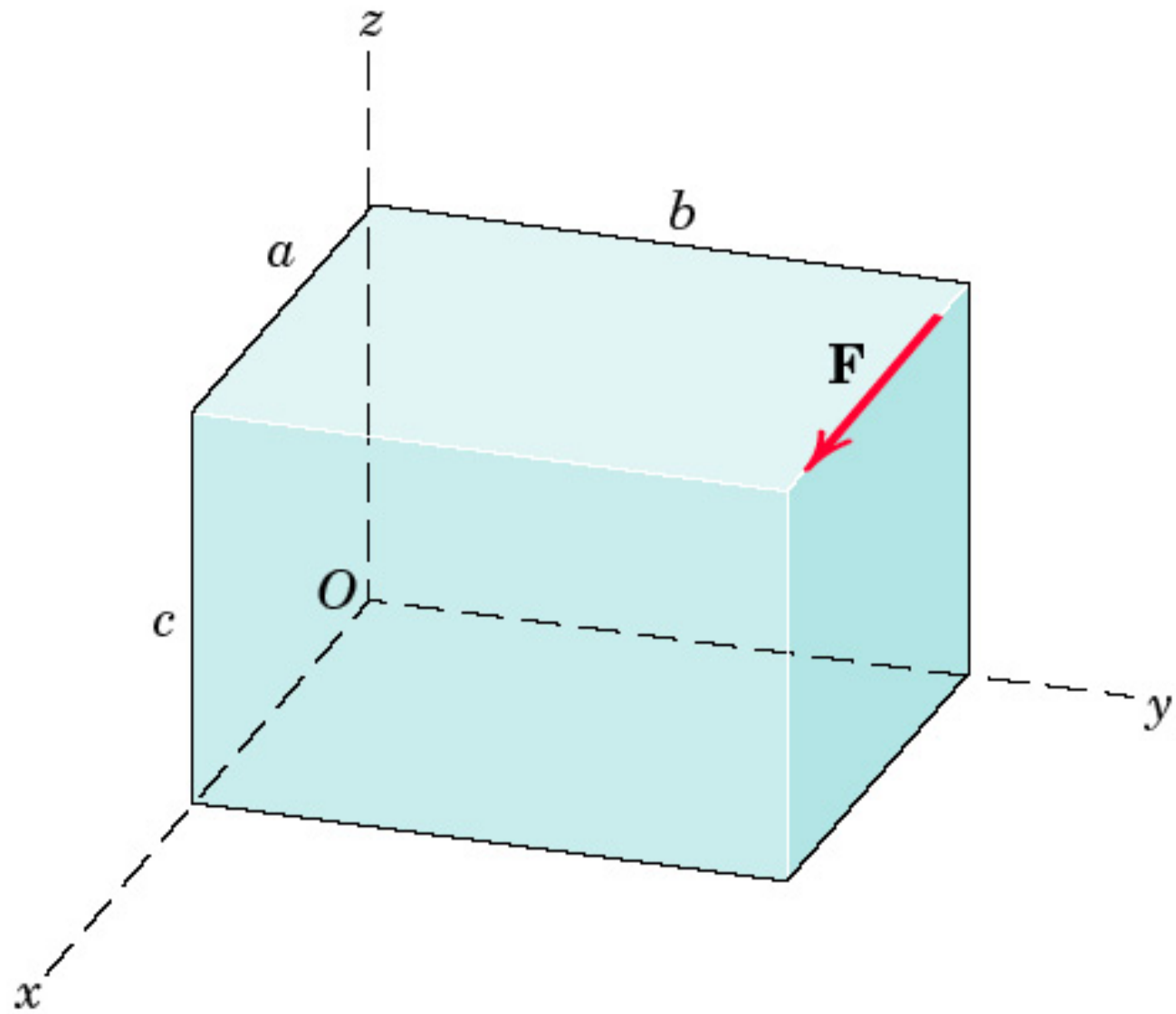
Problem 2/114



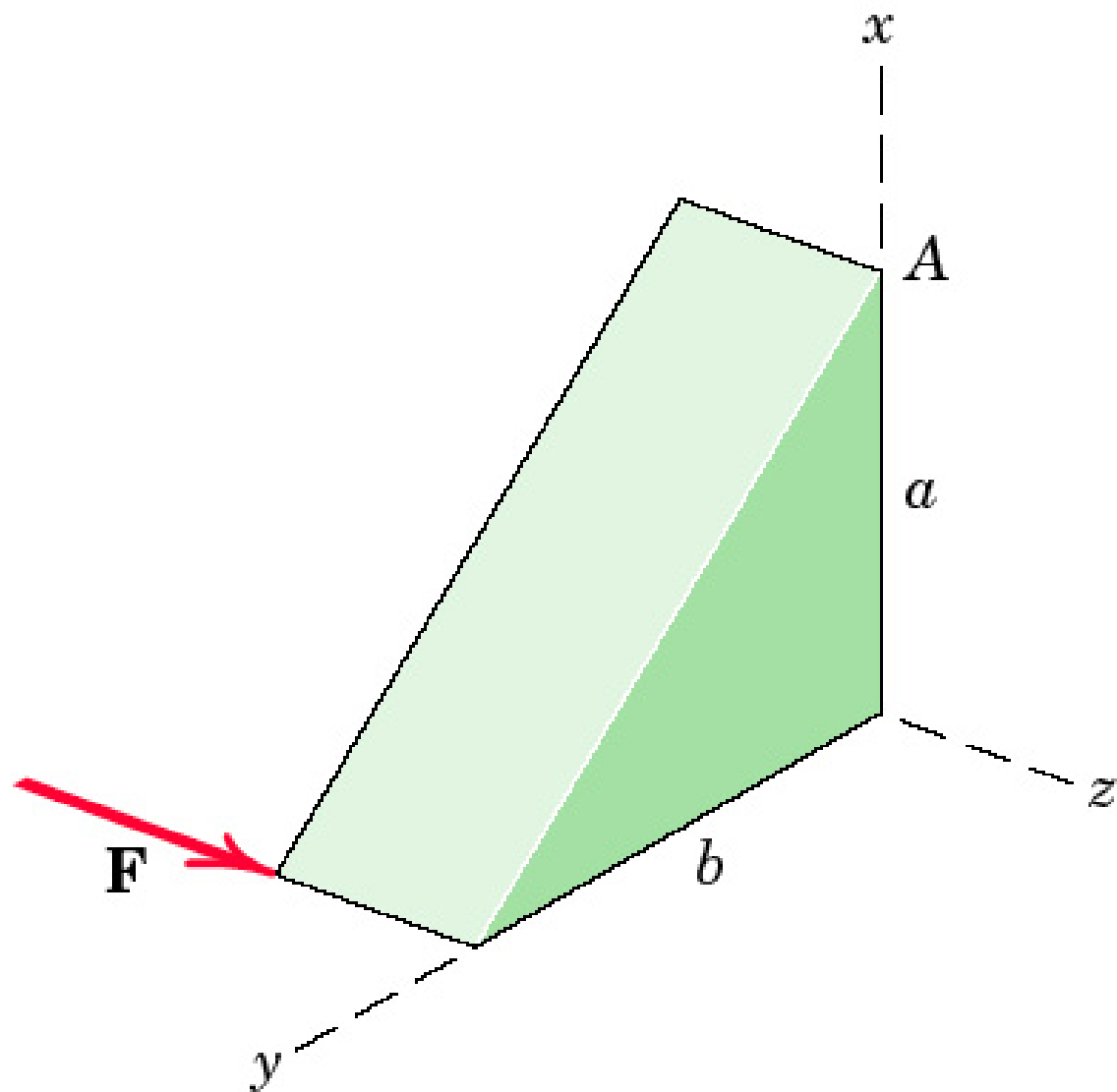
Problem 2/115



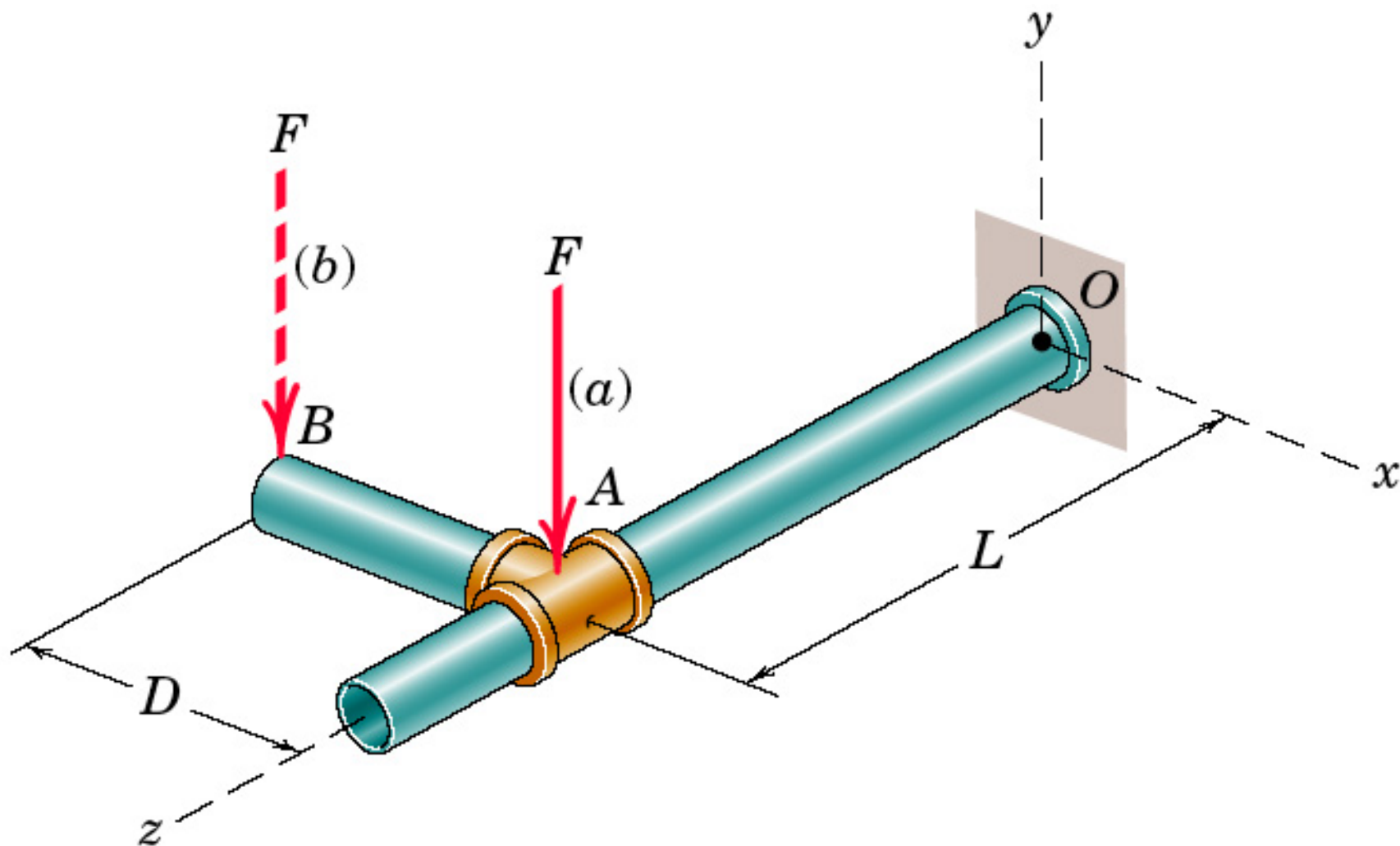
Problem 2/116



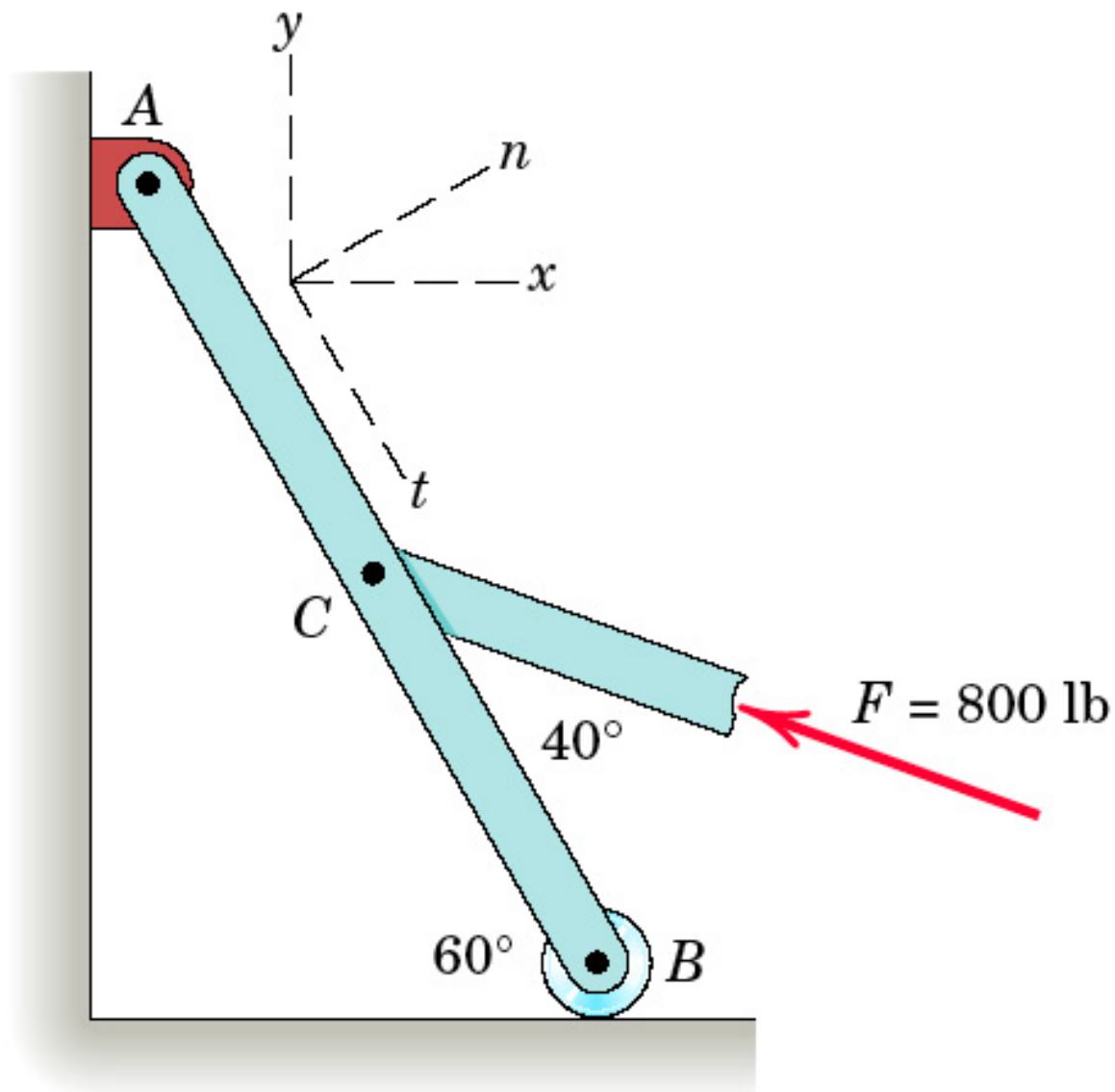
Problem 2/117



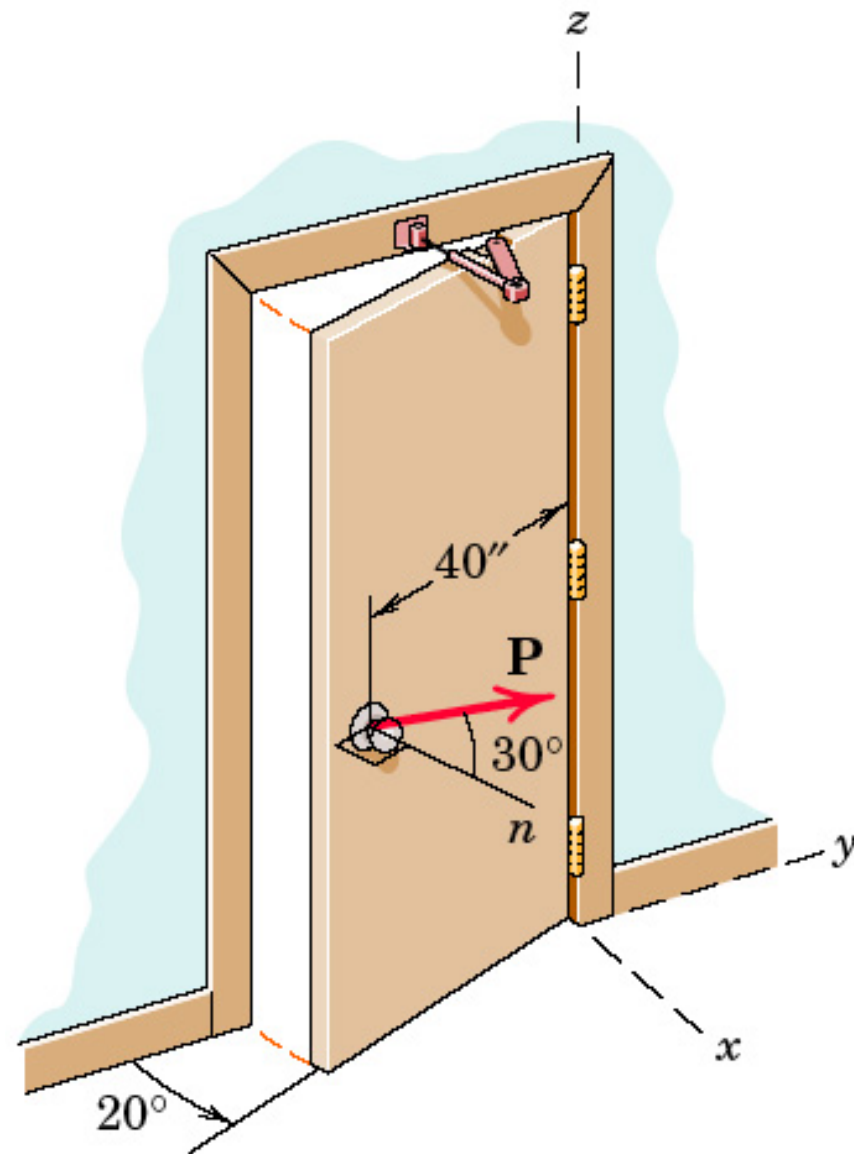
Problem 2/118



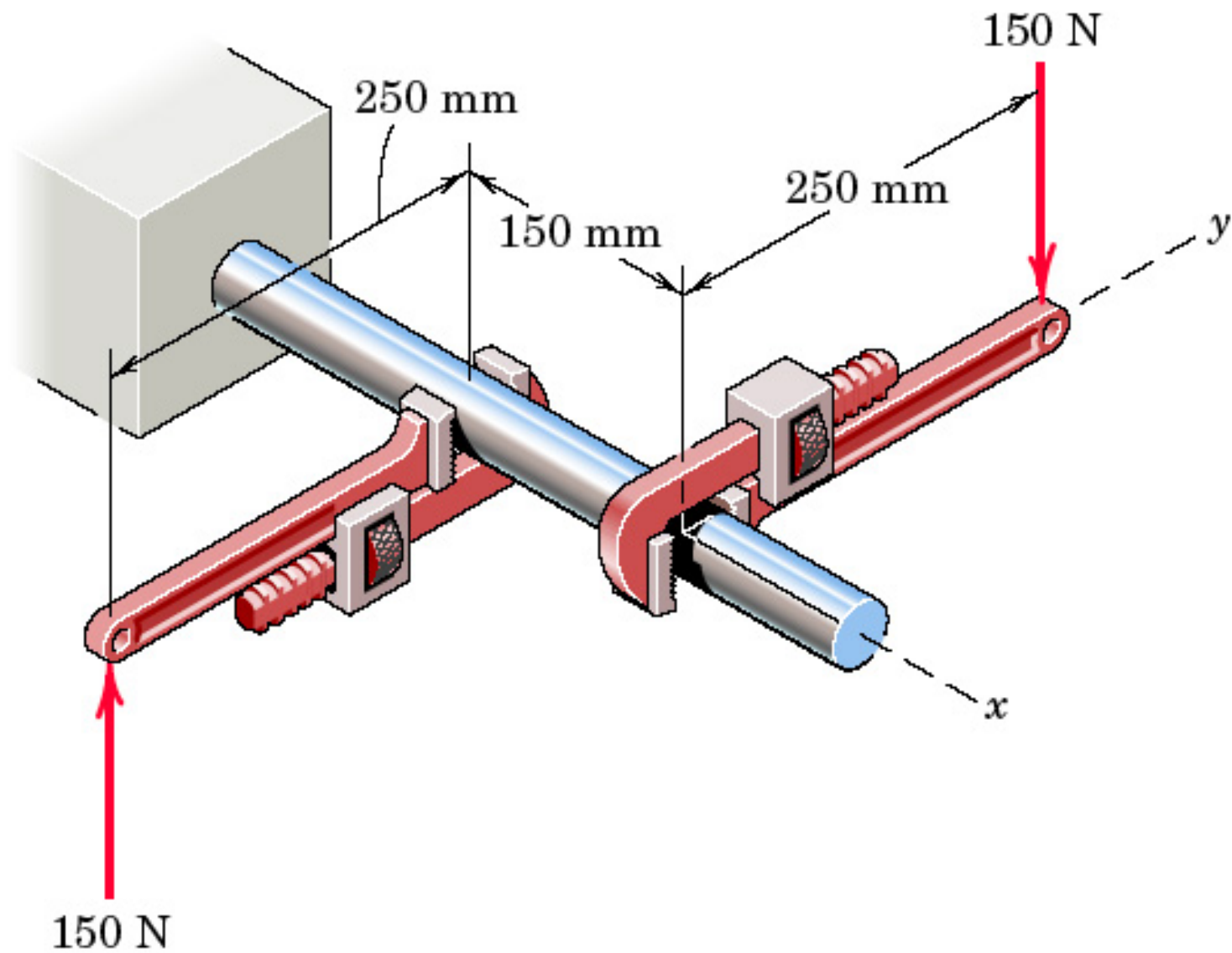
Problem 2/119



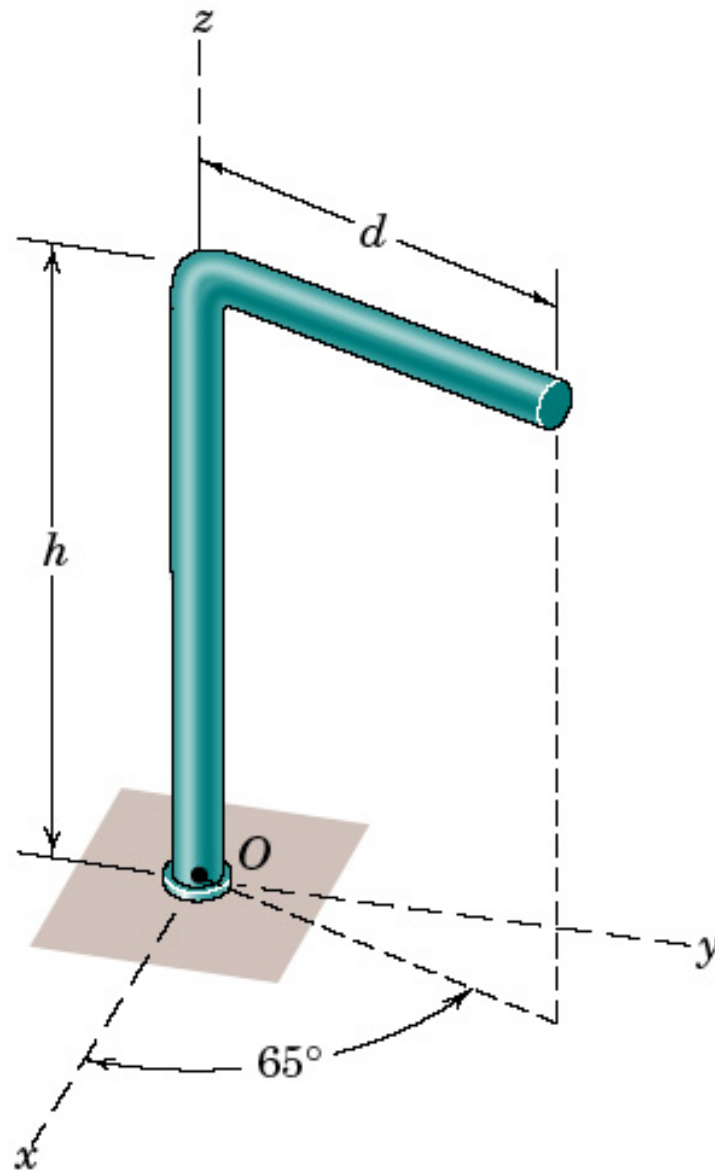
Problem 2/12



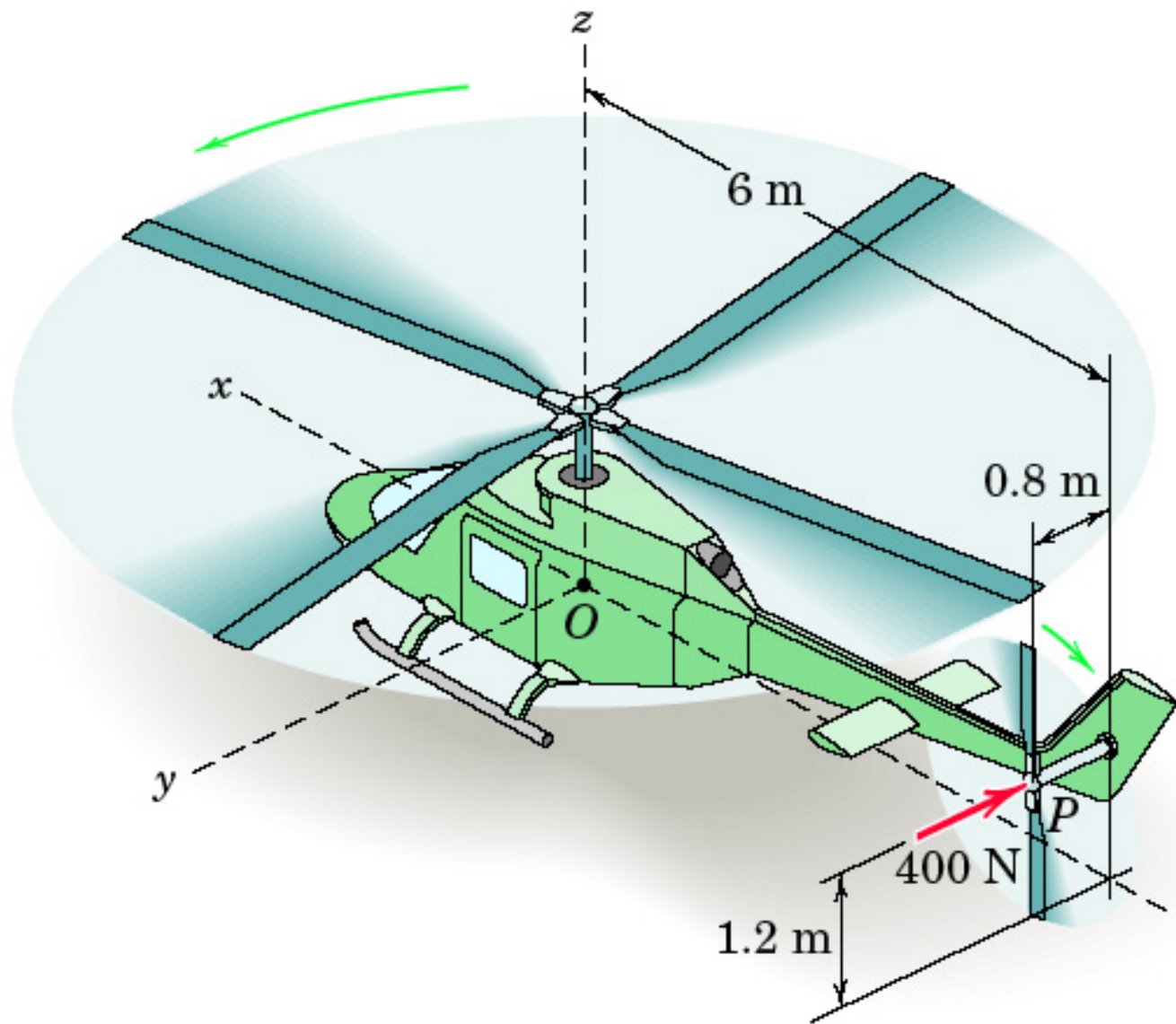
Problem 2/120



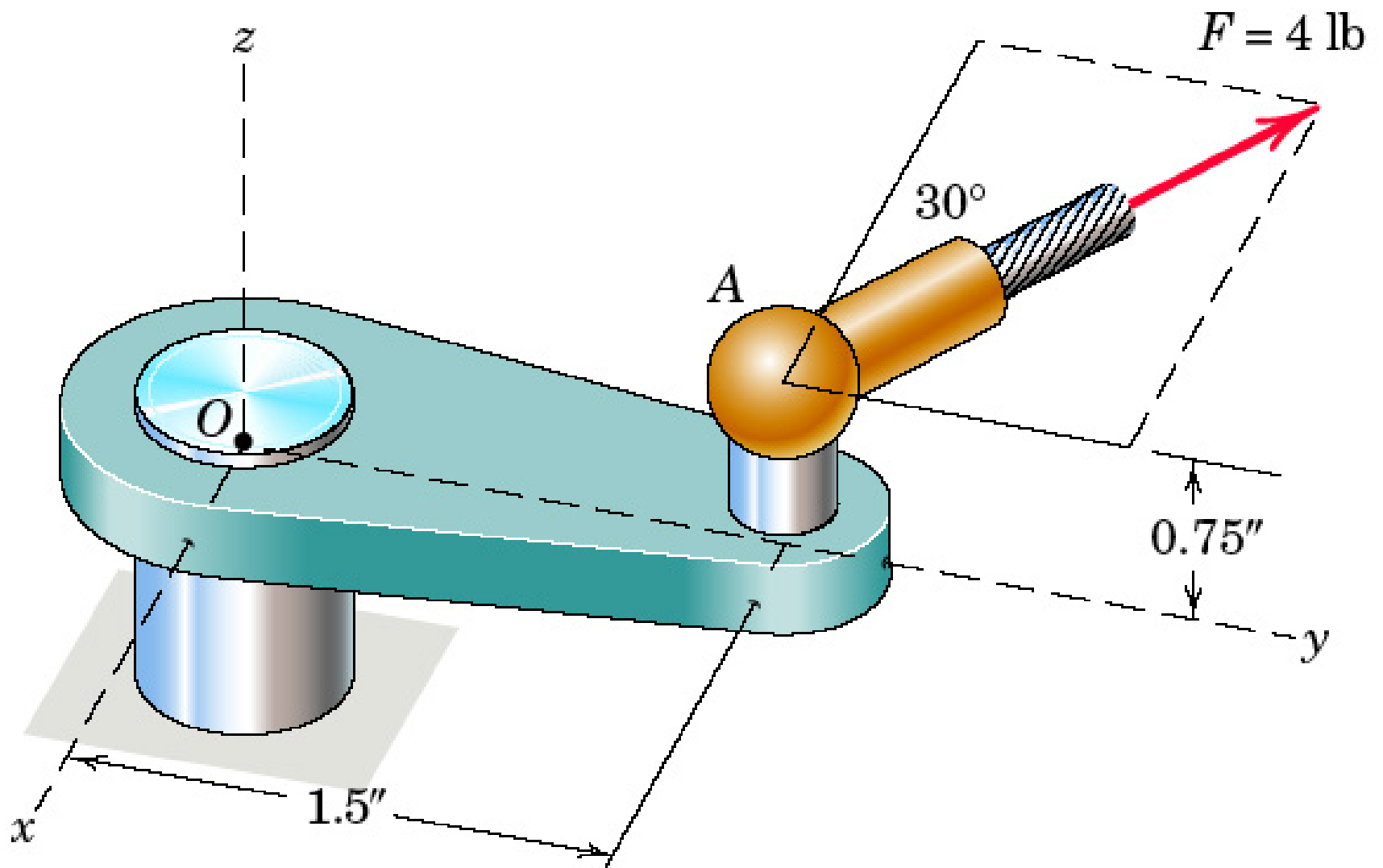
Problem 2/121



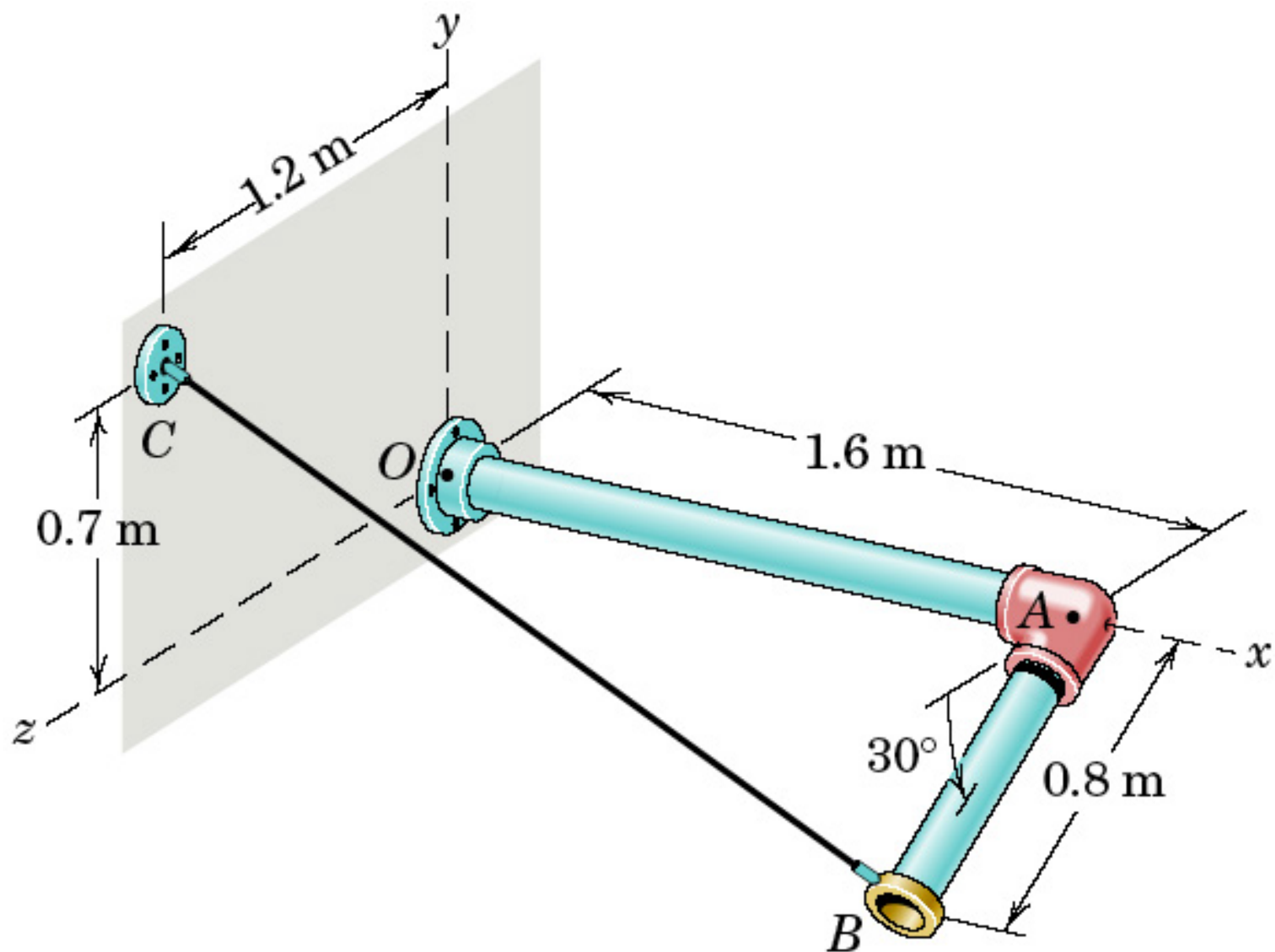
Problem 2/122



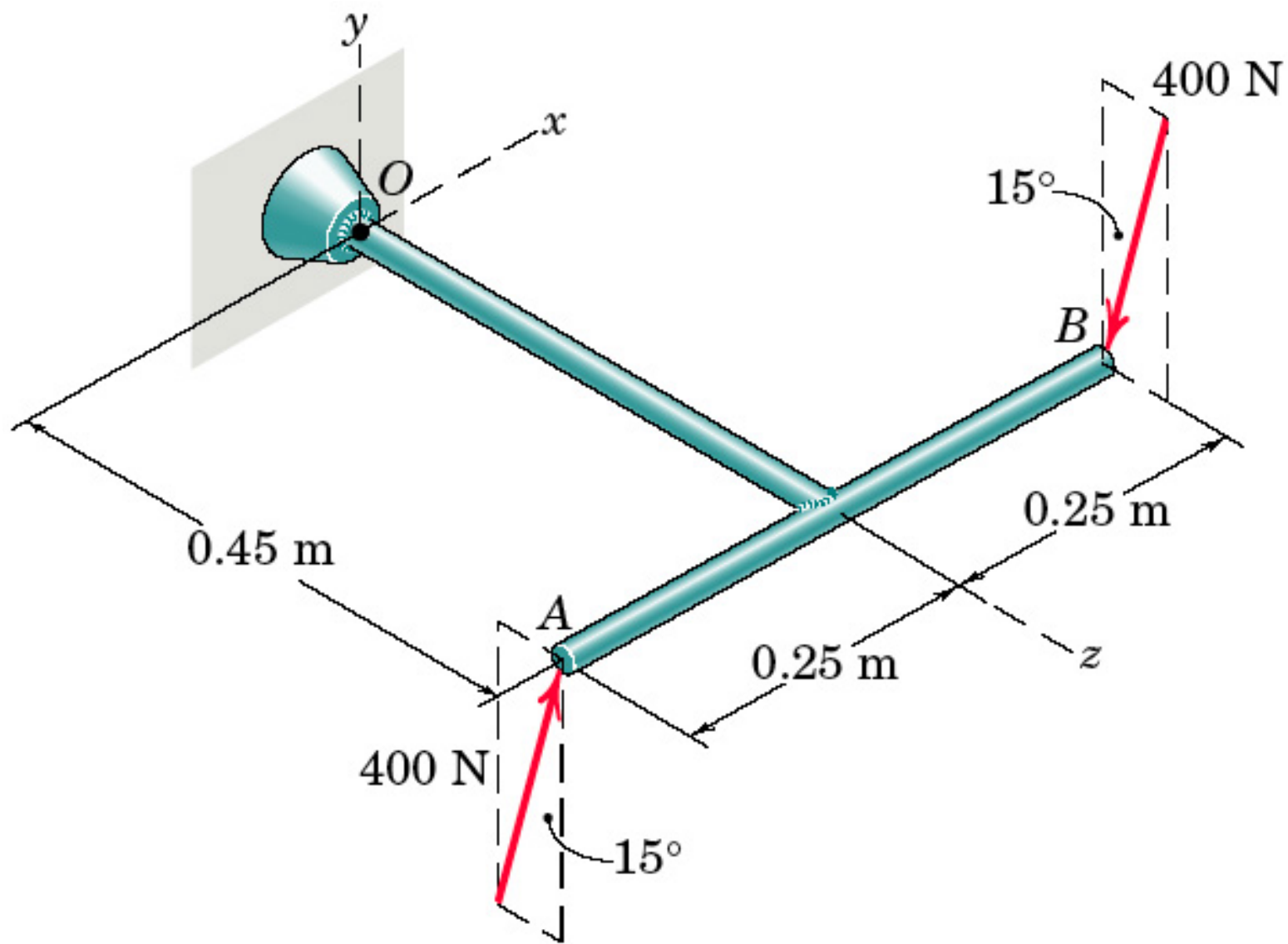
Problem 2/123



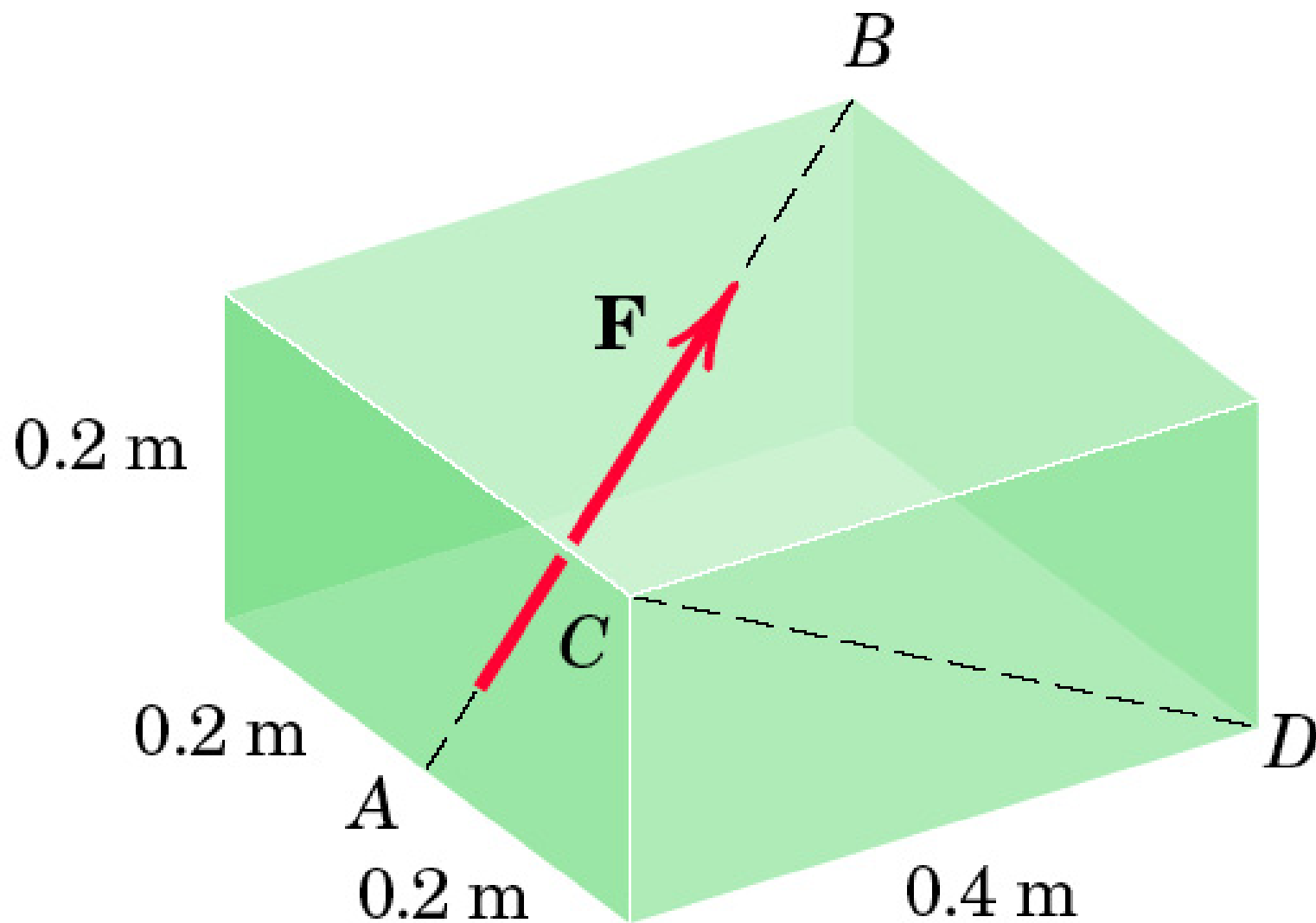
Problem 2/124



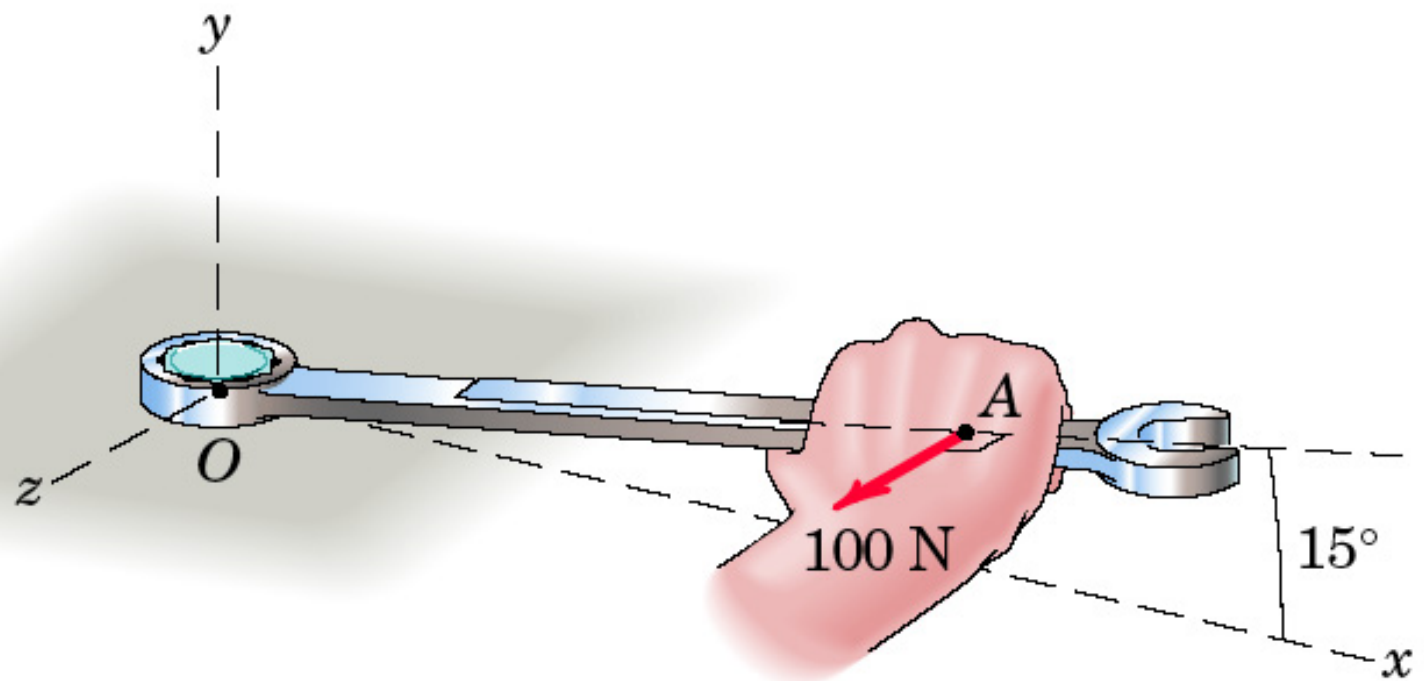
Problem 2/125



Problem 2/126



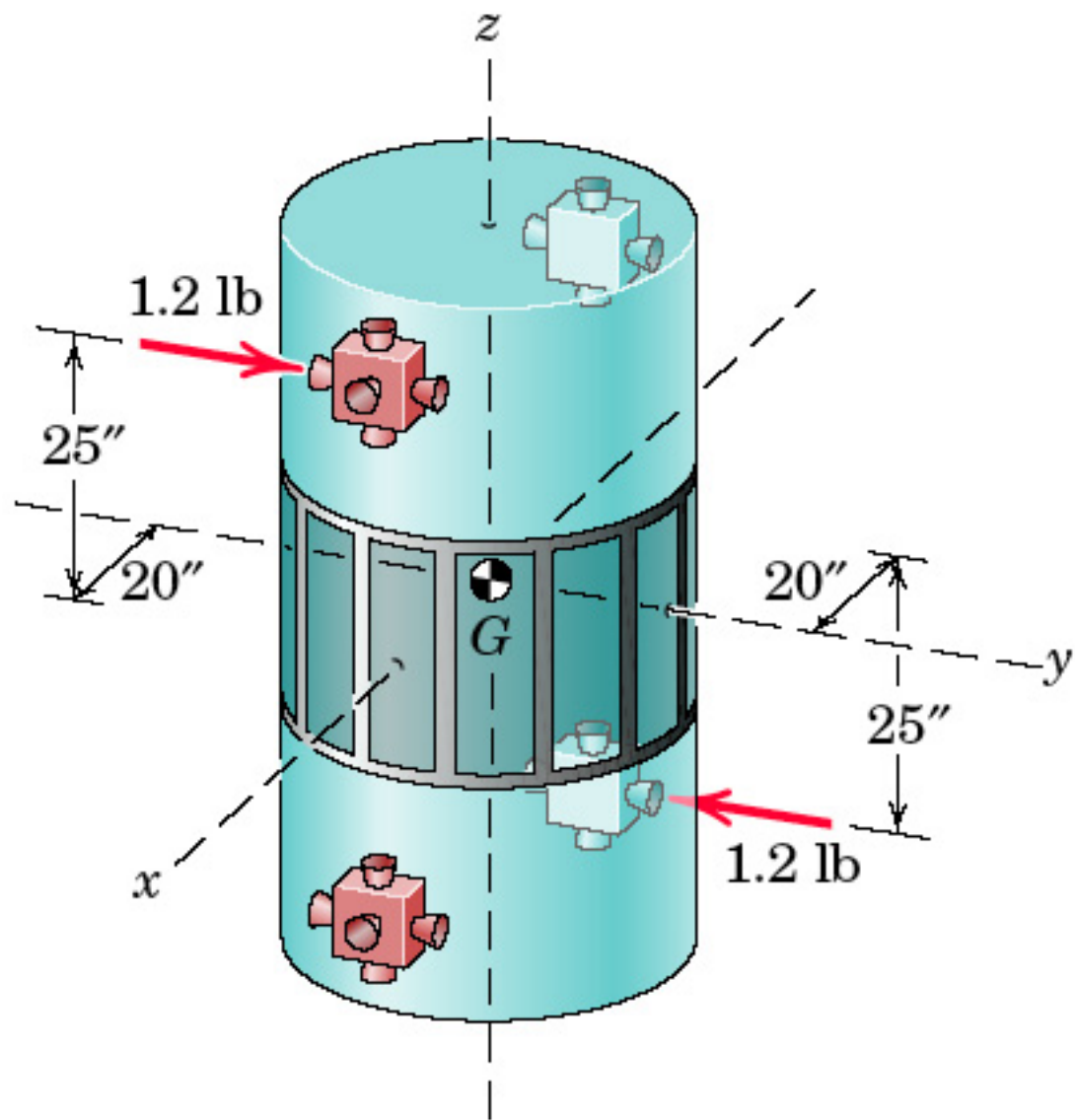
Problem 2/127



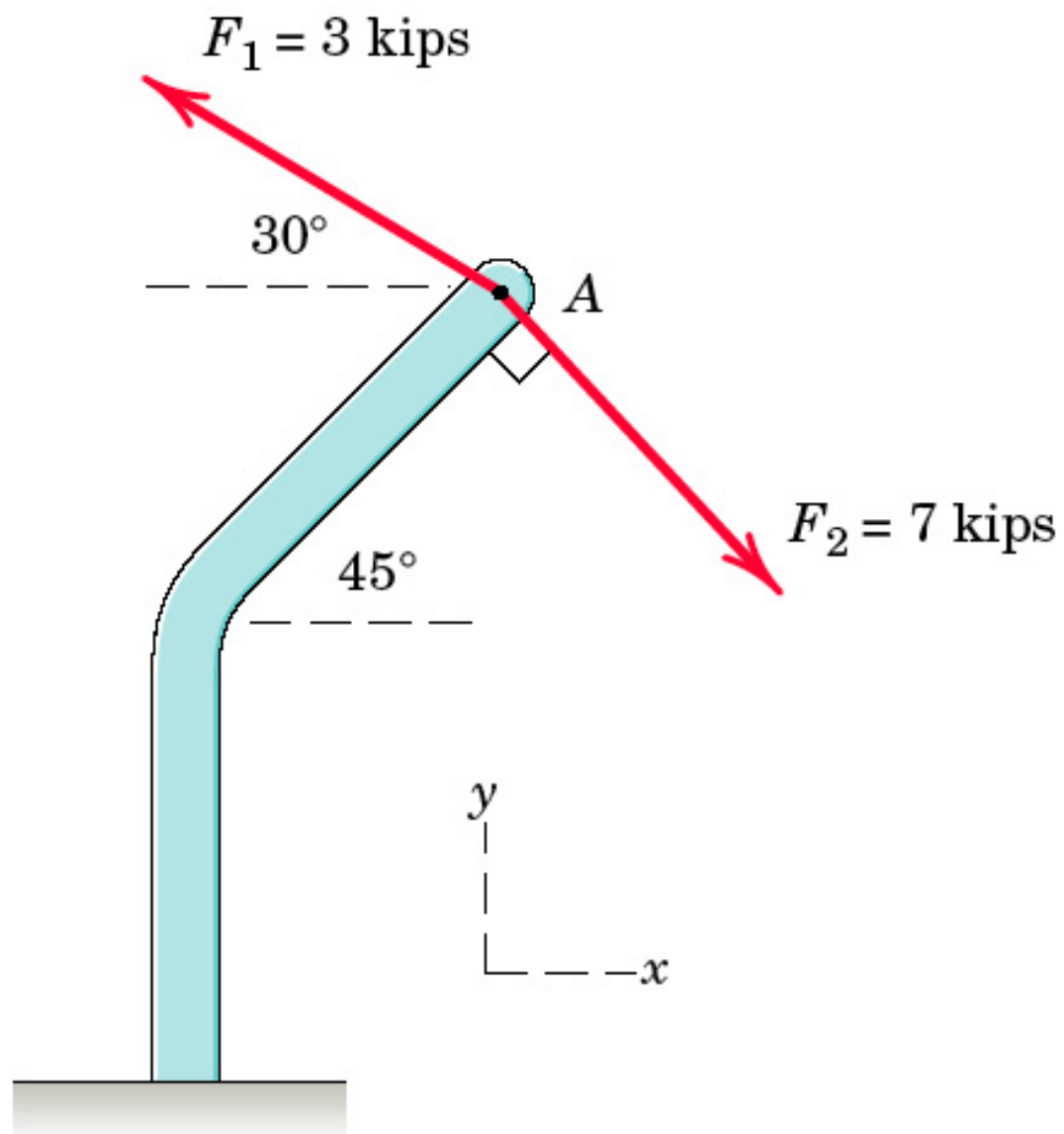
$$\overline{OA} = 185\text{ mm}$$

Problem 2/128

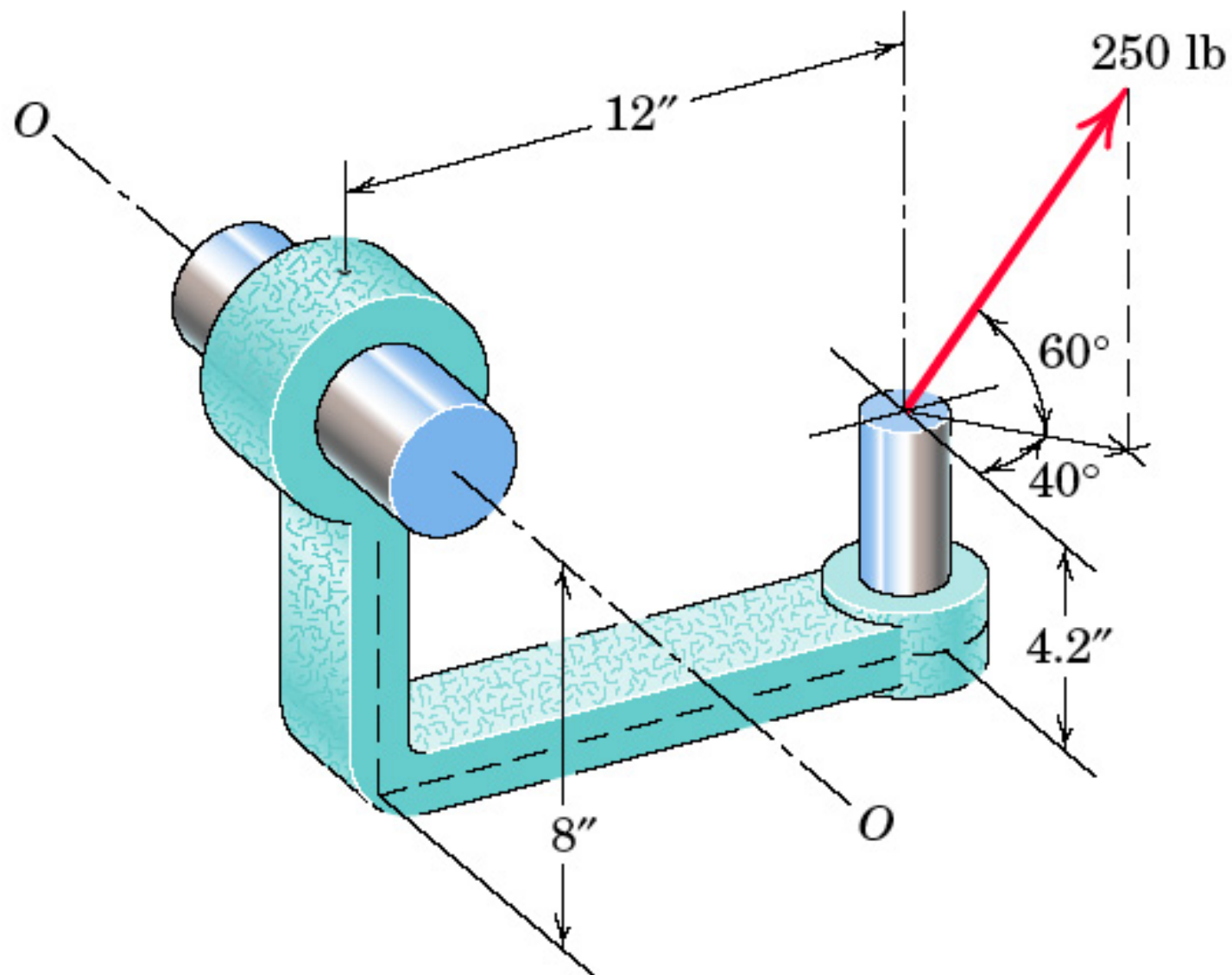
P_02_128



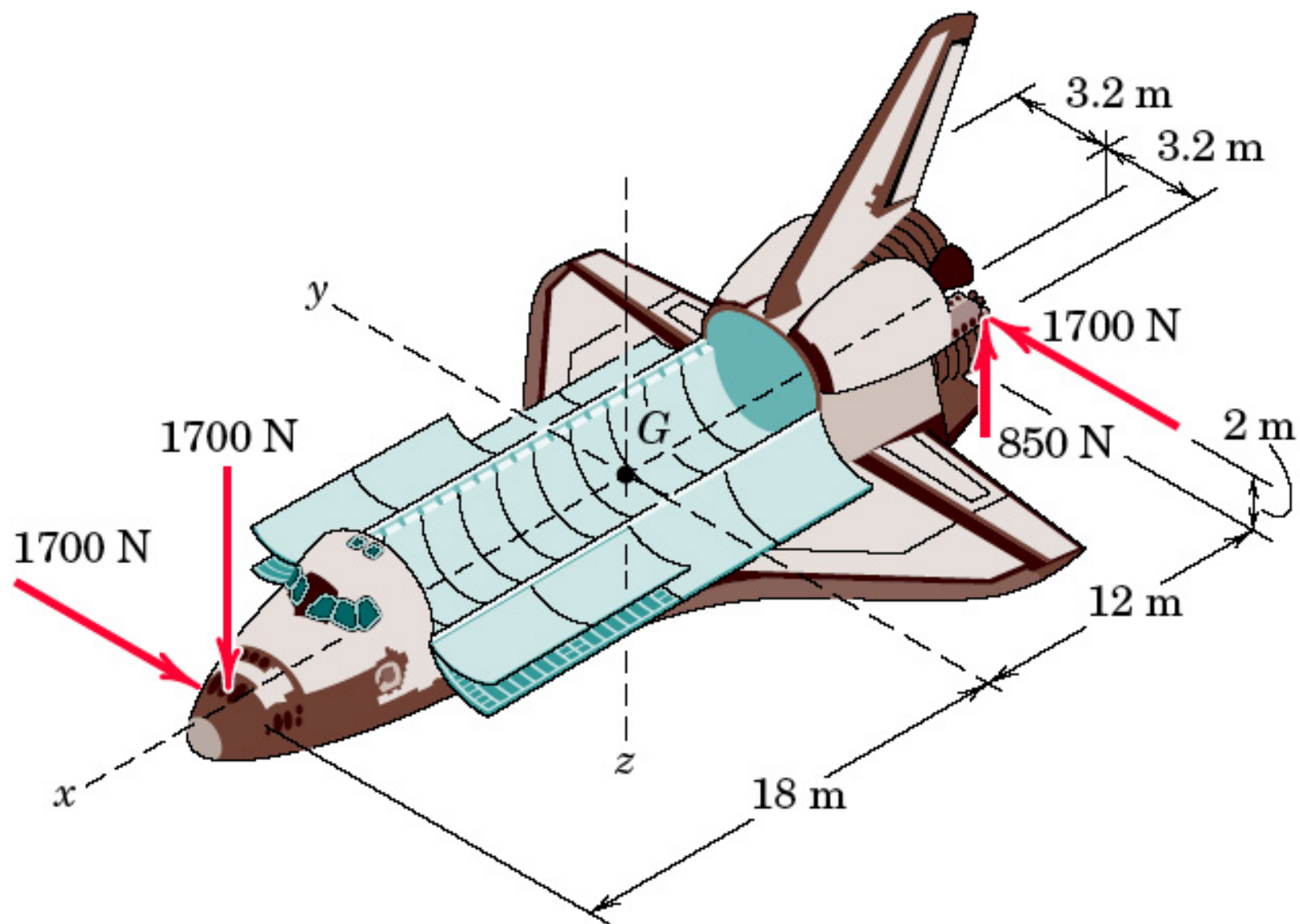
Problem 2/129



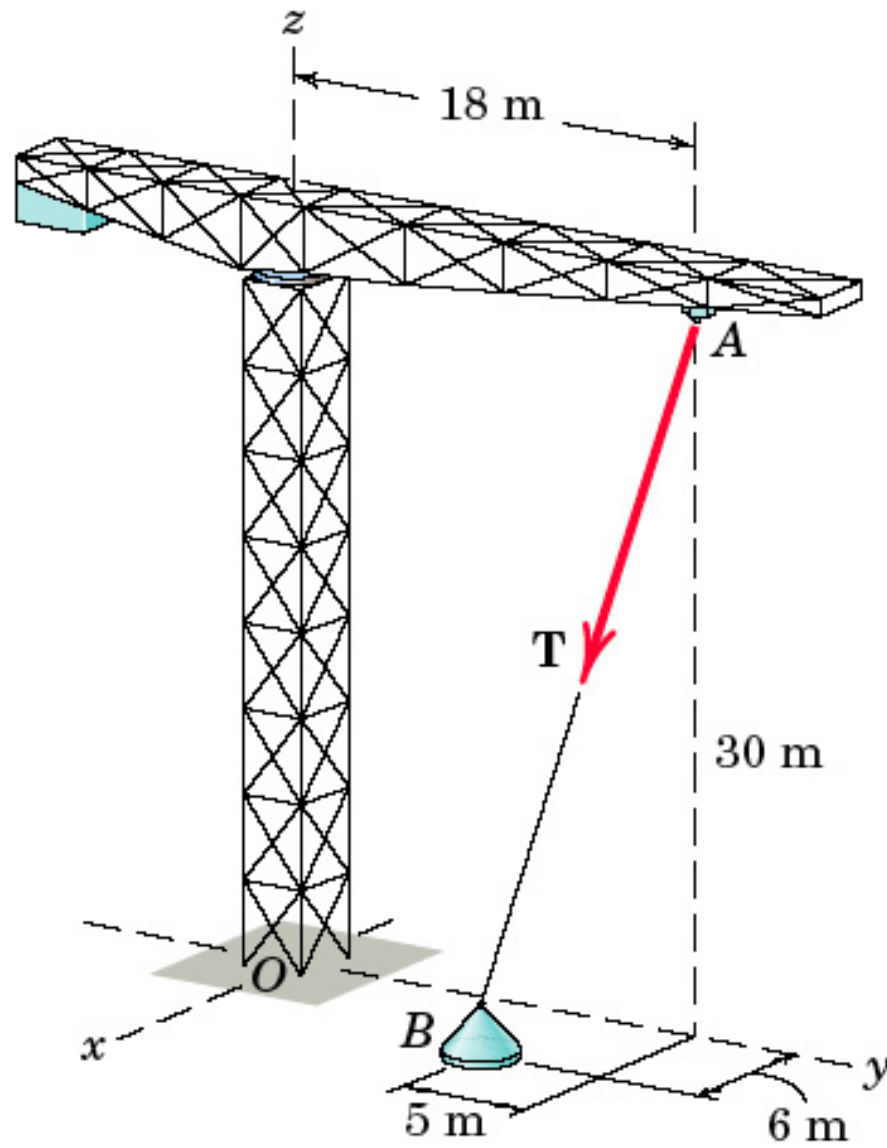
Problem 2/13



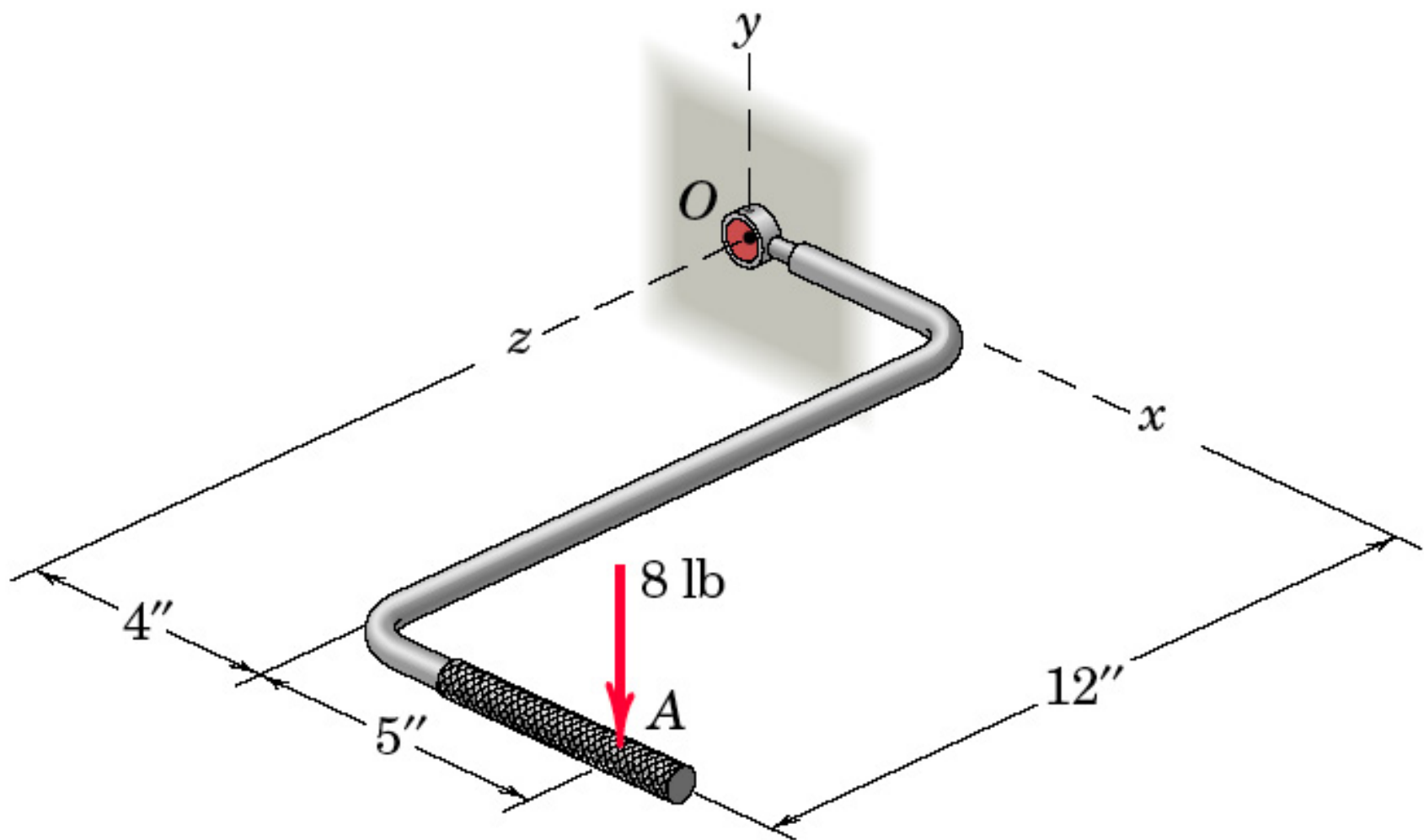
Problem 2/130



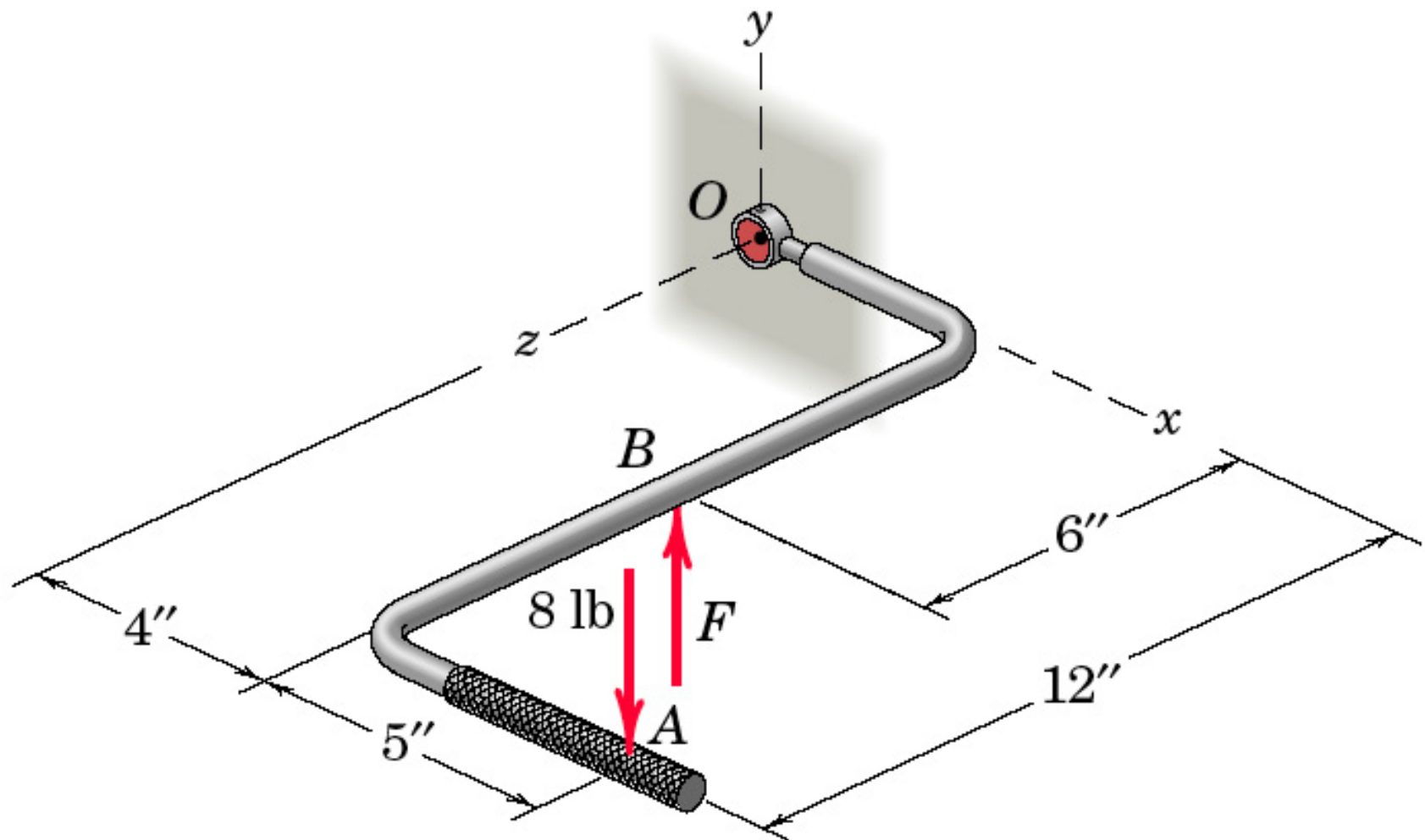
Problem 2/131



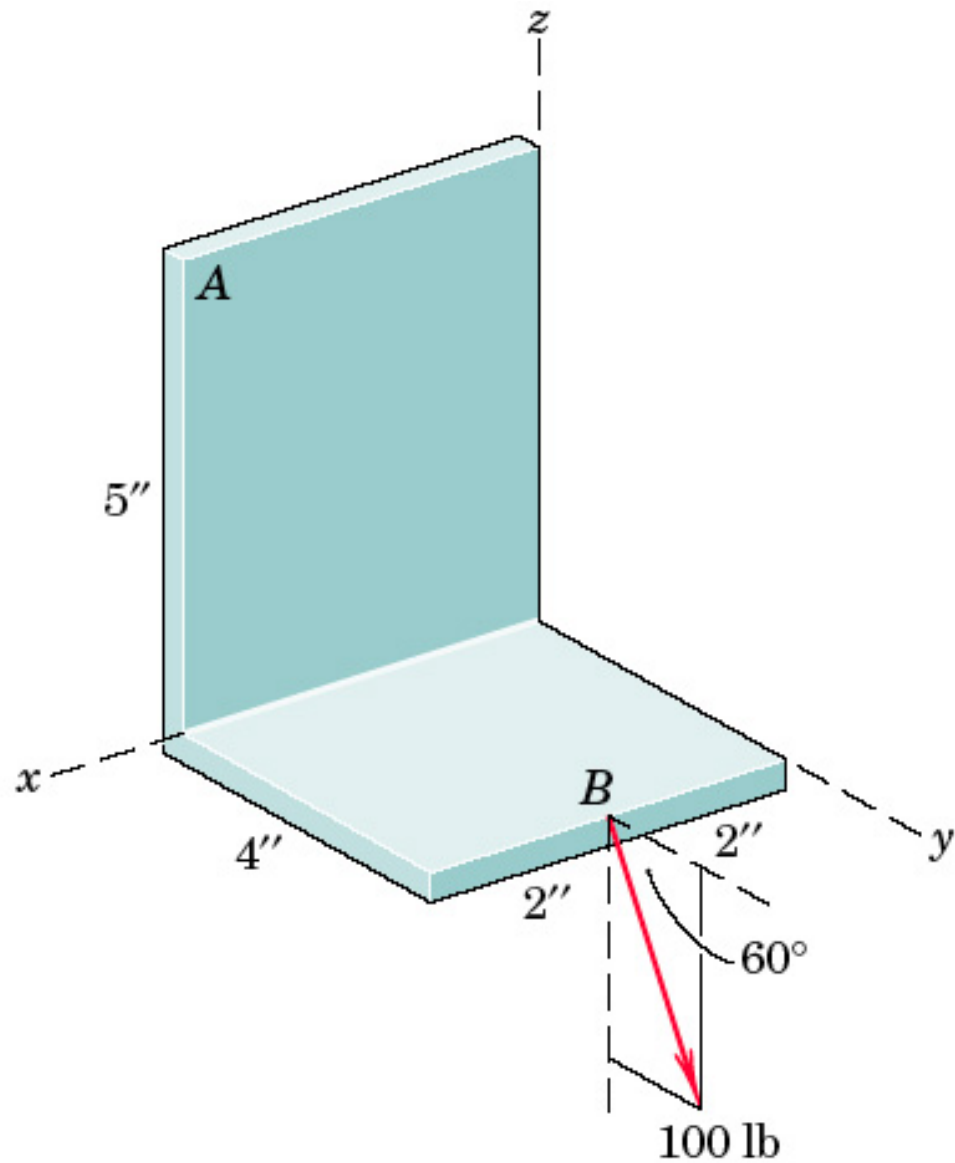
Problem 2/132



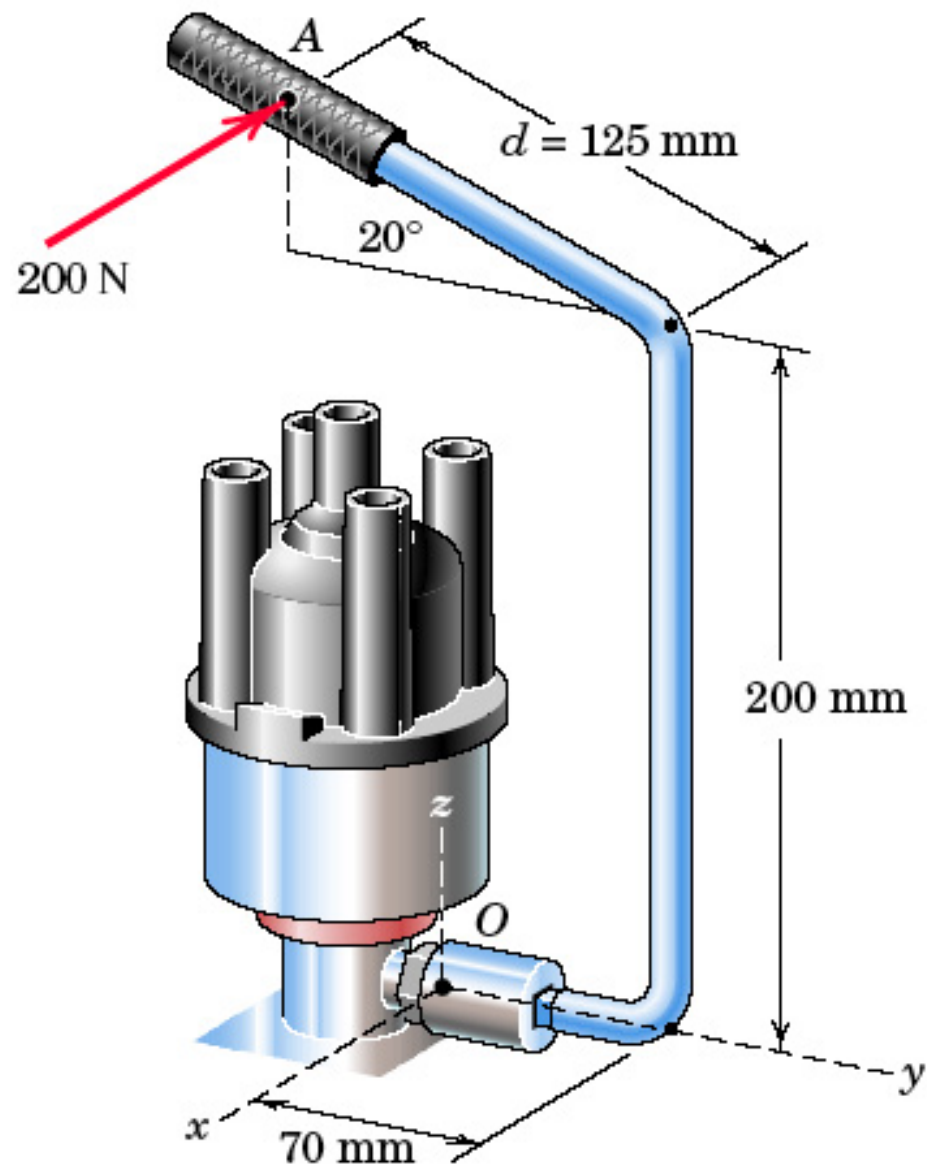
Problem 2/133



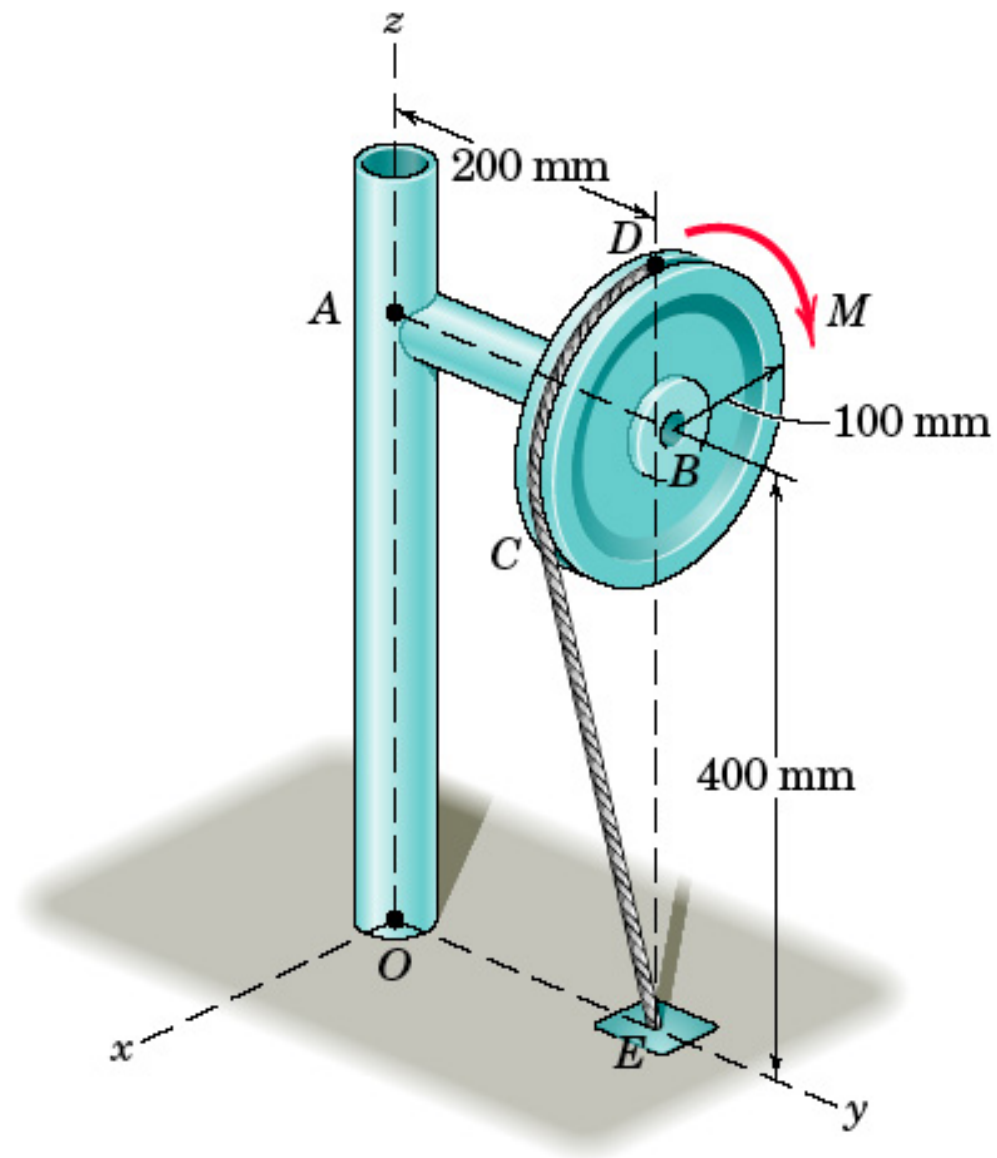
Problem 2/134



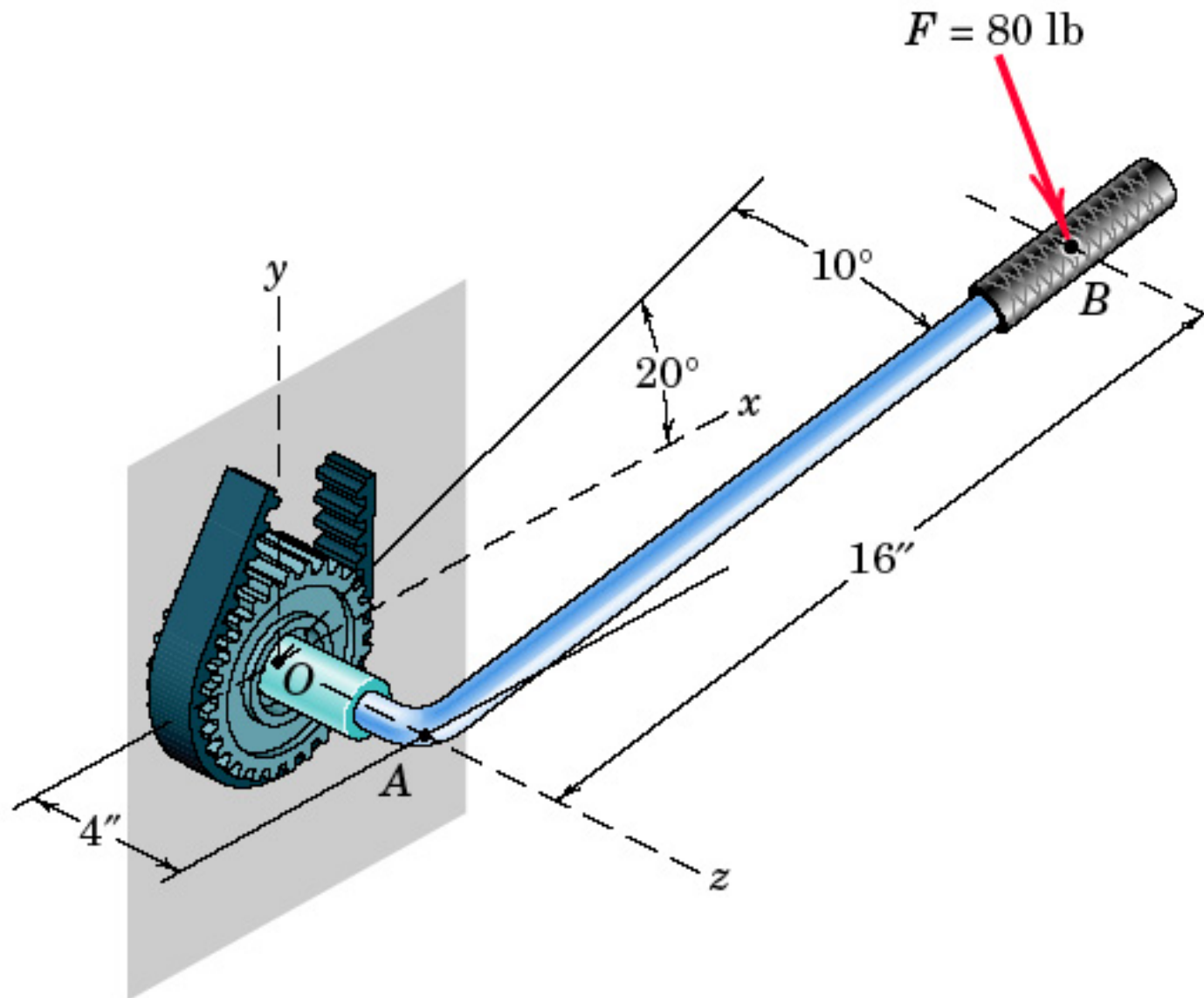
Problem 2/135



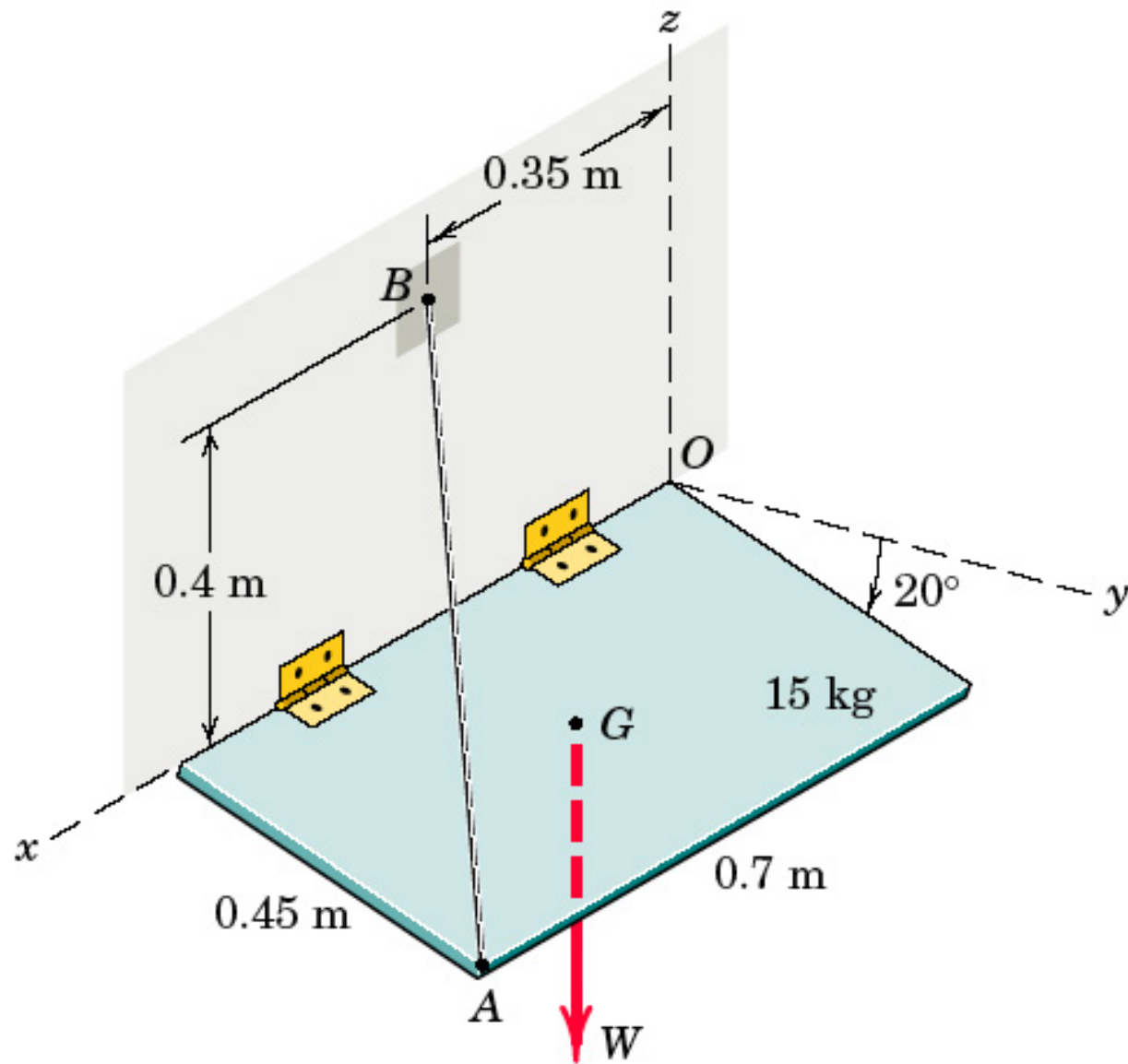
Problem 2/136



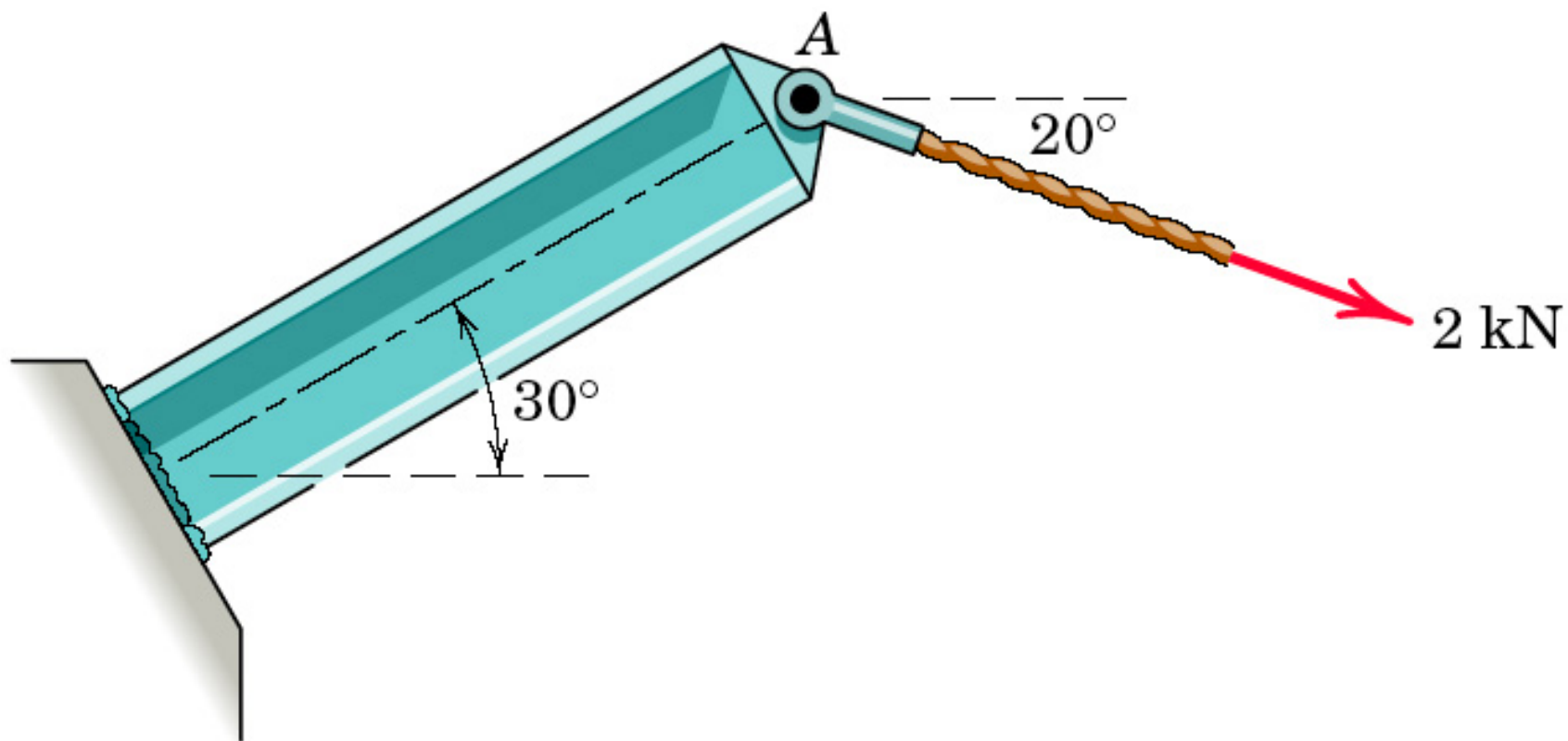
Problem 2/137



Problem 2/138

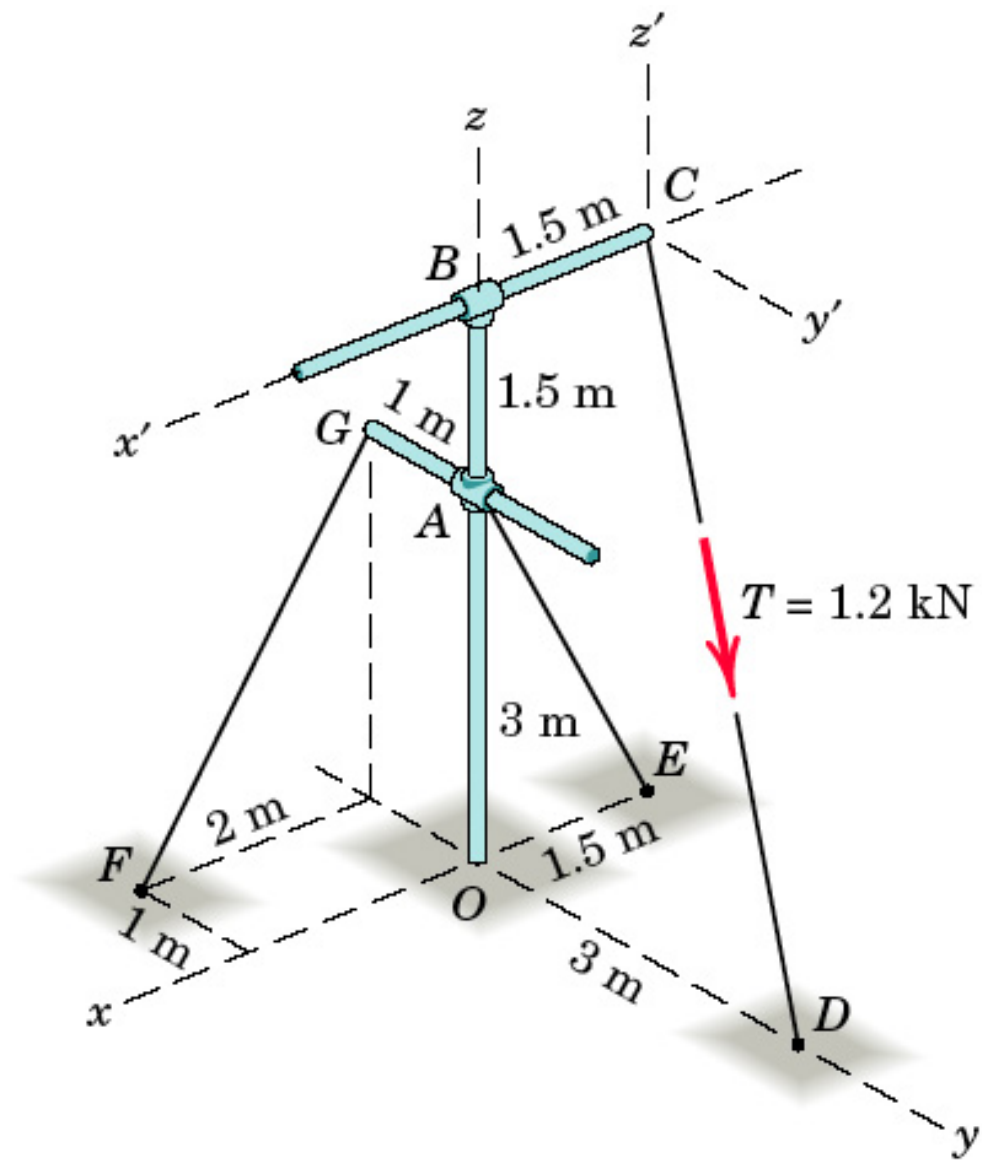


Problem 2/139

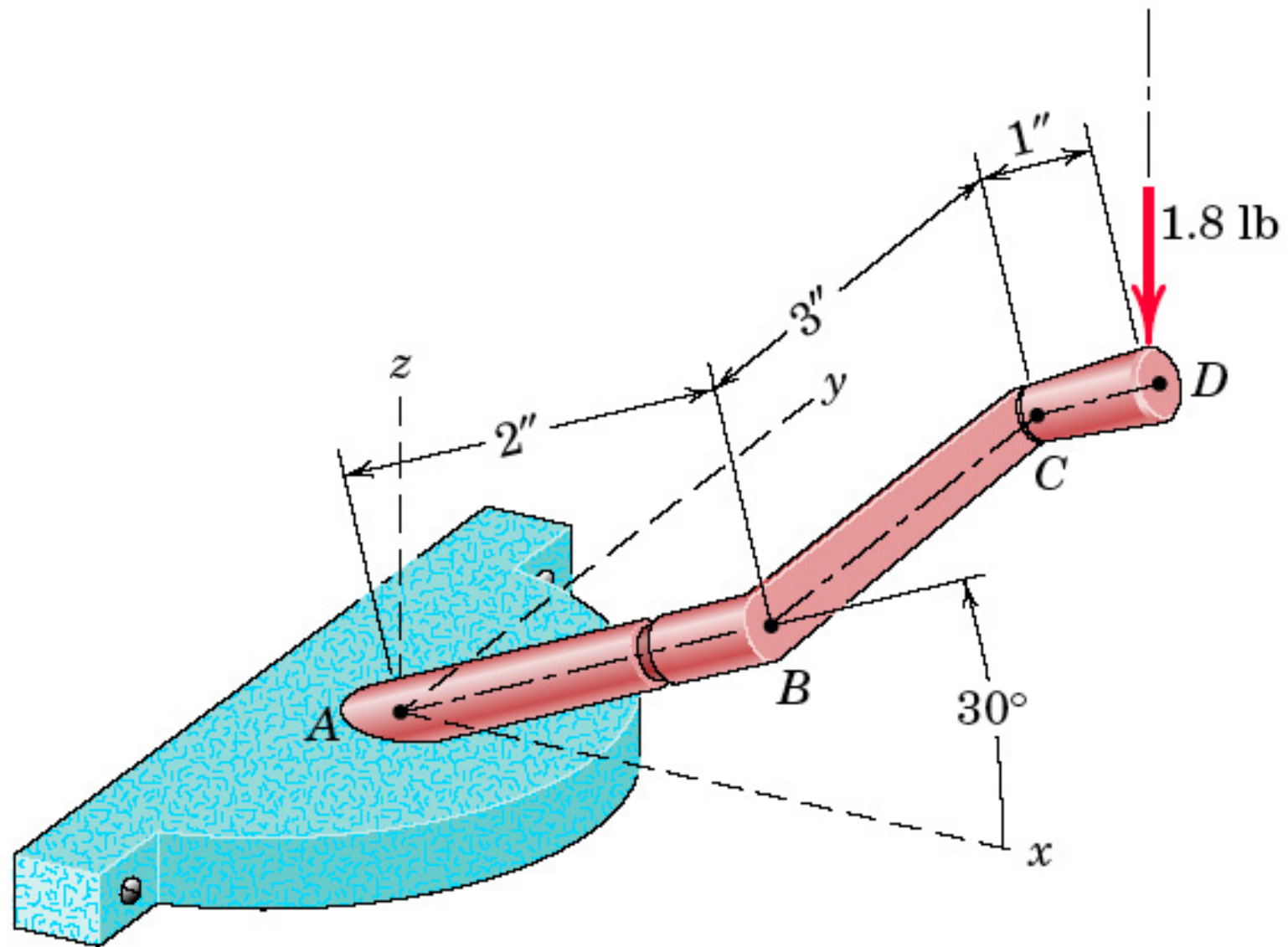


Problem 2/14

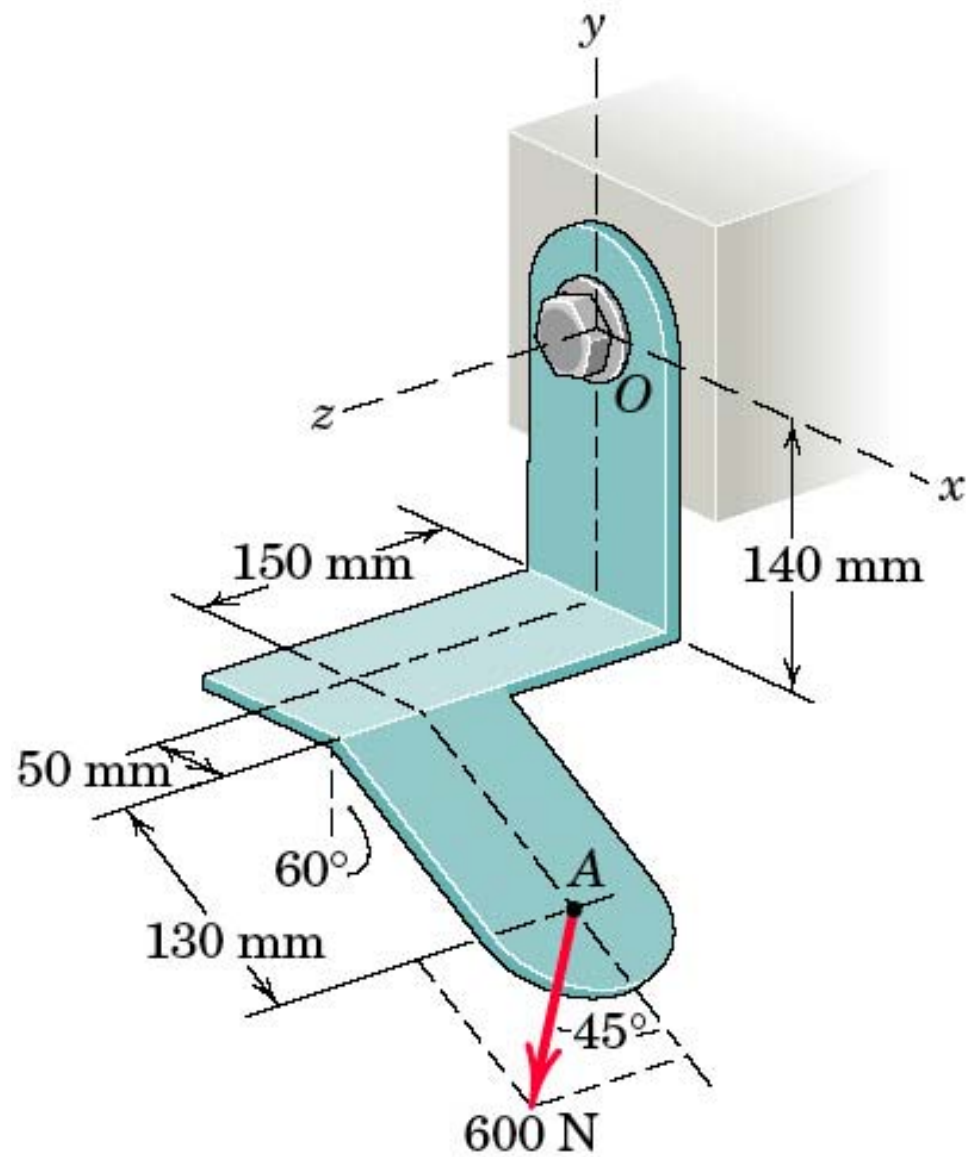
P_02_14



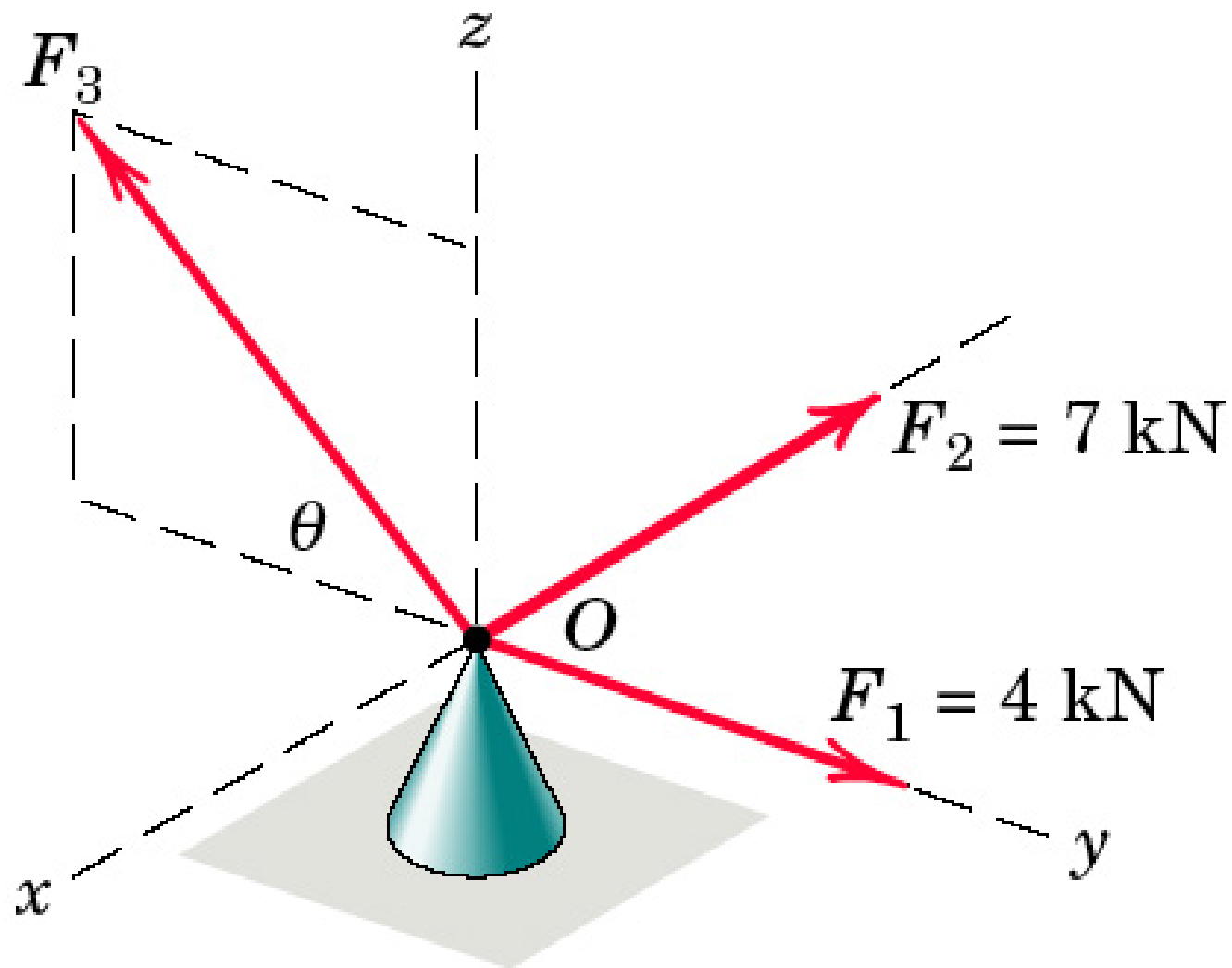
Problem 2/140



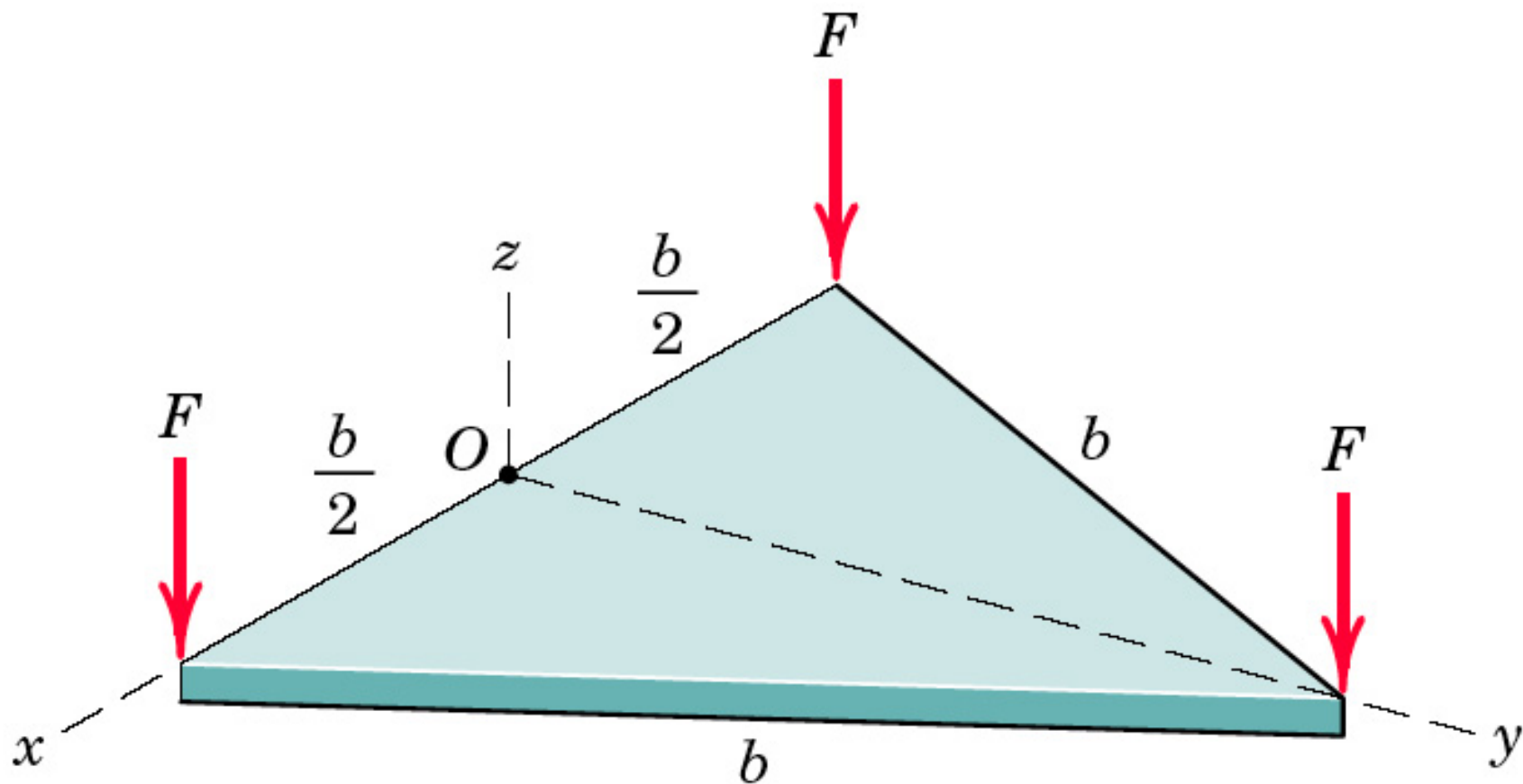
Problem 2/141



Problem 2/142

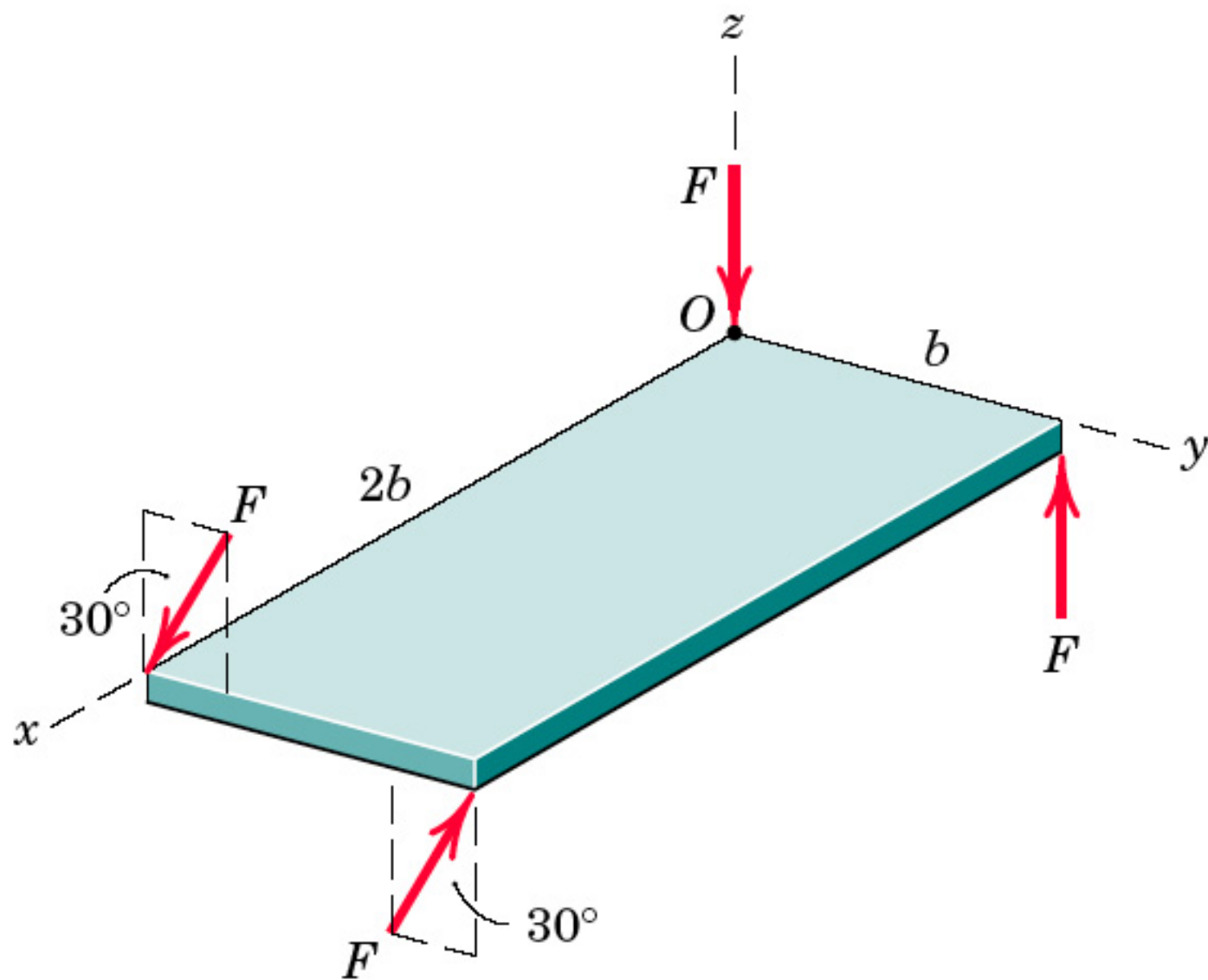


Problem 2/143

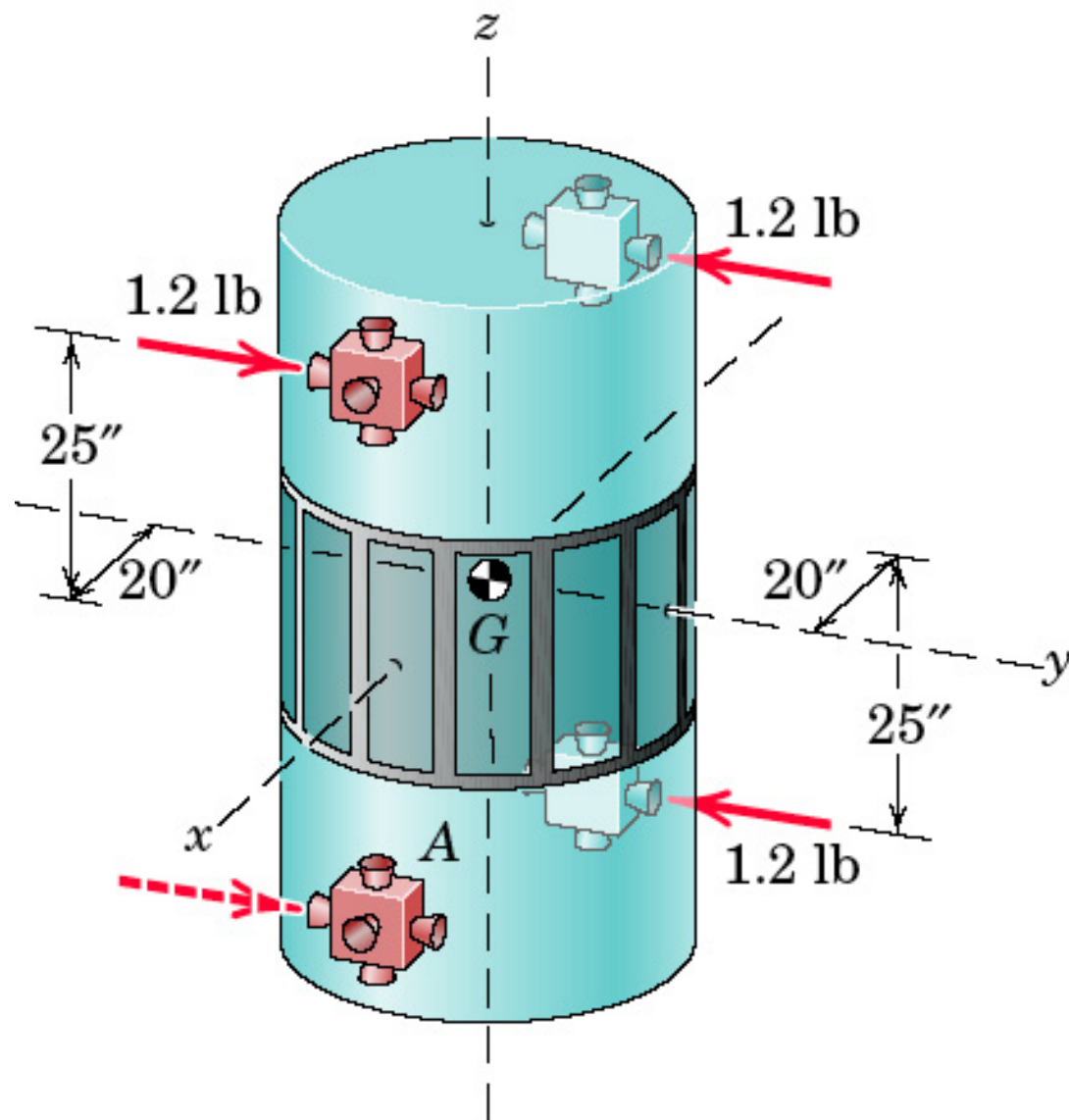


Problem 2/144

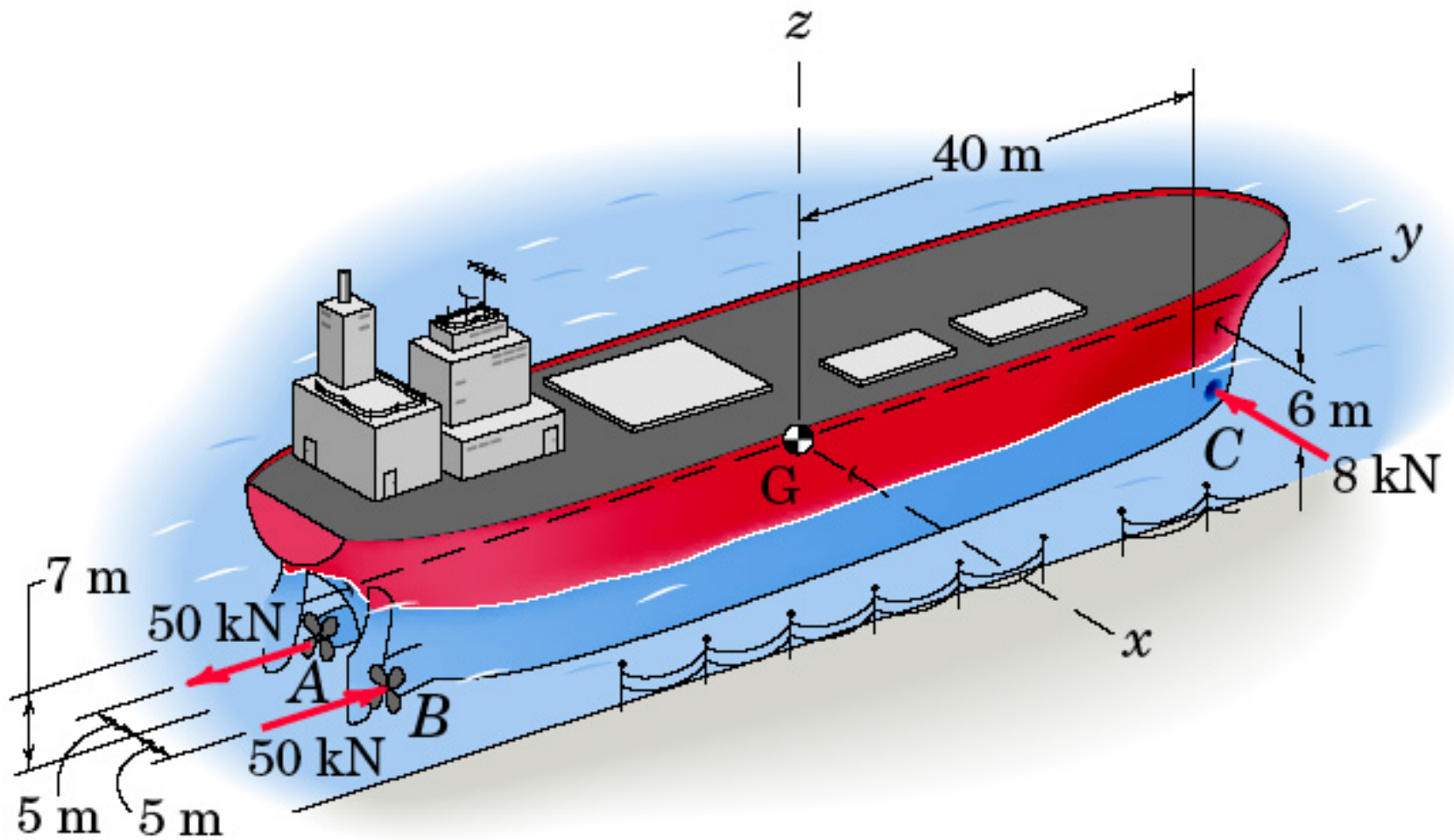
P_02_144



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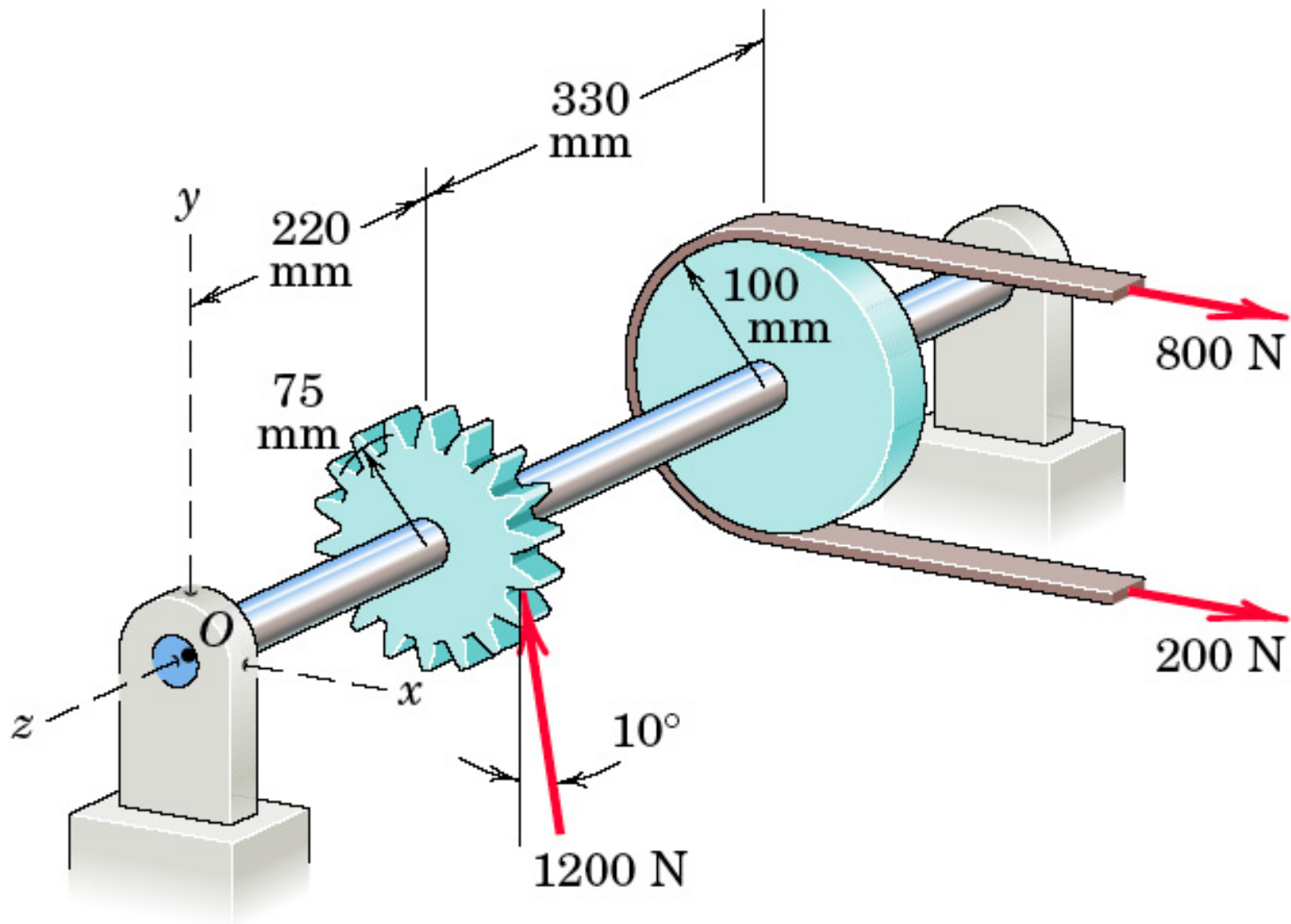


Problem 2/146

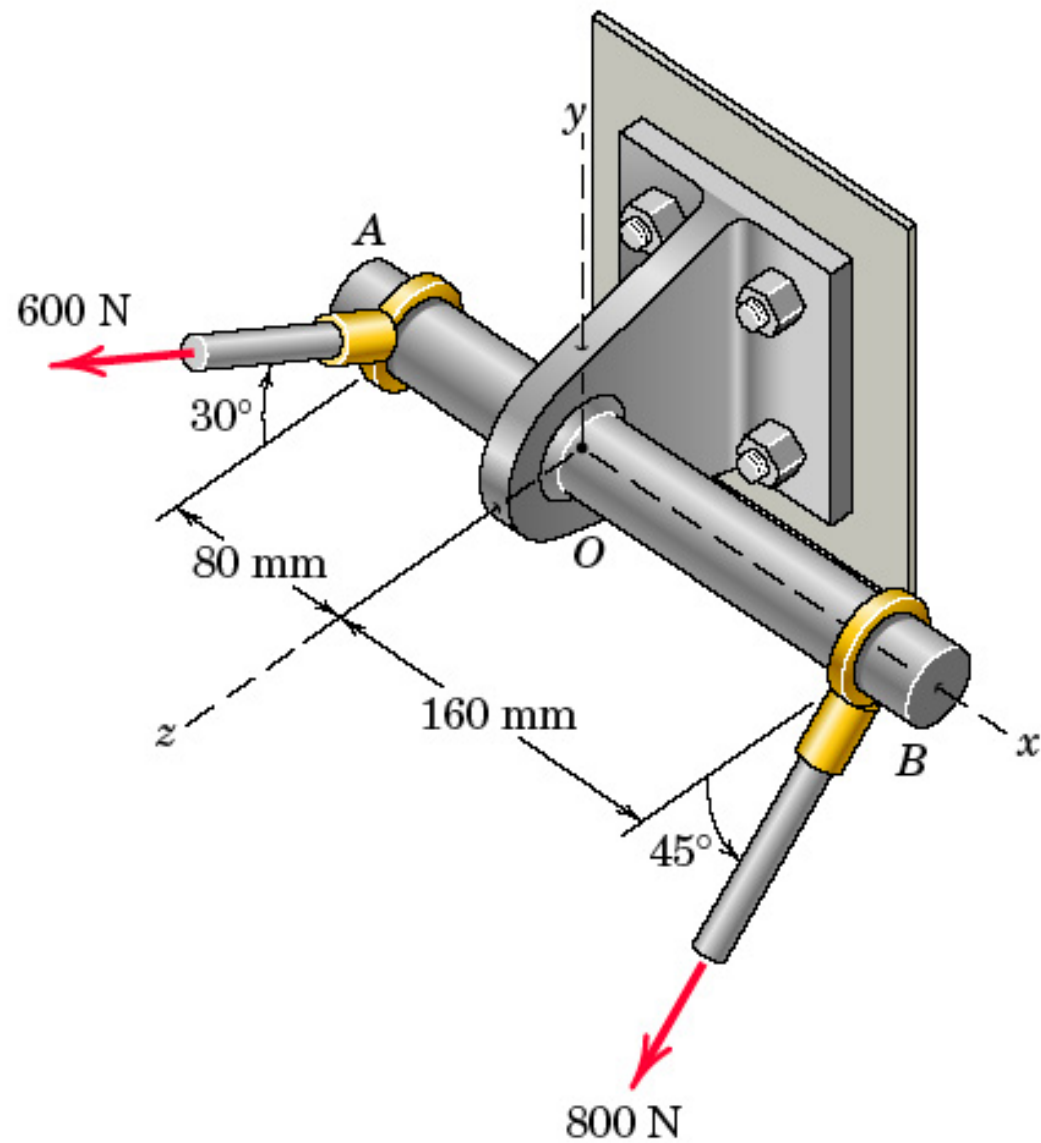


Problem 2/147

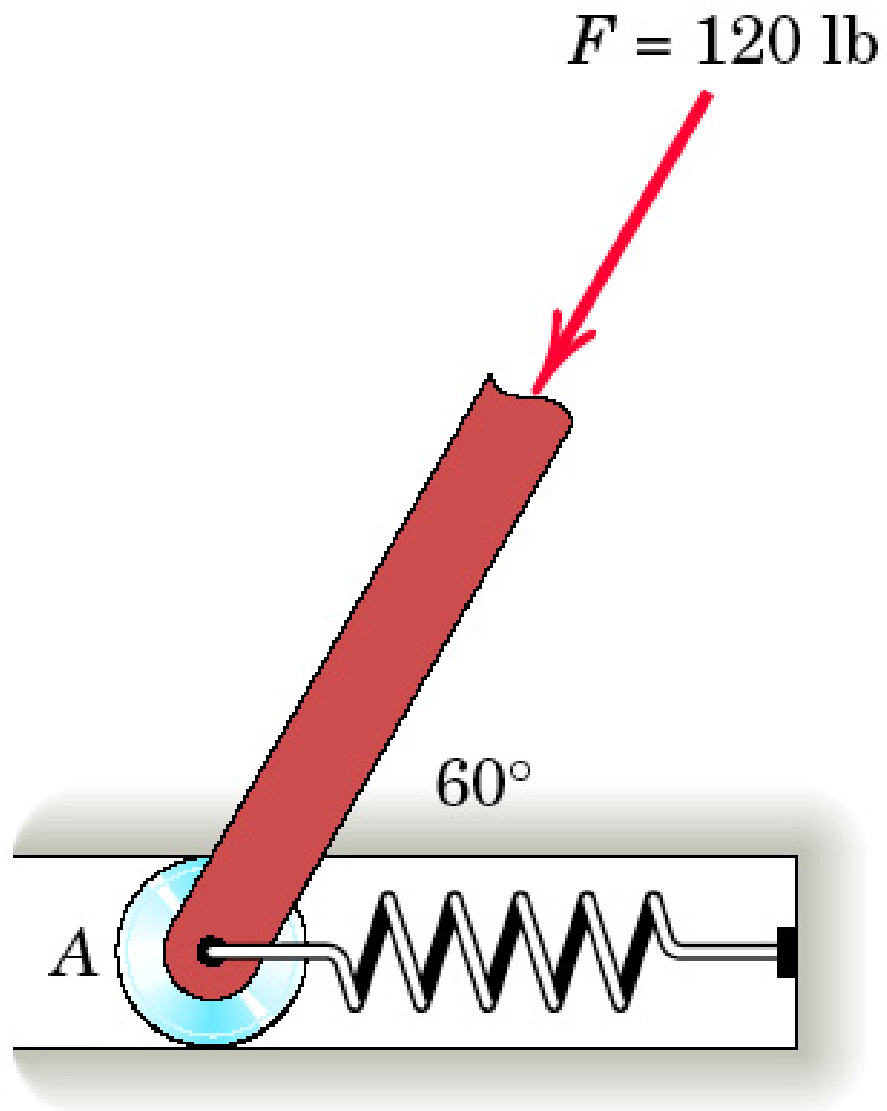
P_02_147



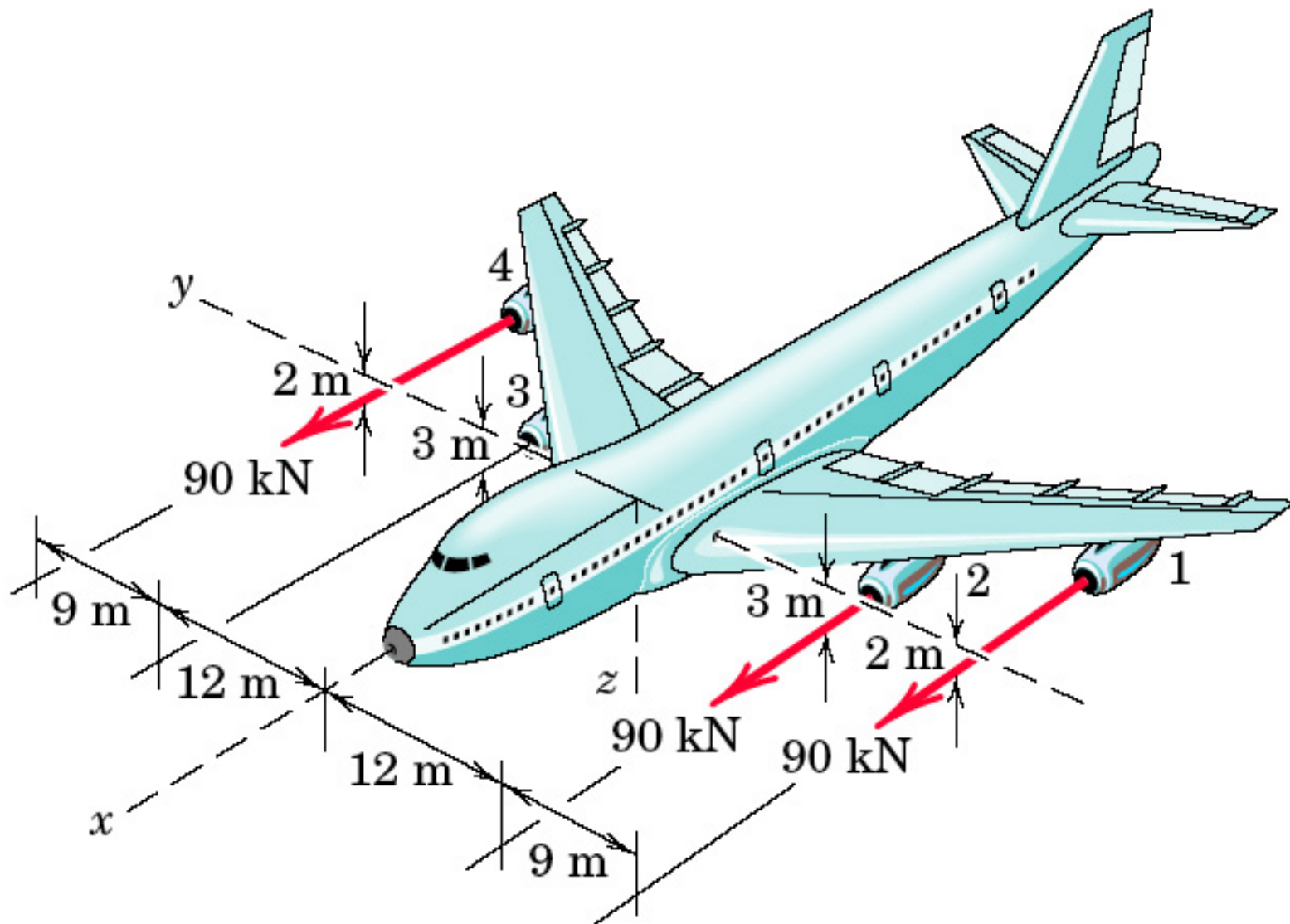
Problem 2/148



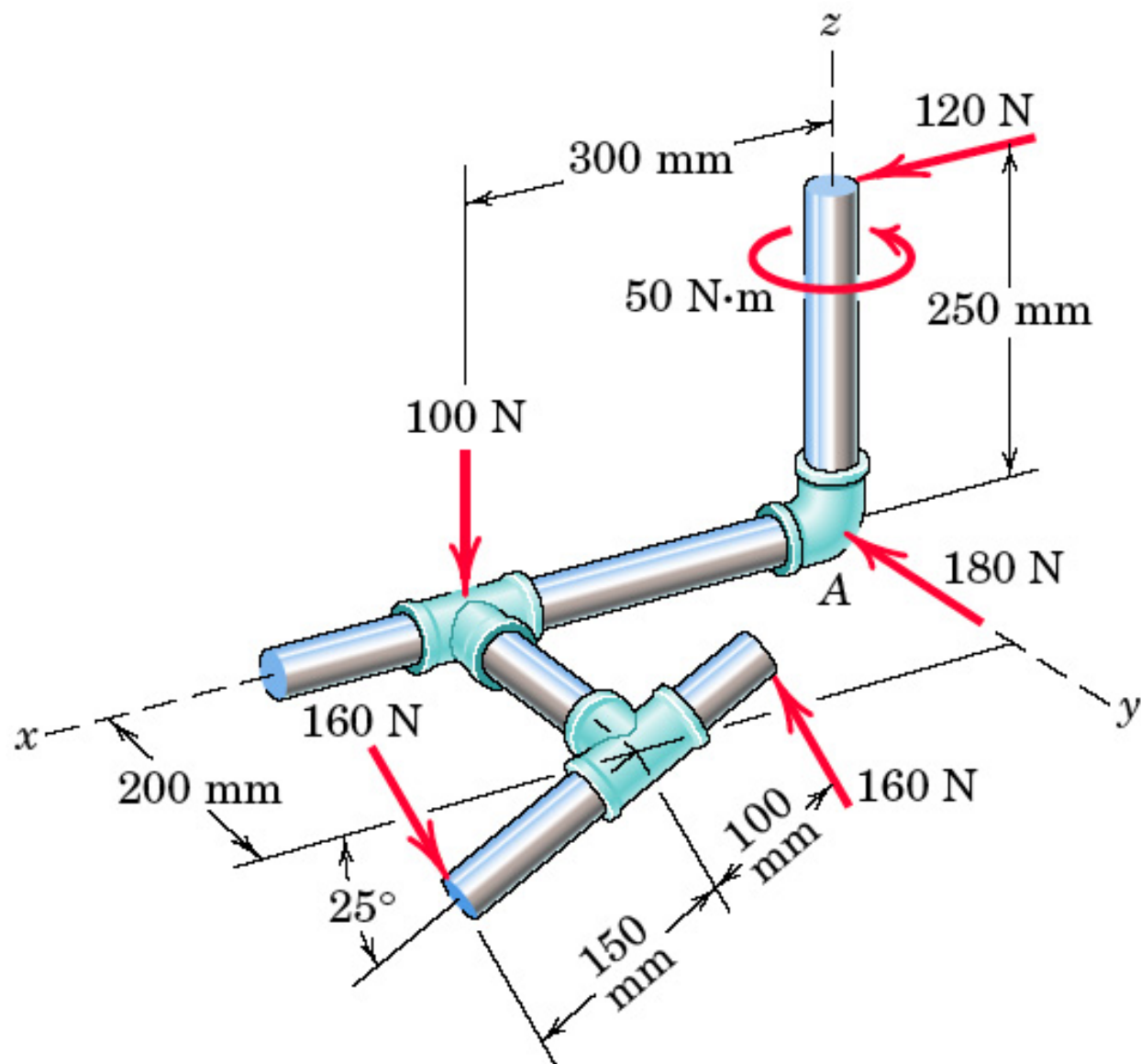
Problem 2/149



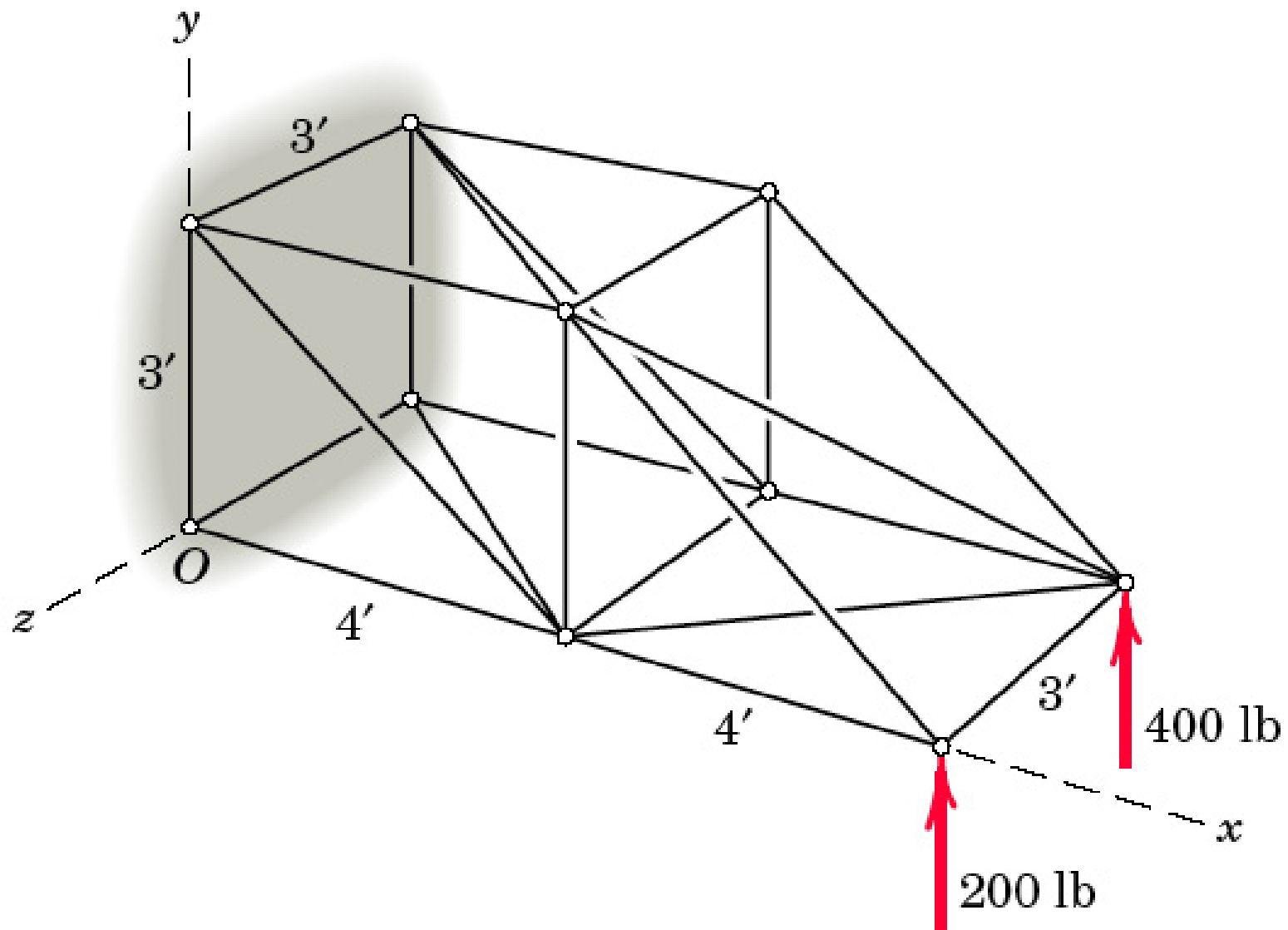
Problem 2/15



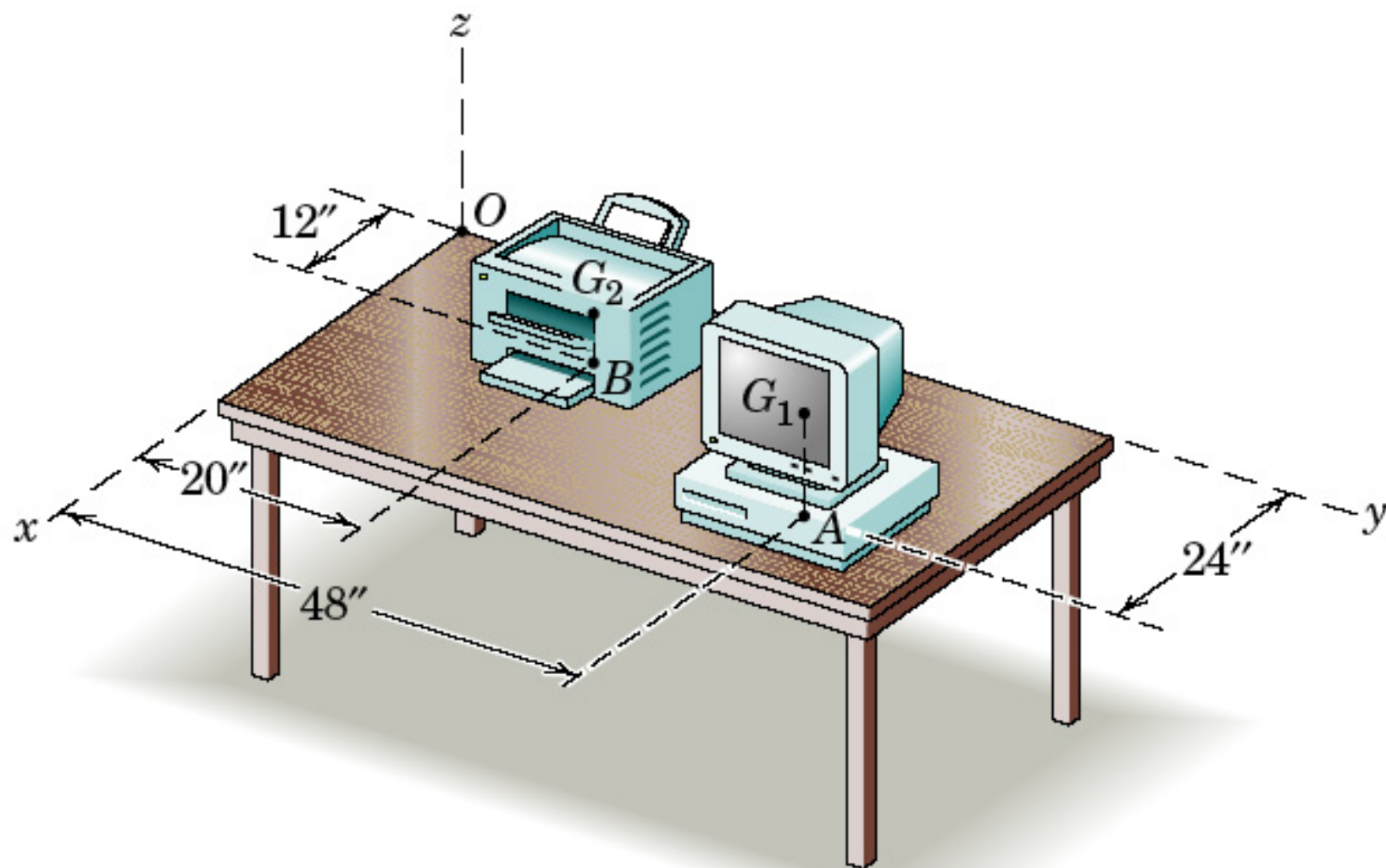
Problem 2/150



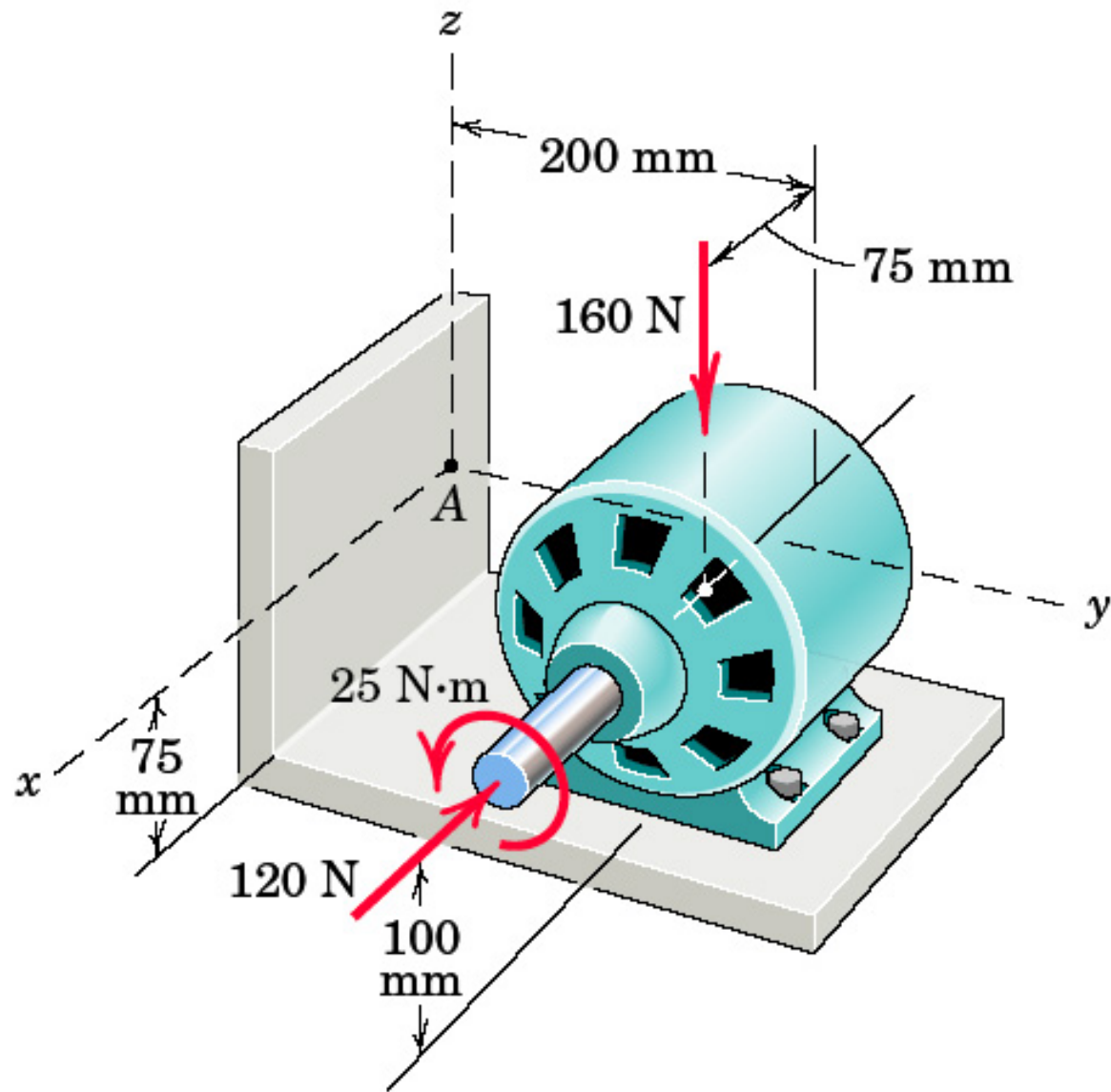
Problem 2/151



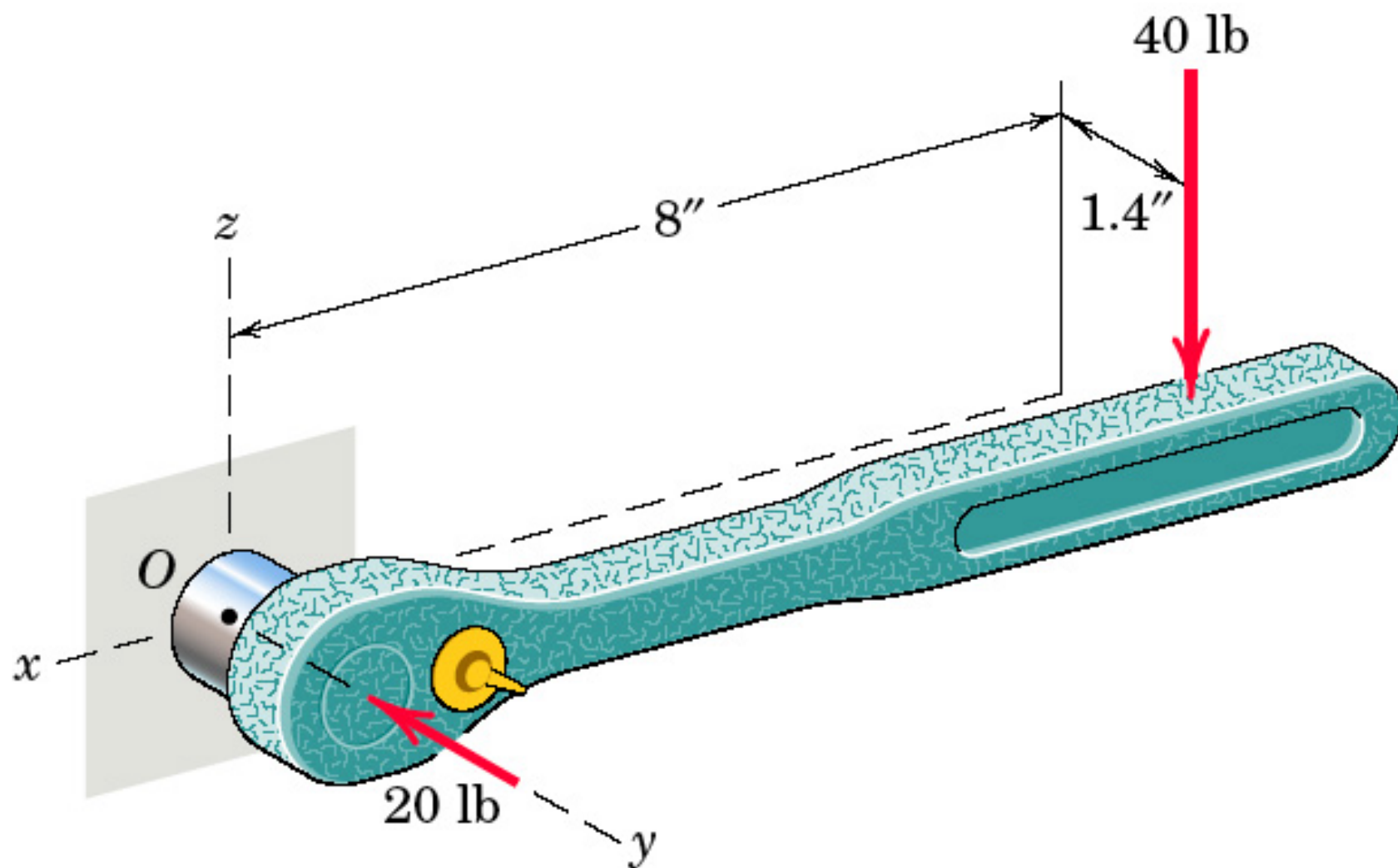
Problem 2/152



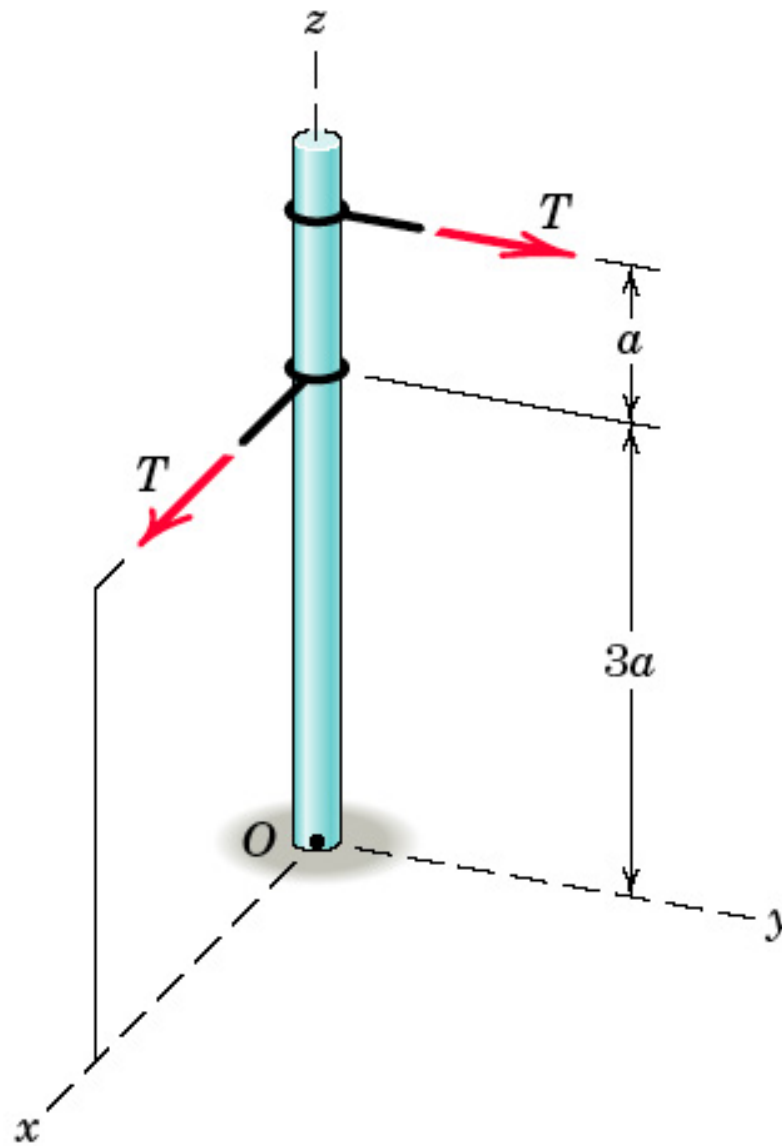
Problem 2/153



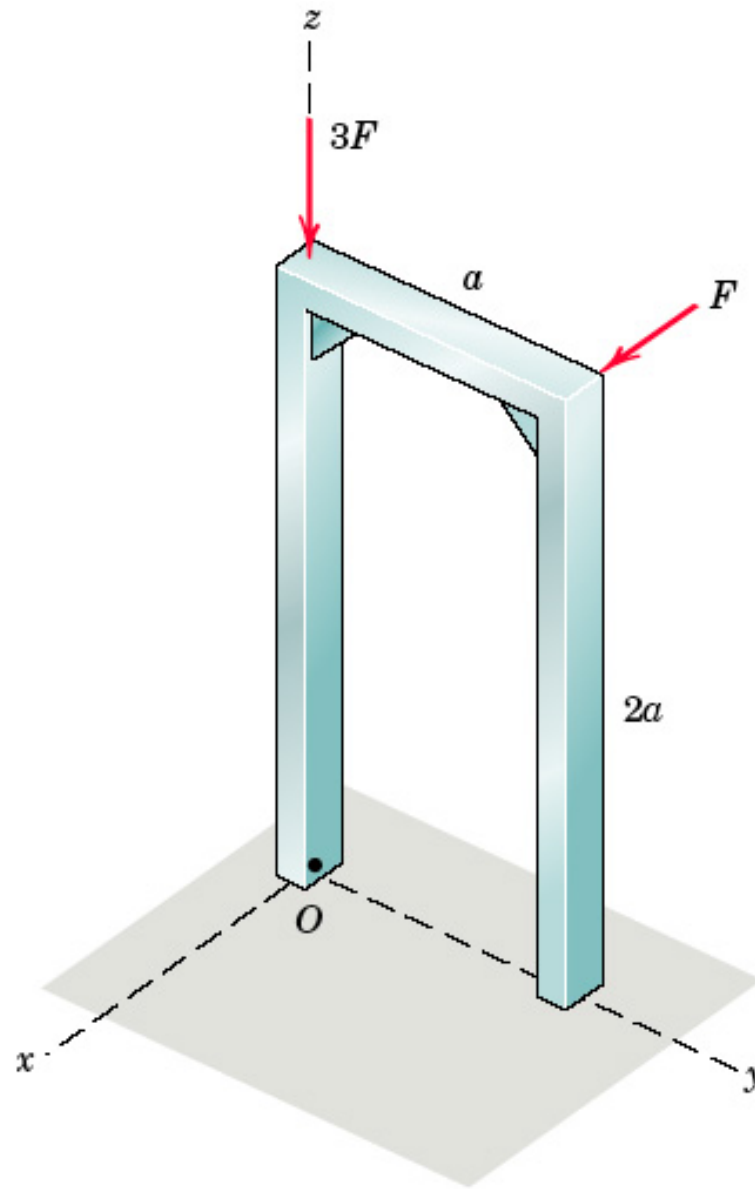
Problem 2/154



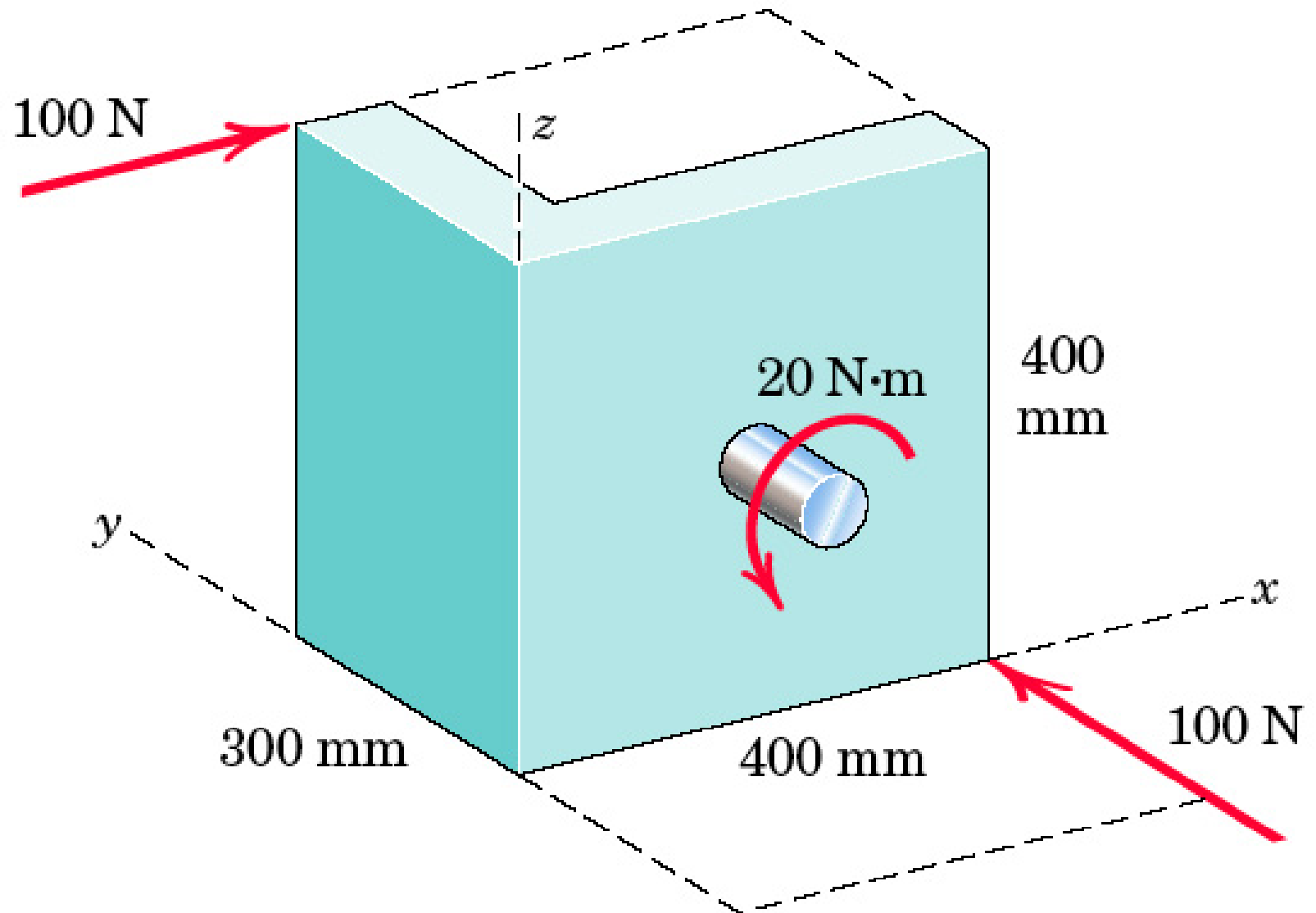
Problem 2/155



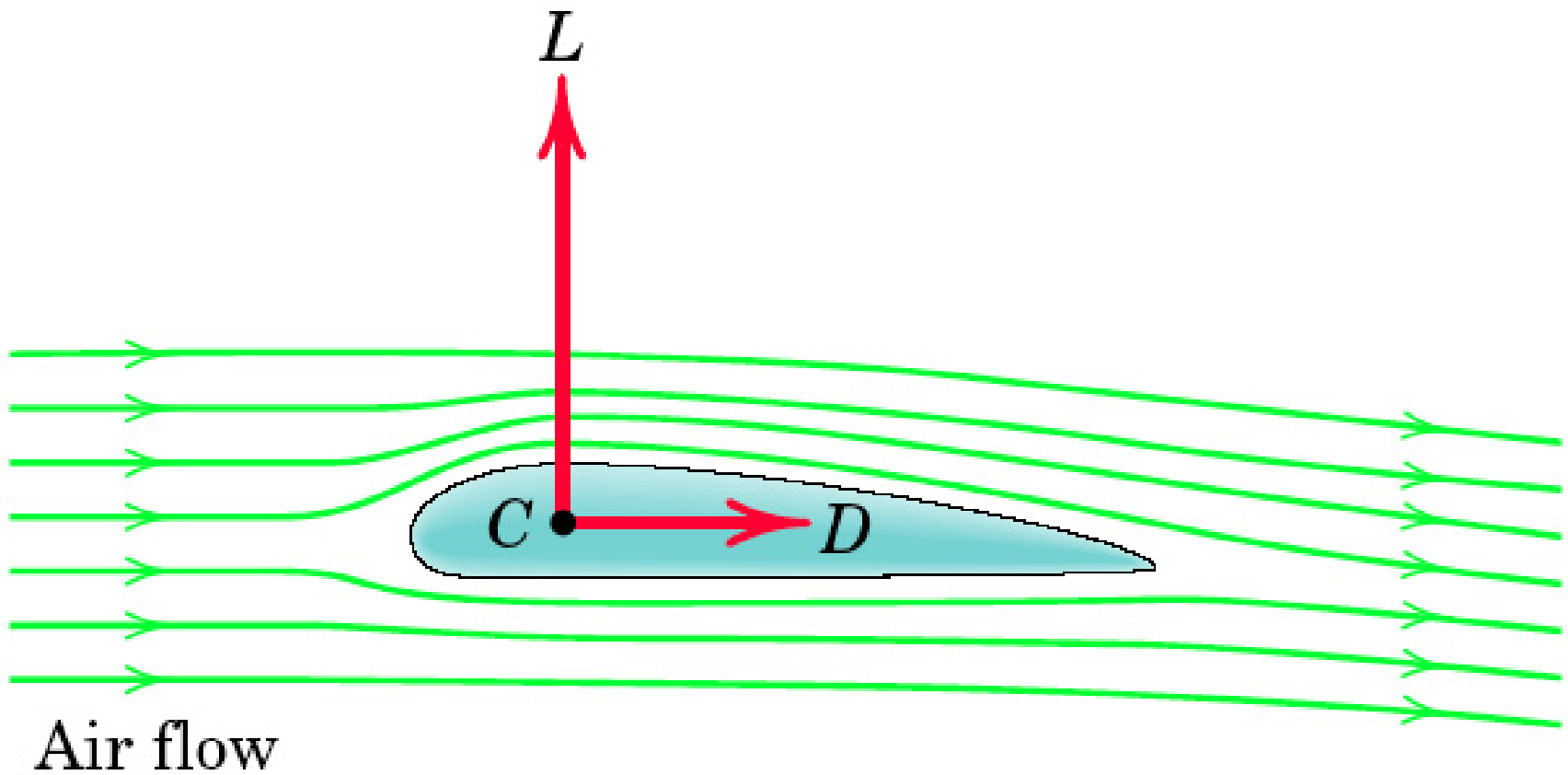
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Problem 2/157

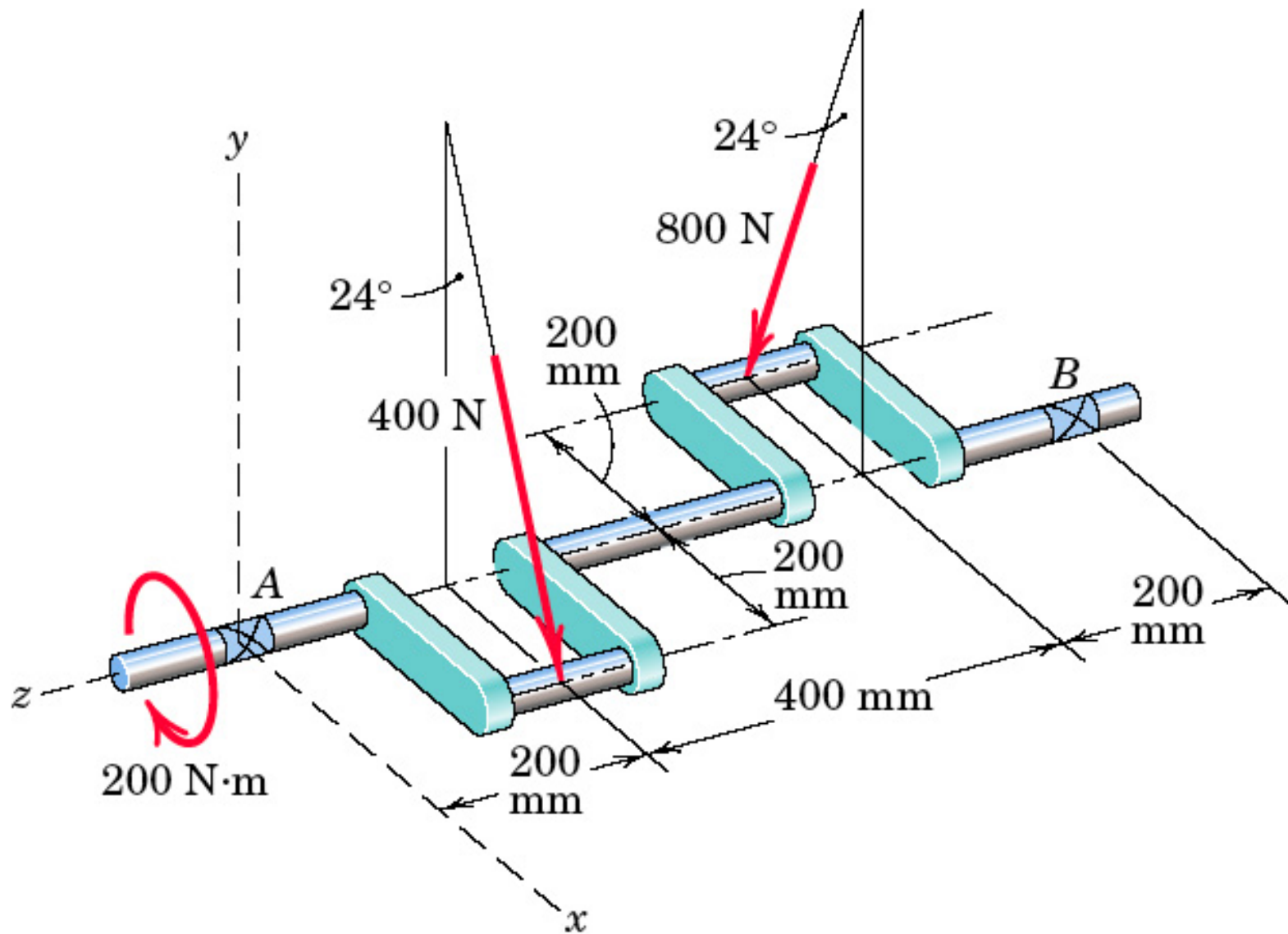


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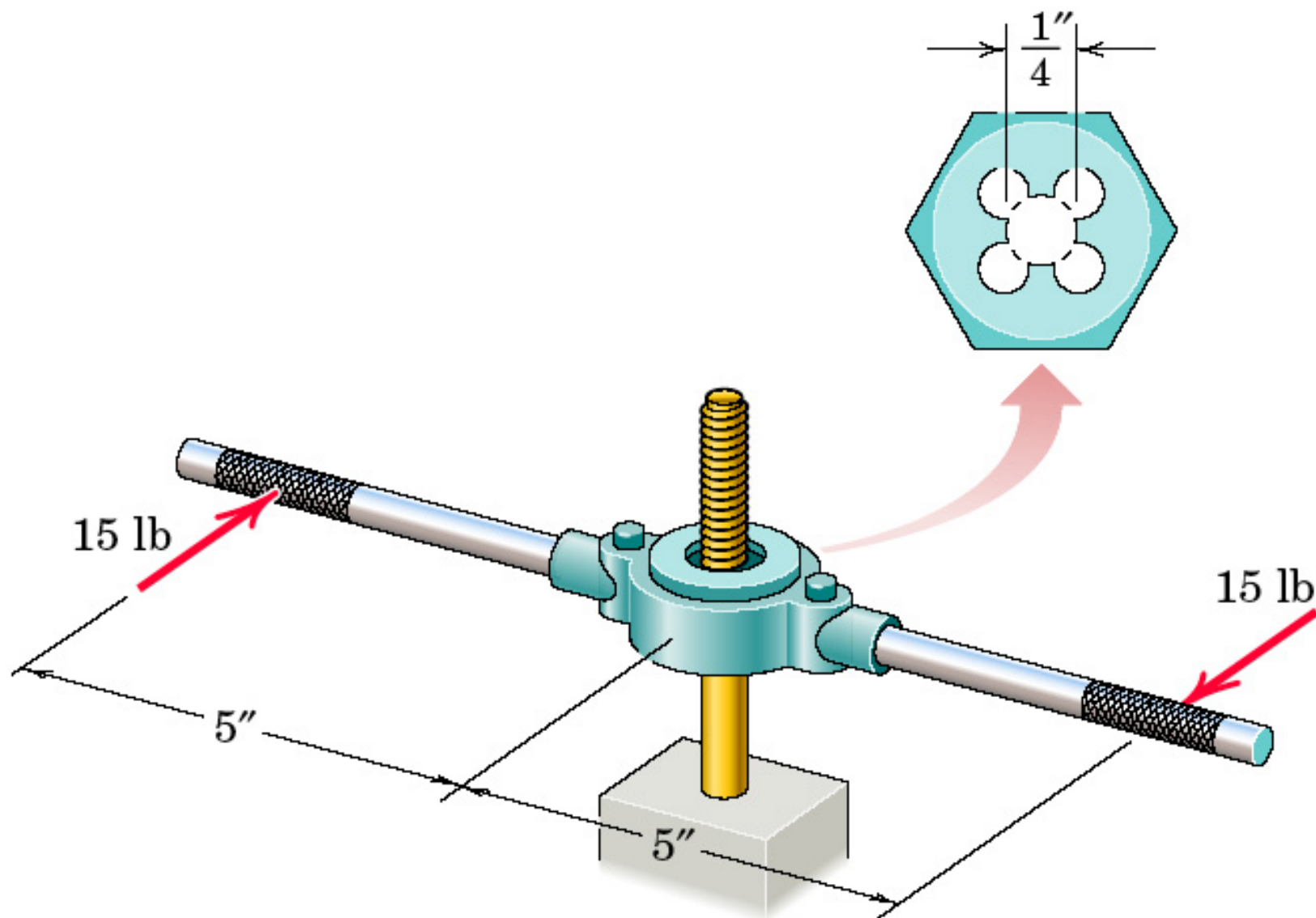


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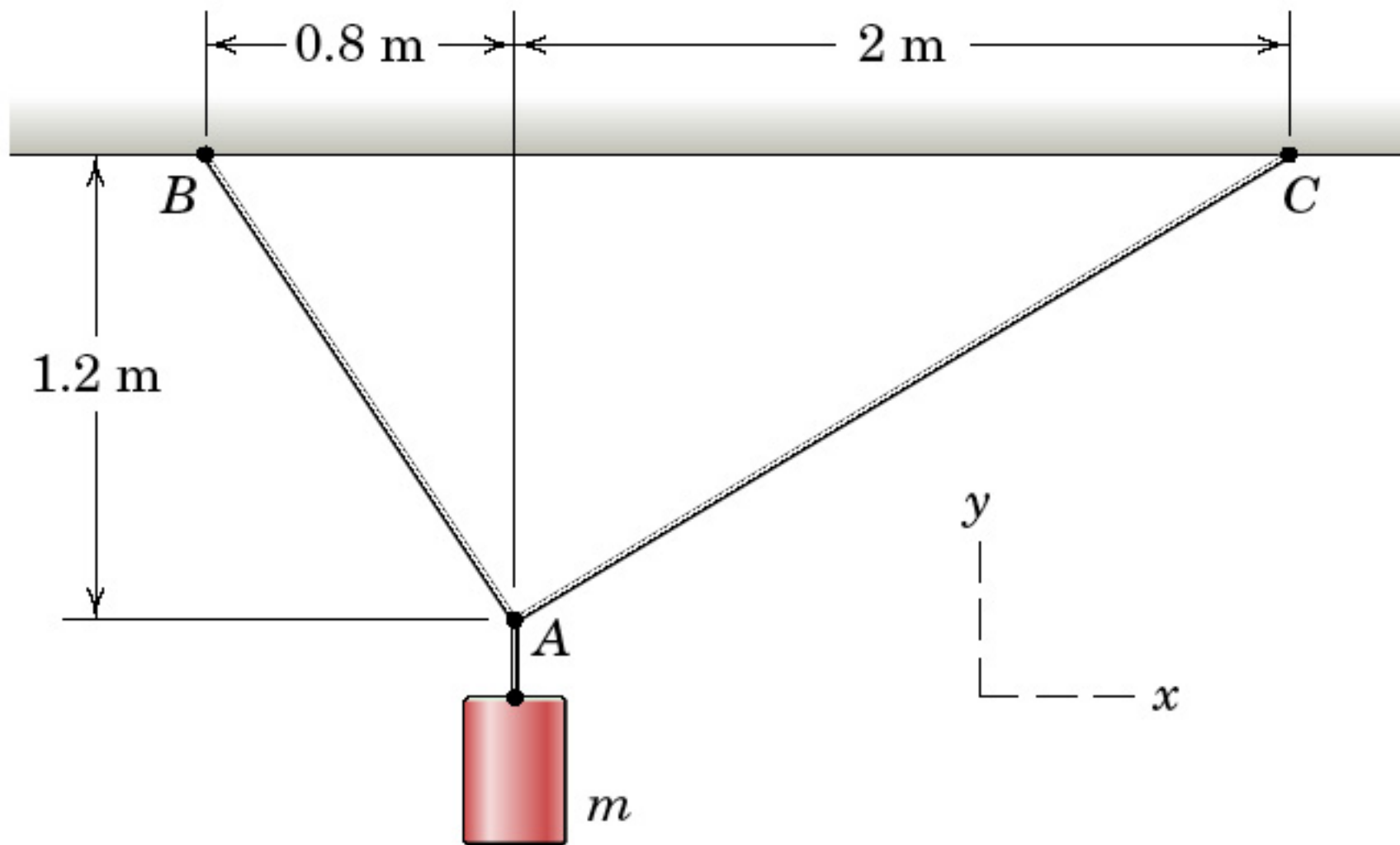
P_02_16



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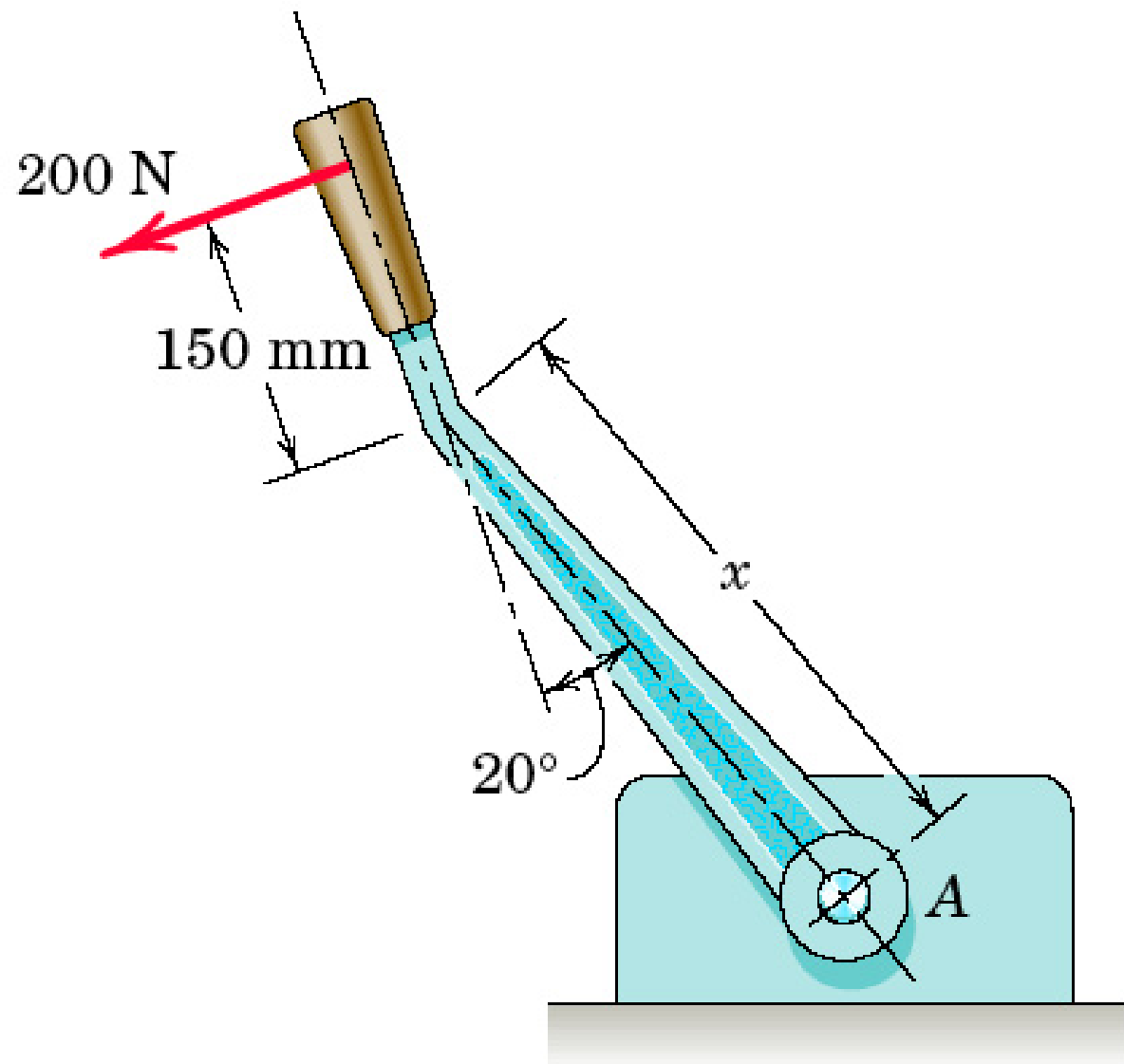


Problem 2/161

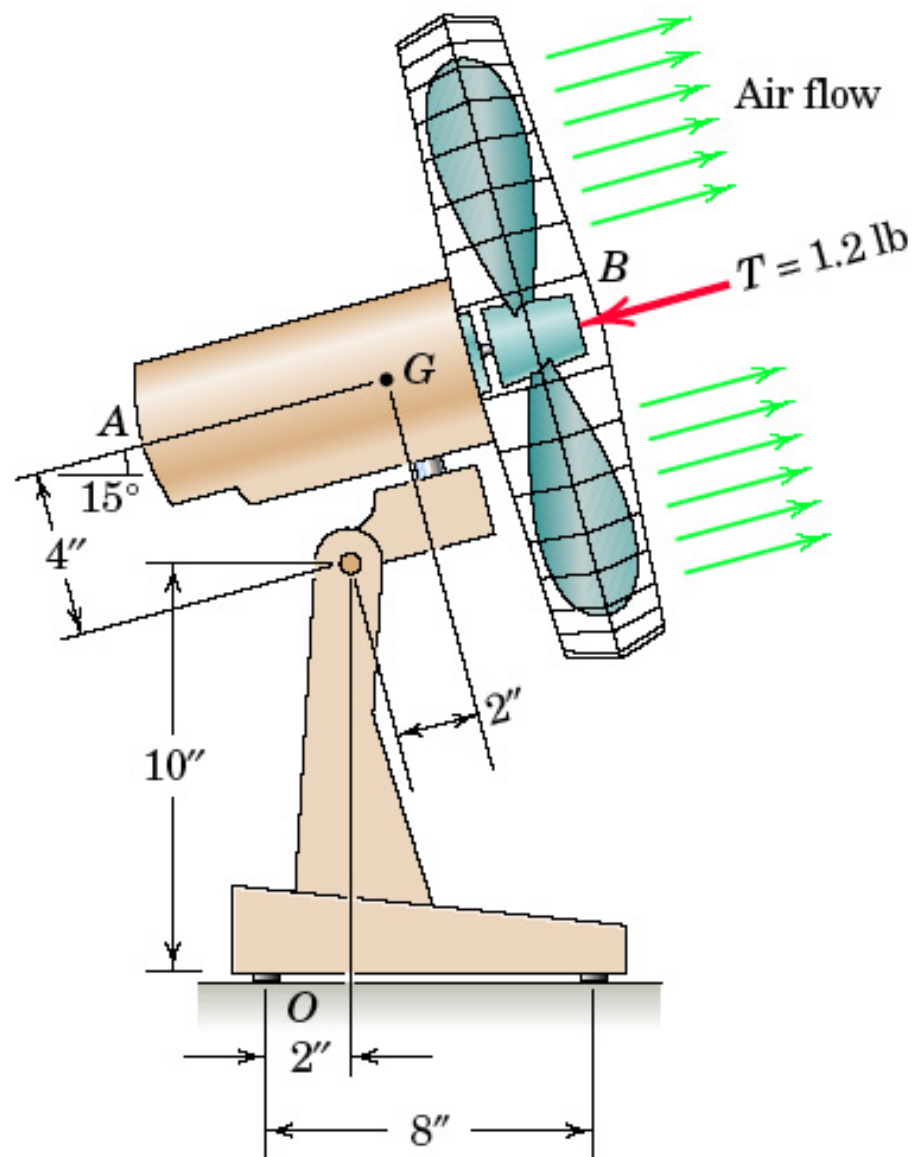


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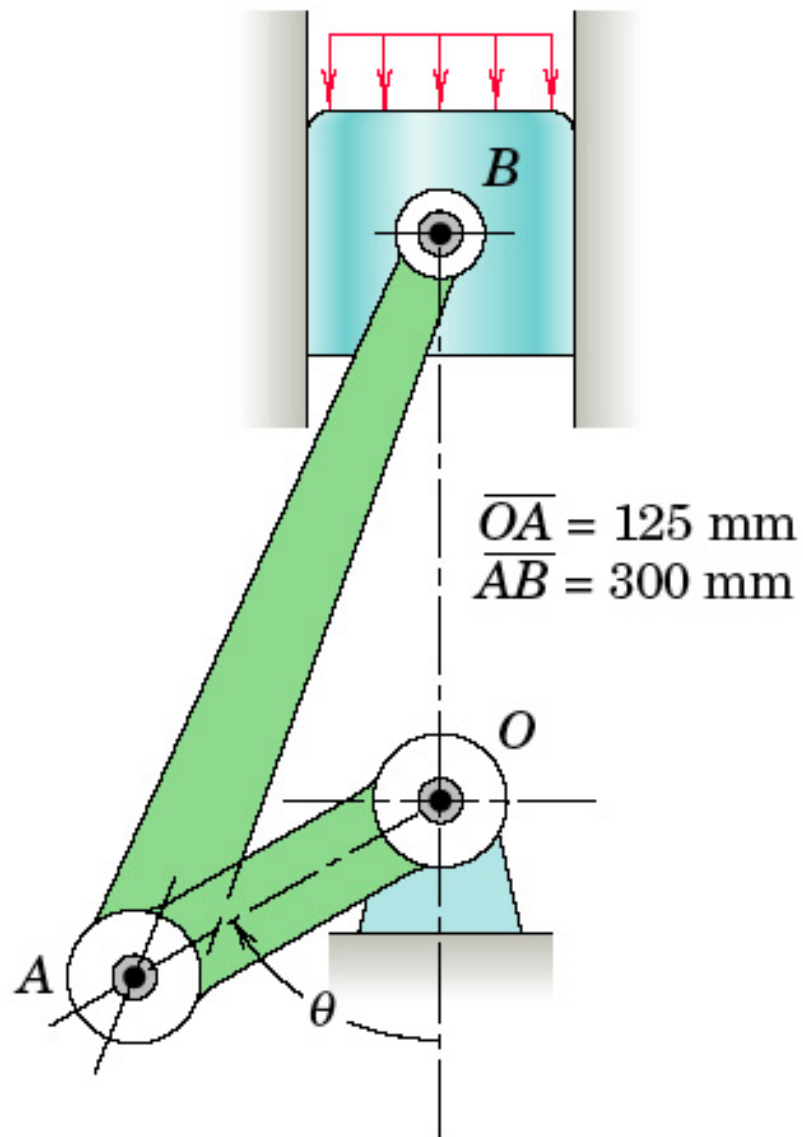
P_02_162



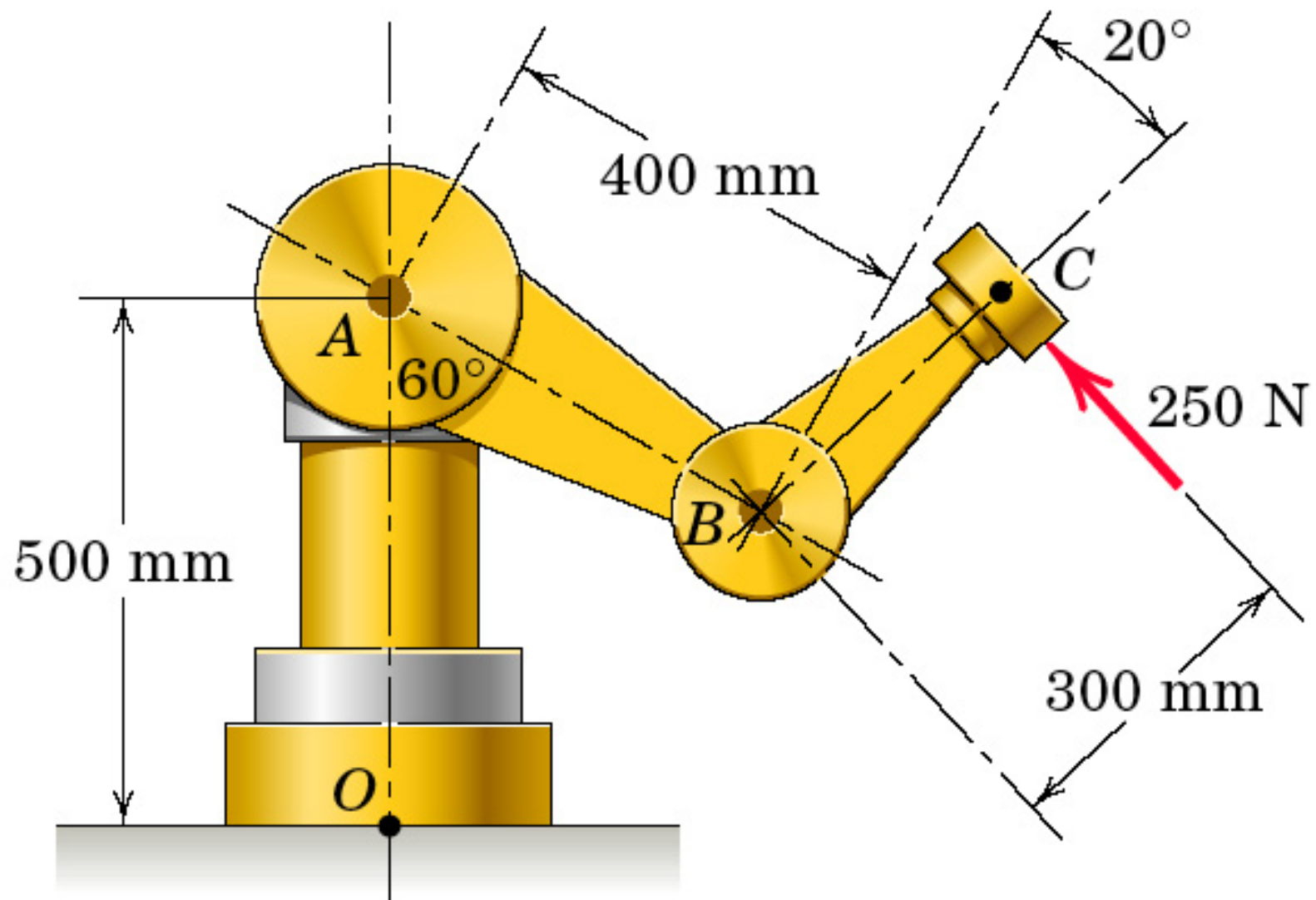
Problem 2/163



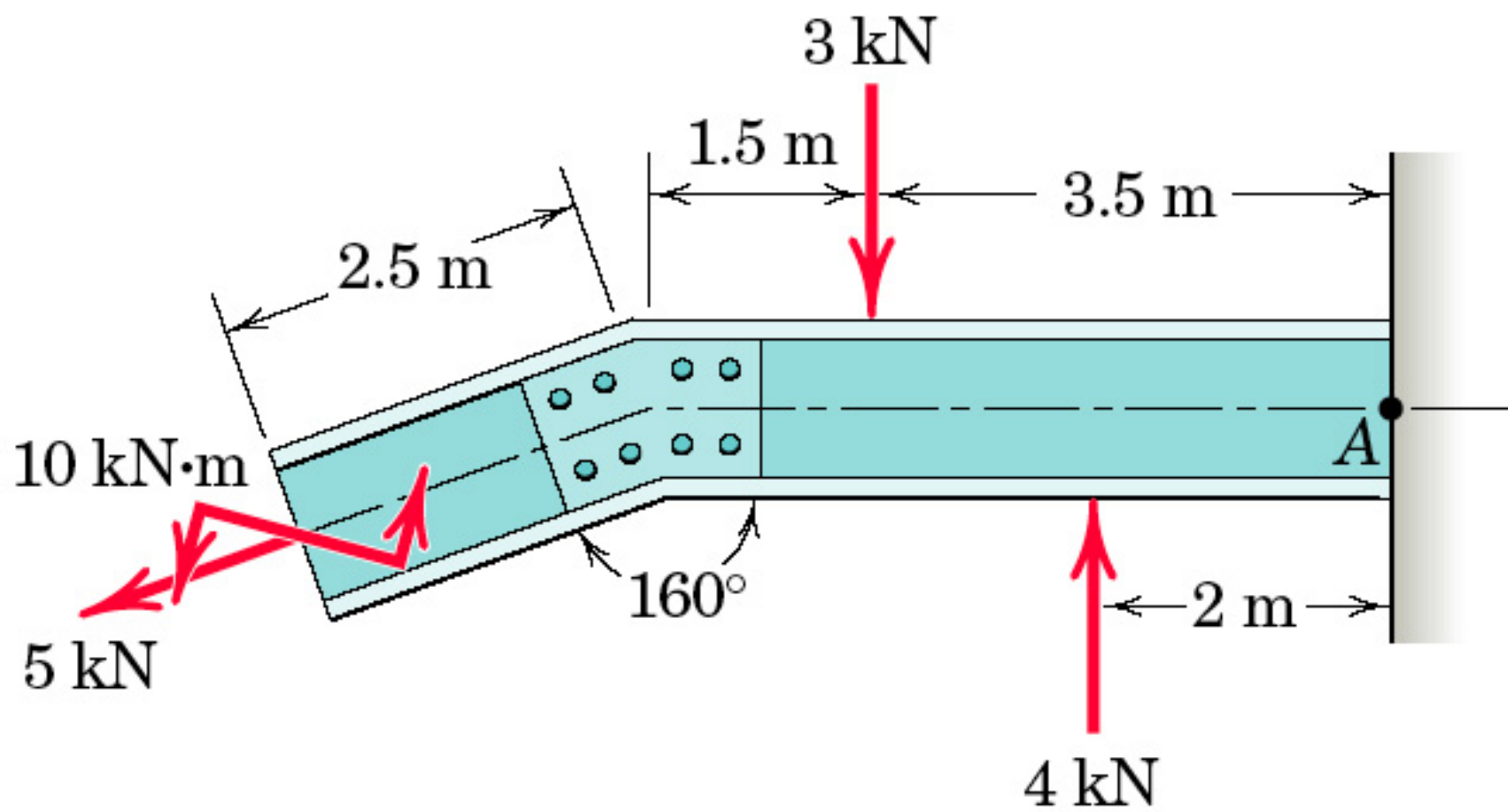
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Problem 2/165

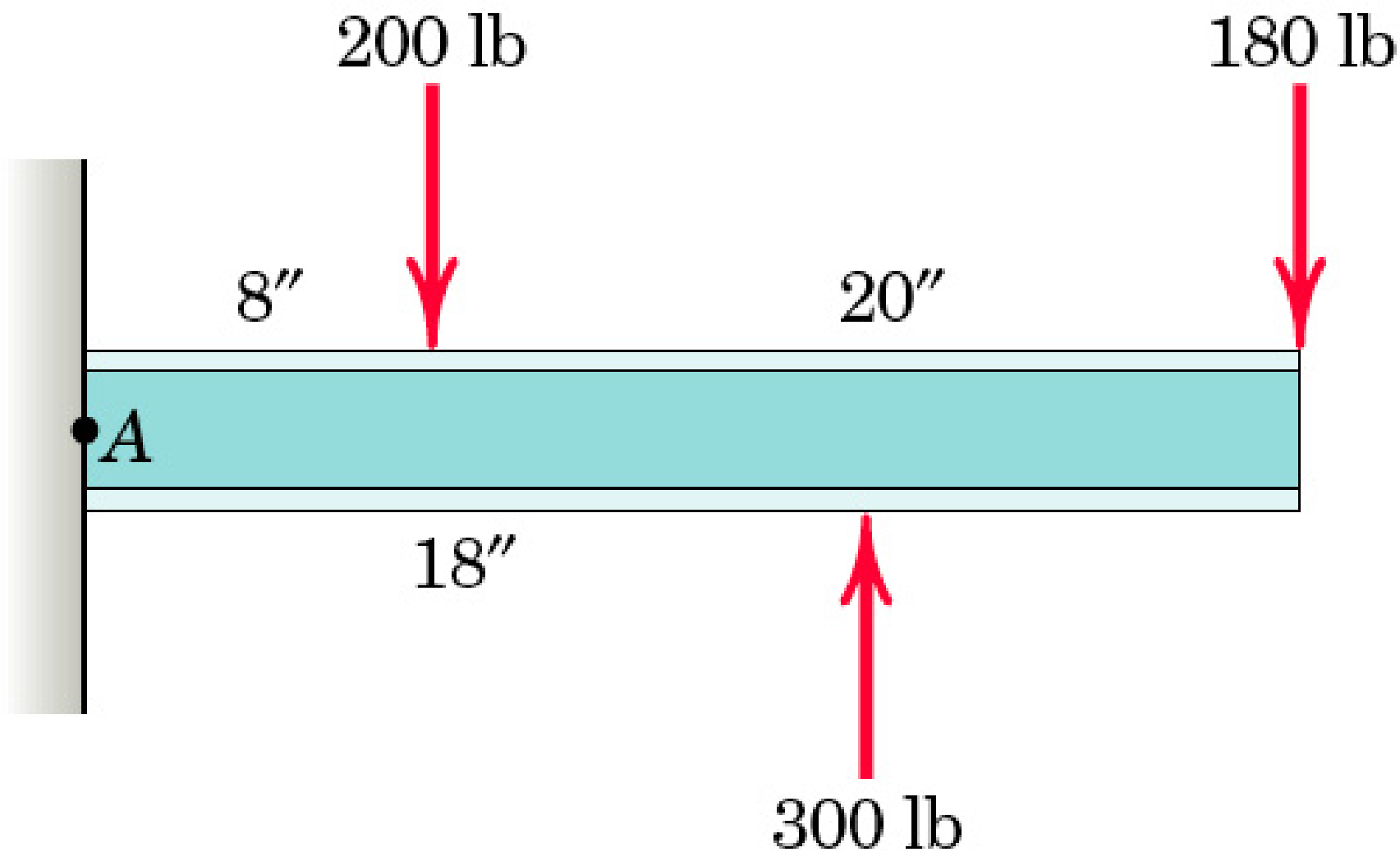


Problem 2/166

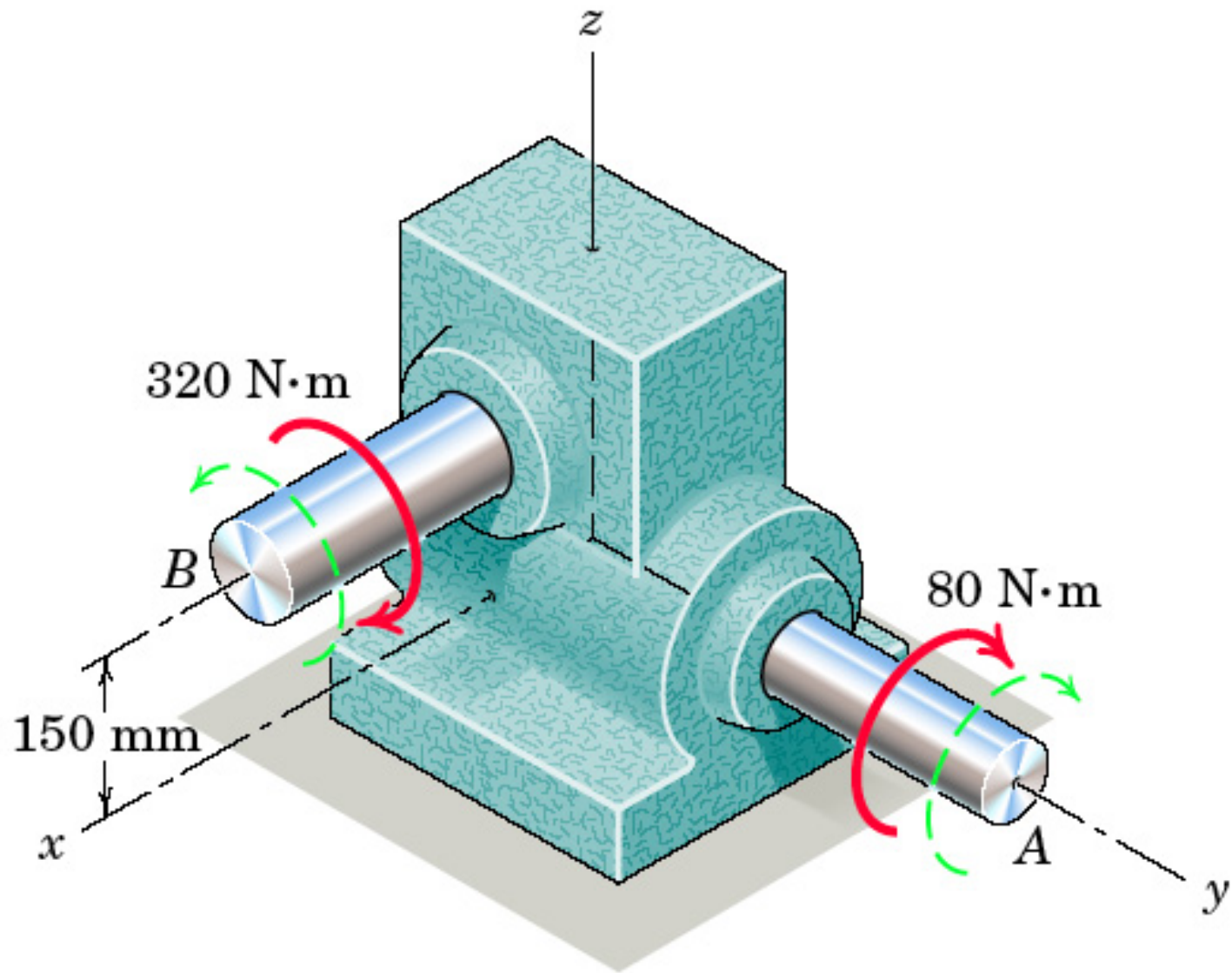


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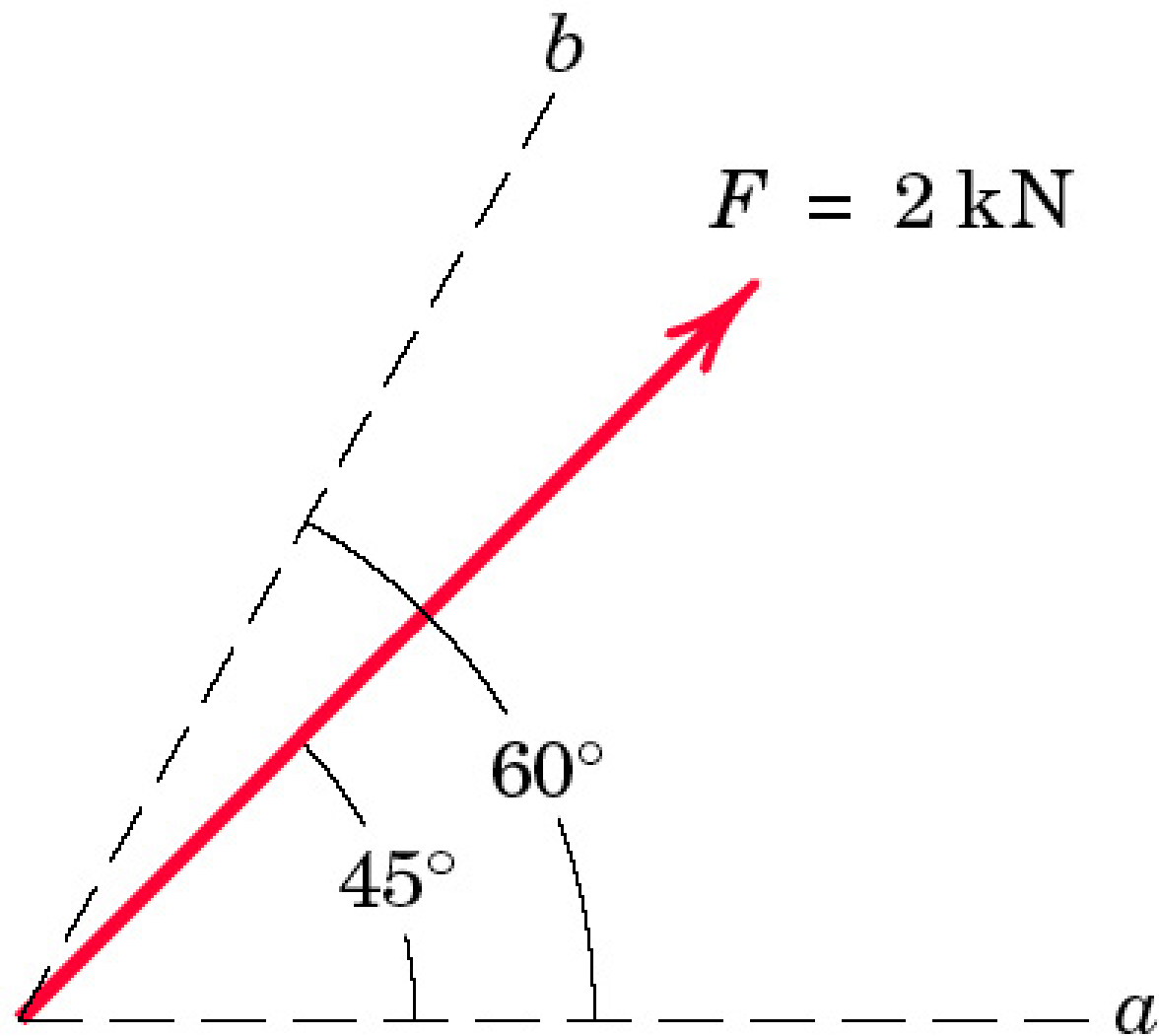
P_02_167



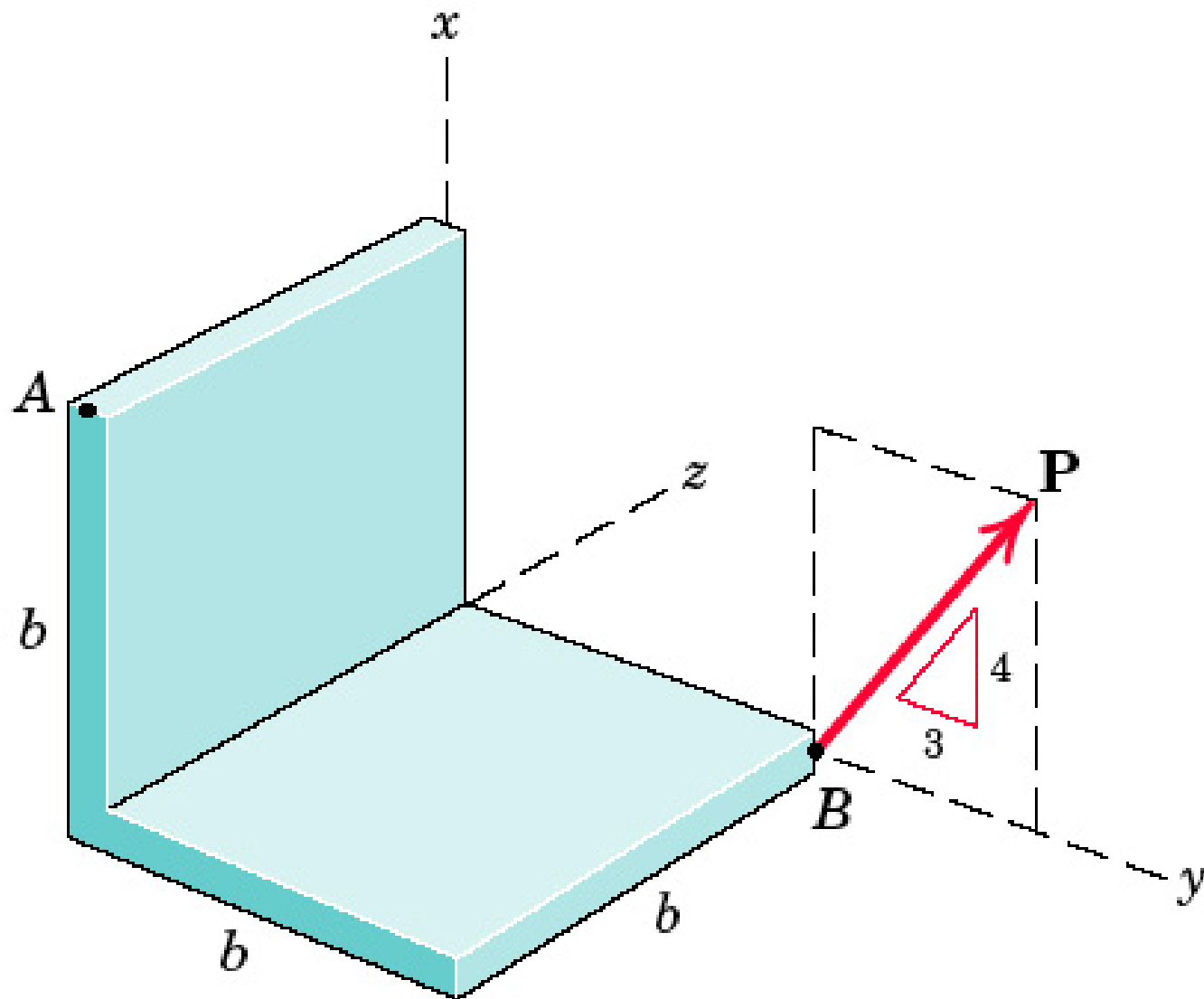
Problem 2/168



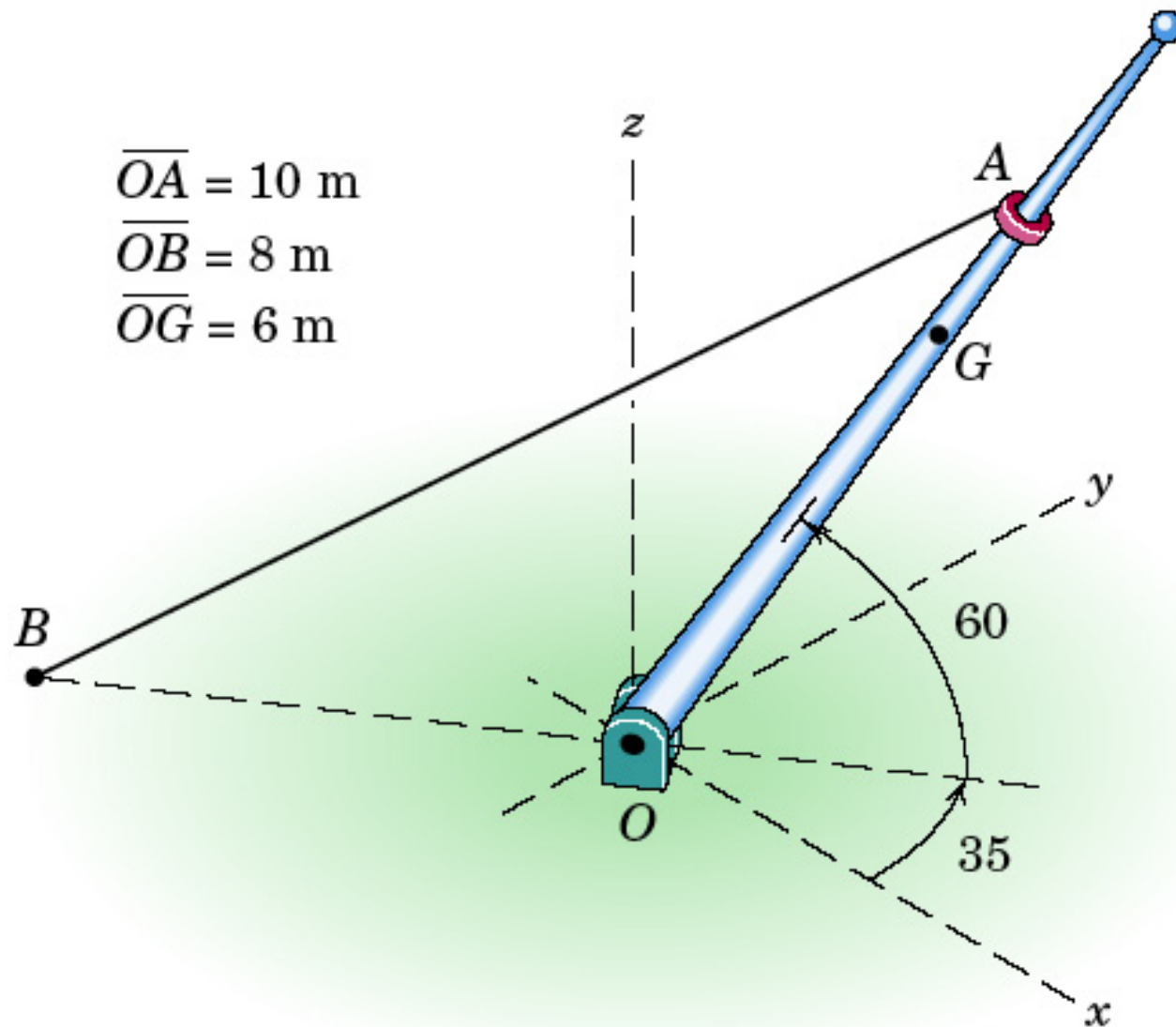
Problem 2/169



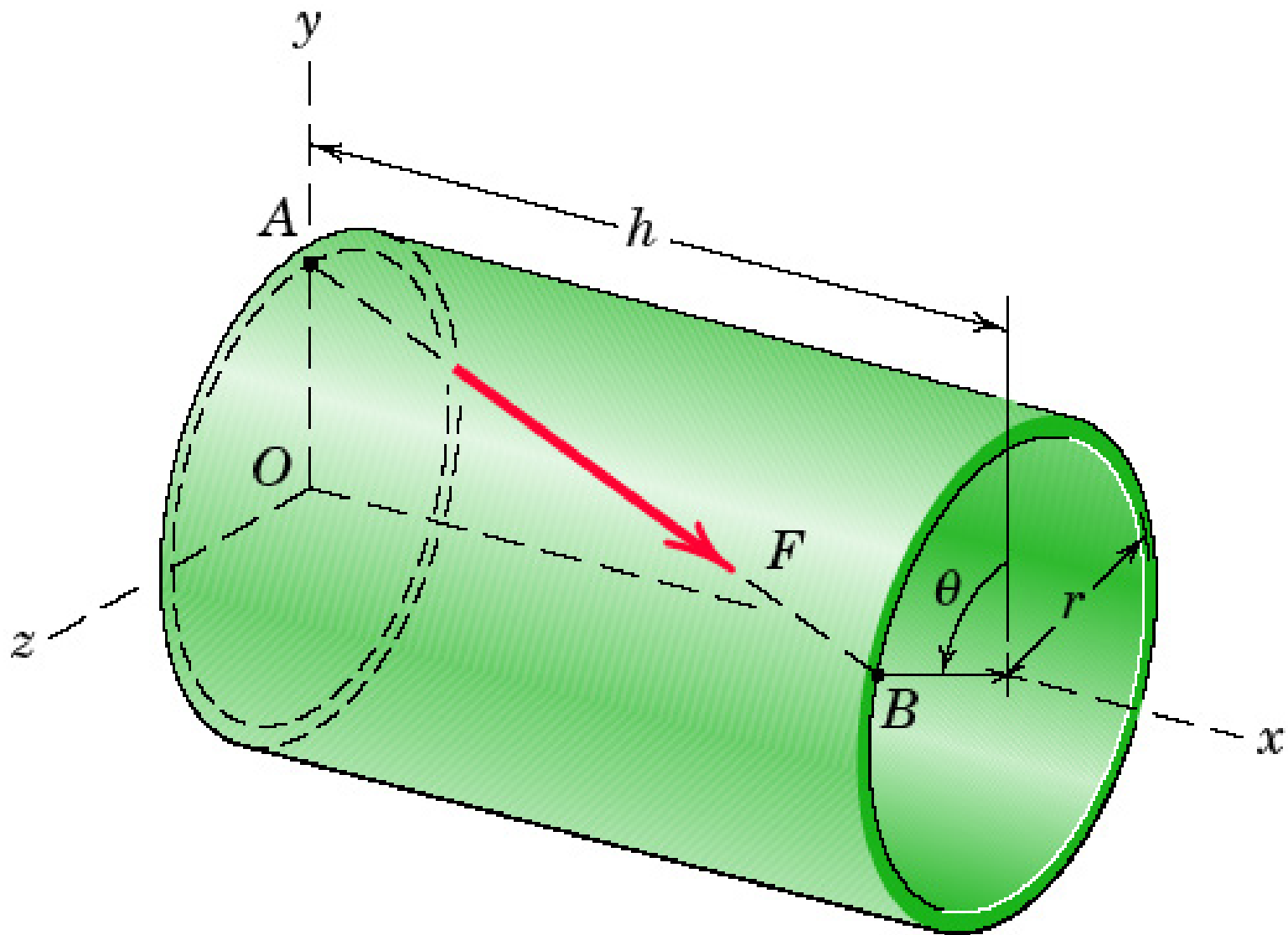
Problem 2/17



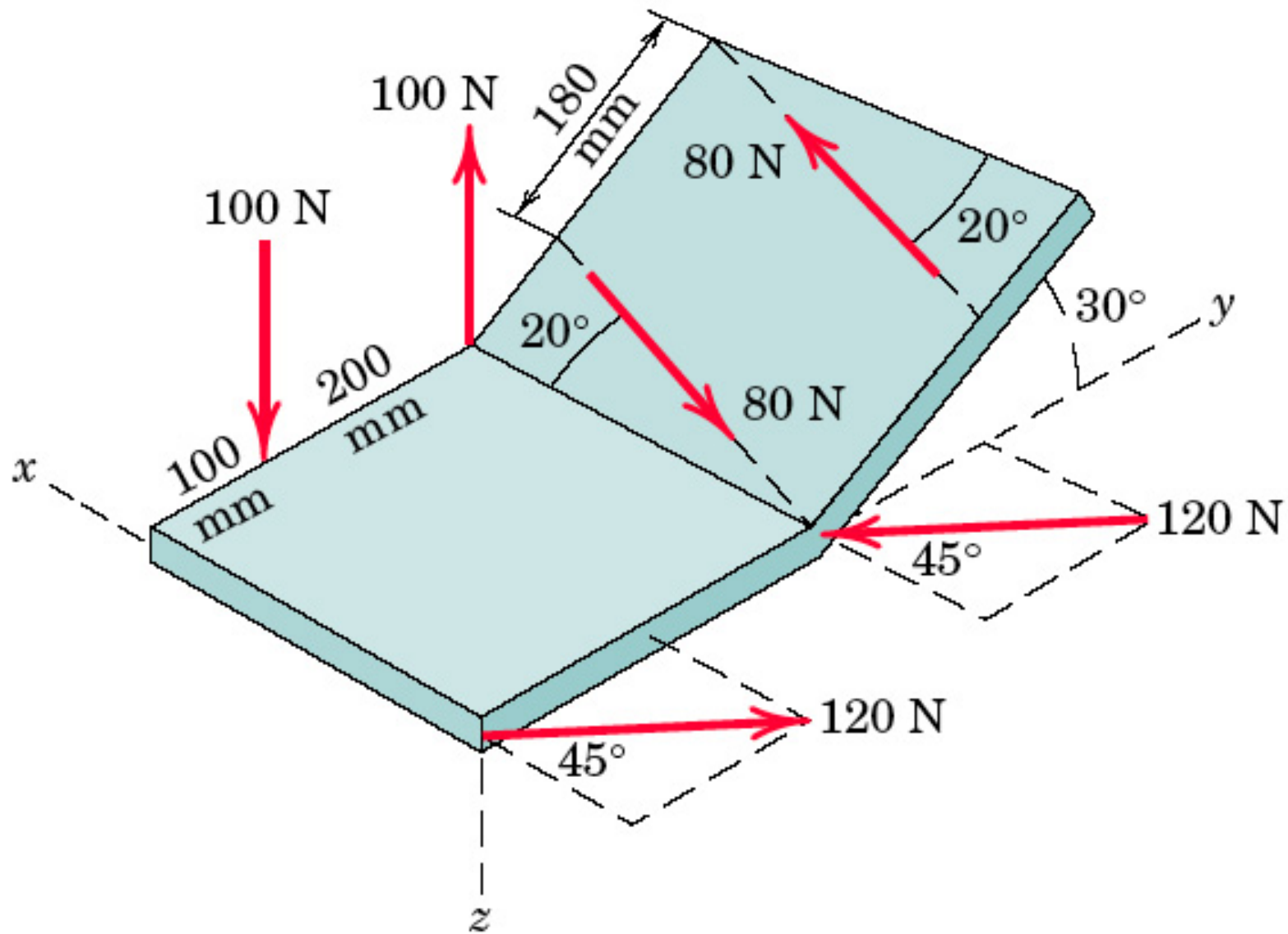
Problem 2/170



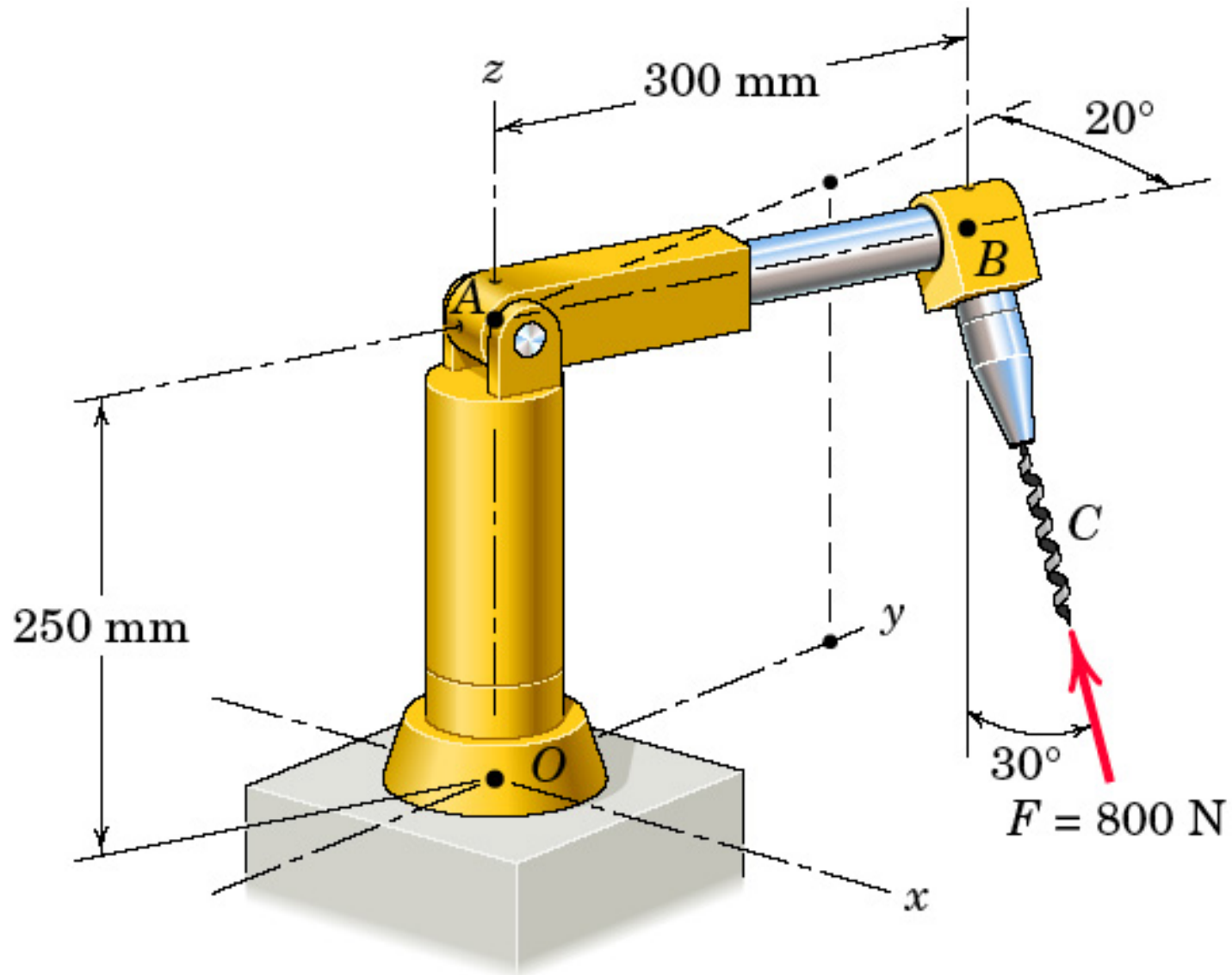
Problem 2/171



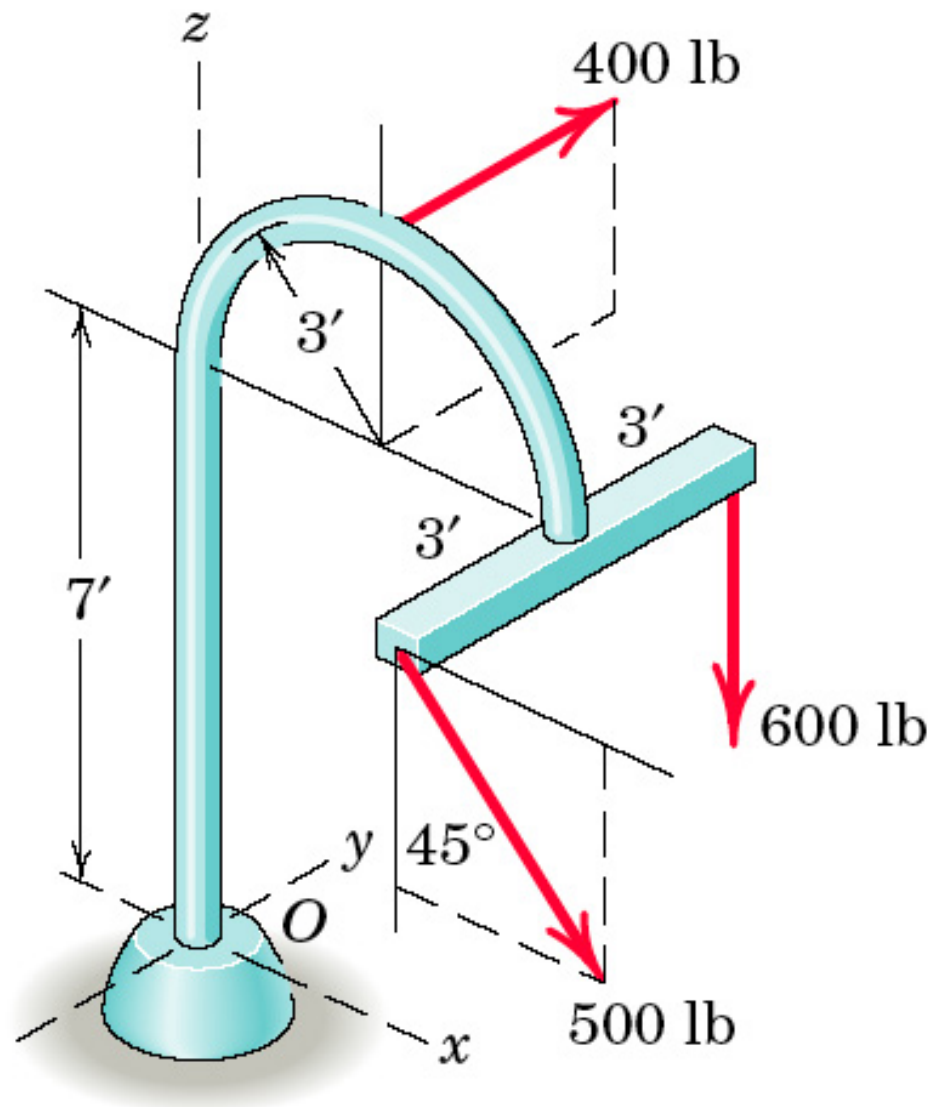
Problem 2/172



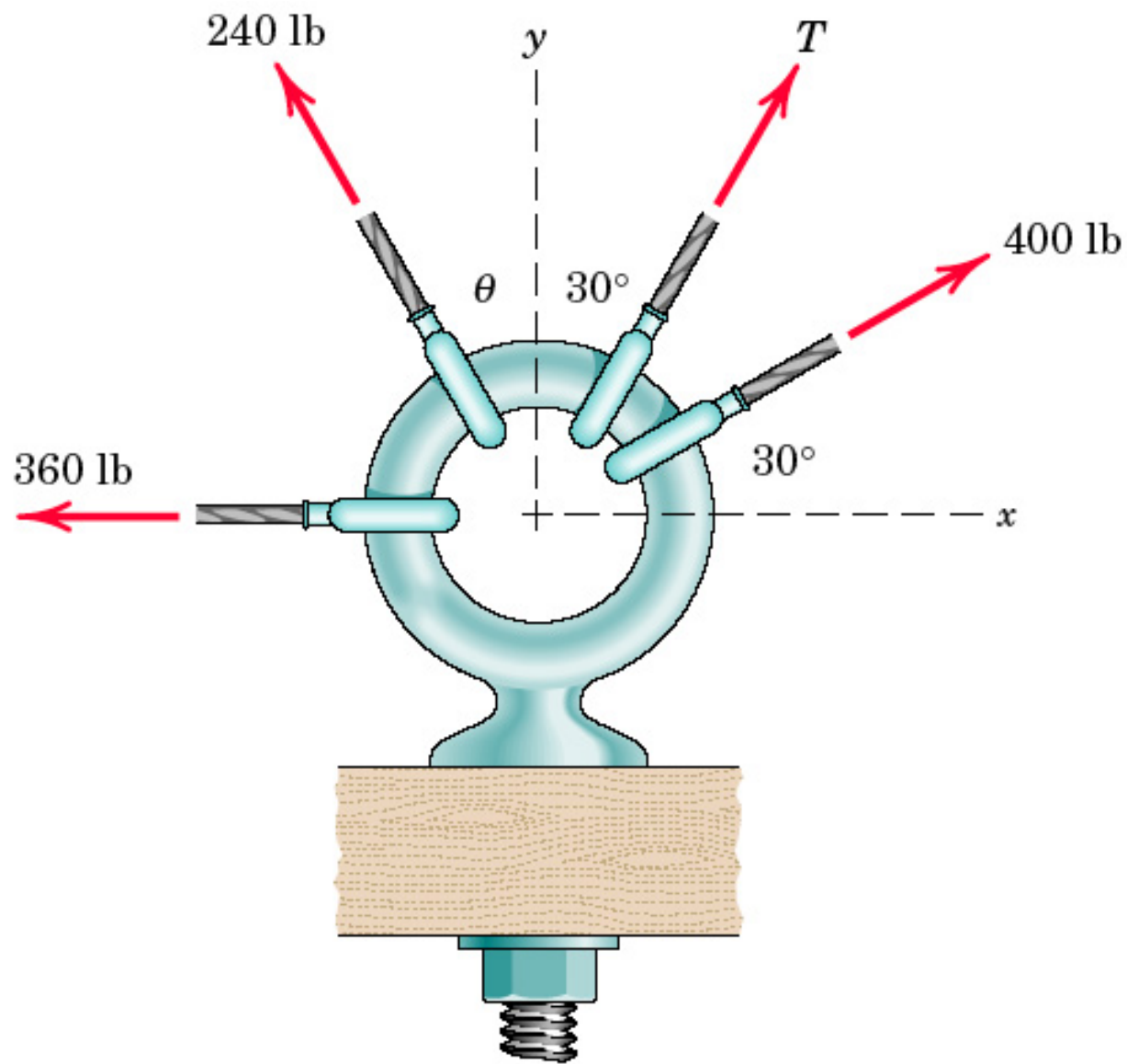
Problem 2/173



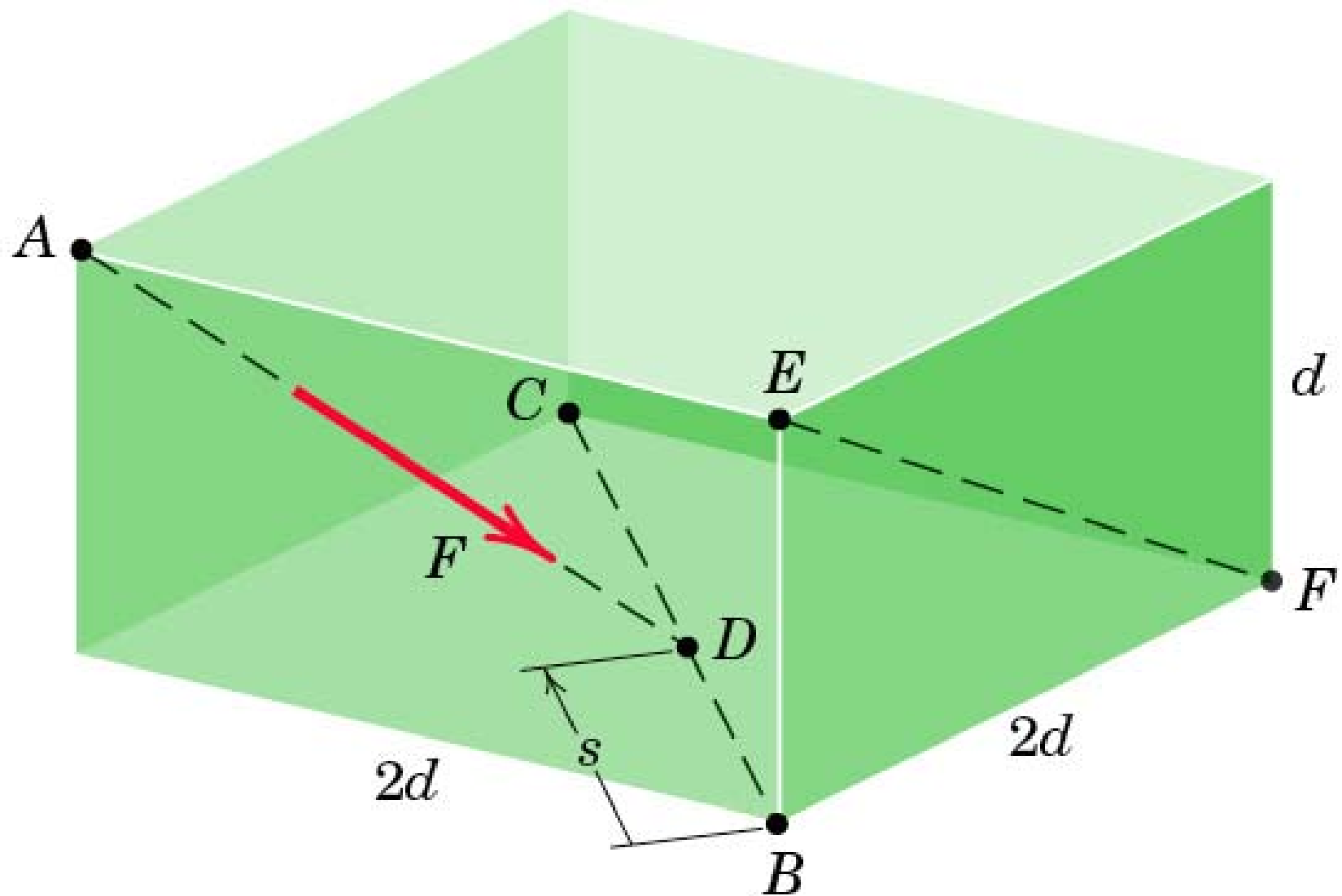
Problem 2/174



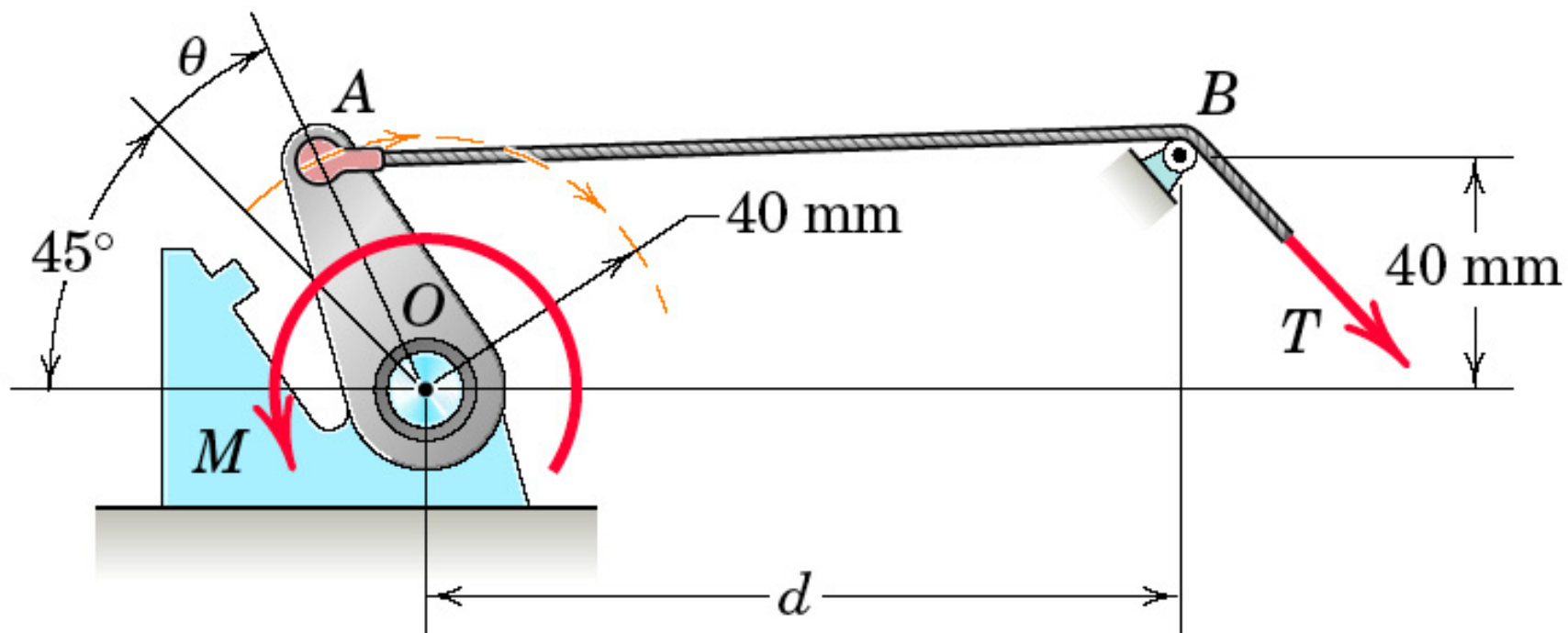
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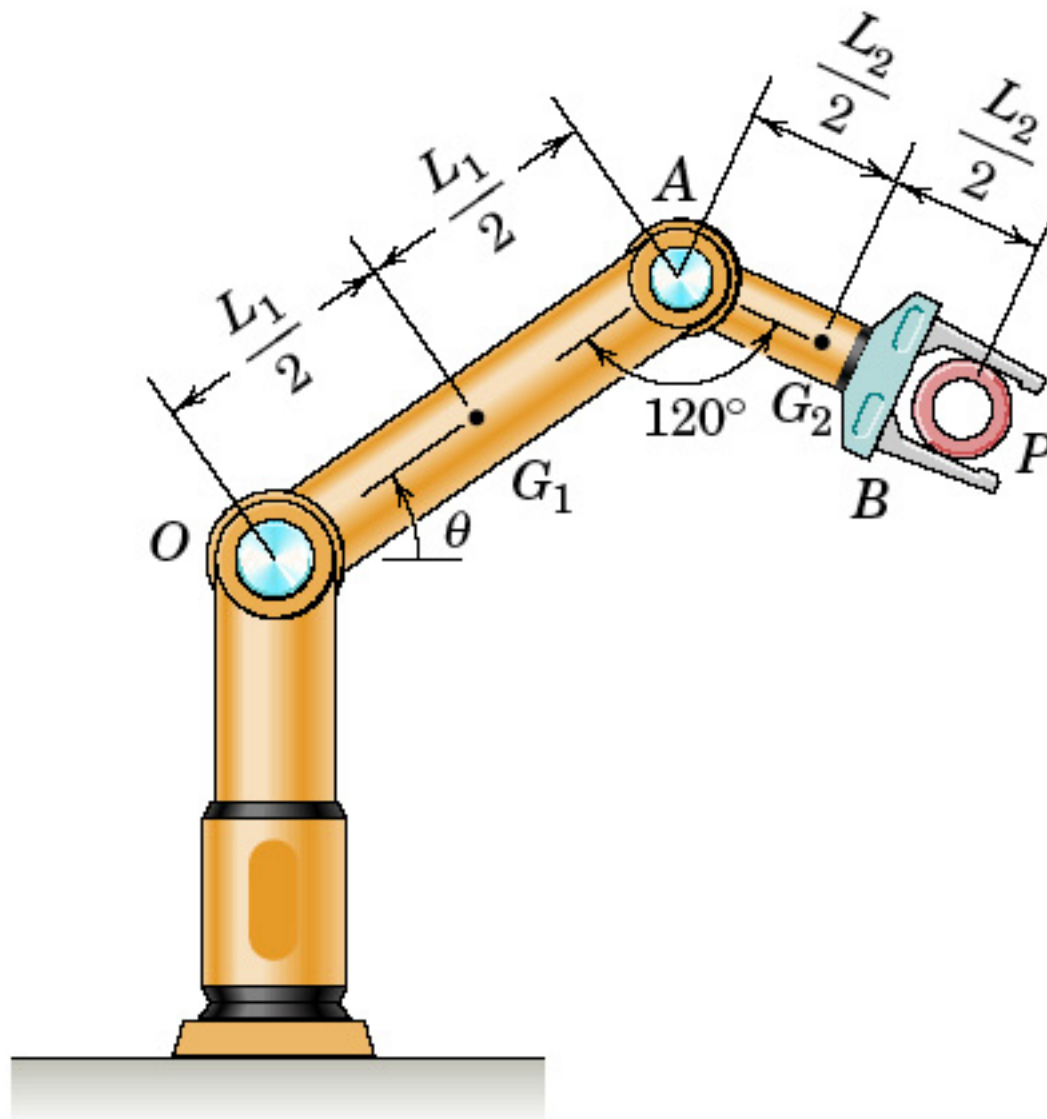


Problem 2/177

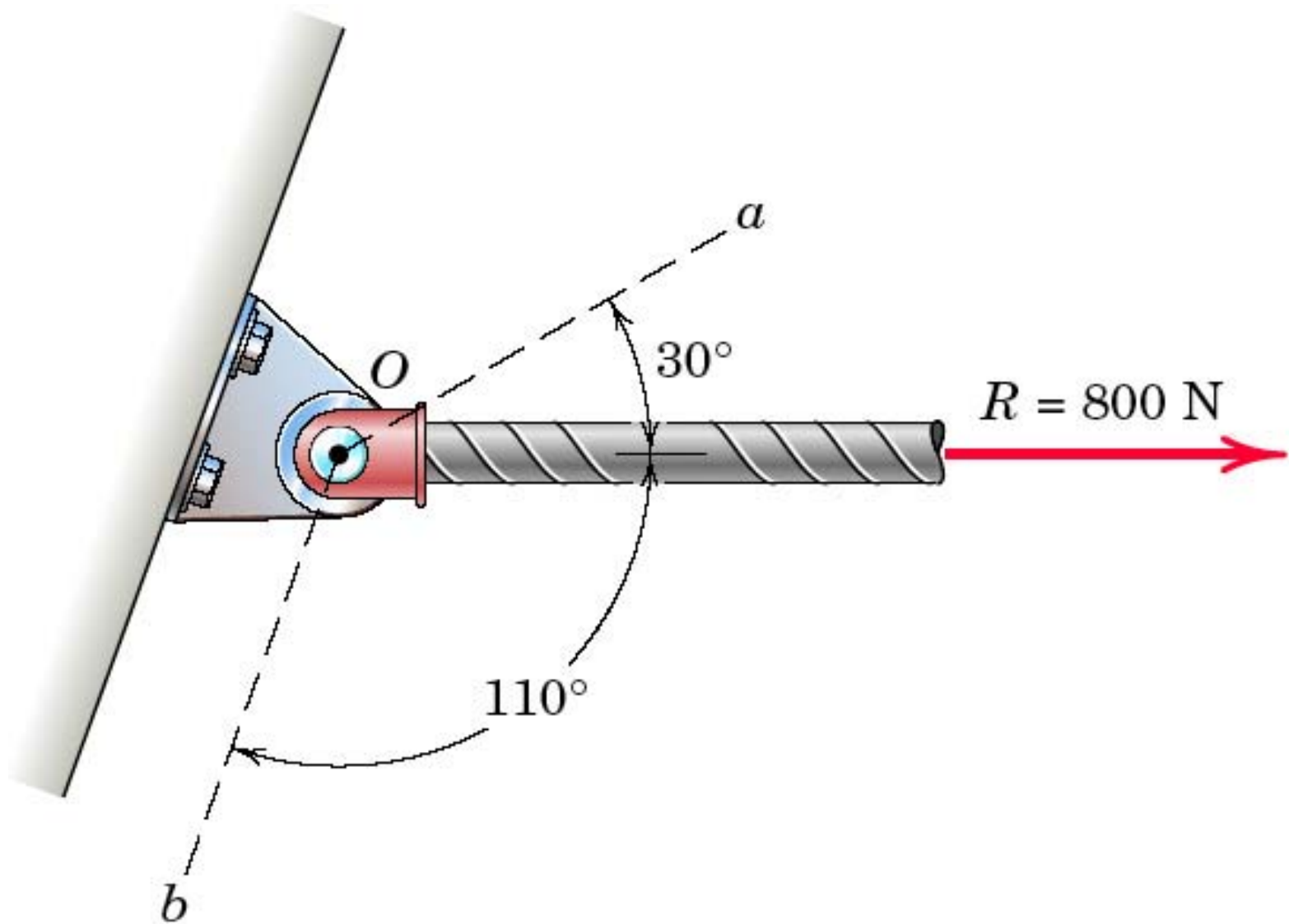


Problem 2/178

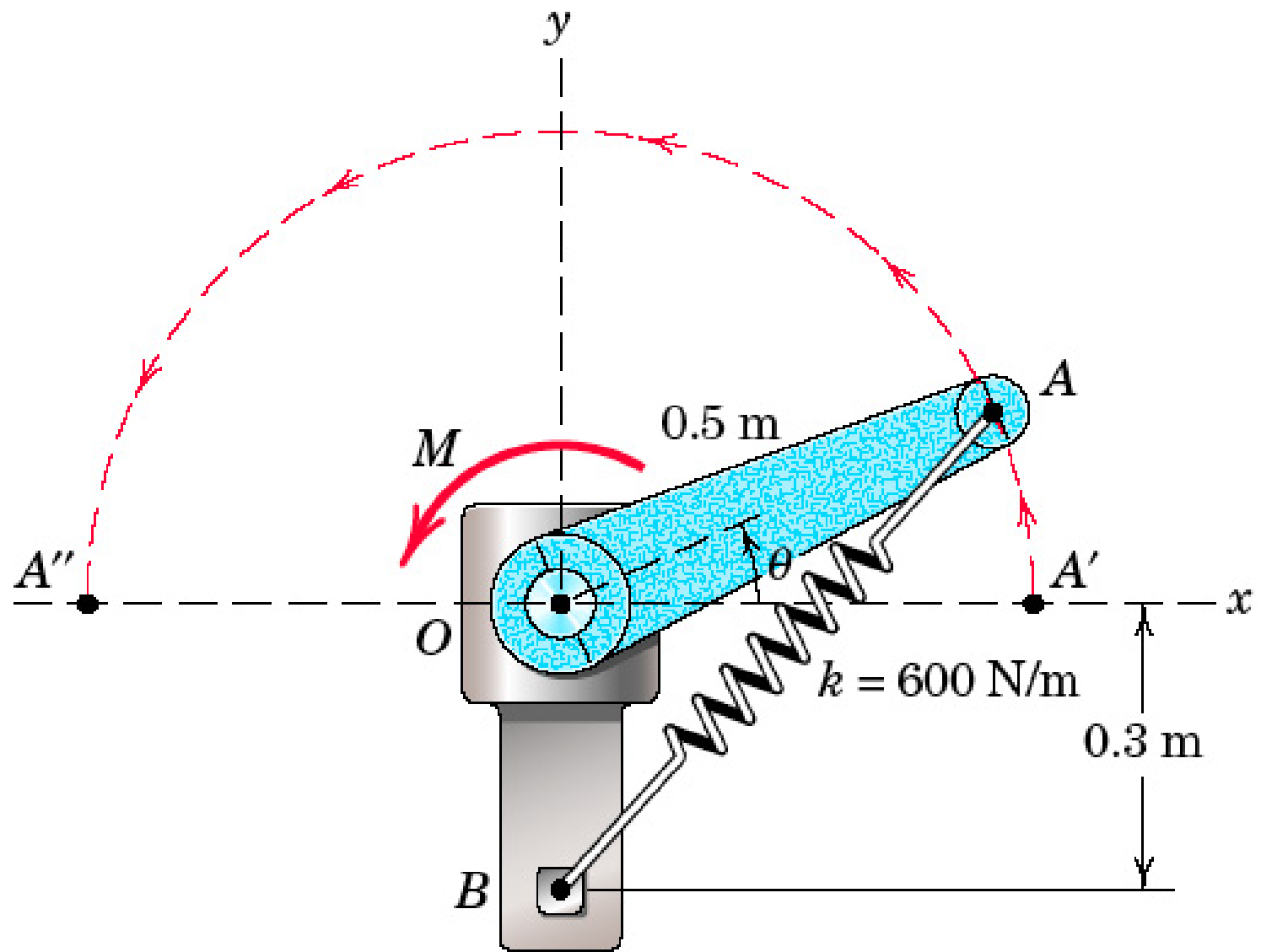
P_02_178



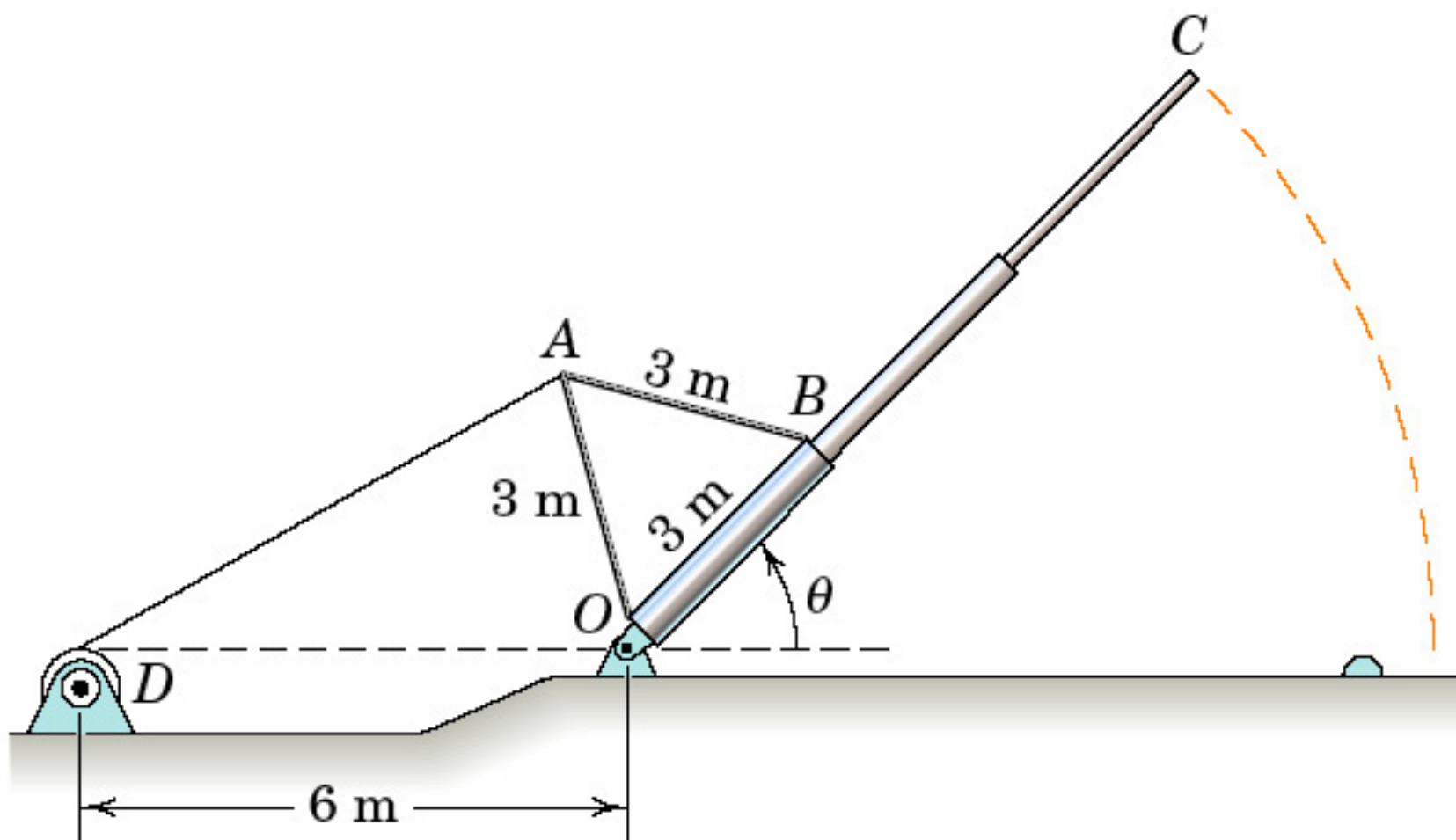
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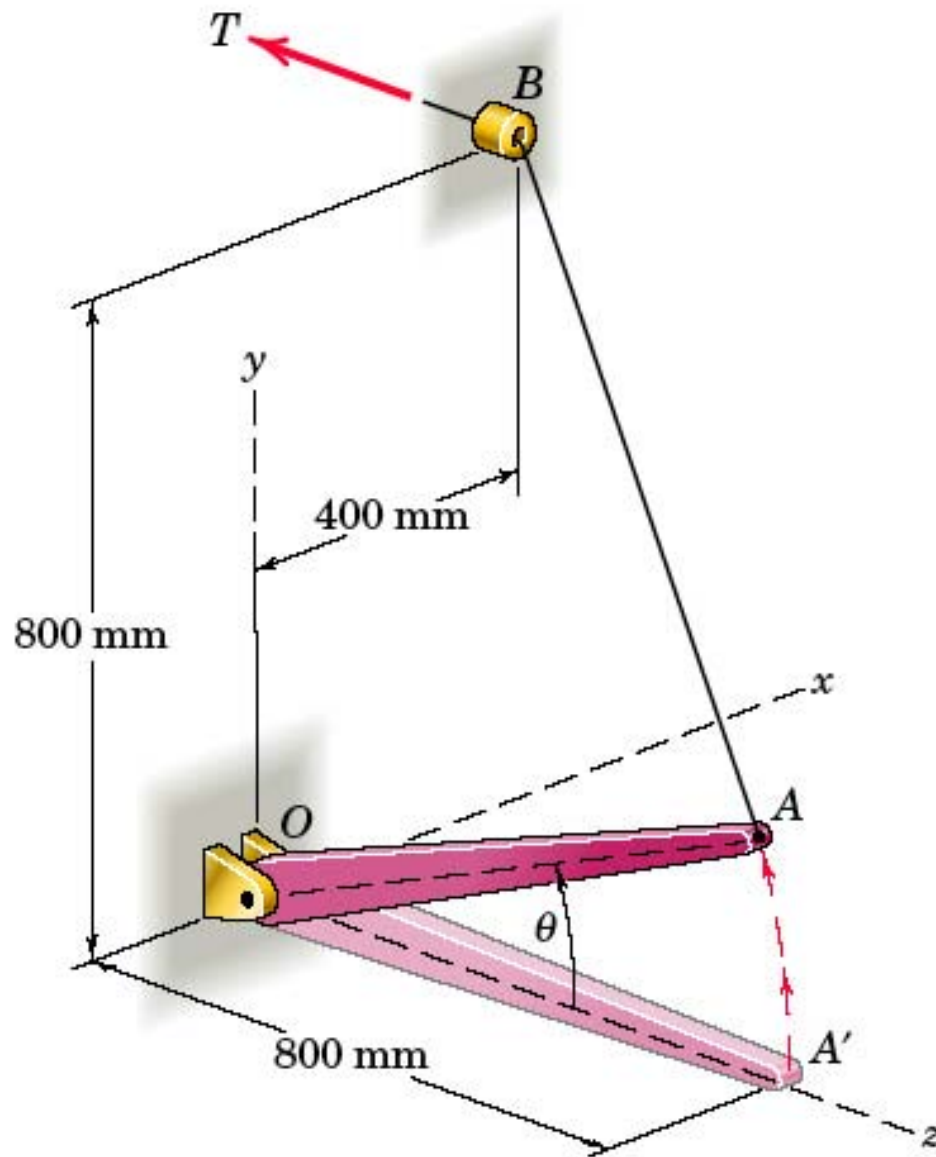


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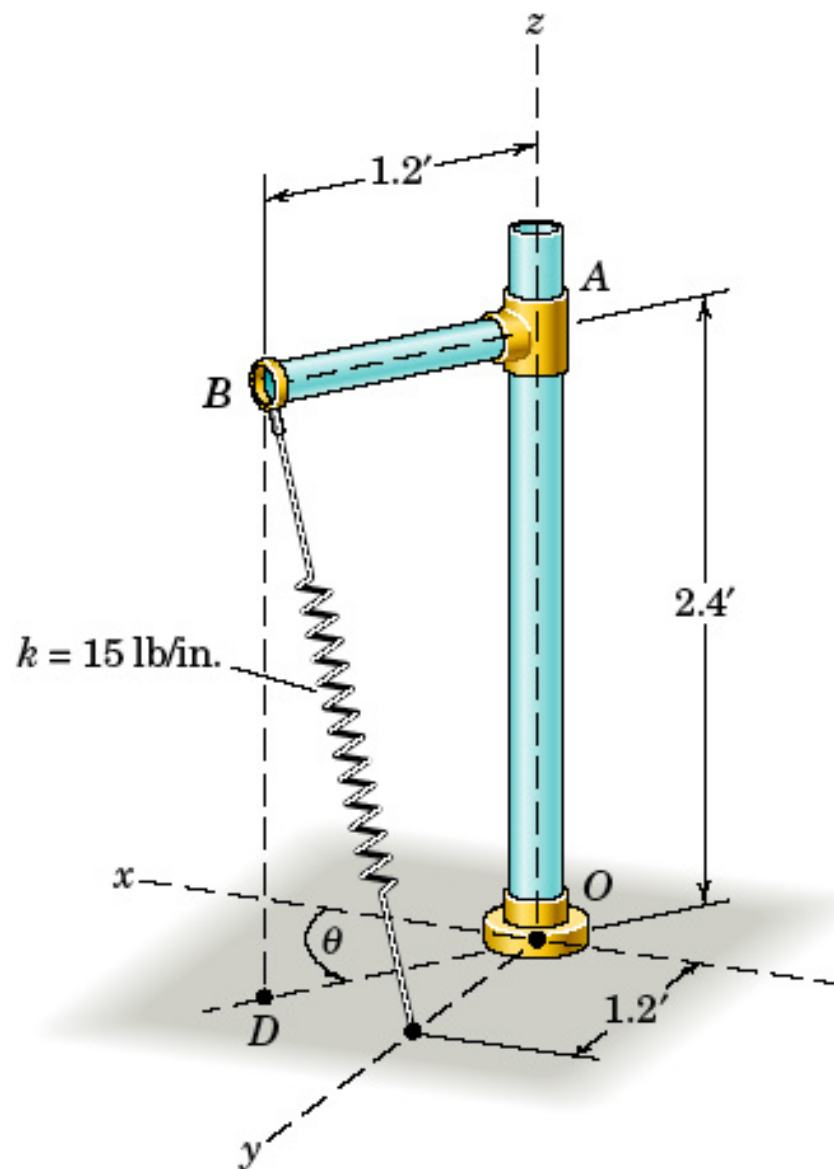


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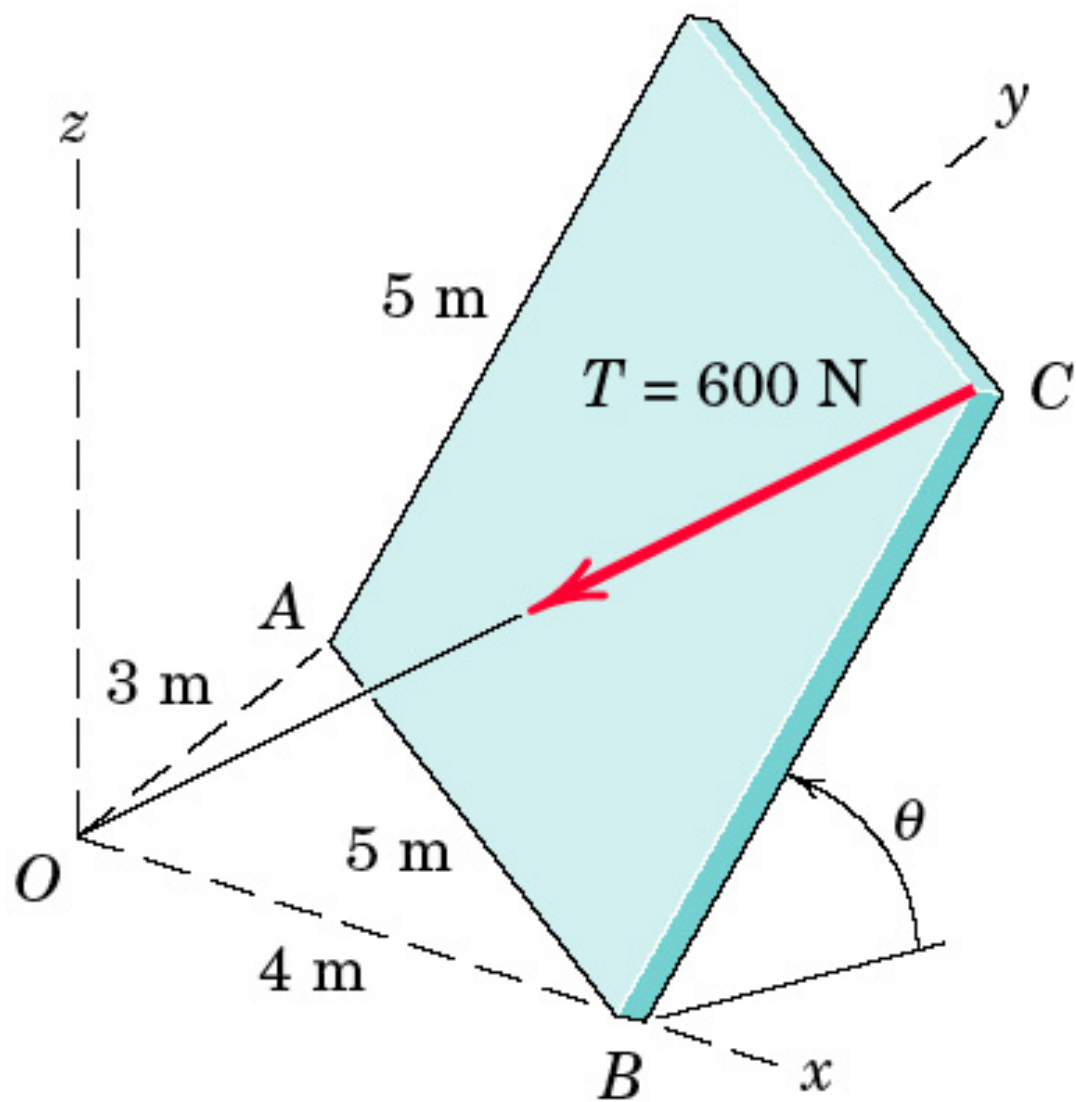
P_02_181



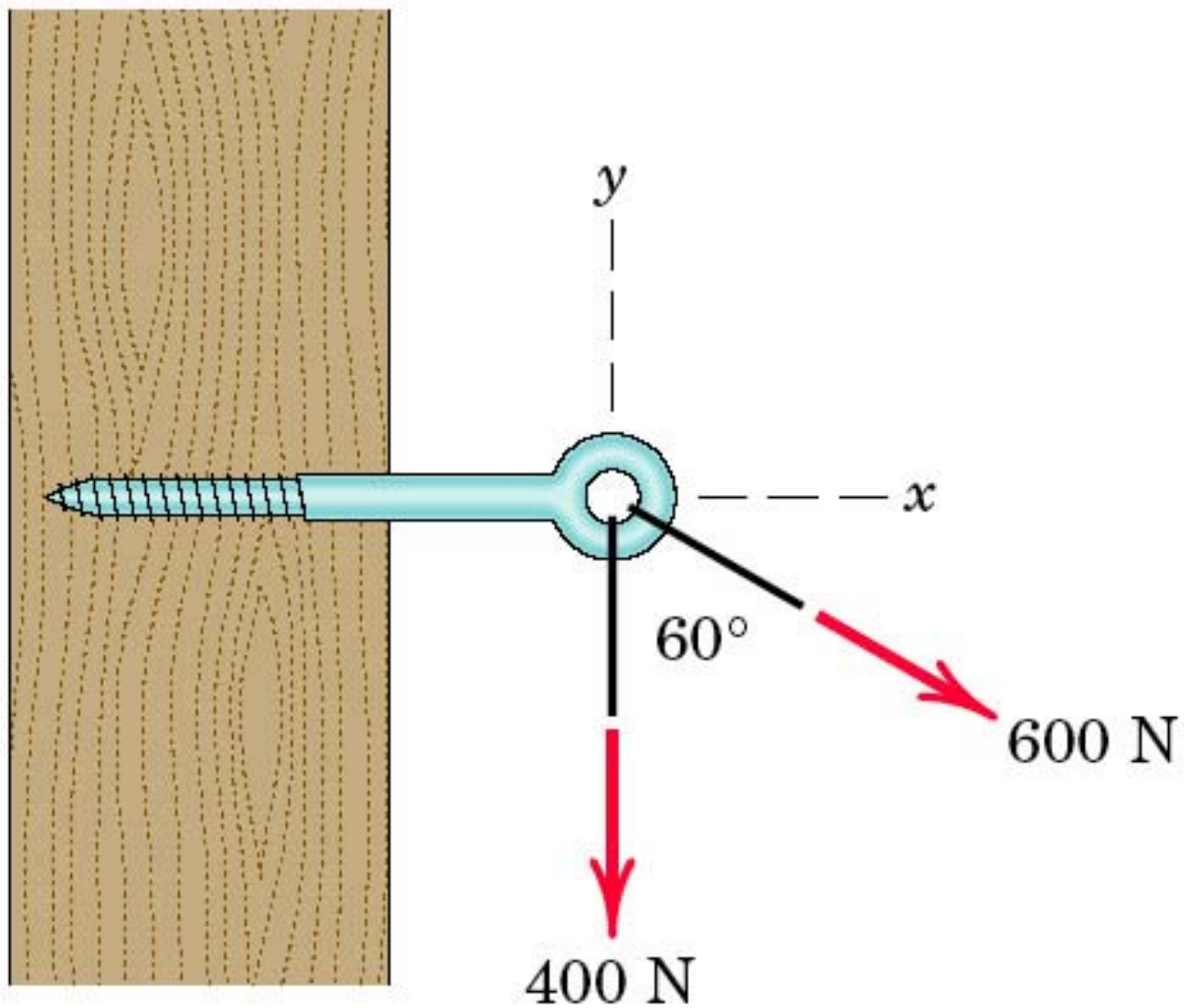
Problem 2/182



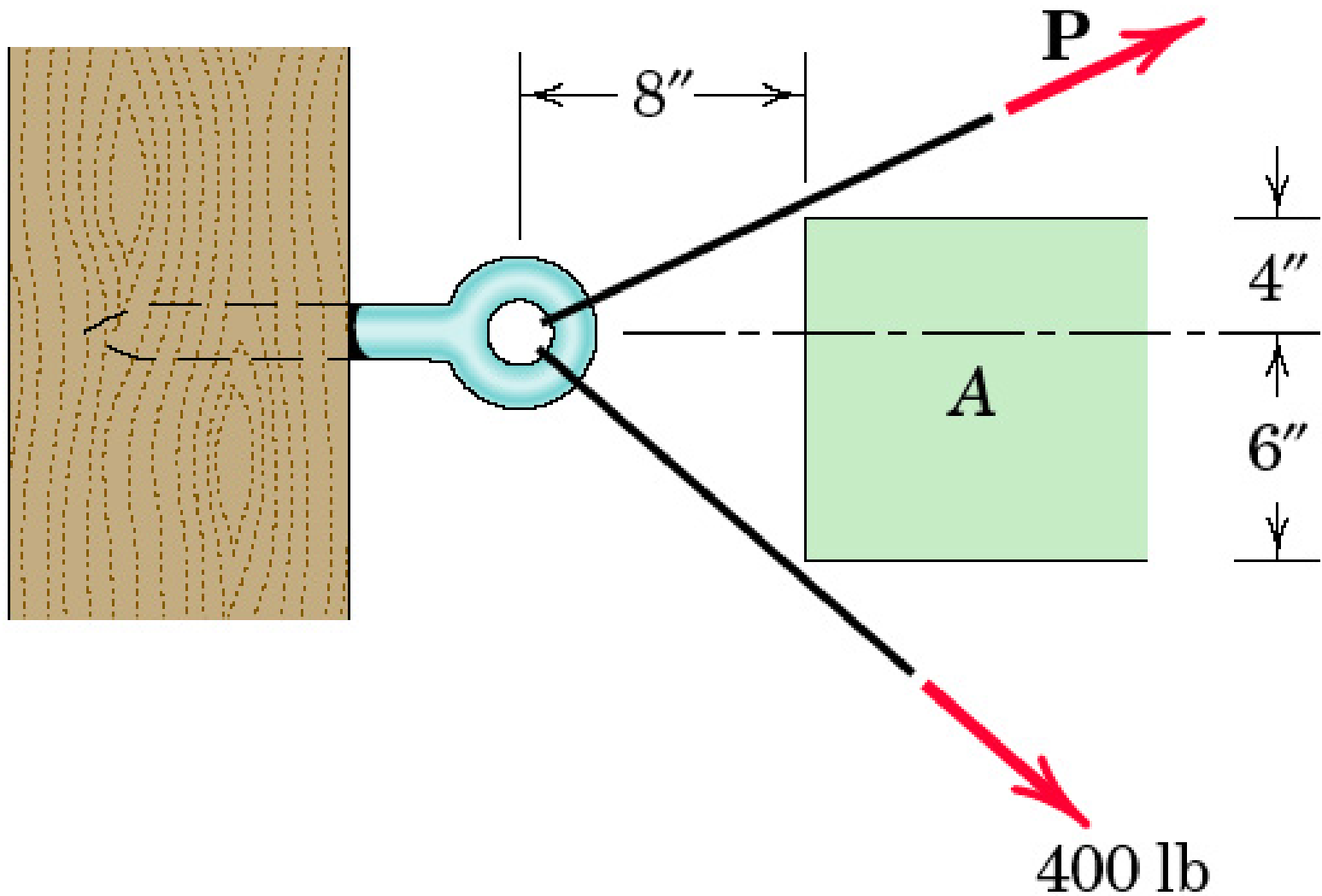
Problem 2/183



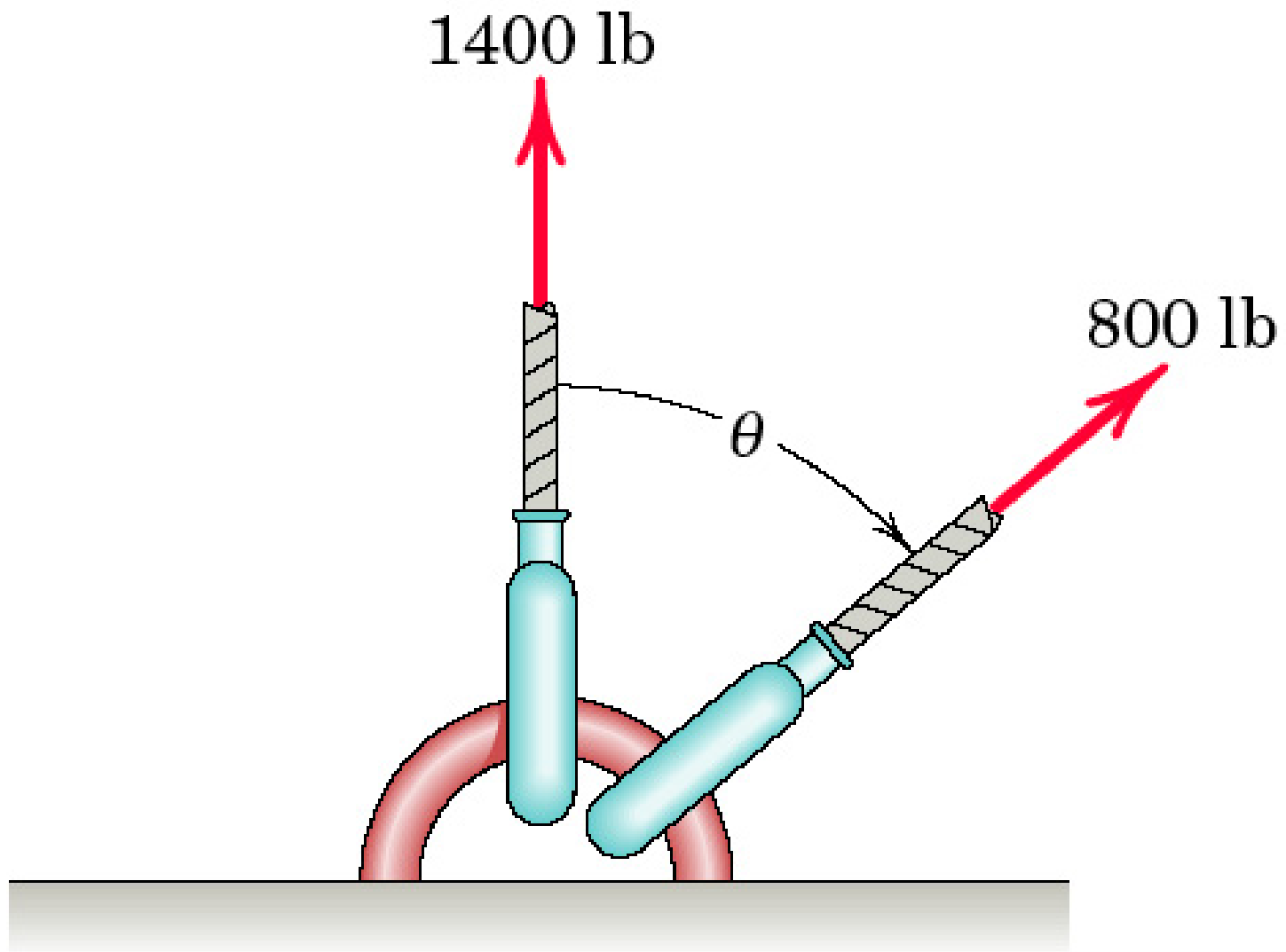
Problem 2/184



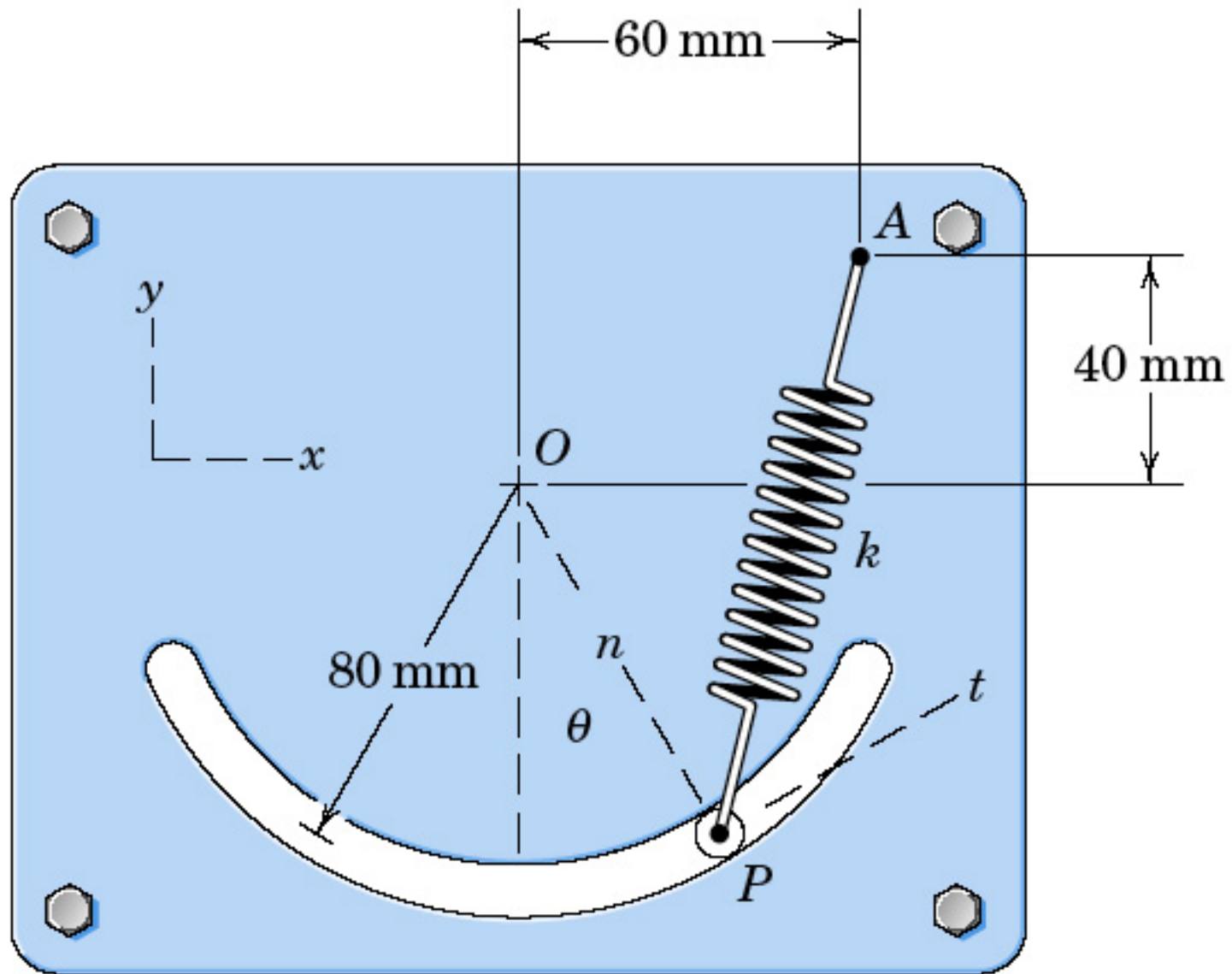
Problem 2/19



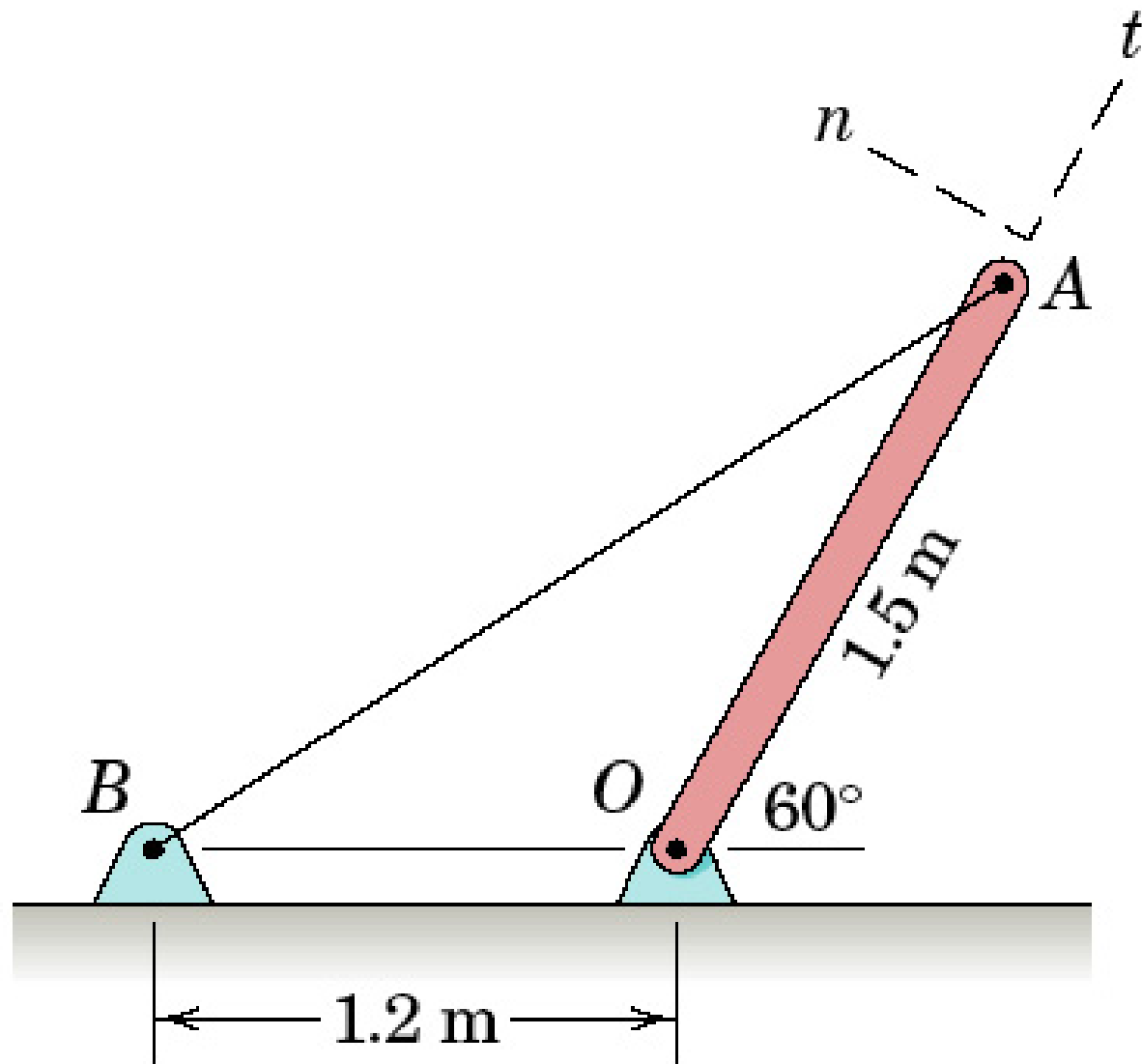
Problem 2/20



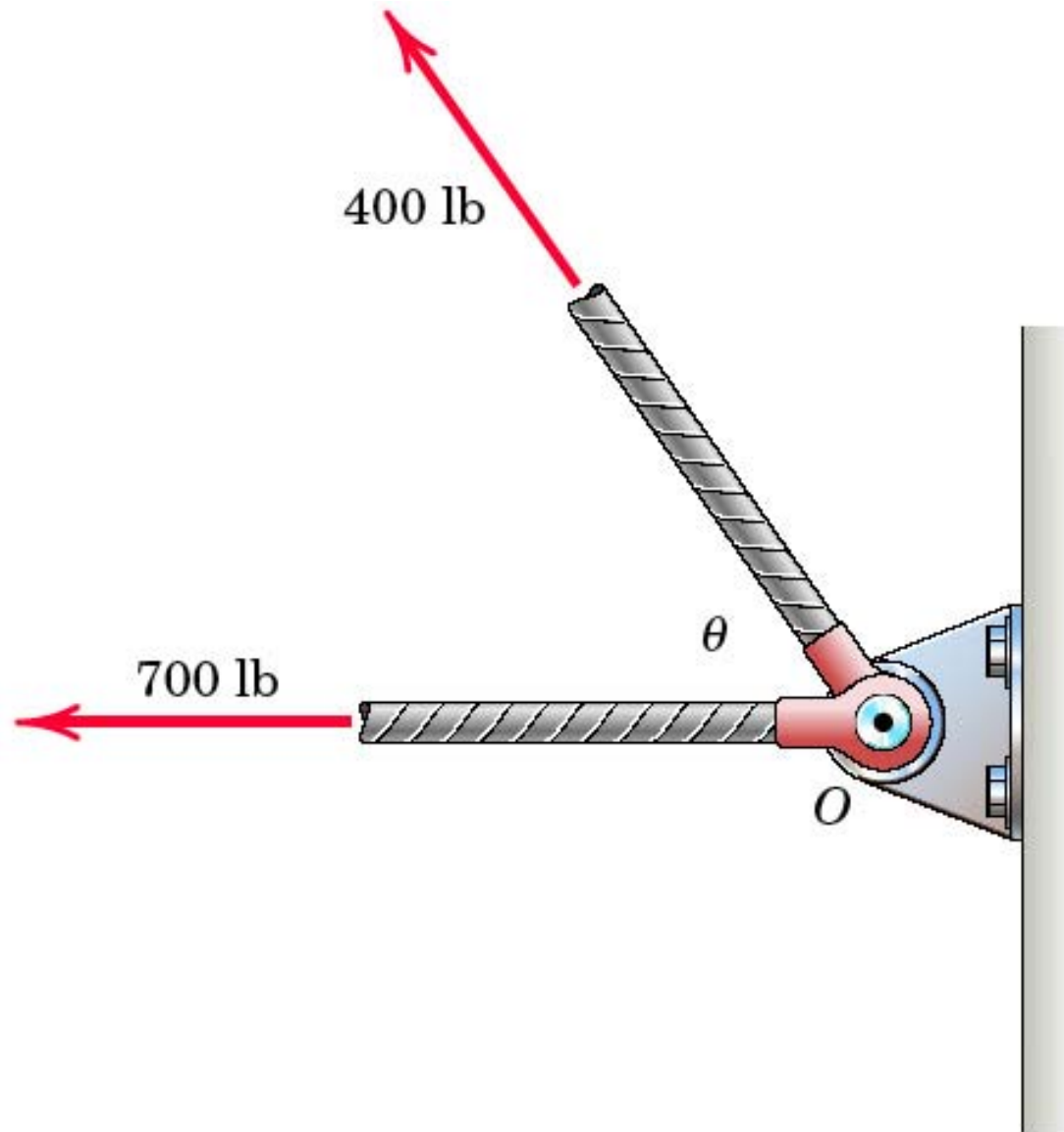
Problem 2/21



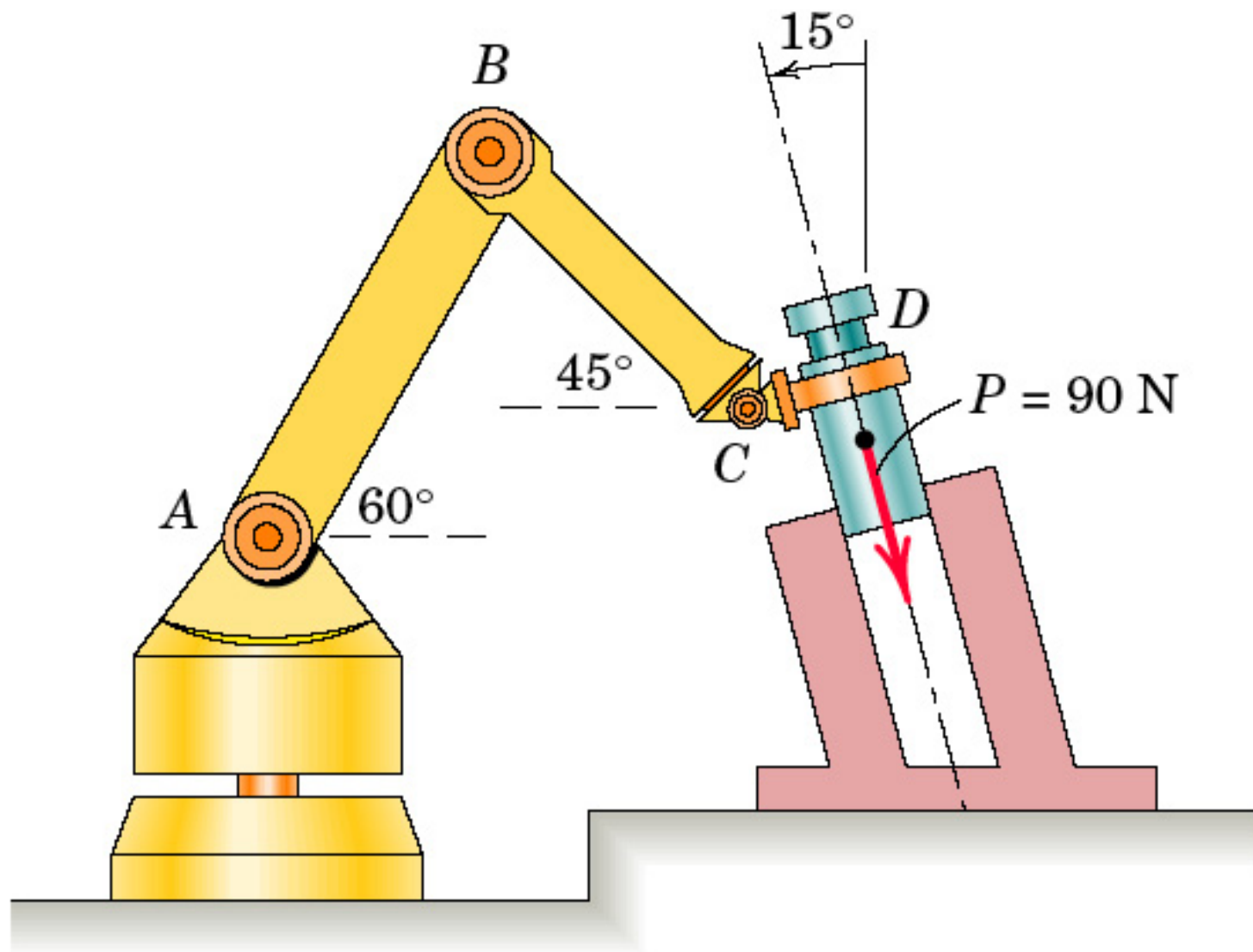
Problem 2/22



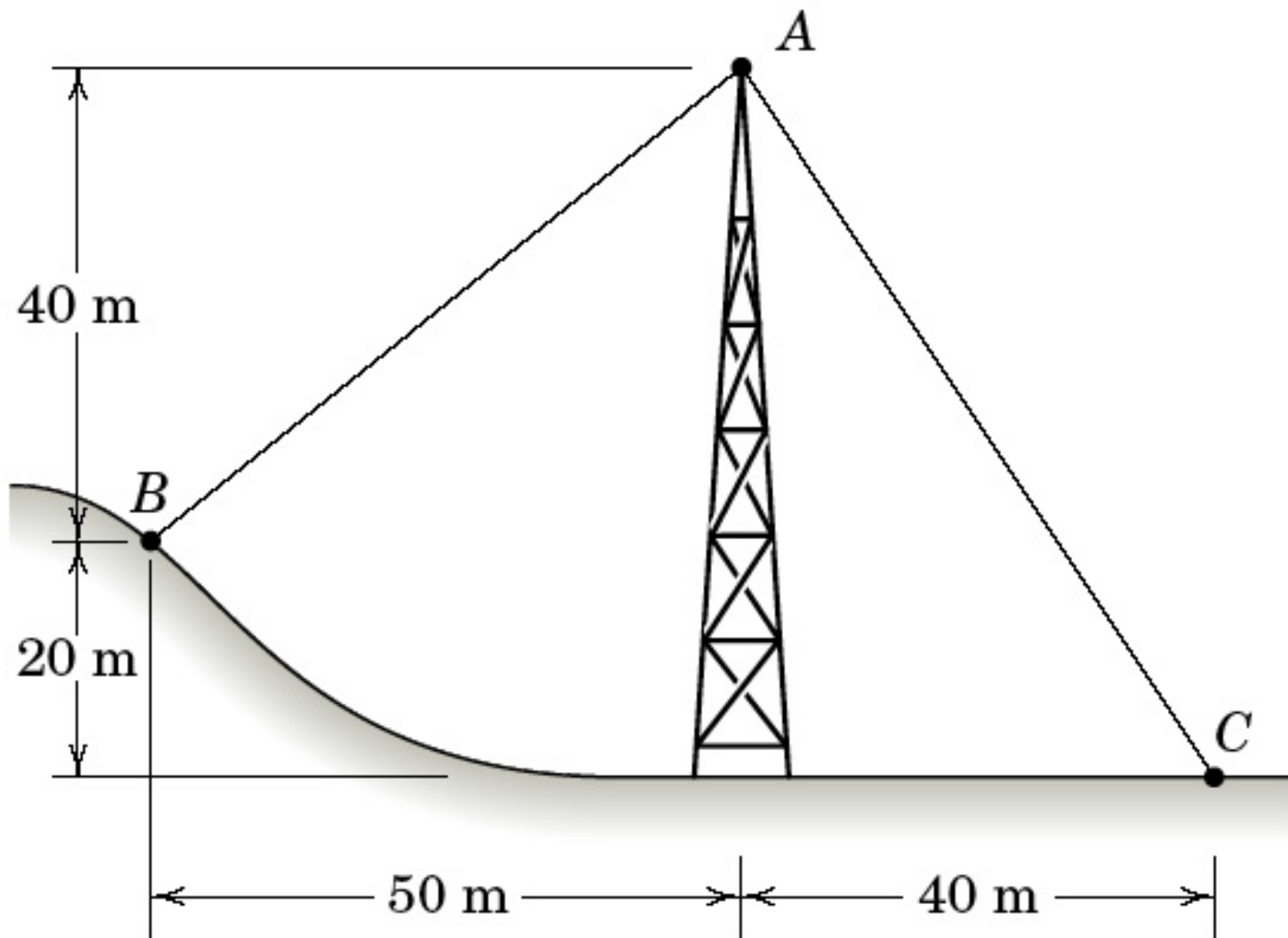
Problem 2/24



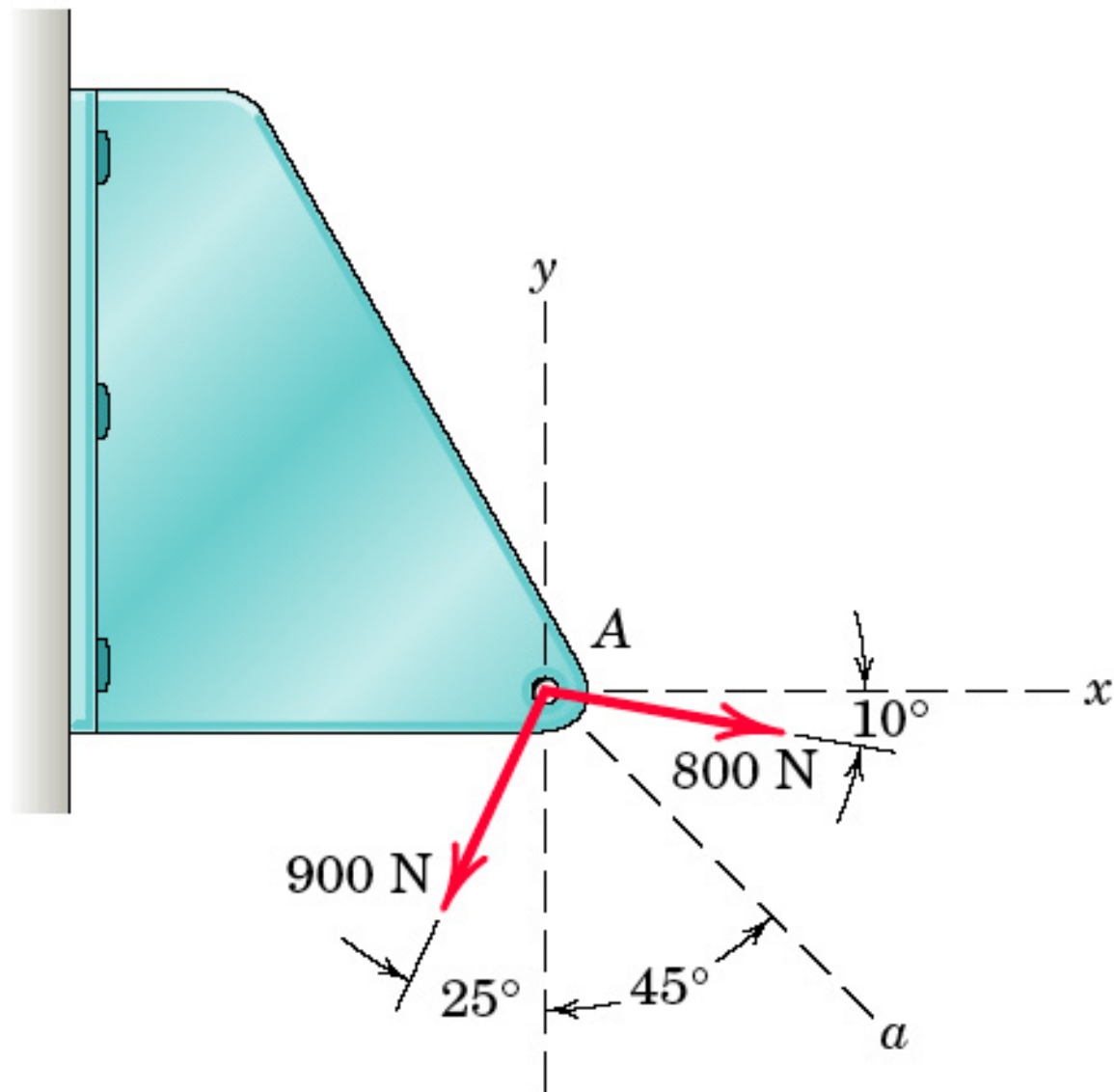
Problem 2/25



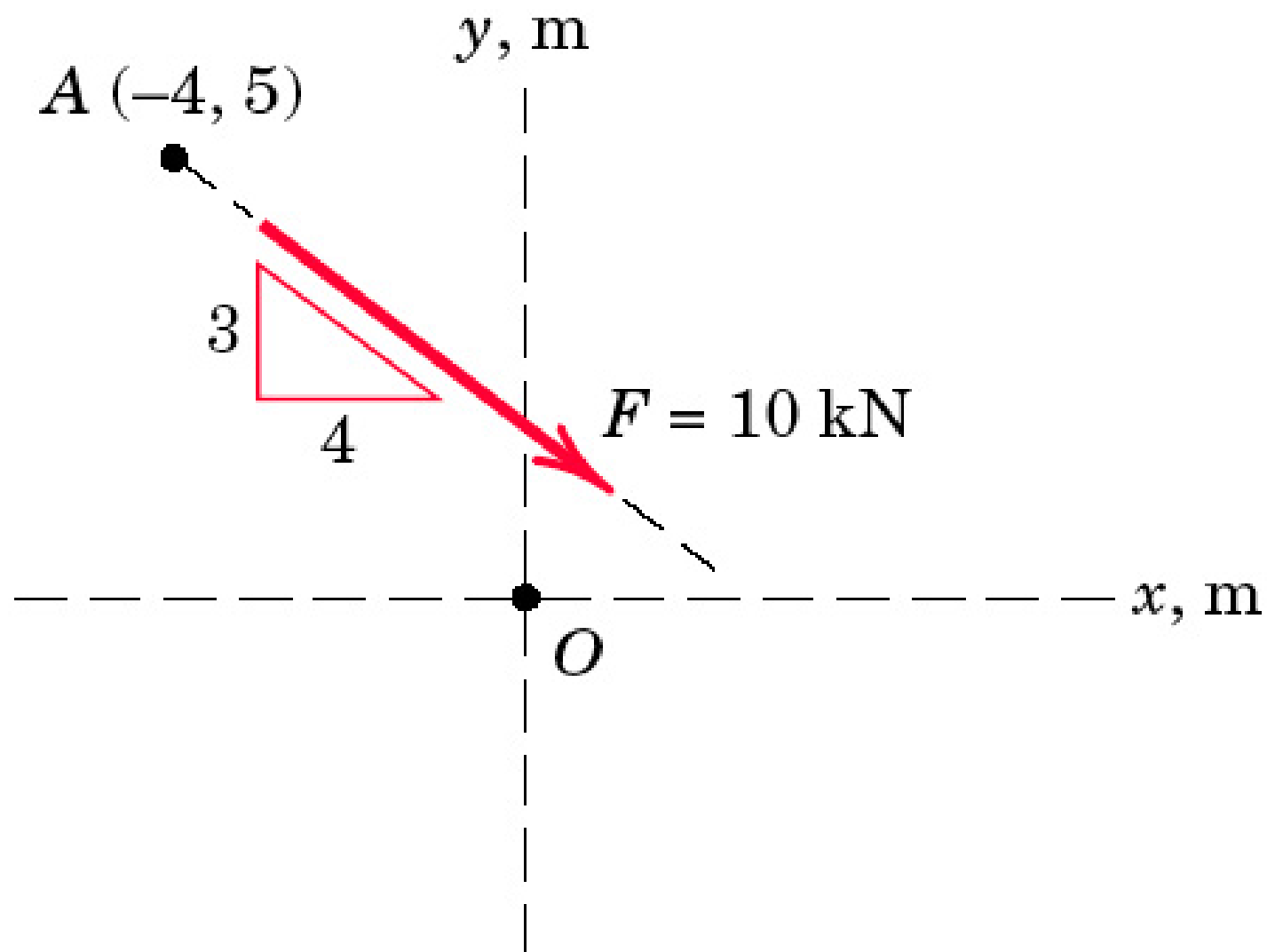
Problem 2/26



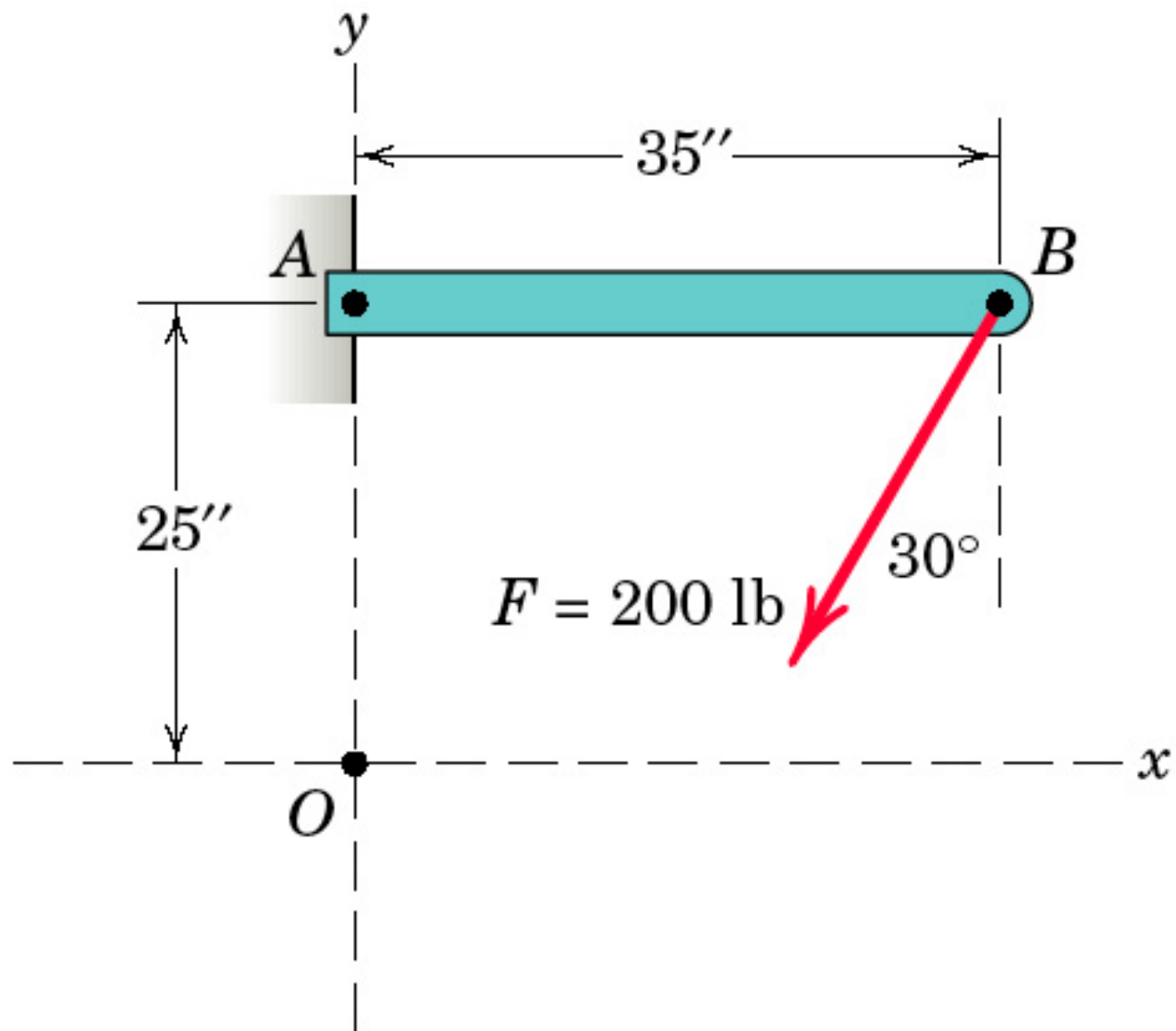
Problem 2/27



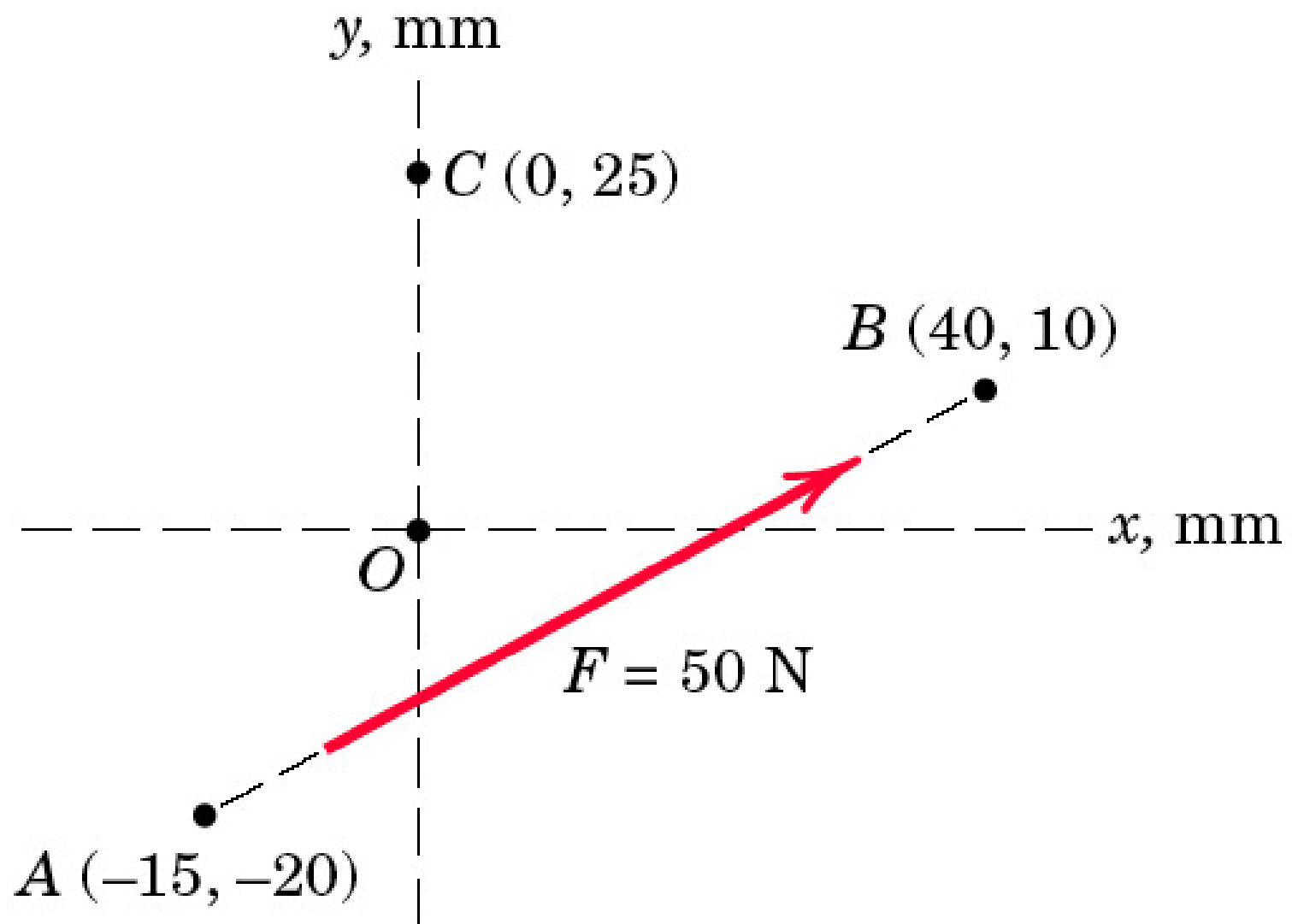
Problem 2/28



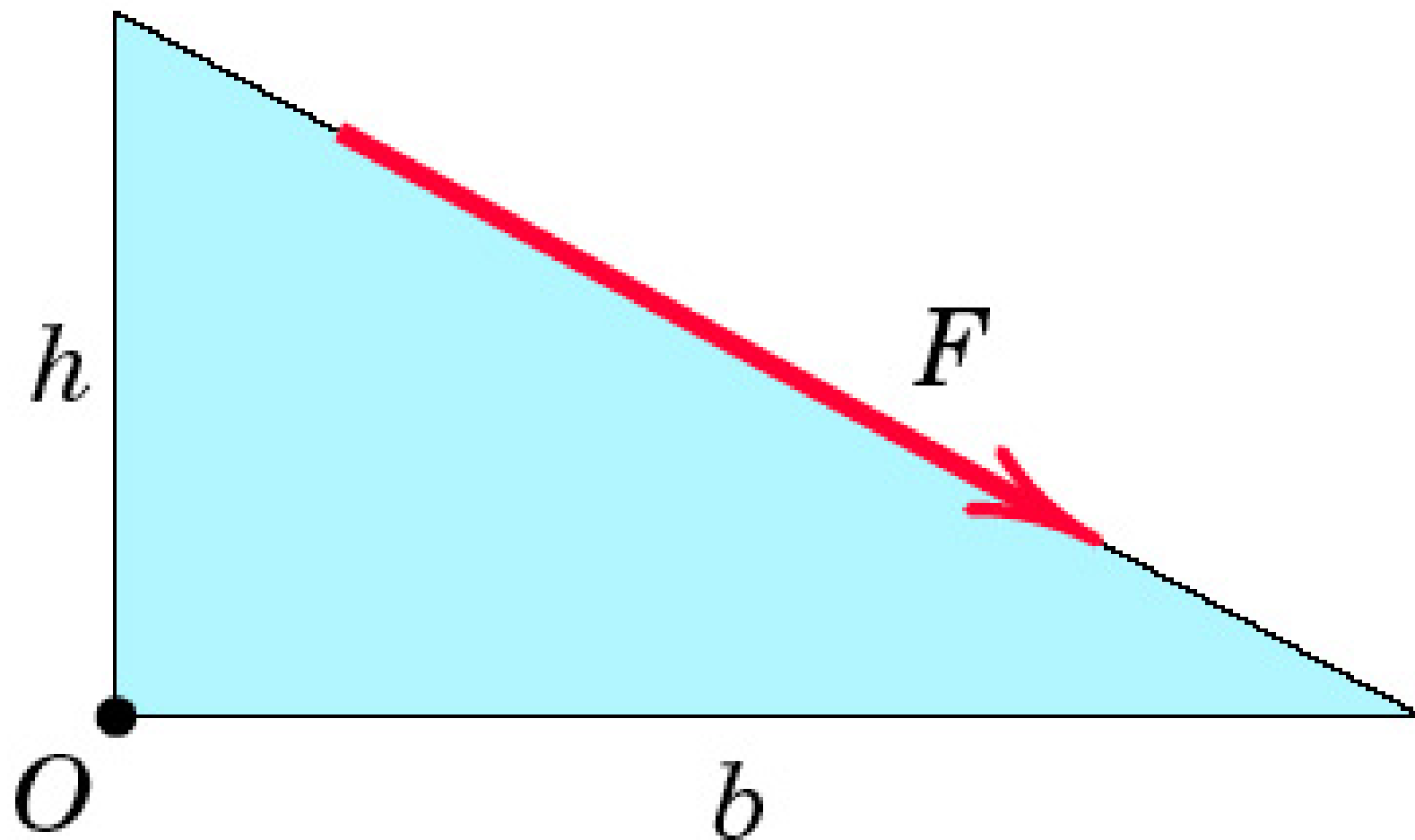
Problem 2/29



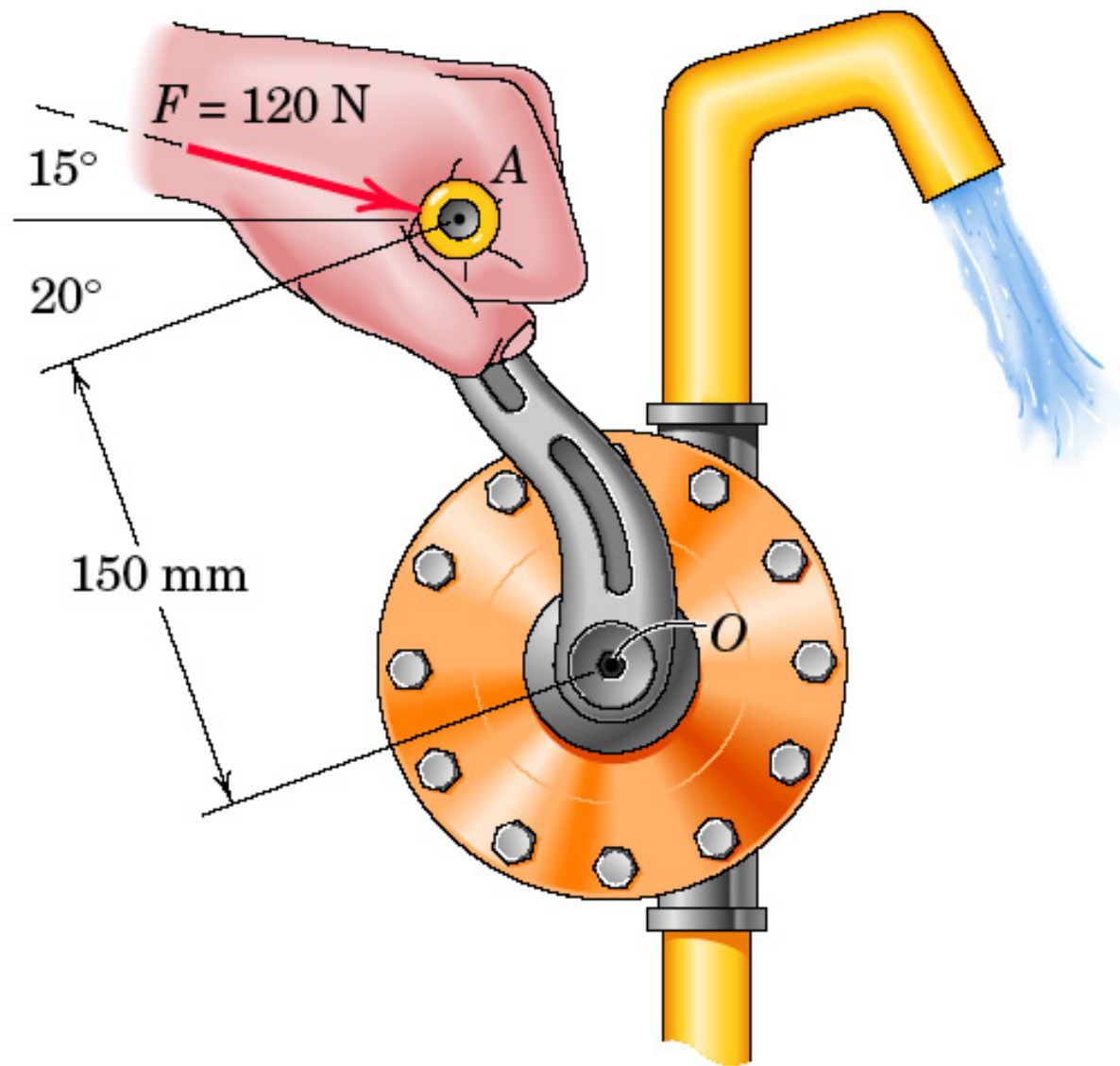
Problem 2/30



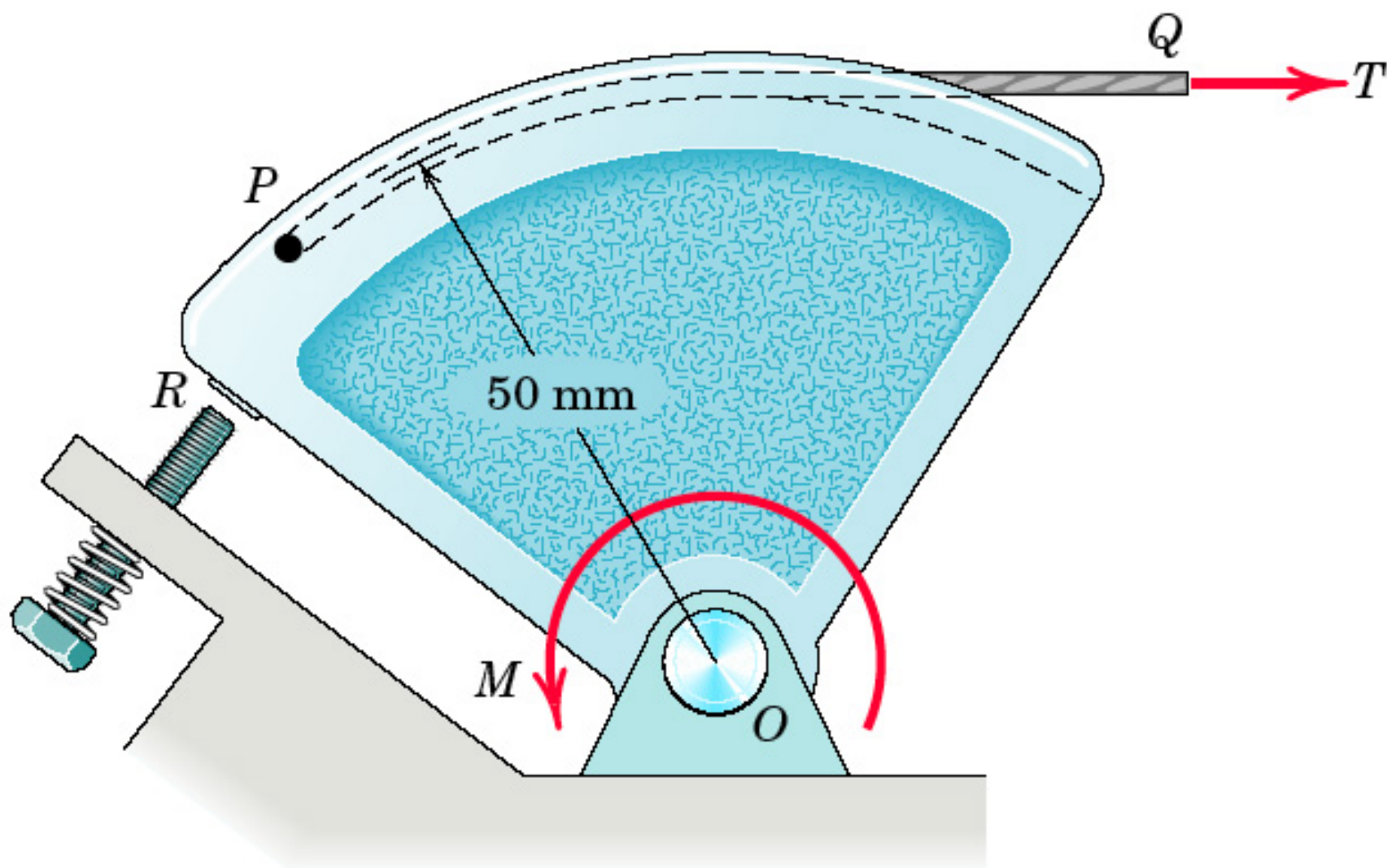
Problem 2/31



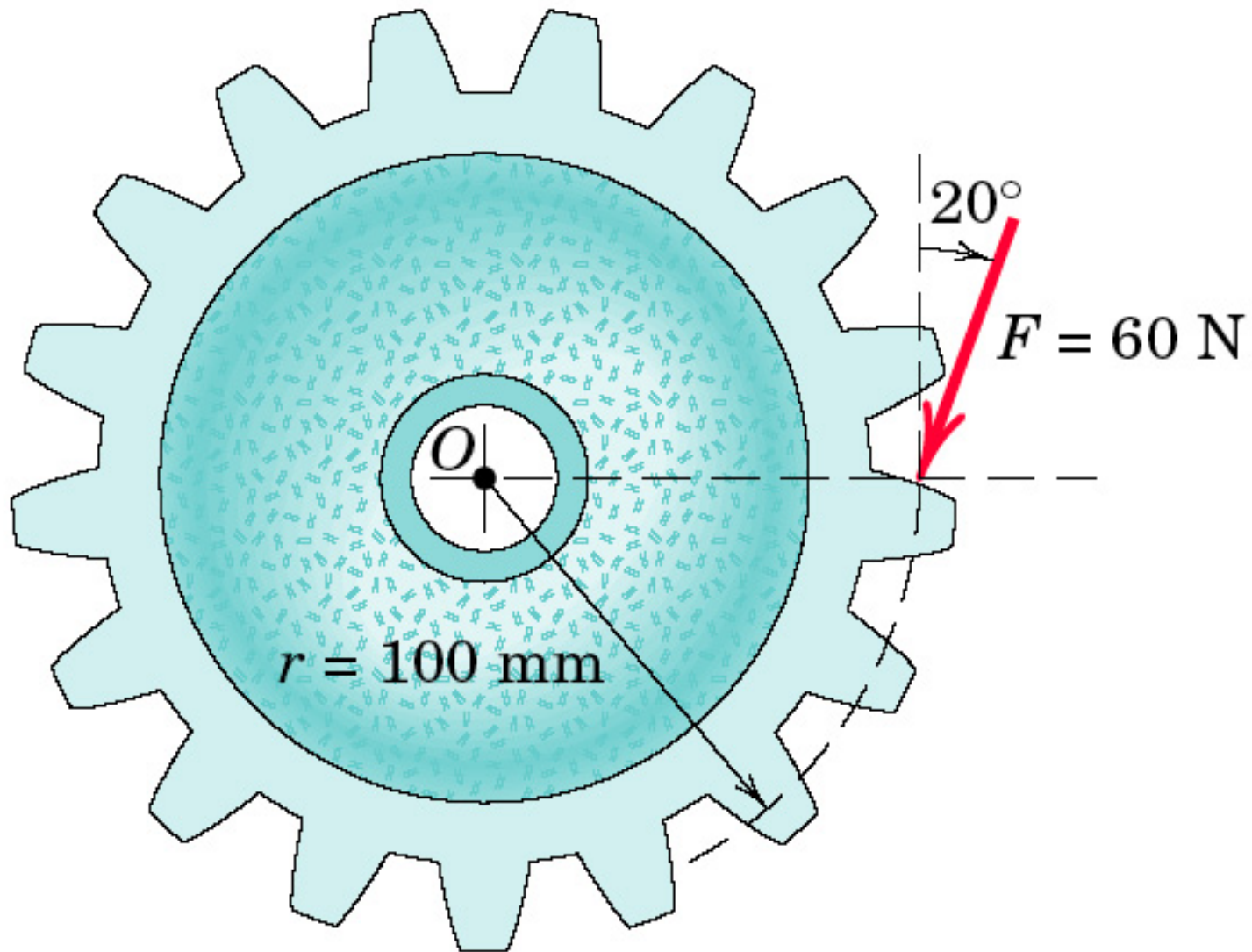
Problem 2/32



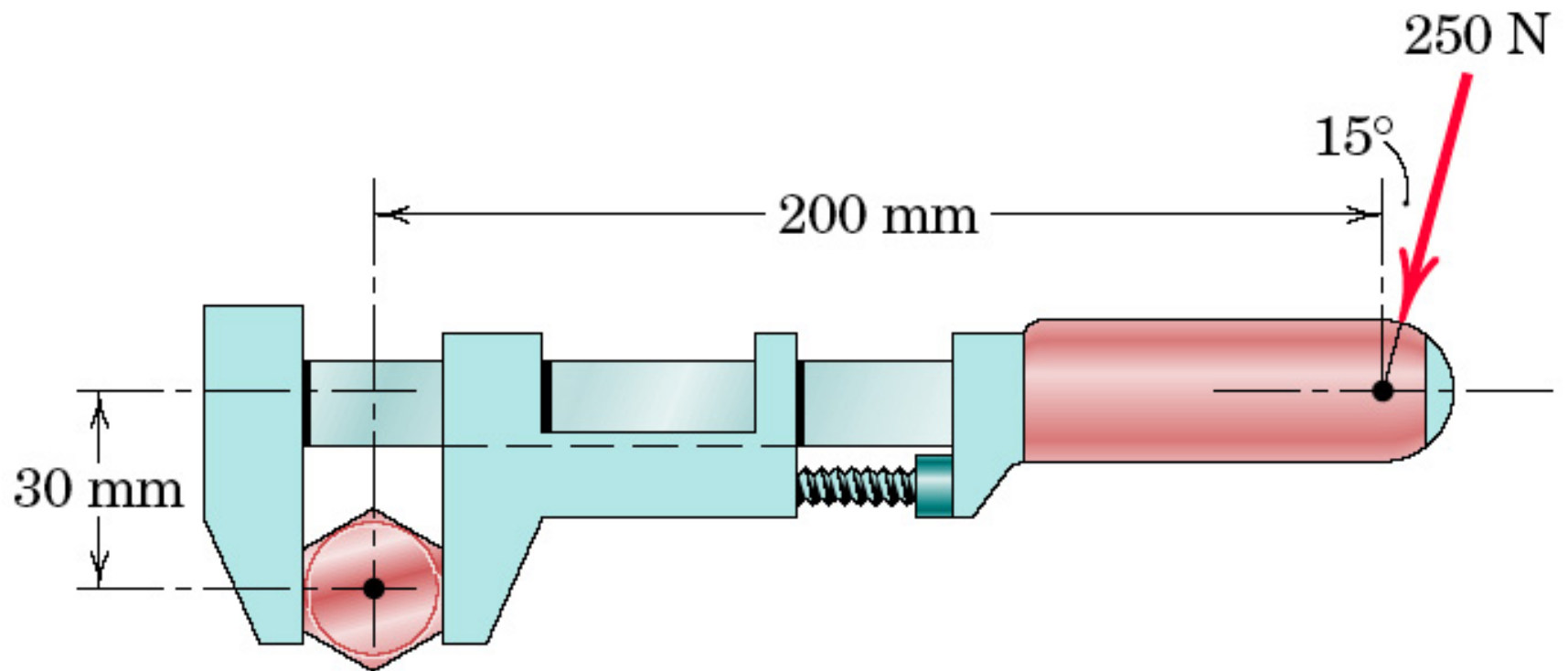
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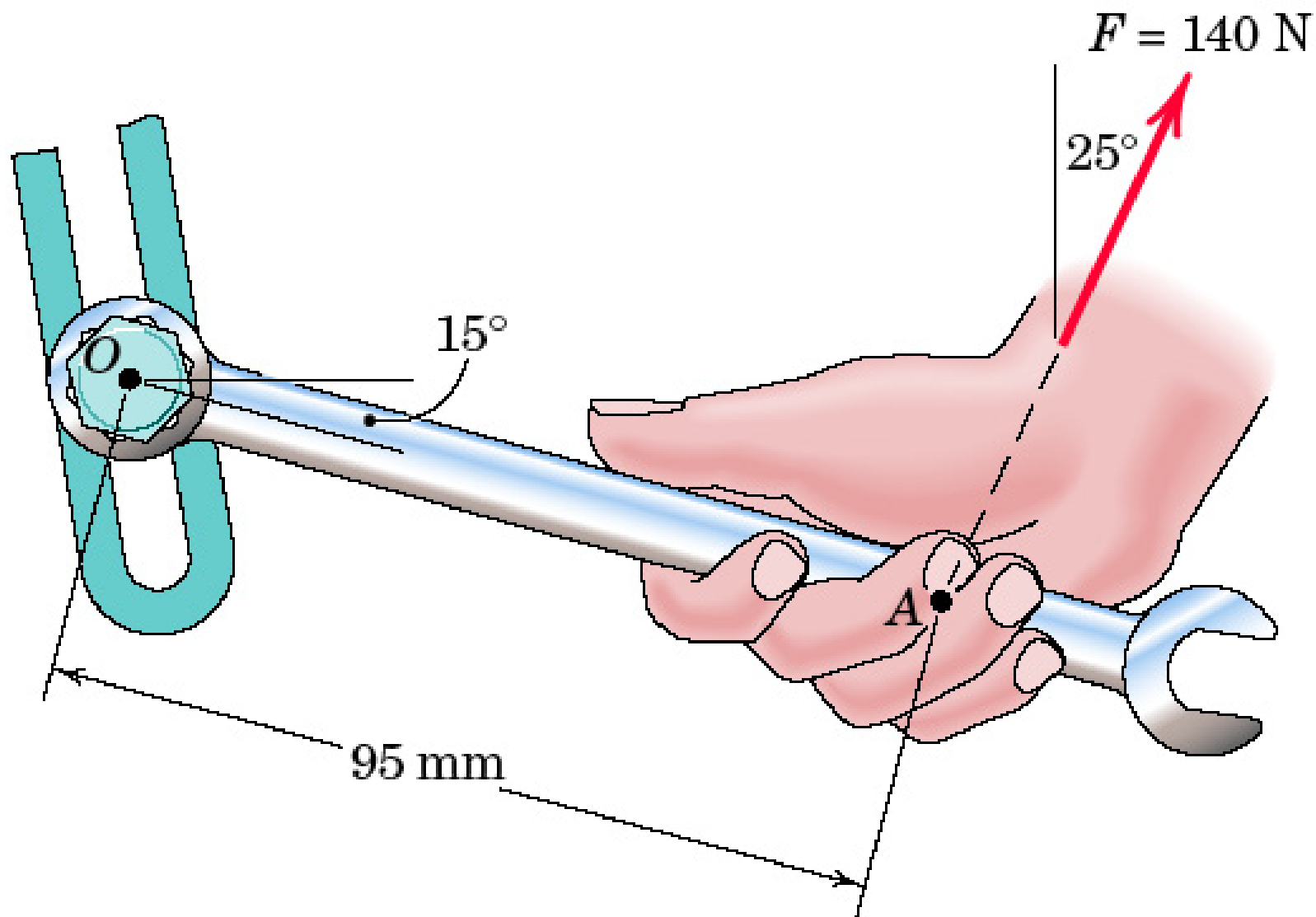


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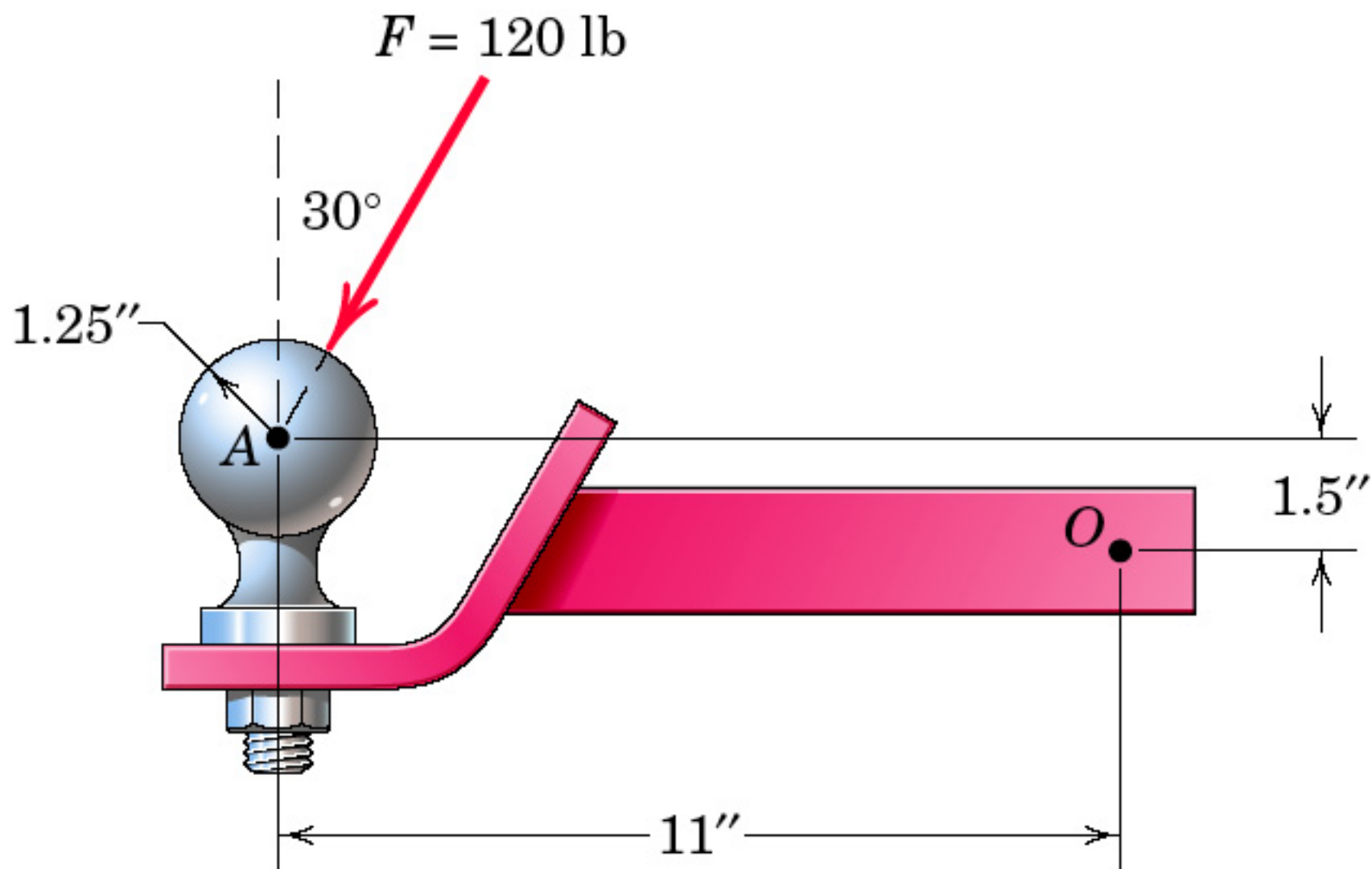


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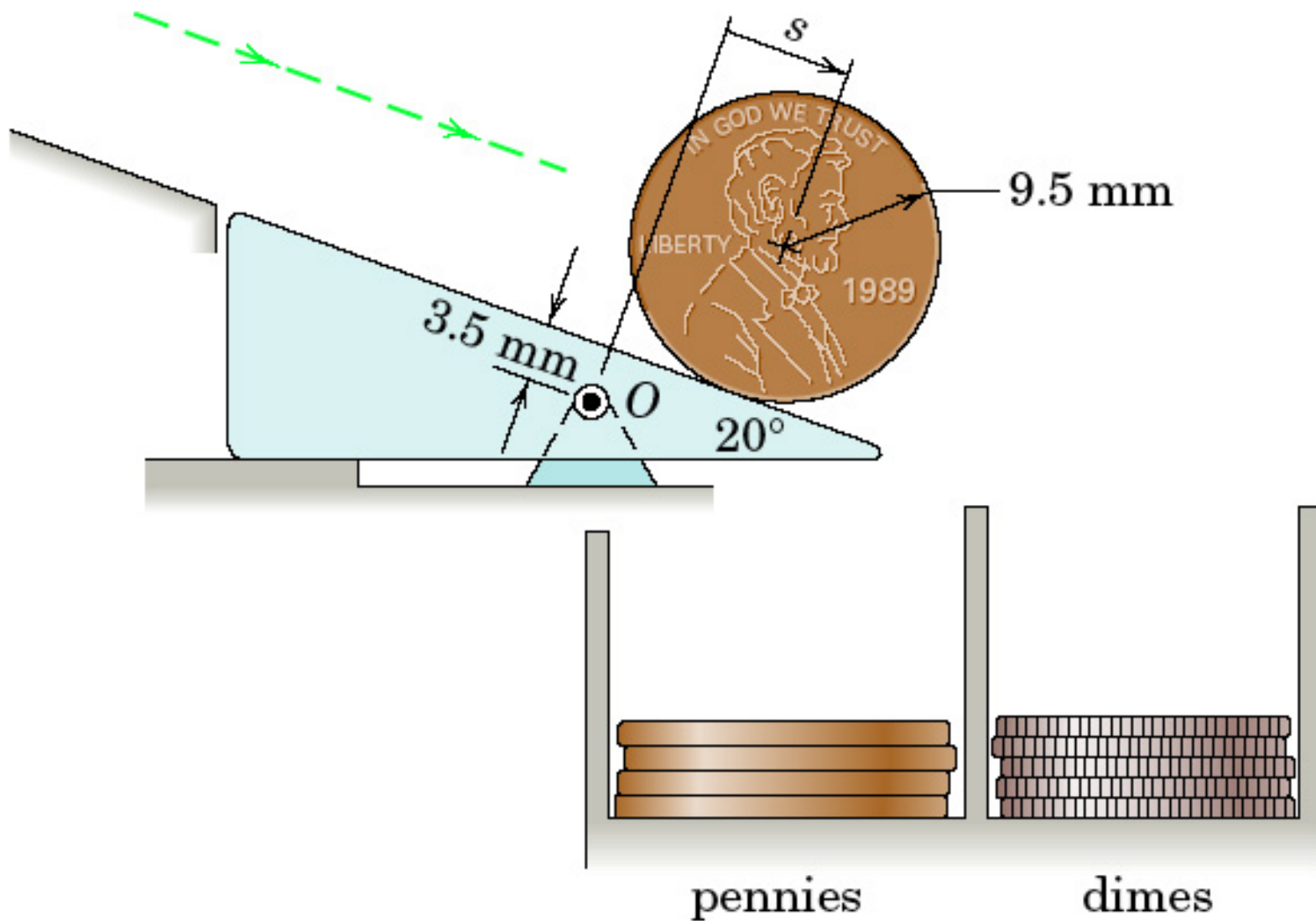
P_02_36



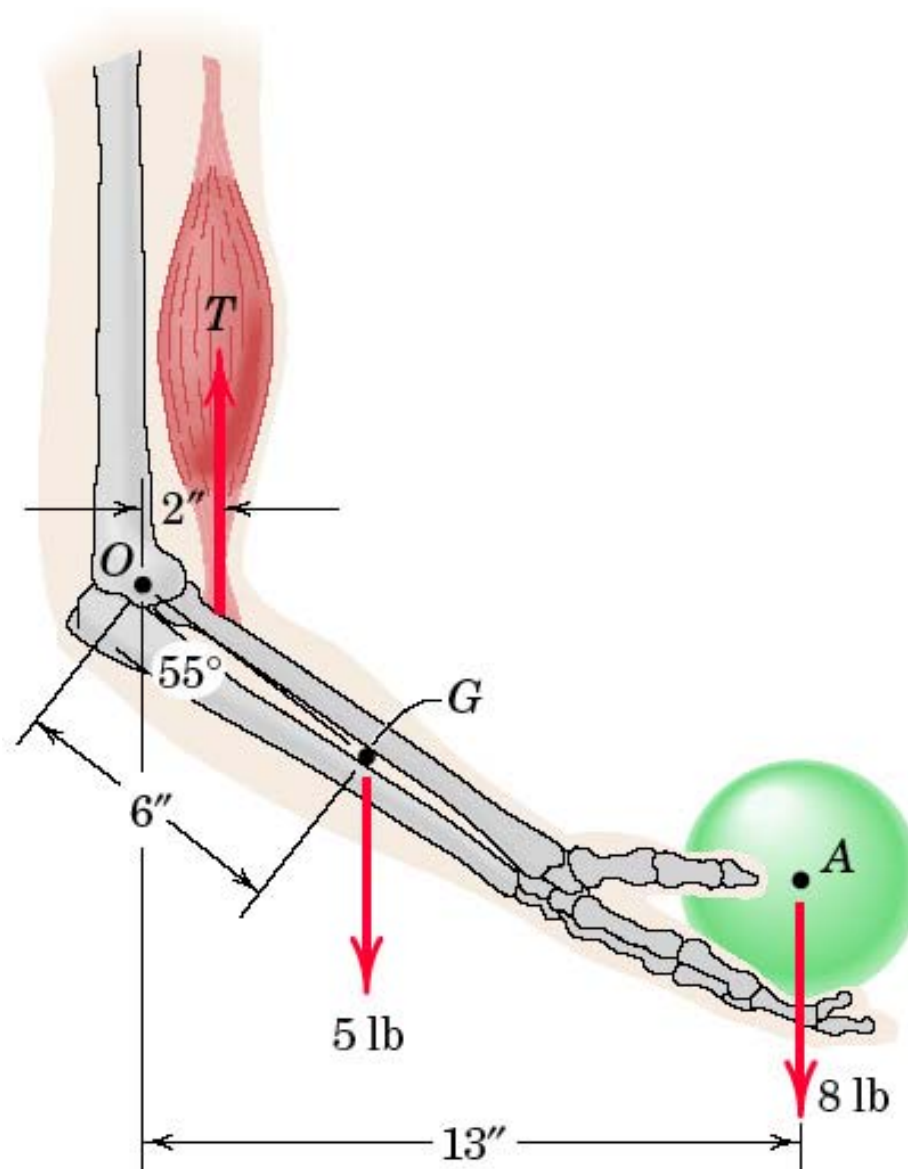
Problem 2/37



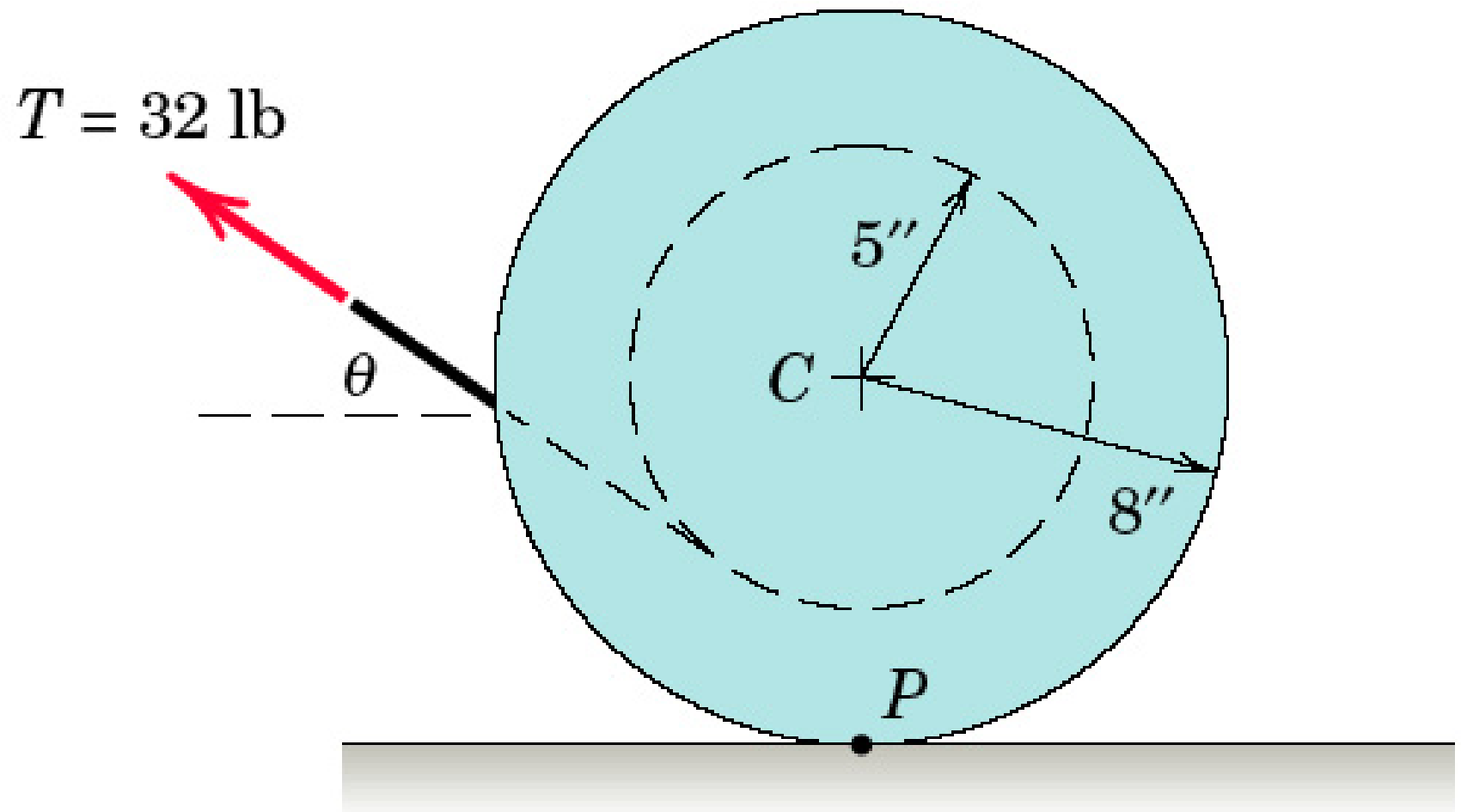
Problem 2/38



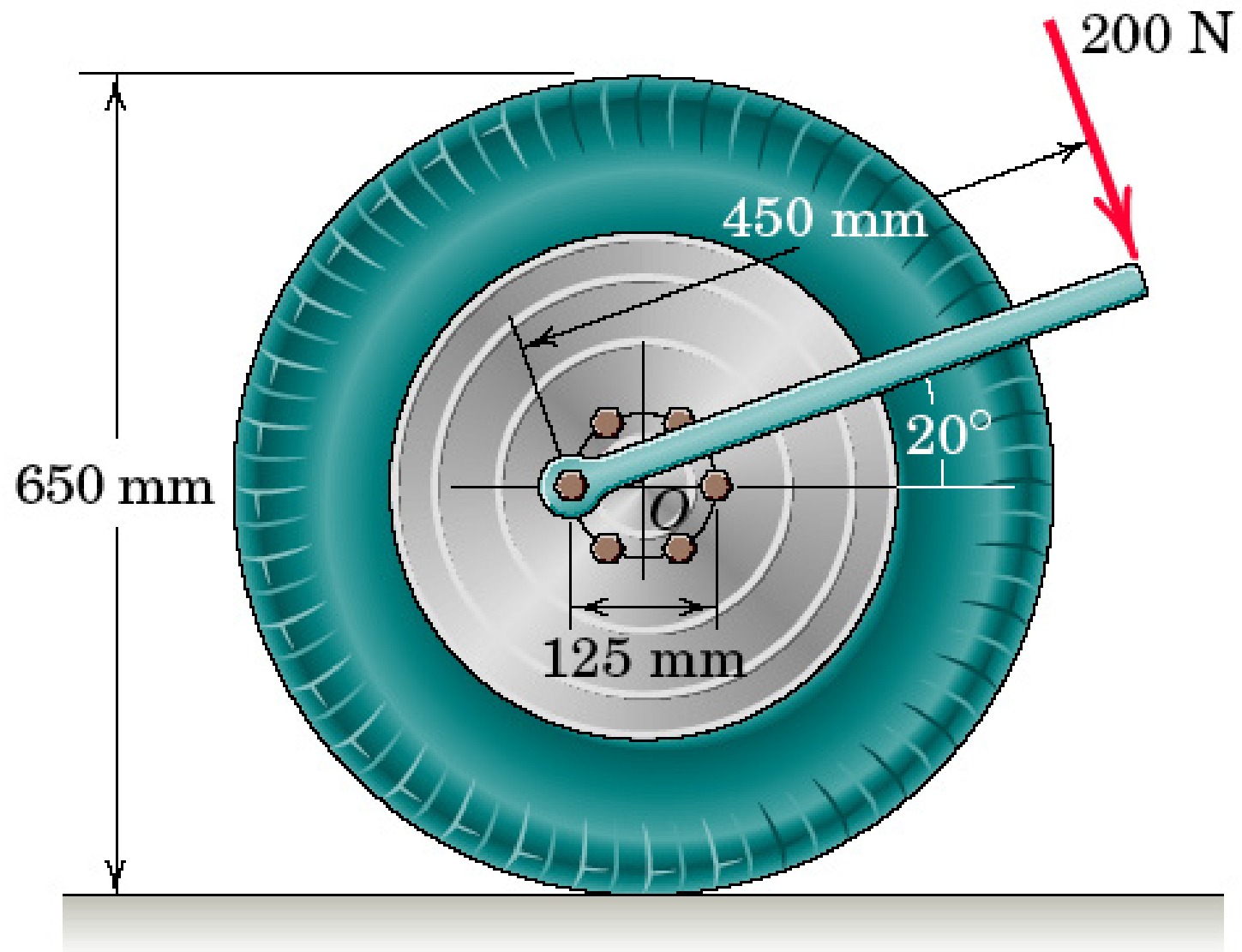
Problem 2/39



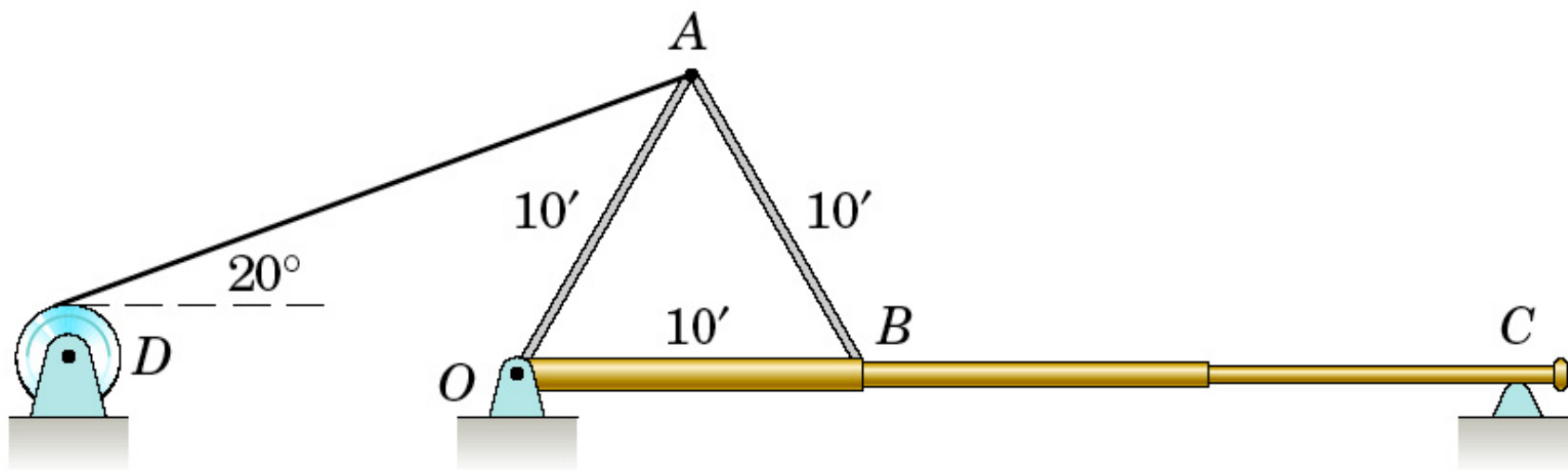
Problem 2/40



Problem 2/41

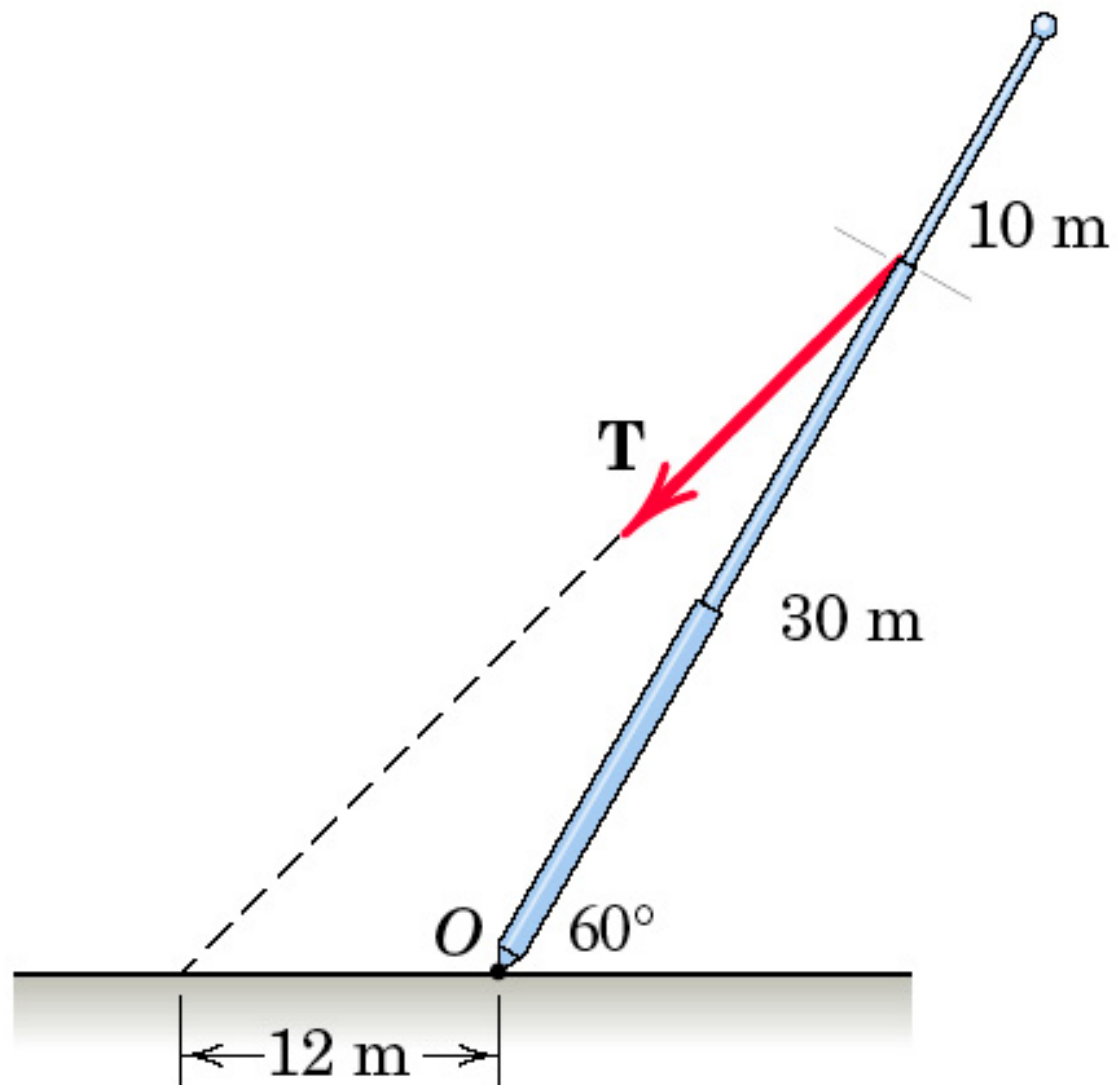


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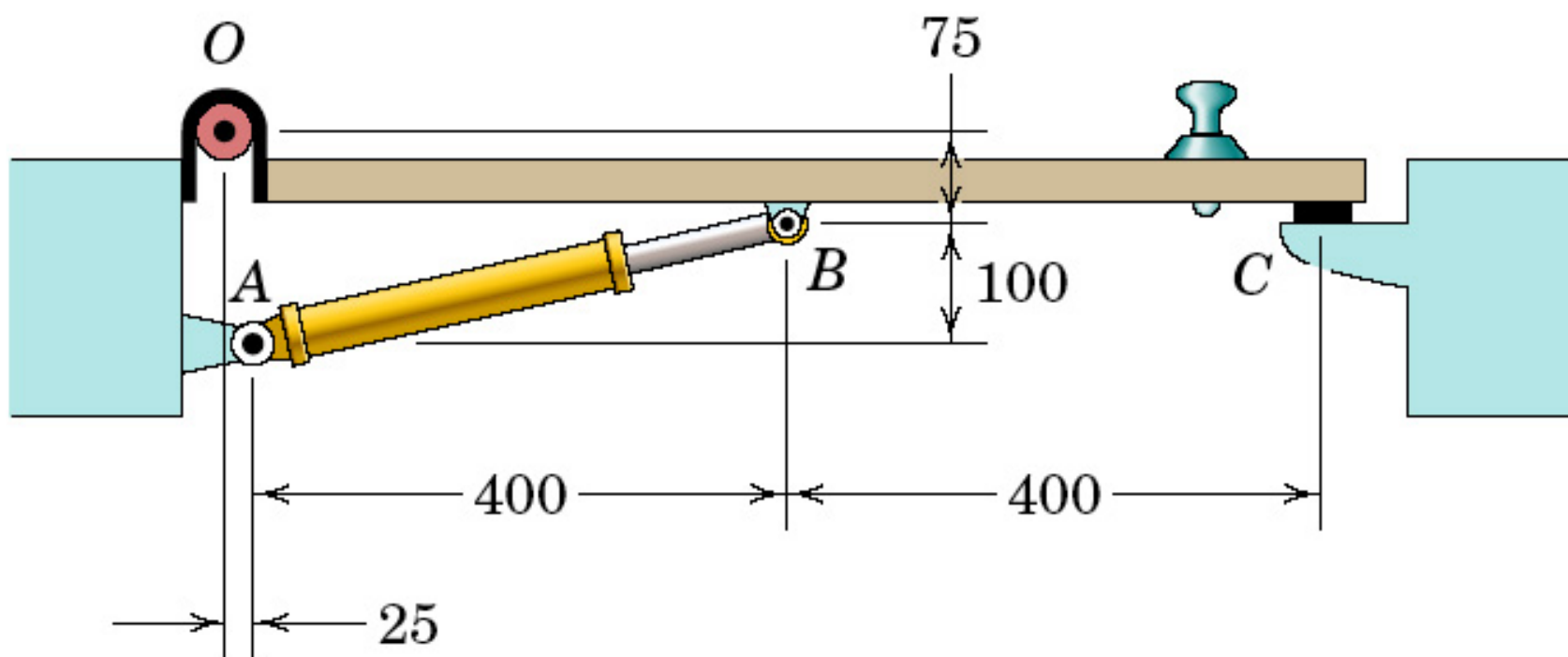


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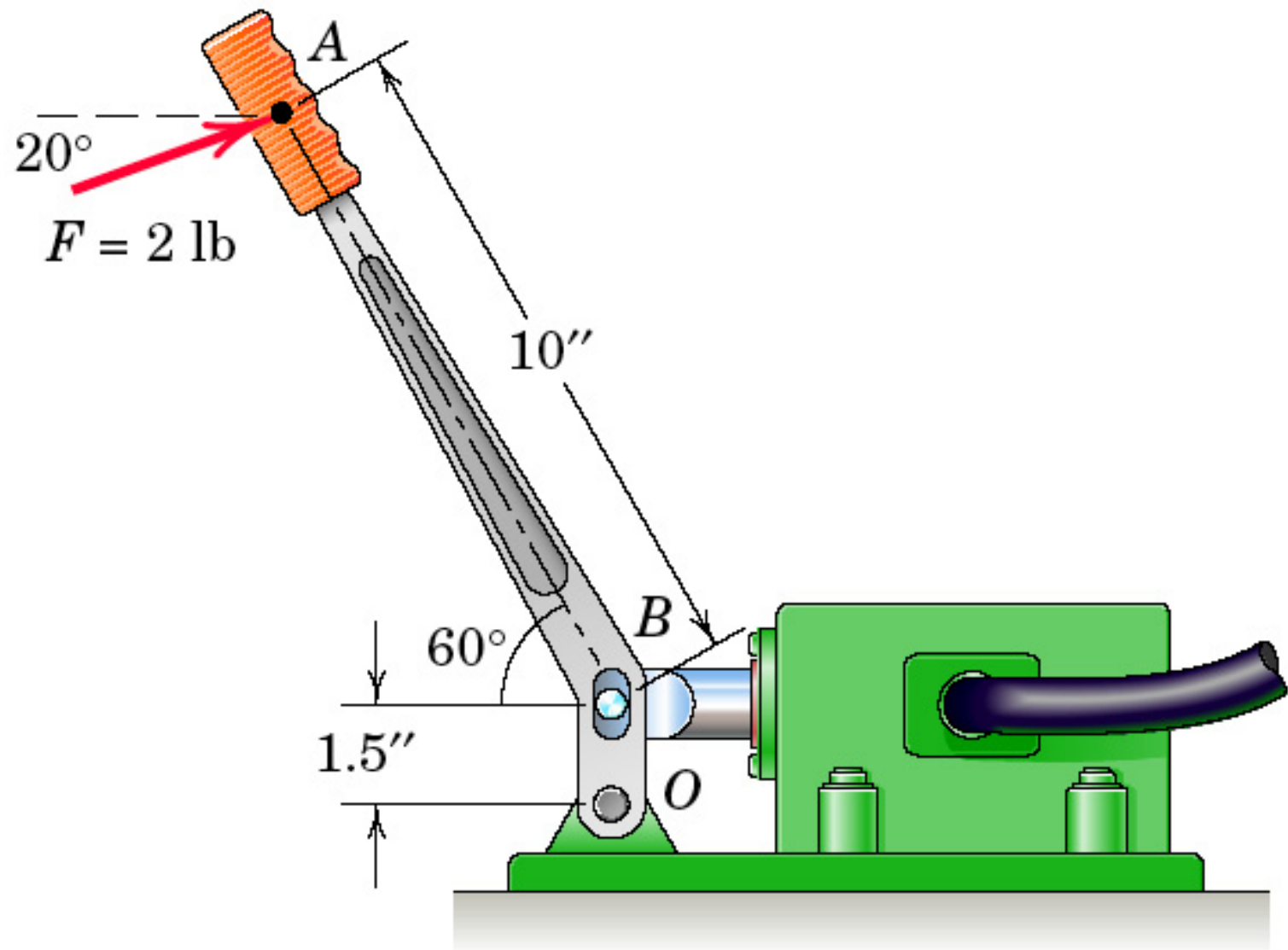
Problem 2/45



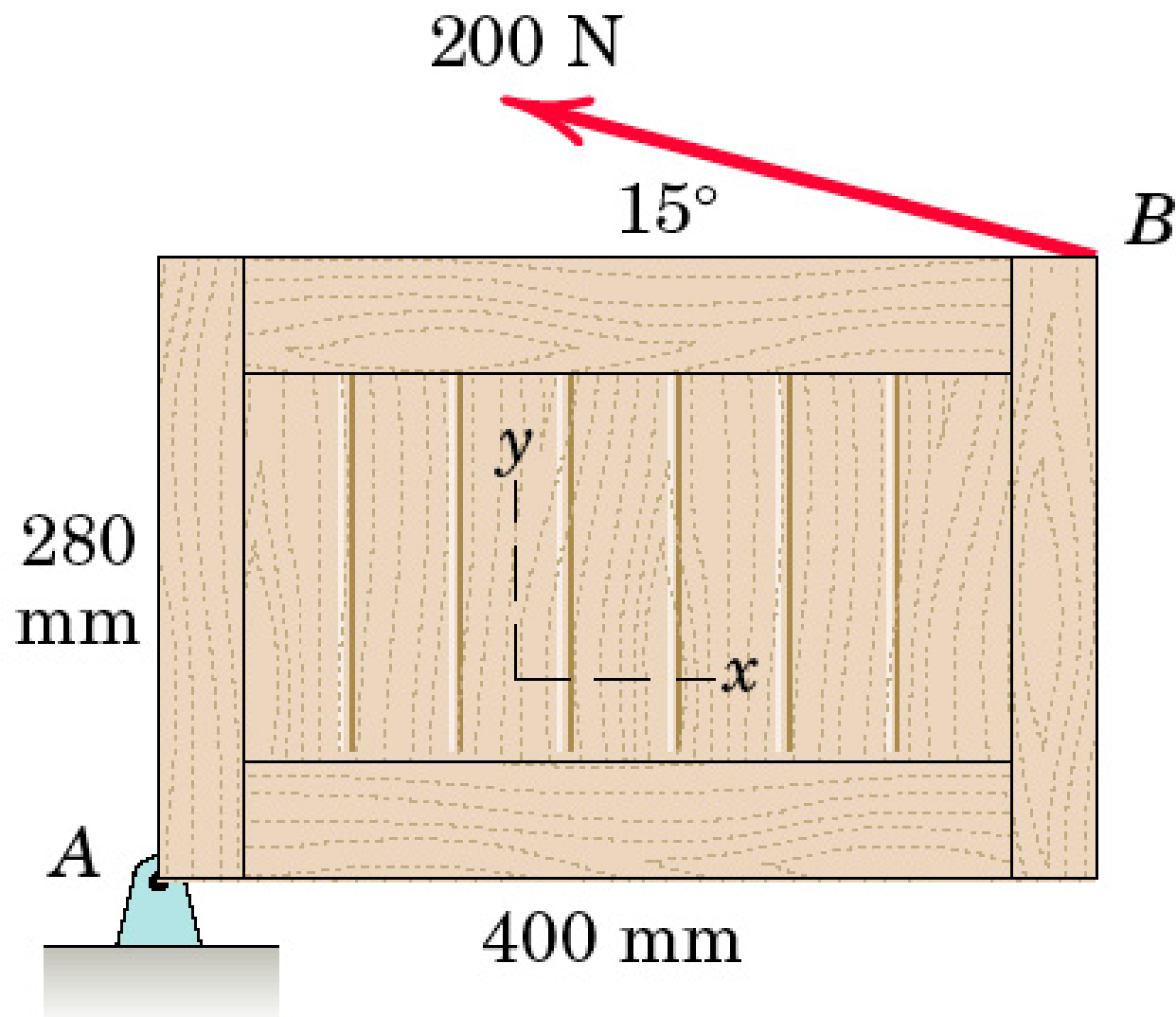
Dimensions in millimeters

Problem 2/46

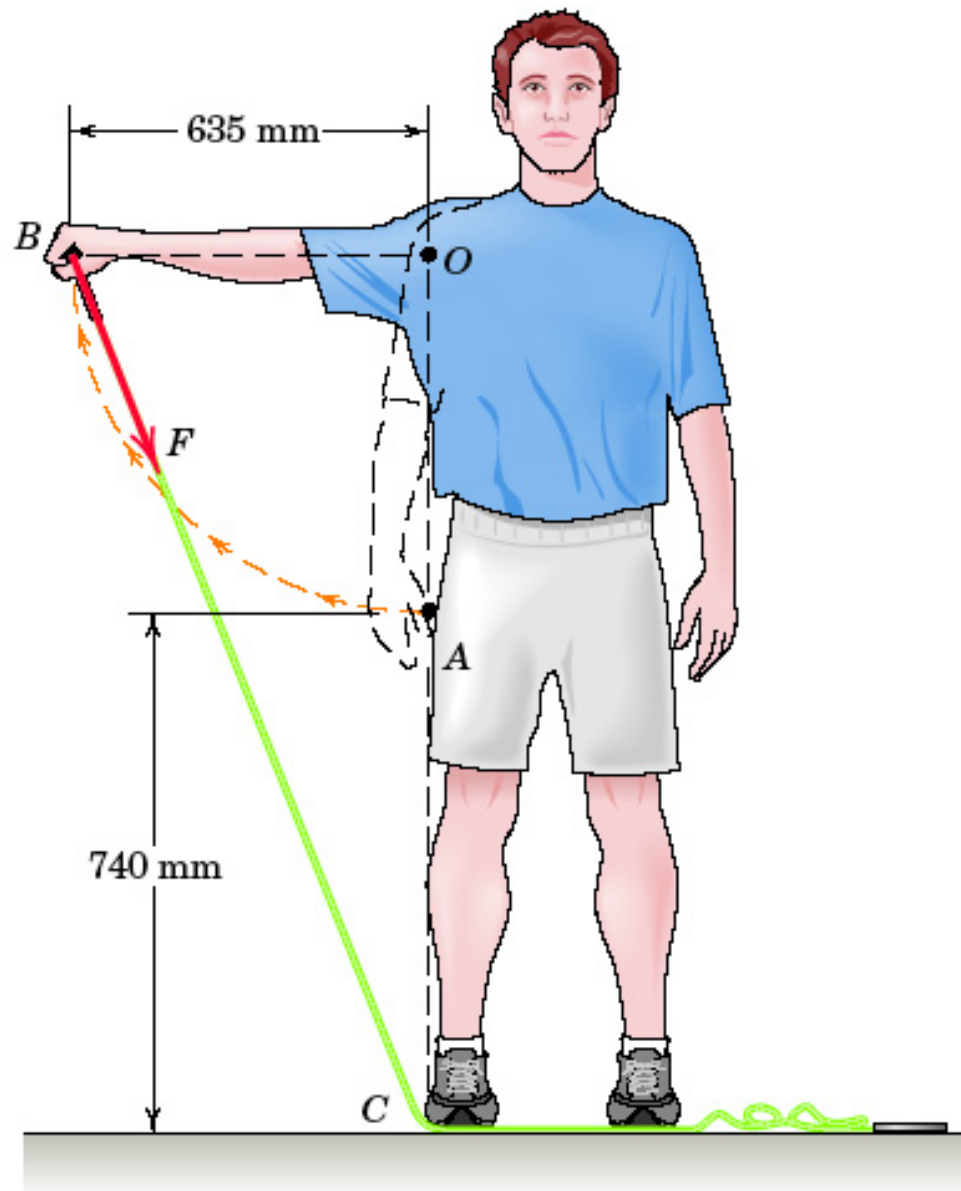
P_02_46



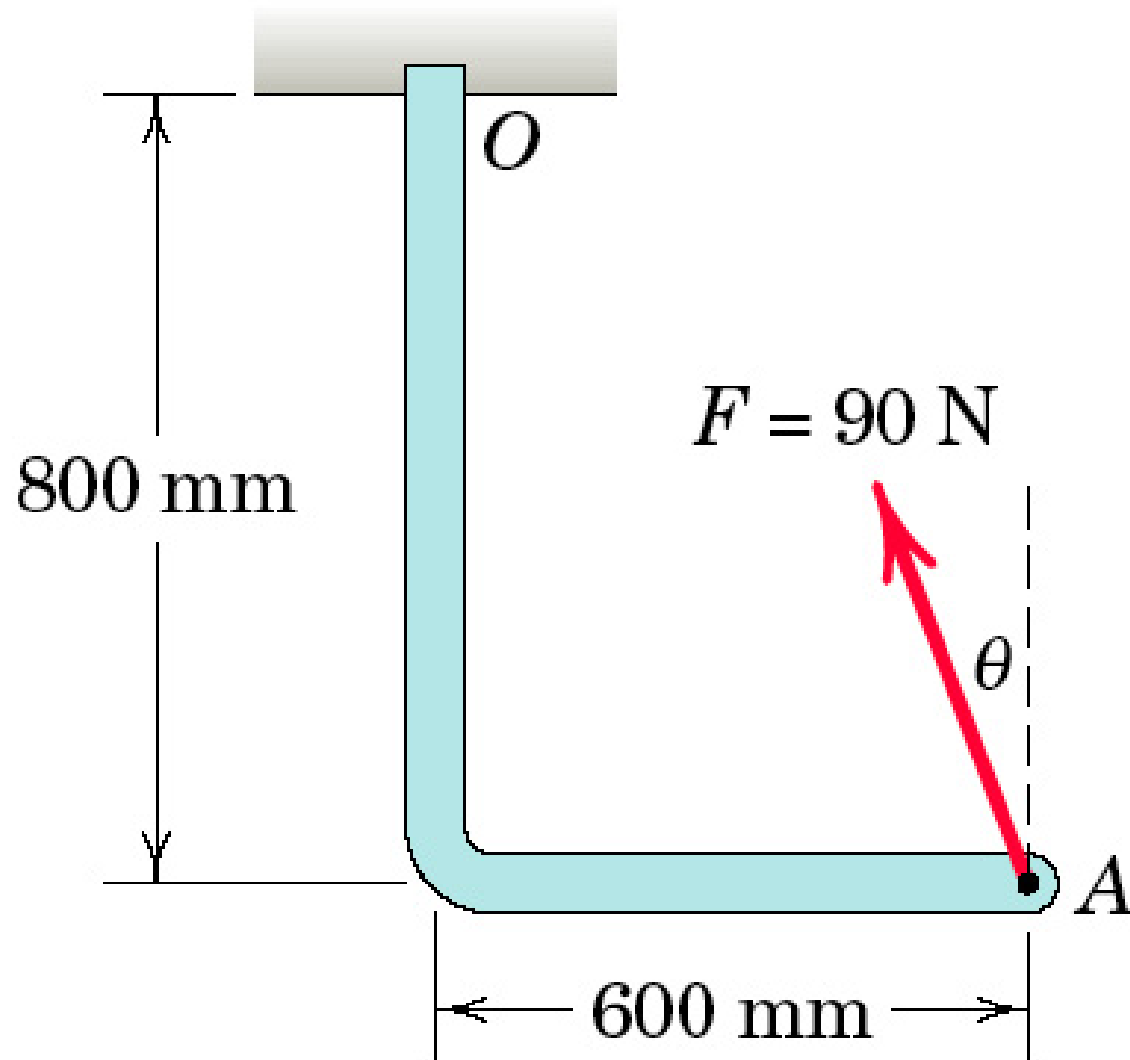
Problem 2/47



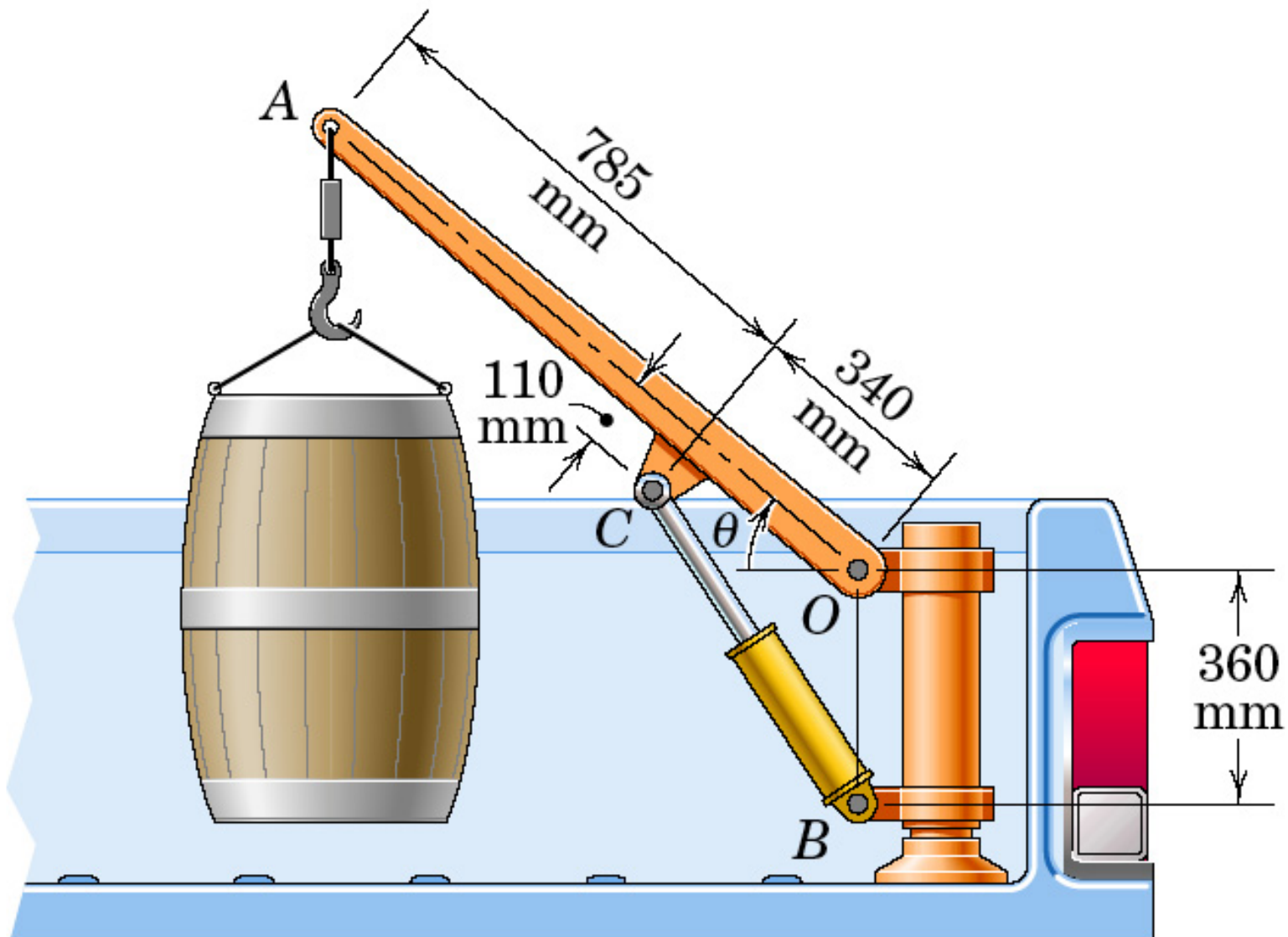
Problem 2/48



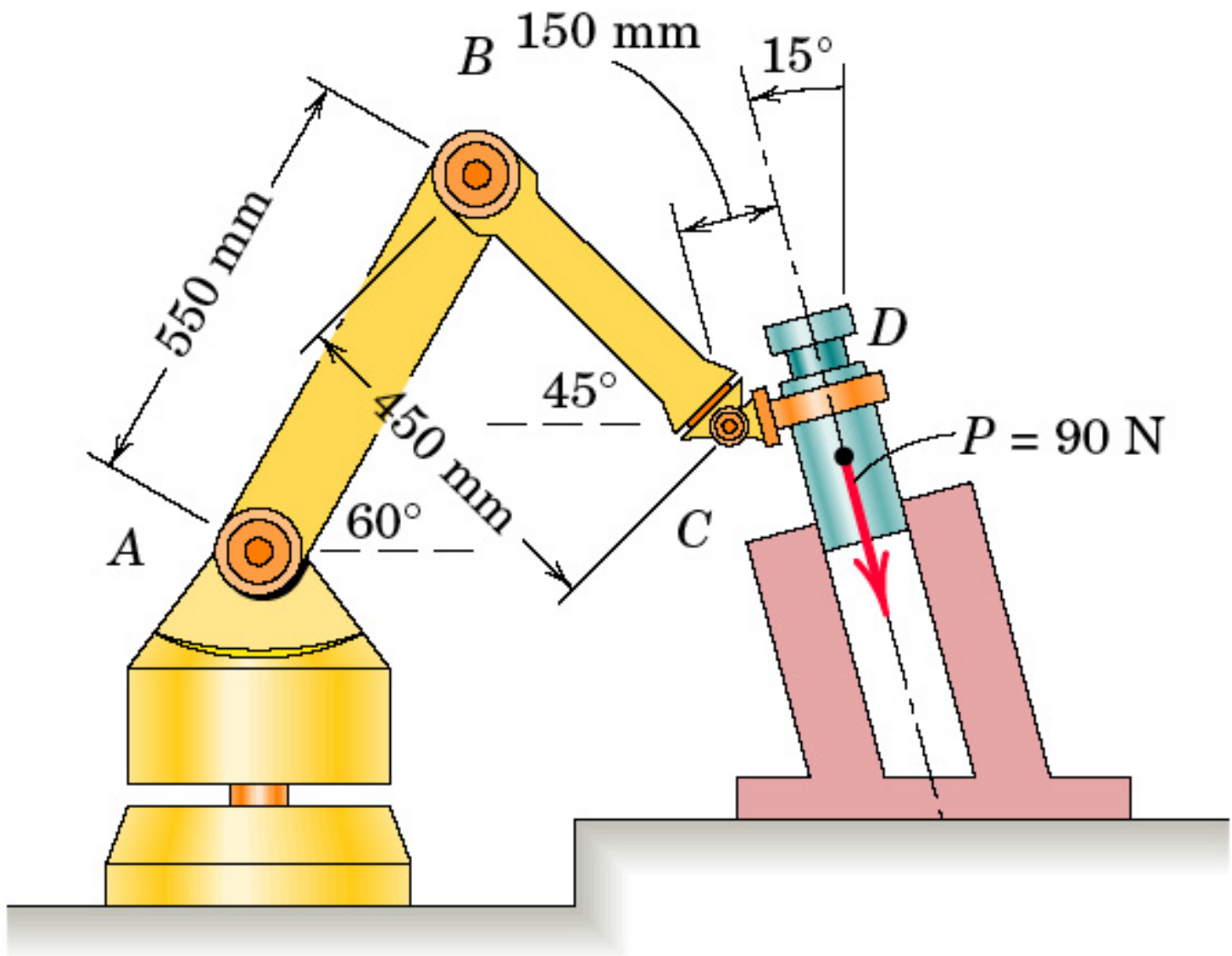
Problem 2/49



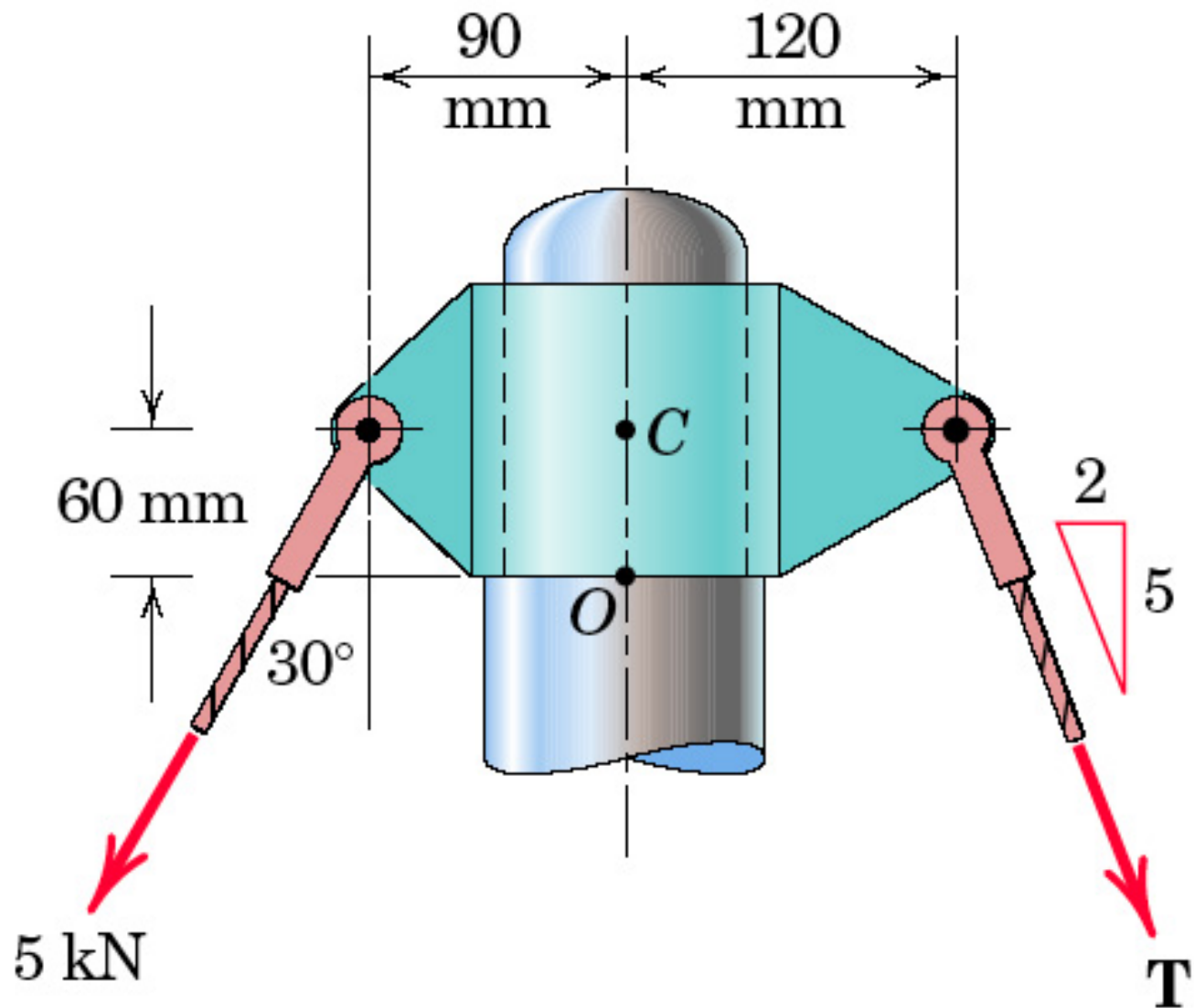
Problem 2/50



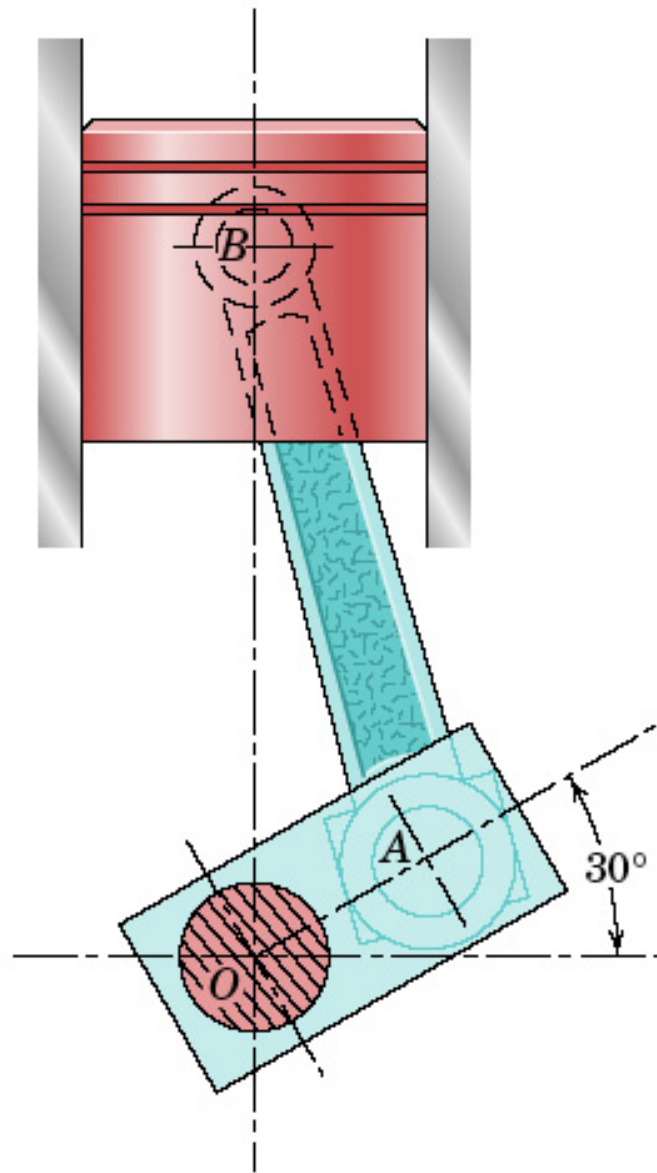
Problem 2/51



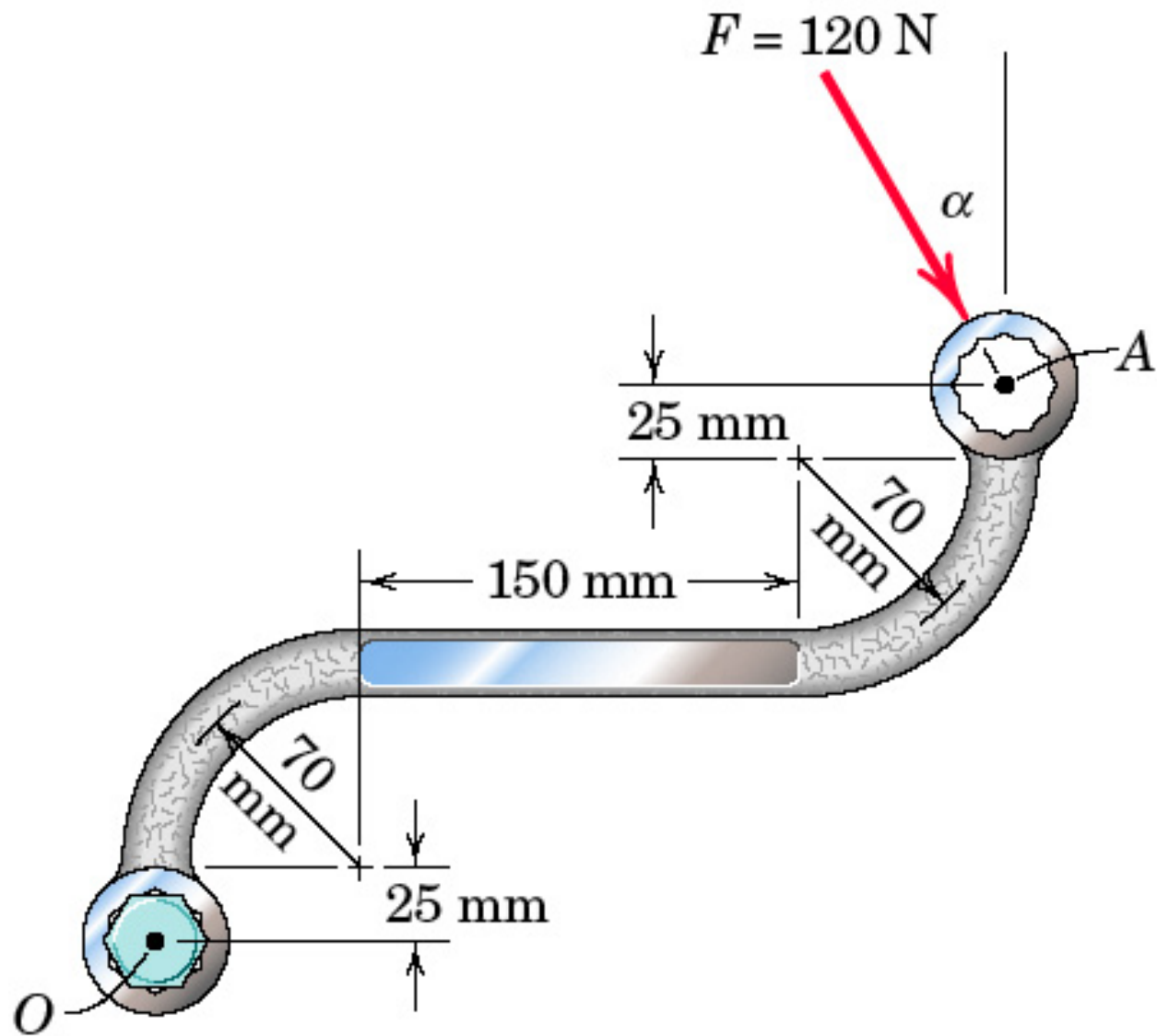
Problem 2/52



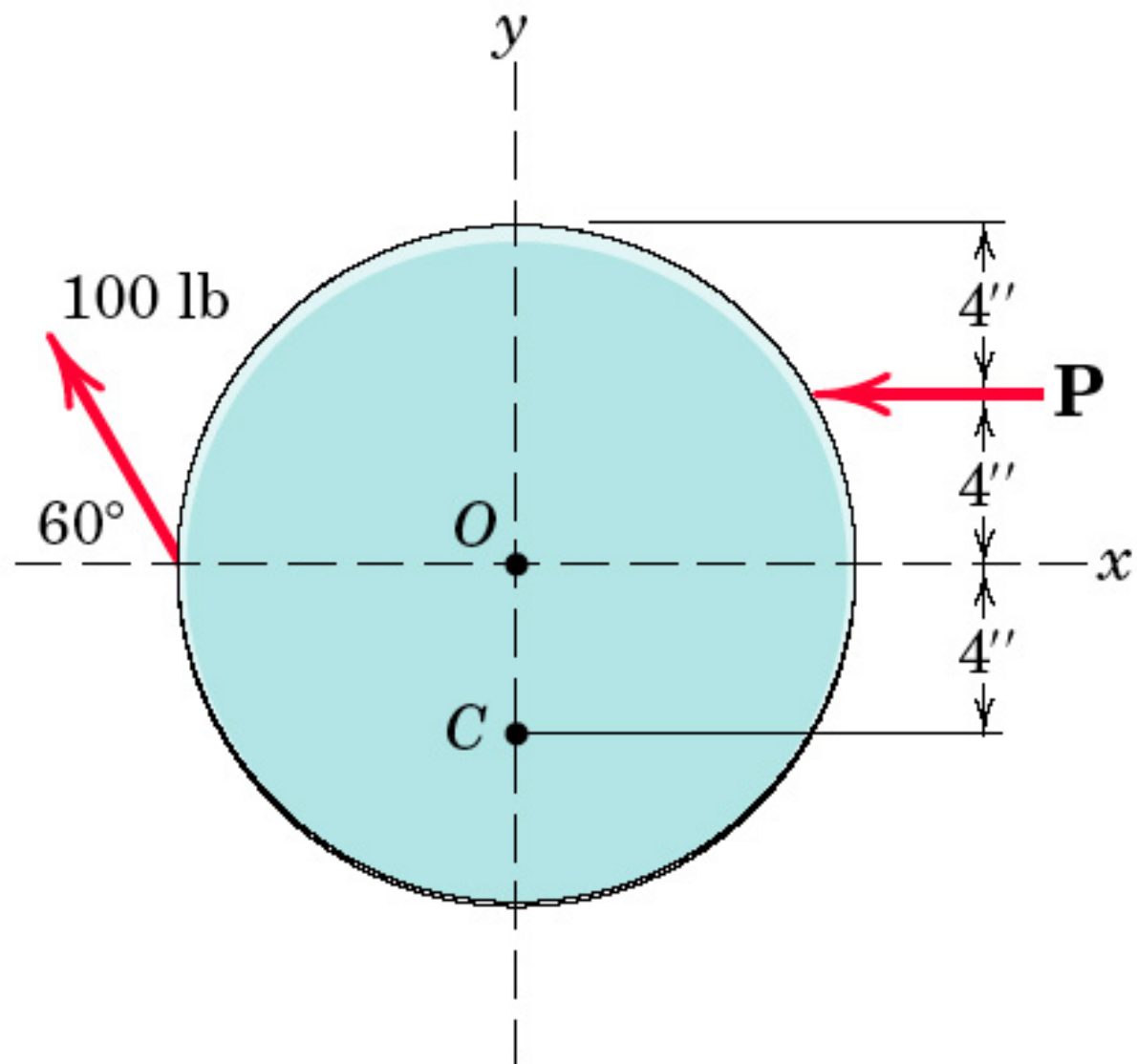
Problem 2/53



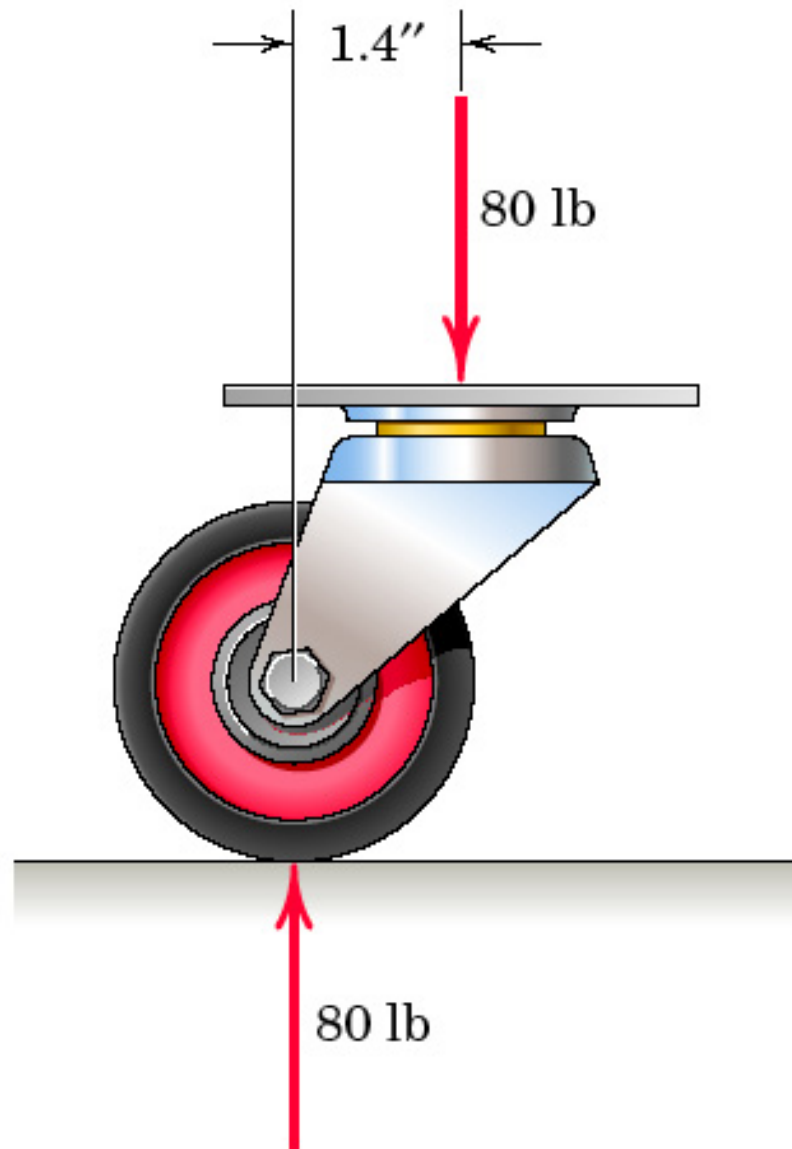
Problem 2/54



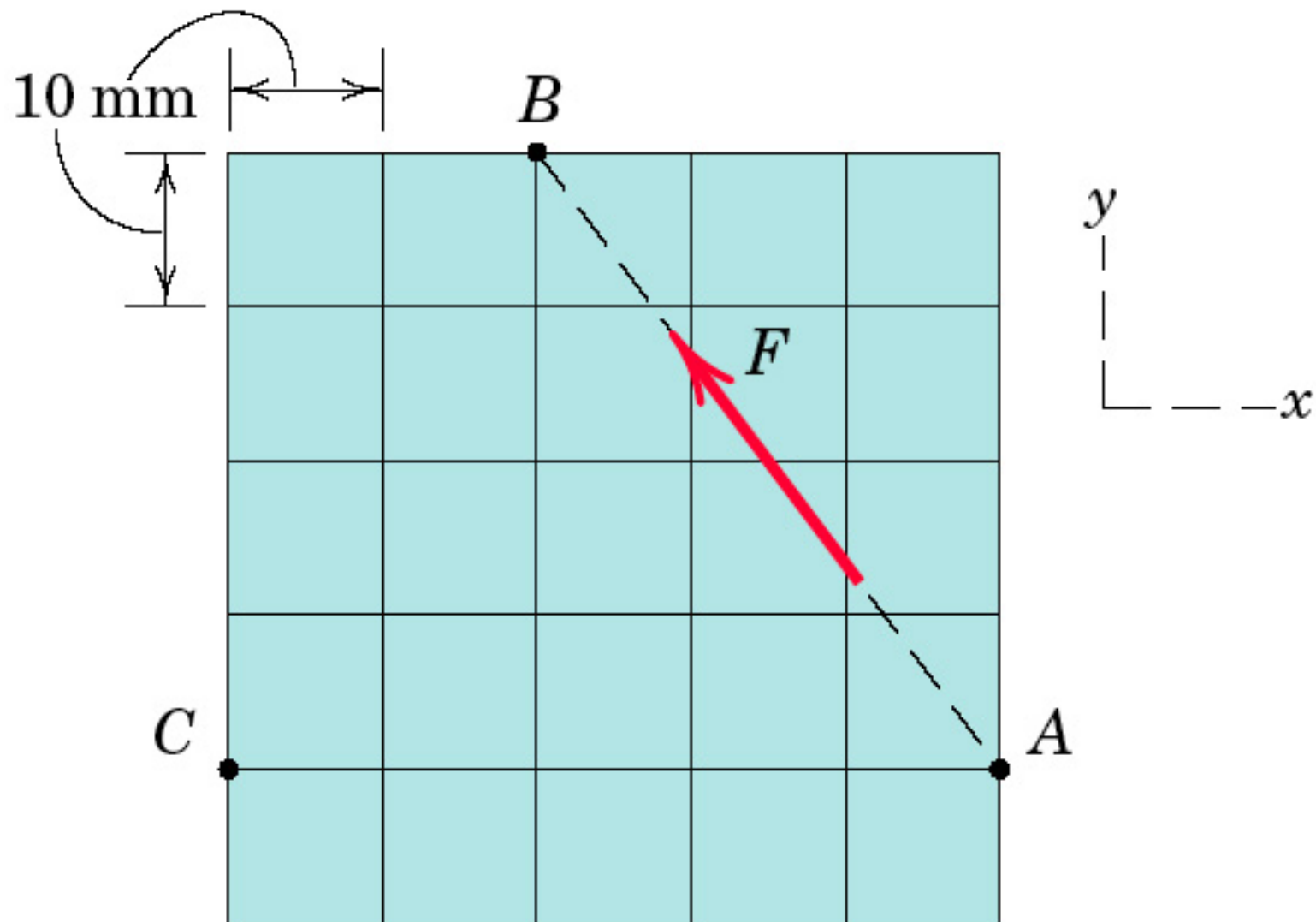
Problem 2/55



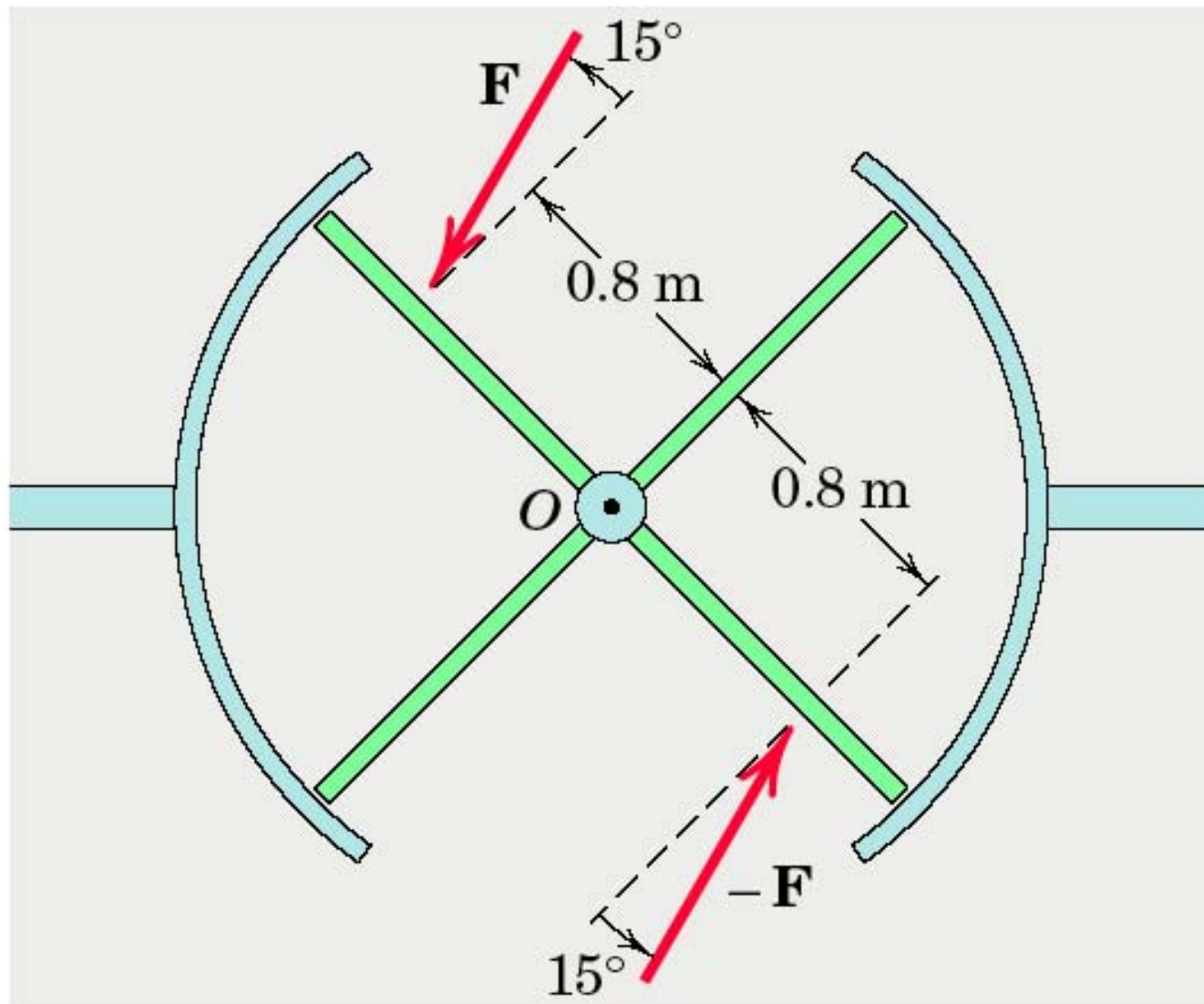
Problem 2/56



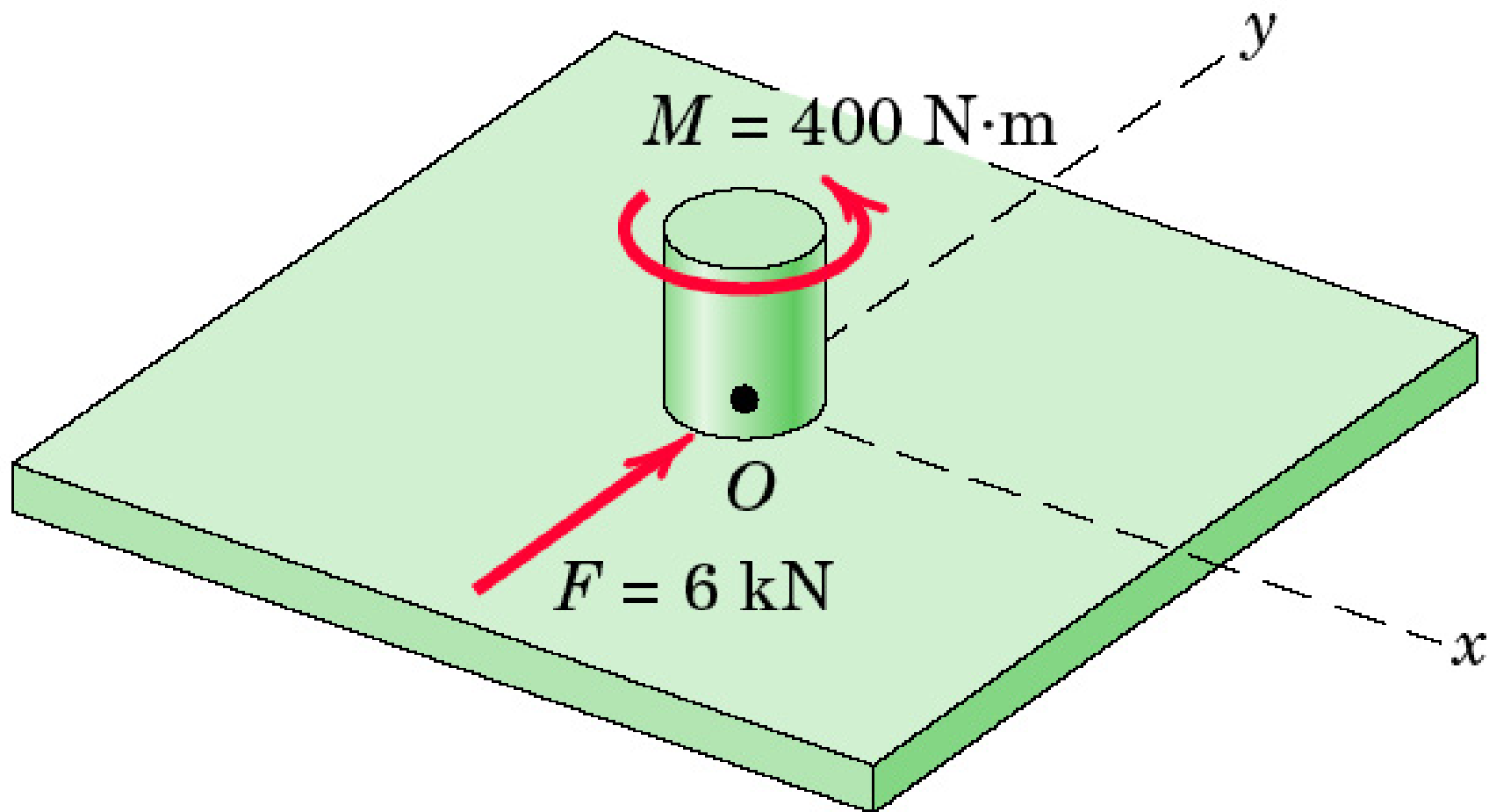
Problem 2/57



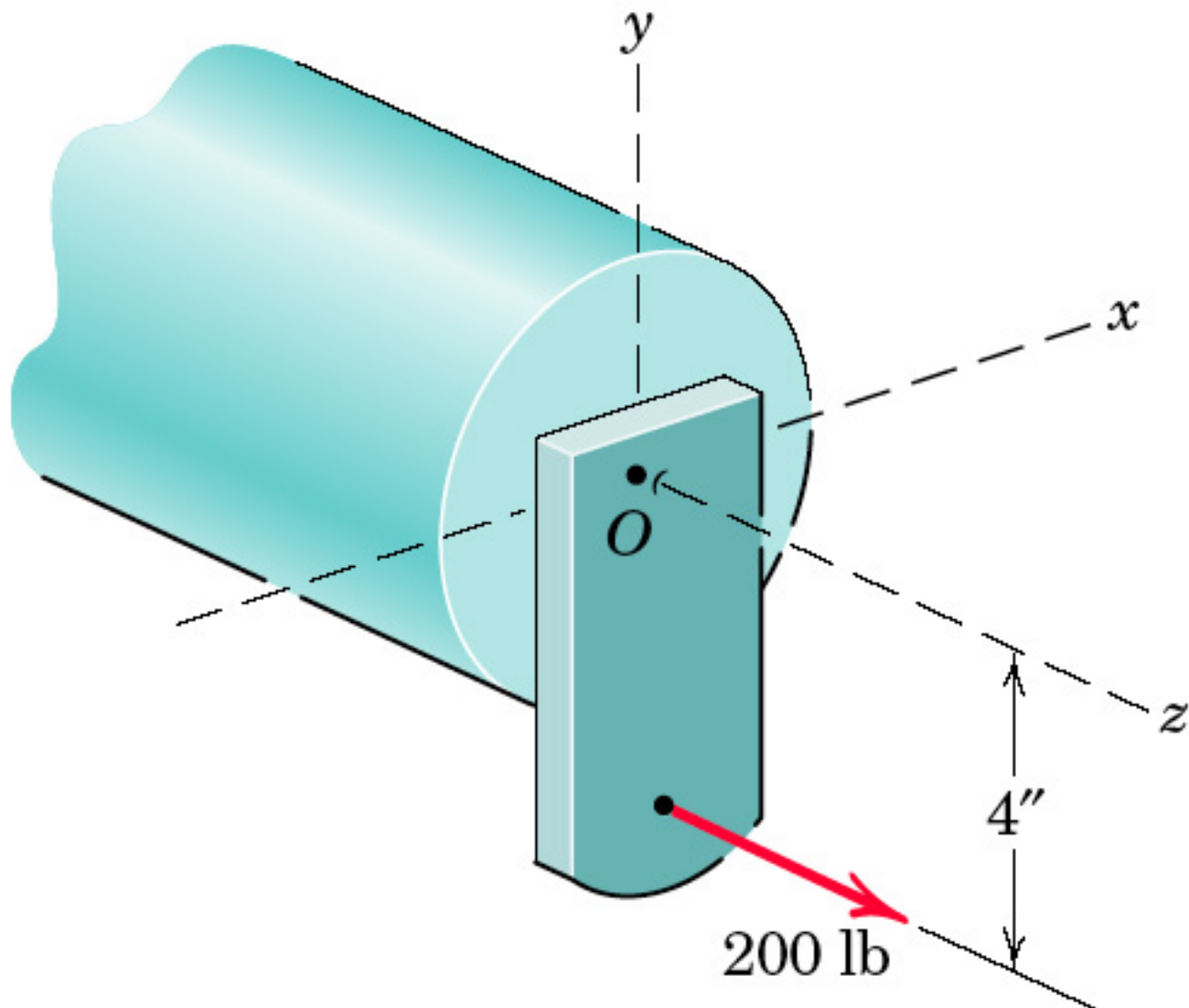
Problem 2/58



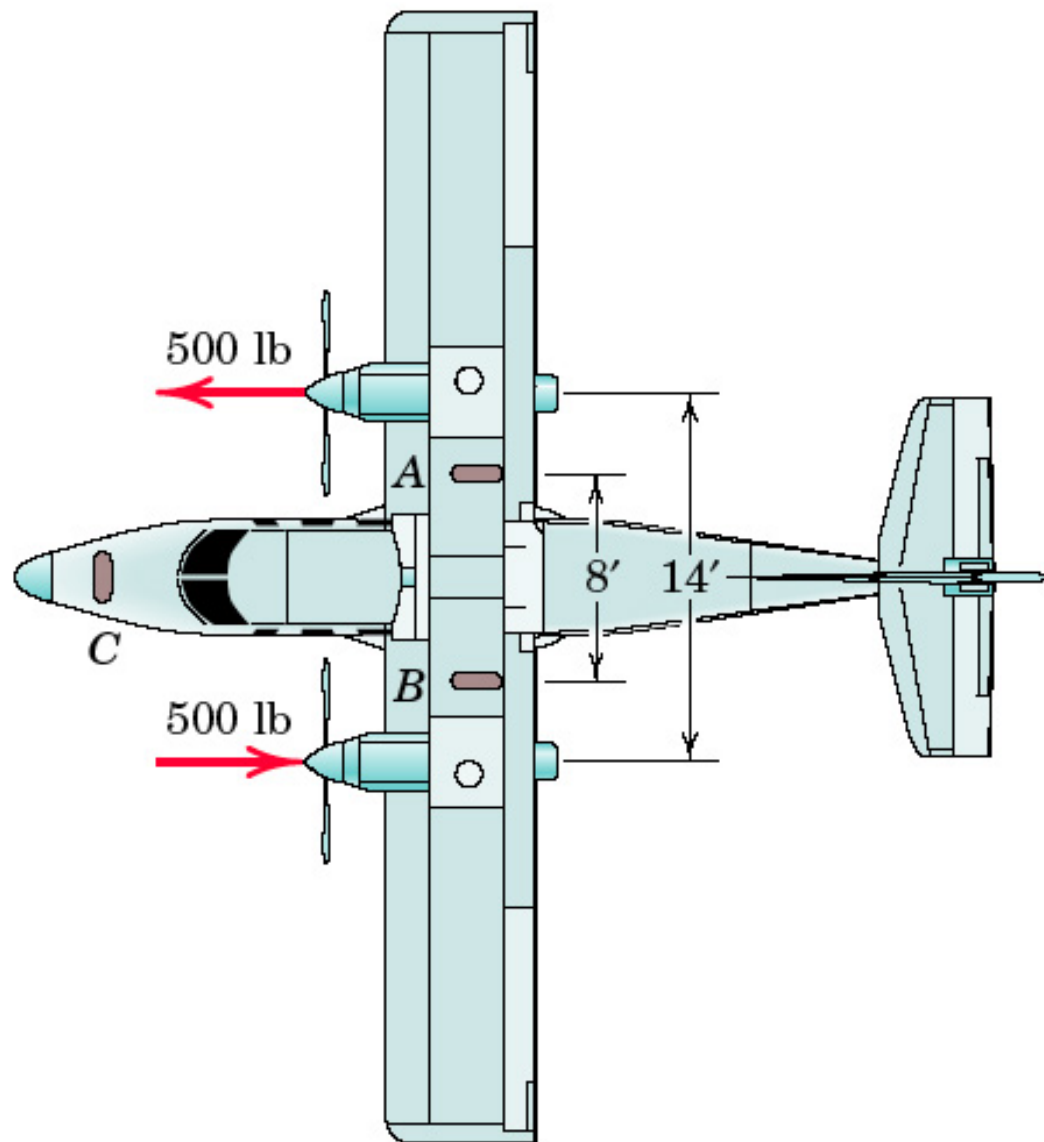
Problem 2/59



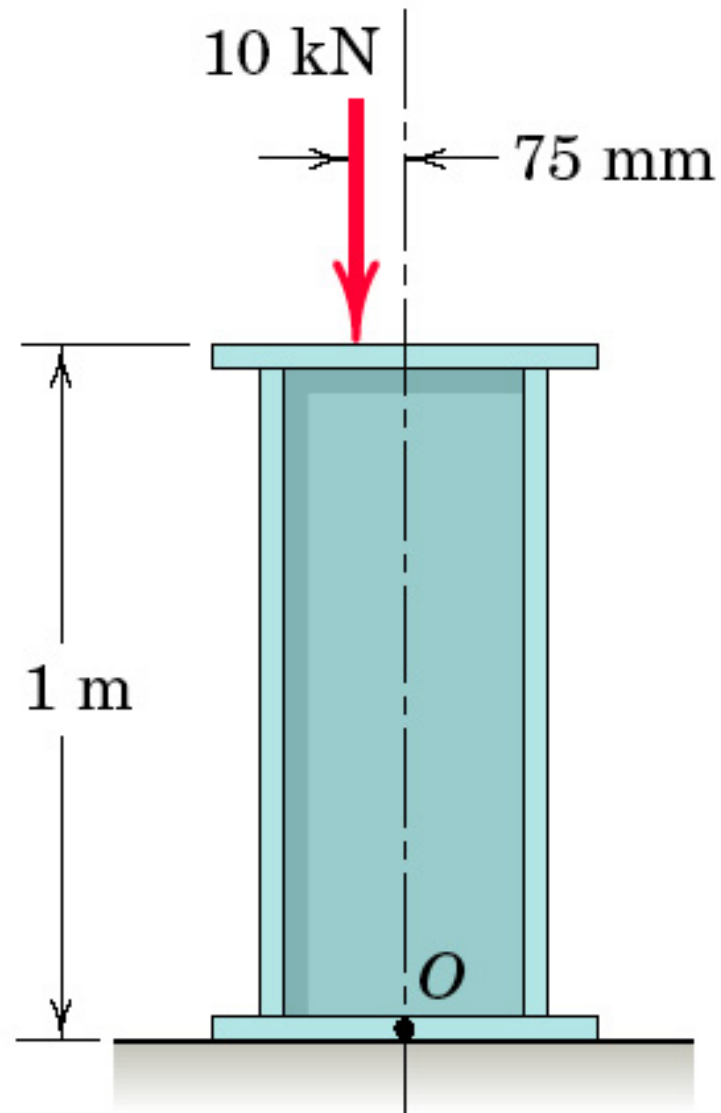
Problem 2/60



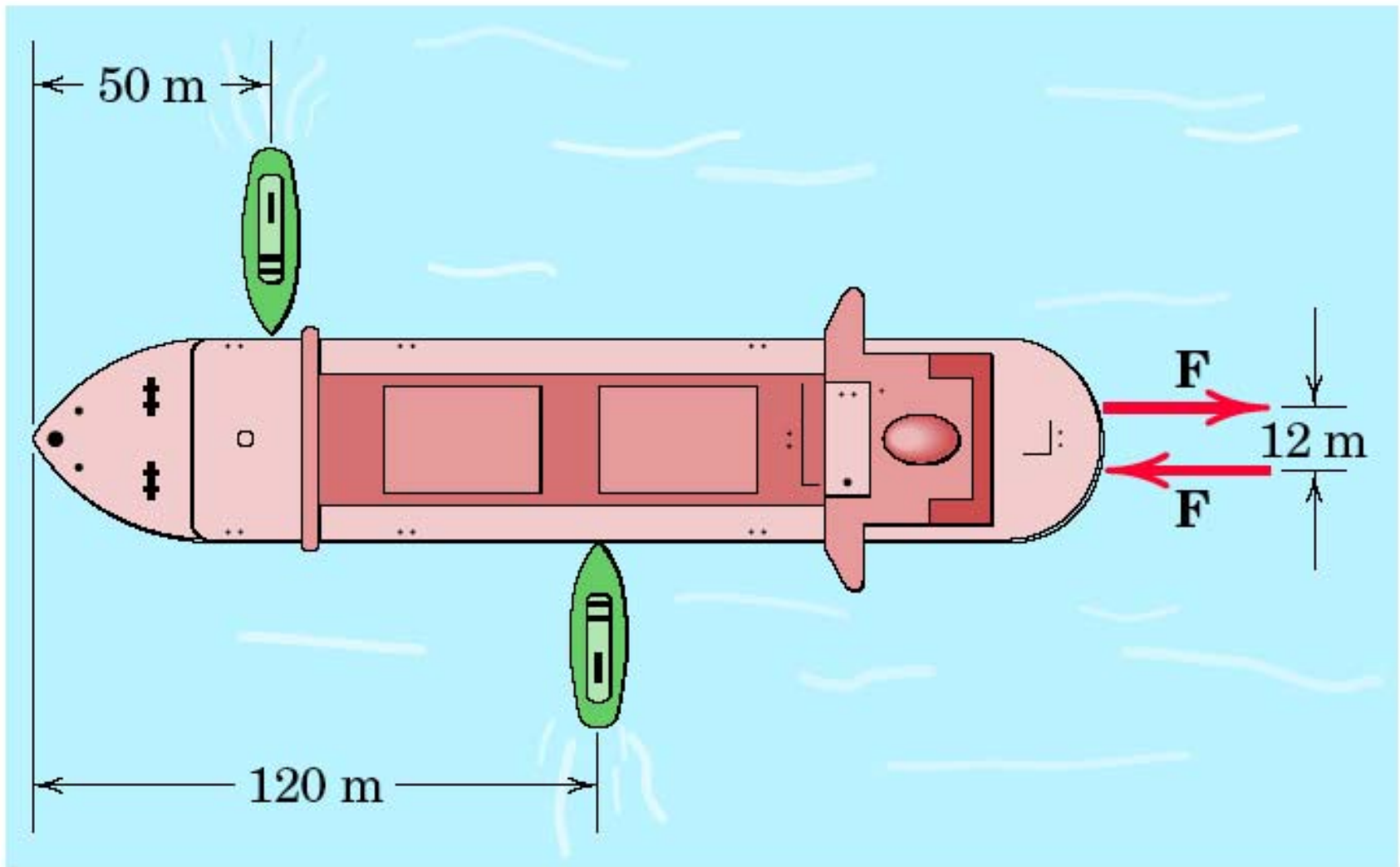
Problem 2/61



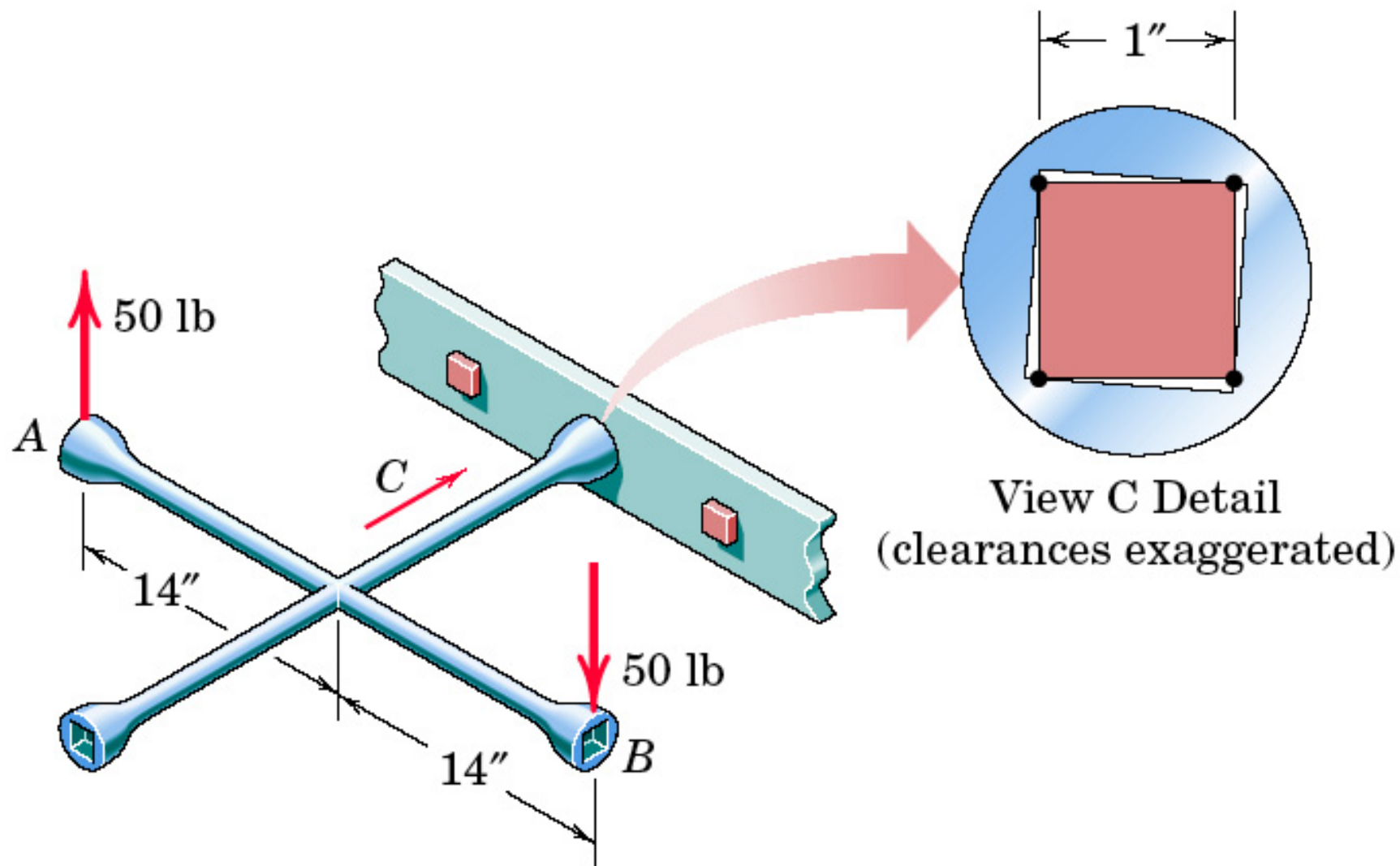
Problem 2/62



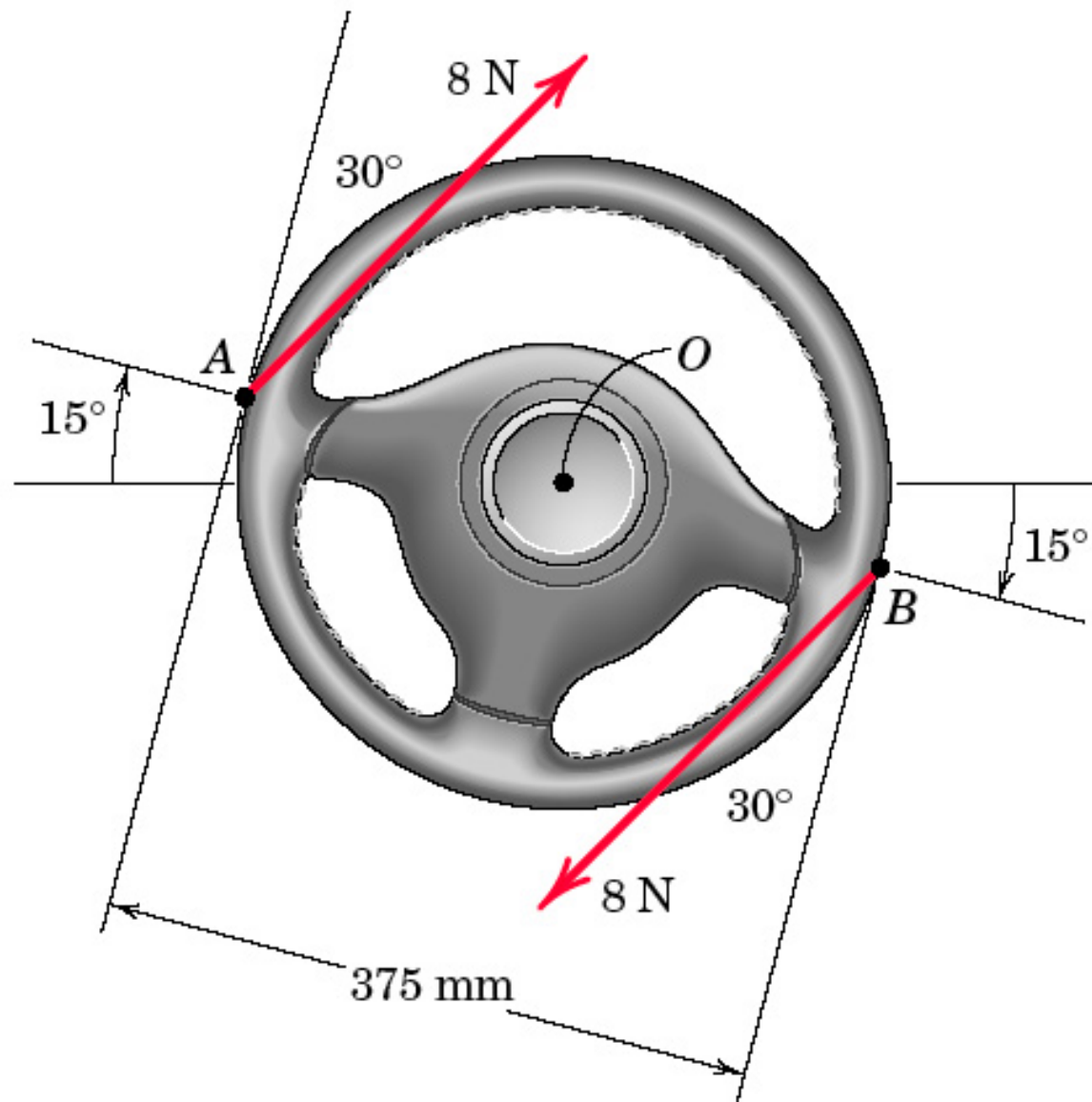
Problem 2/63



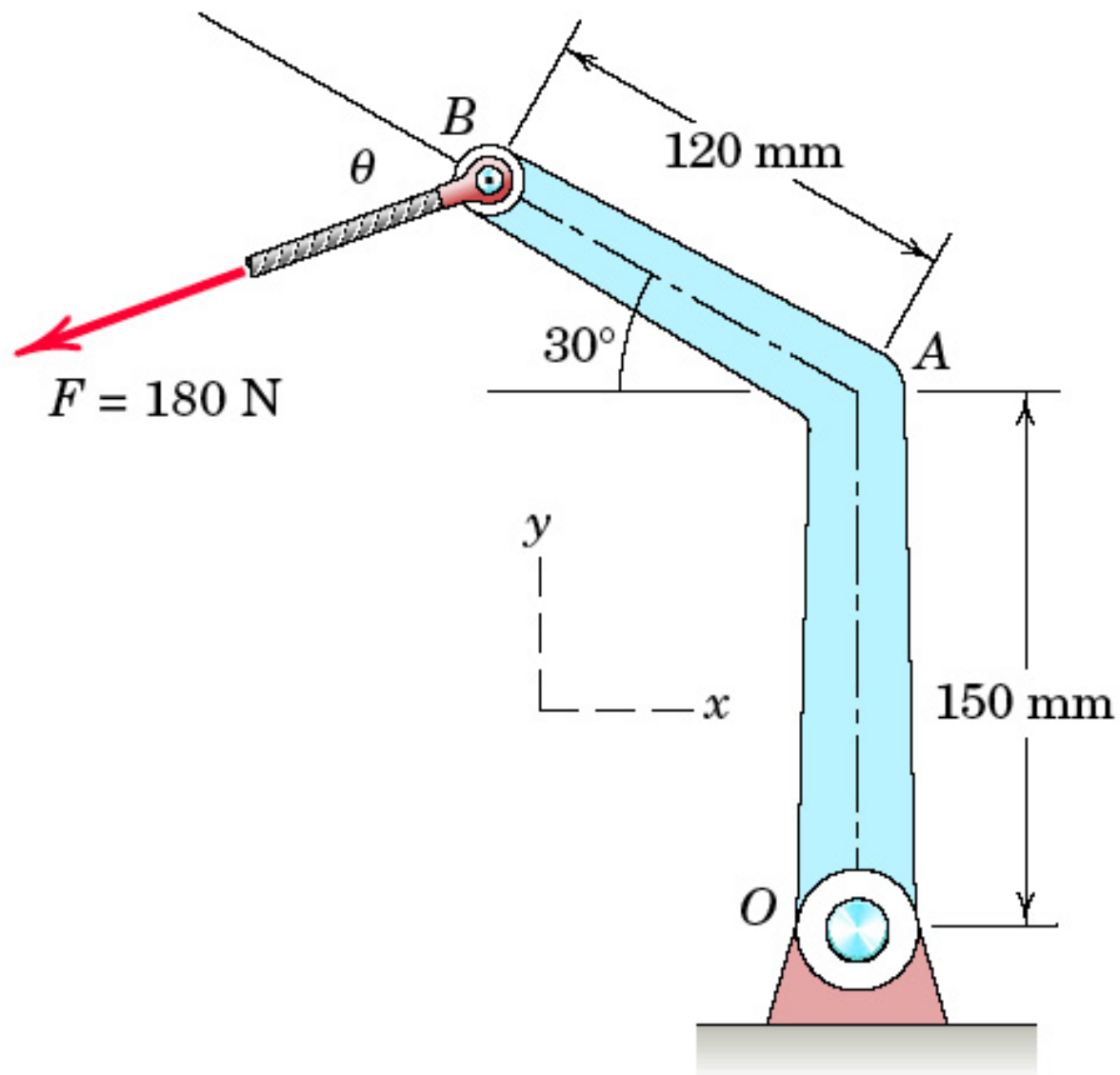
Problem 2/64



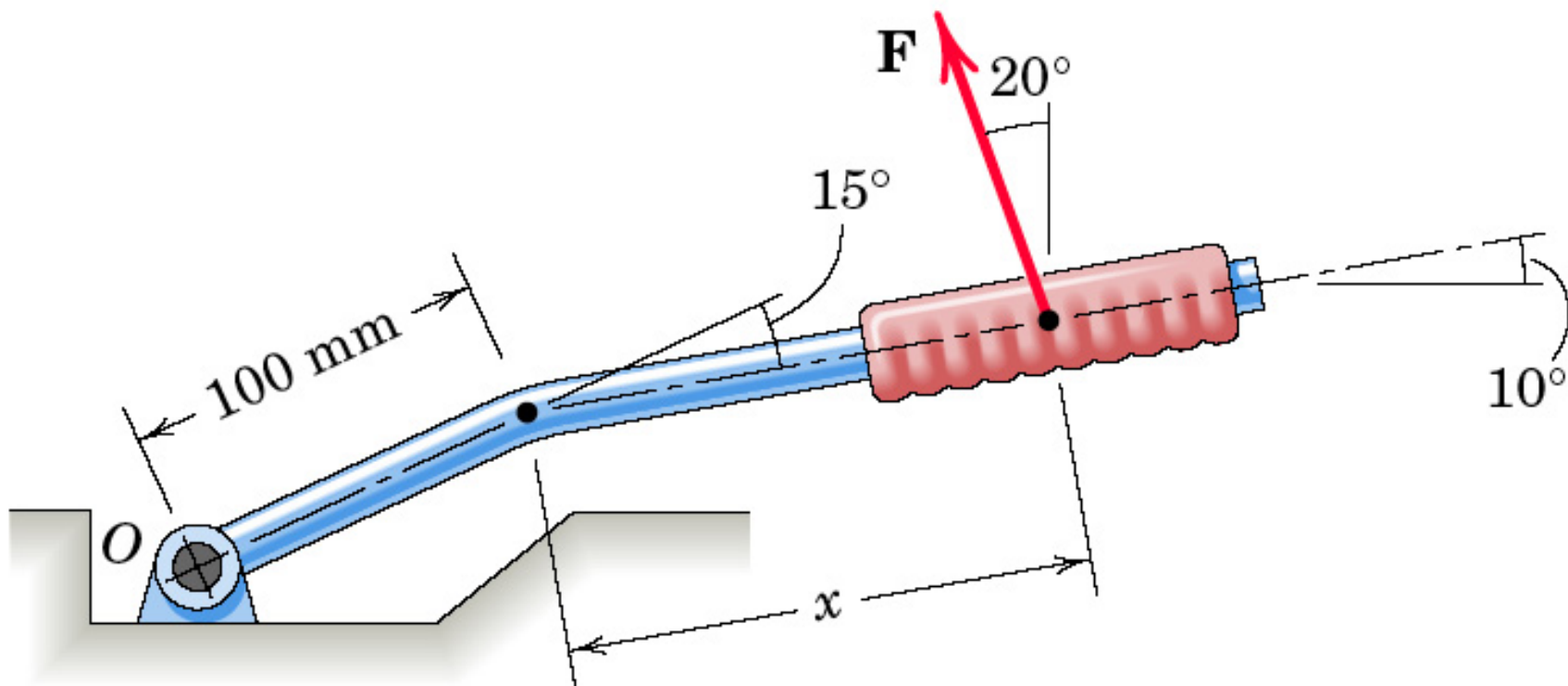
Problem 2/65



Problem 2/66

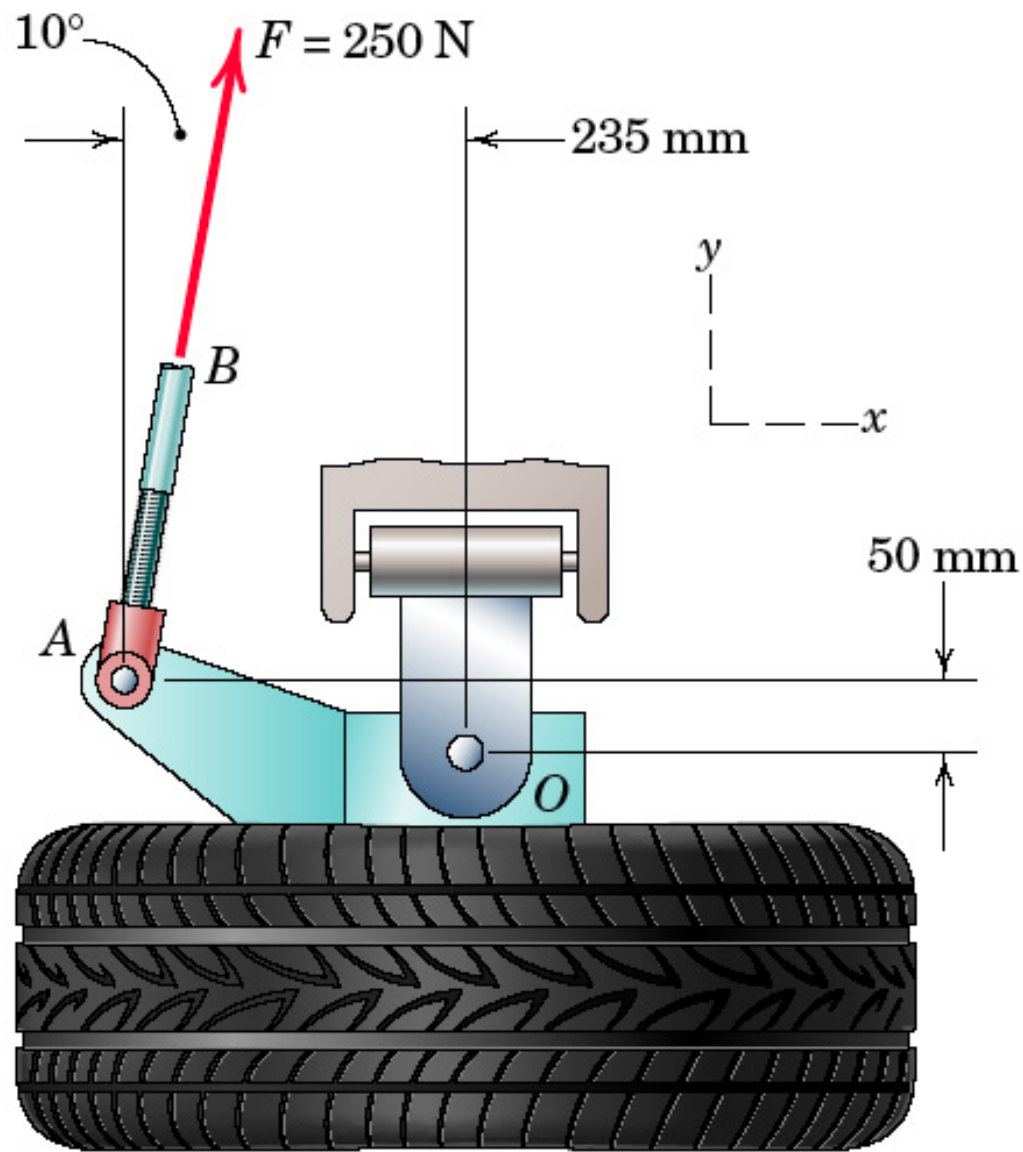


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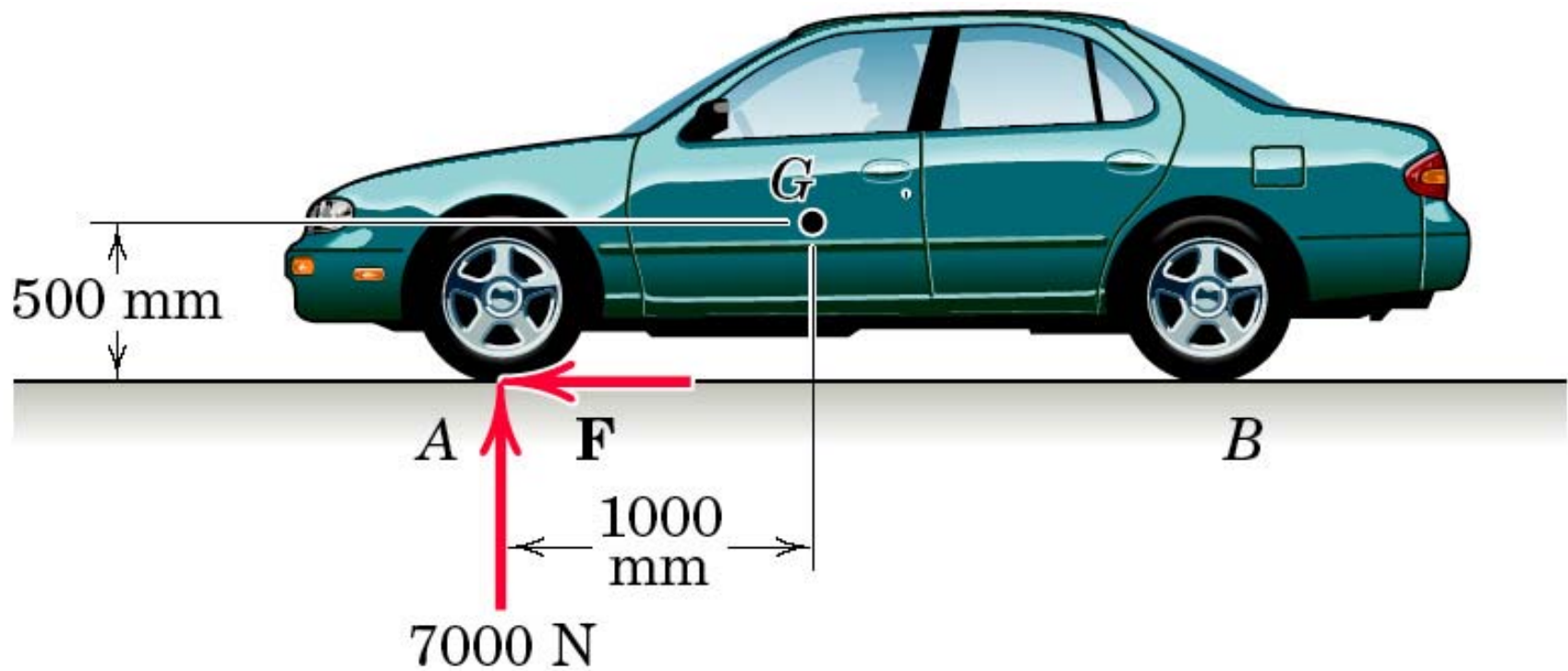


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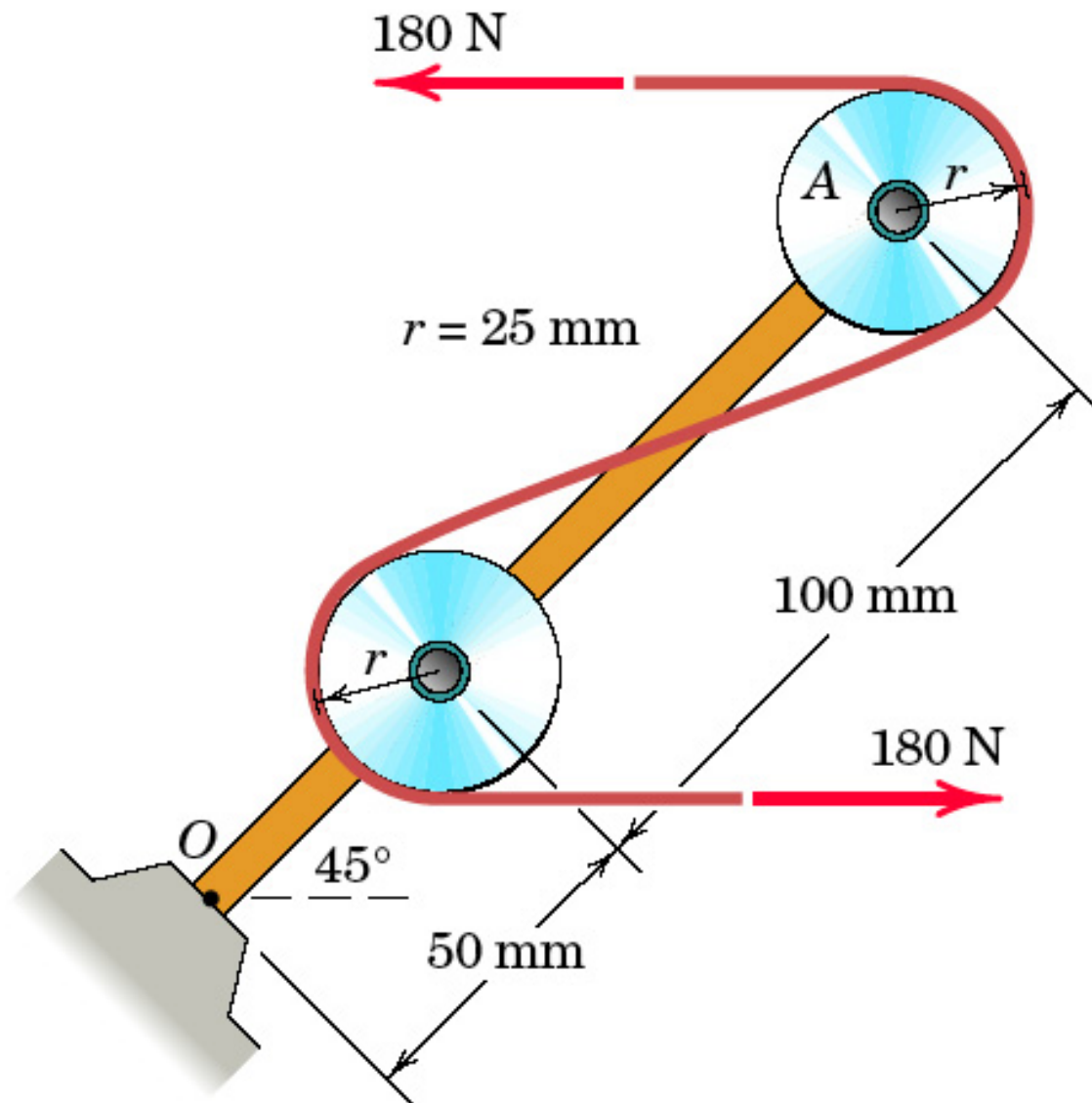


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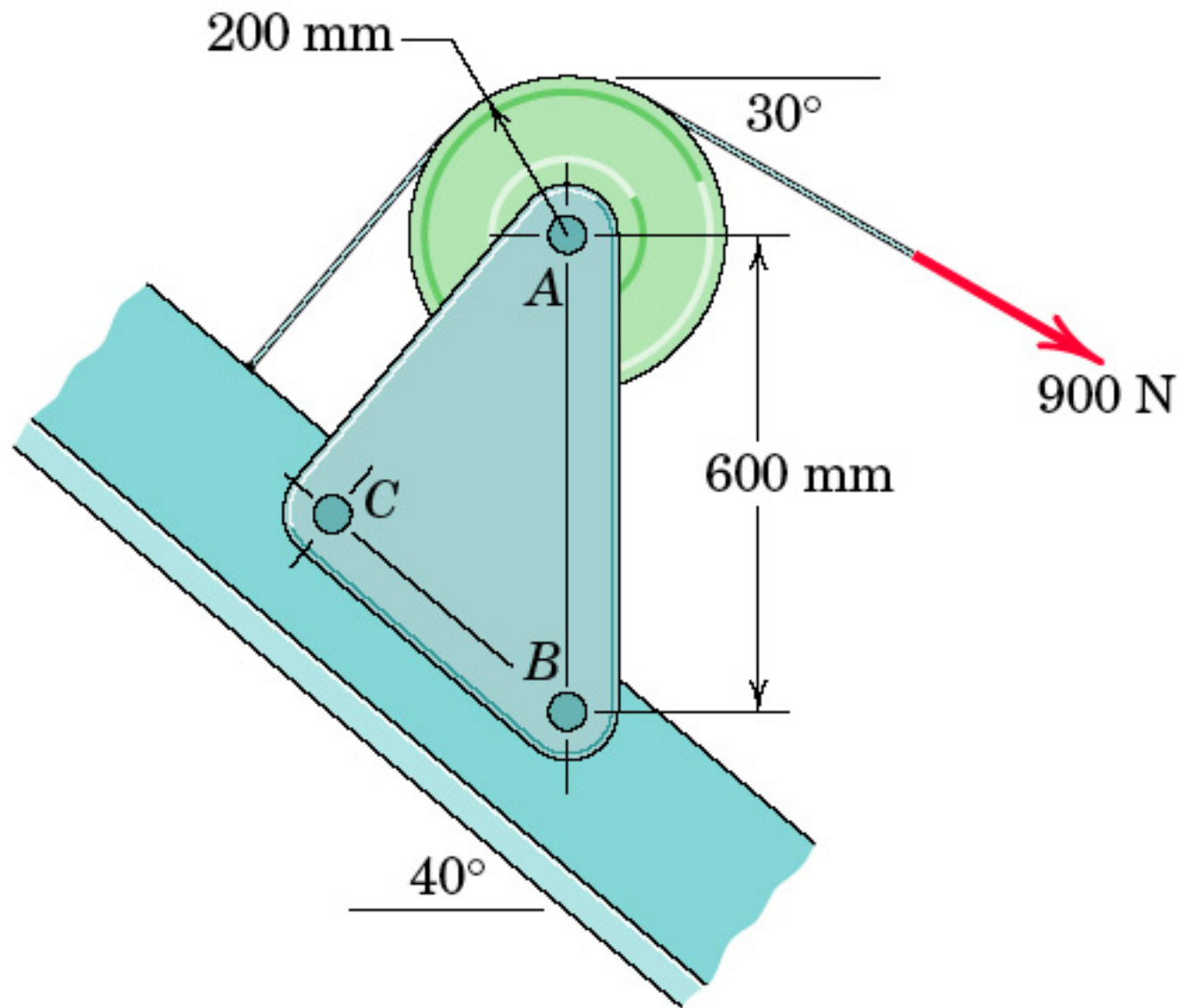


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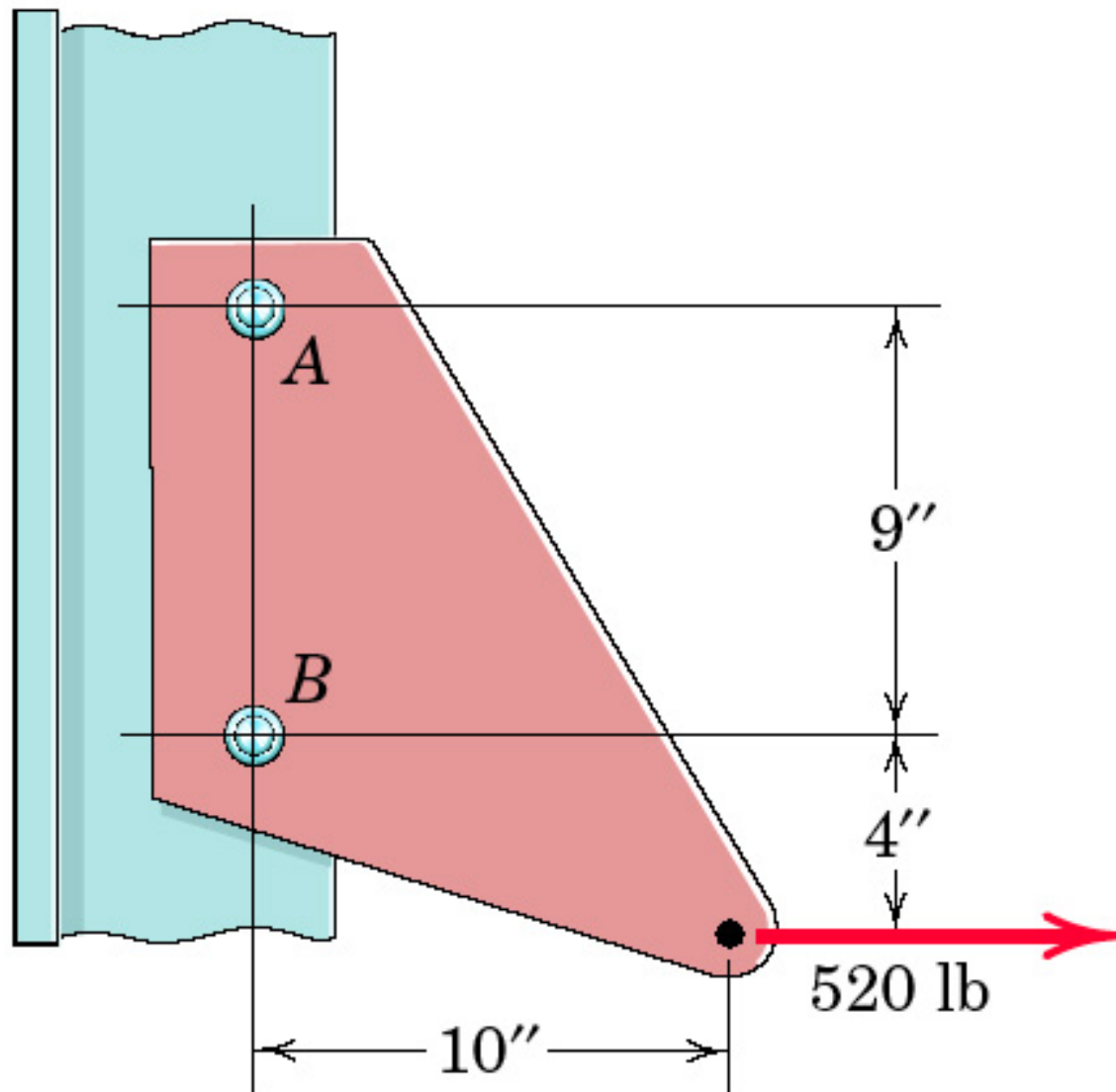
P_02_70



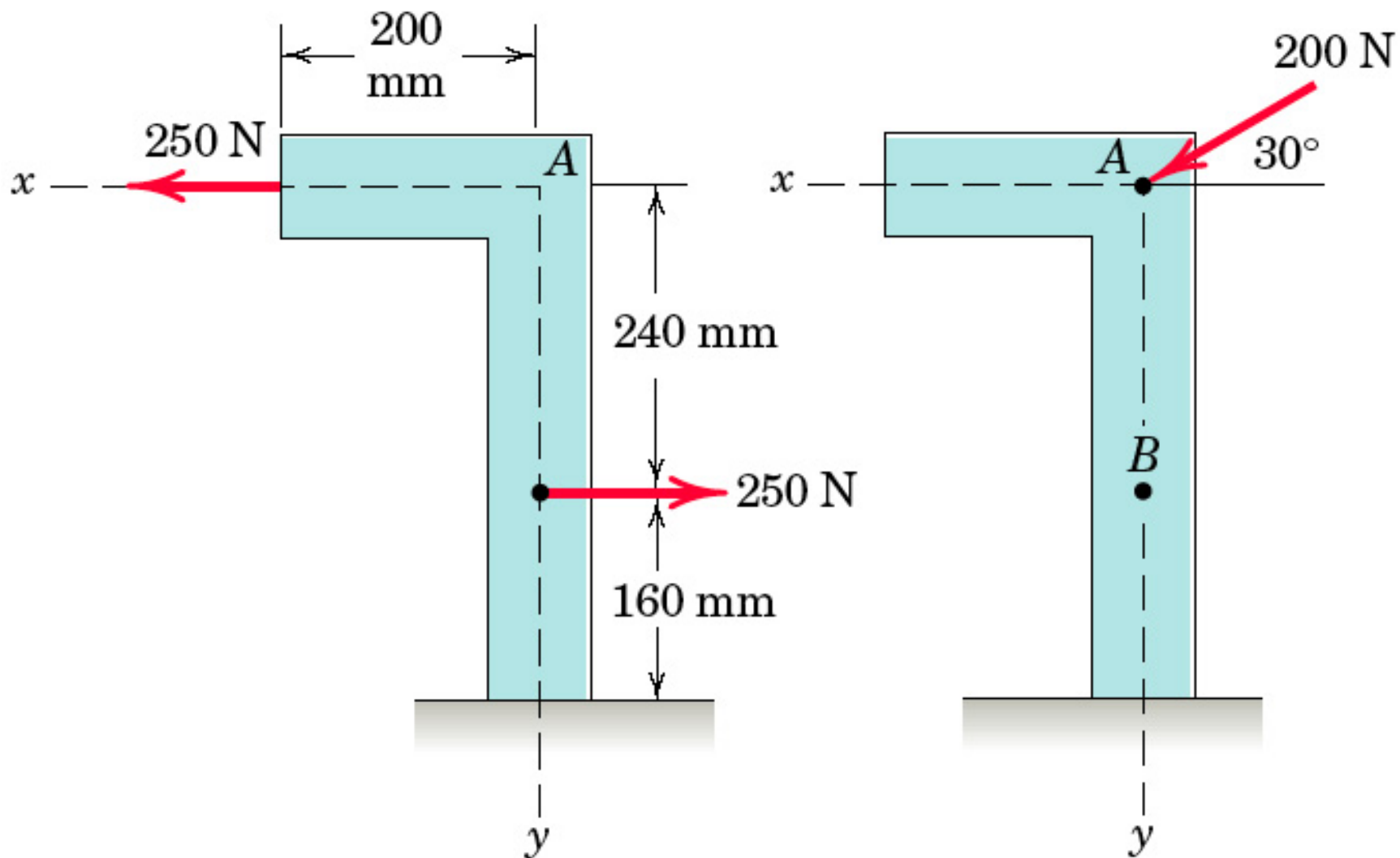
Problem 2/71



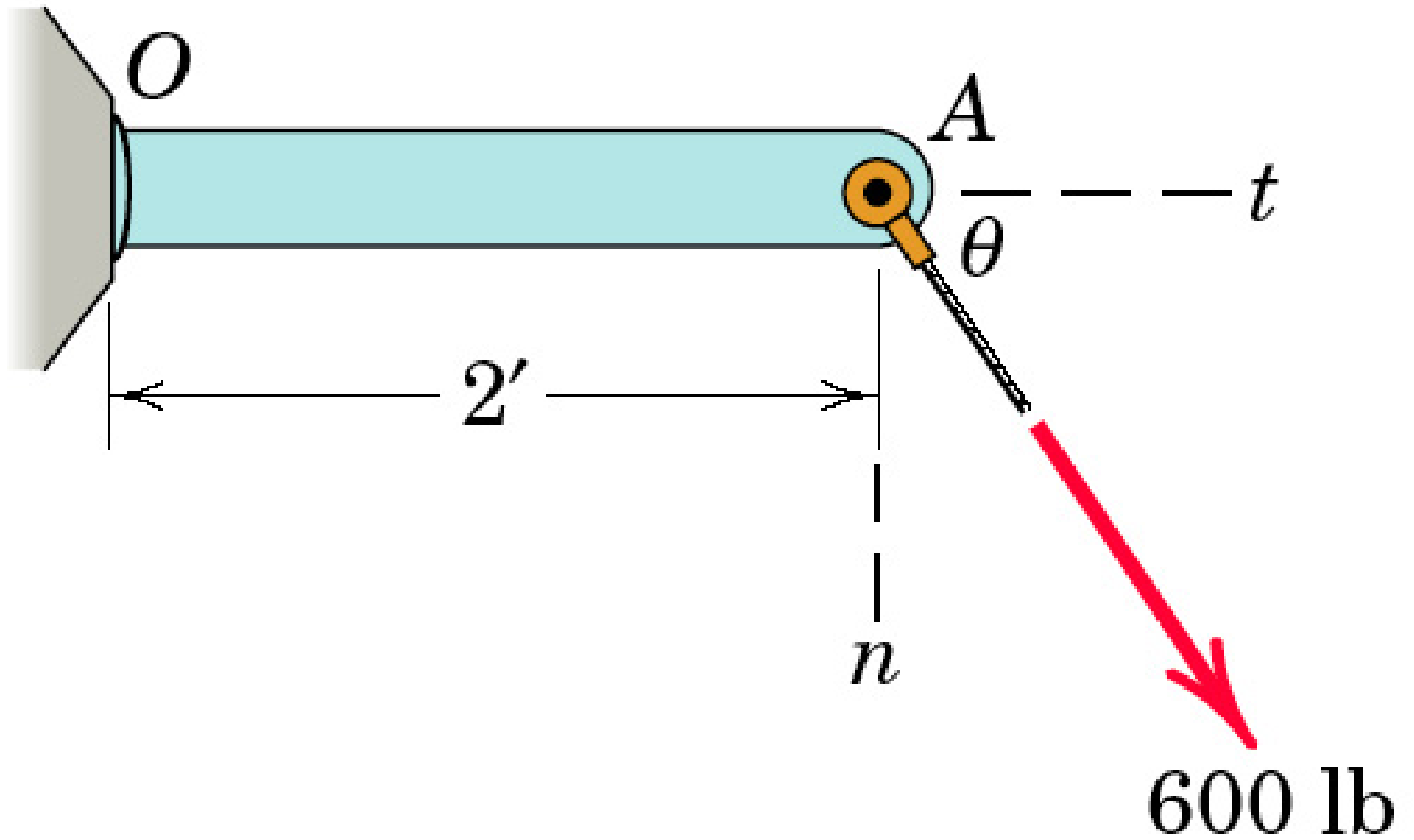
Problem 2/72



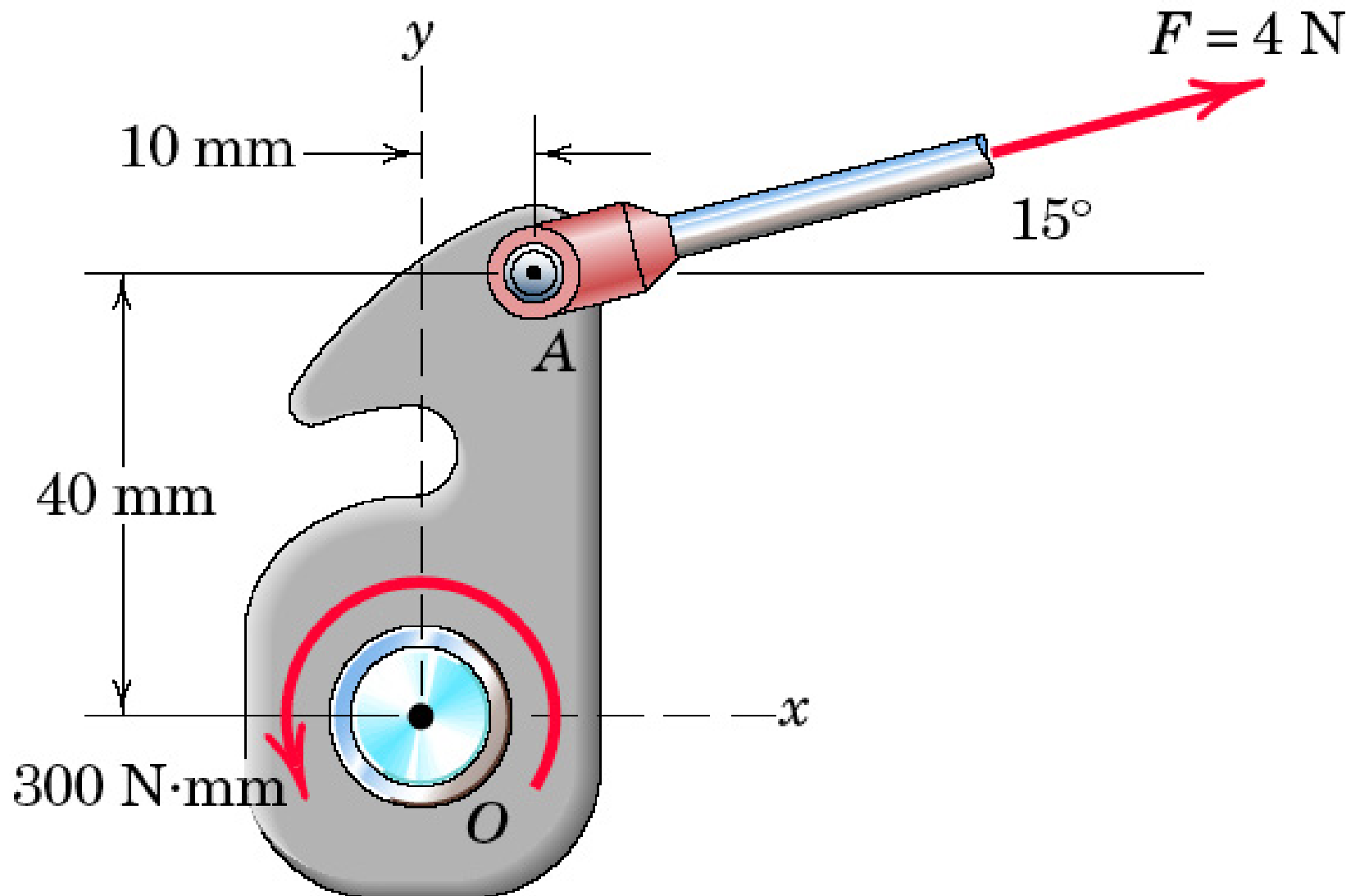
Problem 2/73



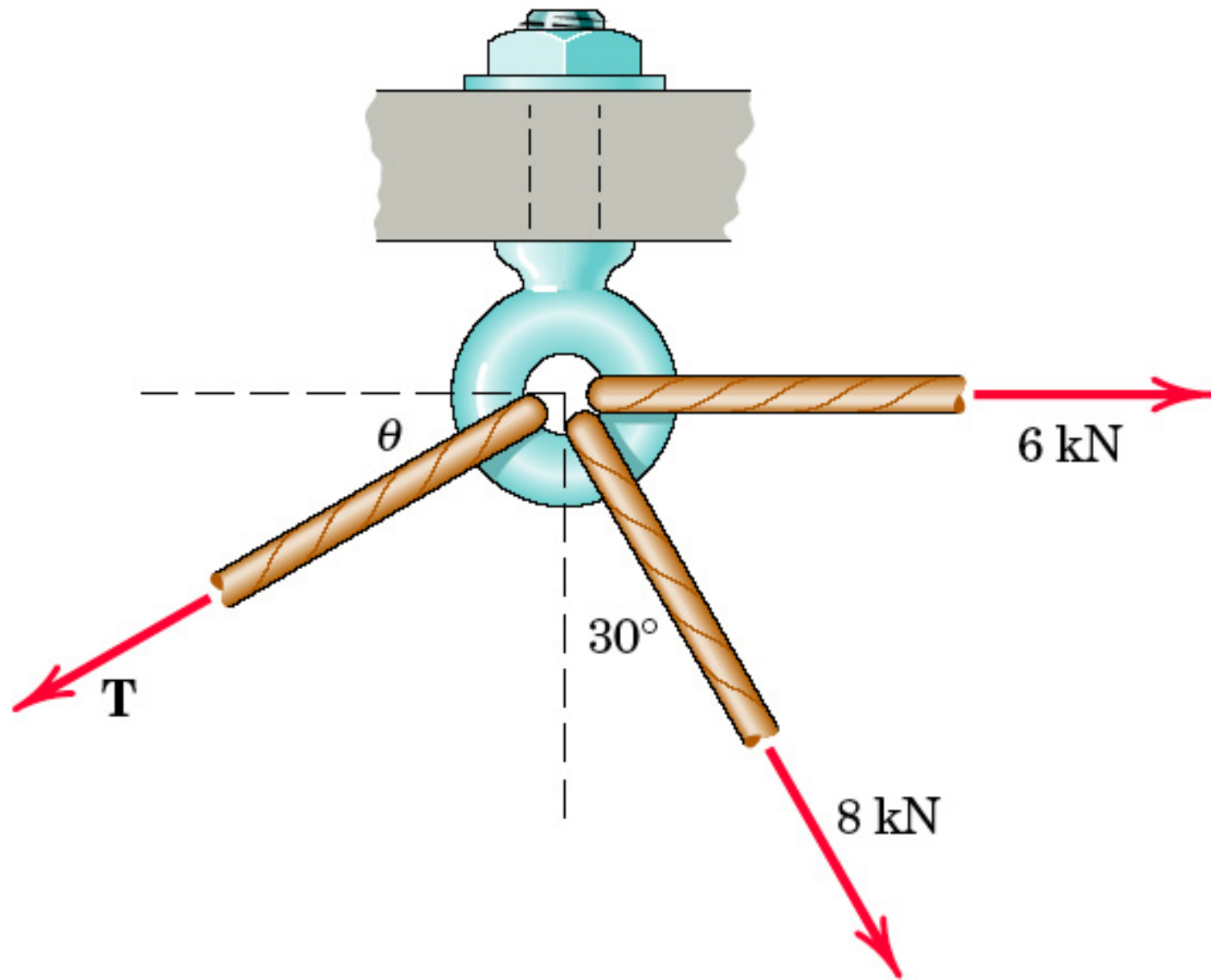
Problem 2/74



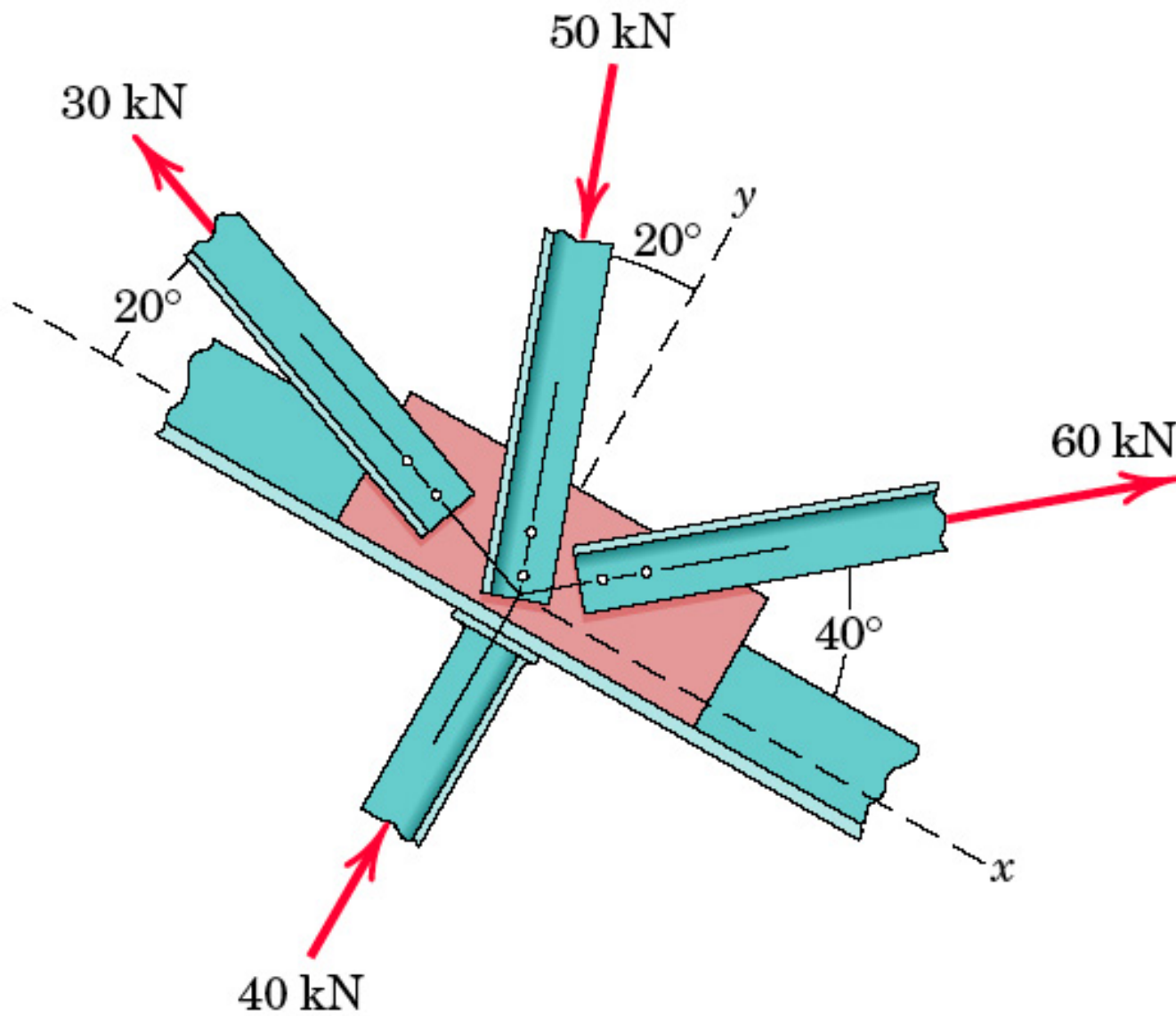
Problem 2/75



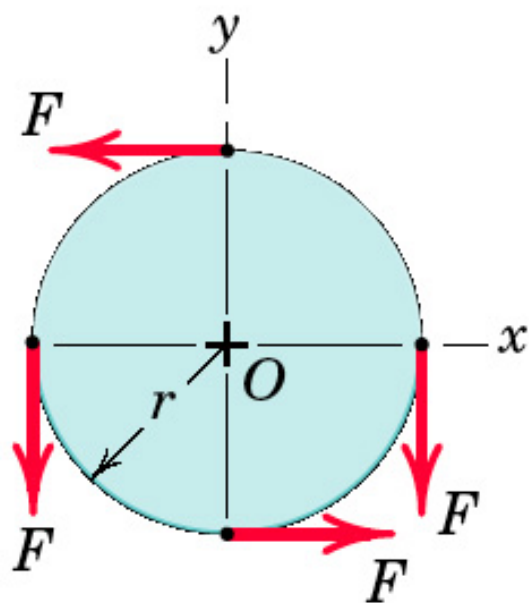
Problem 2/76



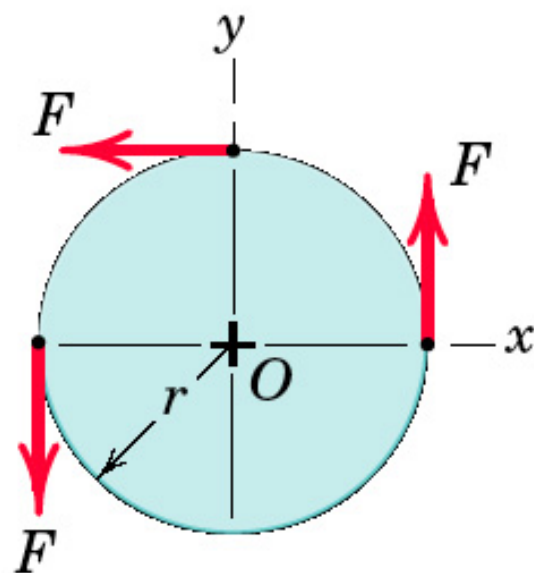
Problem 2/77



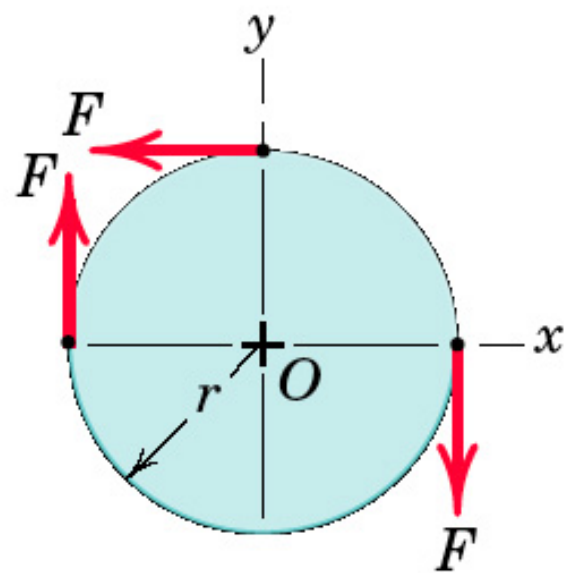
Problem 2/78



(a)



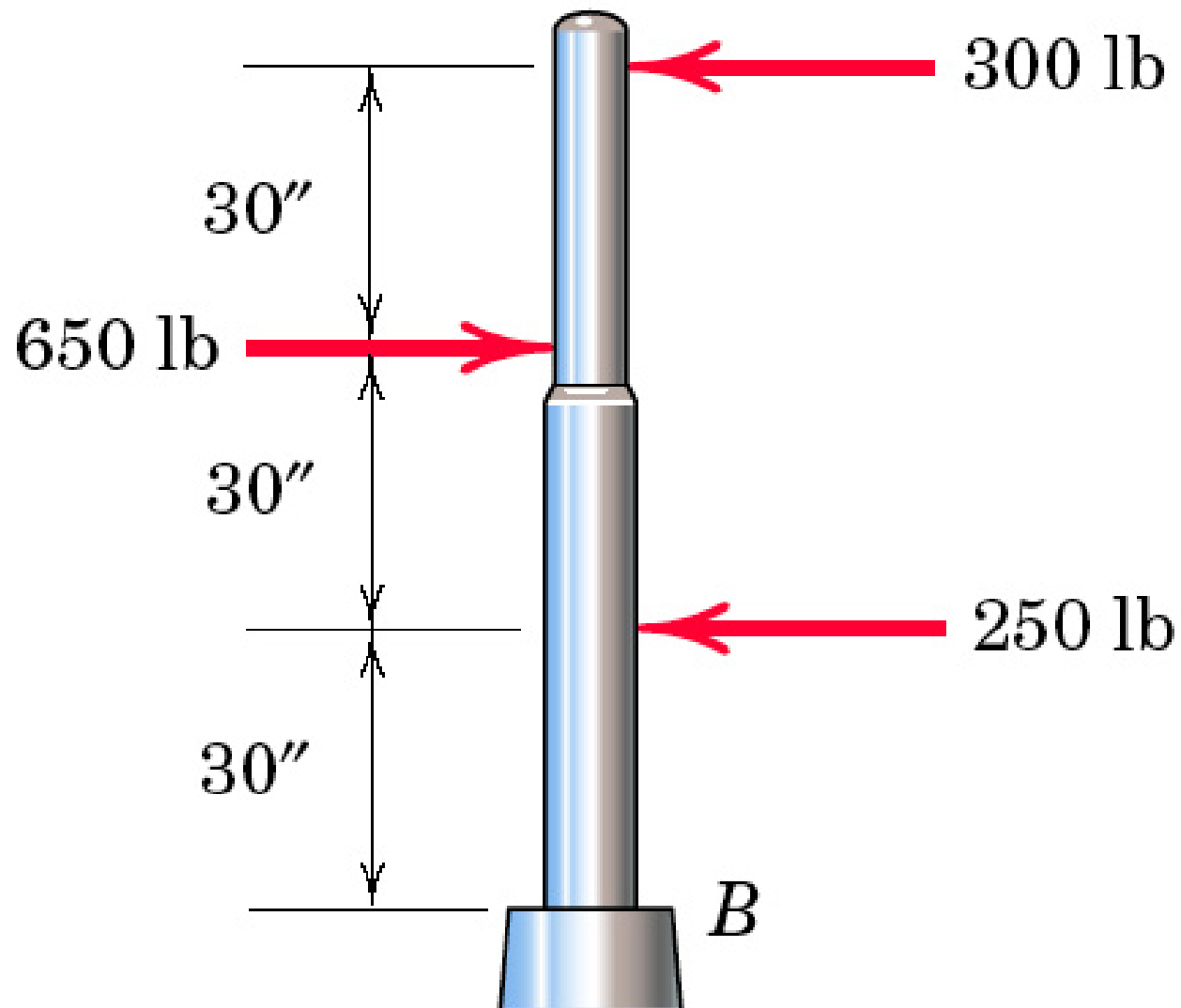
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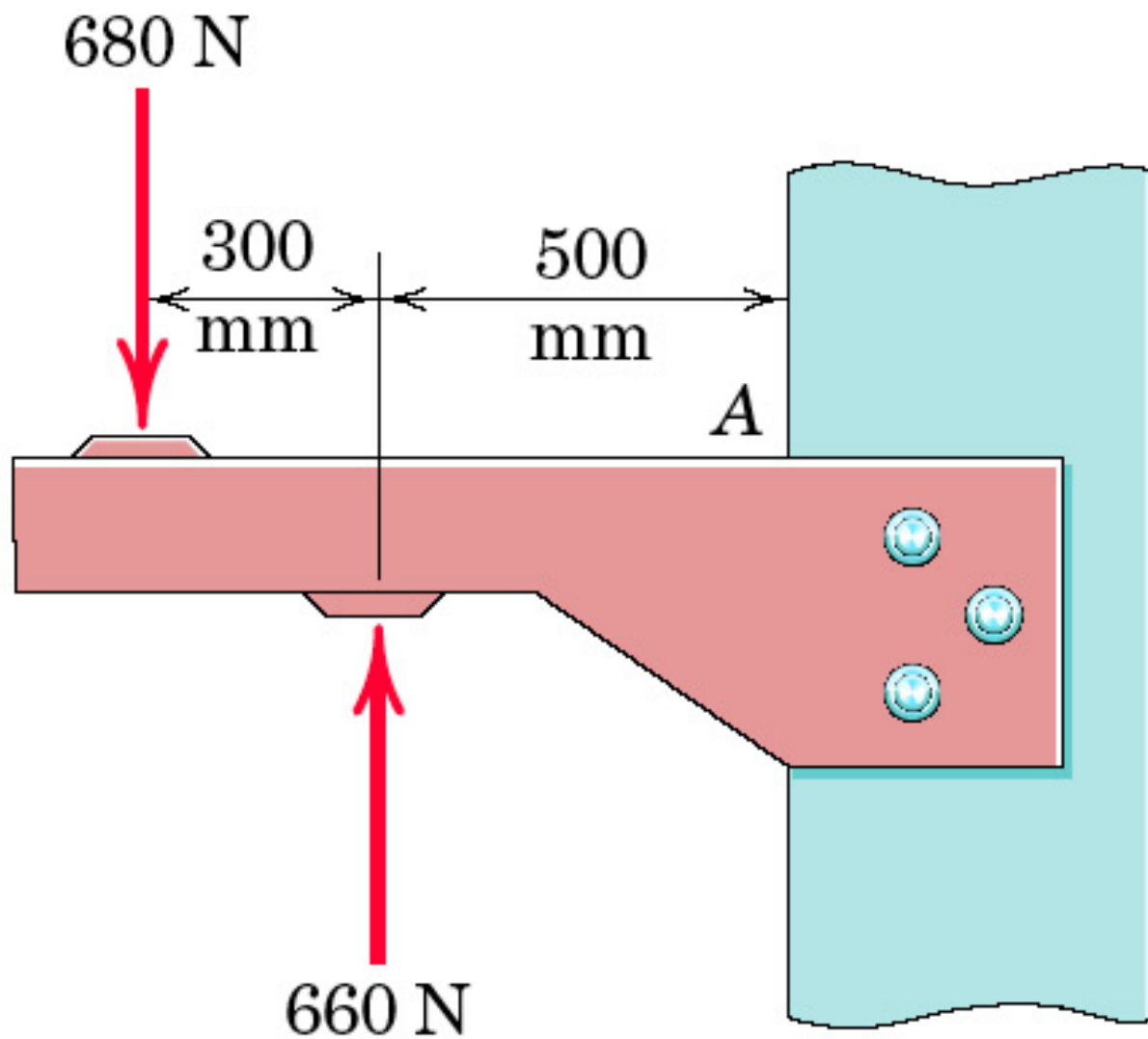
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Problem 2/79

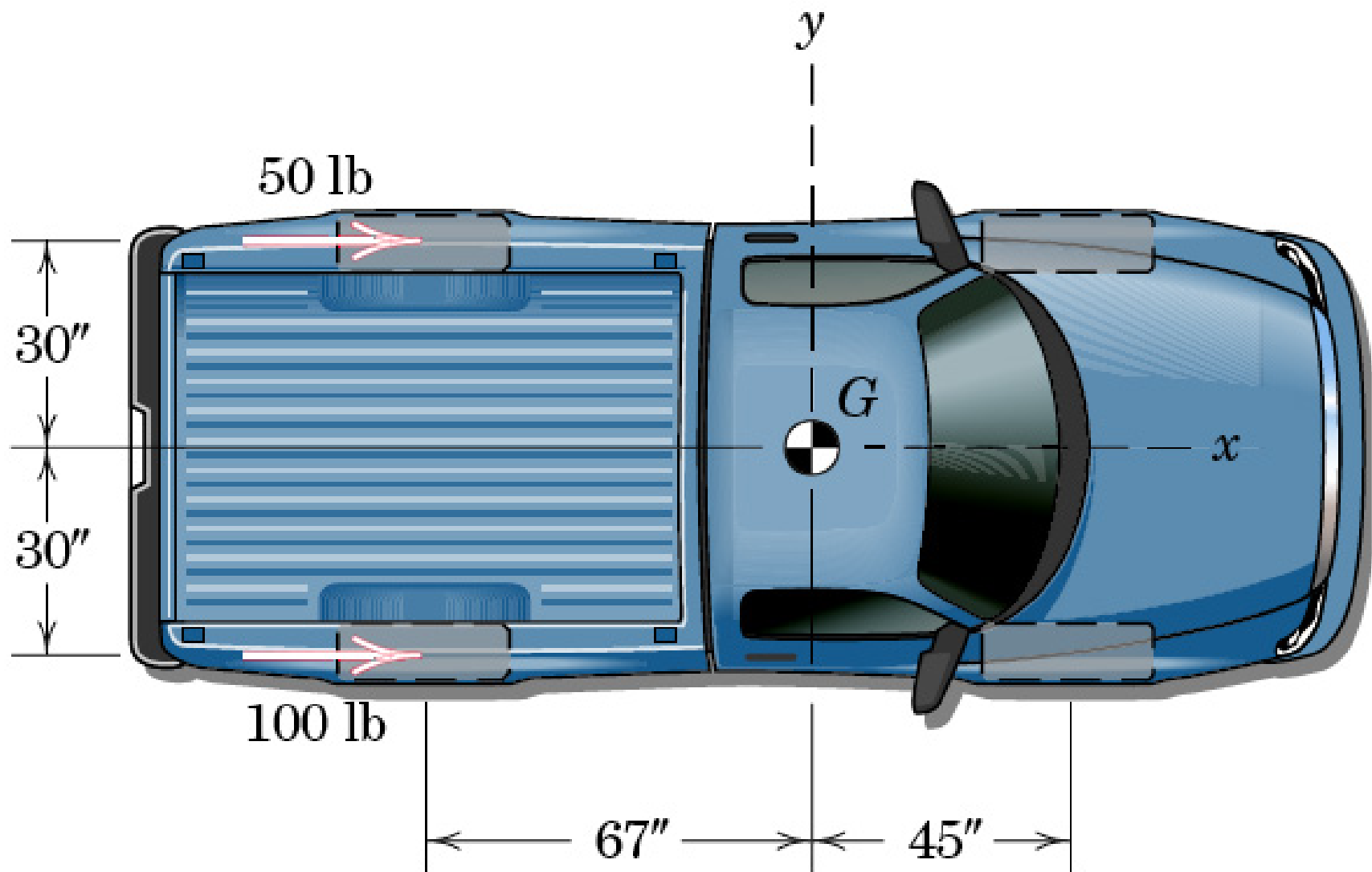
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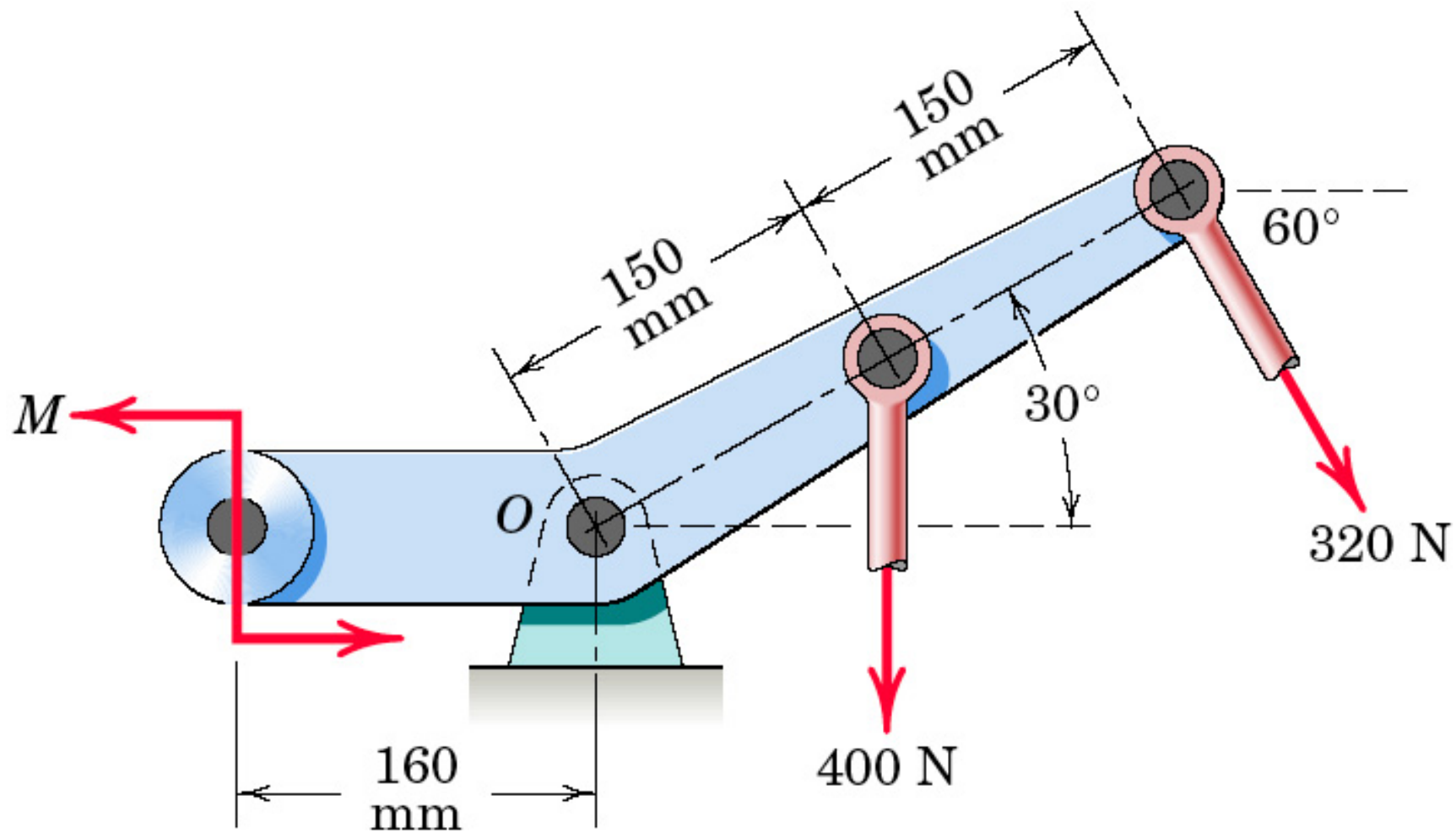
Problem 2/80



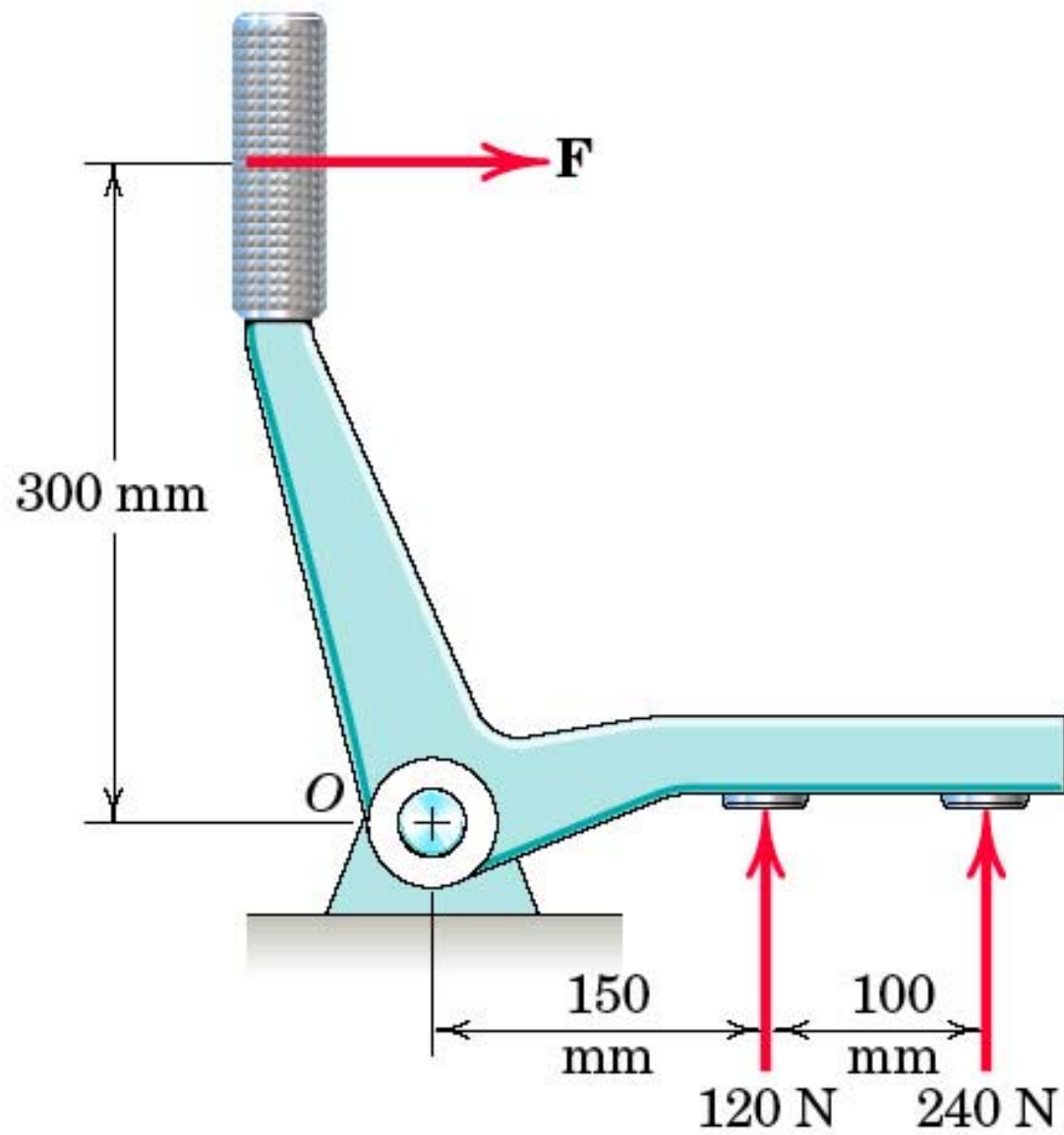
Problem 2/81



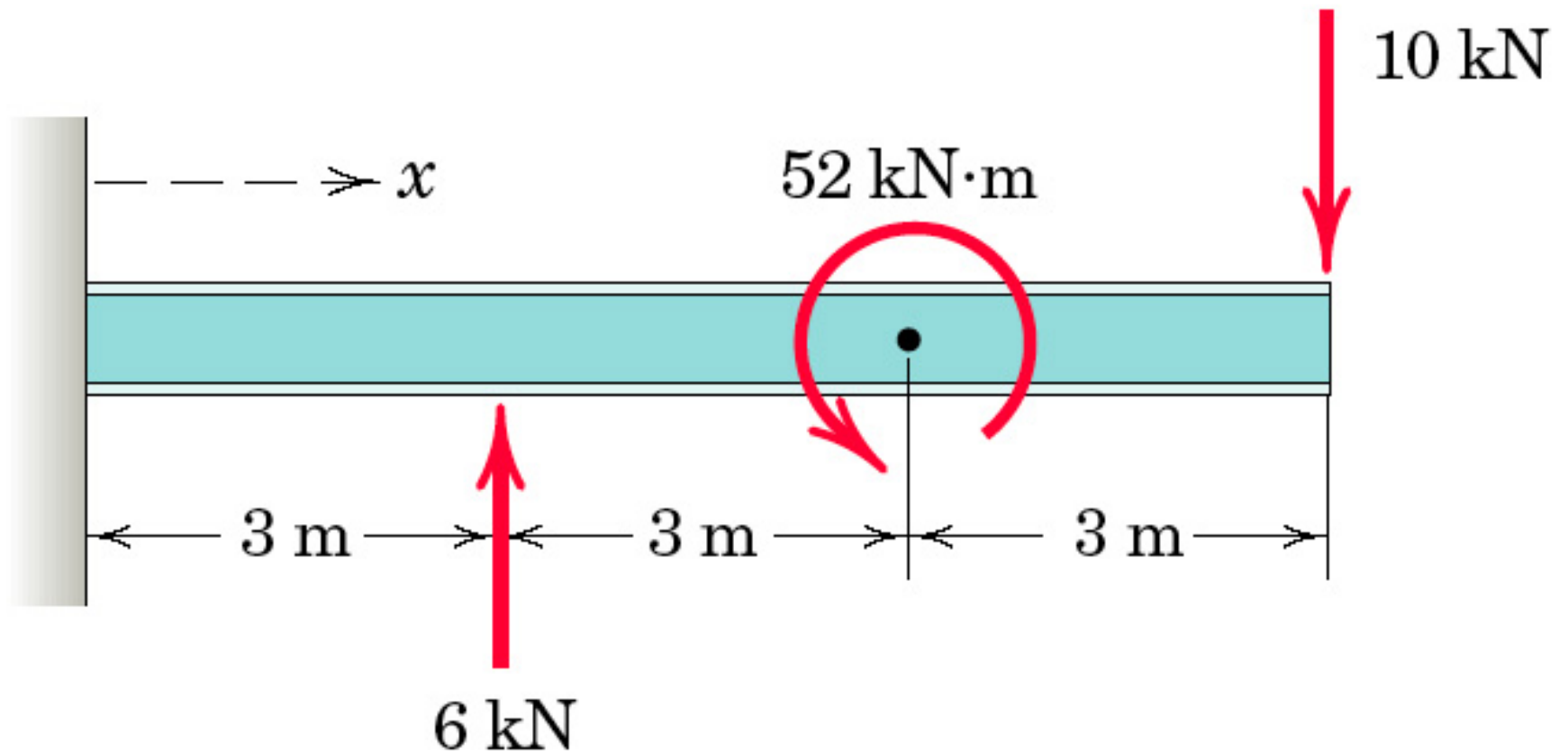
Problem 2/82



Problem 2/83

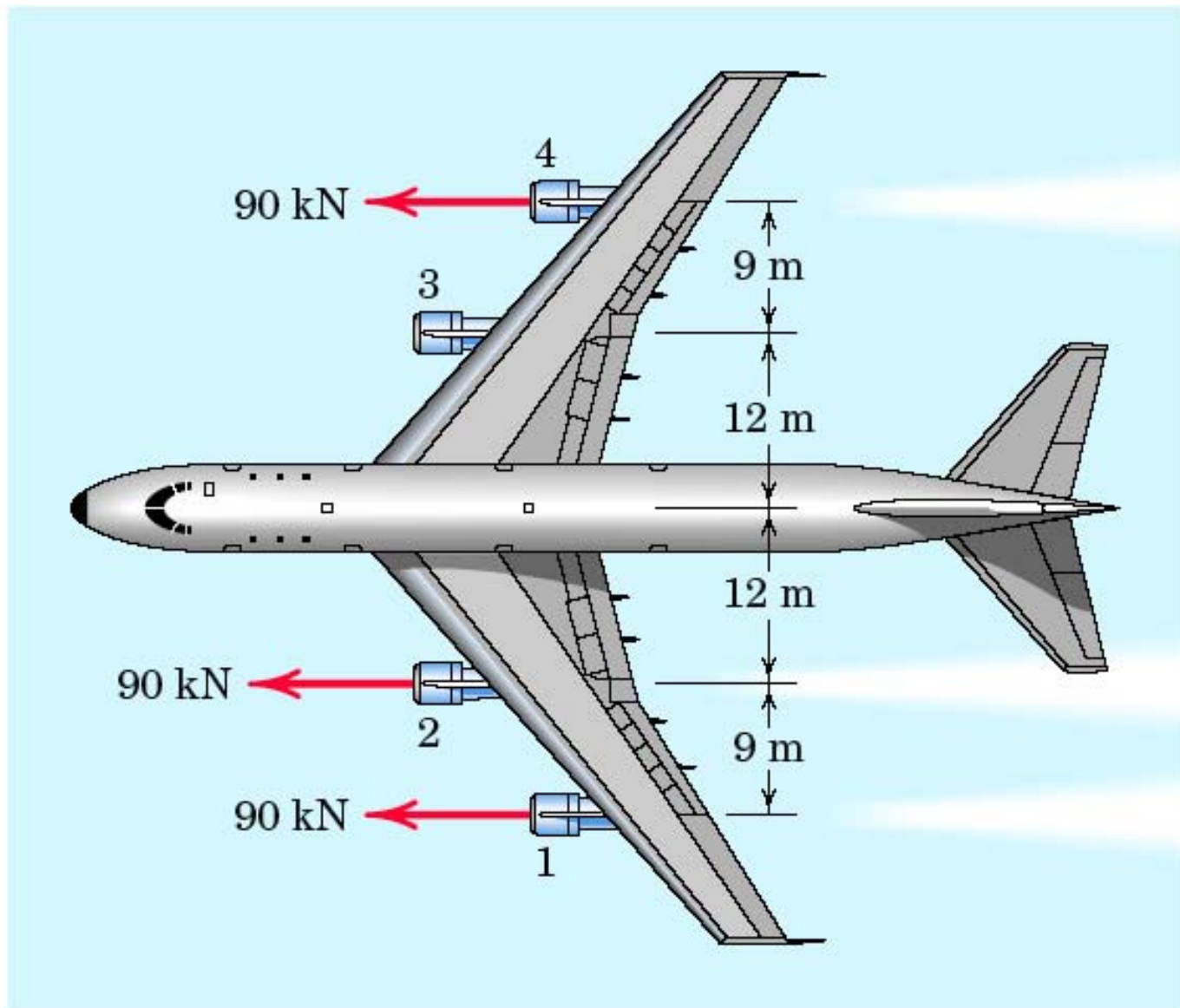


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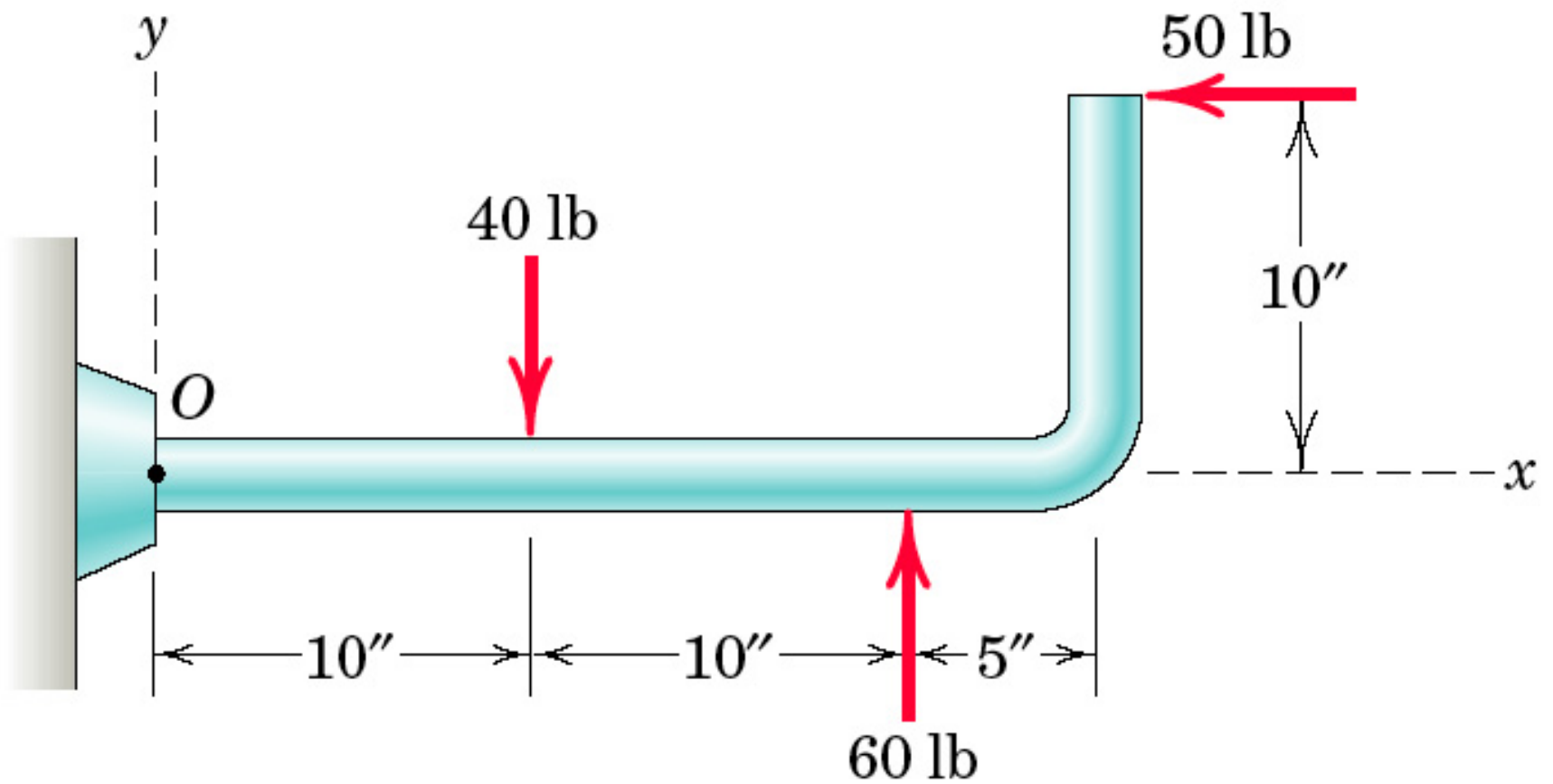


Problem 2/85

P_02_85

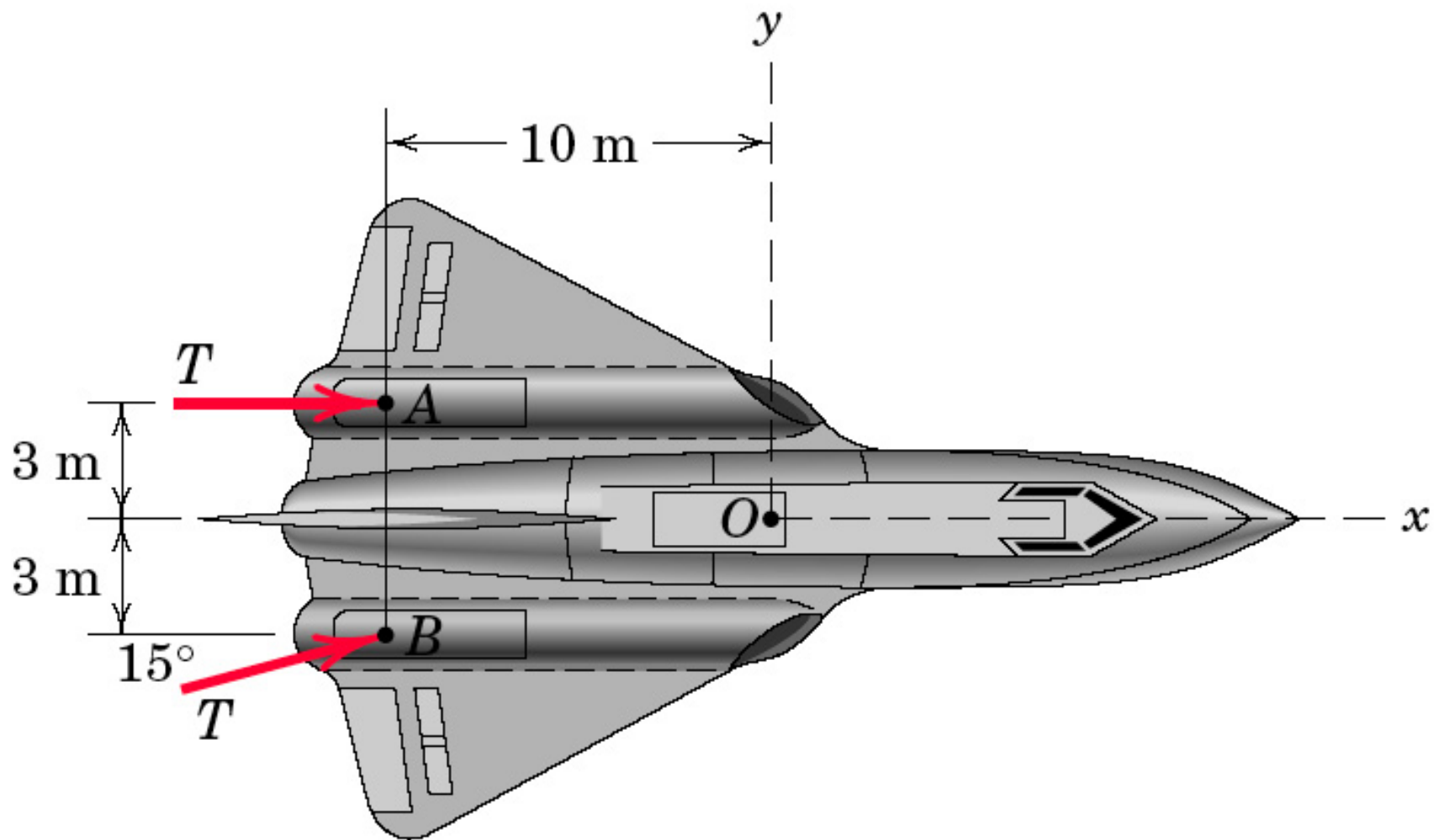


Problem 2/86

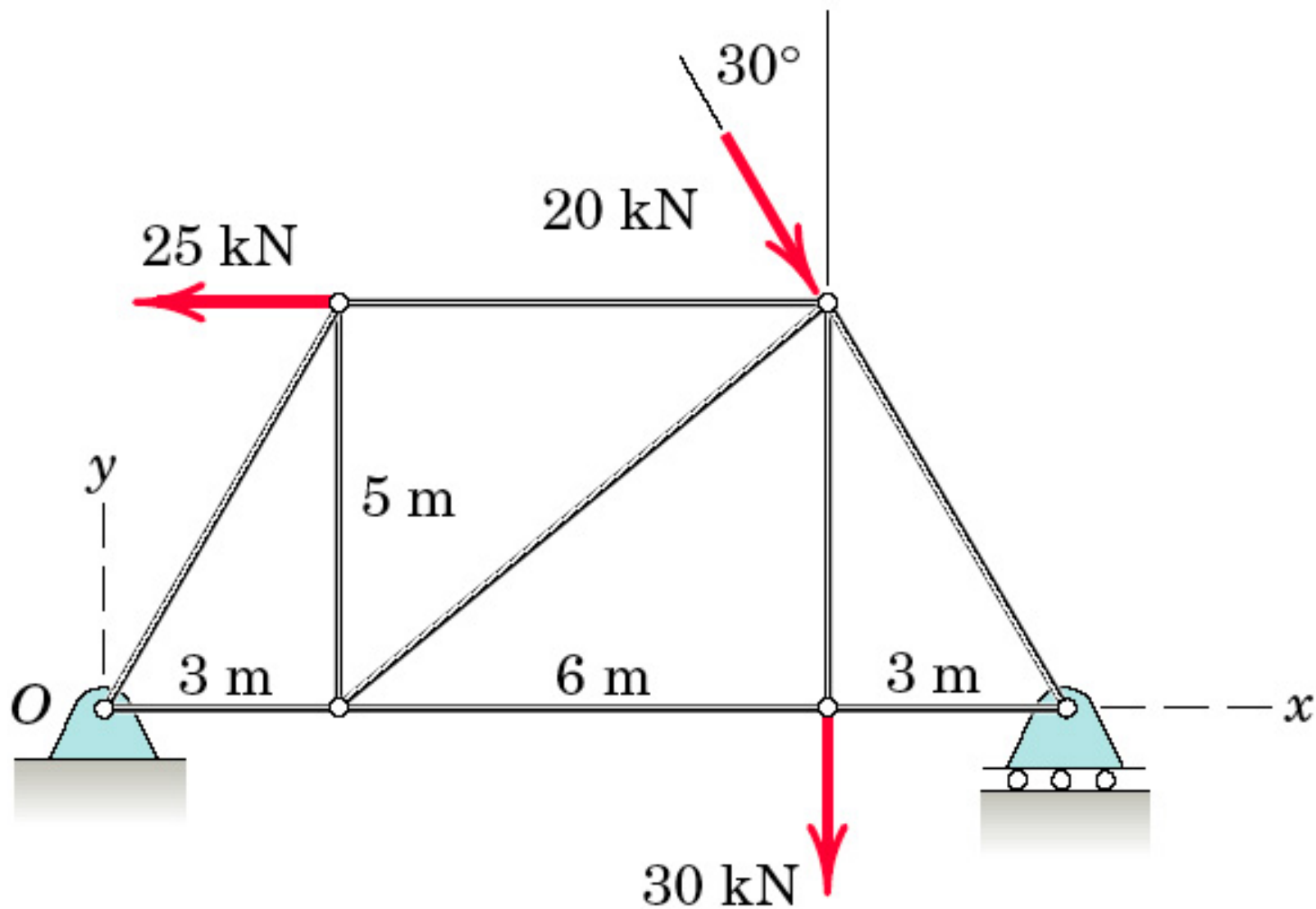


Problem 2/87

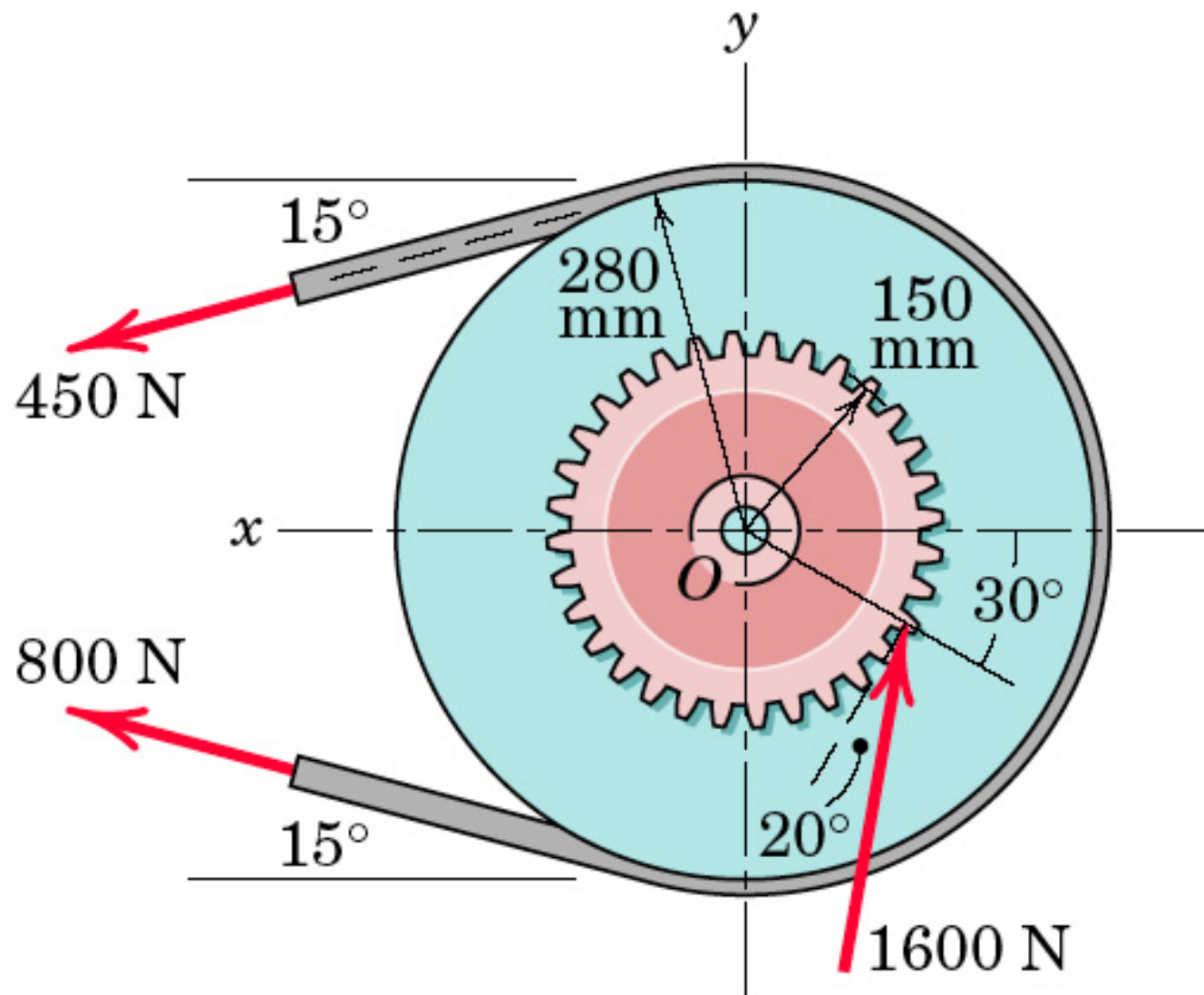
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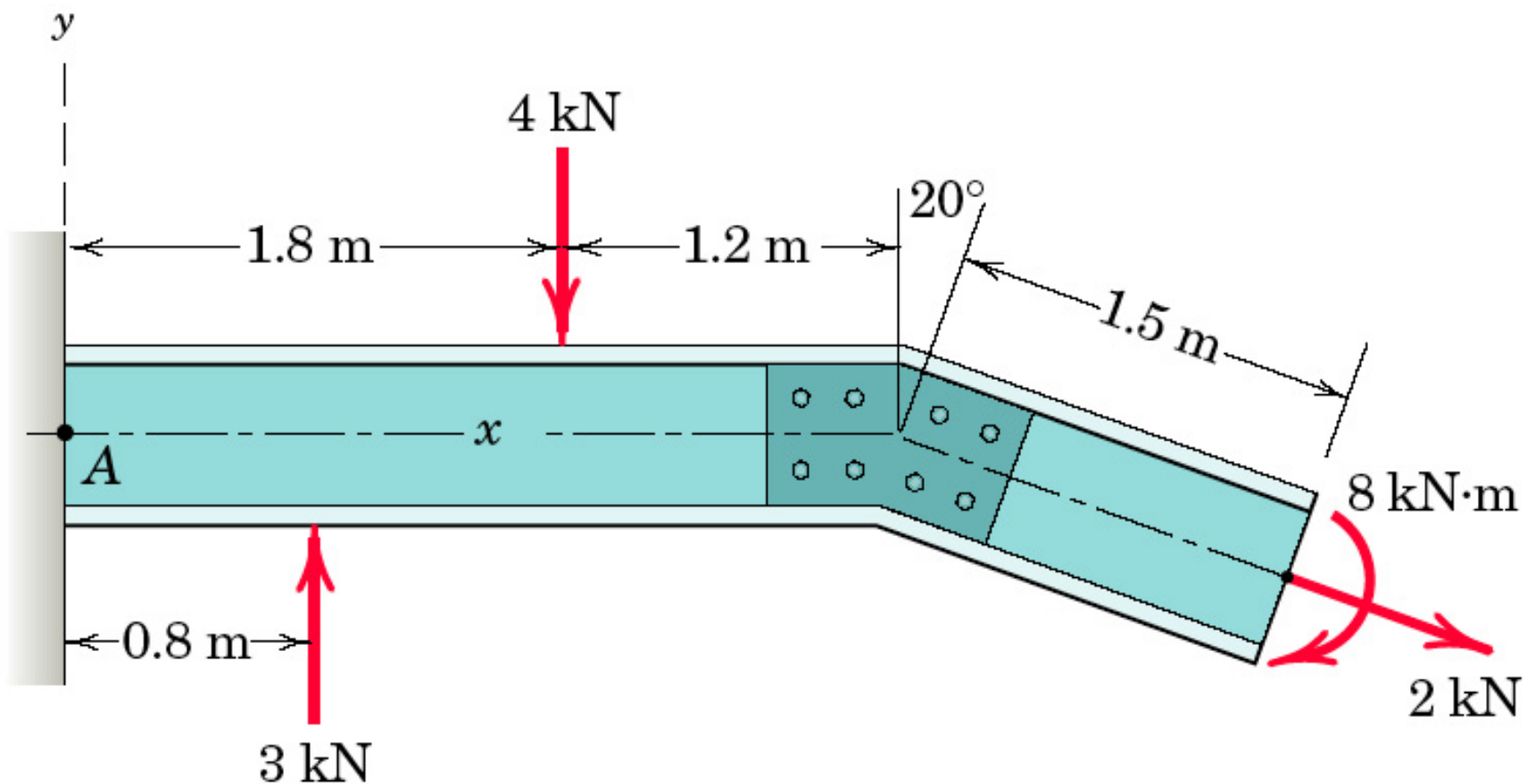
Problem 2/88



Problem 2/89

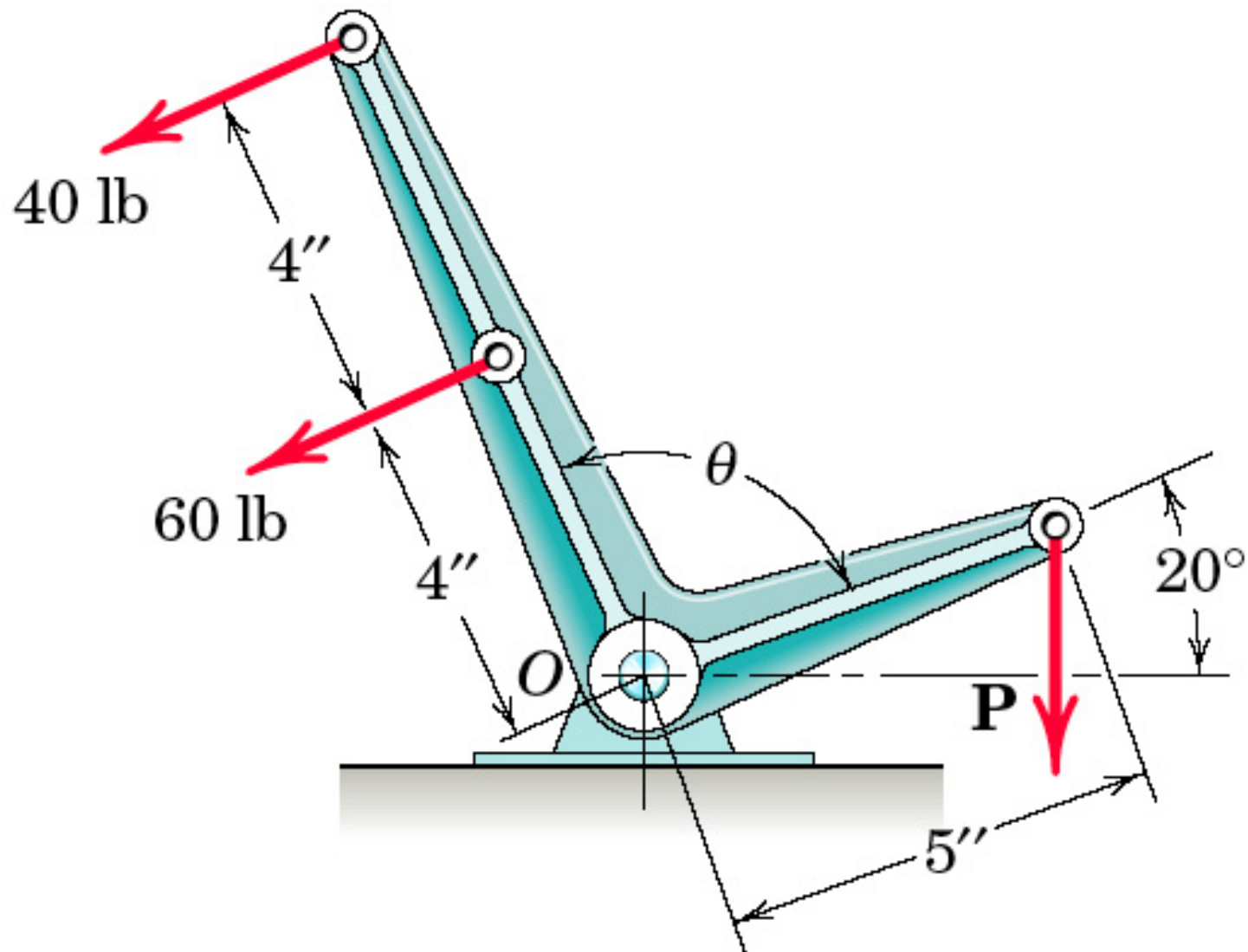


Problem 2/90

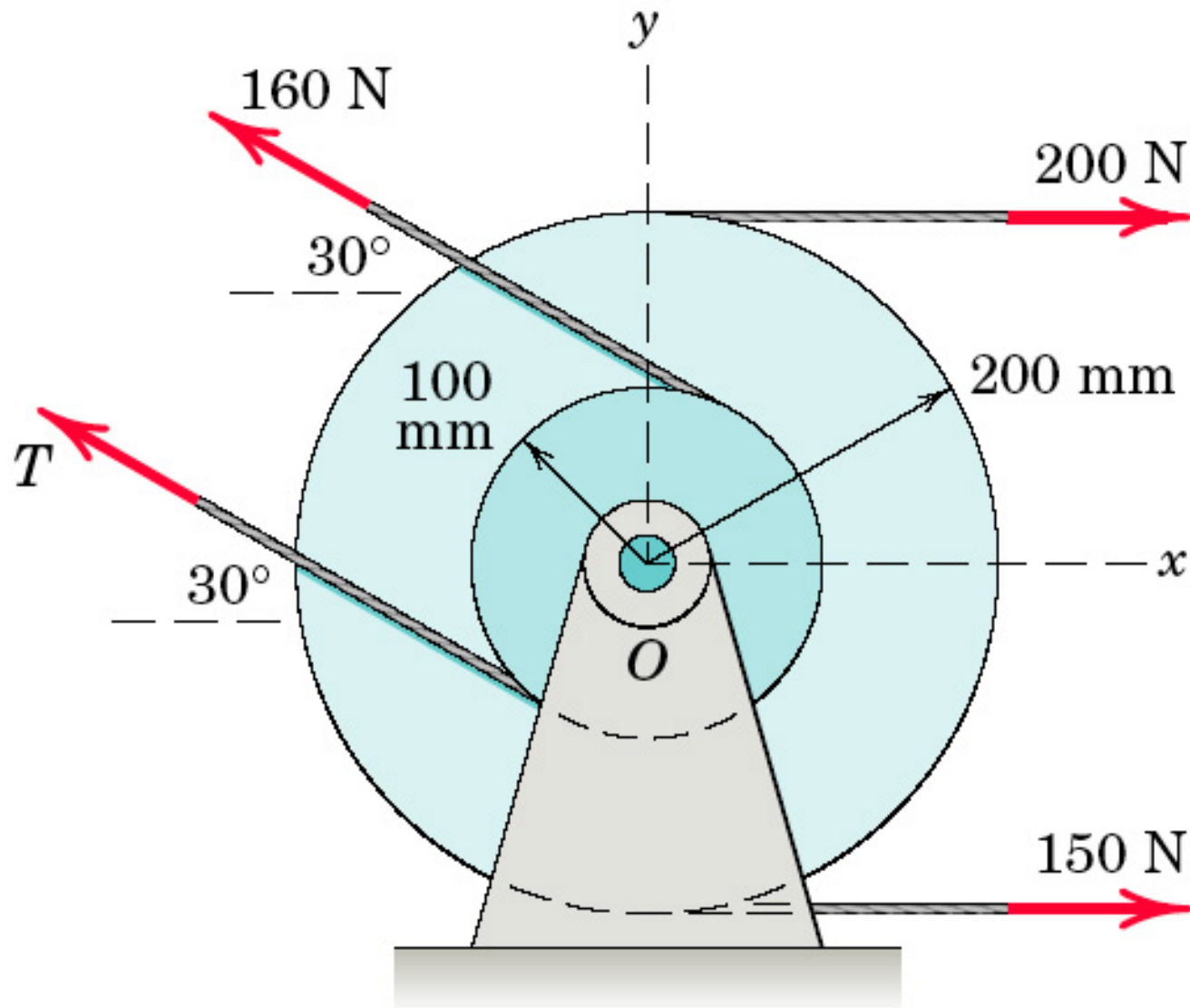


Problem 2/91

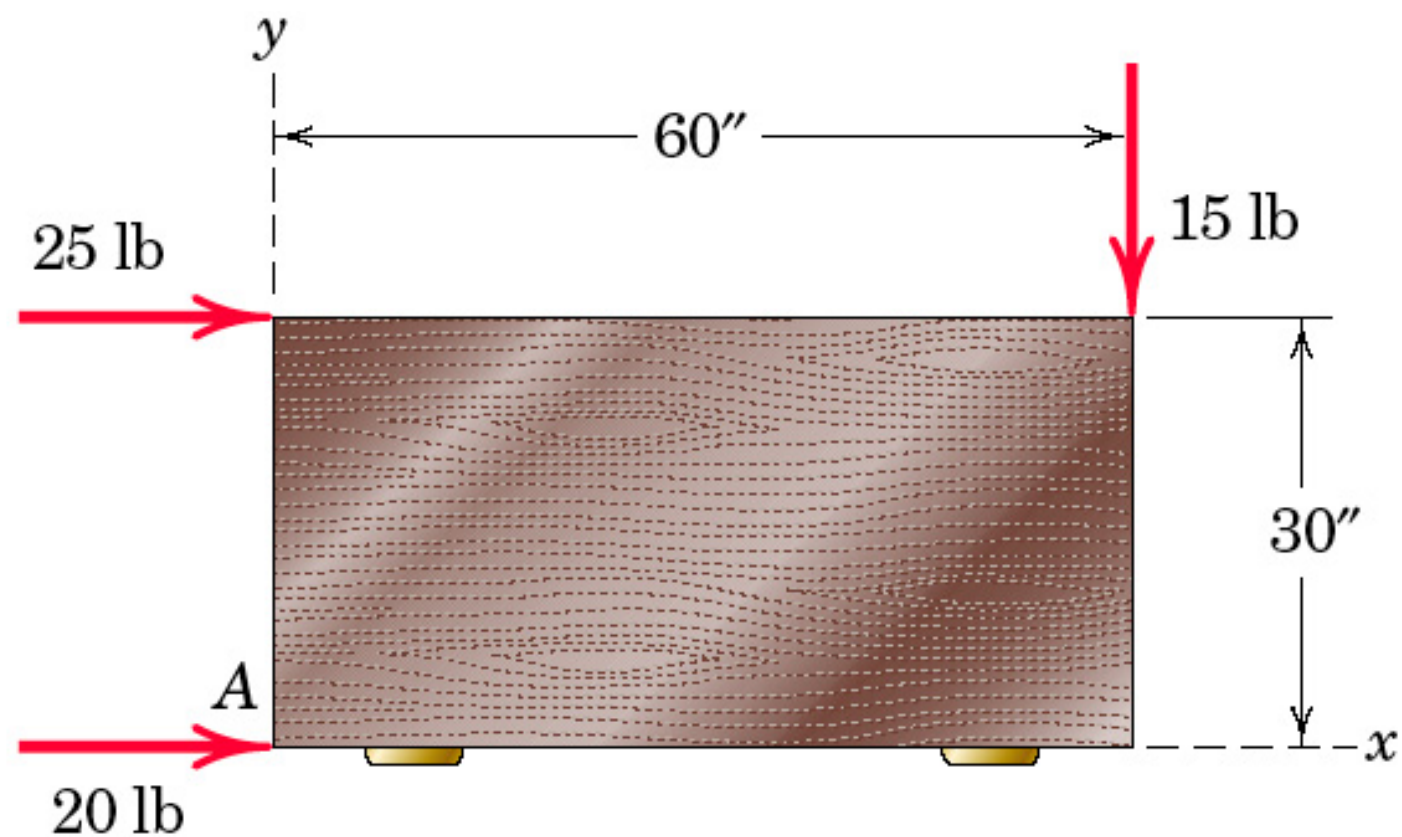
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Problem 2/92

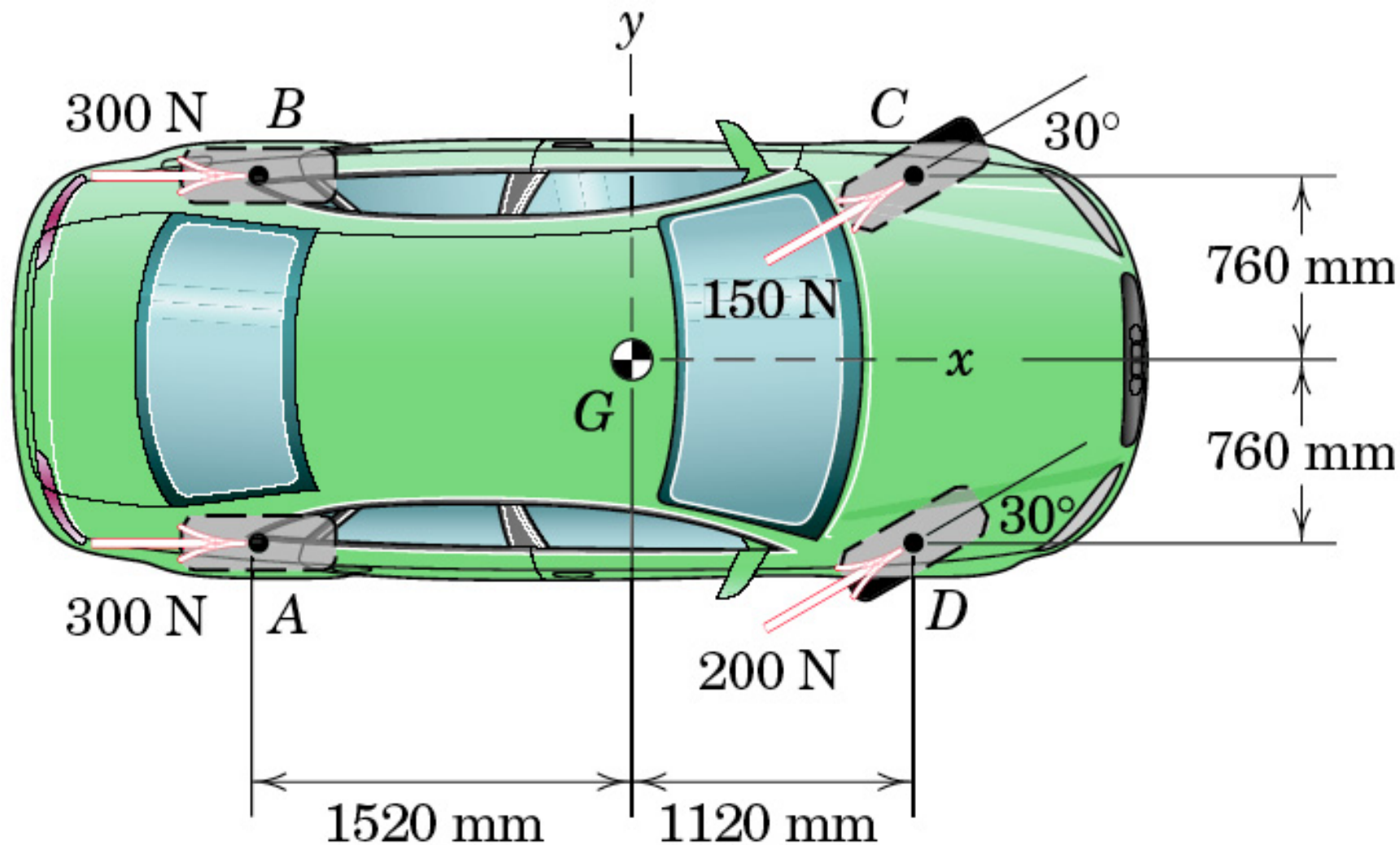


Problem 2/93

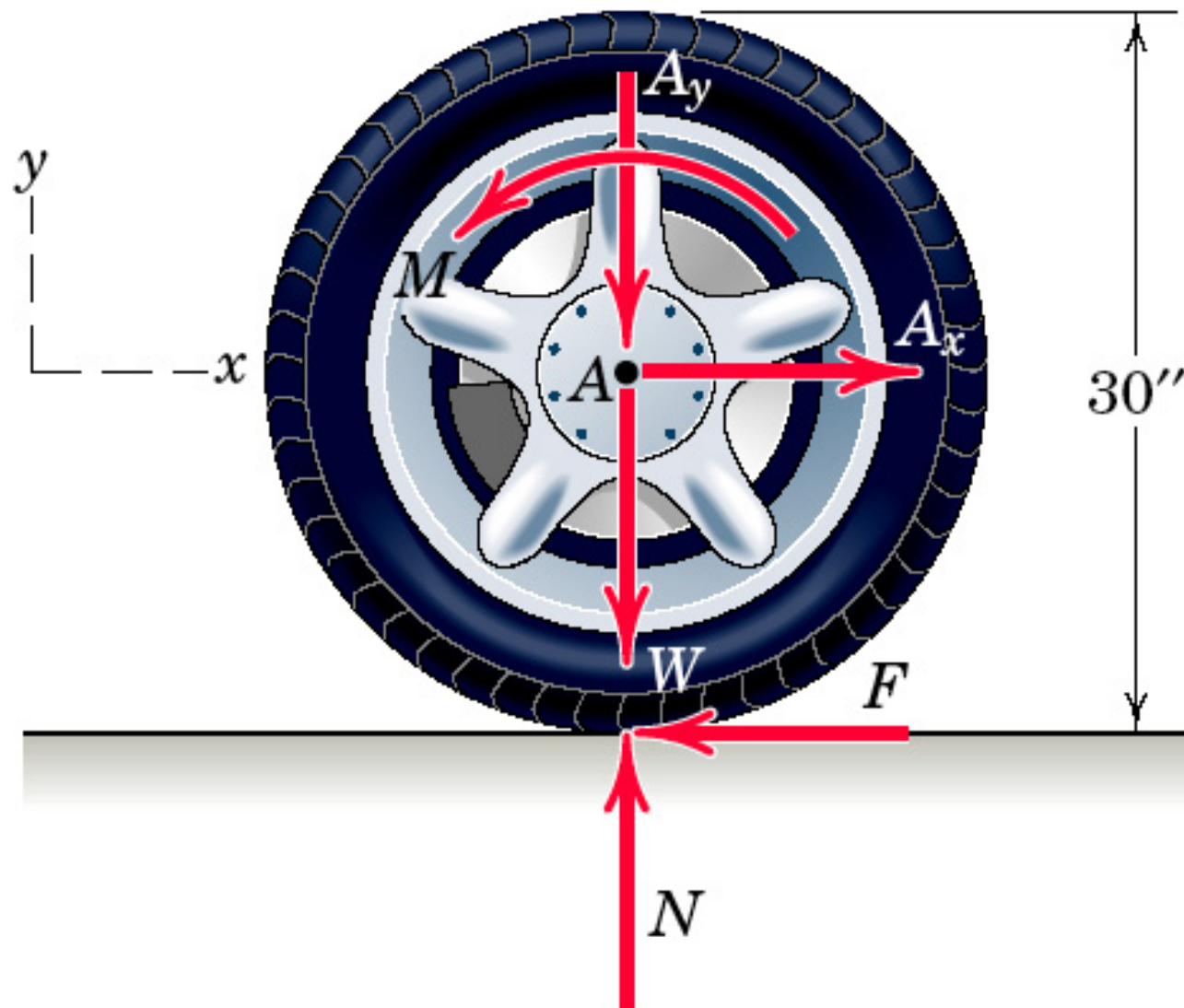


Problem 2/94

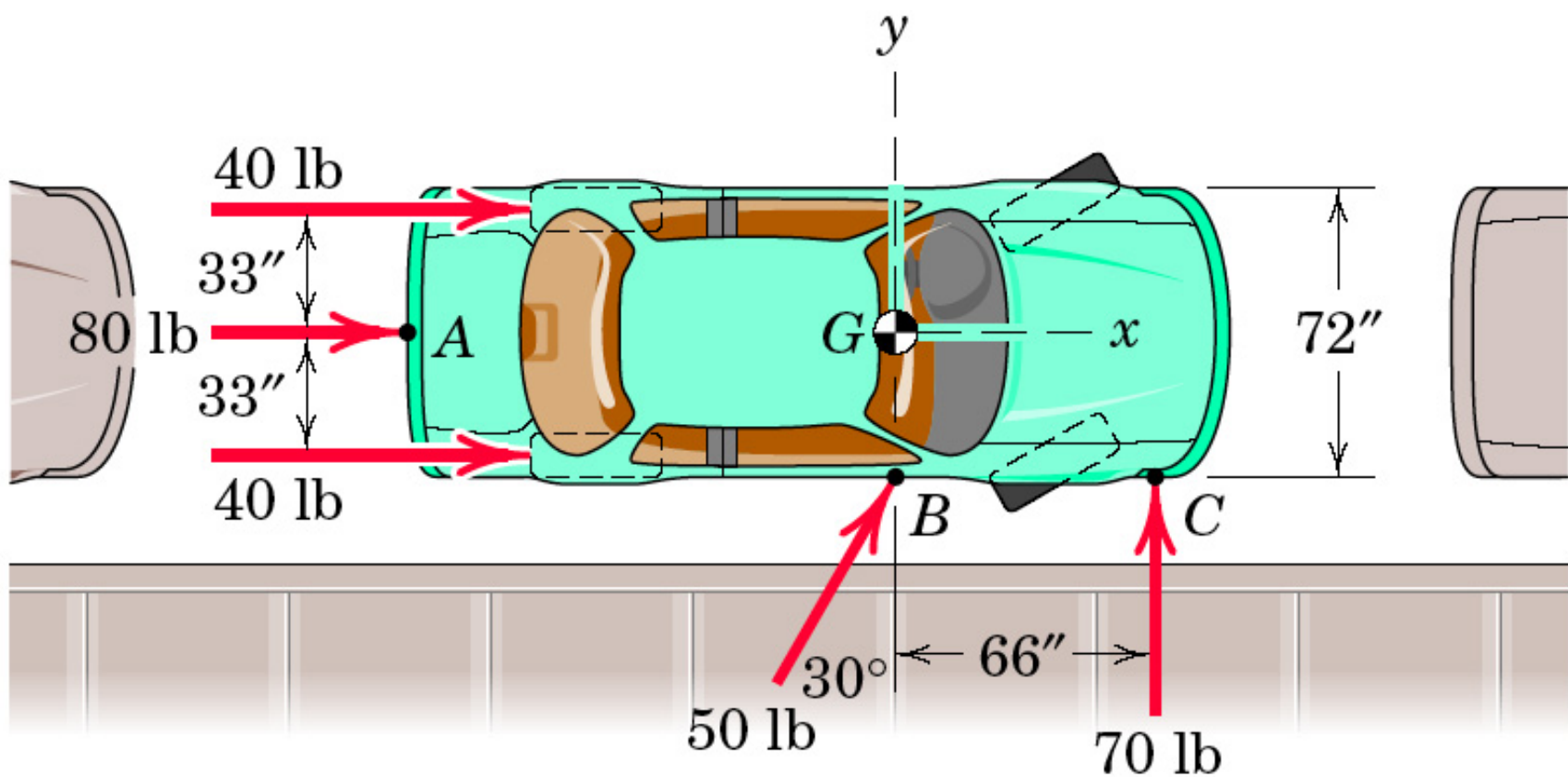
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Problem 2/95

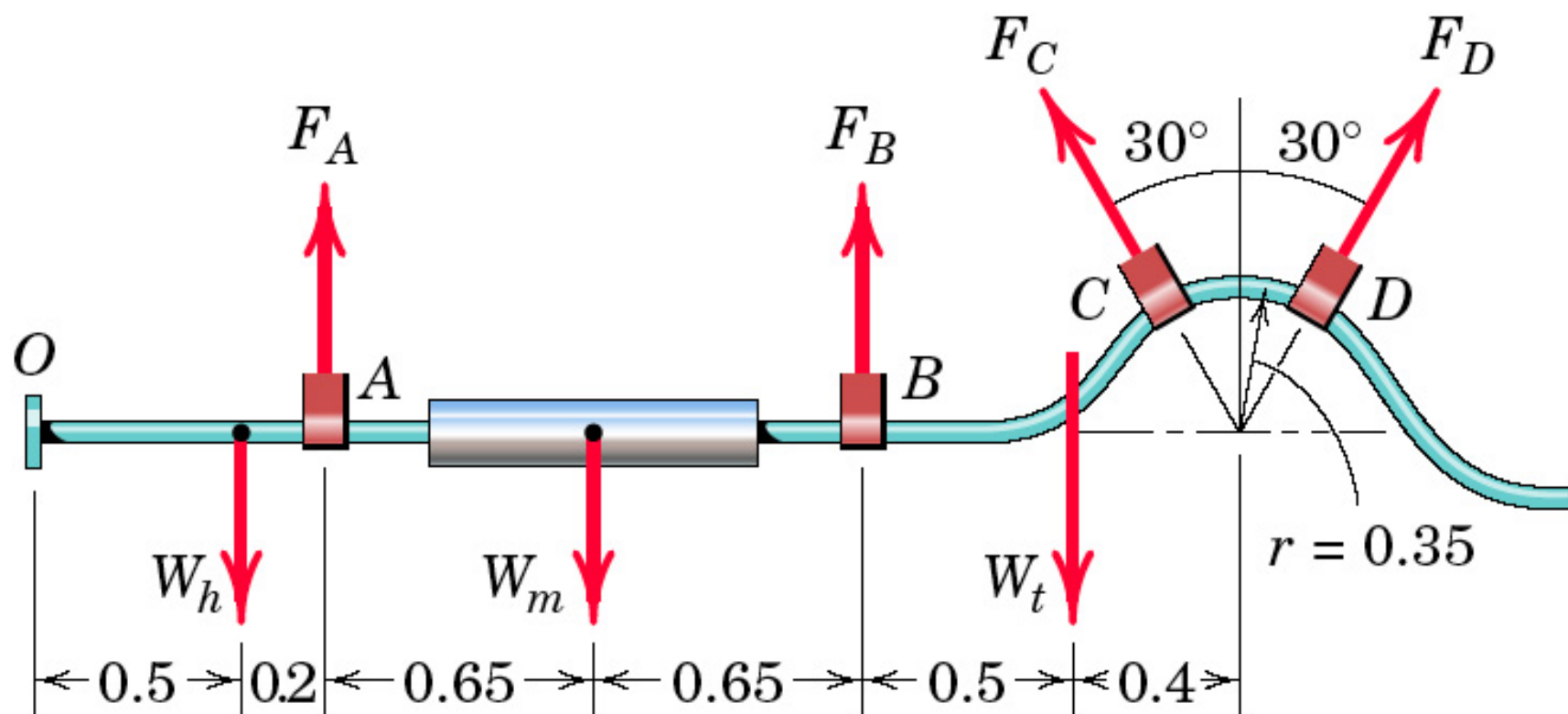


Problem 2/96



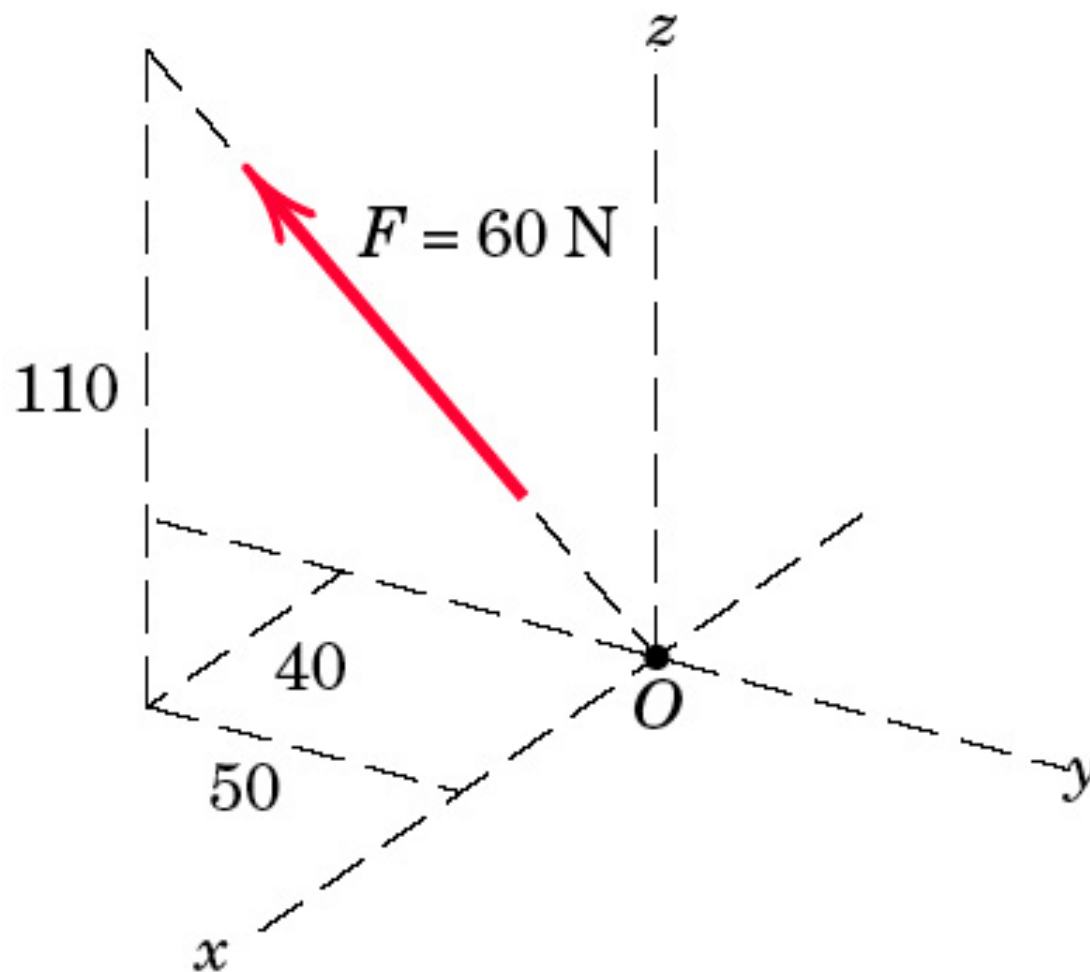
Problem 2/97

P_02_97



Dimensions in meters

Problem 2/98



Dimensions in millimeters

Problem 2/99

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 3 : Equilibrium.


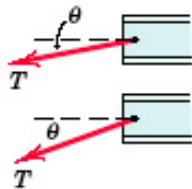



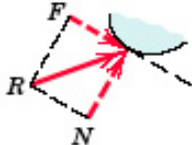
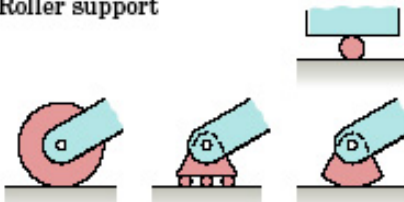
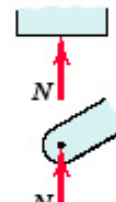
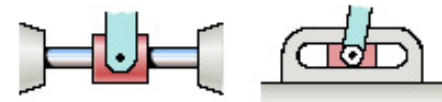
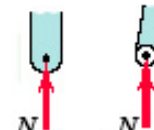
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
<p>1. Flexible cable, belt, chain, or rope</p> <p>Weight of cable negligible</p> <p>Weight of cable not negligible</p> 	 <p>Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.</p>
<p>2. Smooth surfaces</p> 	 <p>Contact force is compressive and is normal to the surface.</p>
<p>3. Rough surfaces</p> 	 <p>Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.</p>
<p>4. Roller support</p> 	 <p>Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.</p>
<p>5. Freely sliding guide</p> 	 <p>Collar or slider free to move along smooth guides; can support force normal to guide only.</p>

Figure 3/1

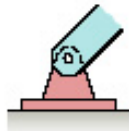



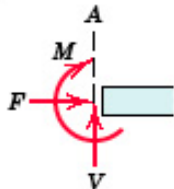
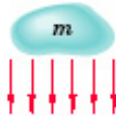
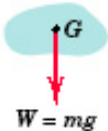
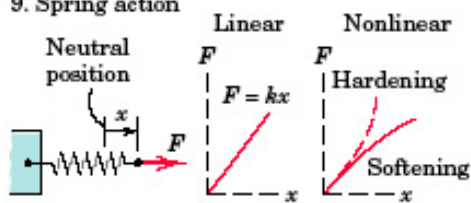

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (cont.)	
Type of Contact and Force Origin	Action on Body to Be Isolated
<p>6. Pin connection</p> 	<p>Pin free to turn</p>  <p>A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y, or a magnitude R and direction θ.</p> <p>Pin not free to turn</p>  <p>A pin not free to turn also supports a couple M.</p>
<p>7. Built-in or fixed support</p> 	 <p>A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.</p>
<p>8. Gravitational attraction</p> 	 <p>The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.</p>
<p>9. Spring action</p>  <p>Linear: $F = kx$</p> <p>Nonlinear: Hardening, Softening</p>	 <p>Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.</p>

Figure 3/1, continued

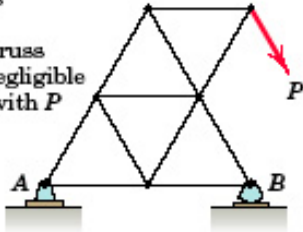
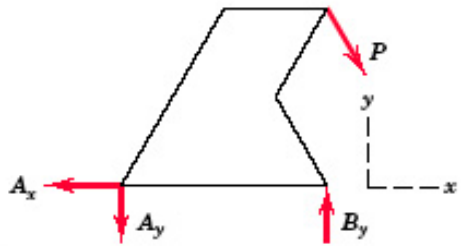
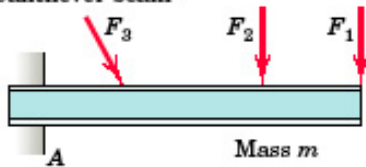
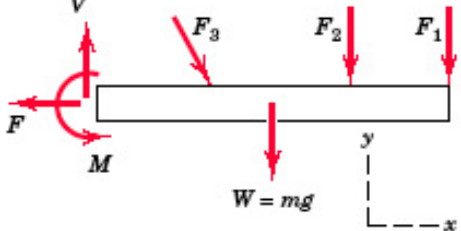
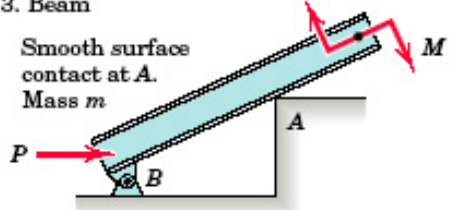
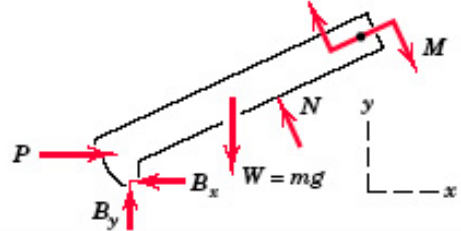
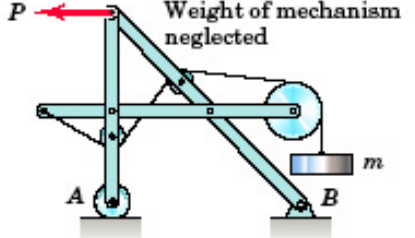
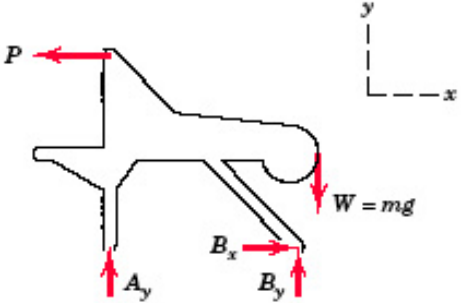
SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	Free-Body Diagram of Isolated Body
<p>1. Plane truss</p> <p>Weight of truss assumed negligible compared with P</p> 	
<p>2. Cantilever beam</p> 	
<p>3. Beam</p> <p>Smooth surface contact at A.</p> <p>Mass m</p> 	
<p>4. Rigid system of interconnected bodies analyzed as a single unit</p> <p>Weight of mechanism neglected</p> 	

Figure 3/2

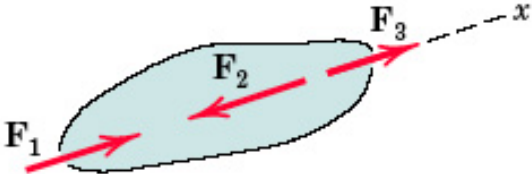
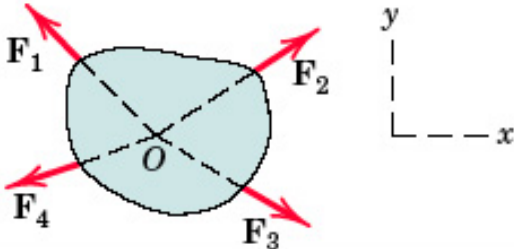
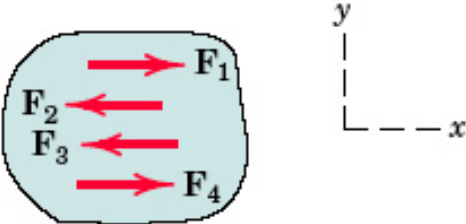
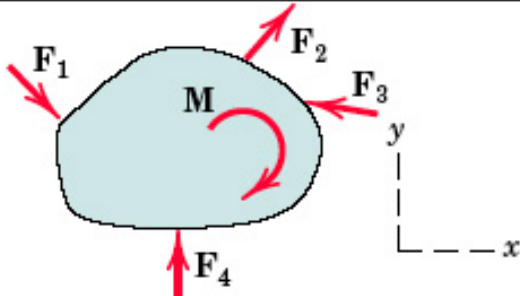
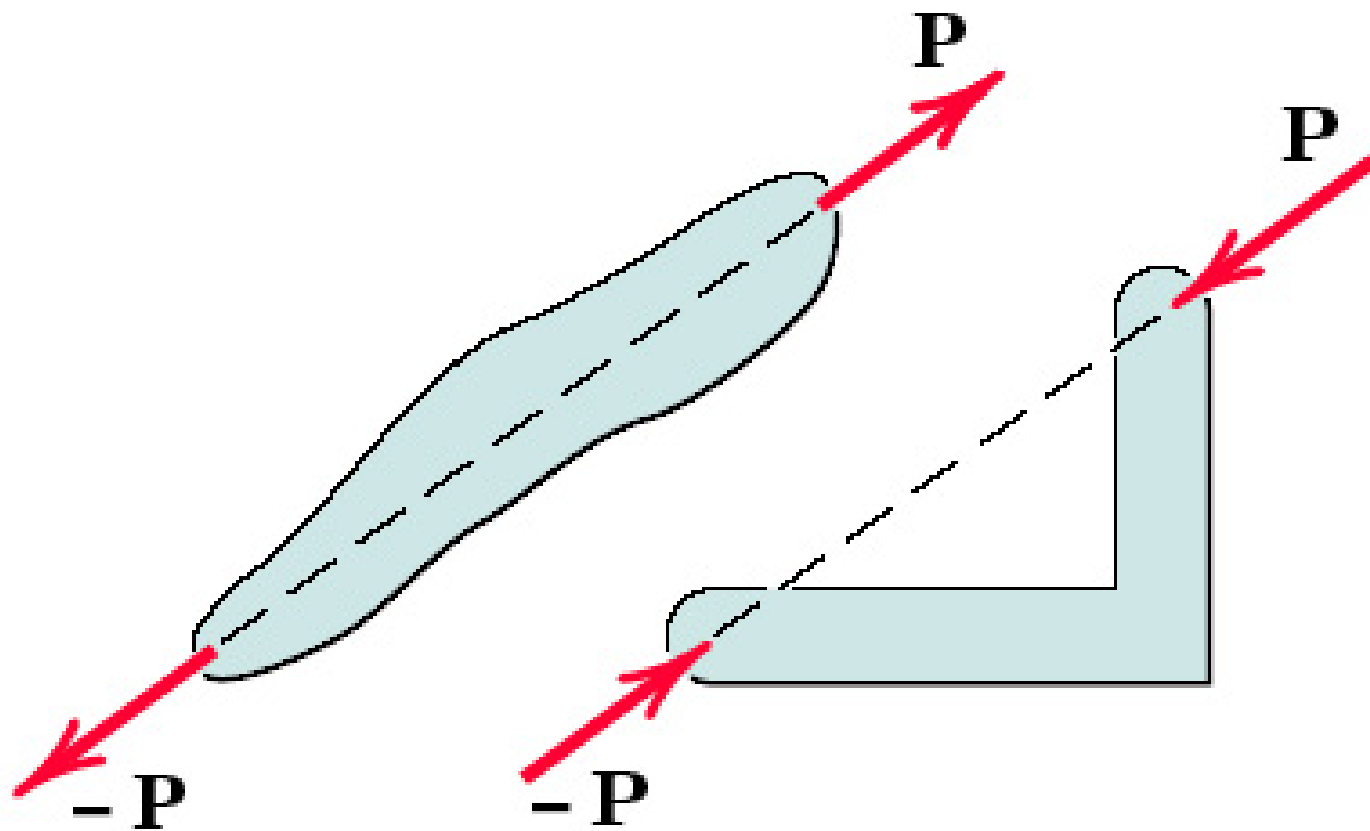
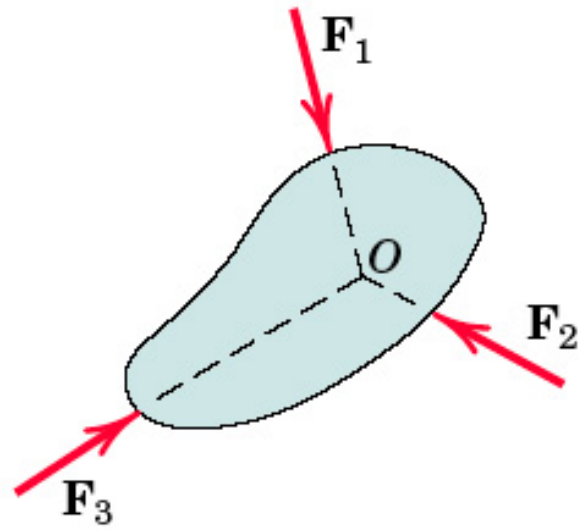
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

Figure 3/3

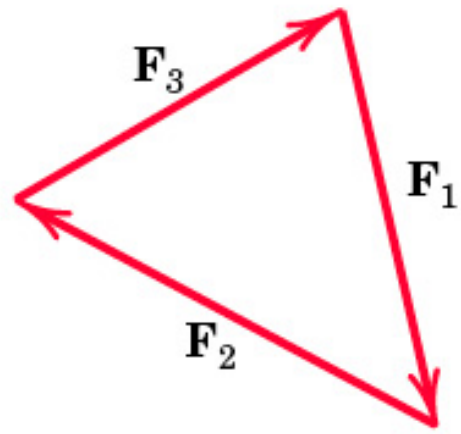


Two-force members

Figure 3/4



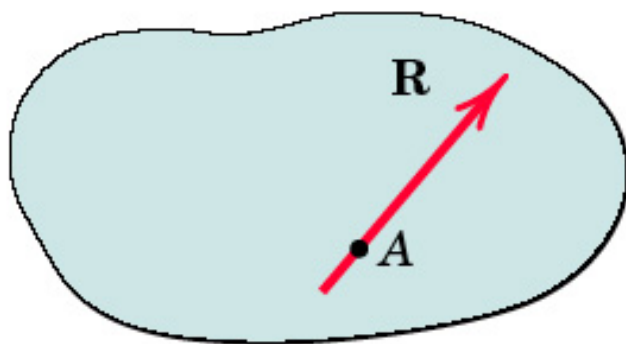
(a) Three-force member



(b) Closed polygon
satisfies $\Sigma \mathbf{F} = \mathbf{0}$

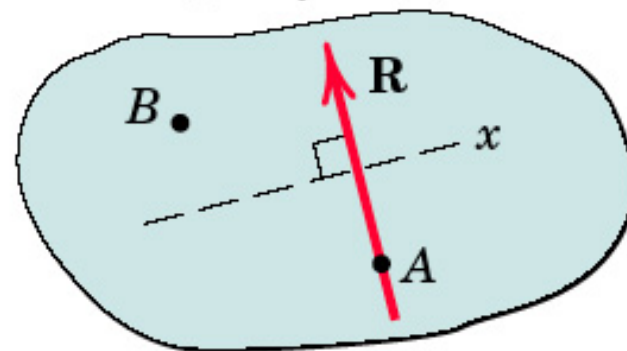
Figure 3/5

$\Sigma M_A = 0$ satisfied



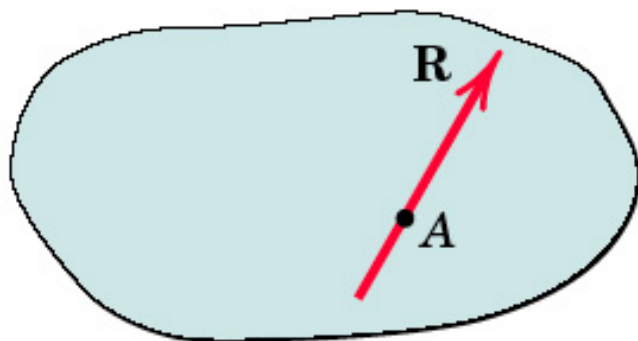
(a)

$\left. \begin{array}{l} \Sigma M_A = 0 \\ \Sigma F_x = 0 \end{array} \right\}$ satisfied



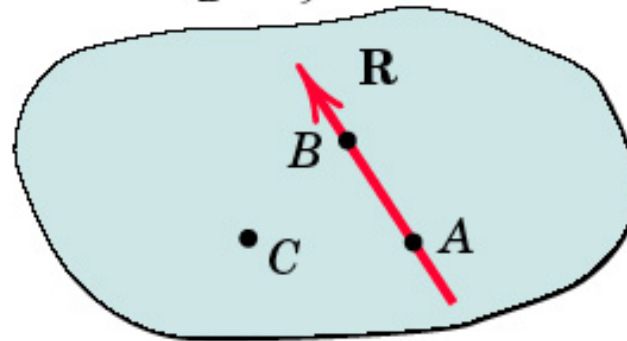
(b)

$\Sigma M_A = 0$ satisfied



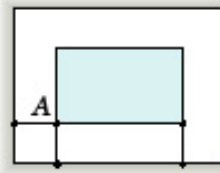
(c)

$\left. \begin{array}{l} \Sigma M_A = 0 \\ \Sigma M_B = 0 \end{array} \right\}$ satisfied

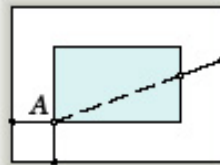


(d)

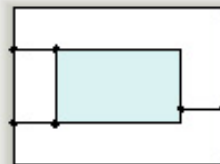
Figure 3/6



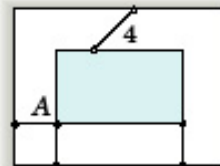
(a) Complete fixity
Adequate constraints



(b) Incomplete fixity
Partial constraints



(c) Incomplete fixity
Partial constraints



(d) Excessive fixity
Redundant constraint

Figure 3/7

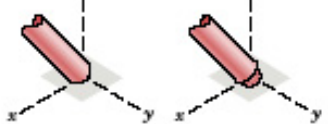
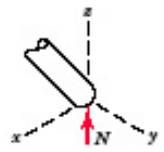
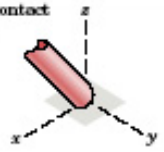

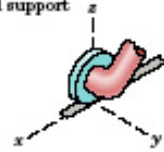
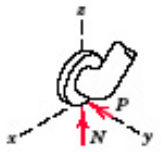
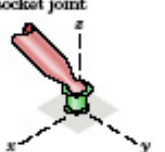
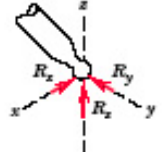
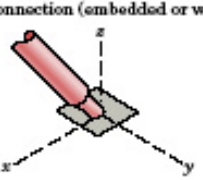
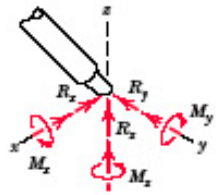
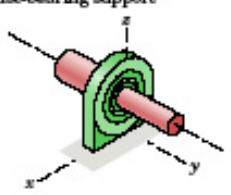
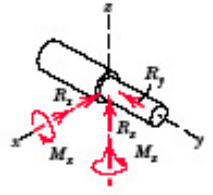
MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
<p>1. Member in contact with smooth surface, or ball-supported member</p> 	 <p>Force must be normal to the surface and directed toward the member.</p>
<p>2. Member in contact with rough surface</p> 	 <p>The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.</p>
<p>3. Roller or wheel support with lateral constraint</p> 	 <p>A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.</p>
<p>4. Ball-and-socket joint</p> 	 <p>A ball-and-socket joint free to pivot about the center of the ball can support a force R with all three components.</p>
<p>5. Fixed connection (embedded or welded)</p> 	 <p>In addition to three components of force, a fixed connection can support a couple M represented by its three components.</p>
<p>6. Thrust-bearing support</p> 	 <p>Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.</p>

Figure 3/8

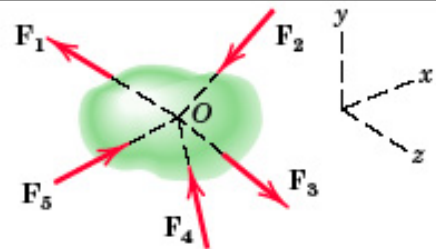
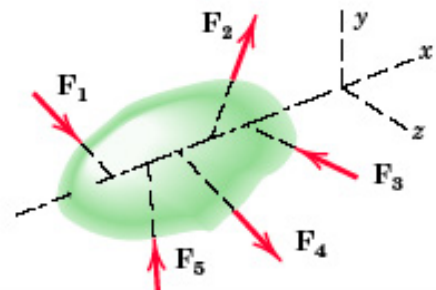
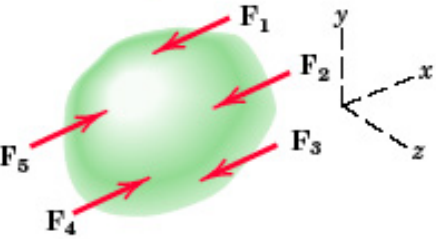
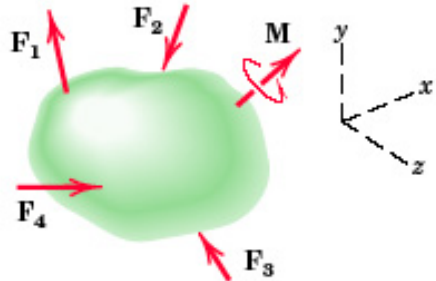
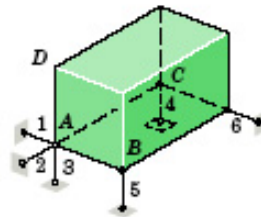
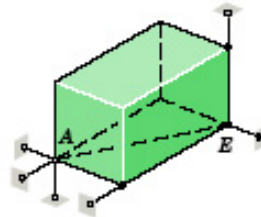
CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma F_y = 0$ $\Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\Sigma M_x = 0$ $\Sigma F_y = 0$ $\Sigma M_y = 0$ $\Sigma F_z = 0$ $\Sigma M_z = 0$

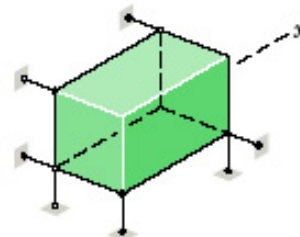
Figure 3/9



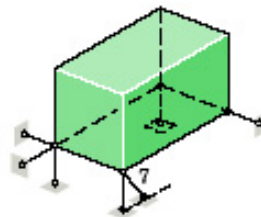
(a) Complete fixity
Adequate constraints



(b) Incomplete fixity
Partial constraints



(c) Incomplete fixity
Partial constraints



(d) Excessive fixity
Redundant constraints

Figure 3/10

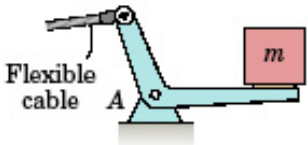
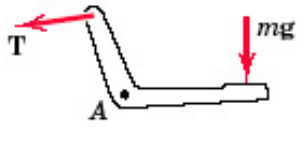
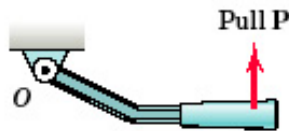

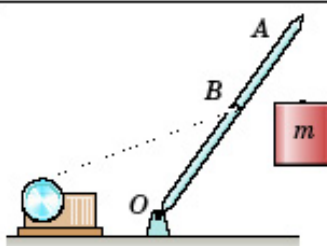
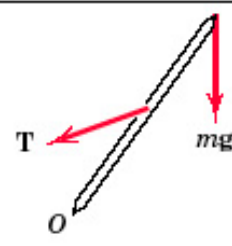
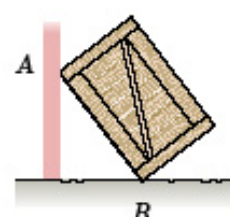
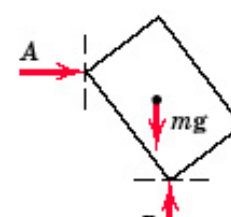
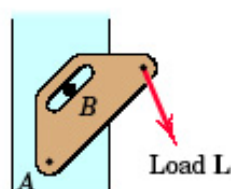
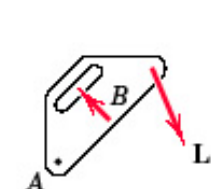
	Body	Incomplete <i>FBD</i>
1. Bell crank supporting mass m with pin support at A .		
2. Control lever applying torque to shaft at O .		
3. Boom OA , of negligible mass compared with mass m . Boom hinged at O and supported by hoisting cable at B .		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B .		

Figure 3/A

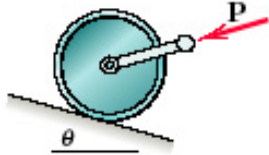

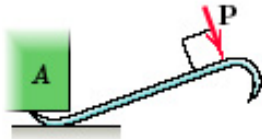
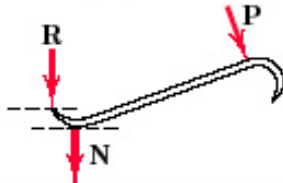
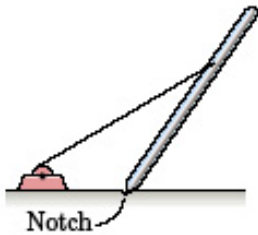
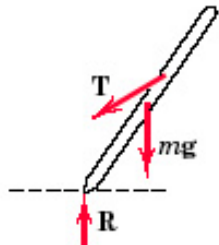
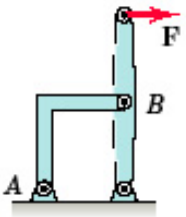
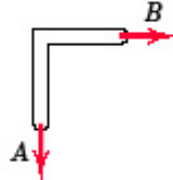
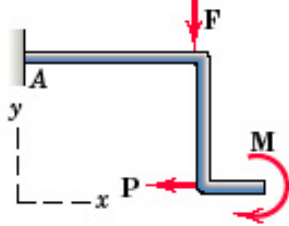
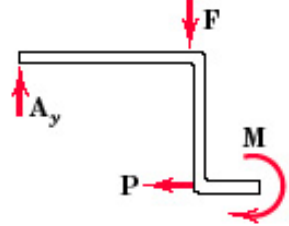
	Body	Wrong or Incomplete <i>FBD</i>
1. Lawn roller of mass m being pushed up incline θ .		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		
4. Supporting angle bracket for frame; pin joints.		
5. Bent rod welded to support at A and subjected to two forces and couple.		

Figure 3/B

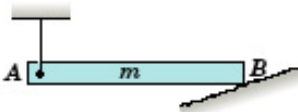
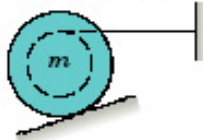
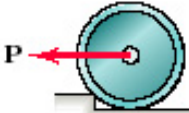
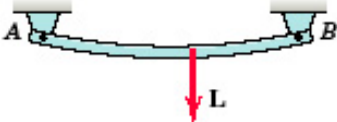
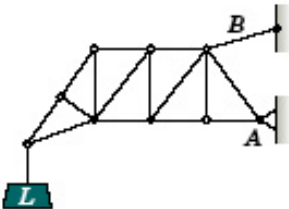
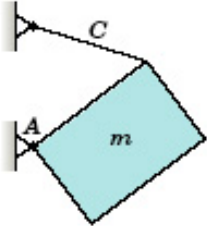
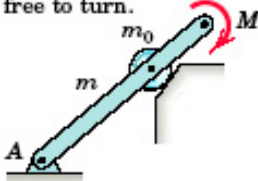
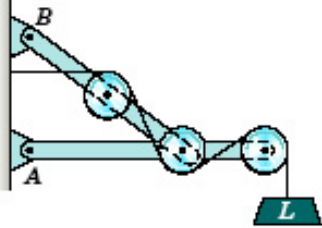
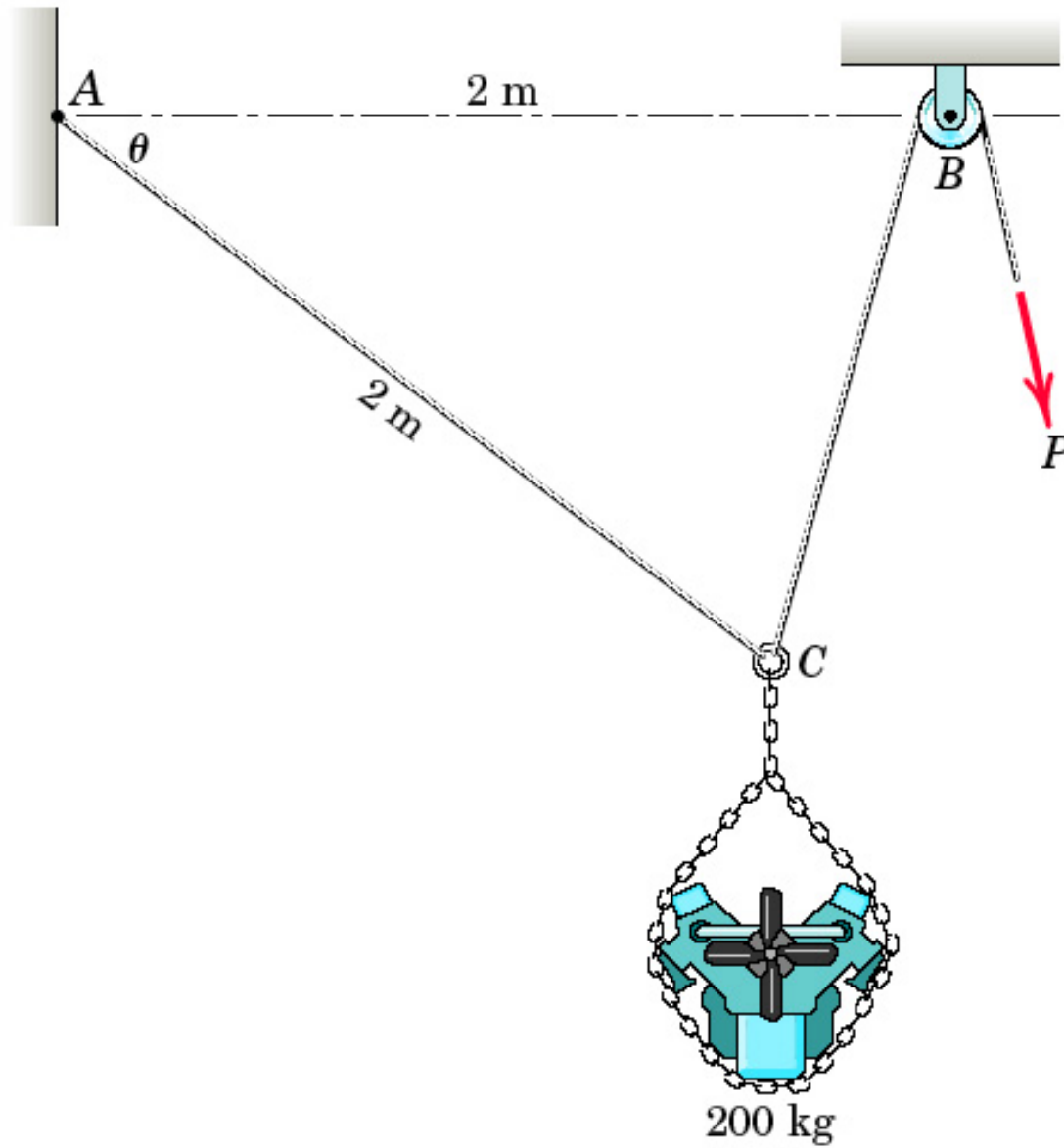
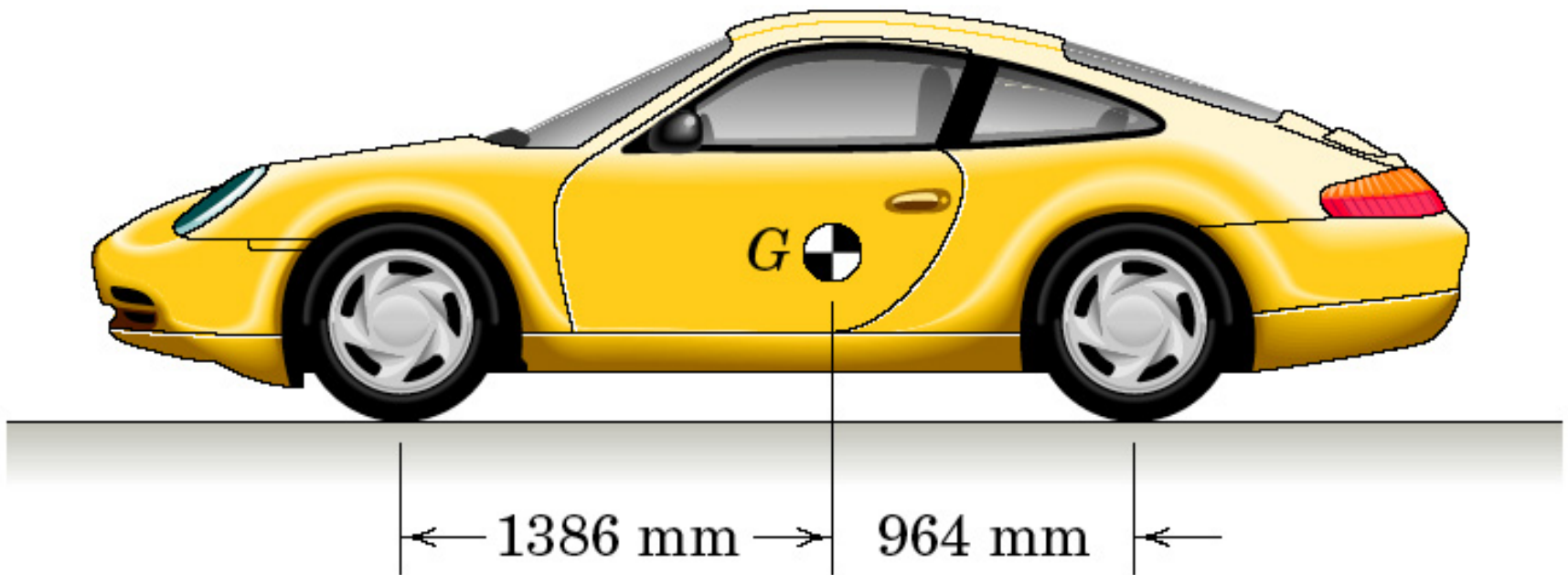
<p>1. Uniform horizontal bar of mass m suspended by vertical cable at A and supported by rough inclined surface at B.</p> 	<p>5. Uniform grooved wheel of mass m supported by a rough surface and by action of horizontal cable.</p> 
<p>2. Wheel of mass m on verge of being rolled over curb by pull P.</p> 	<p>6. Bar, initially horizontal but deflected under load L. Pinned to rigid support at each end.</p> 
<p>3. Loaded truss supported by pin joint at A and by cable at B.</p> 	<p>7. Uniform heavy plate of mass m supported in vertical plane by cable C and hinge A.</p> 
<p>4. Uniform bar of mass m and roller of mass m_0 taken together. Subjected to couple M and supported as shown. Roller is free to turn.</p> 	<p>8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.</p> 

Figure 3/C

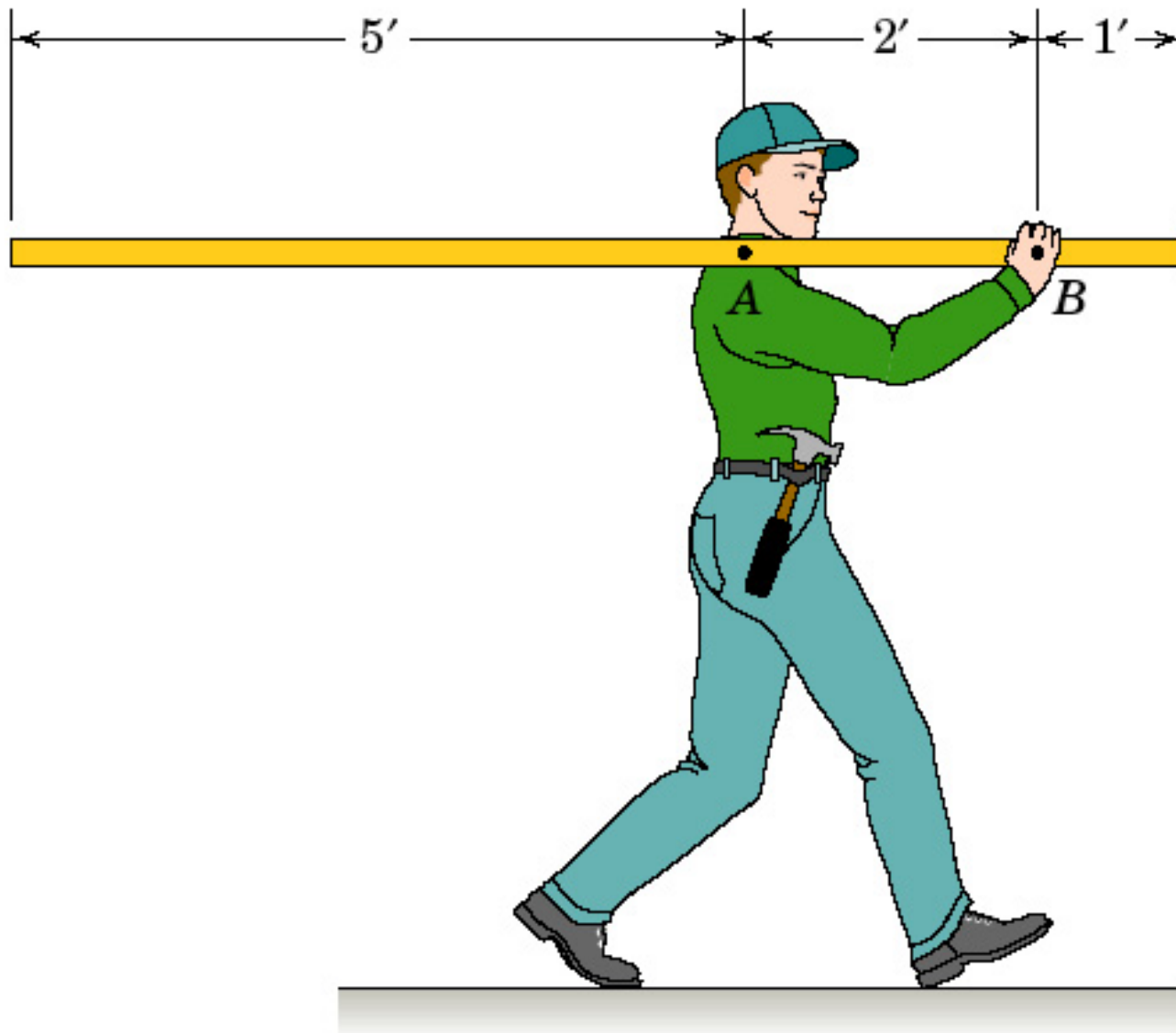


Problem 3/1

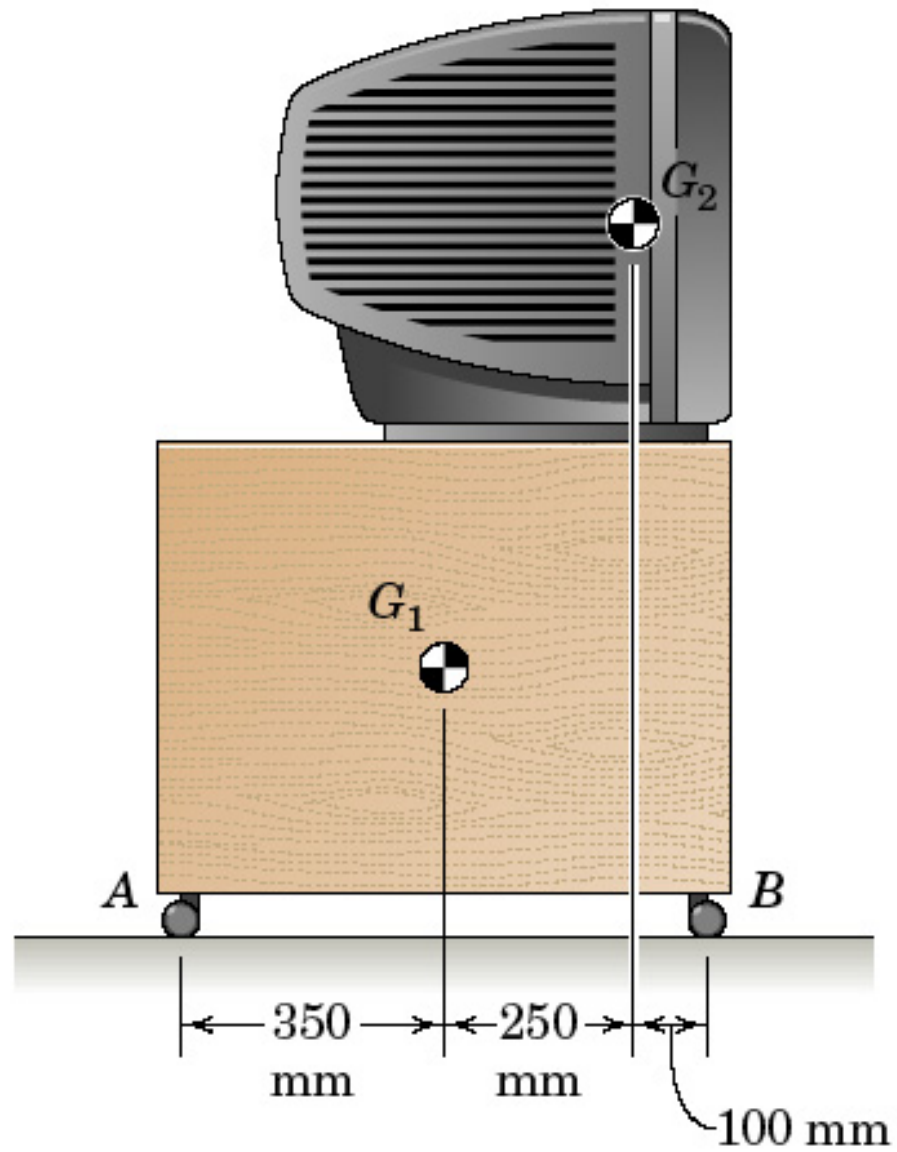


Problem 3/2

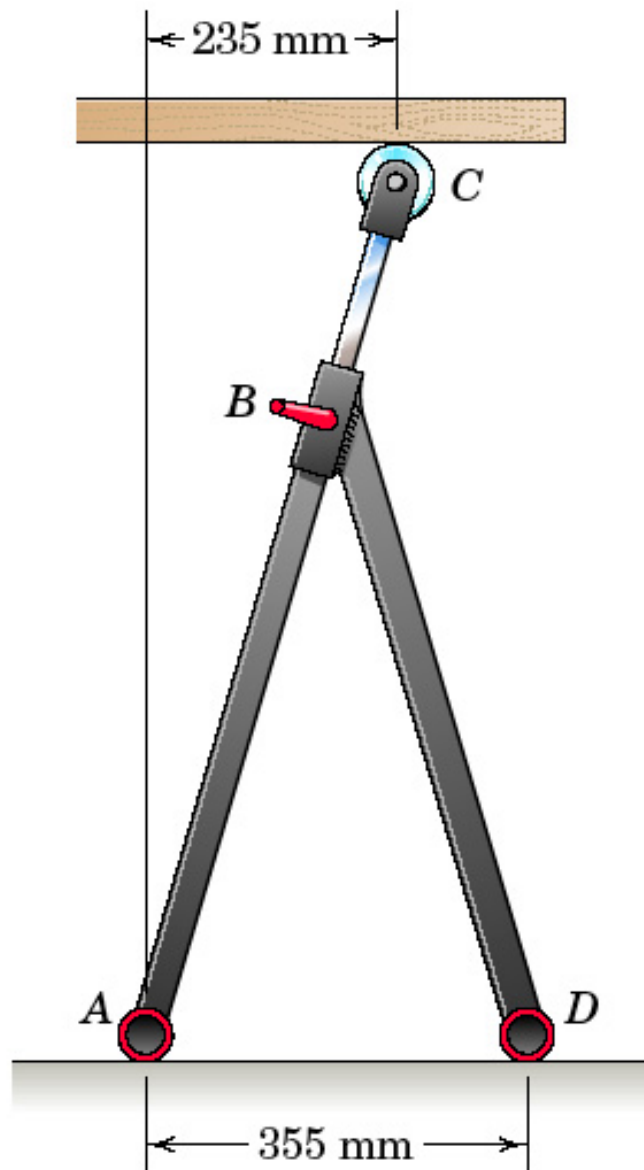
P_03_02



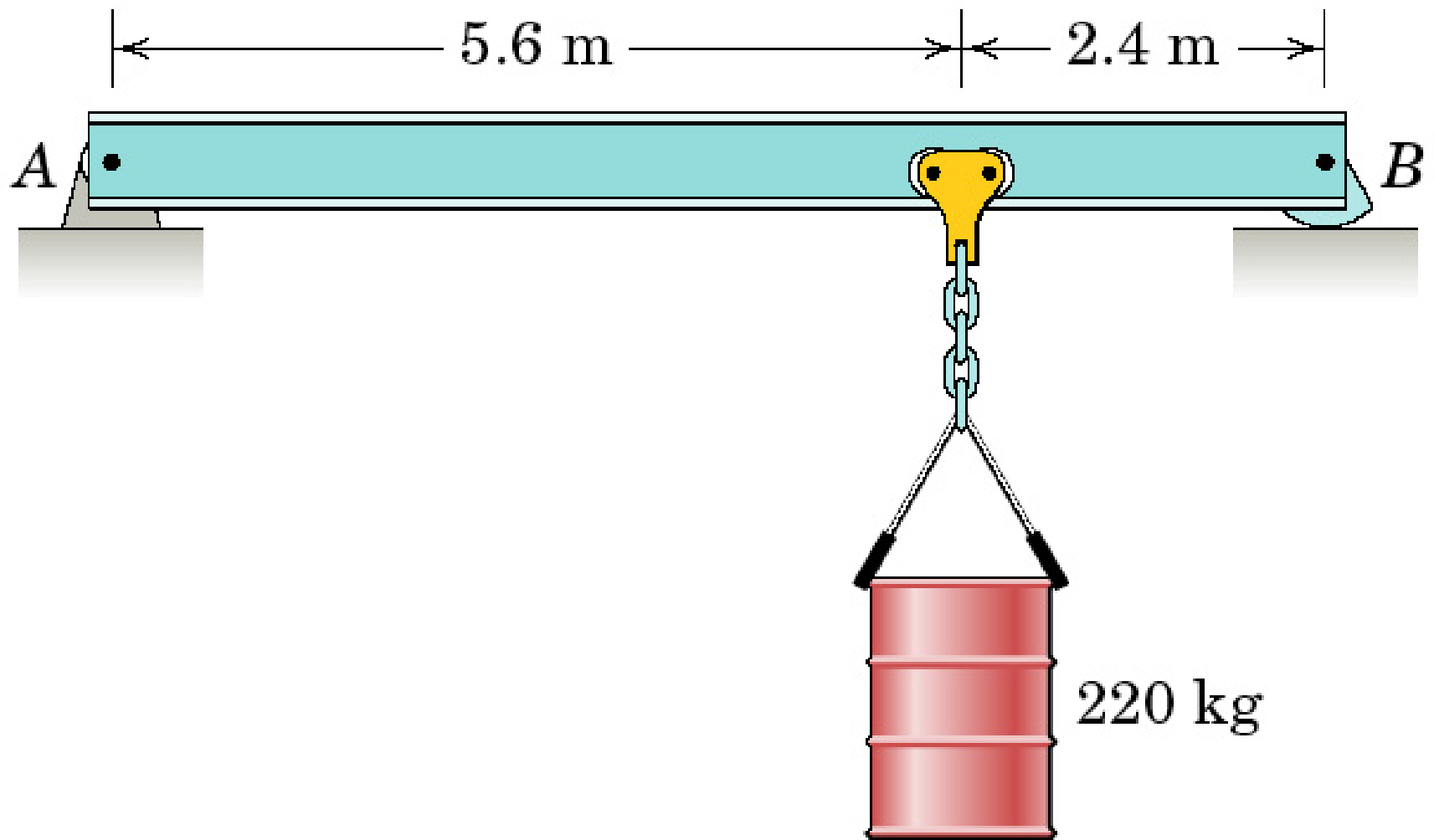
Problem 3/3



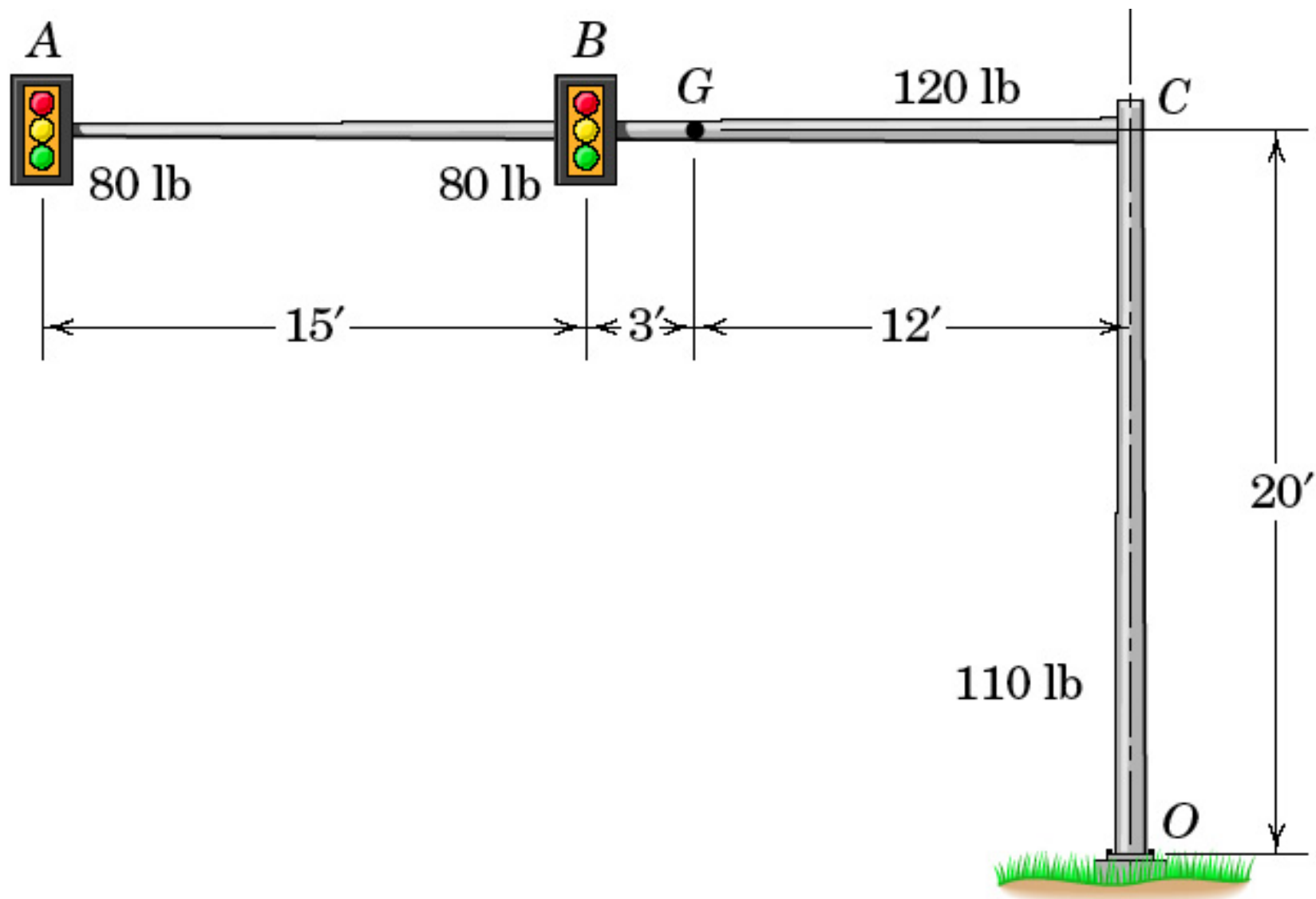
Problem 3/4



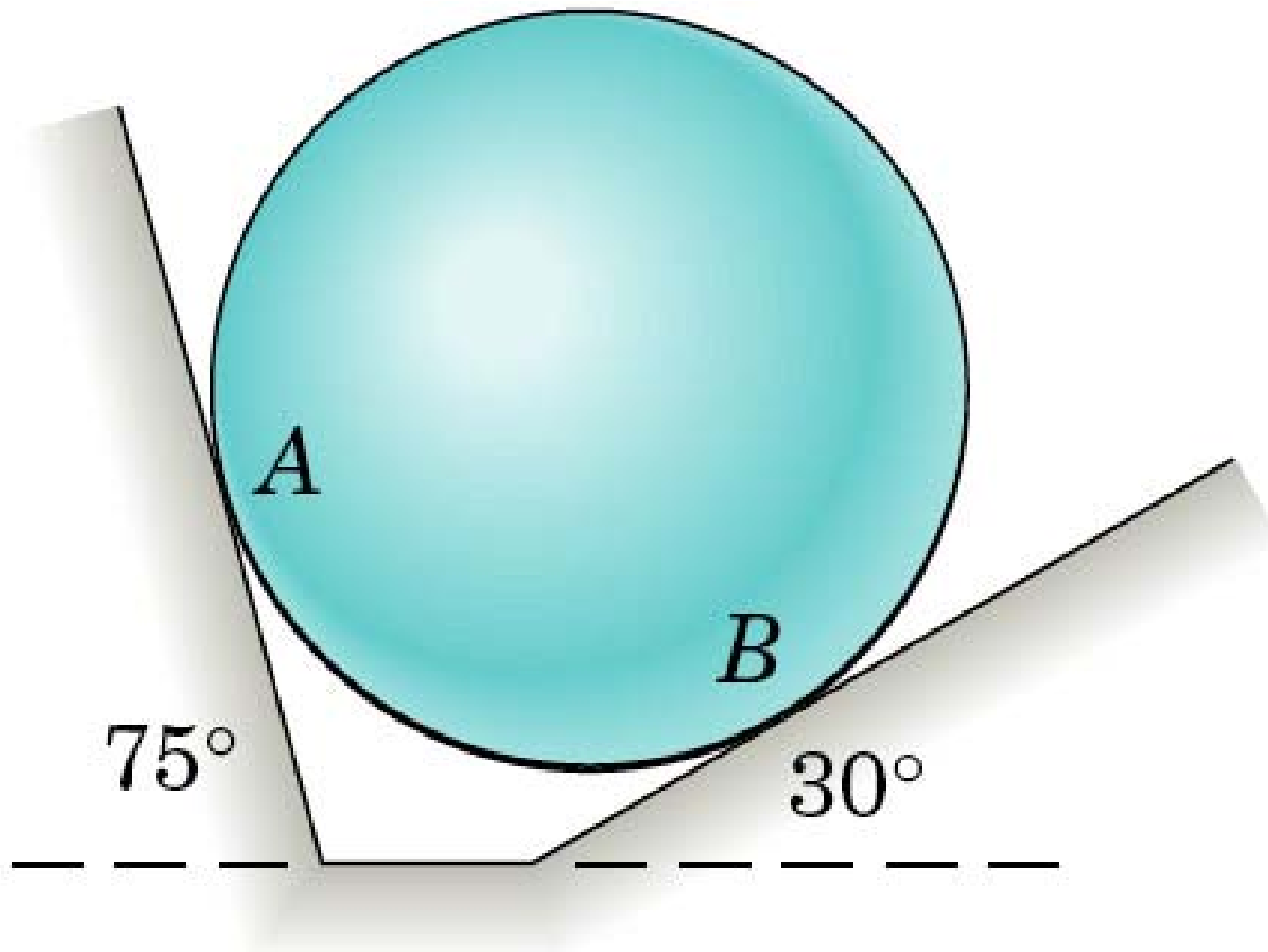
Problem 3/5



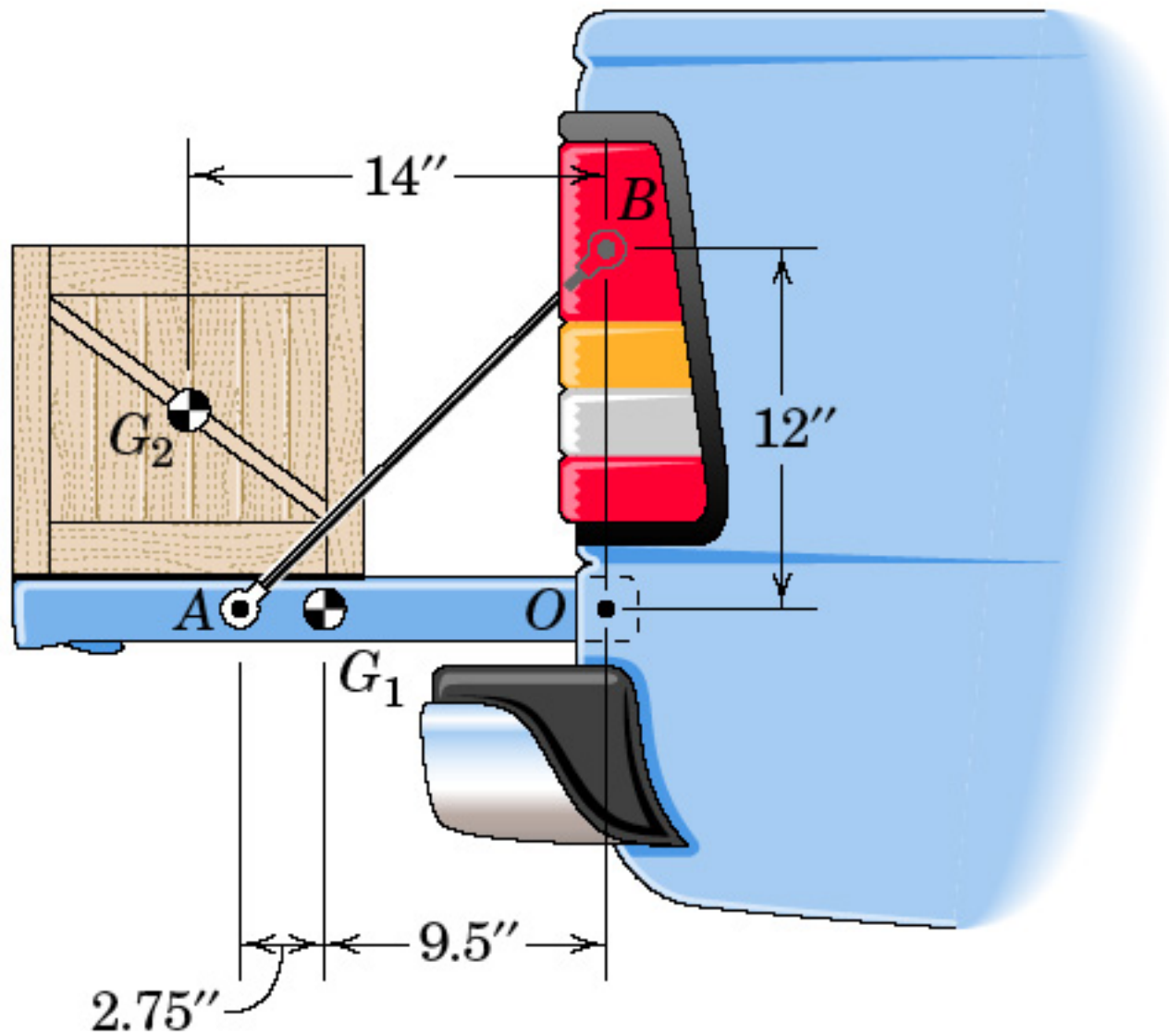
Problem 3/6



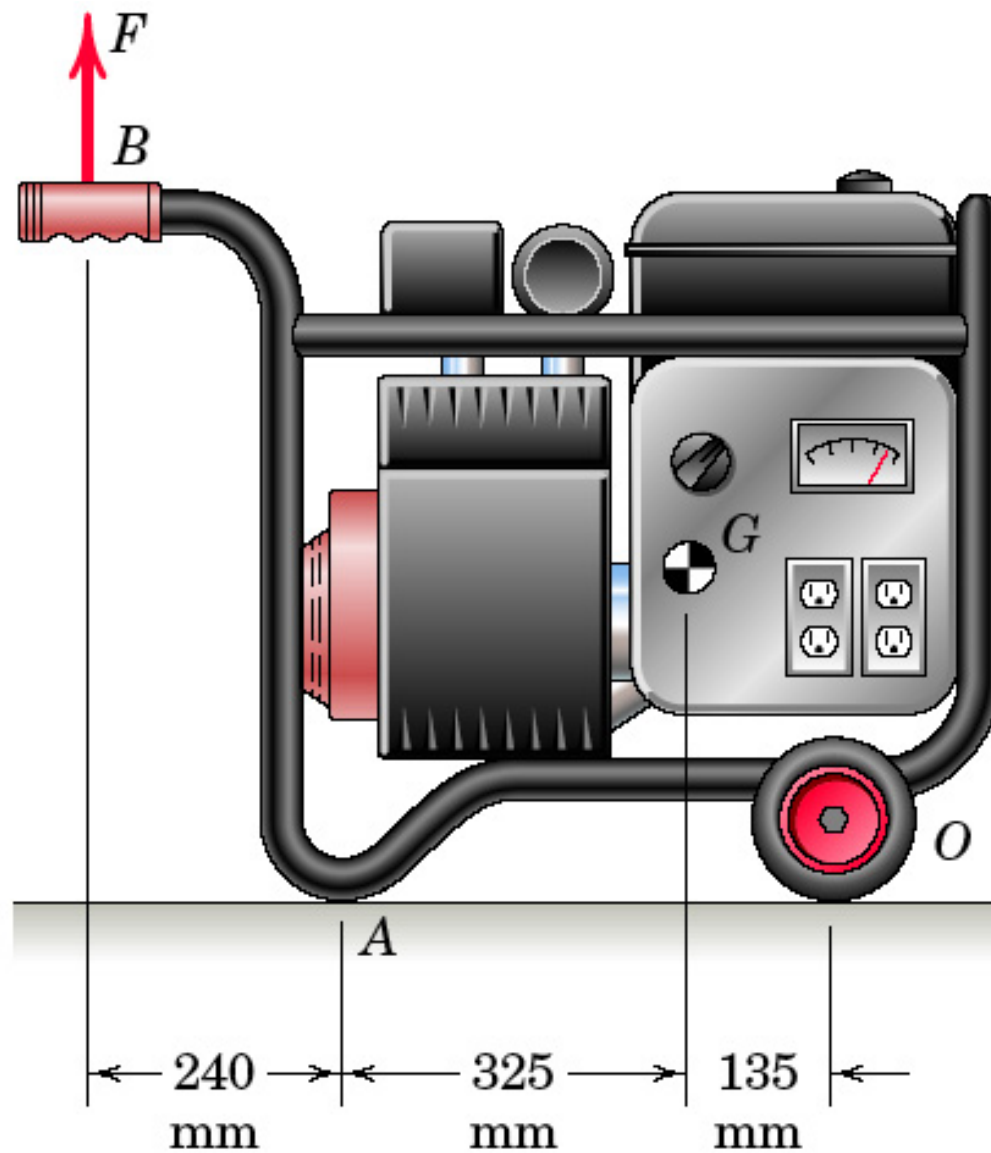
Problem 3/7



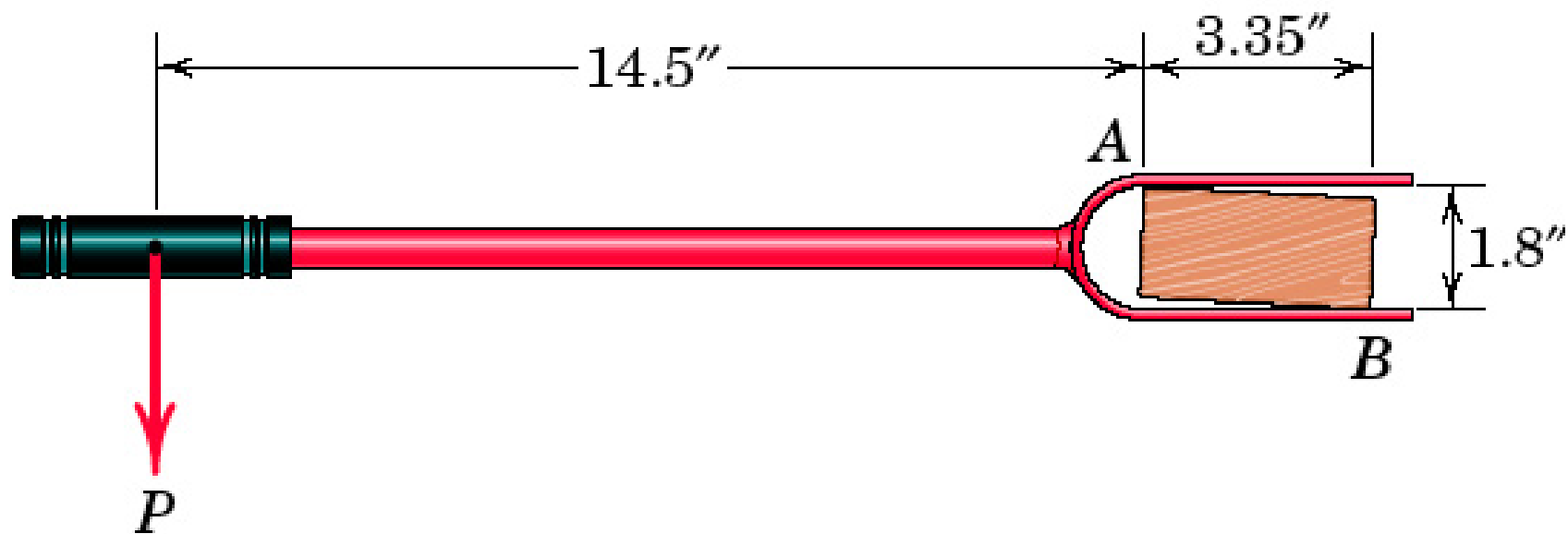
Problem 3/8



Problem 3/9

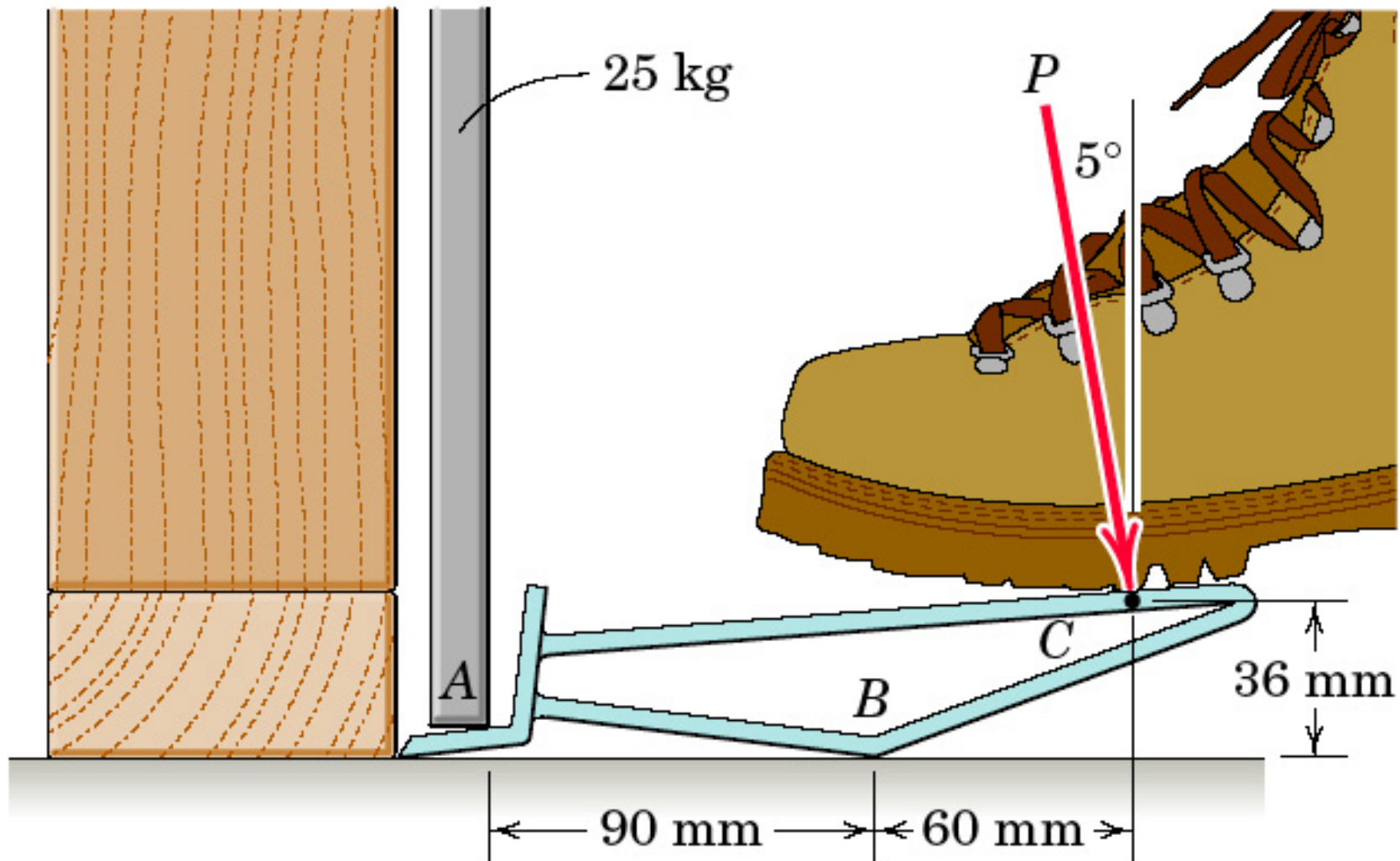


Problem 3/10

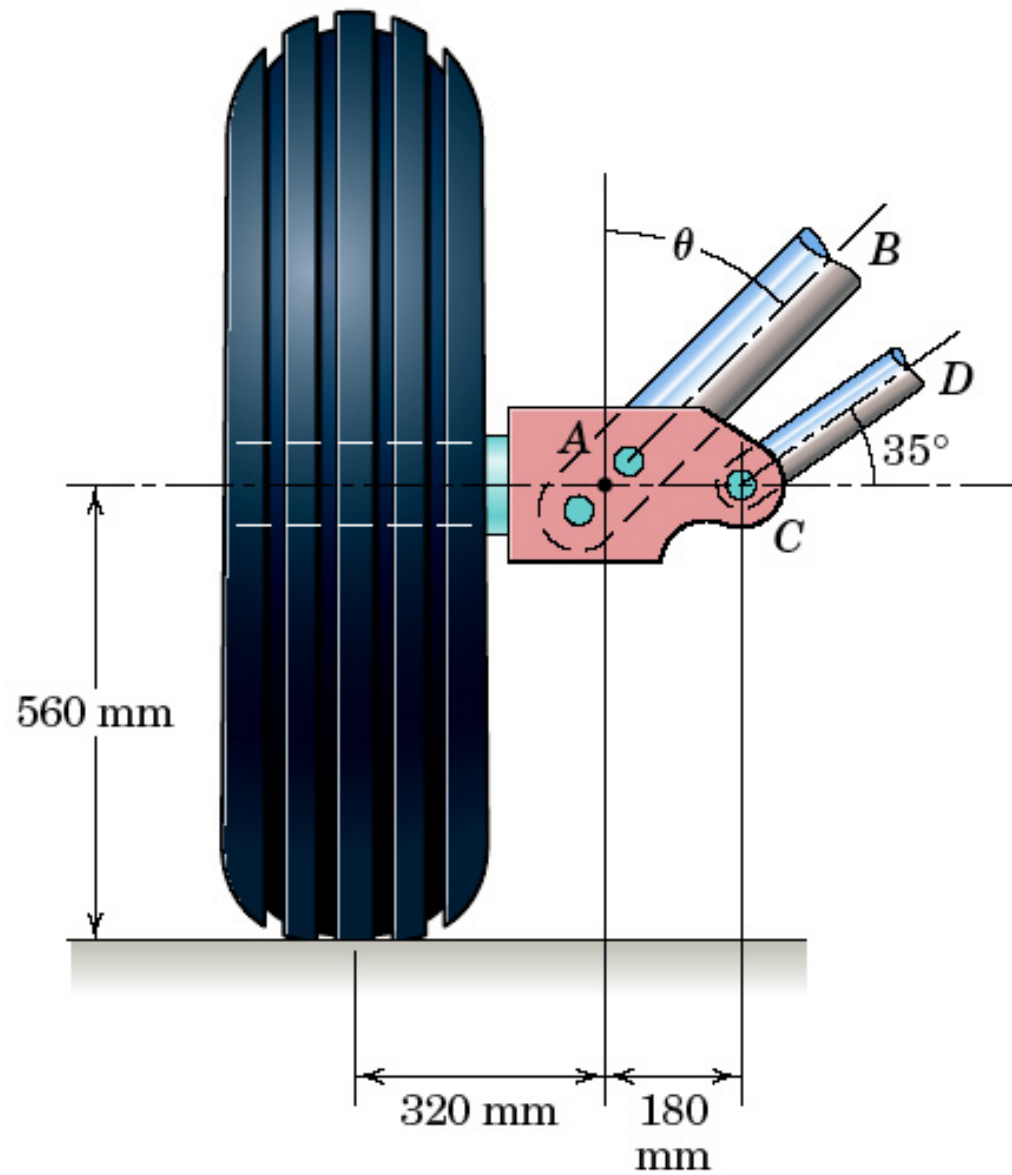


Problem 3/100

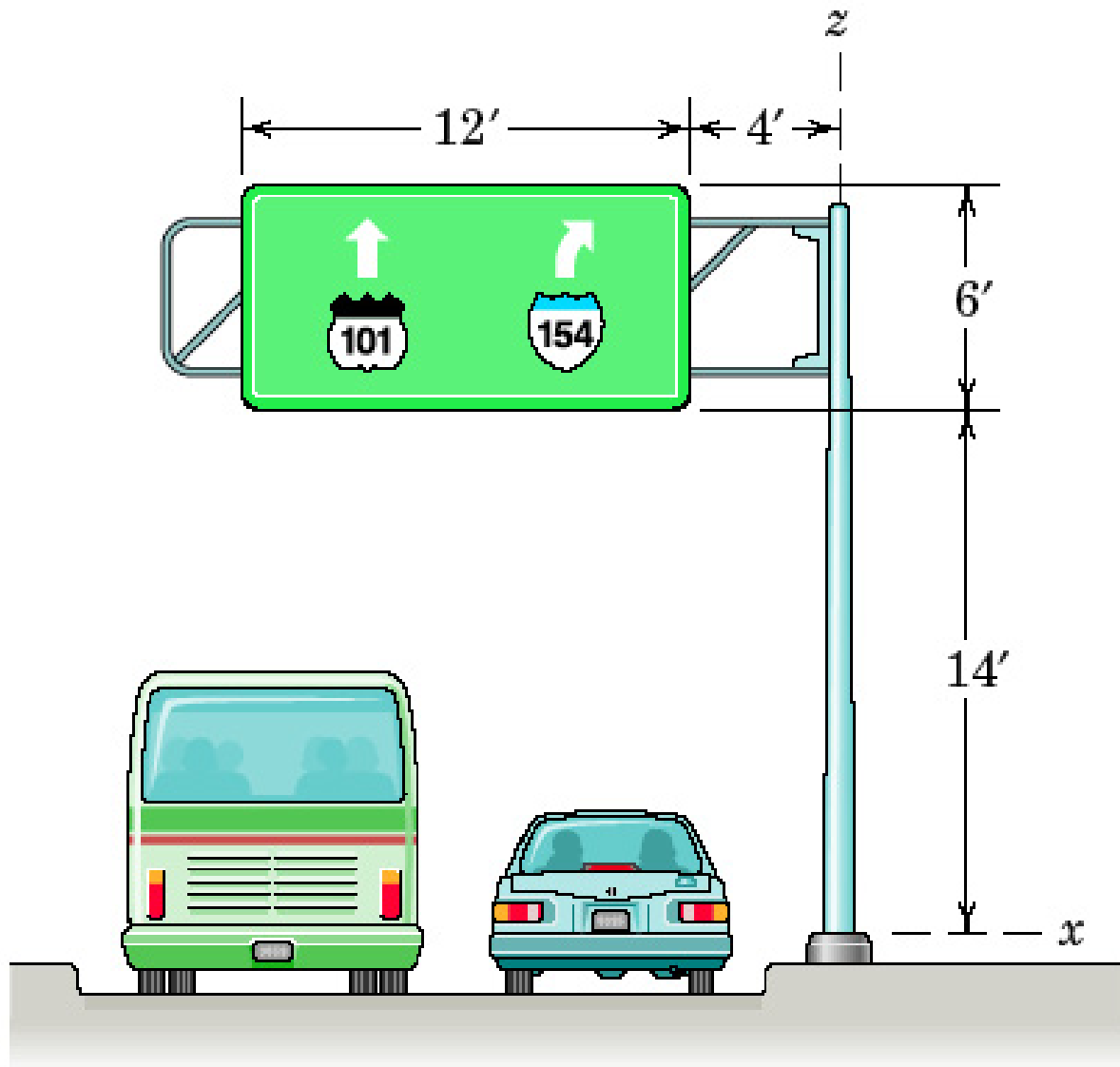
P_03_100



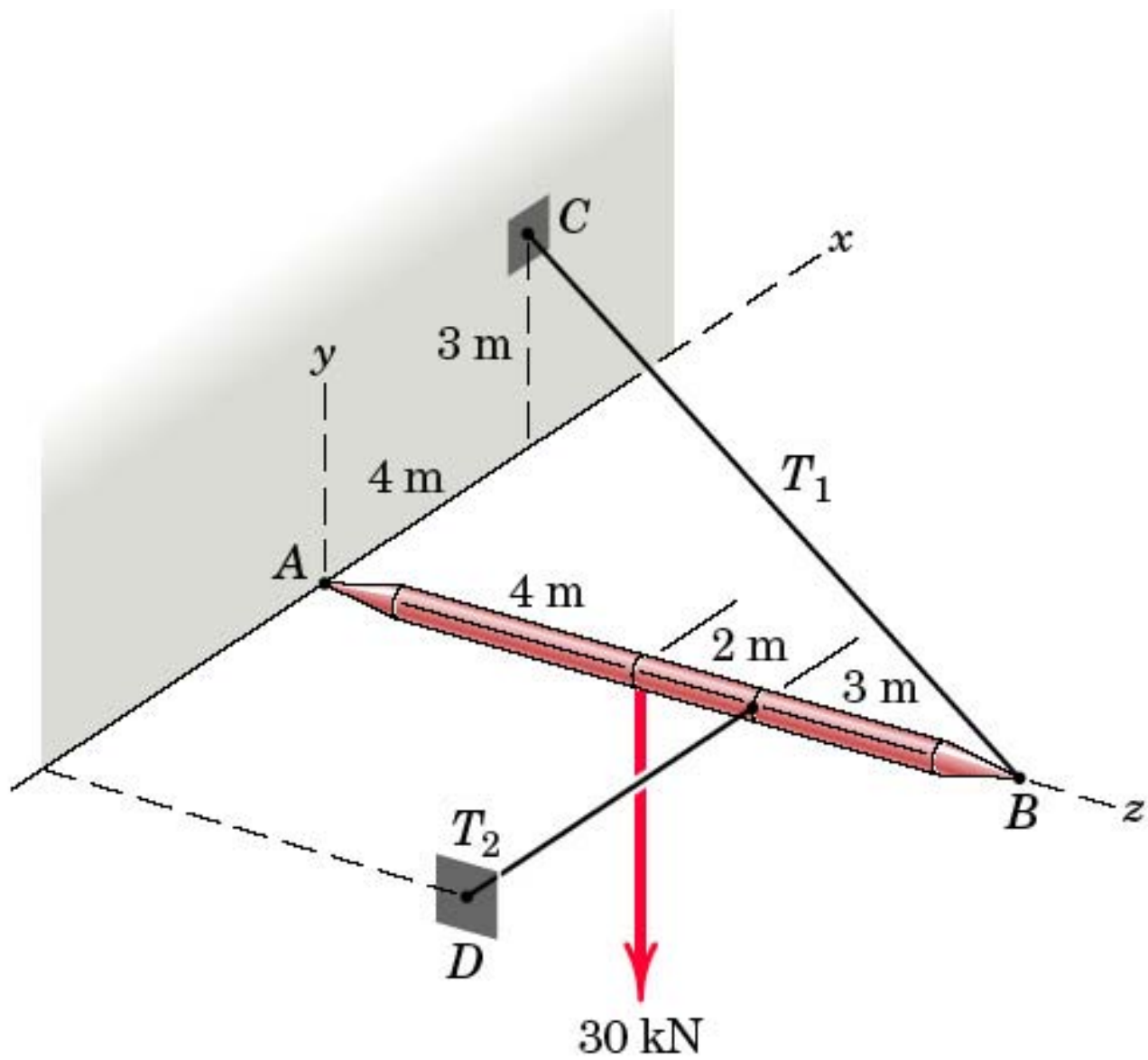
Problem 3/101



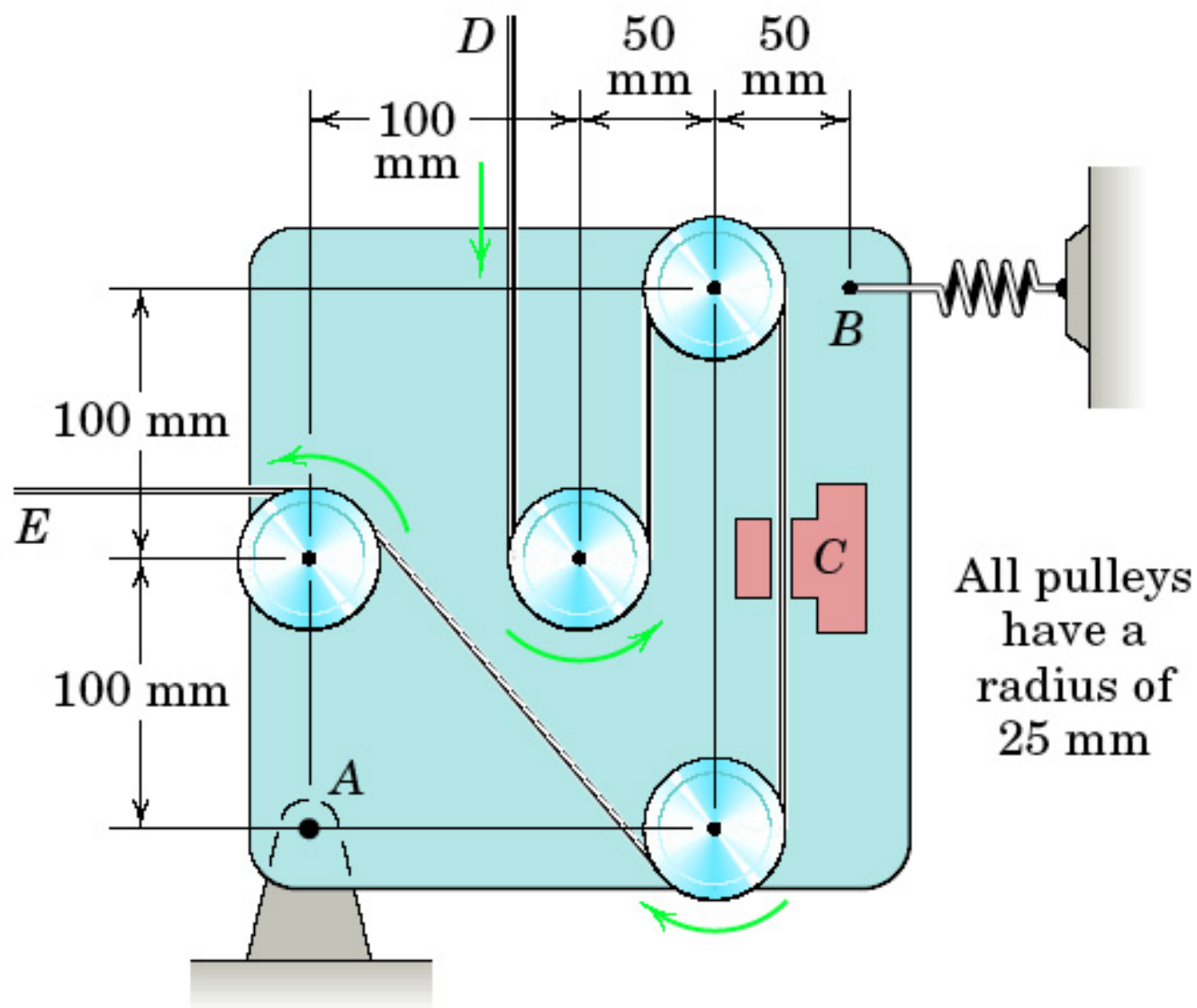
Problem 3/102



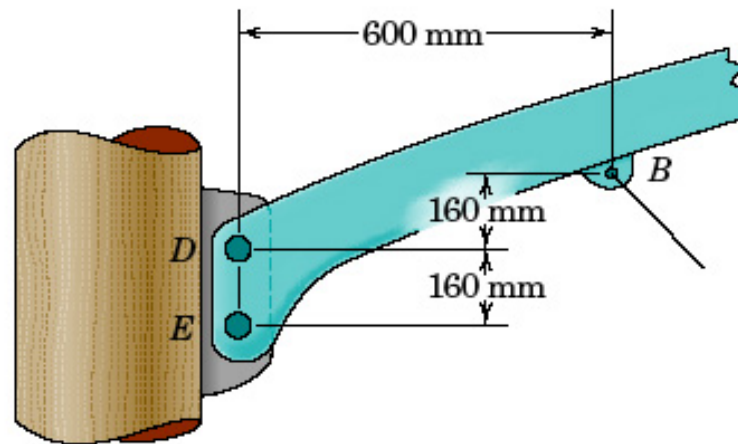
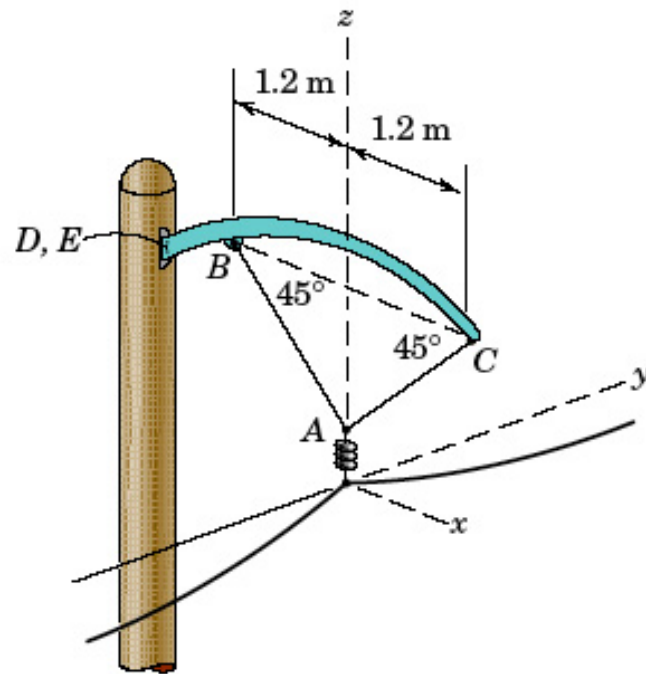
Problem 3/103



Problem 3/104

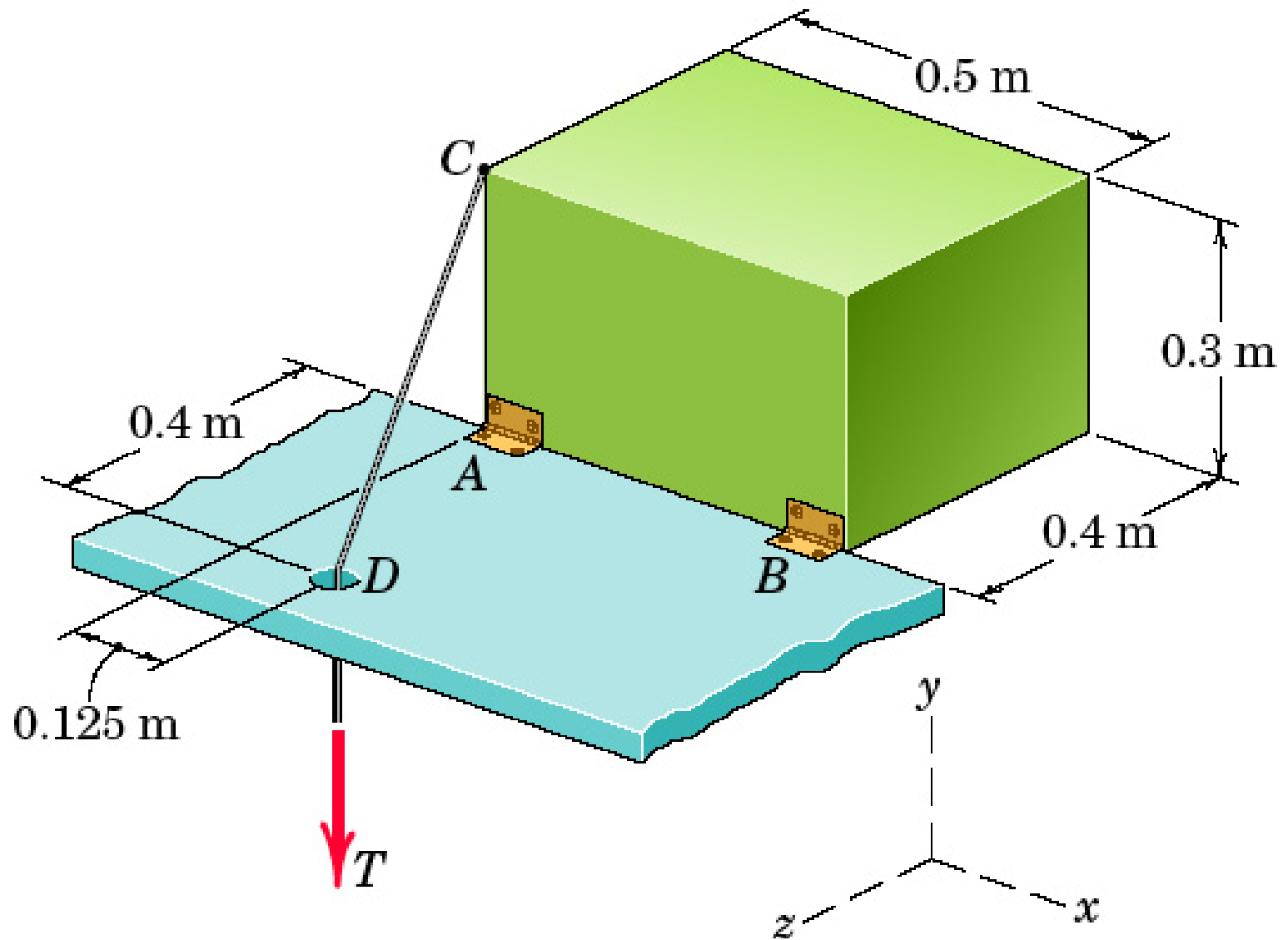


Problem 3/105

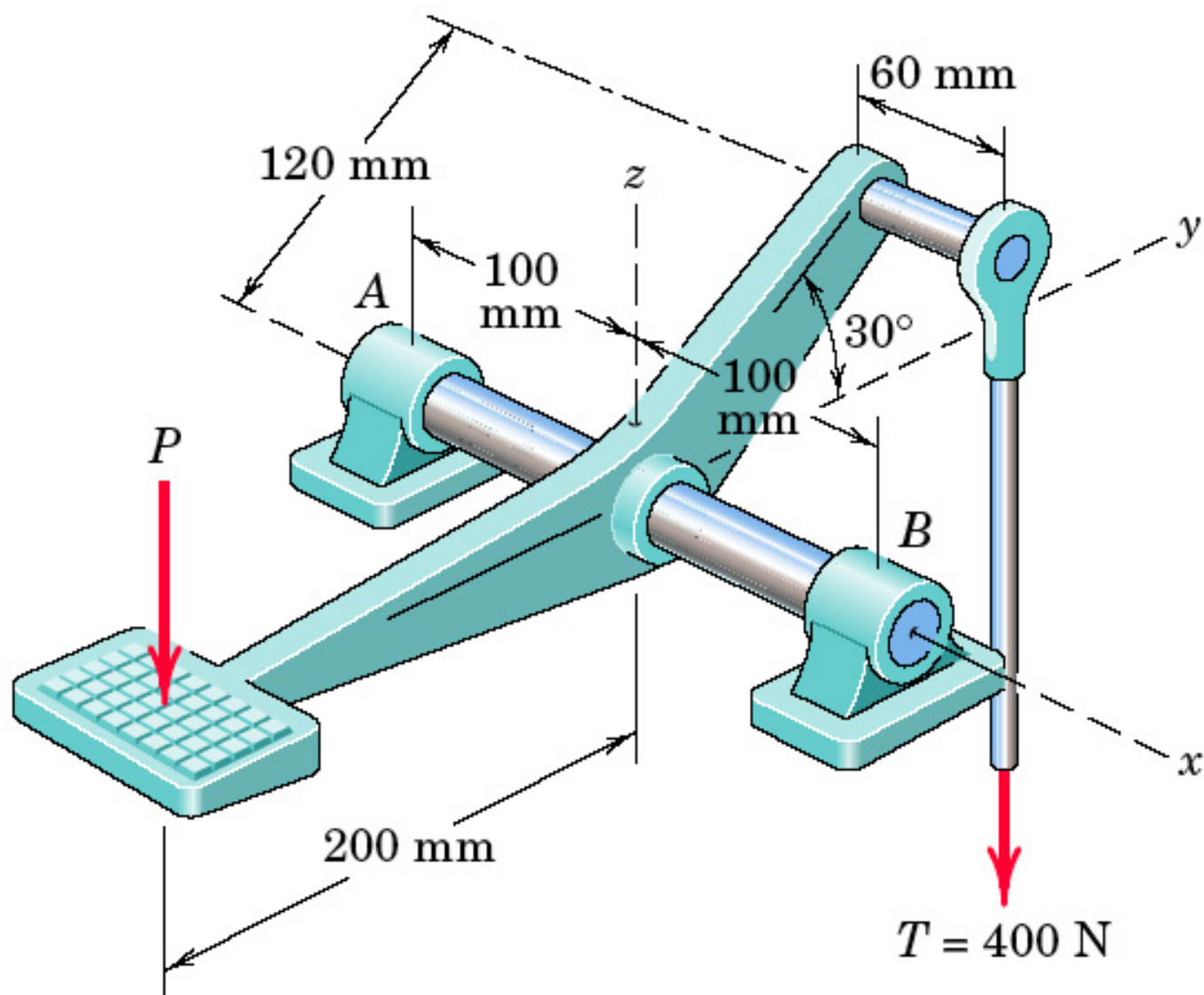


Detail of arm attachment

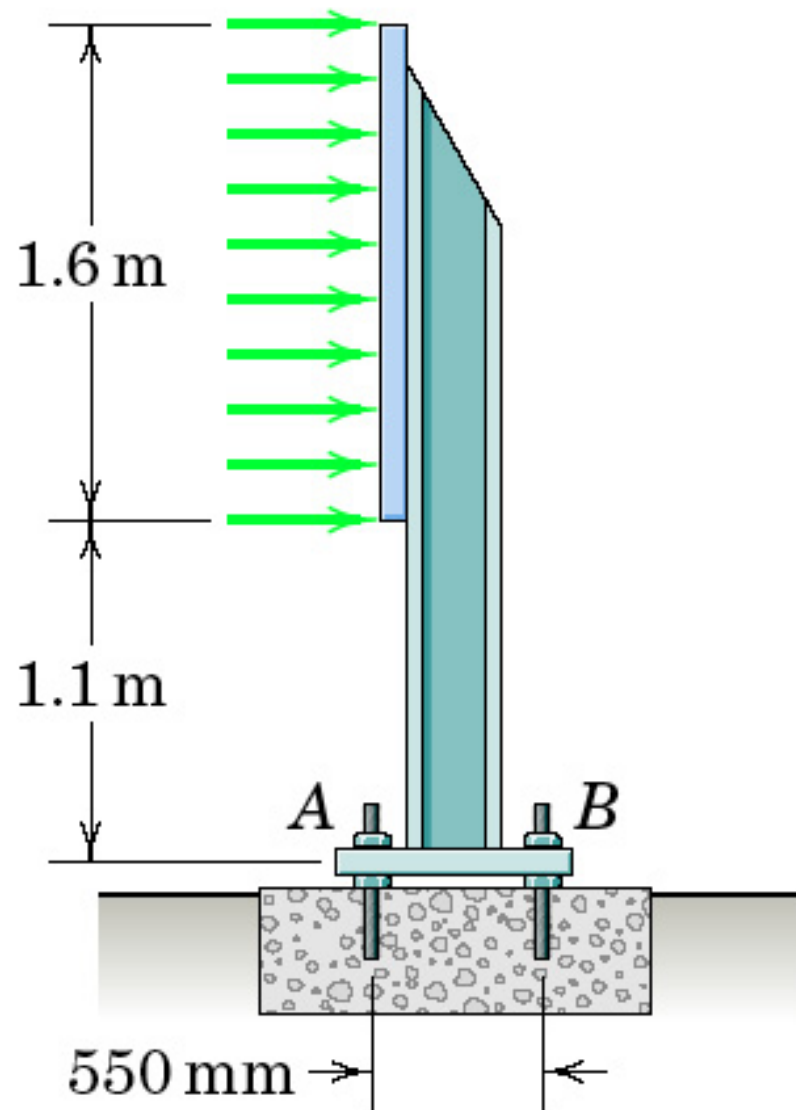
Problem 3/106



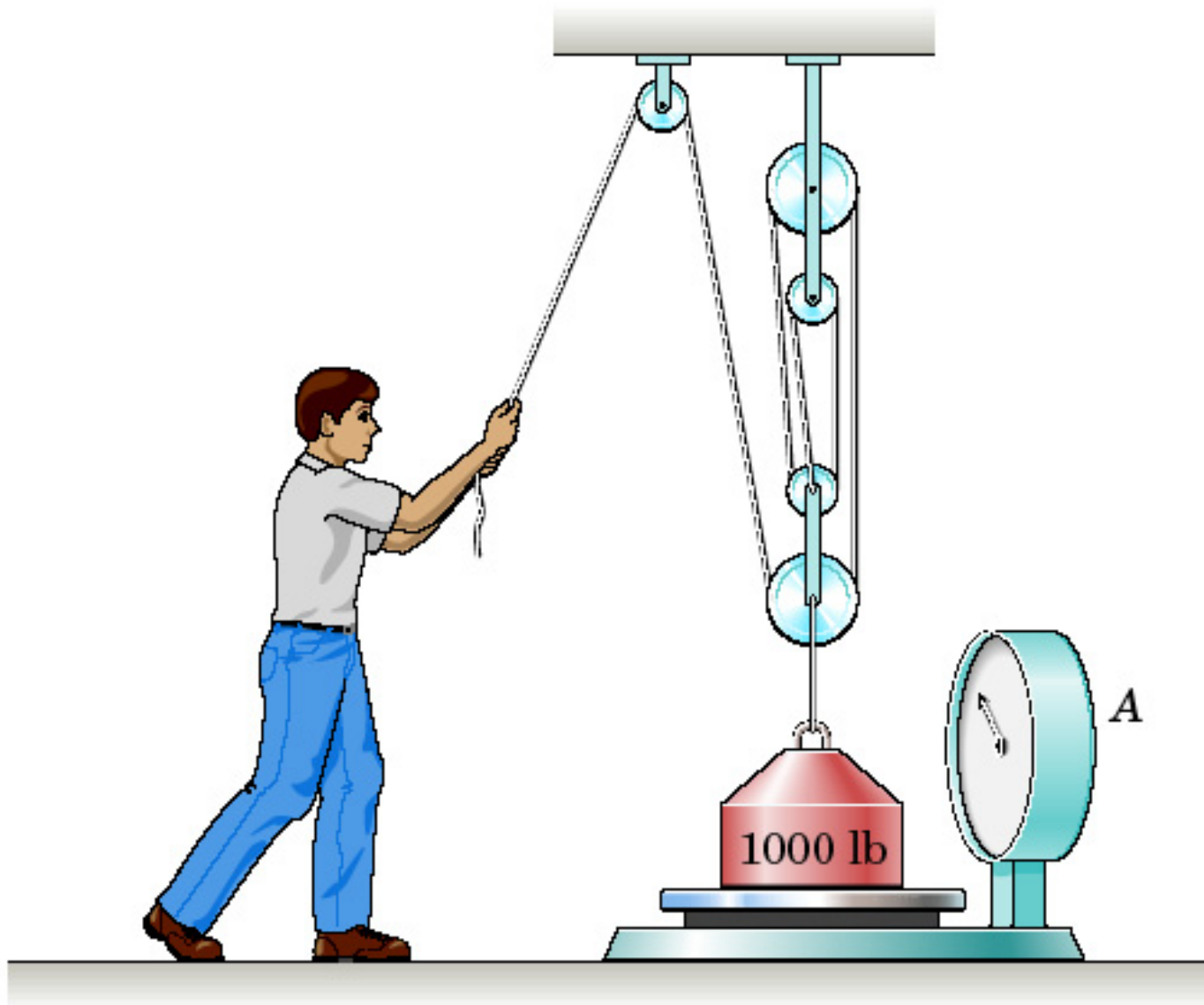
Problem 3/107



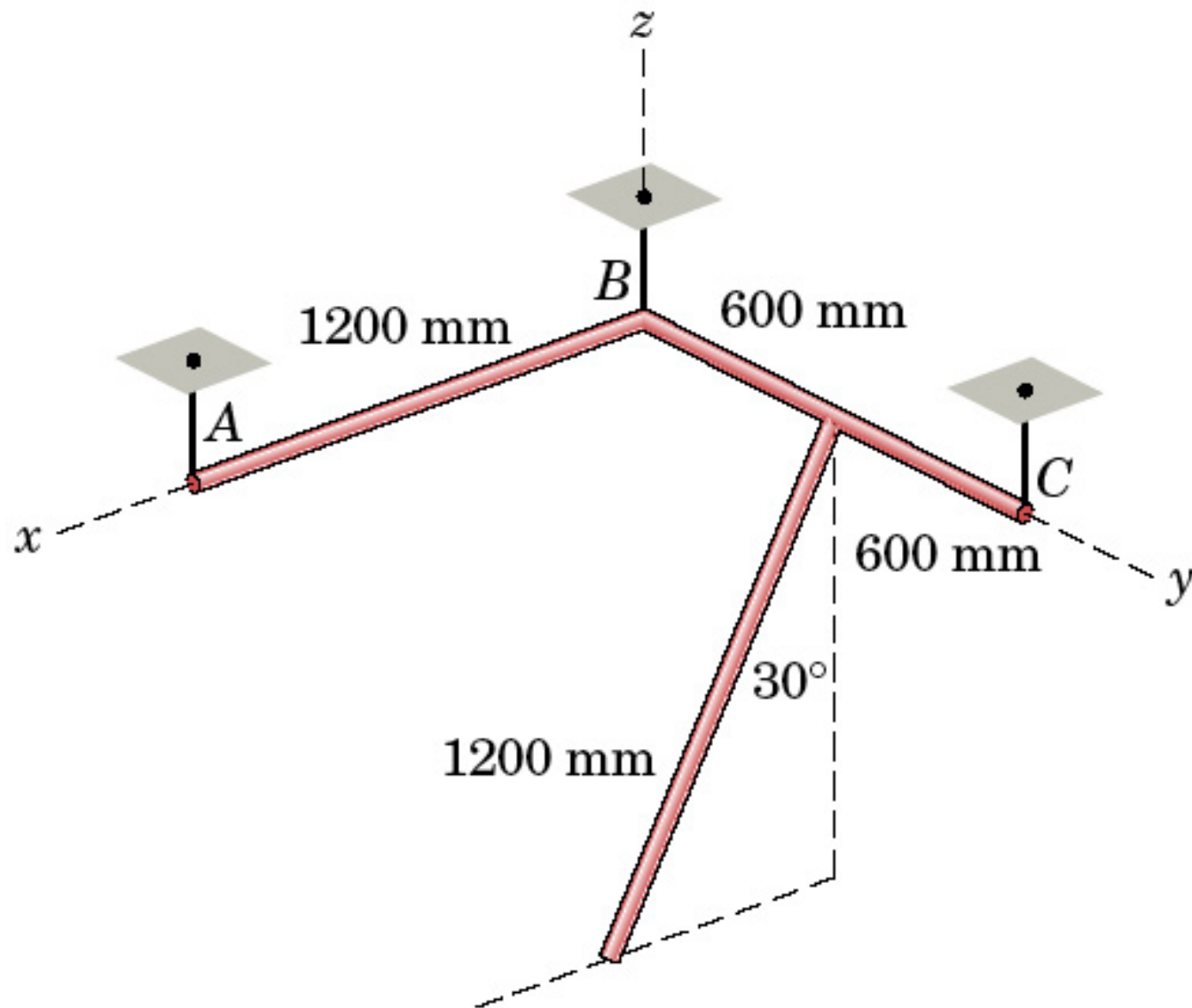
Problem 3/108



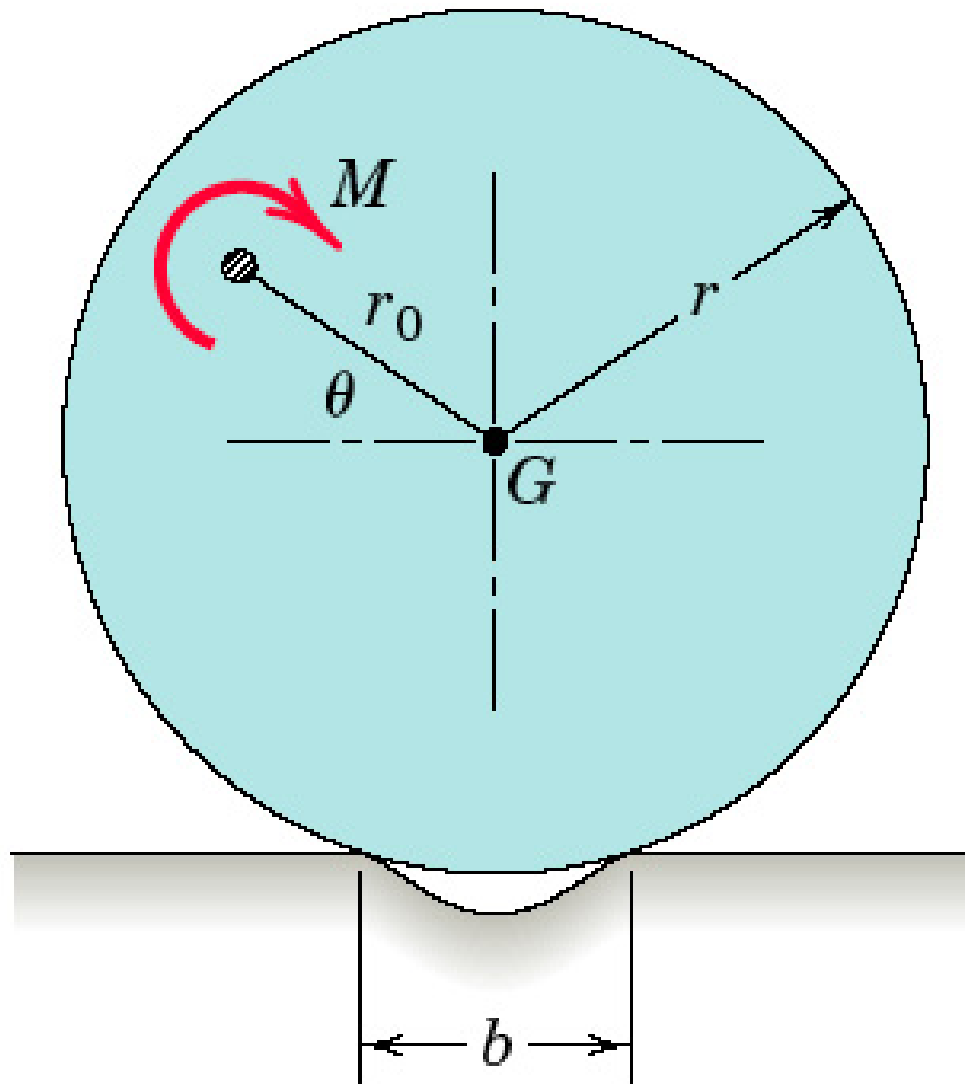
Problem 3/109



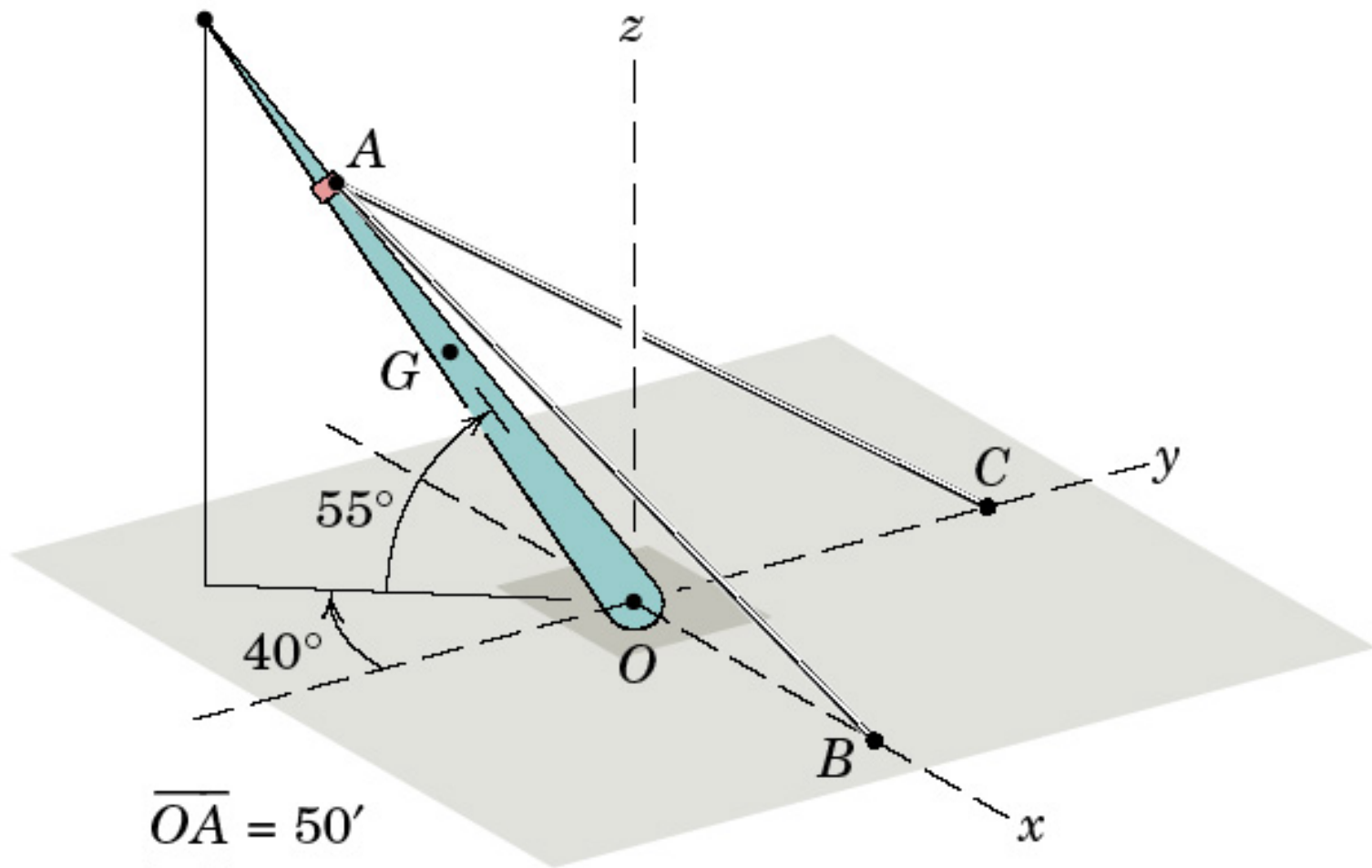
Problem 3/11



Problem 3/110



Problem 3/111

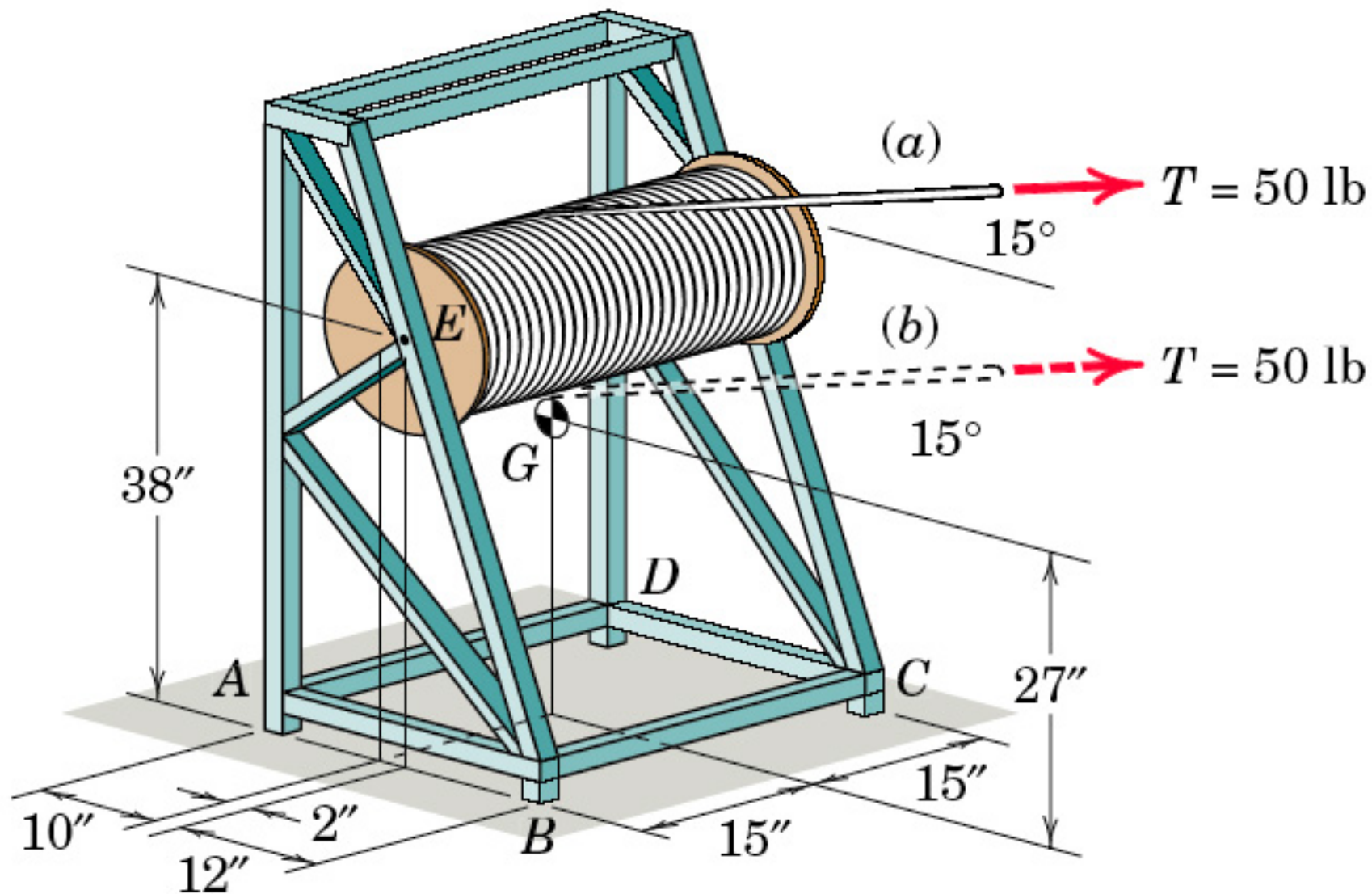


$$\overline{OA} = 50'$$

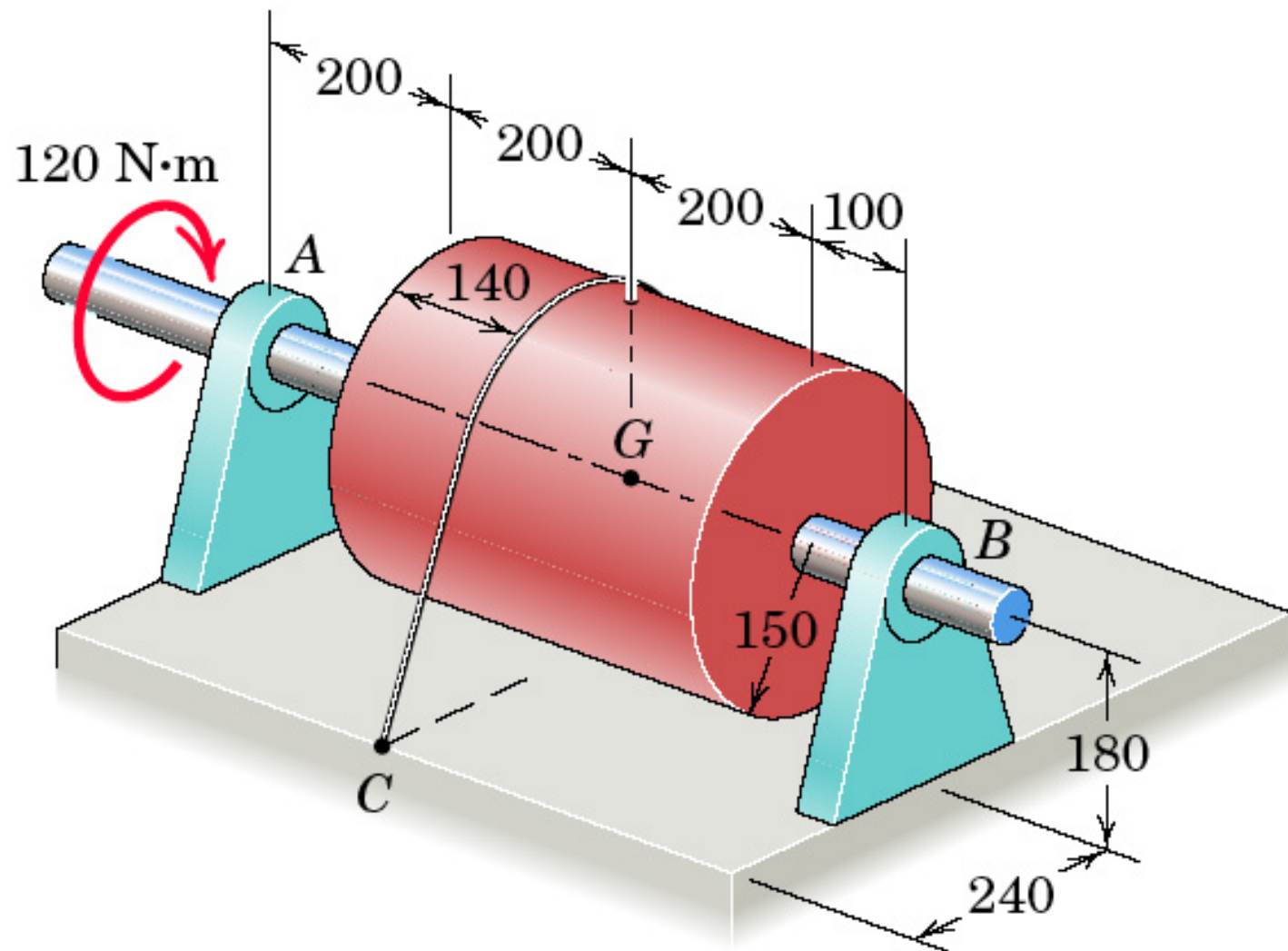
$$\overline{OG} = 30'$$

$$\overline{OB} = \overline{OC} = 40'$$

Problem 3/112

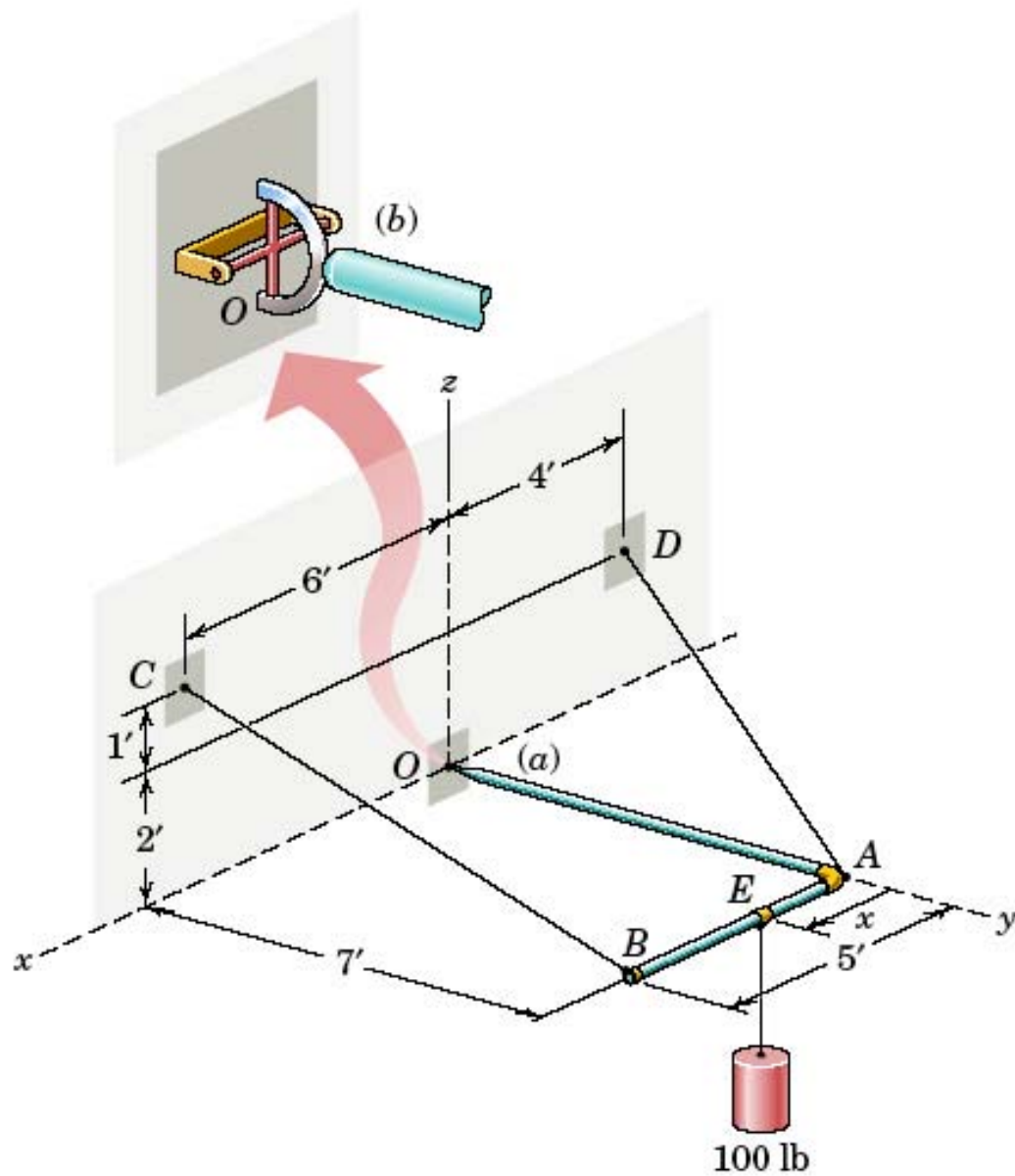


Problem 3/113

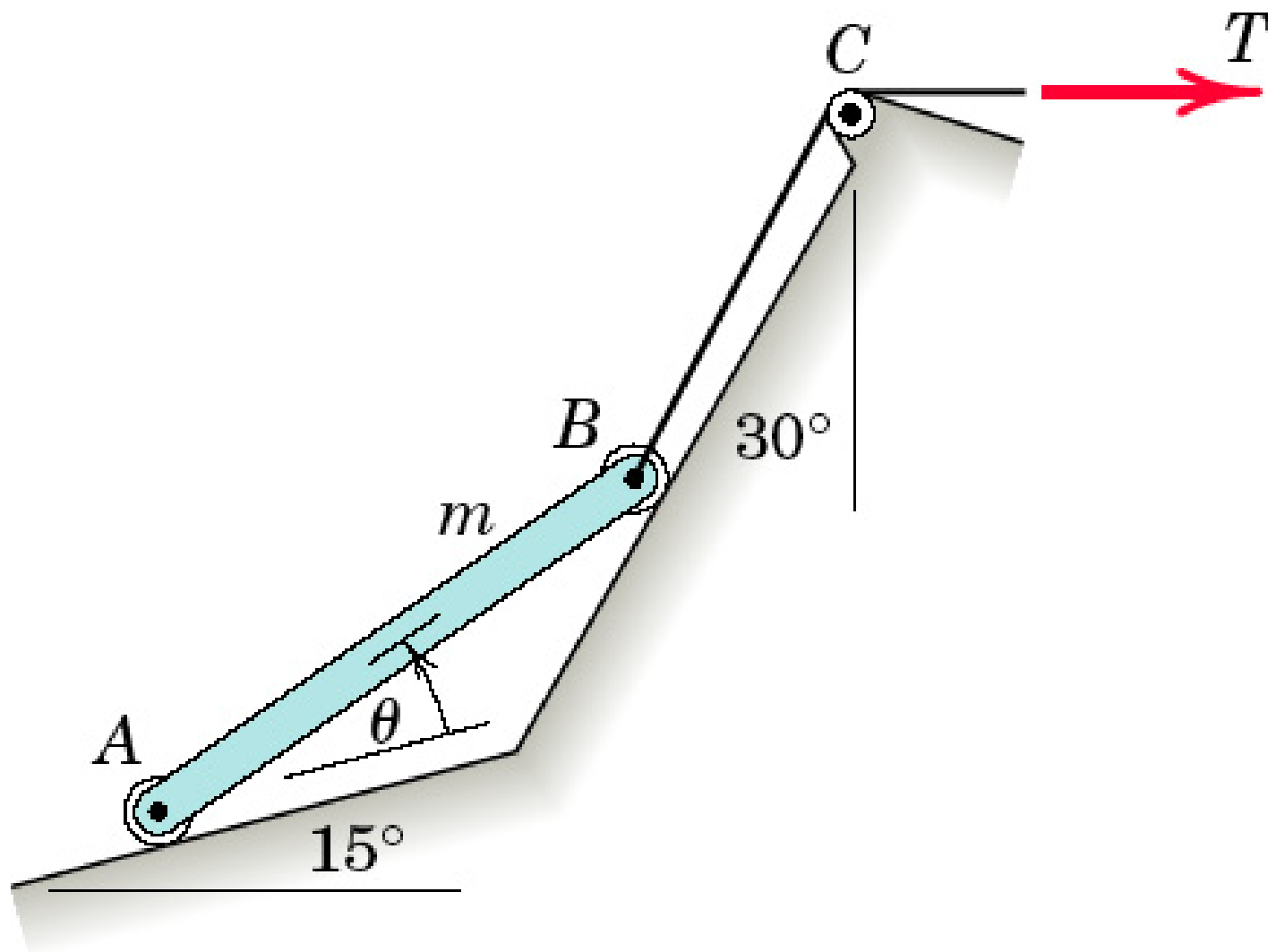


Dimensions in millimeters

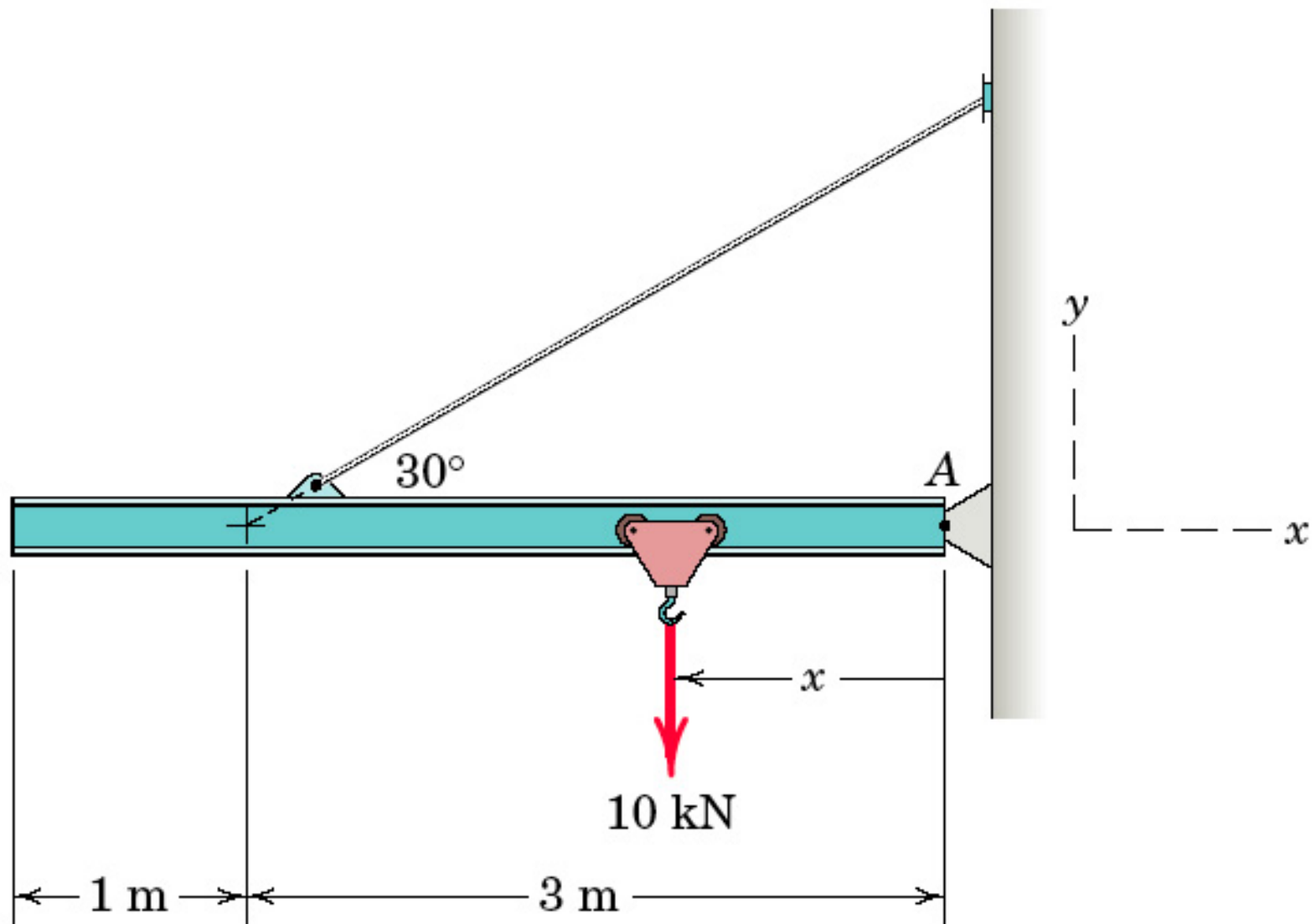
Problem 3/114



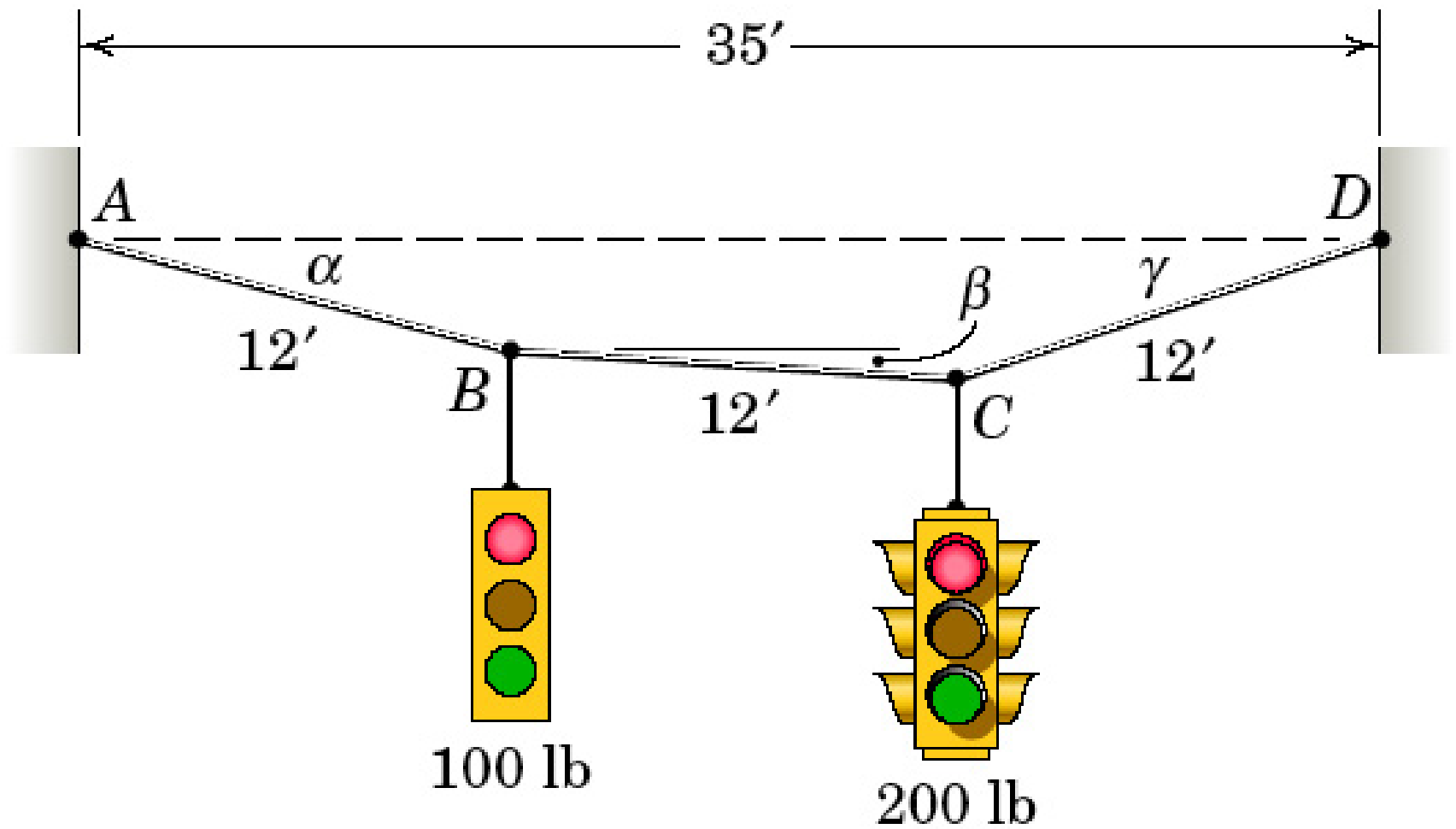
Problem 3/115



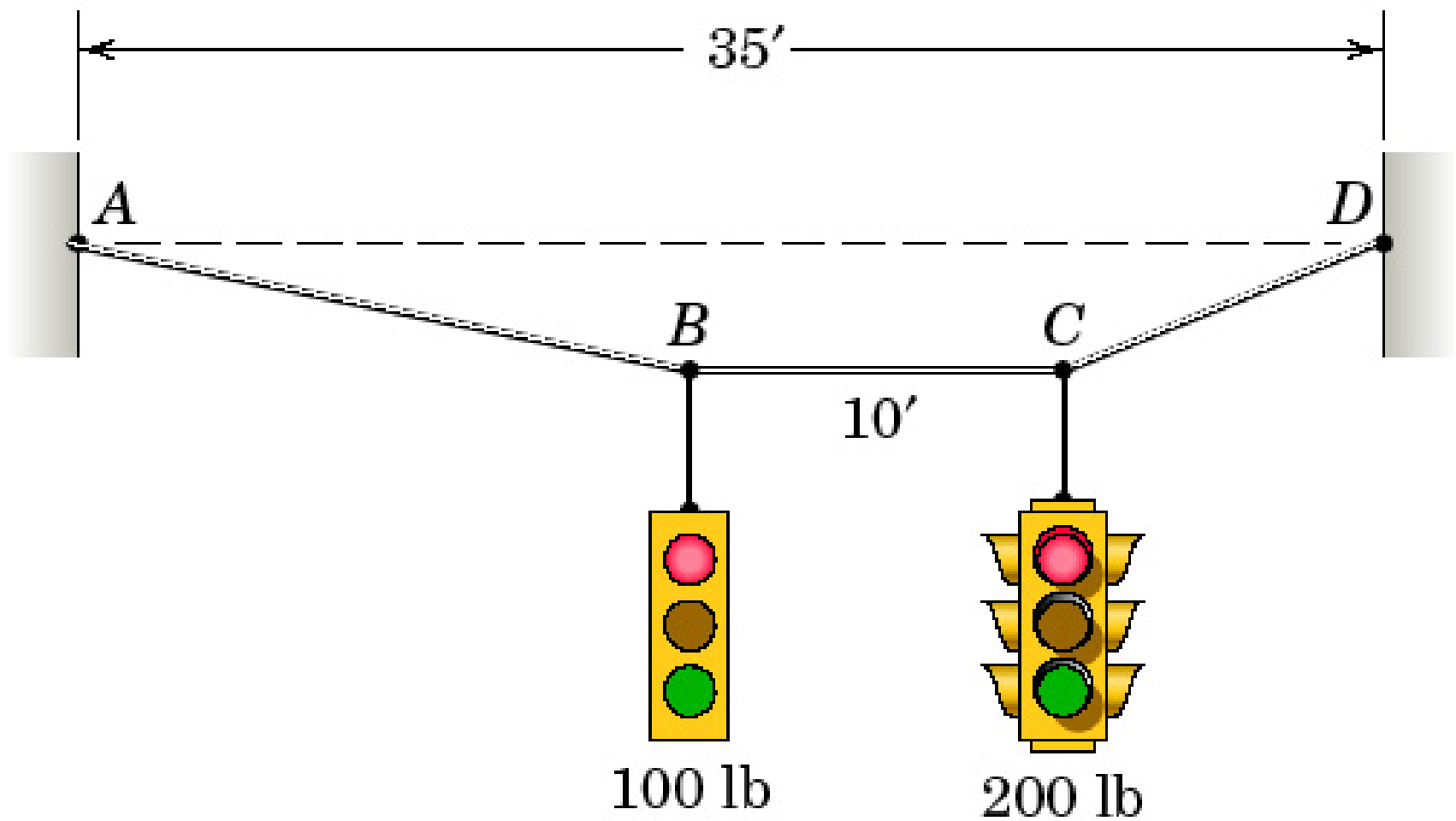
Problem 3/116



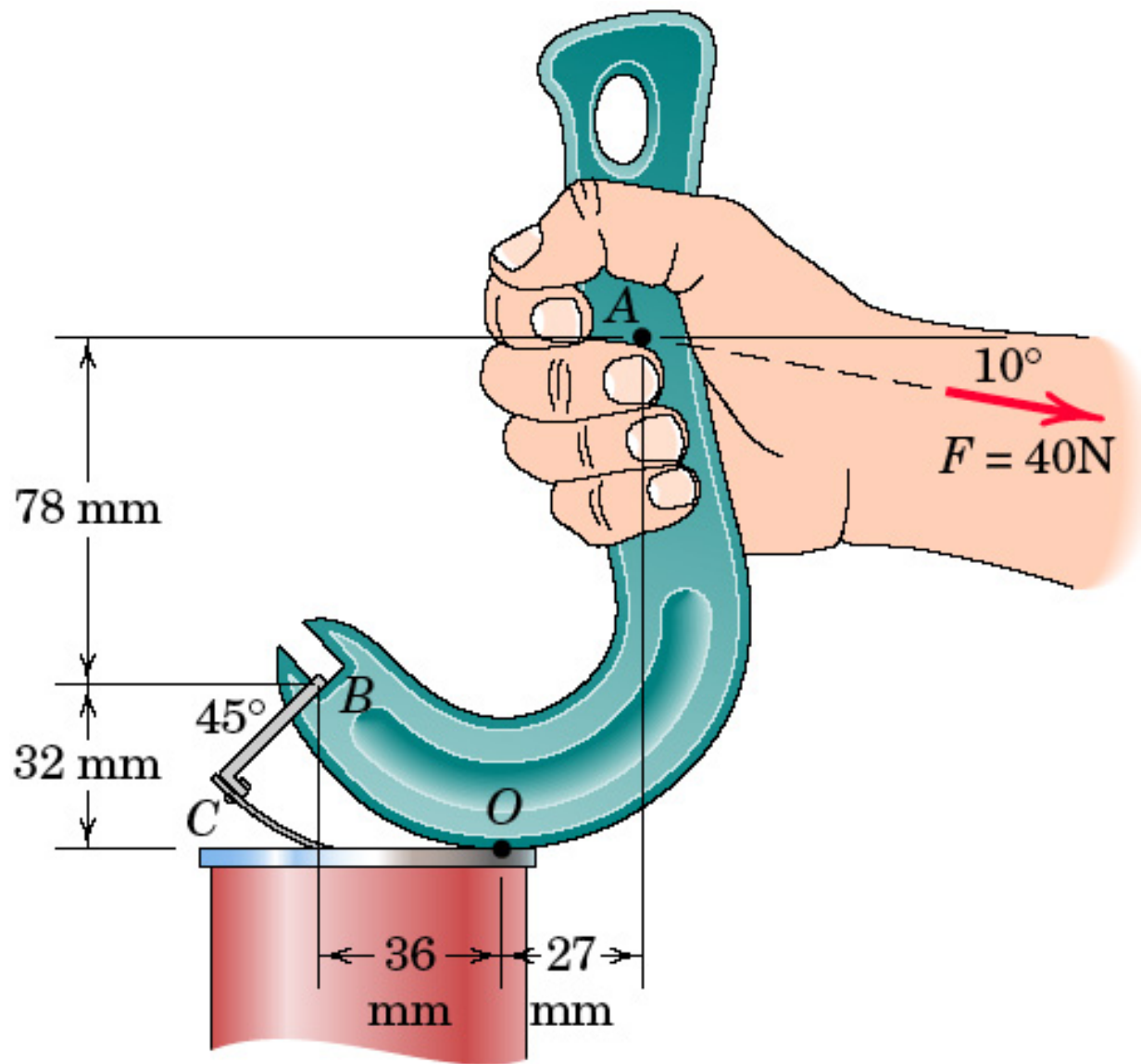
Problem 3/117



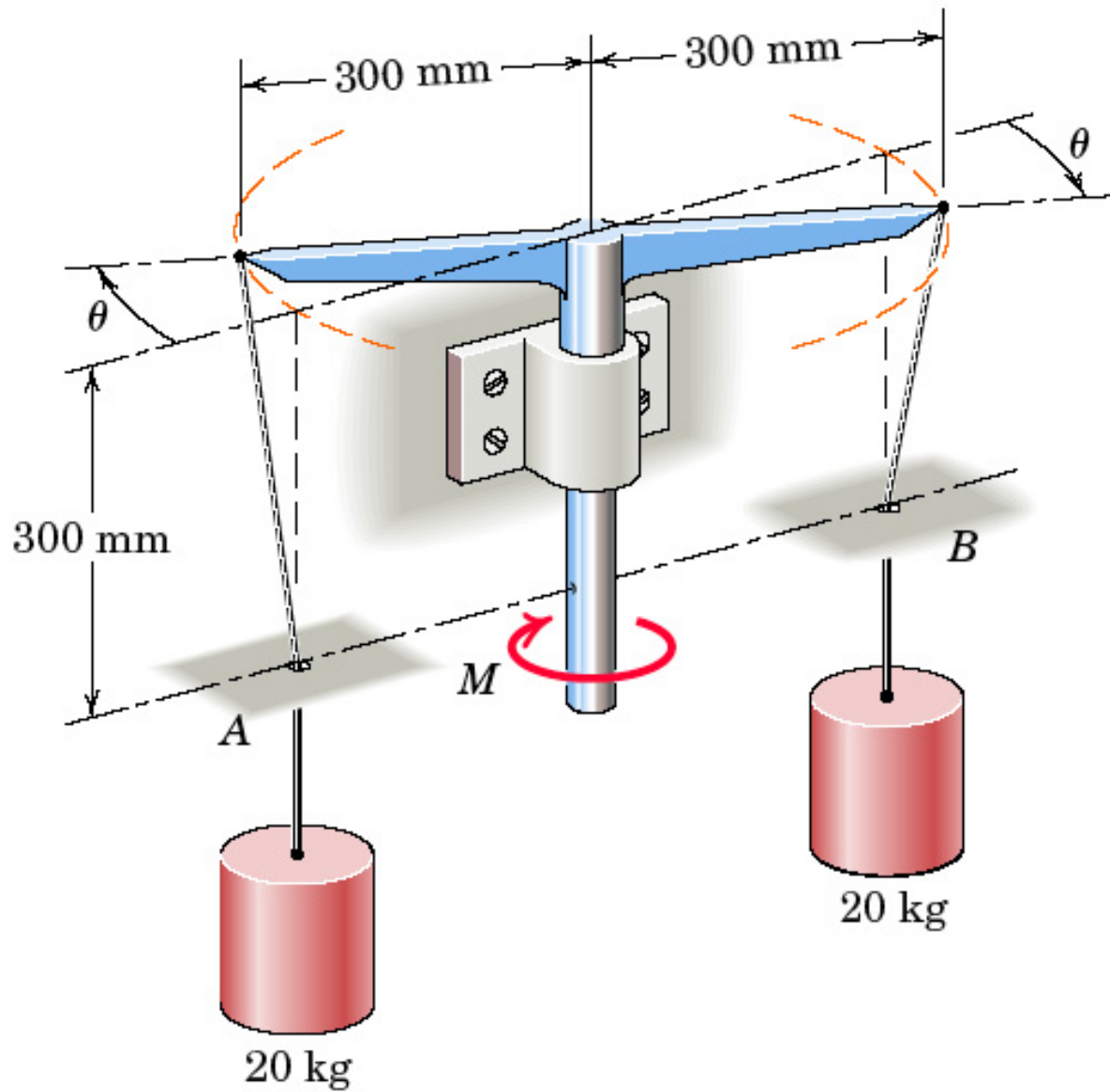
Problem 3/118



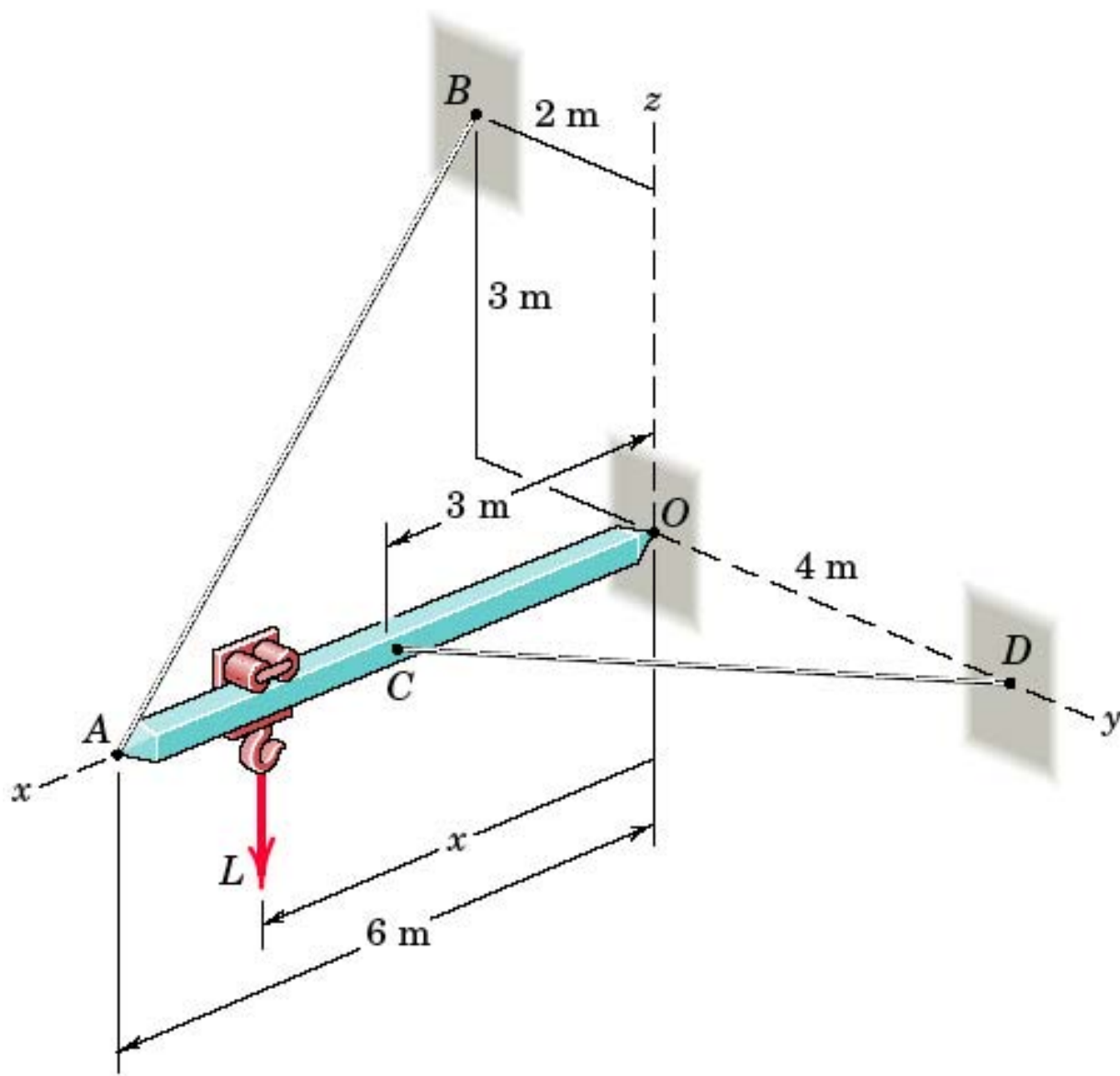
Problem 3/119



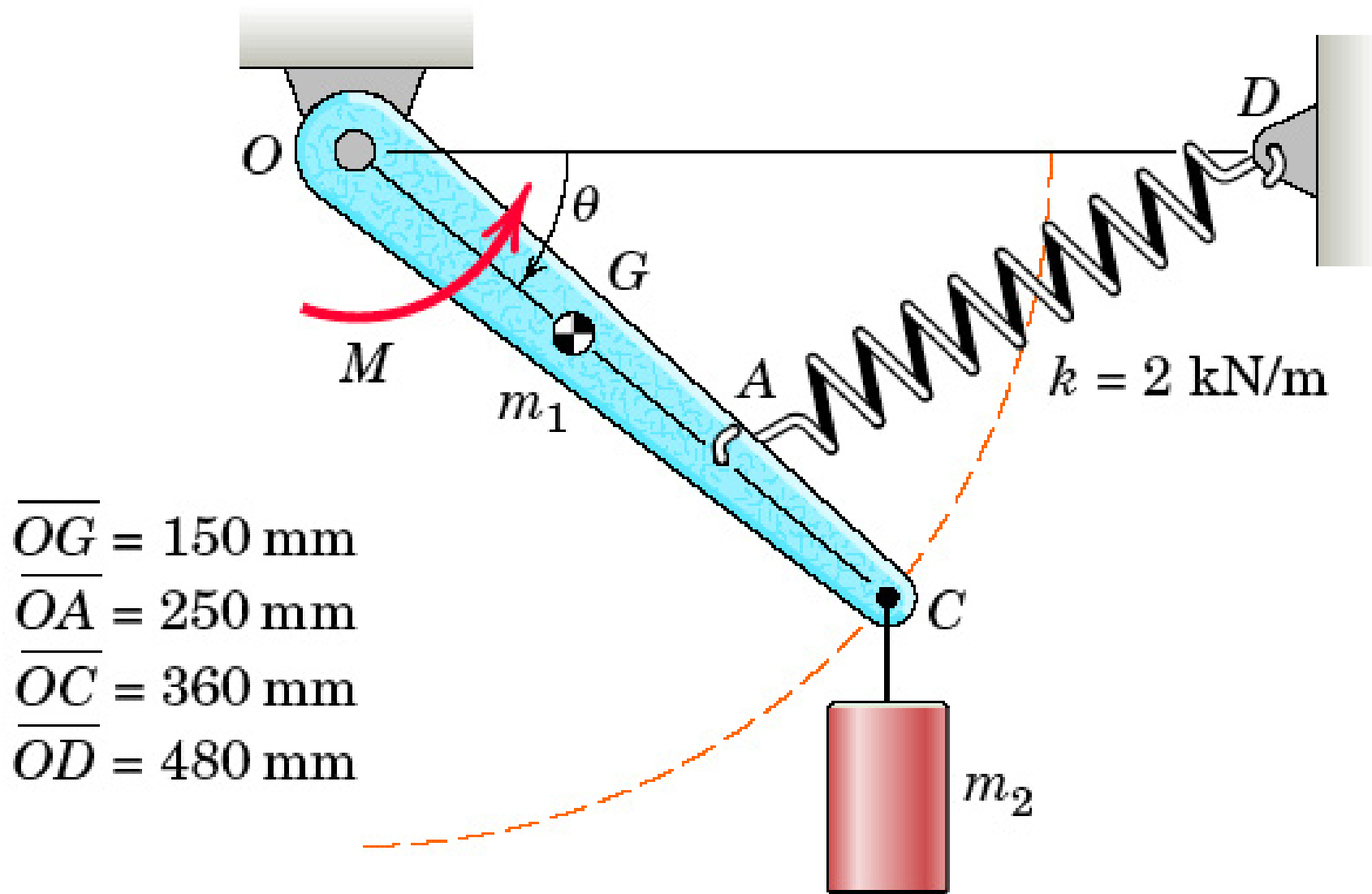
Problem 3/12



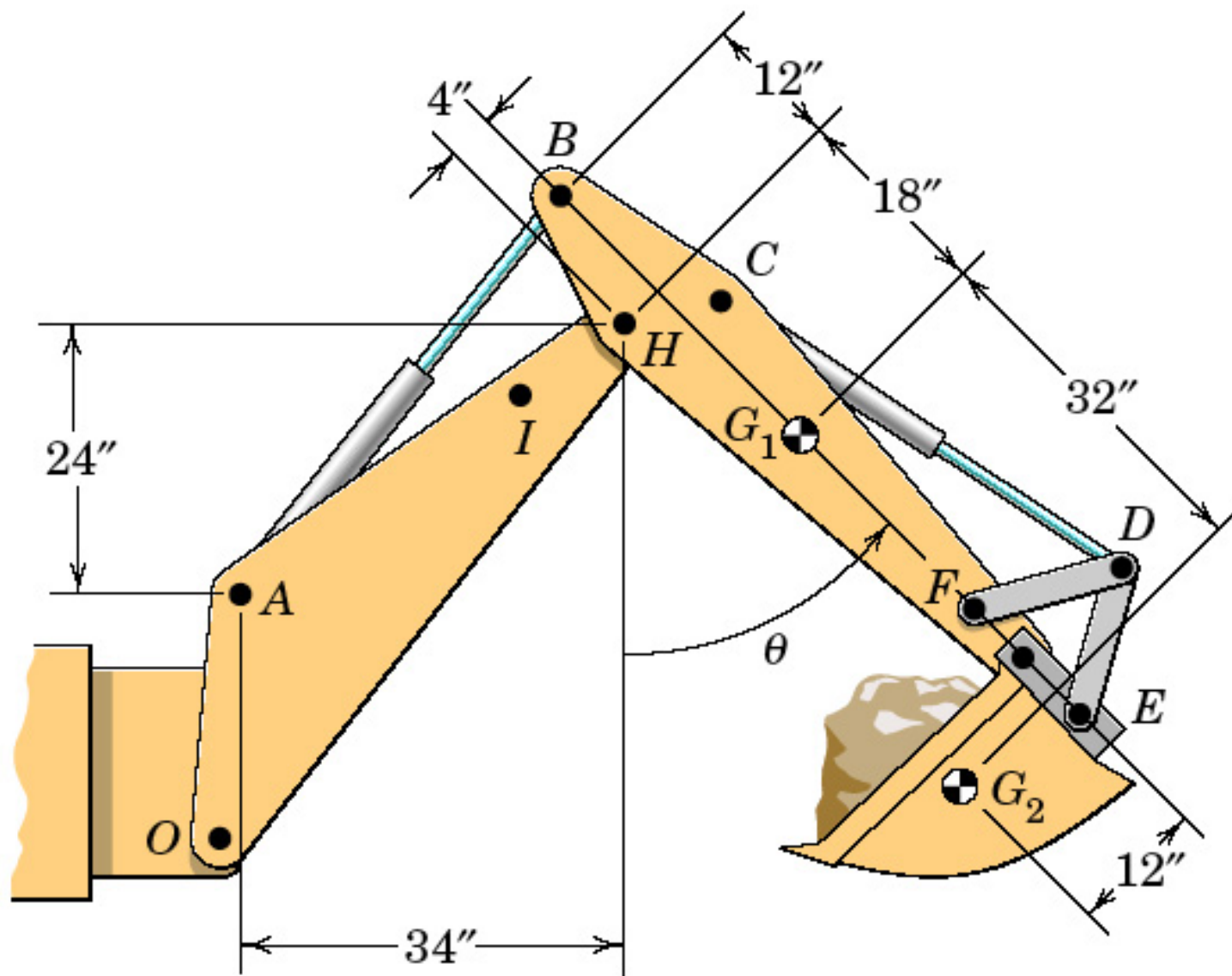
Problem 3/120



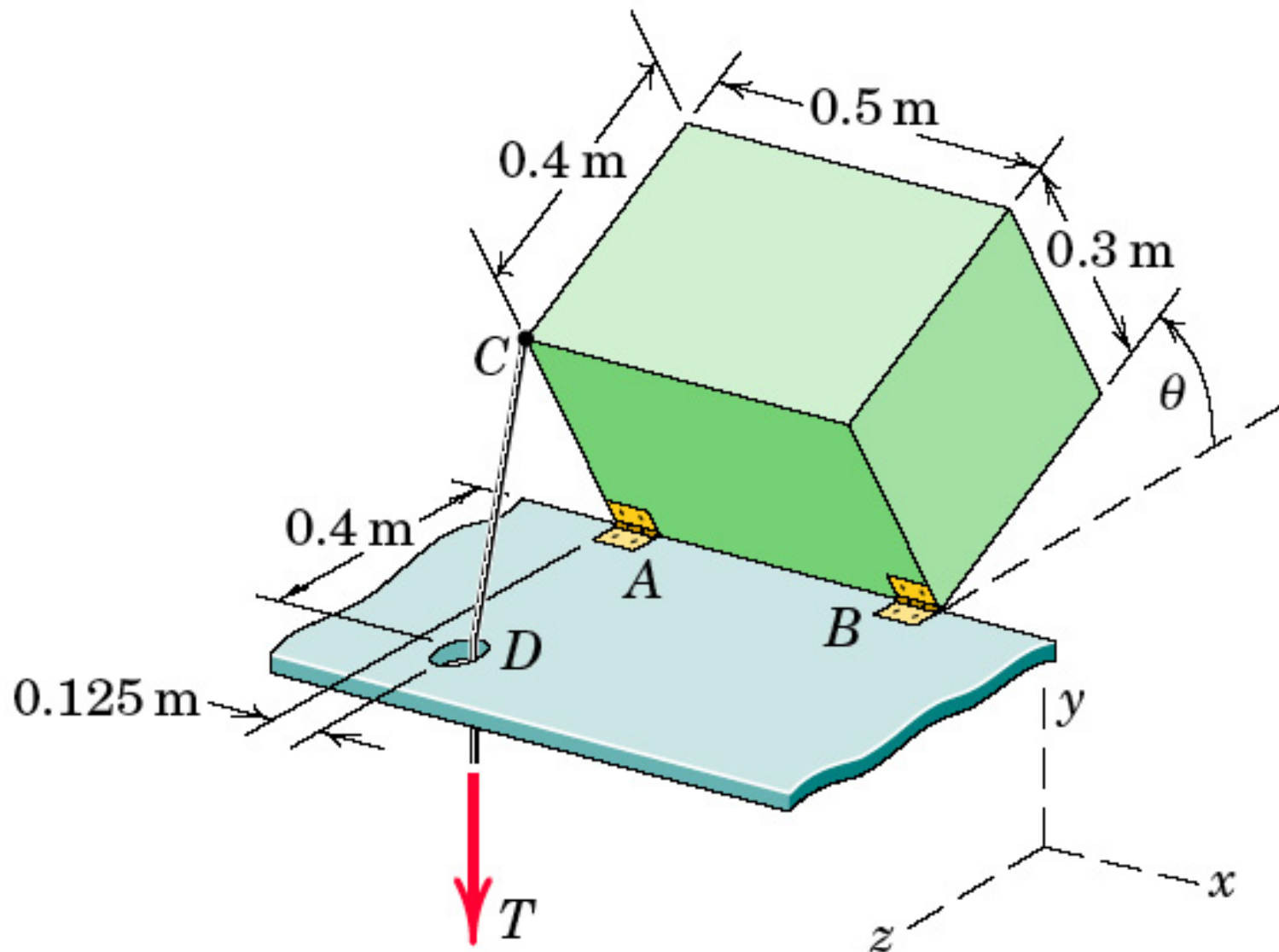
Problem 3/121



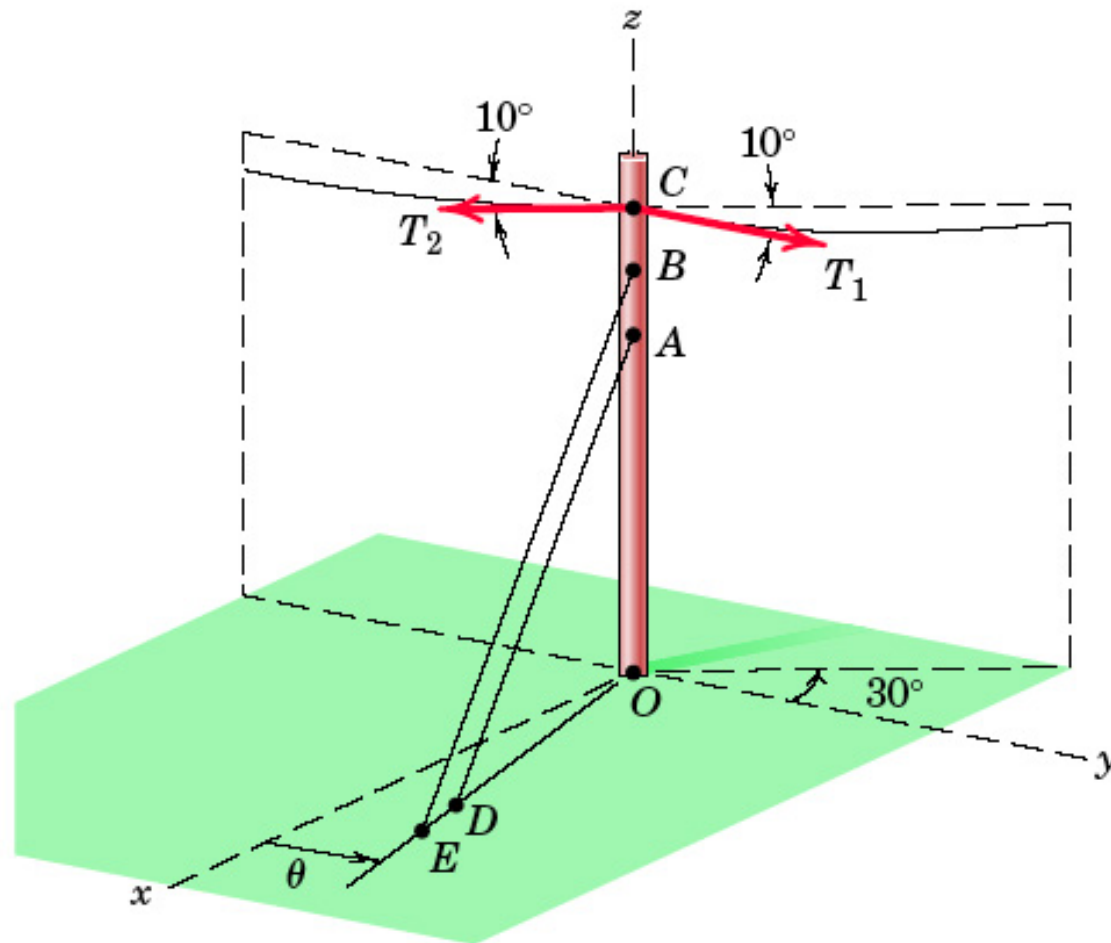
Problem 3/122



Problem 3/123

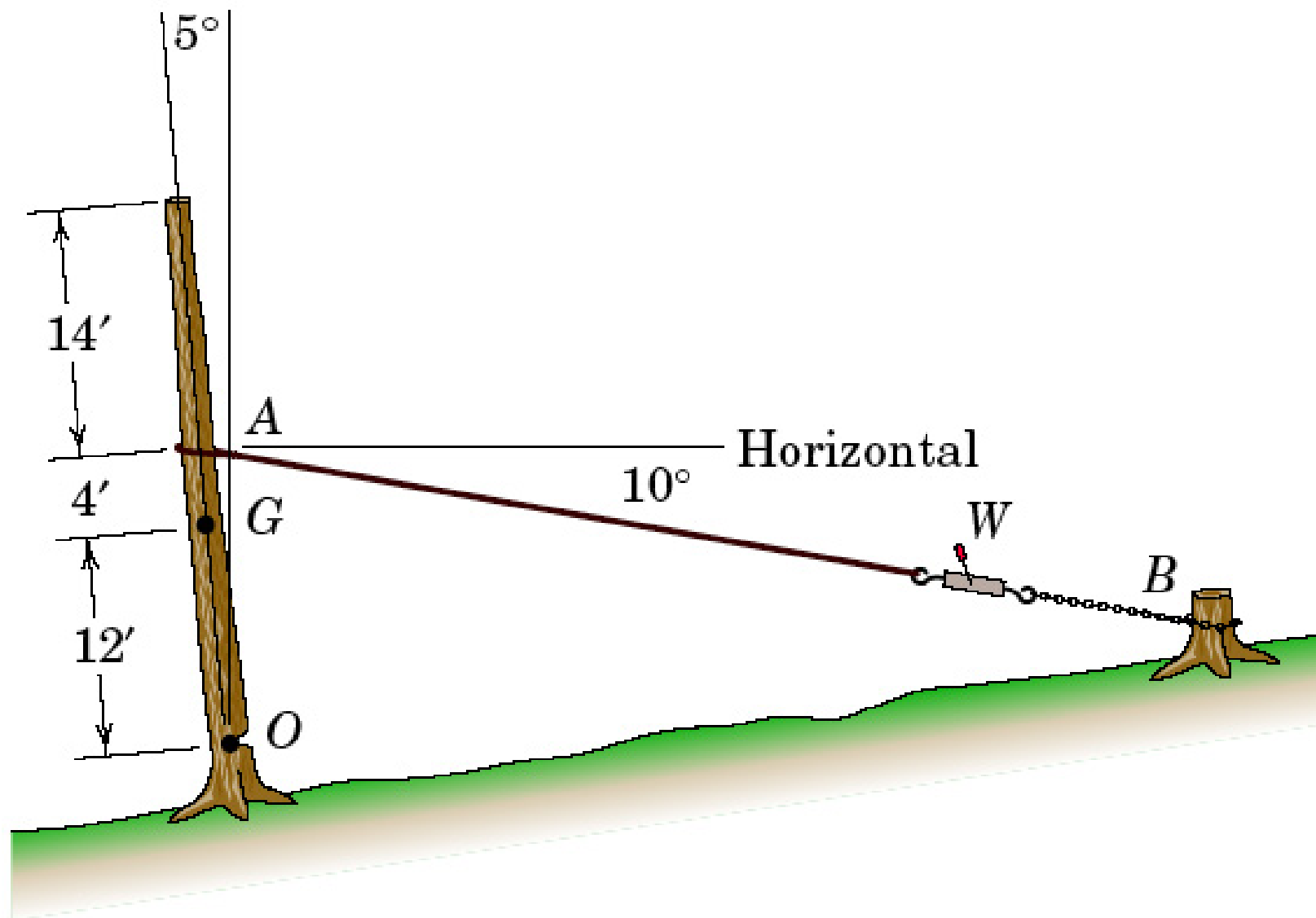


Problem 3/124

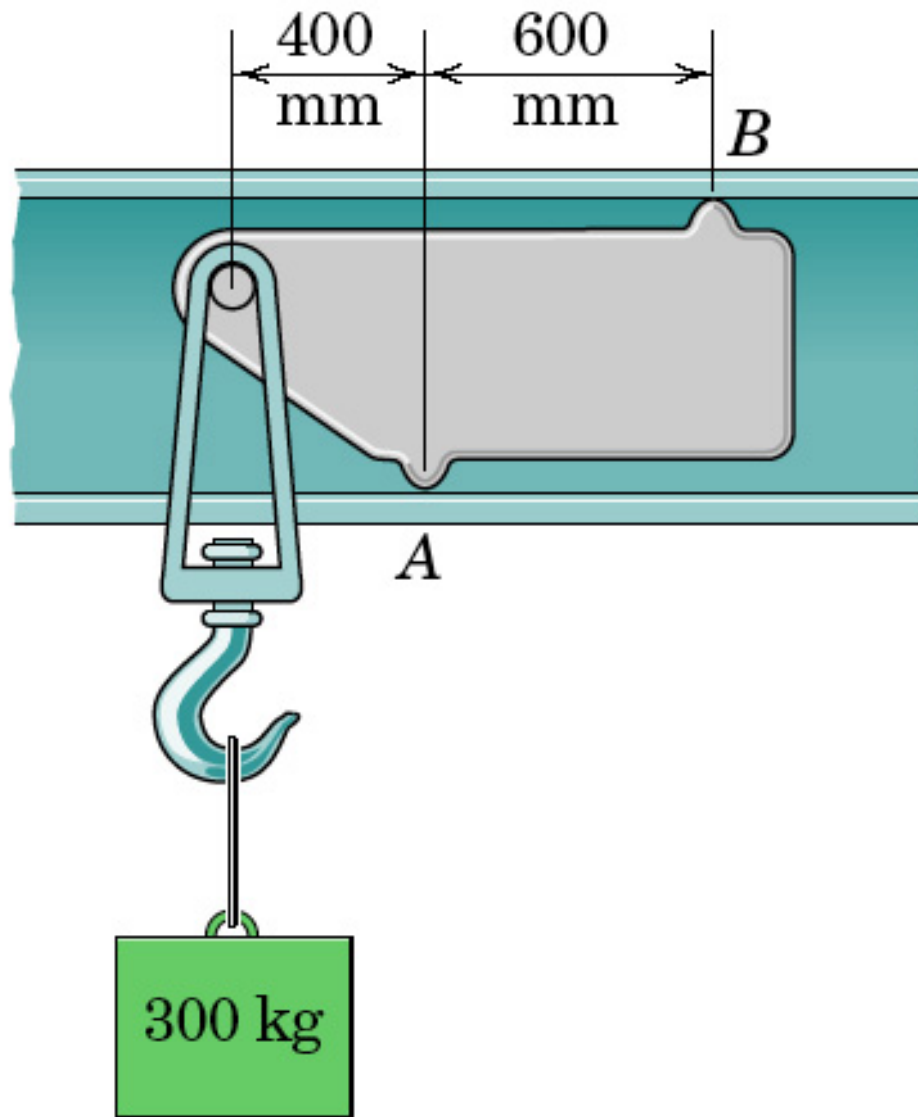


$$\begin{aligned}\overline{OA} &= 9 \text{ m} & \overline{OD} &= 8 \text{ m} \\ \overline{OB} &= 11 \text{ m} & \overline{OE} &= 10 \text{ m} \\ \overline{OC} &= 13 \text{ m}\end{aligned}$$

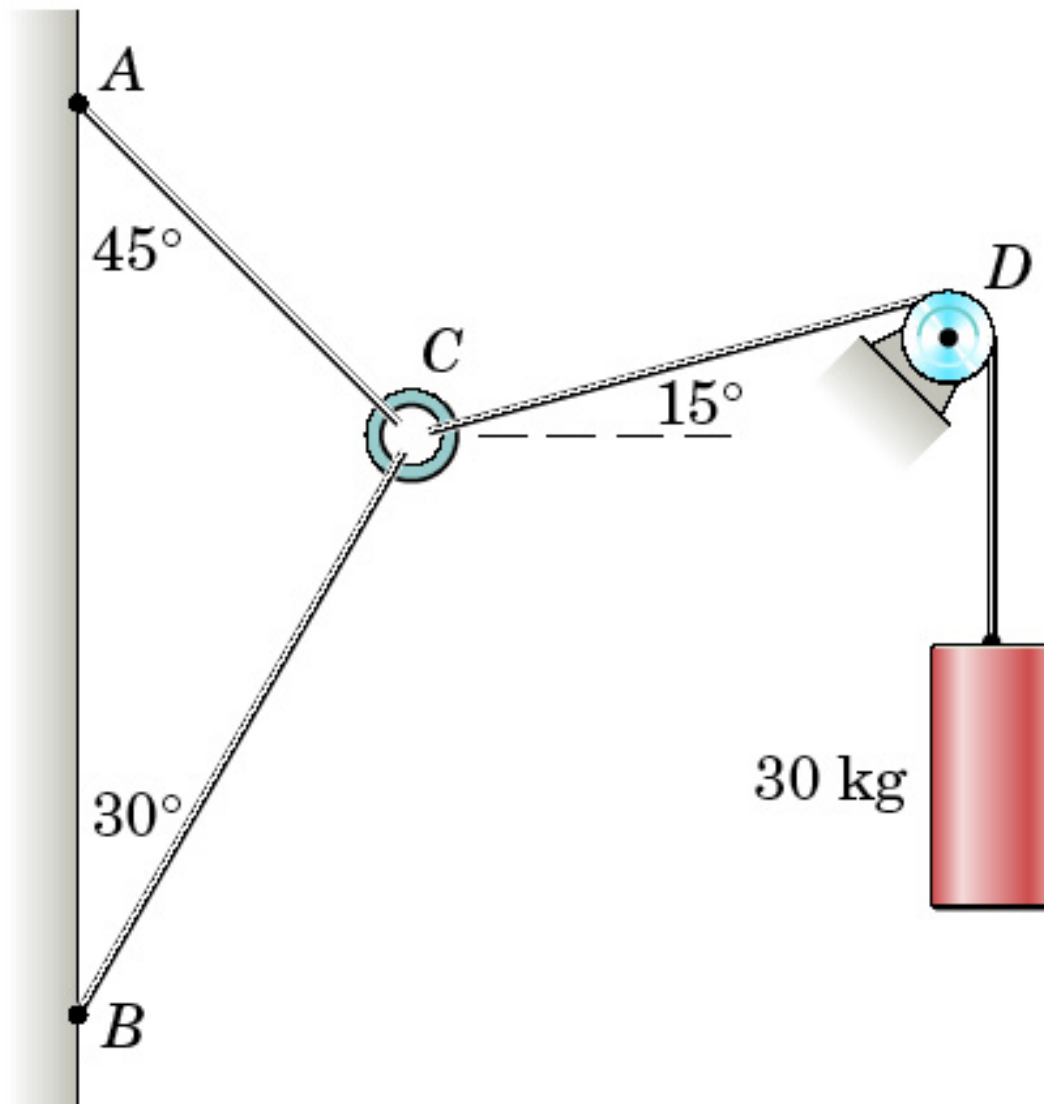
Problem 3/125



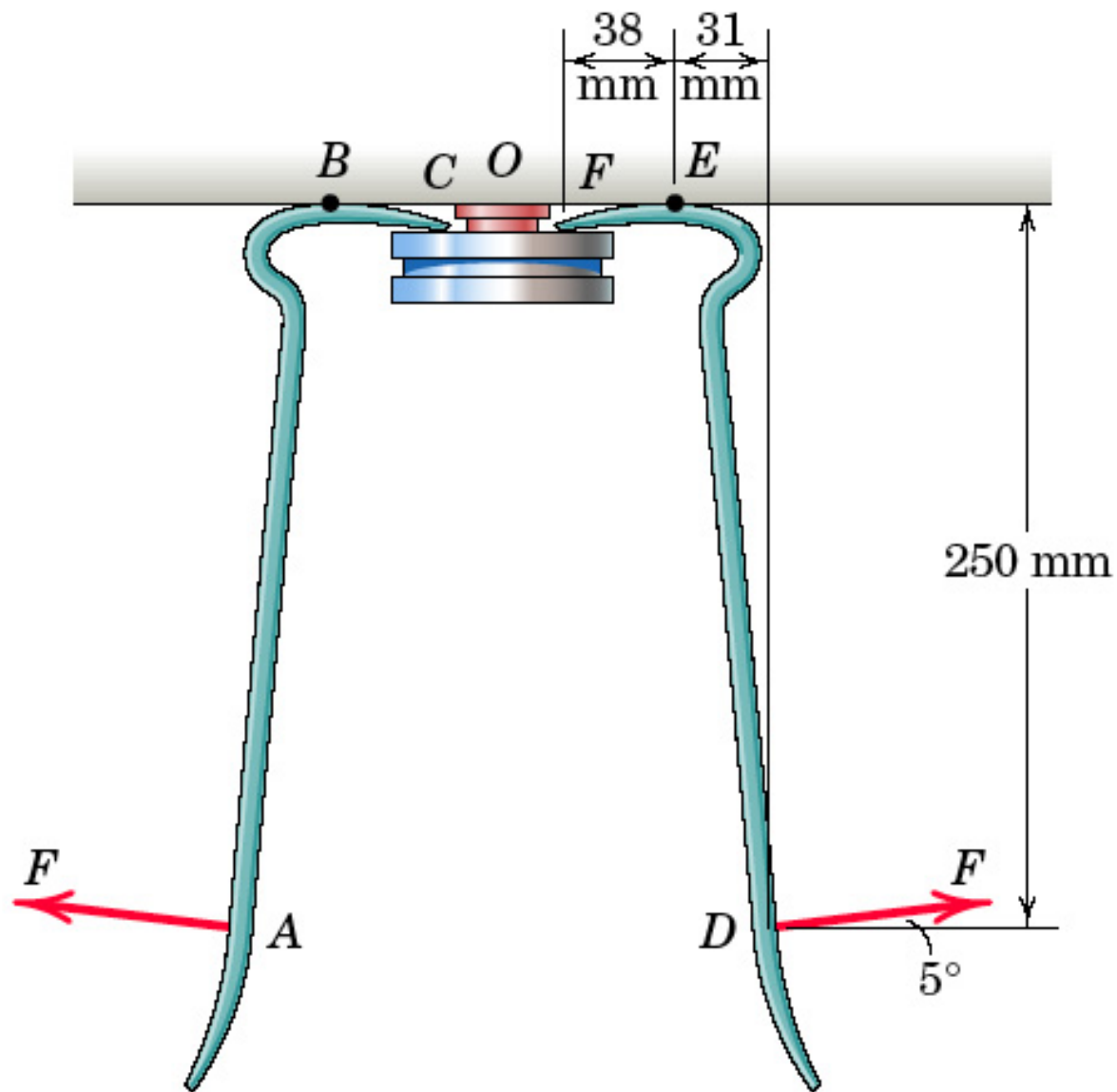
Problem 3/13



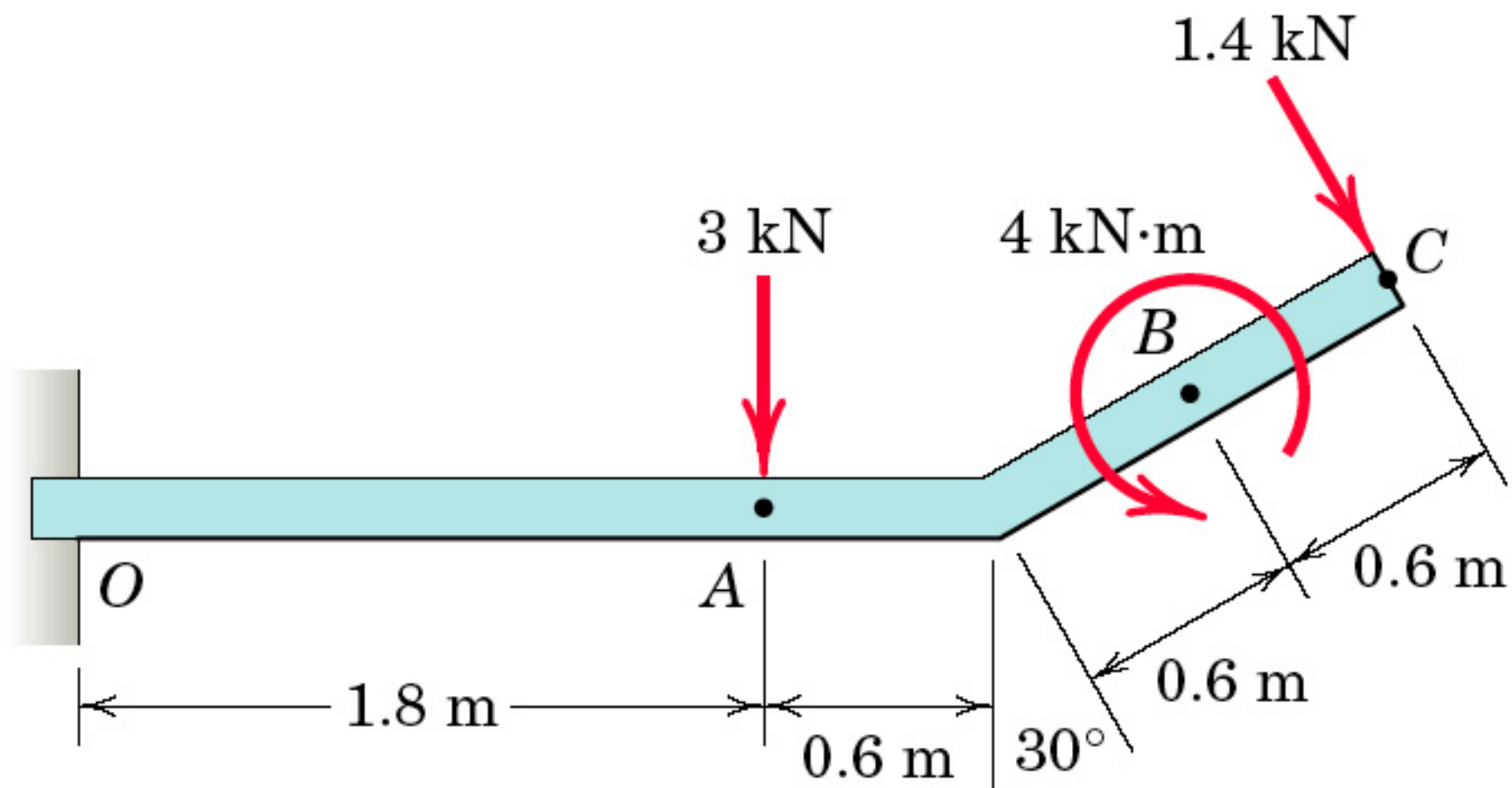
Problem 3/14



Problem 3/15

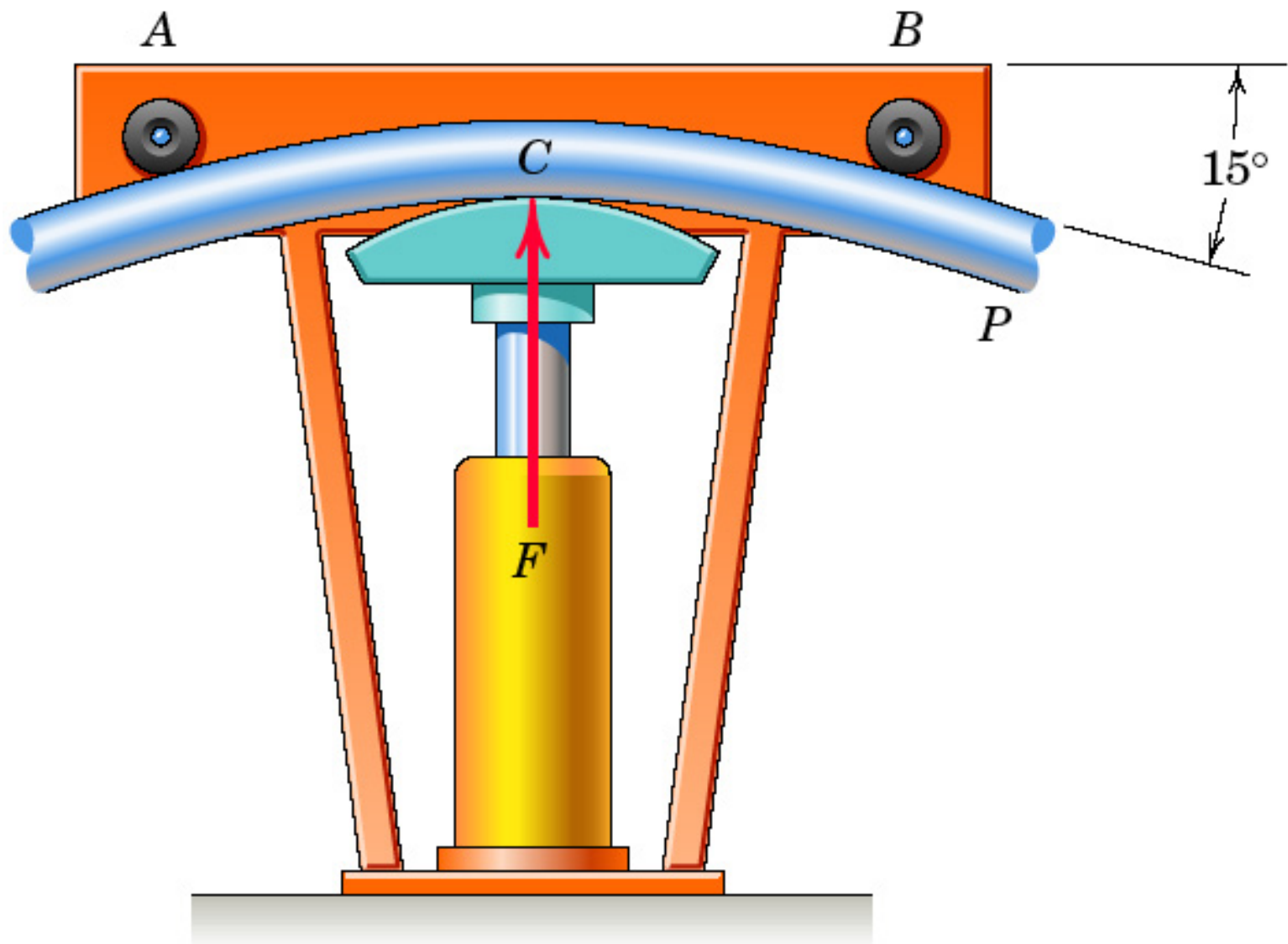


Problem 3/16

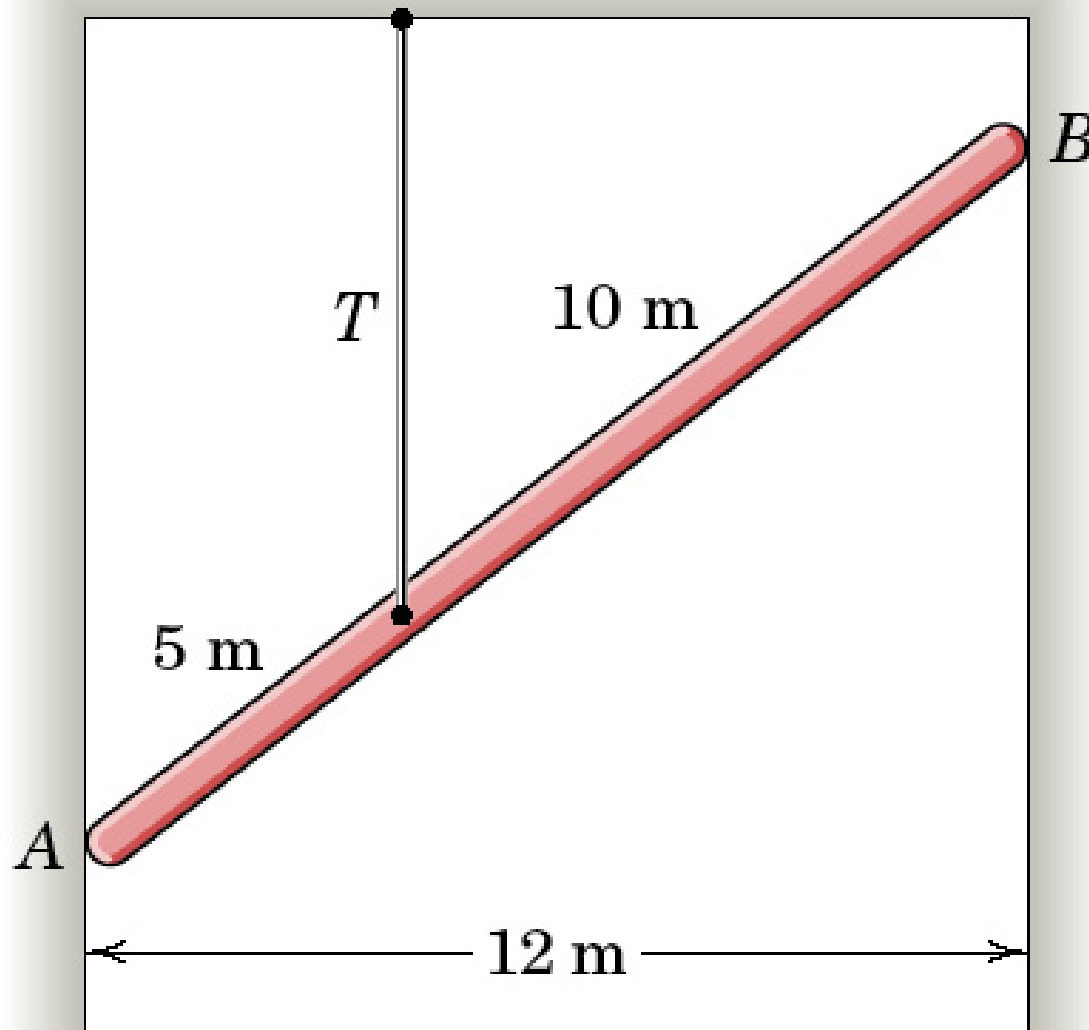


Problem 3/17

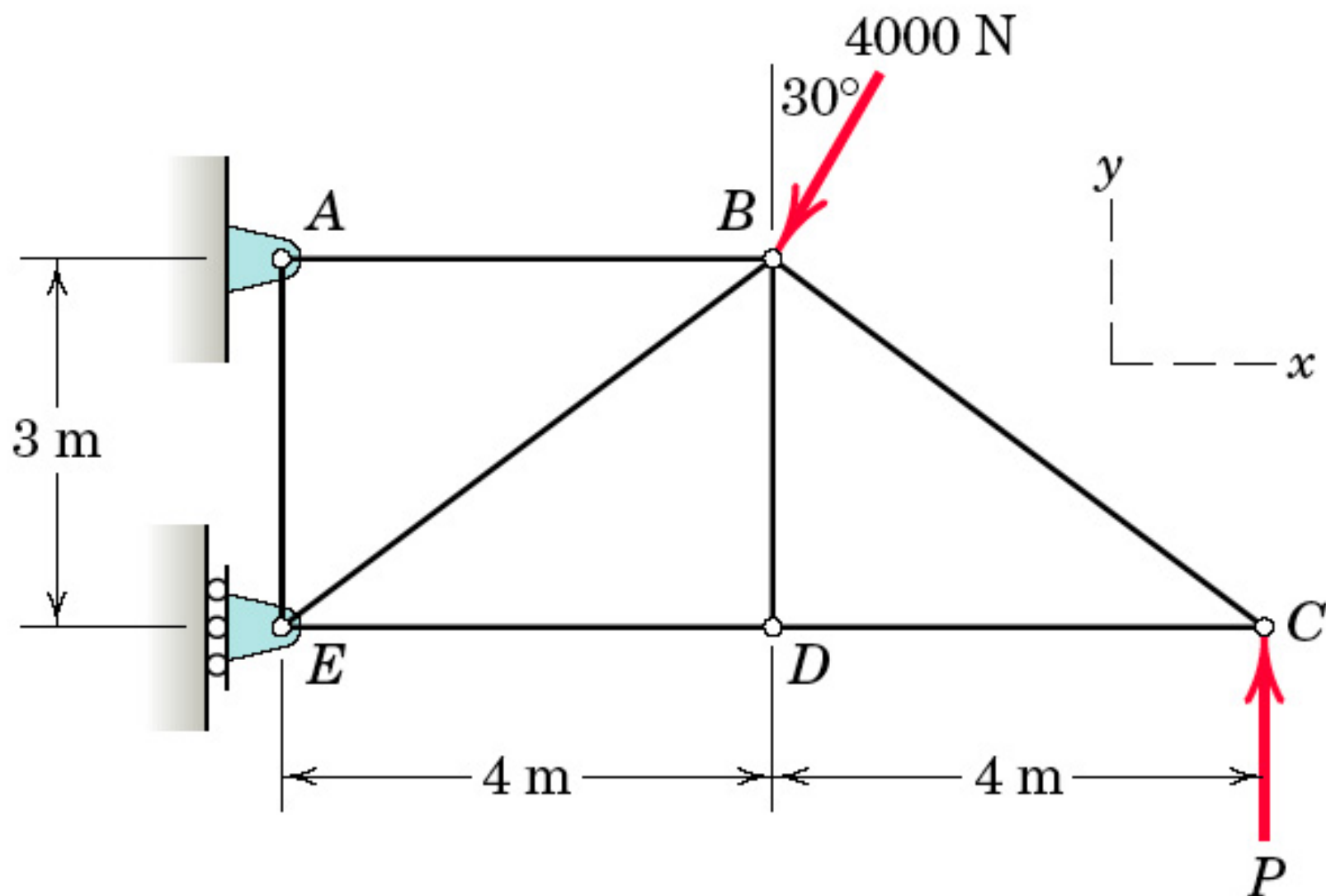
P_03_17



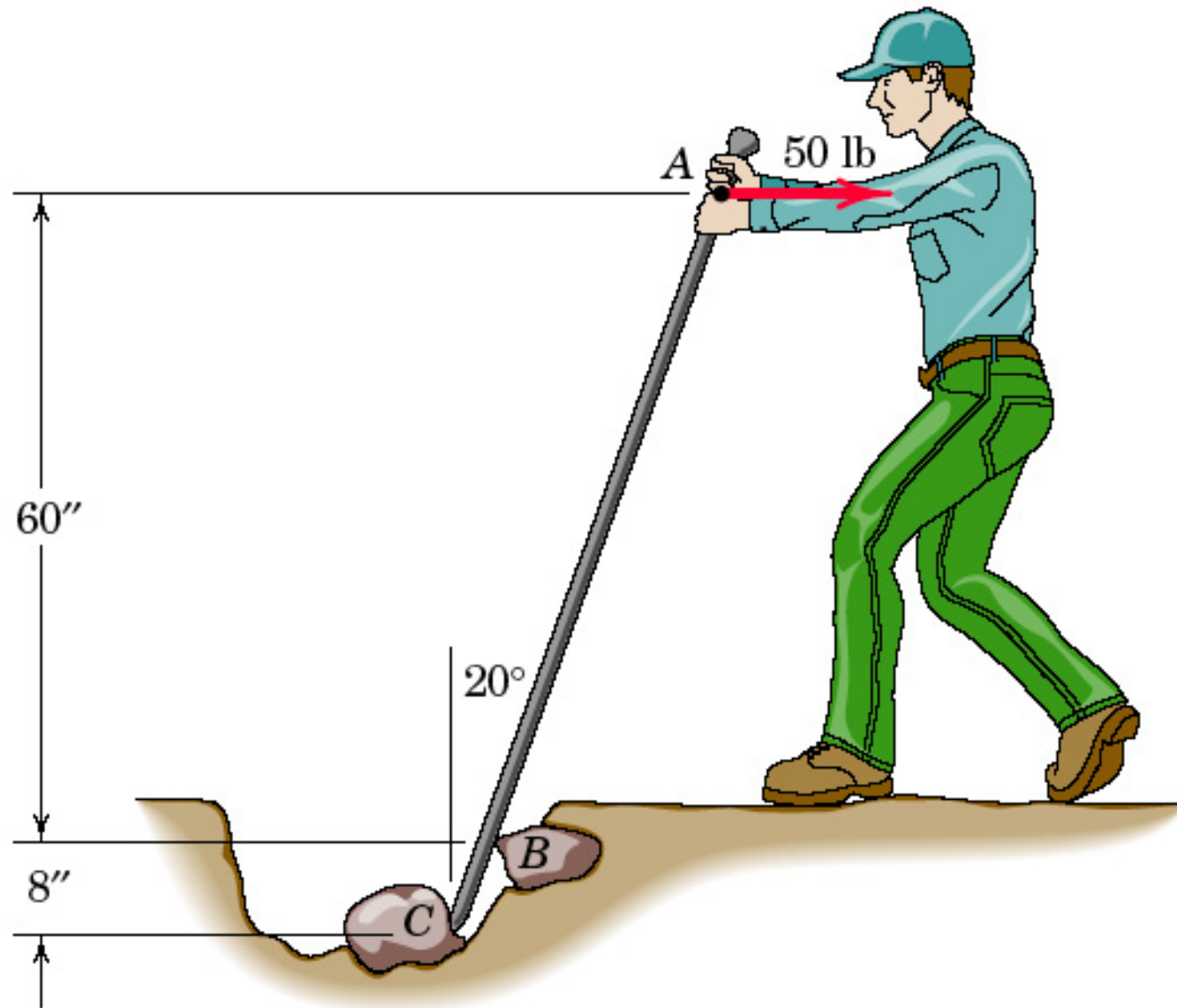
Problem 3/18



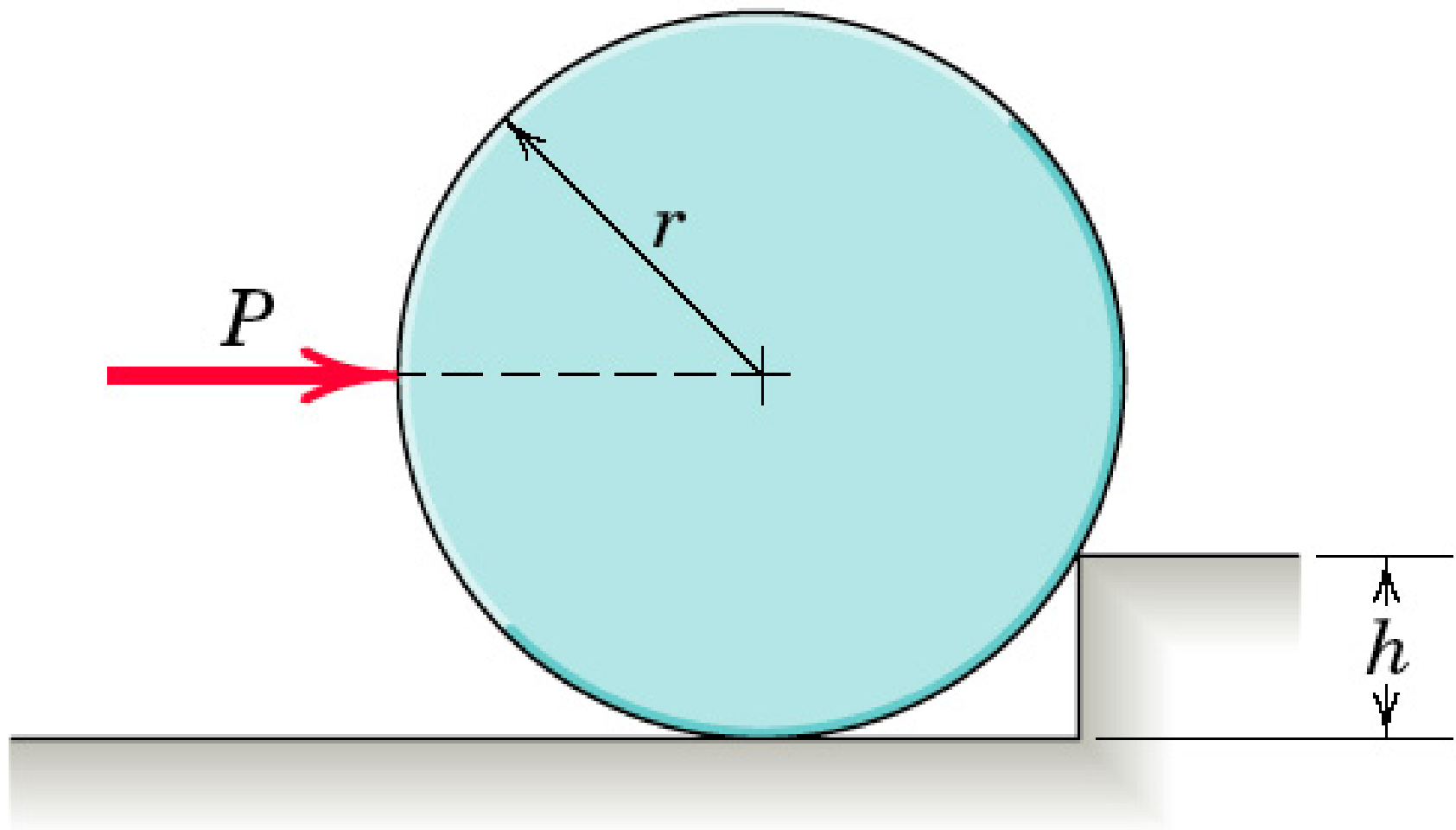
Problem 3/19



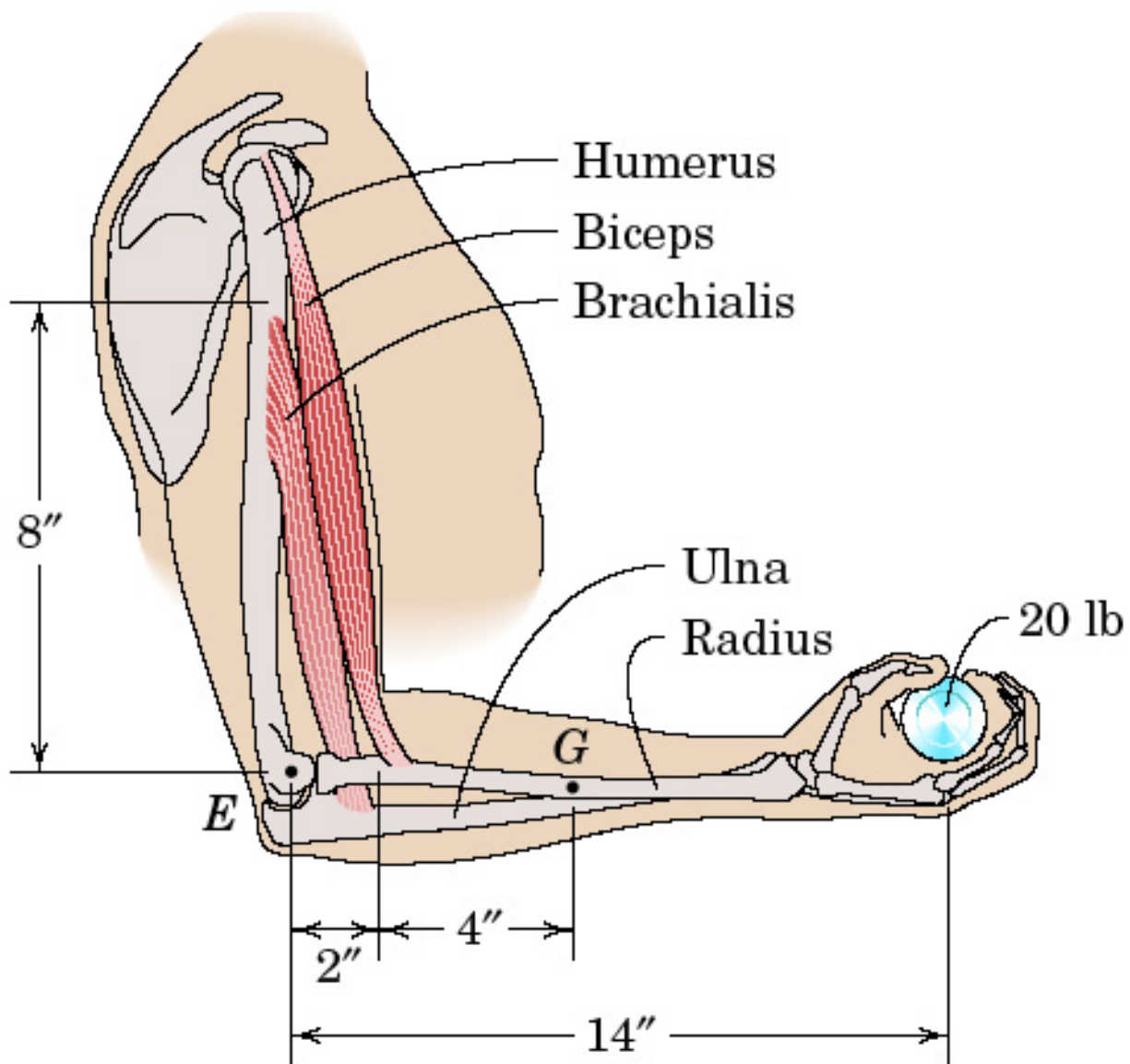
Problem 3/20



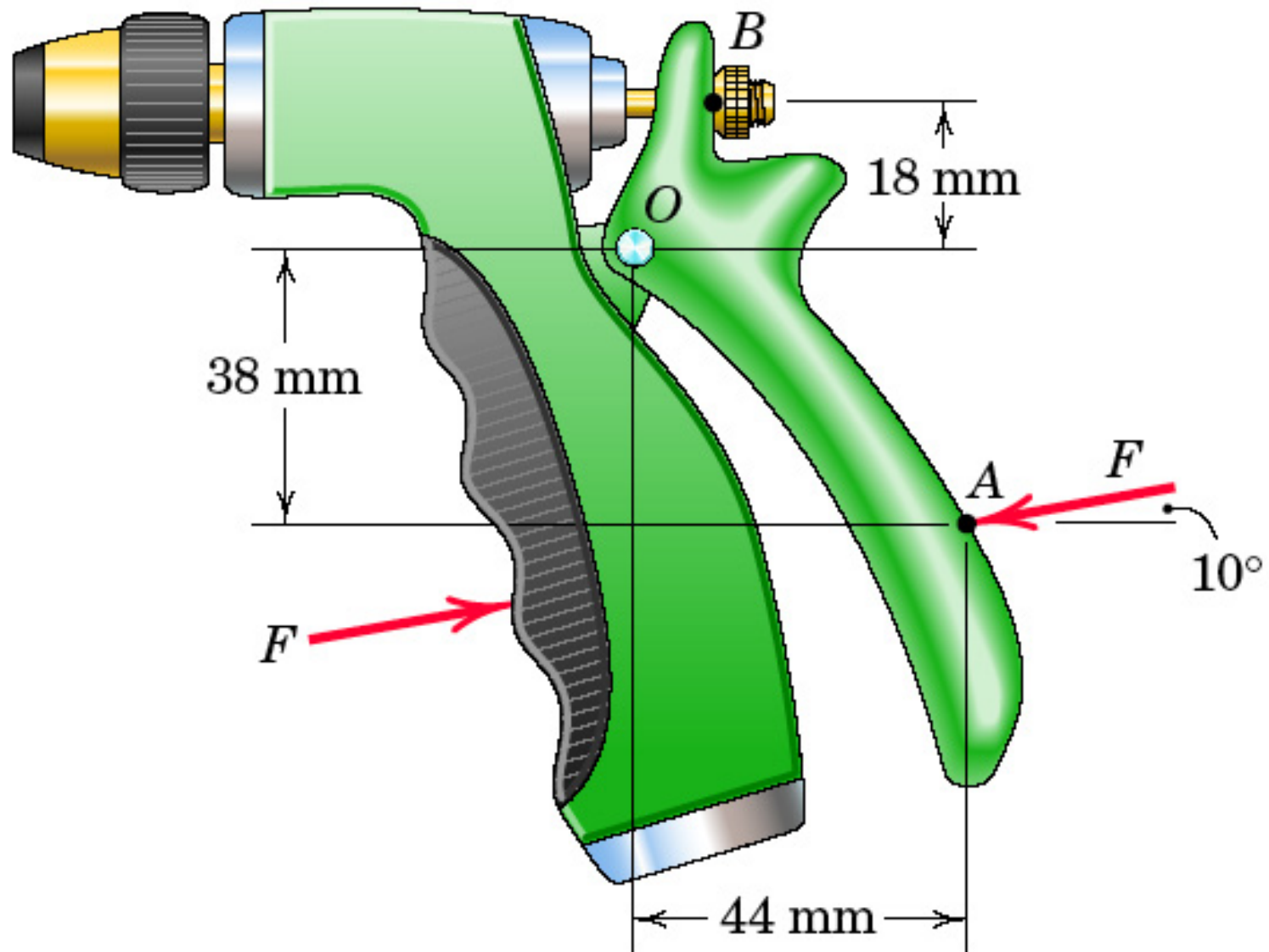
Problem 3/21



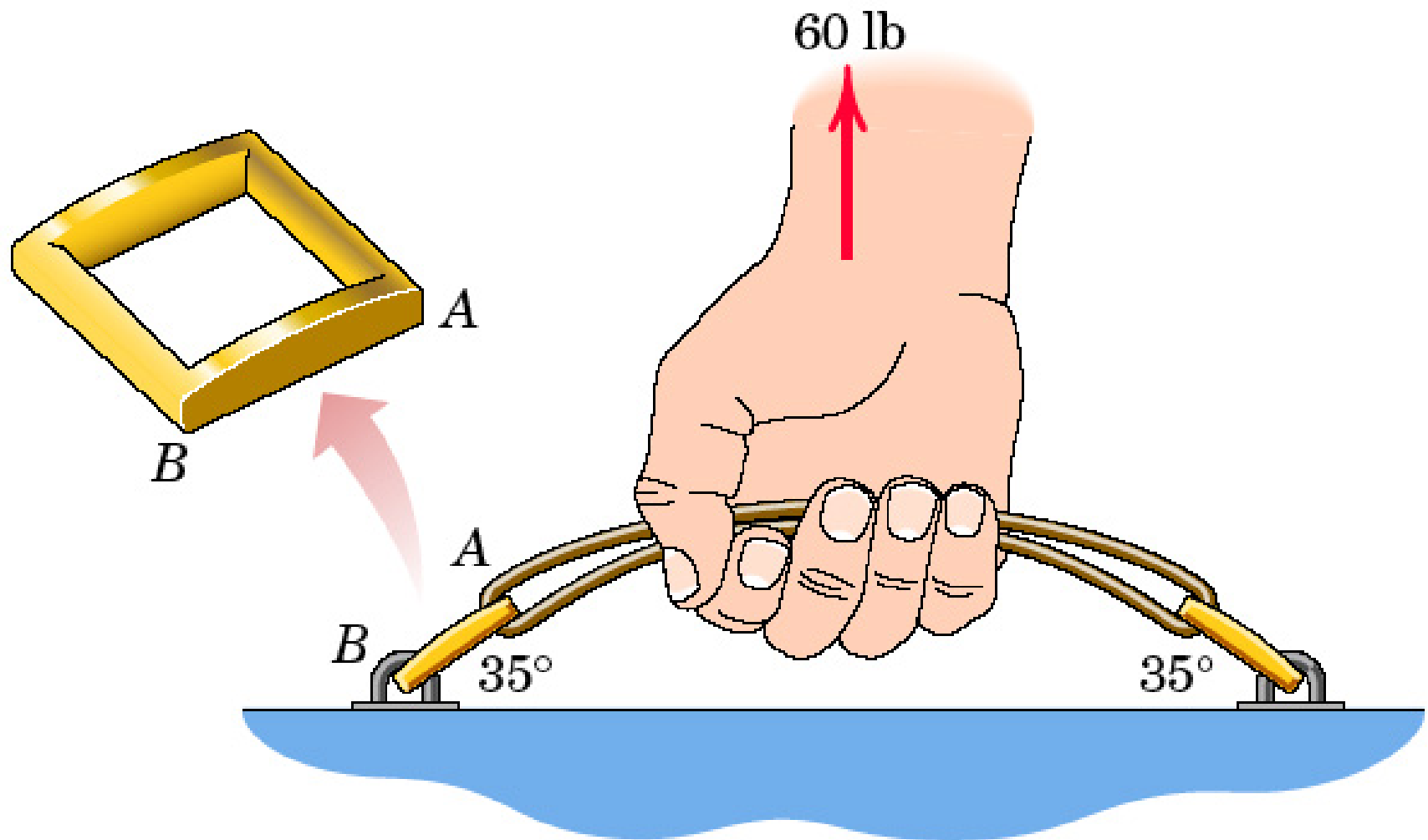
Problem 3/22



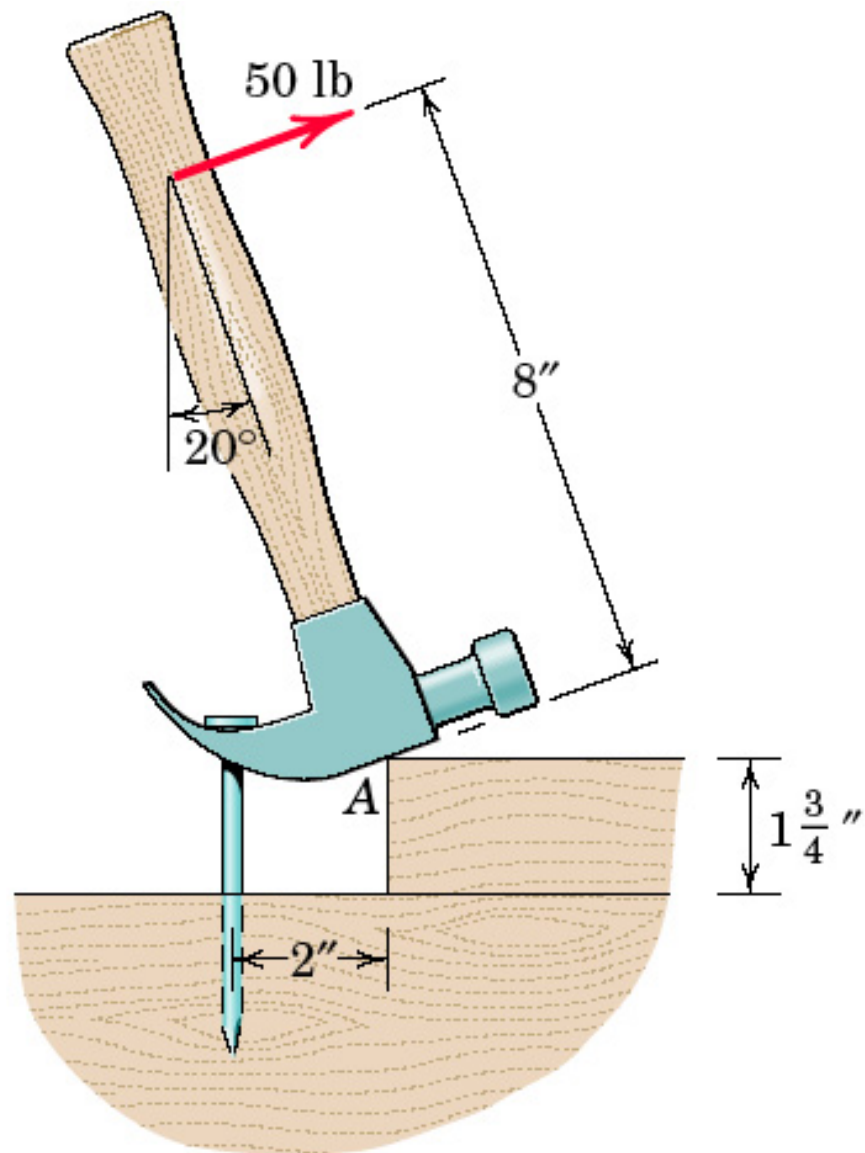
Problem 3/23



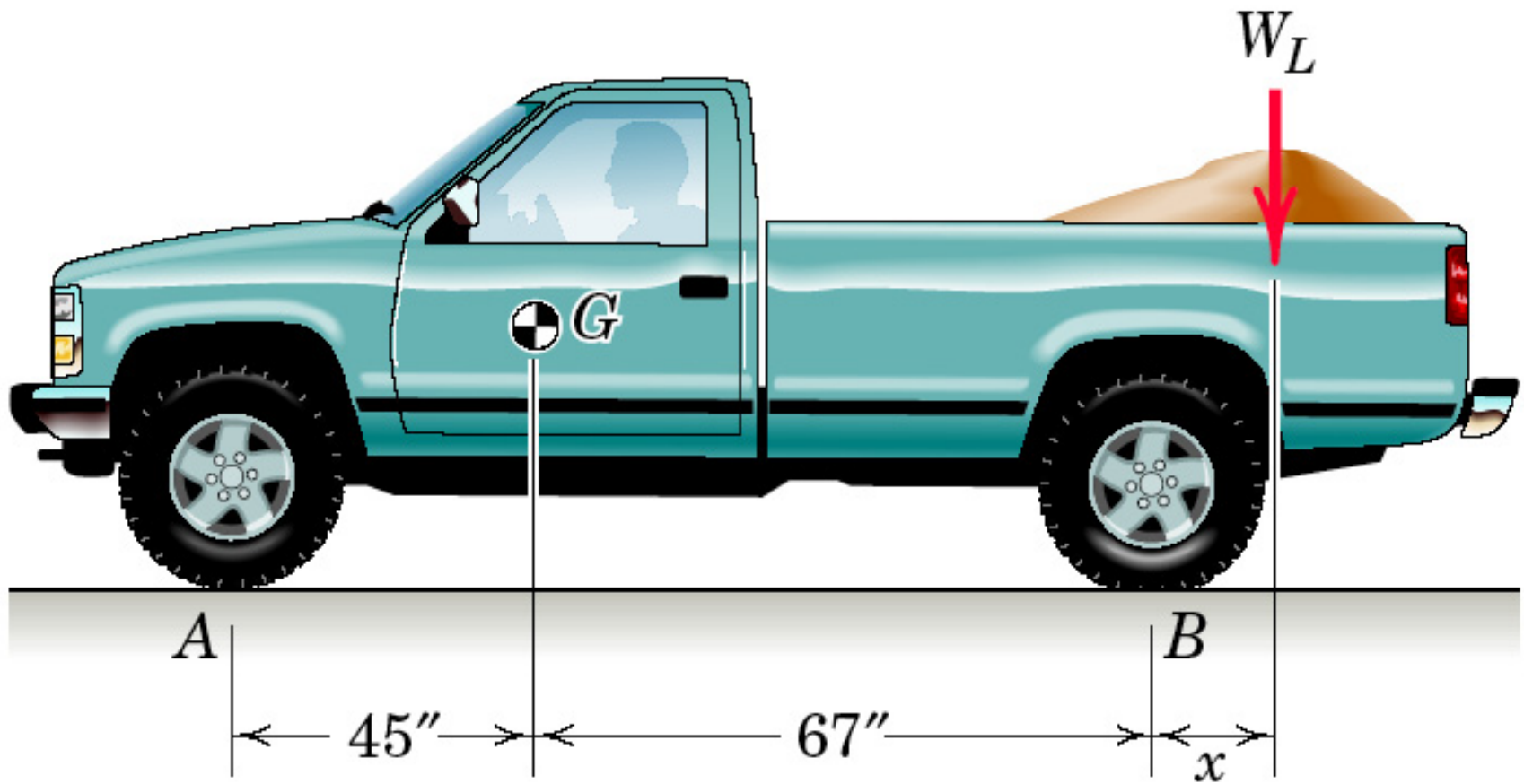
Problem 3/23



Problem 3/24

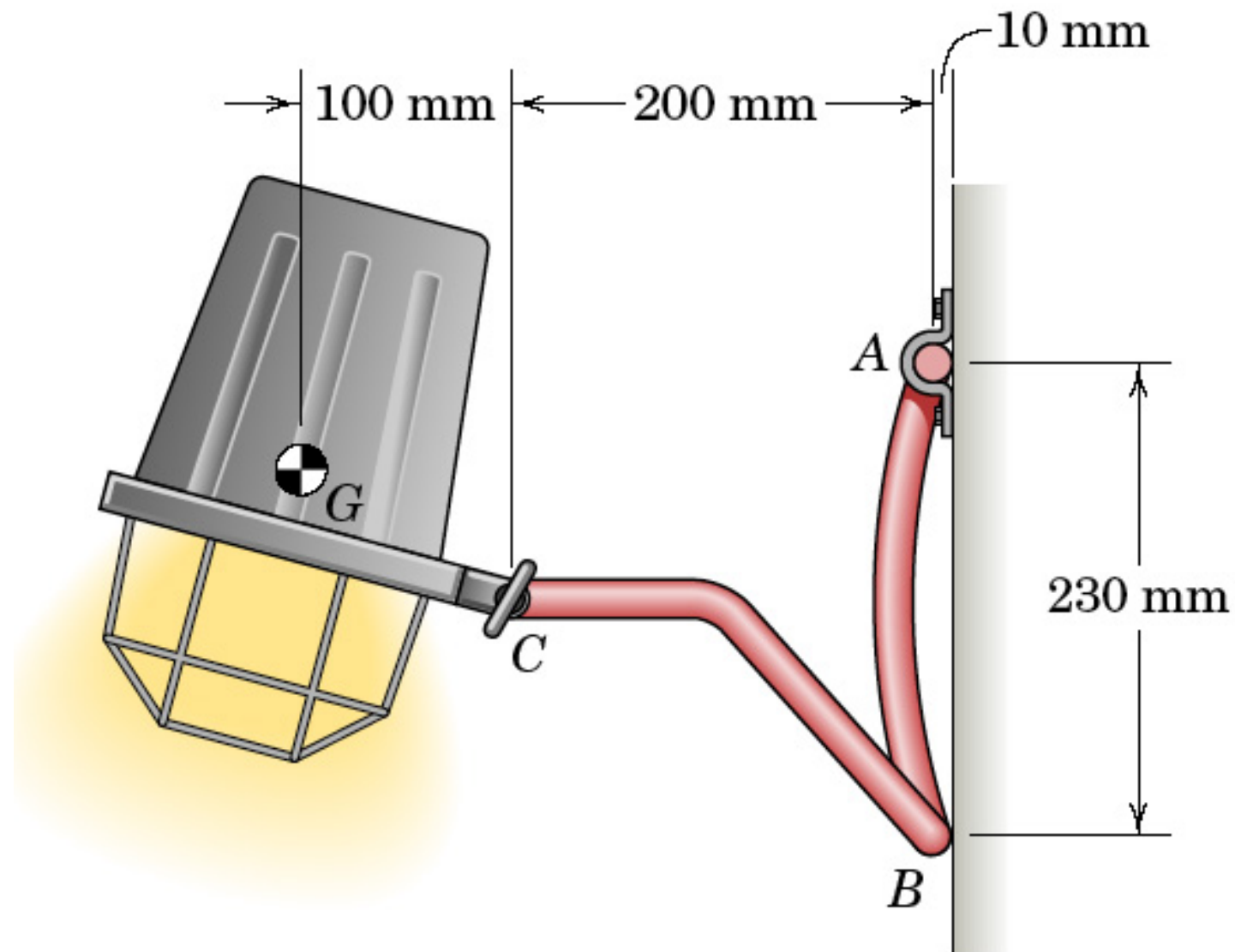


Problem 3/25

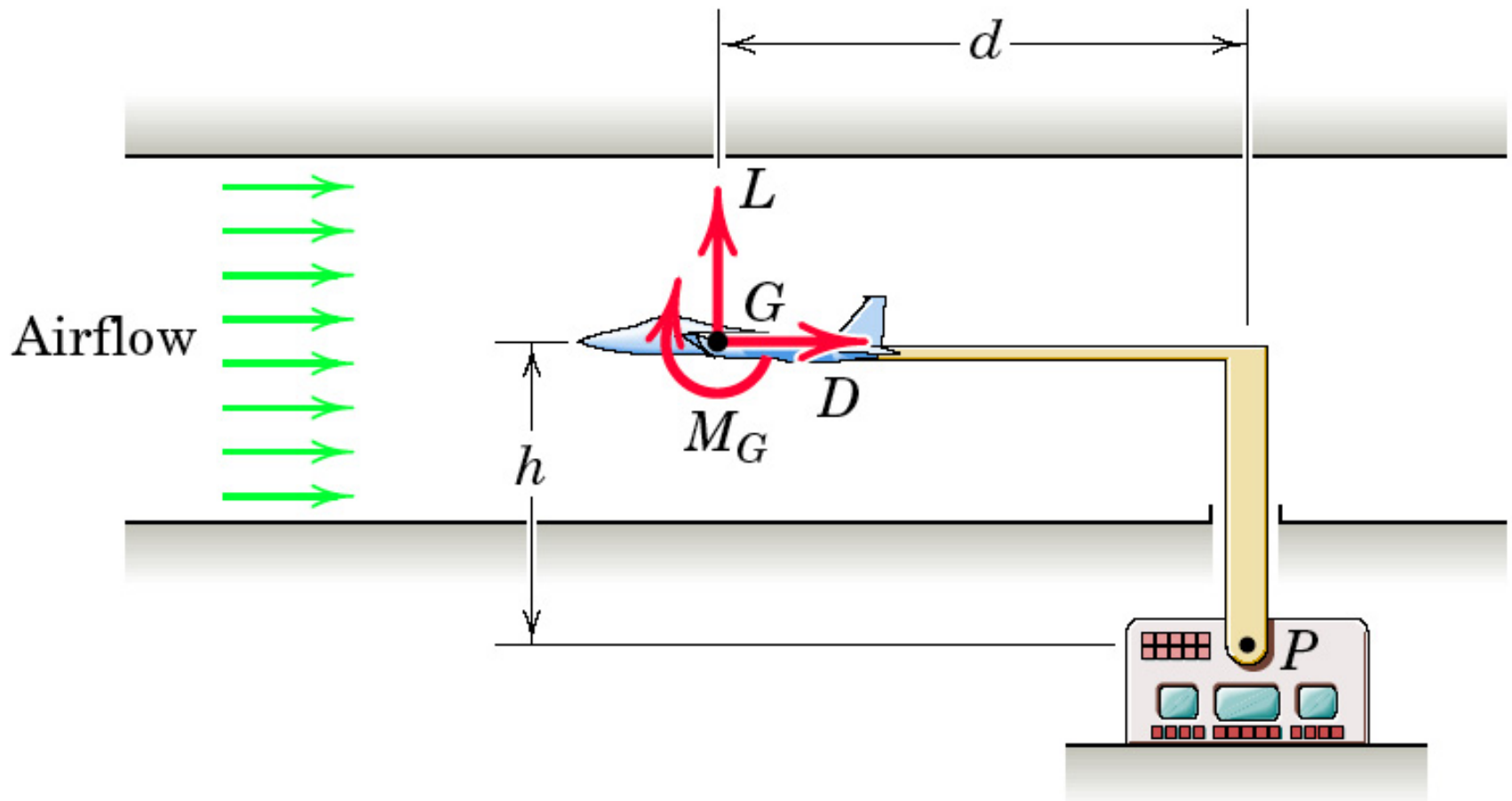


Problem 3/26

P_03_26

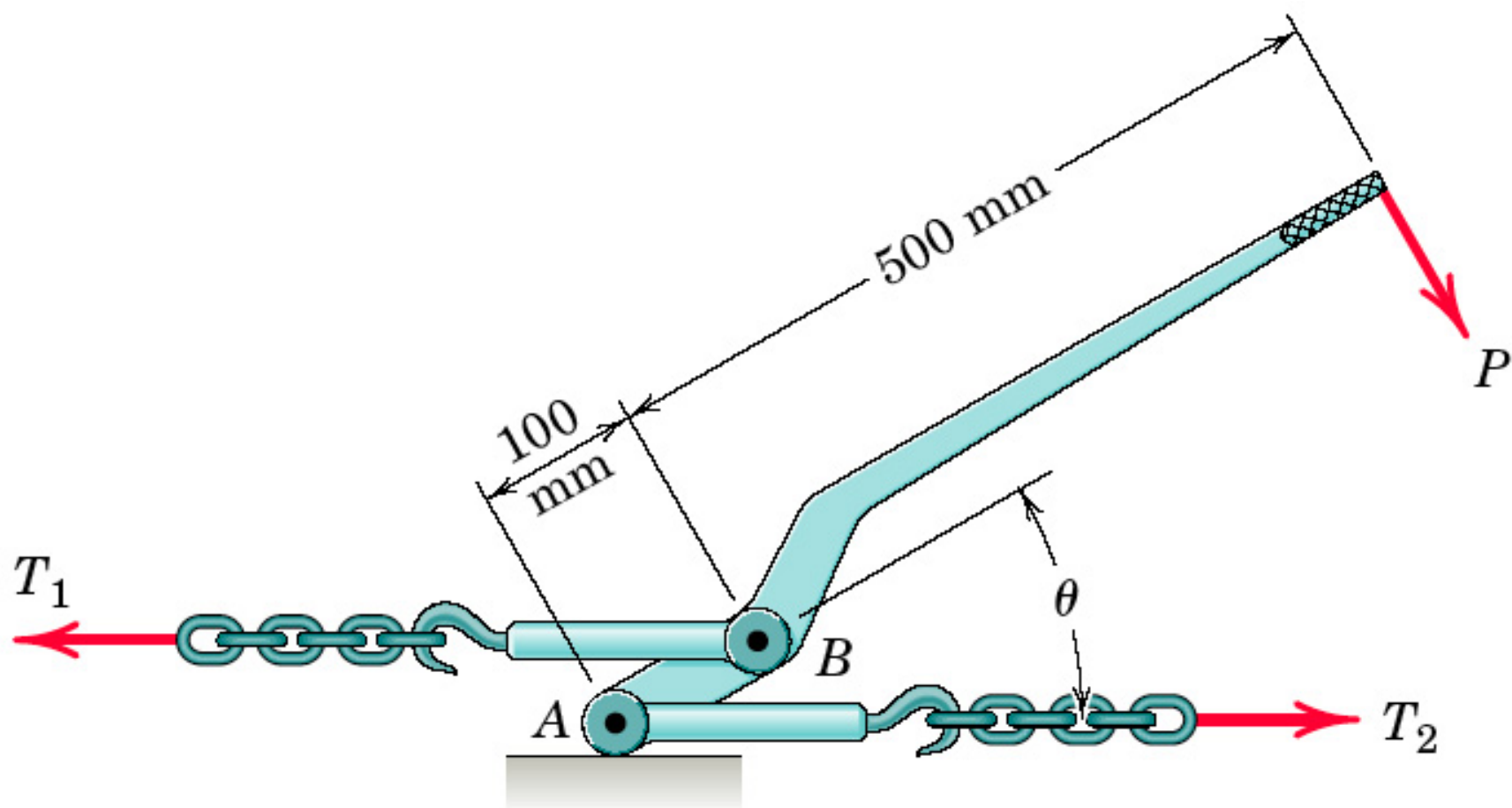


Problem 3/27



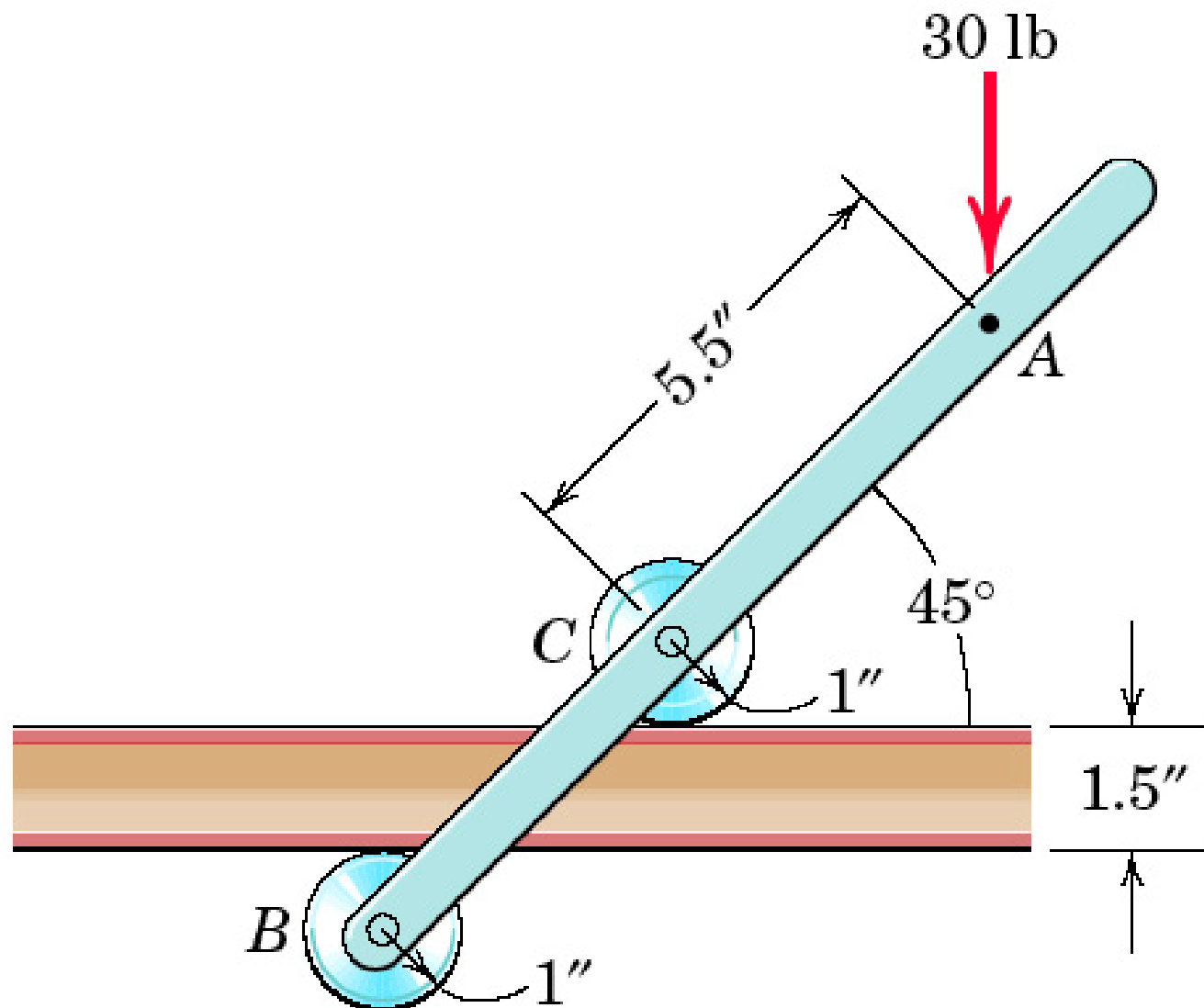
Problem 3/28

P_03_28

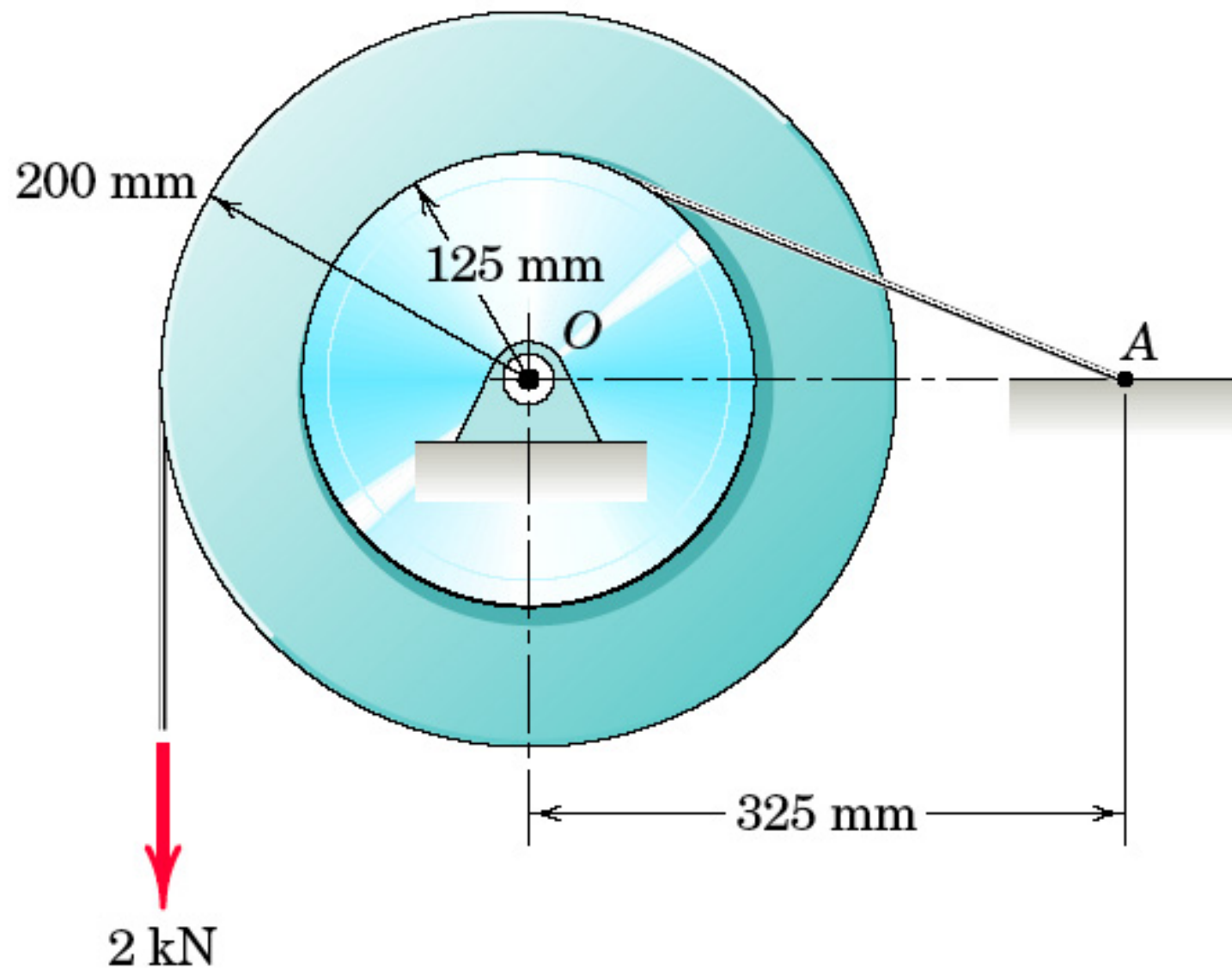


Problem 3/29

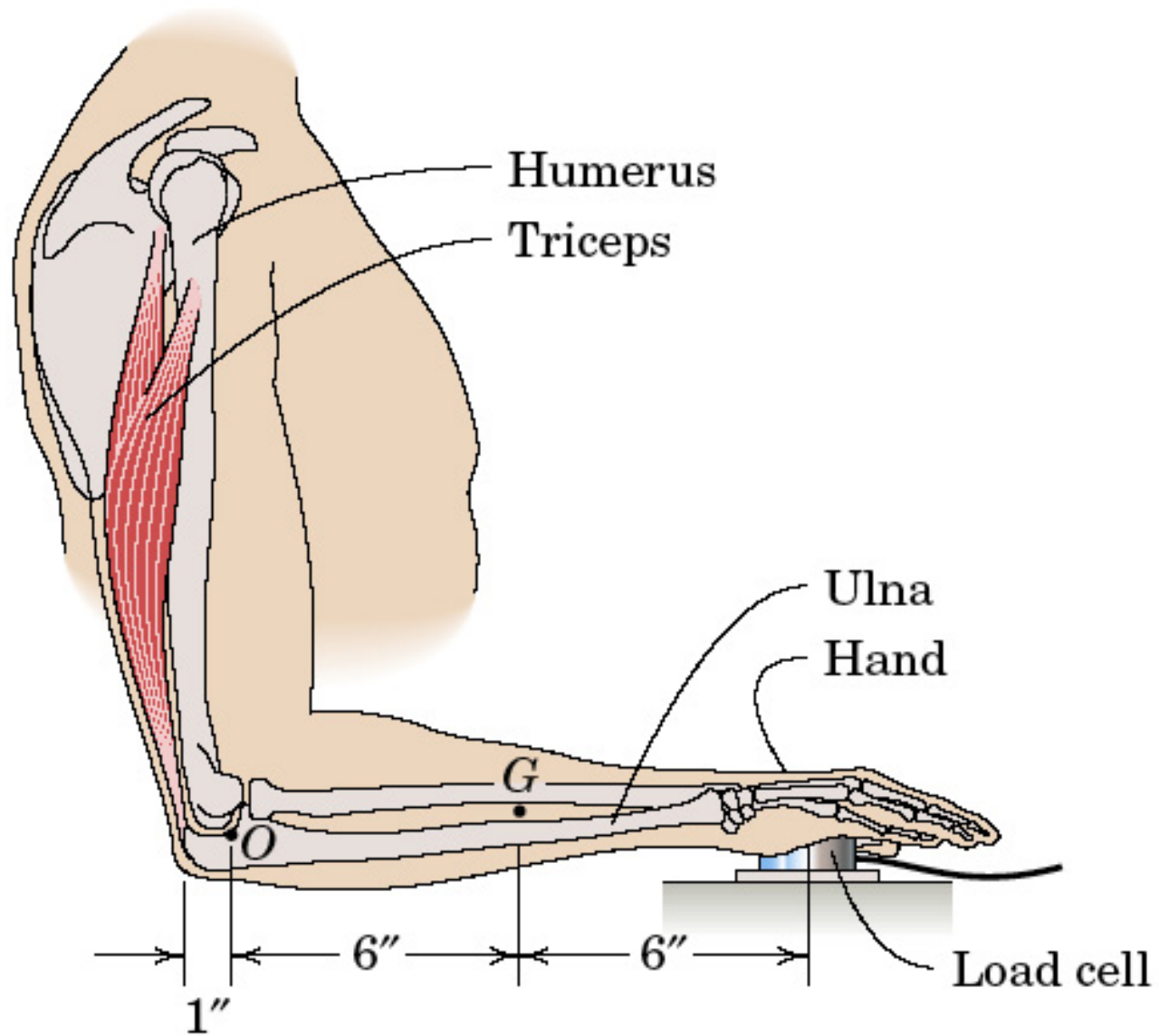
P_03_29



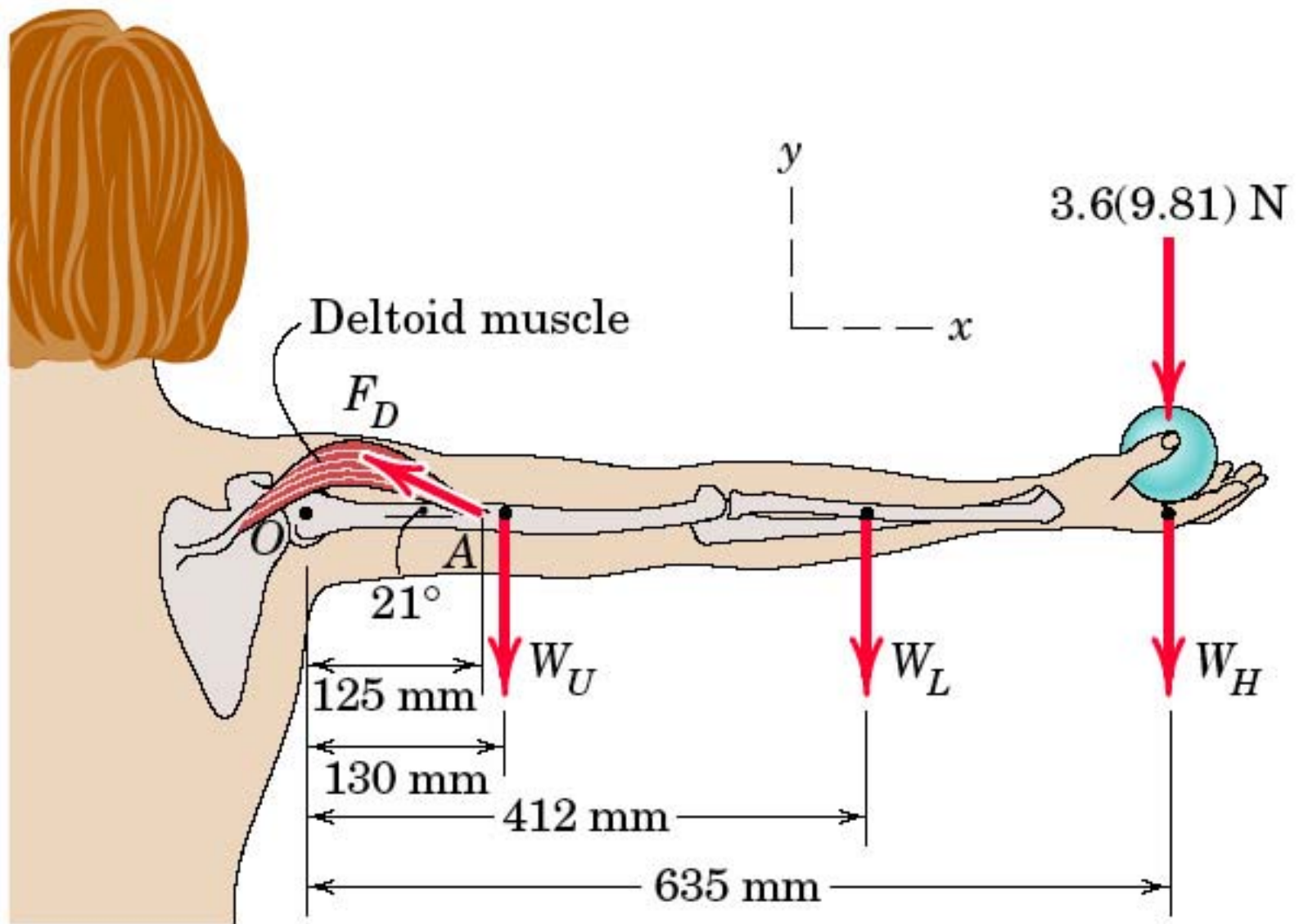
Problem 3/30



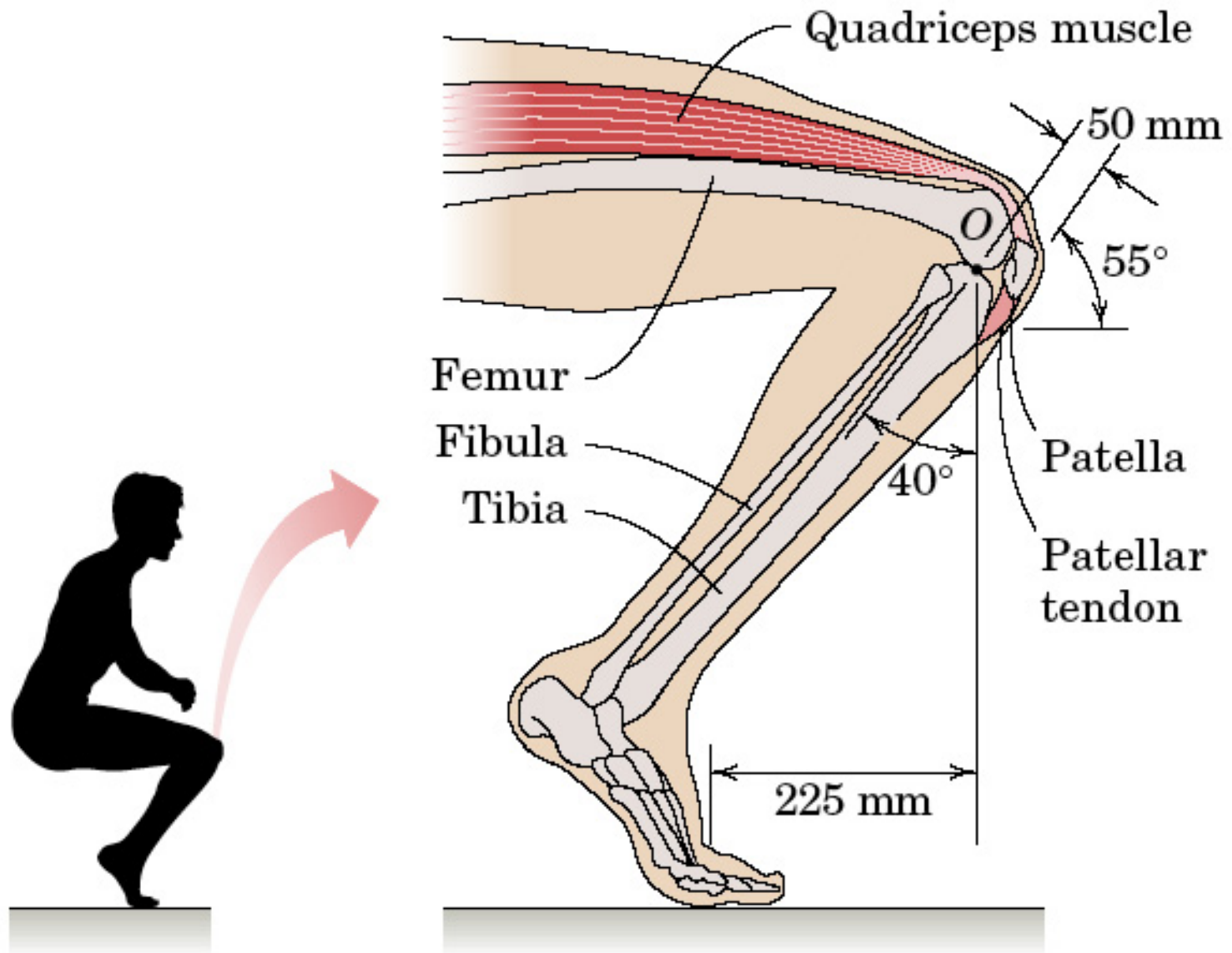
Problem 3/31



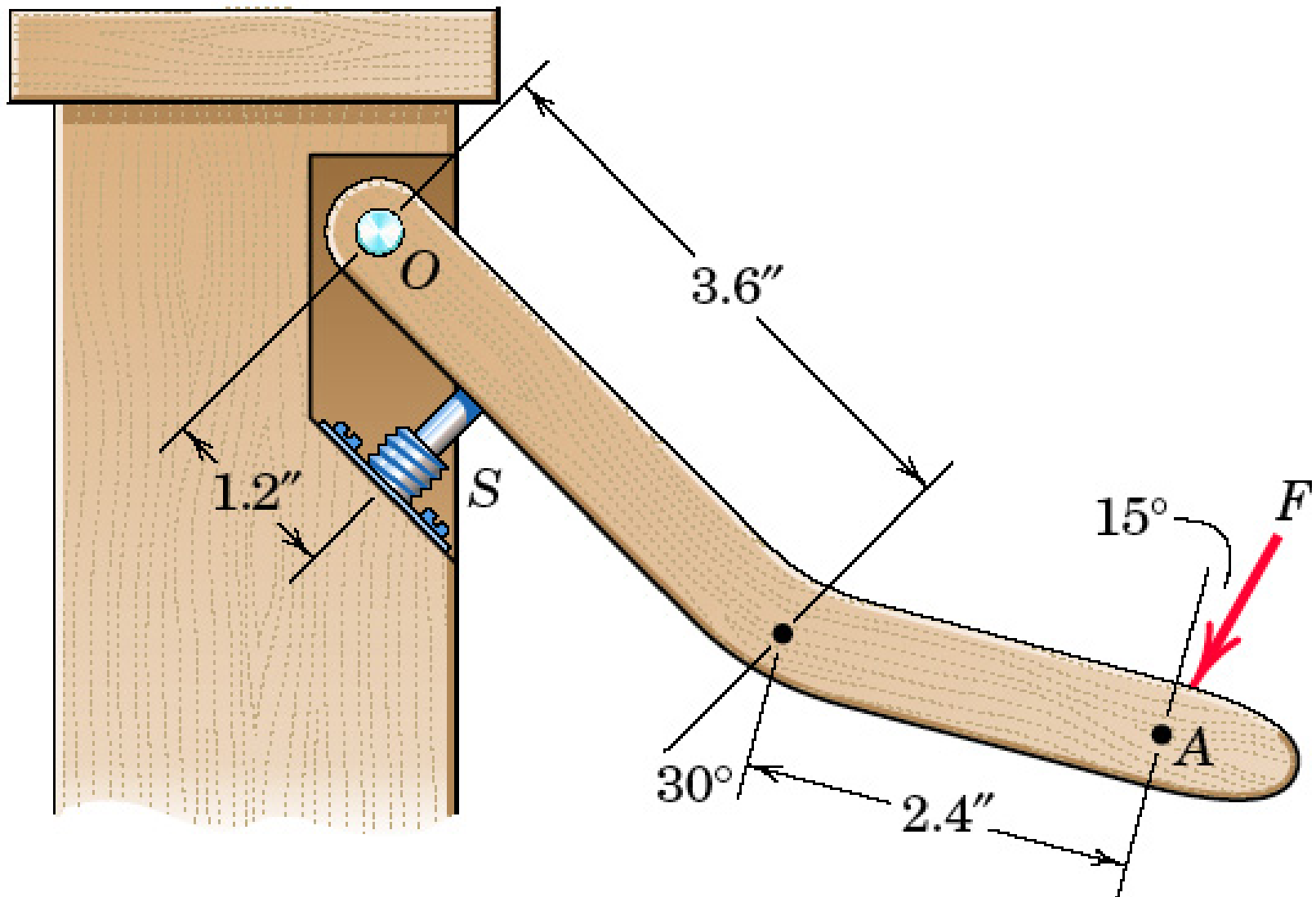
Problem 3/32



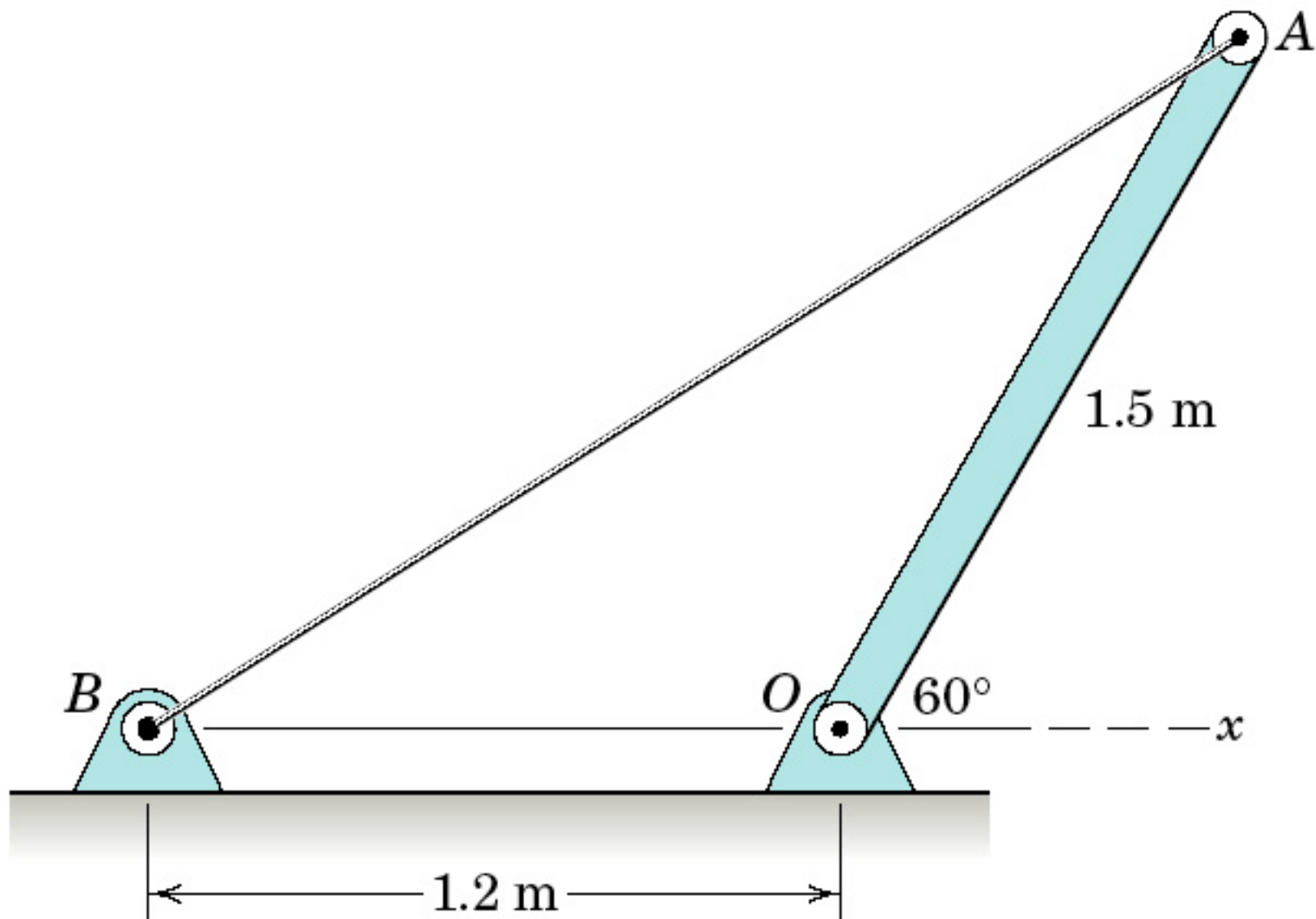
Problem 3/34



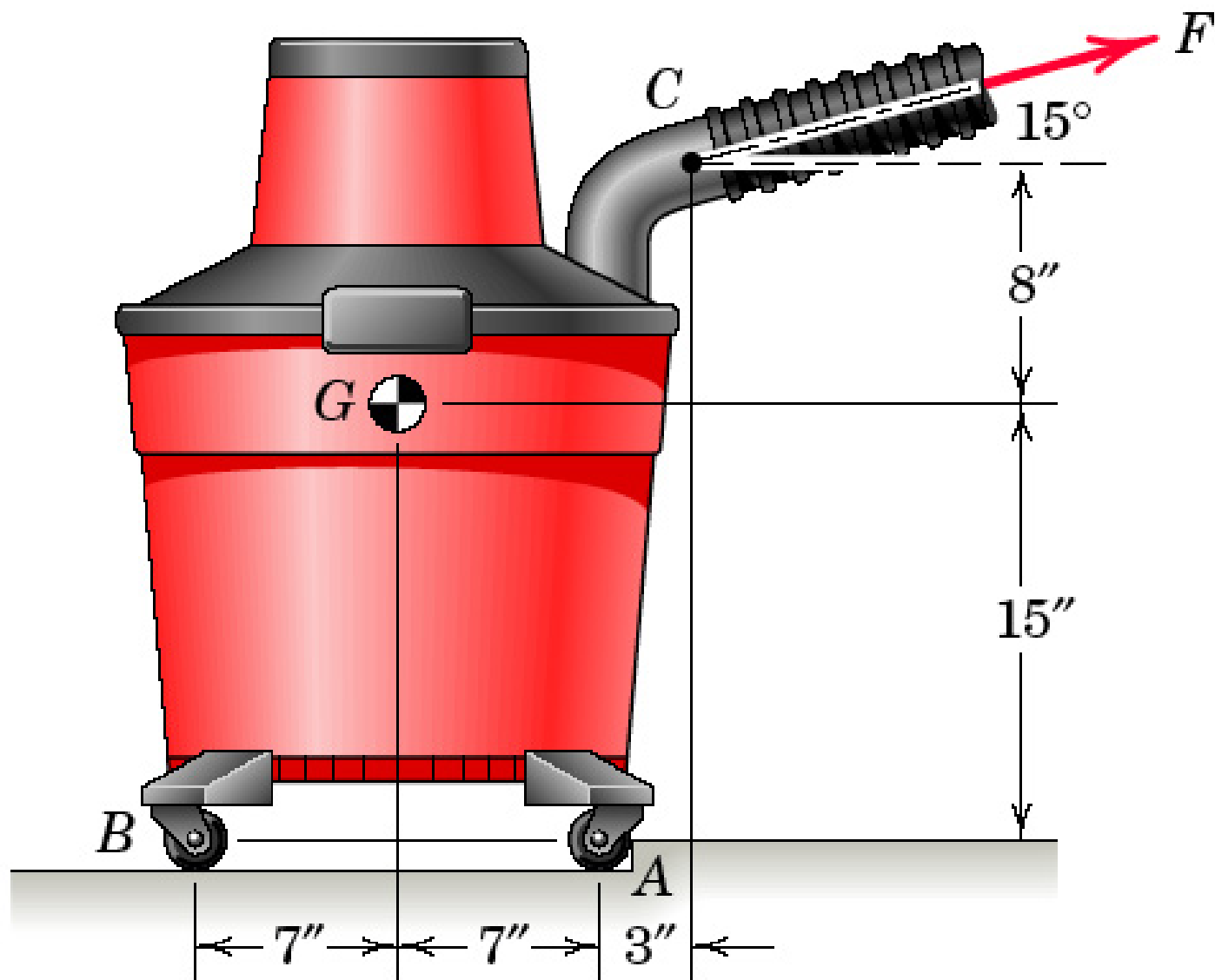
Problem 3/35



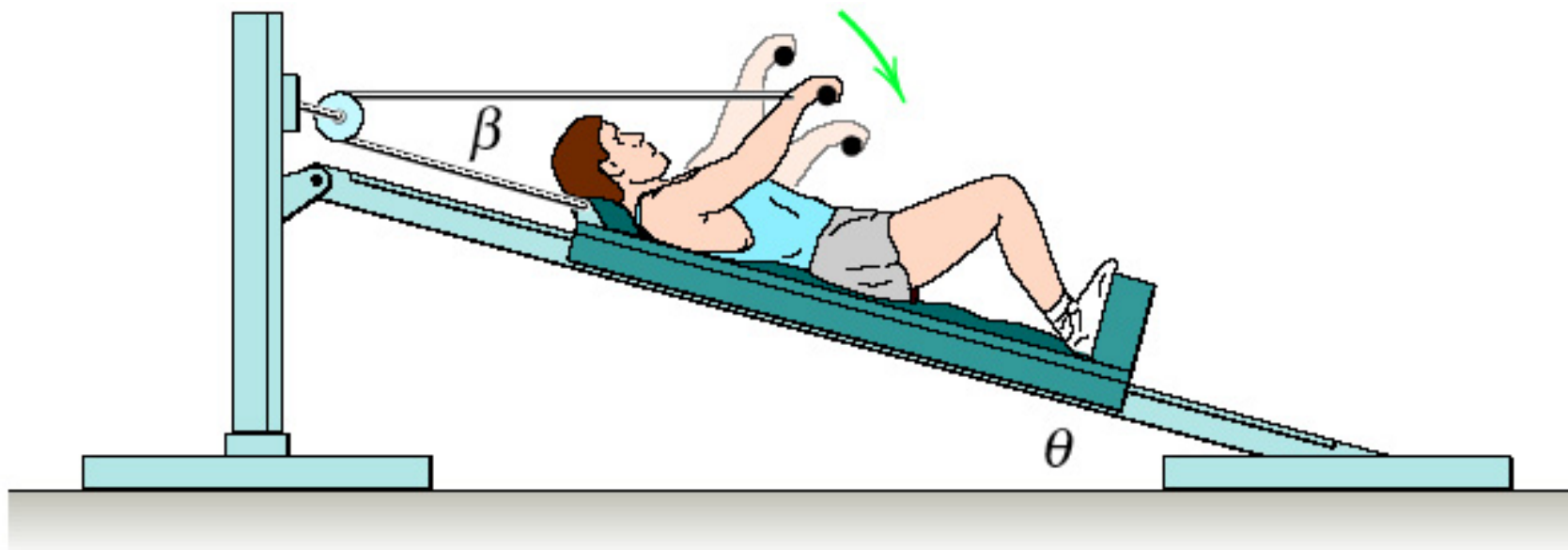
Problem 3/36



Problem 3/37

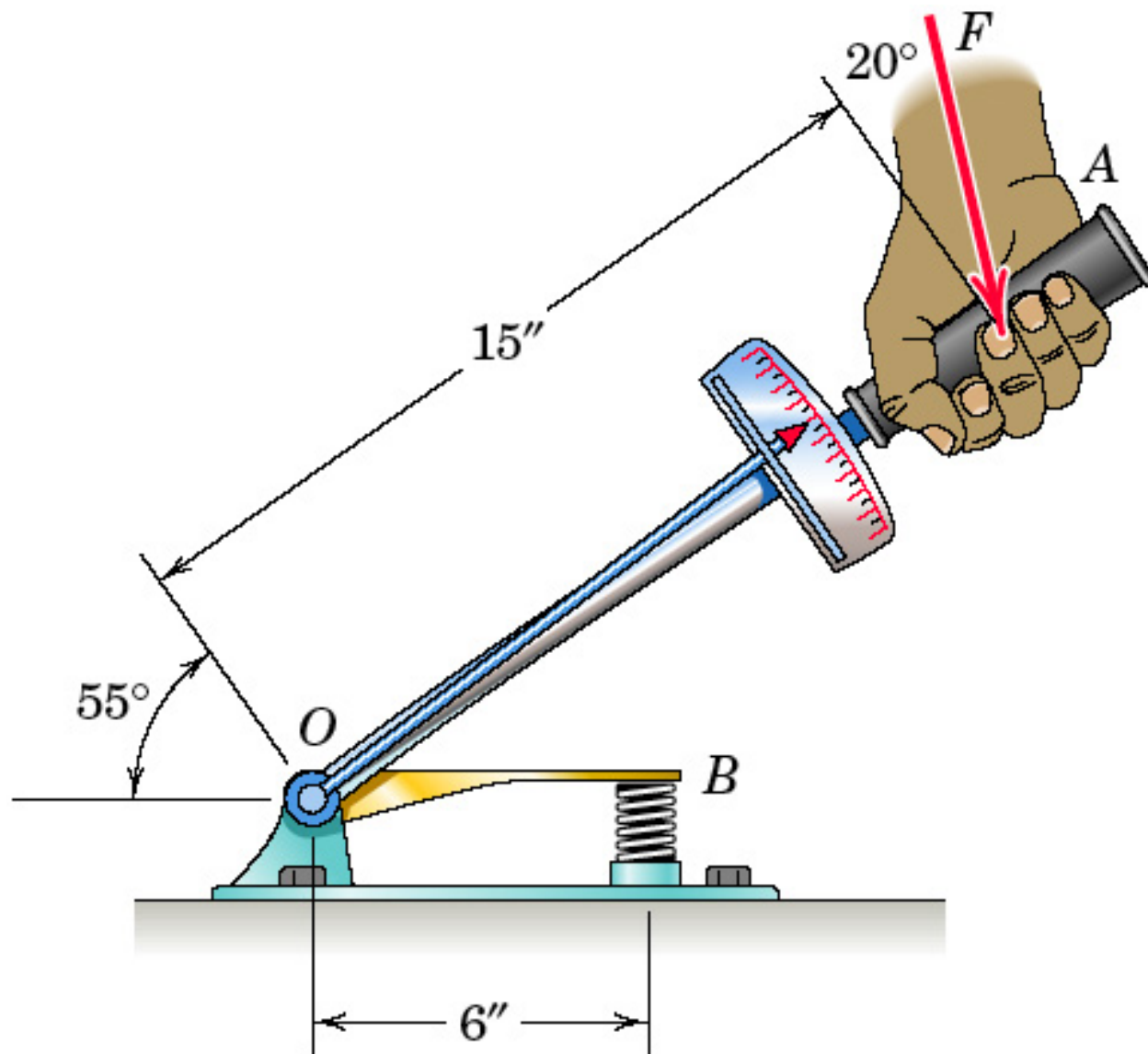


Problem 3/38

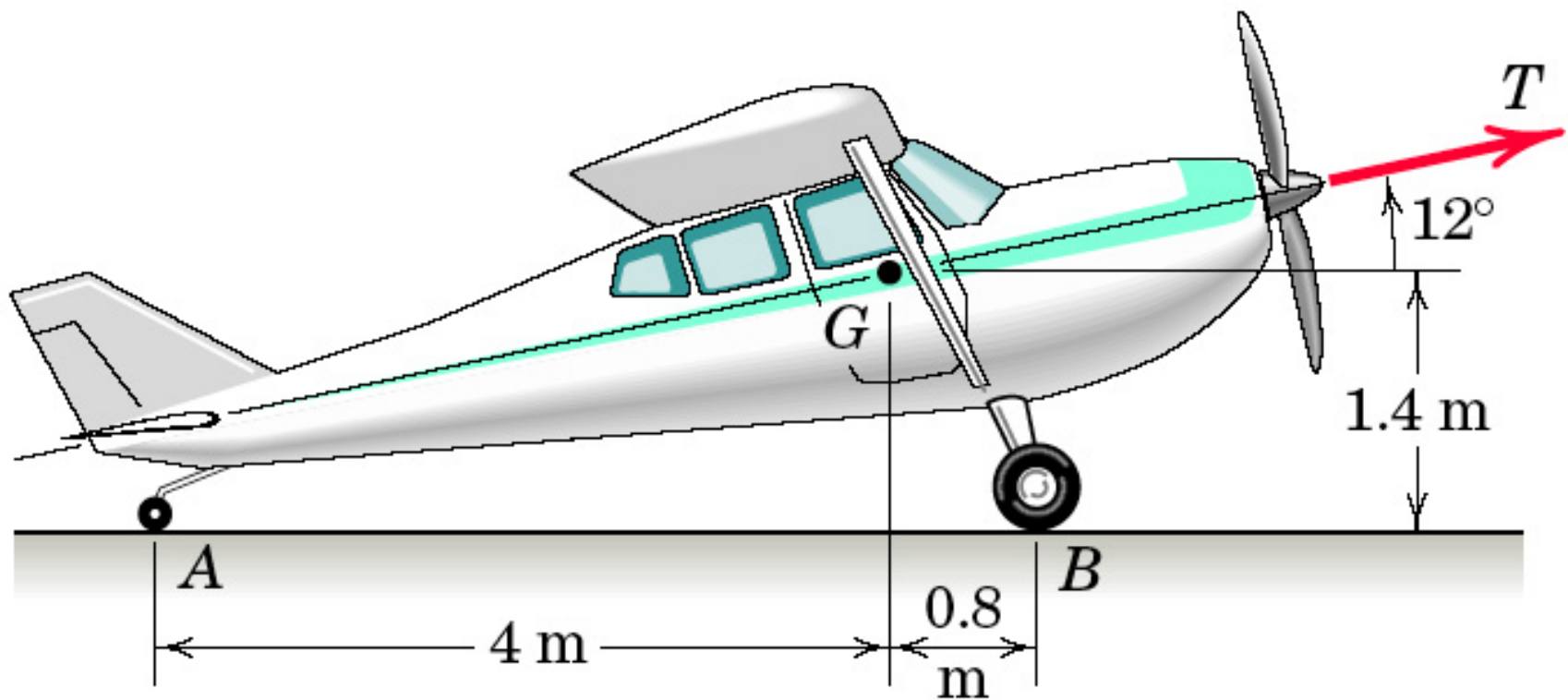


Problem 3/39

P_03_39

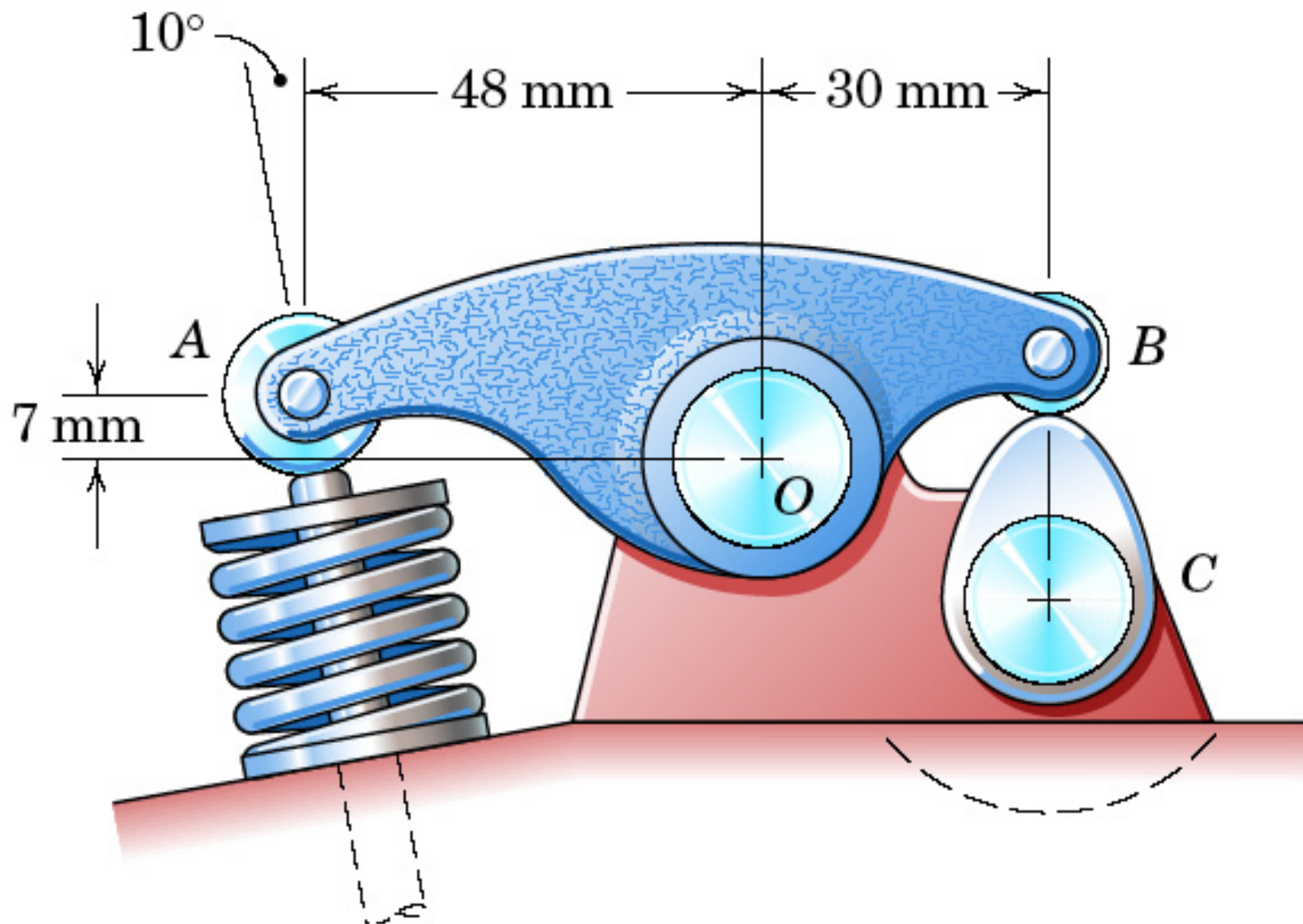


Problem 3/40

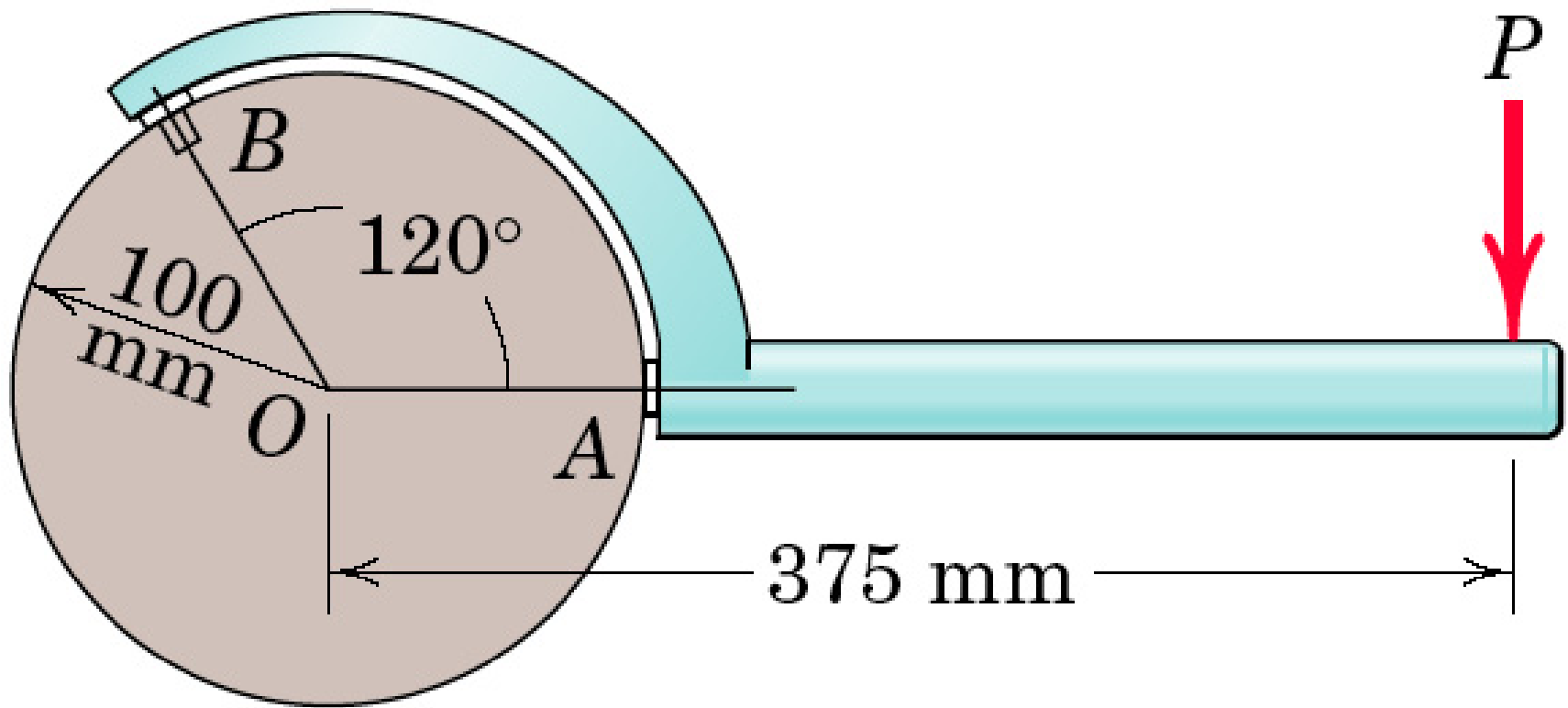


Problem 3/41

P_03_41

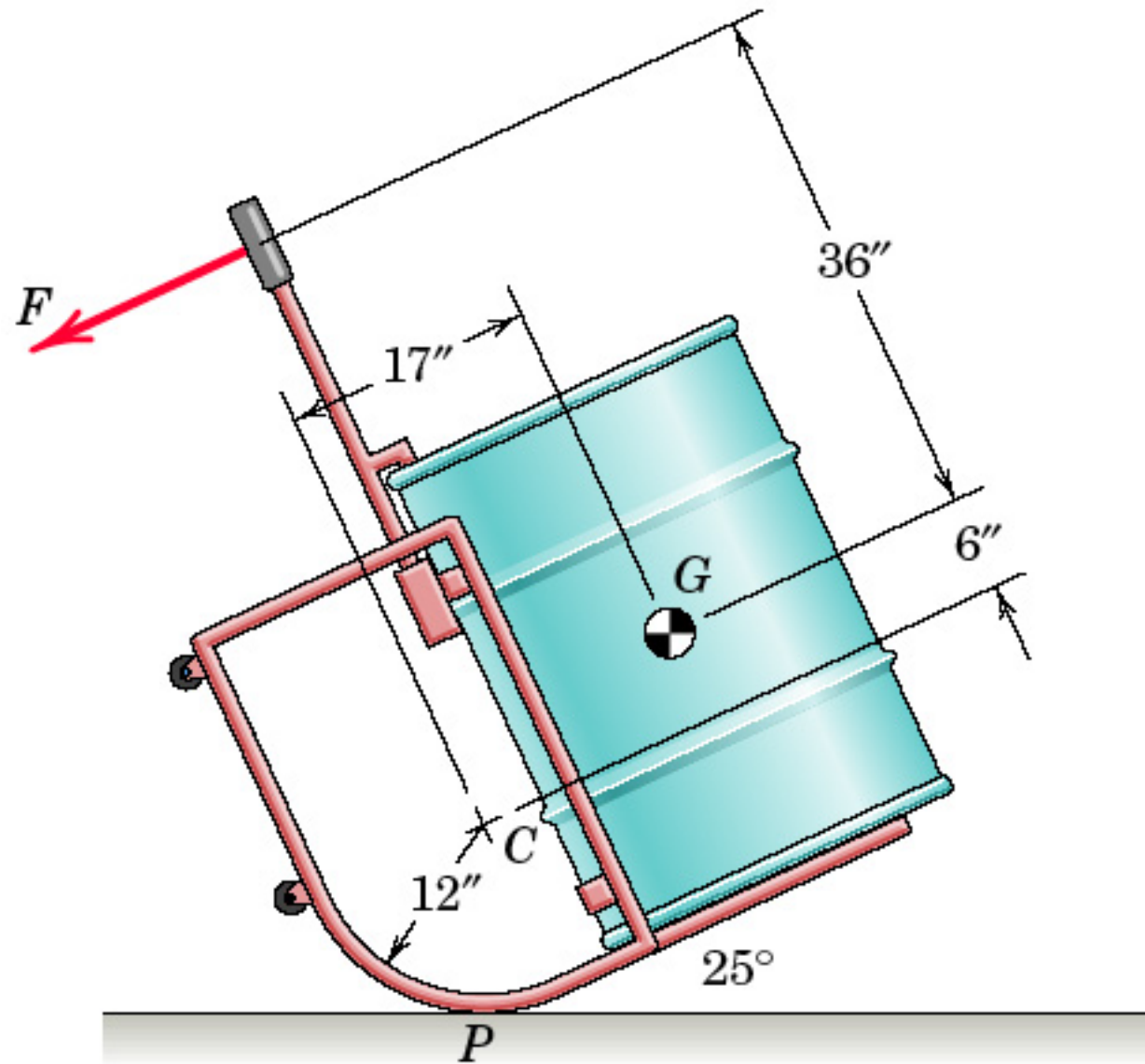


Problem 3/42

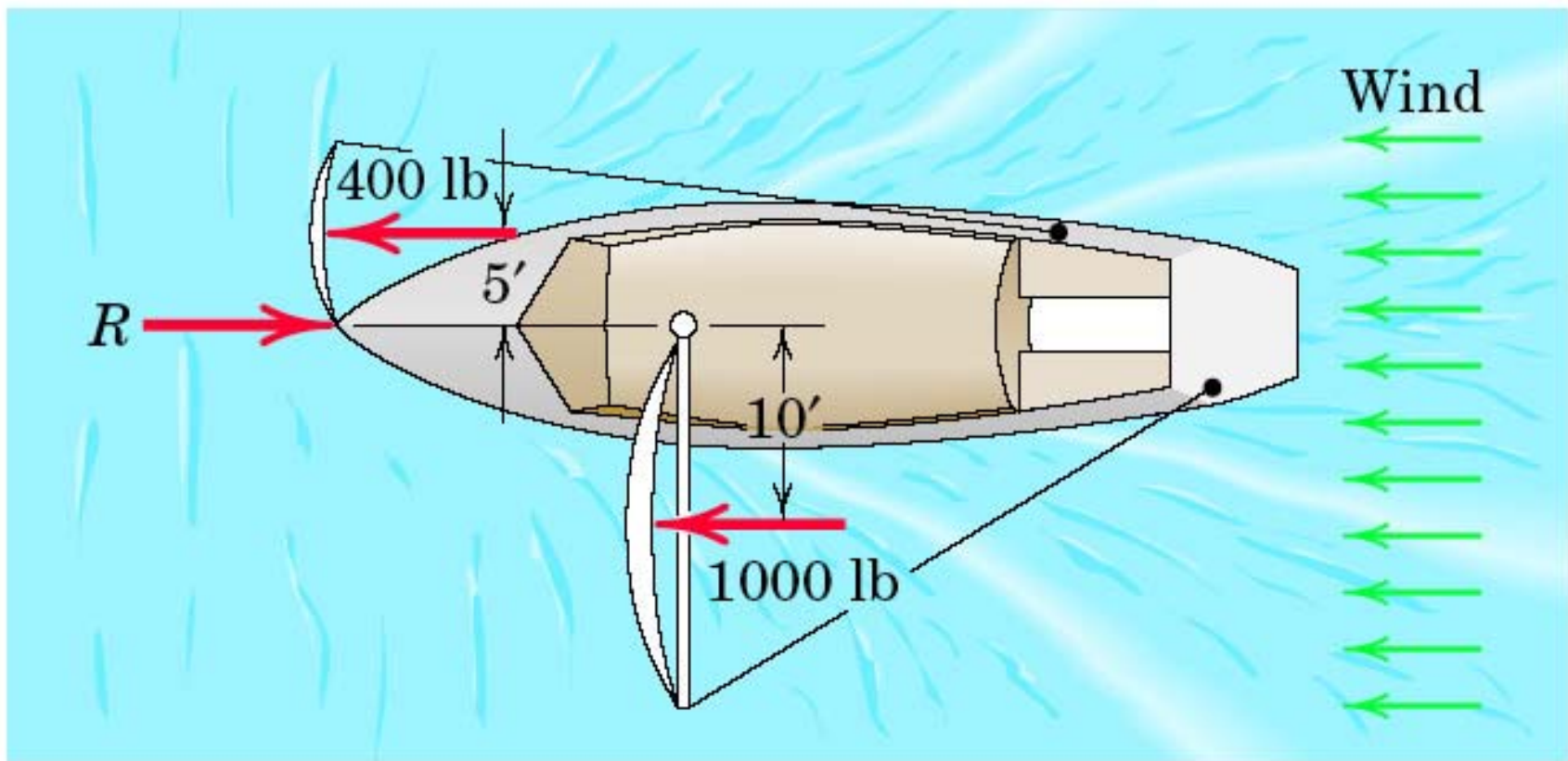


Problem 3/43

P_03_43

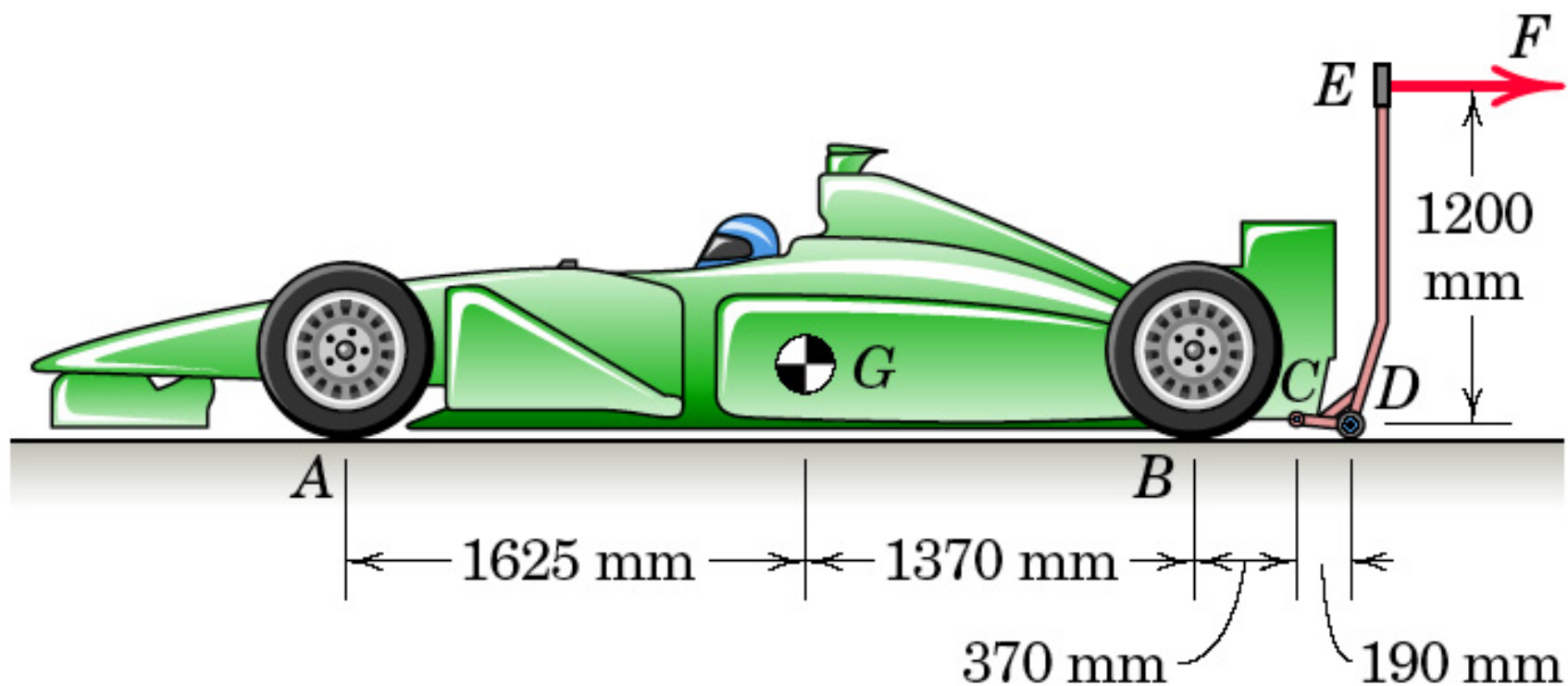


Problem 3/44



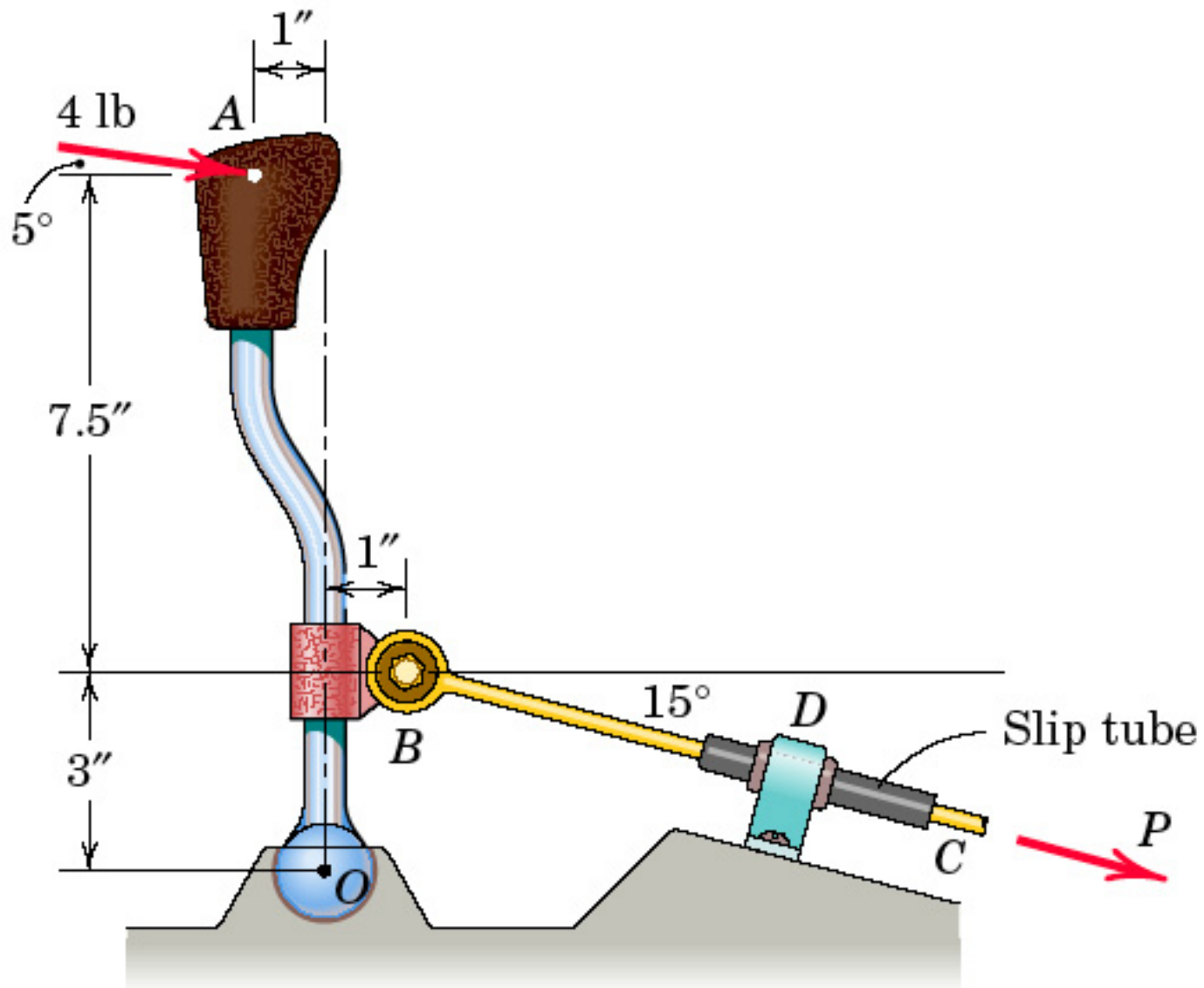
Problem 3/45

P_03_45

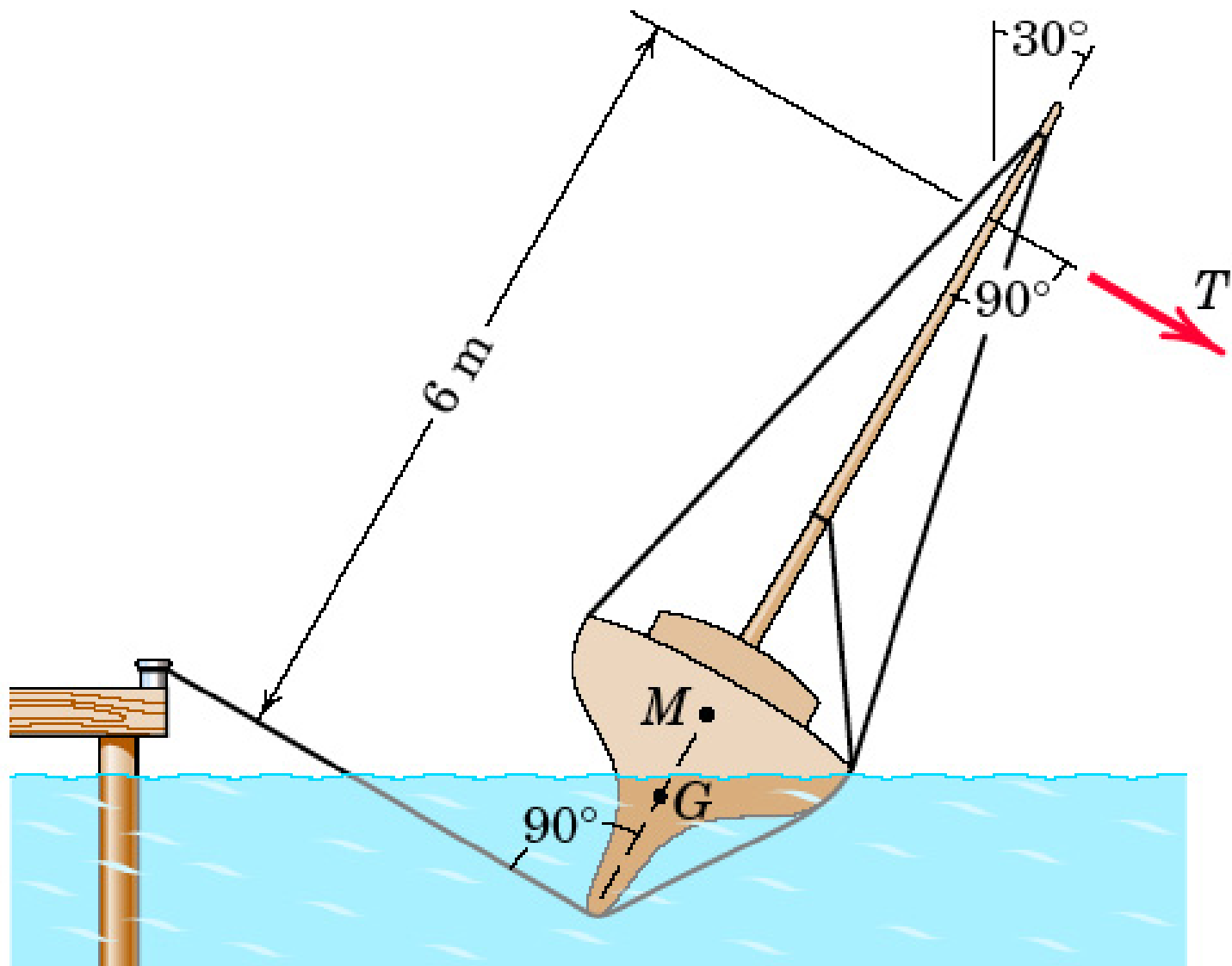


Problem 3/46

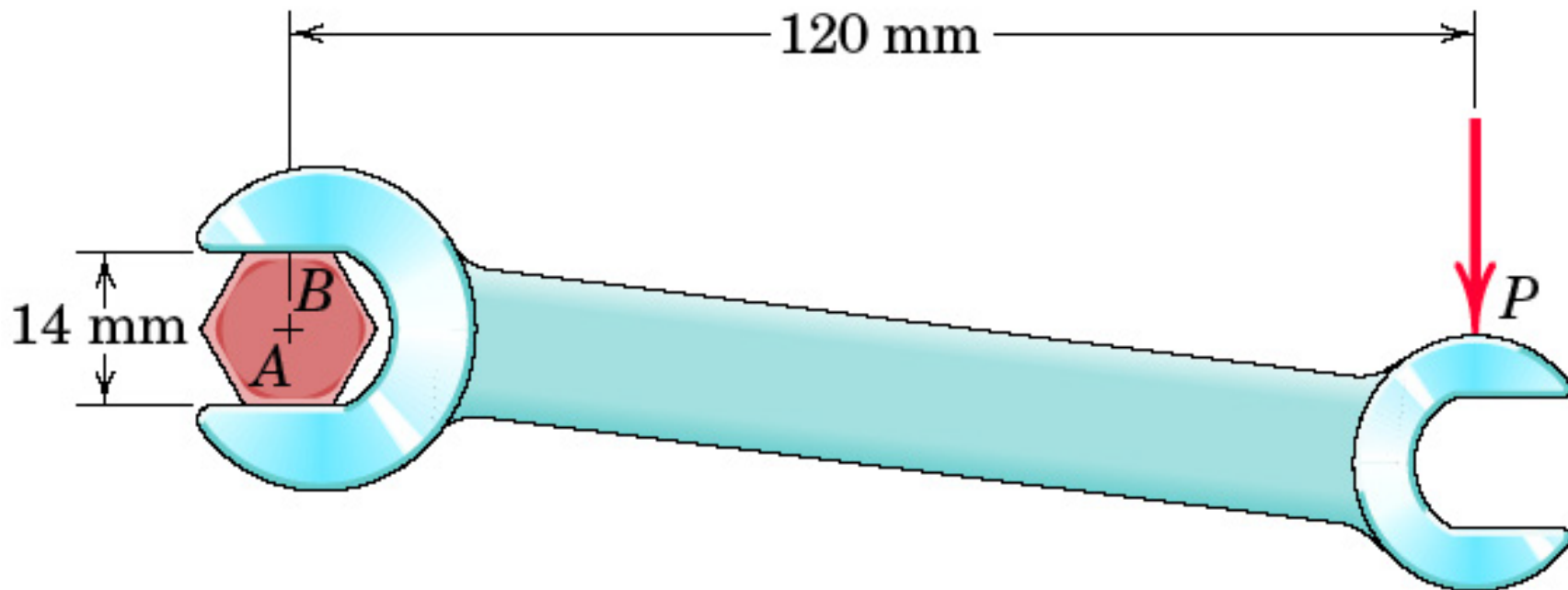
P_03_46



Problem 3/47

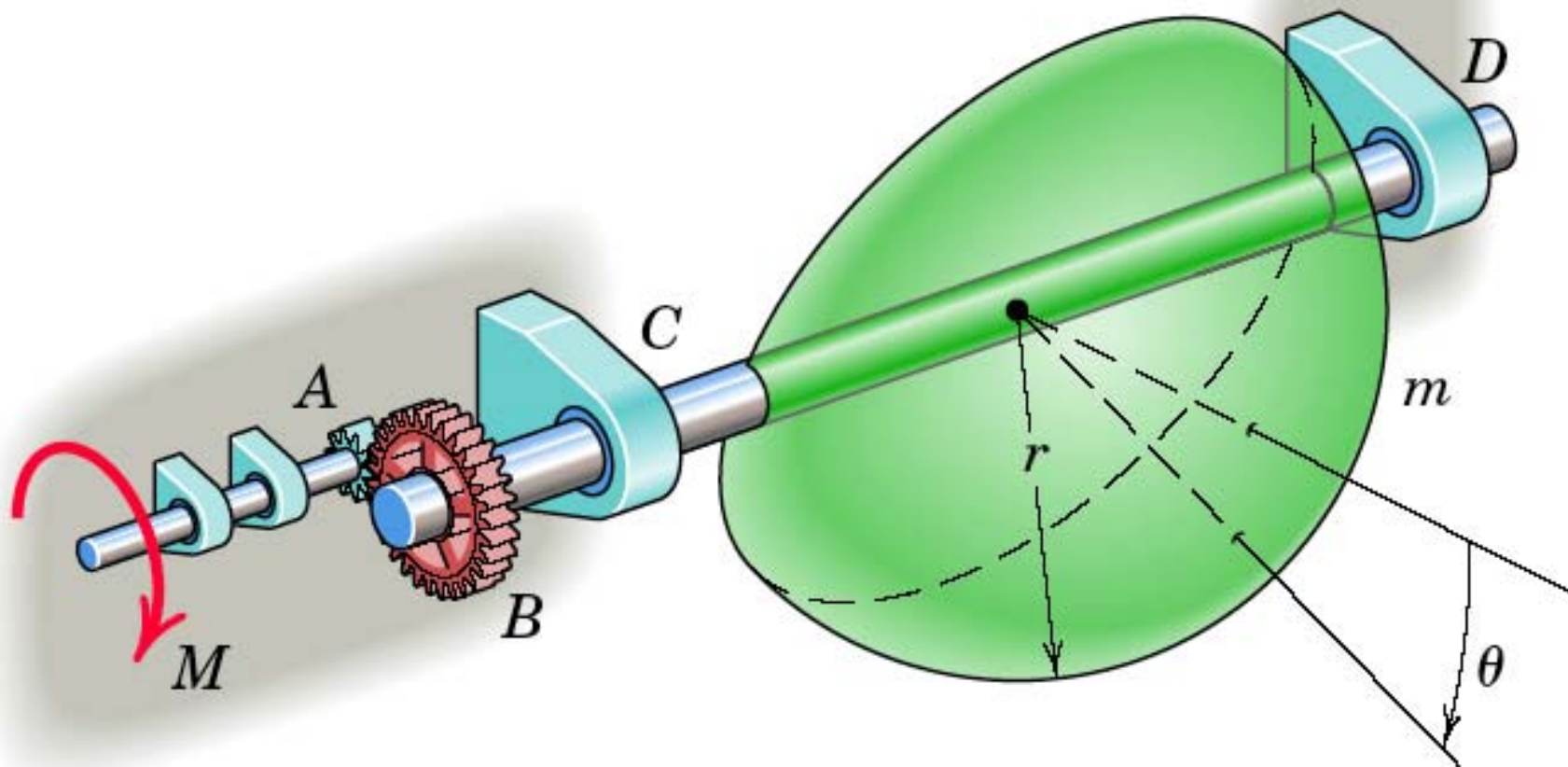


Problem 3/48



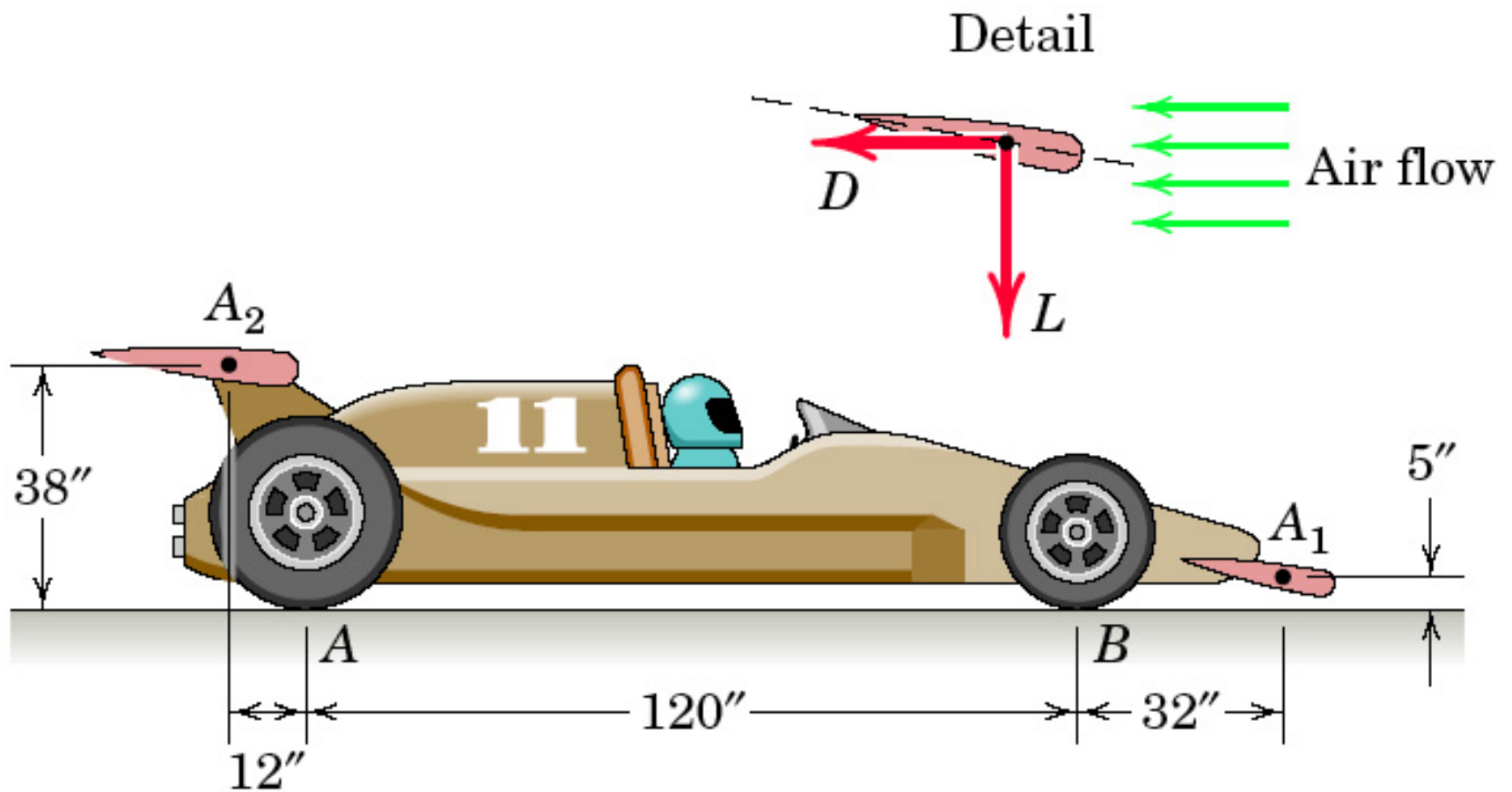
Problem 3/49

P_03_49



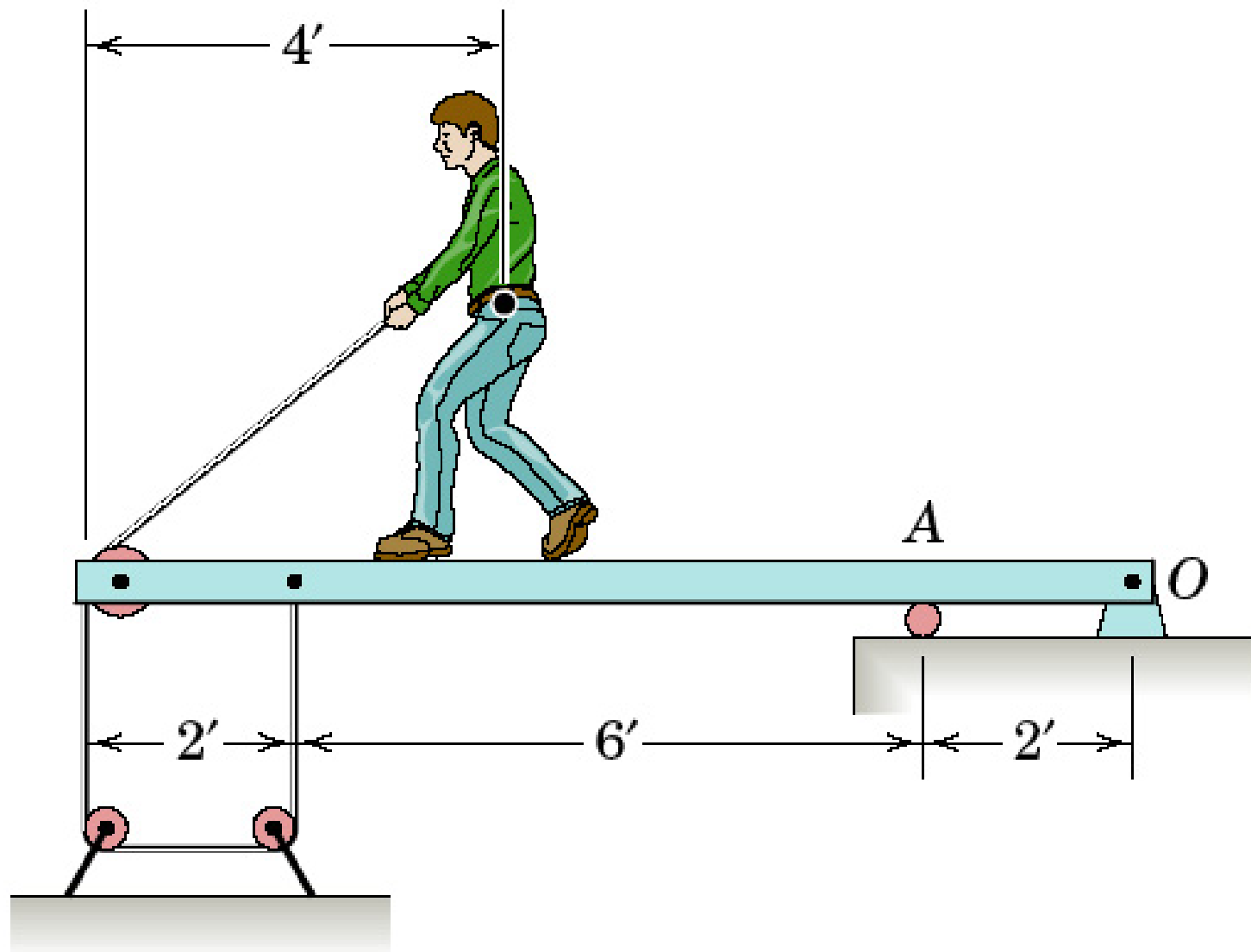
Problem 3/50

P_03_50

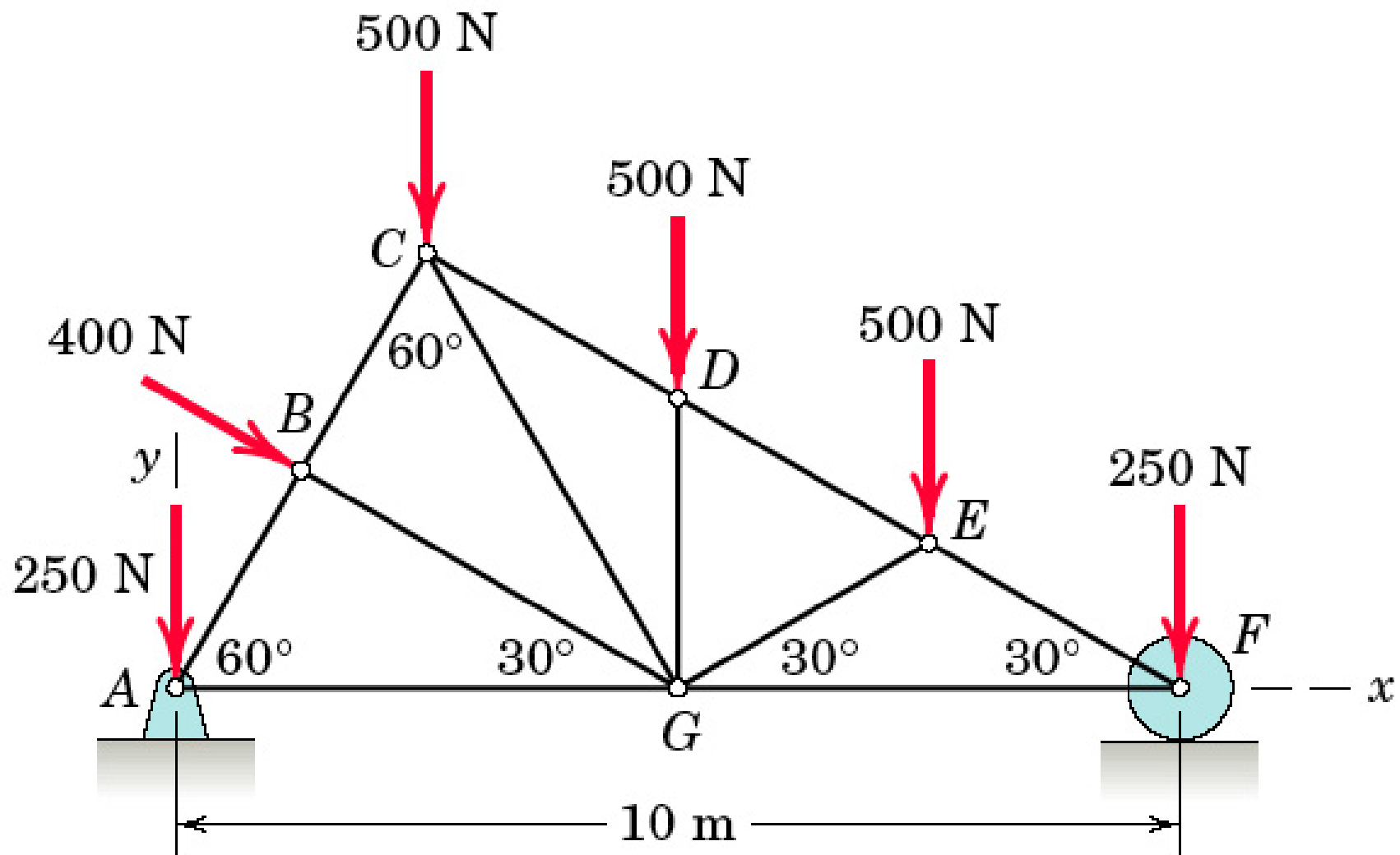


Problem 3/51

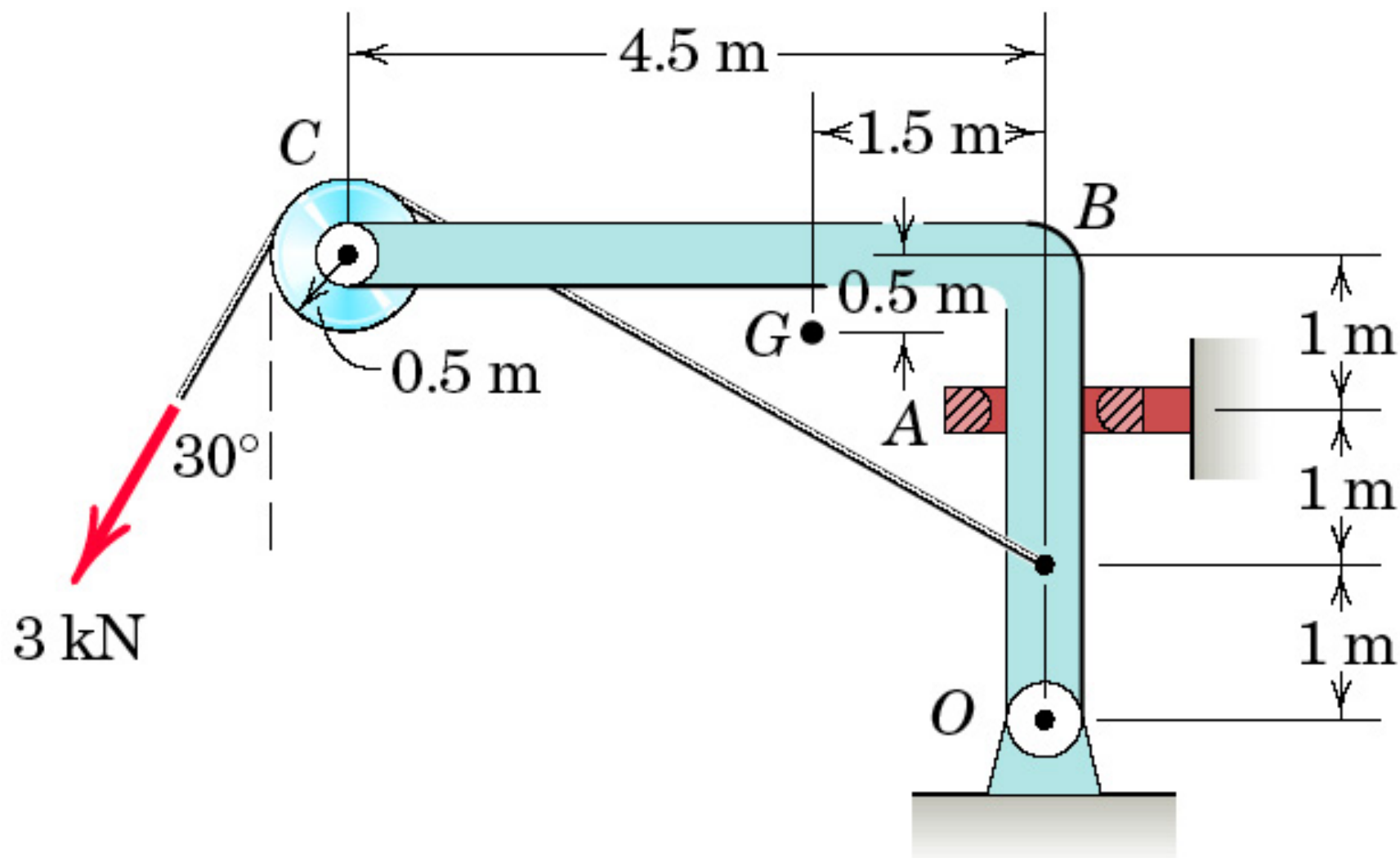
P_03_51



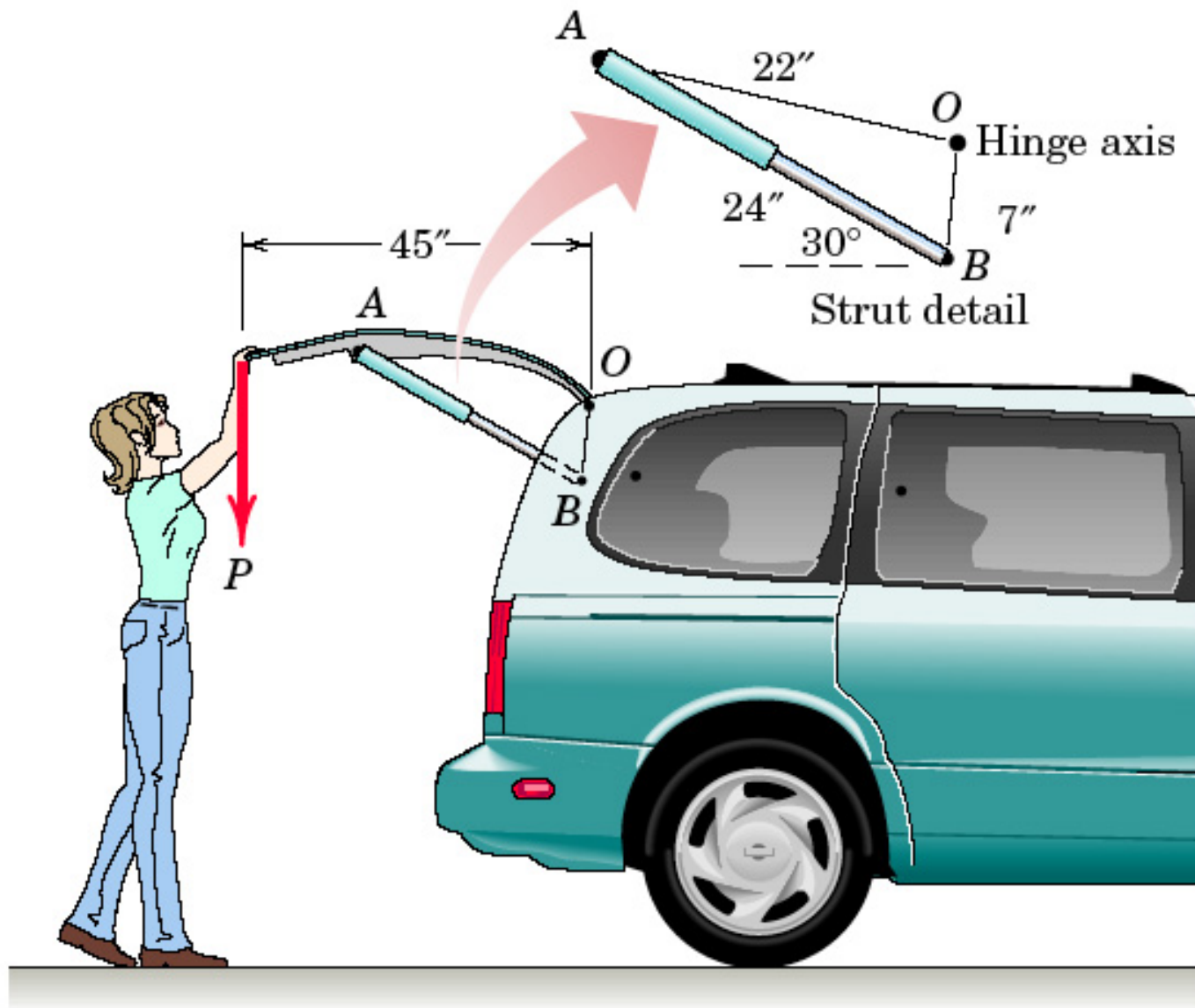
Problem 3/52



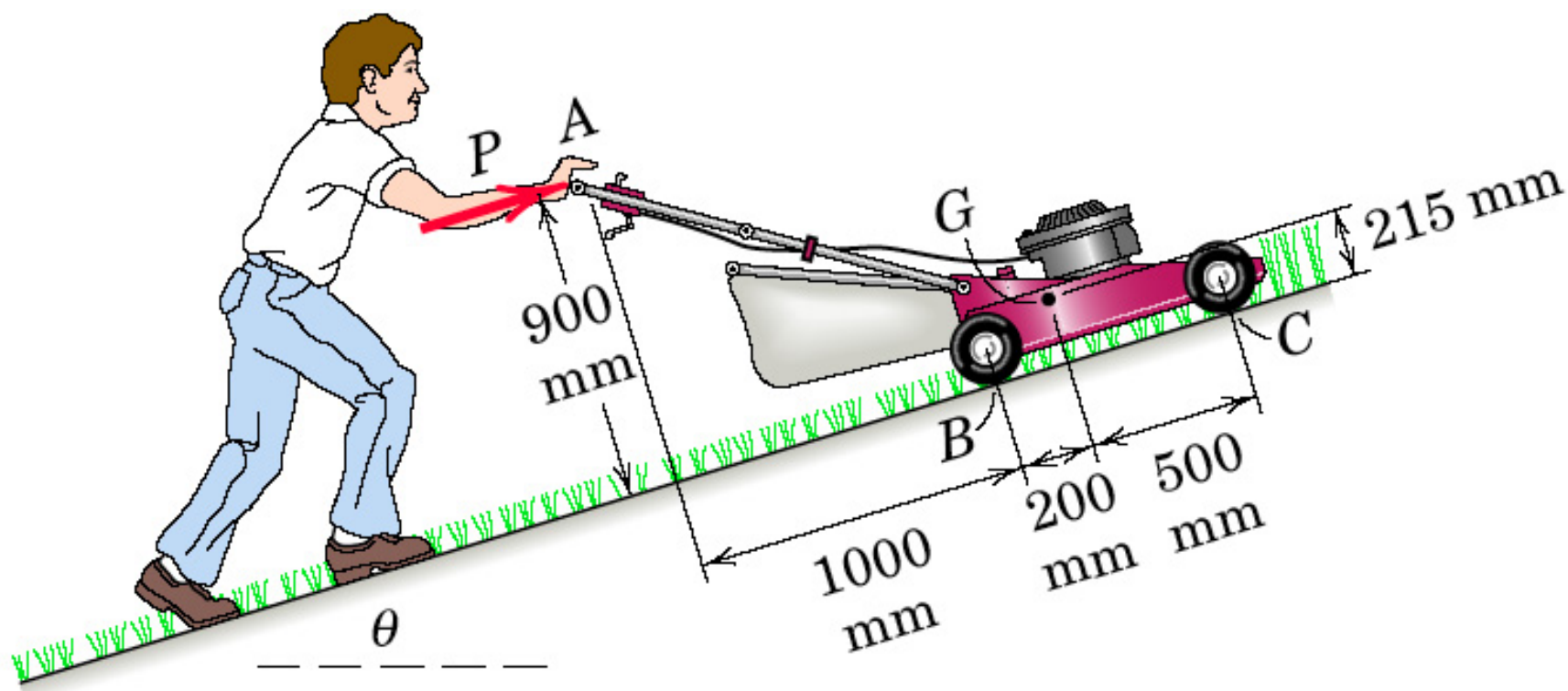
Problem 3/53



Problem 3/54

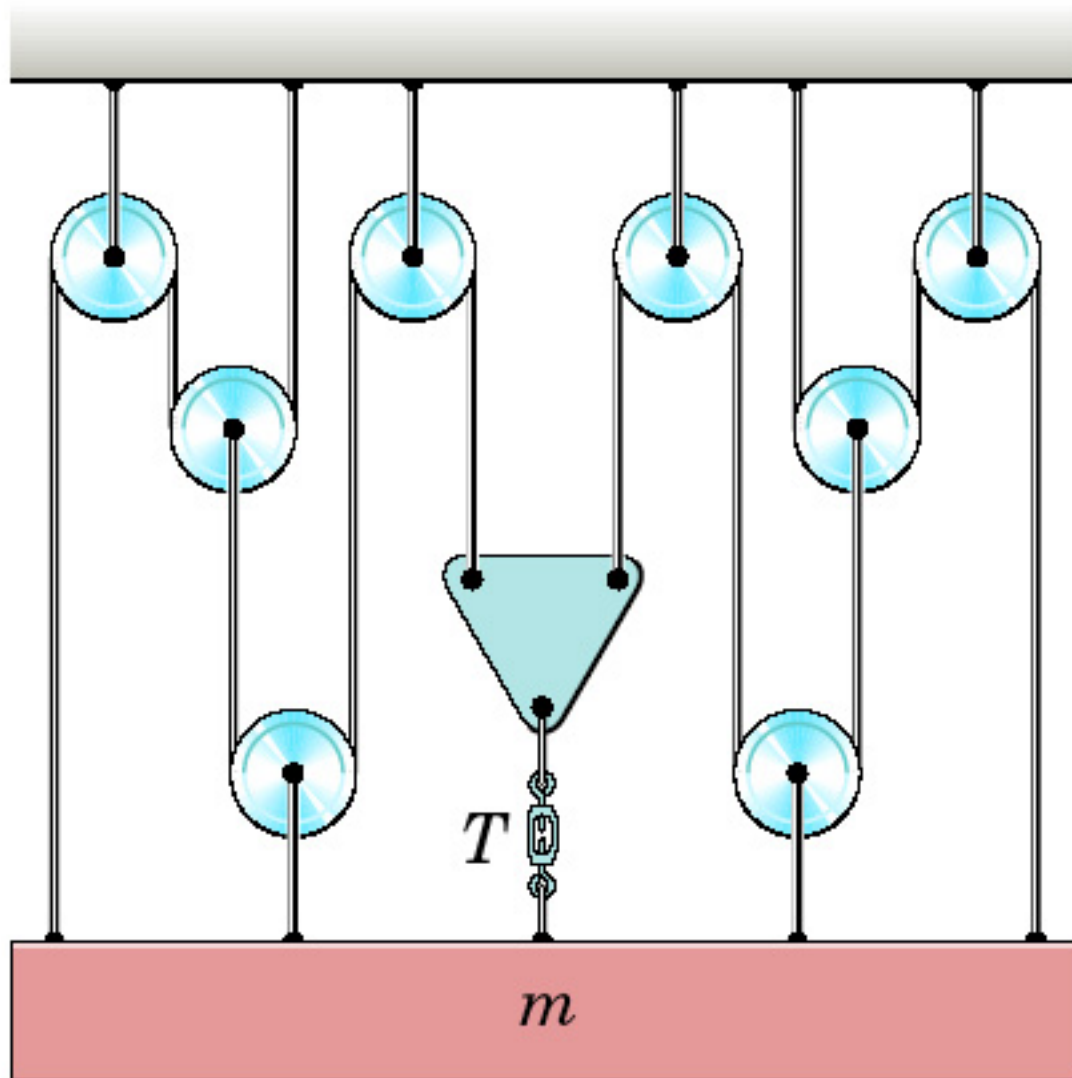


Problem 3/55

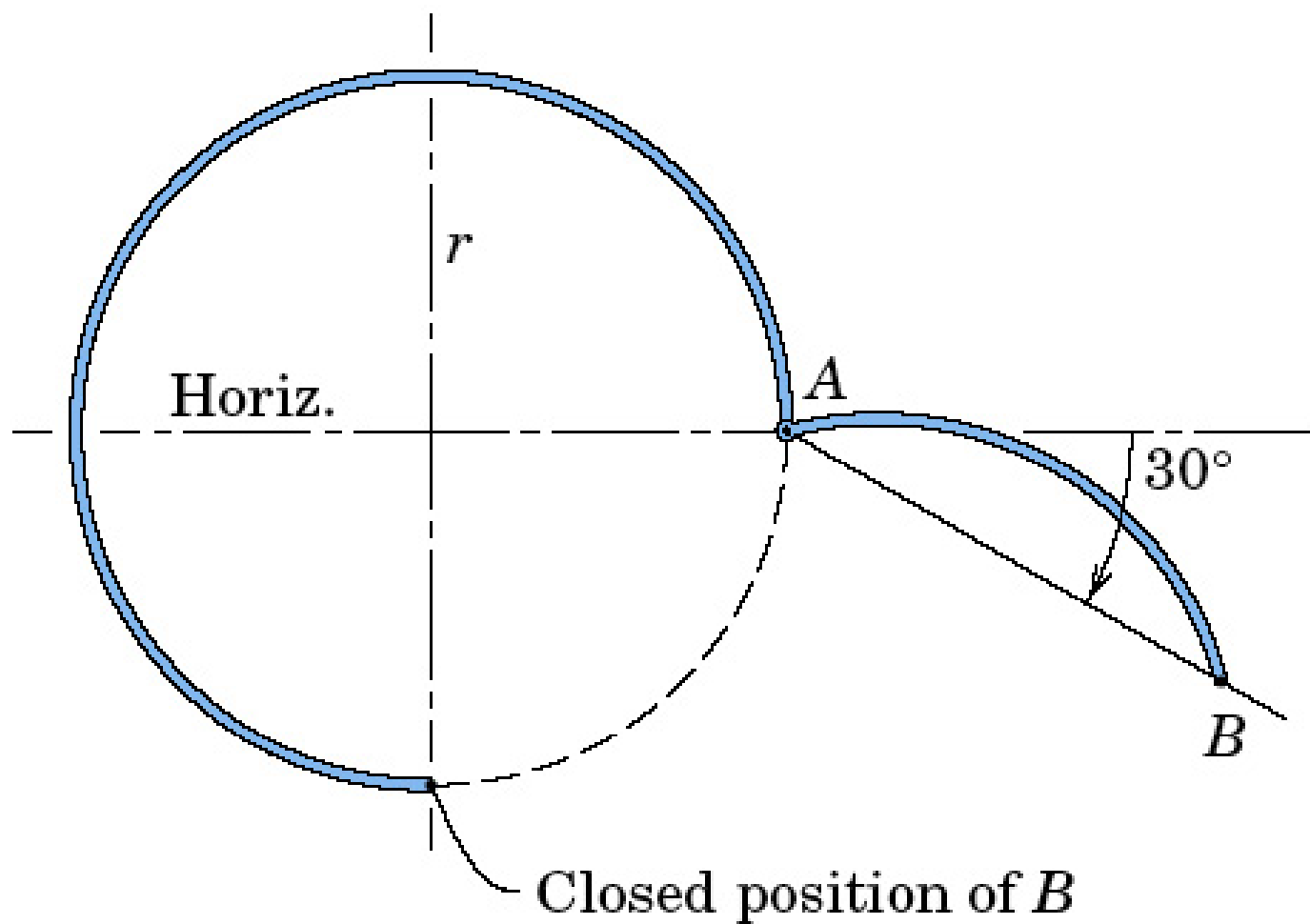


Problem 3/56

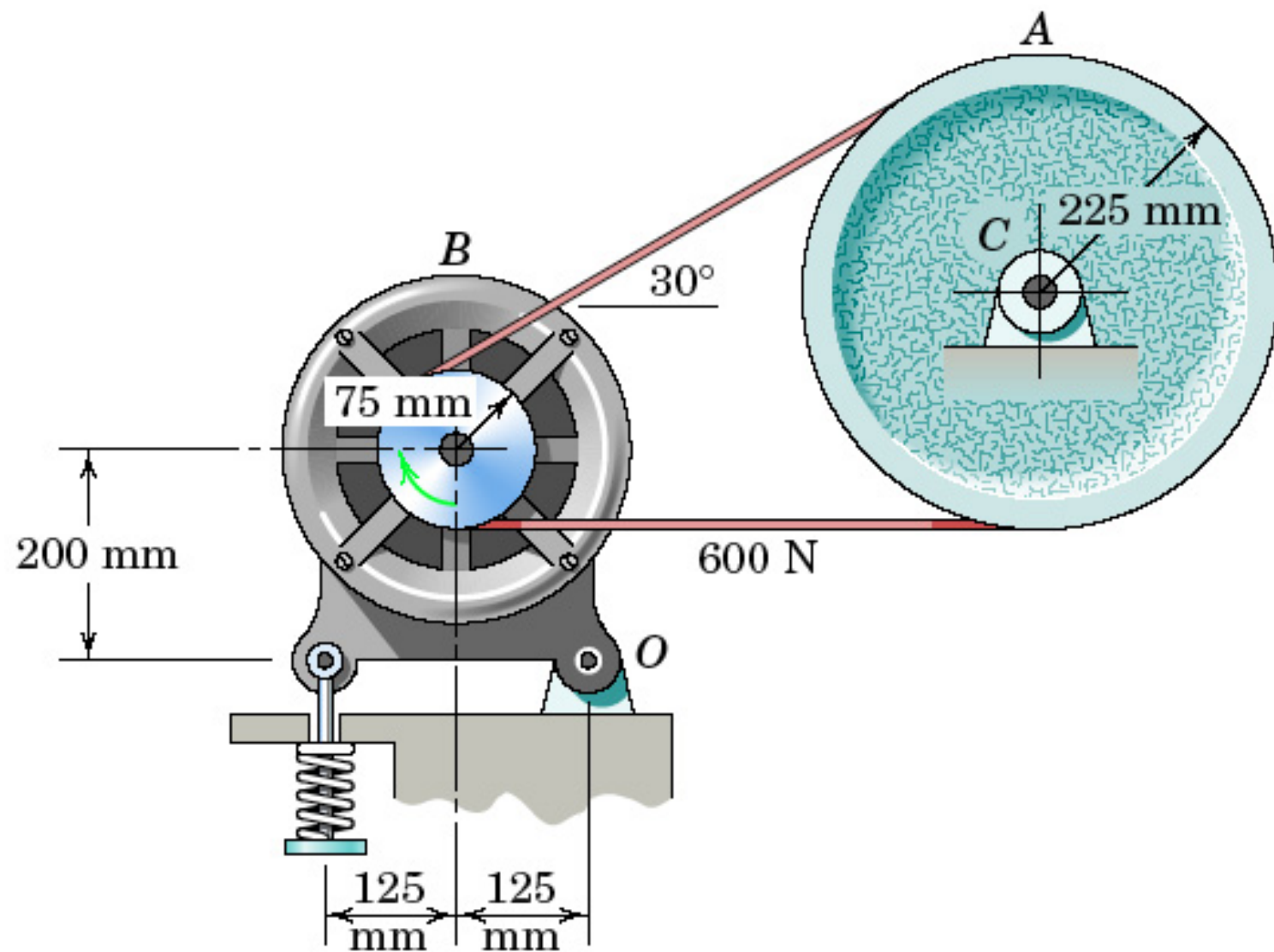
P_03_56



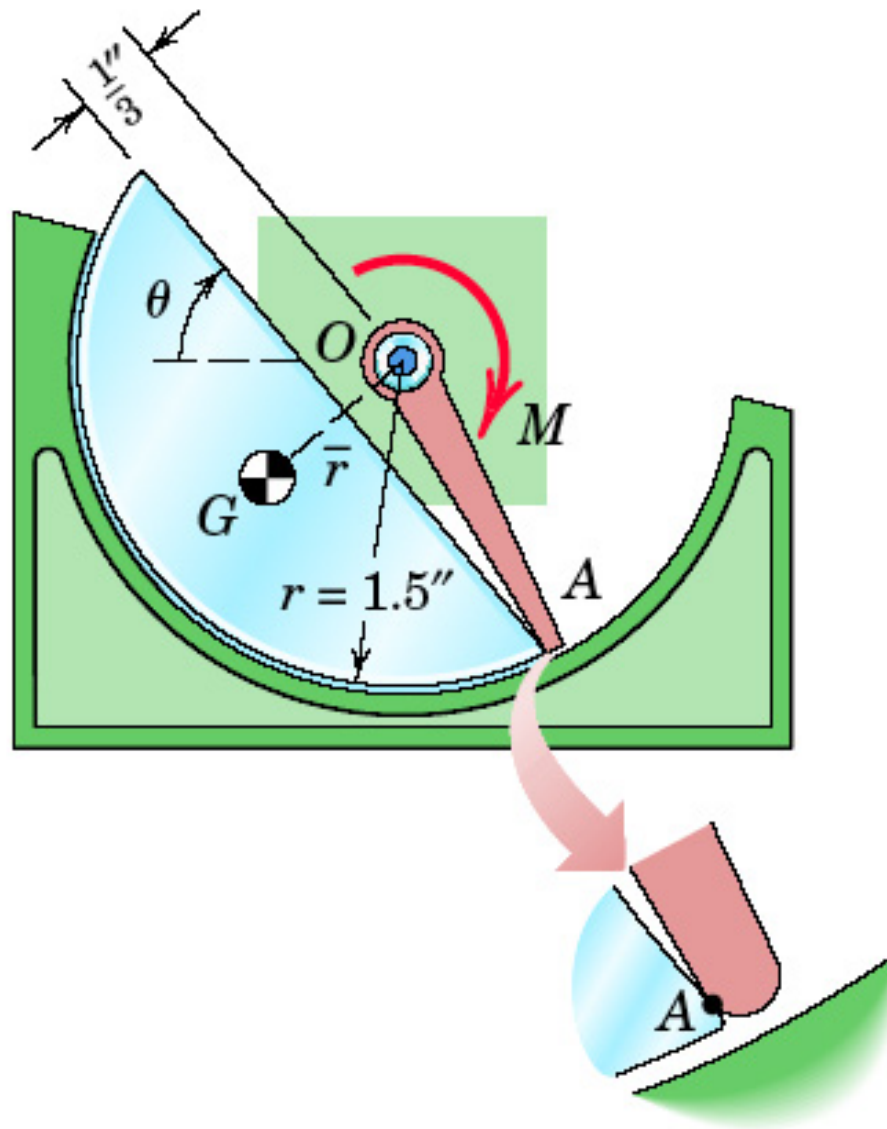
Problem 3/57



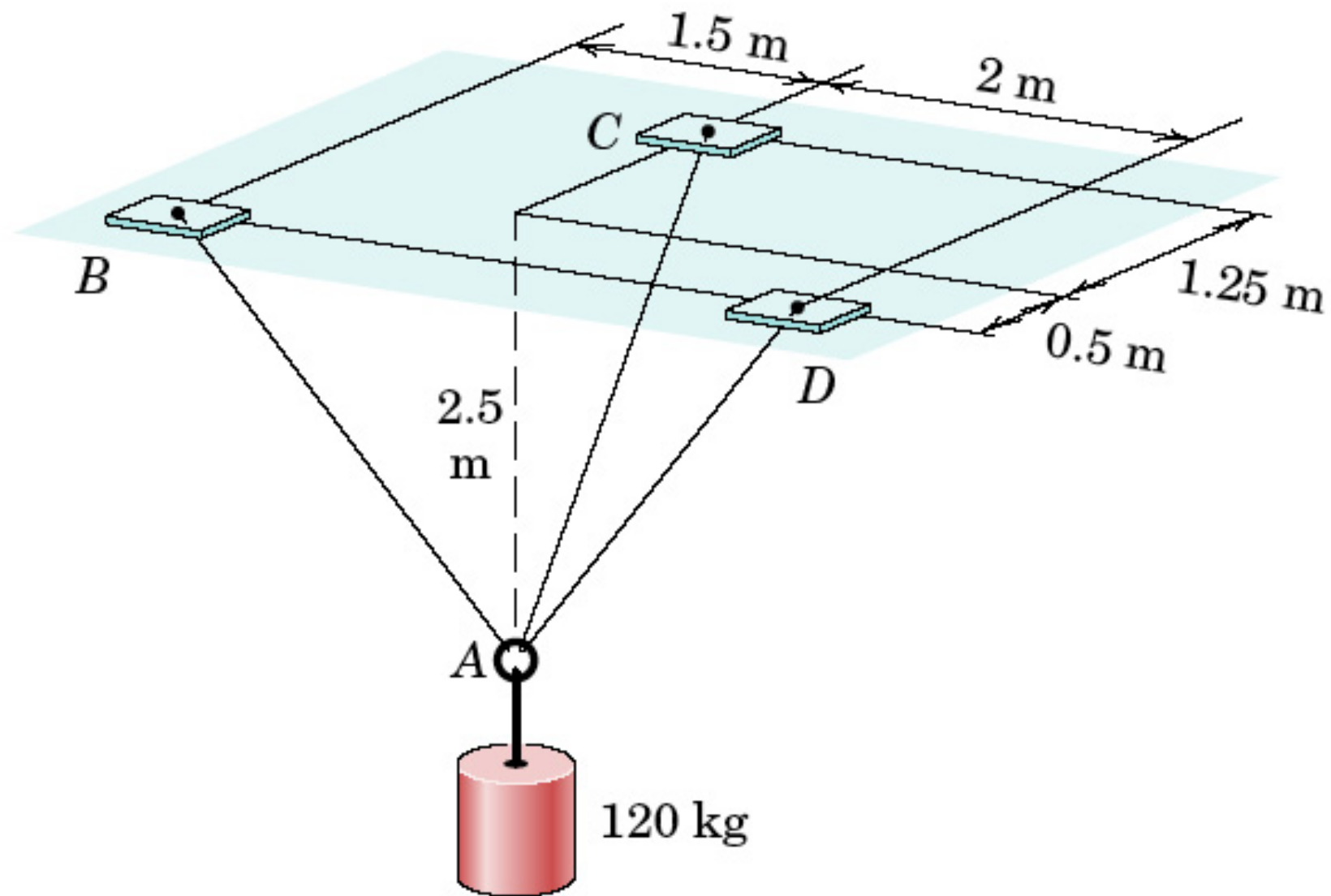
Problem 3/58



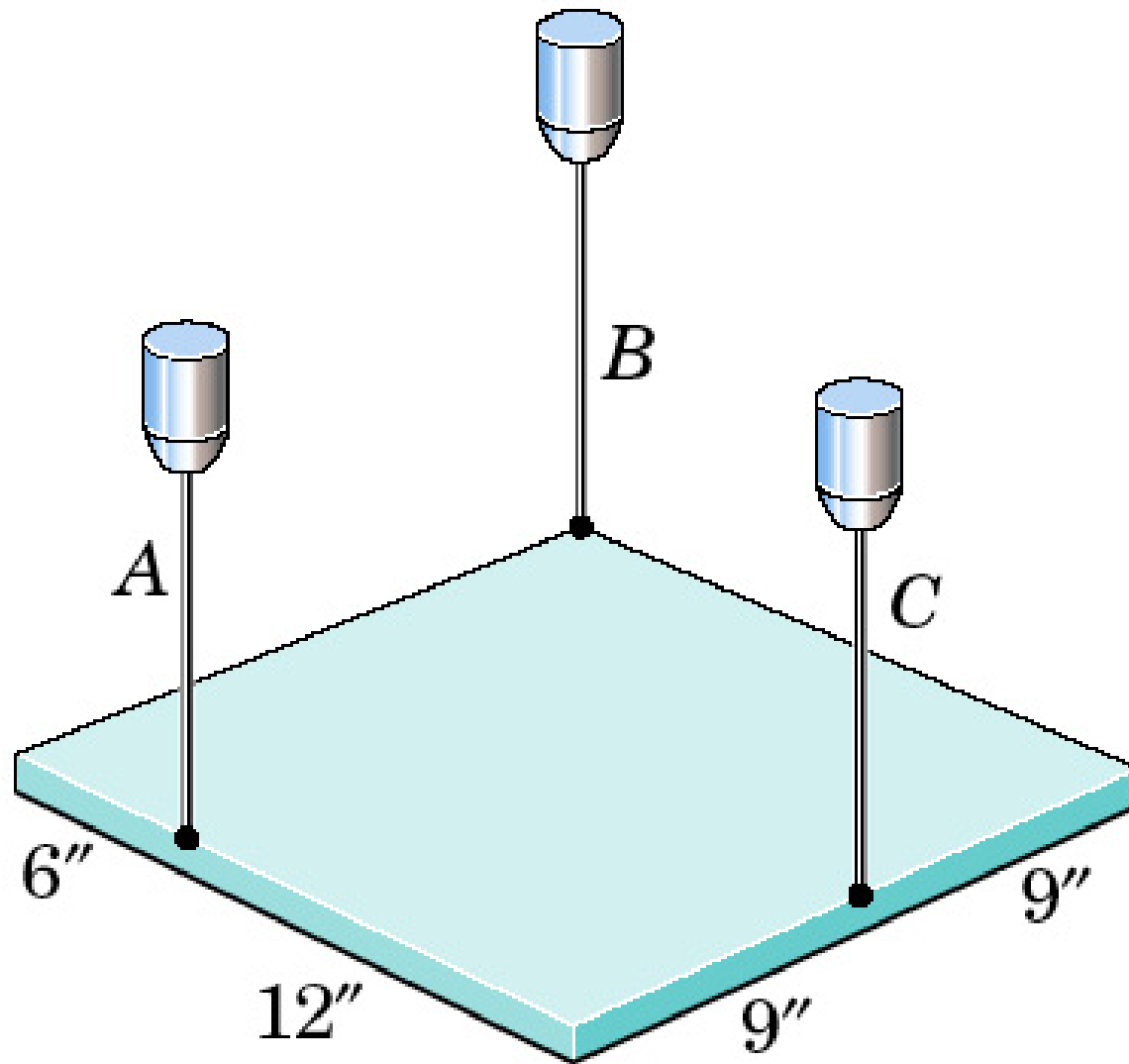
Problem 3/59



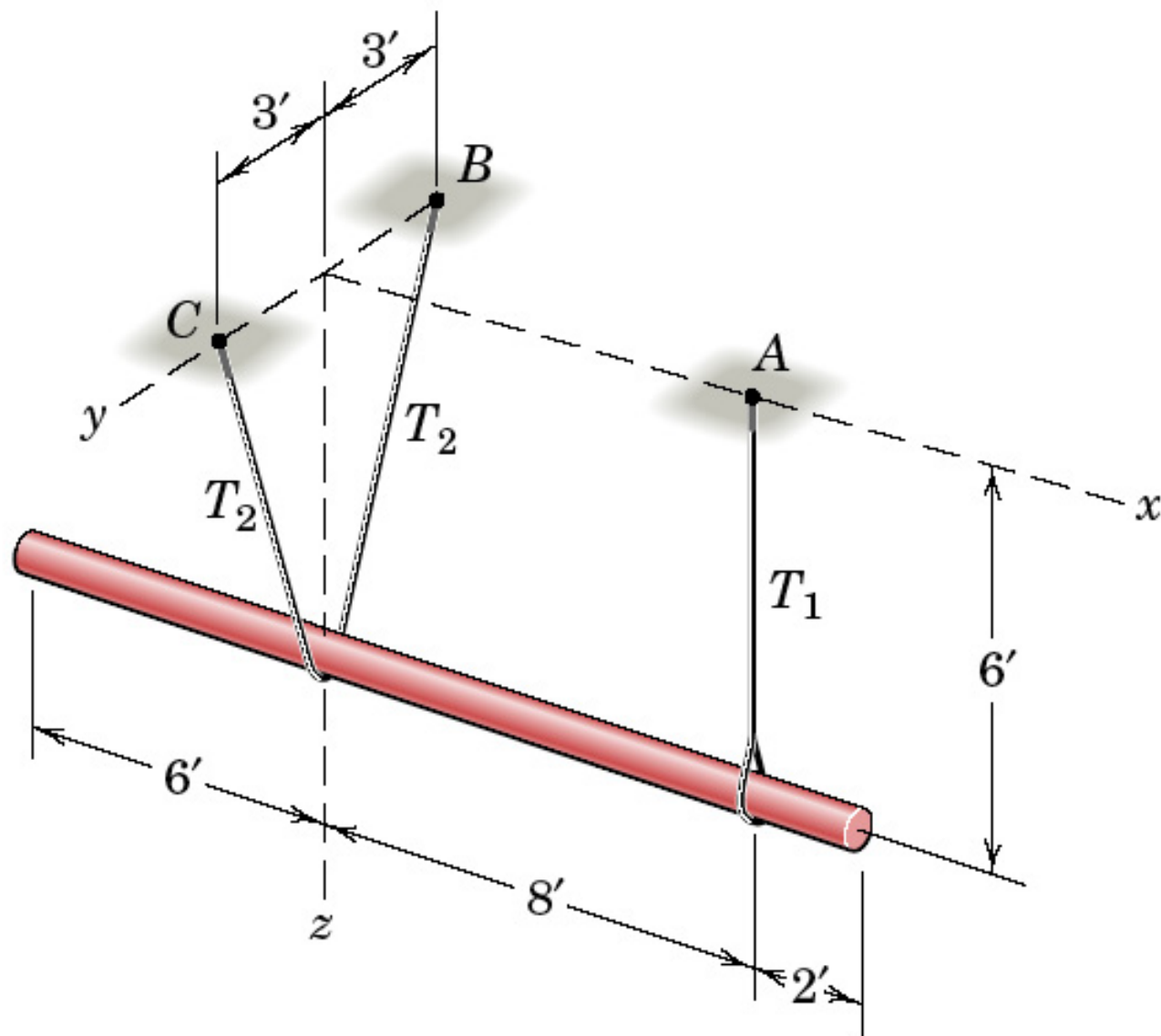
Problem 3/60



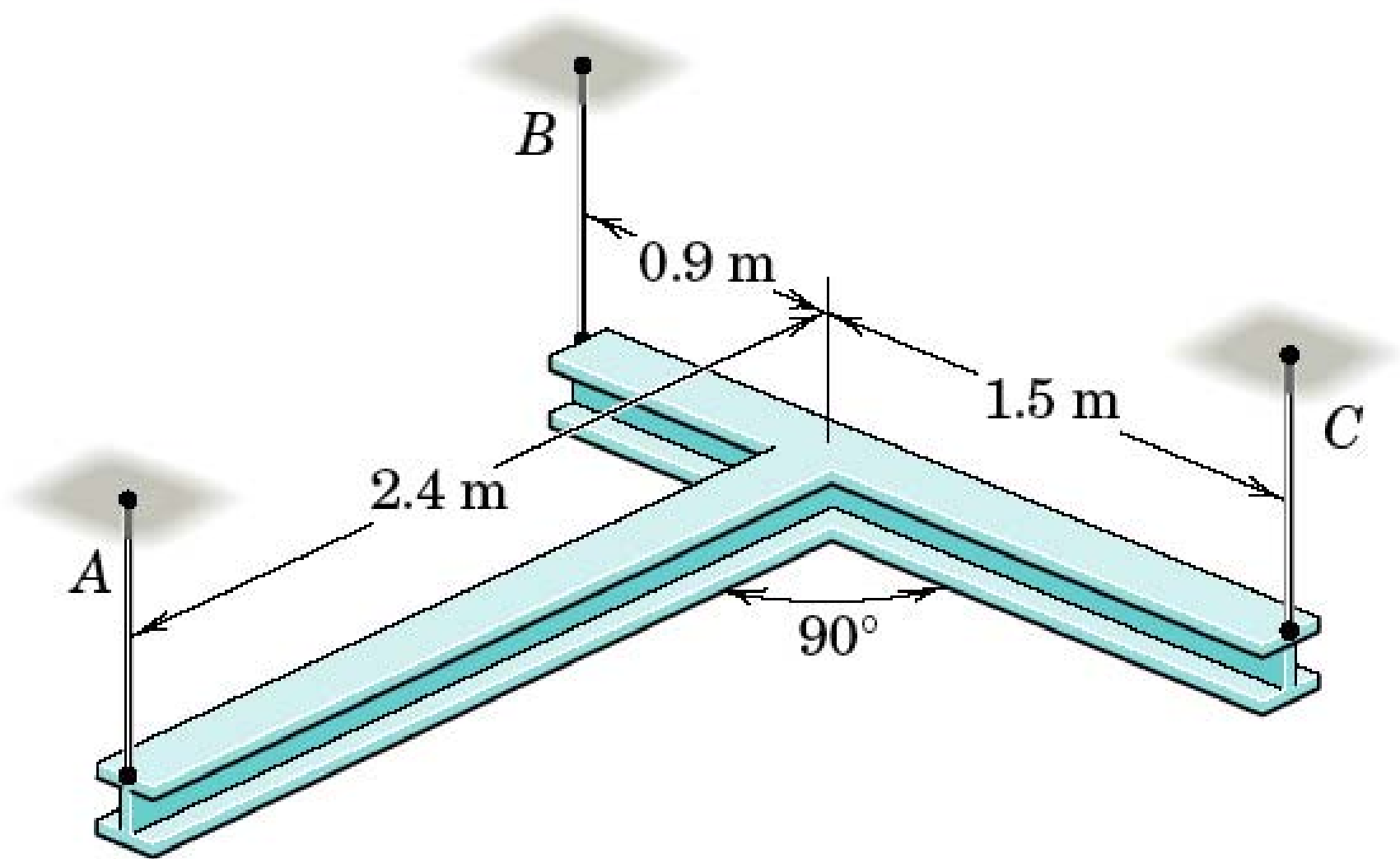
Problem 3/61



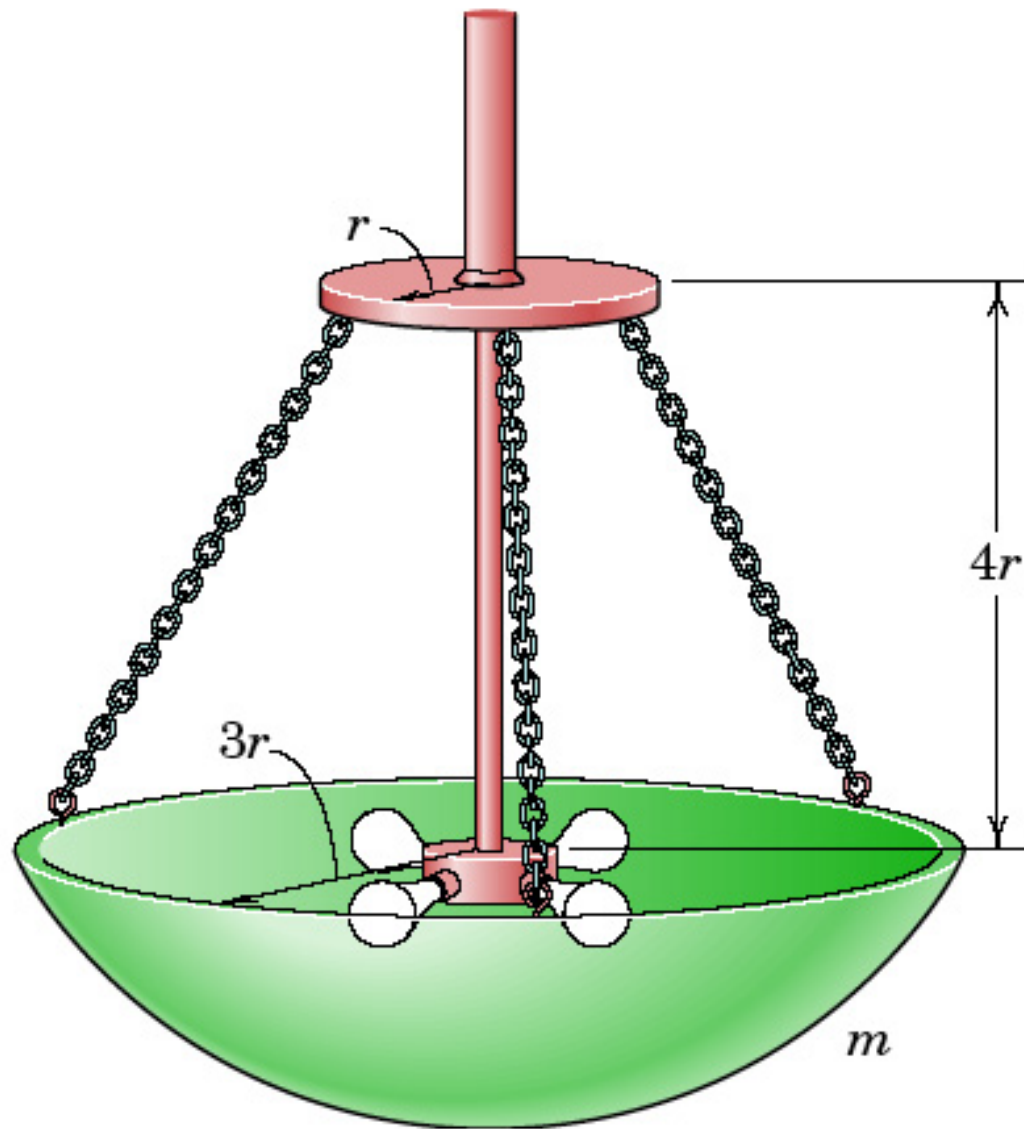
Problem 3/62



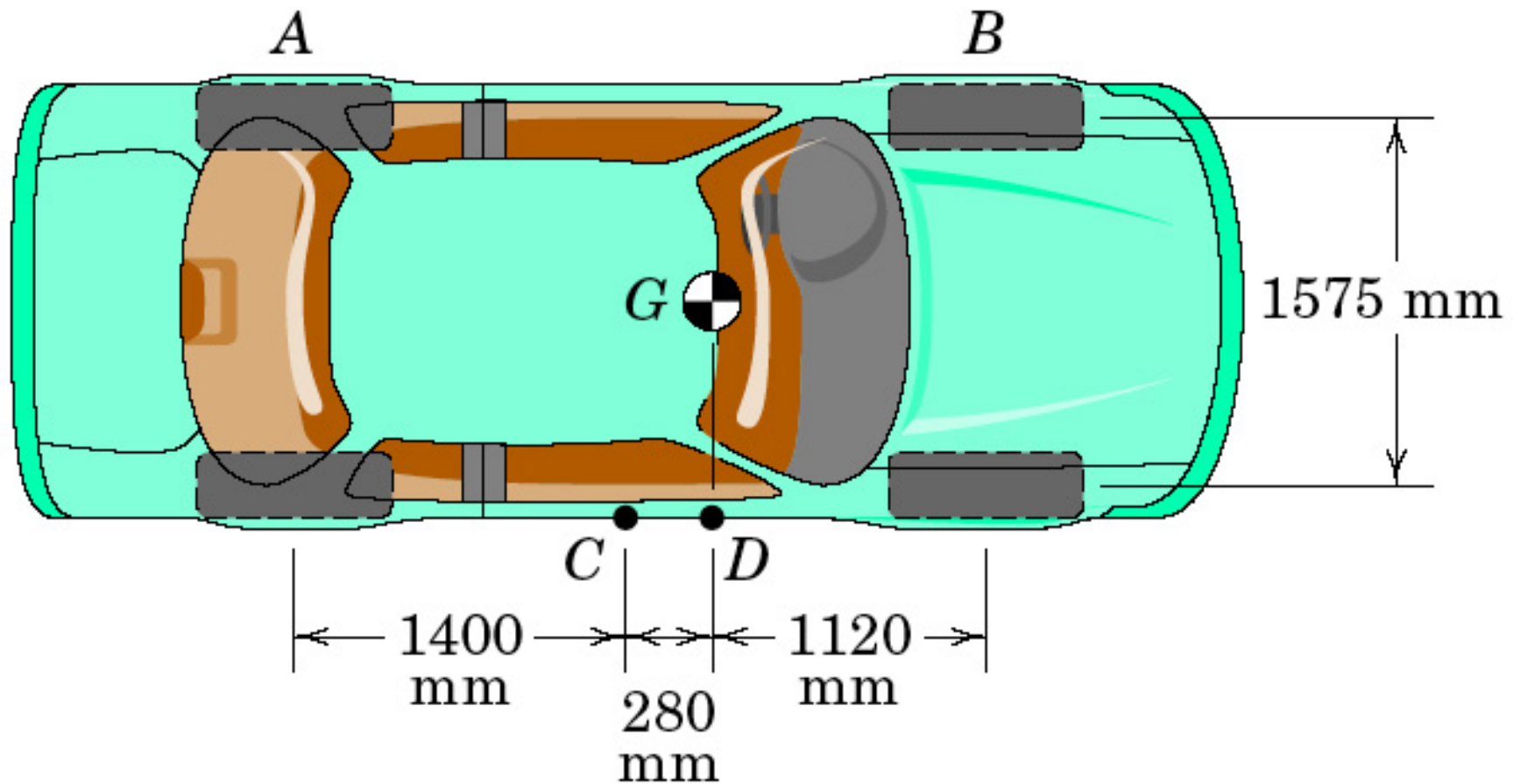
Problem 3/63



Problem 3/64

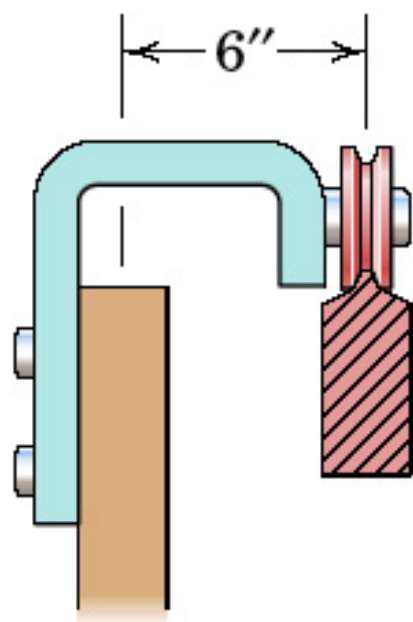


Problem 3/65

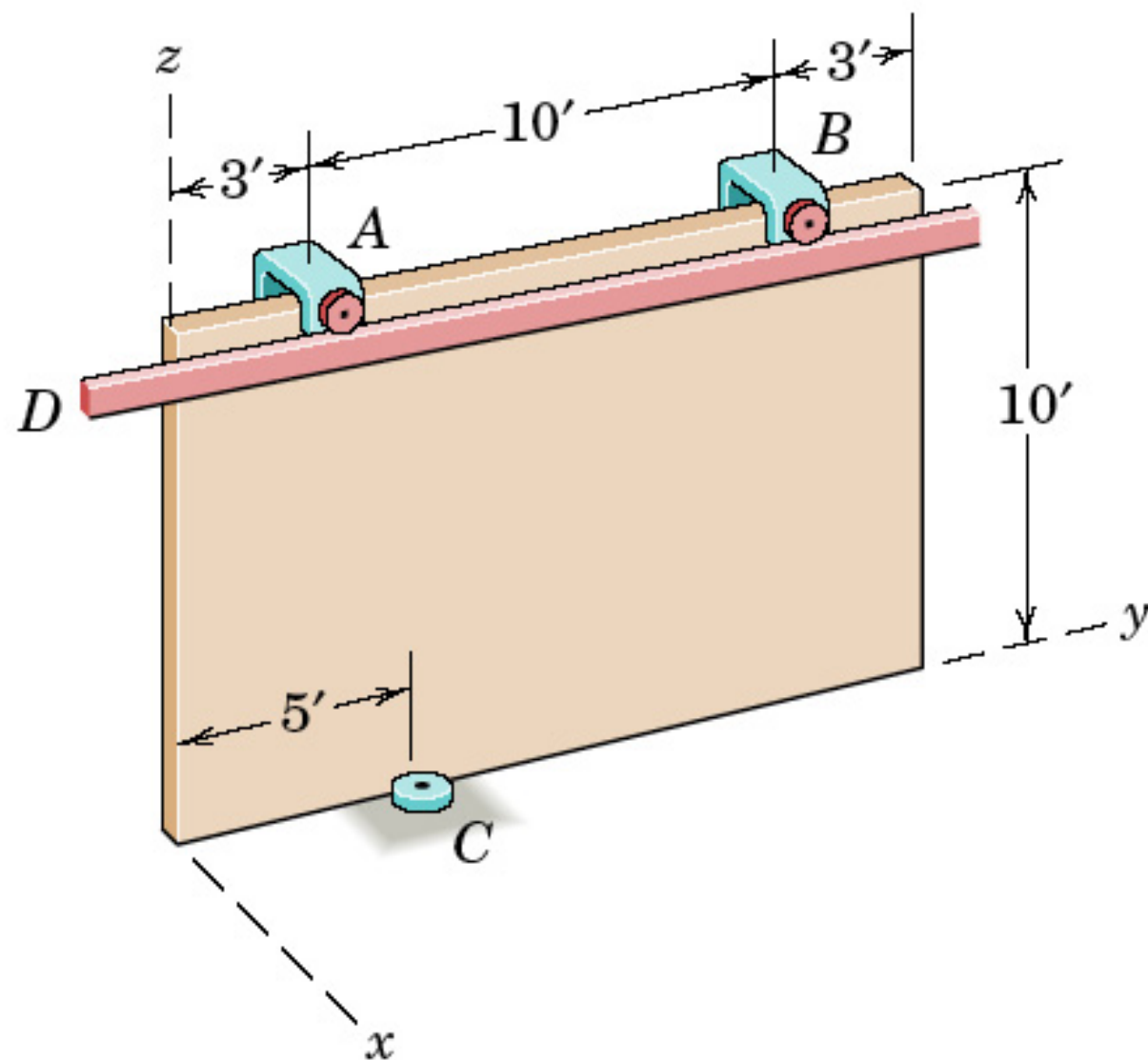


Problem 3/66

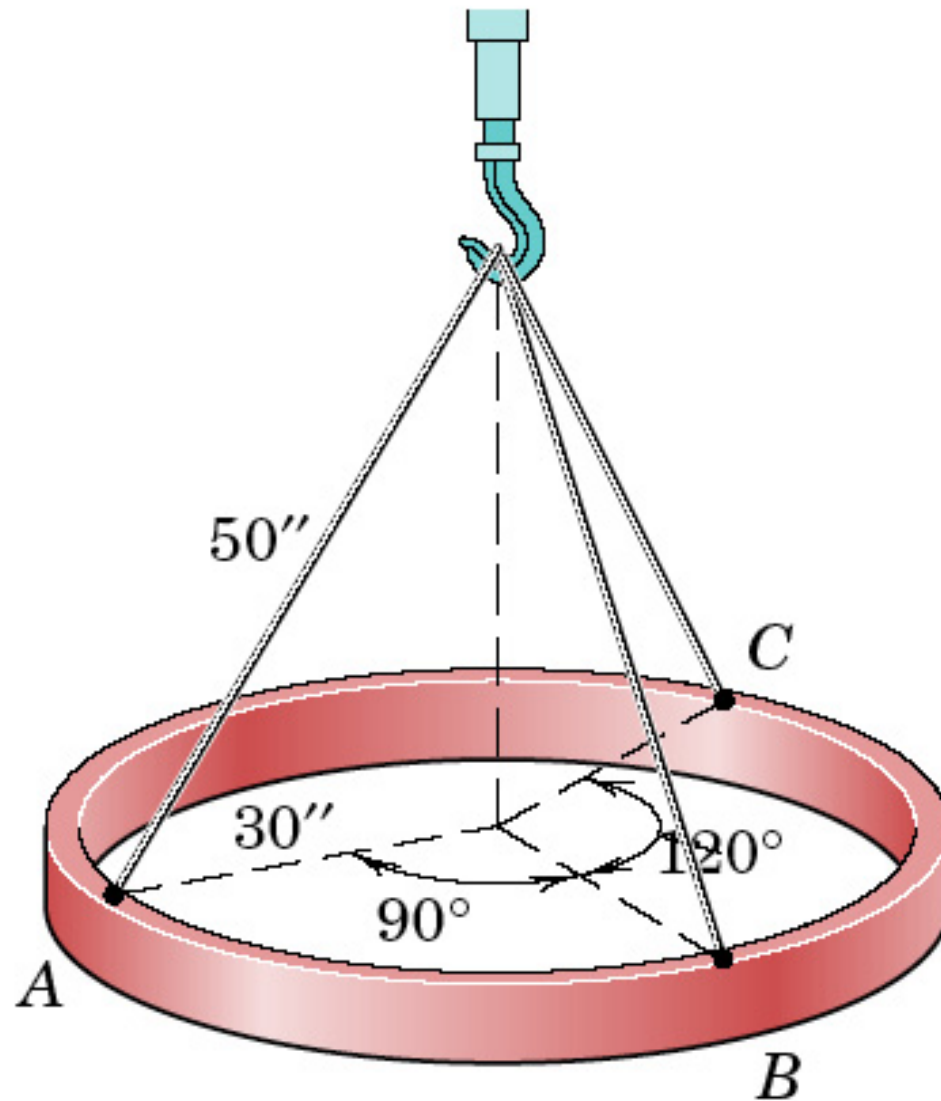
P_03_66



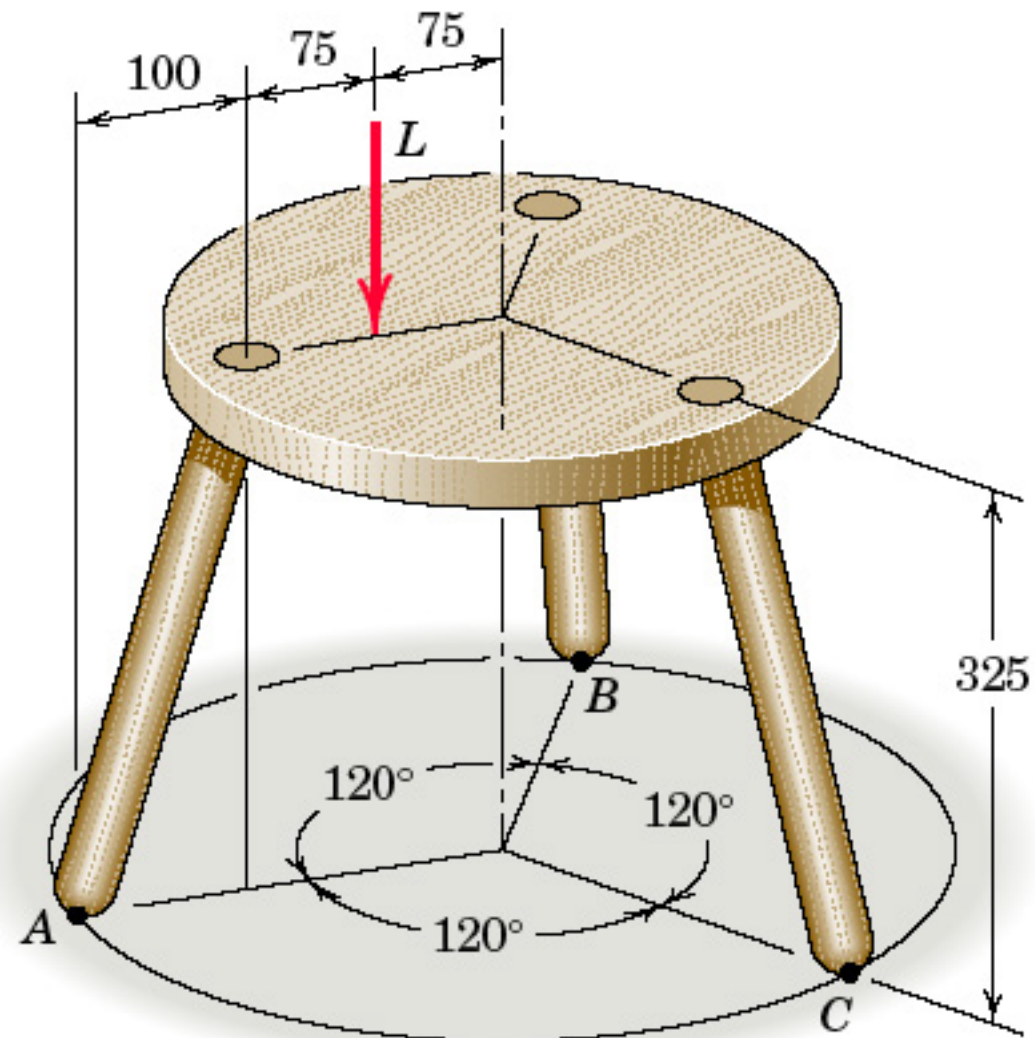
Detail of
door hanger



Problem 3/67

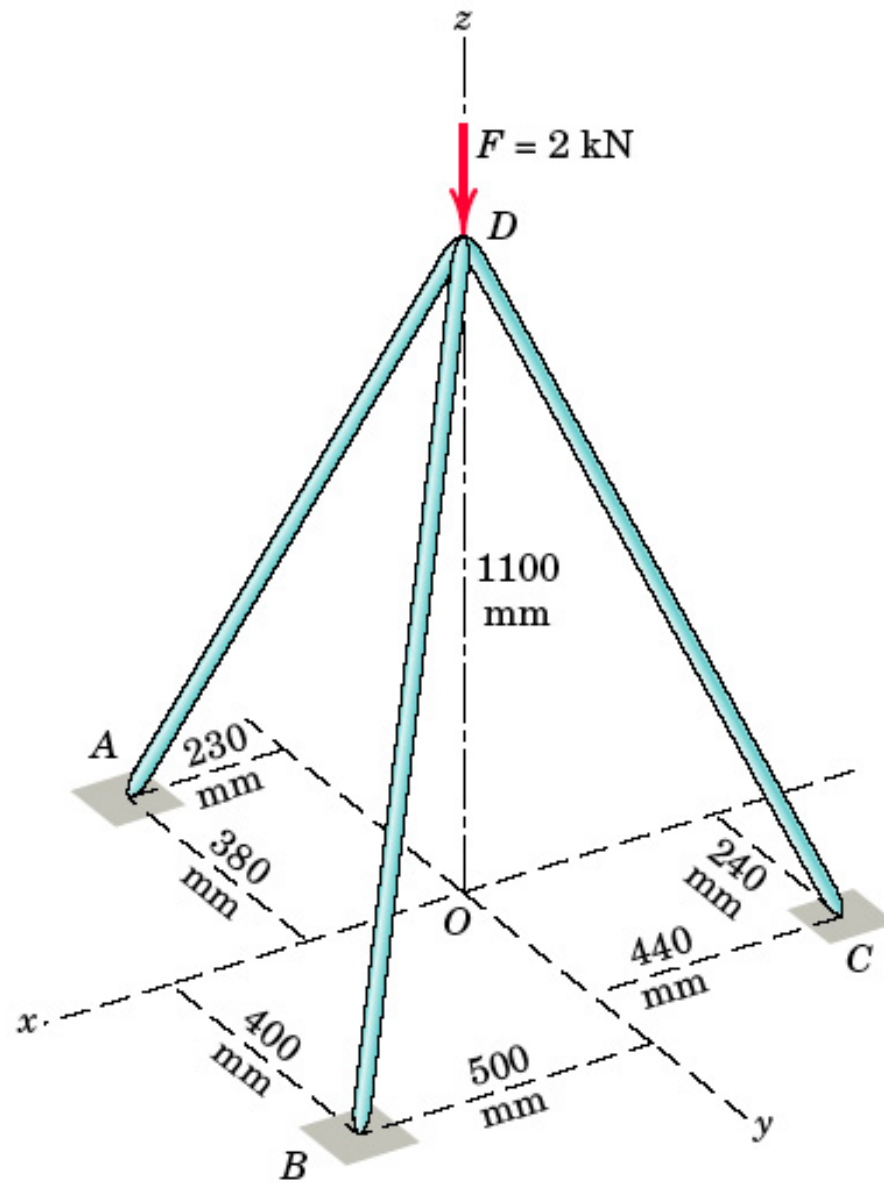


Problem 3/68

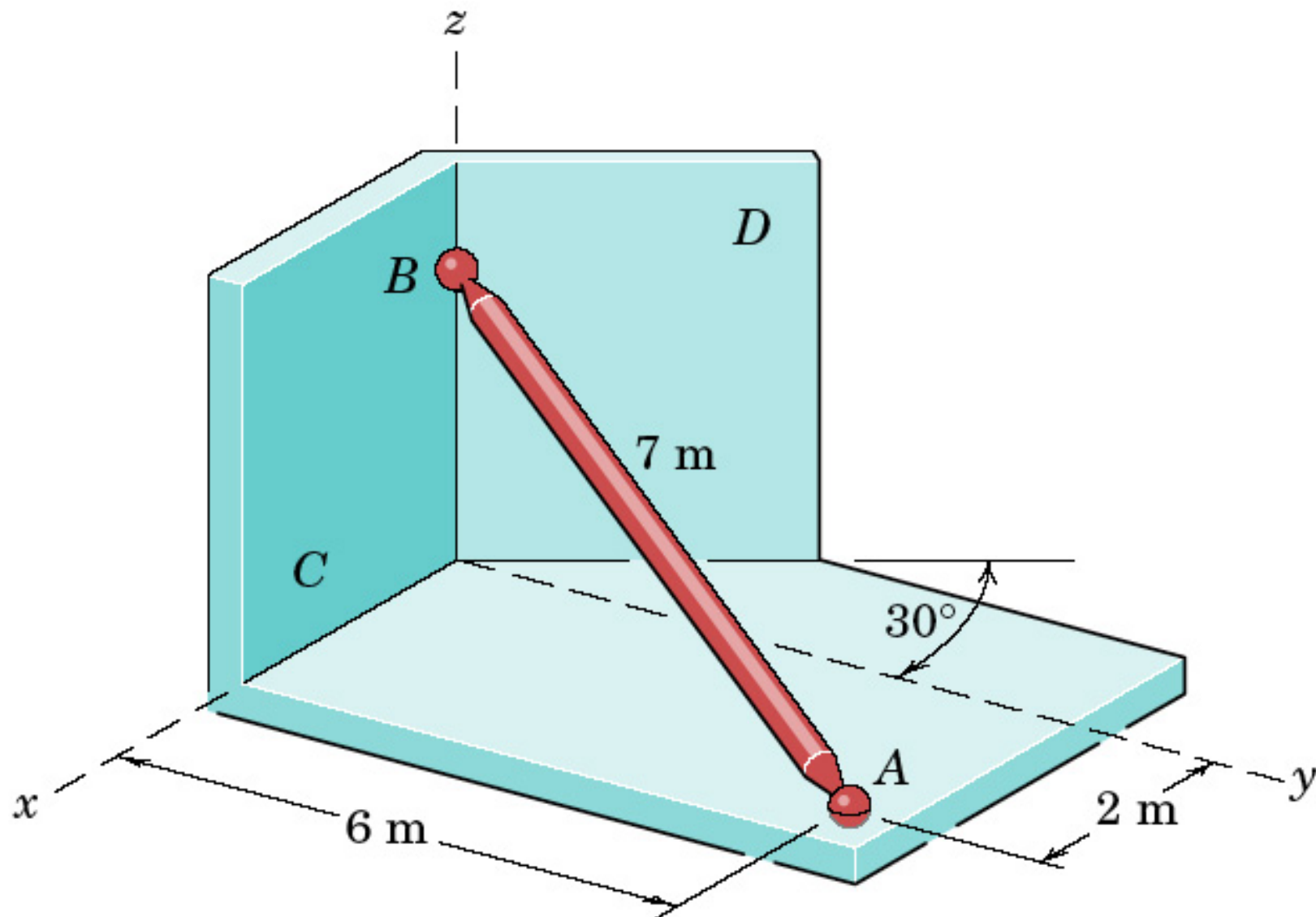


Dimensions in millimeters

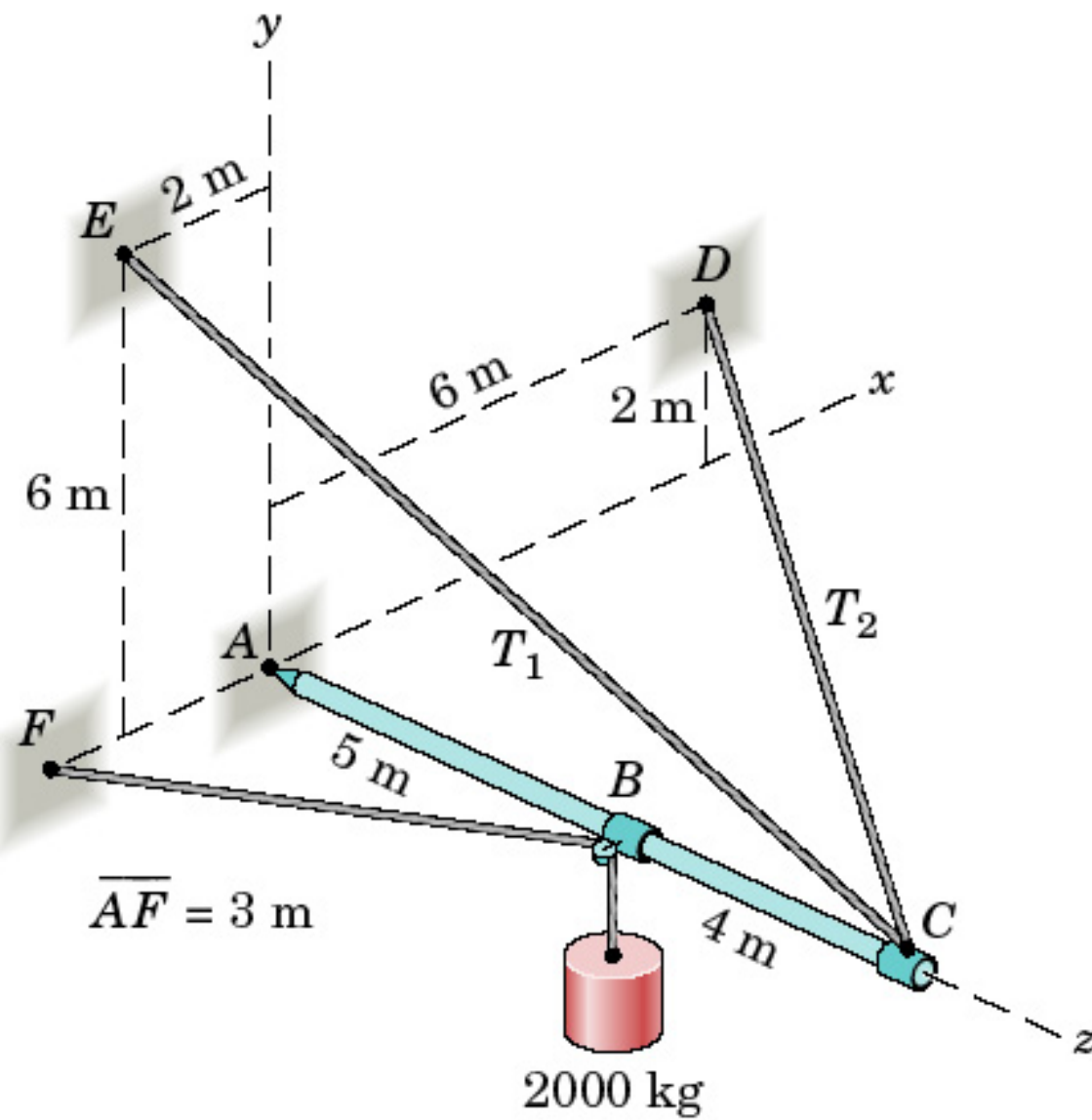
Problem 3/69



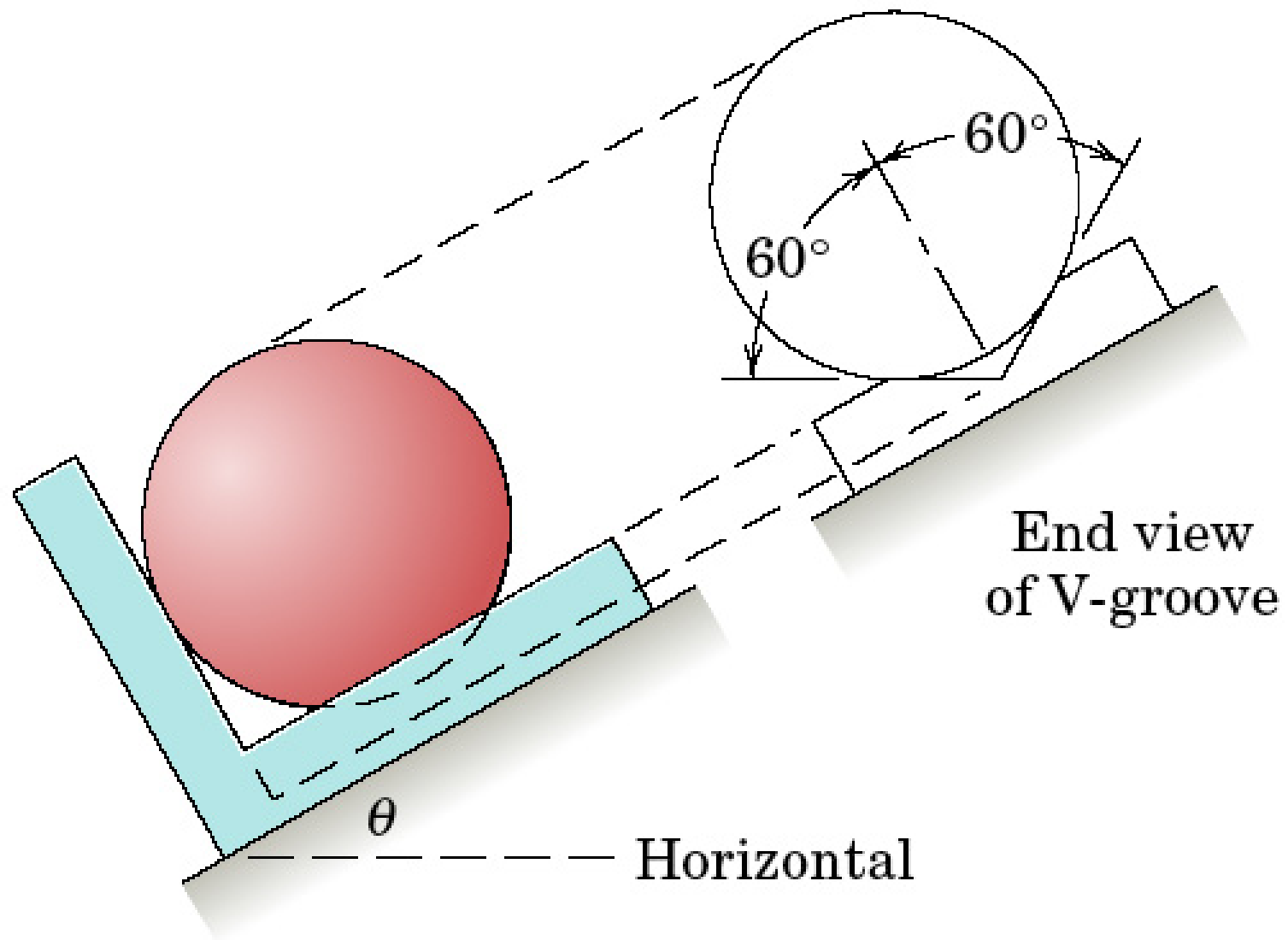
Problem 3/70



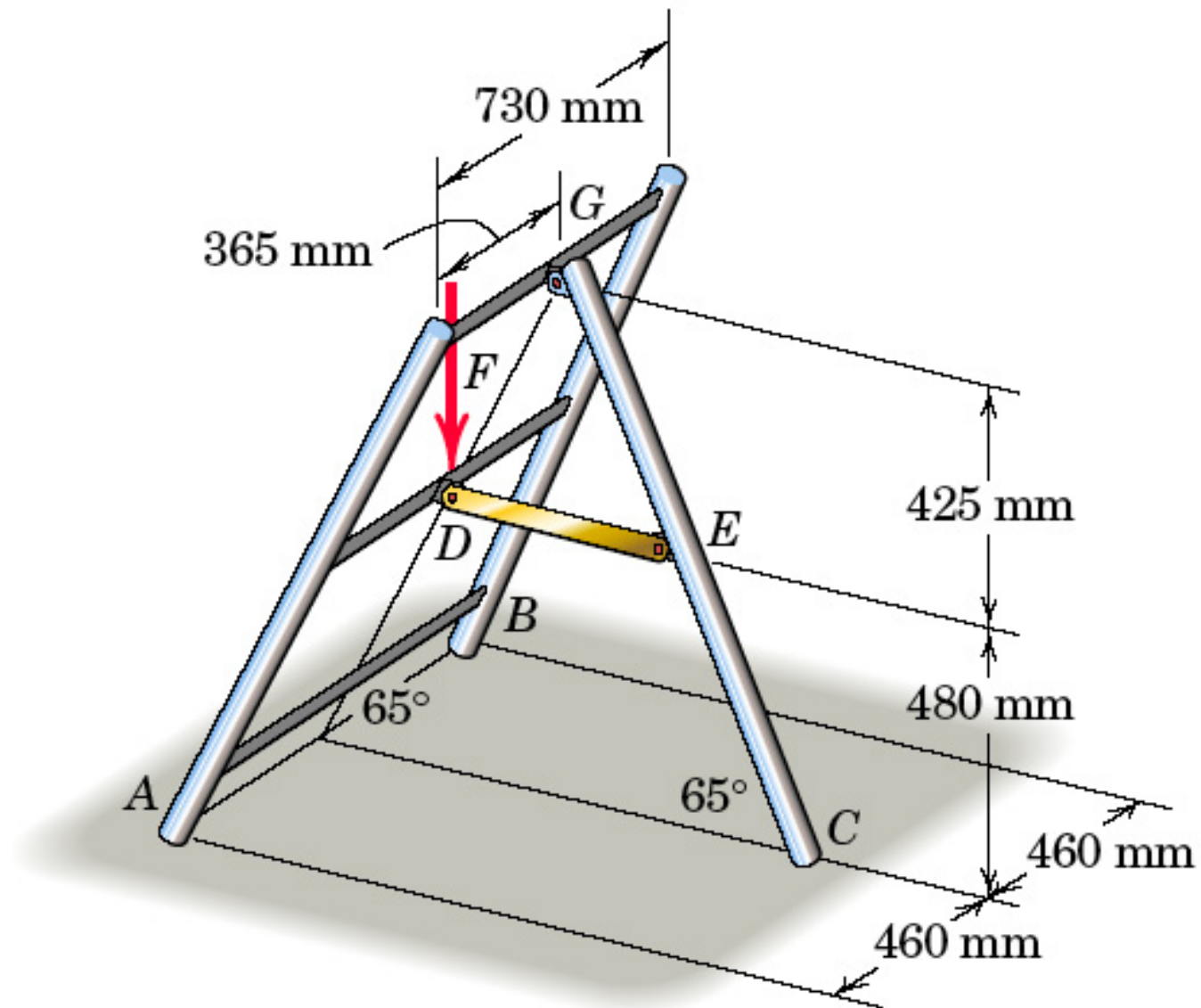
Problem 3/71



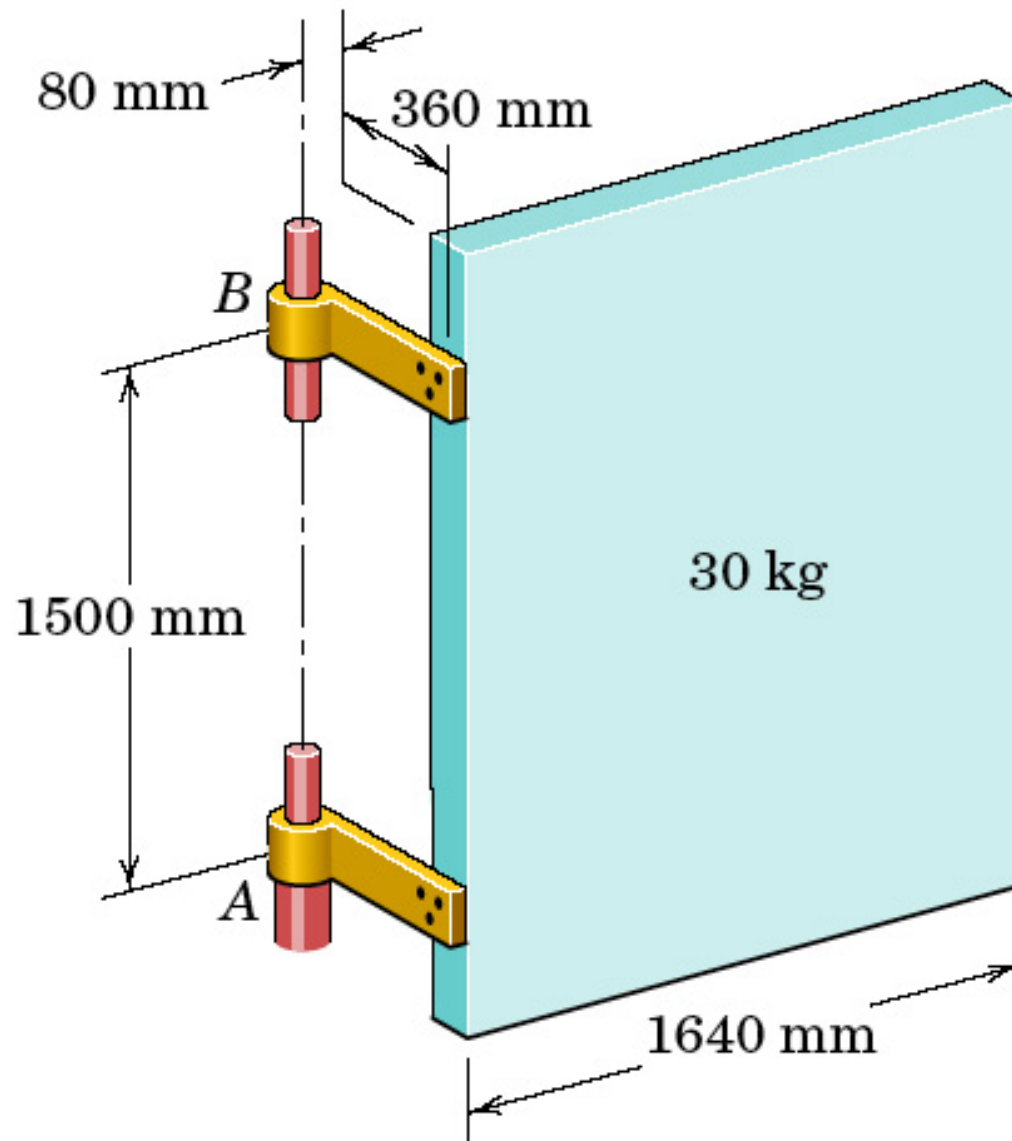
Problem 3/72



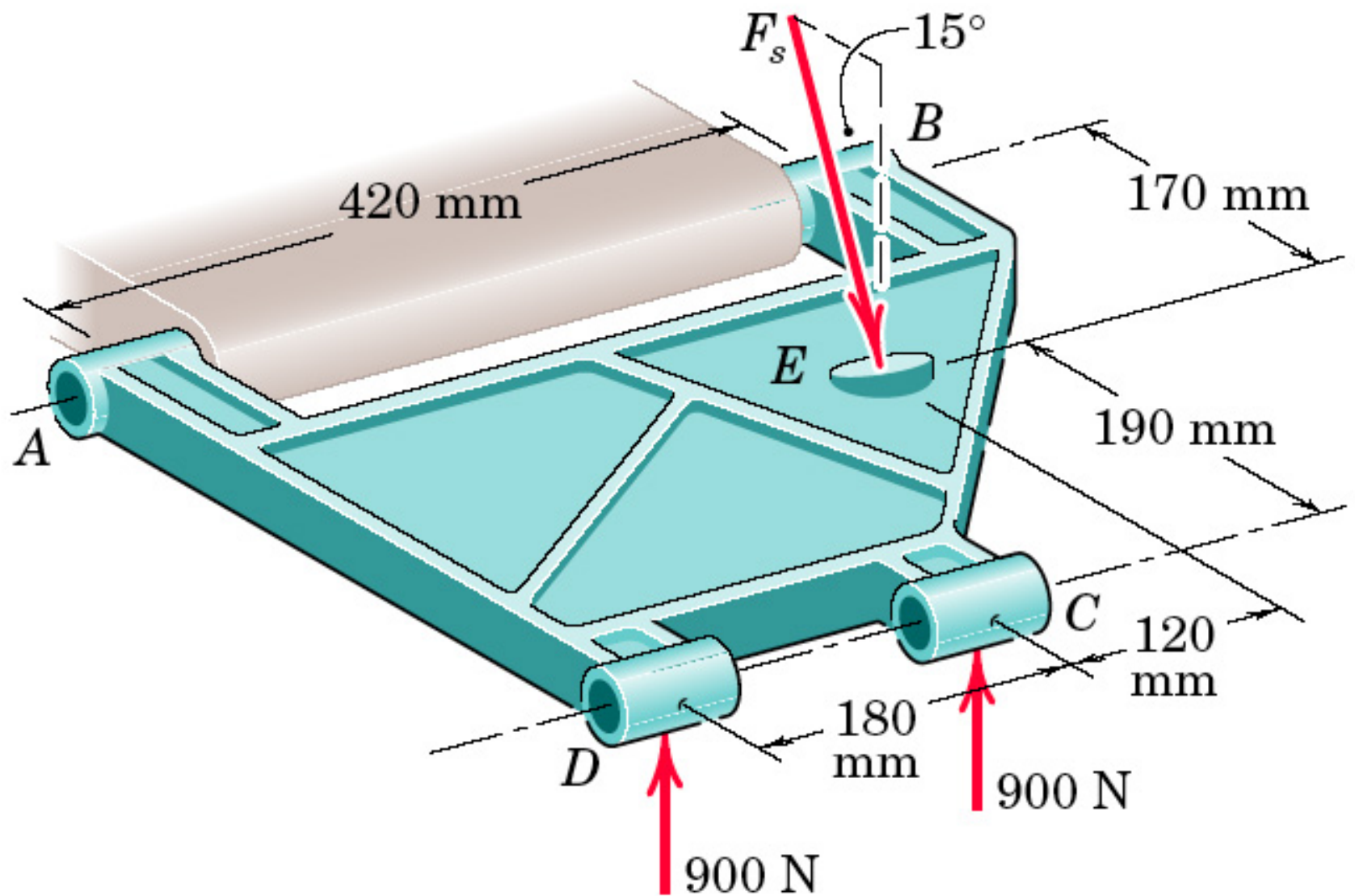
Problem 3/73



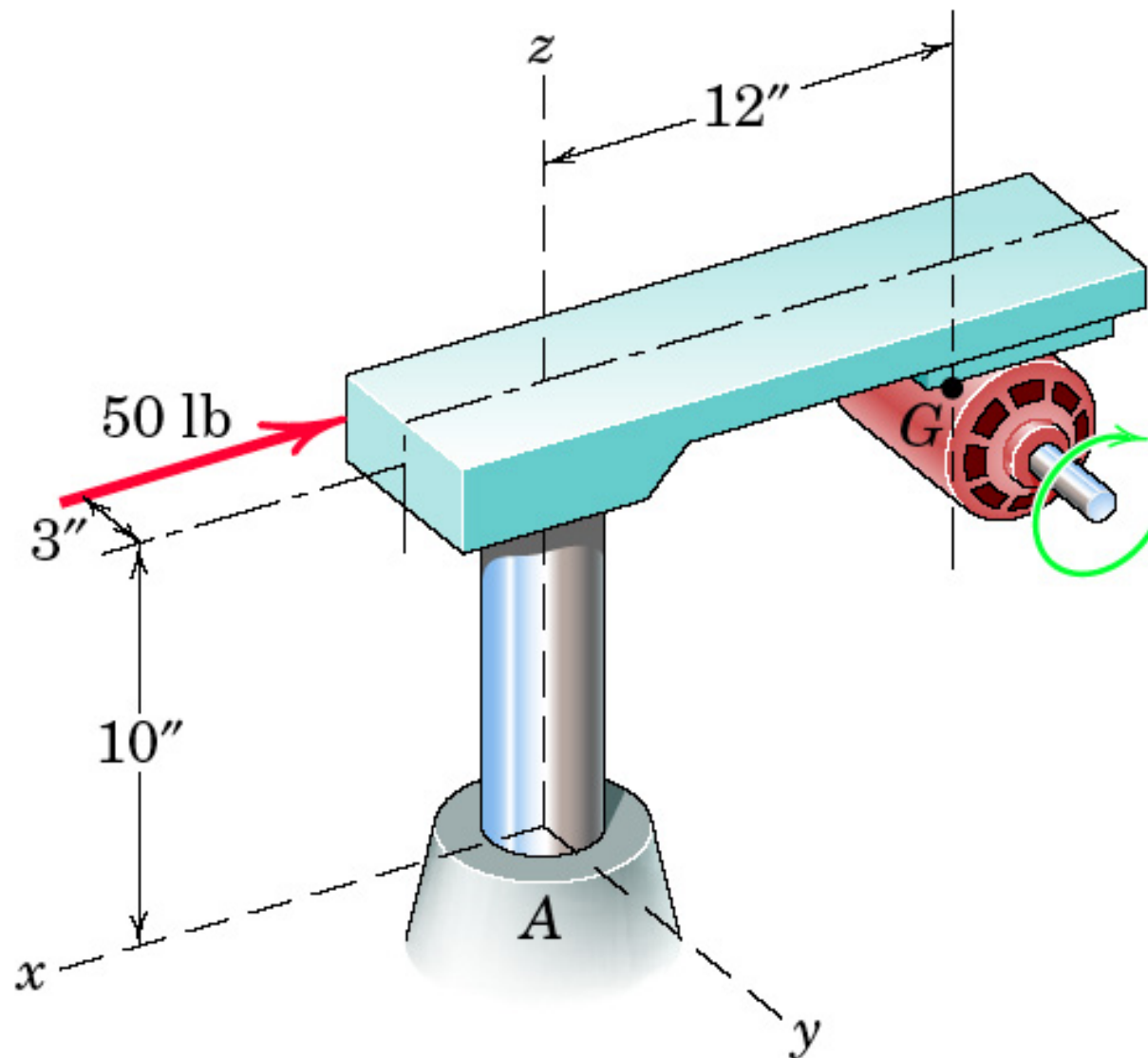
Problem 3/74



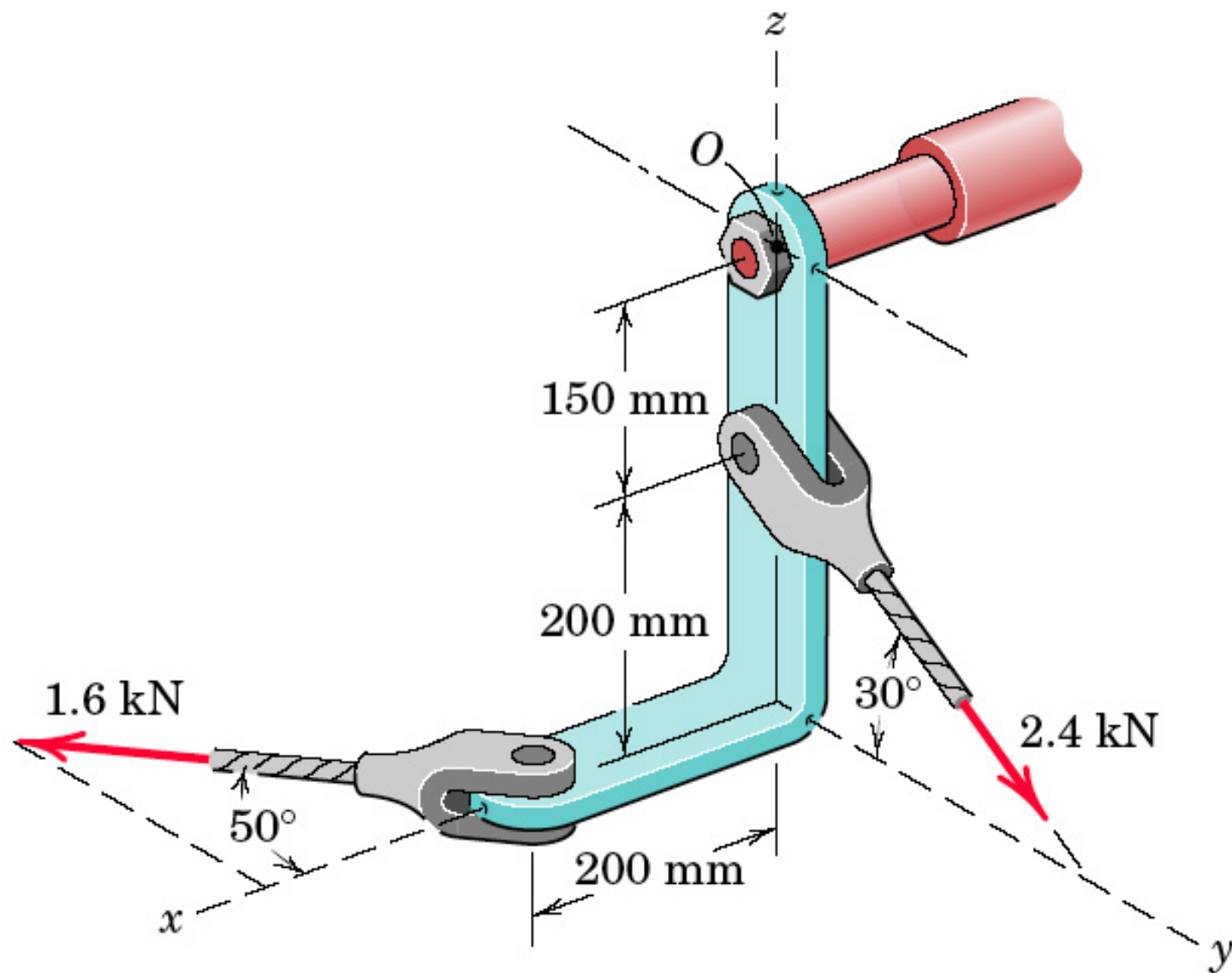
Problem 3/75



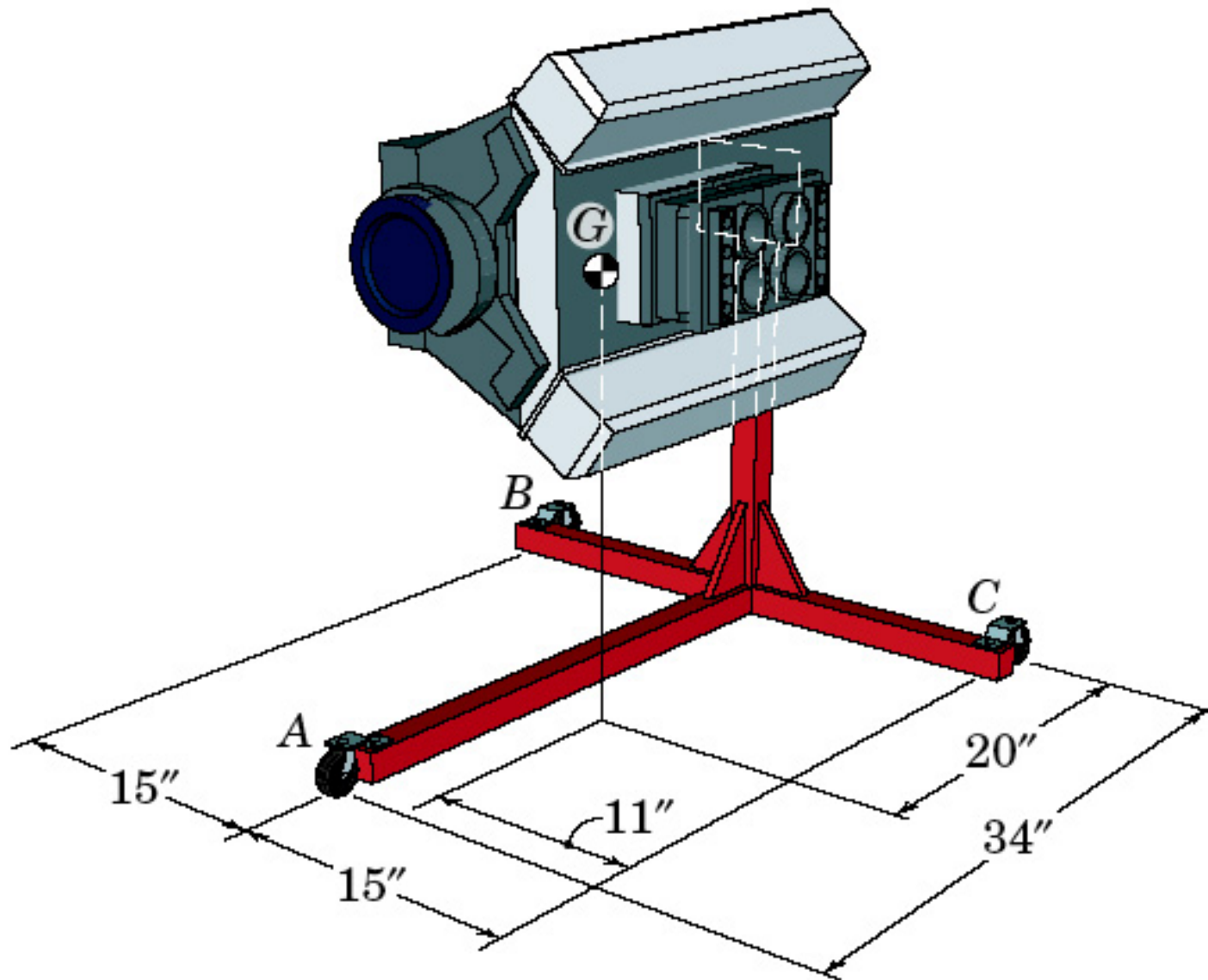
Problem 3/76



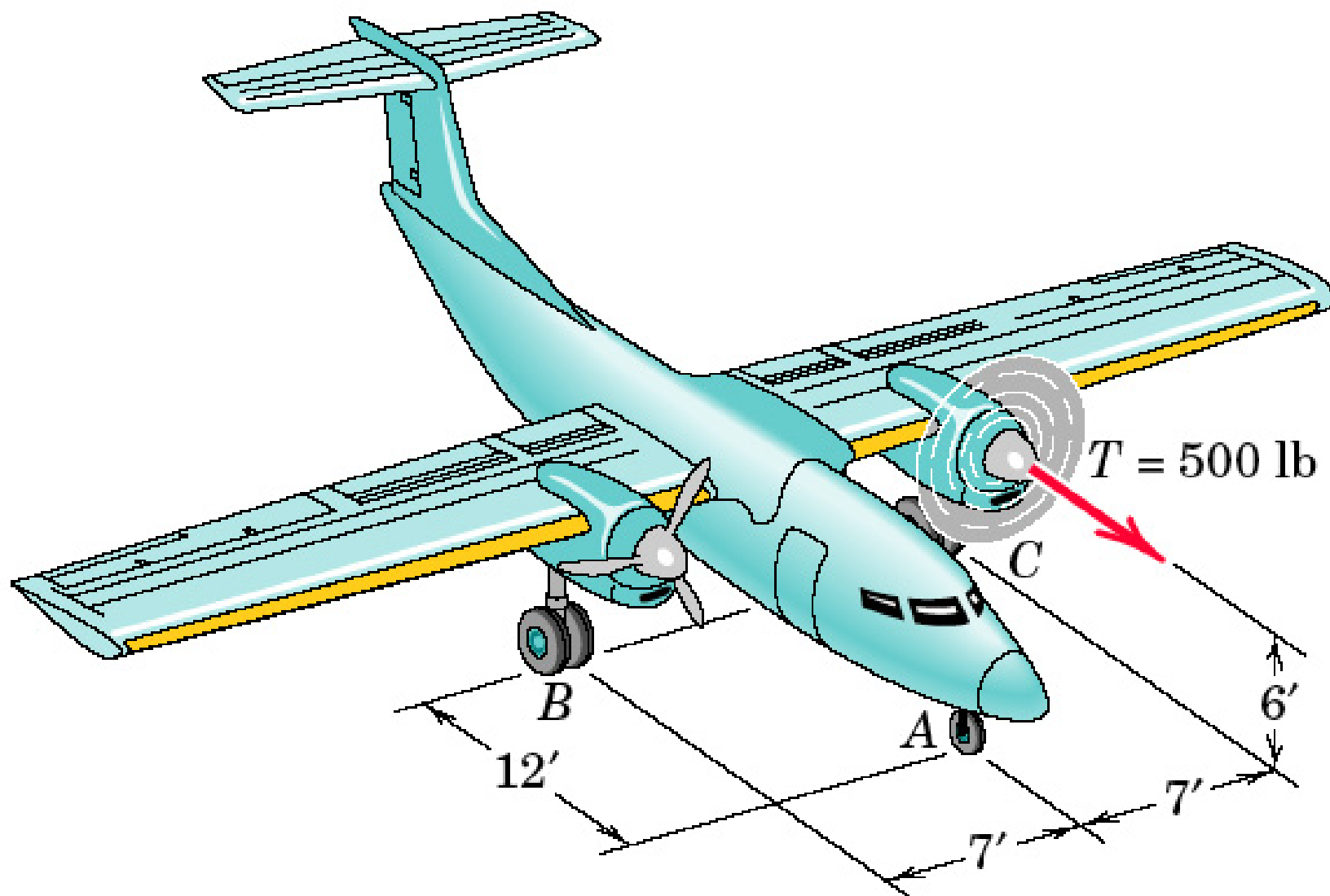
Problem 3/77



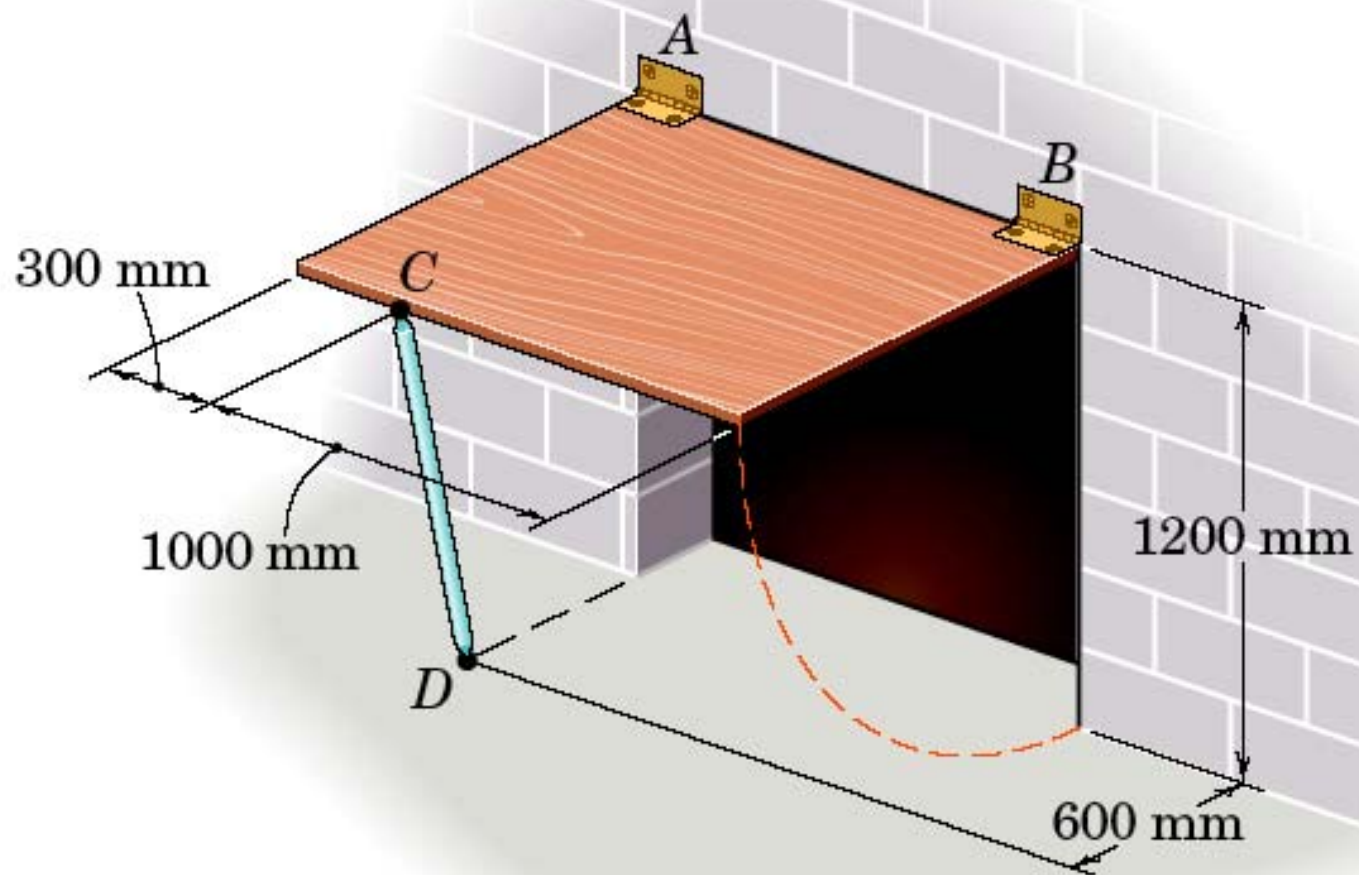
Problem 3/78



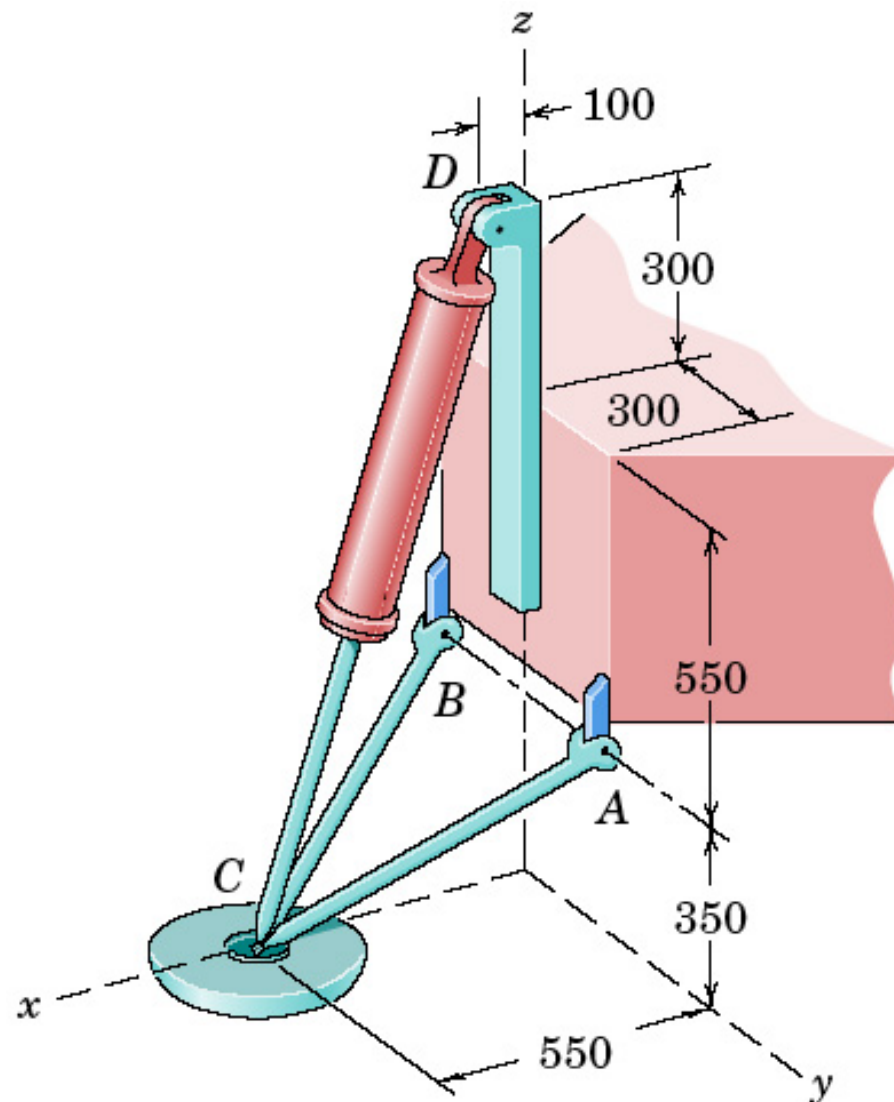
Problem 3/79



Problem 3/80

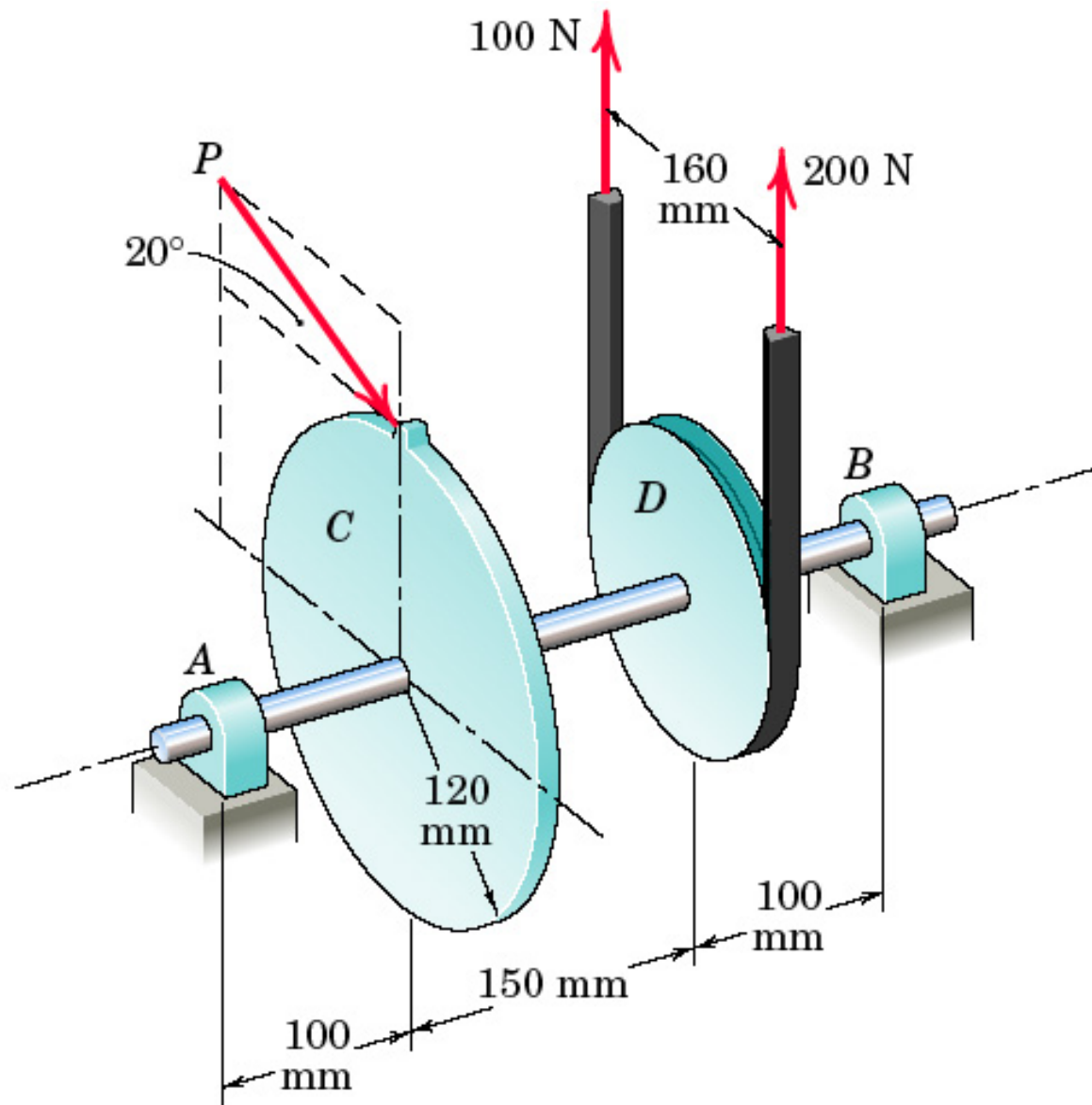


Problem 3/81

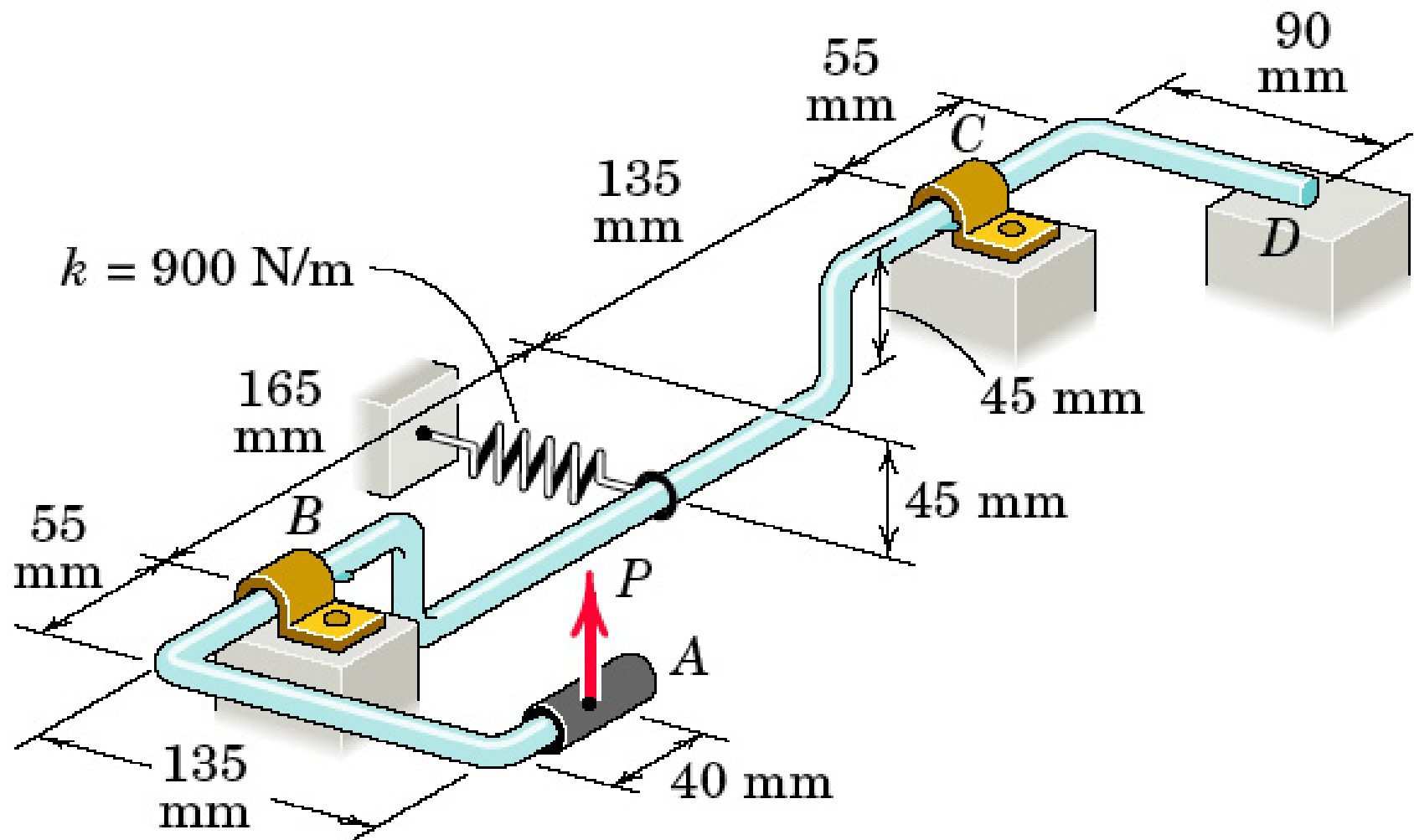


Dimensions in millimeters

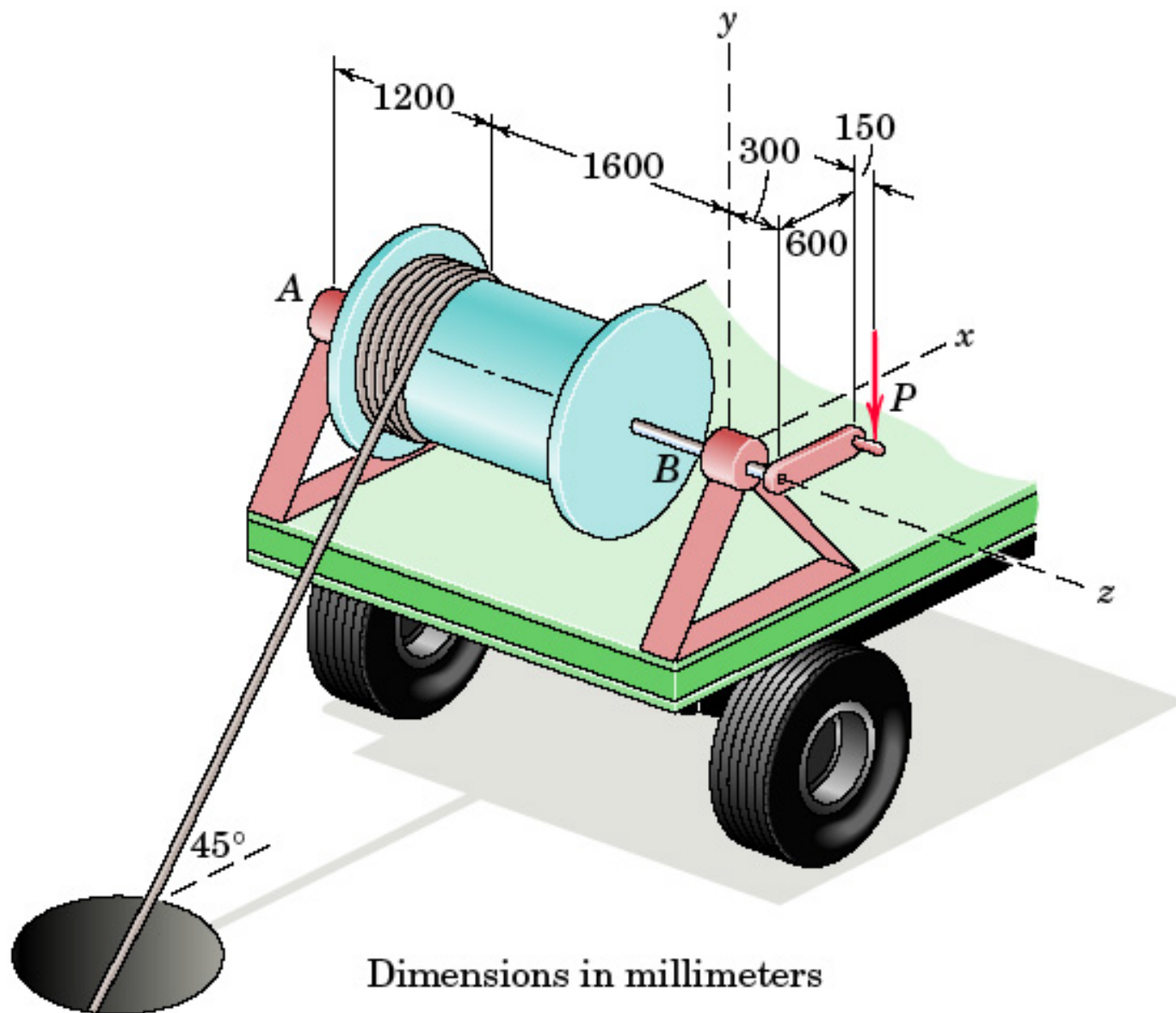
Problem 3/82



Problem 3/83

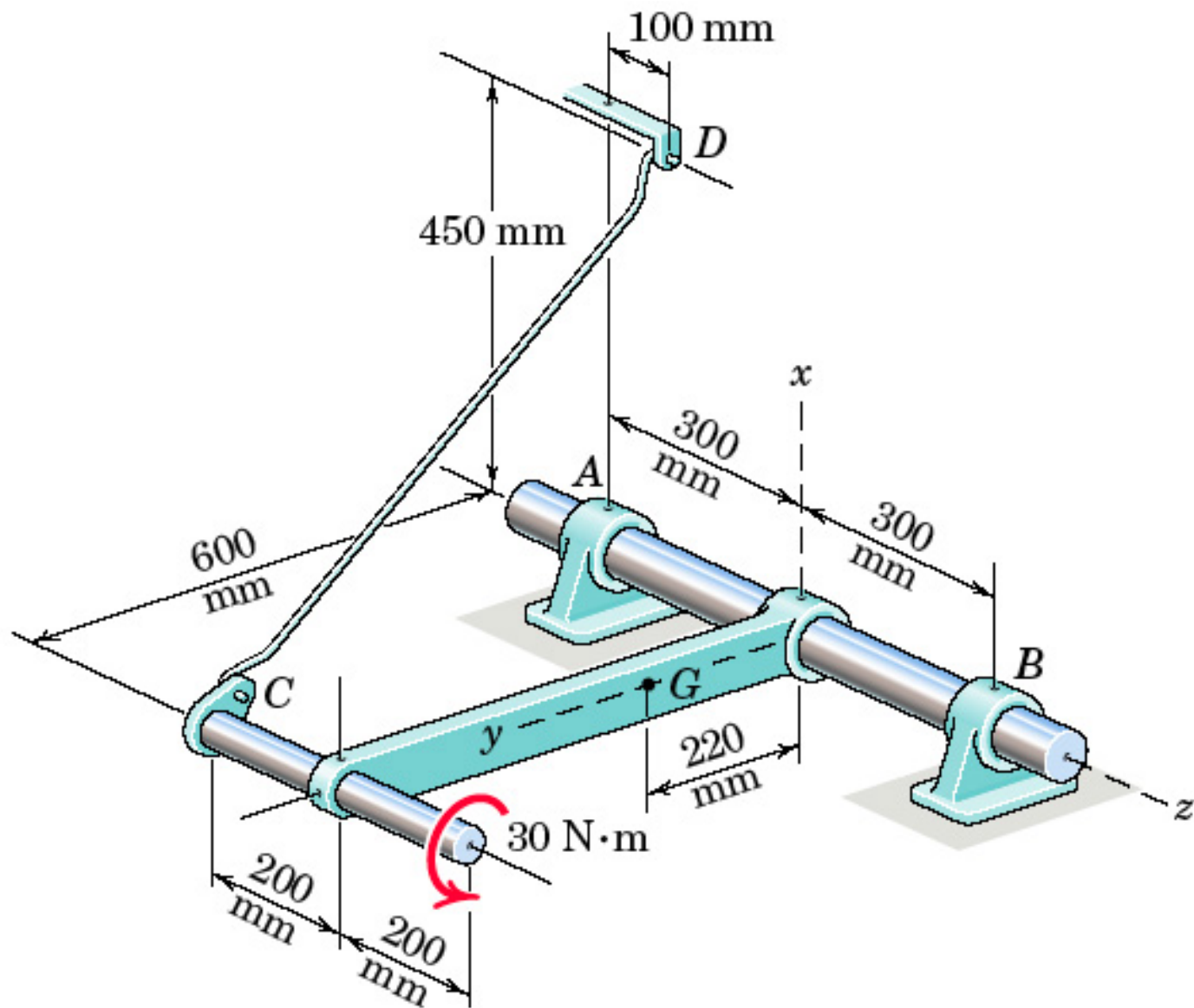


Problem 3/84

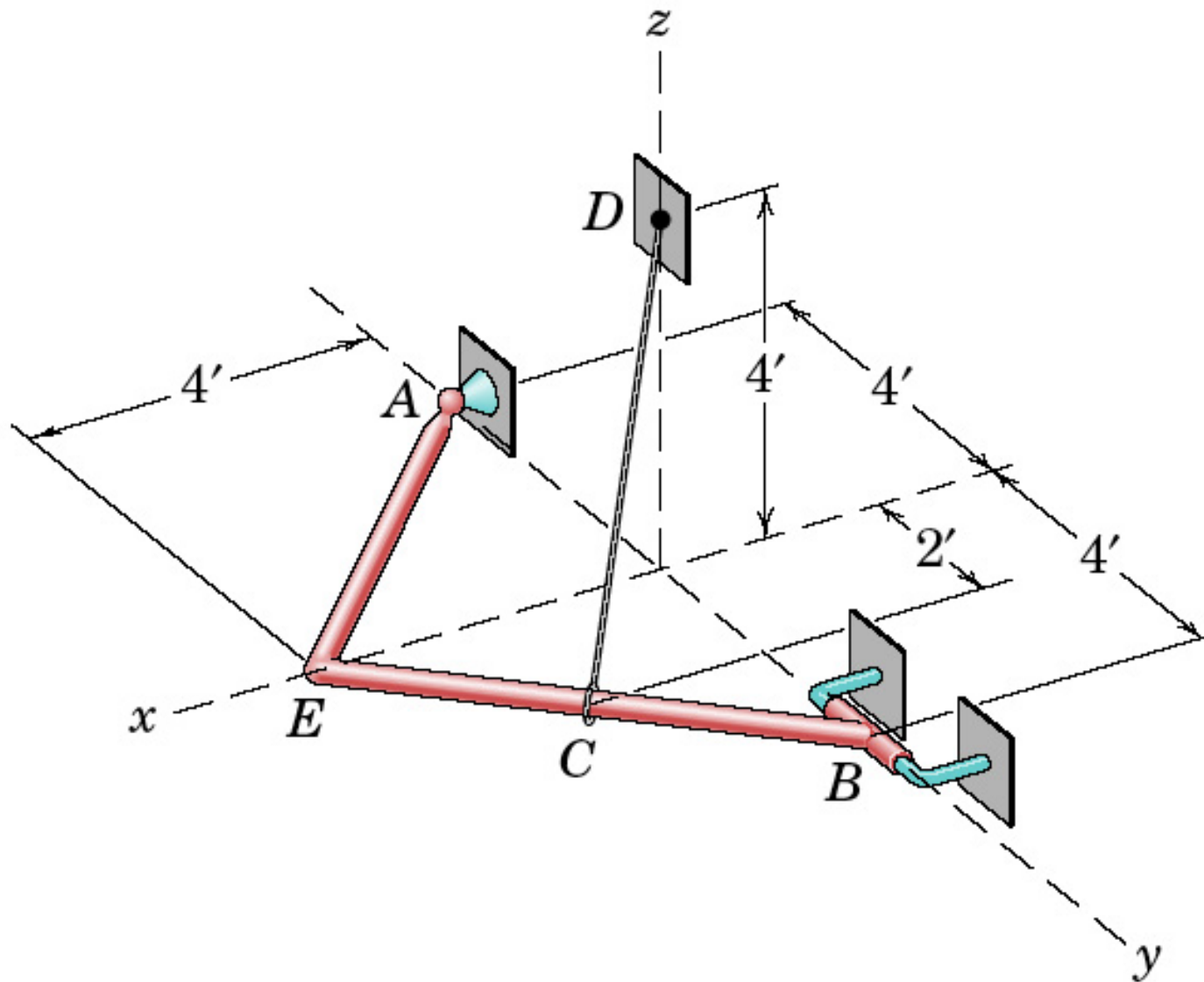


Dimensions in millimeters

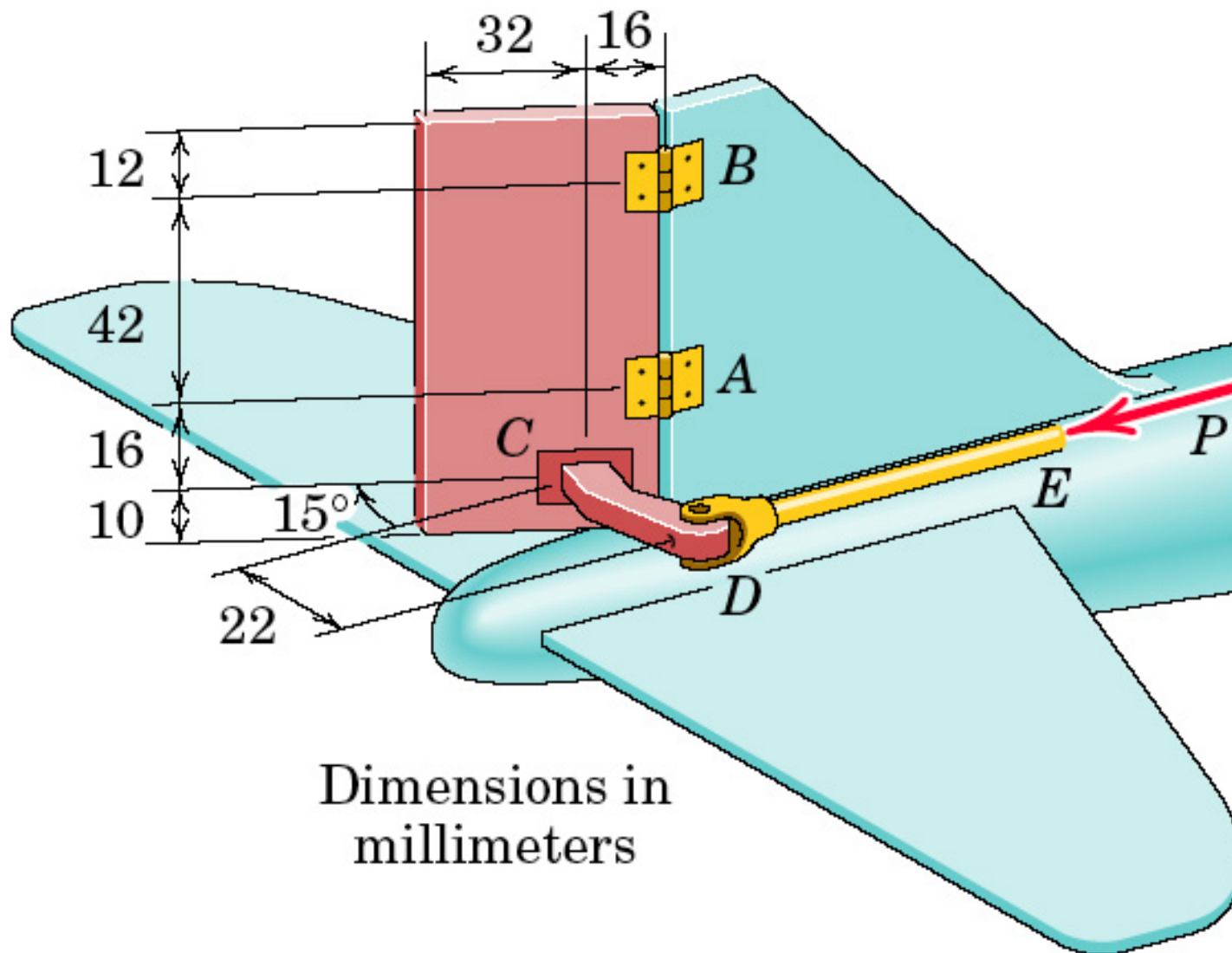
Problem 3/85



Problem 3/86

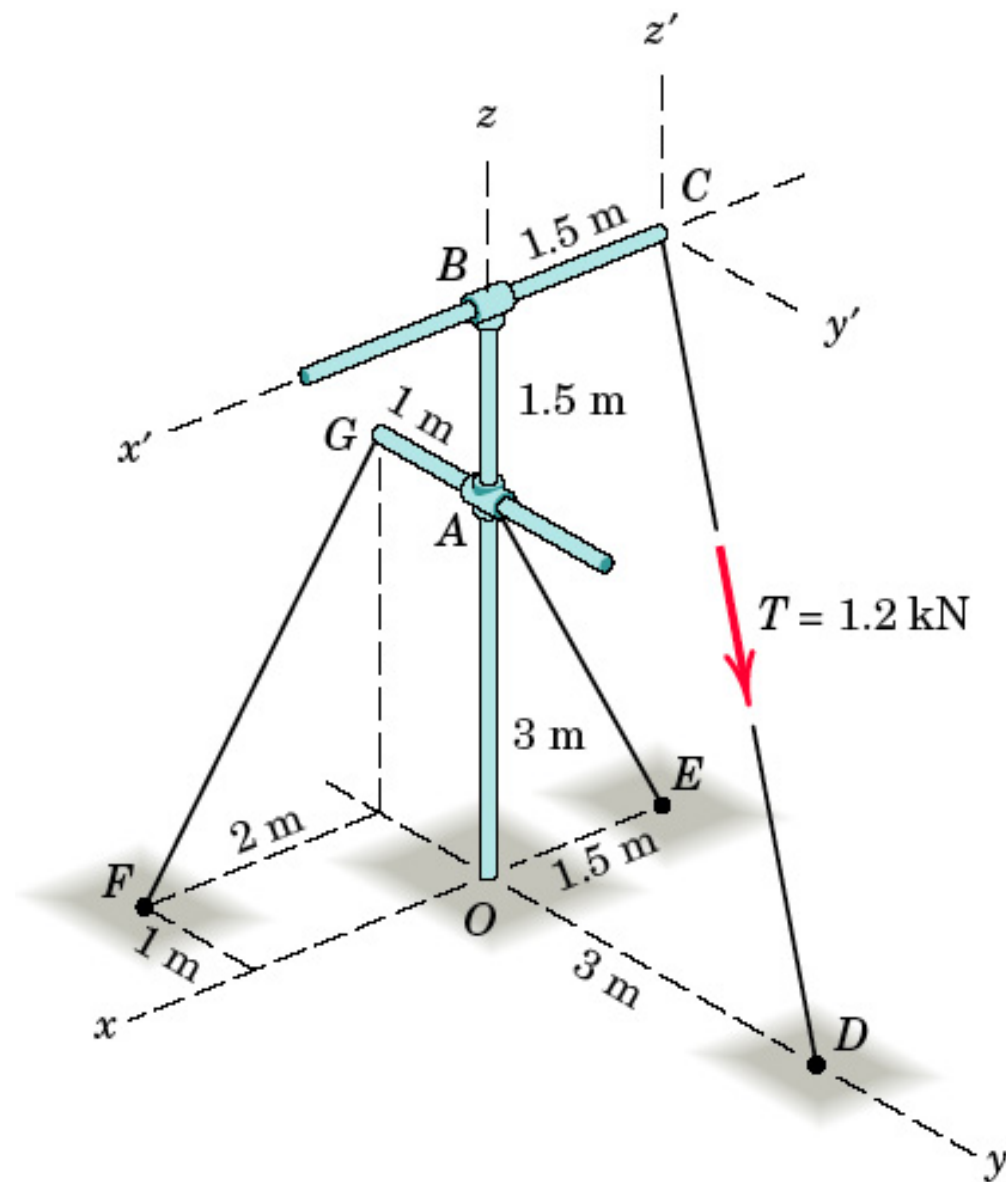


Problem 3/87

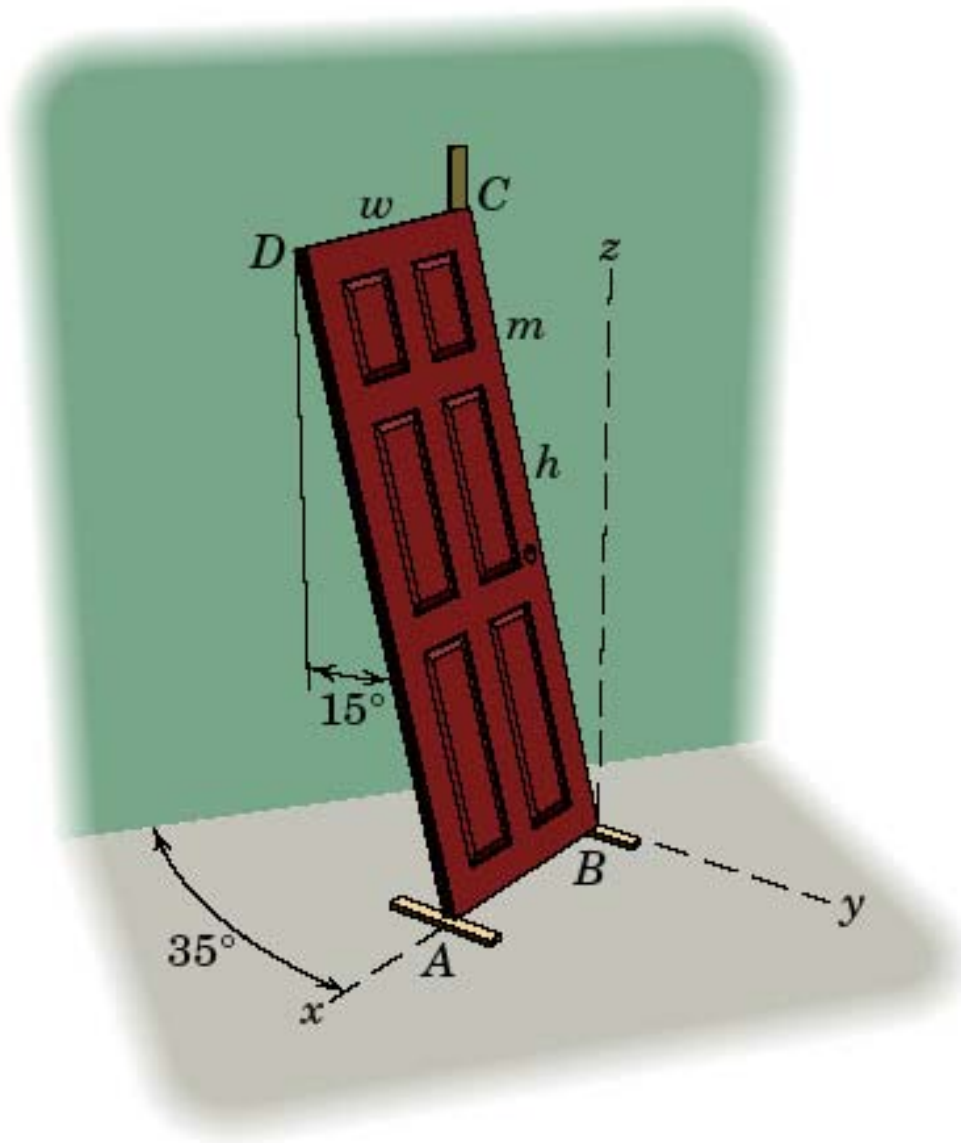


Dimensions in
millimeters

Problem 3/88

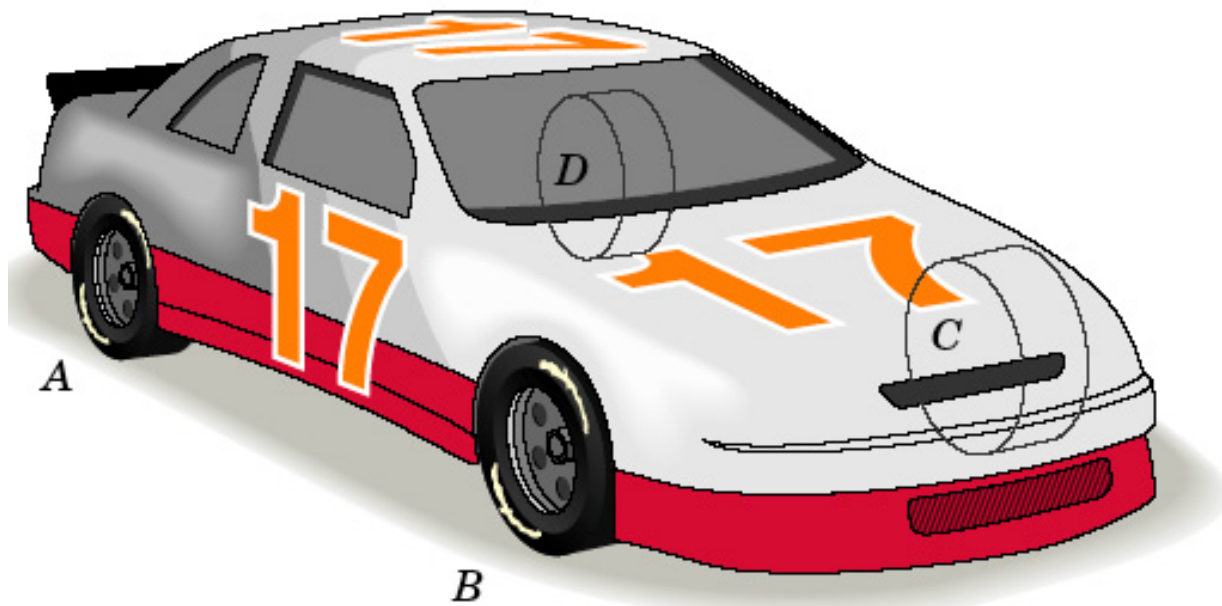
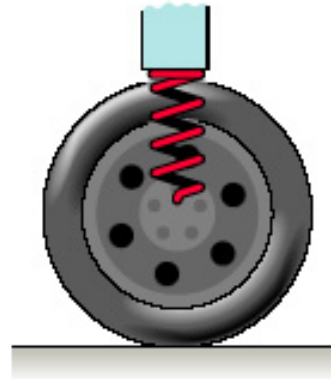


Problem 3/89

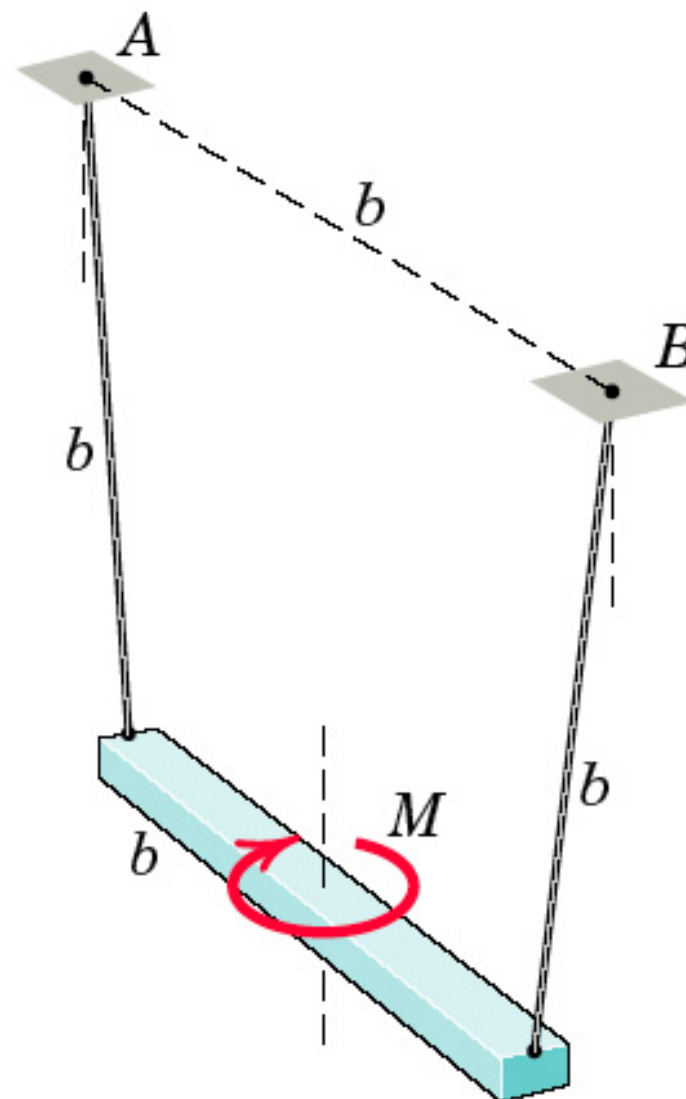


Problem 3/90

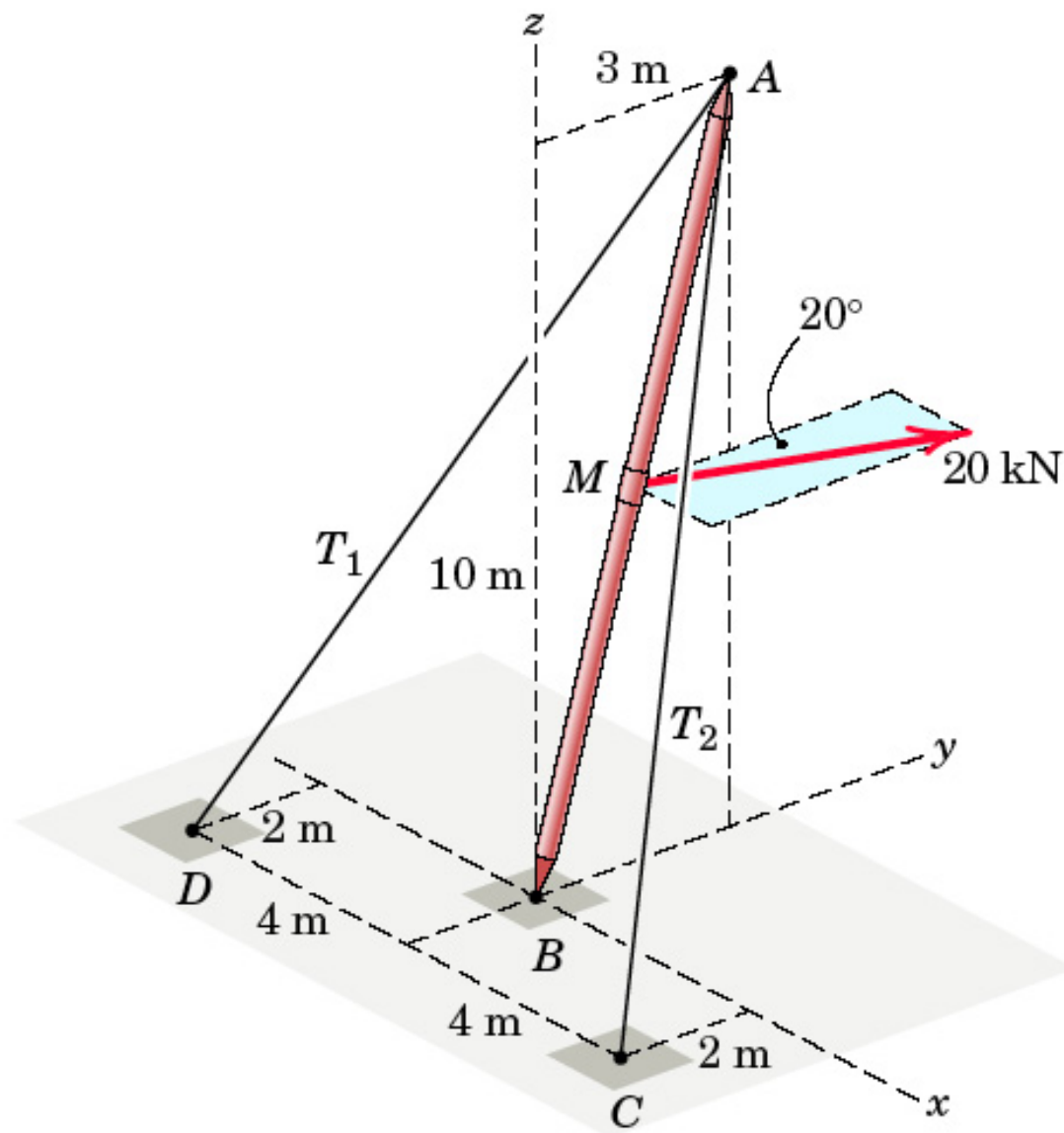
Simplified spring detail



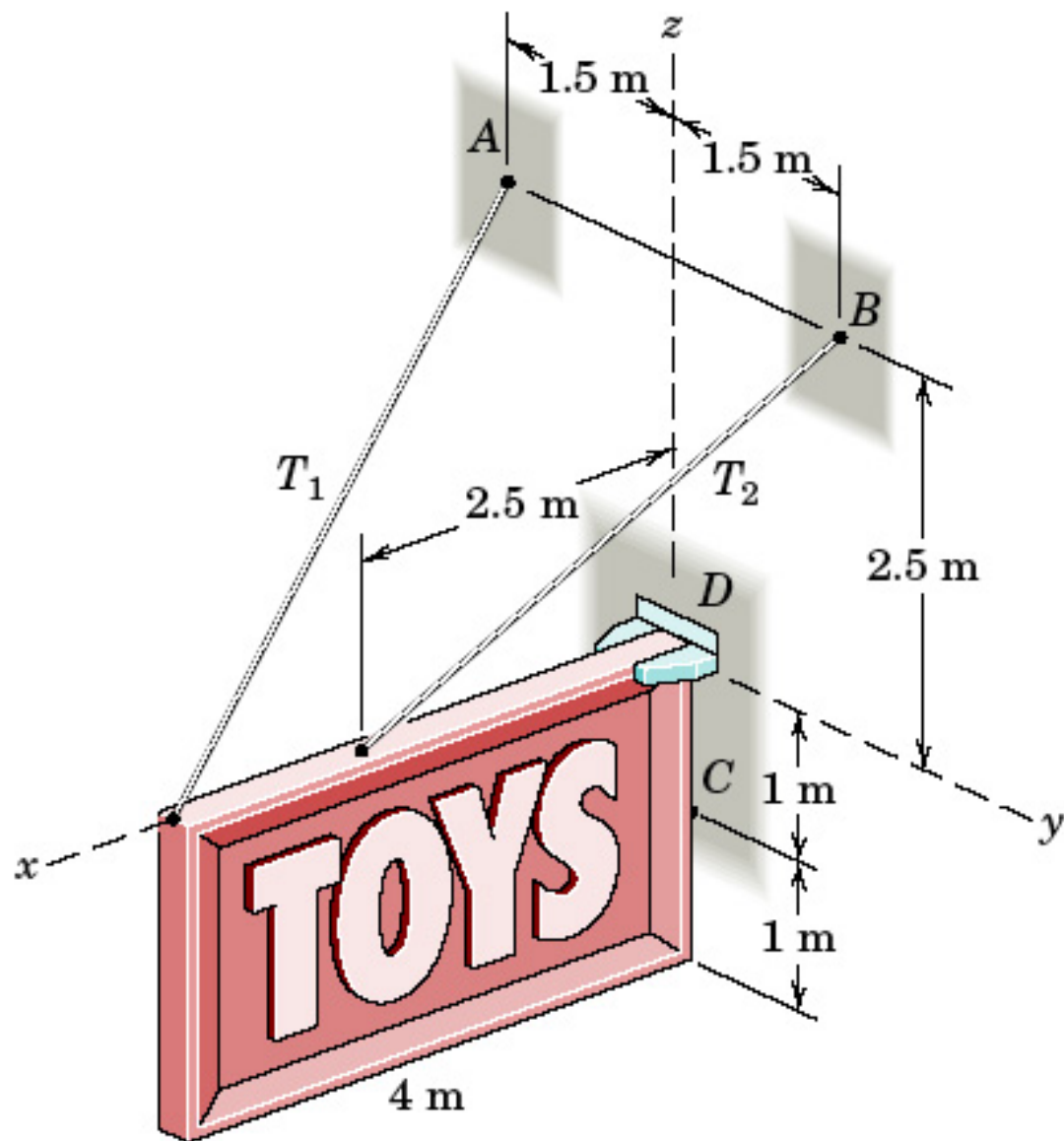
Problem 3/91



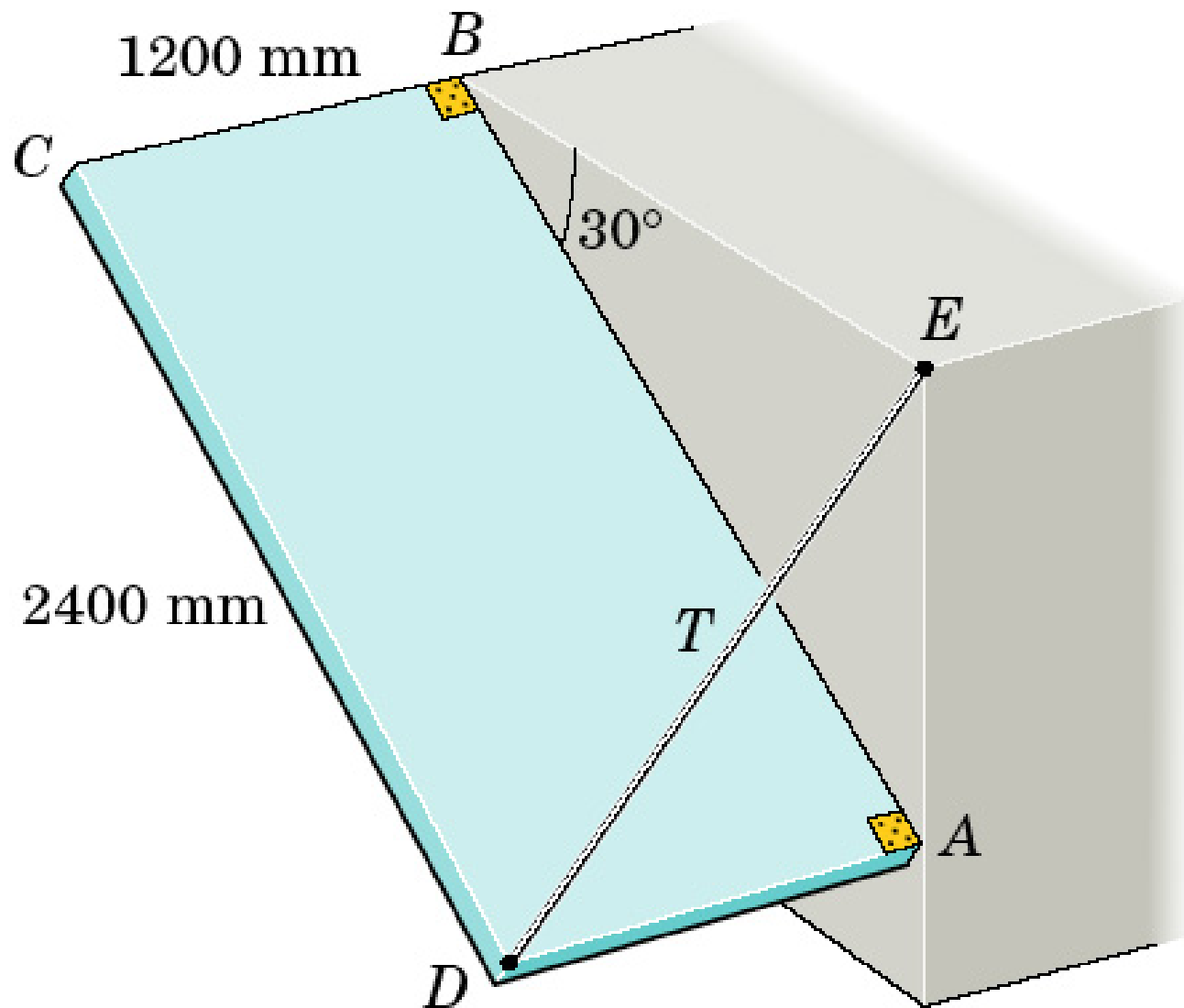
Problem 3/92



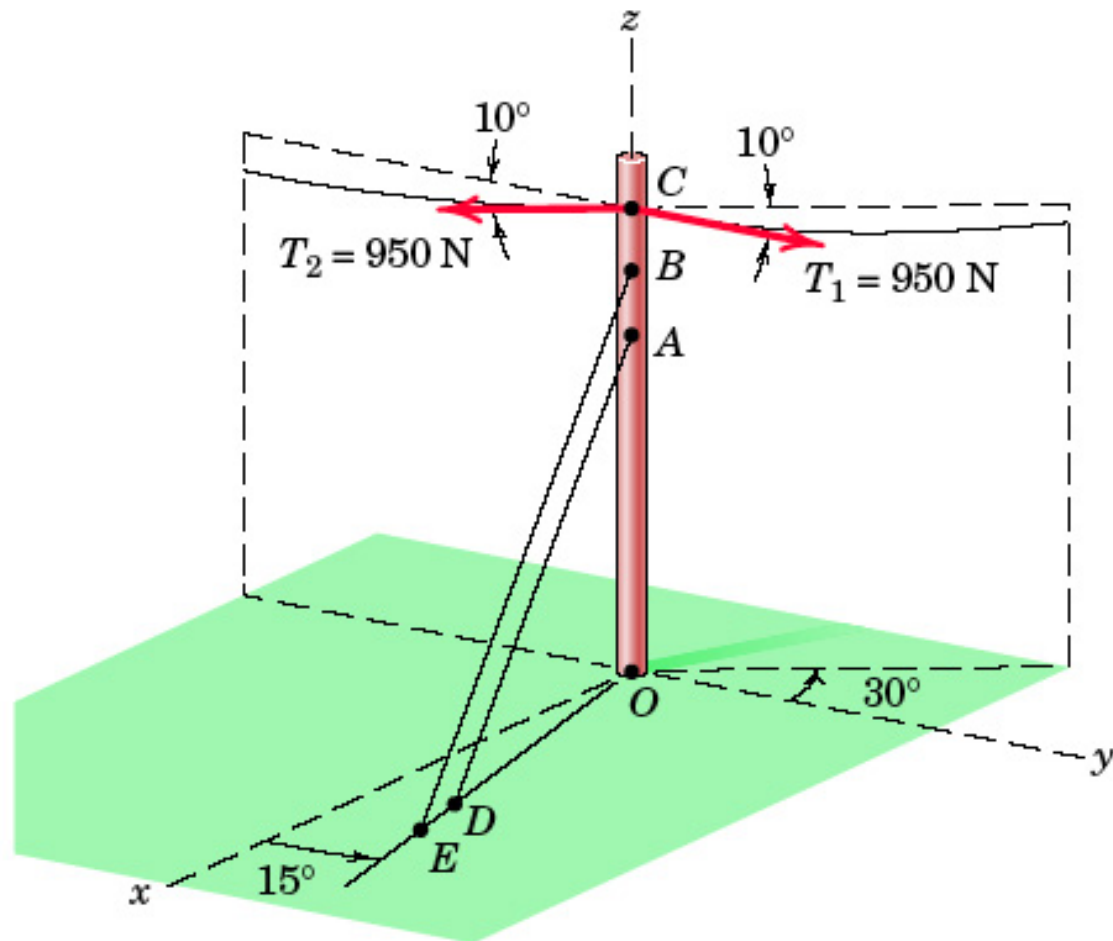
Problem 3/93



Problem 3/94

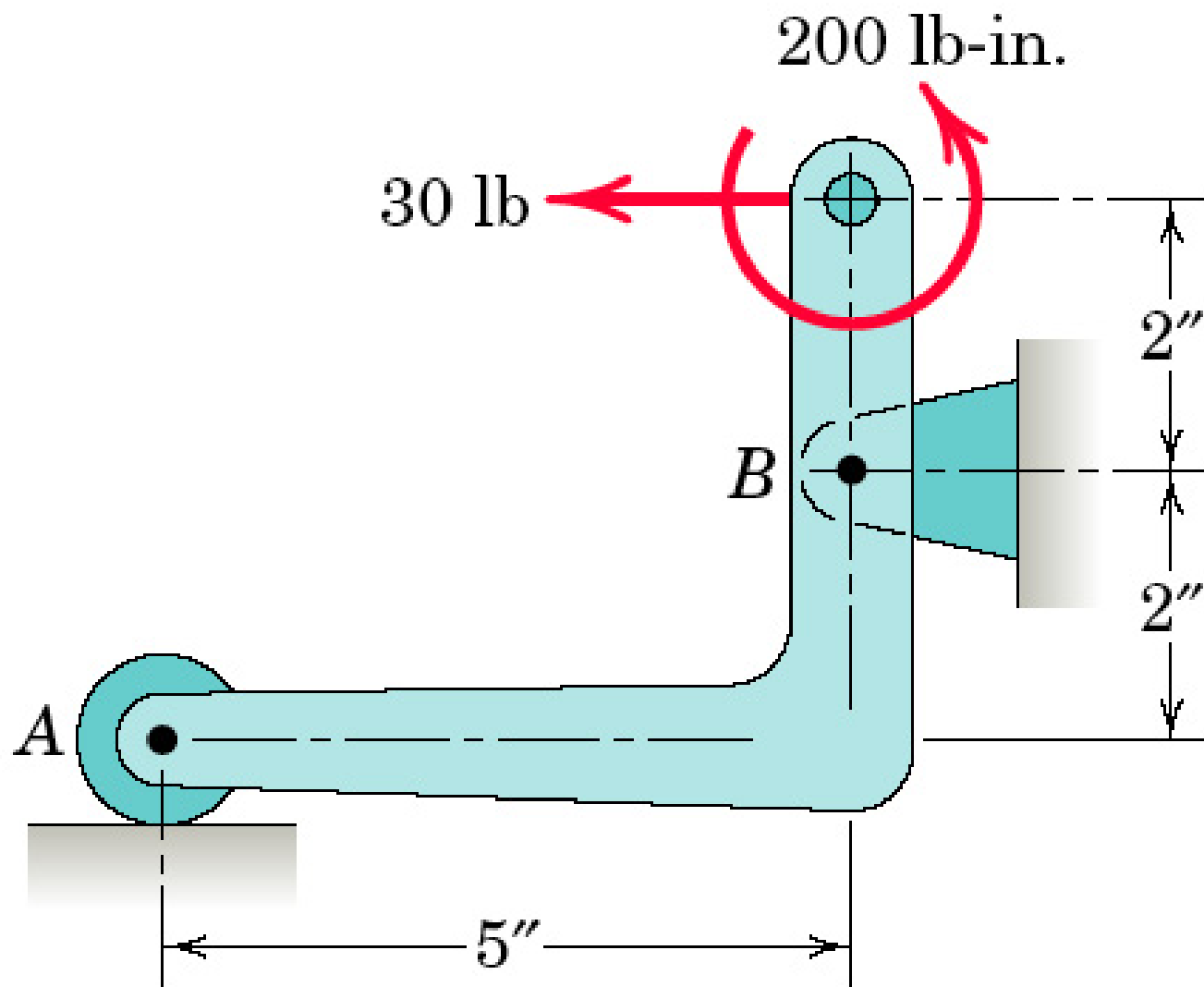


Problem 3/95

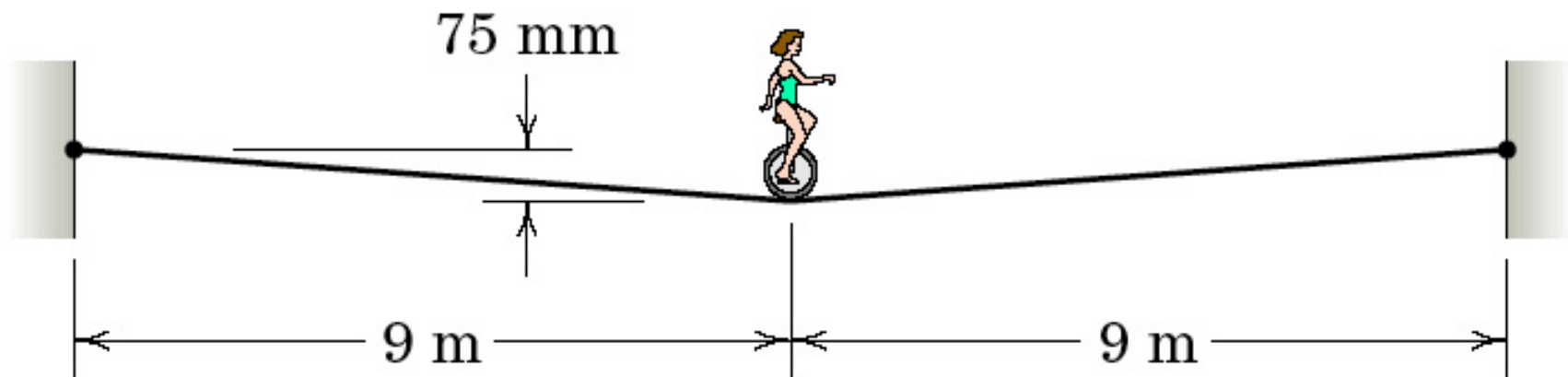


$$\begin{array}{ll} \overline{OA} = 9 \text{ m} & \overline{OD} = 8 \text{ m} \\ \overline{OB} = 11 \text{ m} & \overline{OE} = 10 \text{ m} \\ \overline{OC} = 13 \text{ m} & \end{array}$$

Problem 3/96

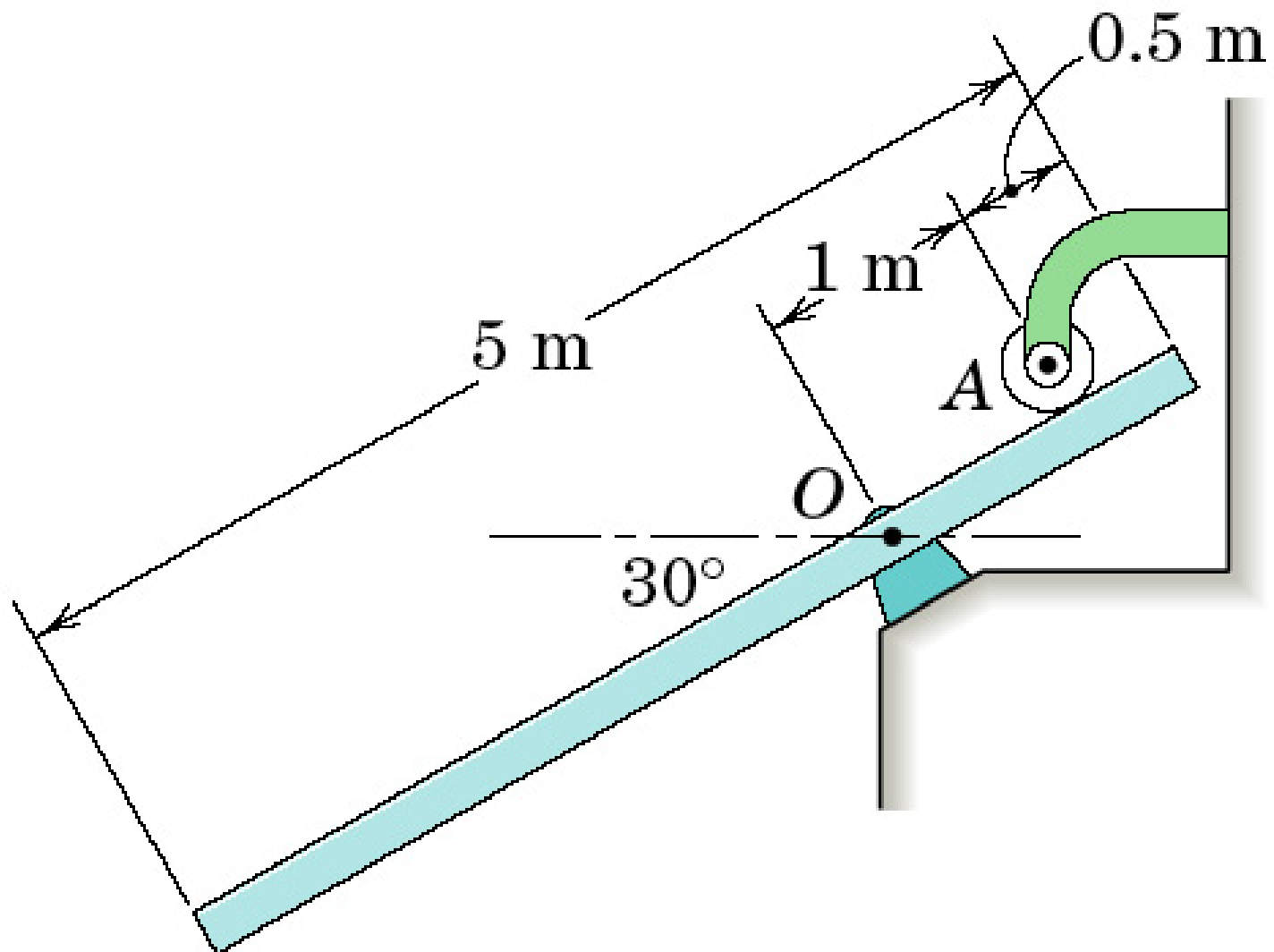


Problem 3/97



Problem 3/98

P_03_98



Problem 3/99

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 4 : Structures.

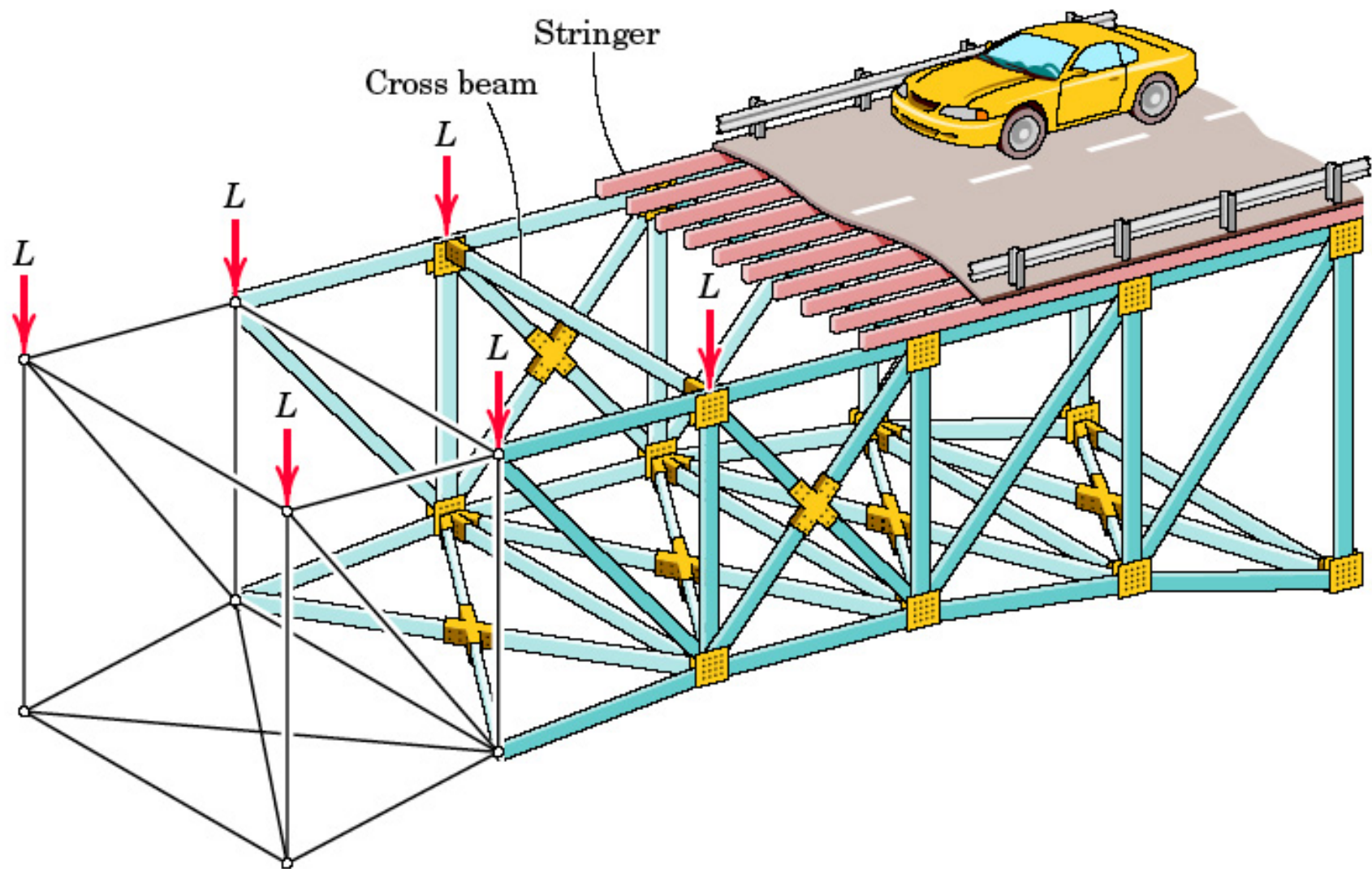
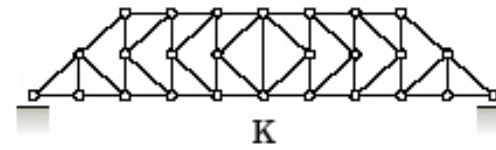
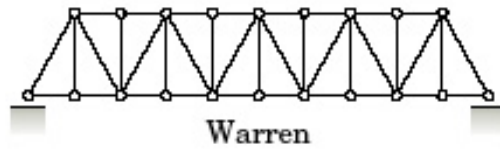
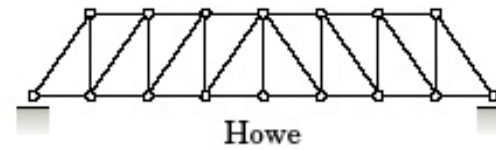
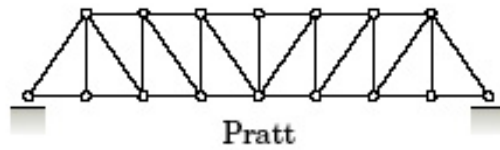
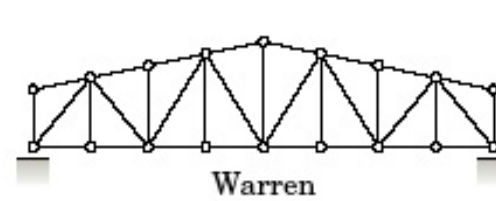
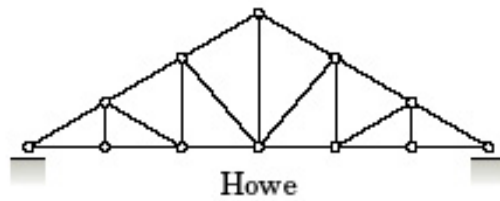
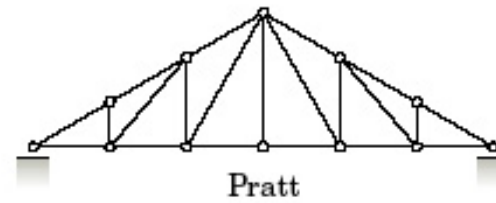
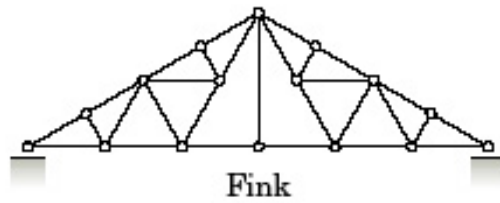


Figure 4/1

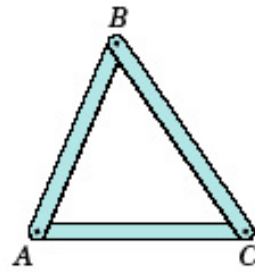


Commonly Used Bridge Trusses

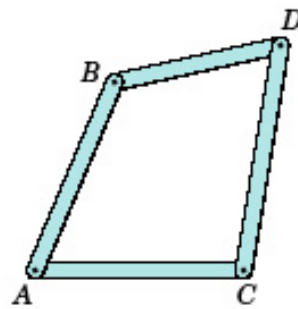


Commonly Used Roof Trusses

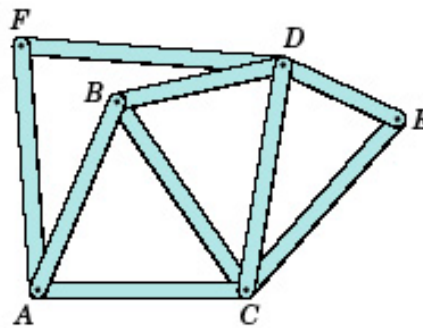
Figure 4/2



(a)

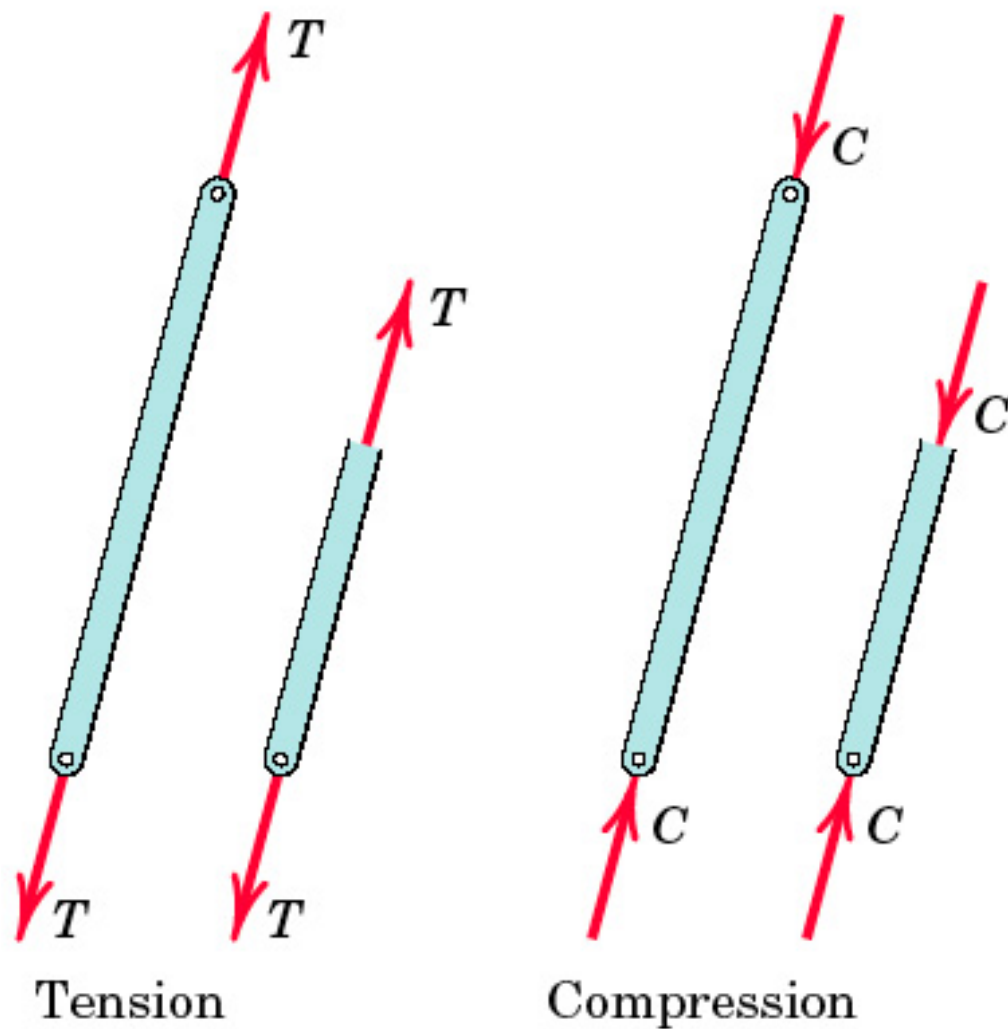


(b)



(c)

Figure 4/3



Two-Force Members

Figure 4/4

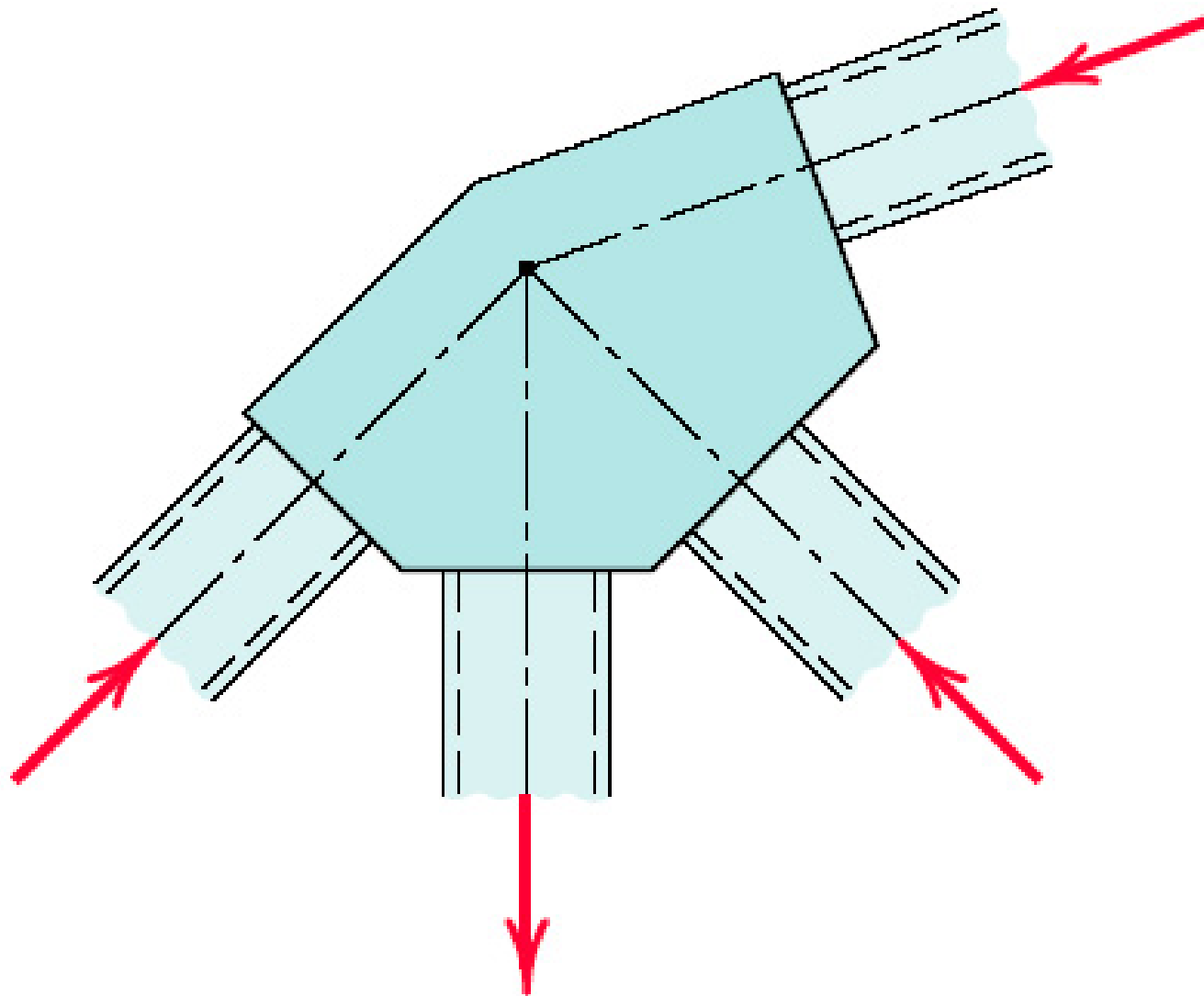
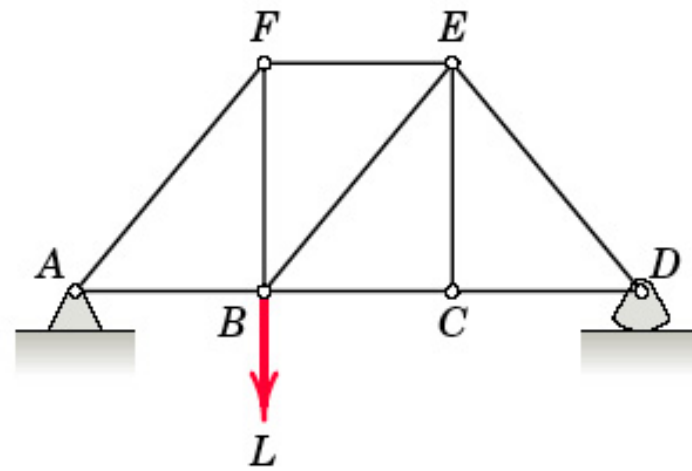
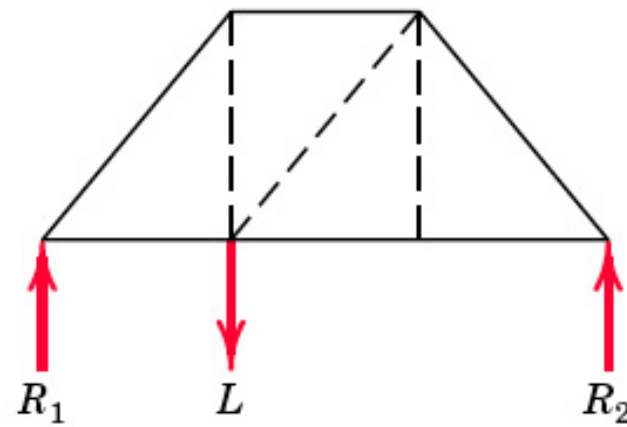


Figure 4/5



(a)



(b)

Figure 4/6

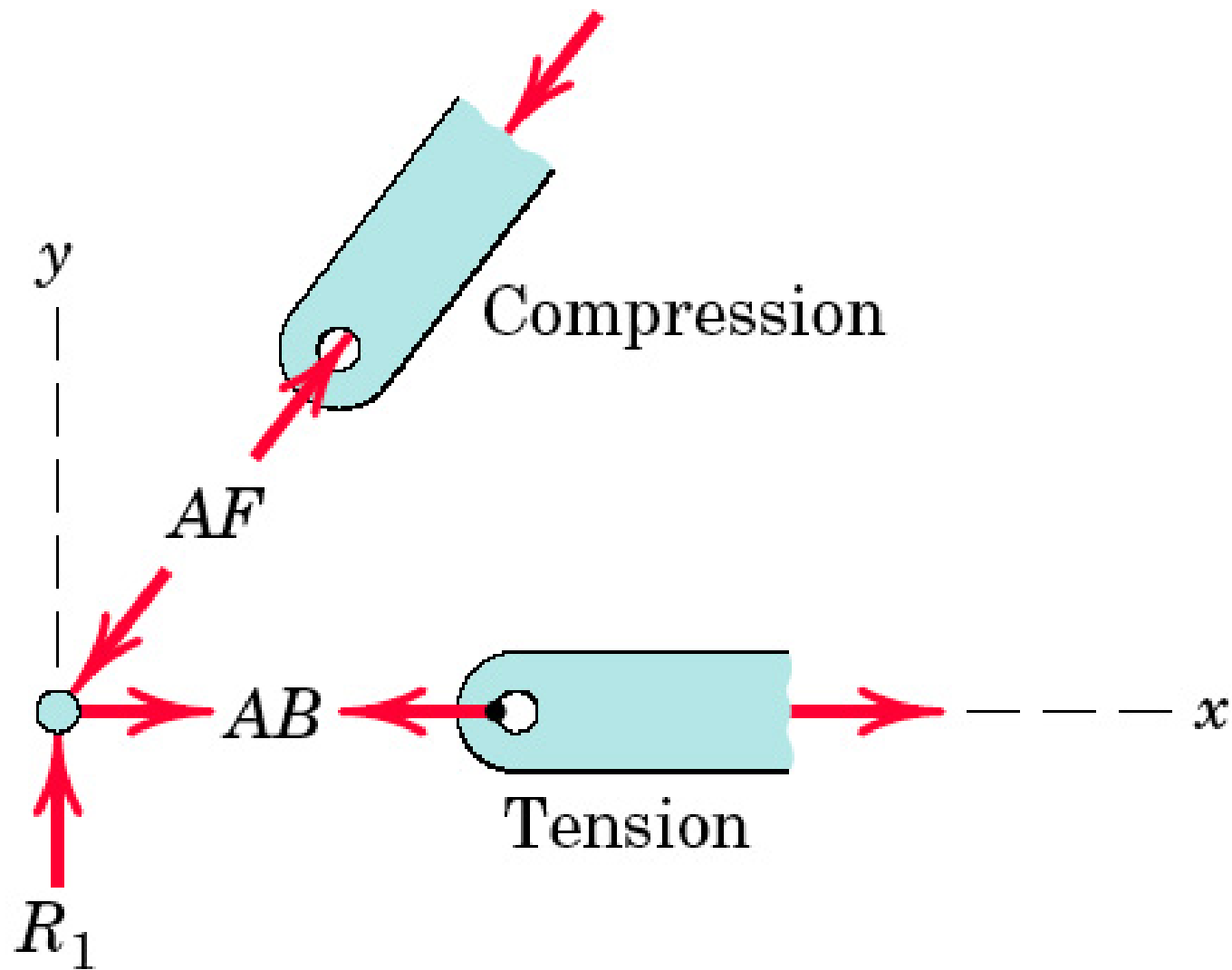


Figure 4/7

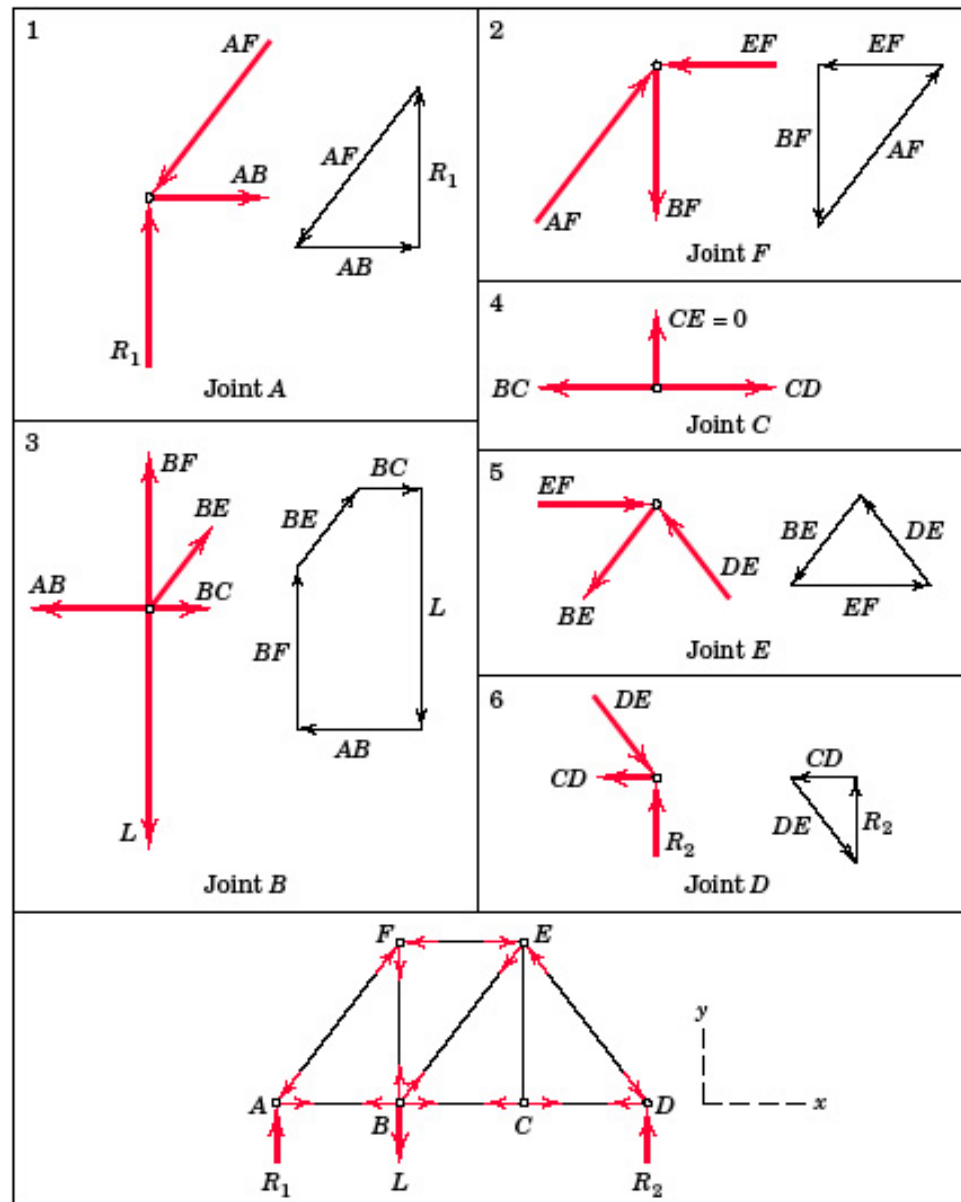
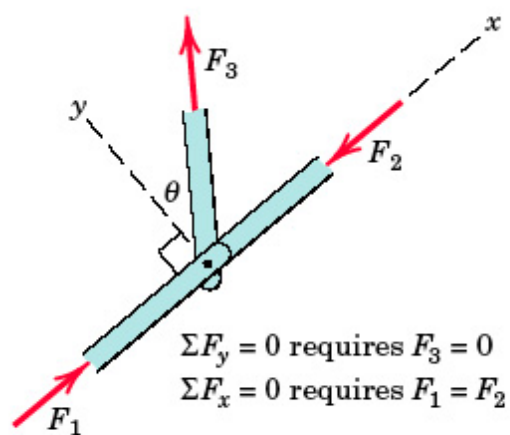
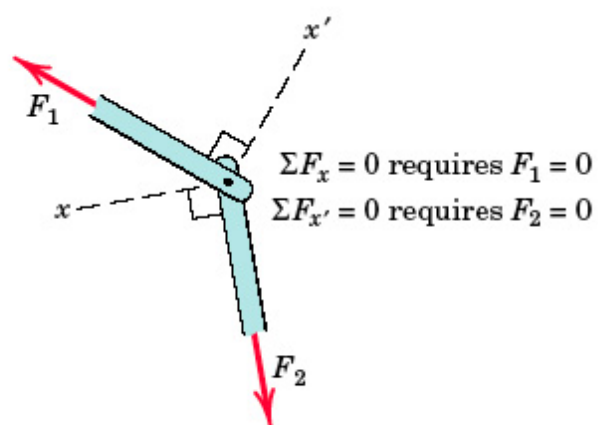


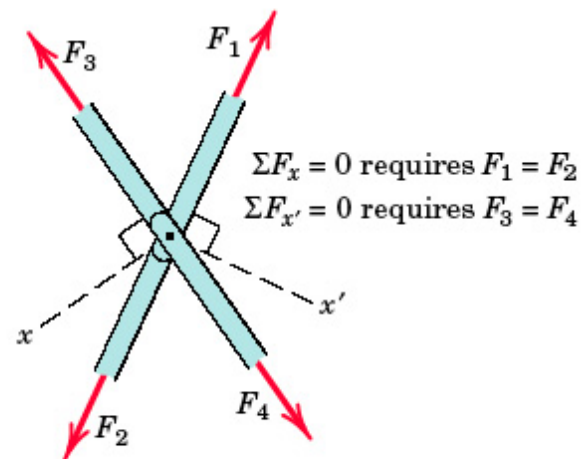
Figure 4/8



(a)

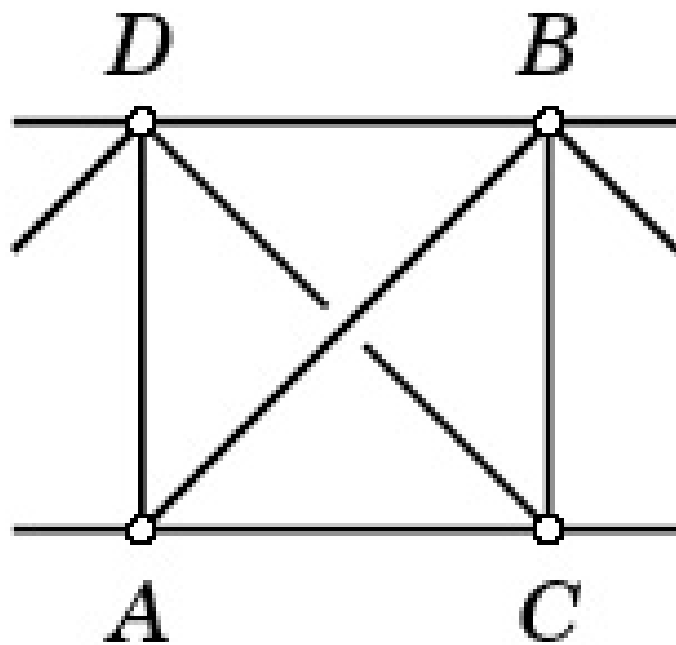


(b)

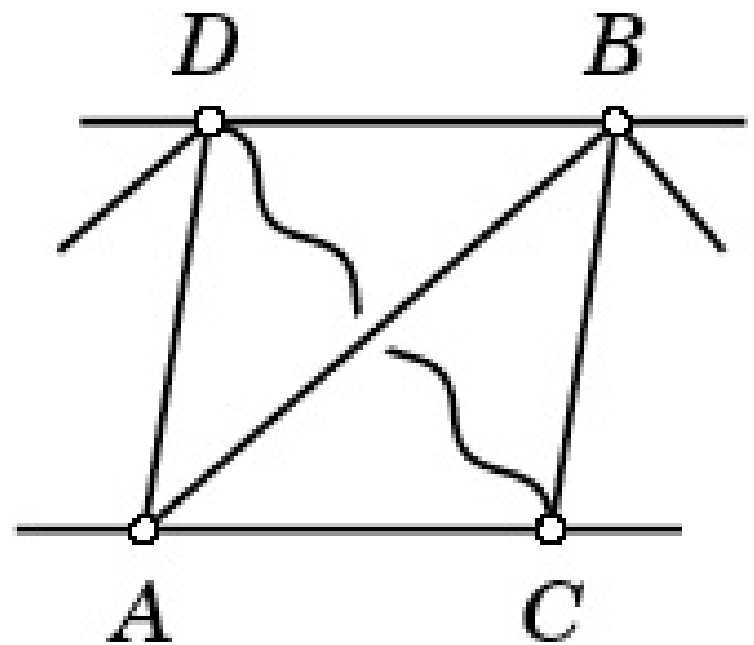


(c)

Figure 4/9



(a)



(b)

Figure 4/10

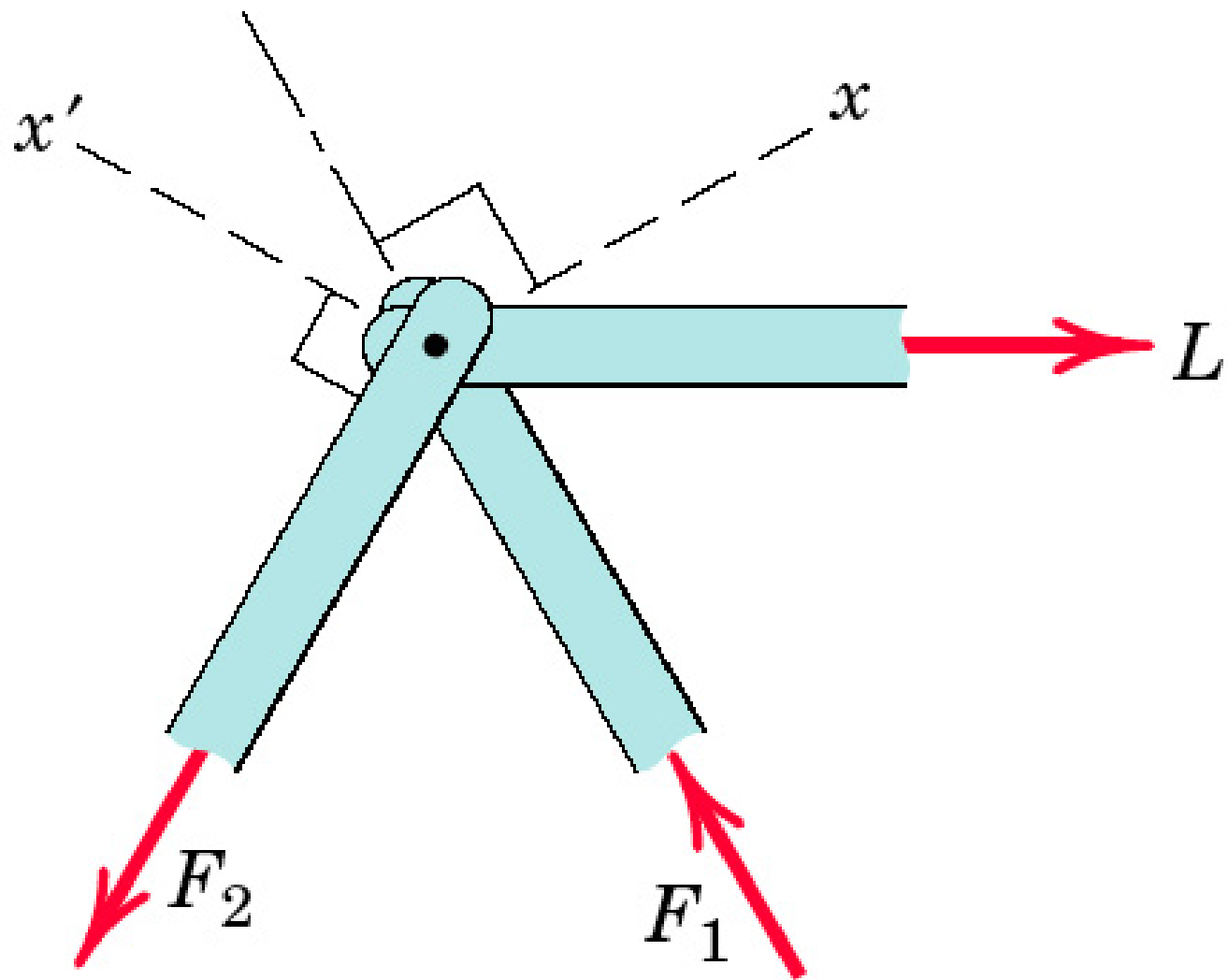
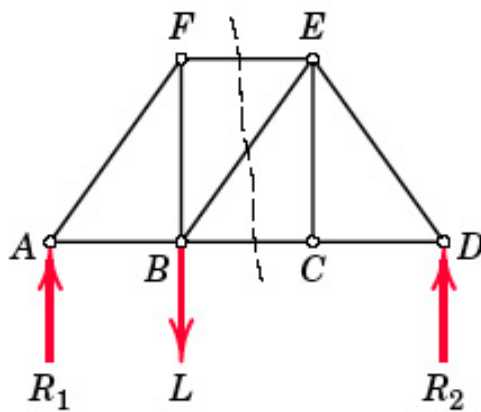
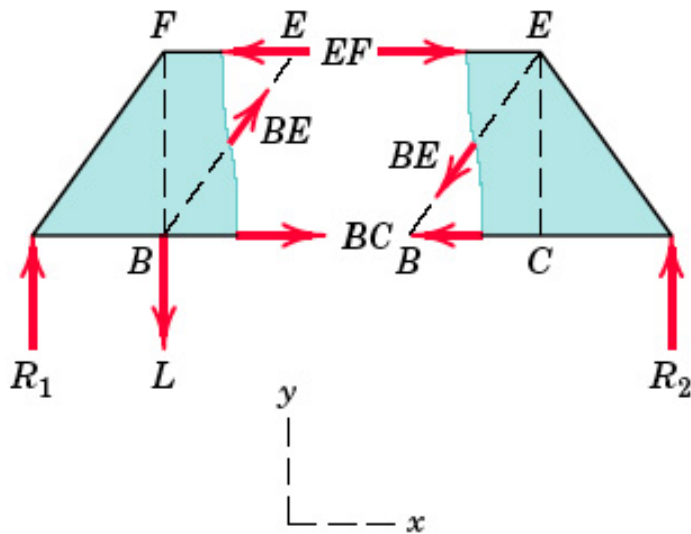


Figure 4/11



(a)



(b)

Figure 4/12

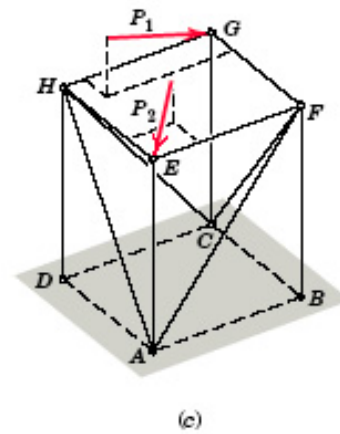
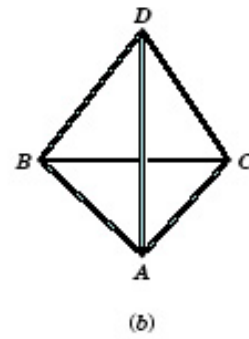
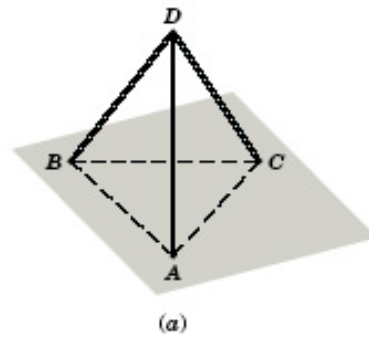


Figure 4/13

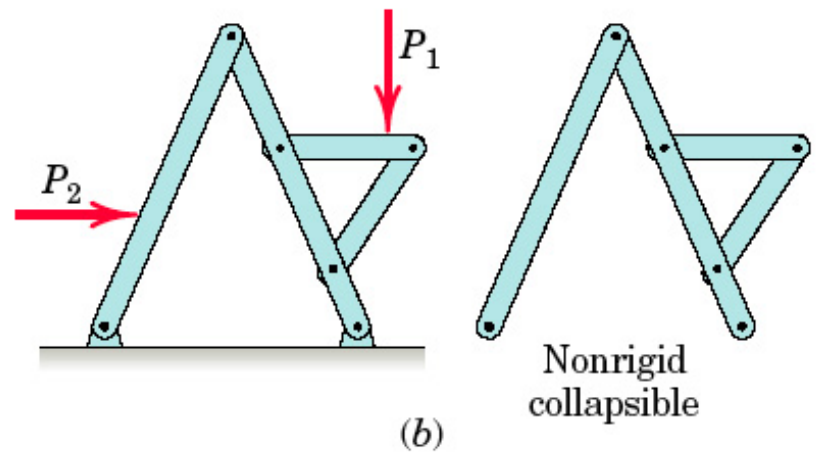
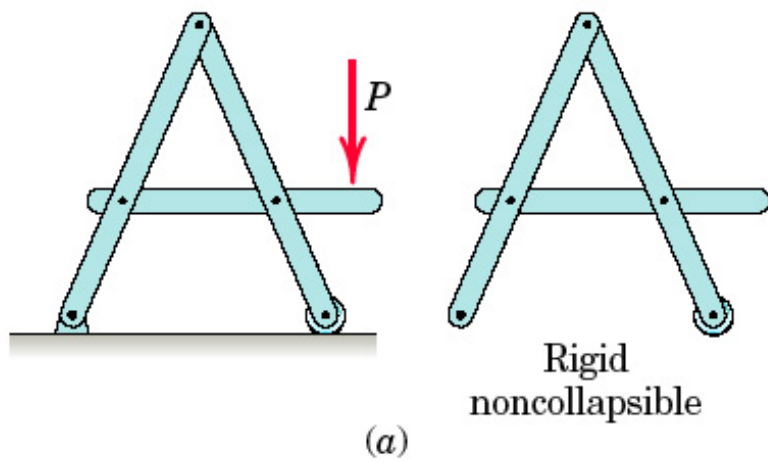


Figure 4/14

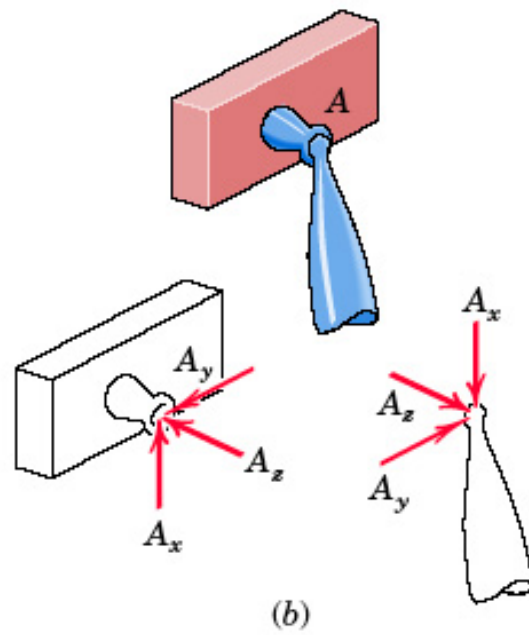
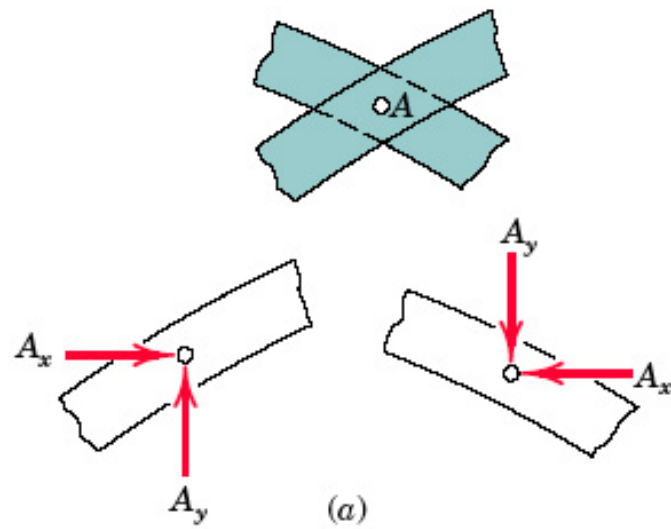


Figure 4/15

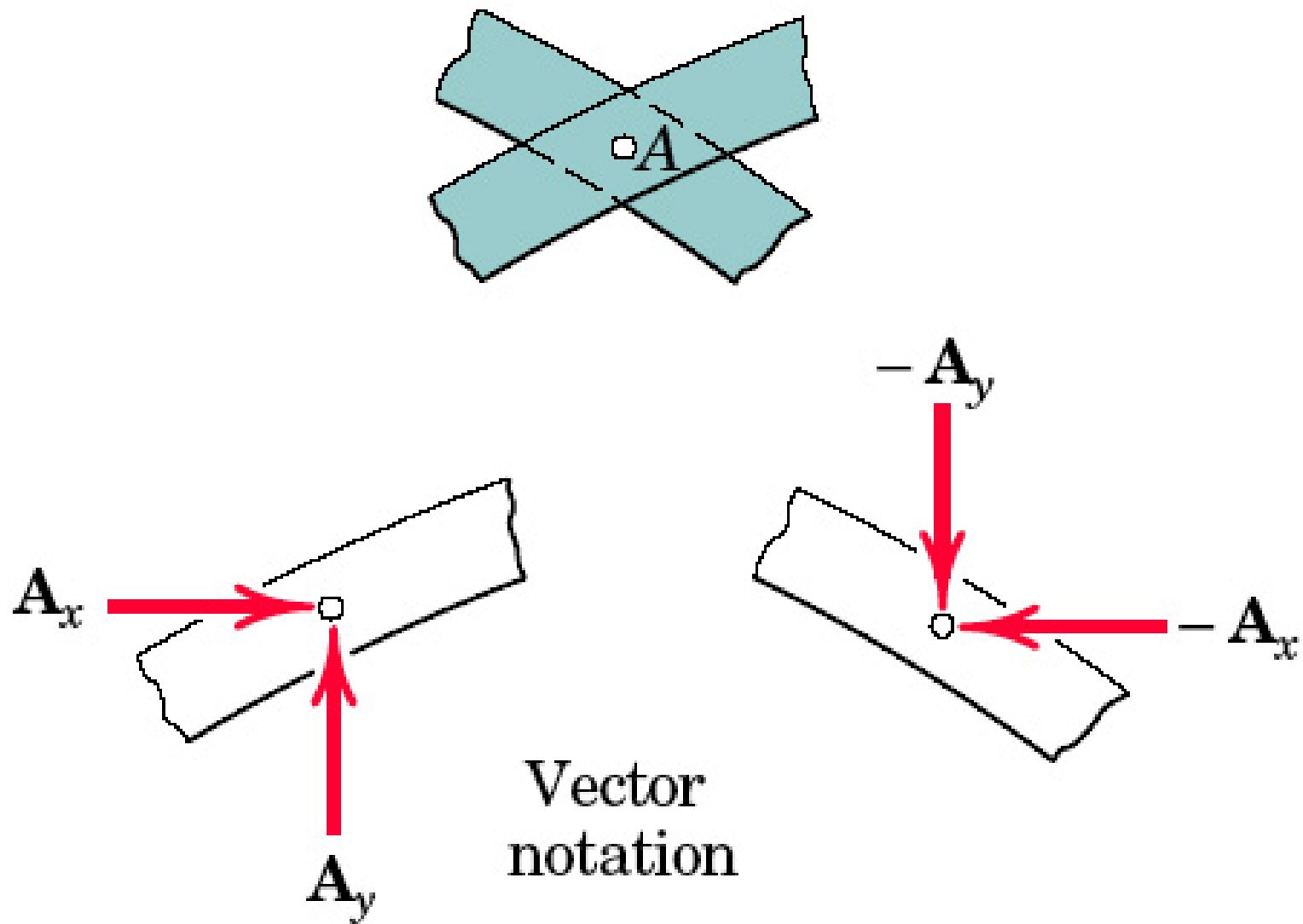
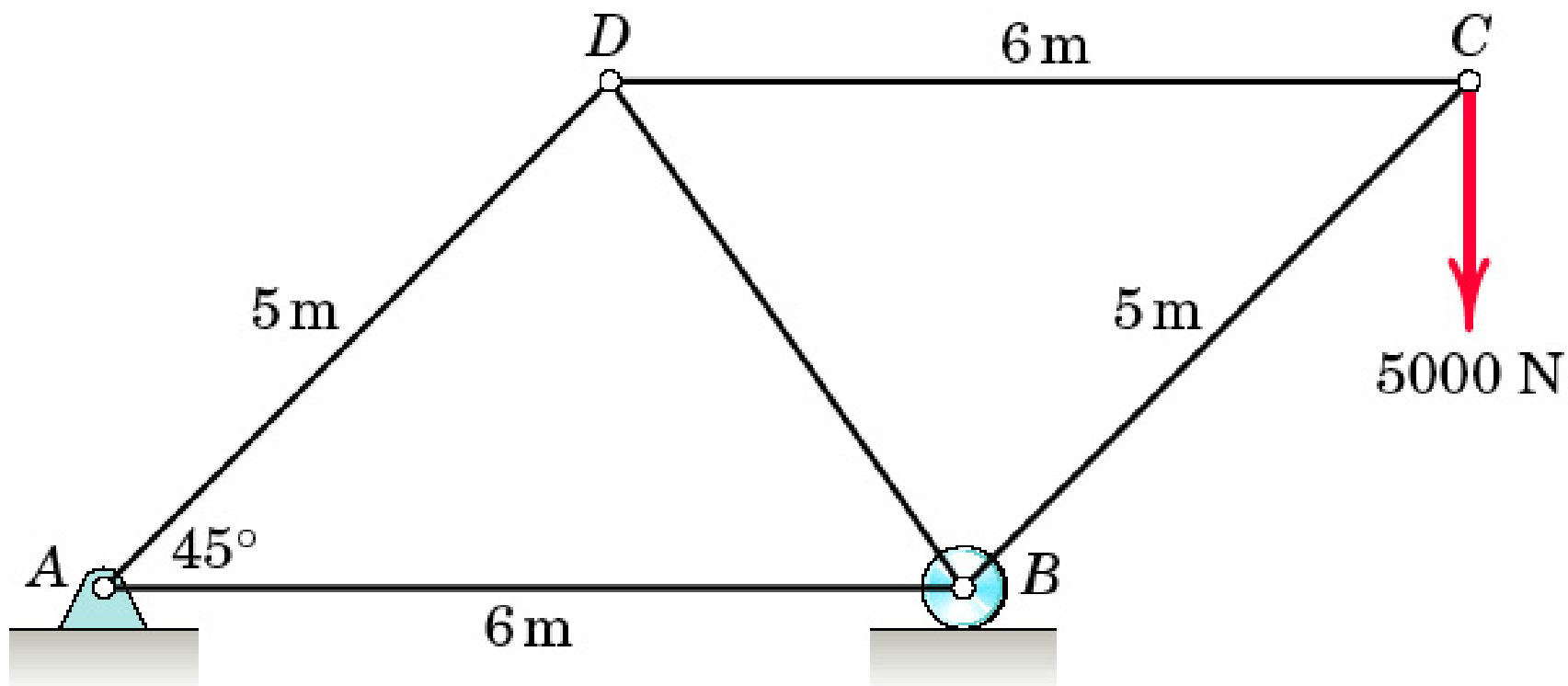
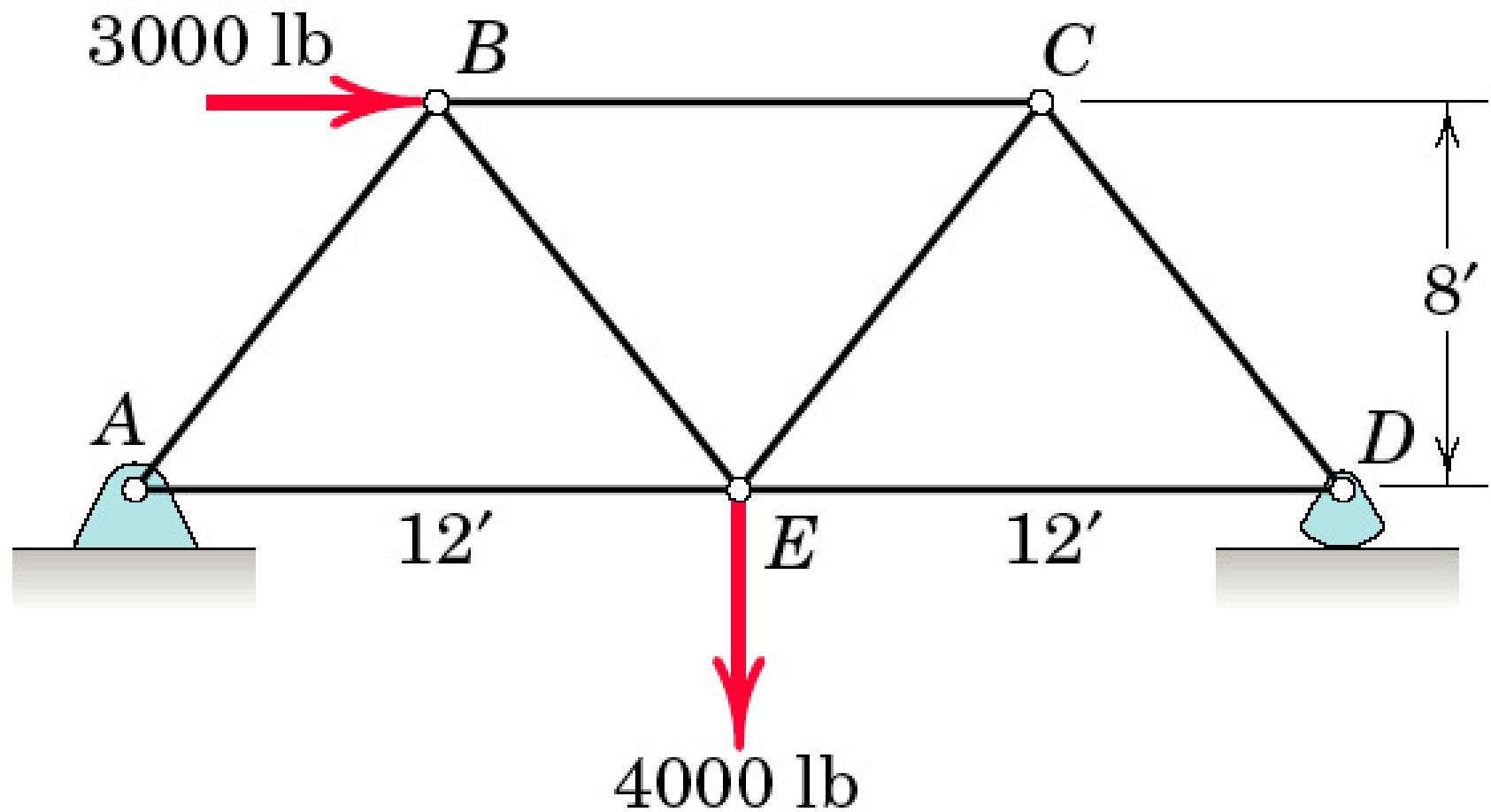


Figure 4/16

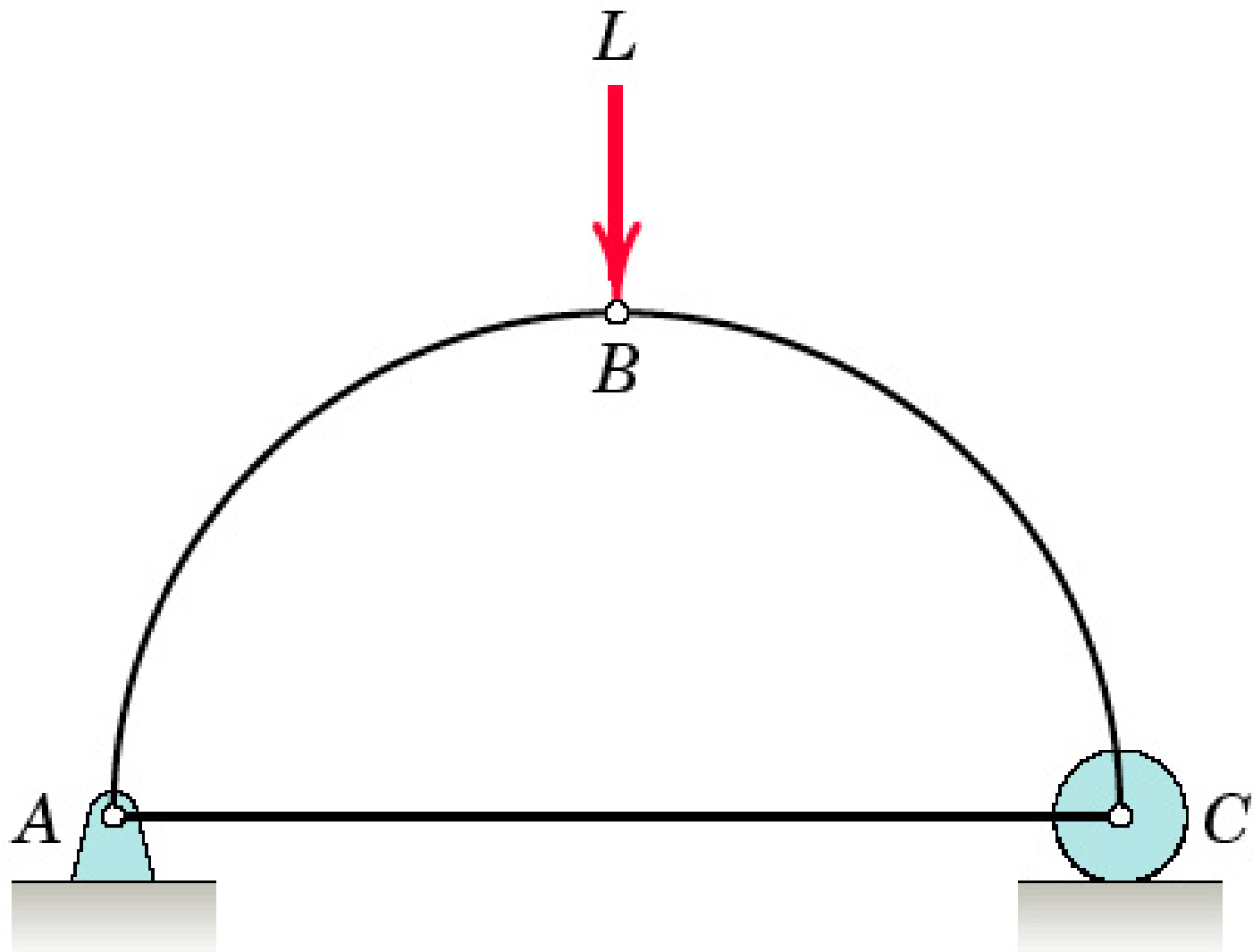


Problem 4/4

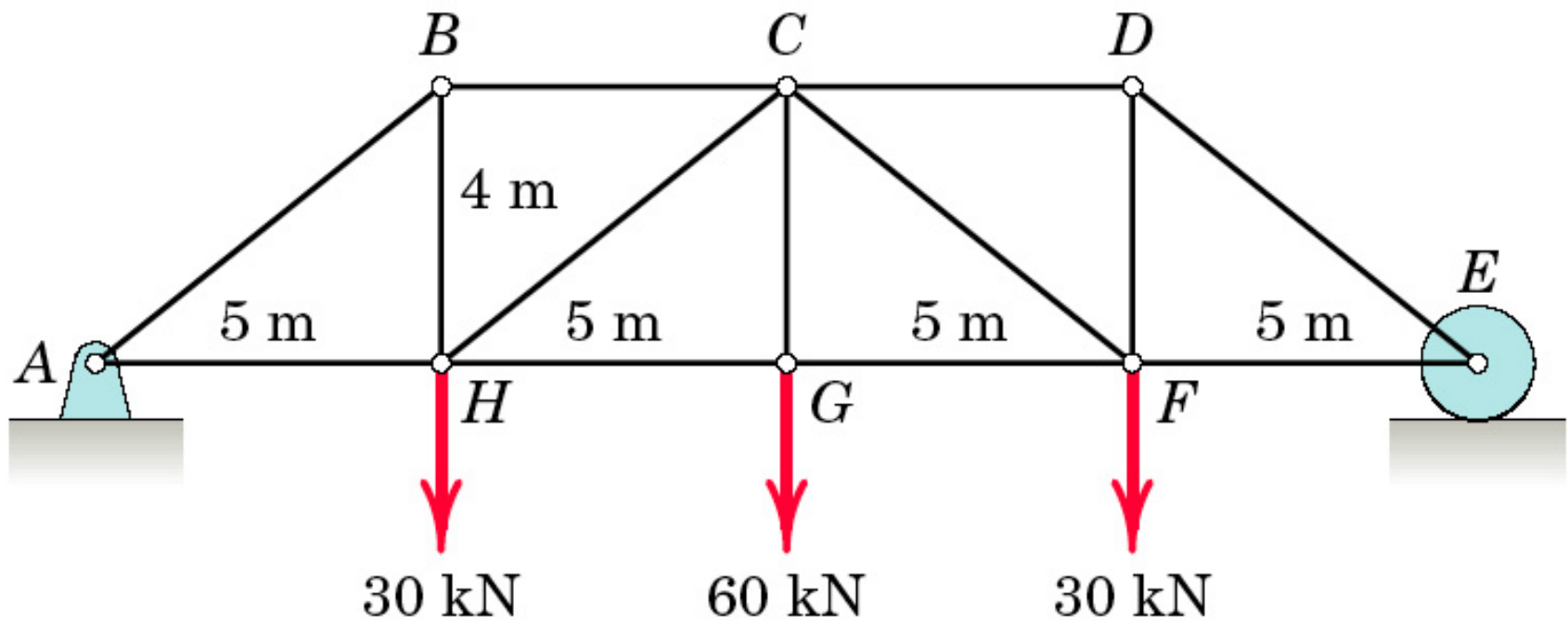
P_04_04



Problem 4/6

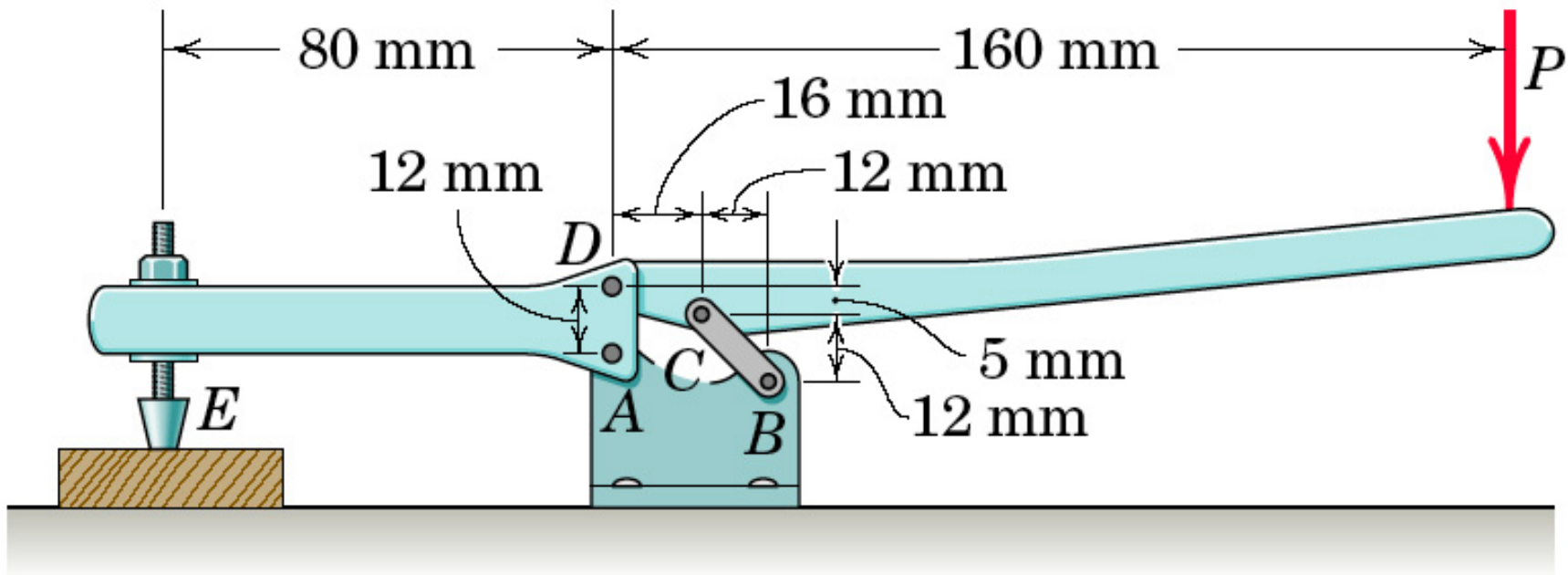


Problem 4/7



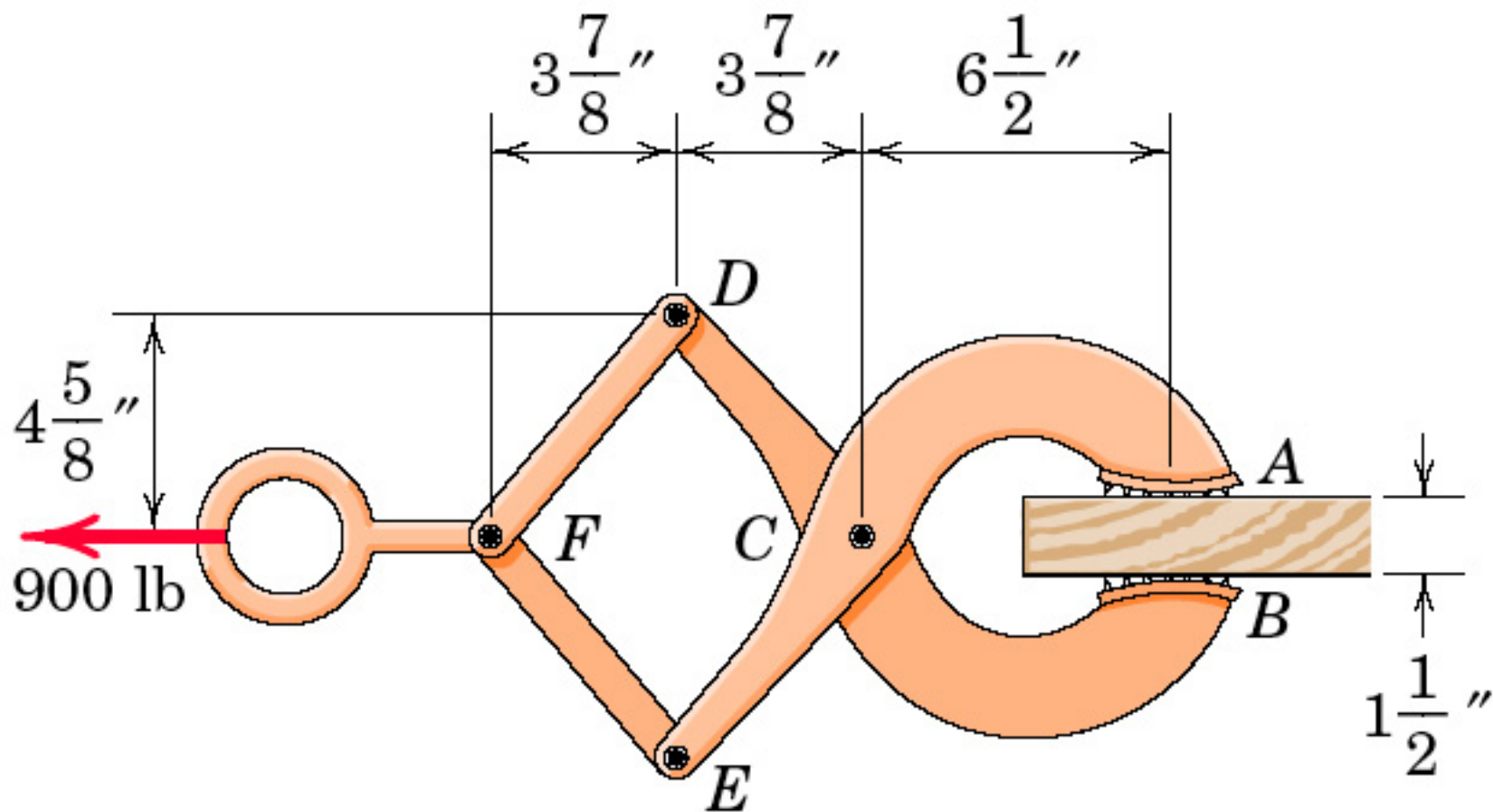
Problem 4/8

P_04_08

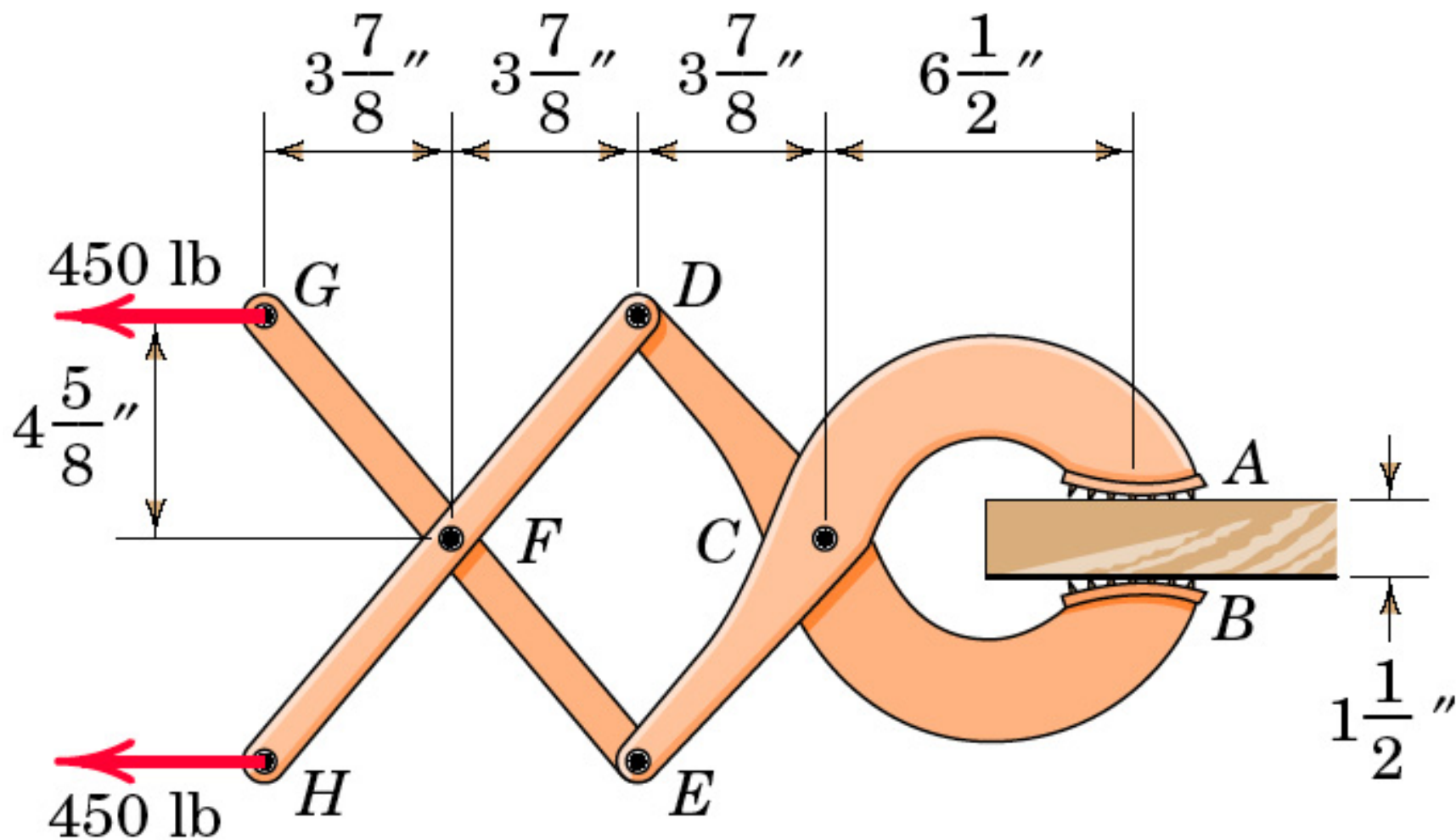


Problem 4/100

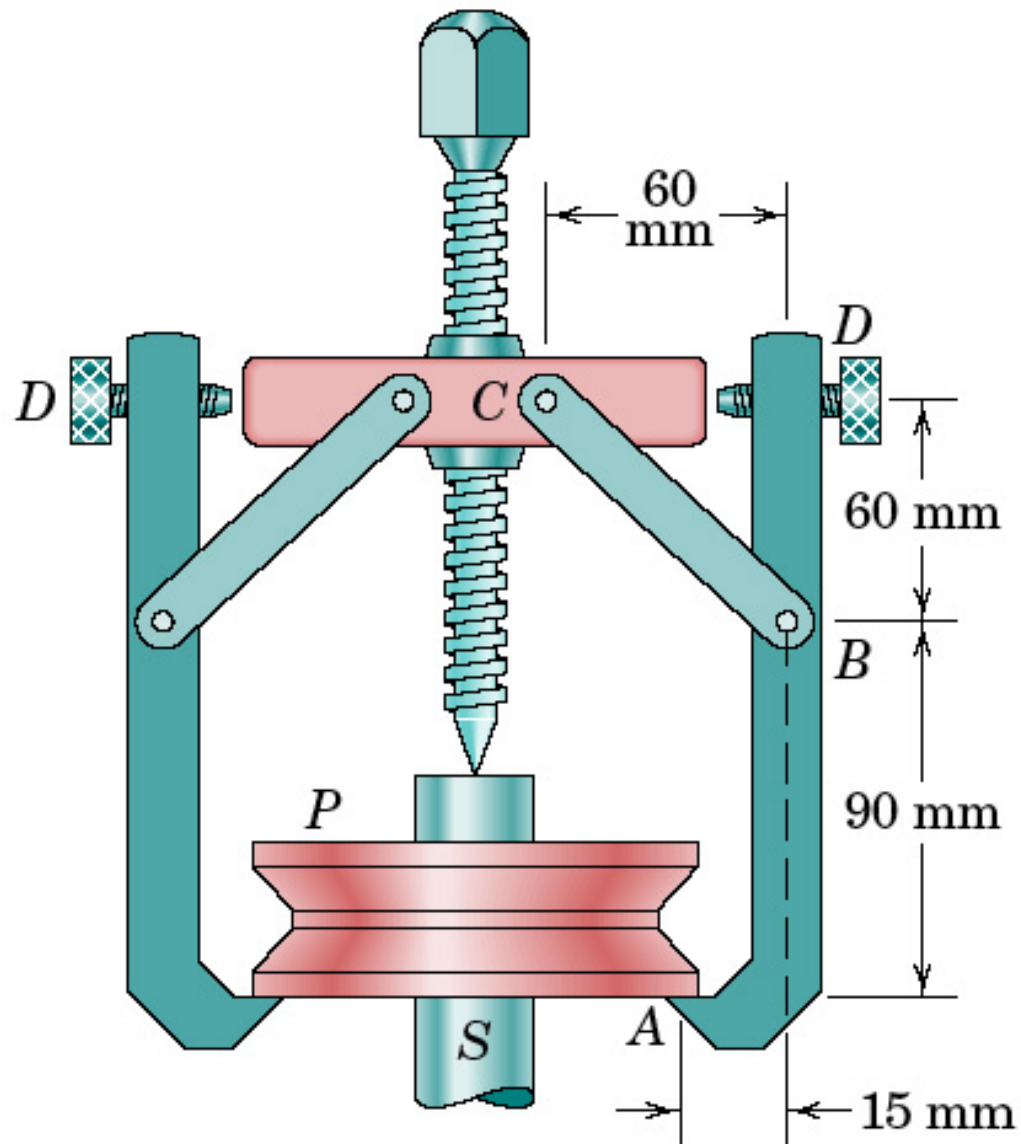
P_04_100



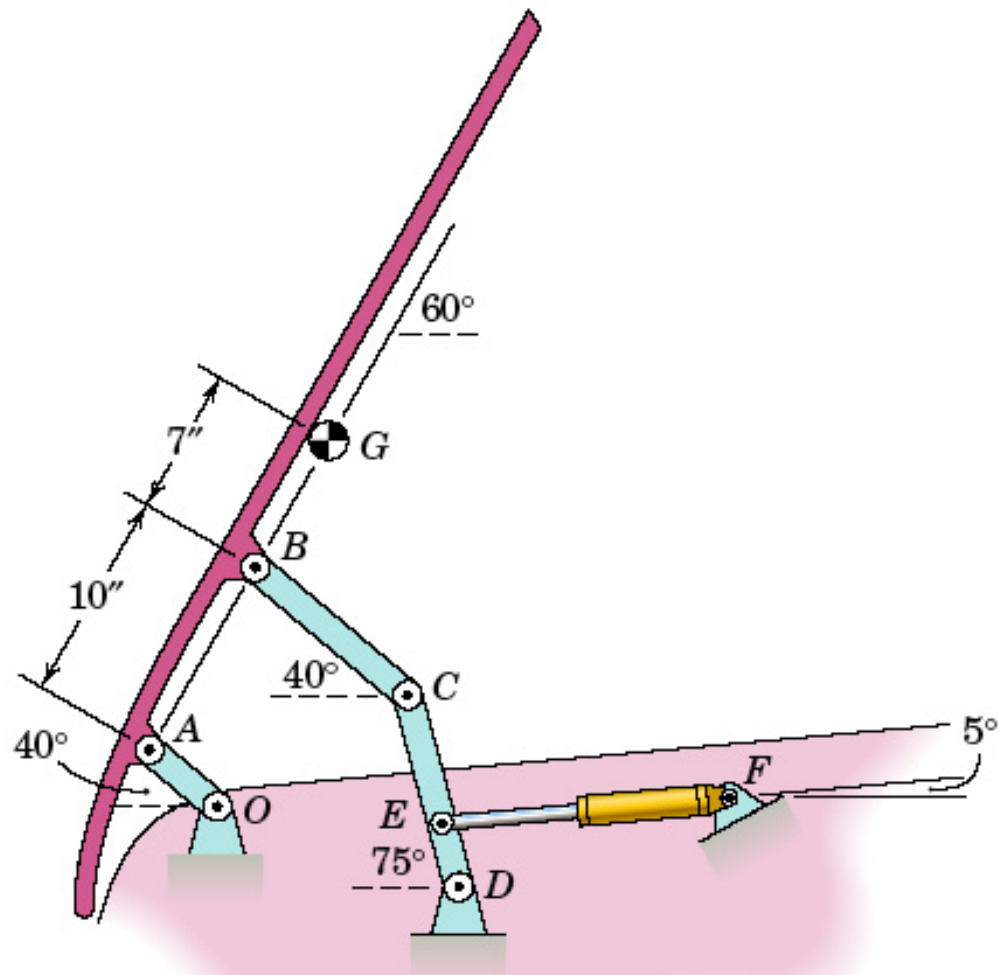
Problem 4/101



Problem 4/102



Problem 4/103

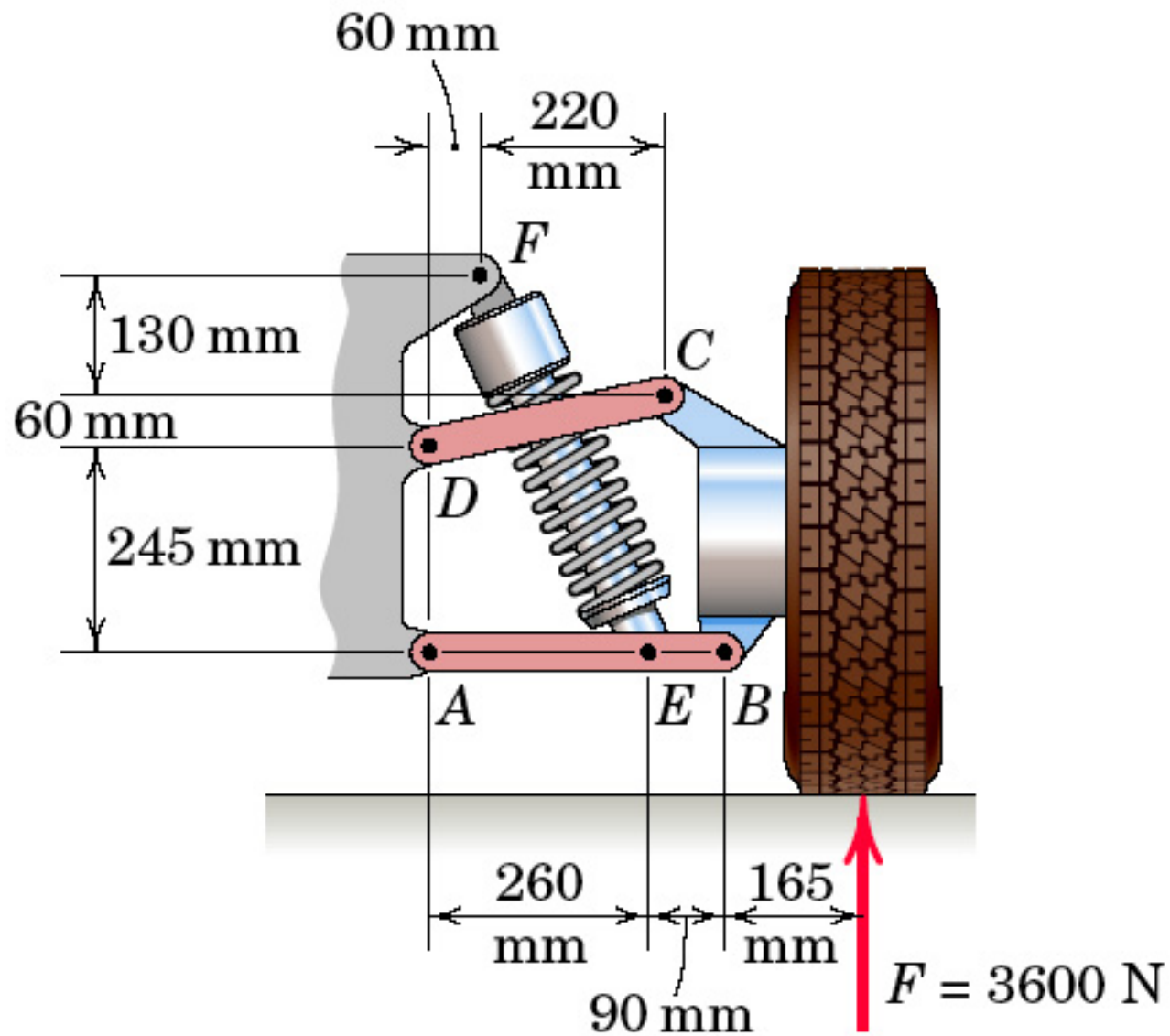


$$\overline{AO} = 4''$$

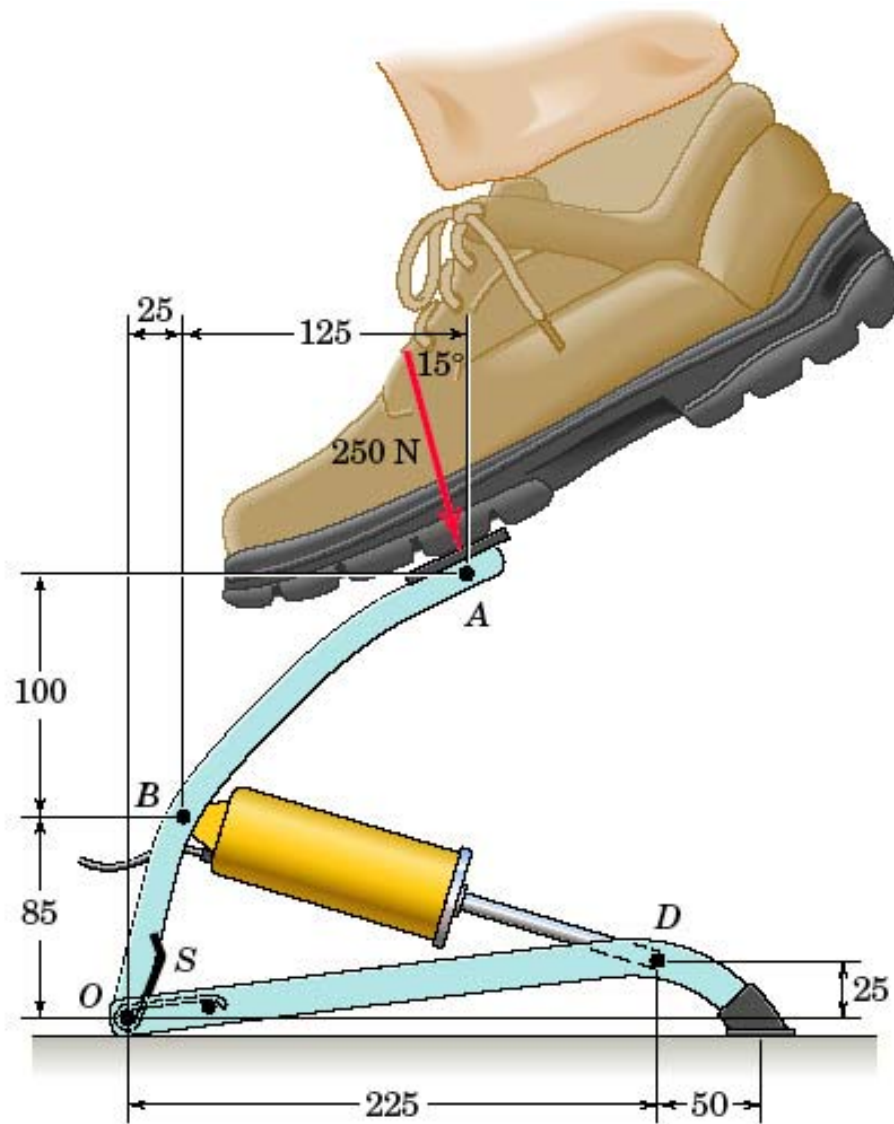
$$\overline{DE} = 3''$$

$$\overline{BC} = \overline{CD} = 9''$$

Problem 4/104

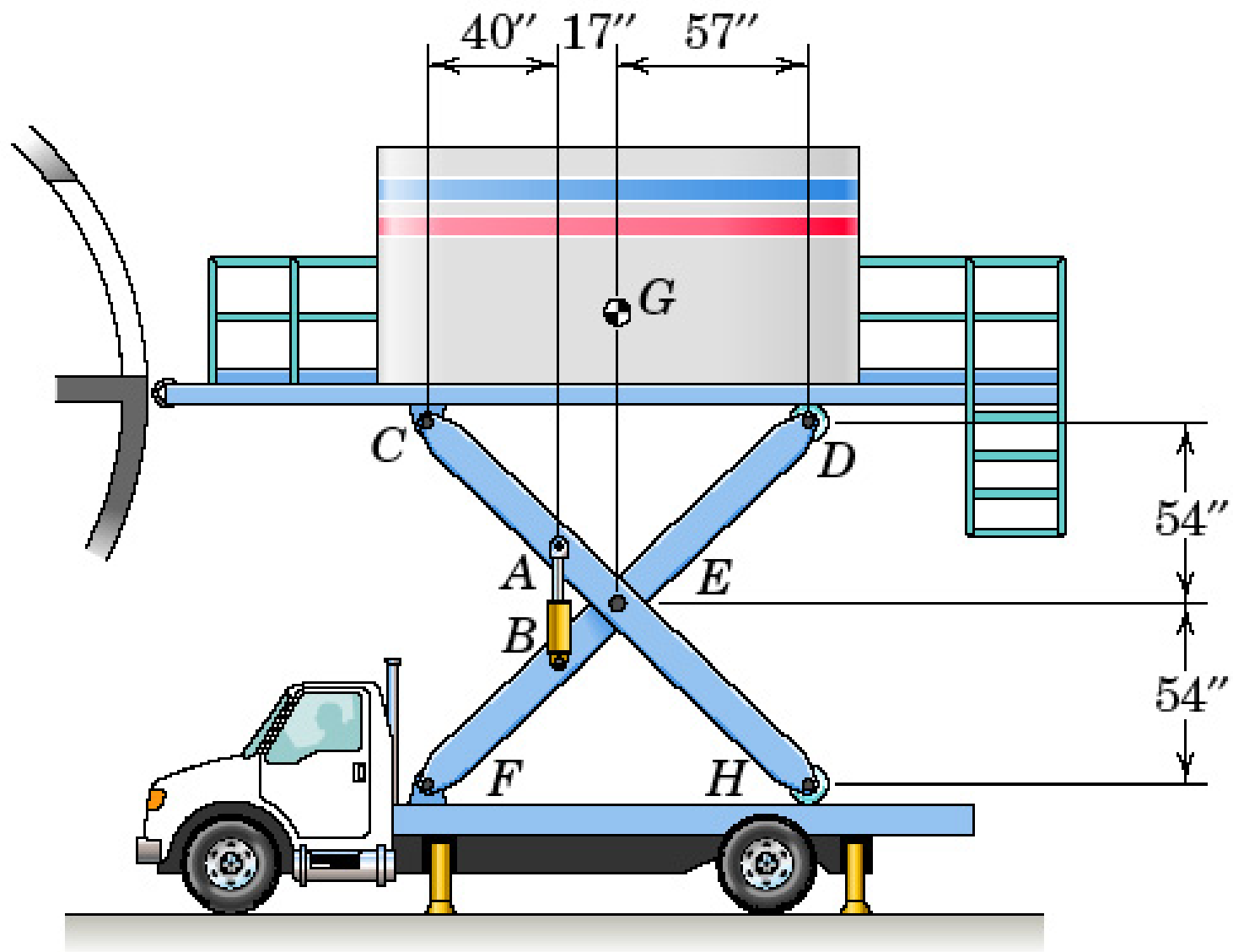


Problem 4/105

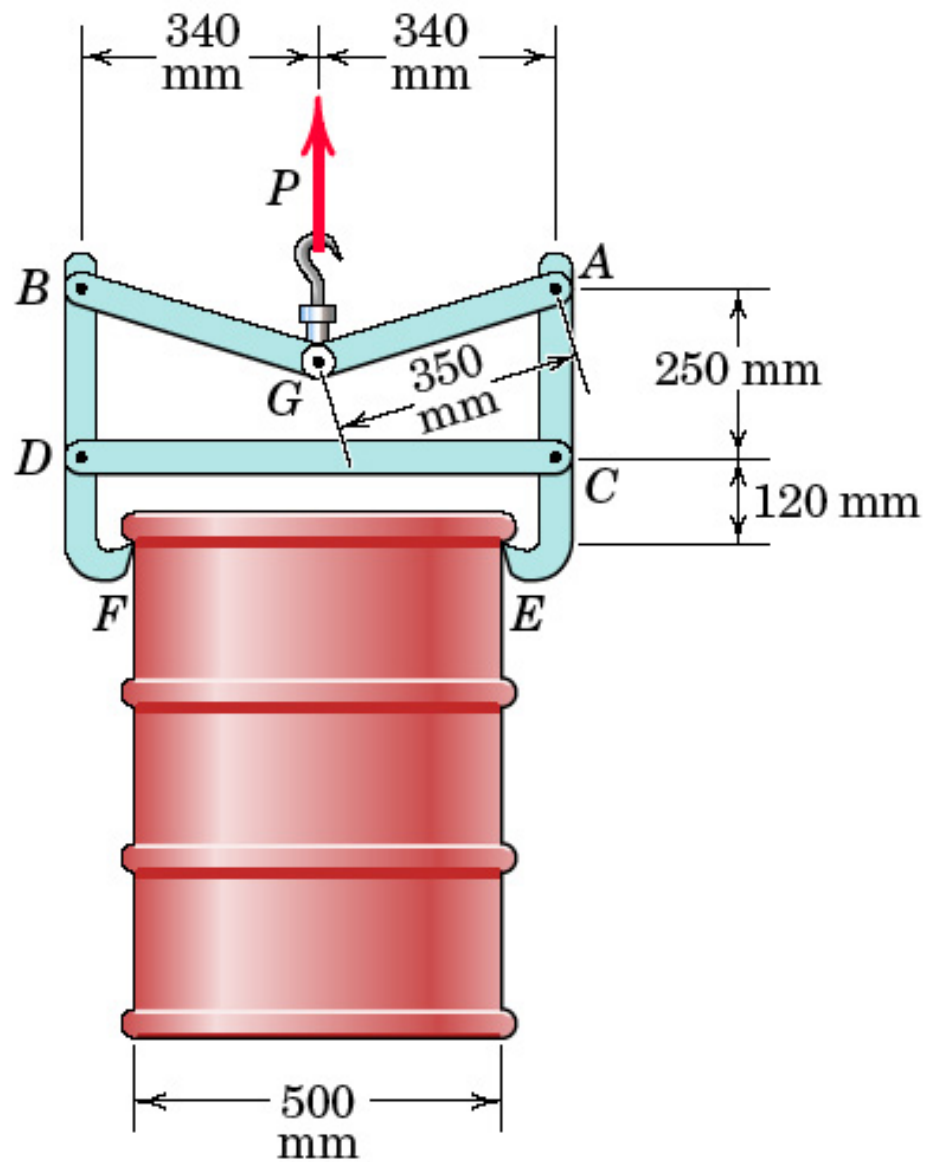


Dimensions in millimeters

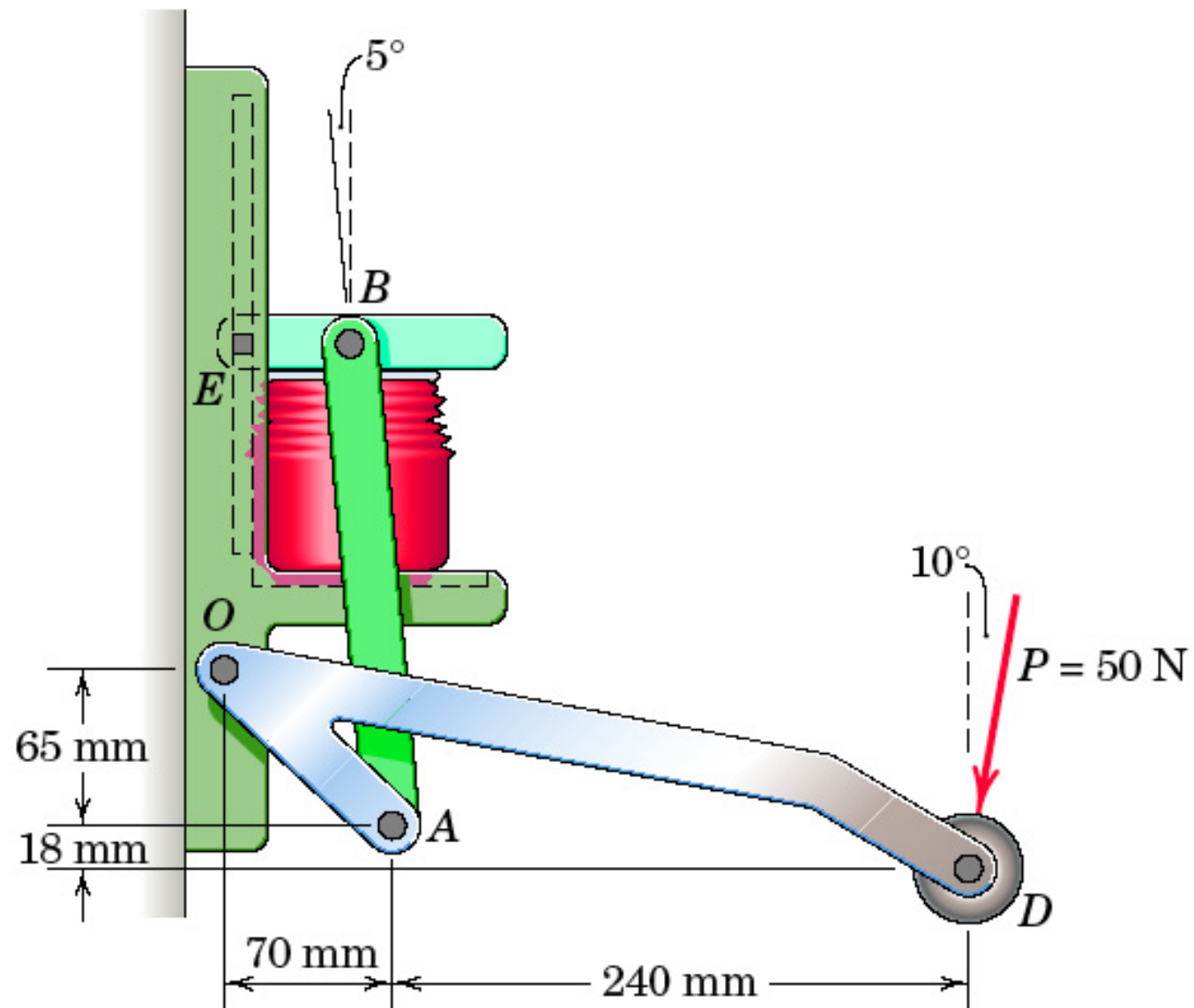
Problem 4/106



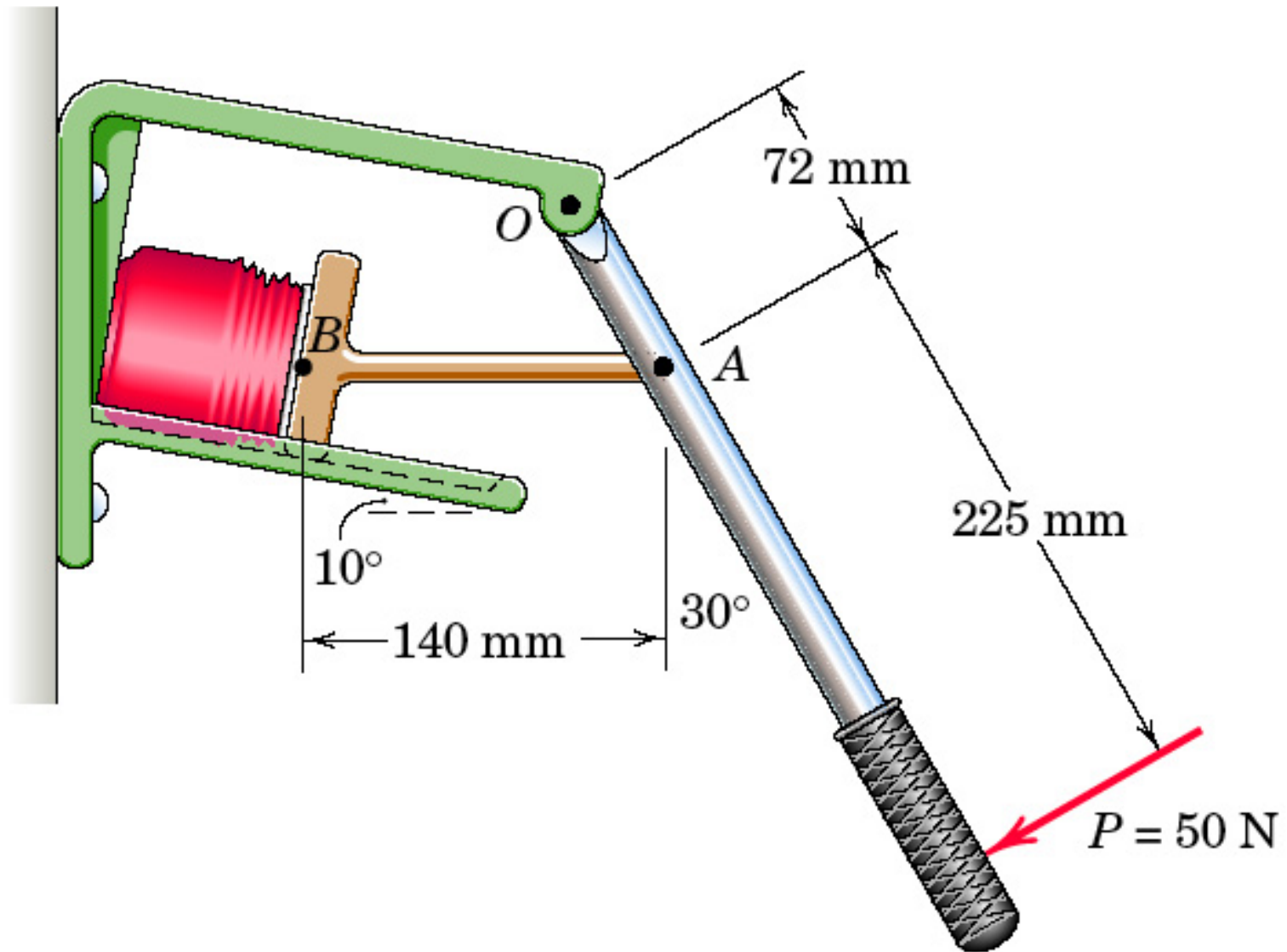
Problem 4/107



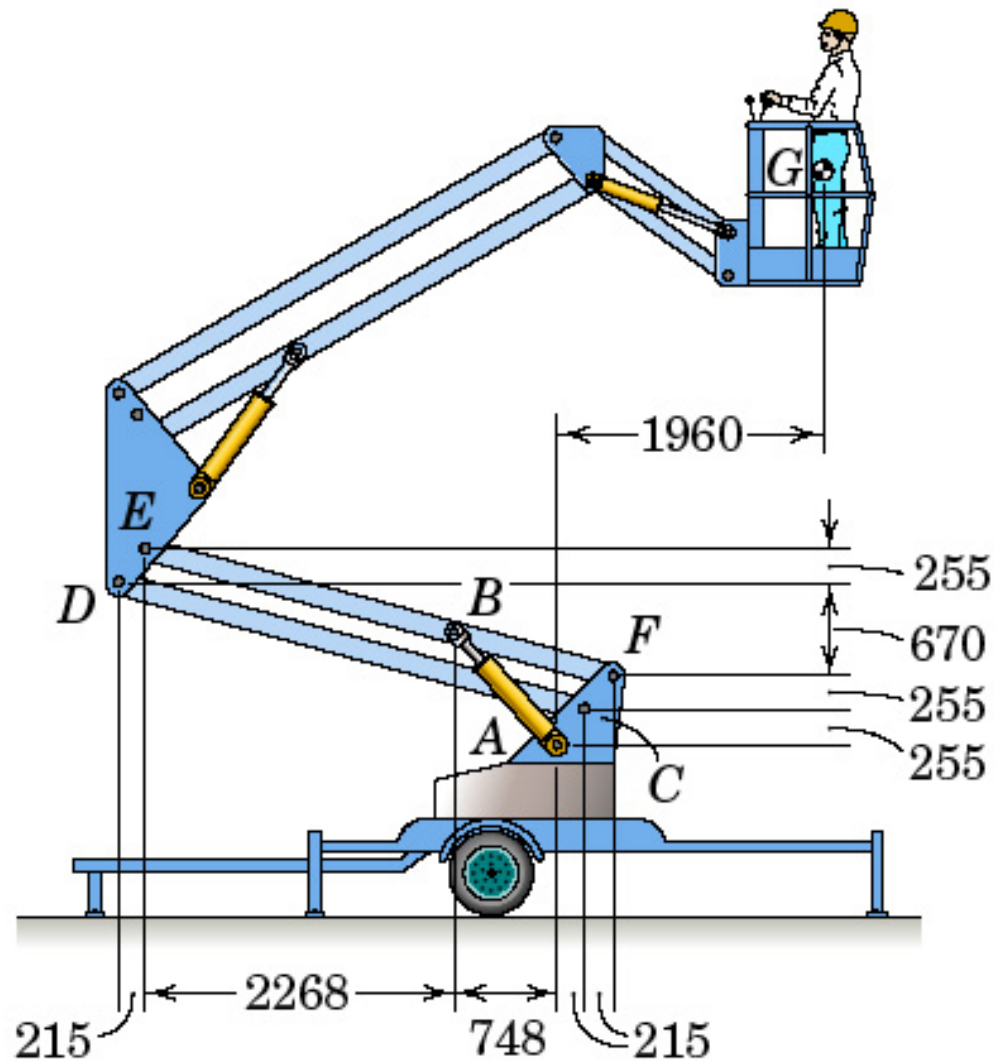
Problem 4/108



Problem 4/109

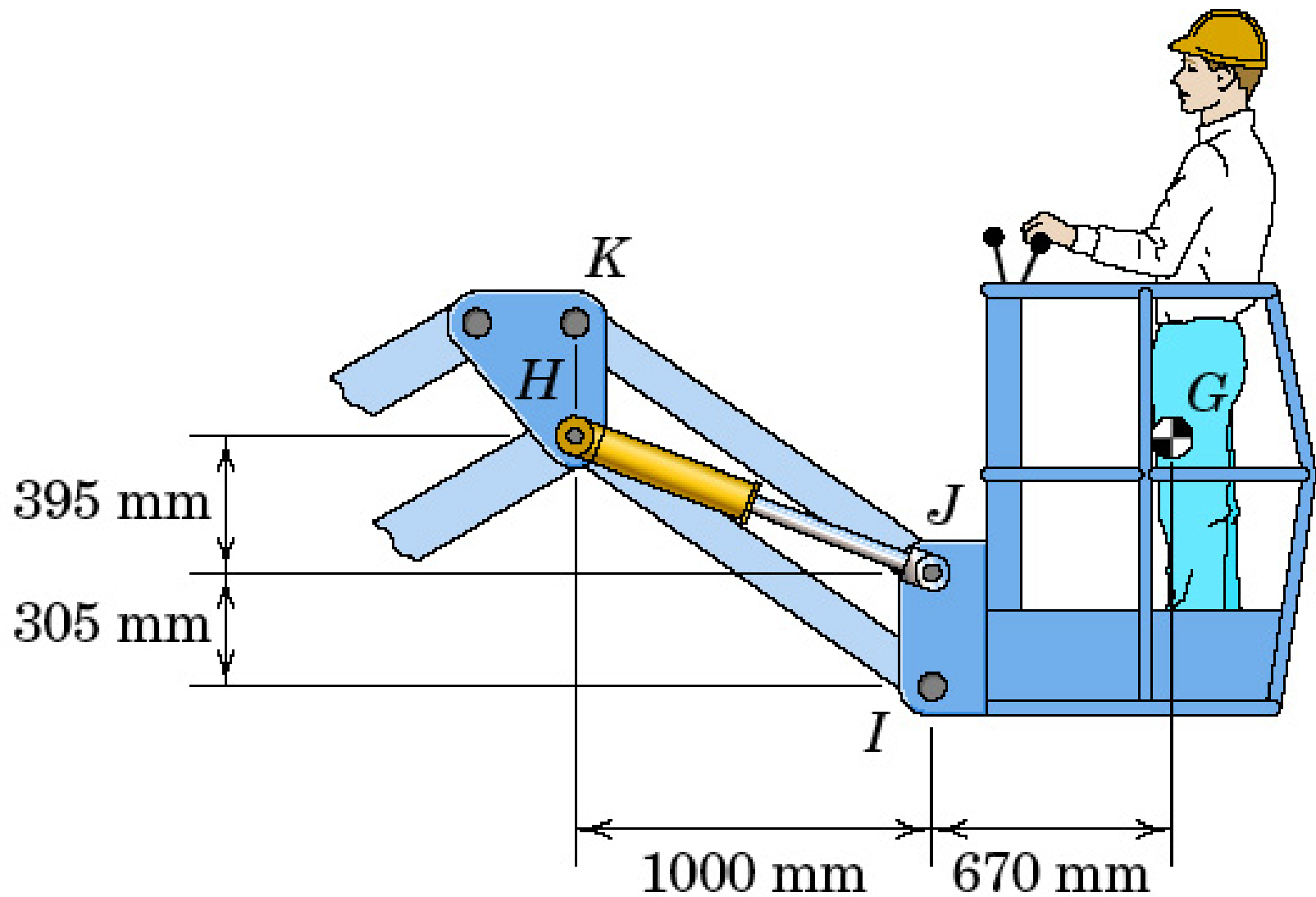


Problem 4/110

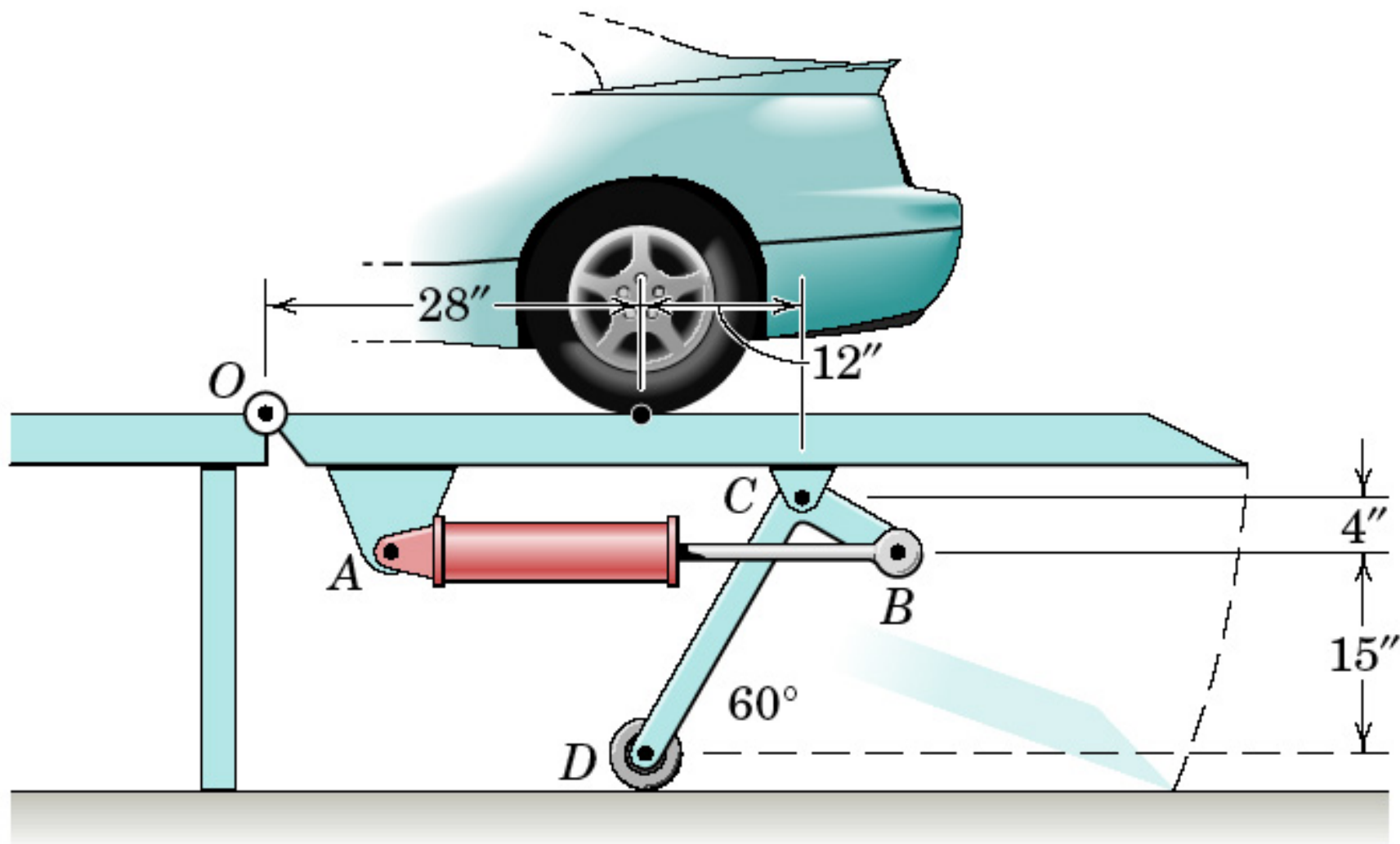


Dimensions in millimeters

Problem 4/111

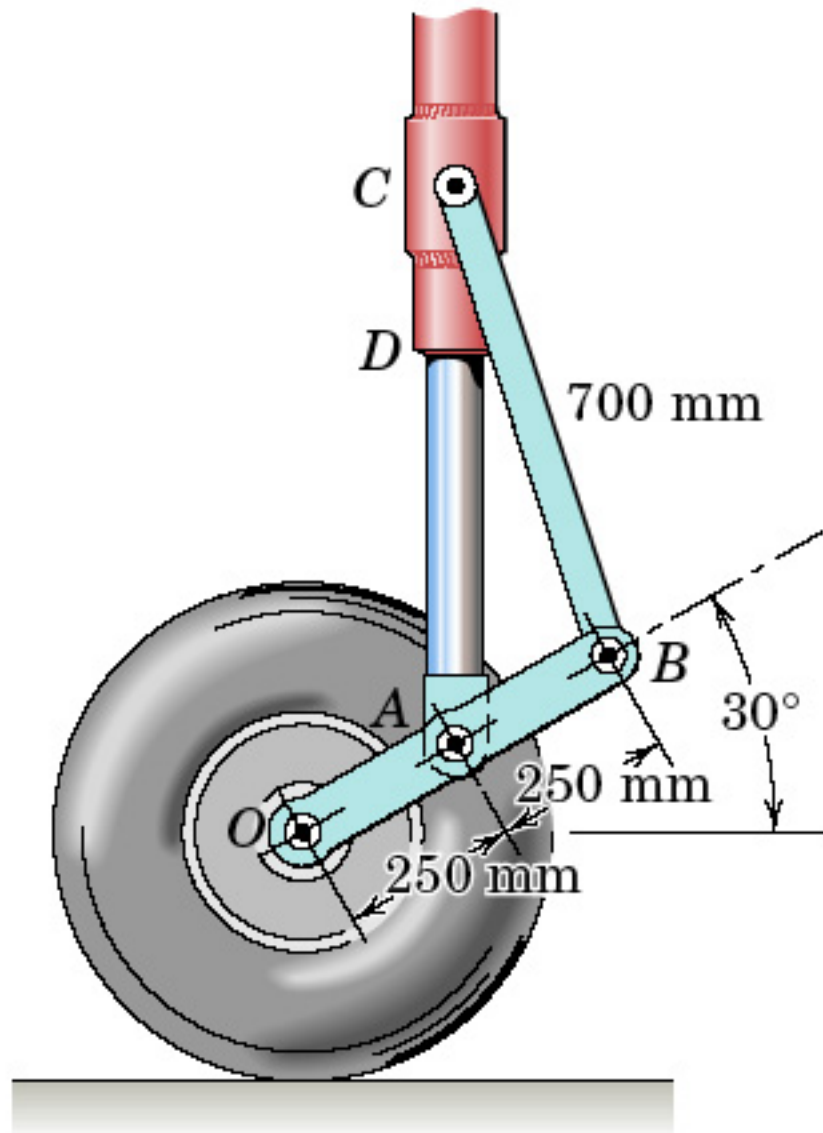


Problem 4/112

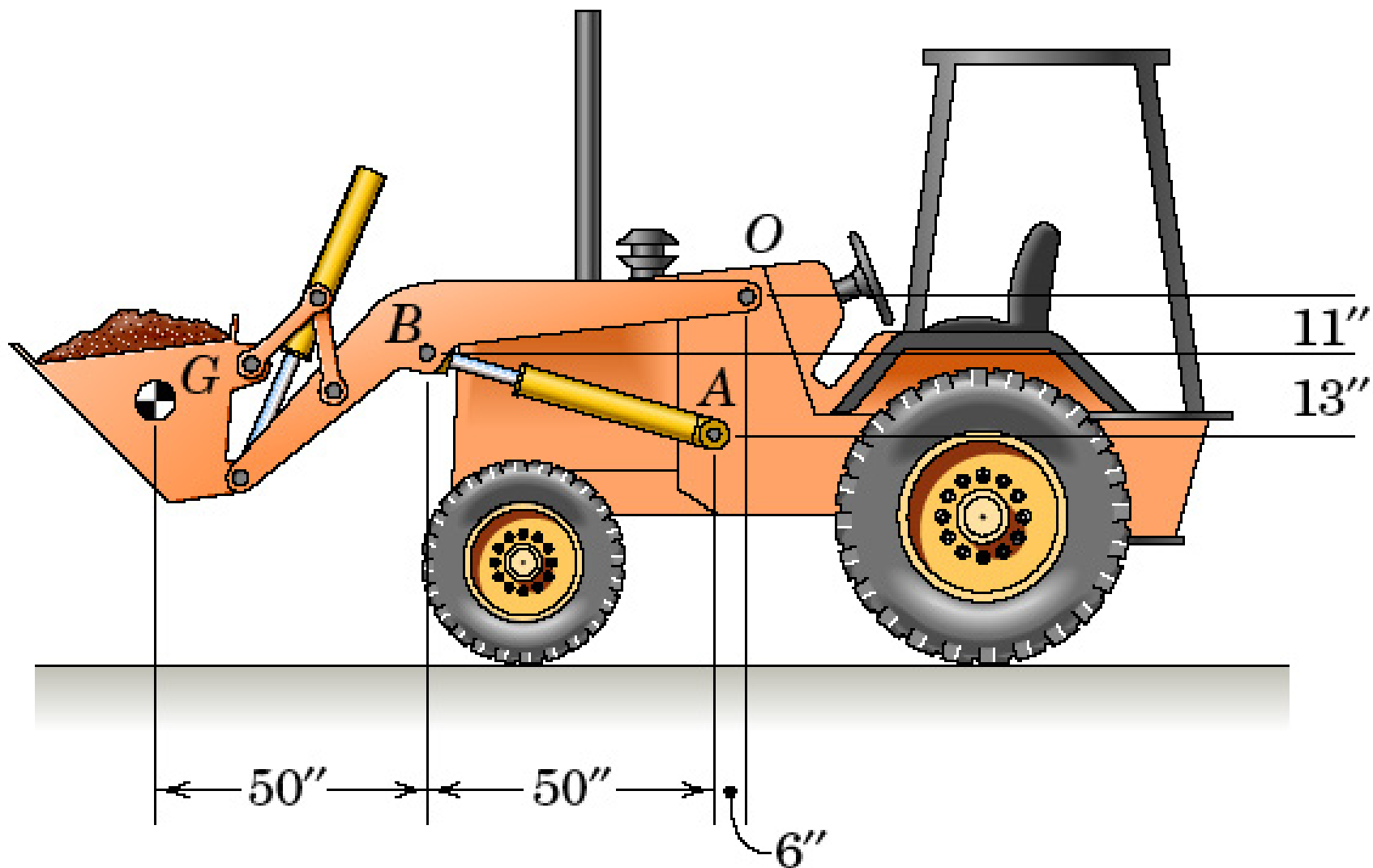


Problem 4/113

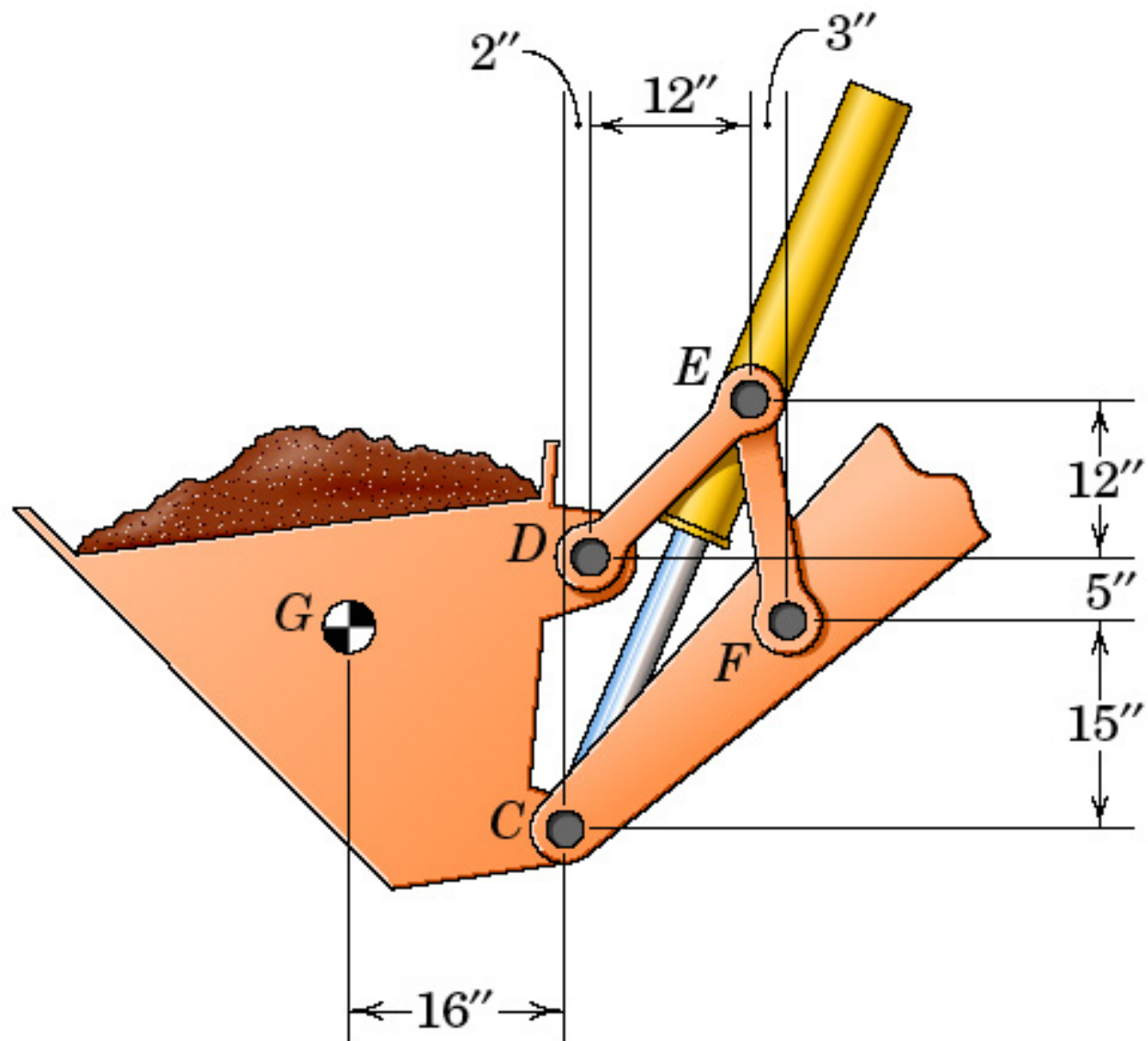
P_04_113



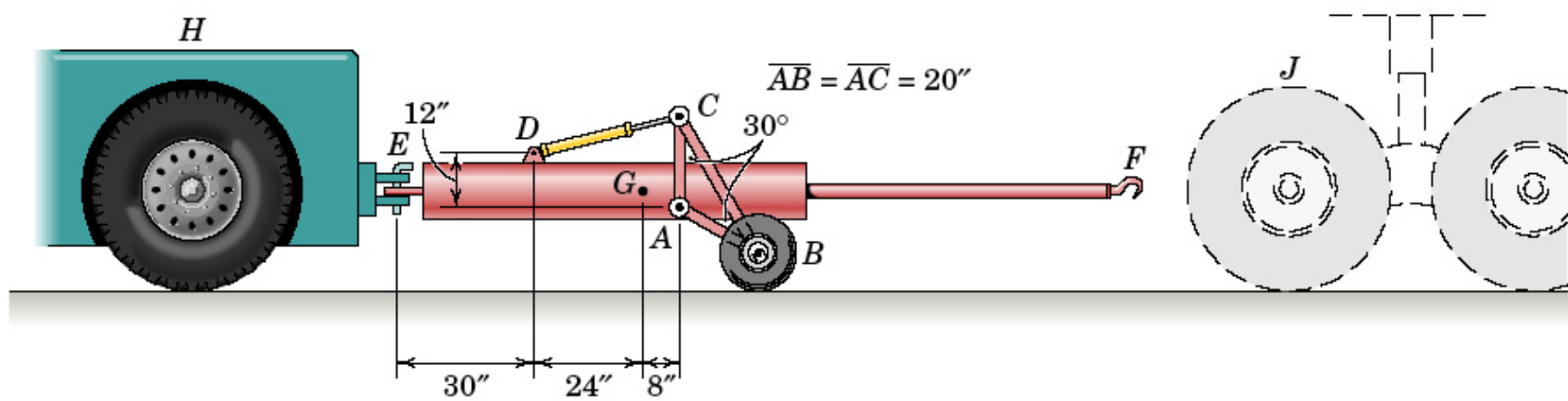
Problem 4/114



Problem 4/115

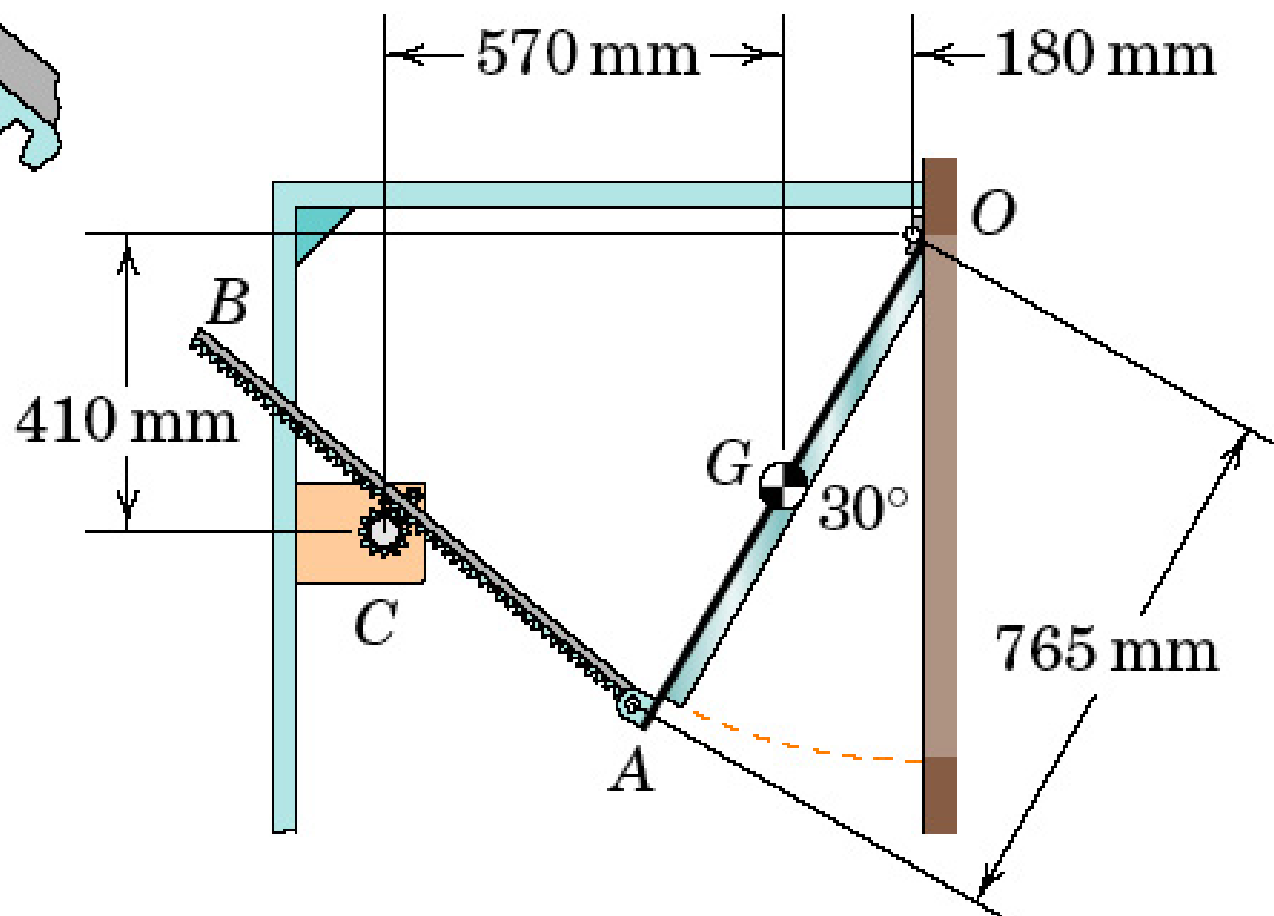
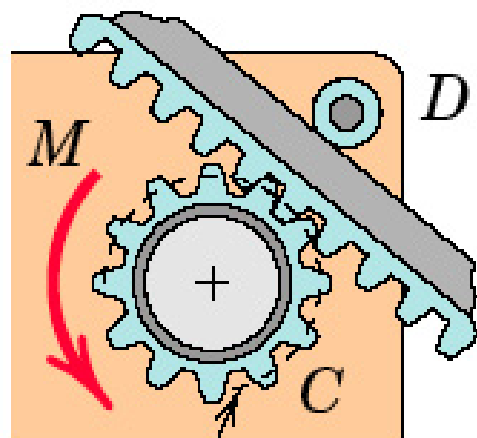


Problem 4/116

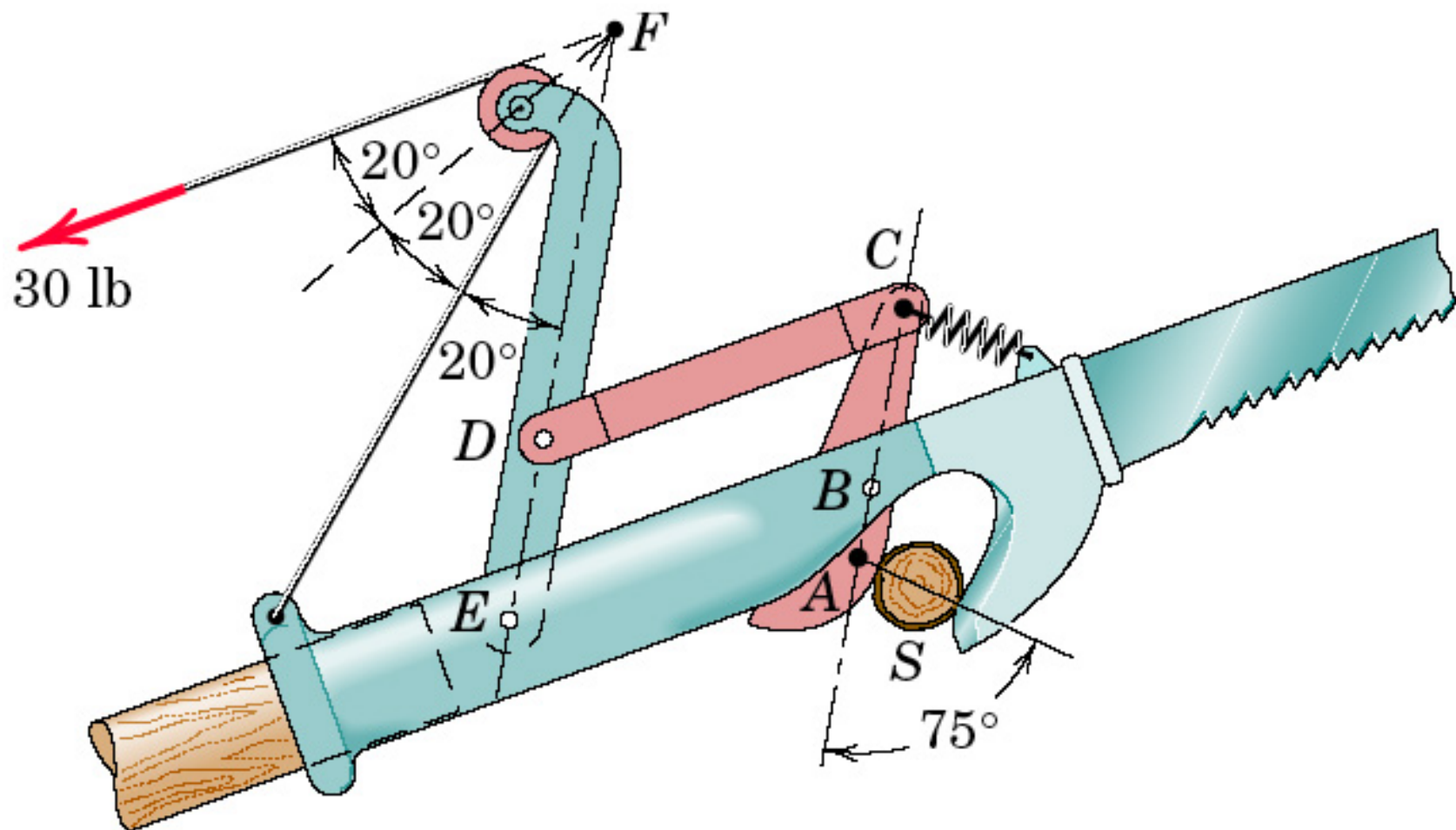


Problem 4/117

P_04_117

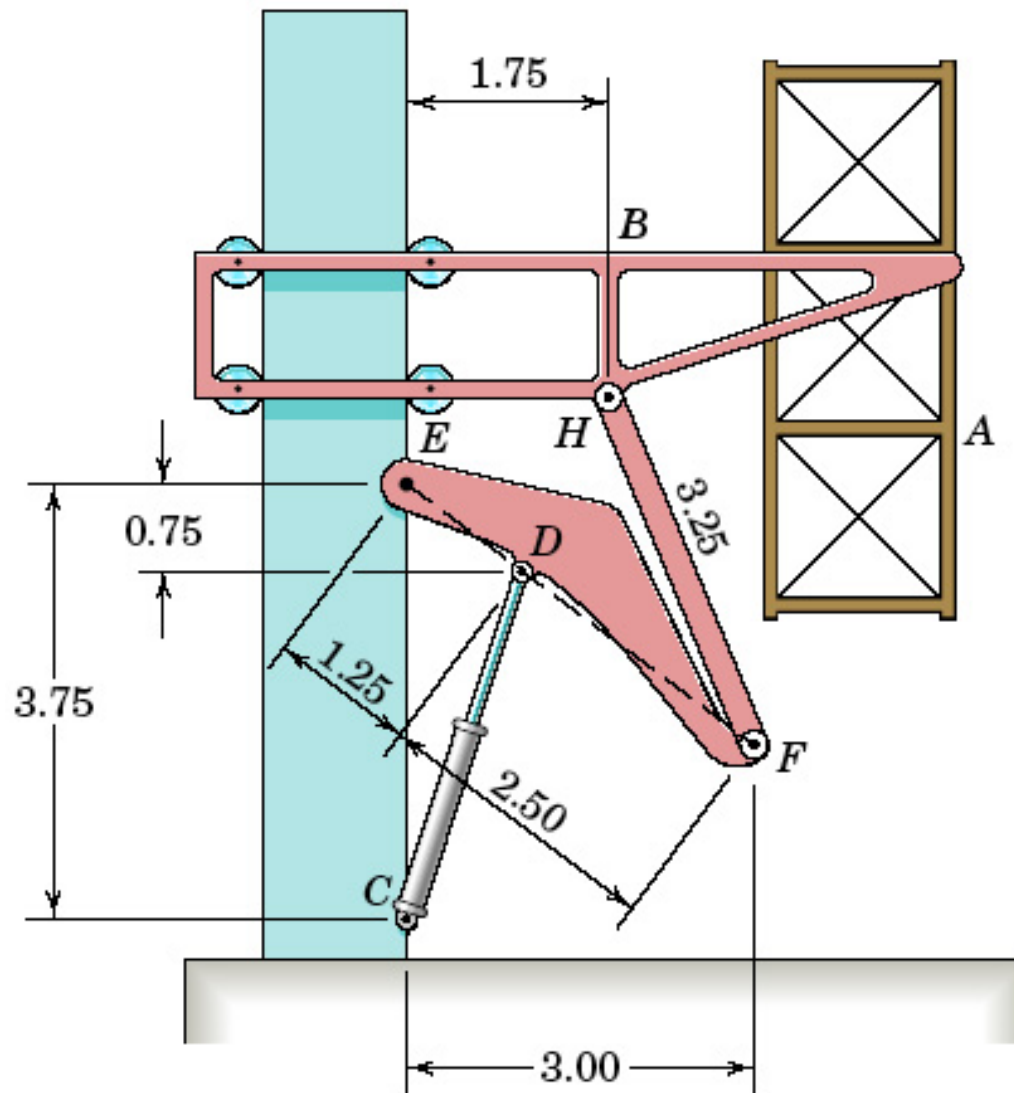


Problem 4/118



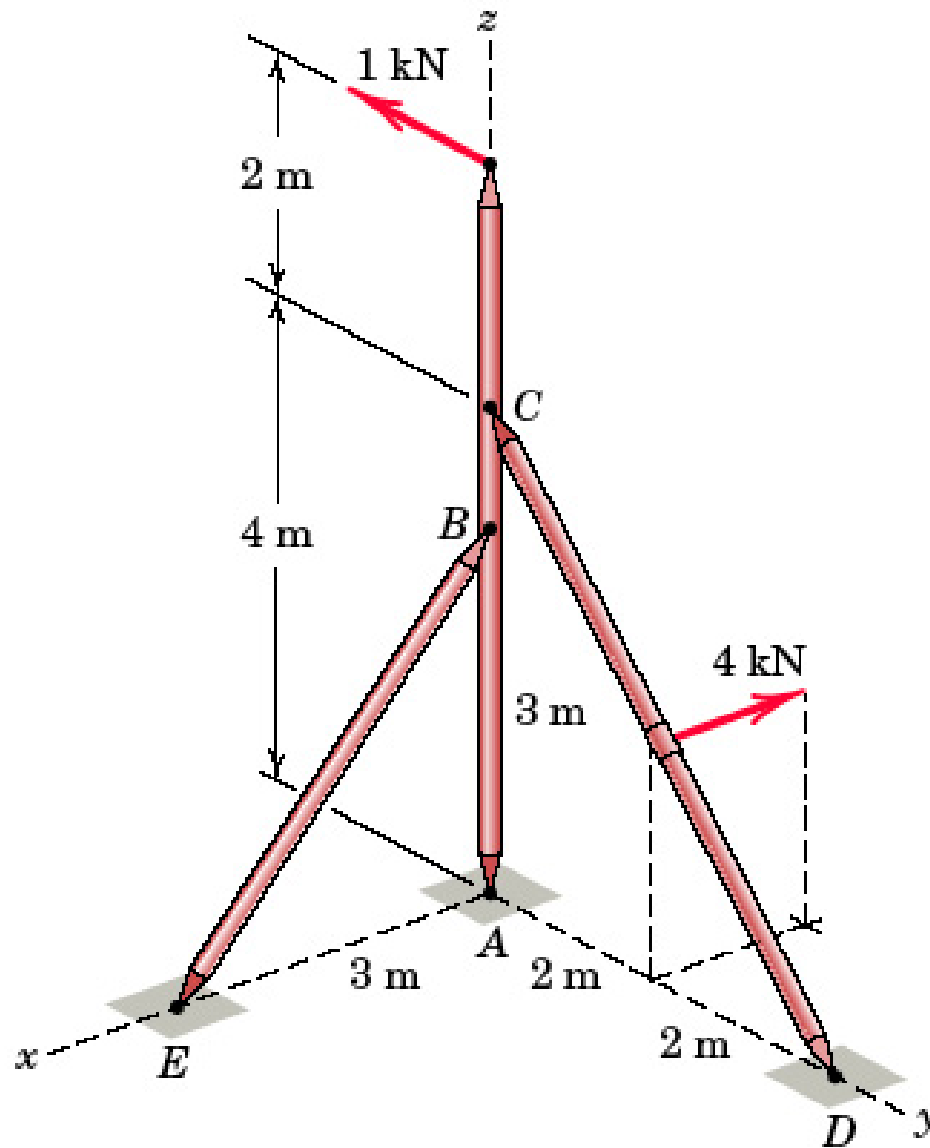
$$\overline{AB} = 1'', \overline{BC} = \overline{ED} = 3'', \overline{EB} = \overline{DC} = 4\frac{1}{2}'', \overline{DF} = 6''$$

Problem 4/119

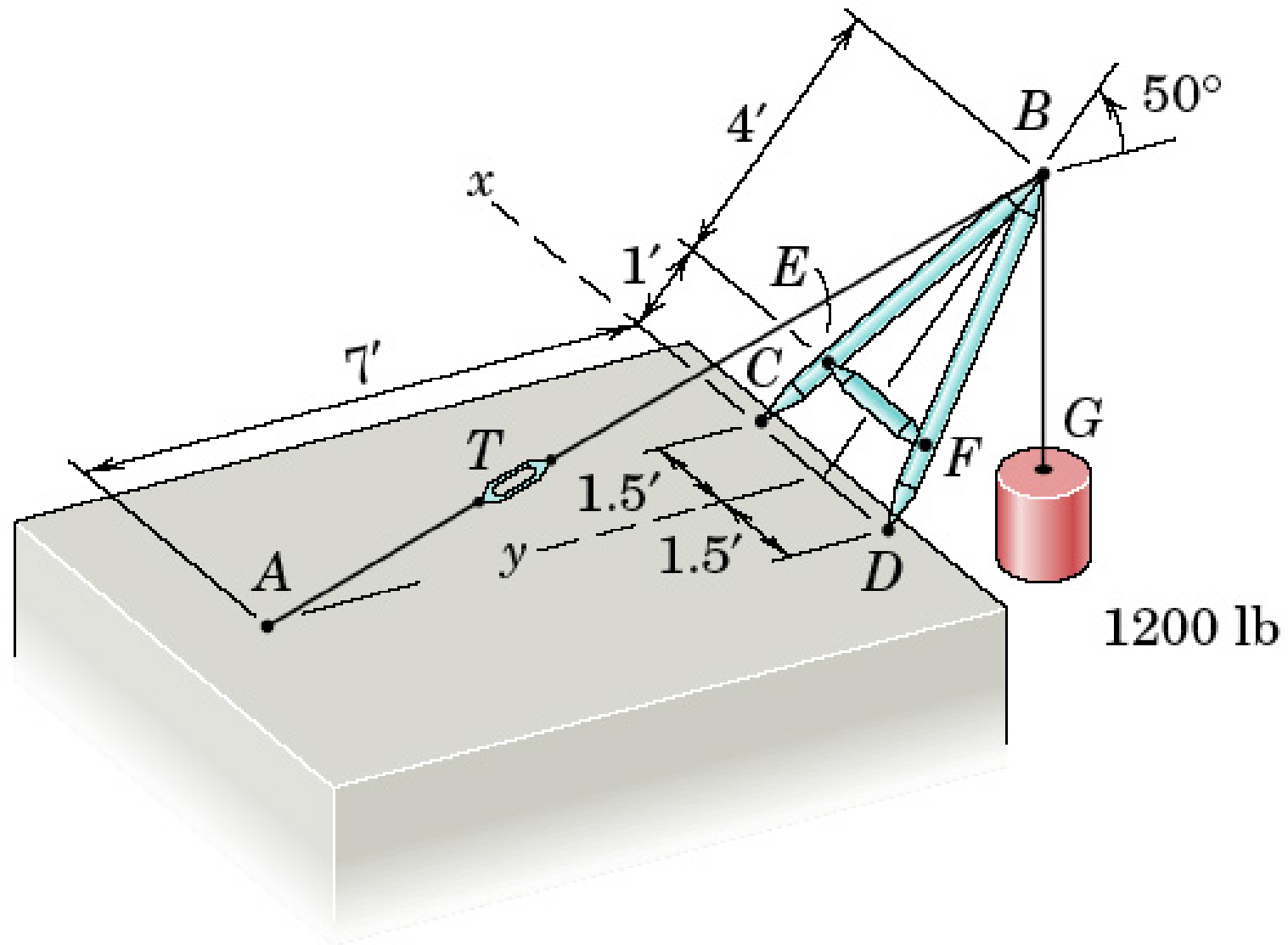


Dimensions in meters

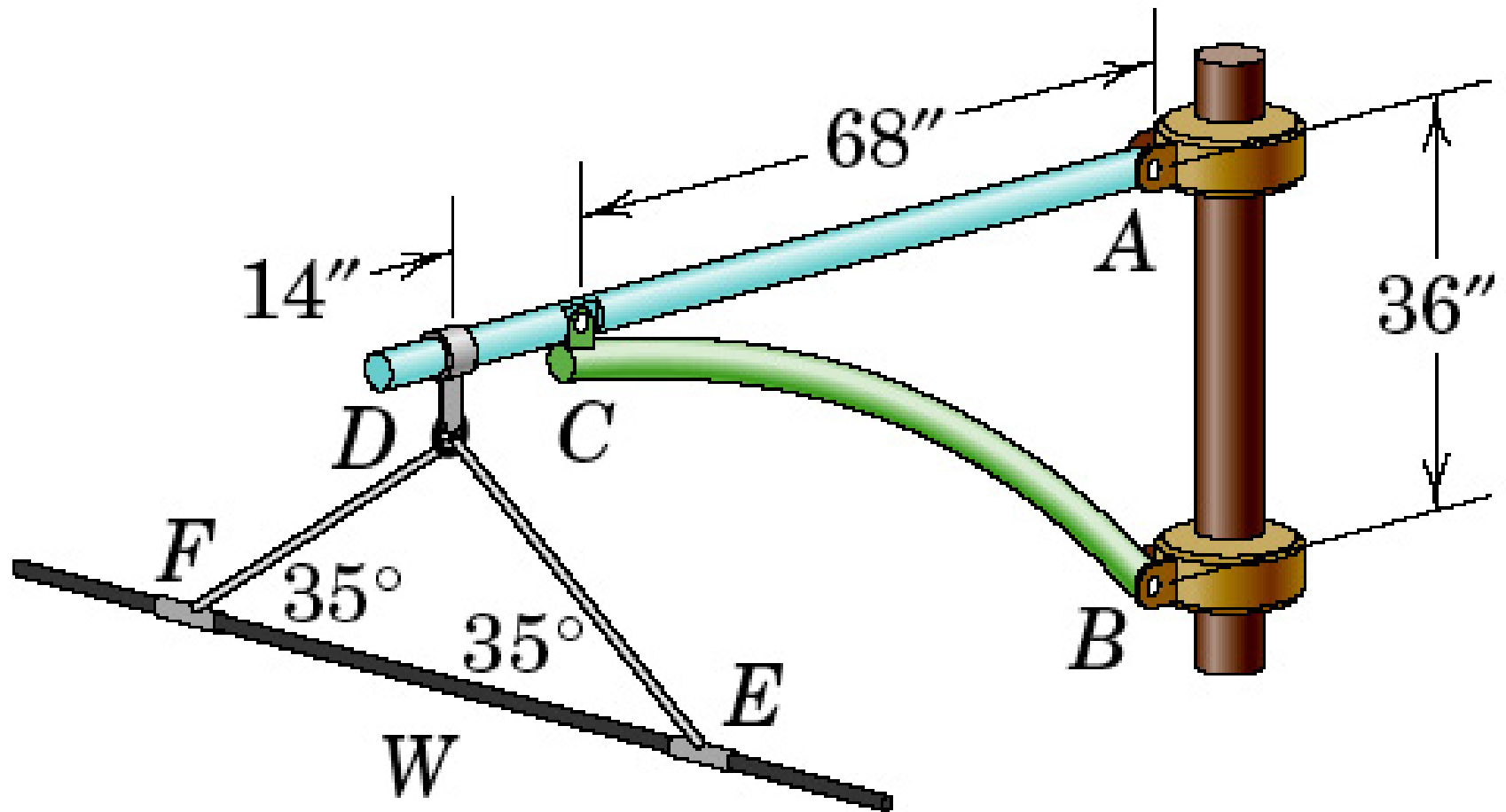
Problem 4/120



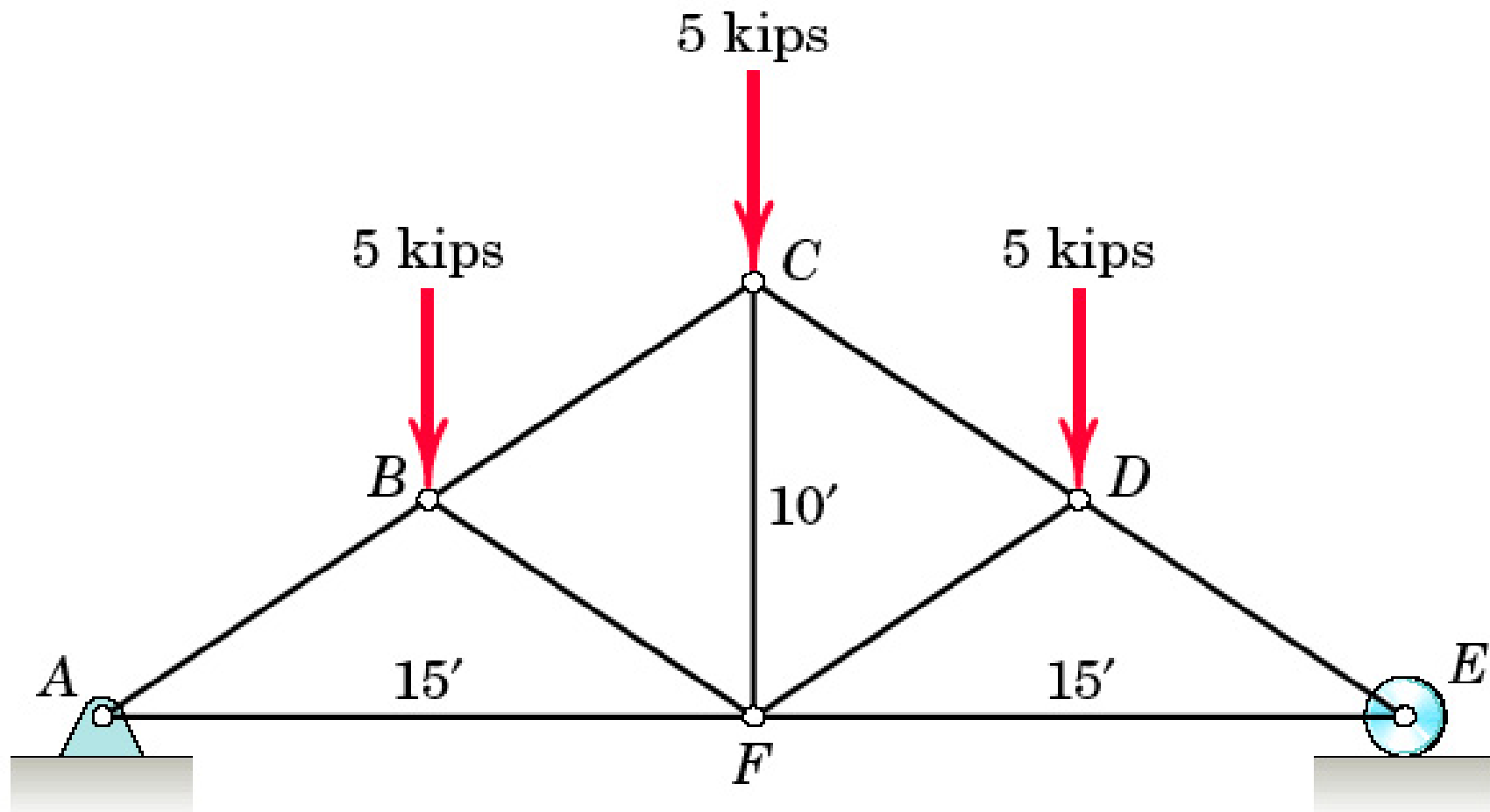
Problem 4/121



Problem 4/122

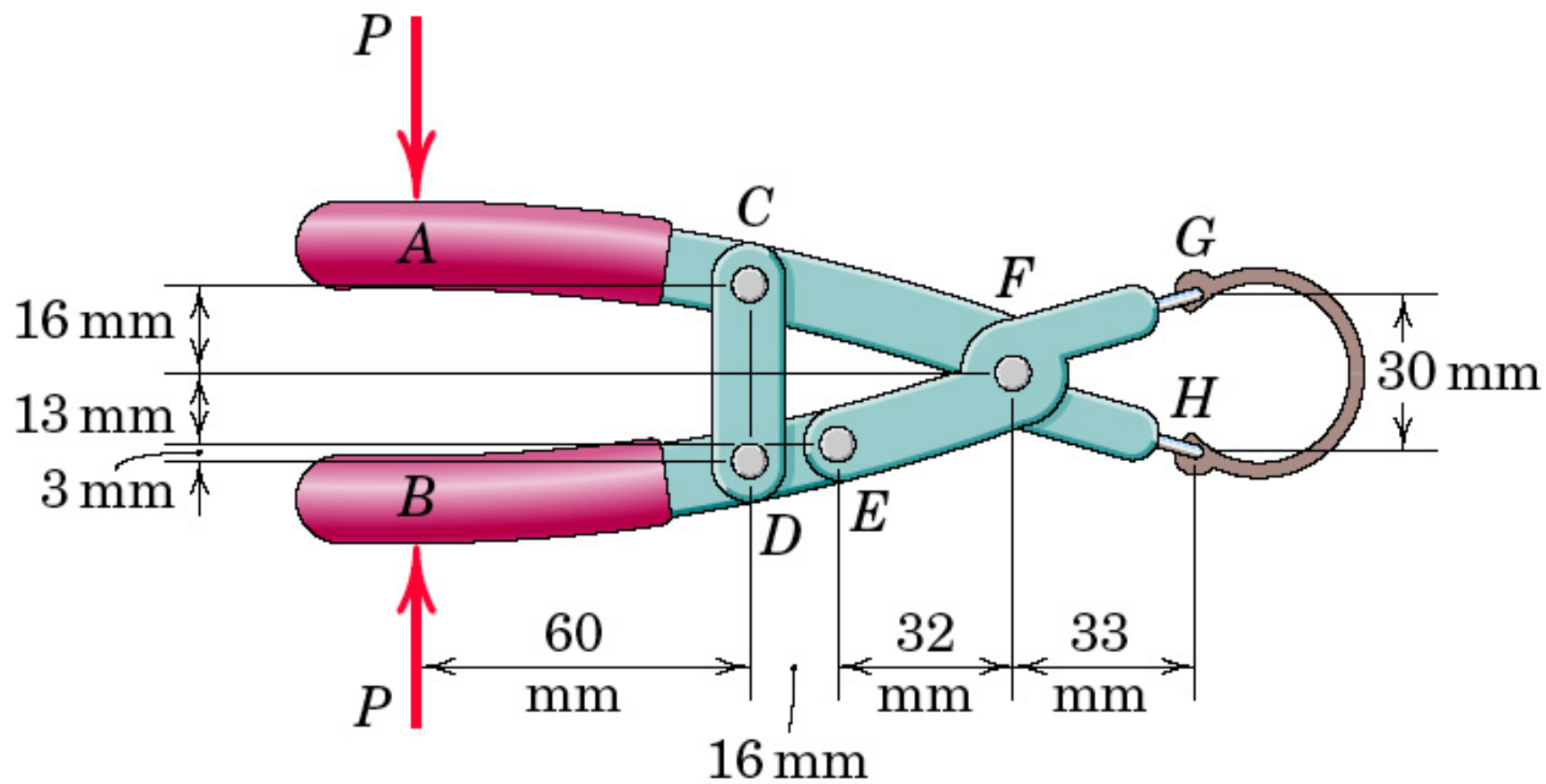


Problem 4/123



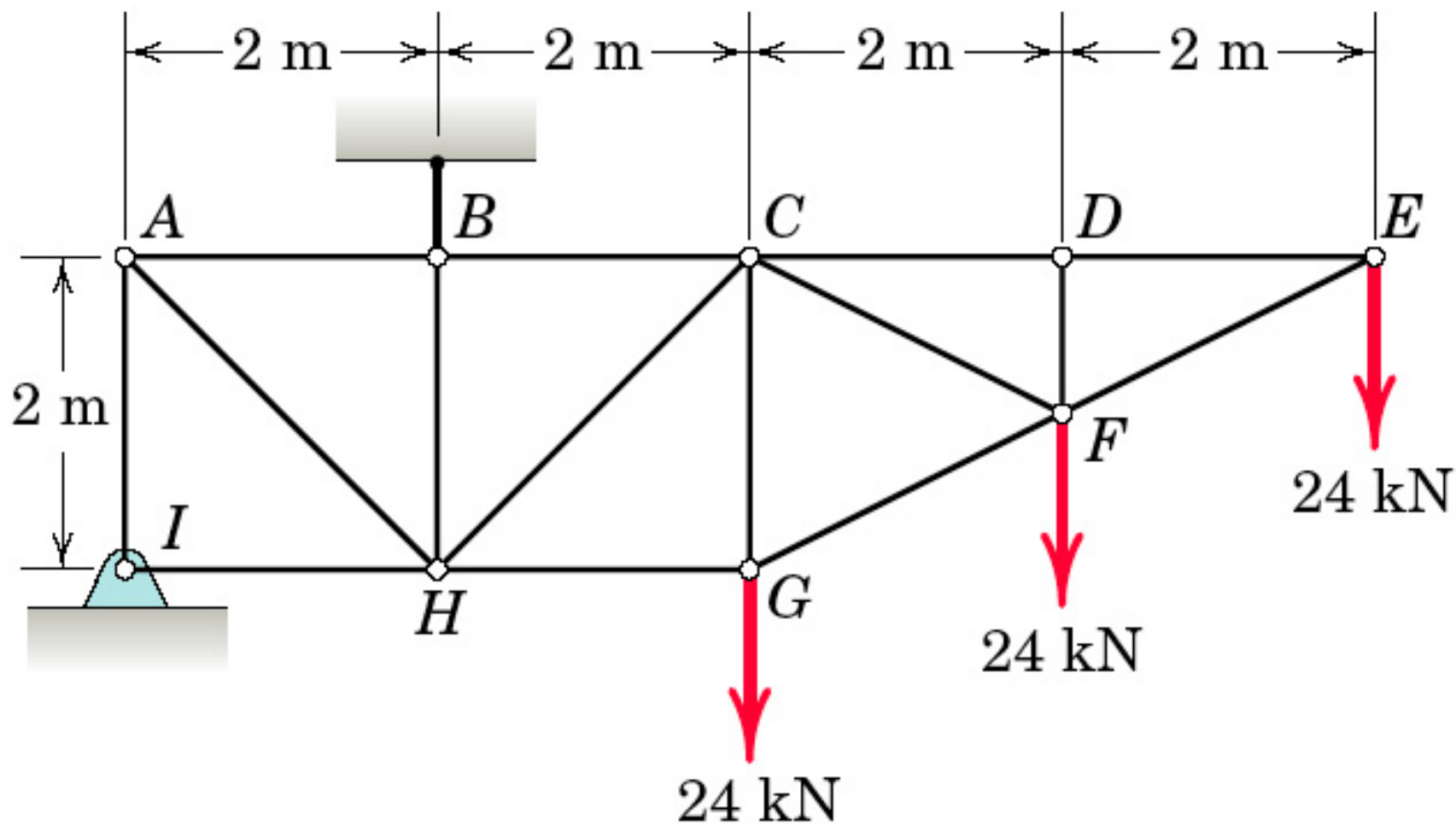
$$\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$$

Problem 4/124



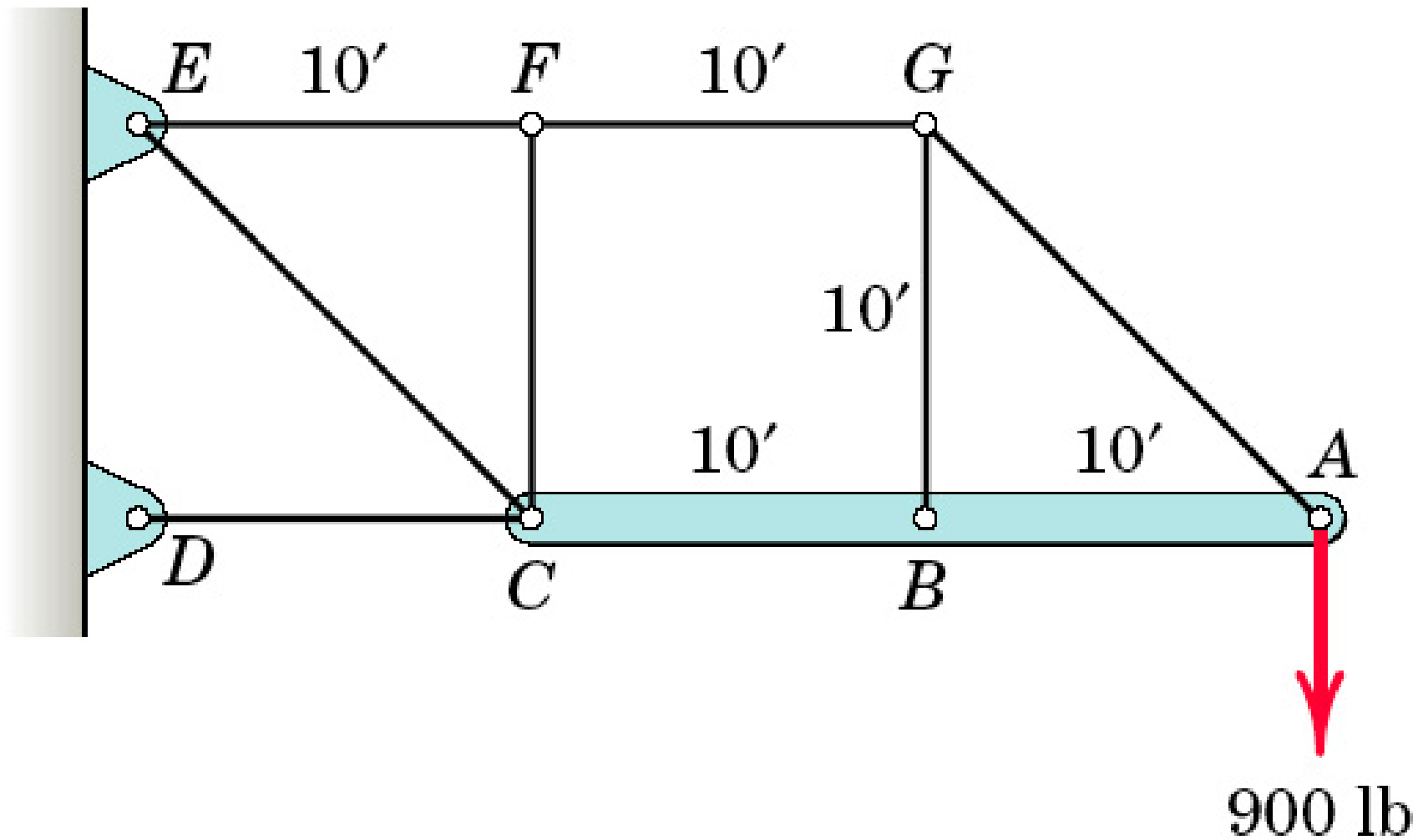
Problem 4/125

P_04_125

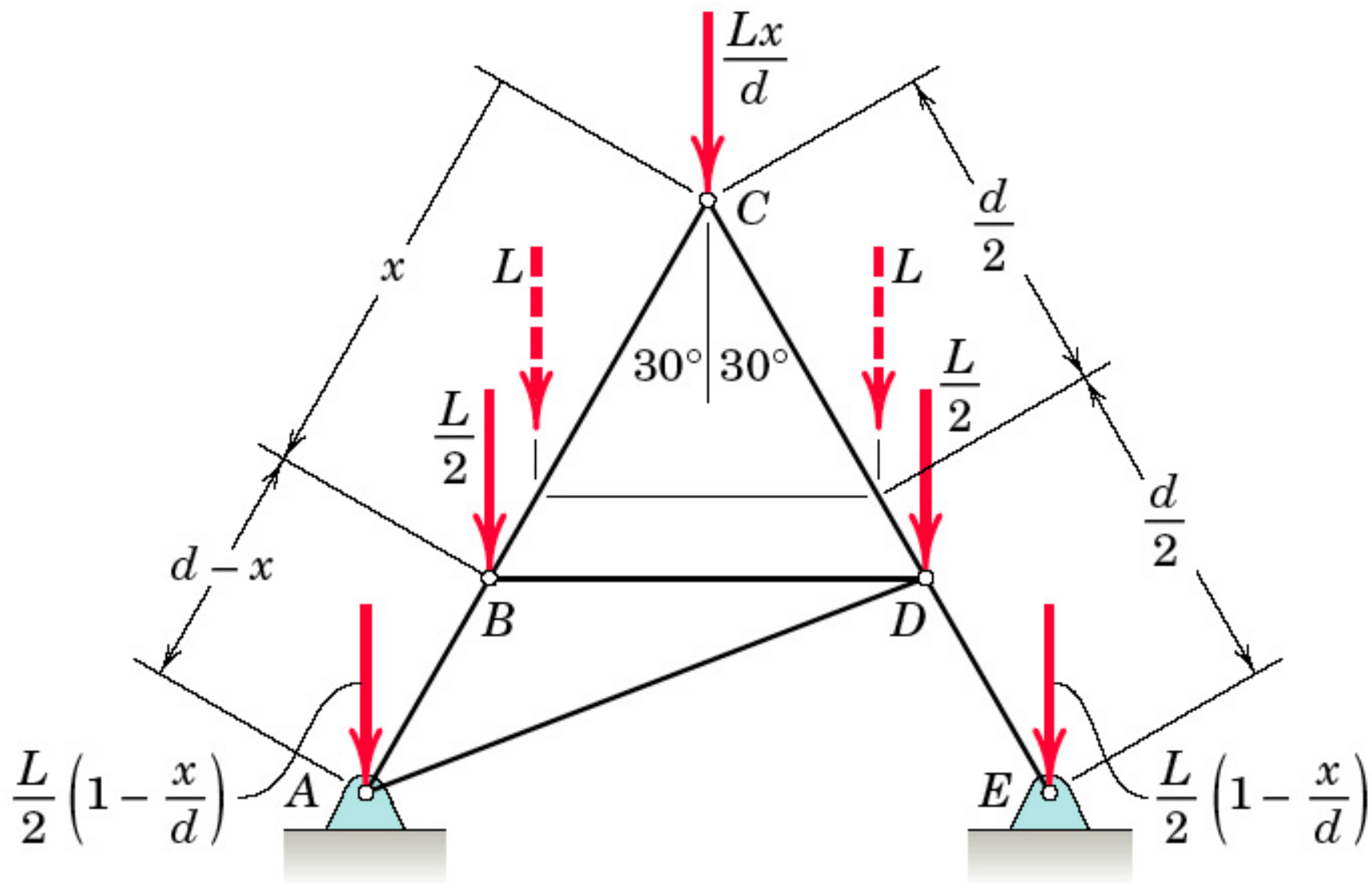


Problem 4/126

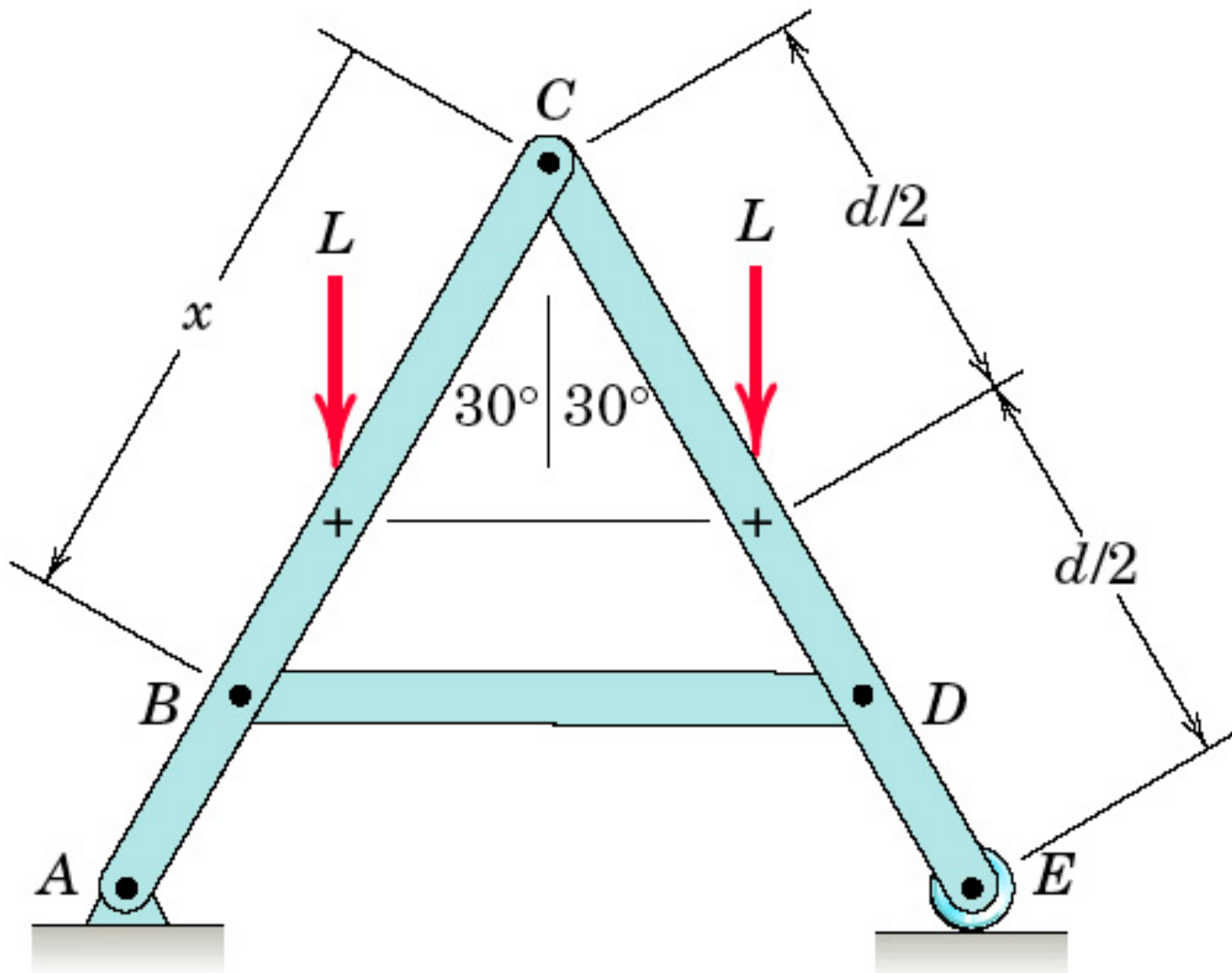
P_04_126



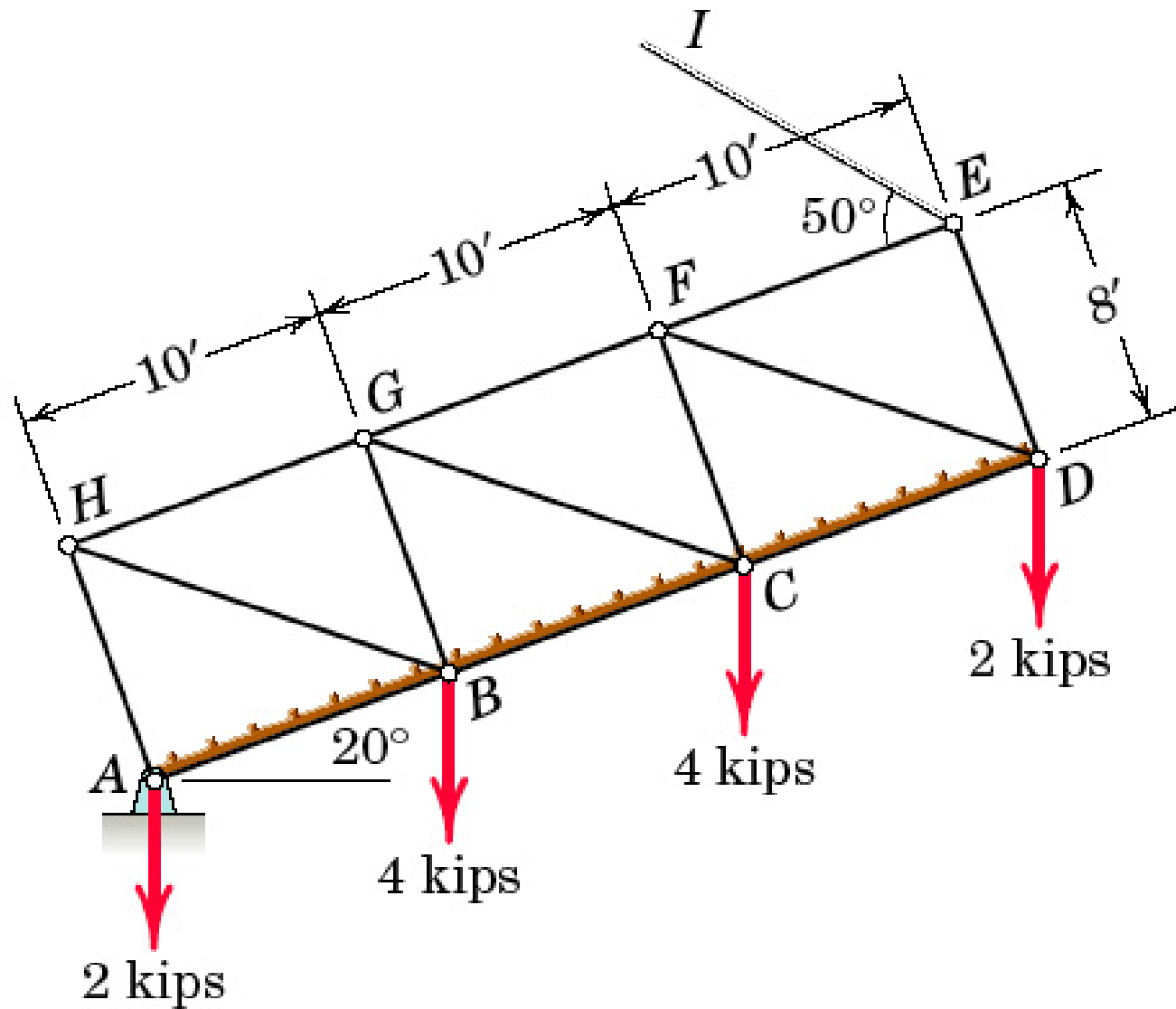
Problem 4/127



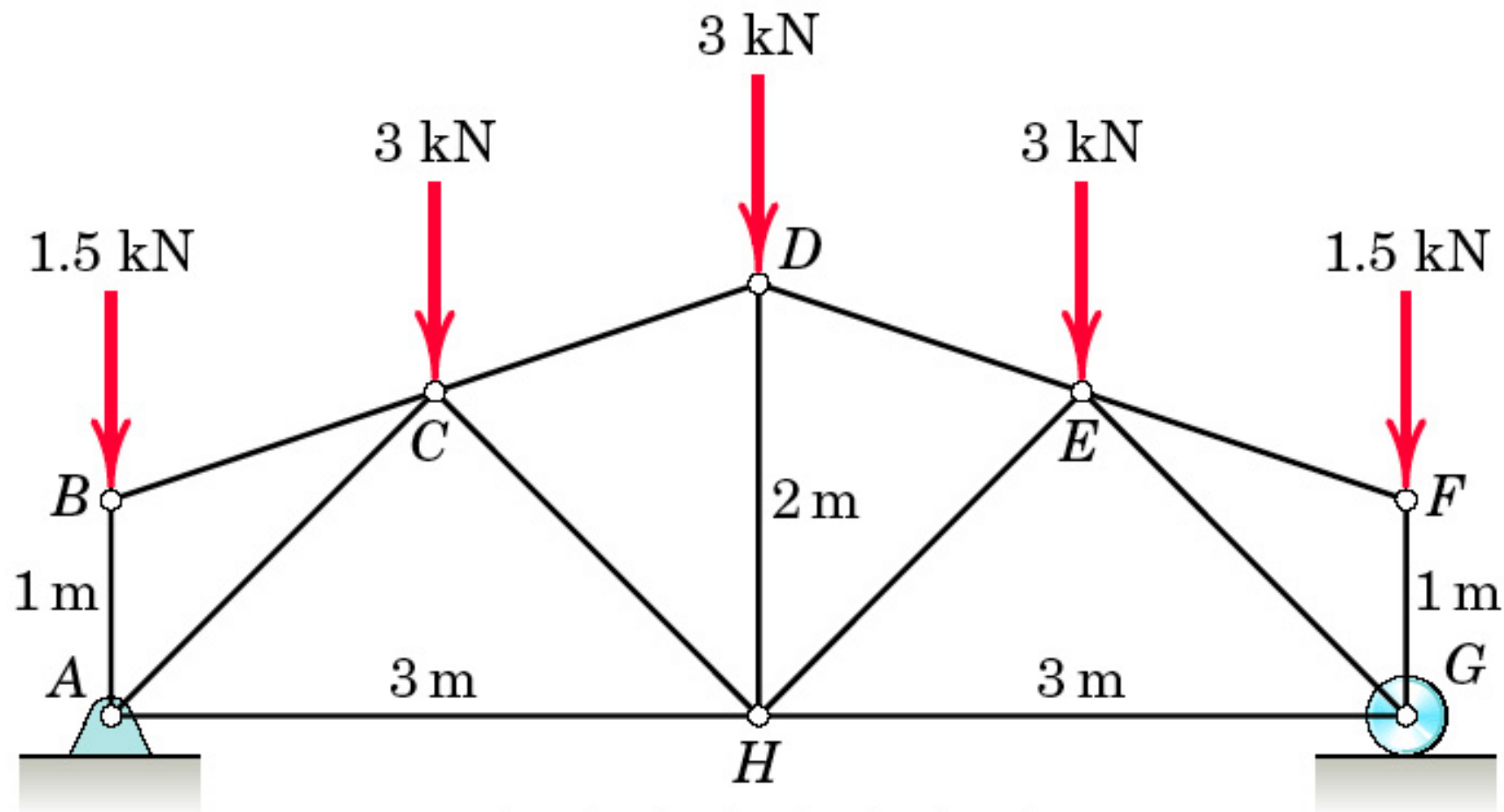
Problem 4/128



Problem 4/129

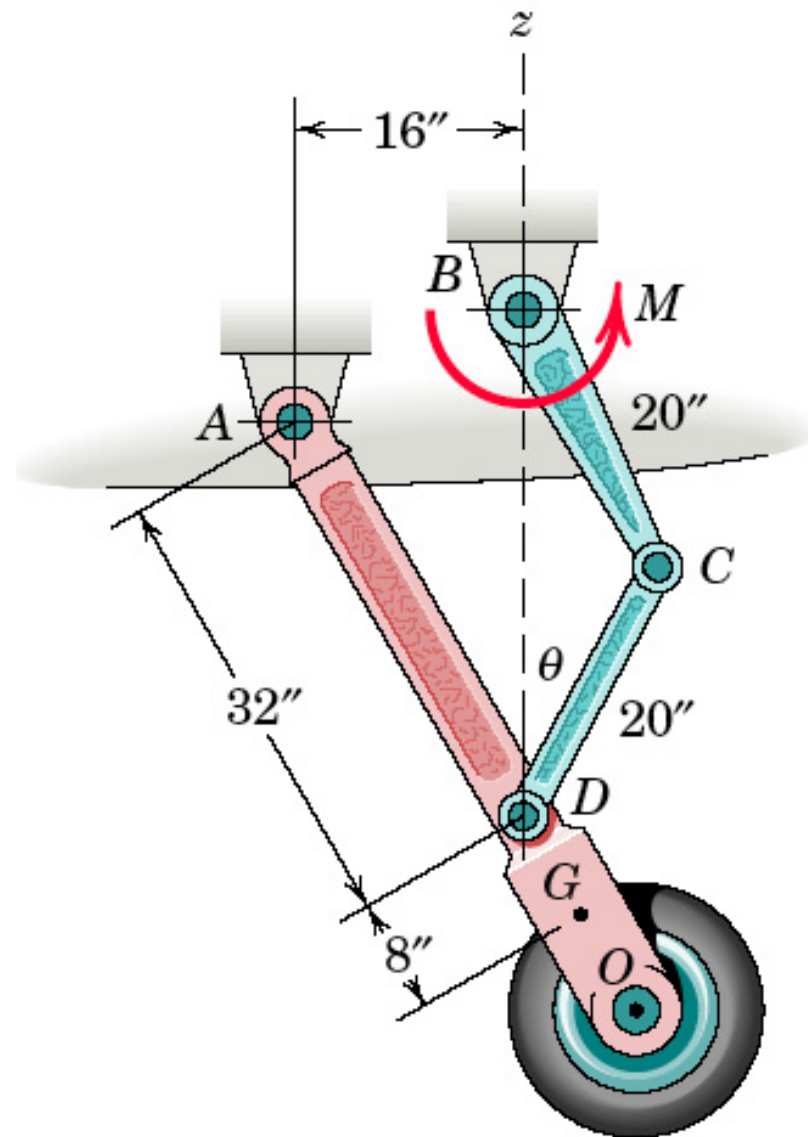


Problem 4/13

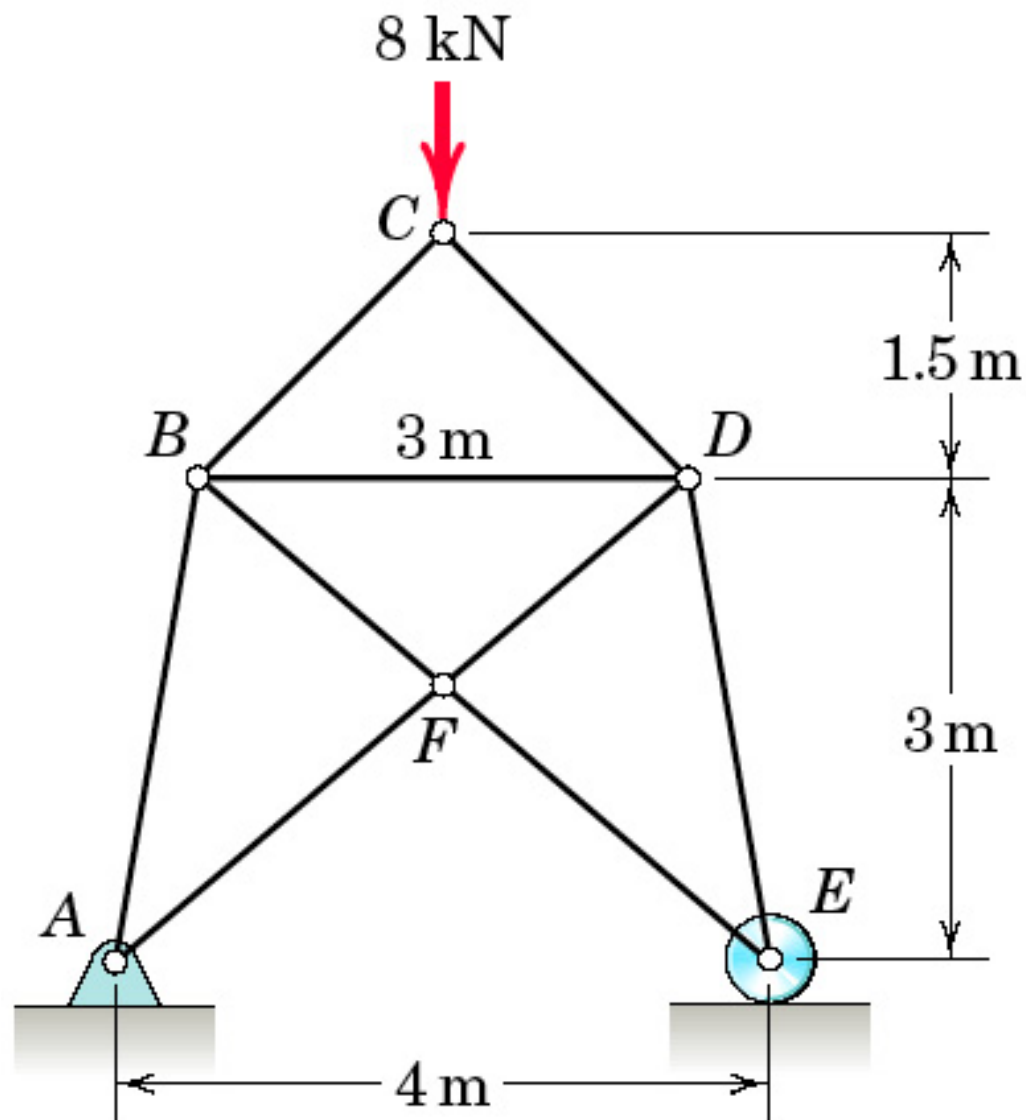


$$\overline{BC} = \overline{CD} = \overline{DE} = \overline{EF}$$

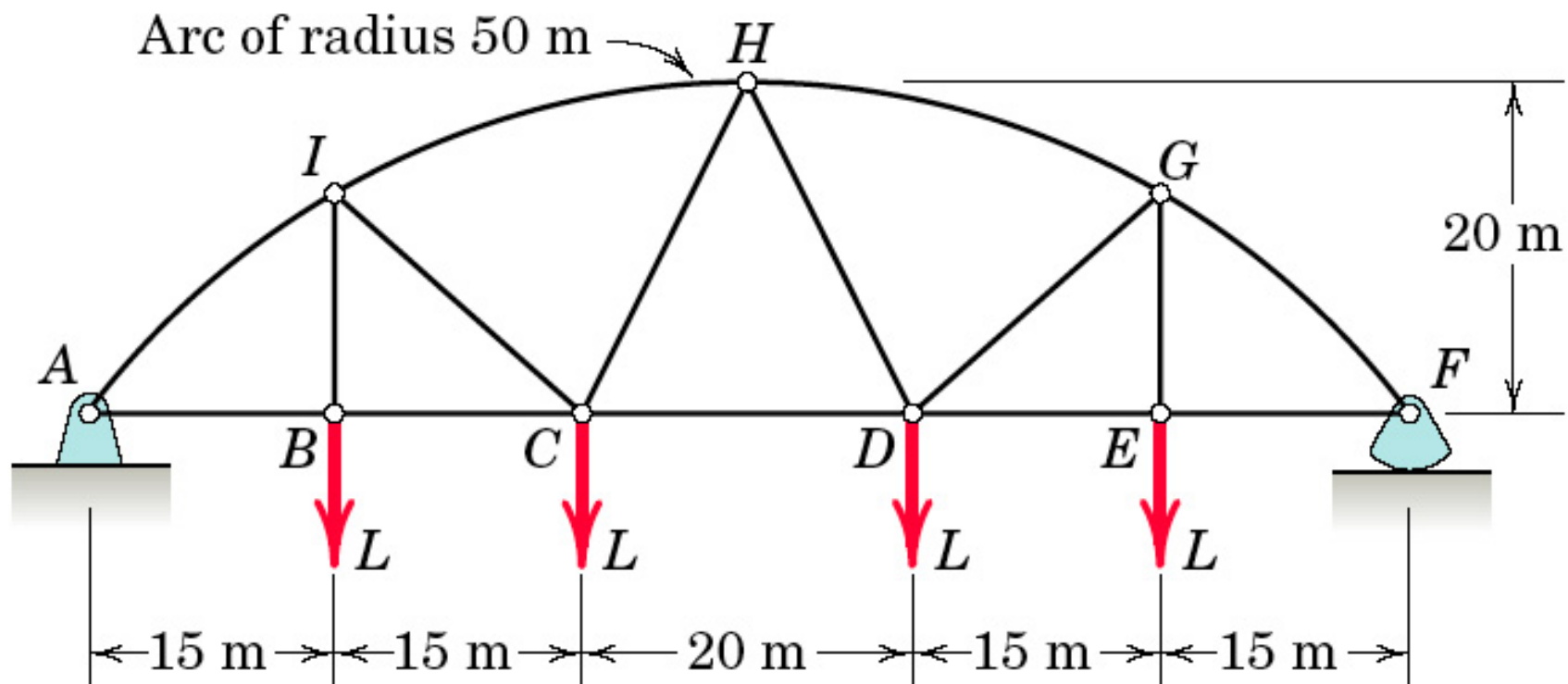
Problem 4/130



Problem 4/131

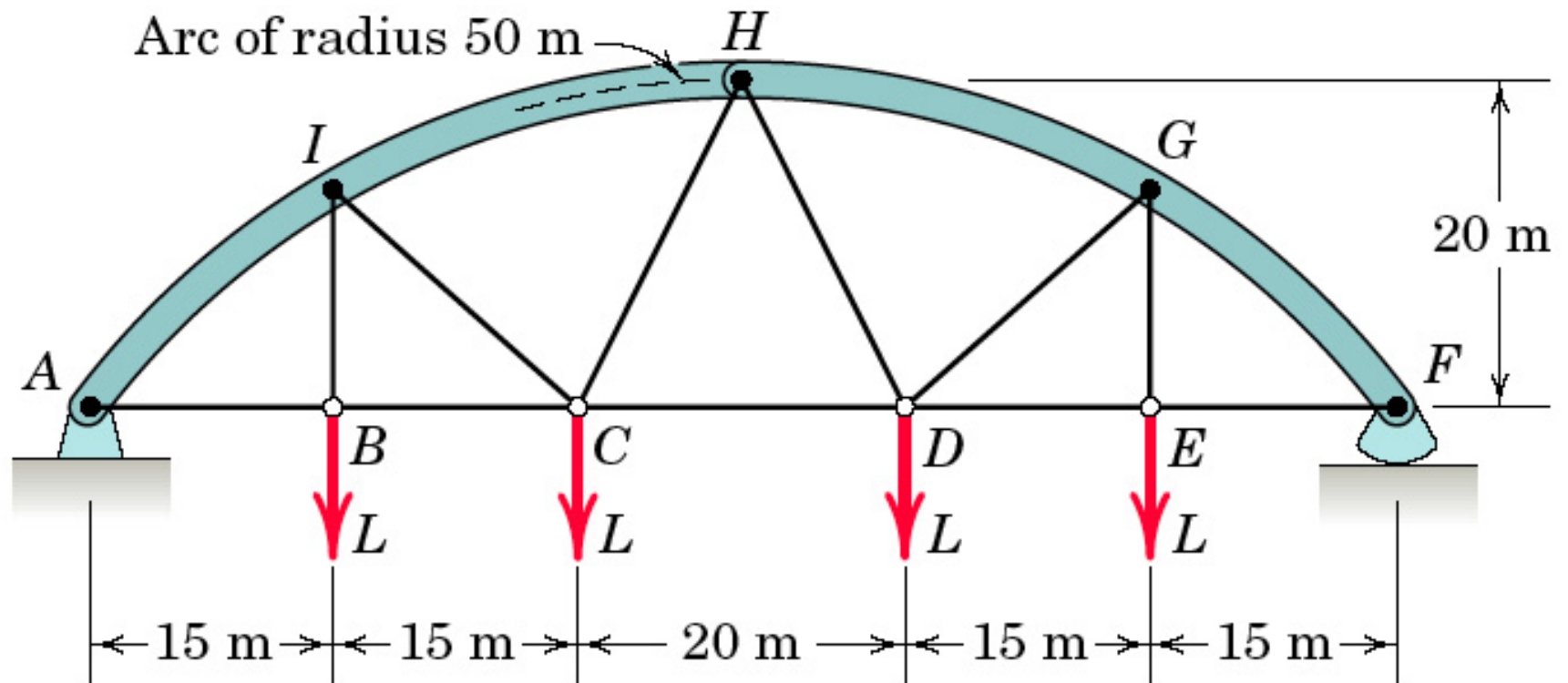


Problem 4/132



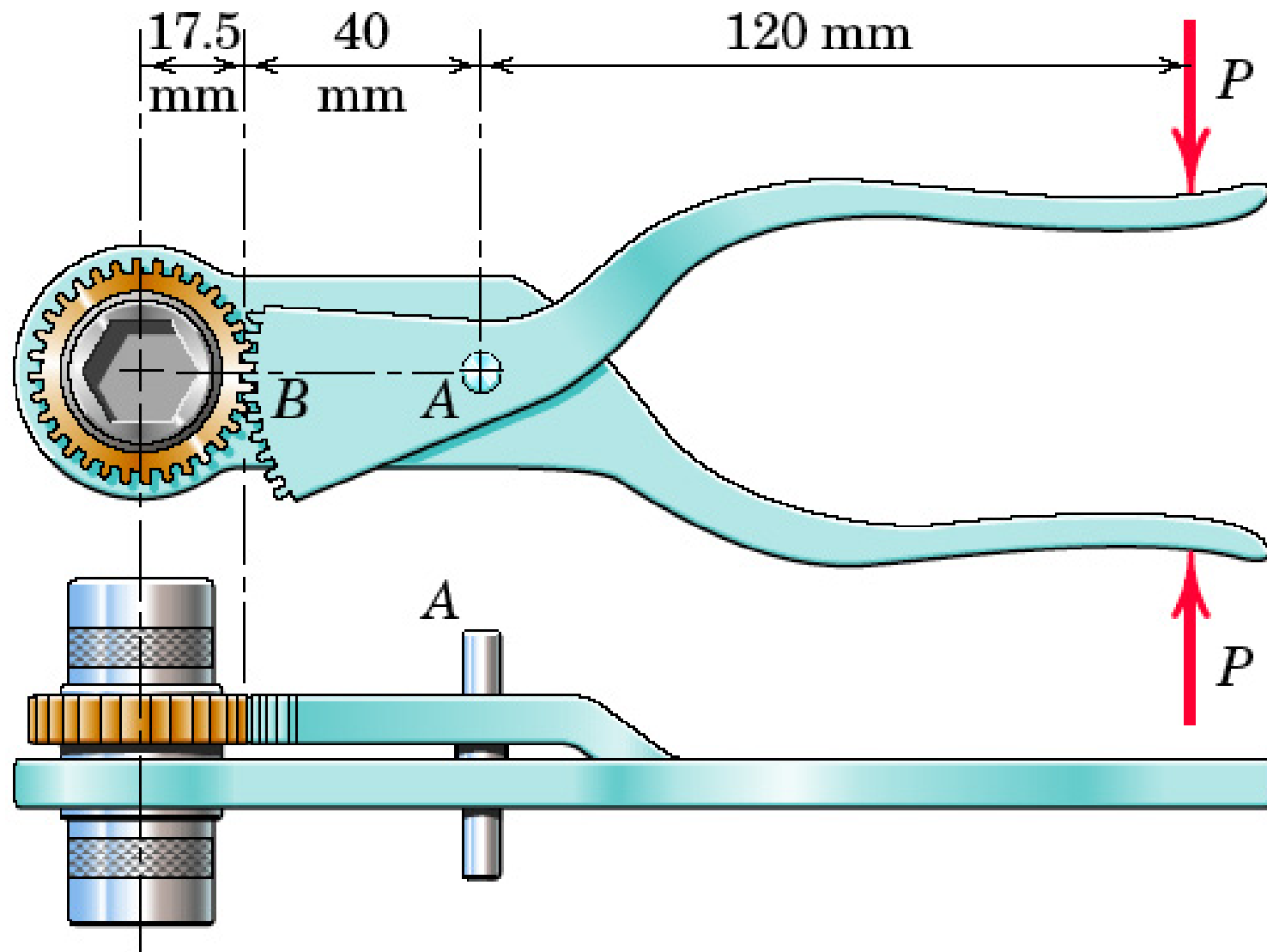
Problem 4/133

P_04_133

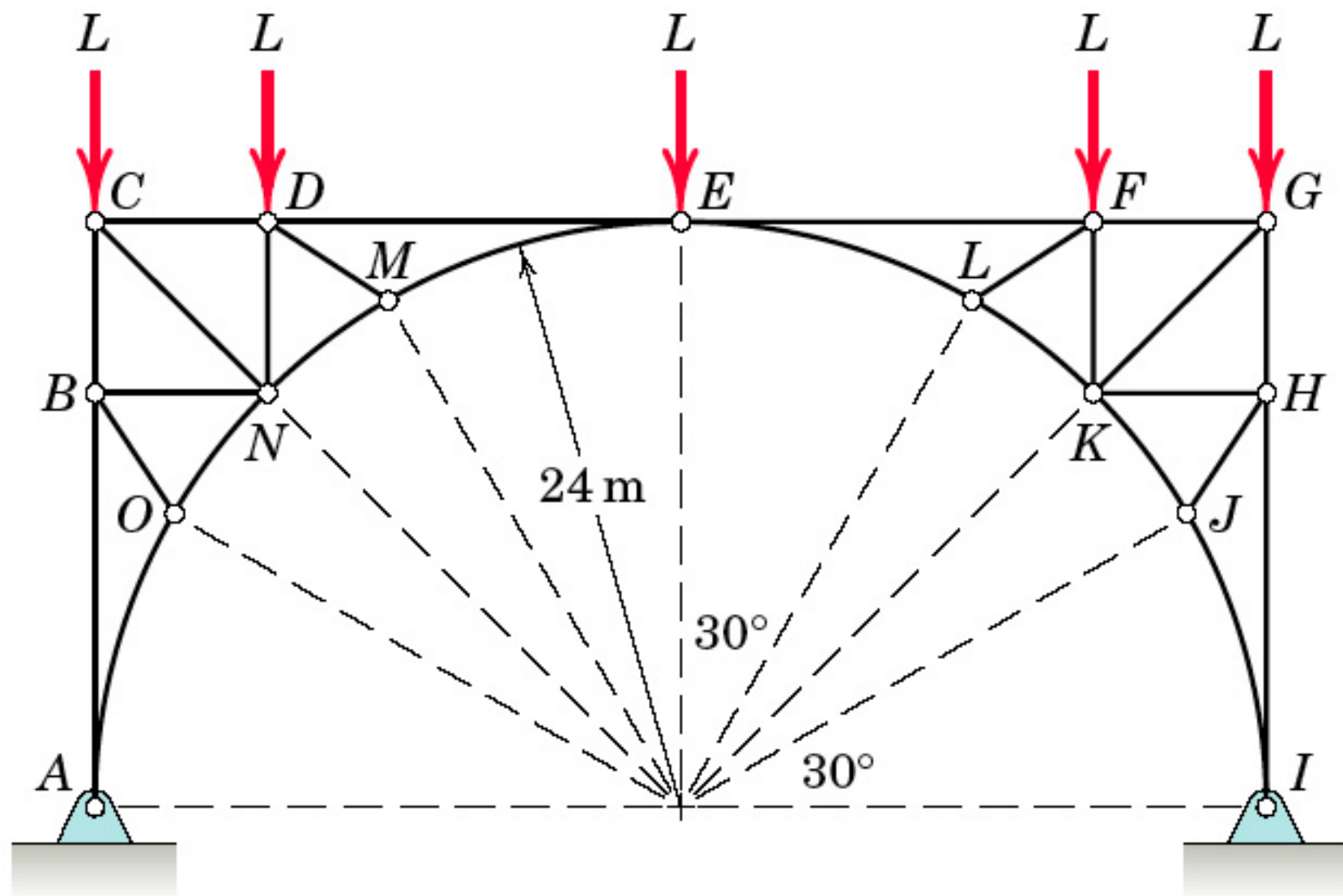


Problem 4/134

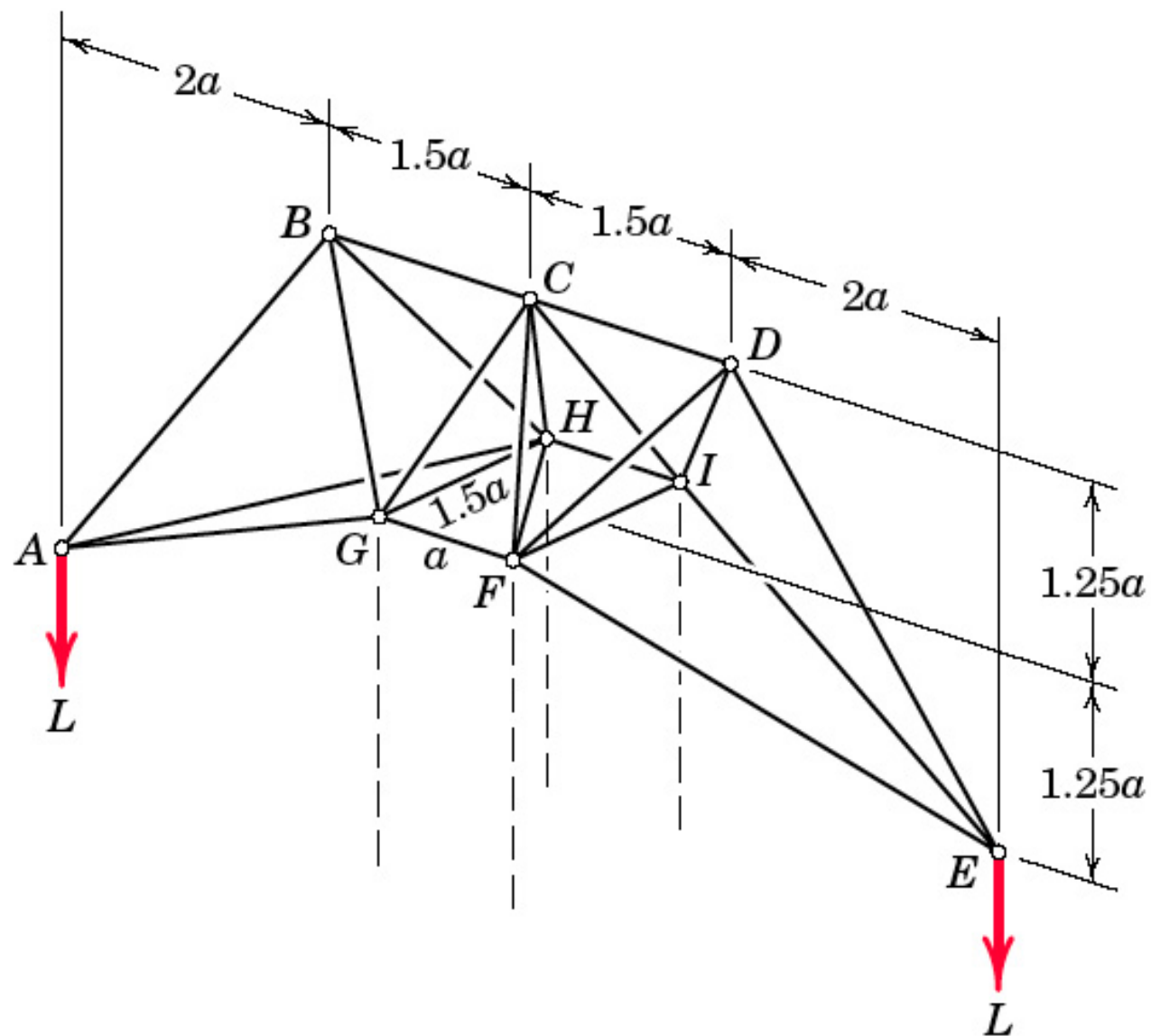
P_04_134



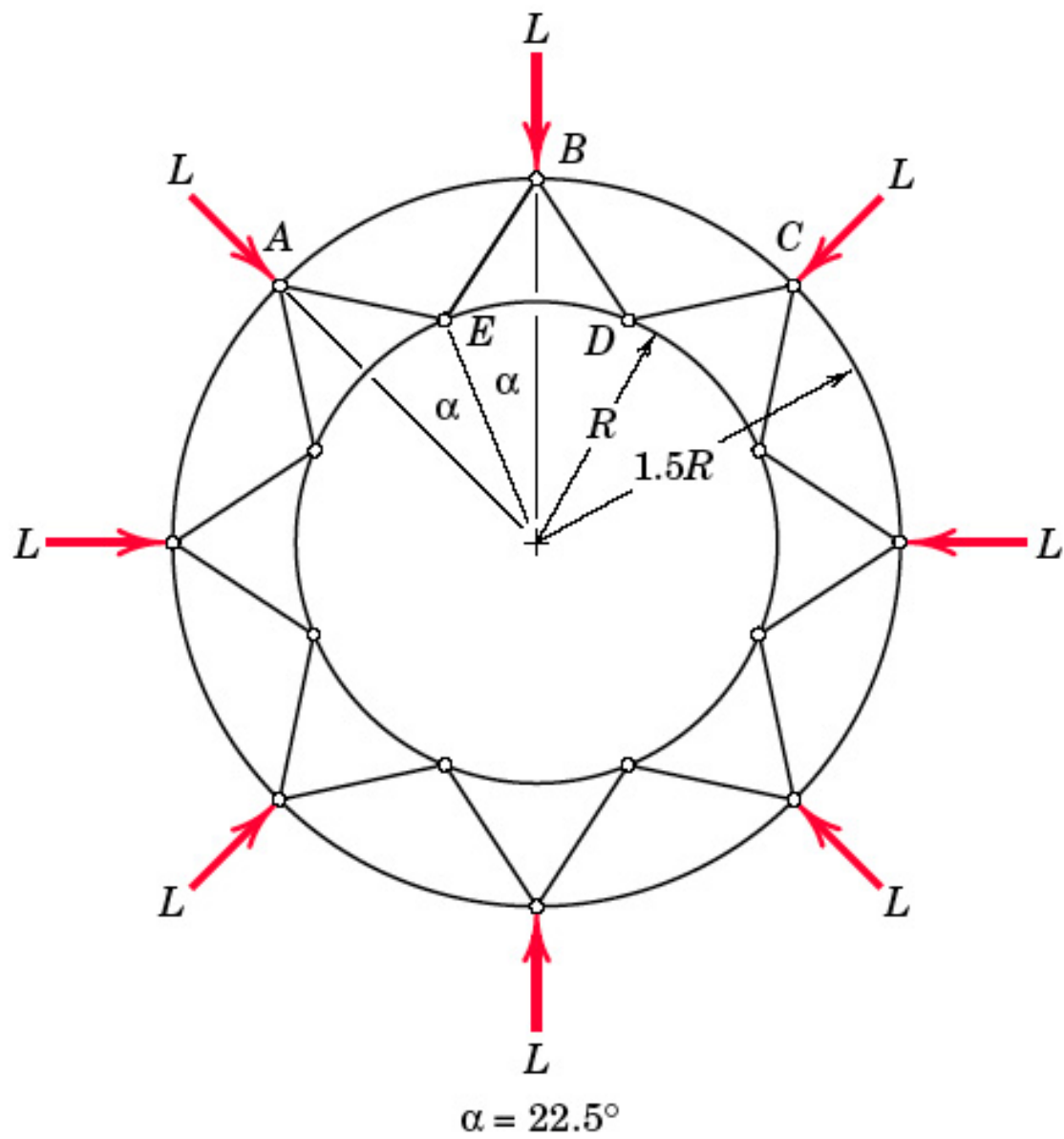
Problem 4/135



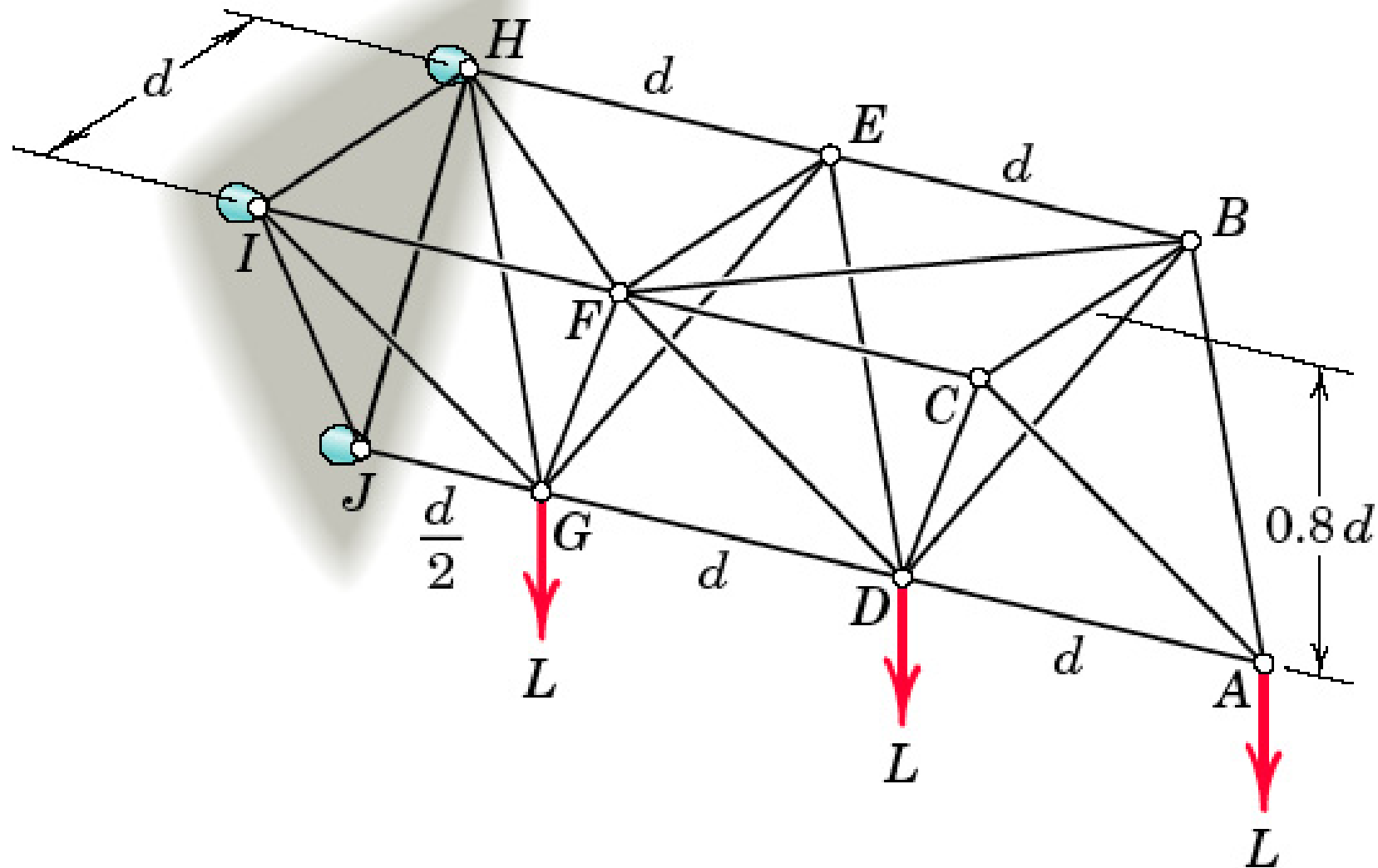
Problem 4/136



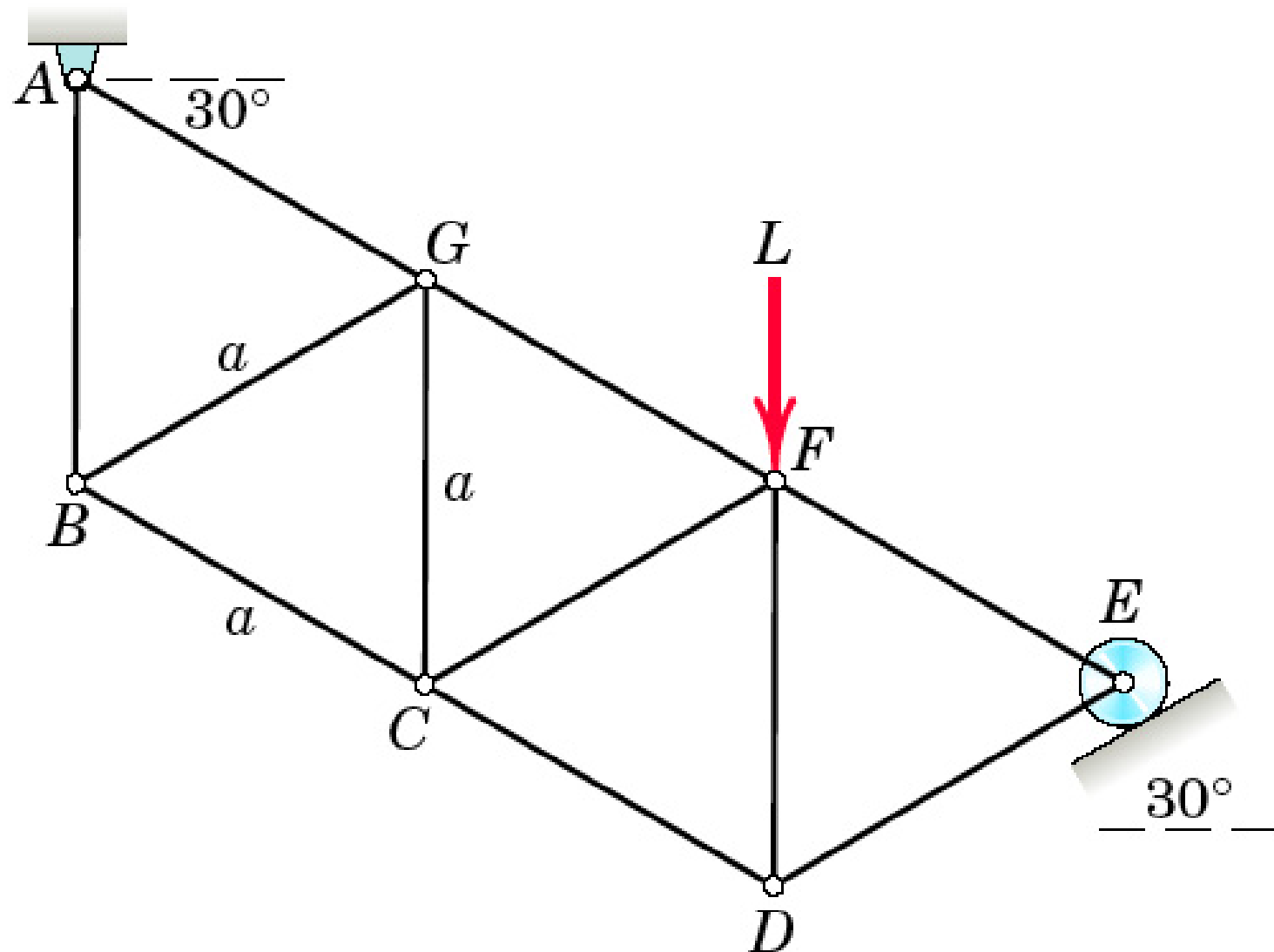
Problem 4/137



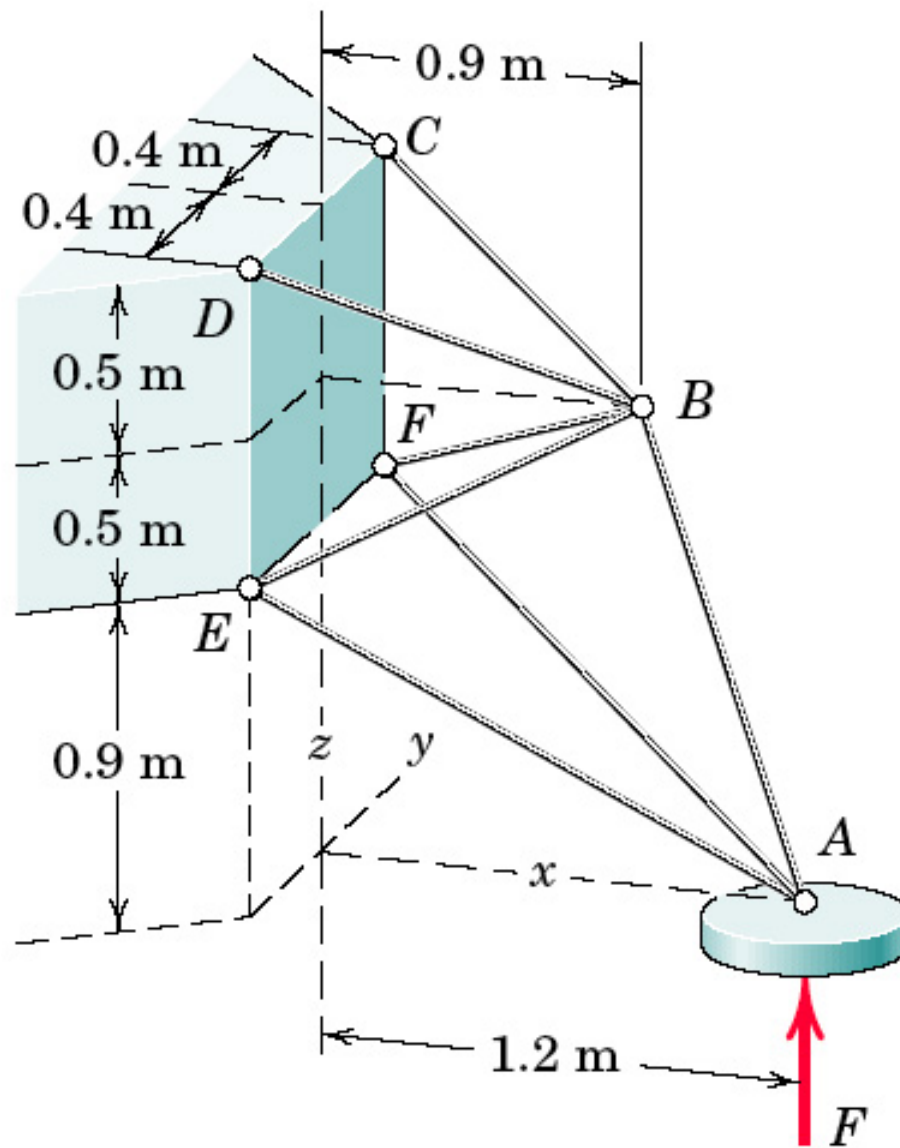
Problem 4/138



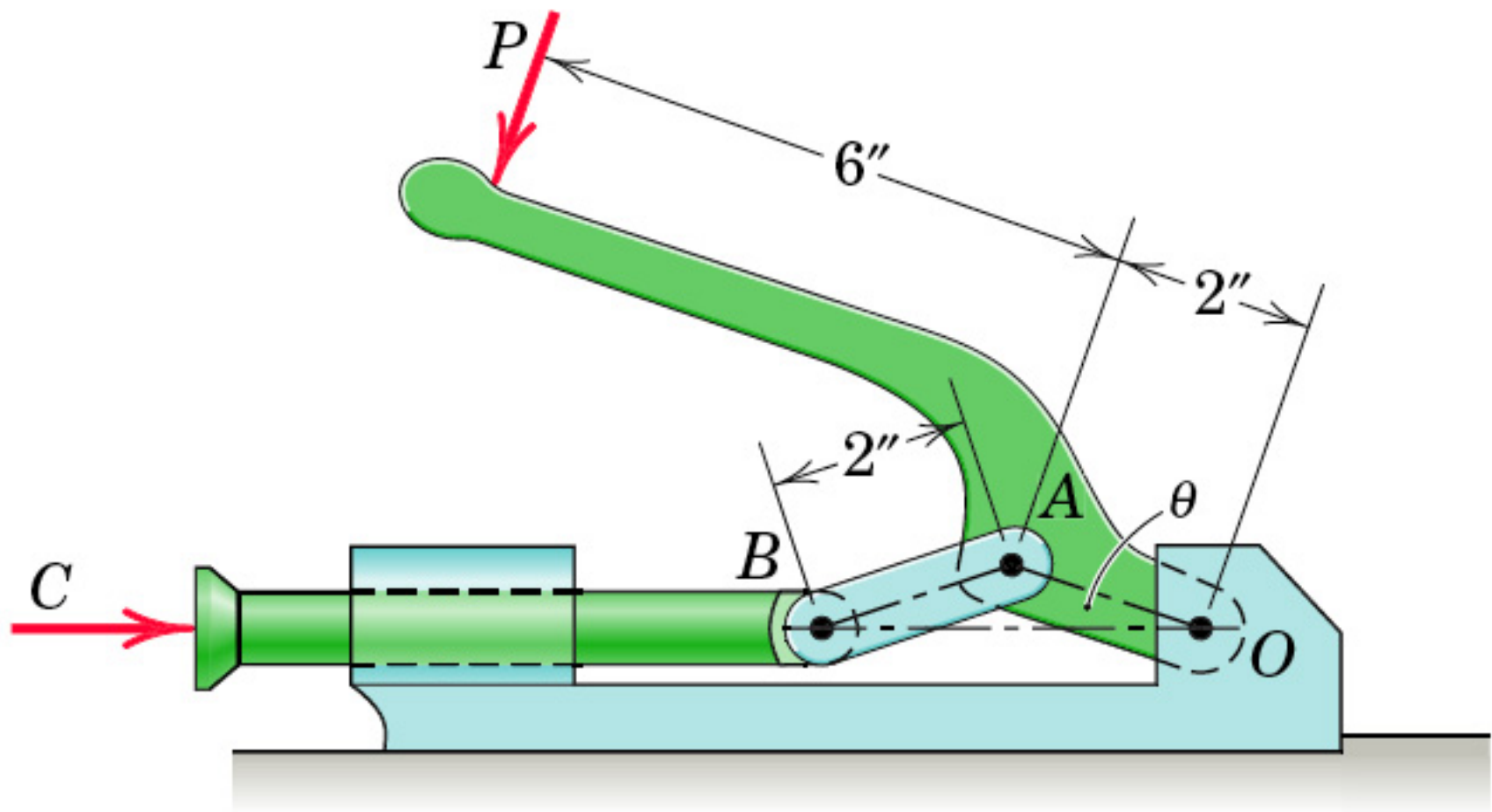
Problem 4/139



Problem 4/14

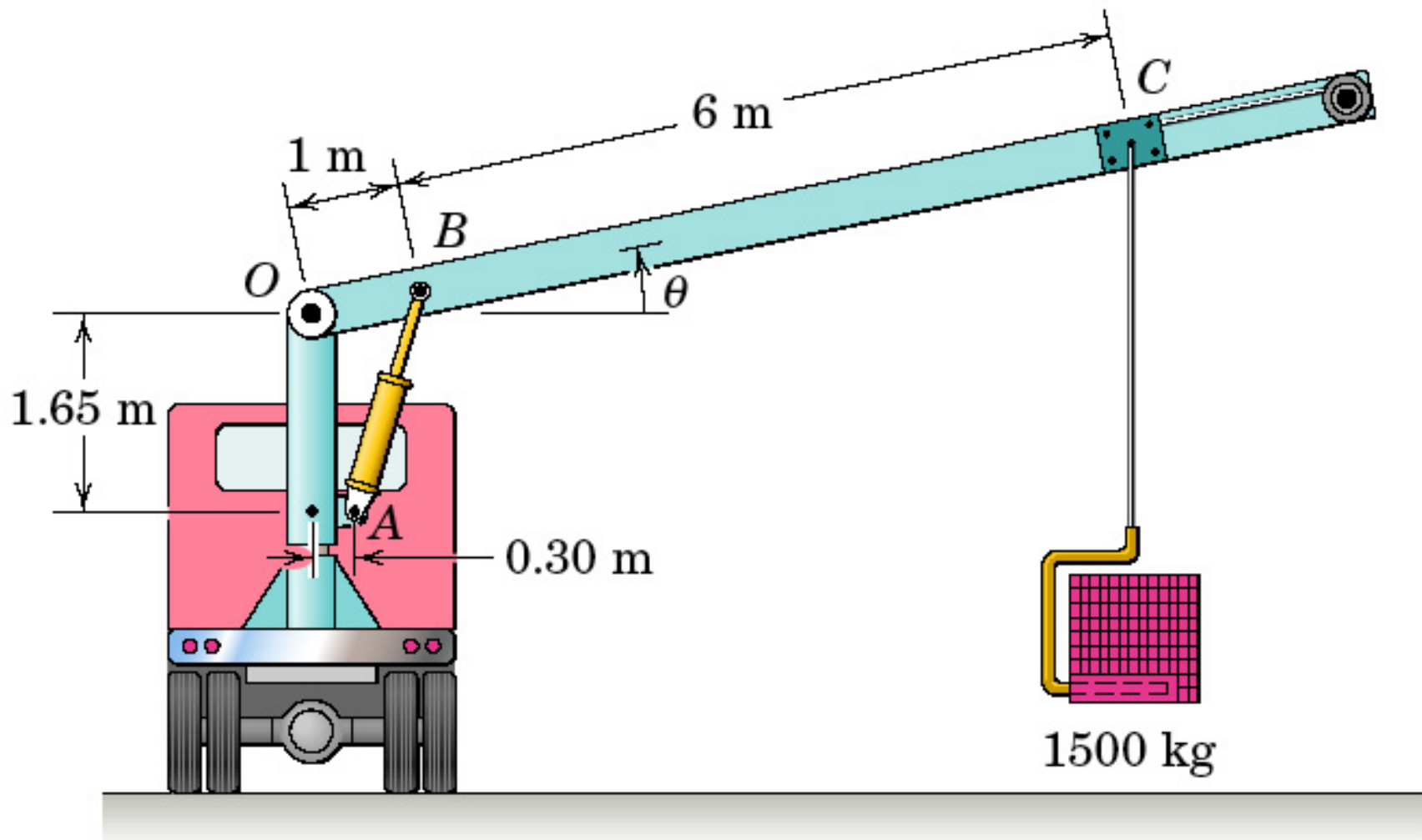


Problem 4/140



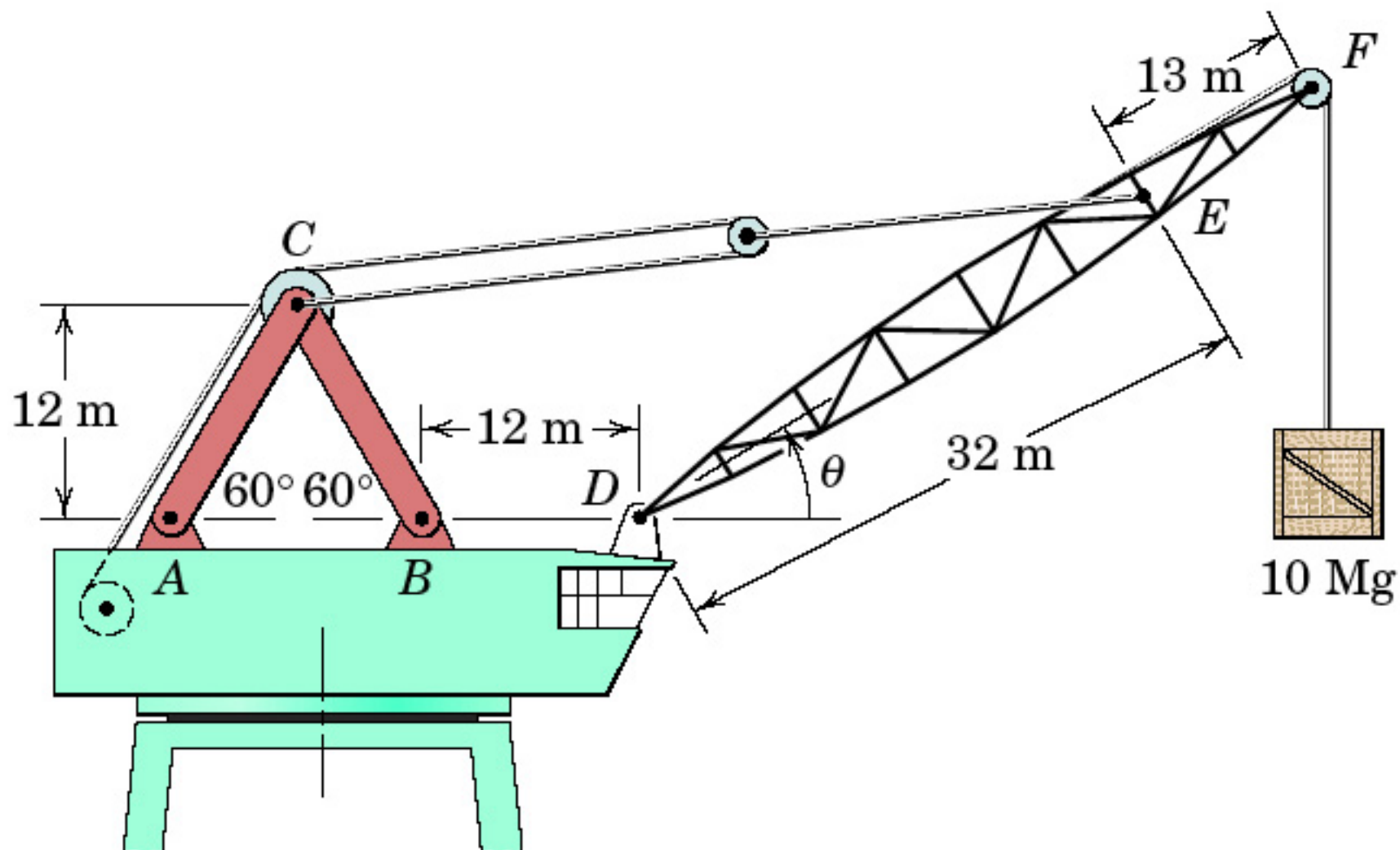
Problem 4/141

P_04_141

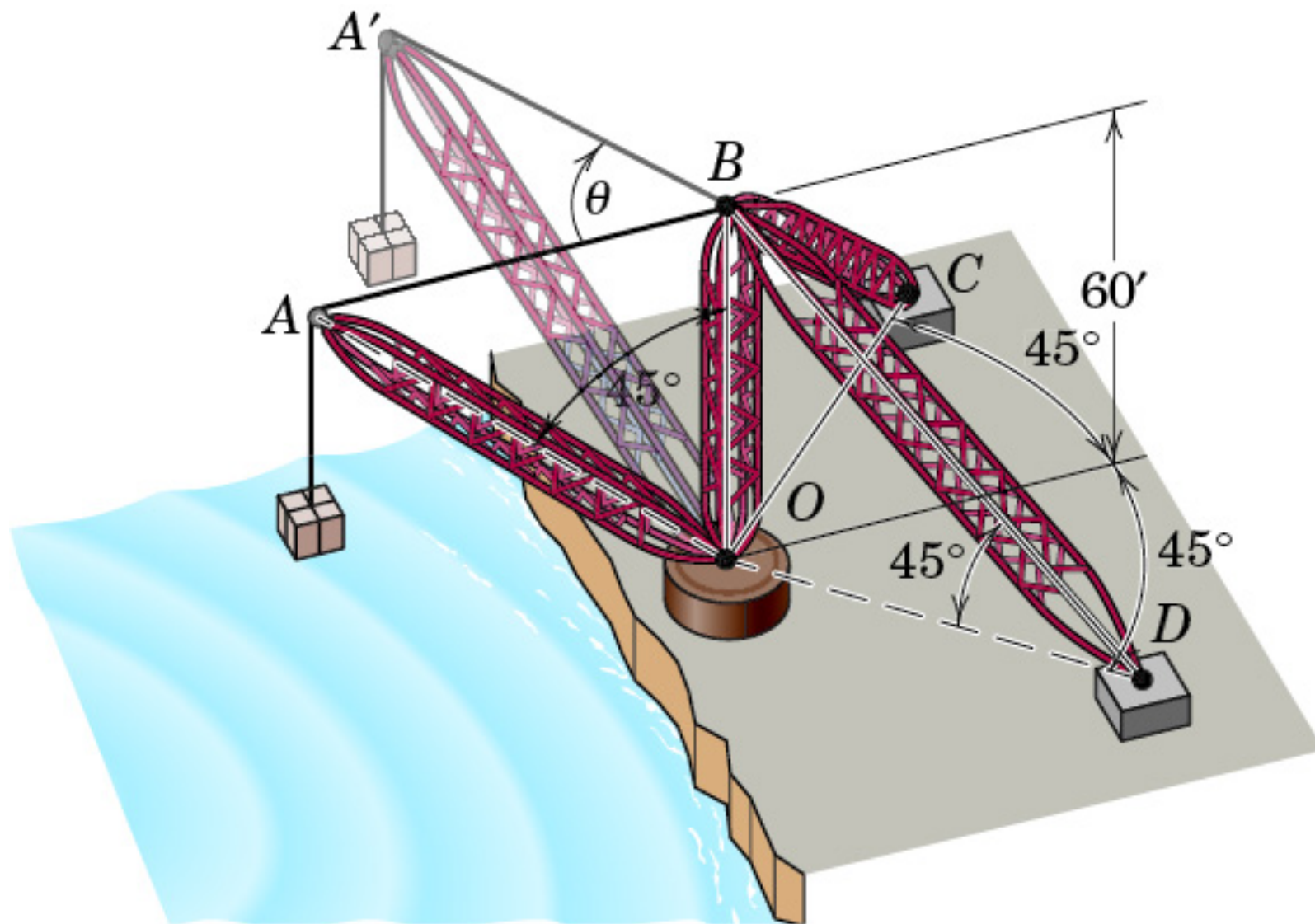


Problem 4/142

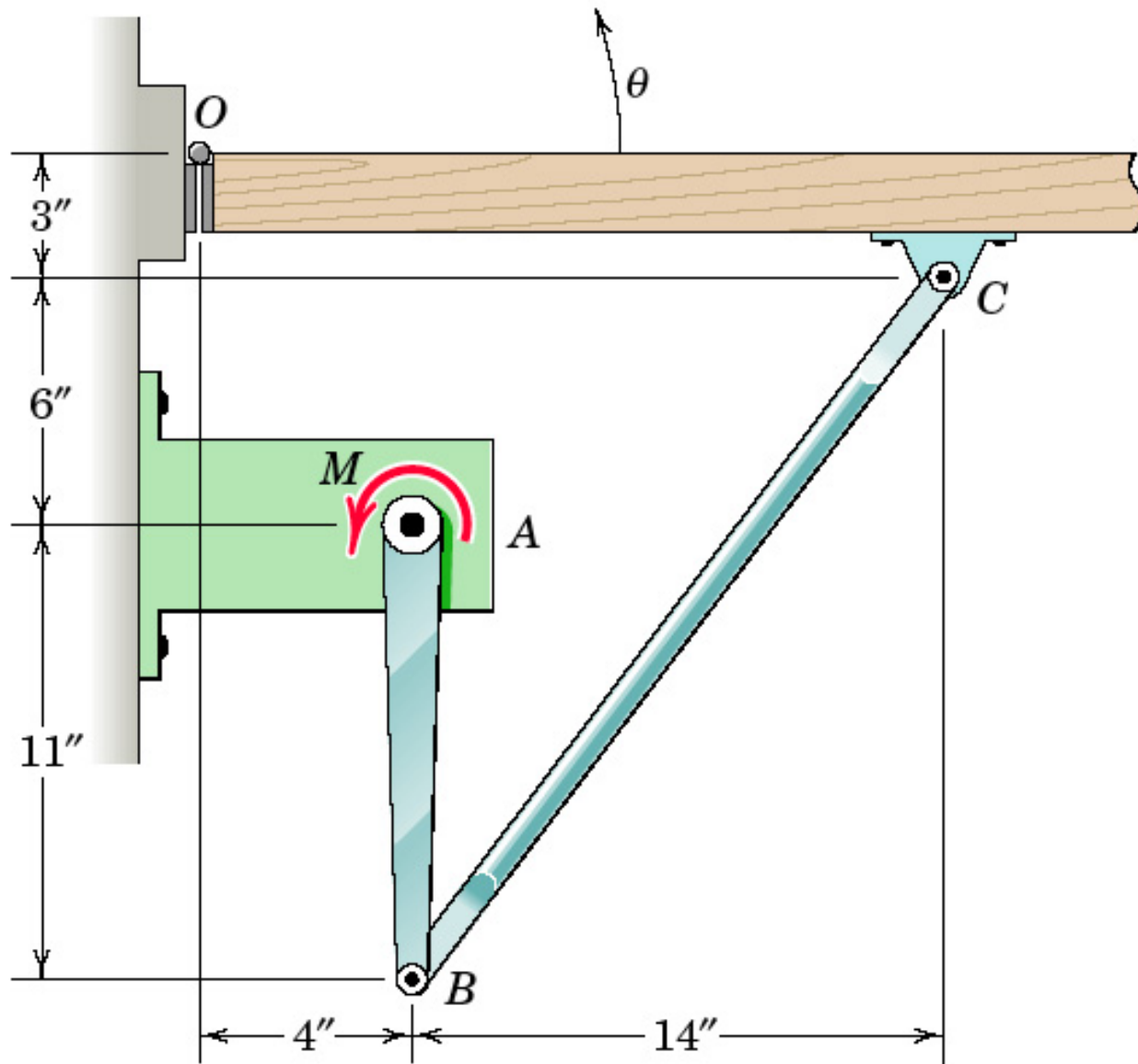
P_04_142



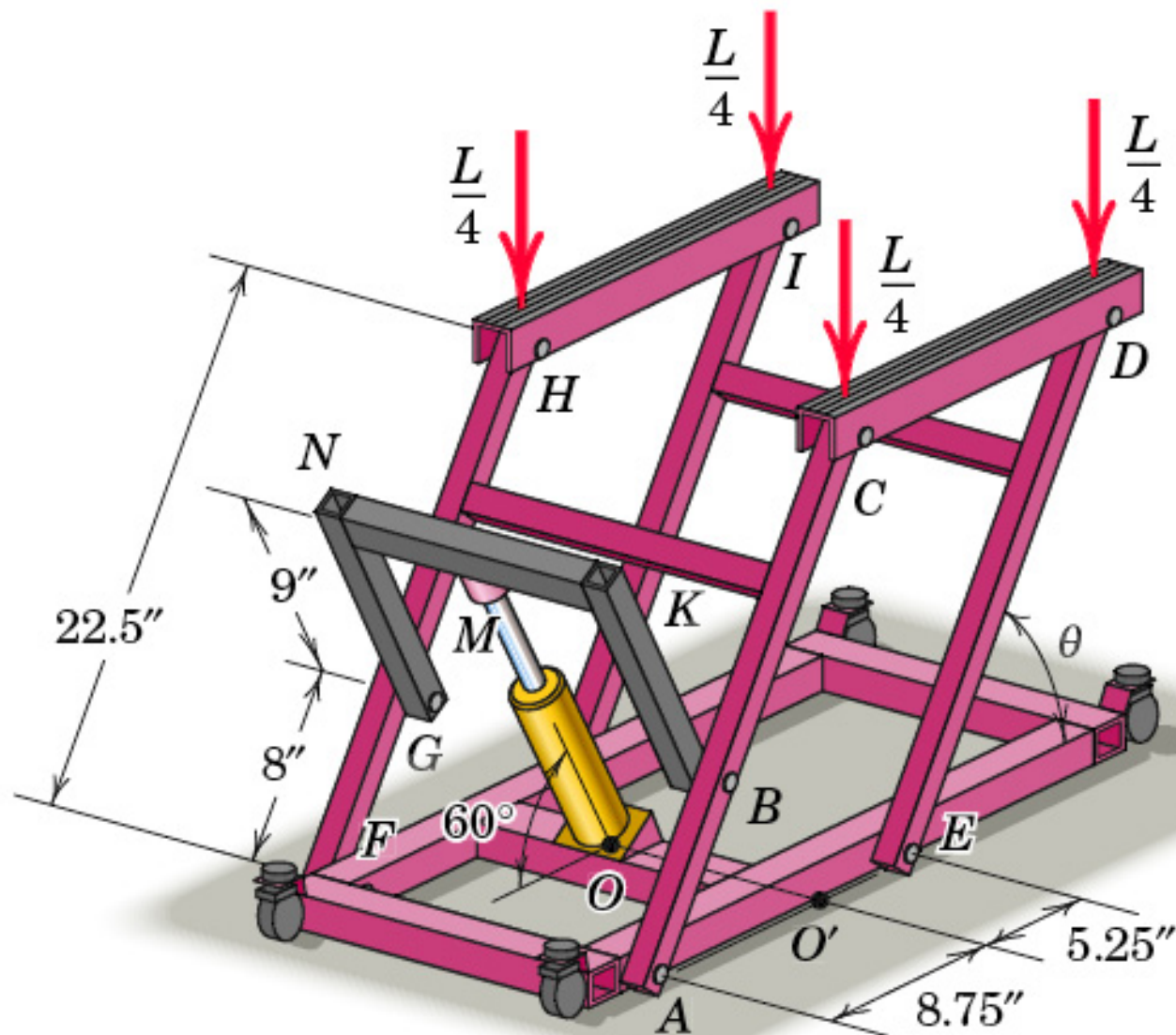
Problem 4/143



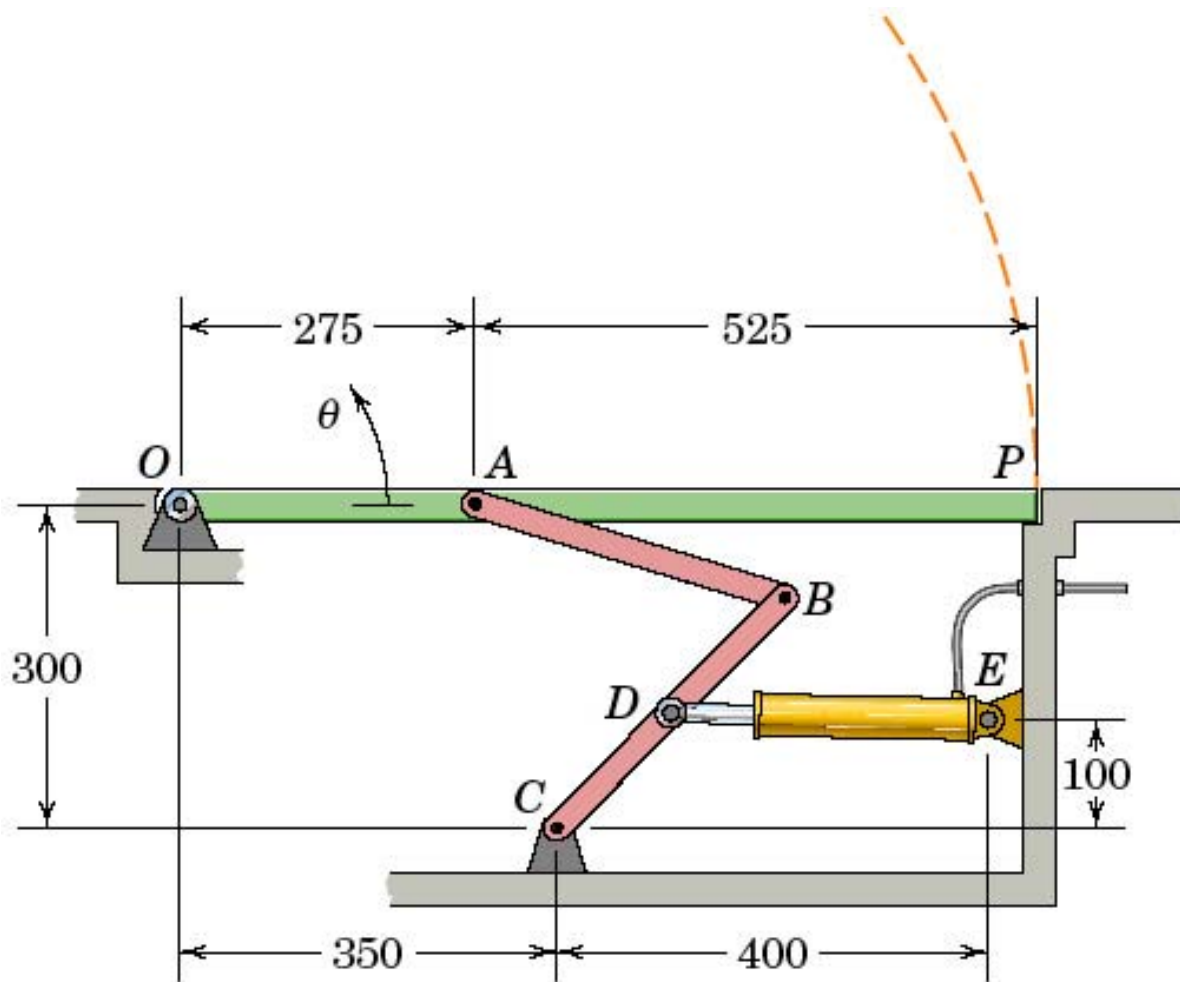
Problem 4/144



Problem 4/145



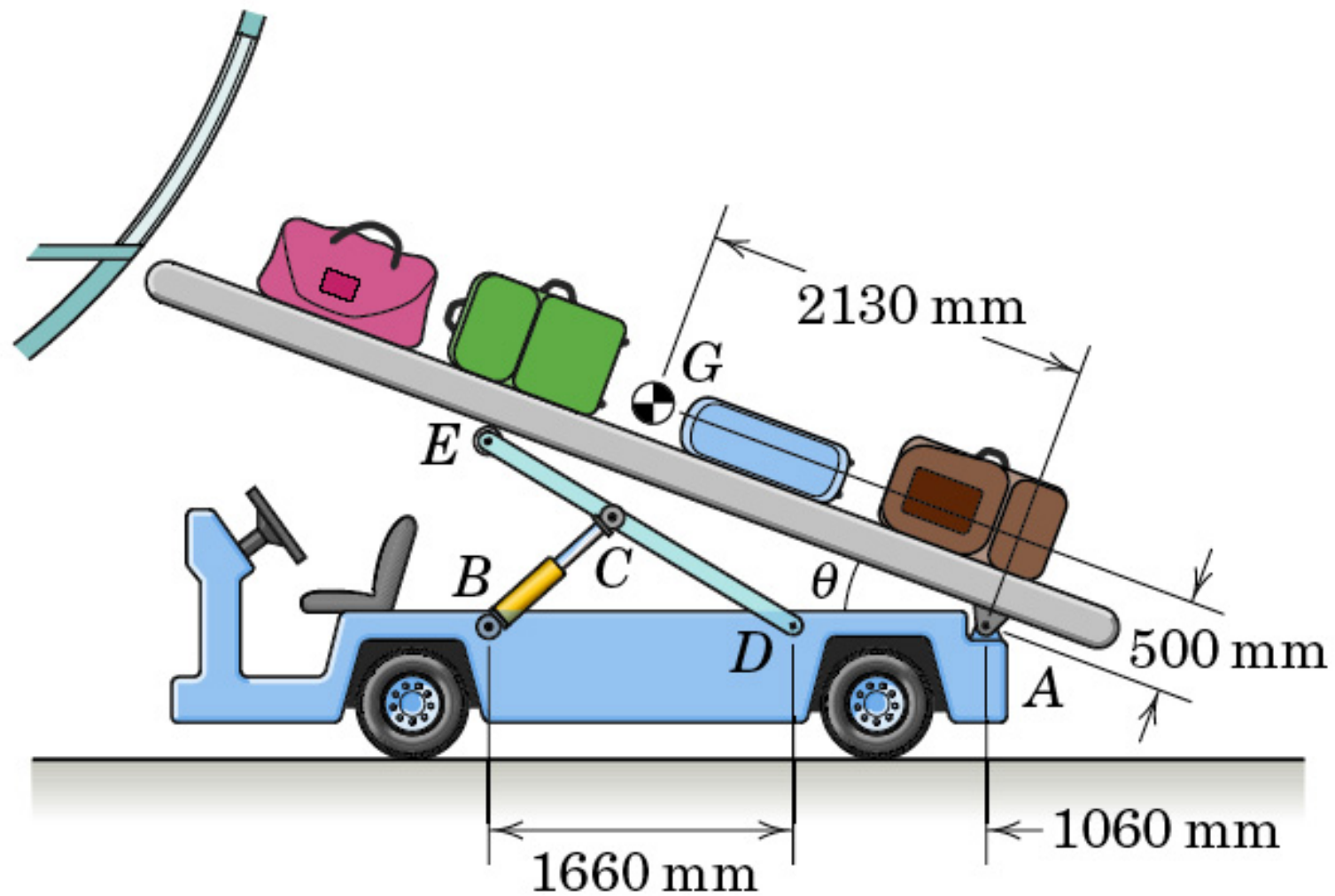
Problem 4/146



$$\overline{AB} = 300 \quad \overline{CD} = \overline{DB} = 150$$

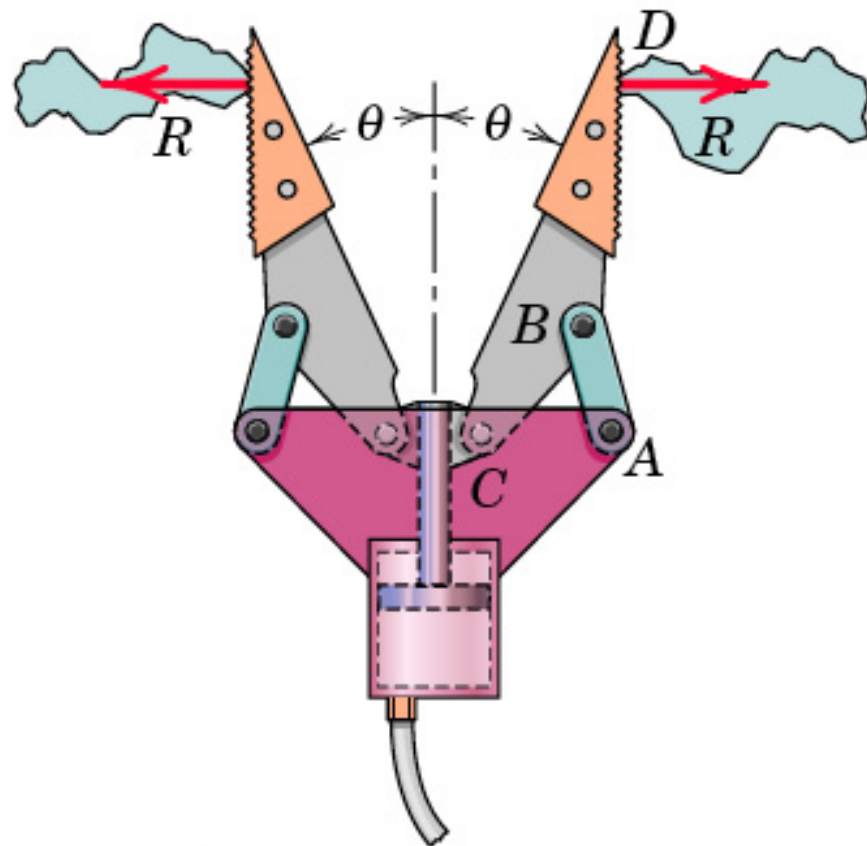
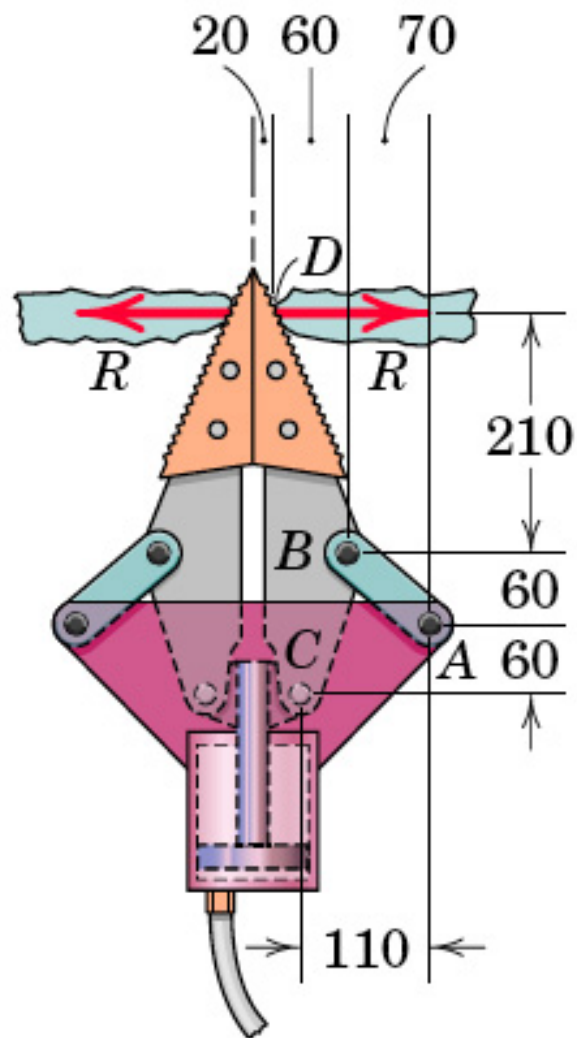
Dimensions in millimeters

Problem 4/147



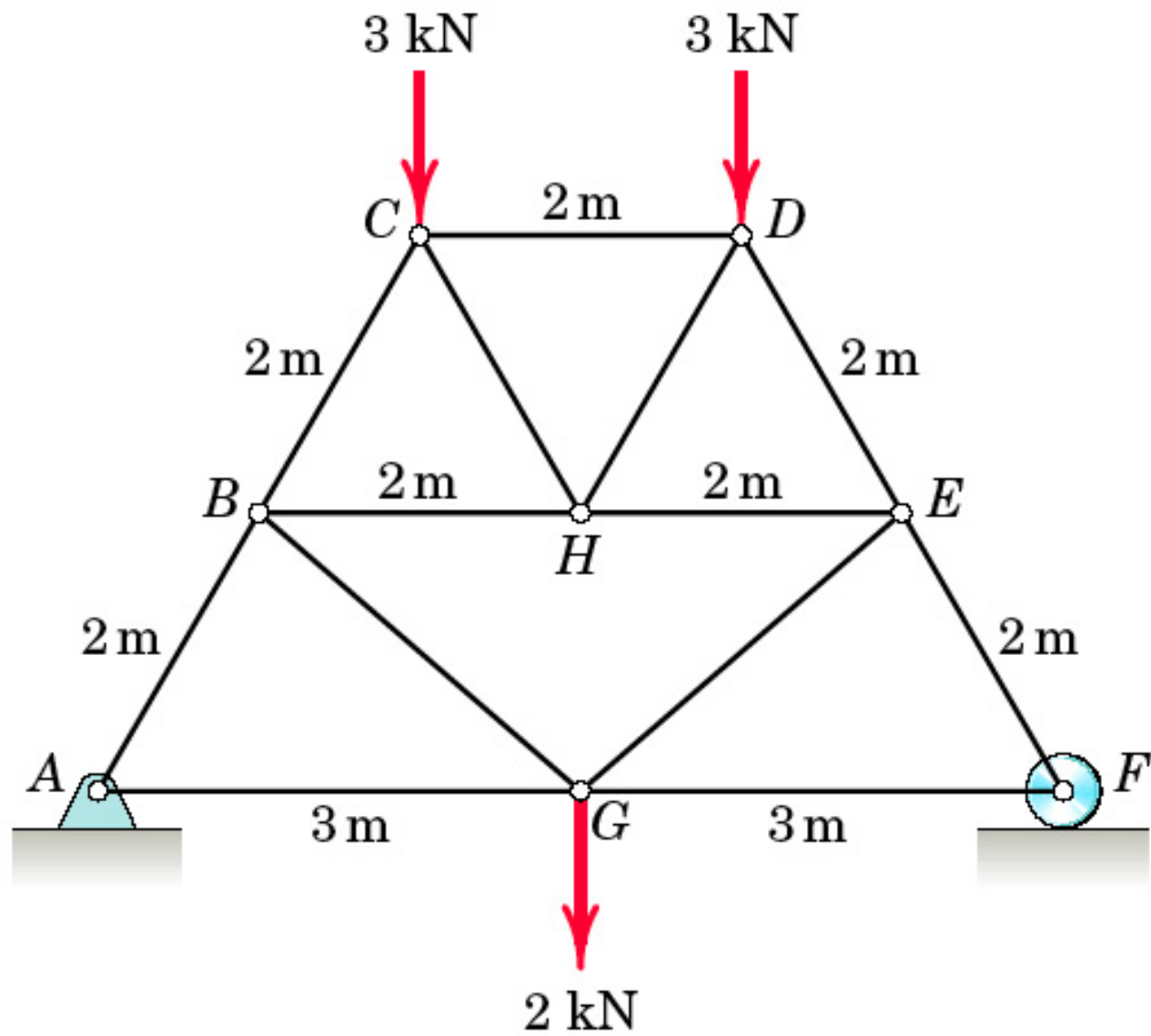
$$\overline{DE} = 1945 \text{ mm} \quad \overline{CD} = 1150 \text{ mm}$$

Problem 4/148

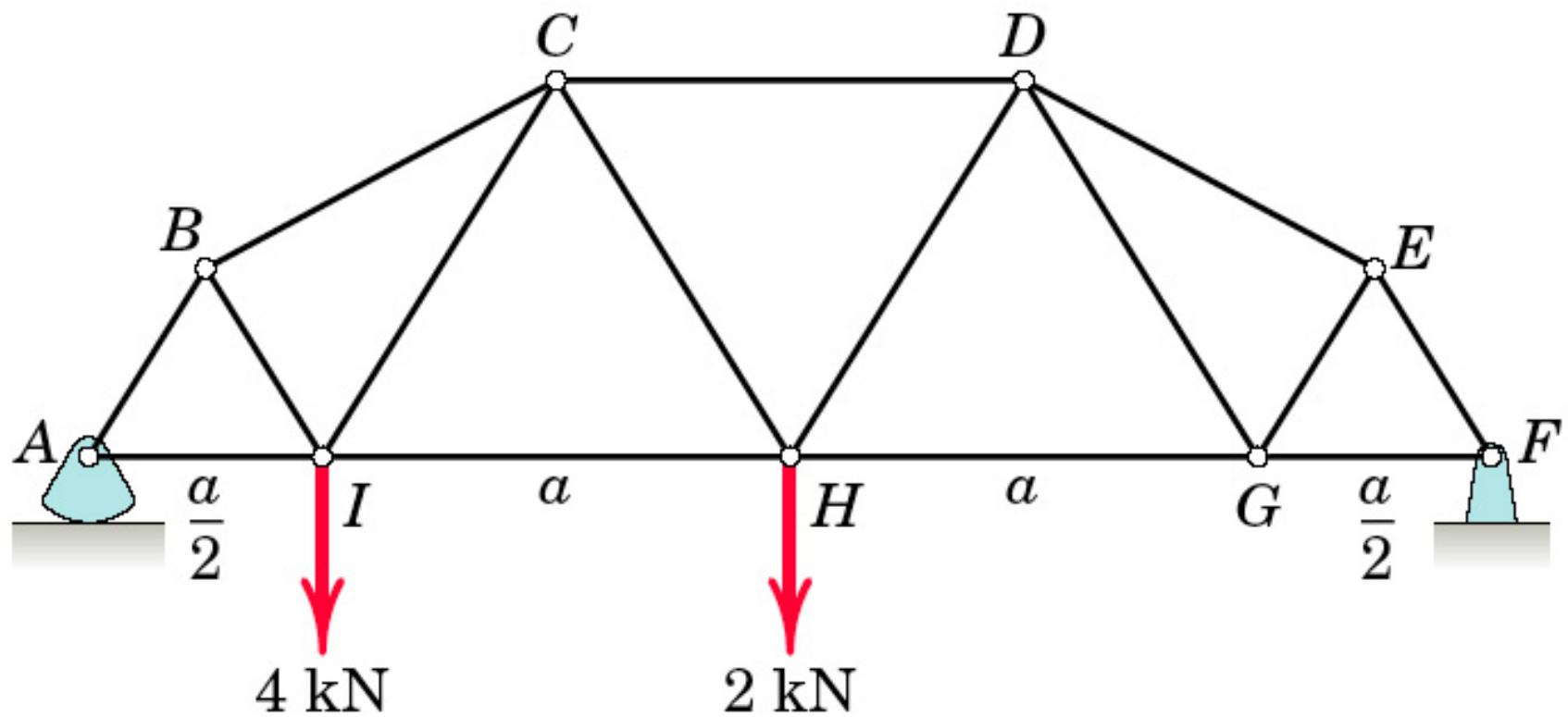


Dimensions in millimeters

Problem 4/149

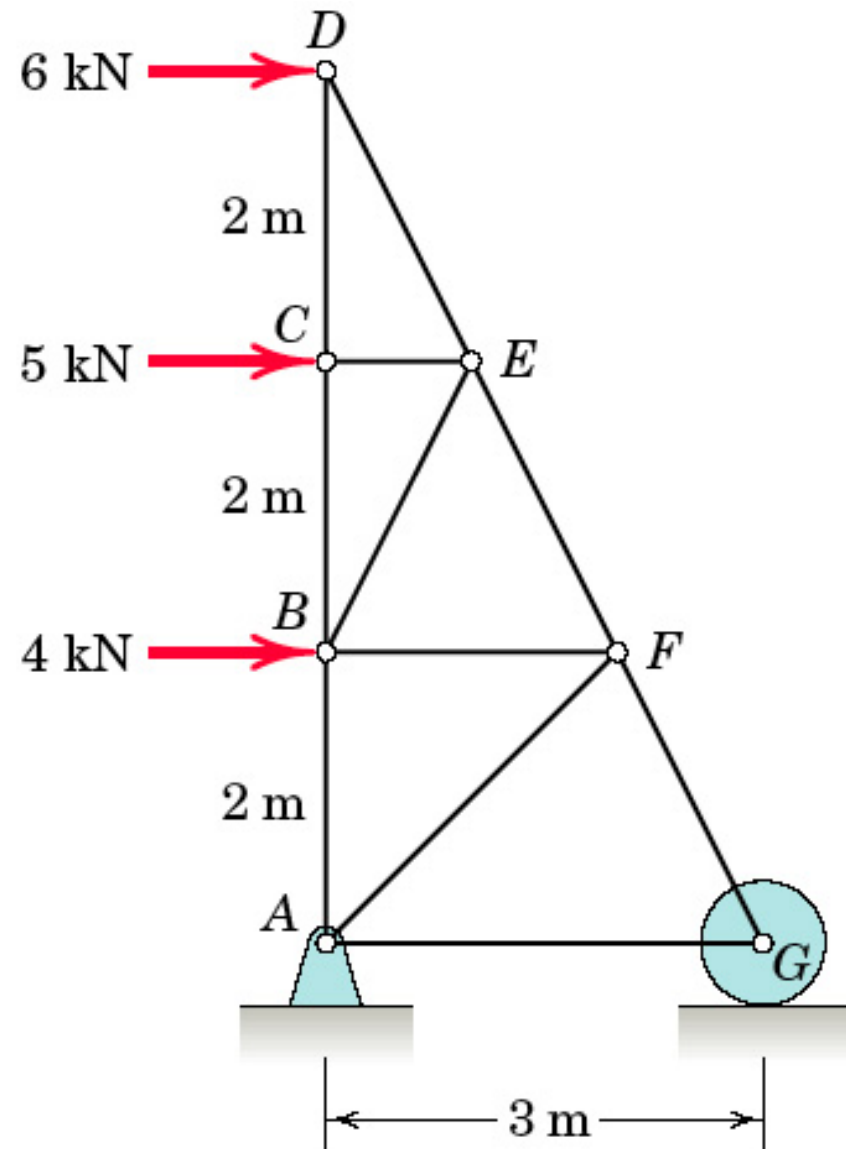


Problem 4/15

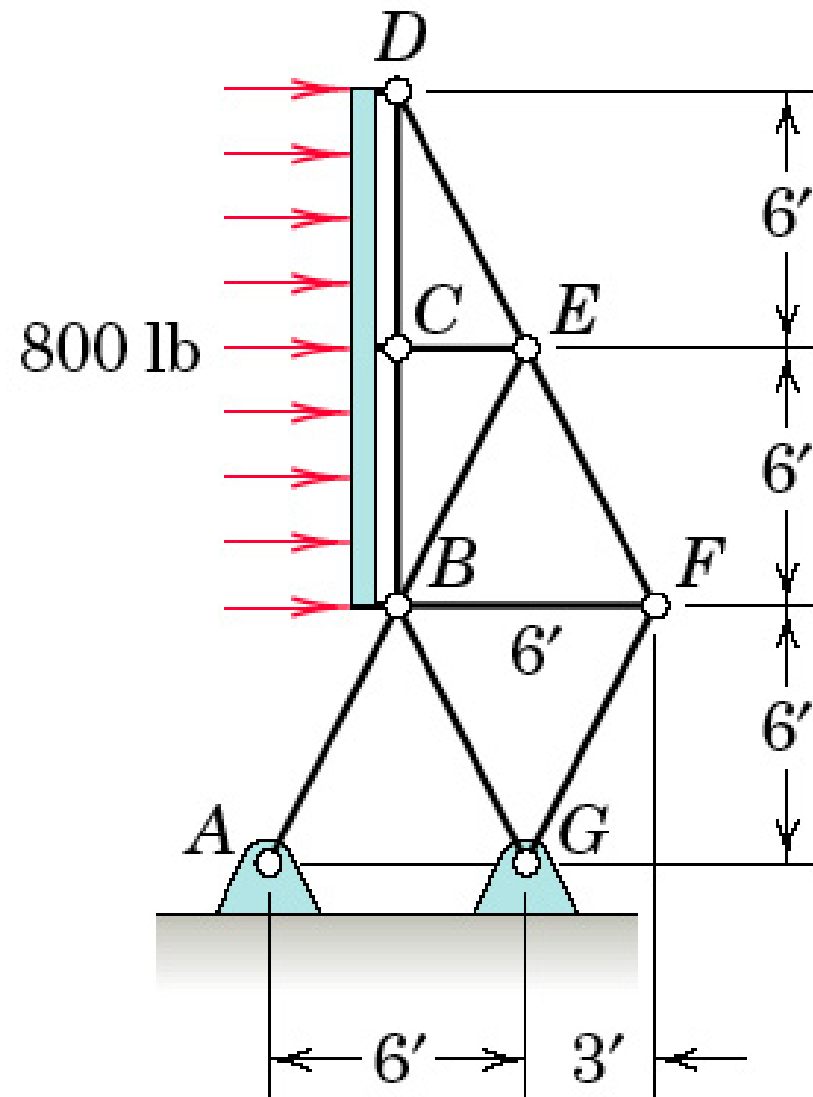


Problem 4/16

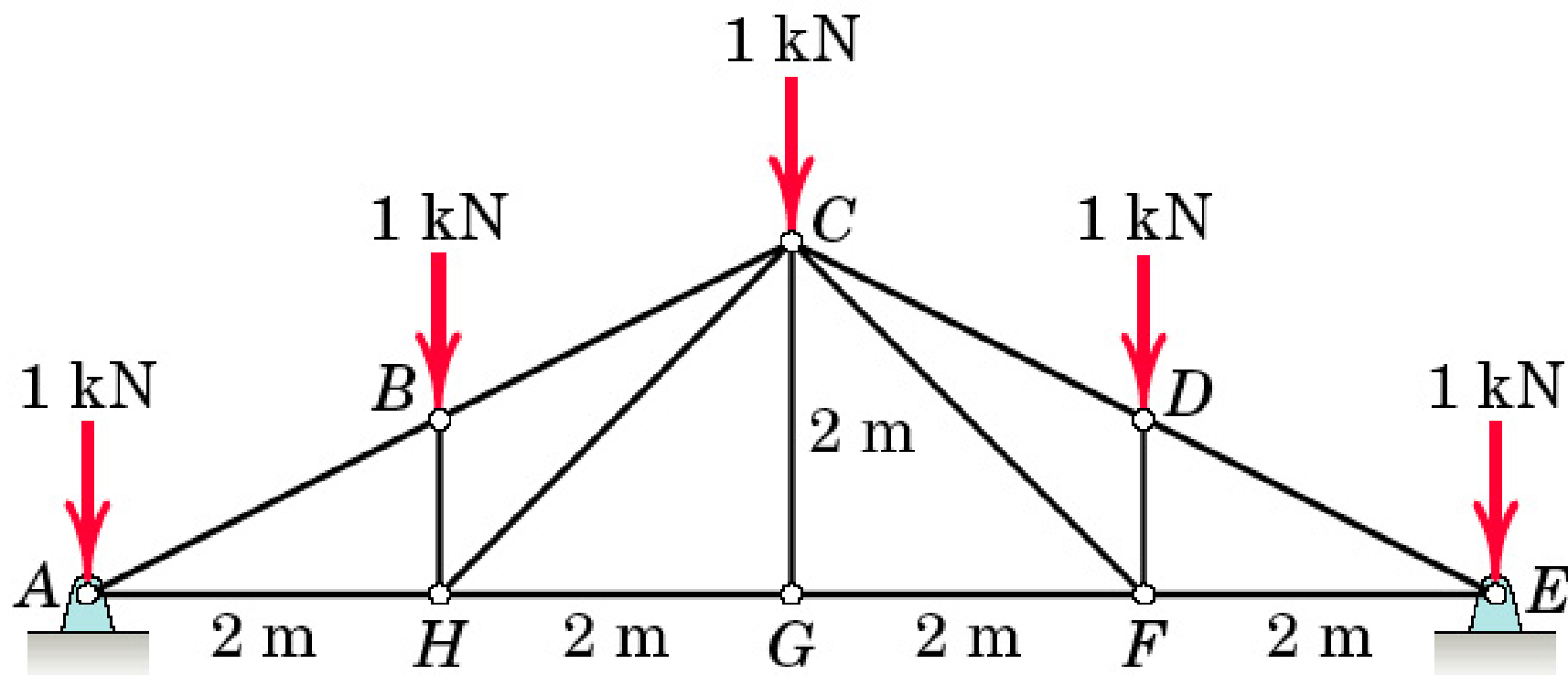
P_04_16



Problem 4/17

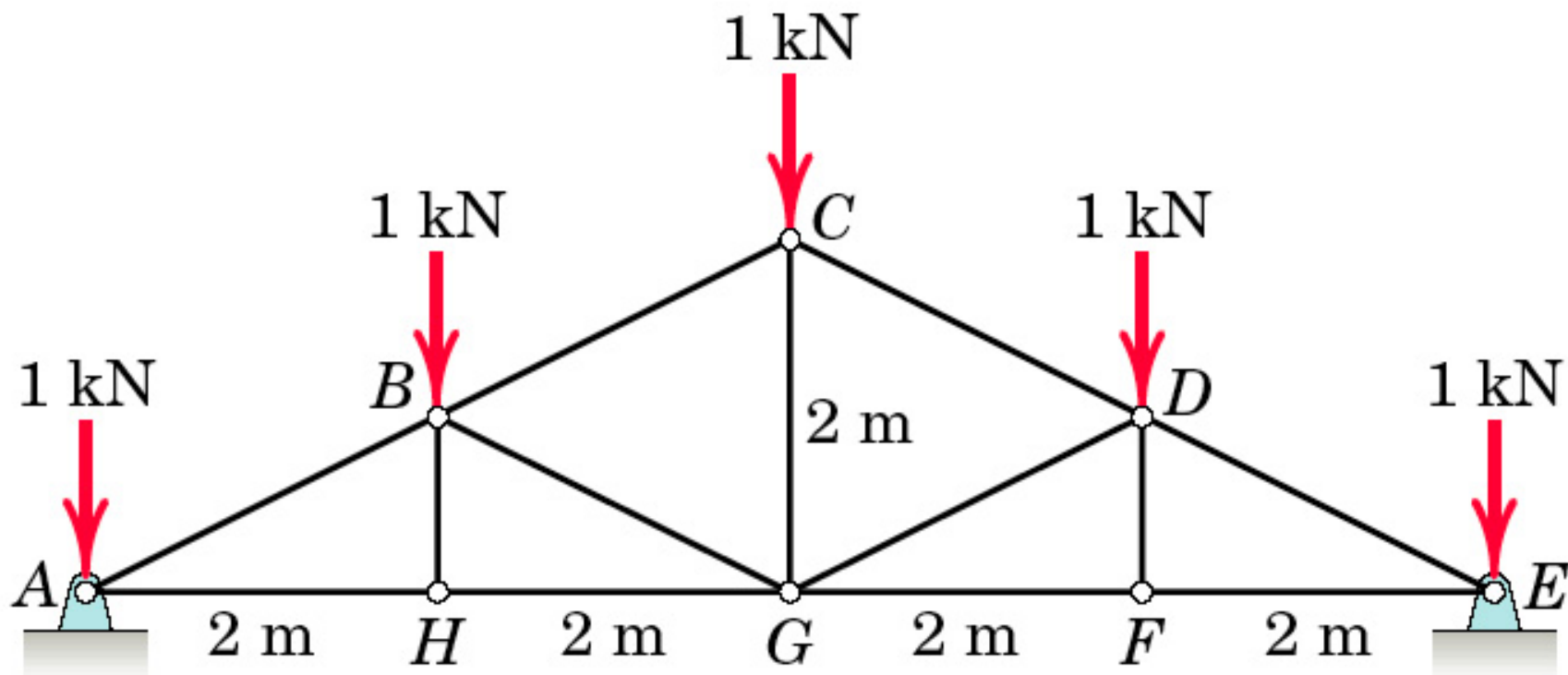


Problem 4/18



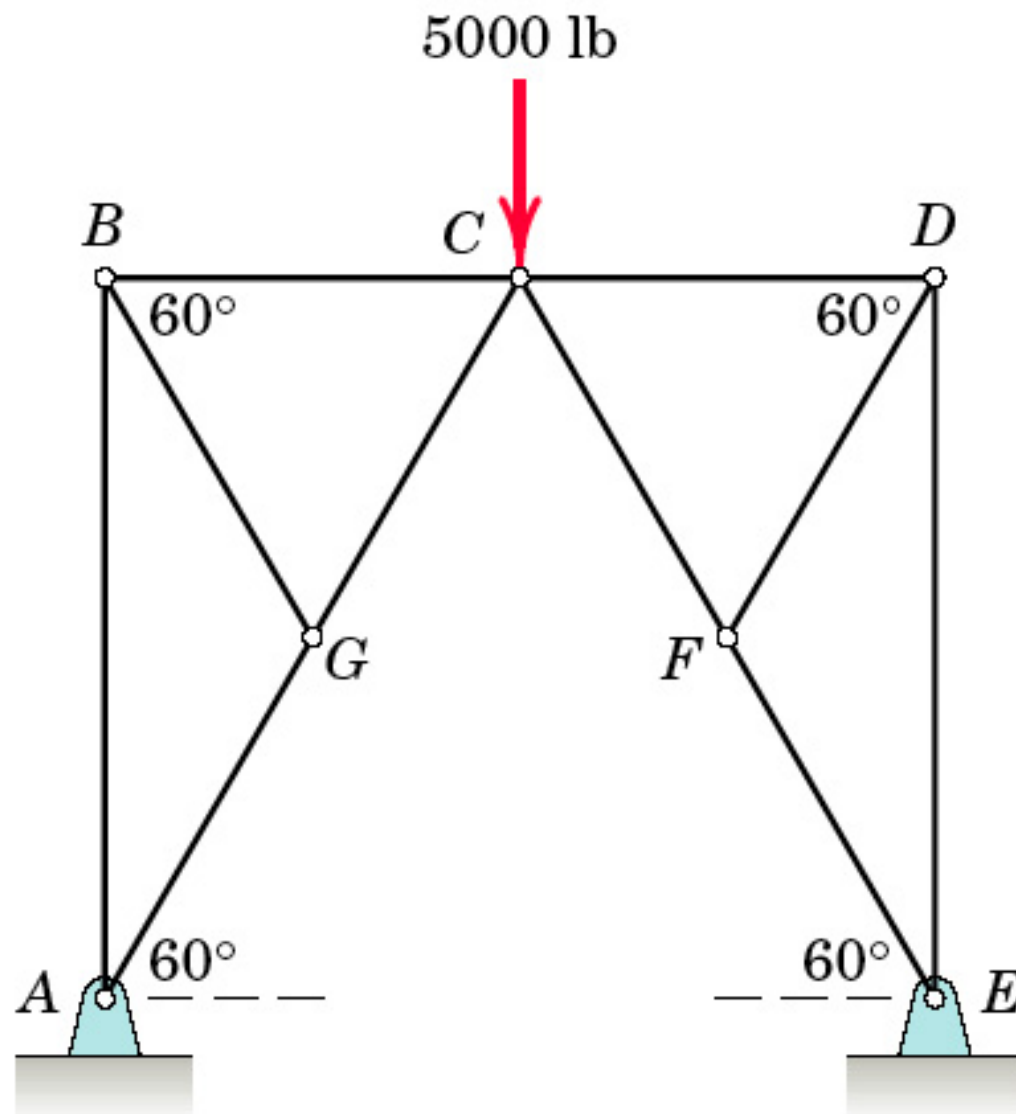
Problem 4/19

P_04_19

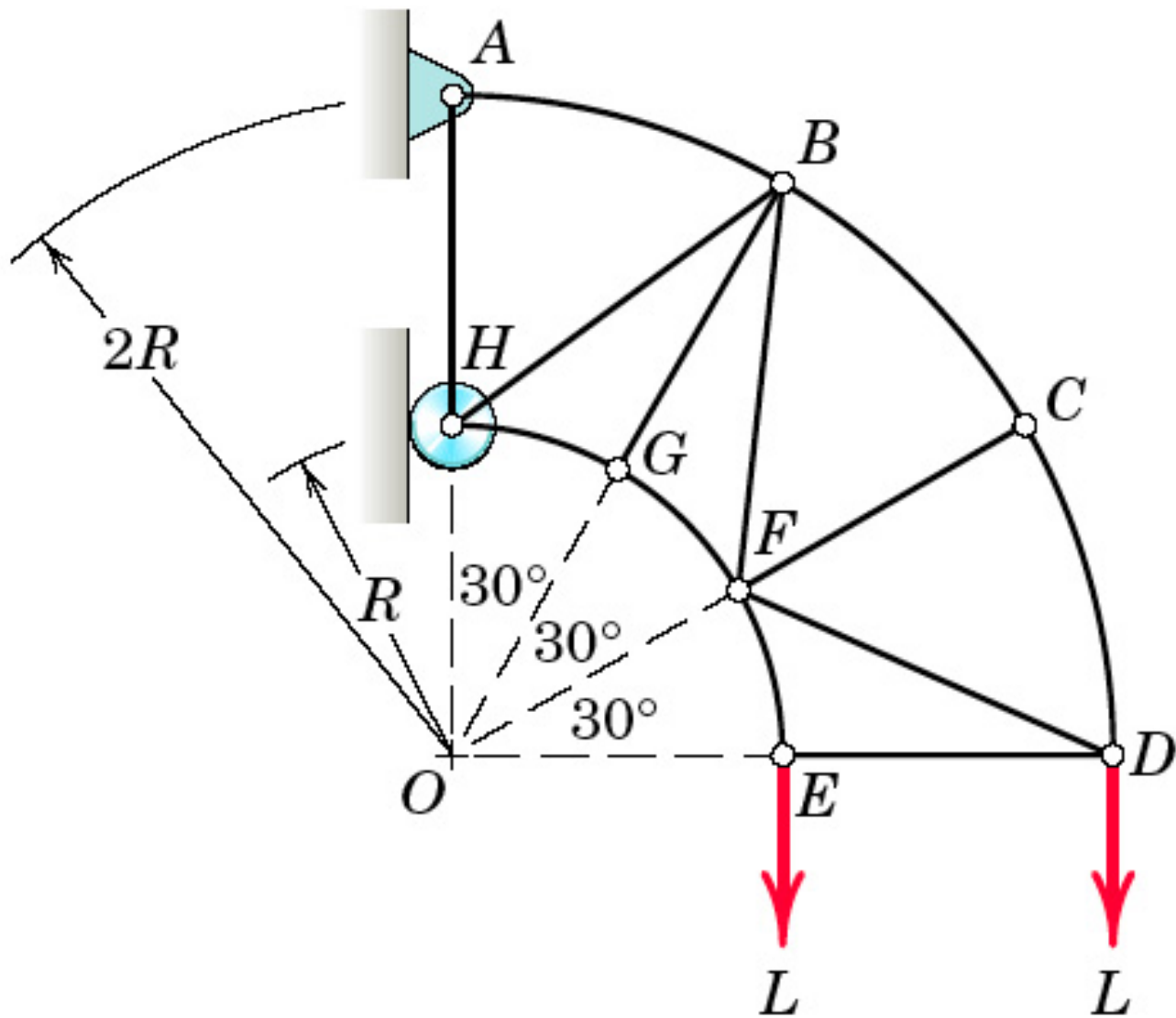


Problem 4/20

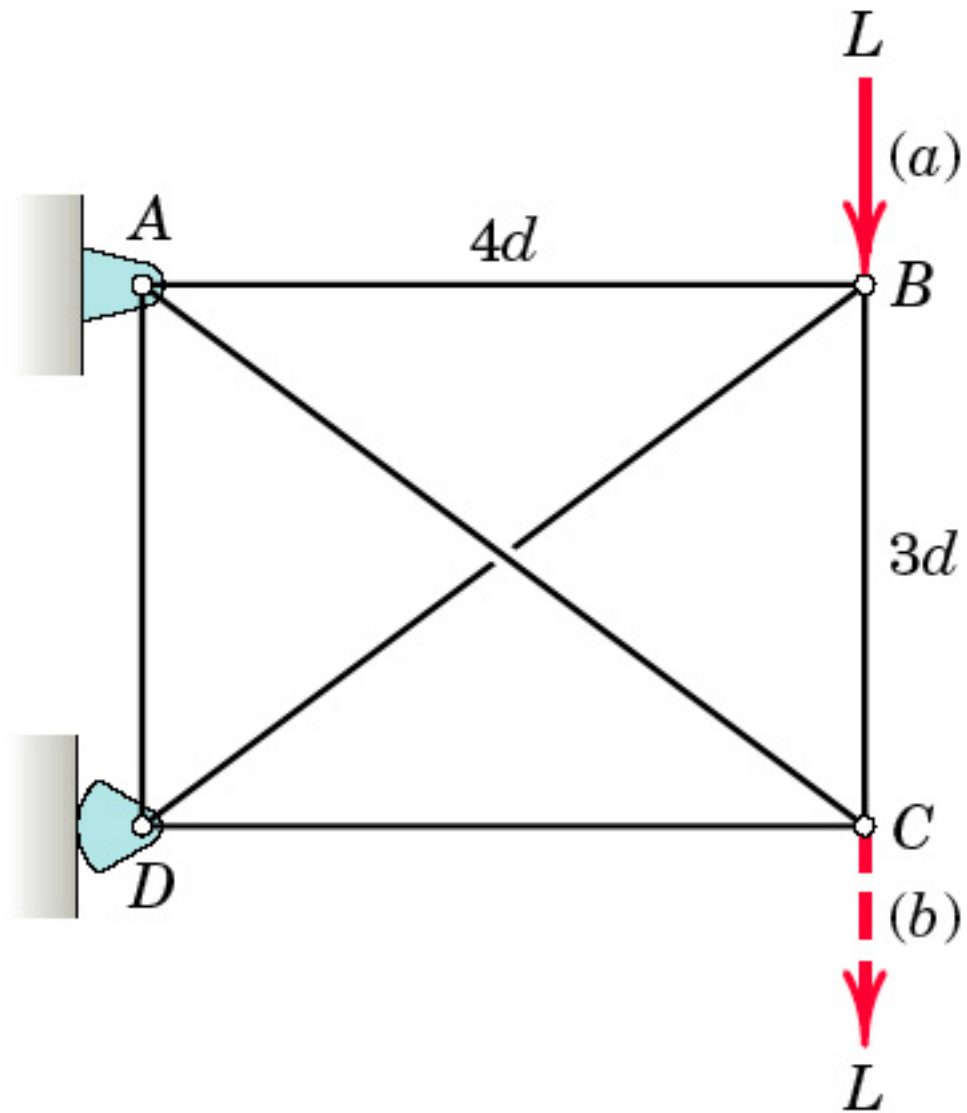
P_04_20



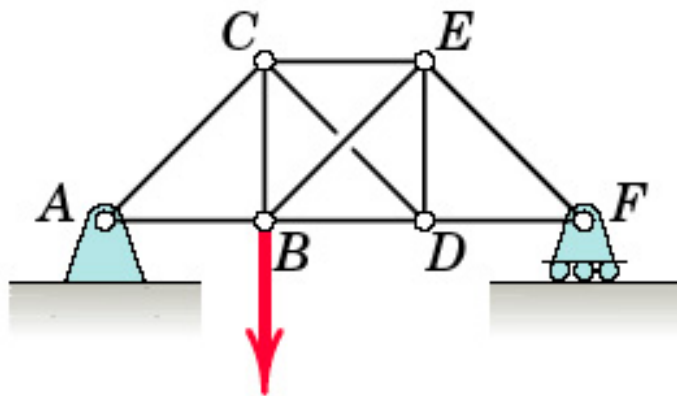
Problem 4/21



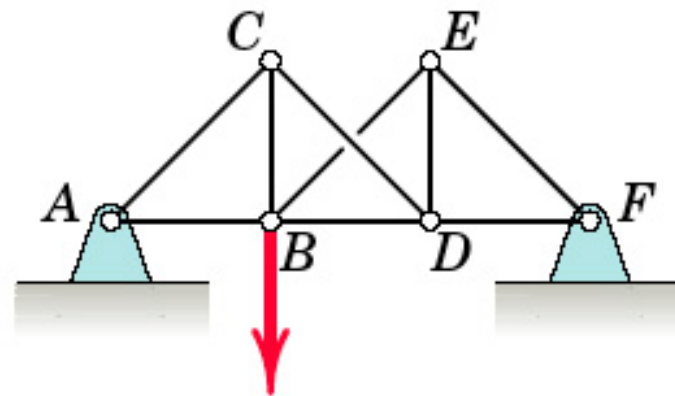
Problem 4/22



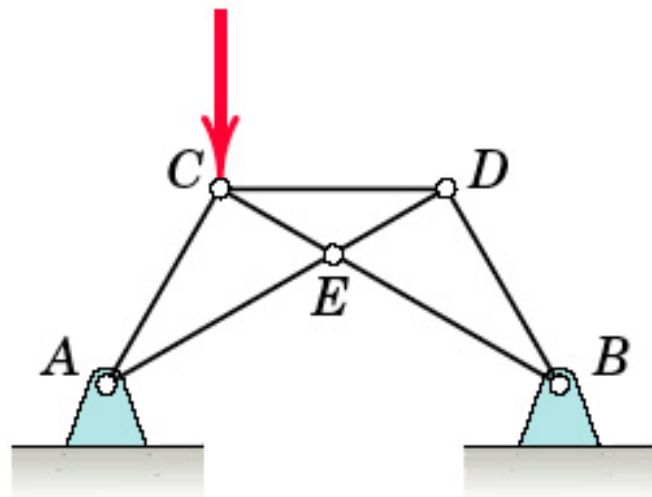
Problem 4/23



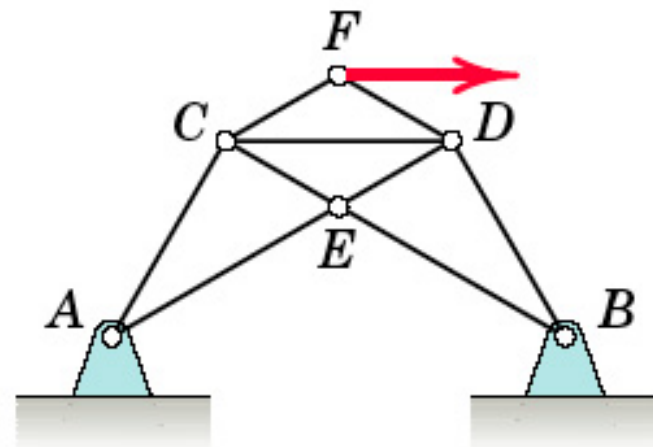
(a)



(b)

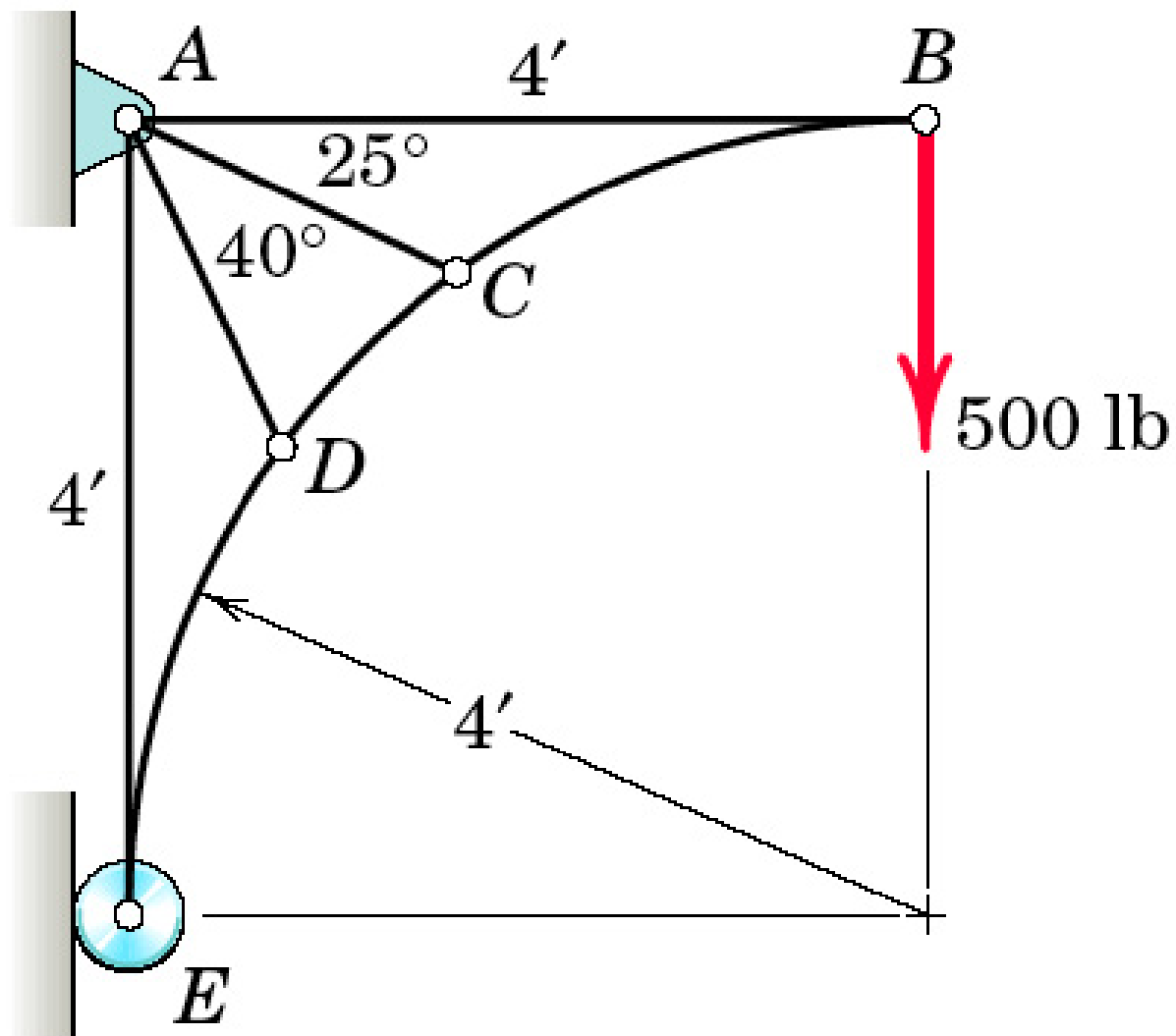


(c)

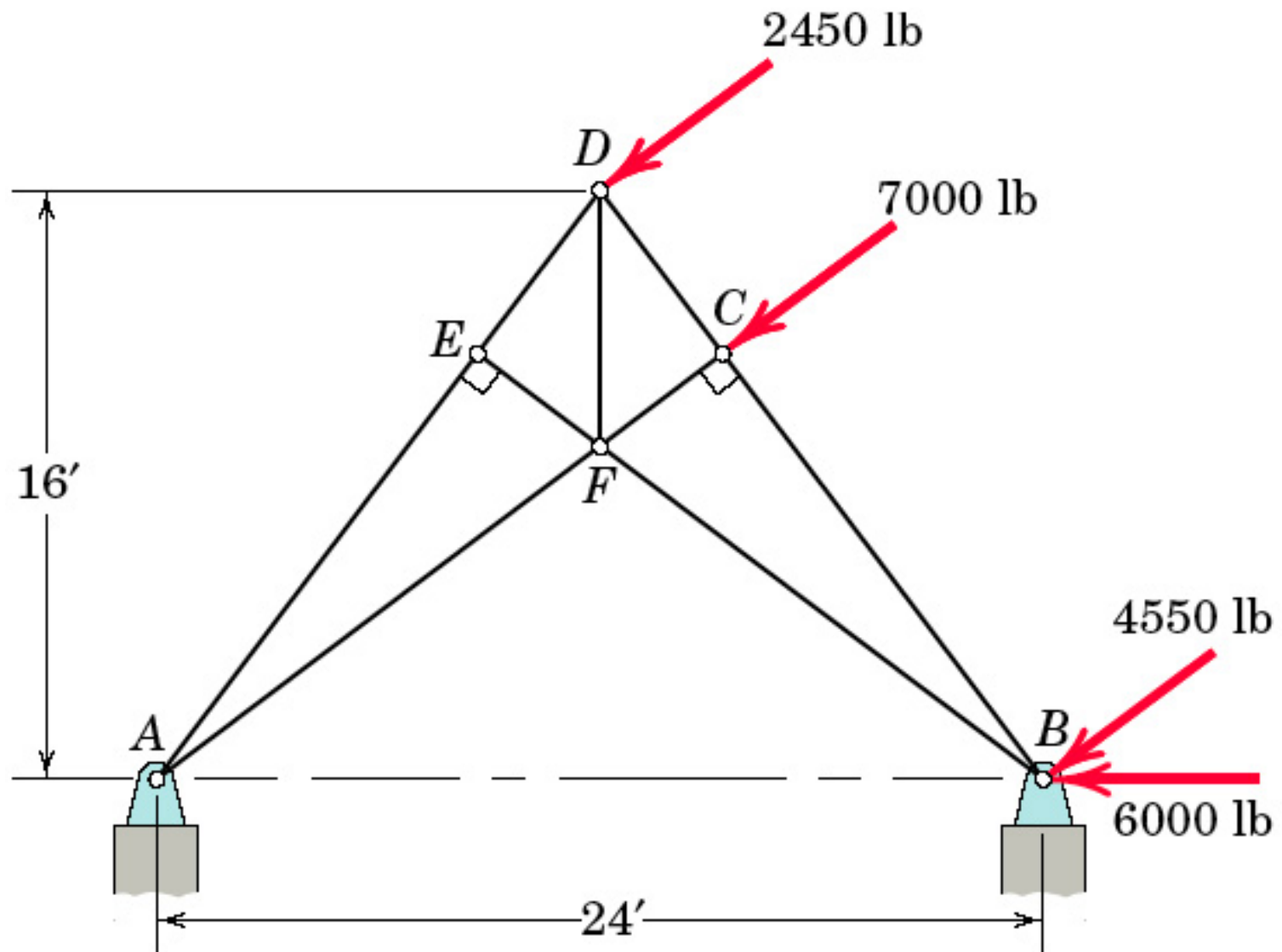


(d)

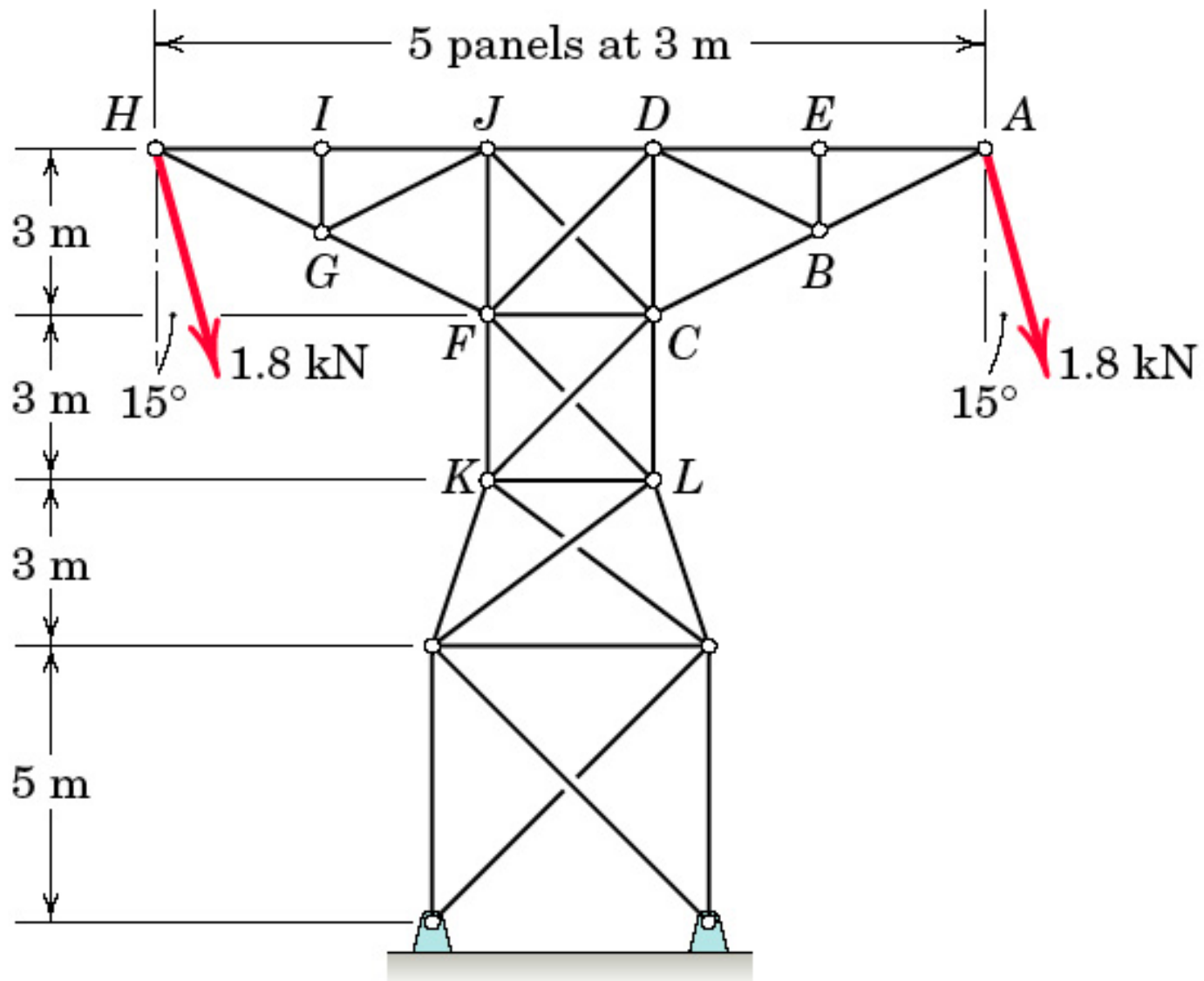
Problem 4/24



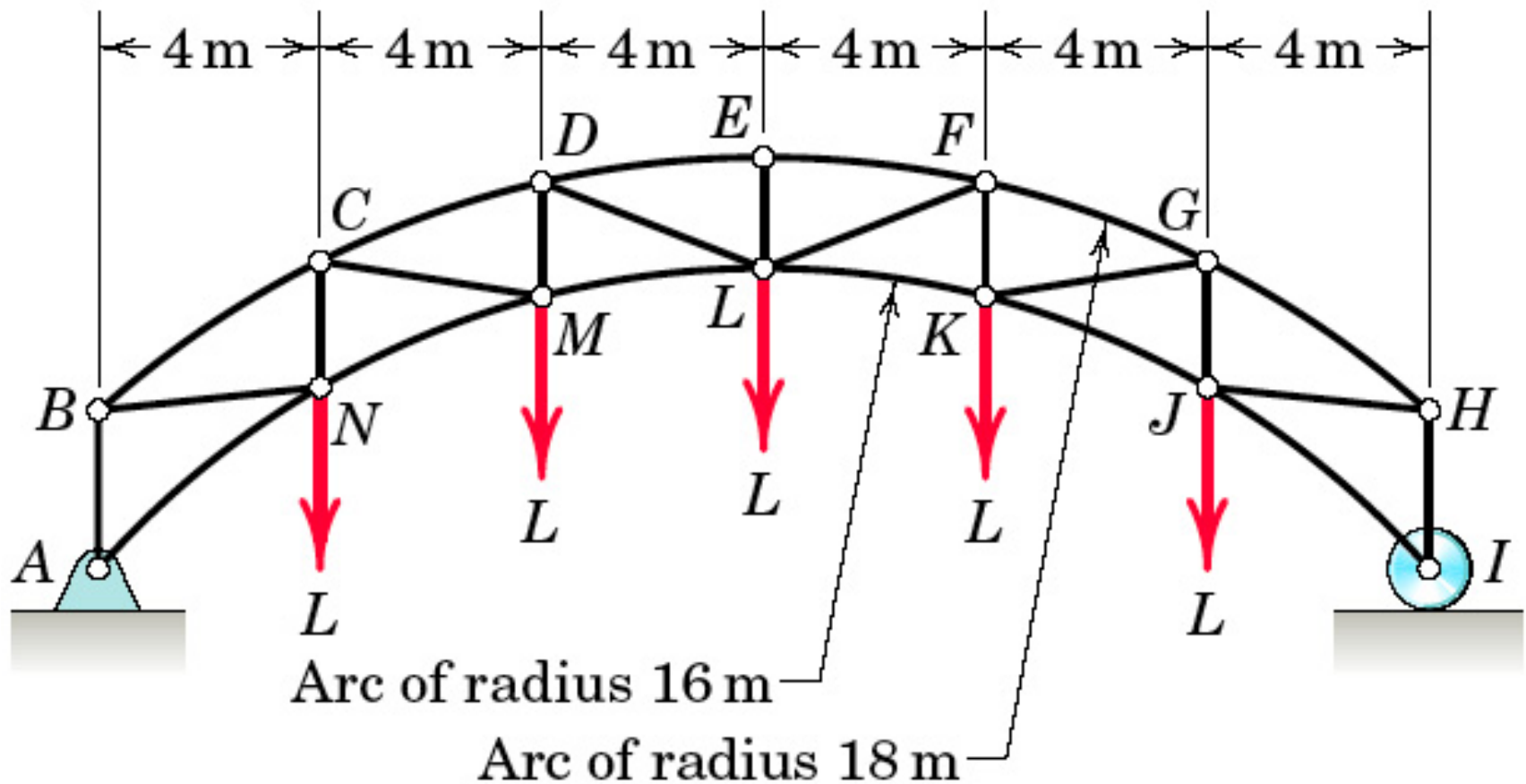
Problem 4/25



Problem 4/26

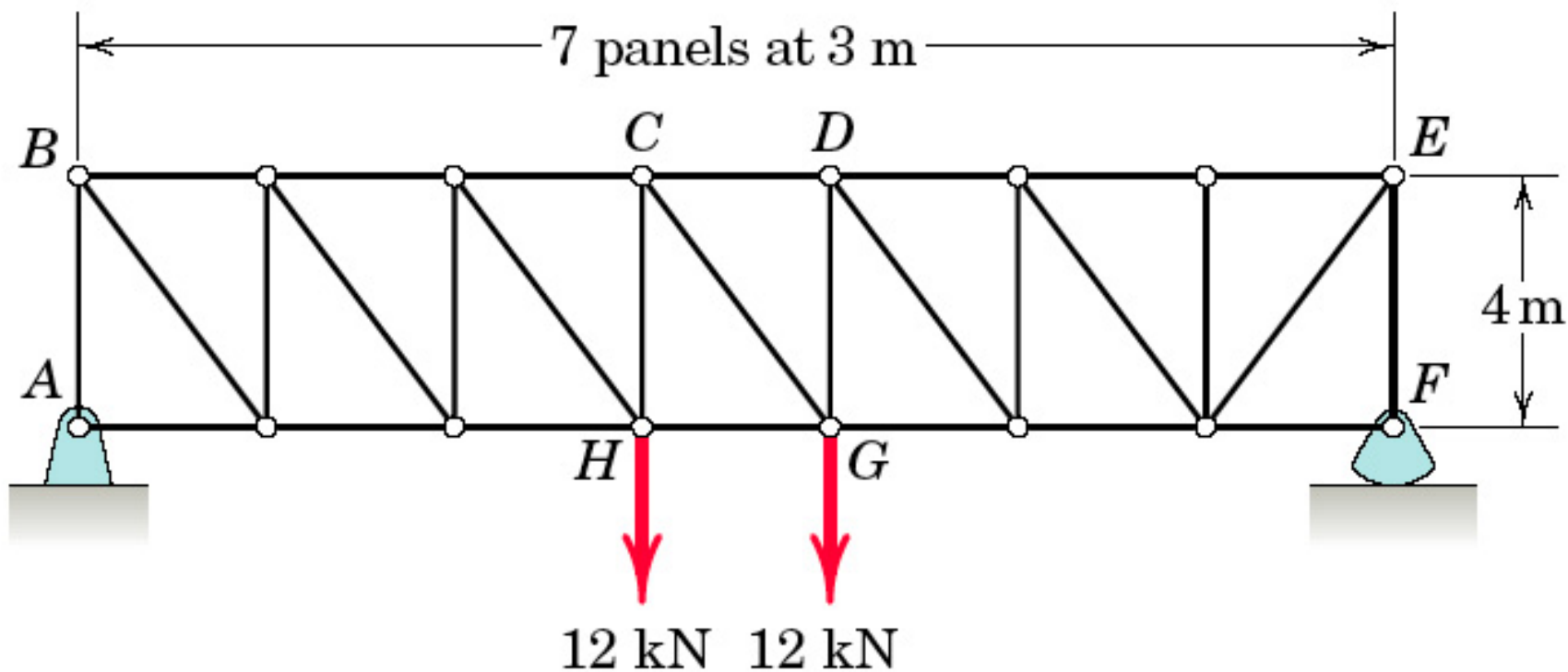


Problem 4/27



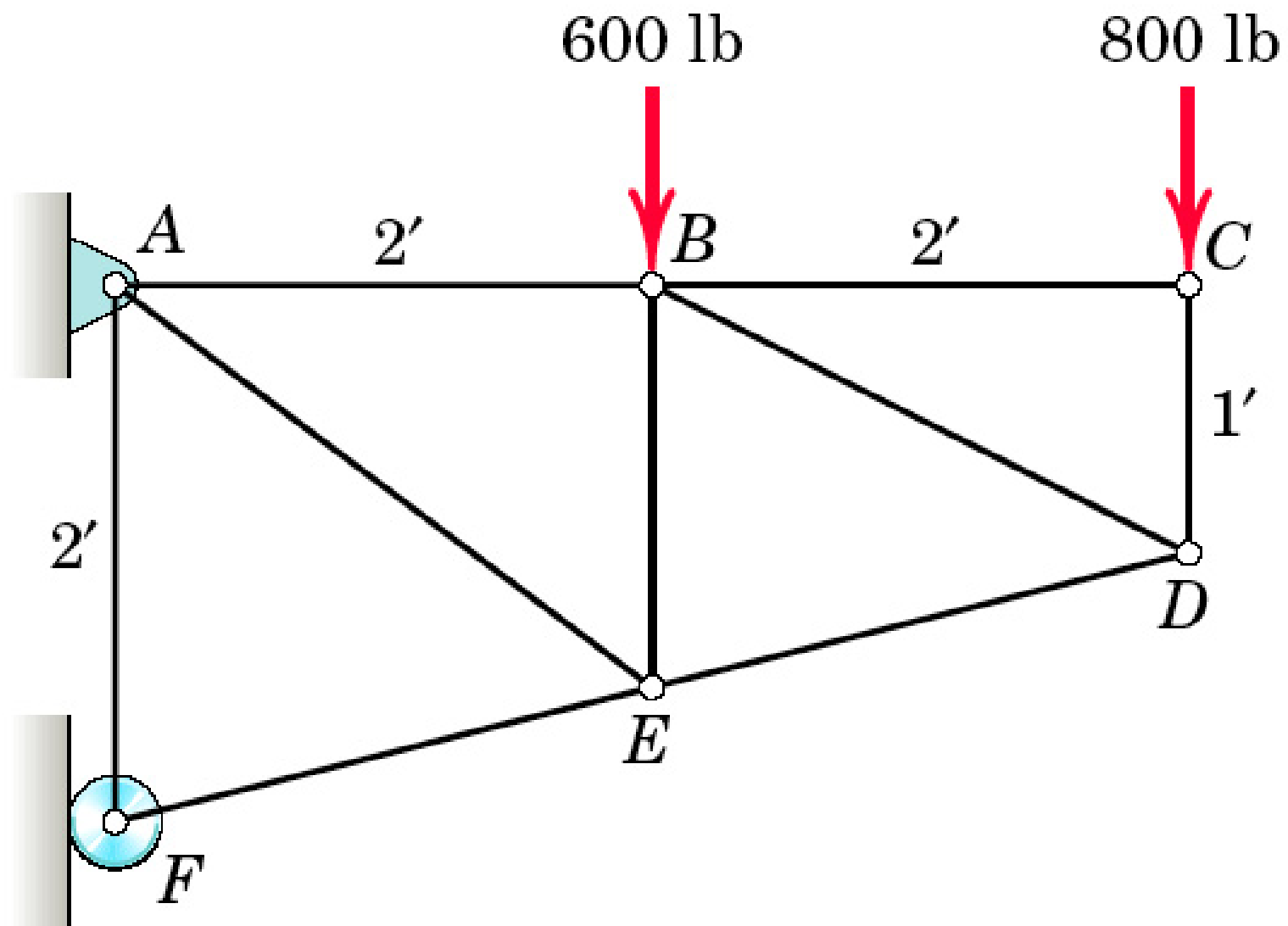
Problem 4/28

P_04_28

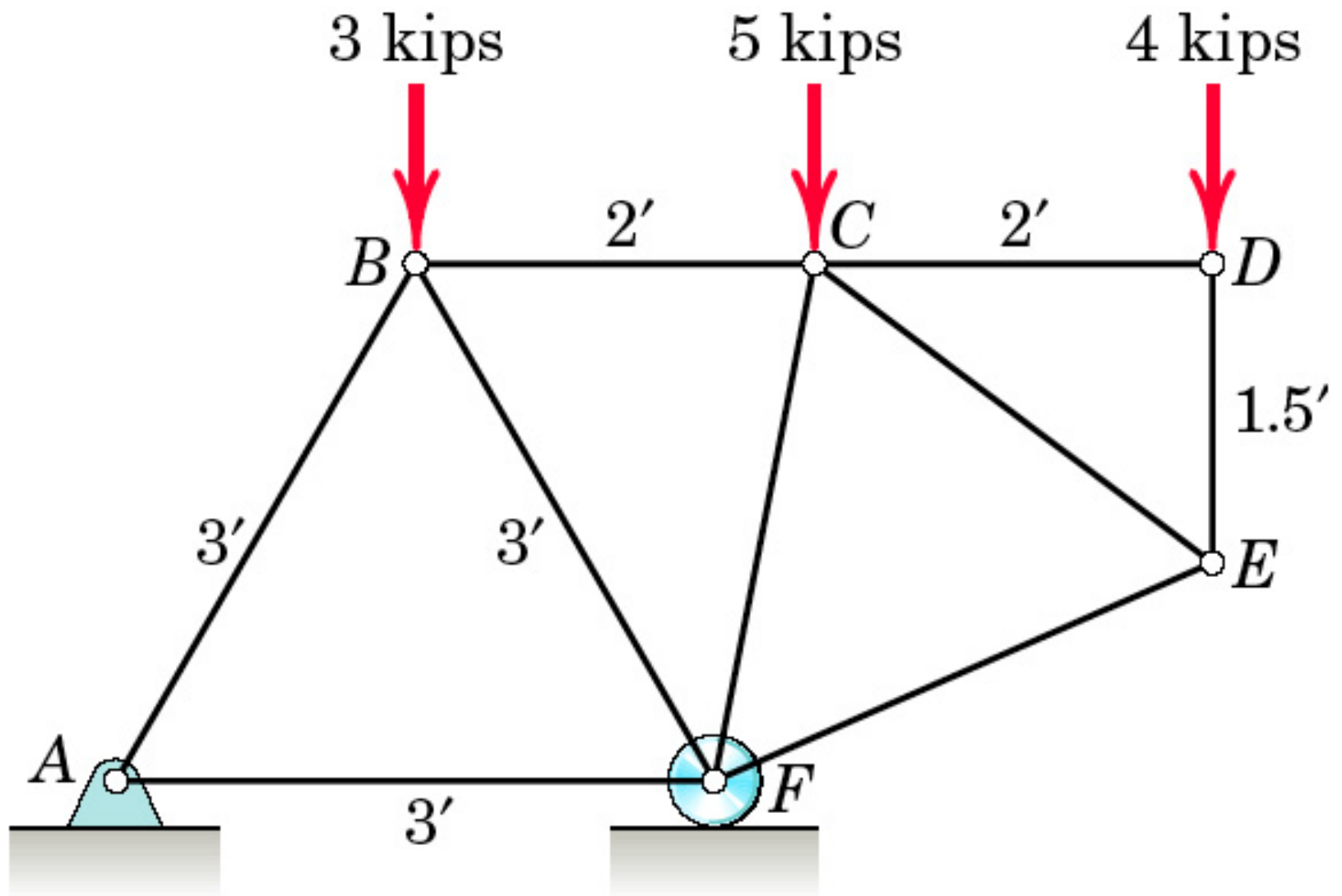


Problem 4/29

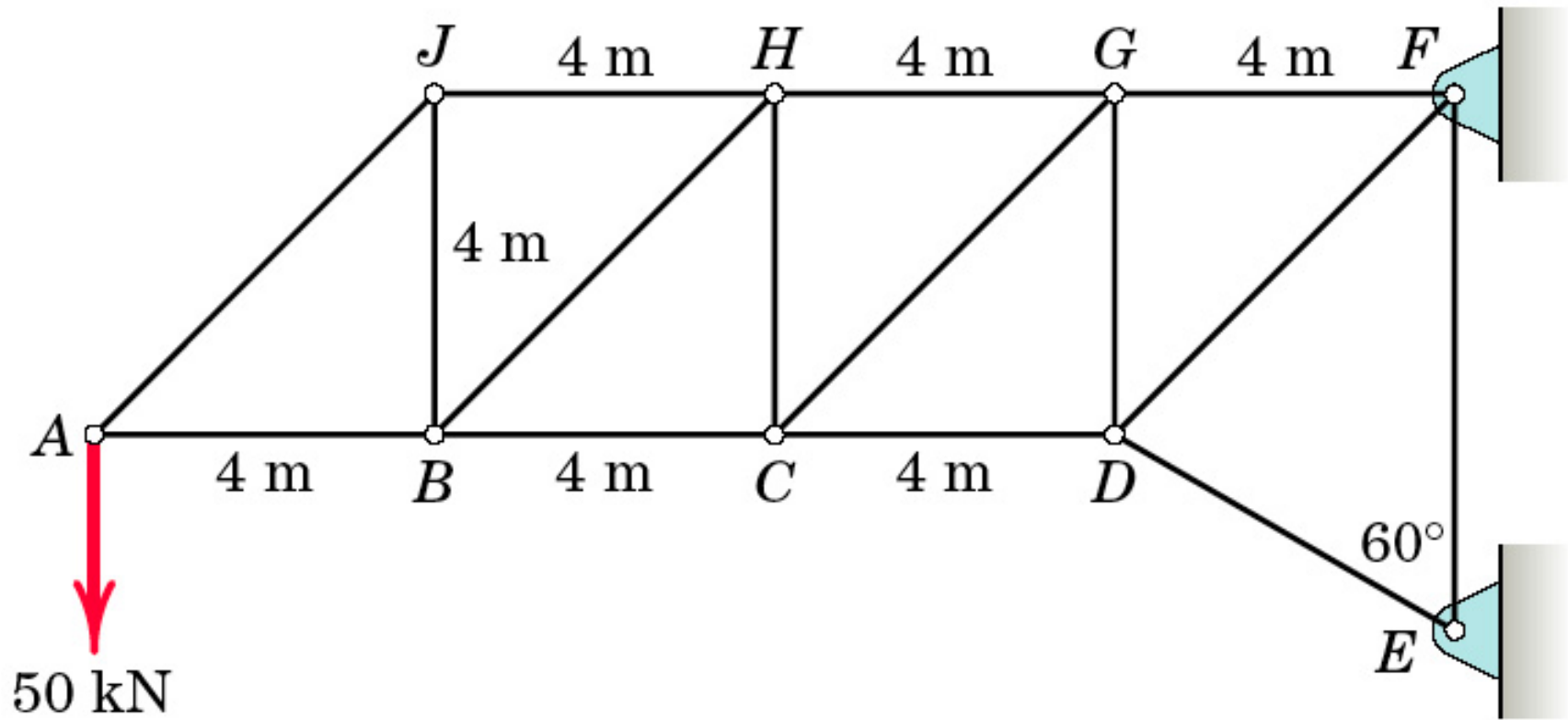
P_04_29



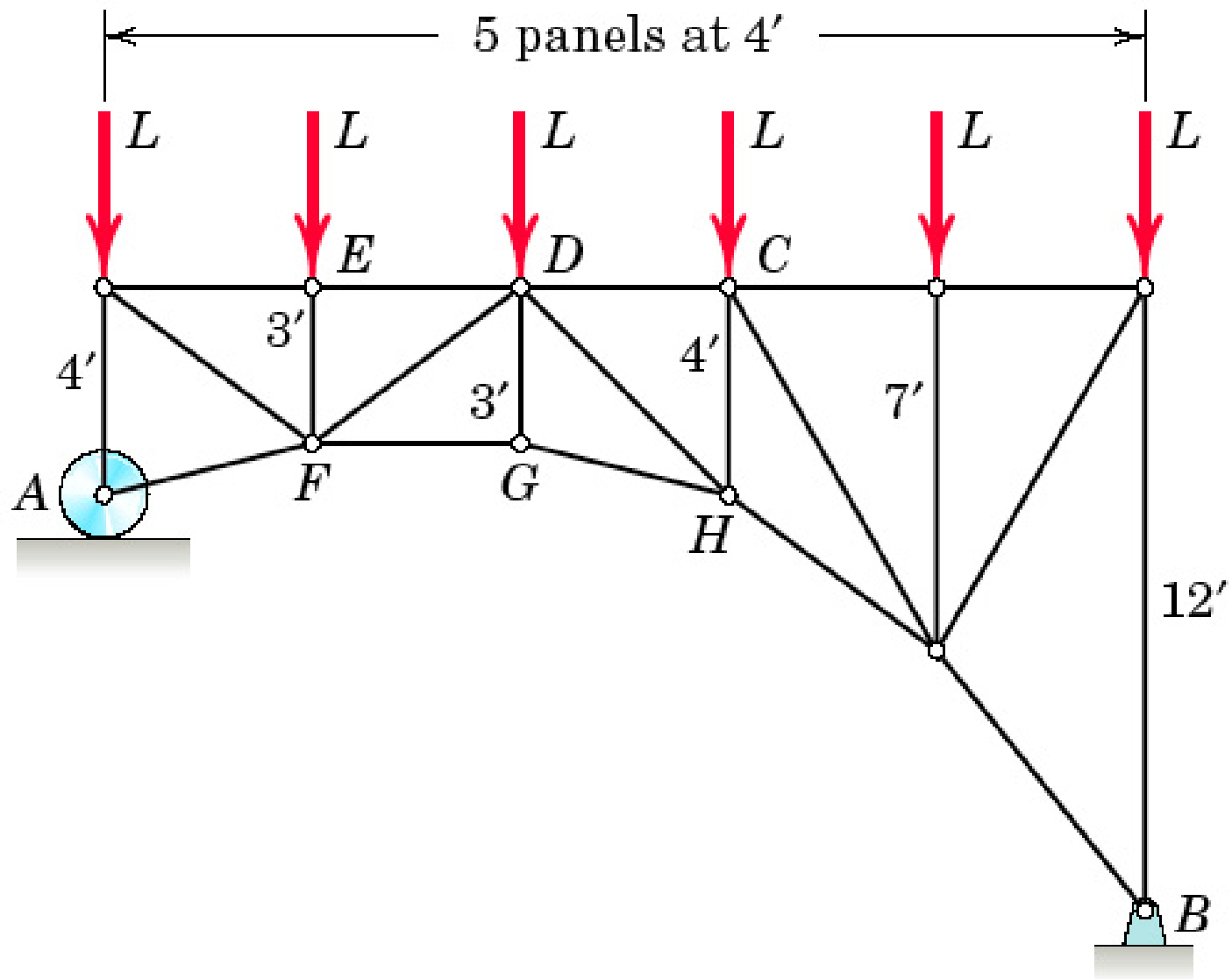
Problem 4/30



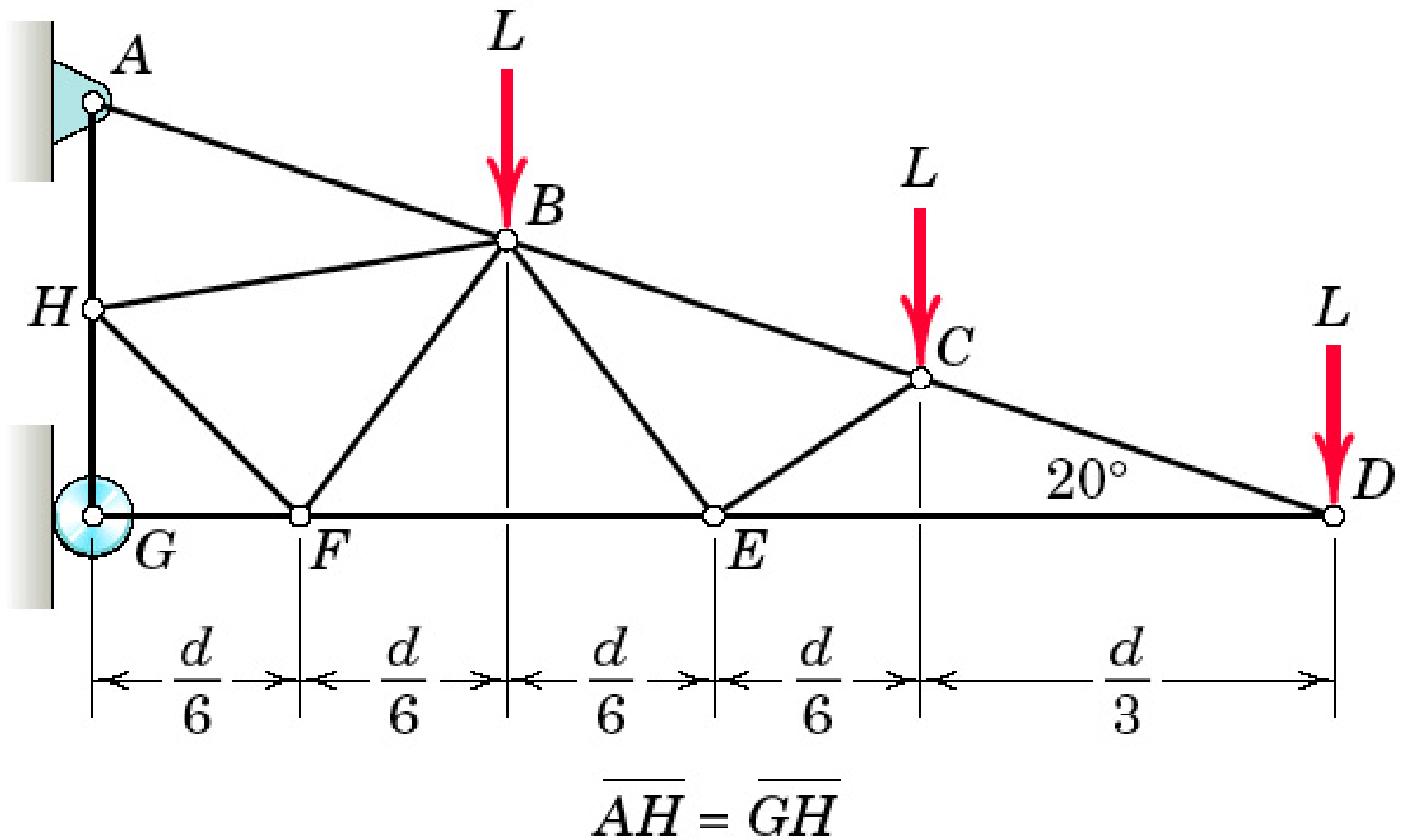
Problem 4/31



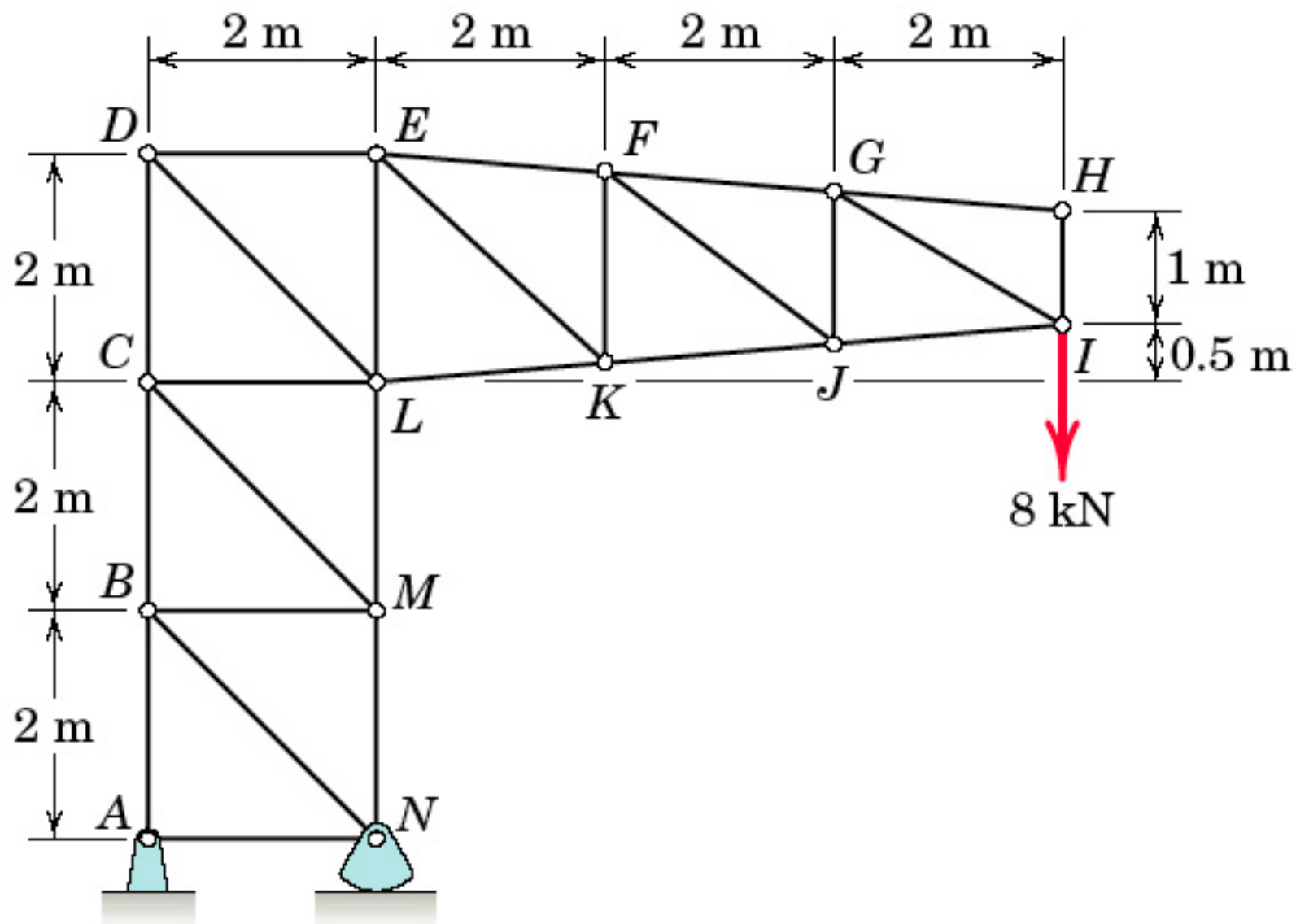
P_04_32



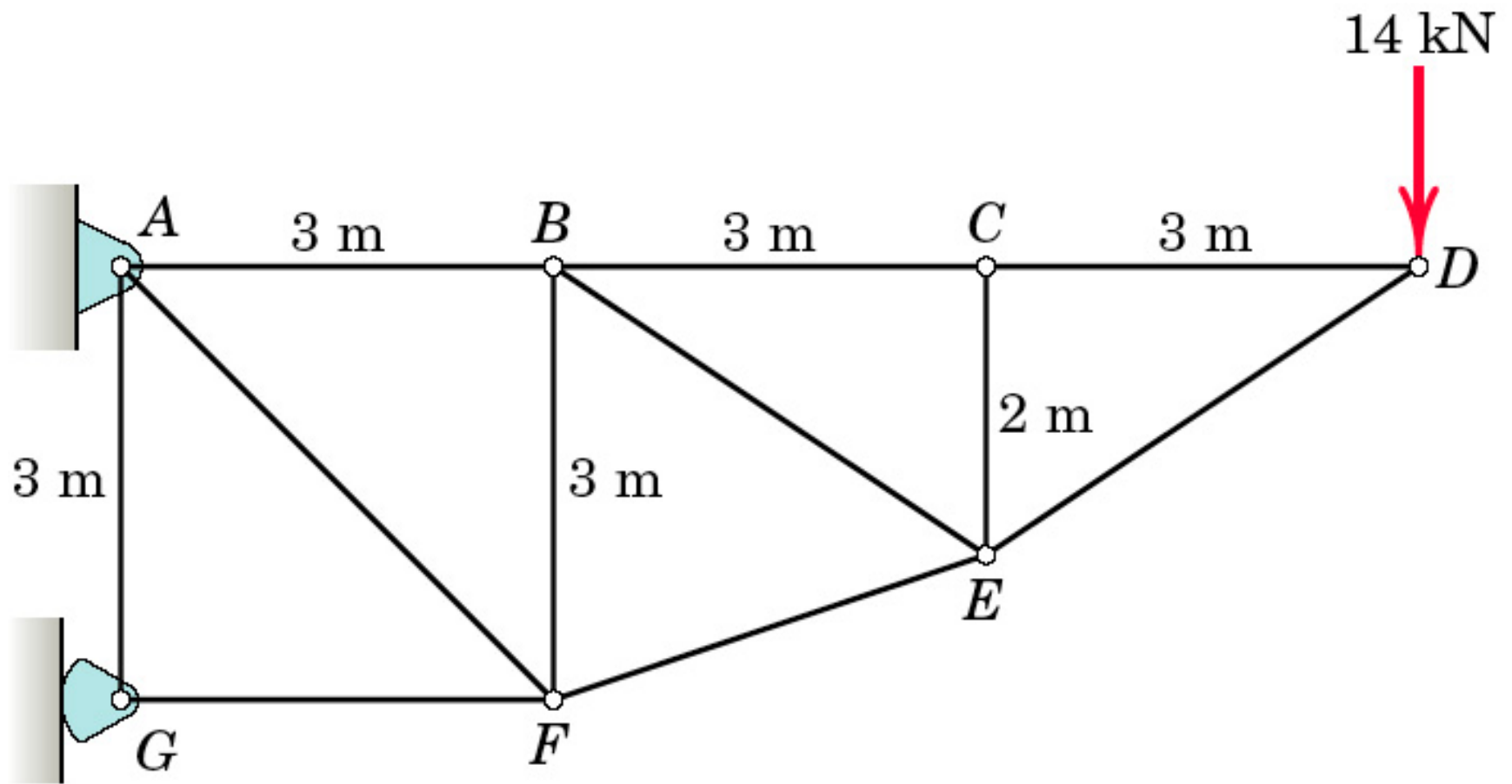
Problem 4/33



Problem 4/34

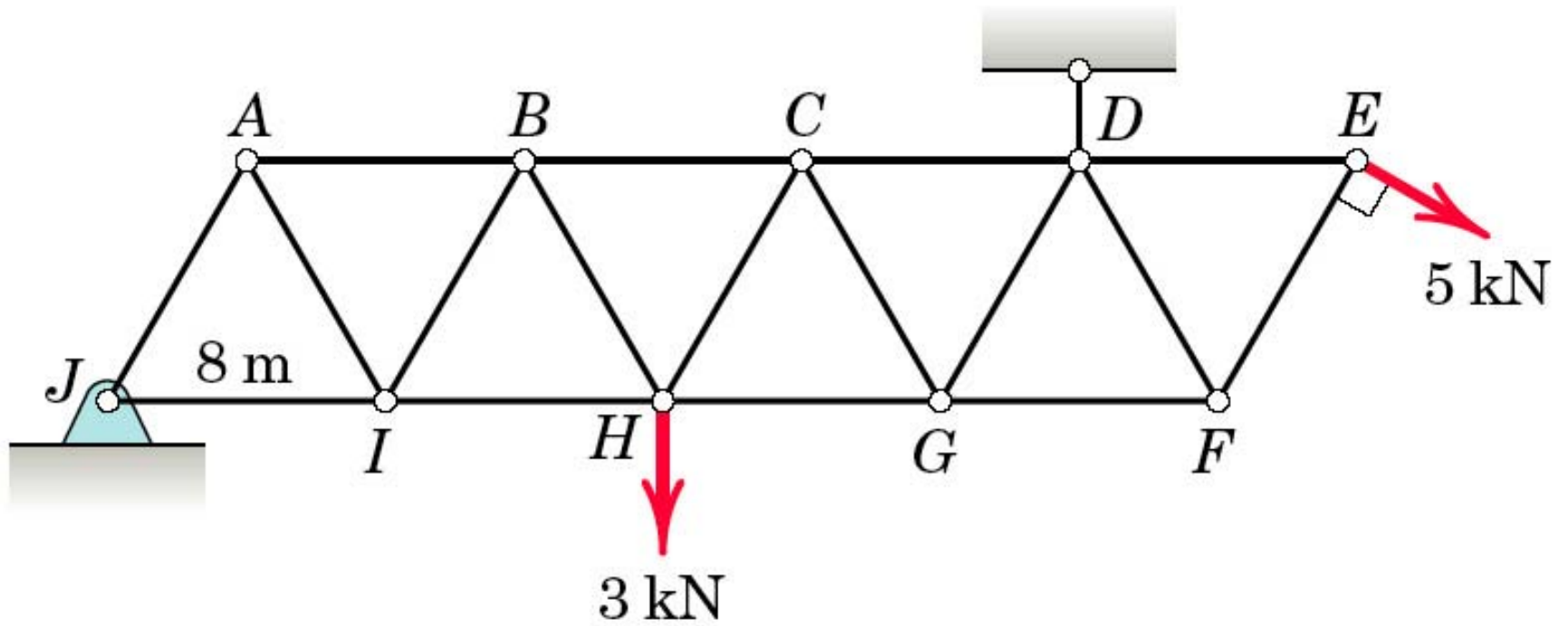


Problem 4/35



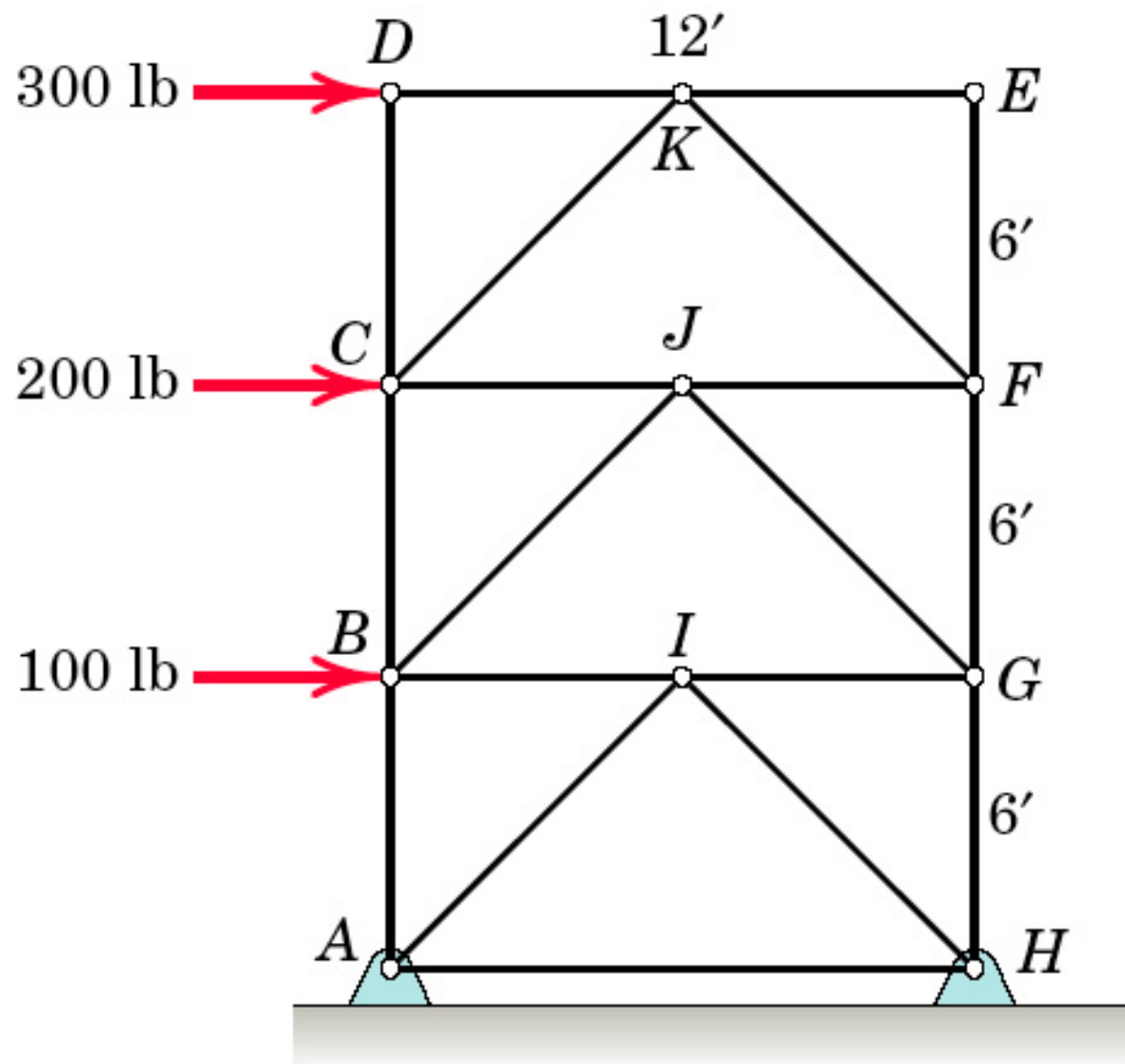
Problem 4/36

P_04_36

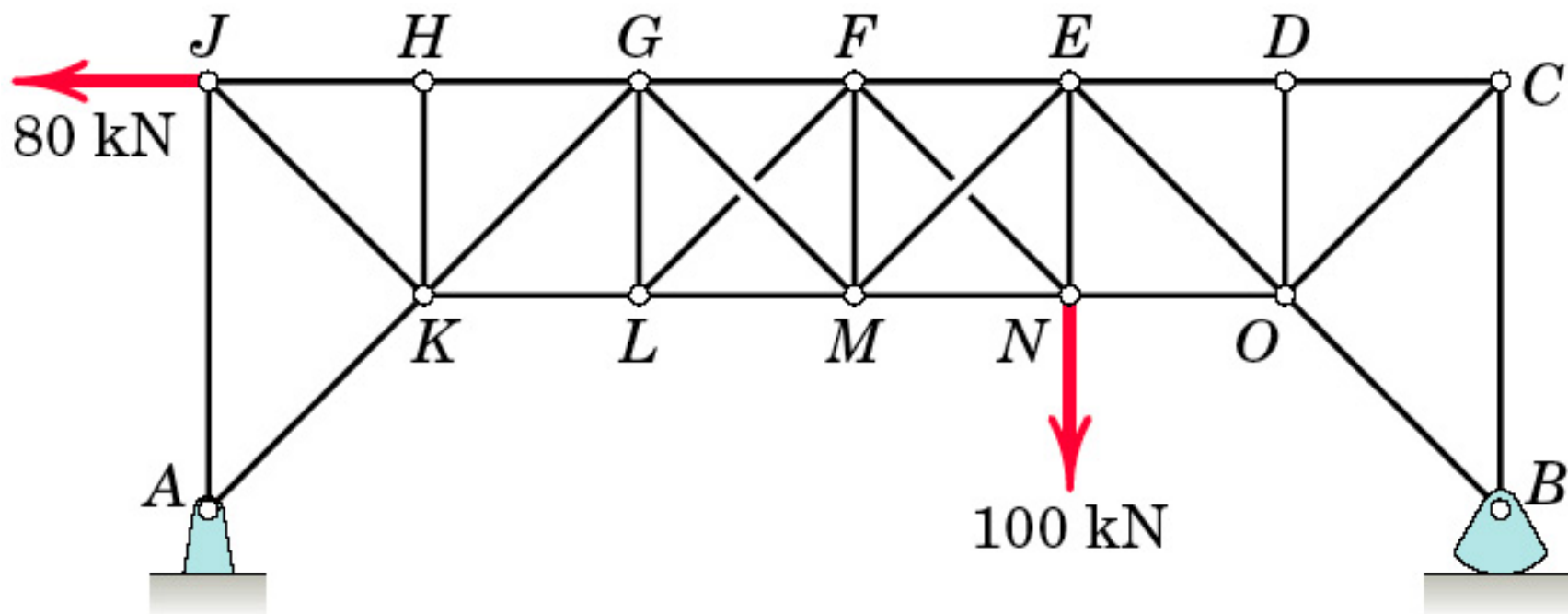


Problem 4/37

P_04_37

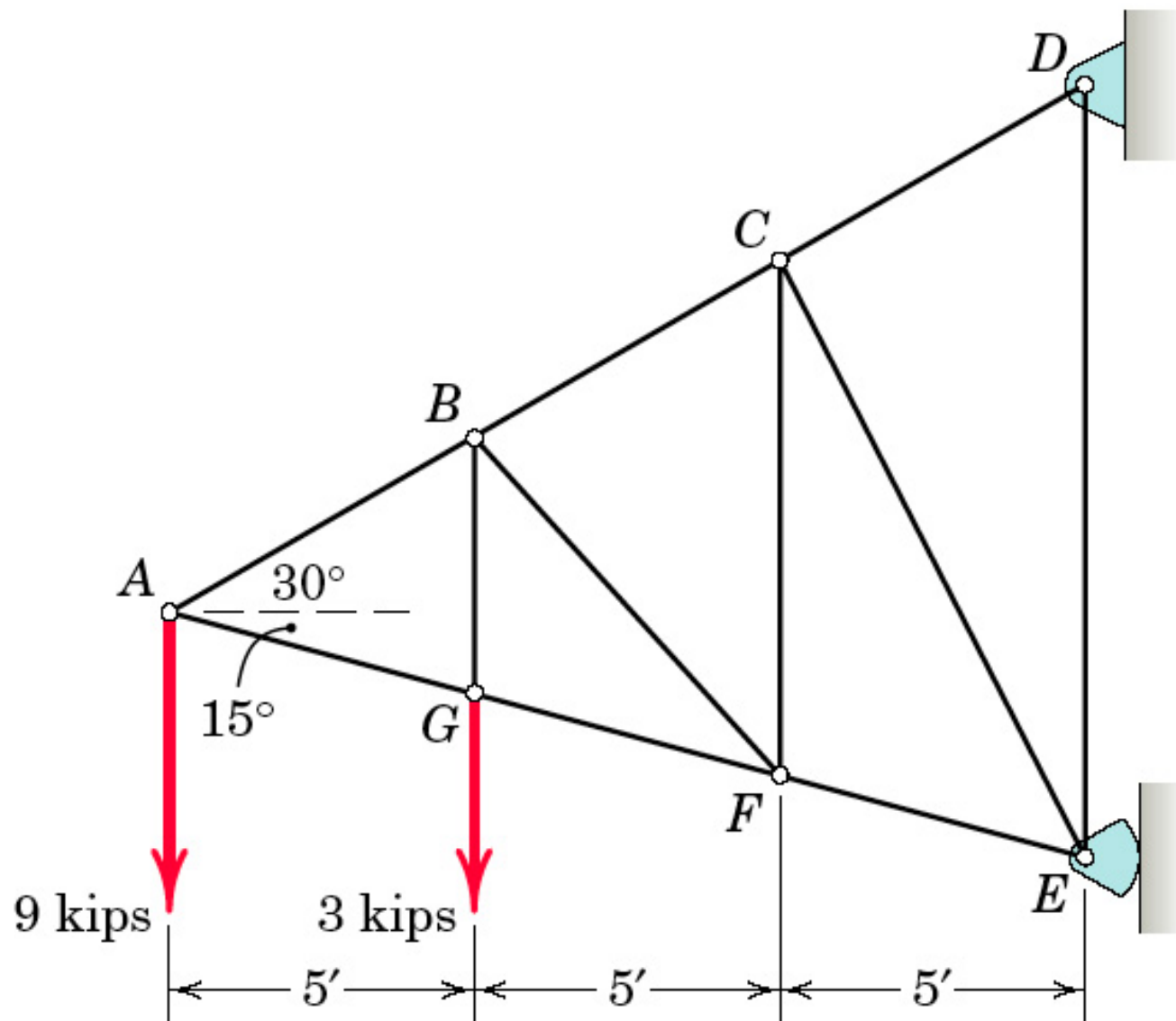


Problem 4/38

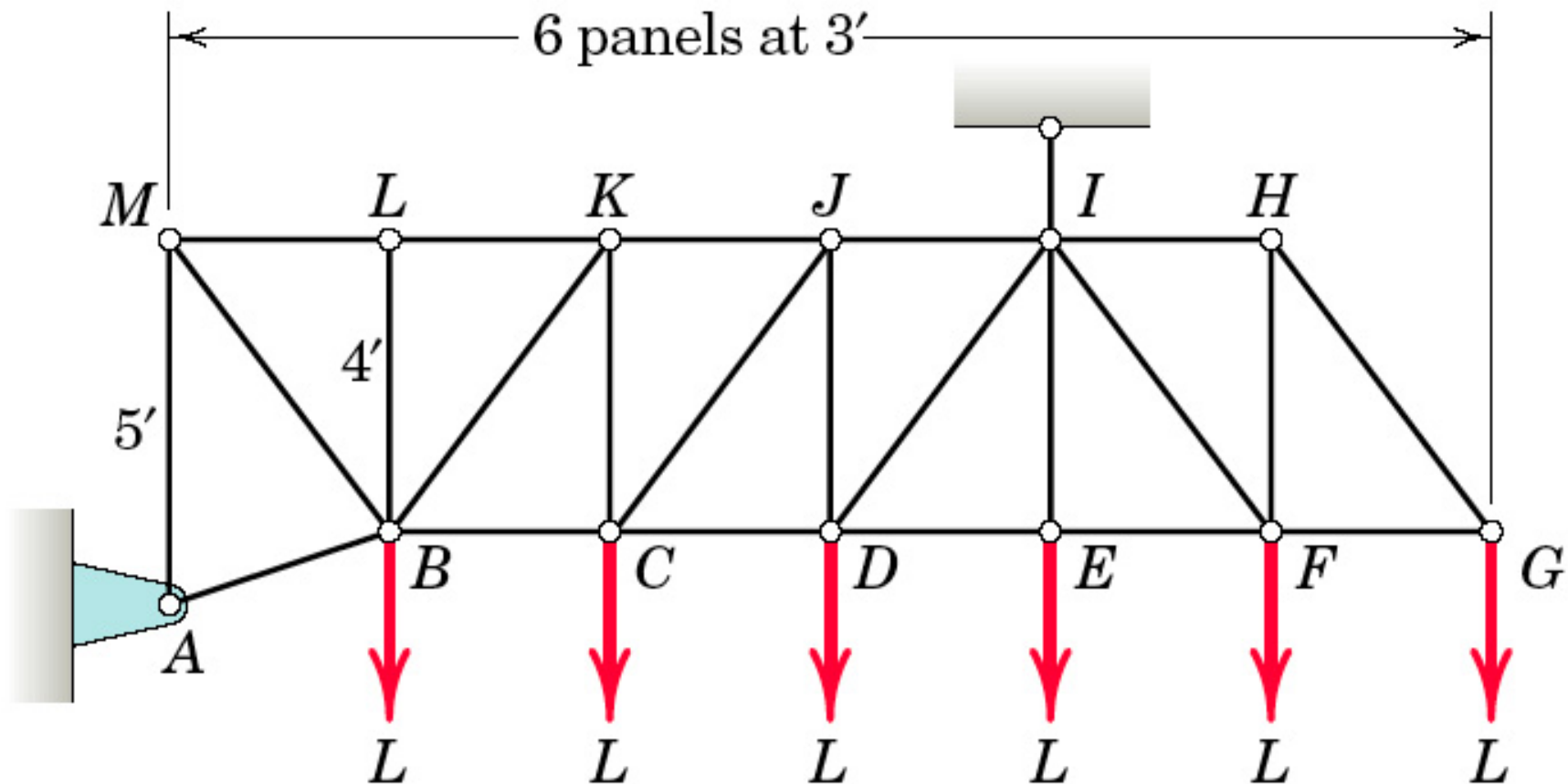


Problem 4/39

P_04_39

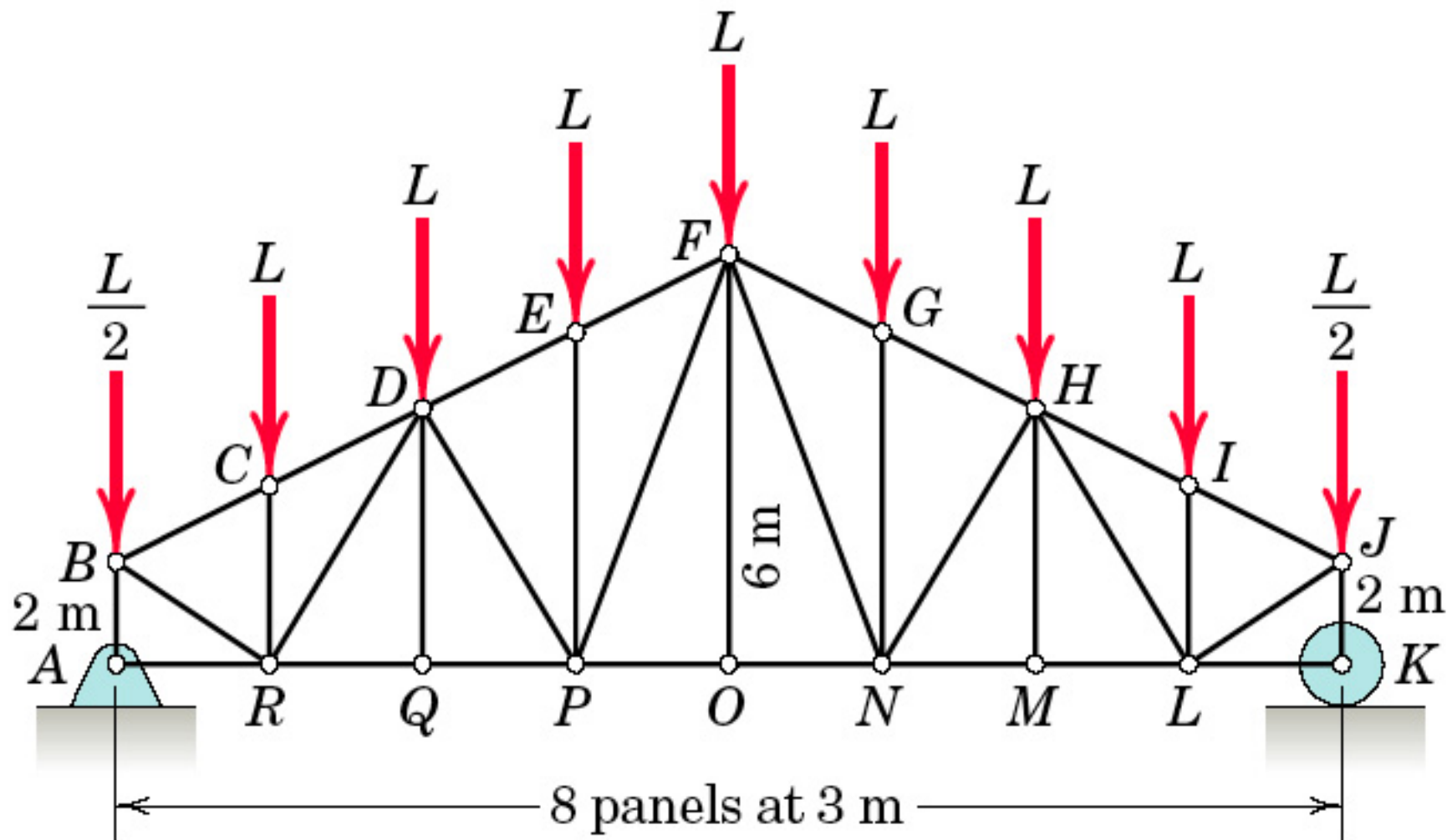


Problem 4/40

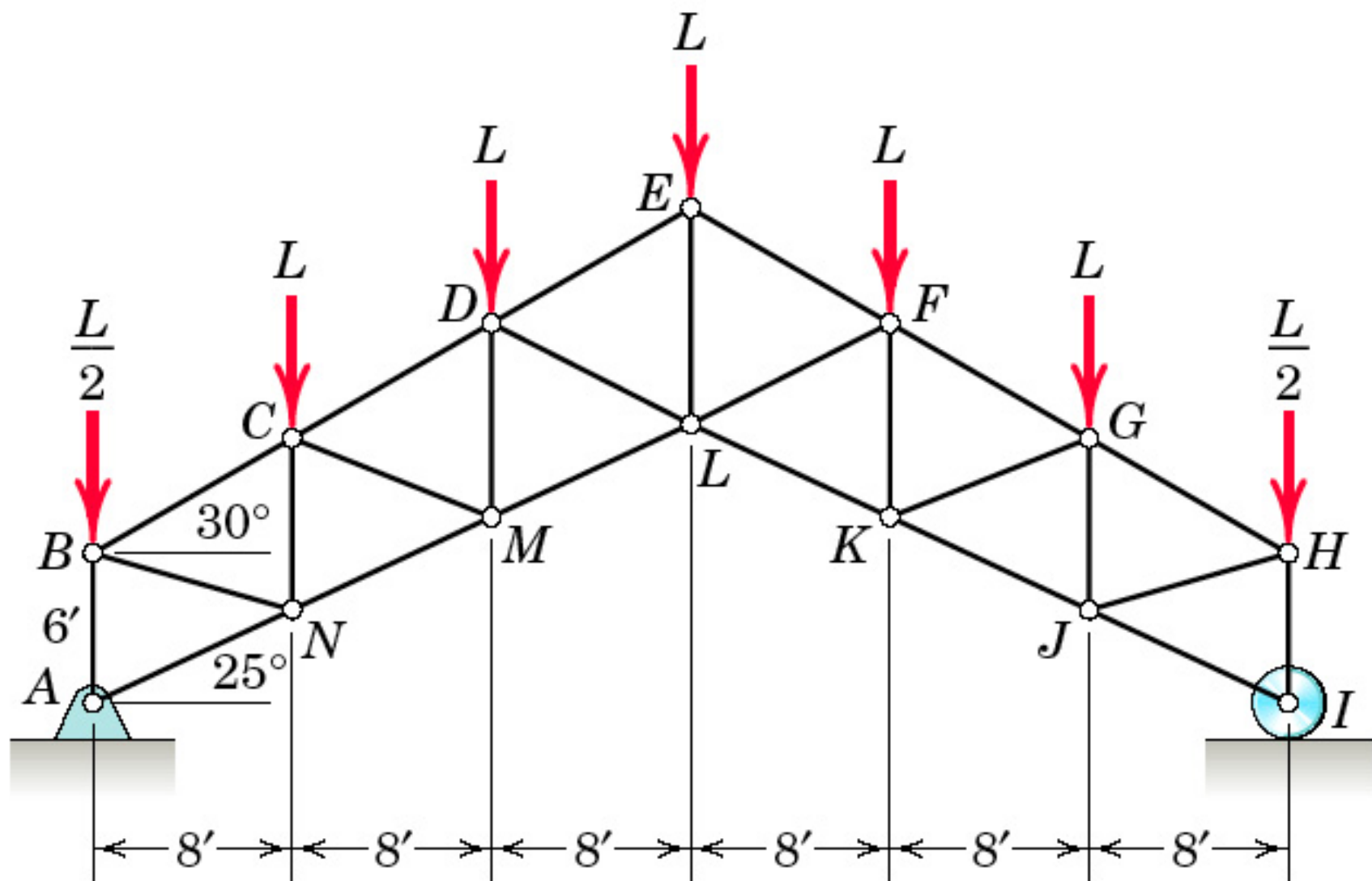


Problem 4/41

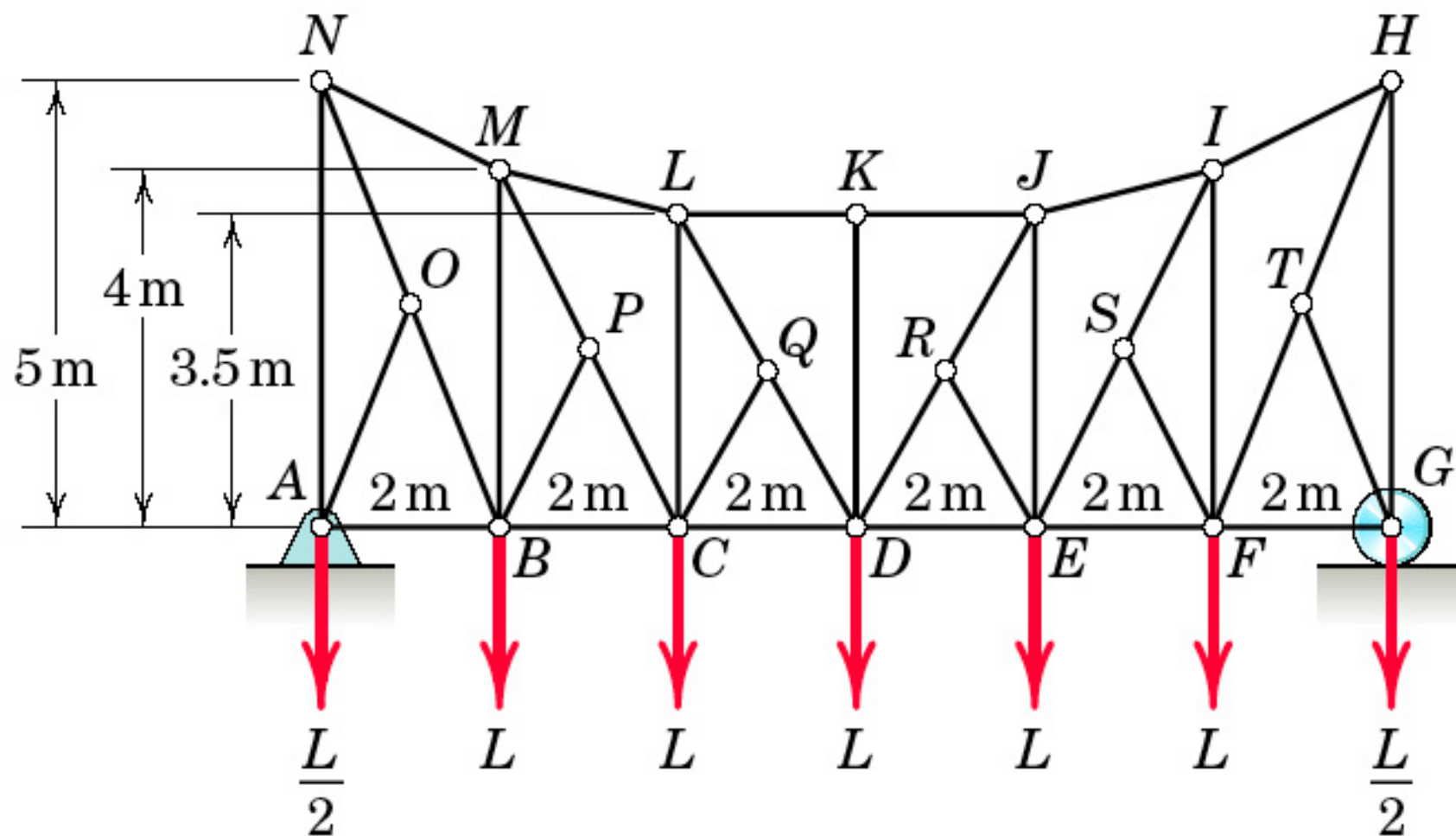
P_04_41



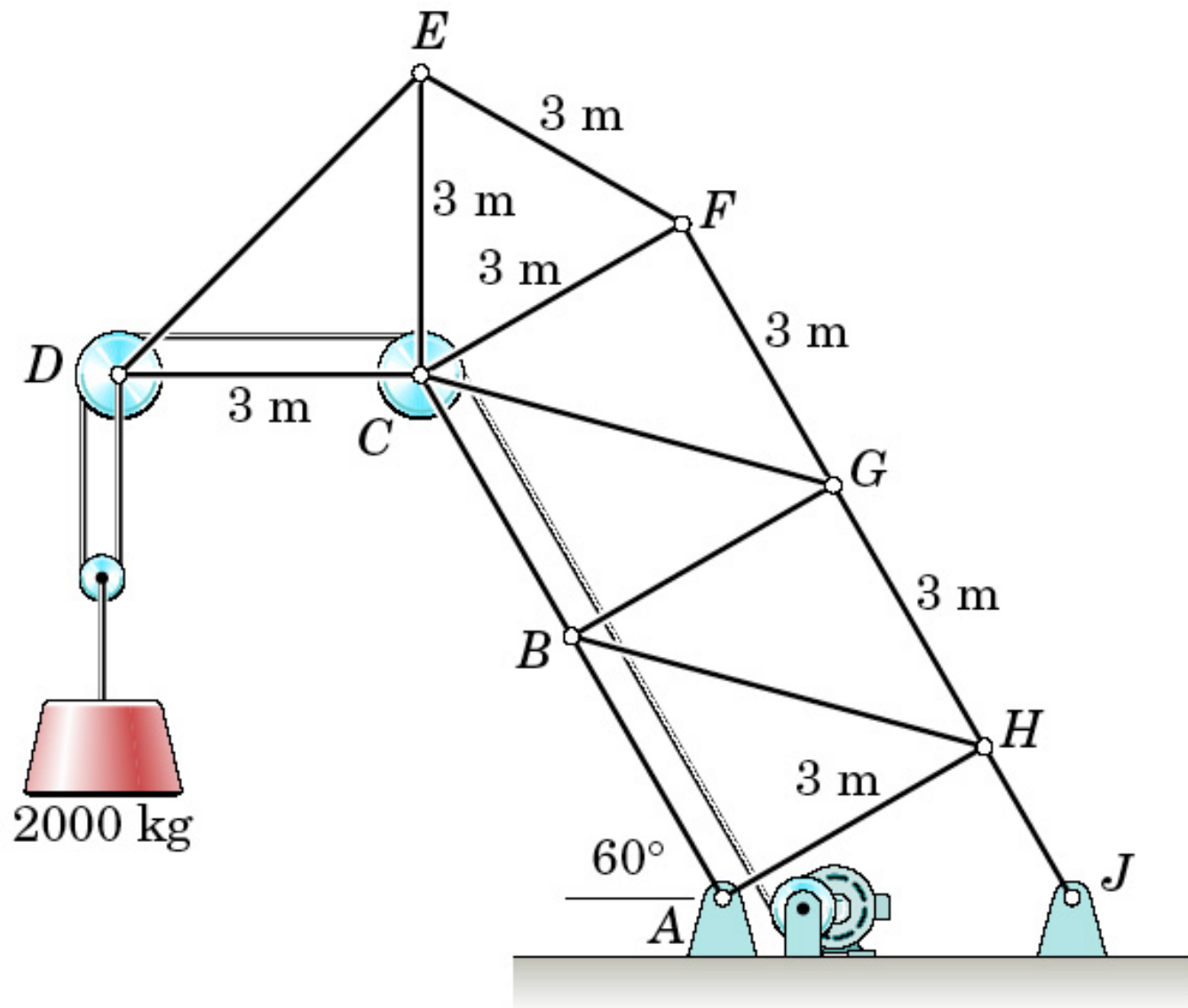
Problem 4/42



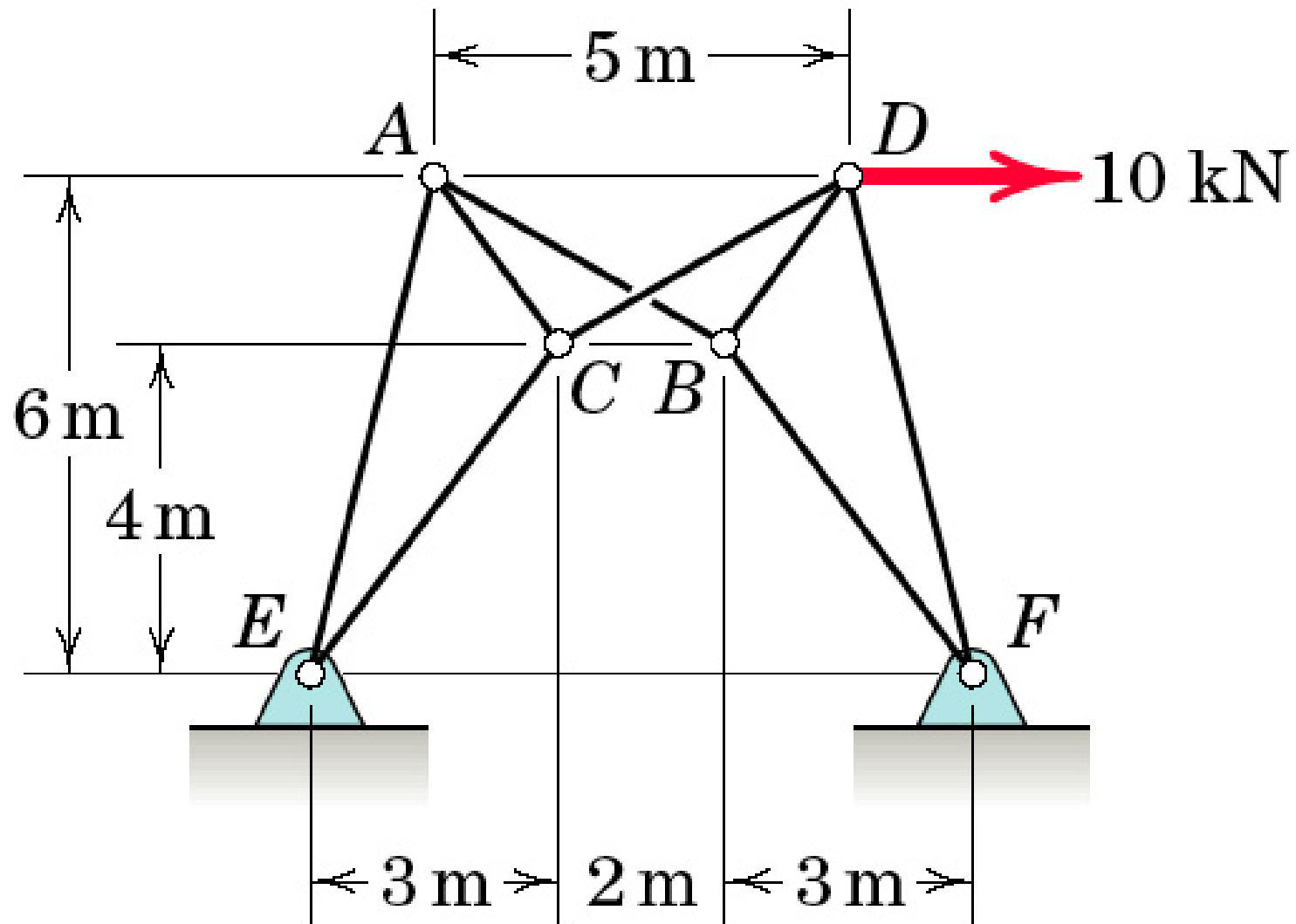
Problem 4/43



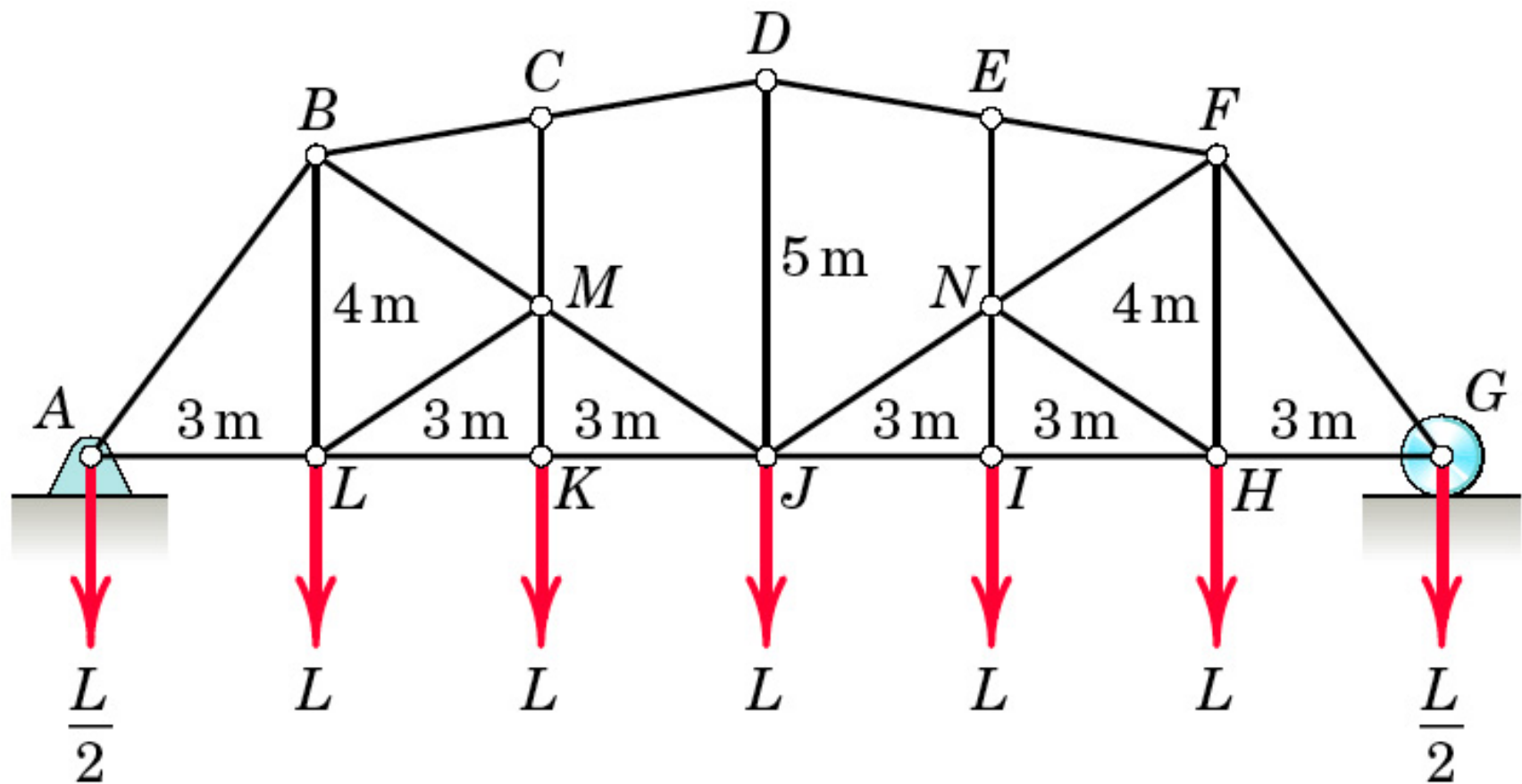
Problem 4/44



Problem 4/45

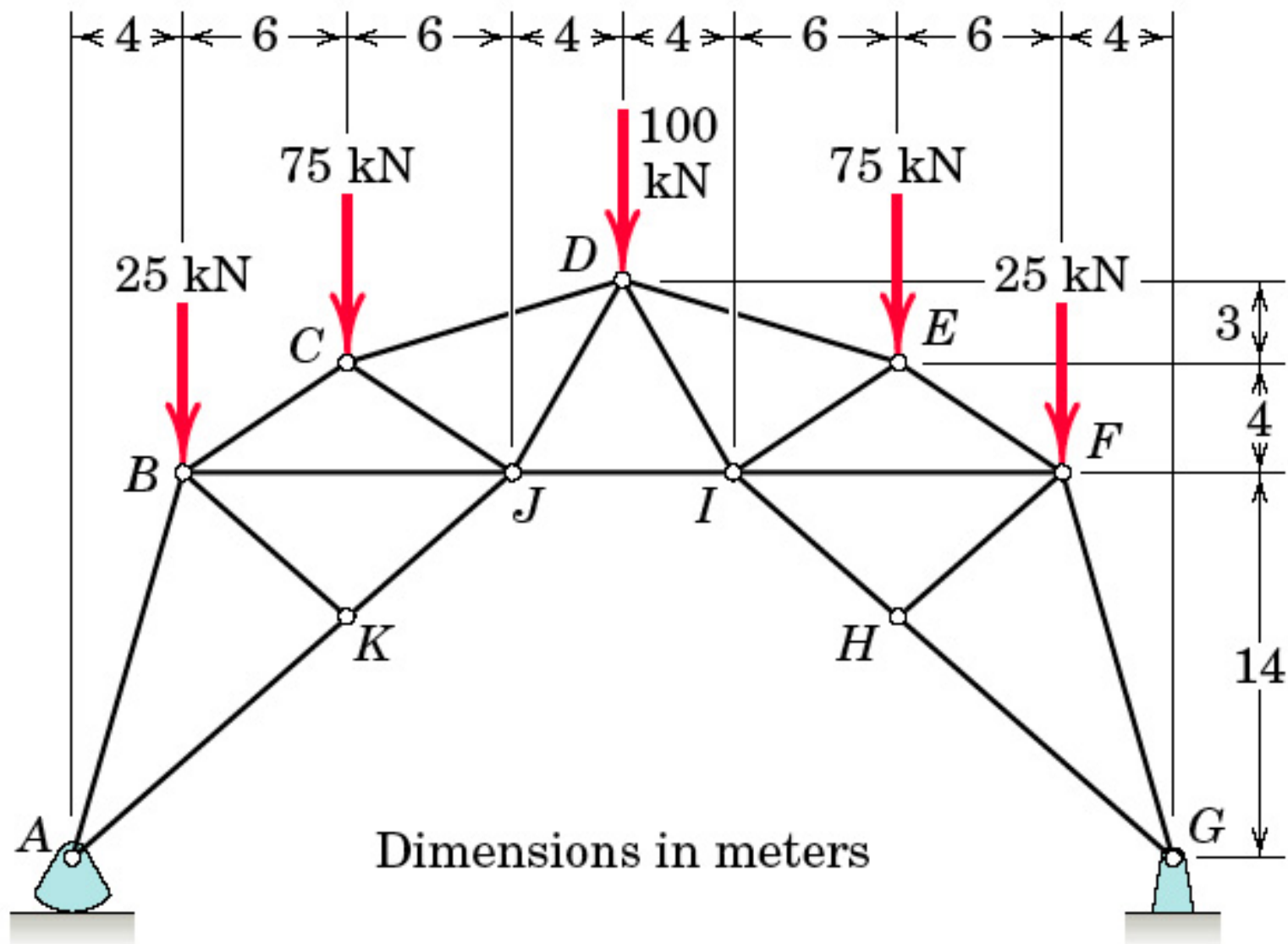


Problem 4/46



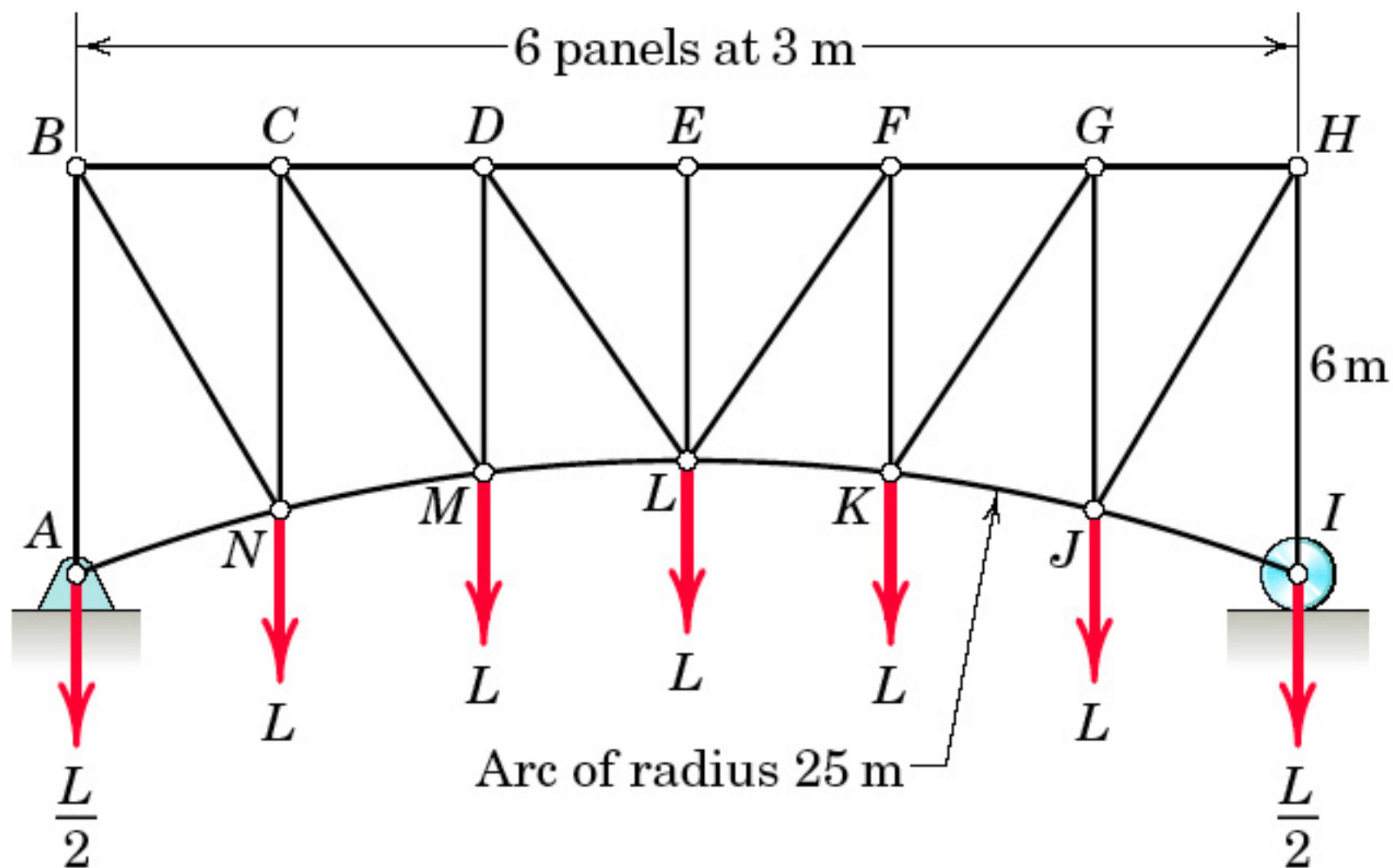
Problem 4/47

P_04_47

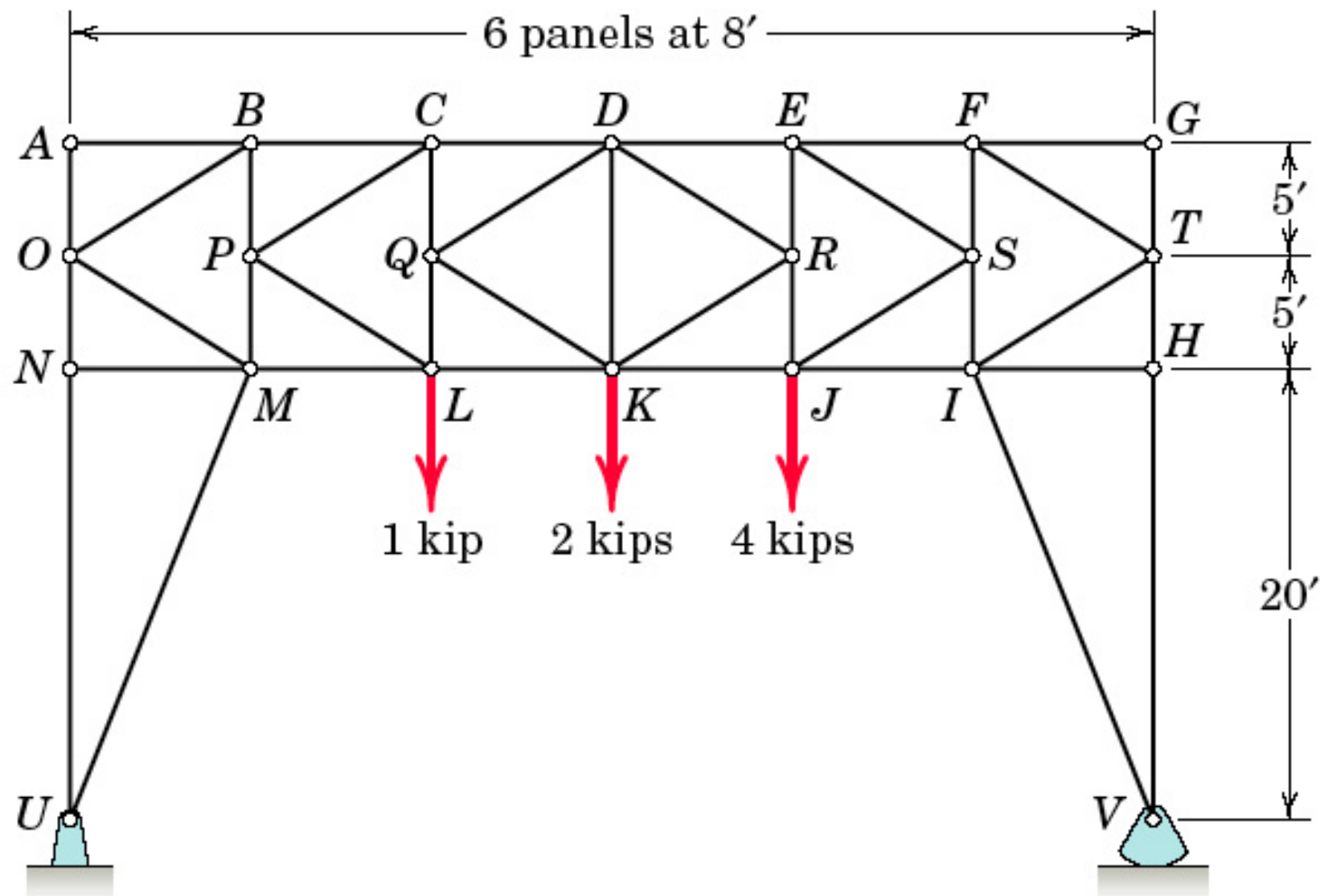


Dimensions in meters

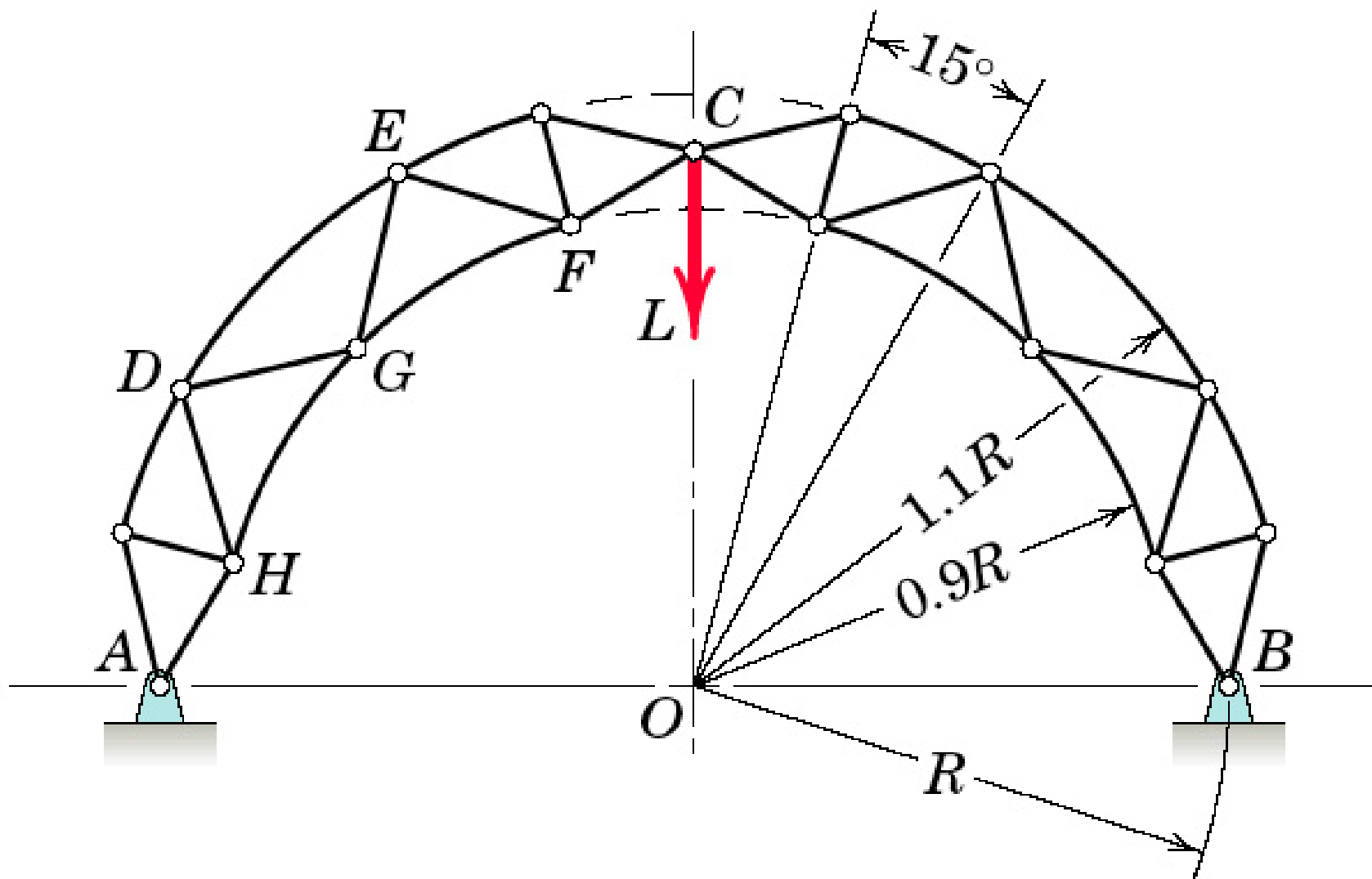
Problem 4/48



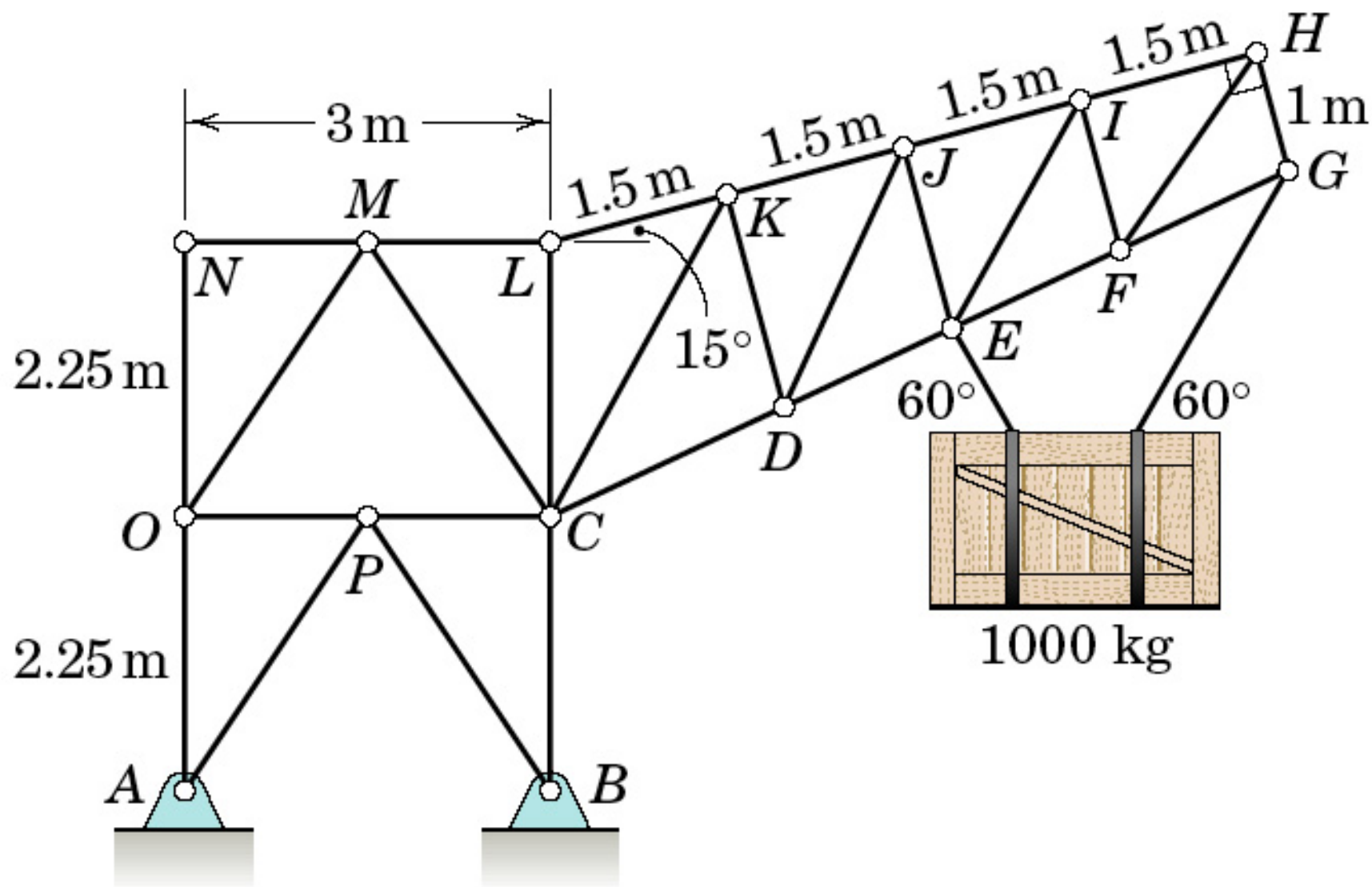
Problem 4/49



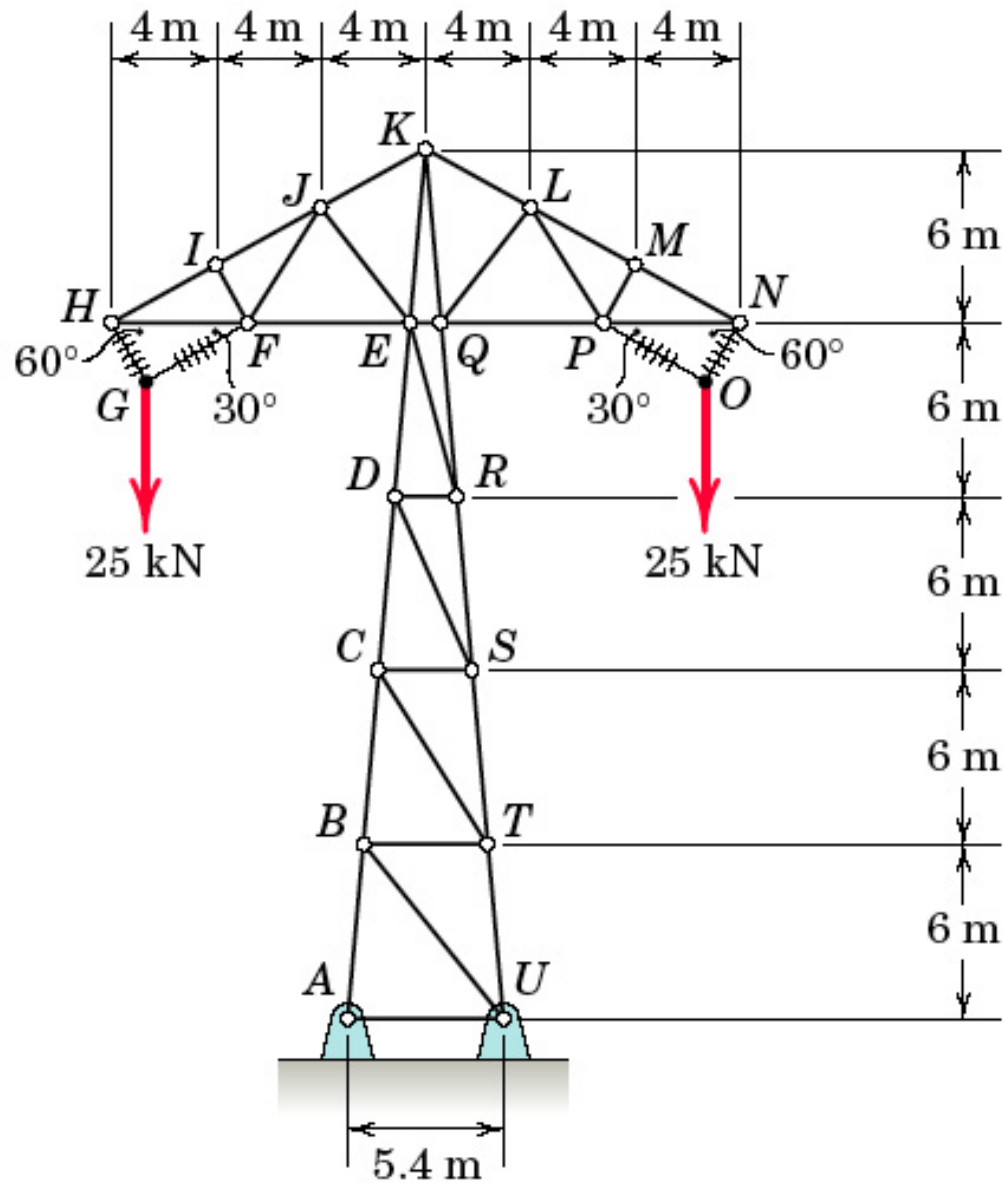
Problem 4/51



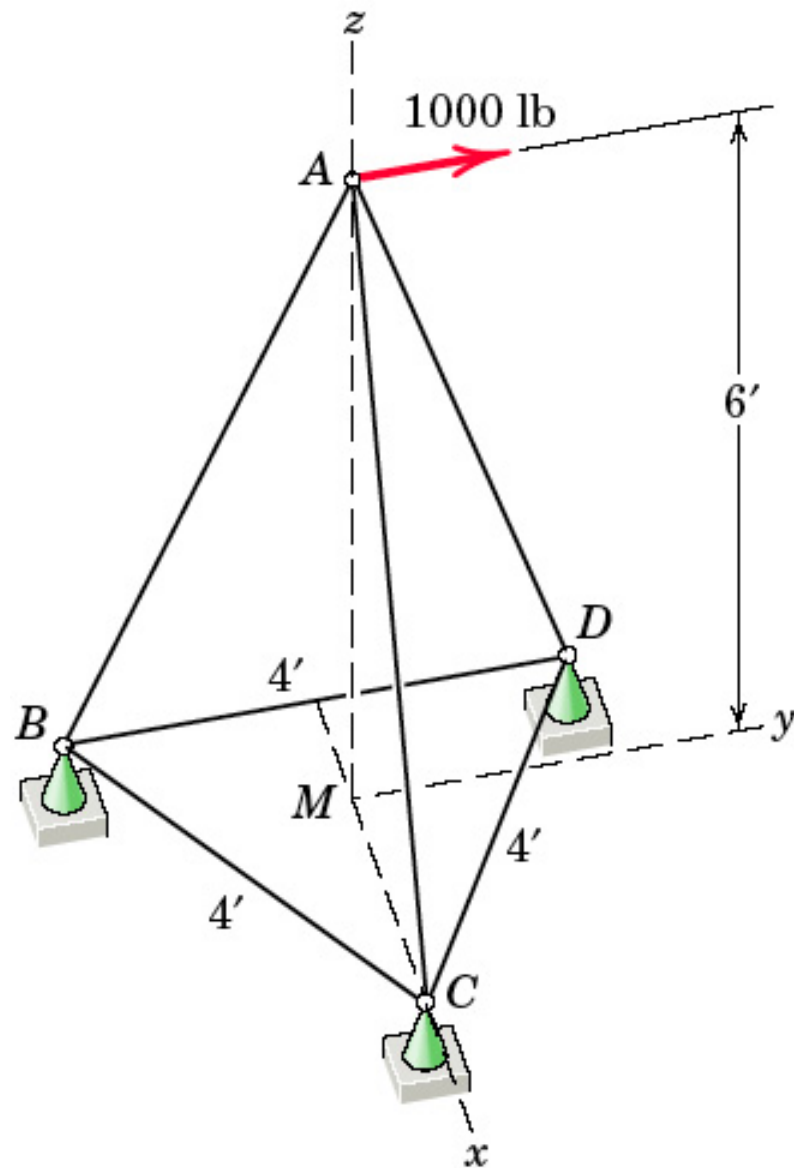
Problem 4/52



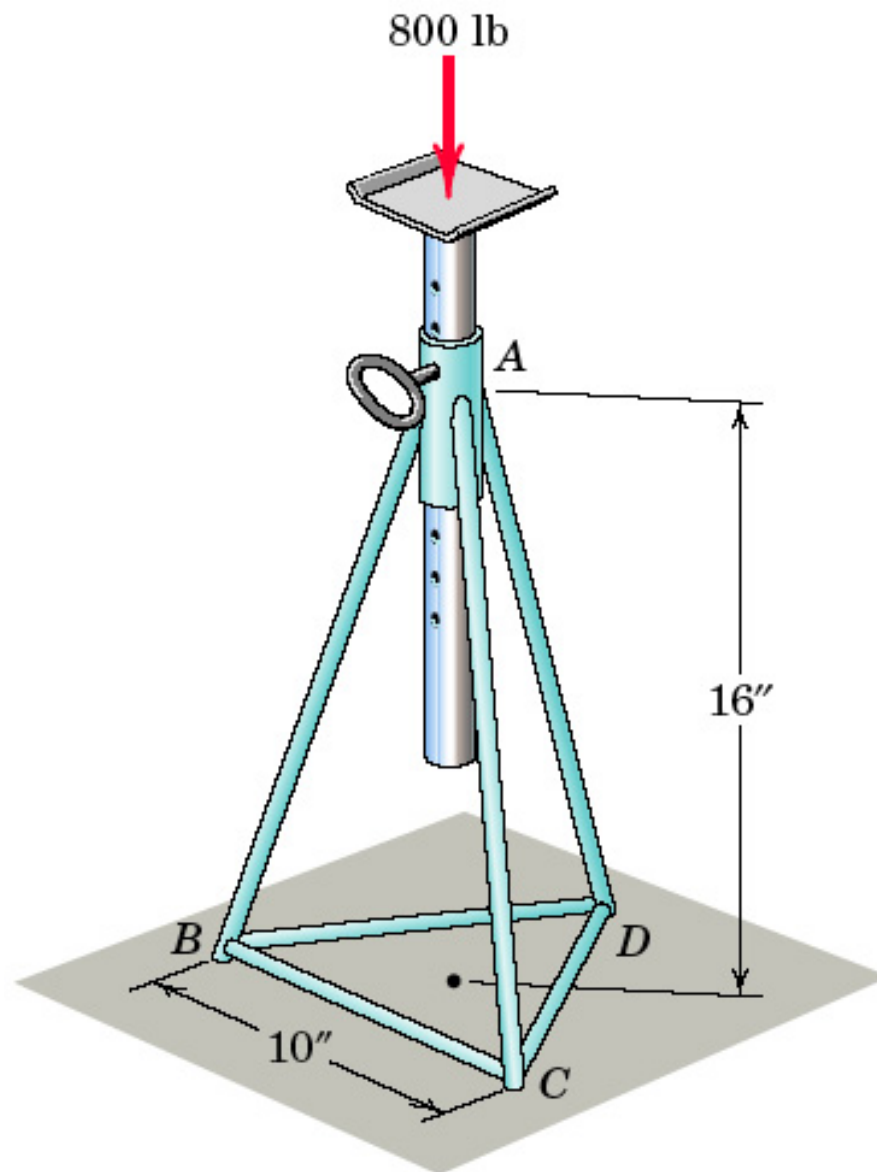
Problem 4/53



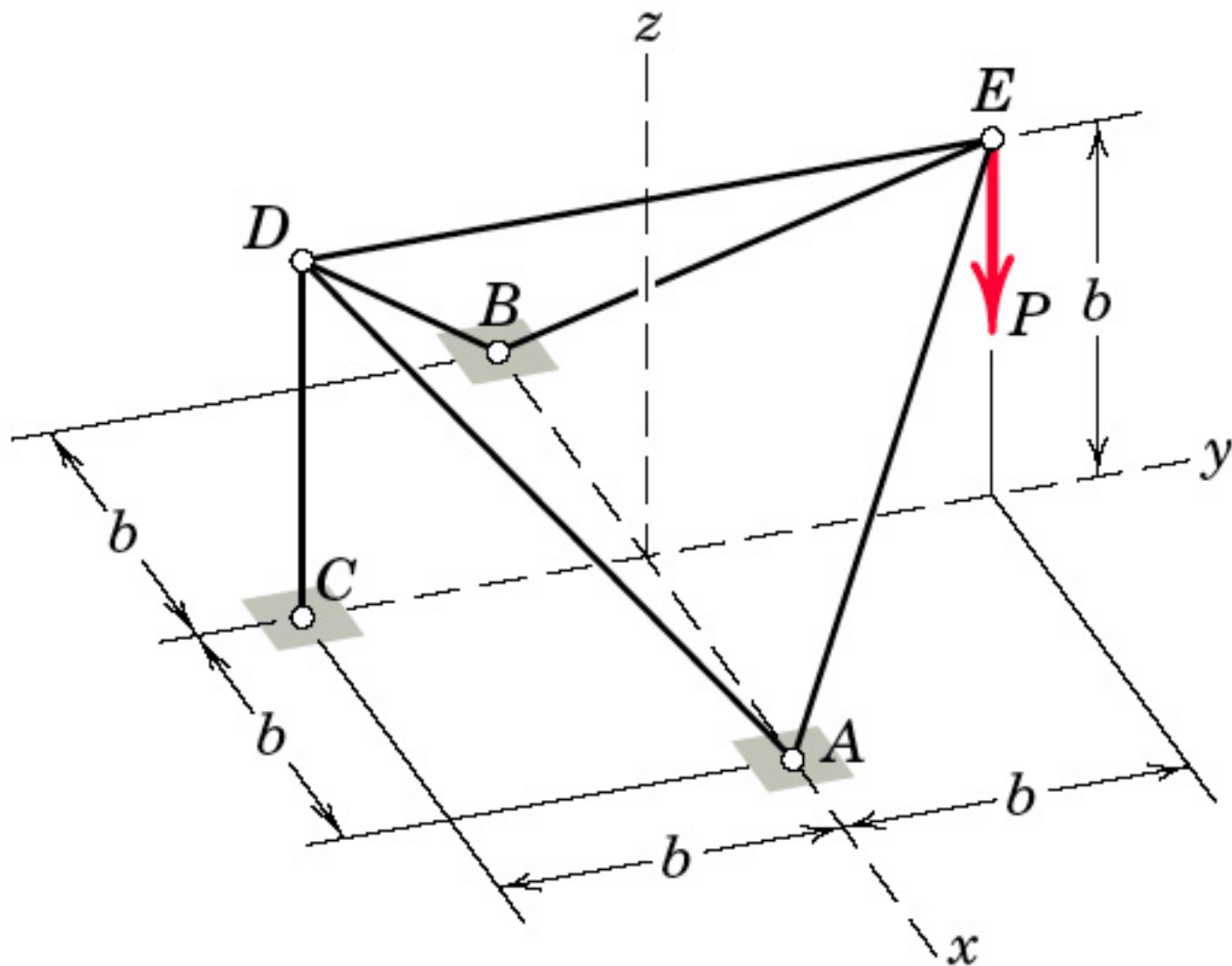
Problem 4/54



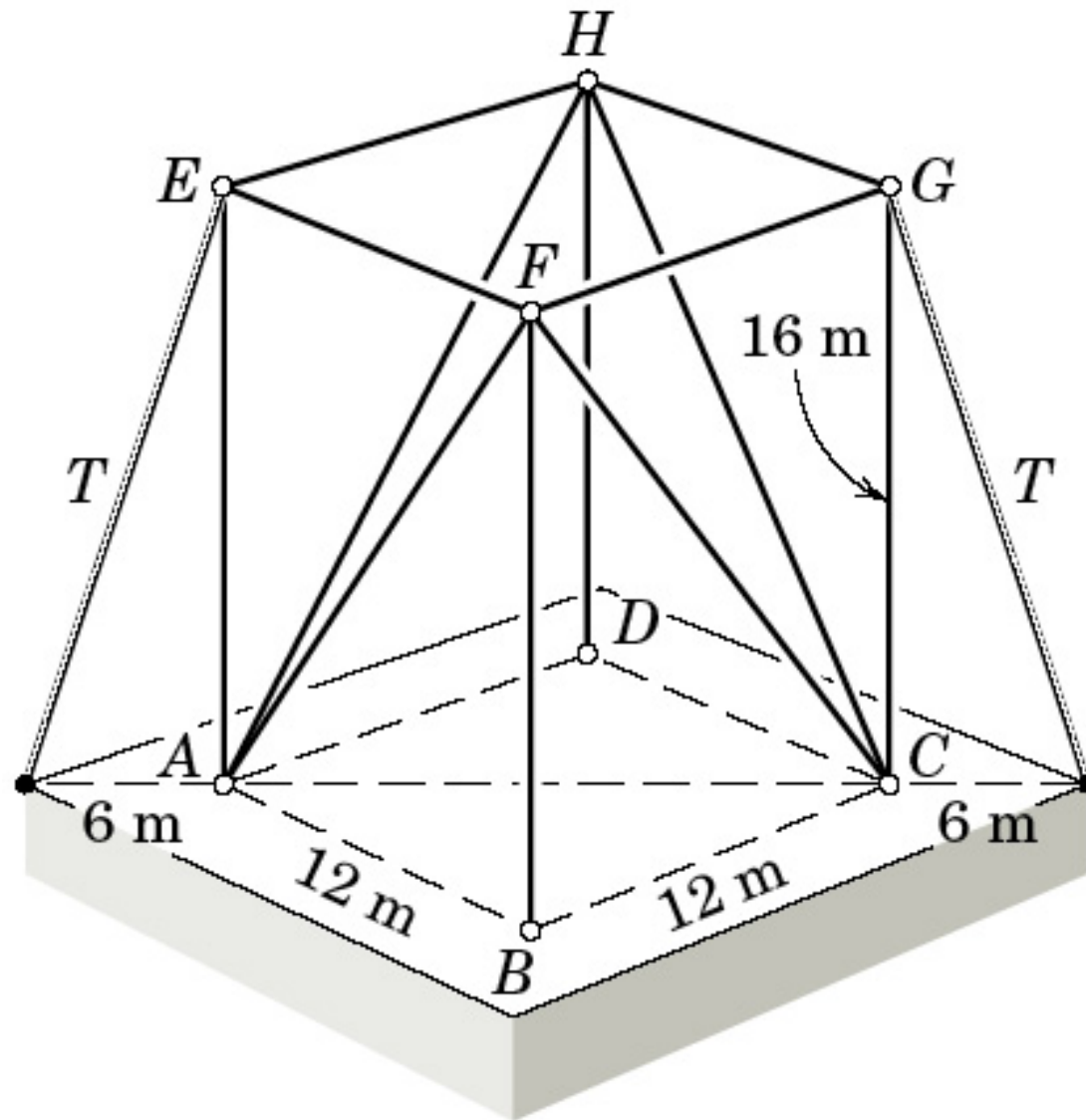
Problem 4/55



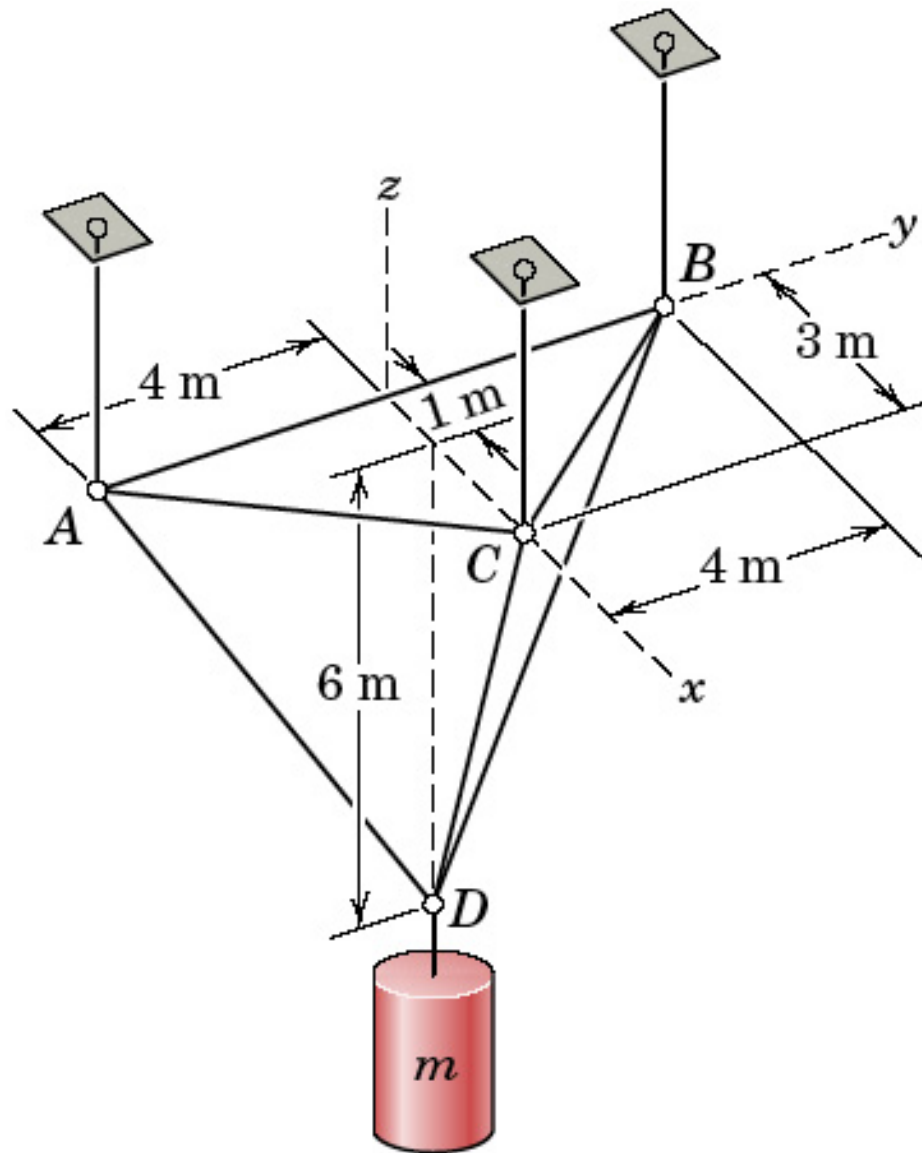
Problem 4/56



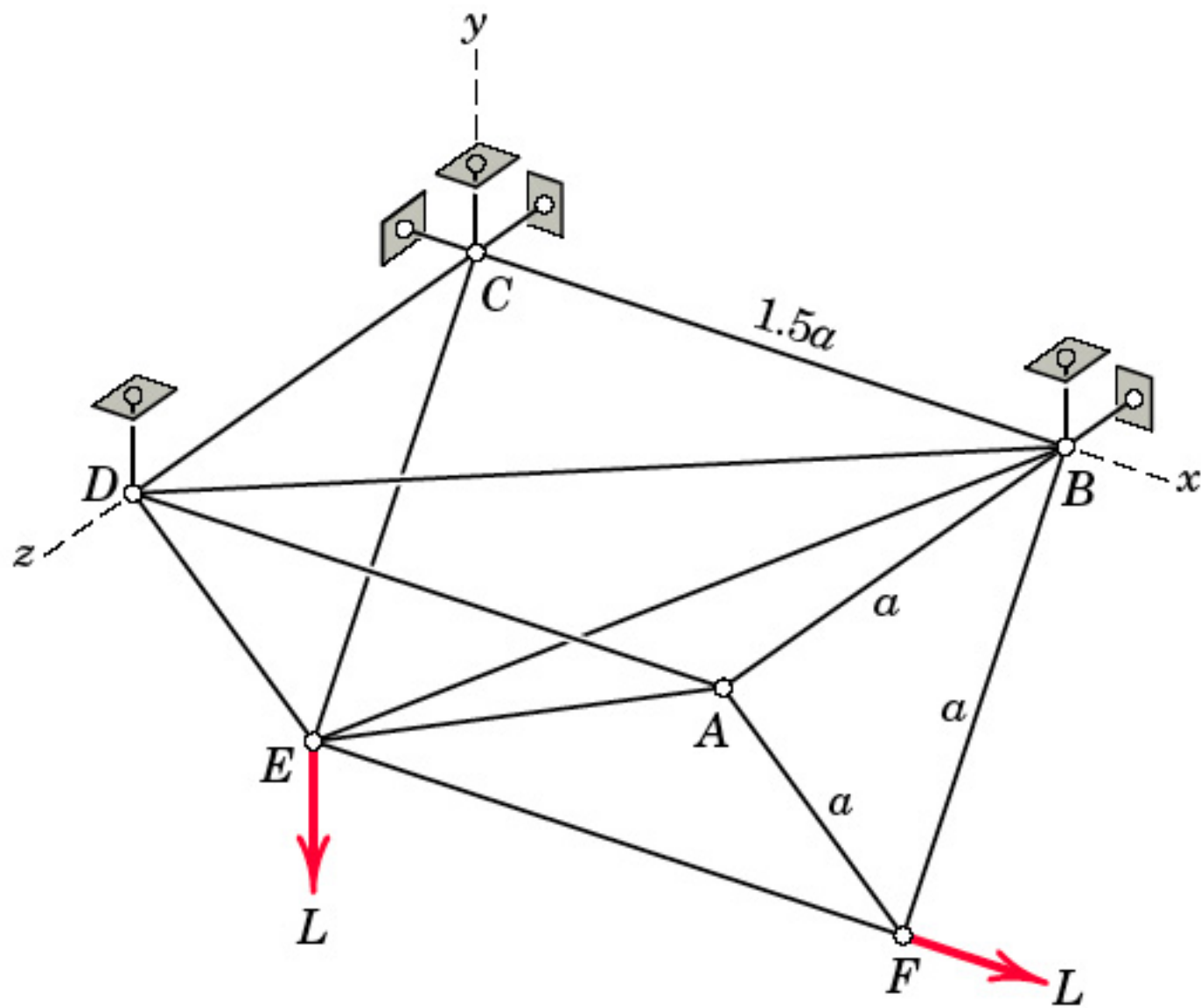
Problem 4/57



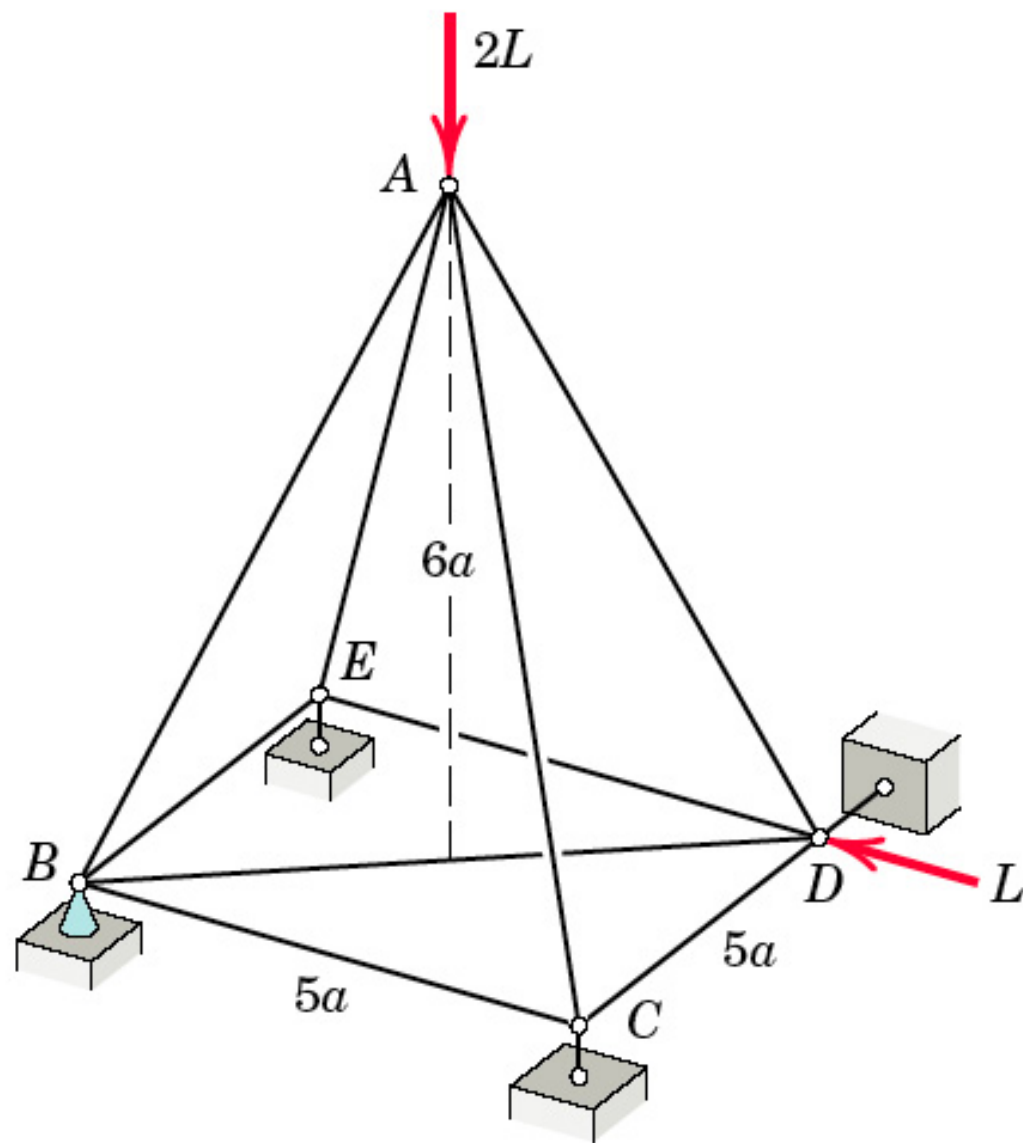
Problem 4/58



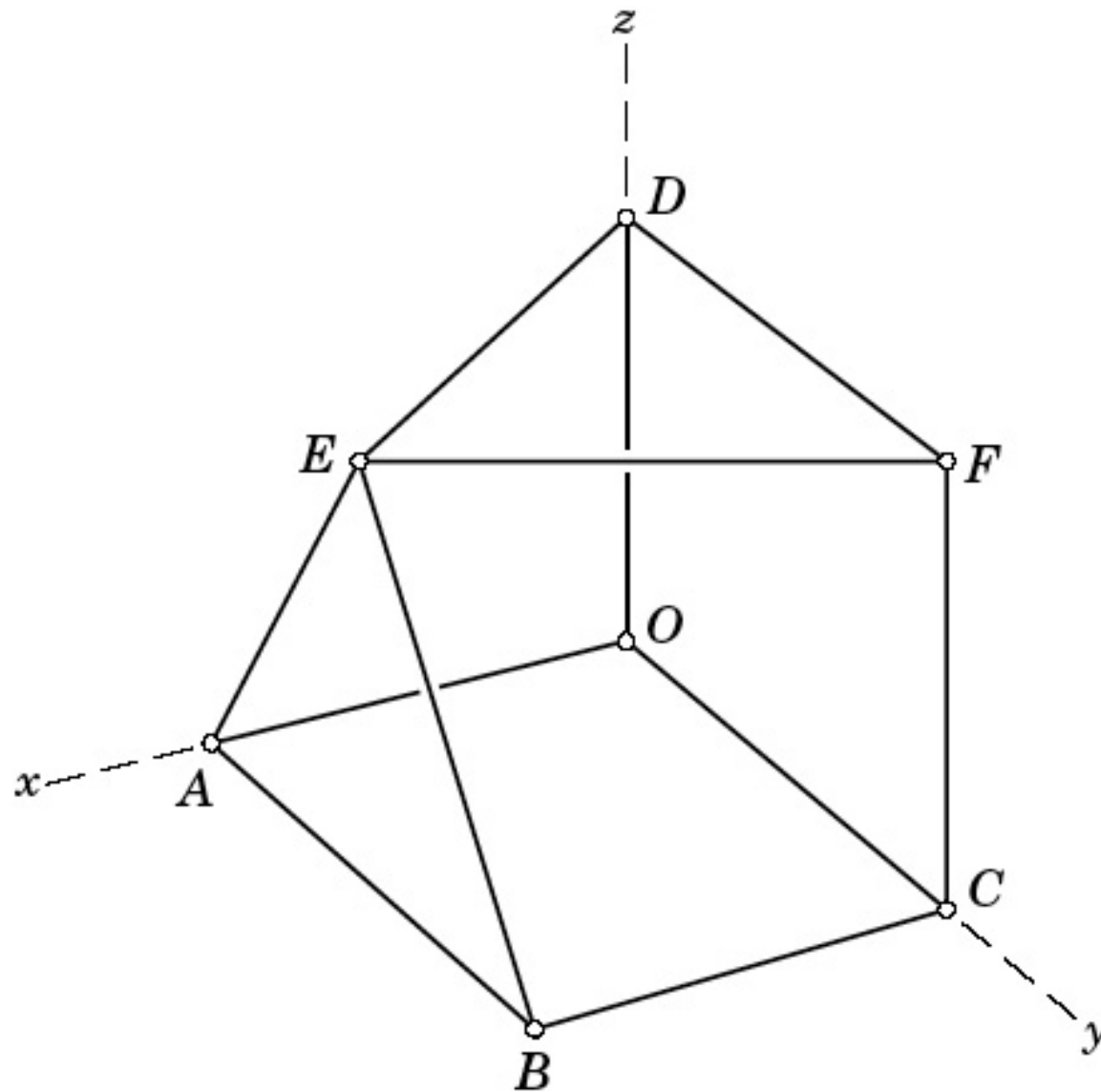
Problem 4/59



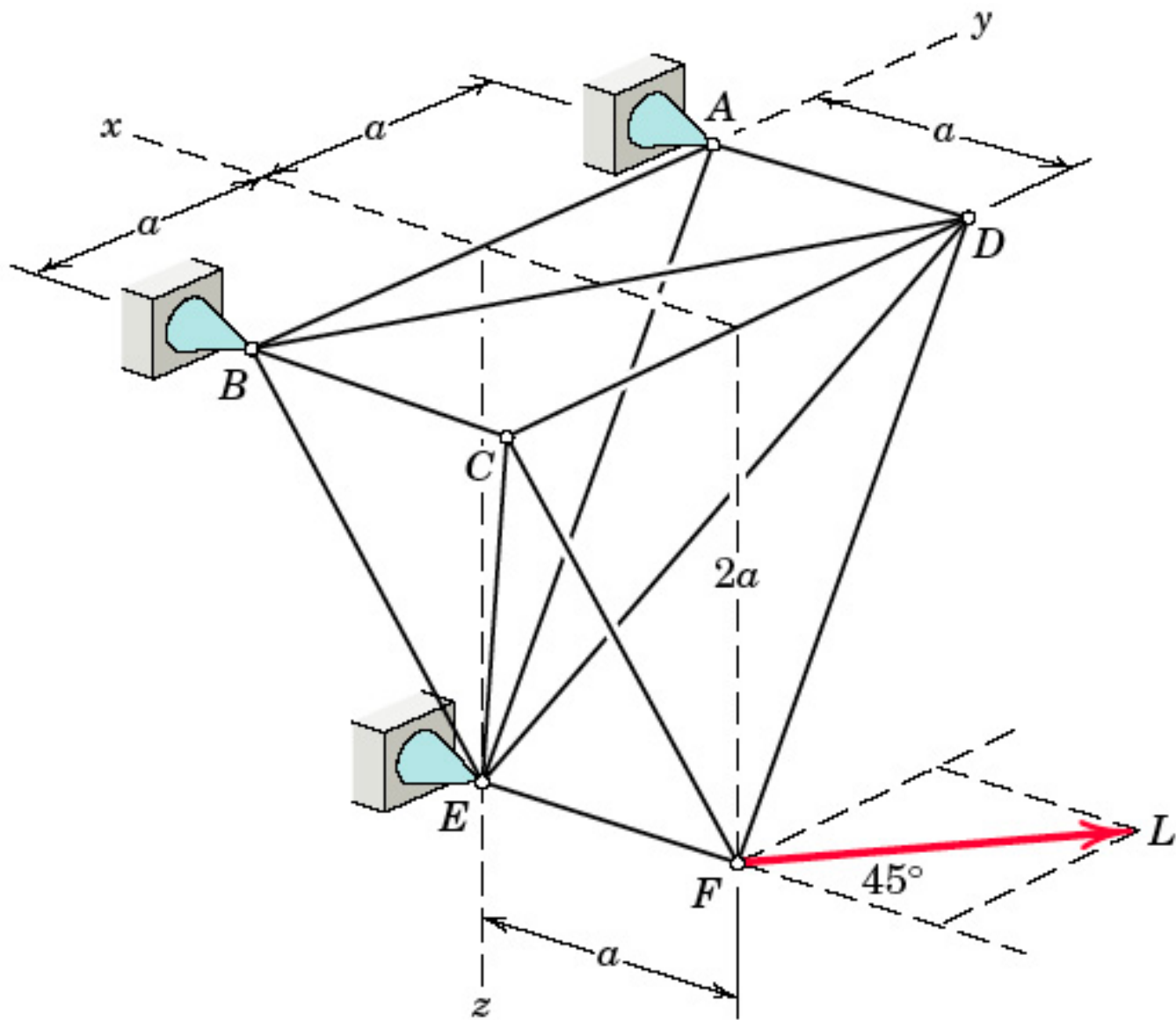
Problem 4/60



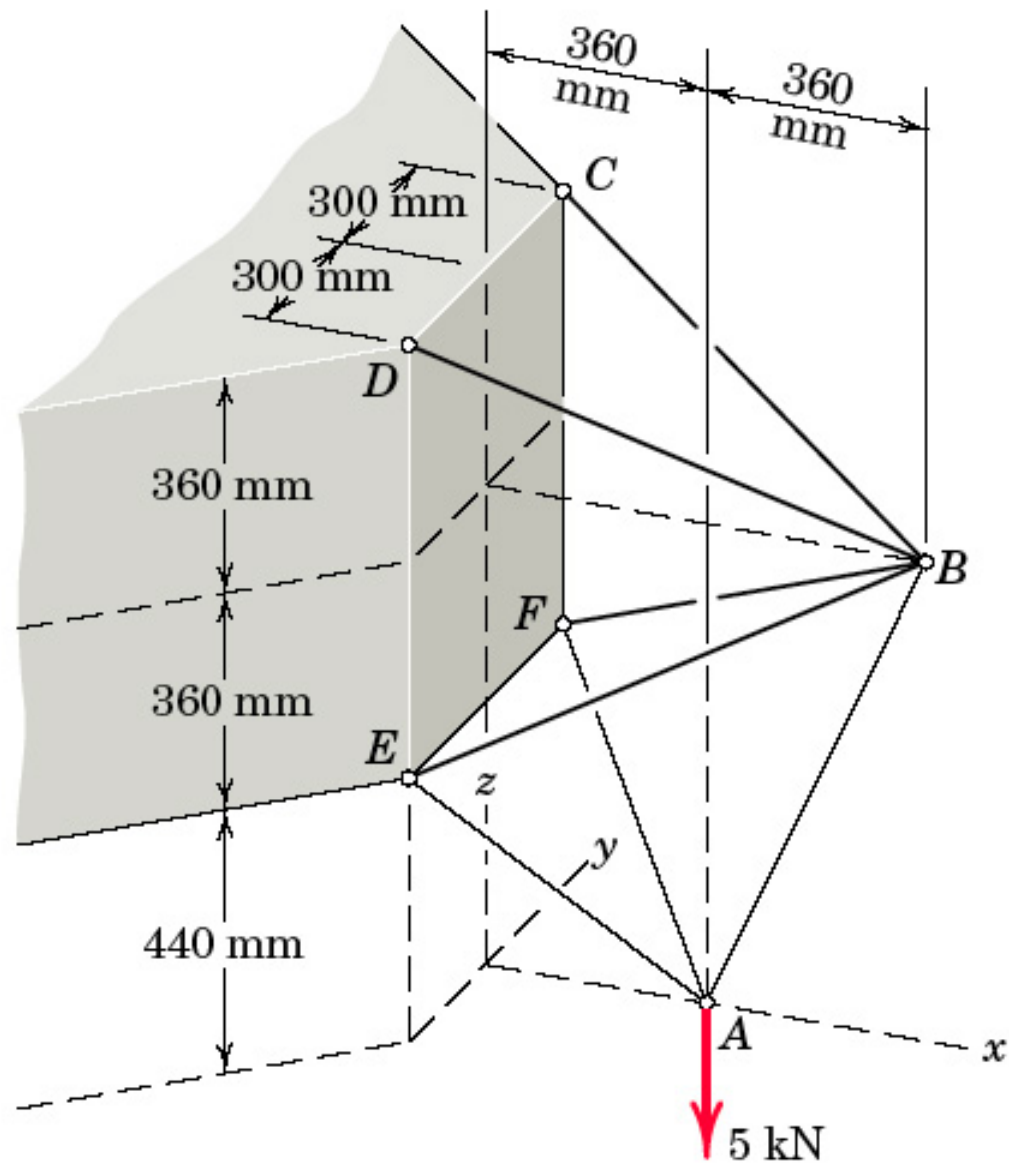
Problem 4/61



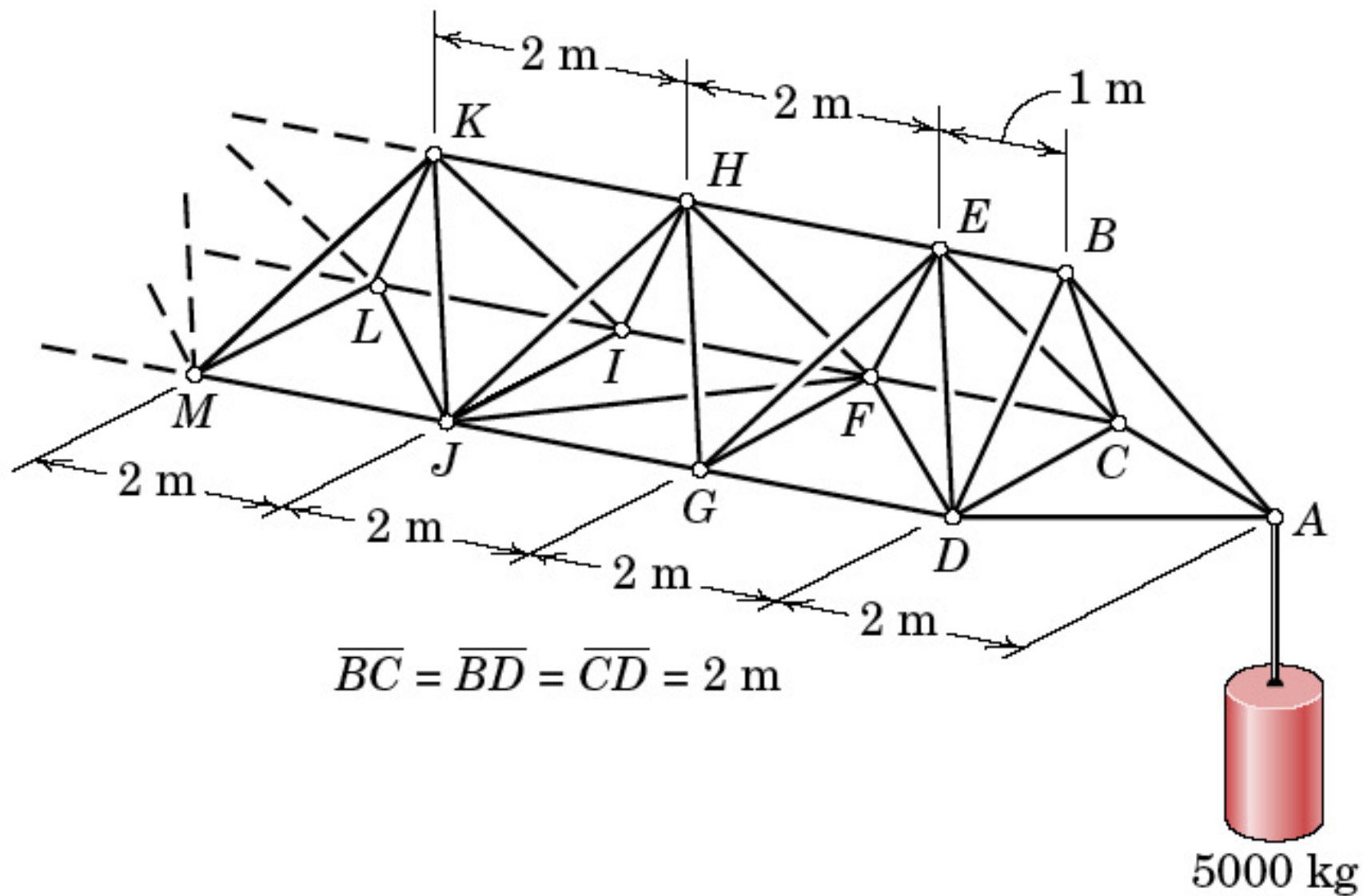
Problem 4/62



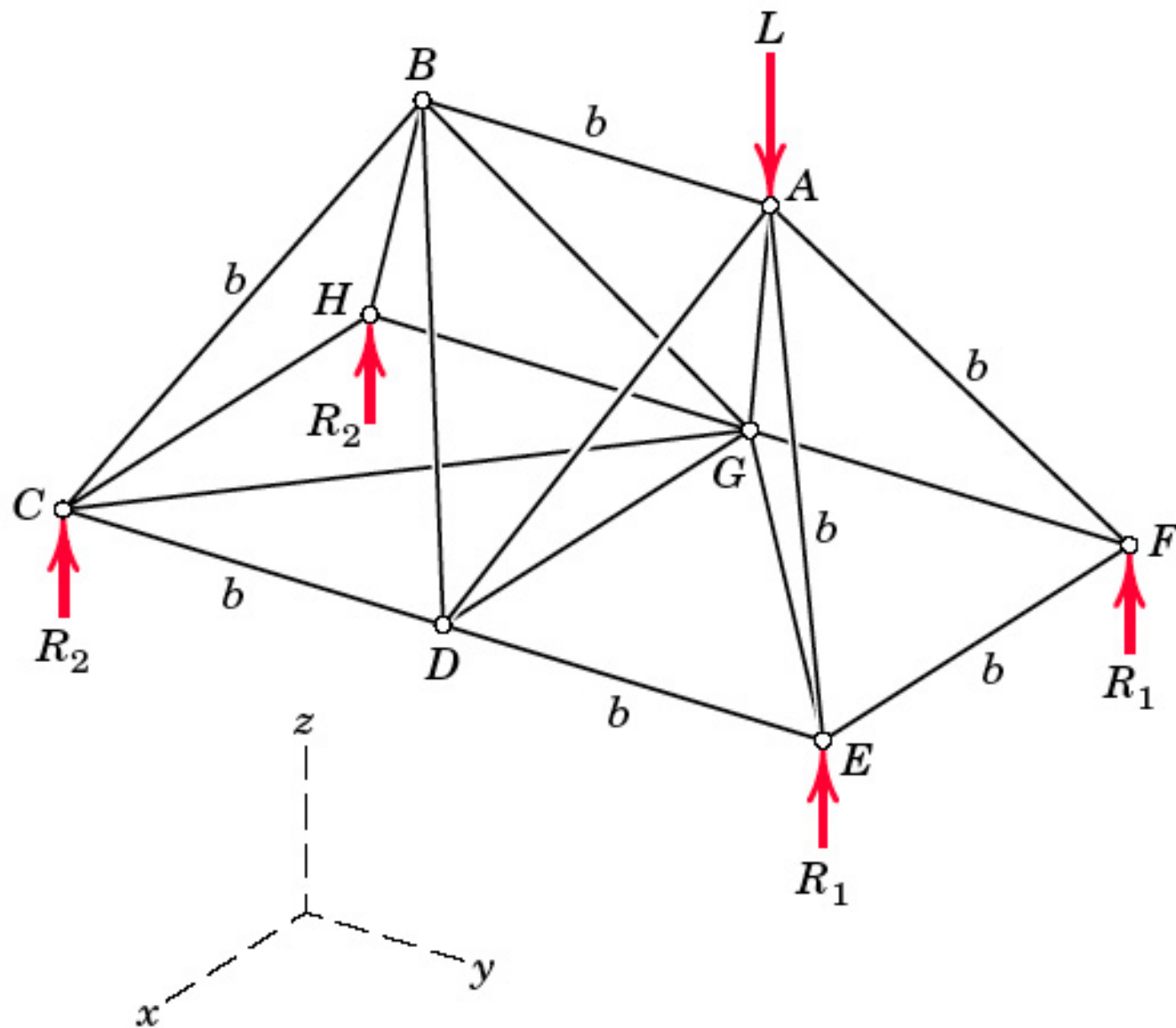
Problem 4/63



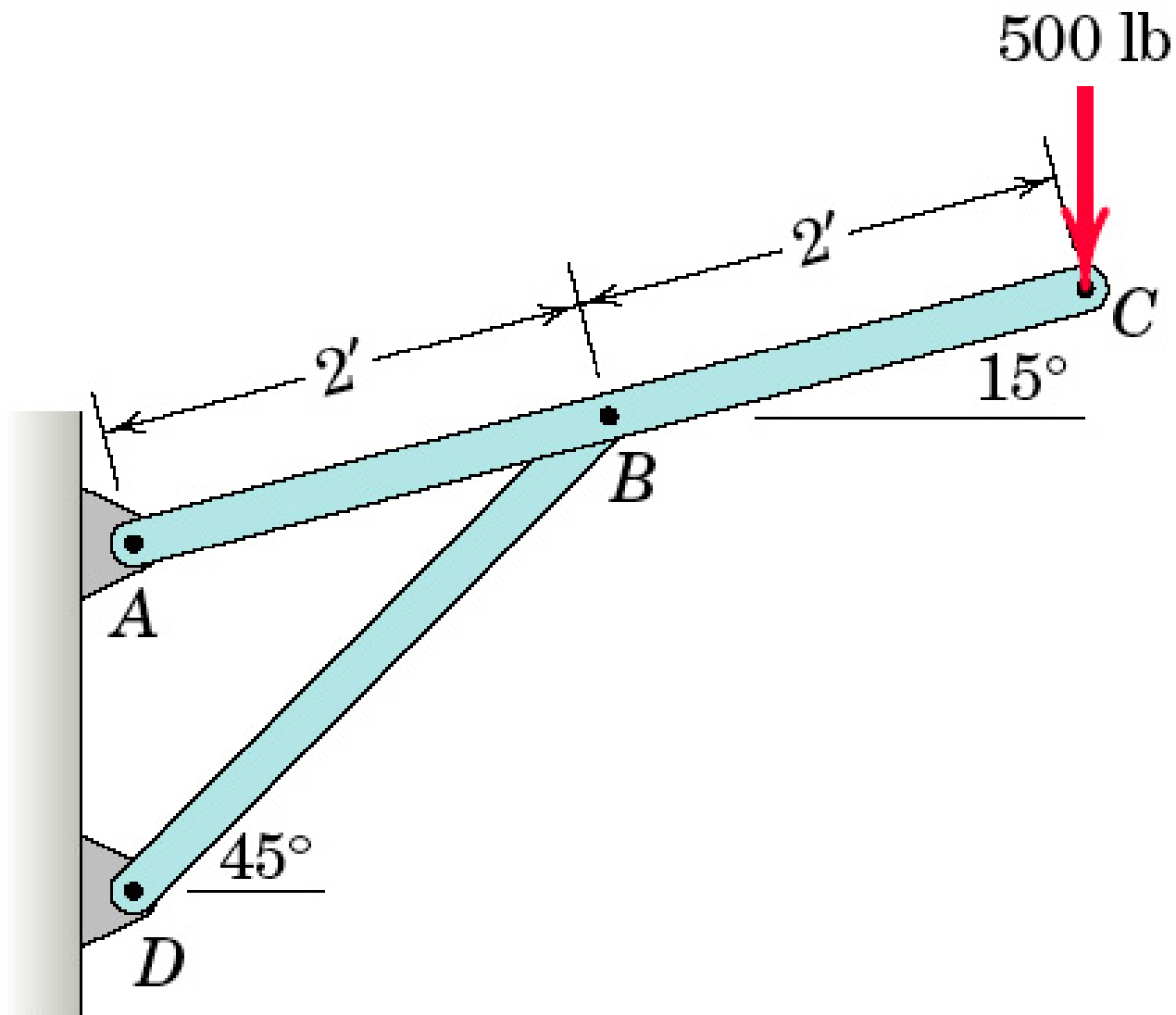
Problem 4/64



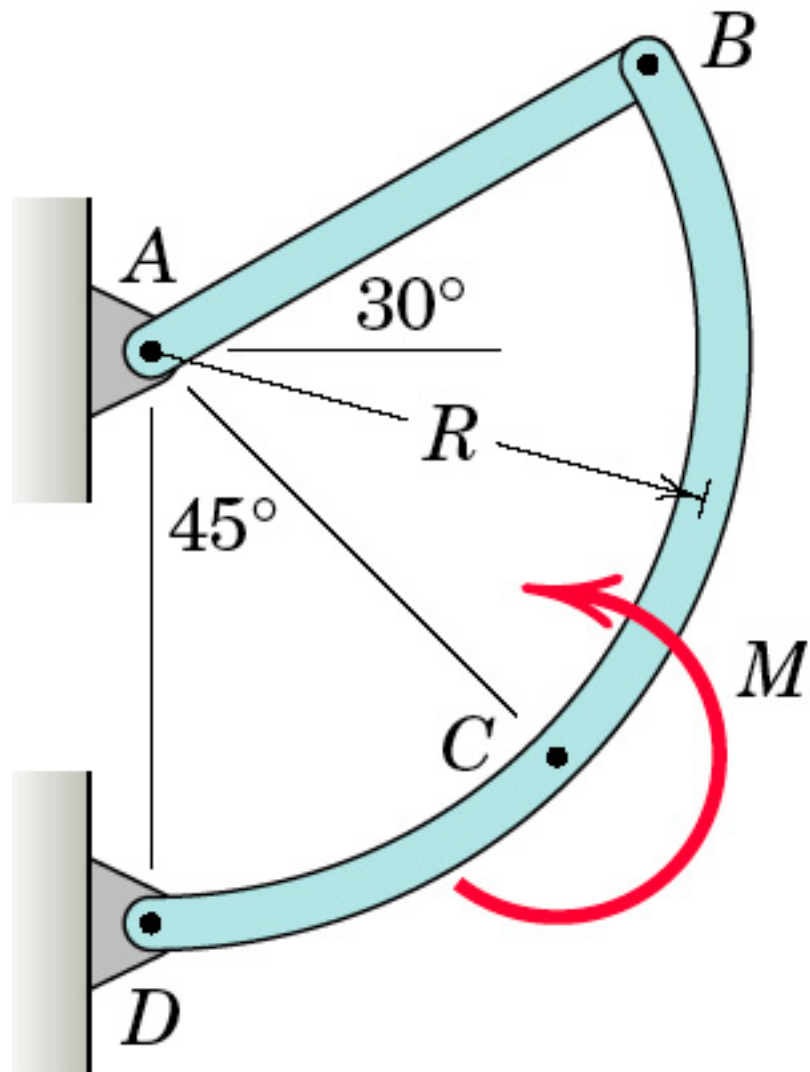
Problem 4/65



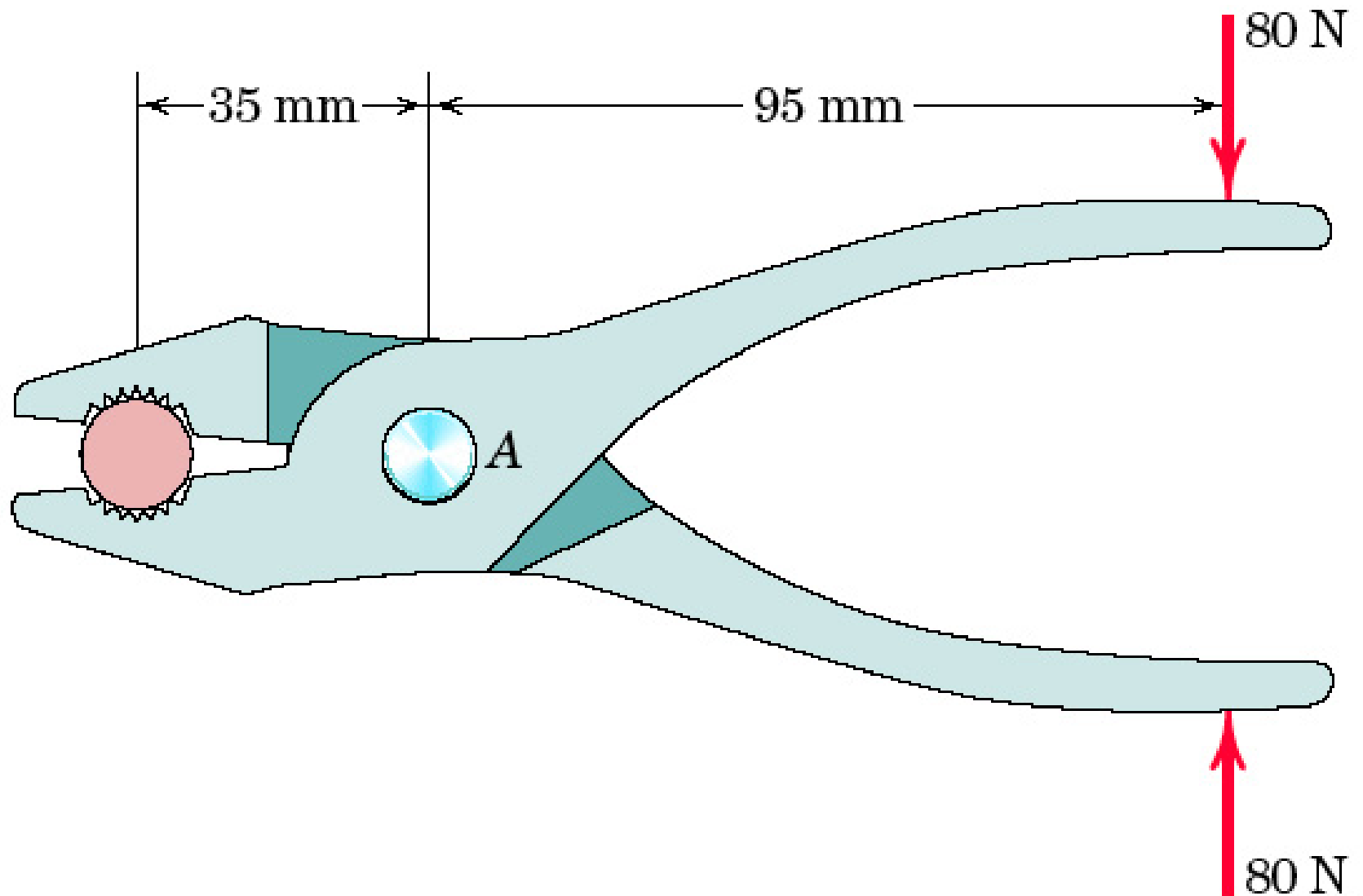
Problem 4/66



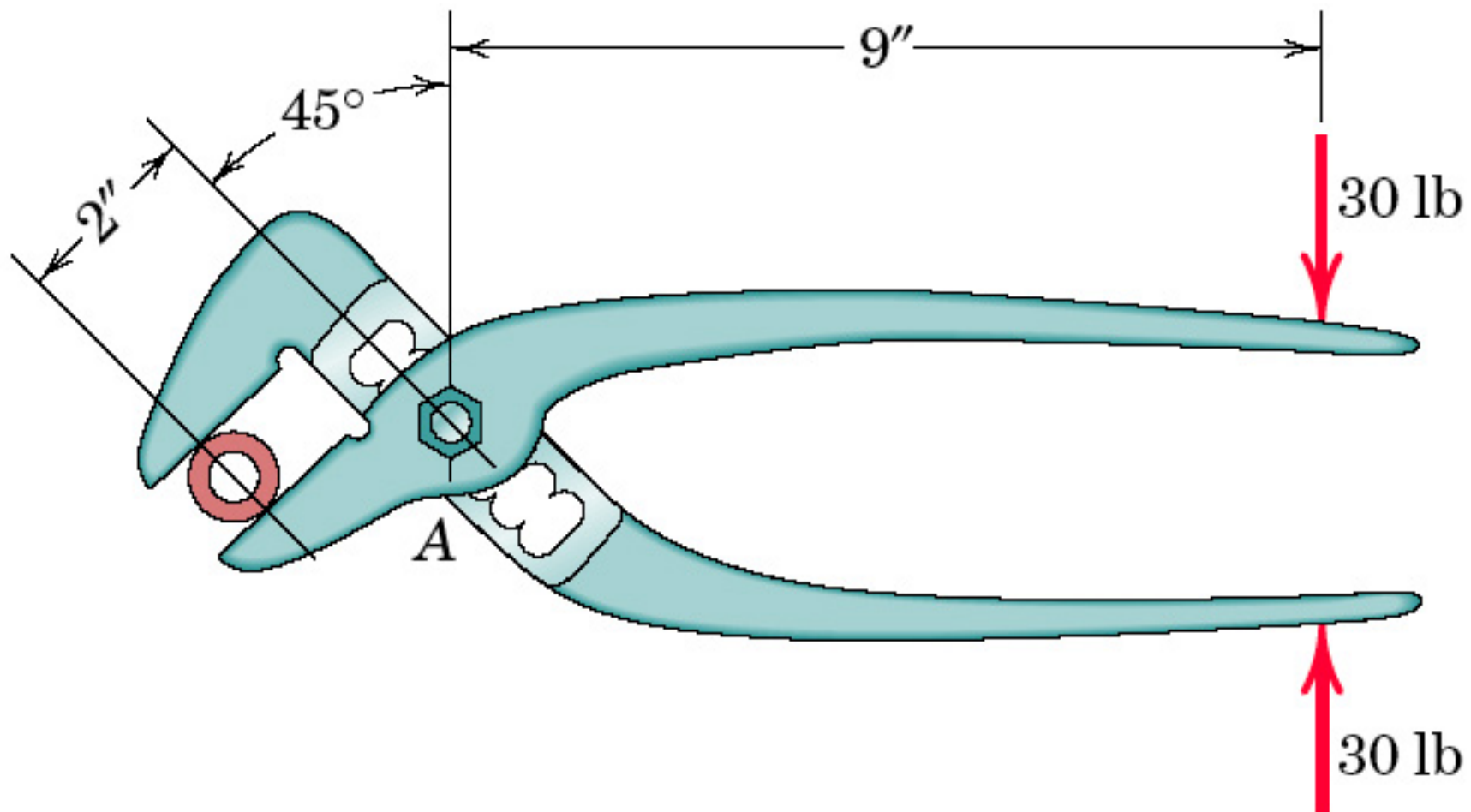
Problem 4/67



Problem 4/68

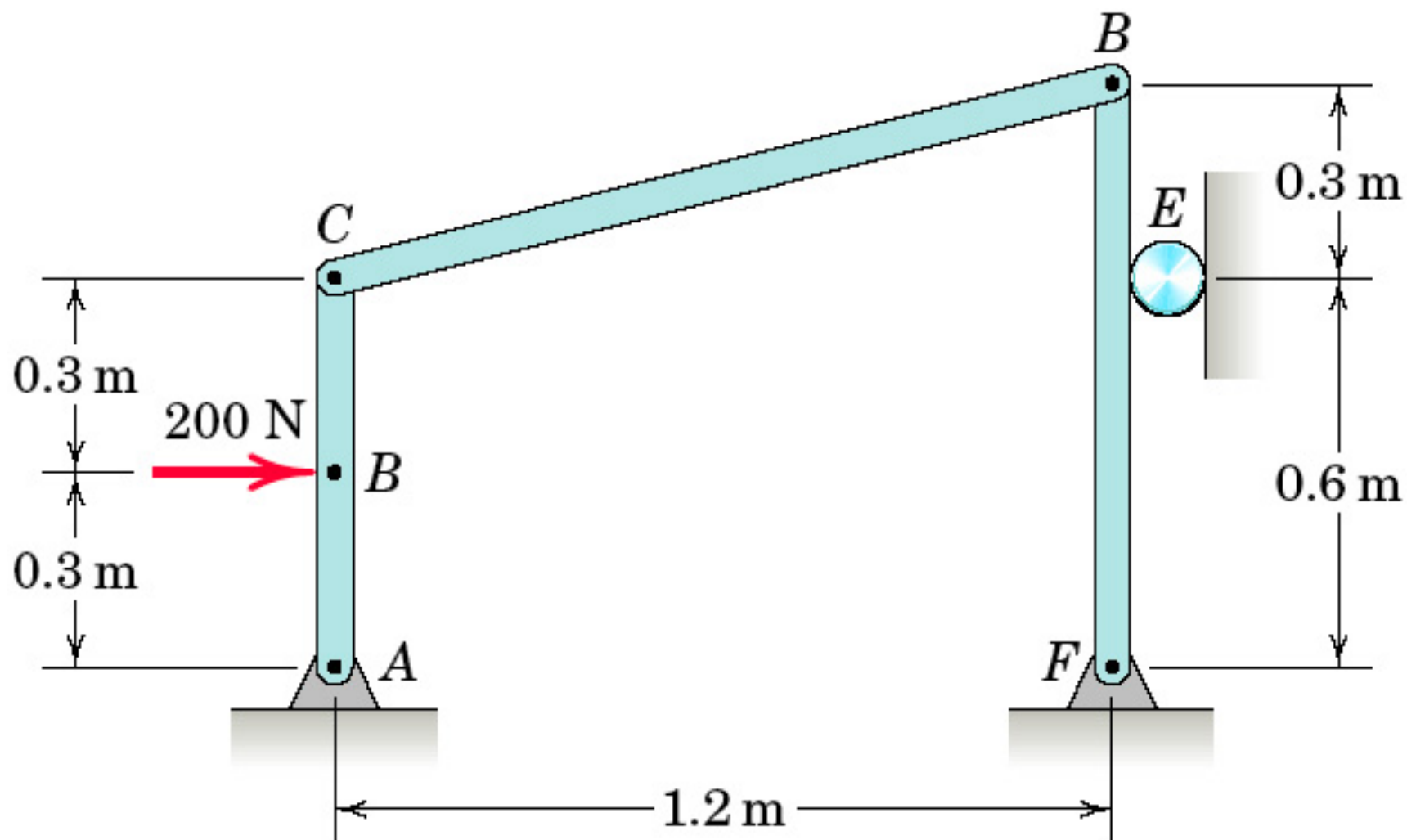


Problem 4/69

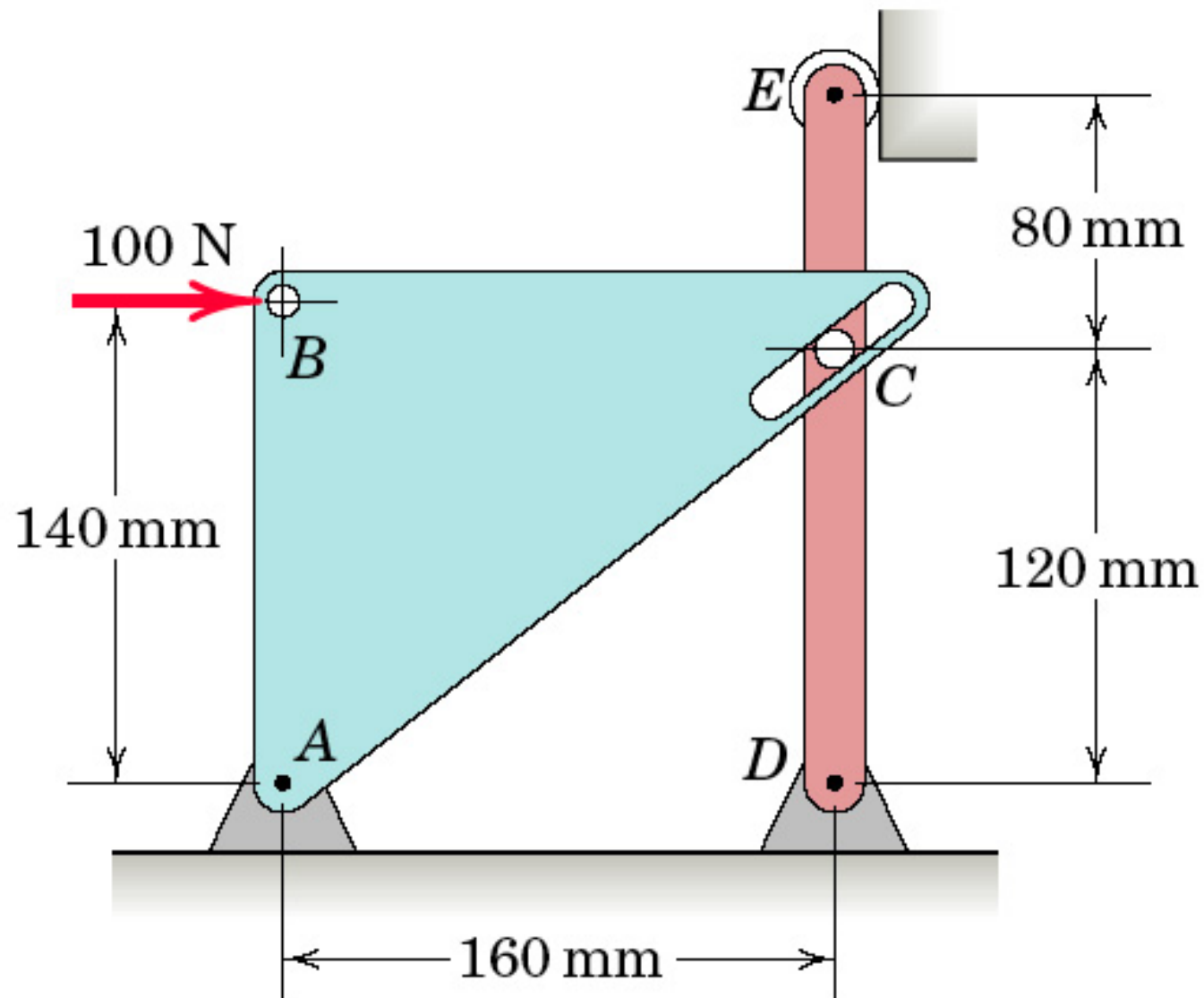


Problem 4/70

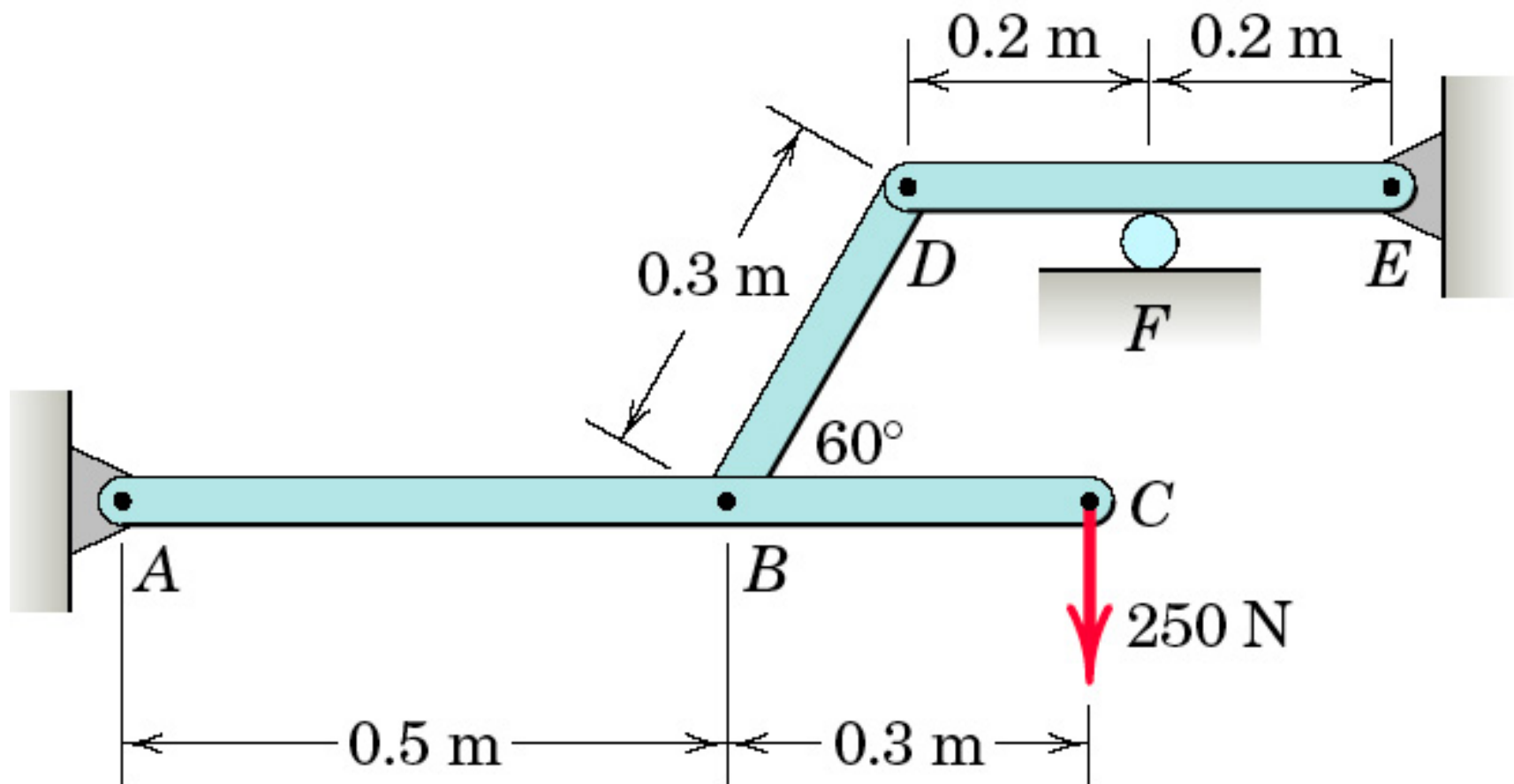
P_04_70



Problem 4/71

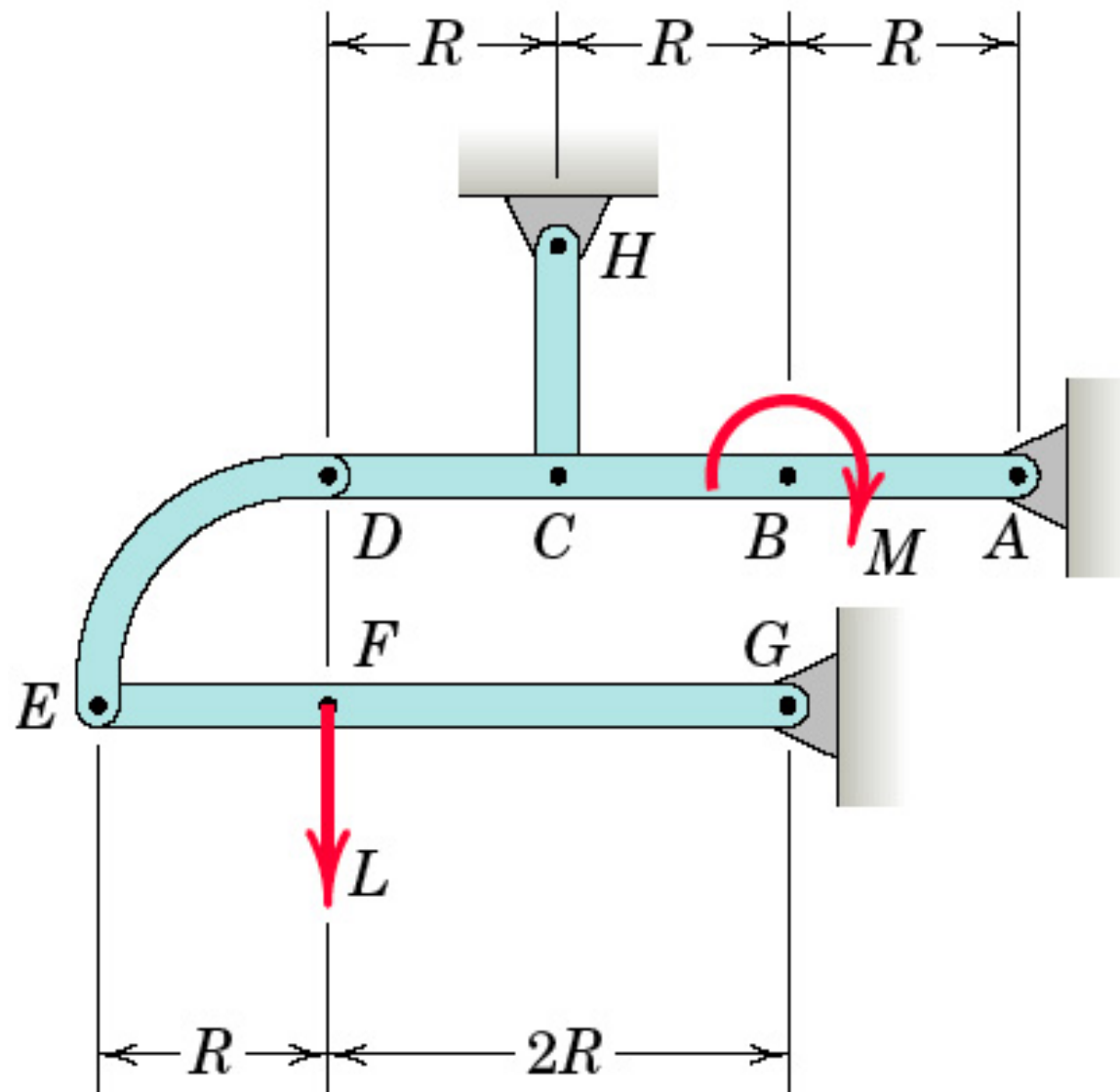


Problem 4/72

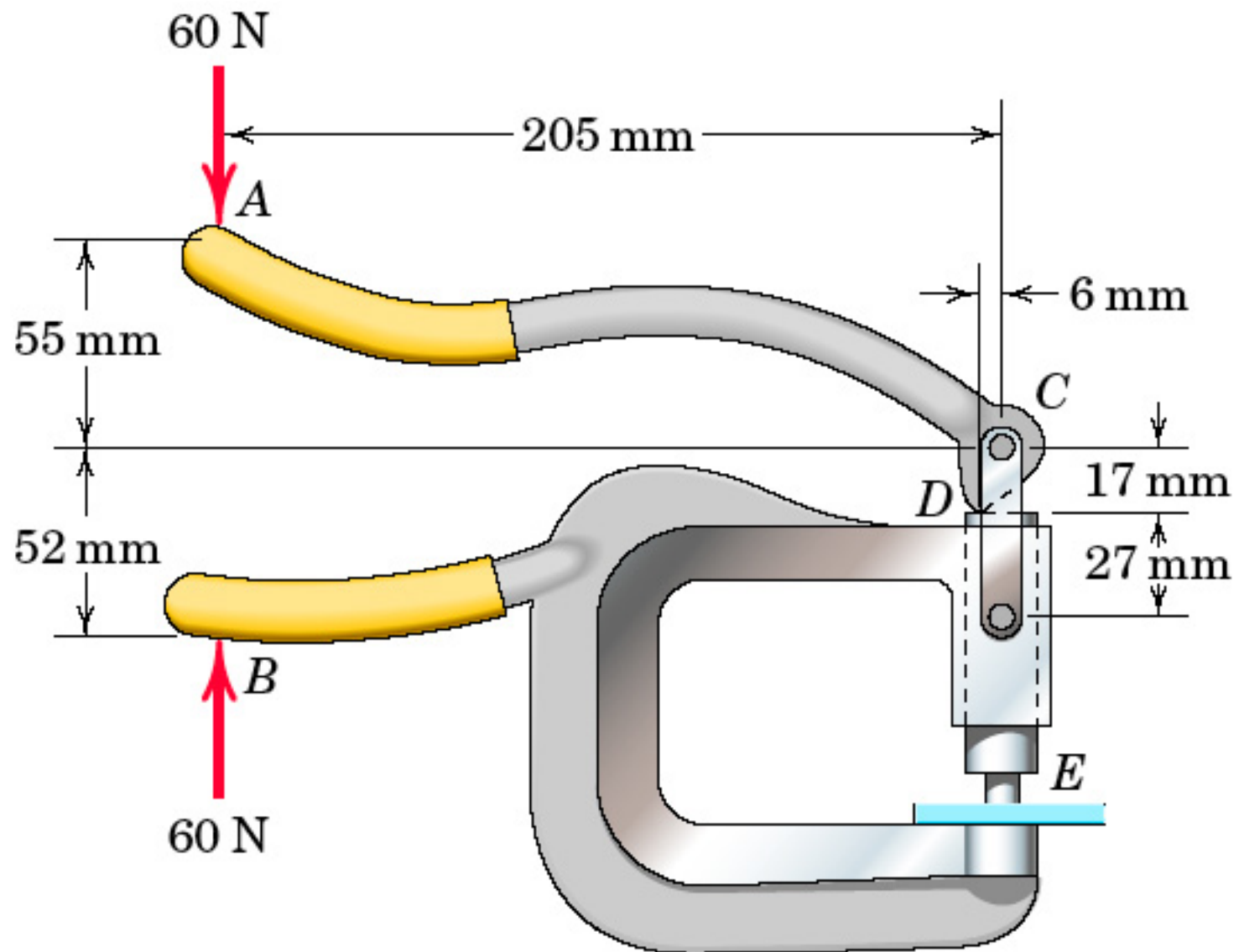


Problem 4/73

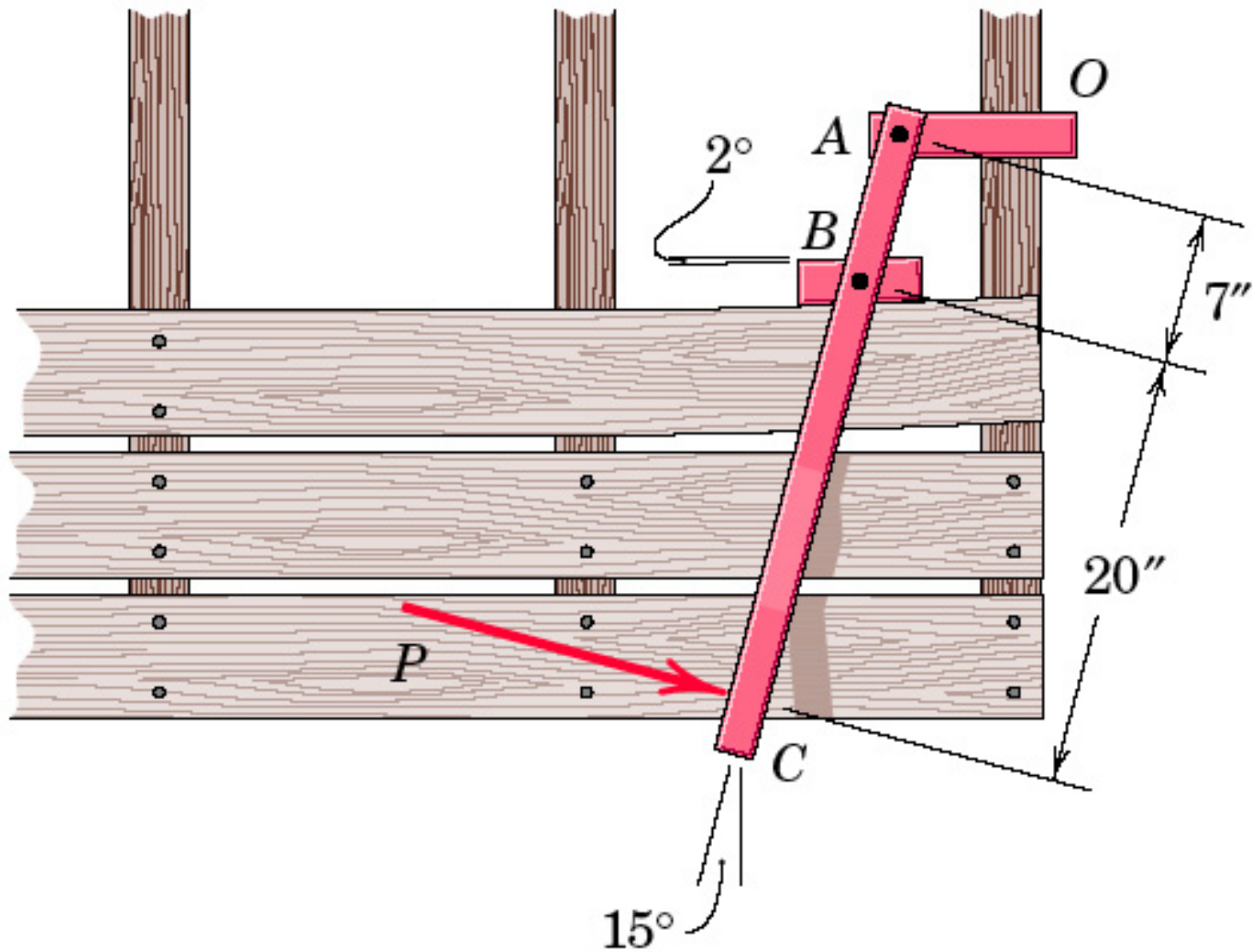
P_04_73



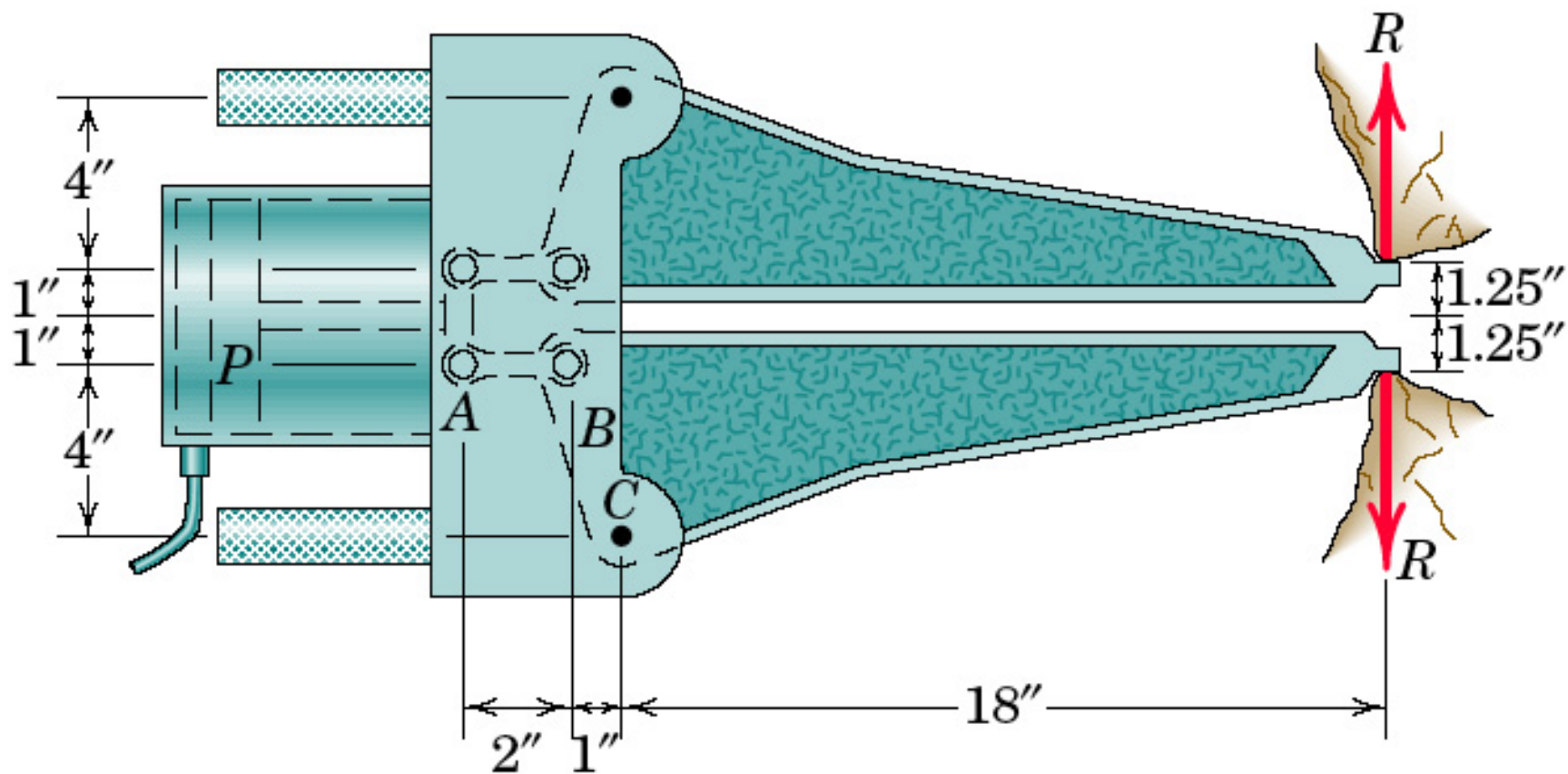
Problem 4/74



Problem 4/75

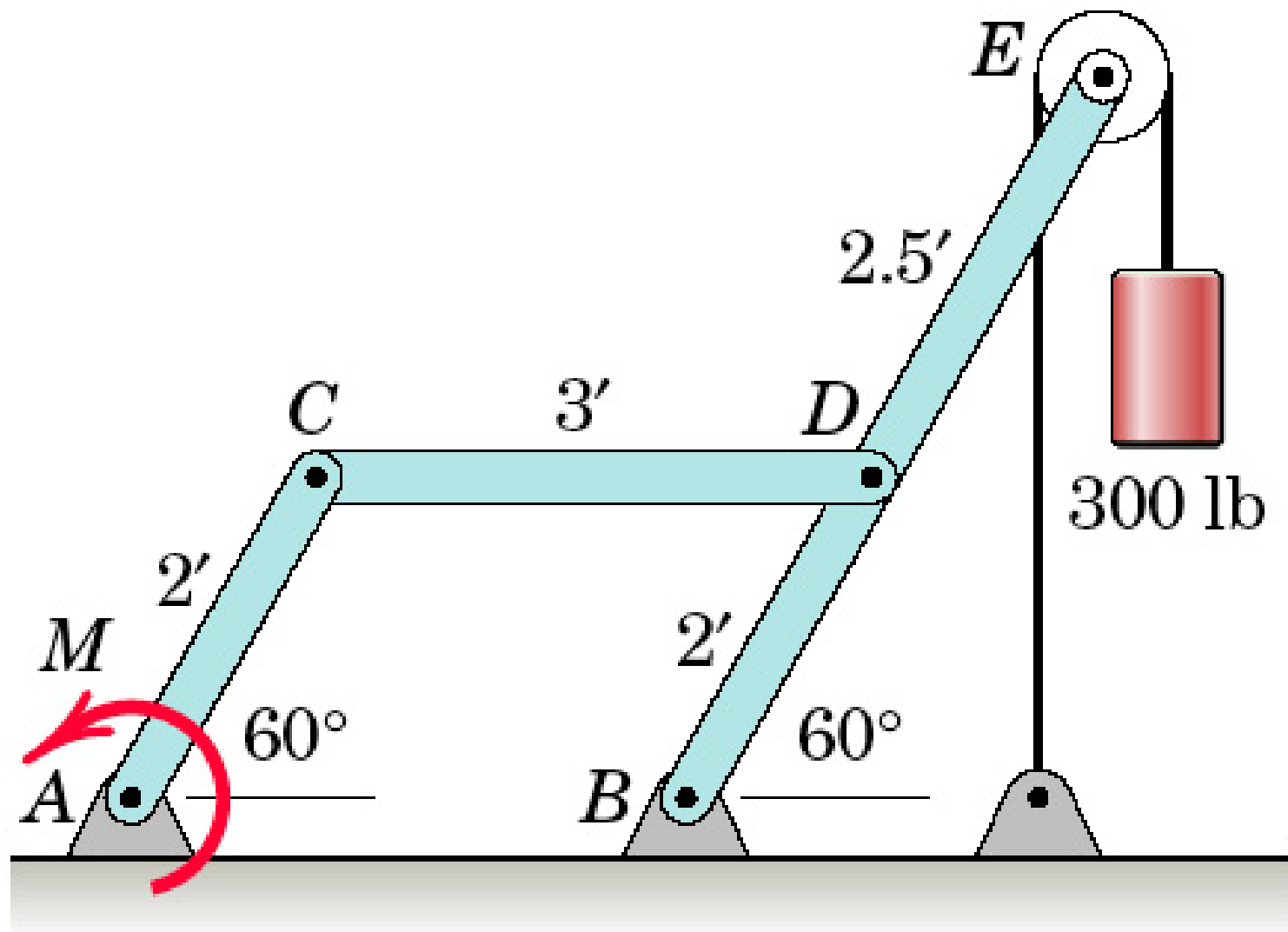


Problem 4/76

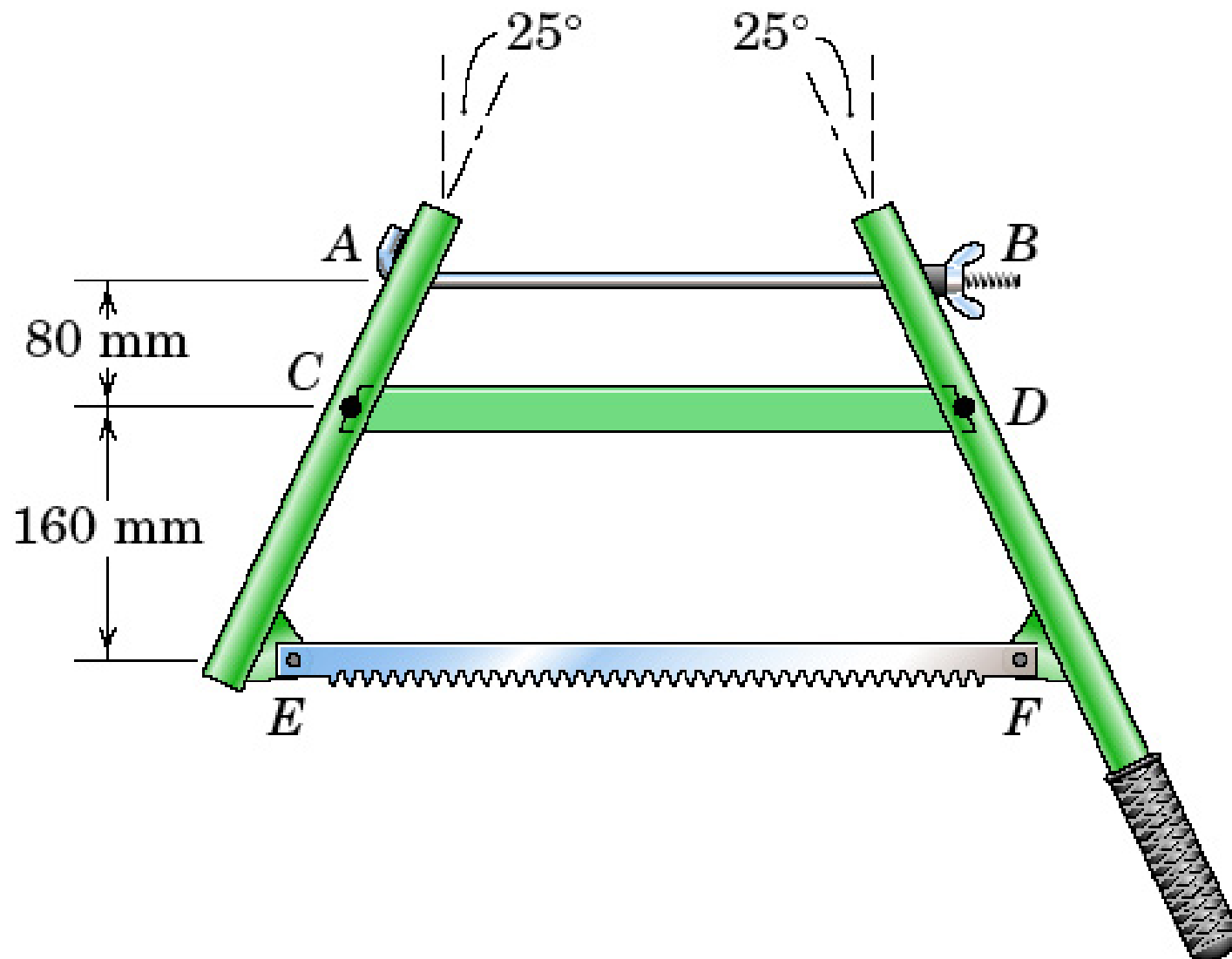


Problem 4/77

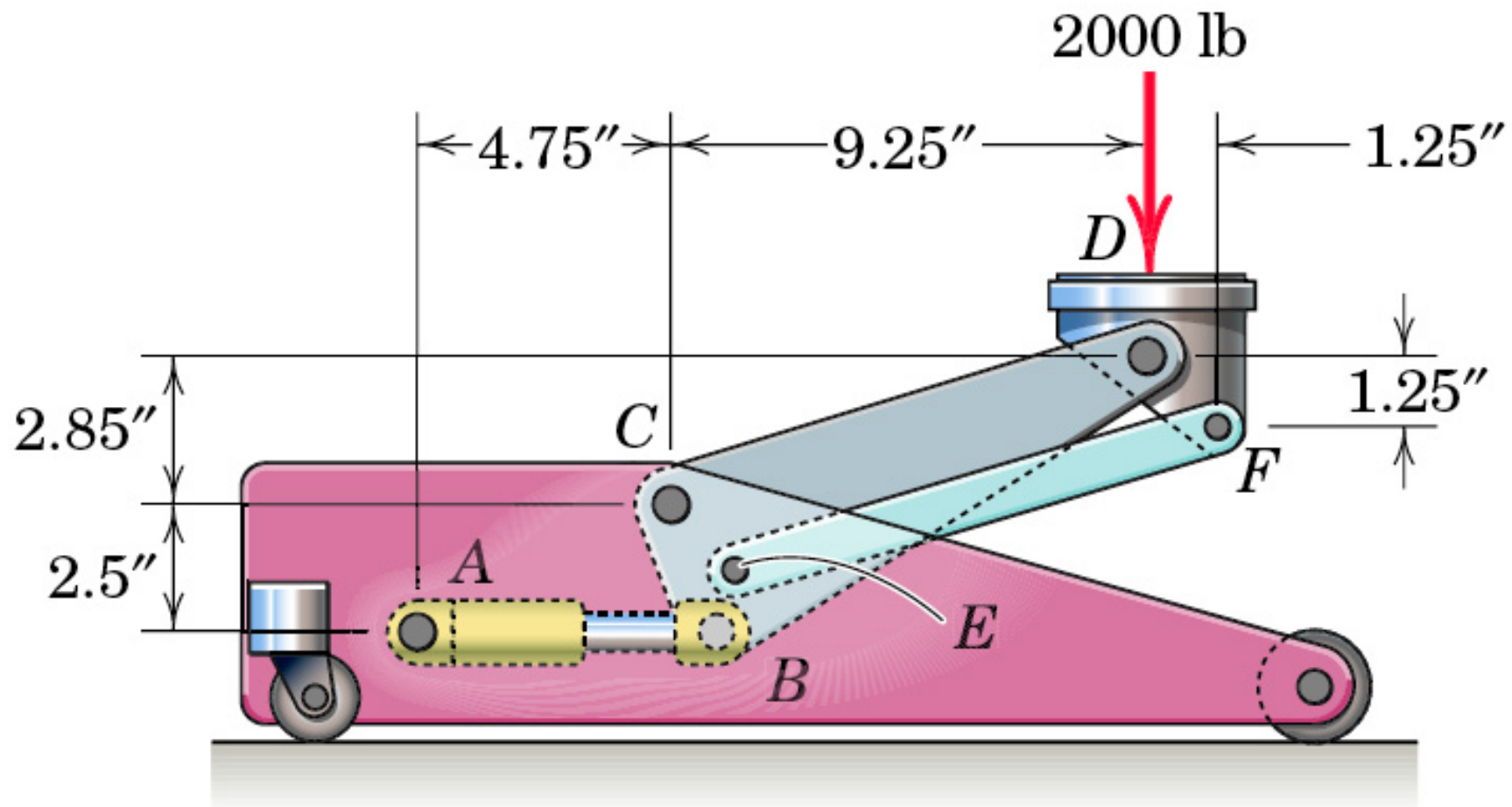
P_04_77



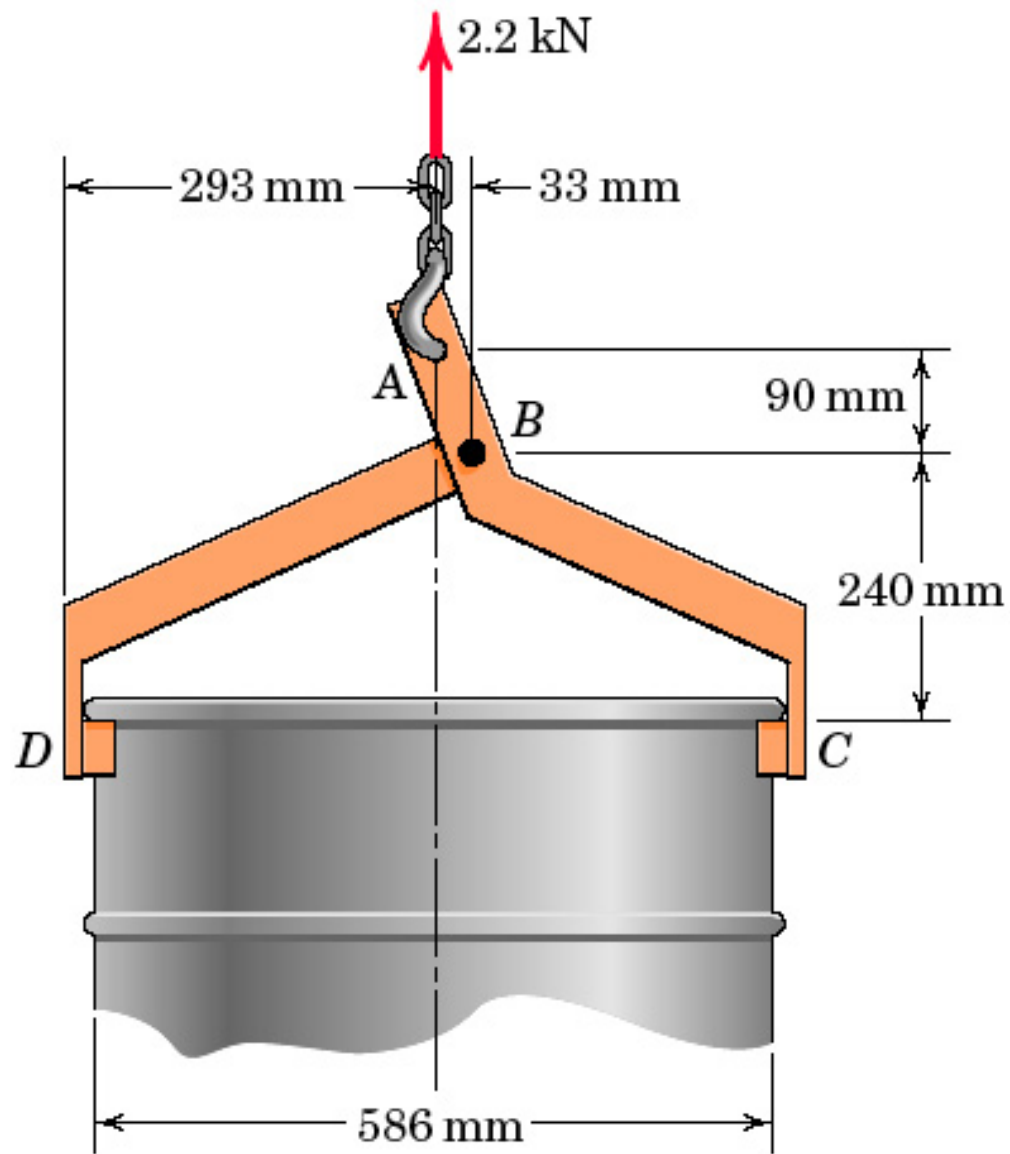
Problem 4/78



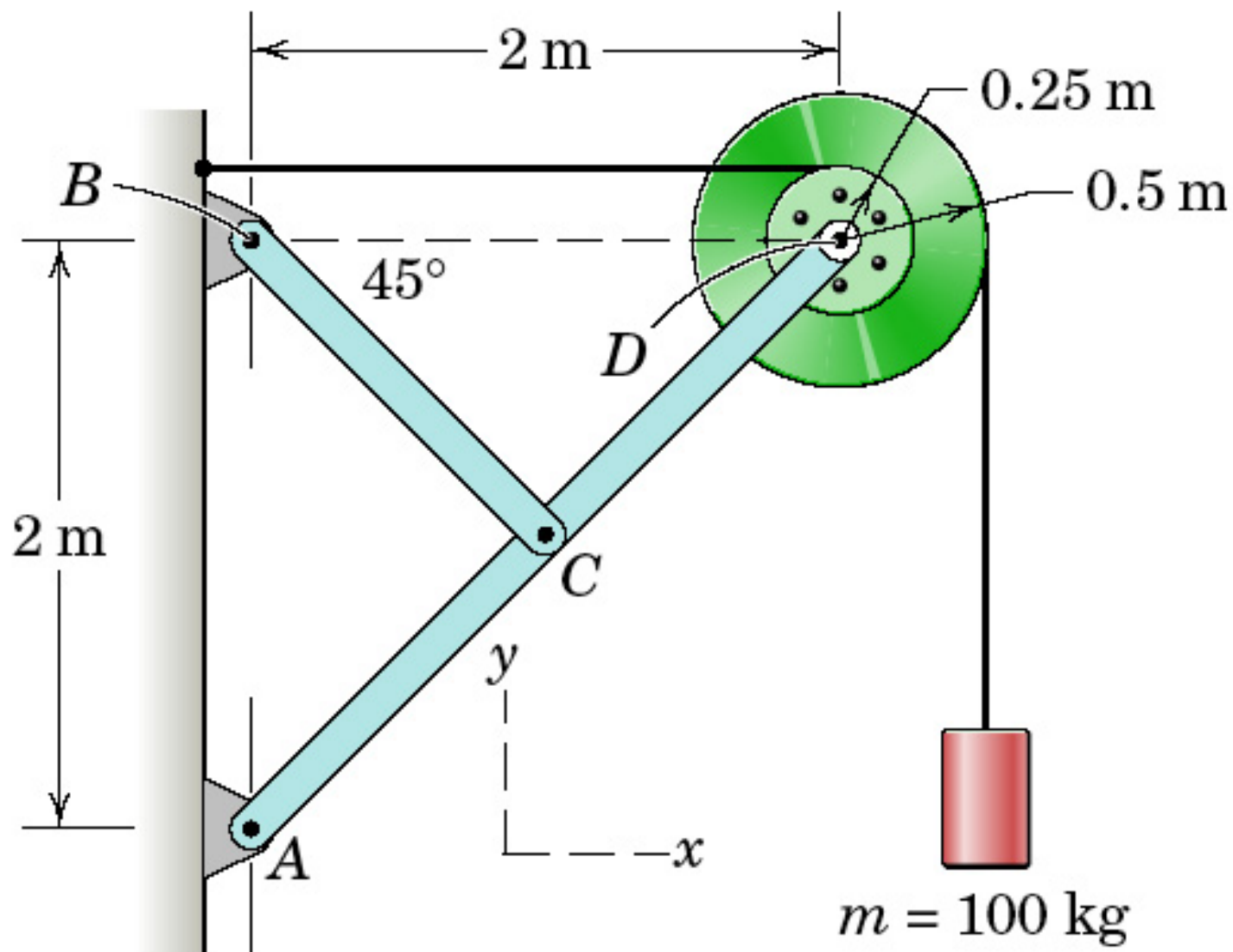
Problem 4/79



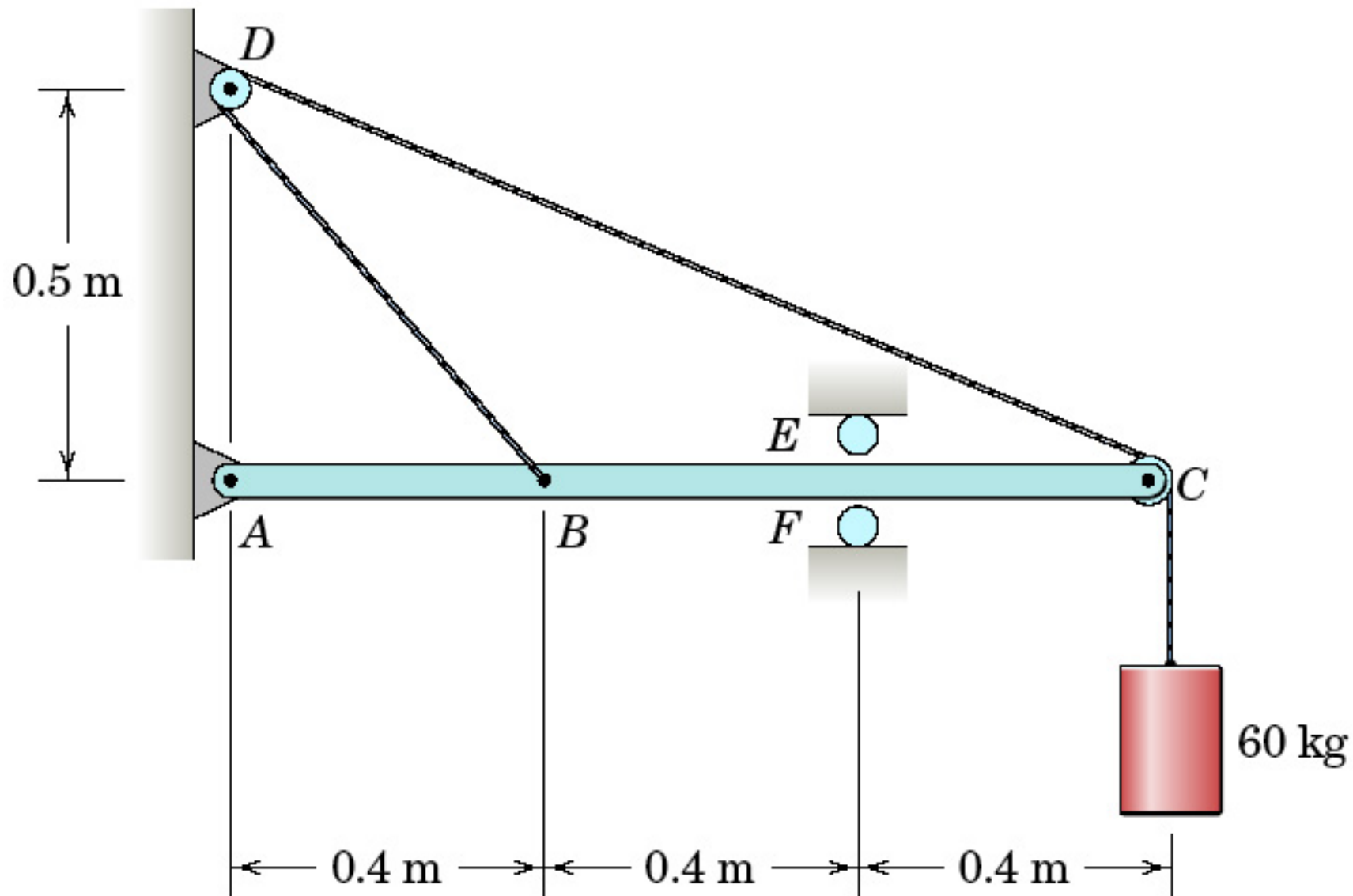
Problem 4/80



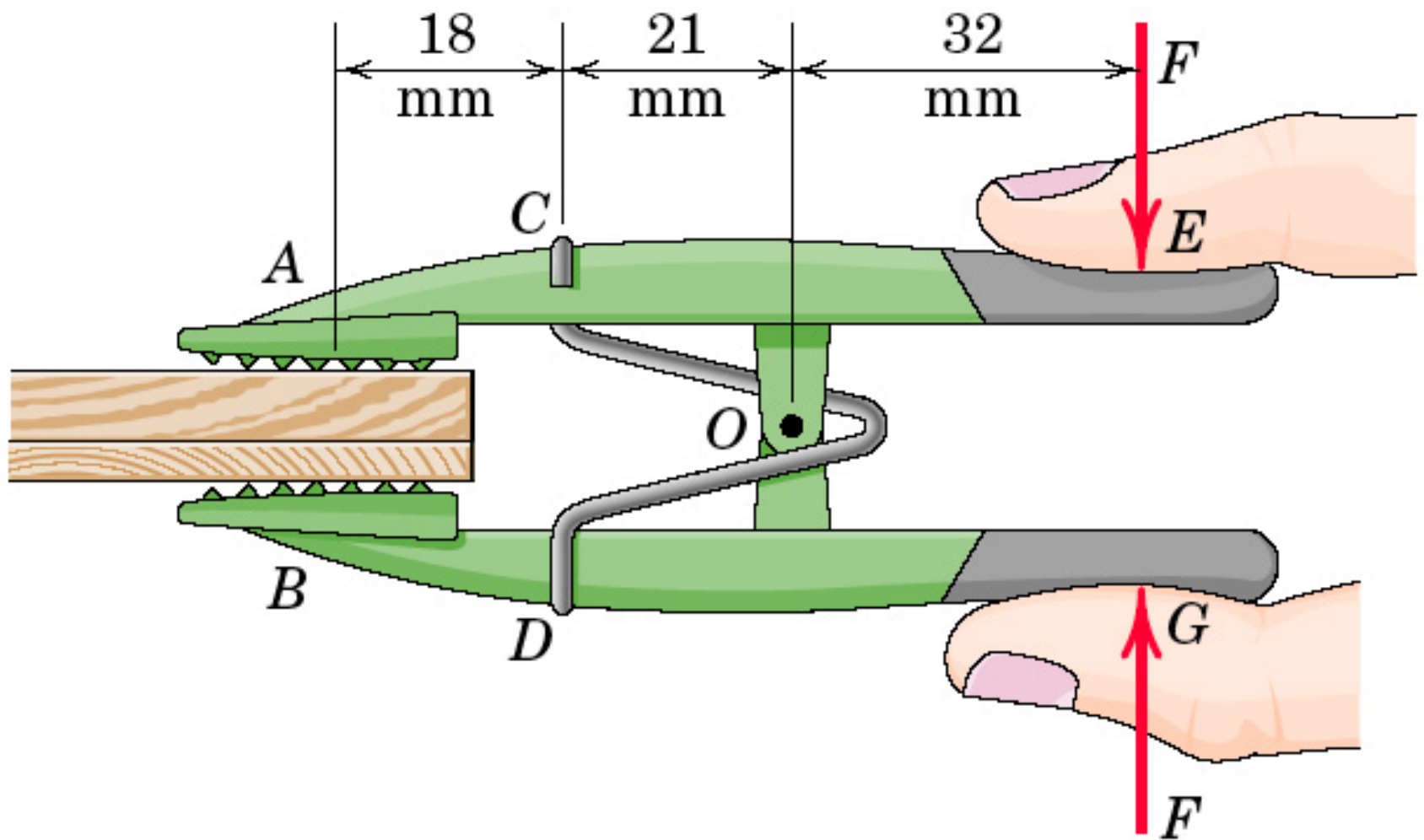
Problem 4/81



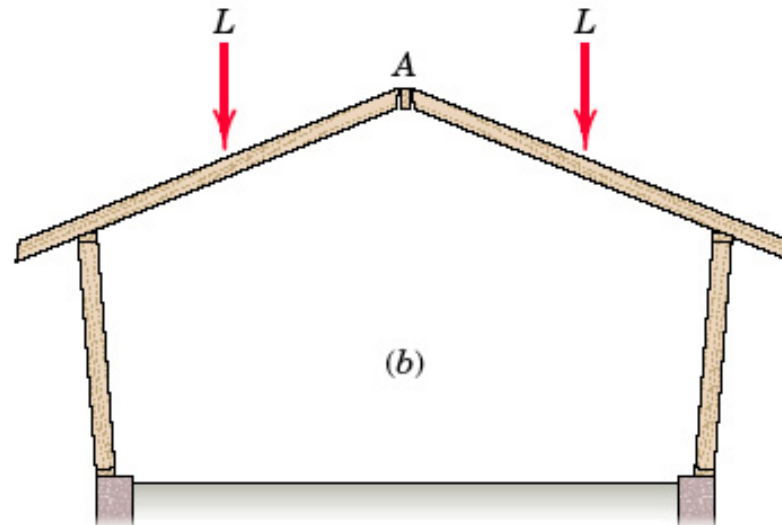
Problem 4/82



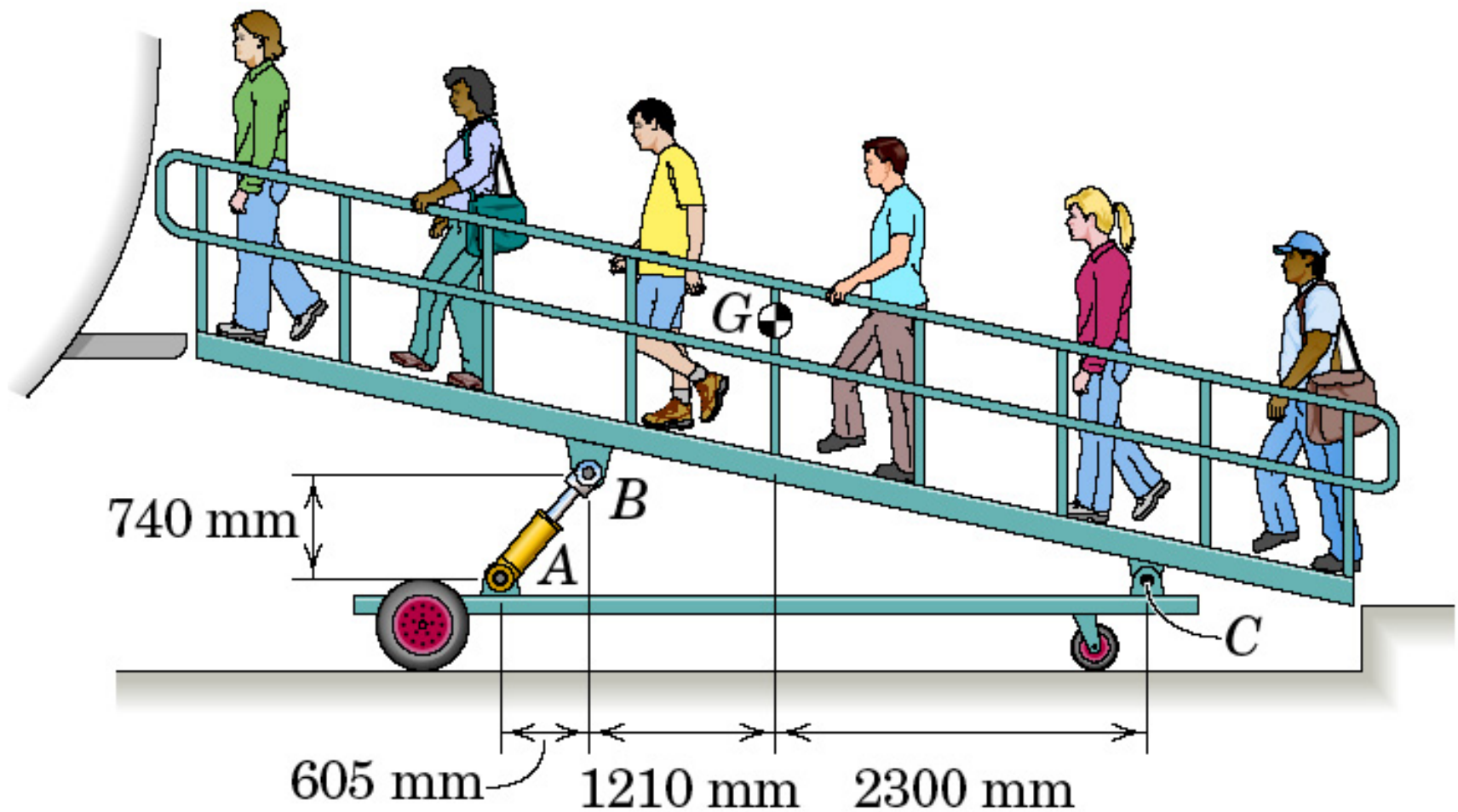
Problem 4/83



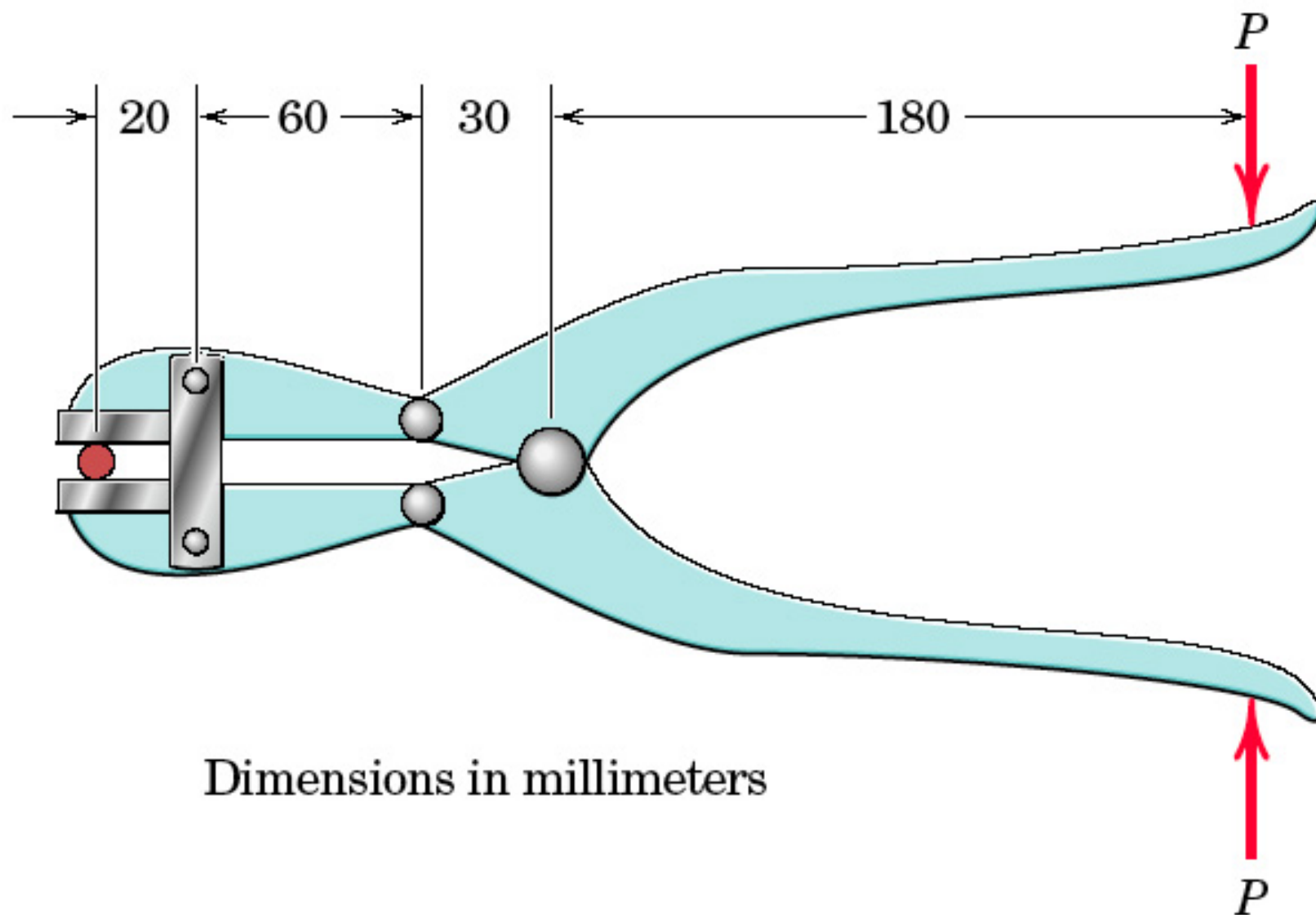
Problem 4/84



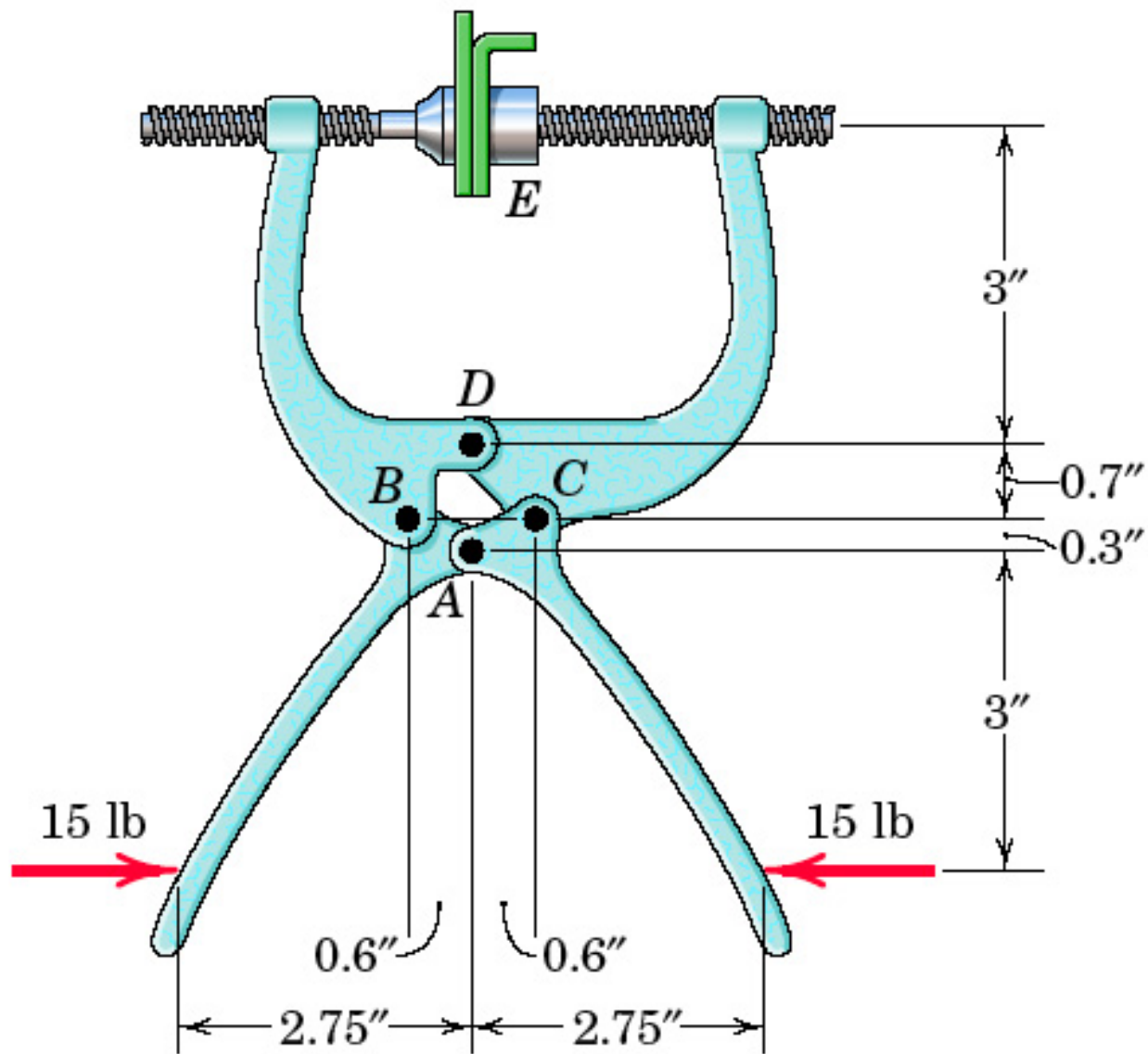
P 04 85



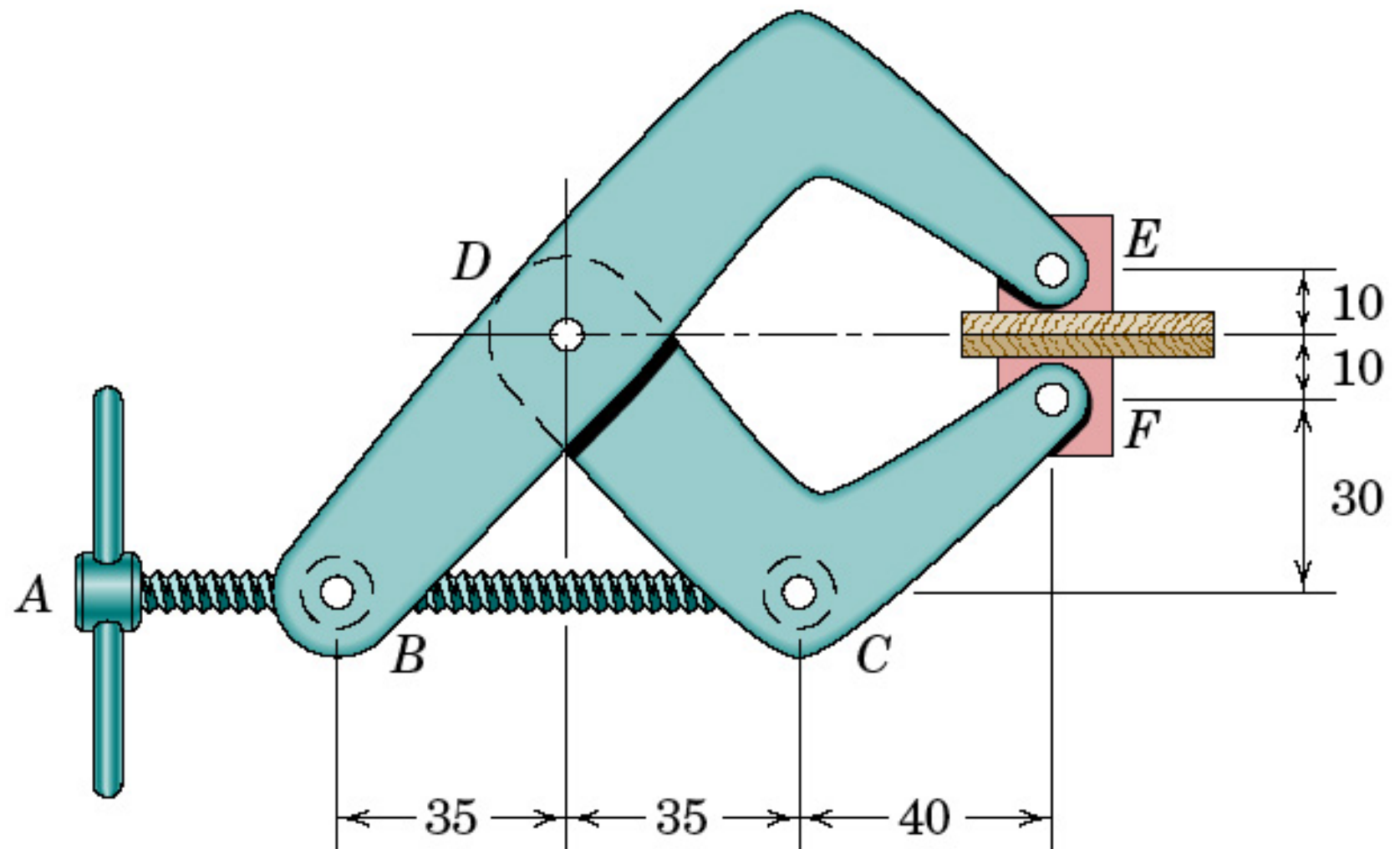
Problem 4/86



Problem 4/87

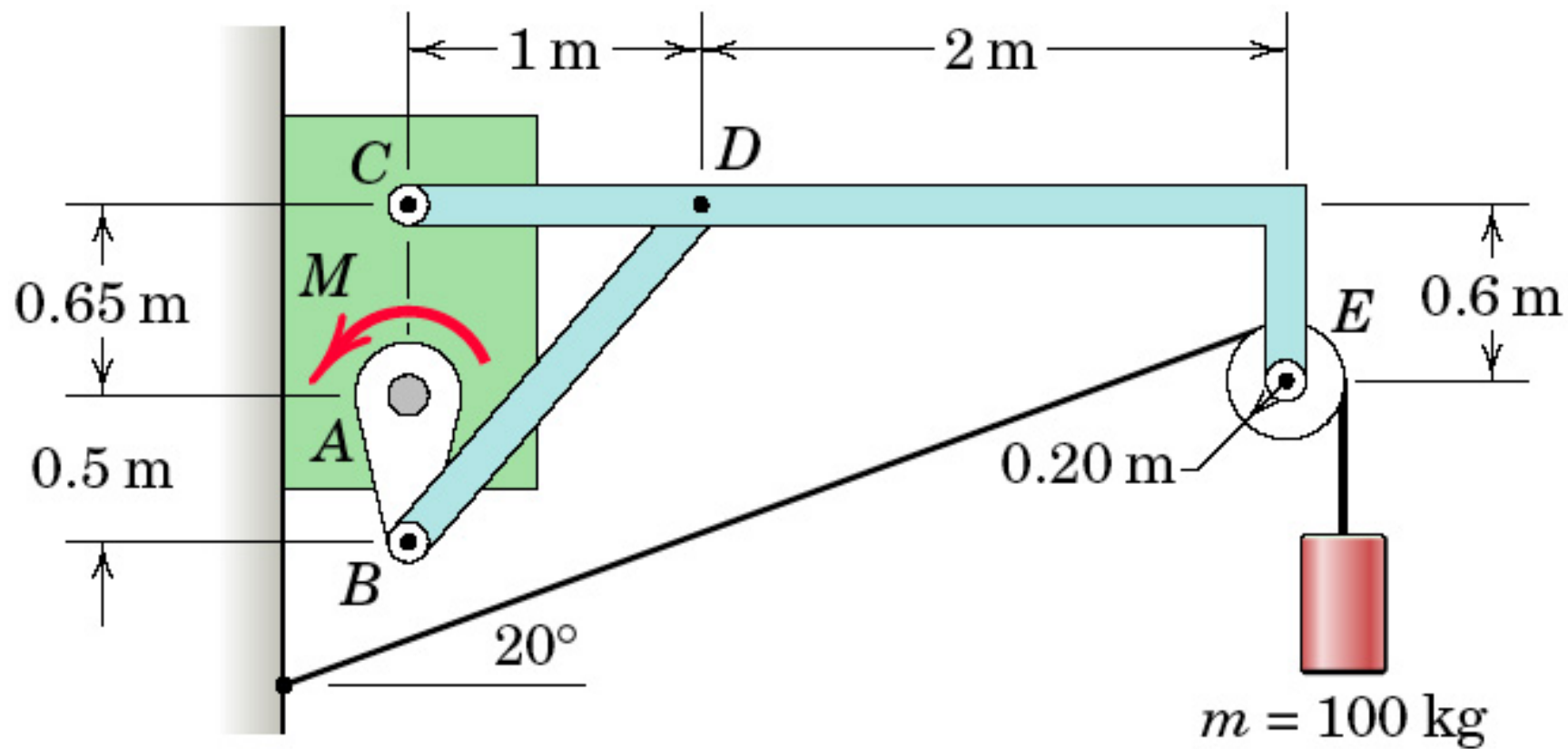


Problem 4/88

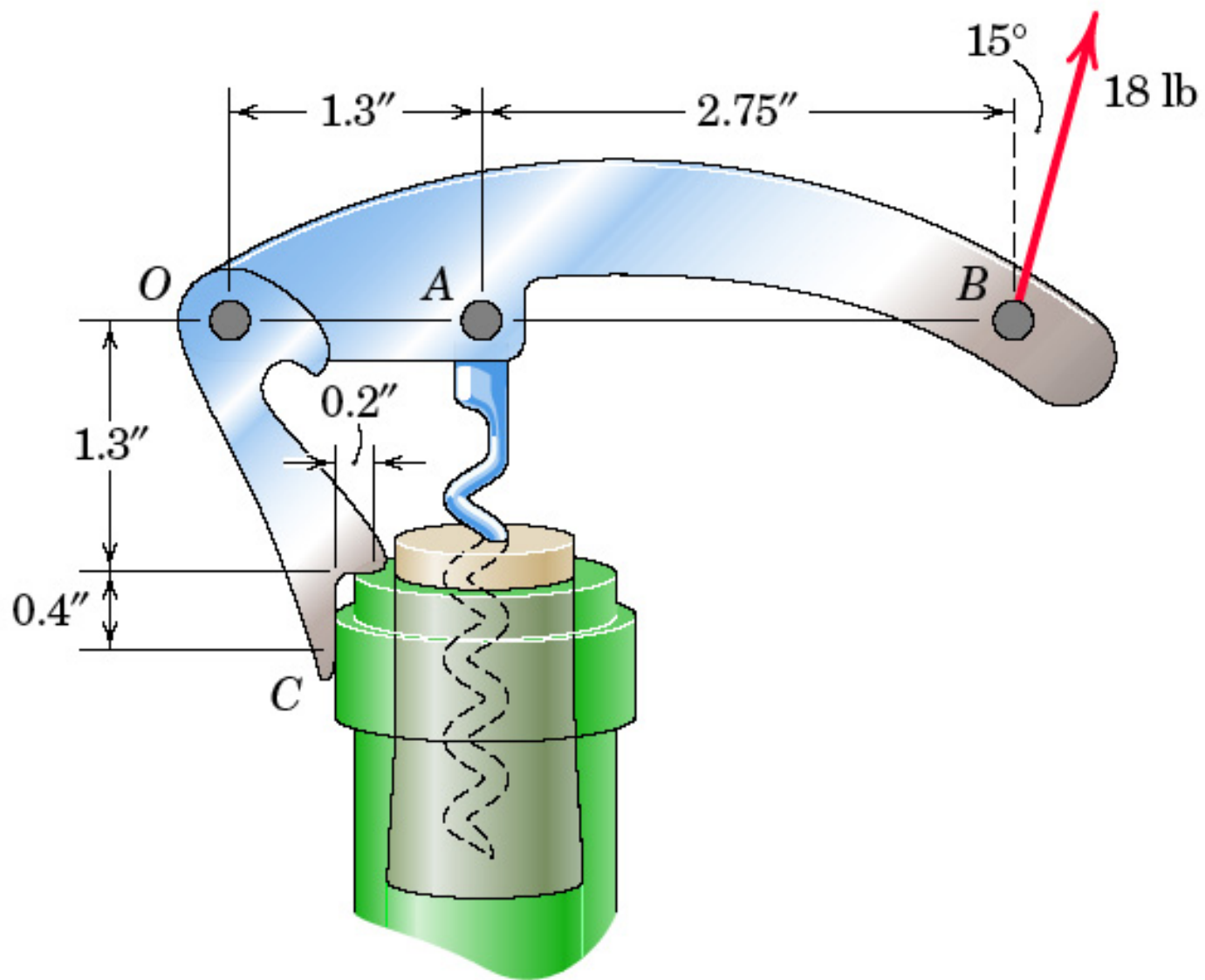


Dimensions in millimeters

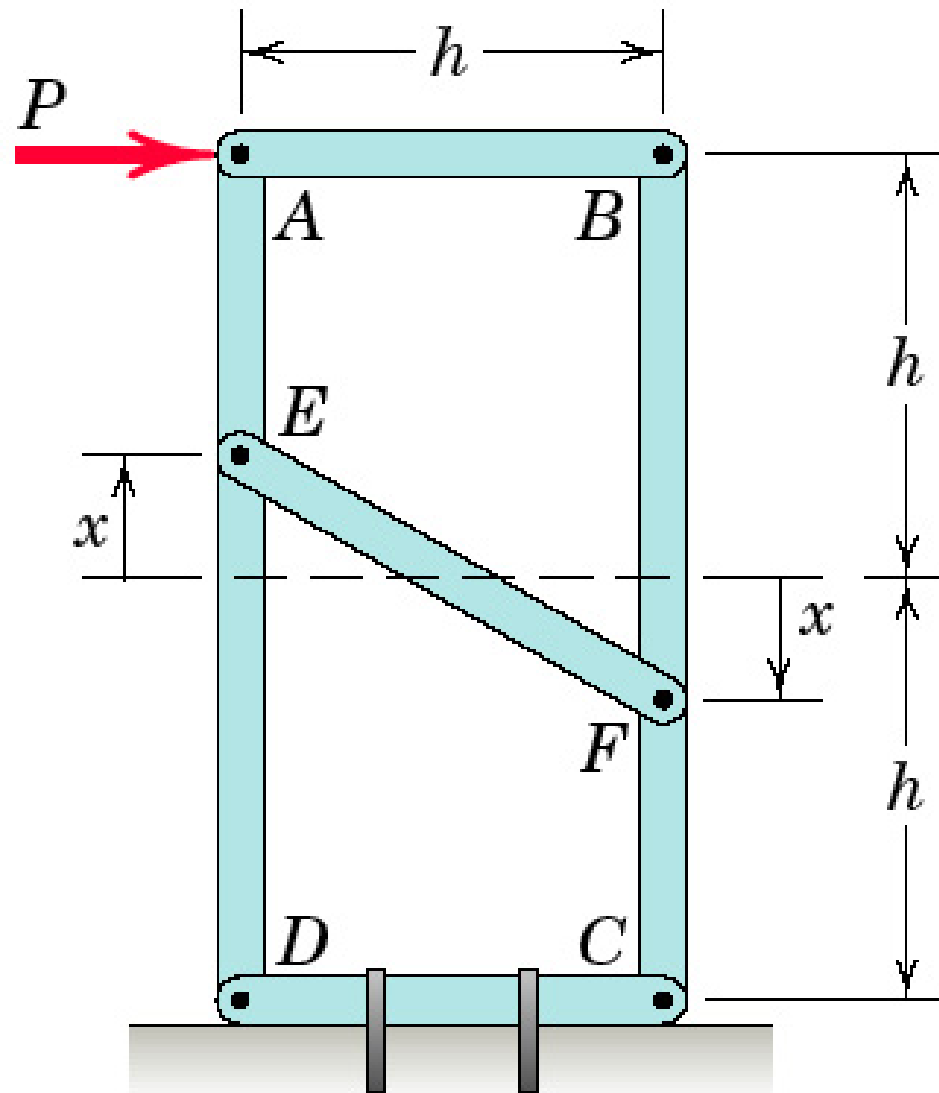
Problem 4/89



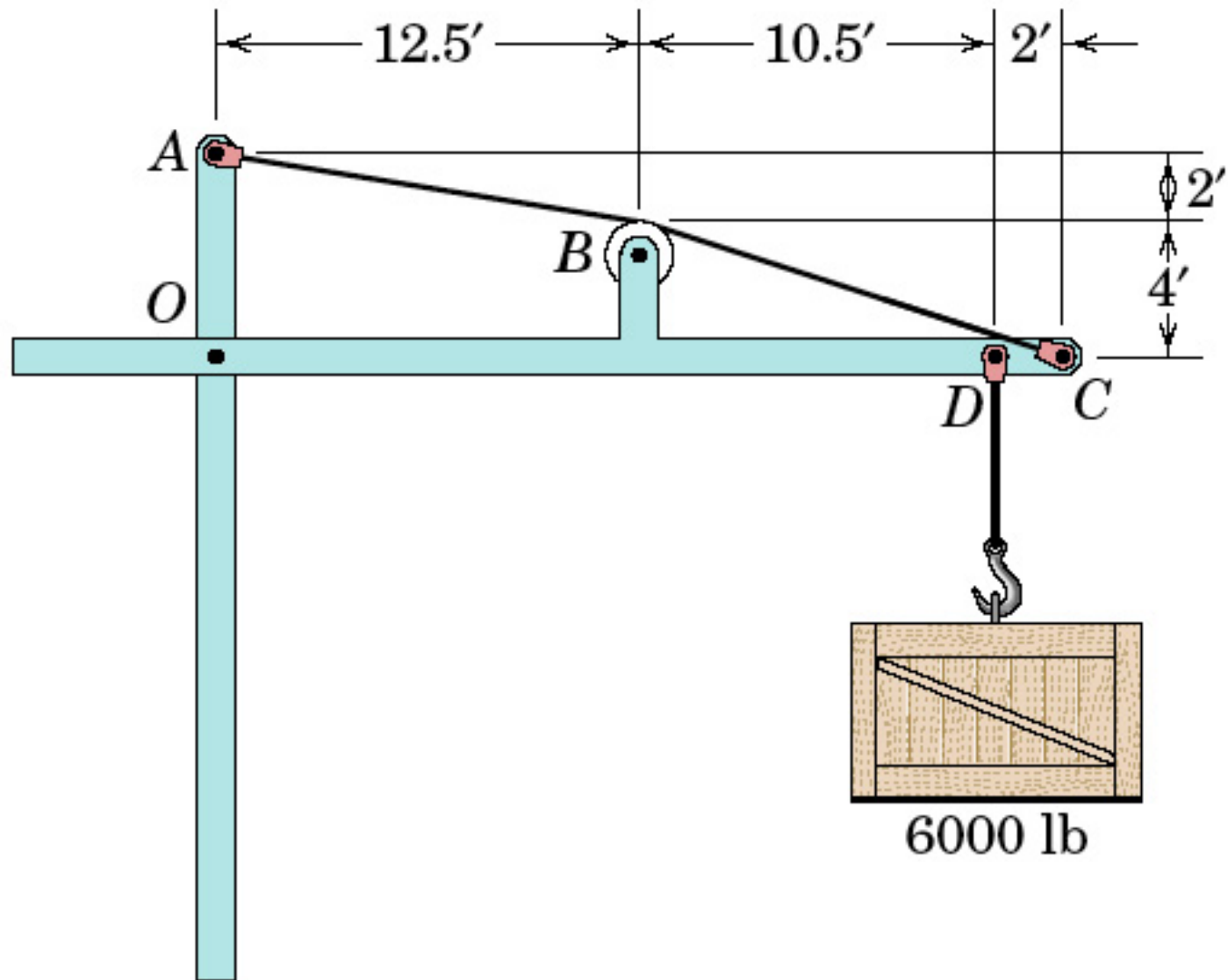
P_04_90



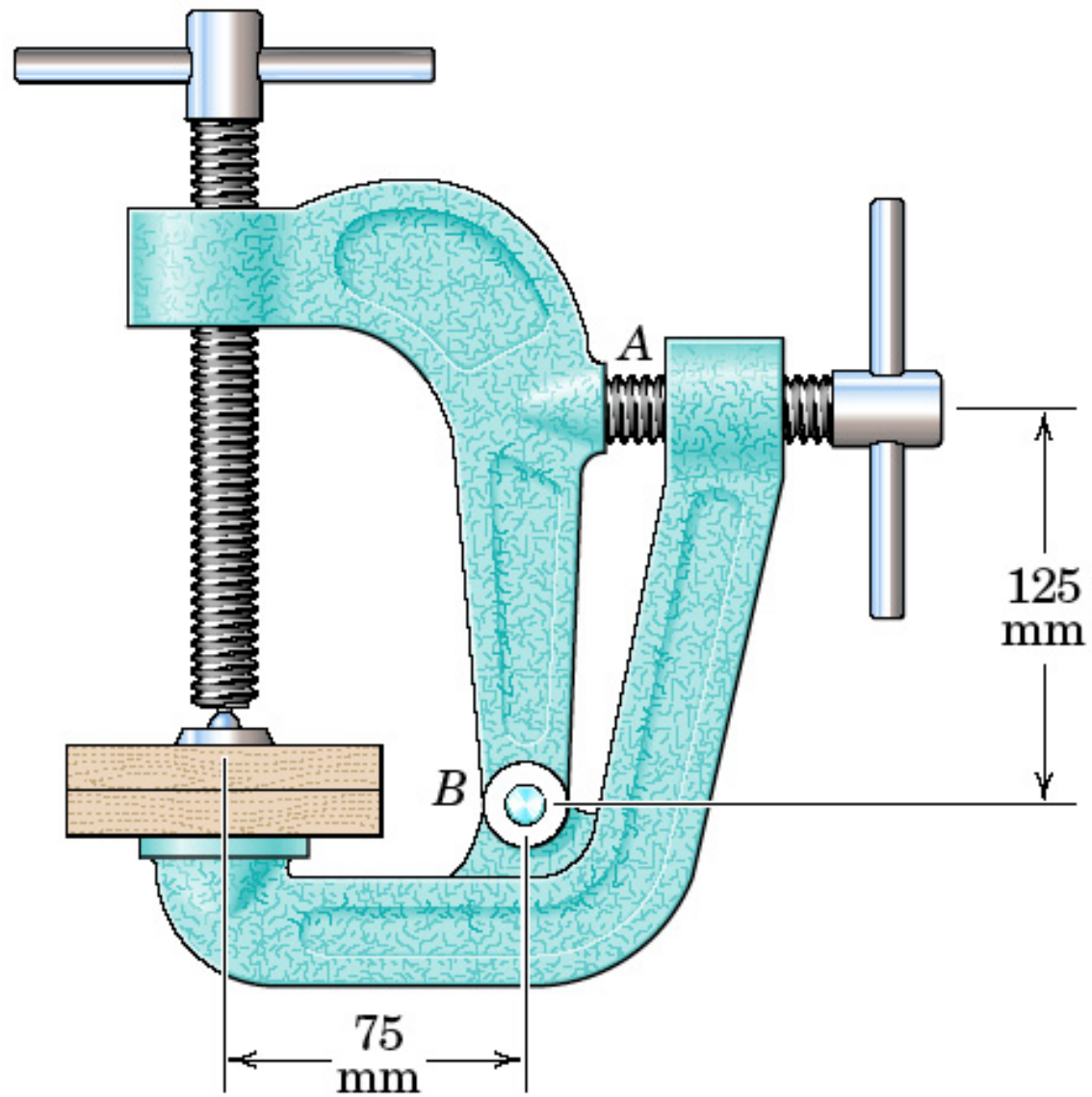
Problem 4/91



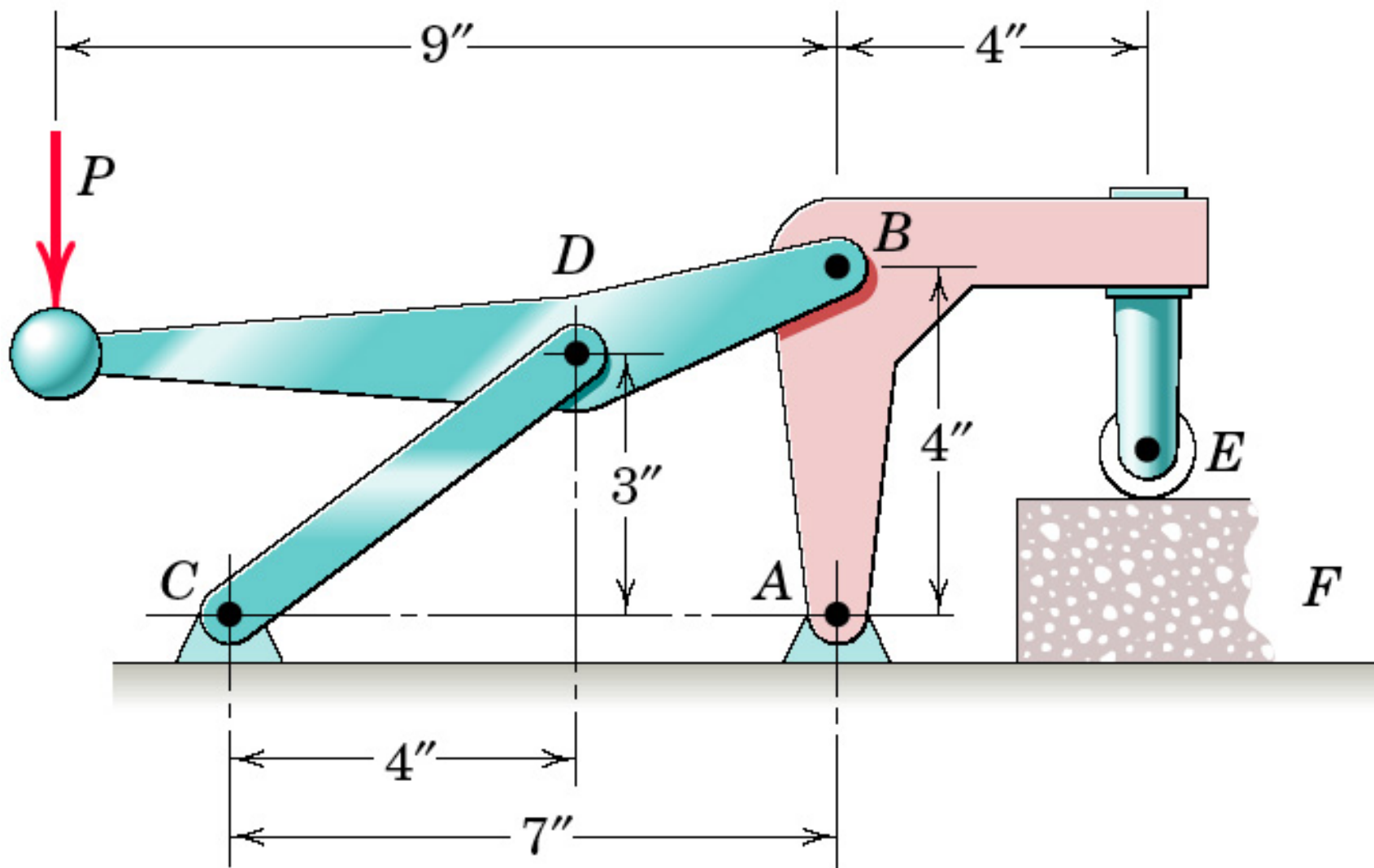
Problem 4/92



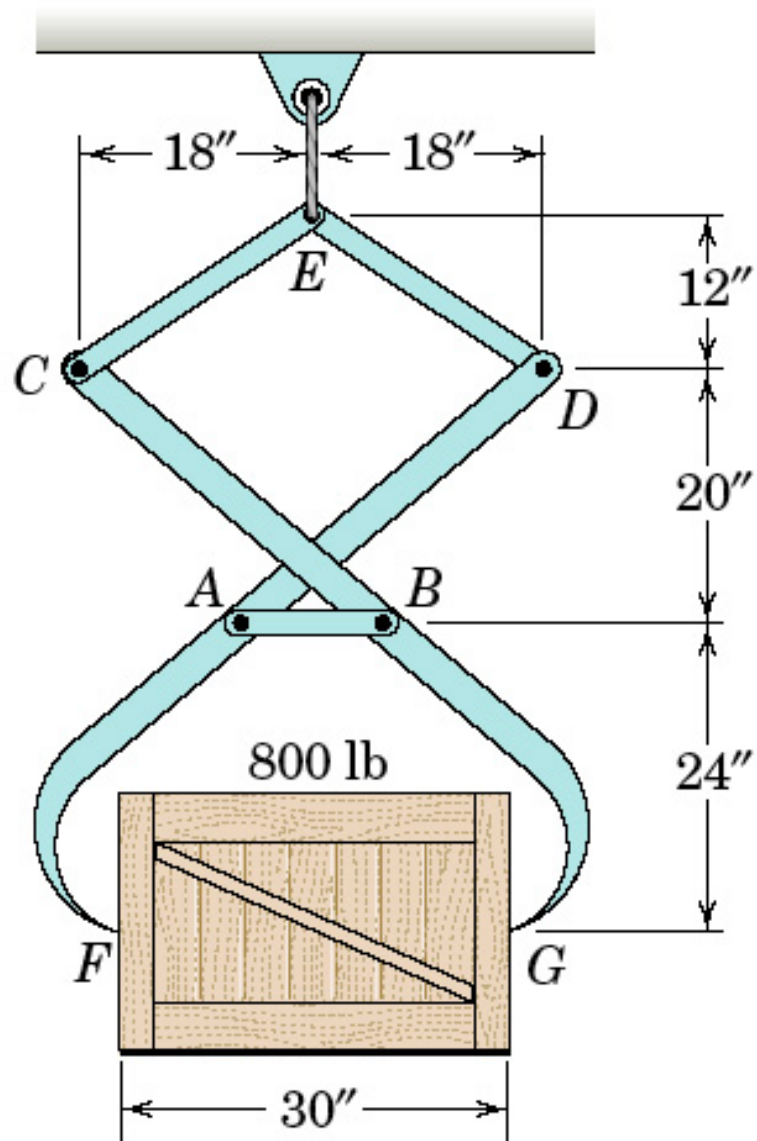
Problem 4/93



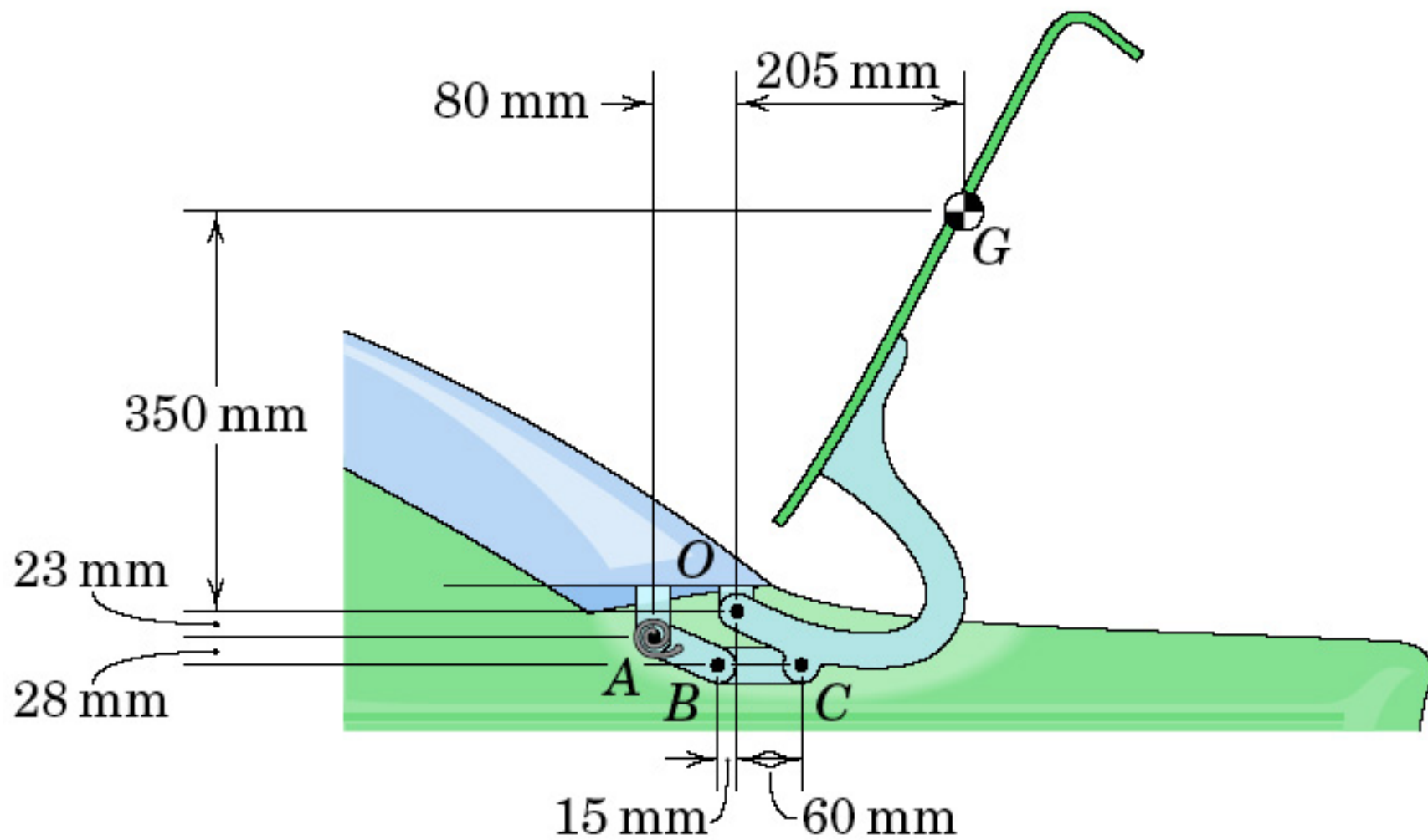
Problem 4/94



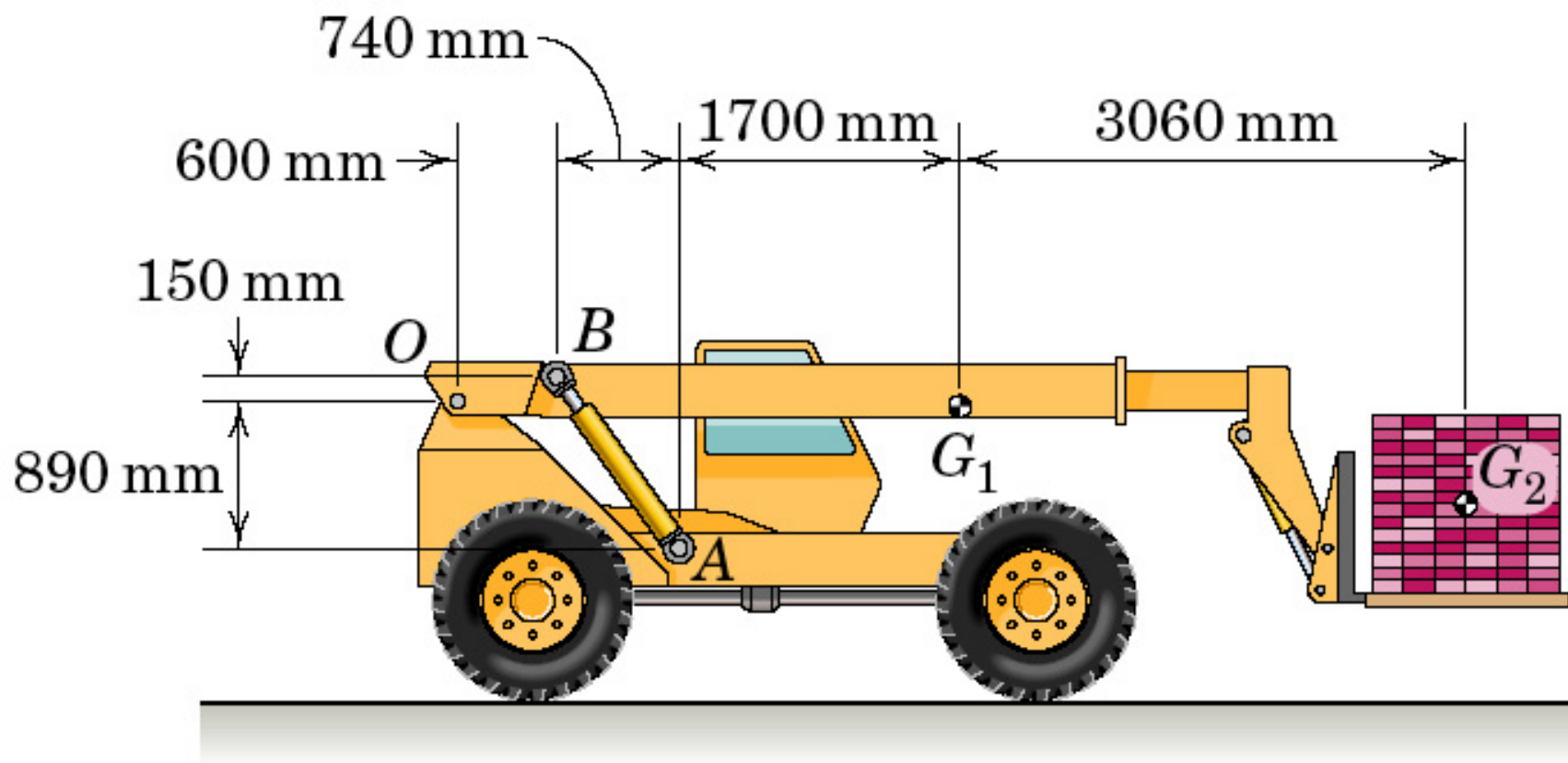
Problem 4/95



Problem 4/96

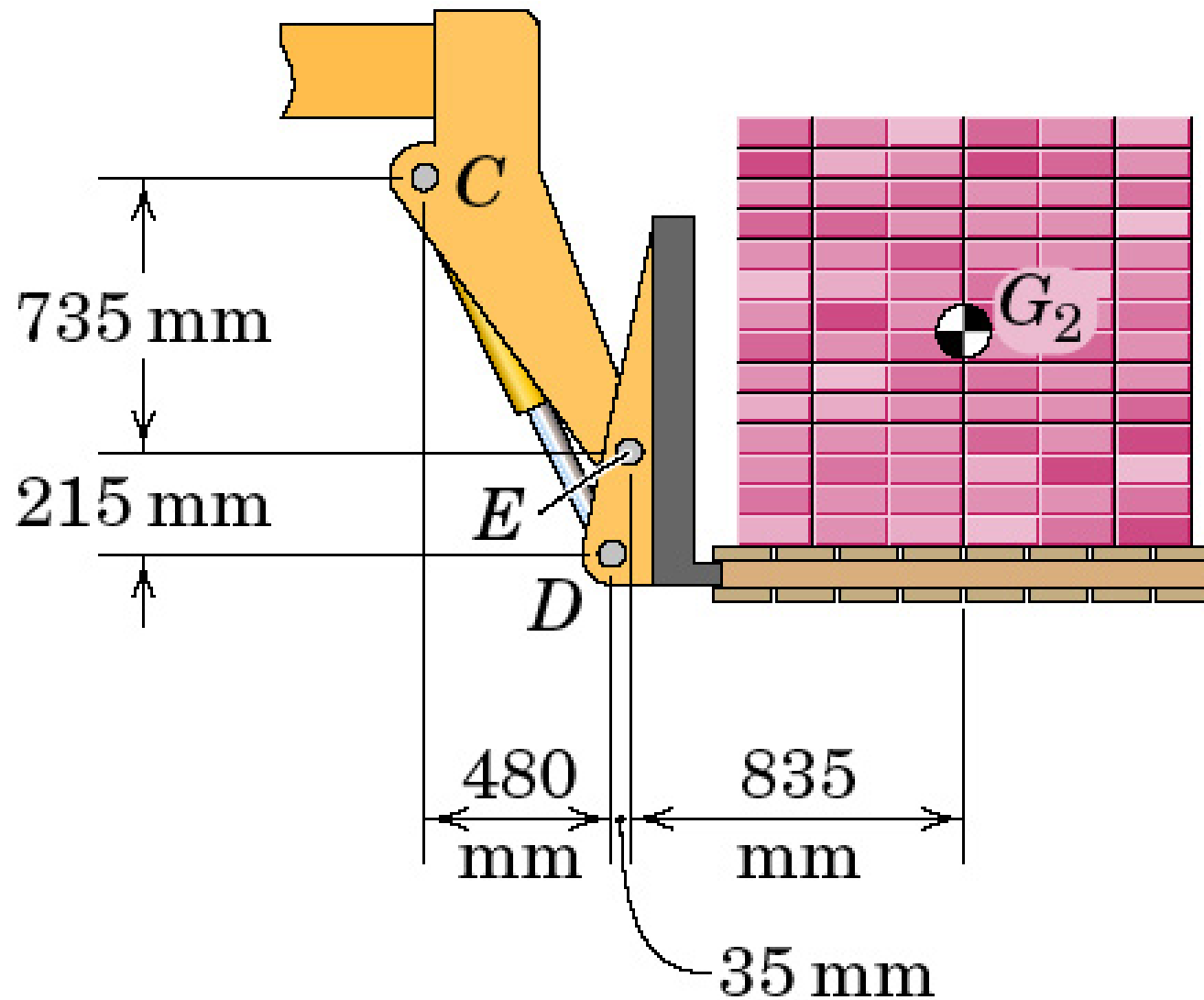


Problem 4/97



Problem 4/98

P_04_98



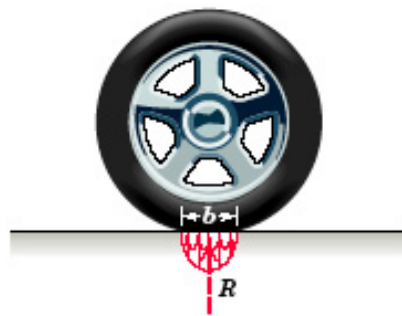
Problem 4/99

Text Illustrations

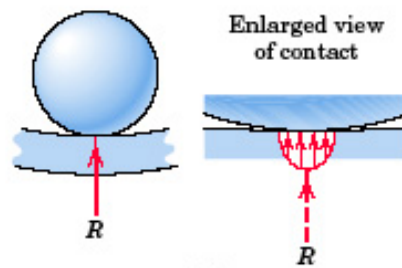
To Accompany

Meriam/Kraige – Statics 6e

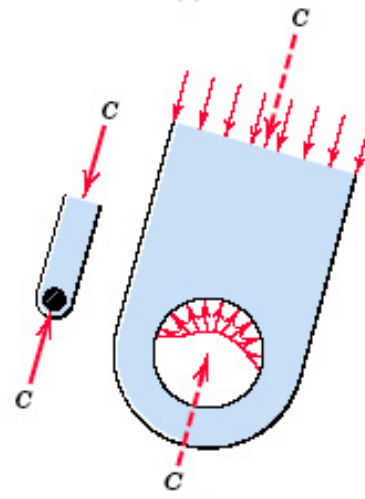
Chapter 5 : Distributed Forces.



(a)



(b)



(c)

Figure 5/1

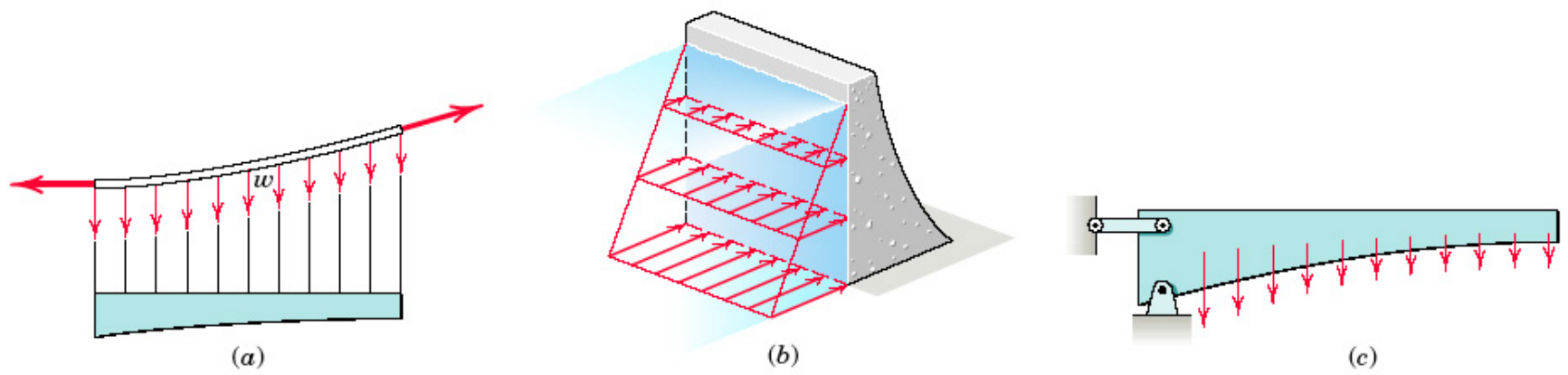
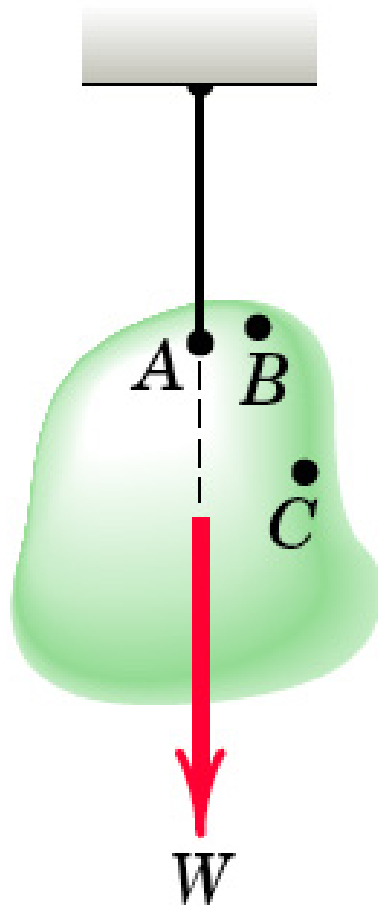
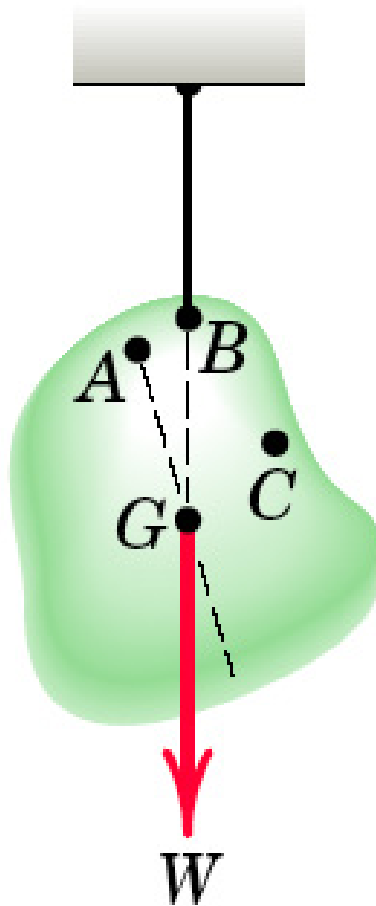


Figure 5/2

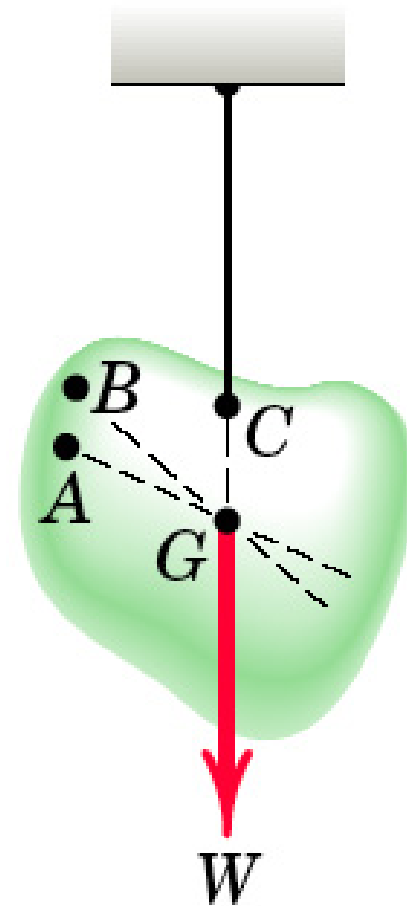
05_02



(a)

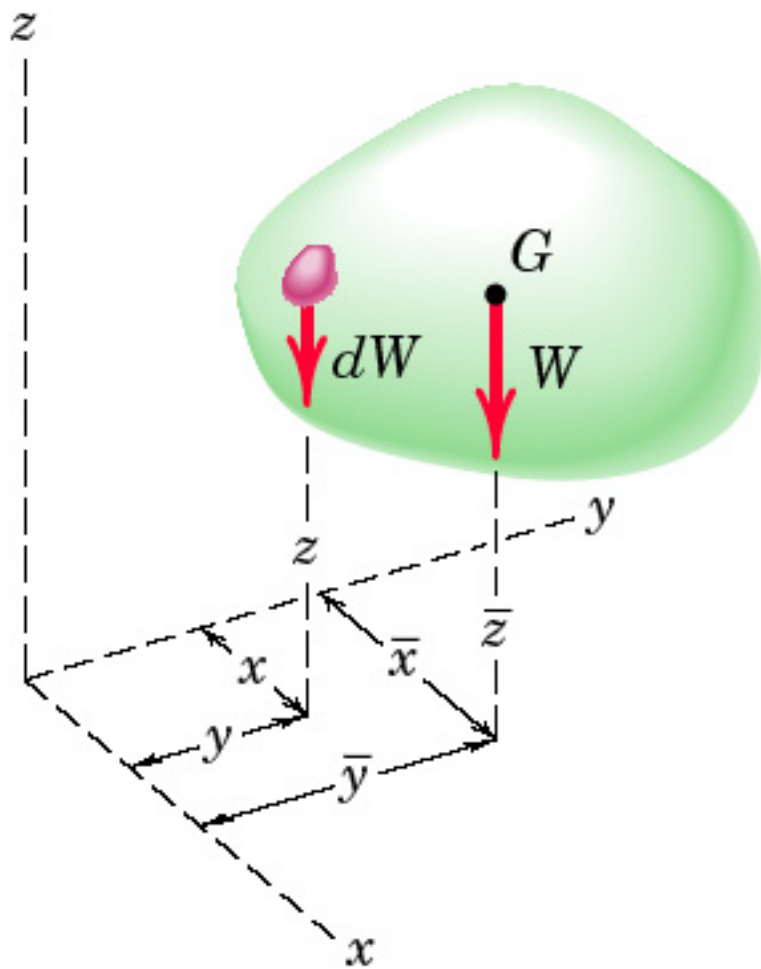


(b)

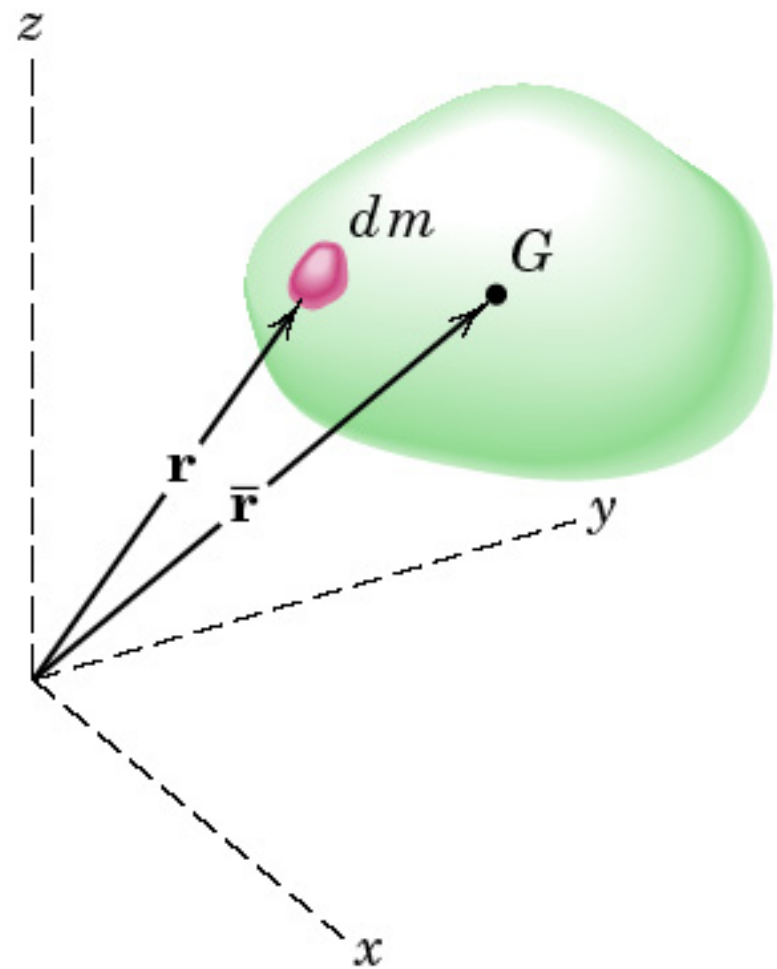


(c)

Figure 5/3



(a)



(b)

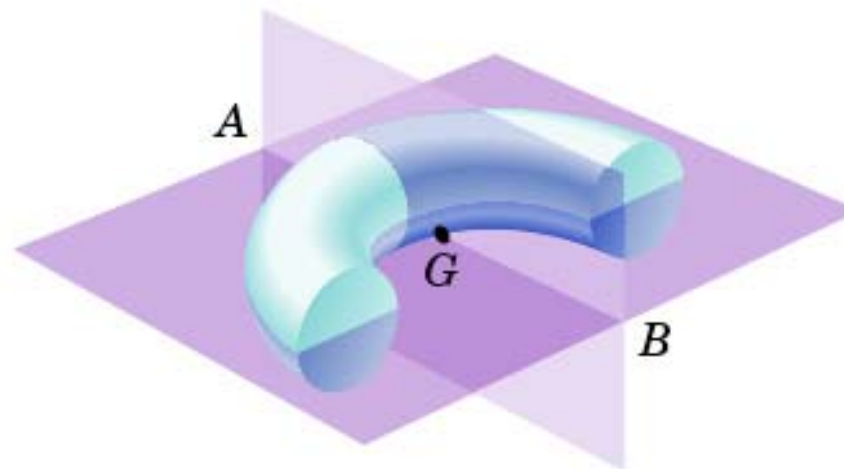
Figure 5/4



(a)



(b)



(c)

Figure 5/5

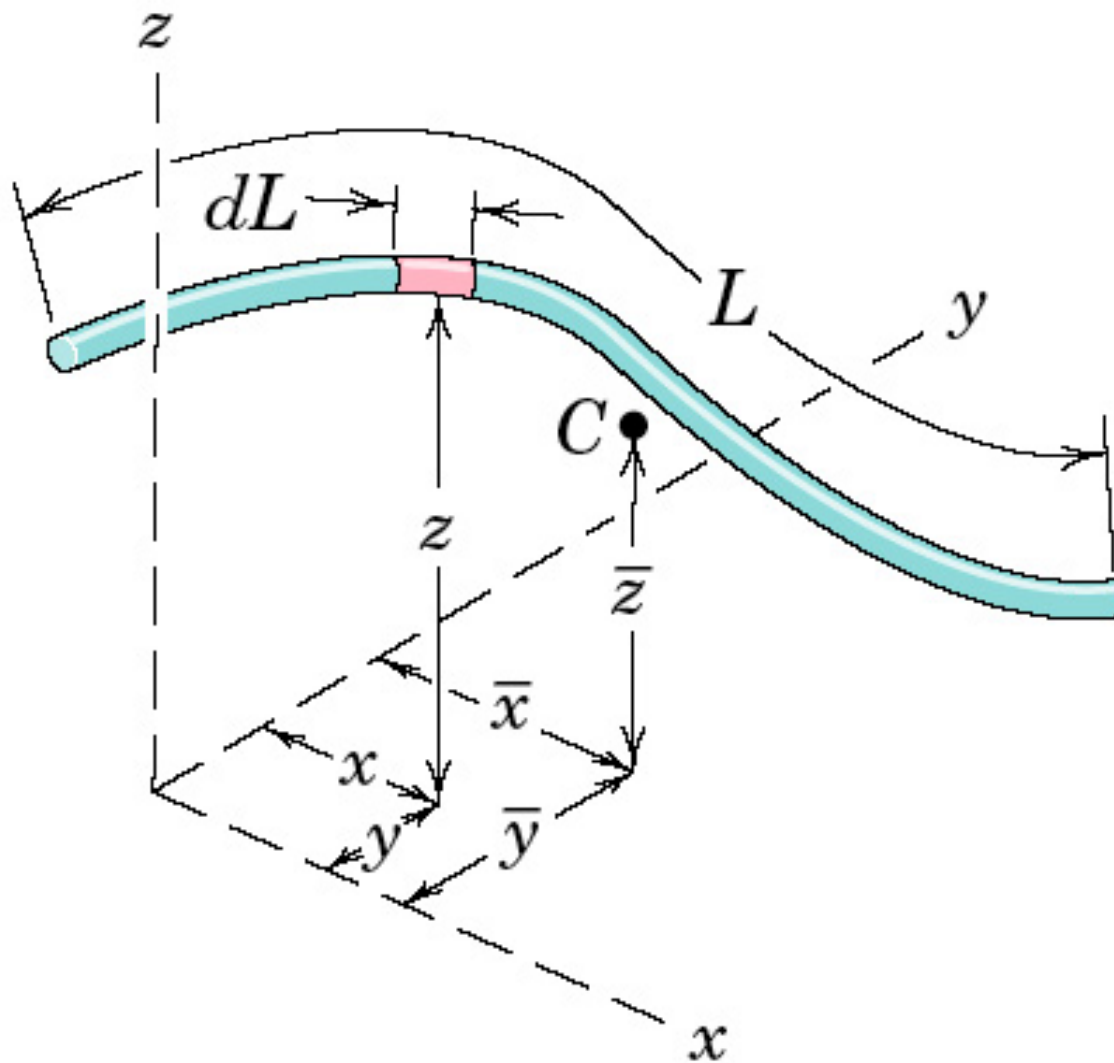


Figure 5/6

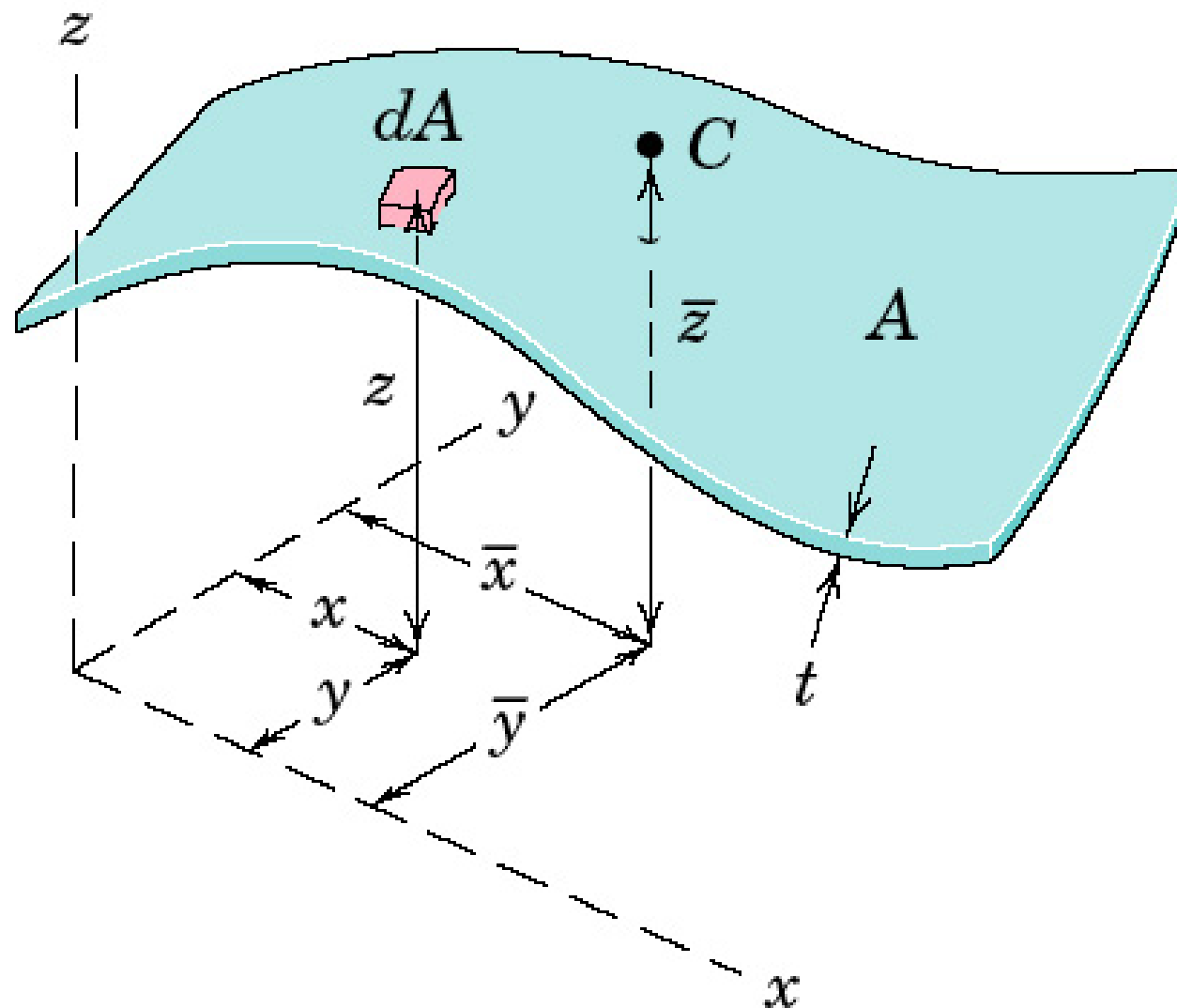
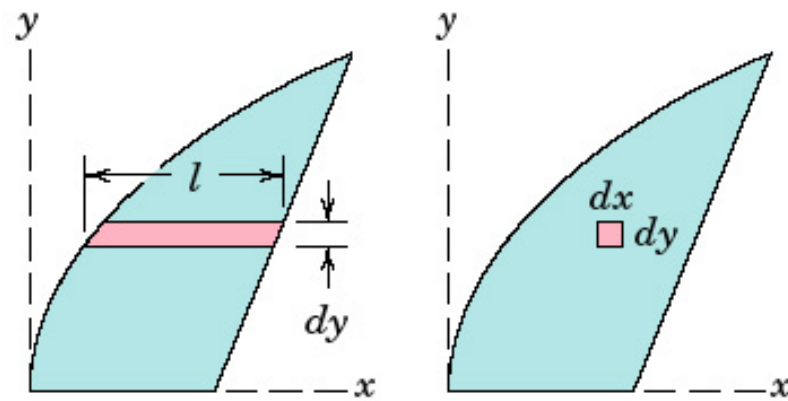
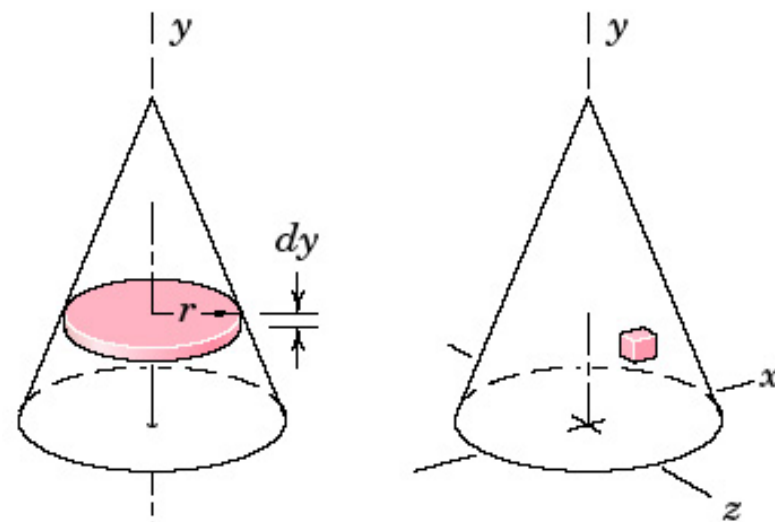


Figure 5/7



(a)



(b)

Figure 5/8

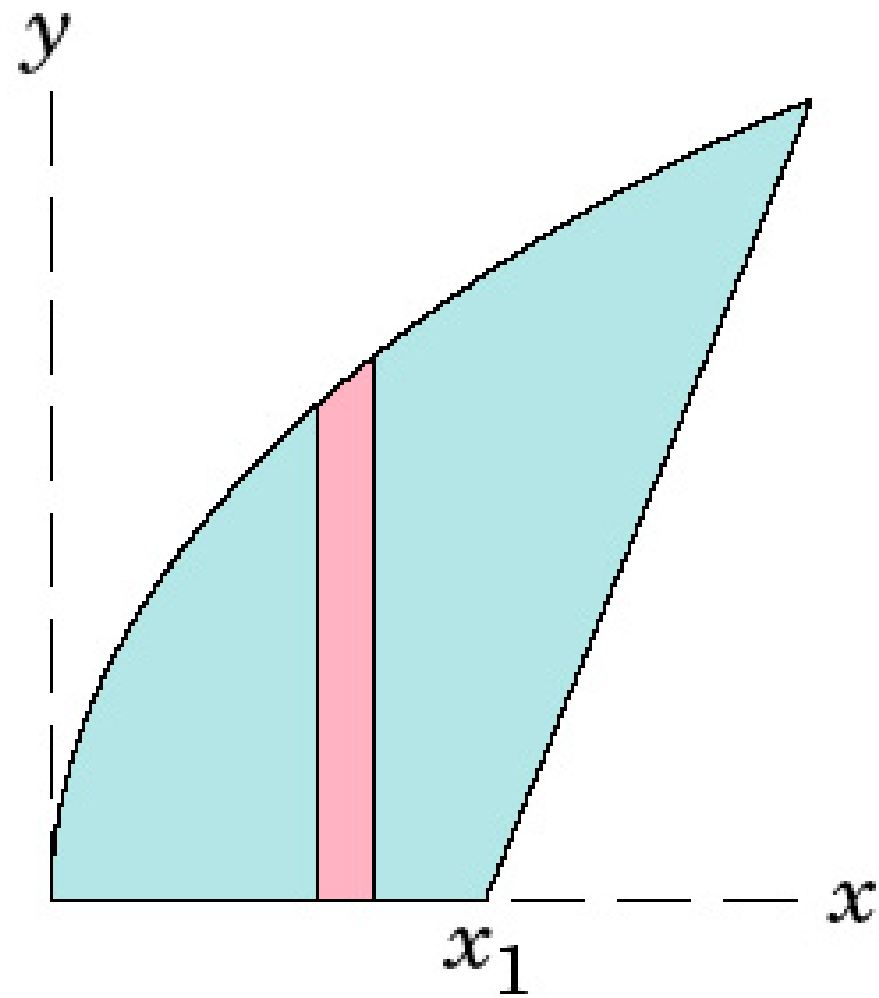


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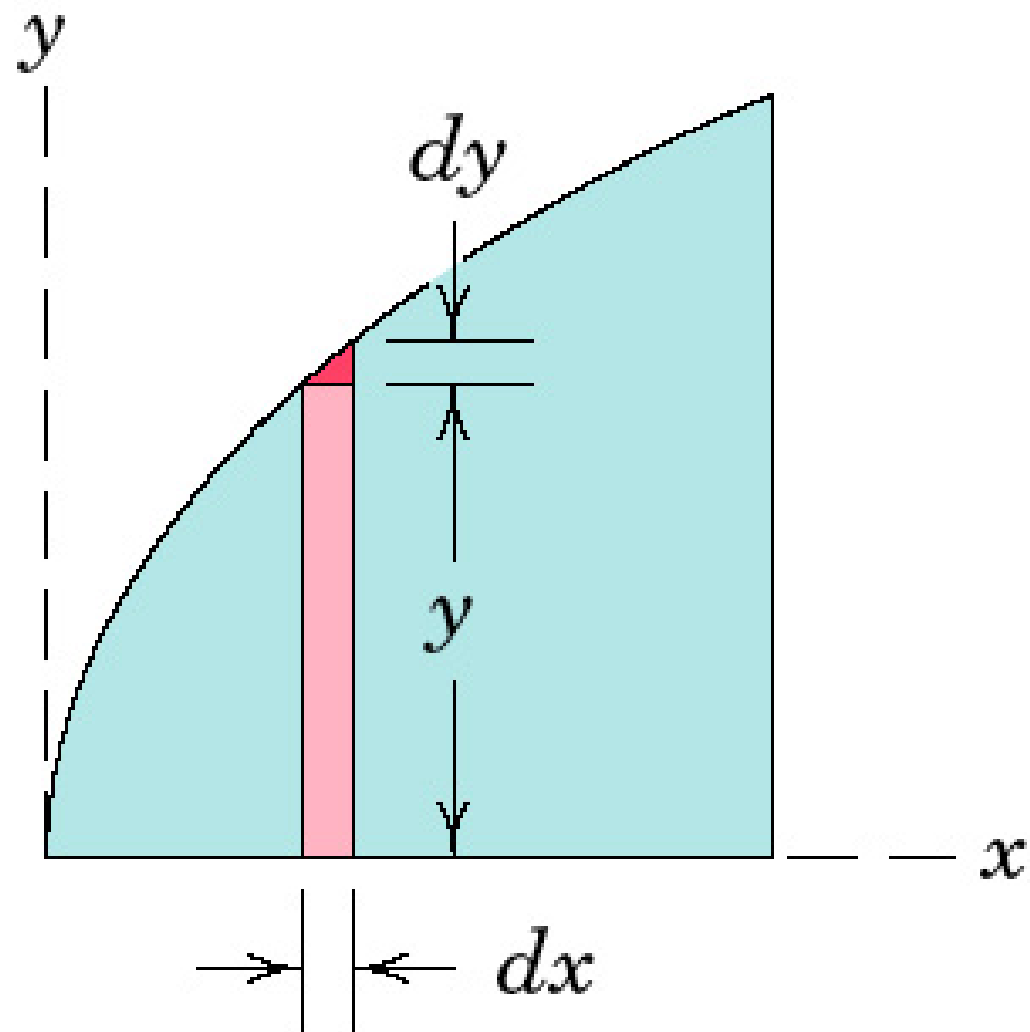


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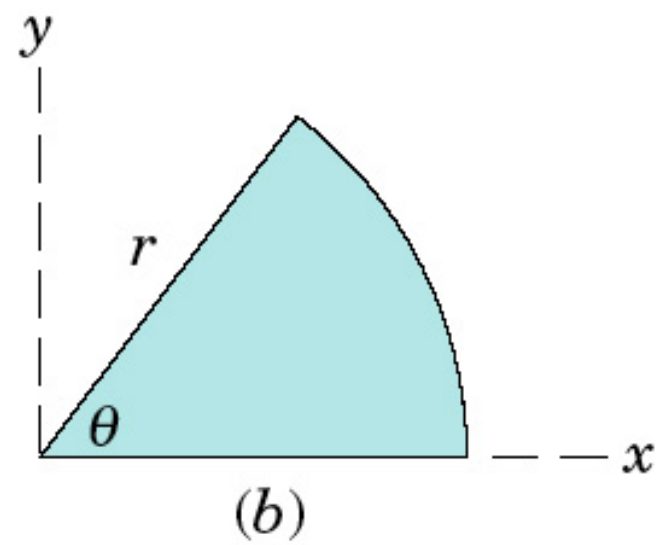
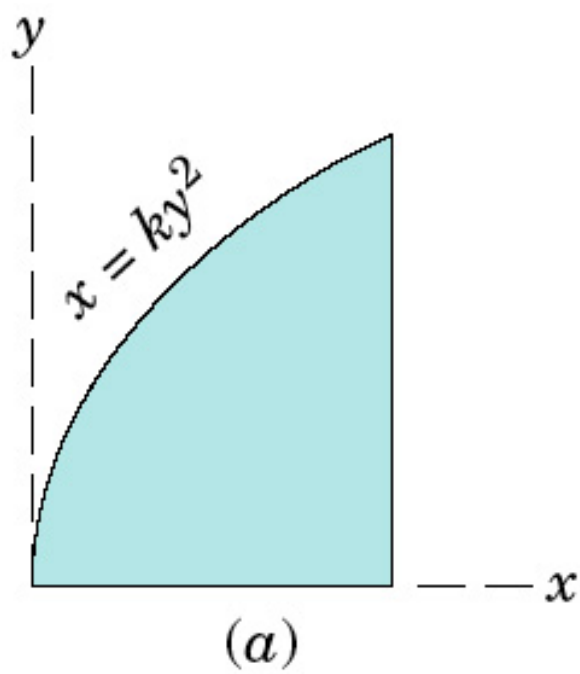


Figure 5/1 1

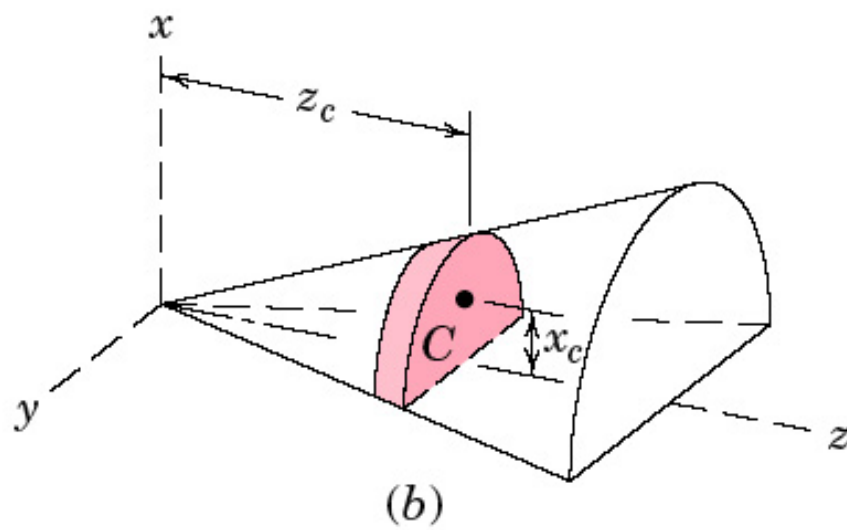
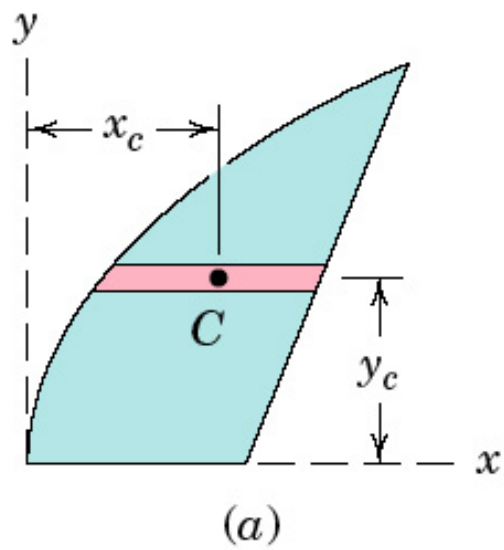


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05_12

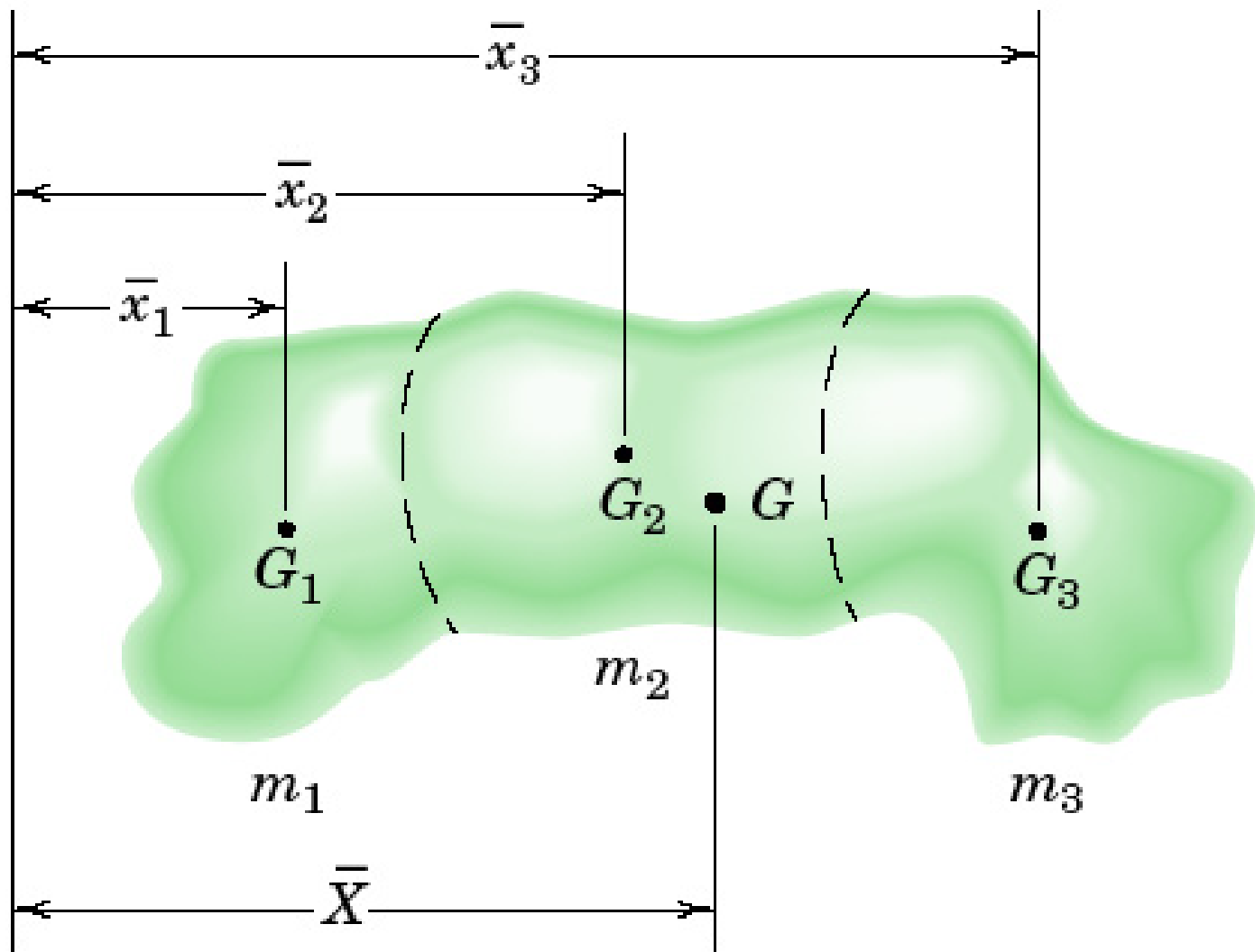


Figure 5/13

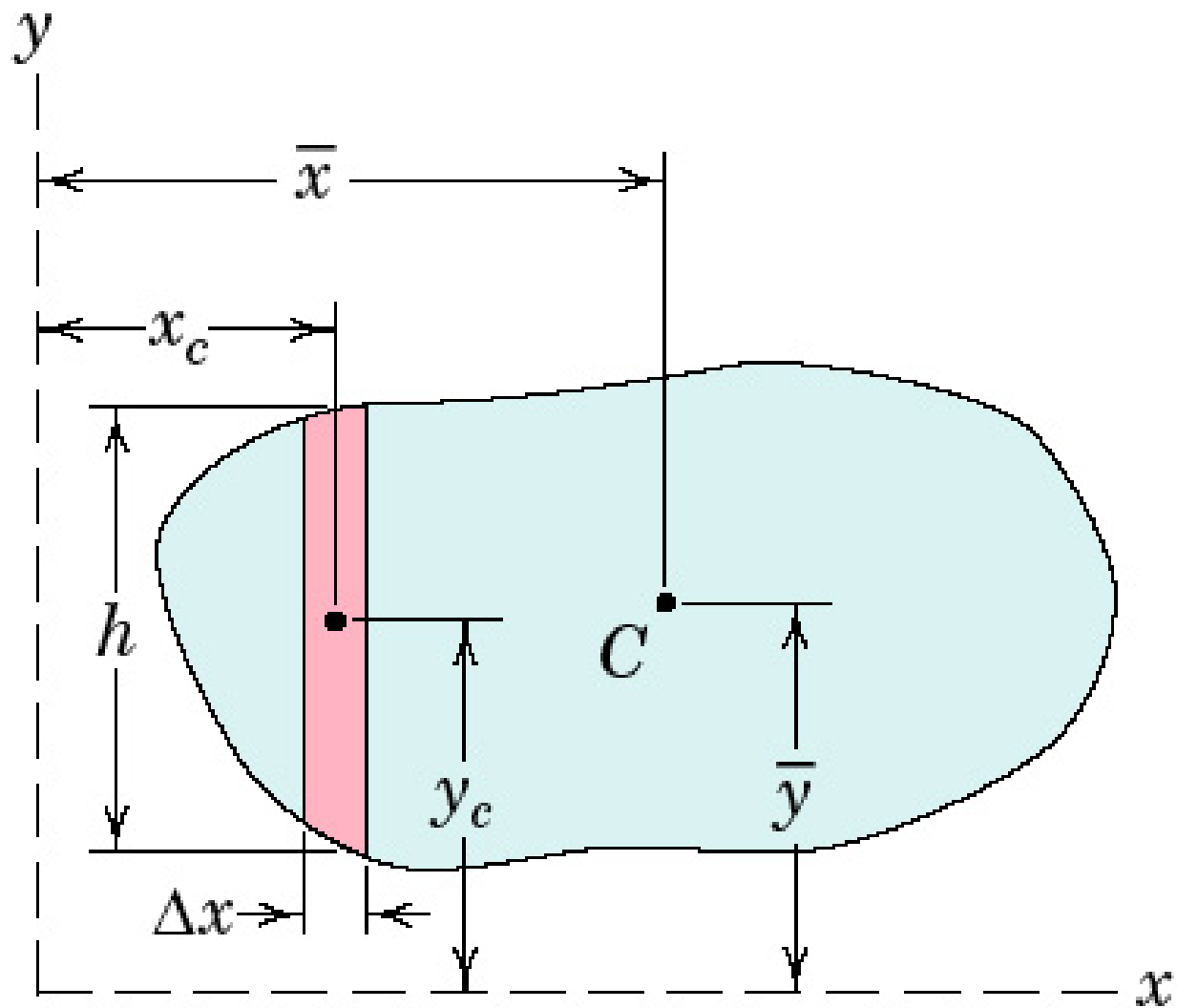


Figure 5/14

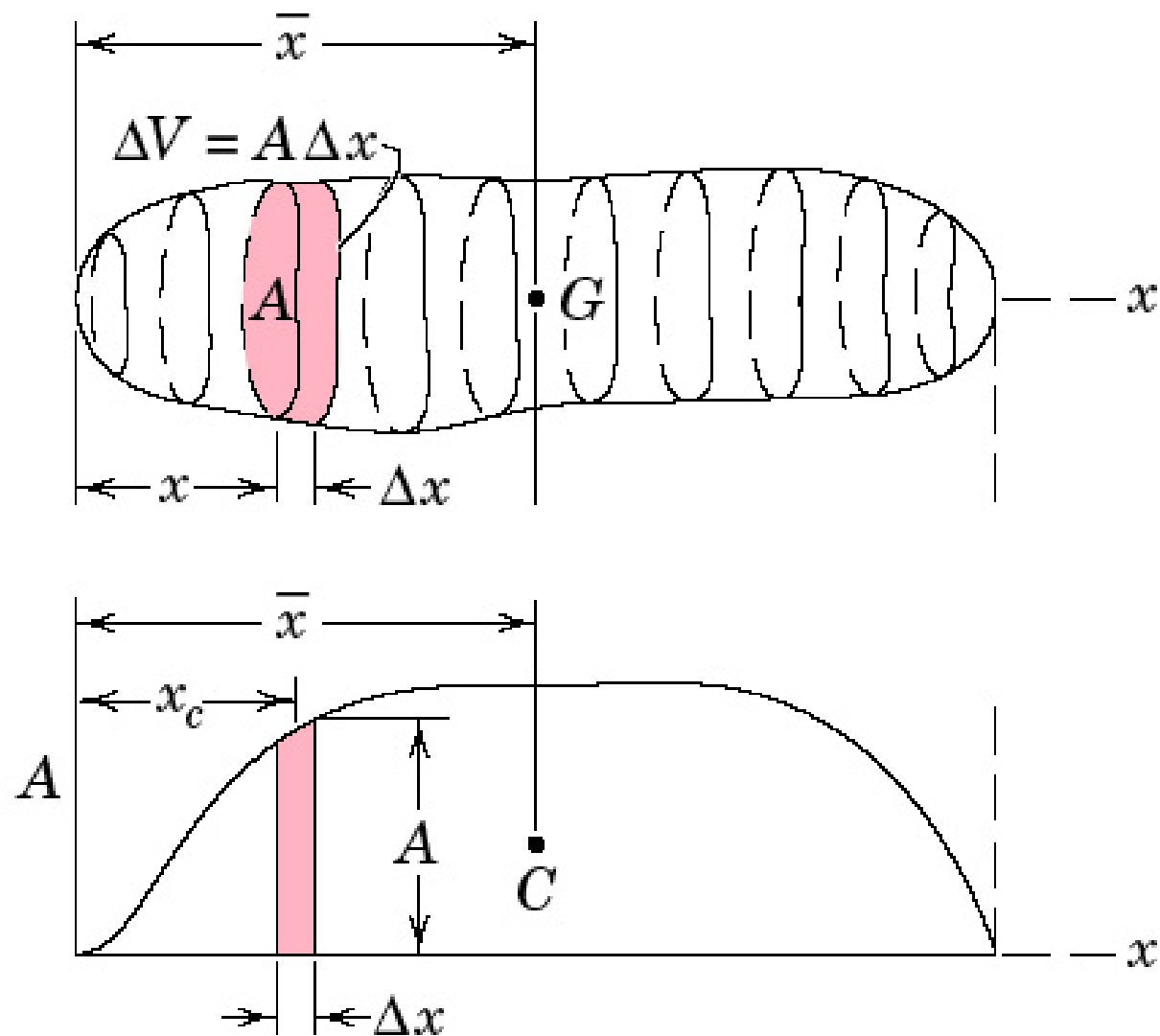


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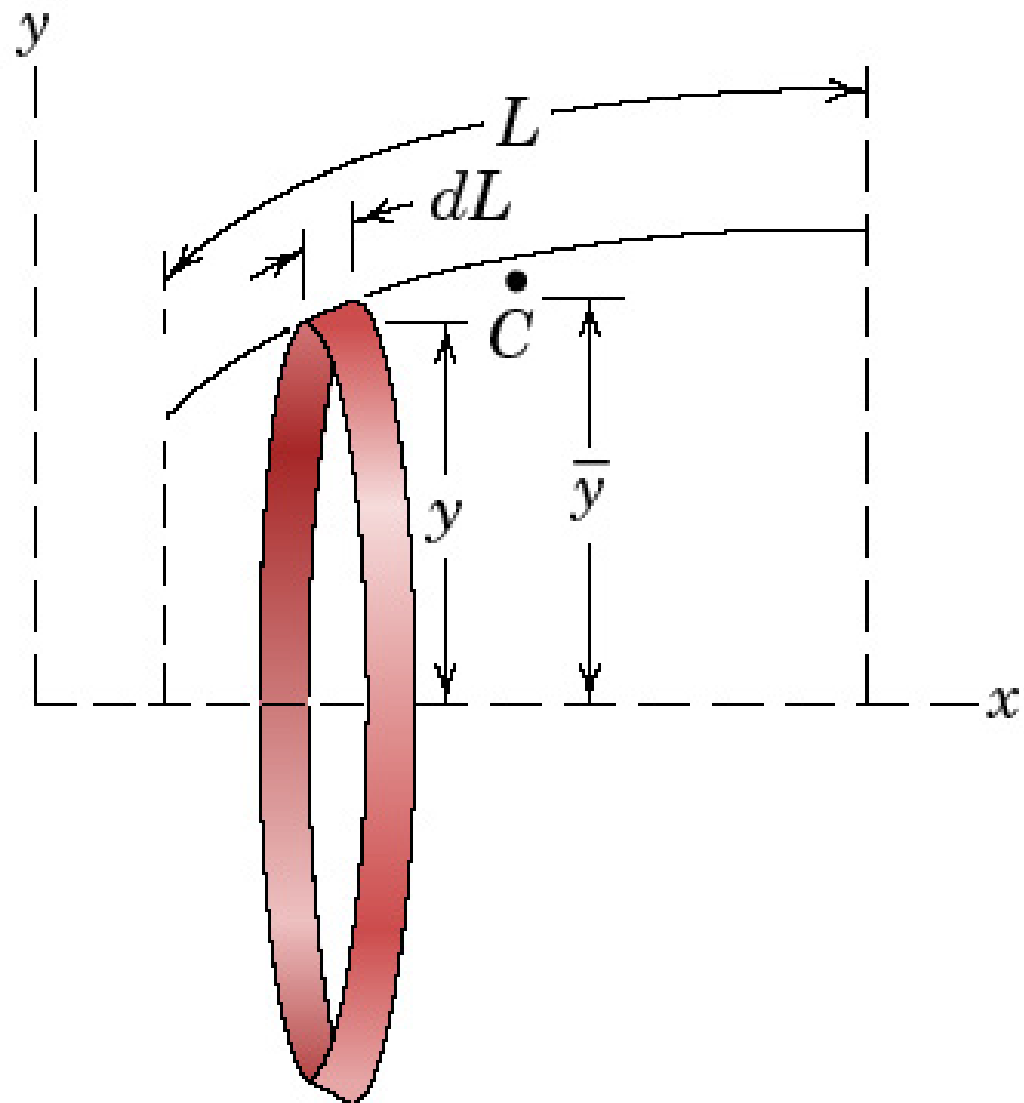


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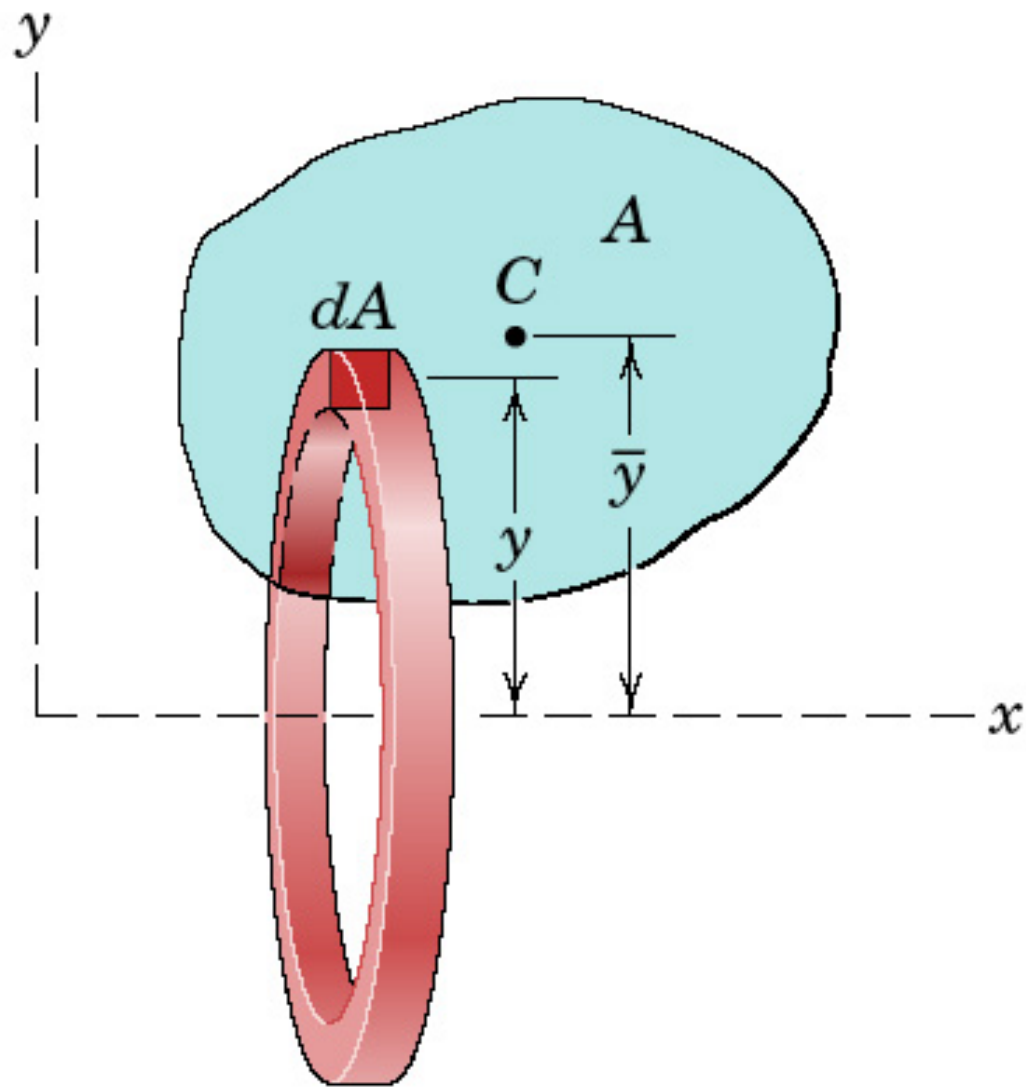
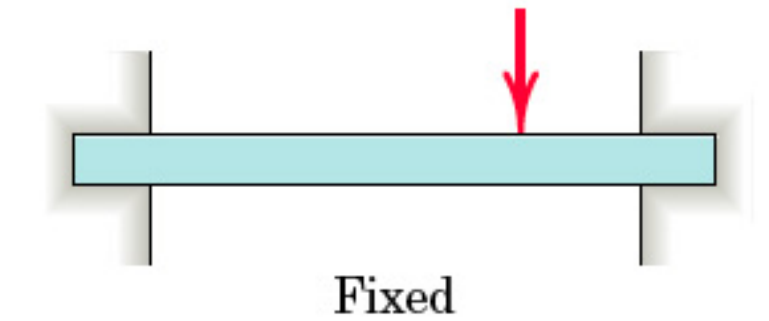
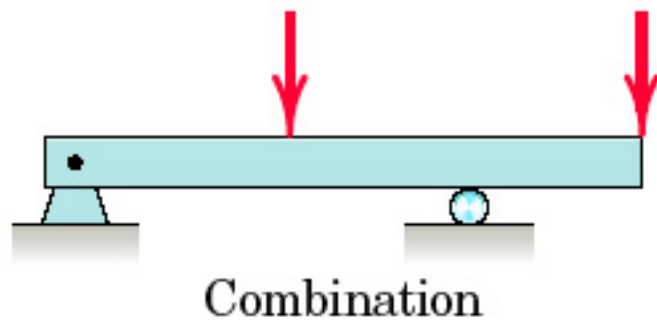
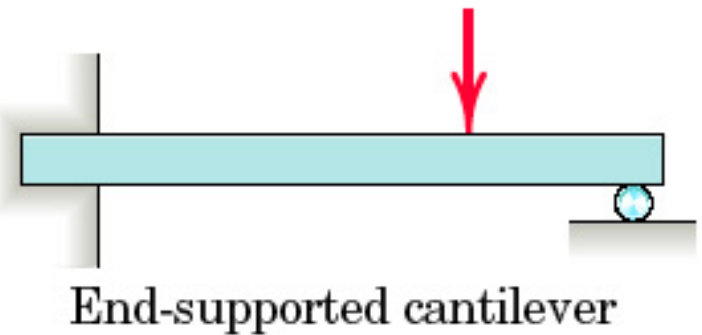
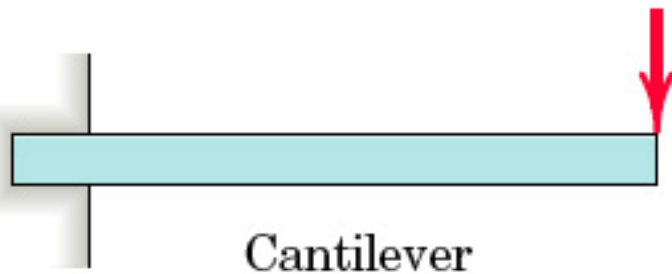
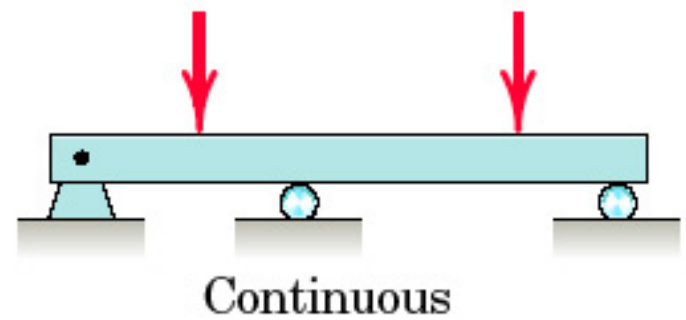
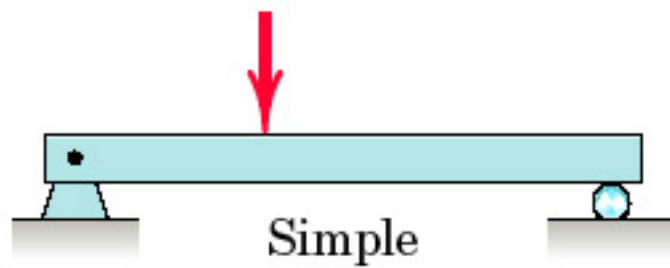


Figure 5/17



Statically determinate beams

Statically indeterminate beams

Figure 5/18

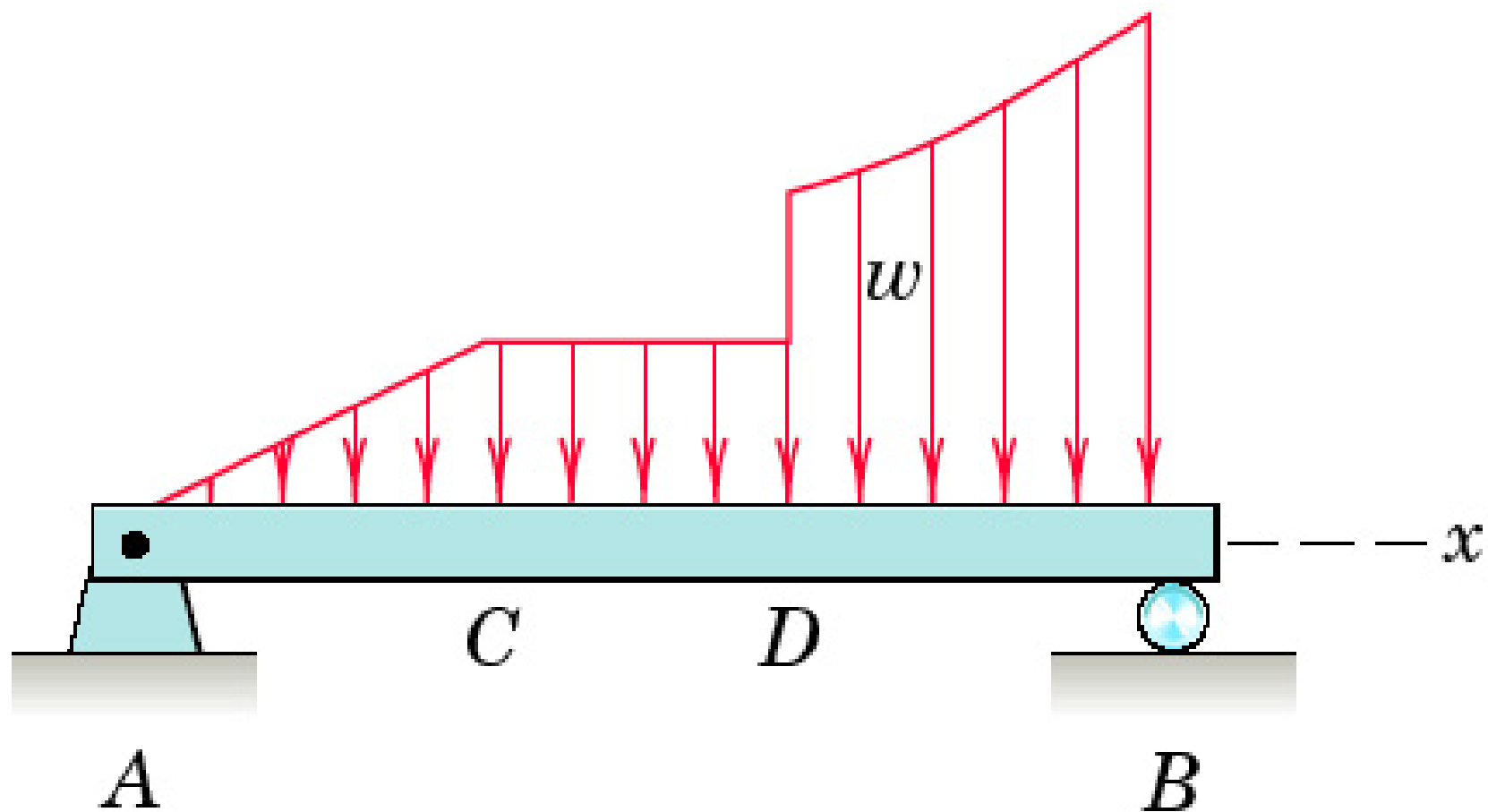


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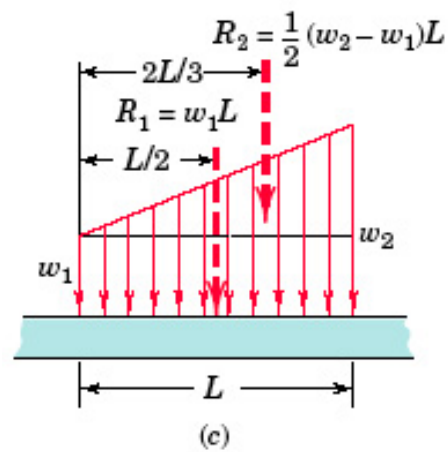
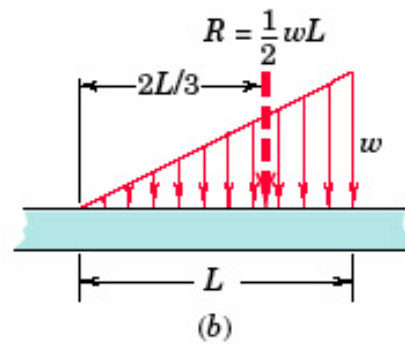
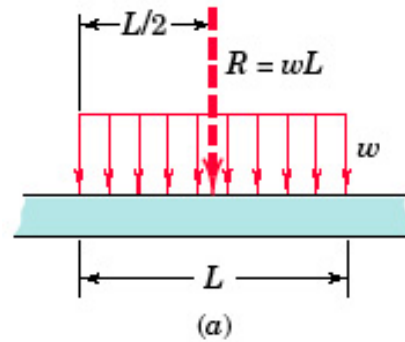


Figure 5/20

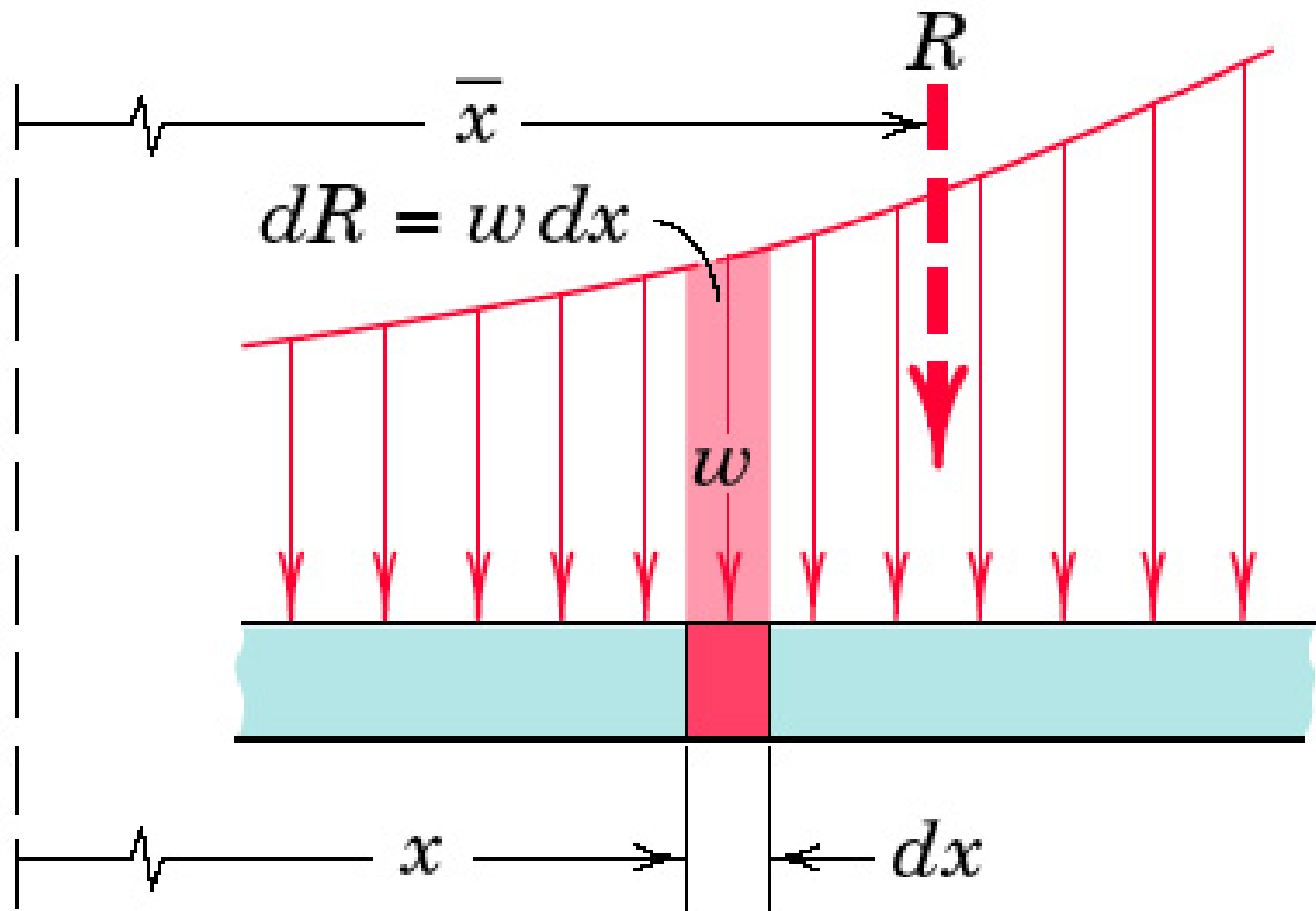


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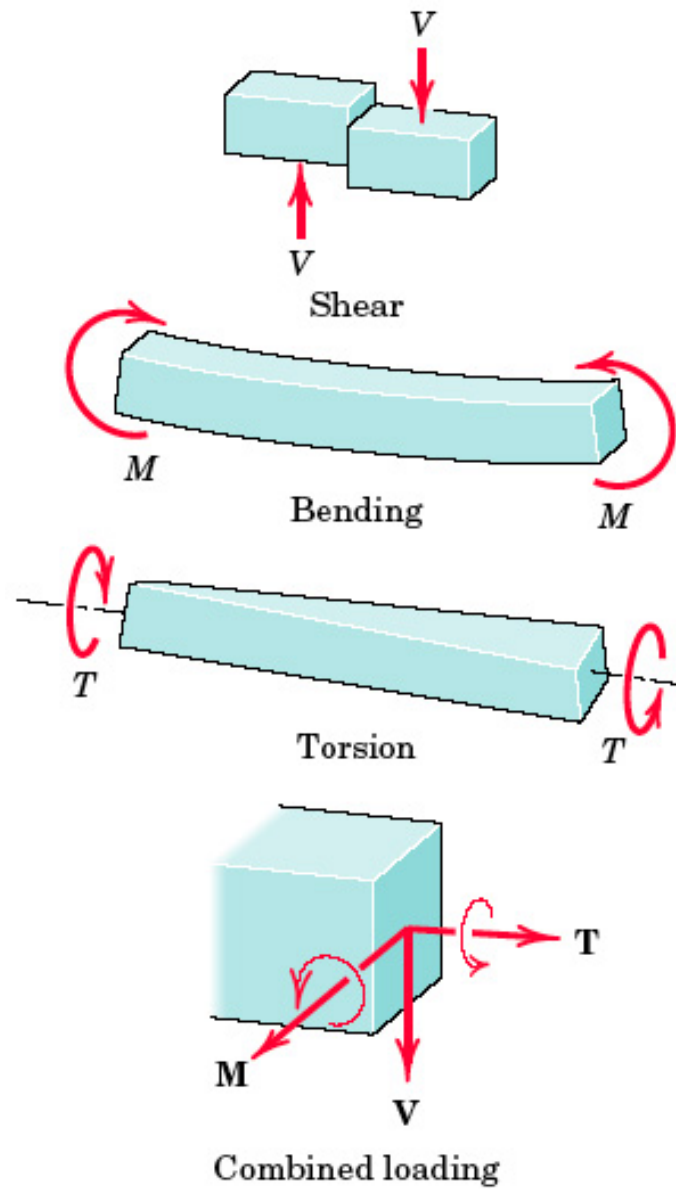


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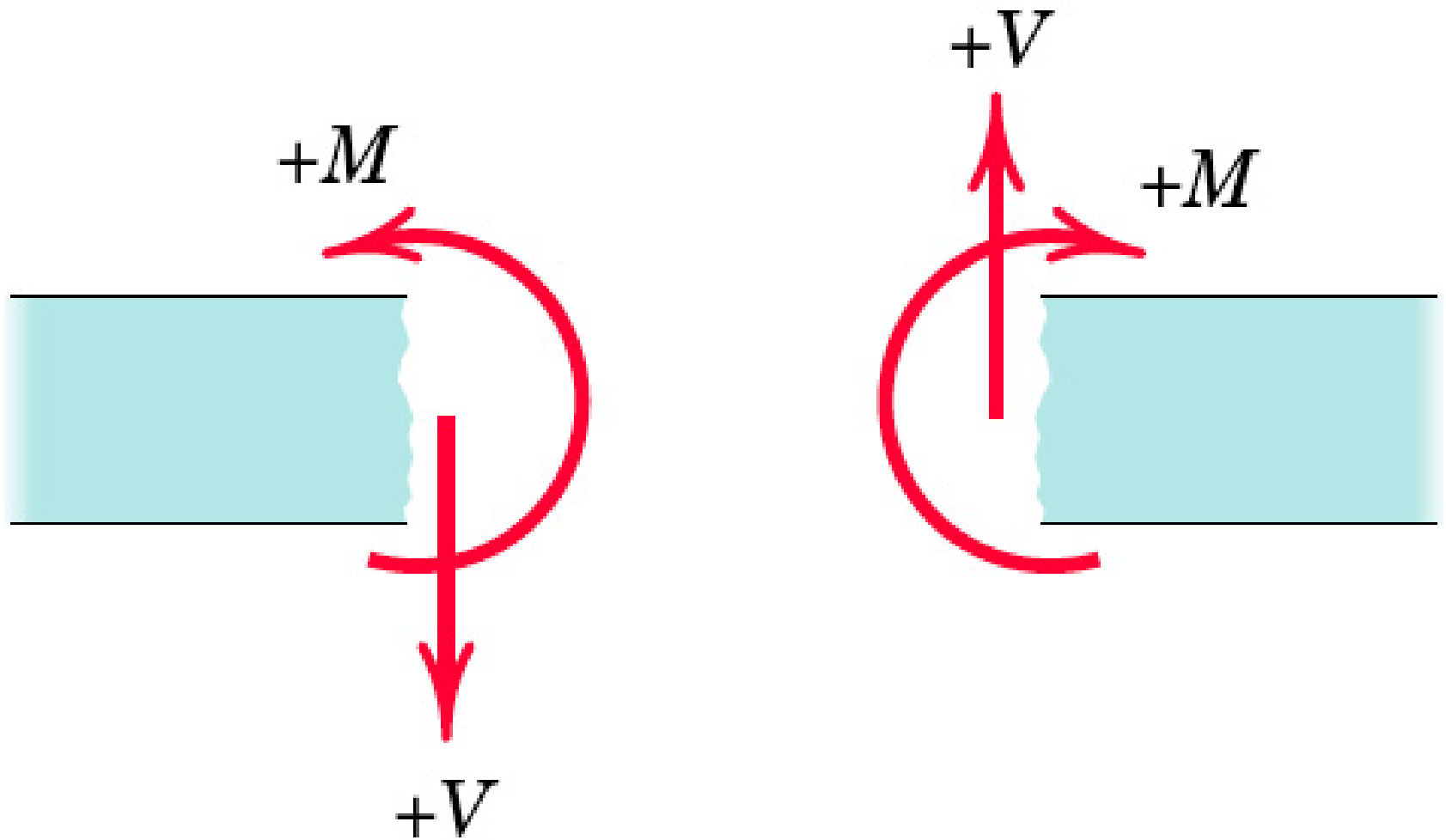


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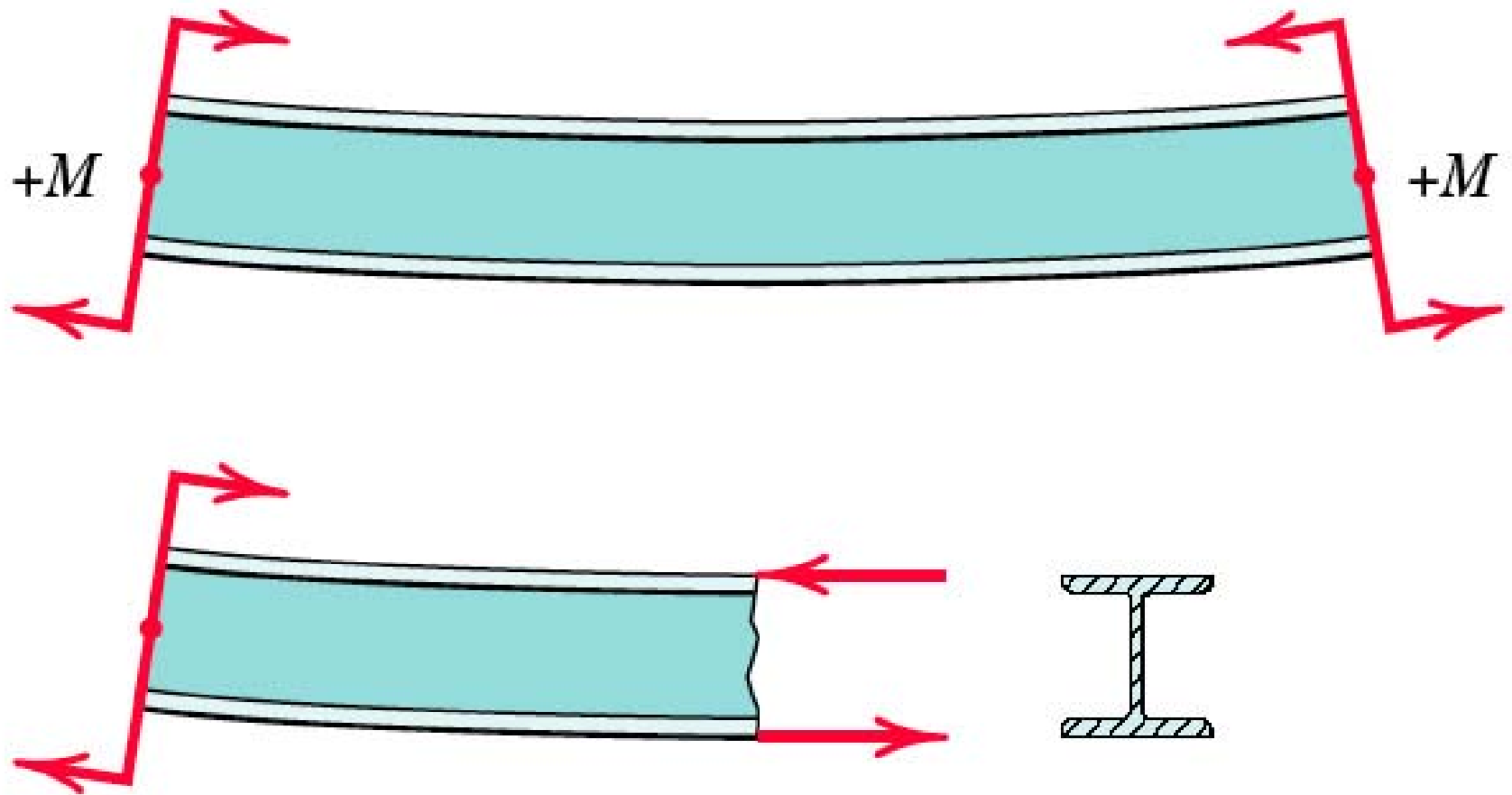


Figure 5/24

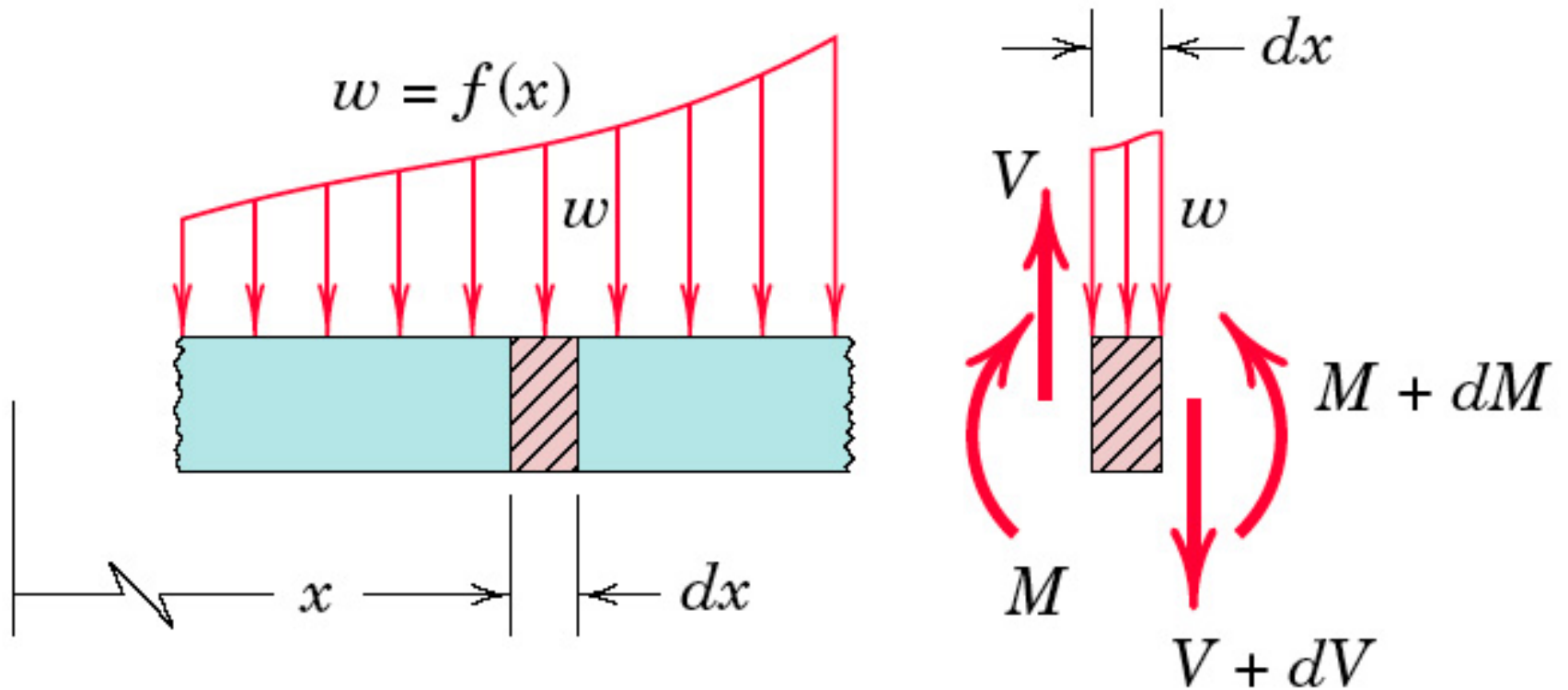


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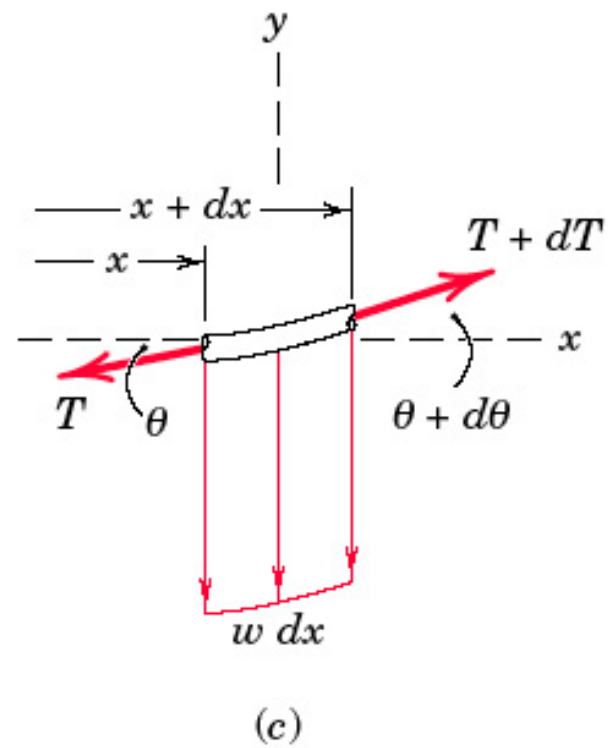
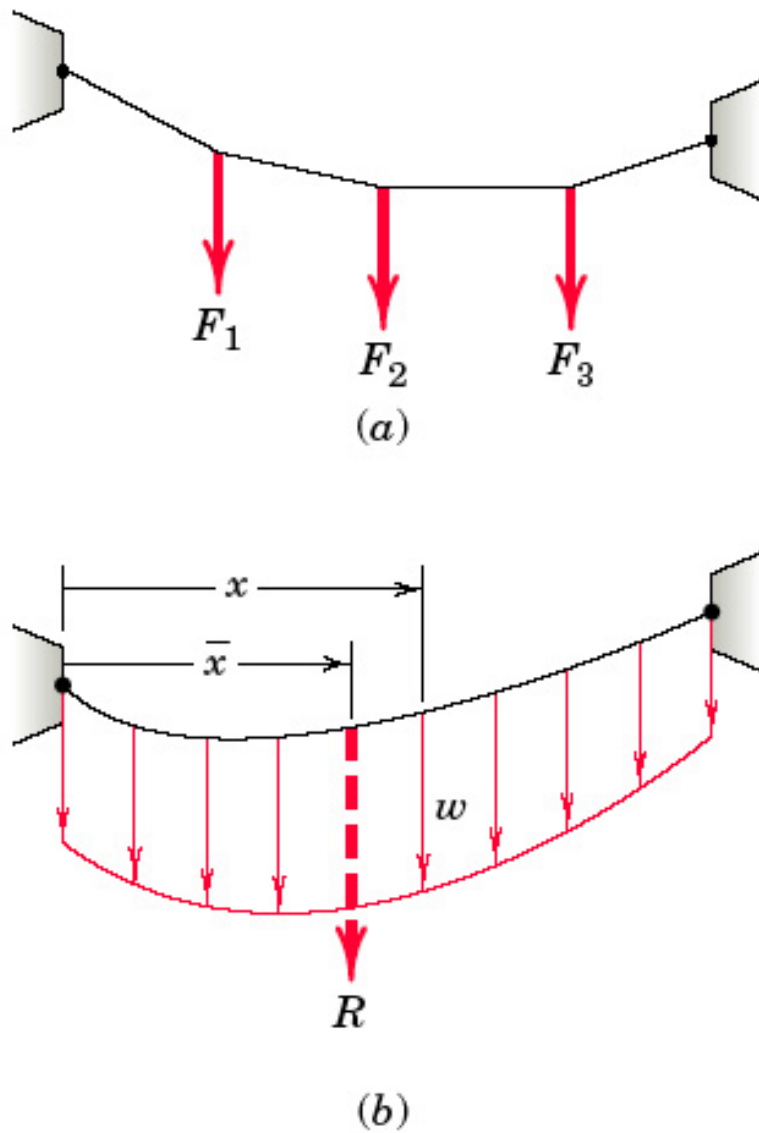
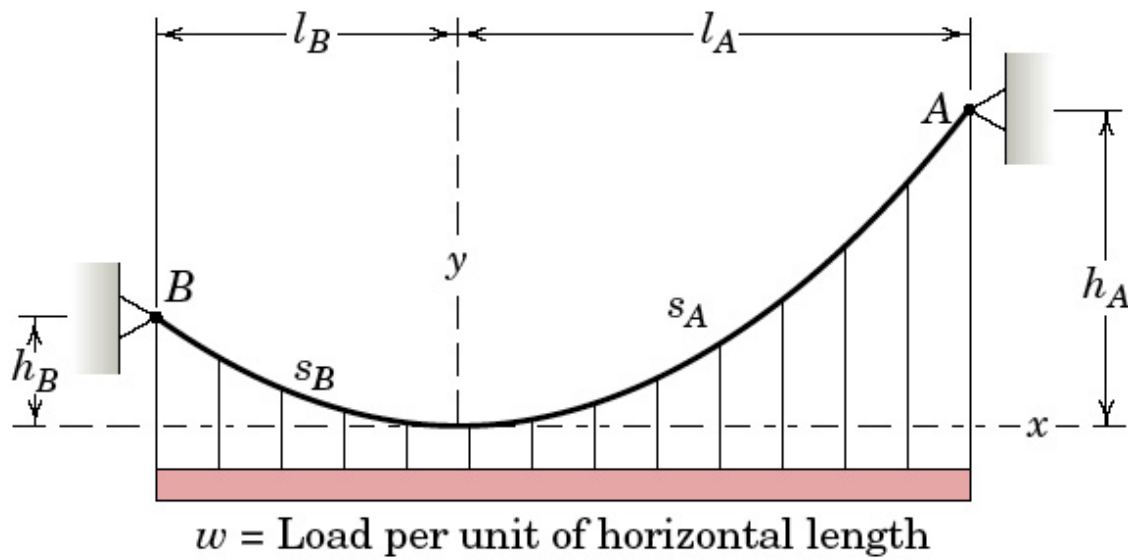
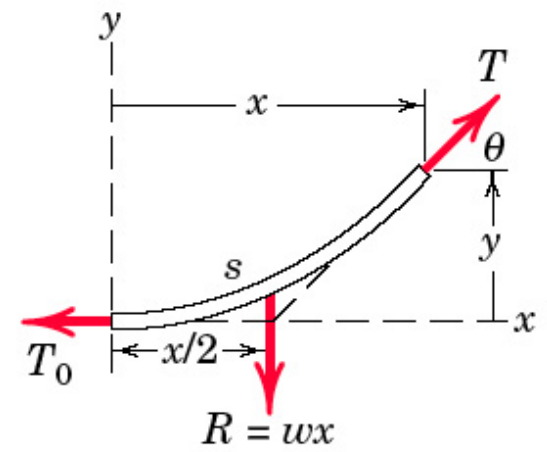


Figure 5/26



(a)



(b)

Figure 5/27

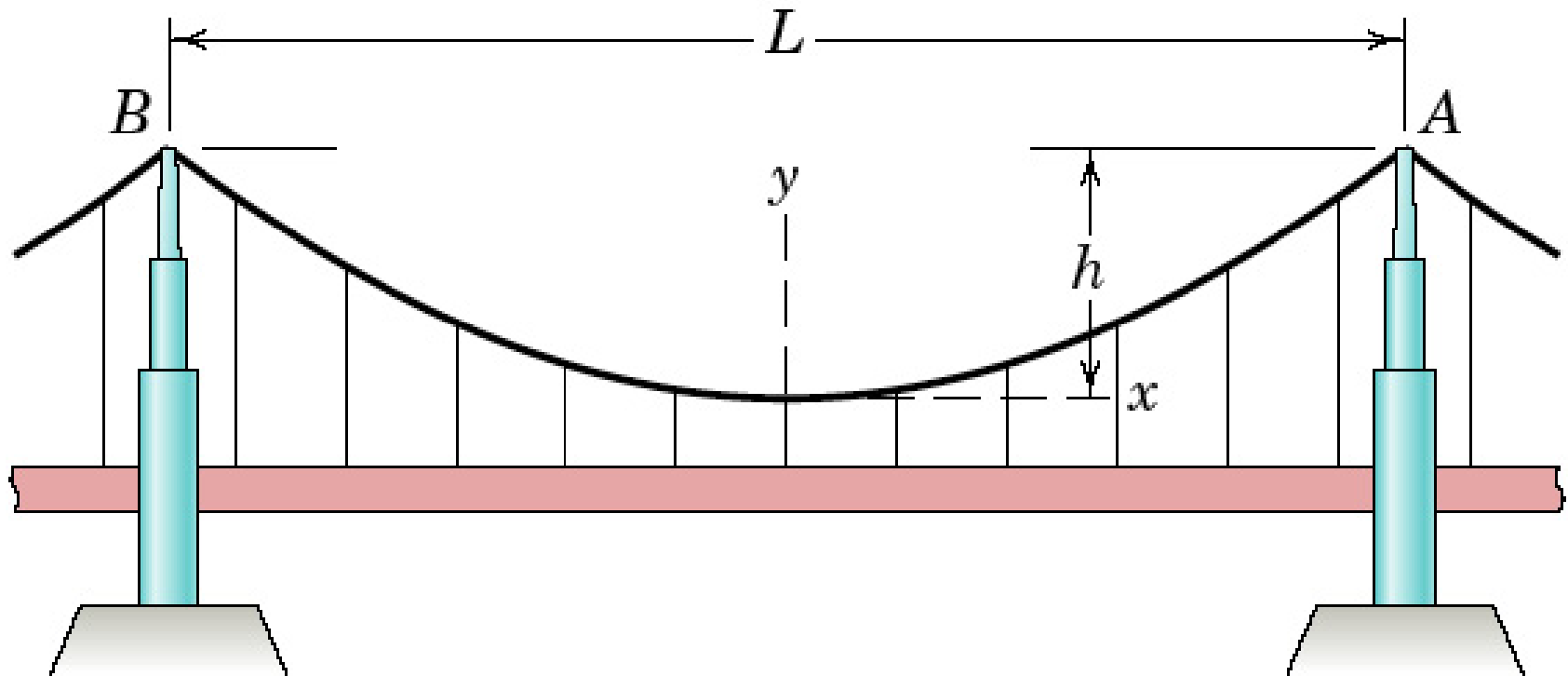
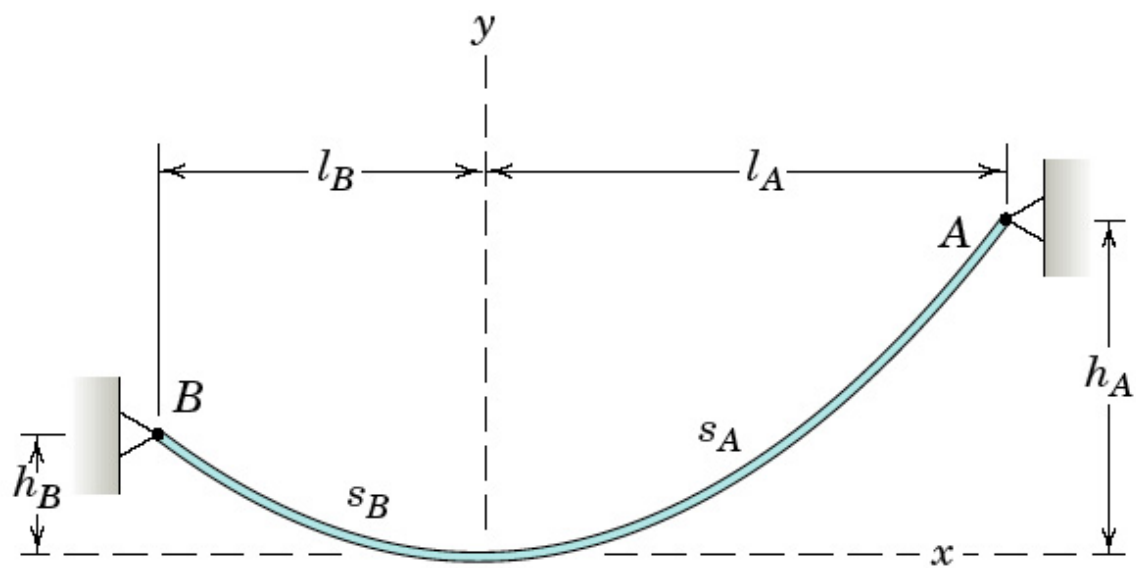
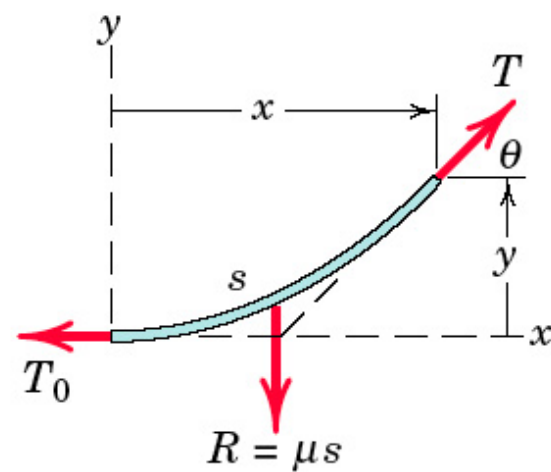


Figure 5/28



(a)



(b)

Figure 5/29

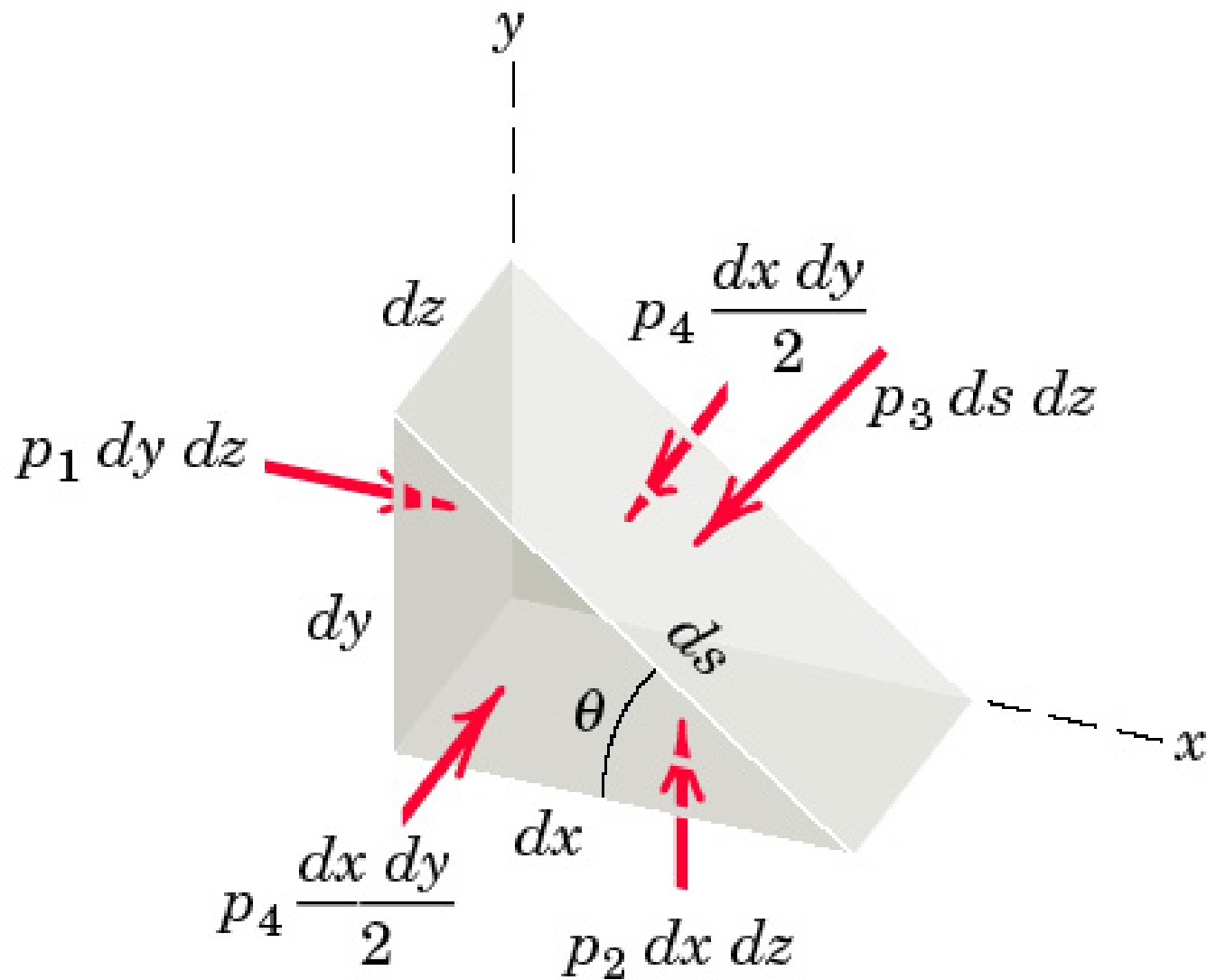


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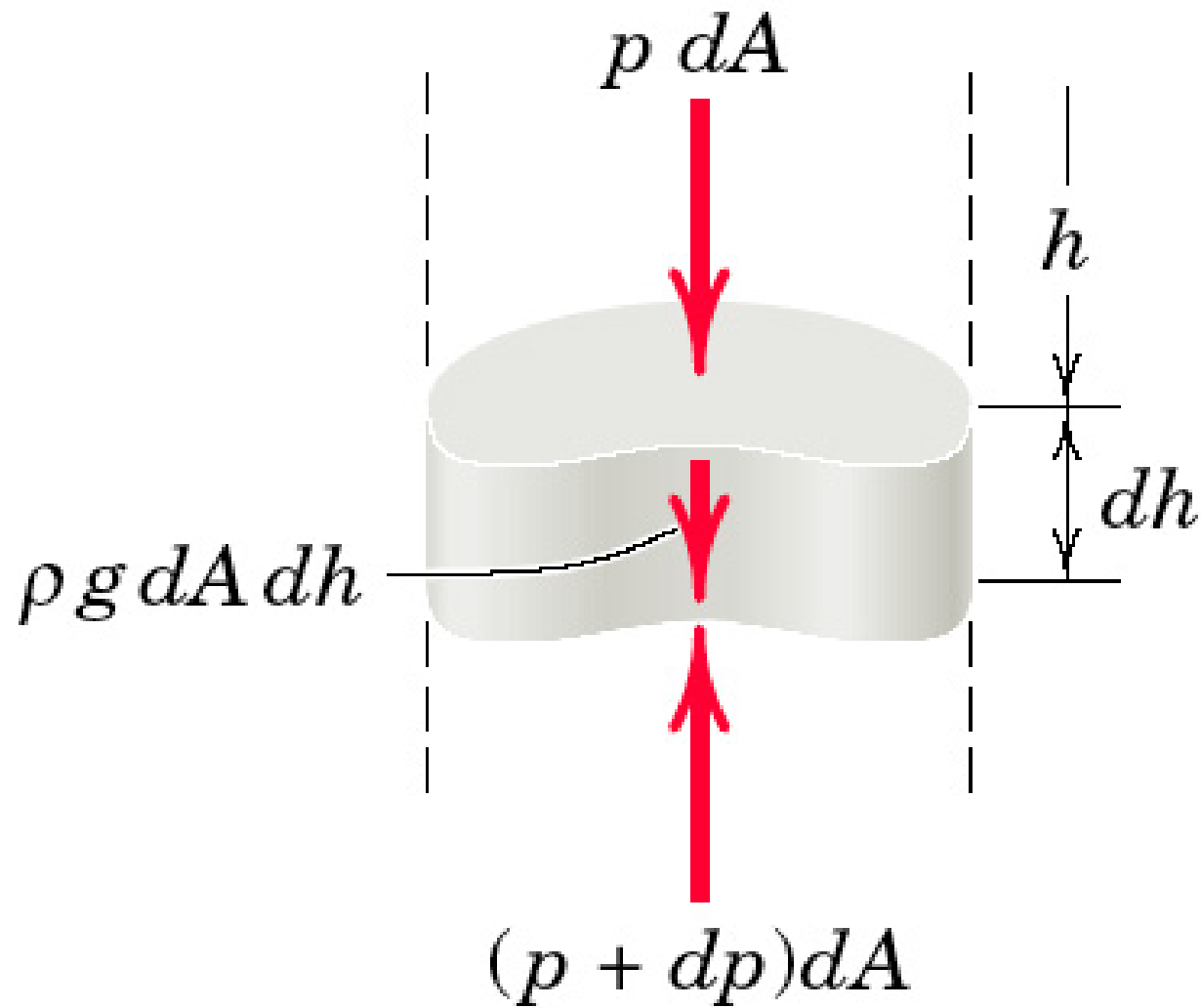
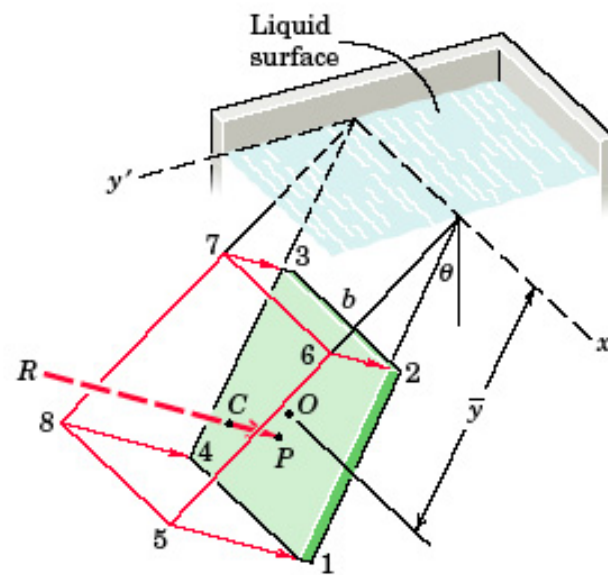
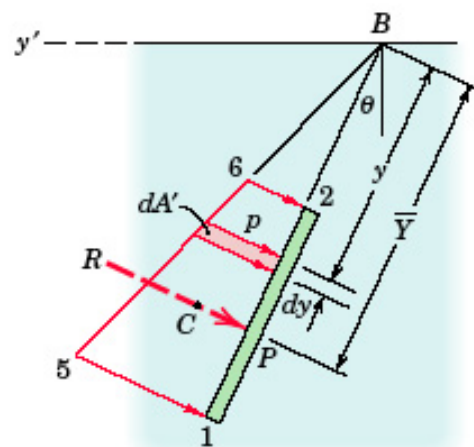


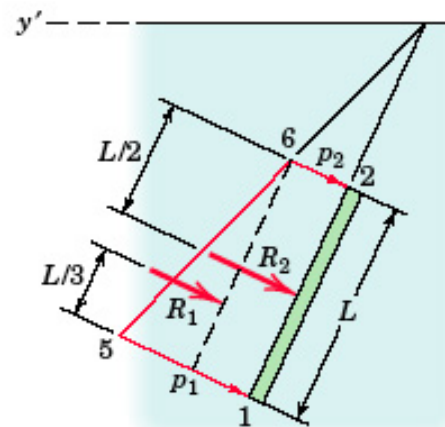
Figure 5/31



(a)

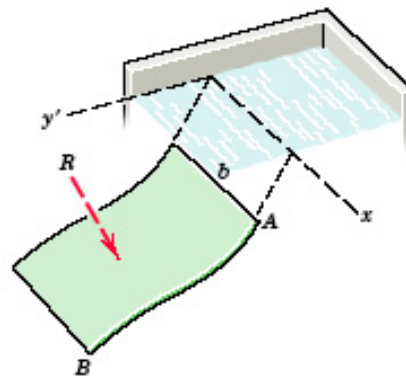


(b)

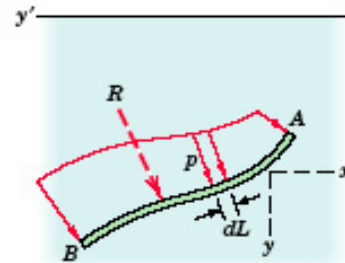


(c)

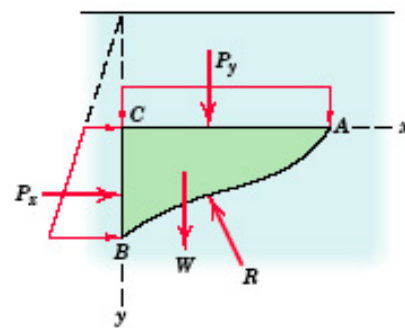
Figure 5/32



(a)

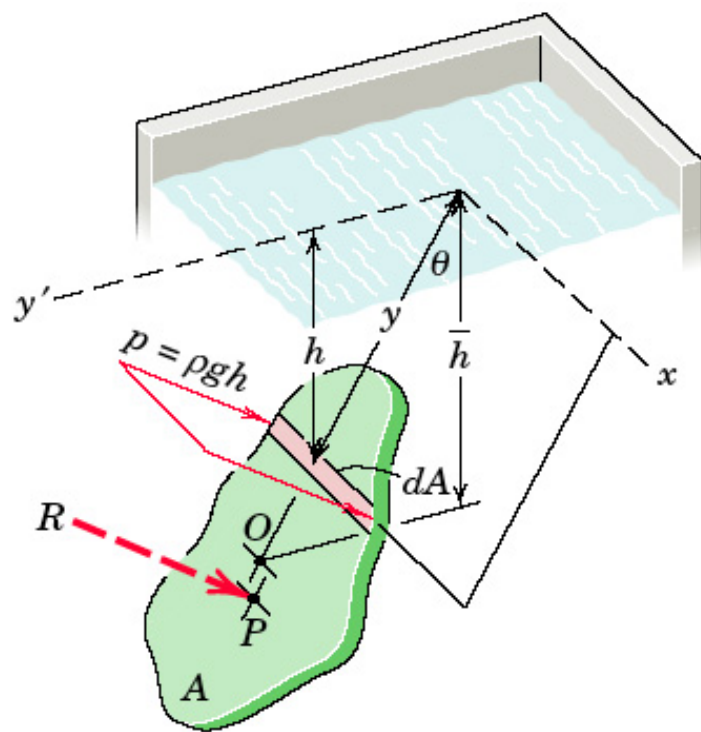


(b)

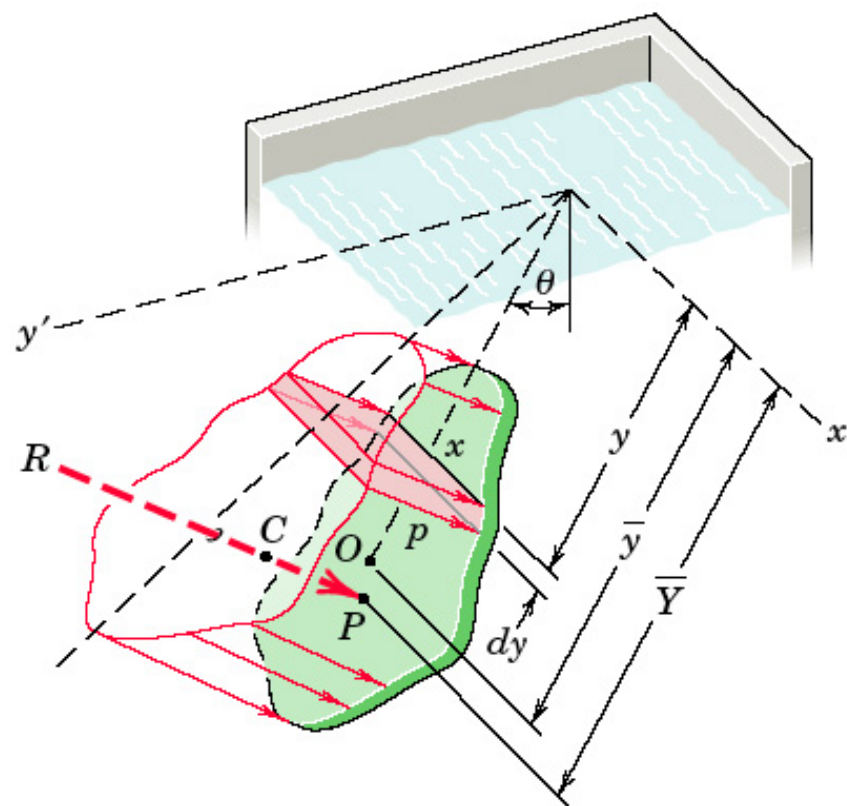


(c)

Figure 5/33



(a)



(b)

Figure 5/34

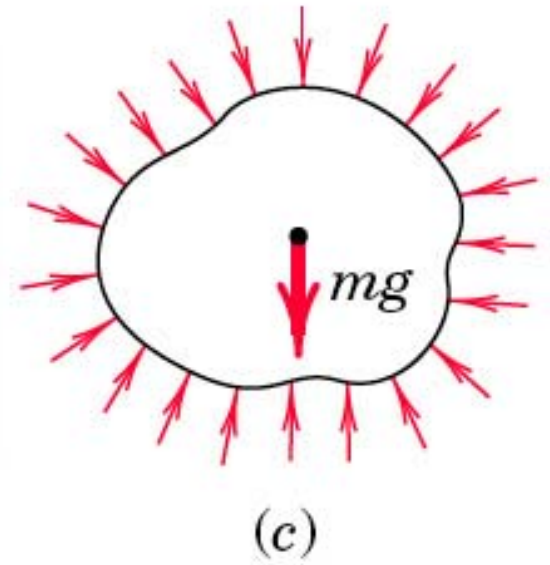
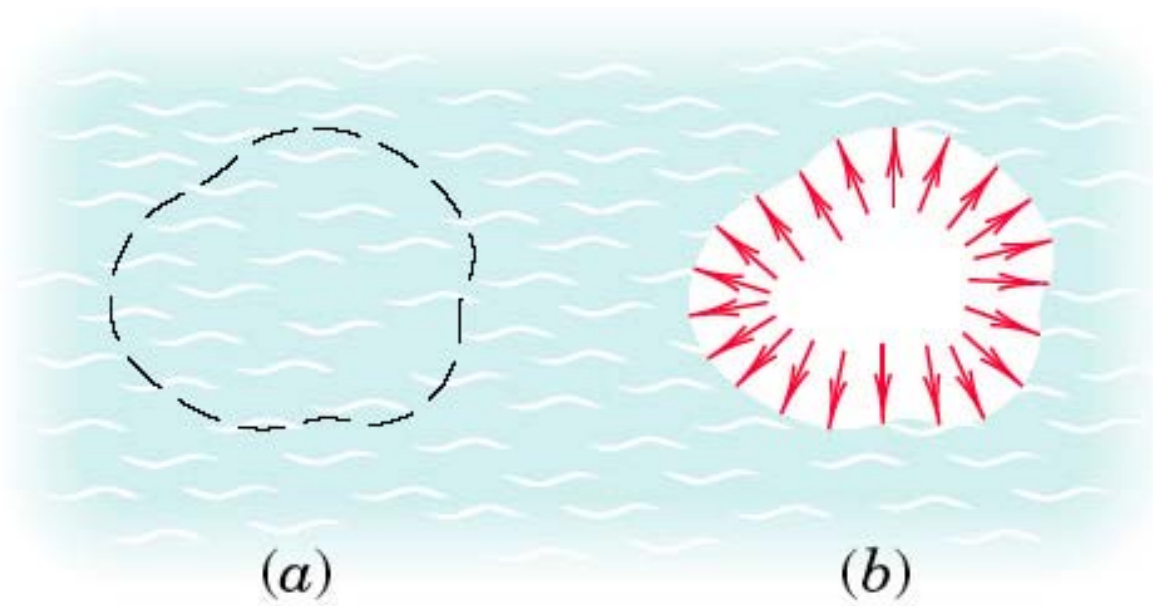


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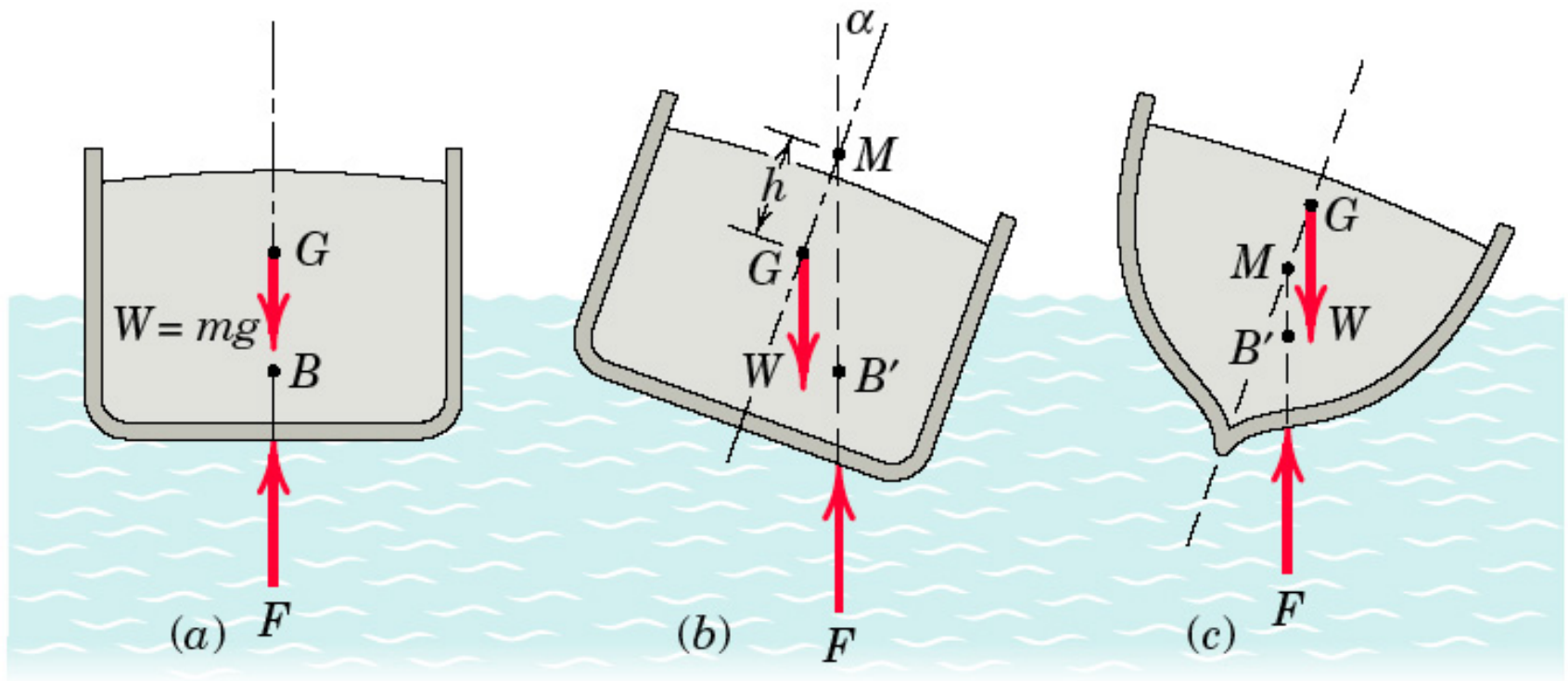
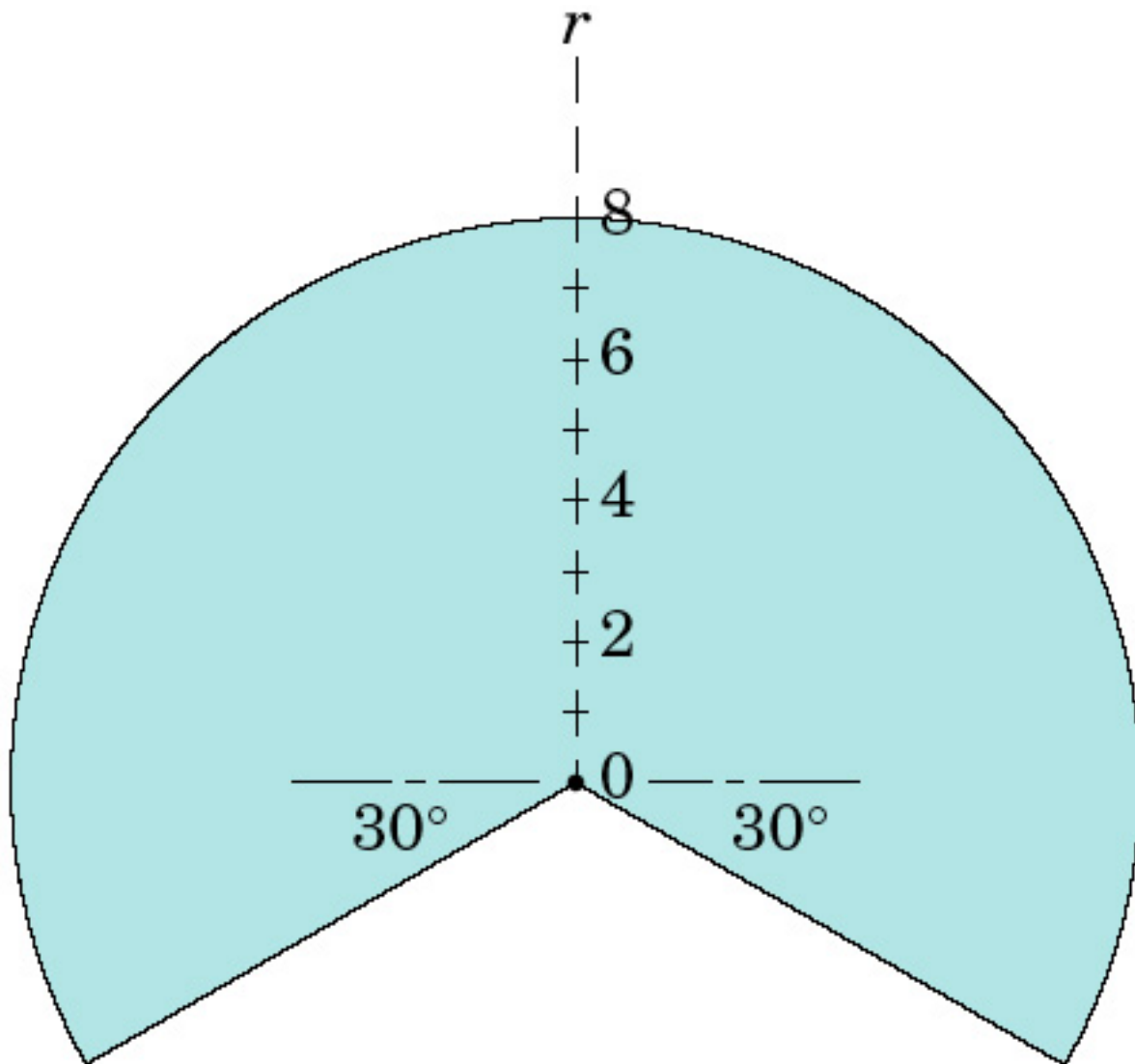
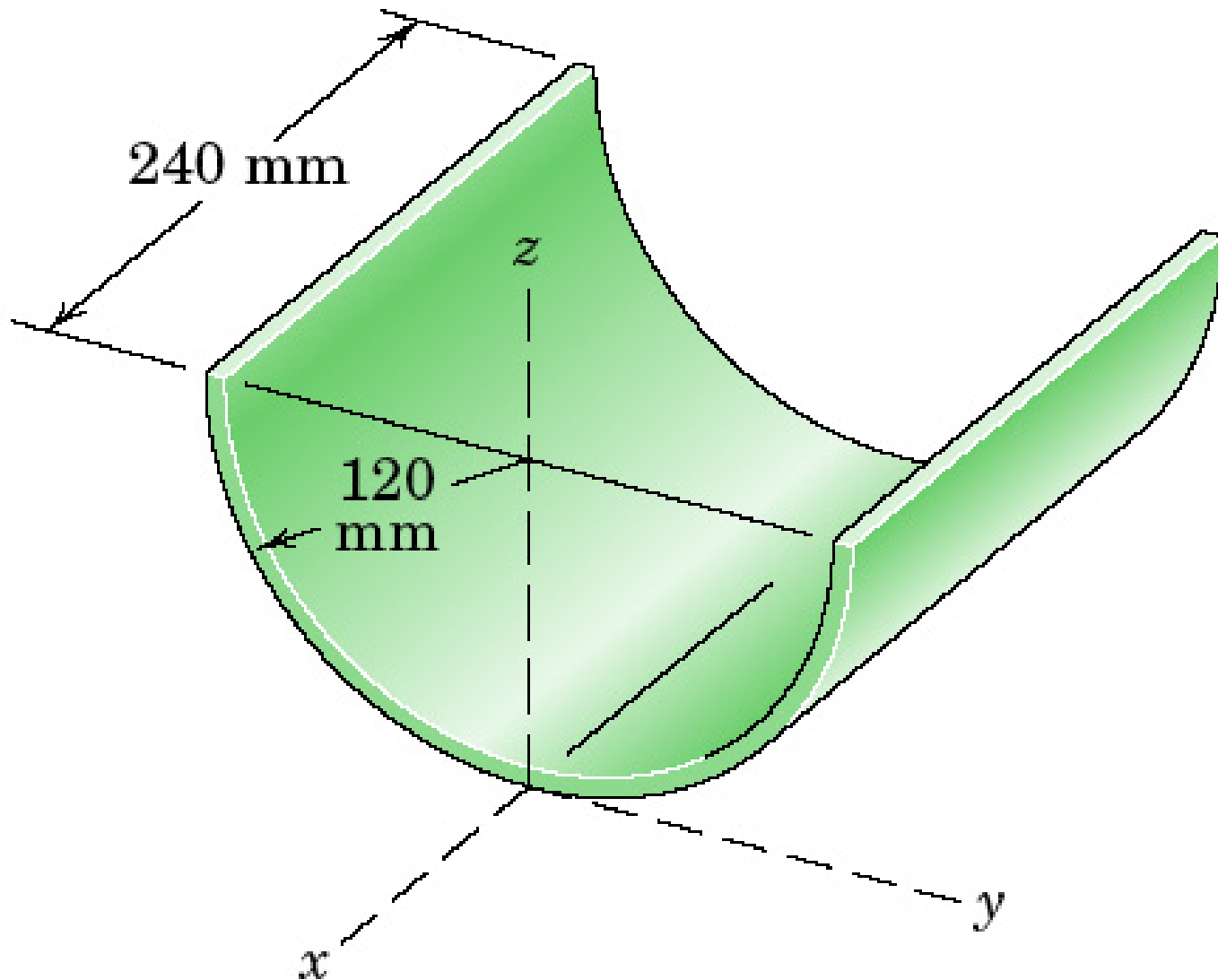


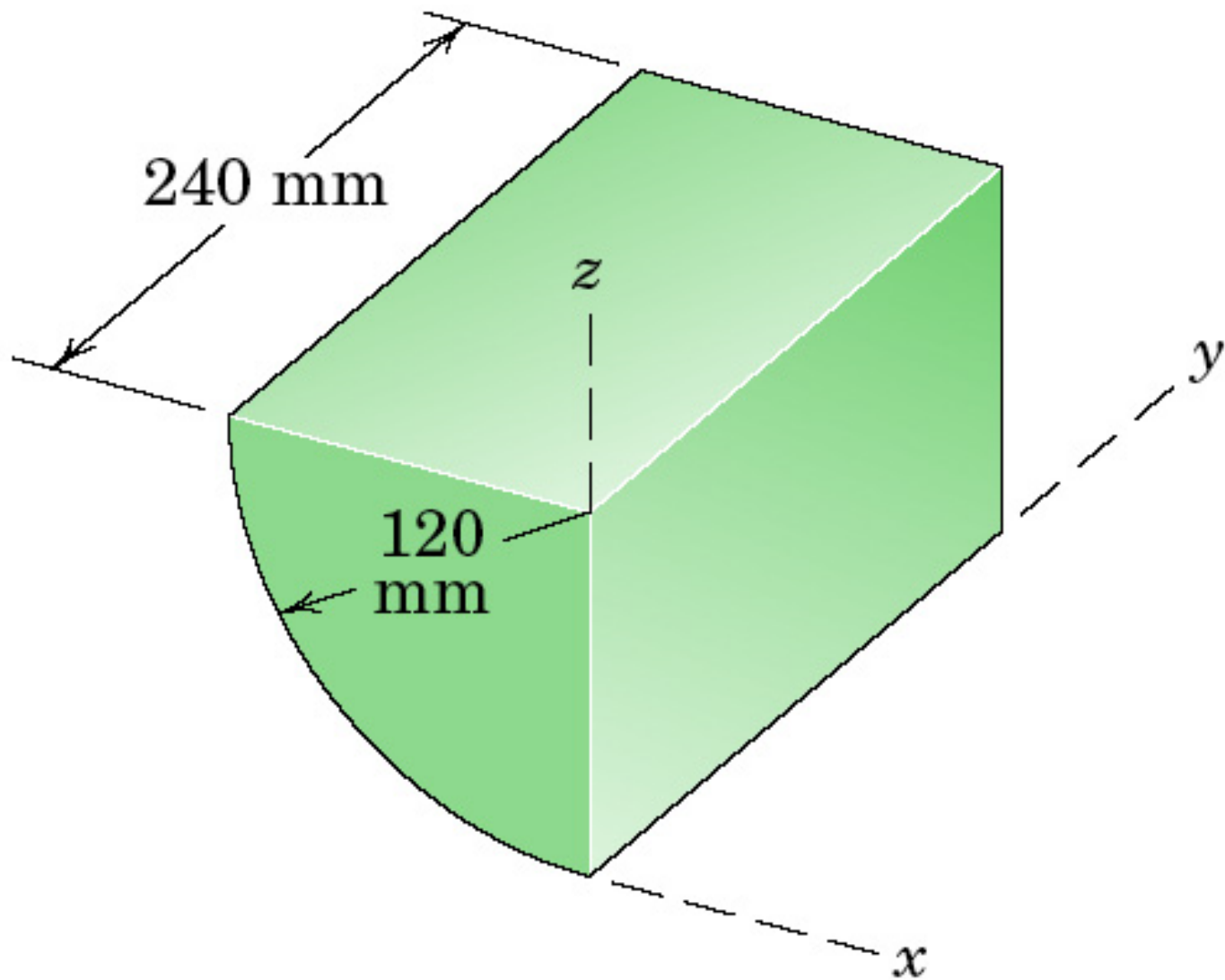
Figure 5/36



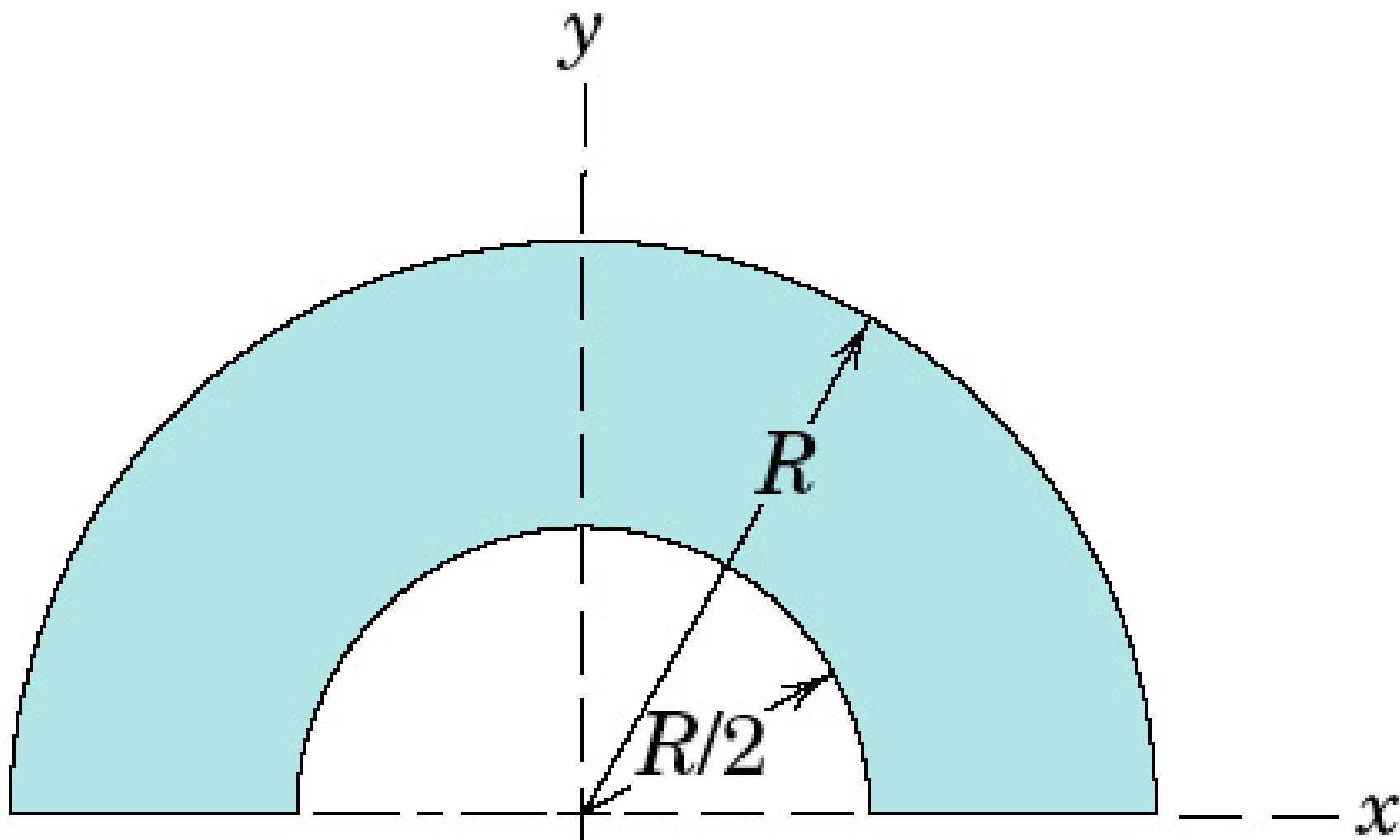
Problem 5/2



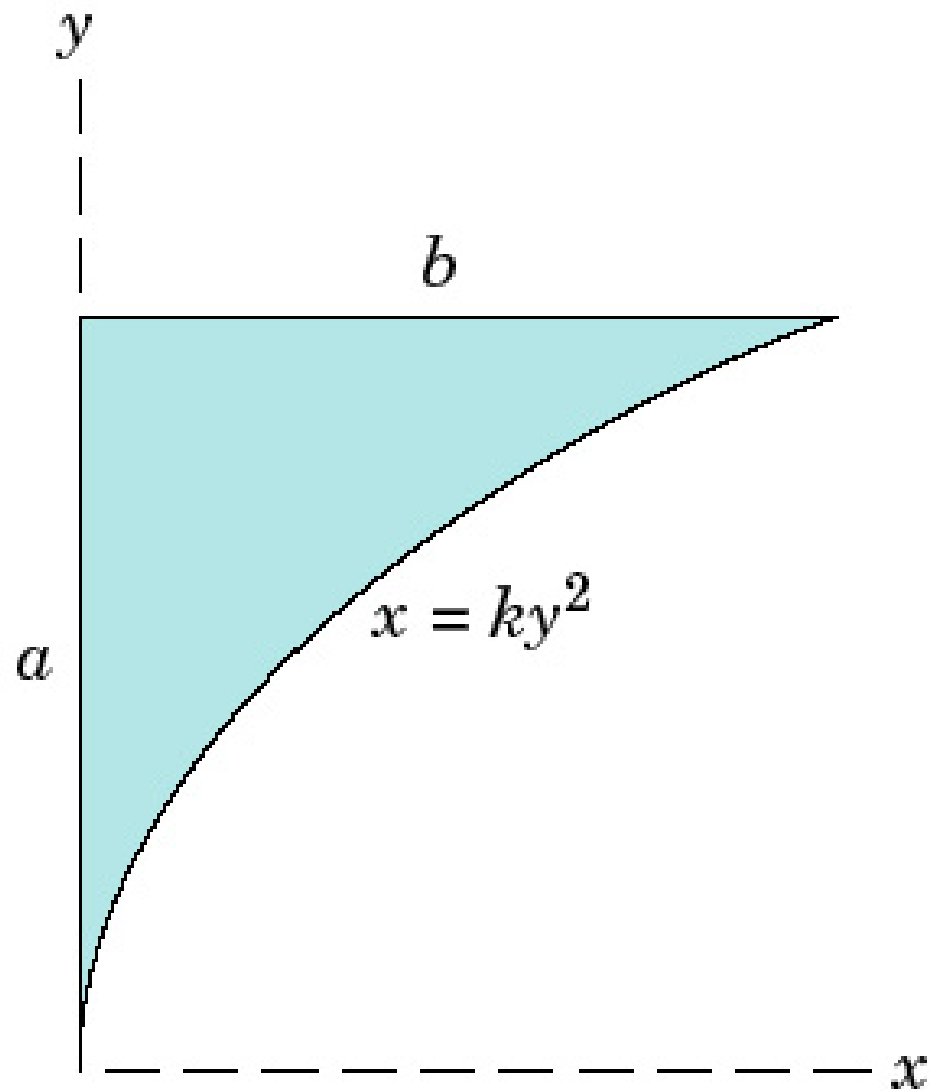
Problem 5/3



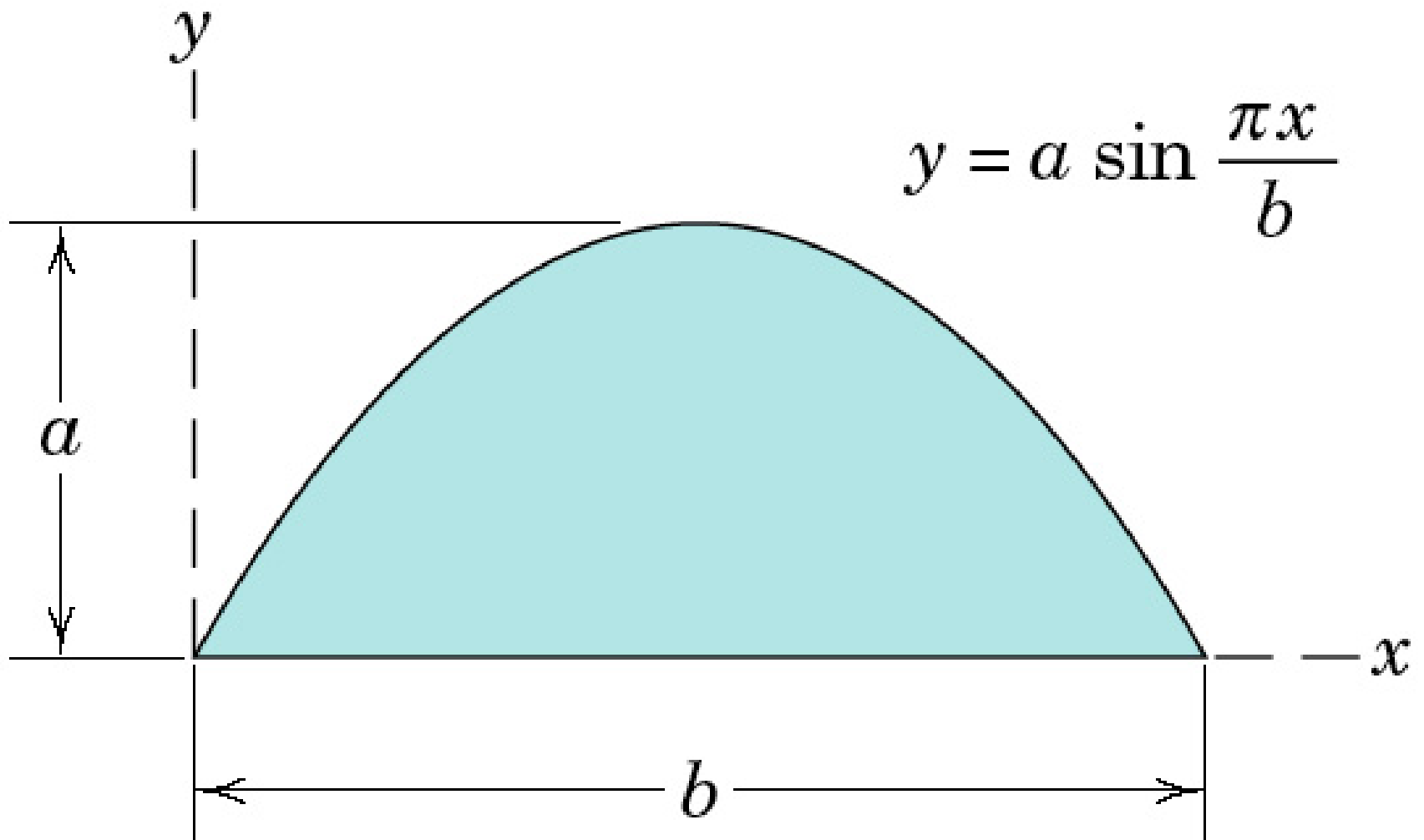
Problem 5/4



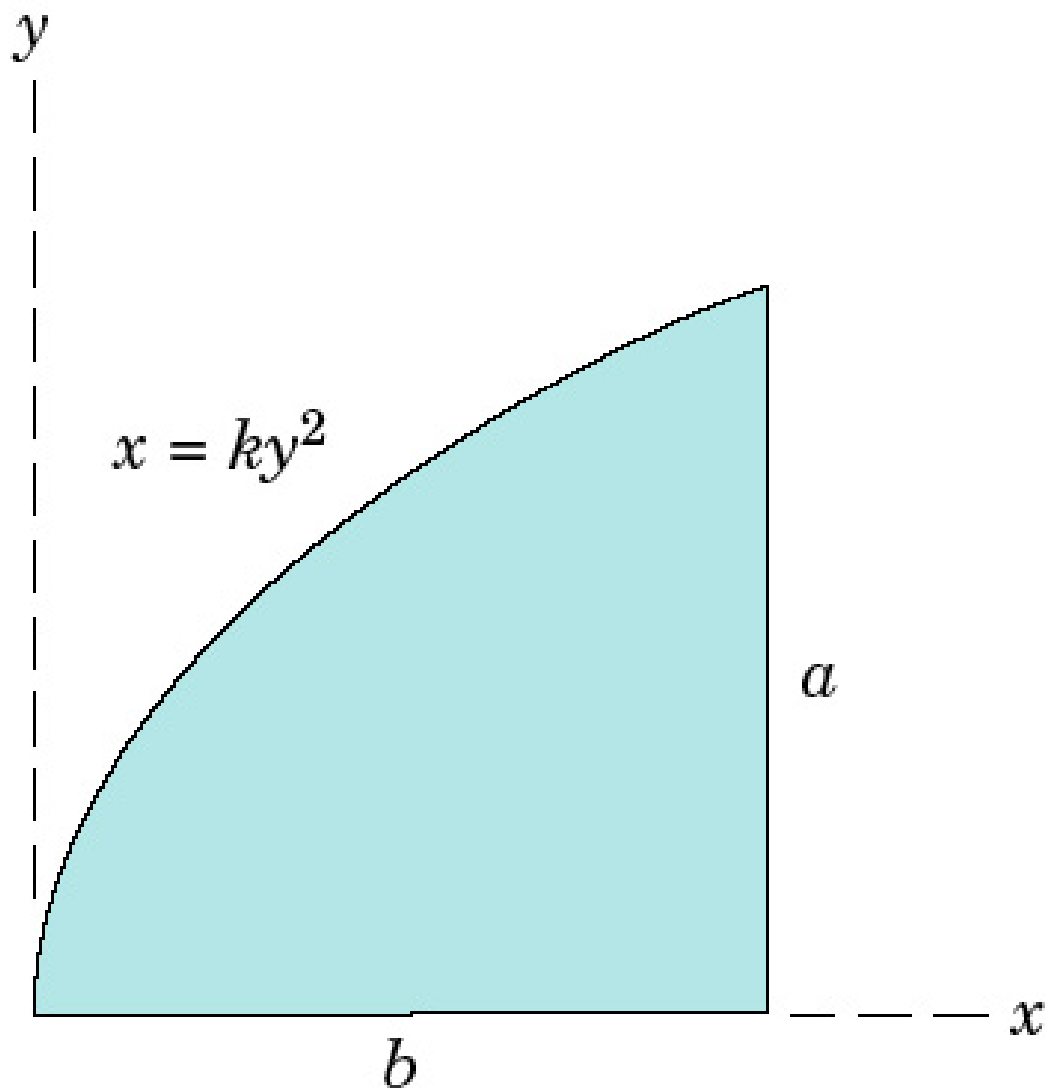
Problem 5/5



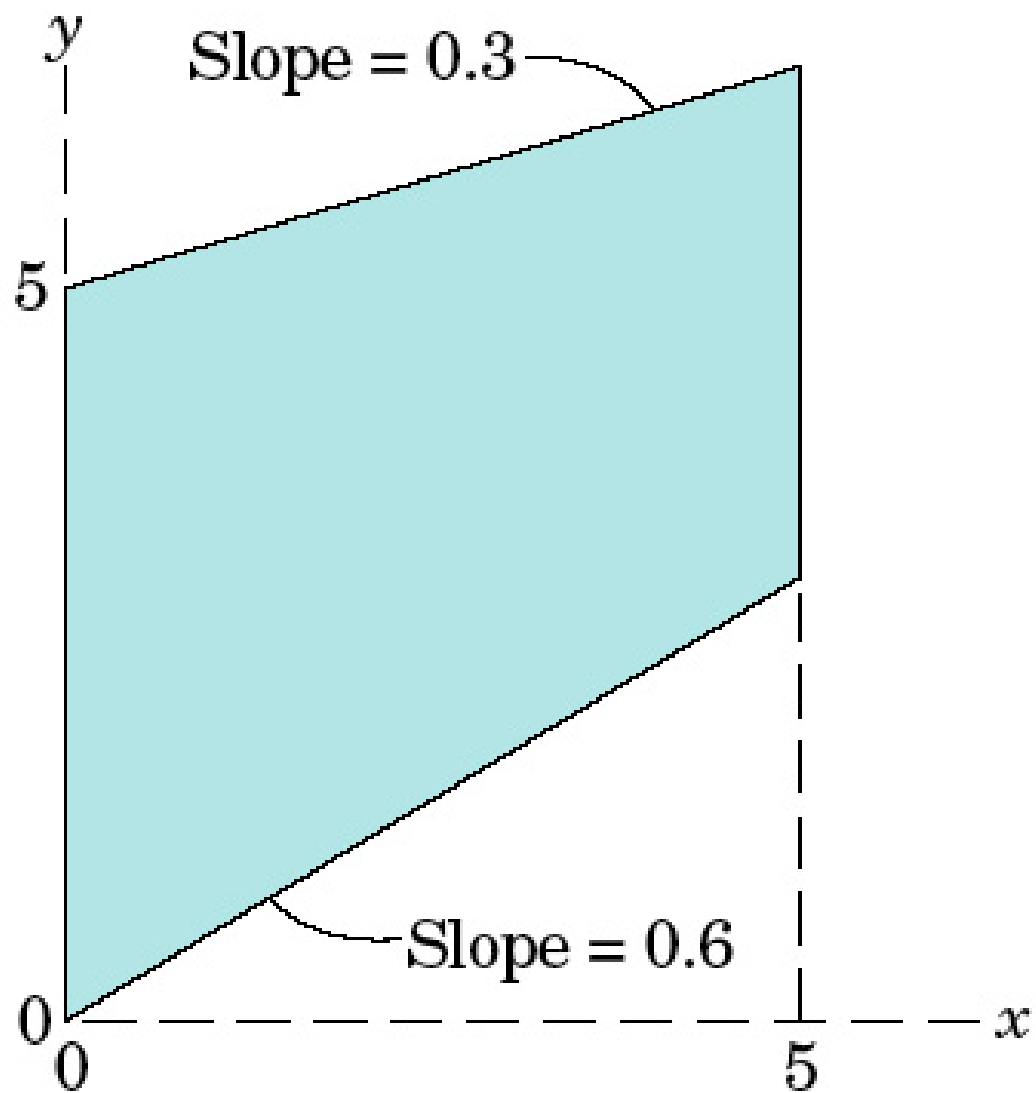
Problem 5/6



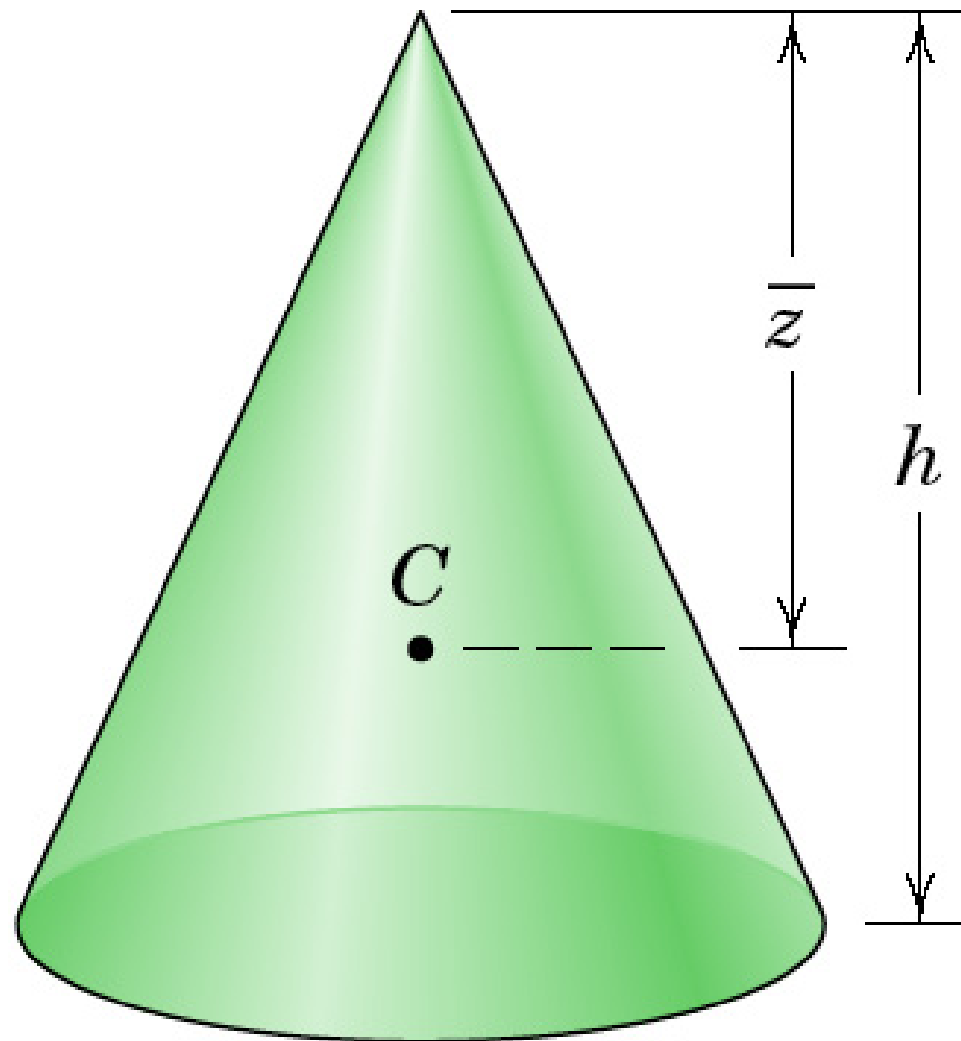
Problem 5/7



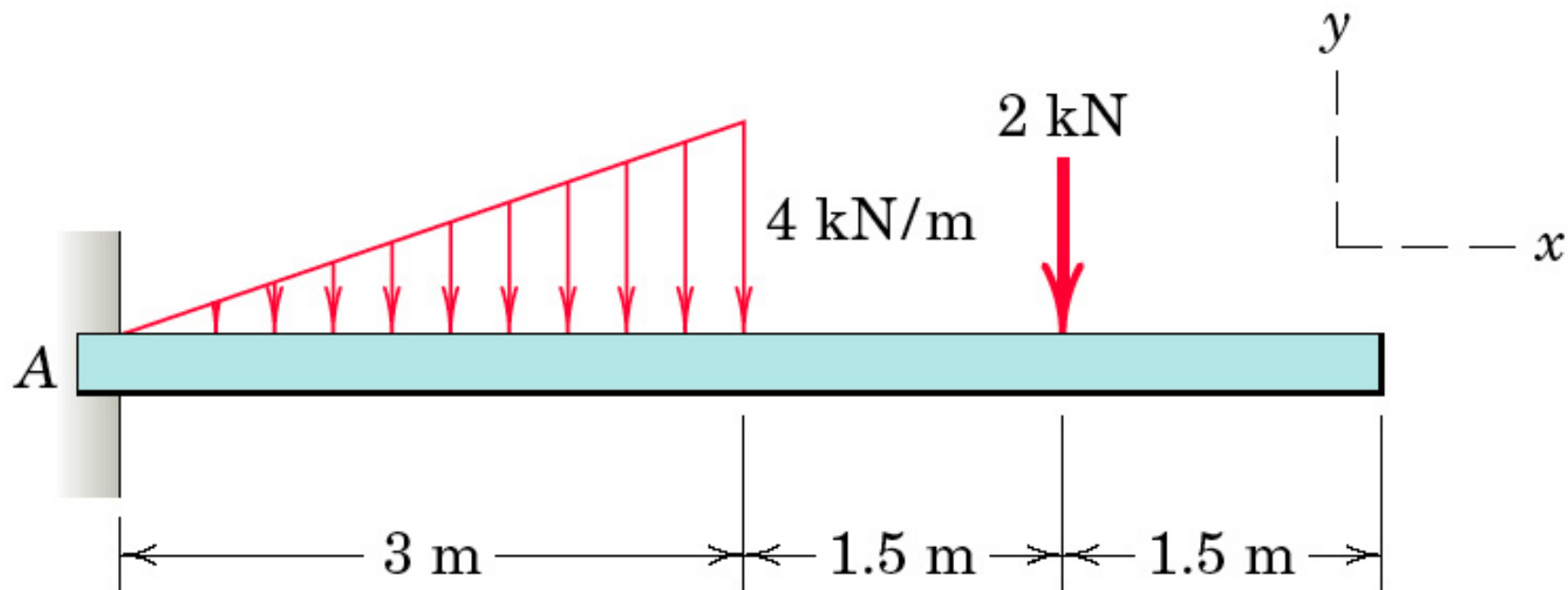
Problem 5/8



Problem 5/9

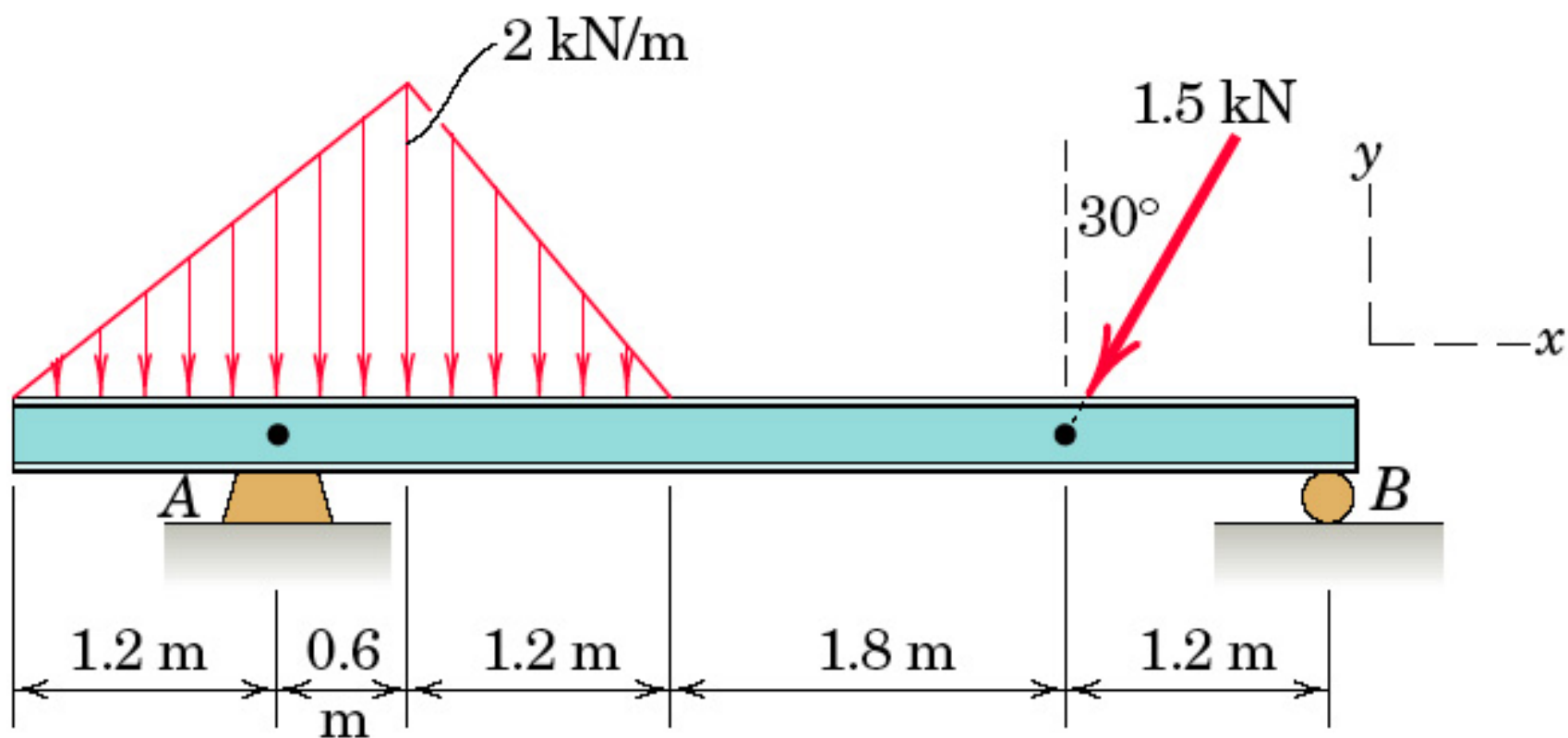


Problem 5/10



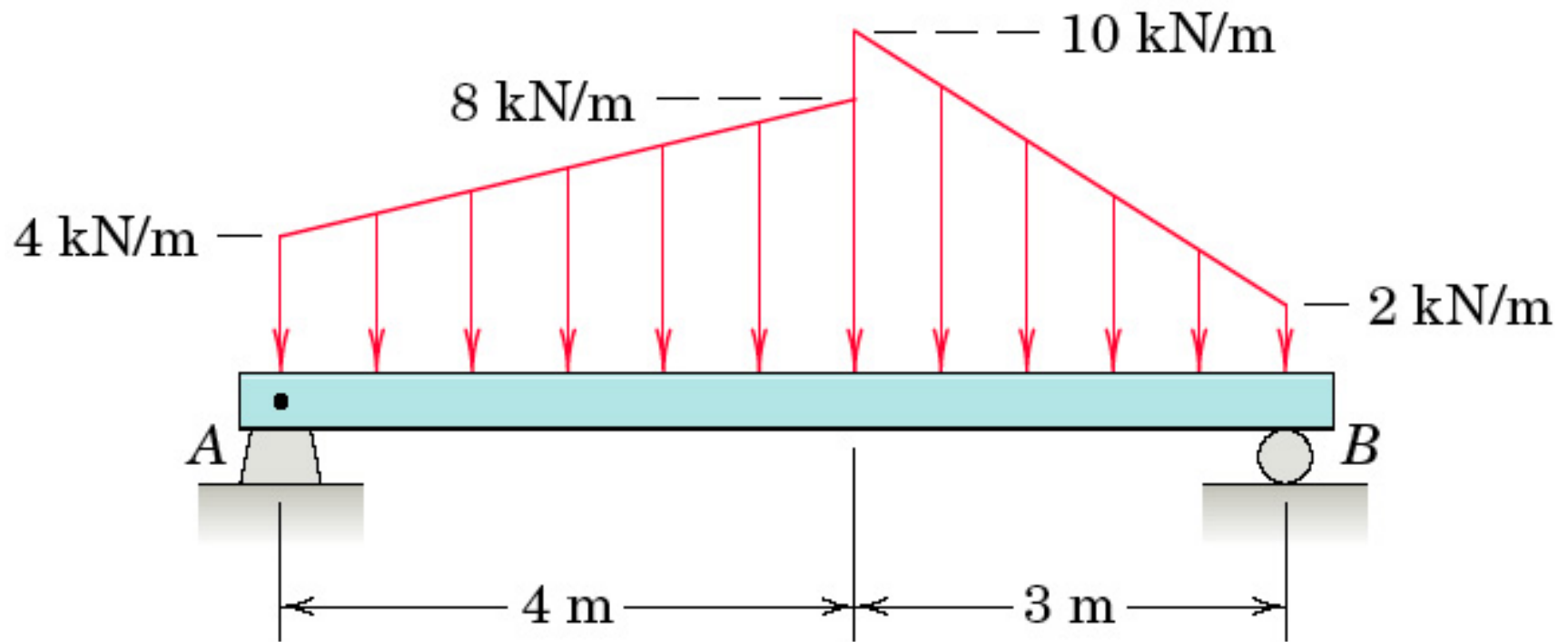
Problem 5/100

P_05_100



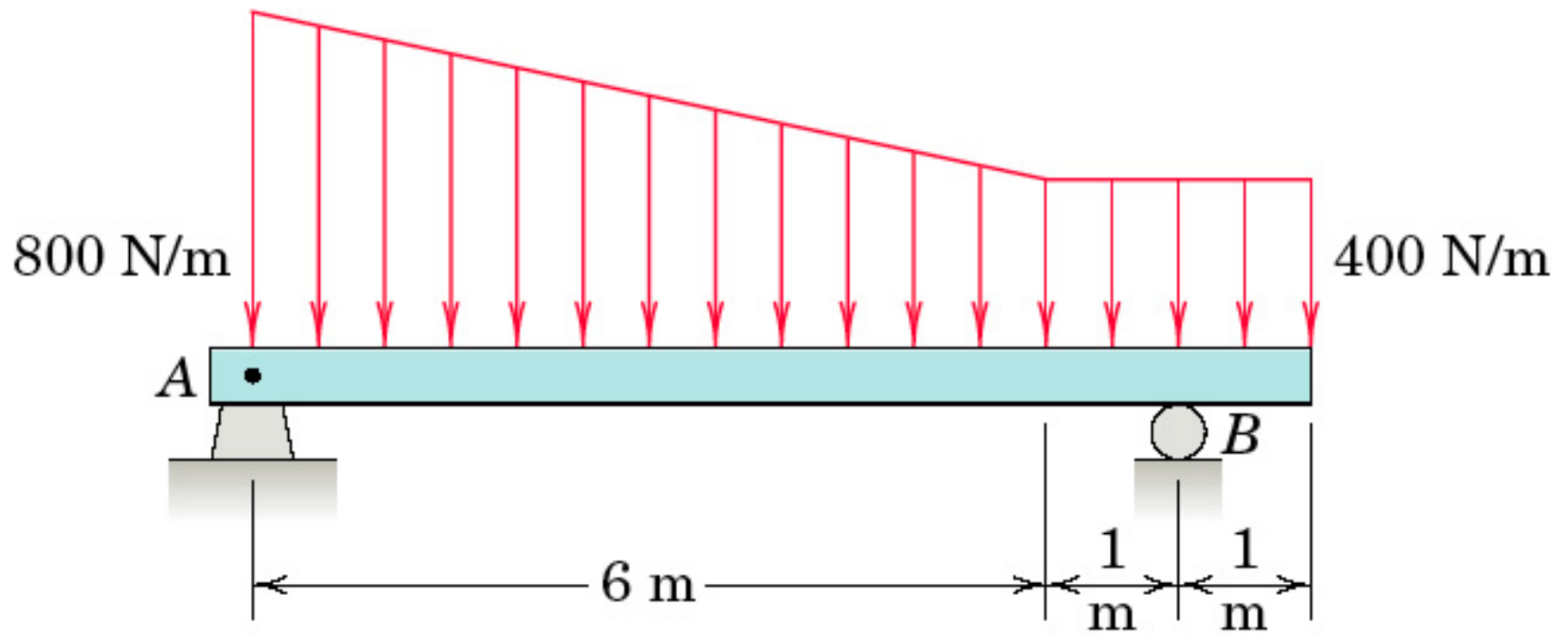
Problem 5/101

P_05_101



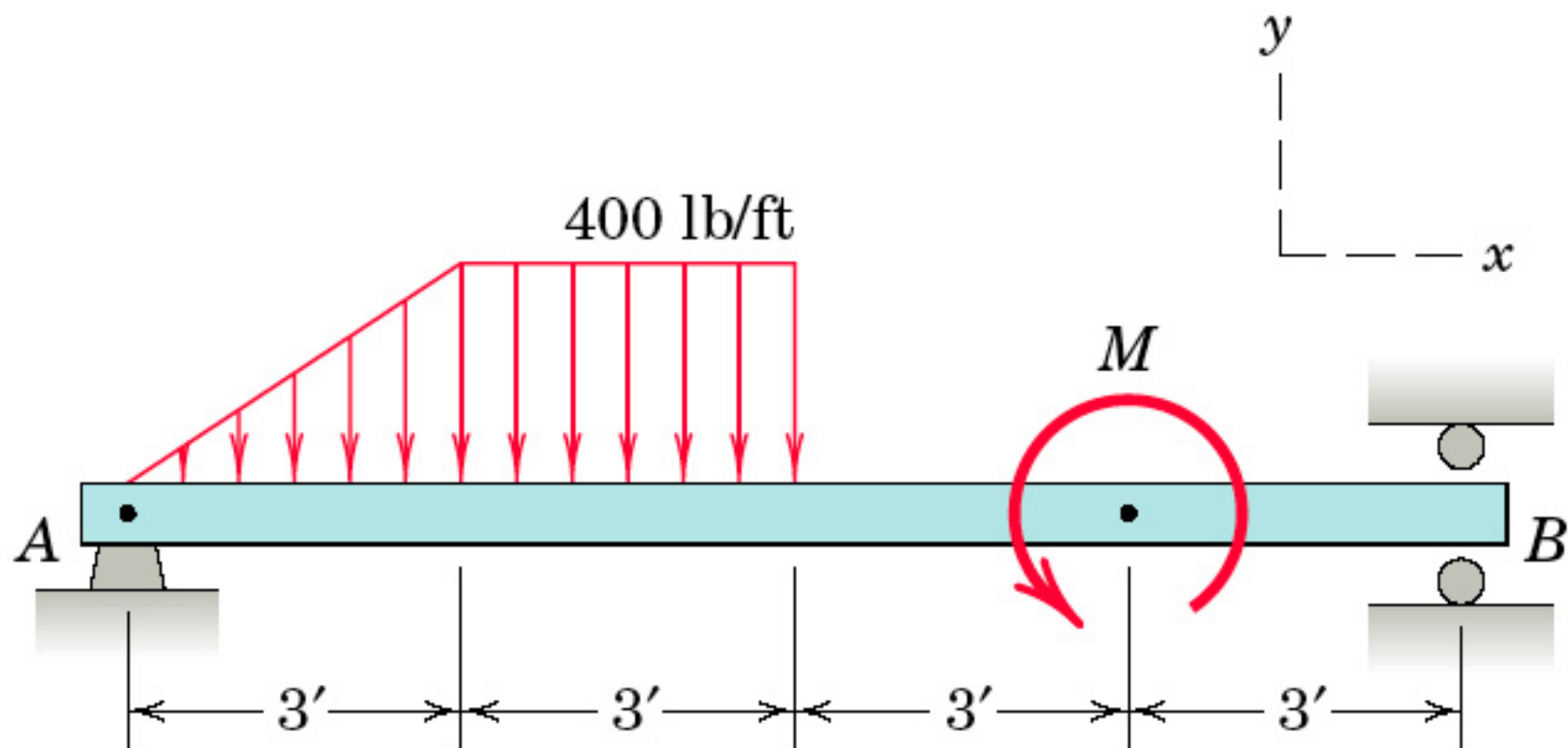
Problem 5/102

P_05_102



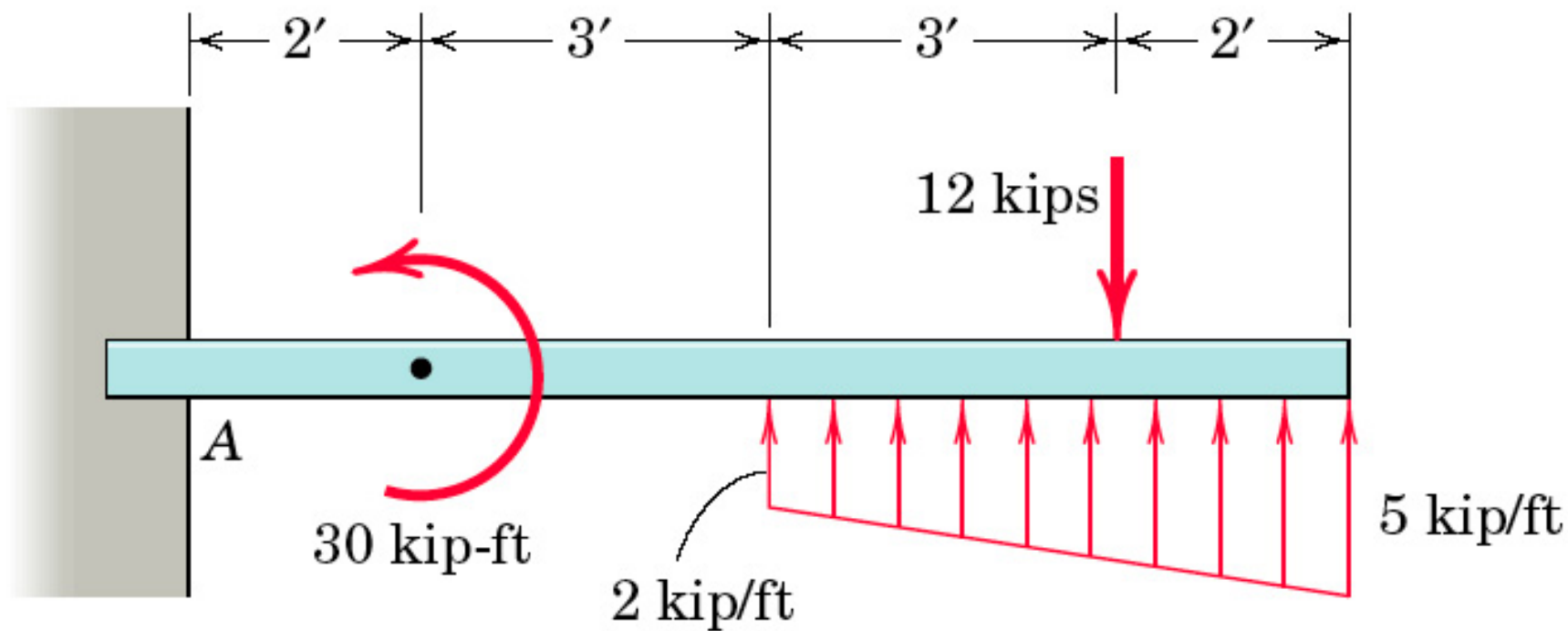
Problem 5/103

P_05_103



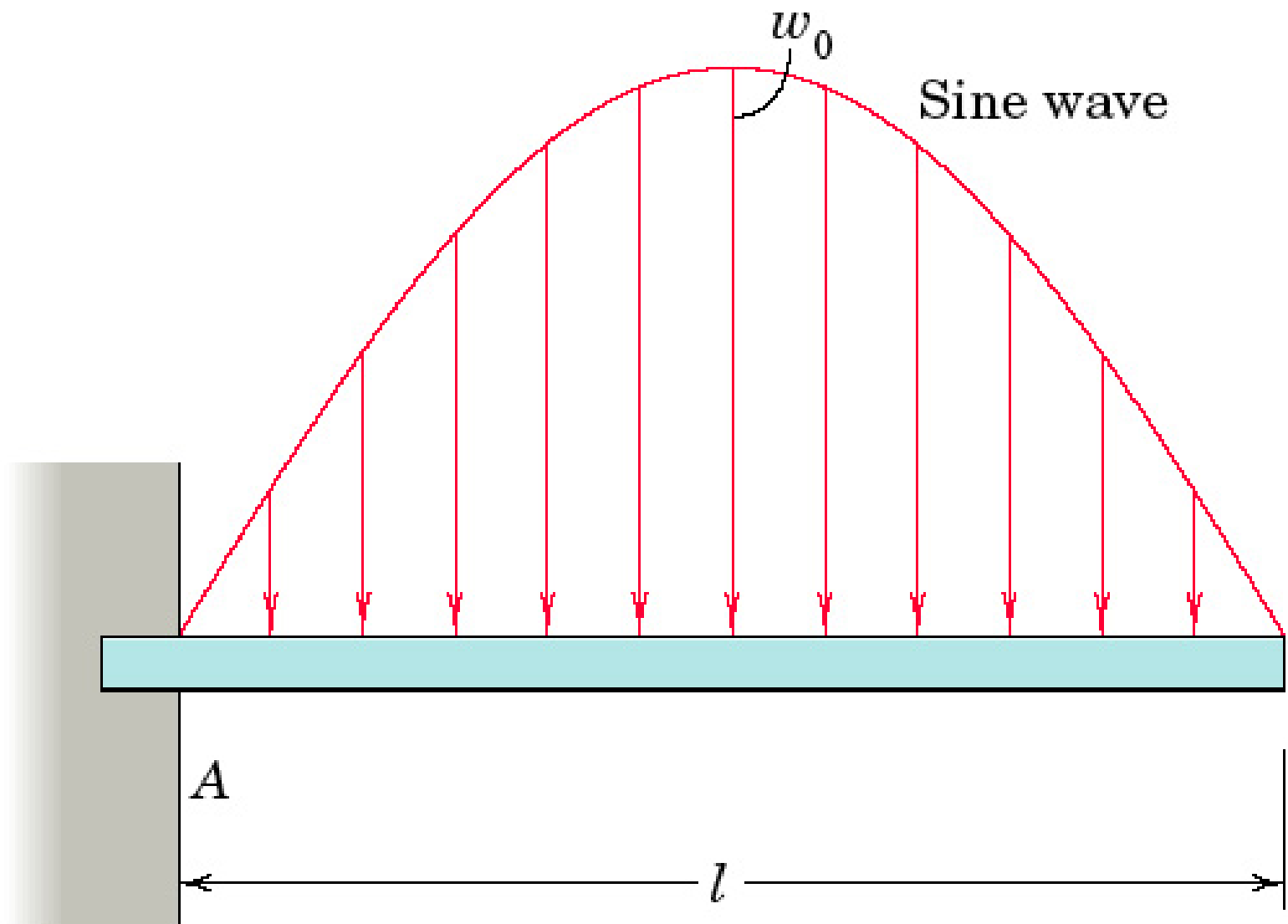
Problem 5/104

P_05_104

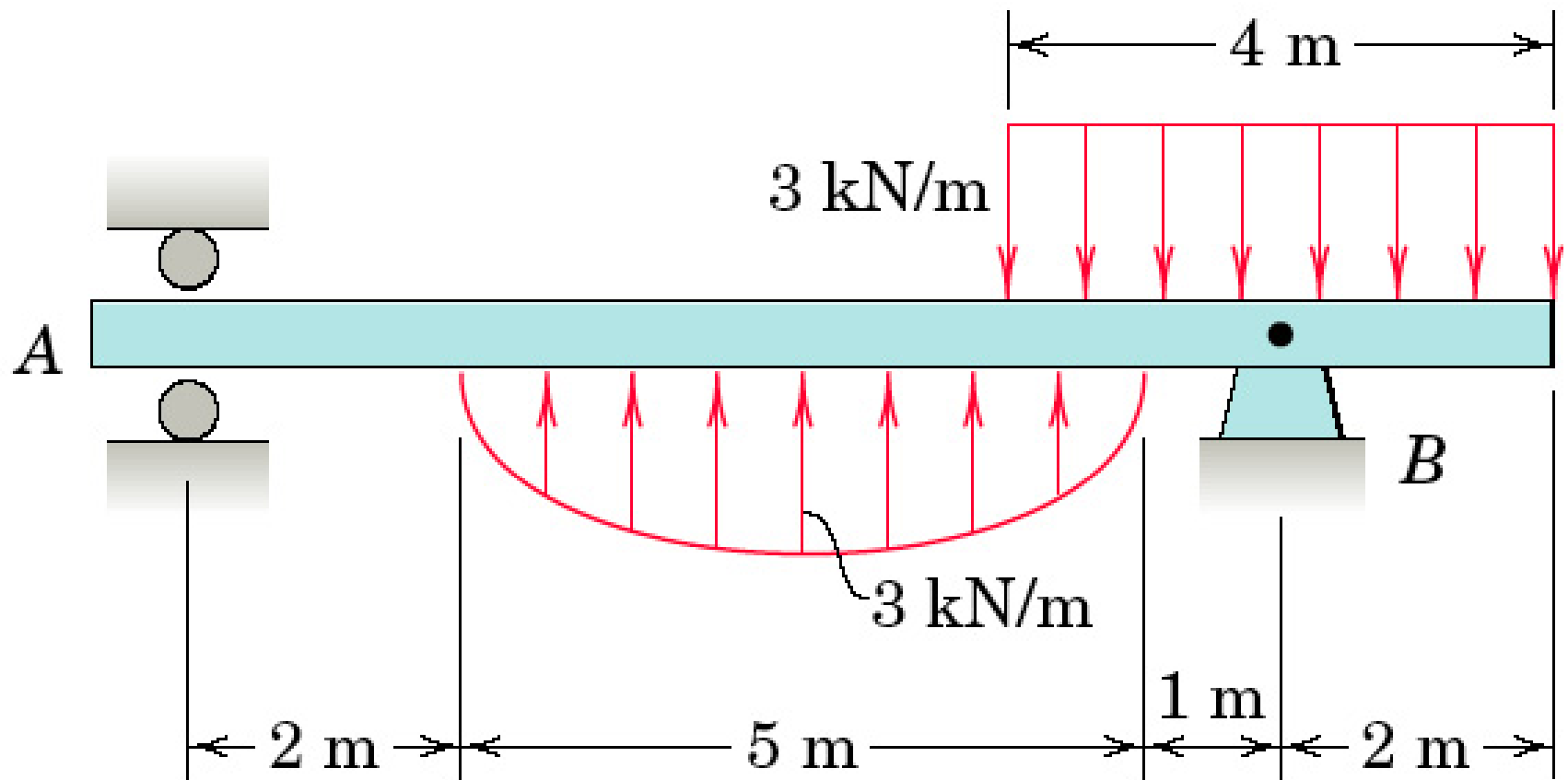


Problem 5/105

P_05_105

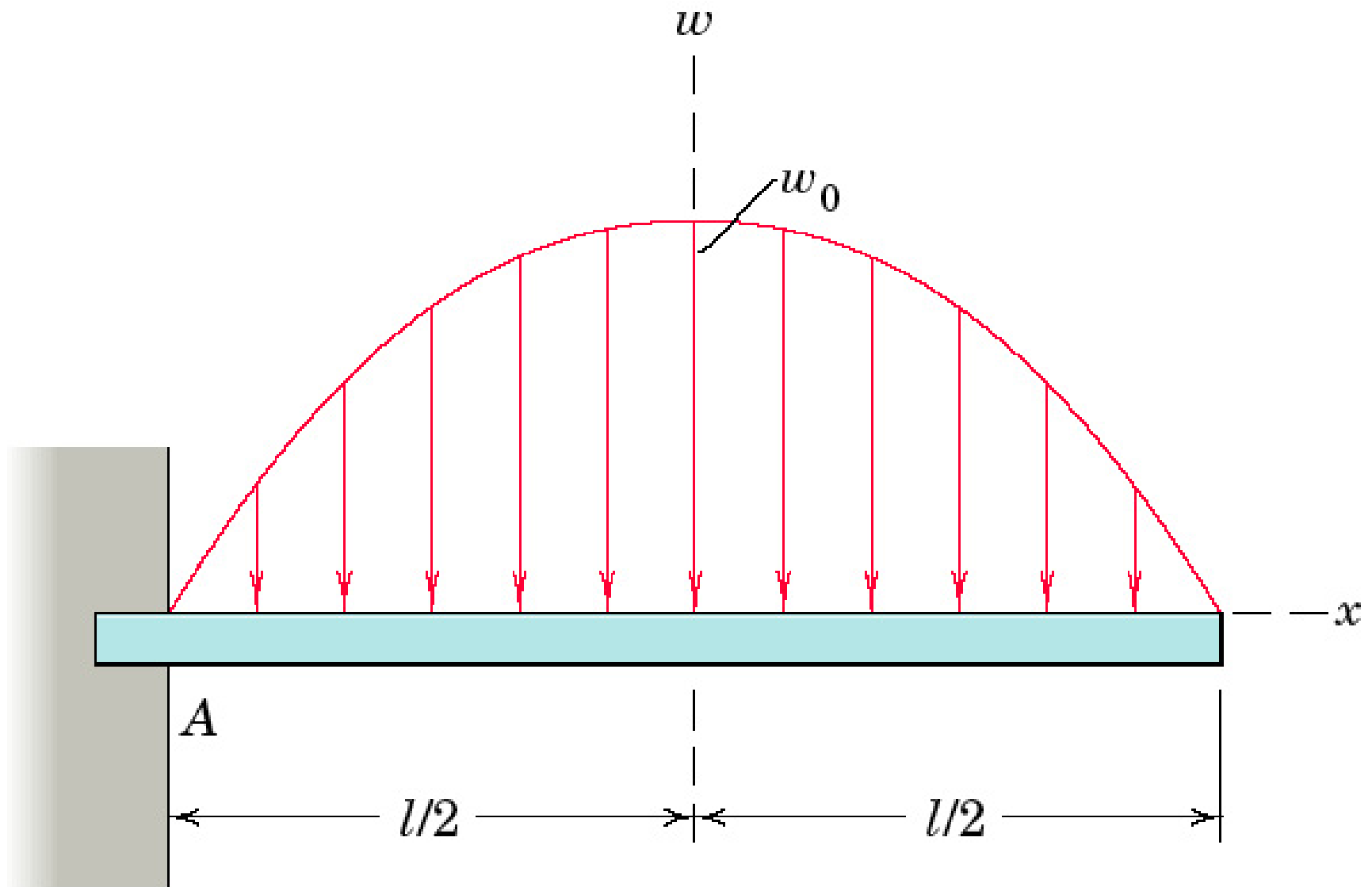


Problem 5/106

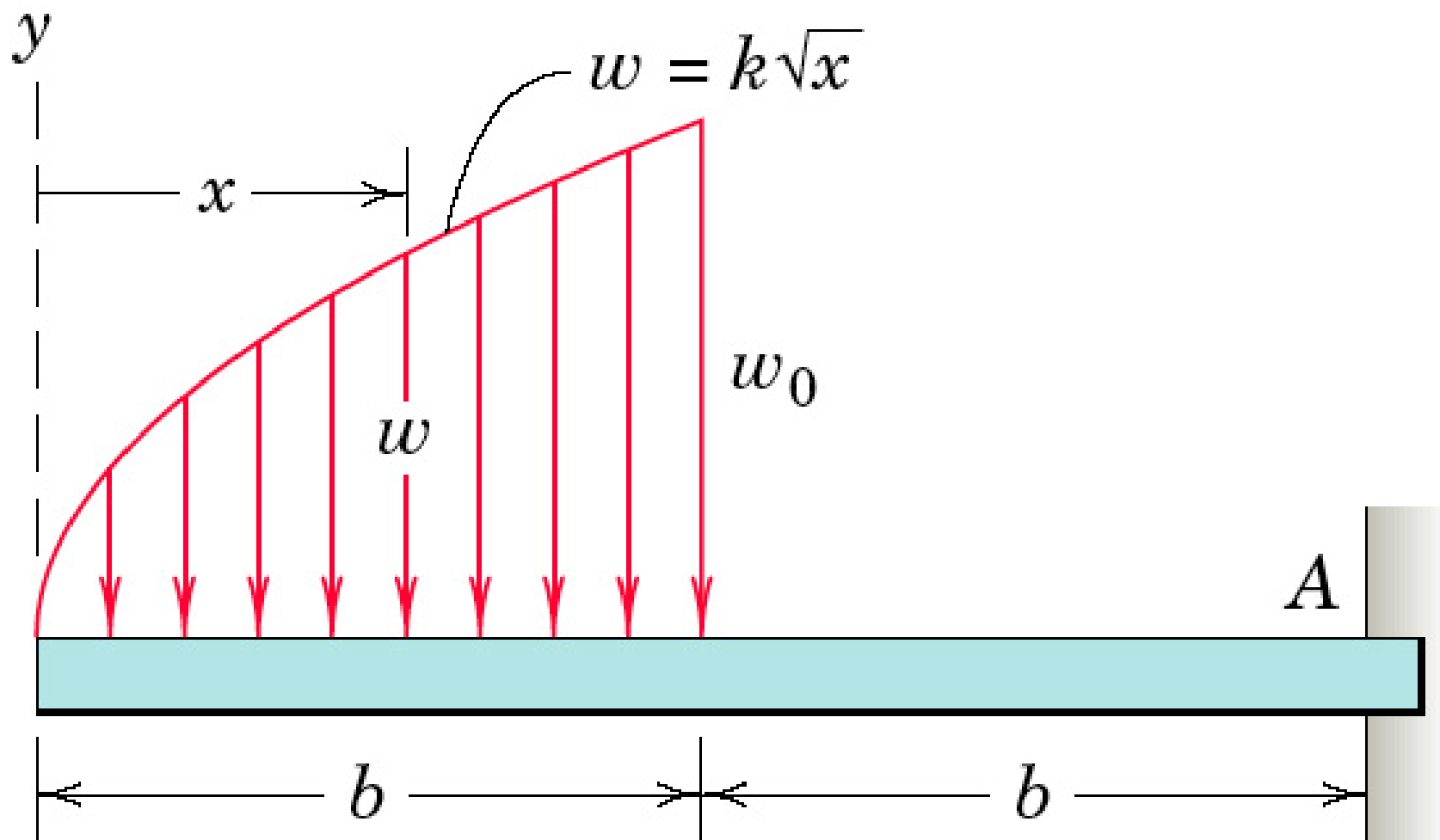


Problem 5/107

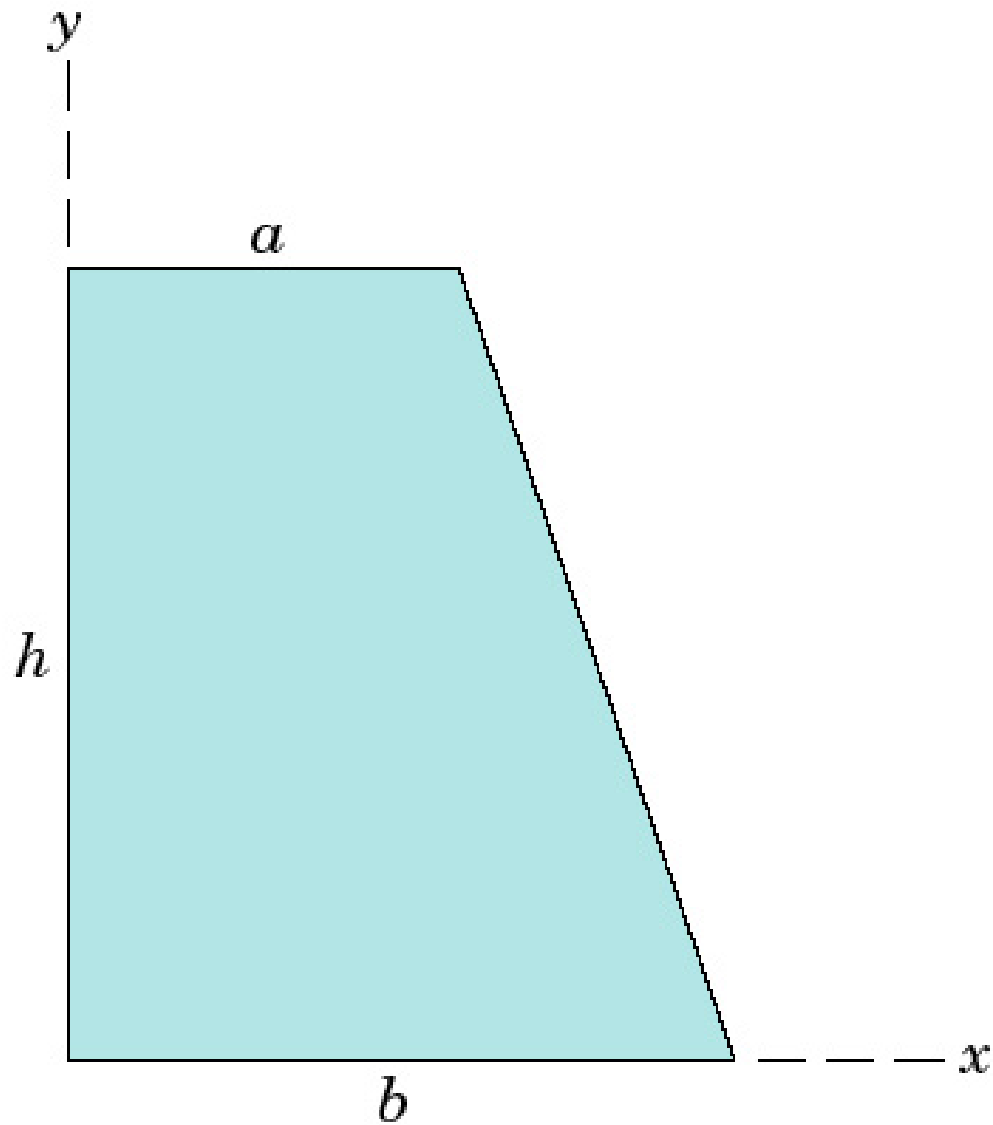
P_05_107



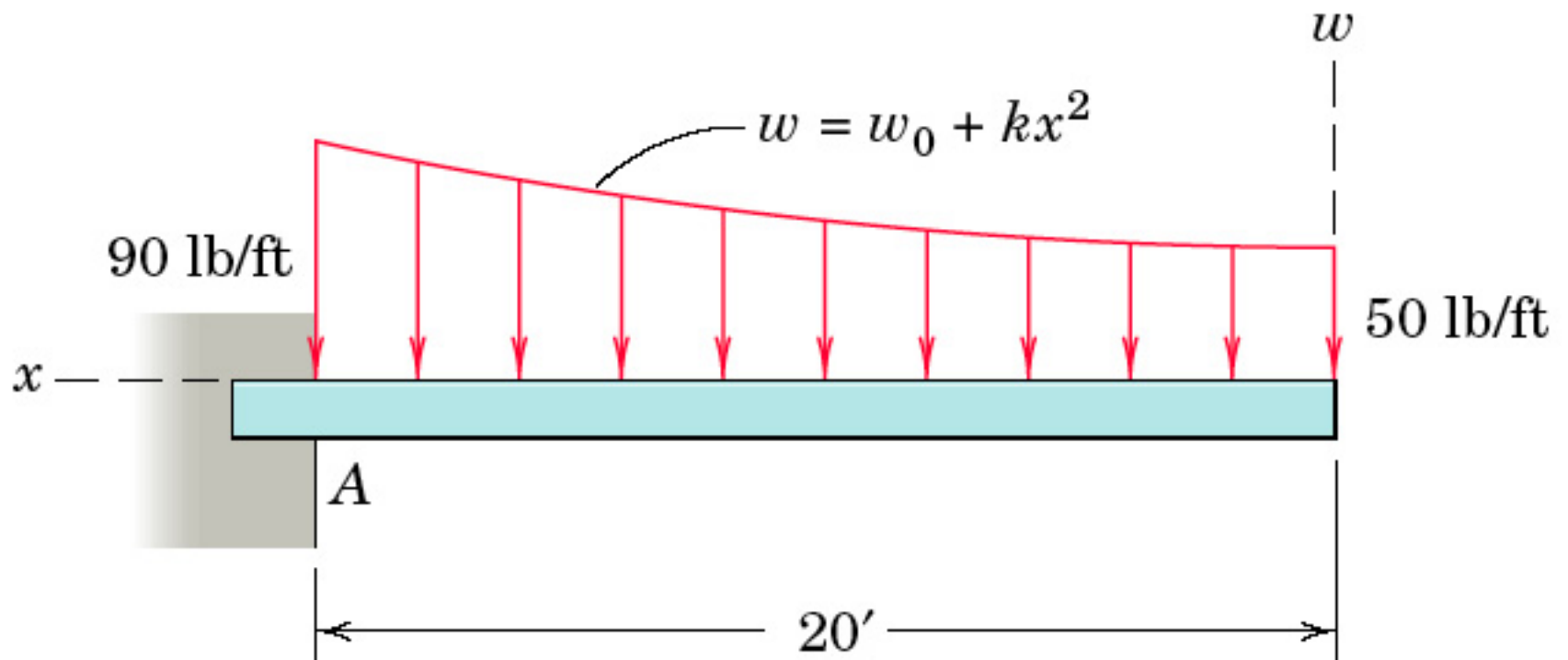
Problem 5/108



Problem 5/109

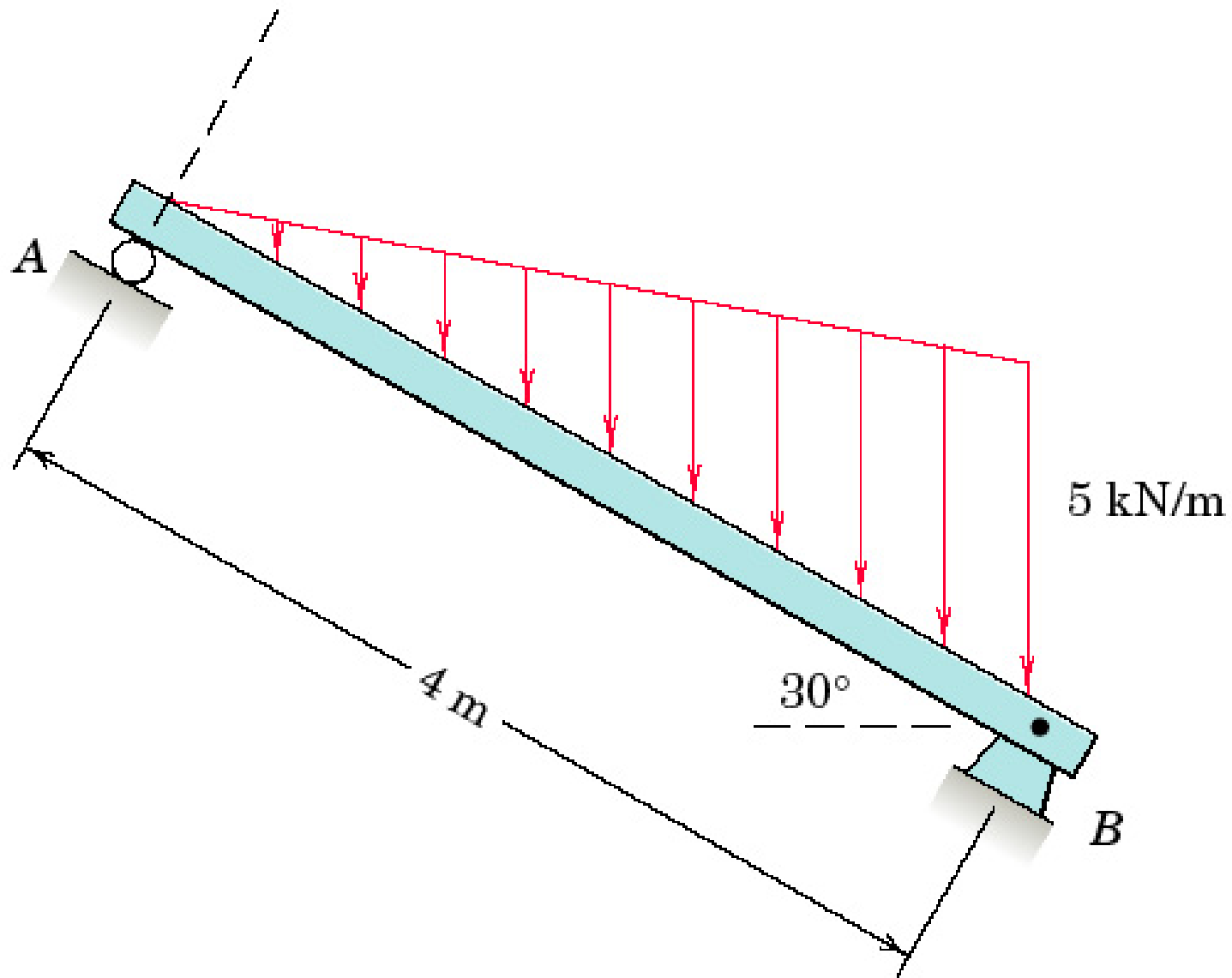


Problem 5/11

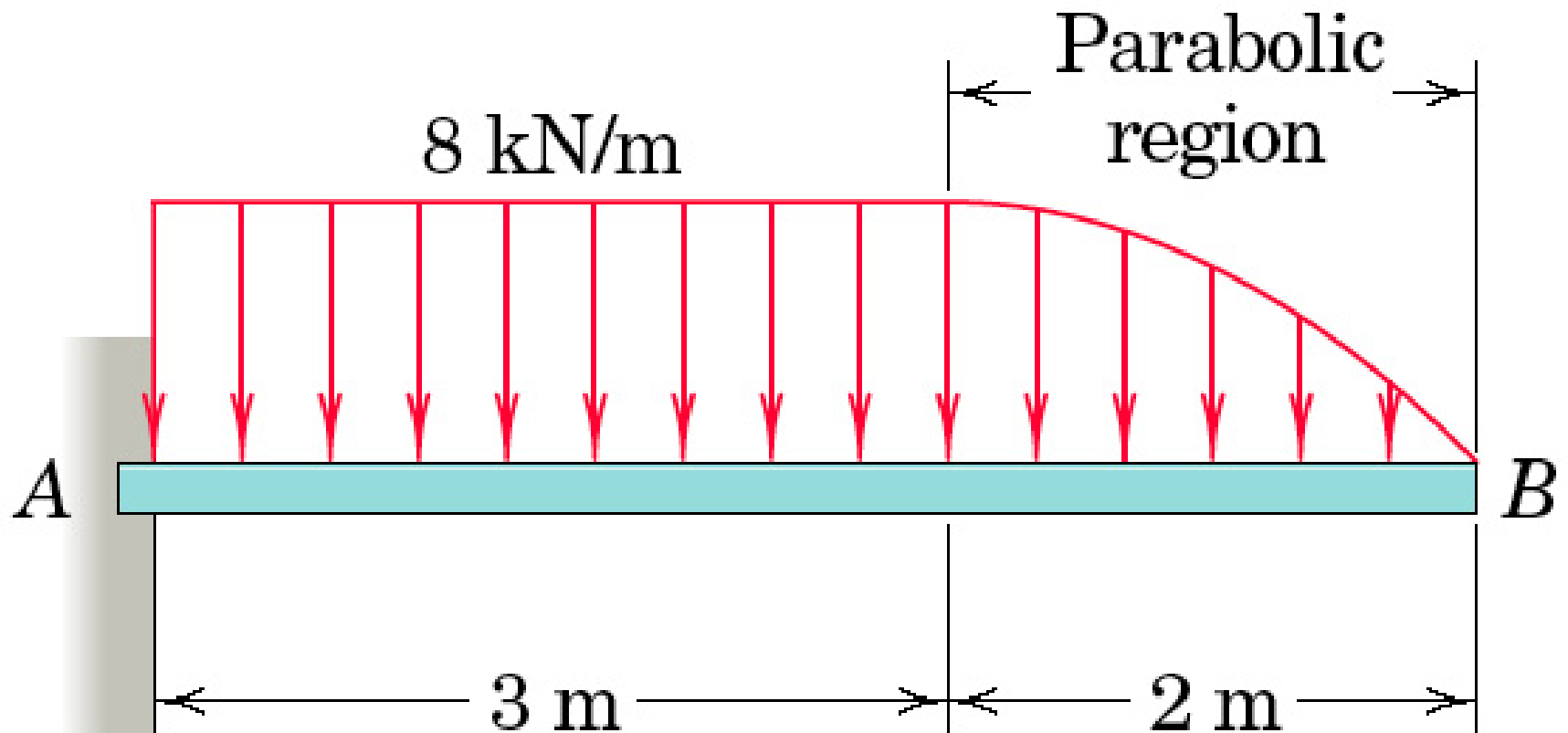


Problem 5/110

P_05_110

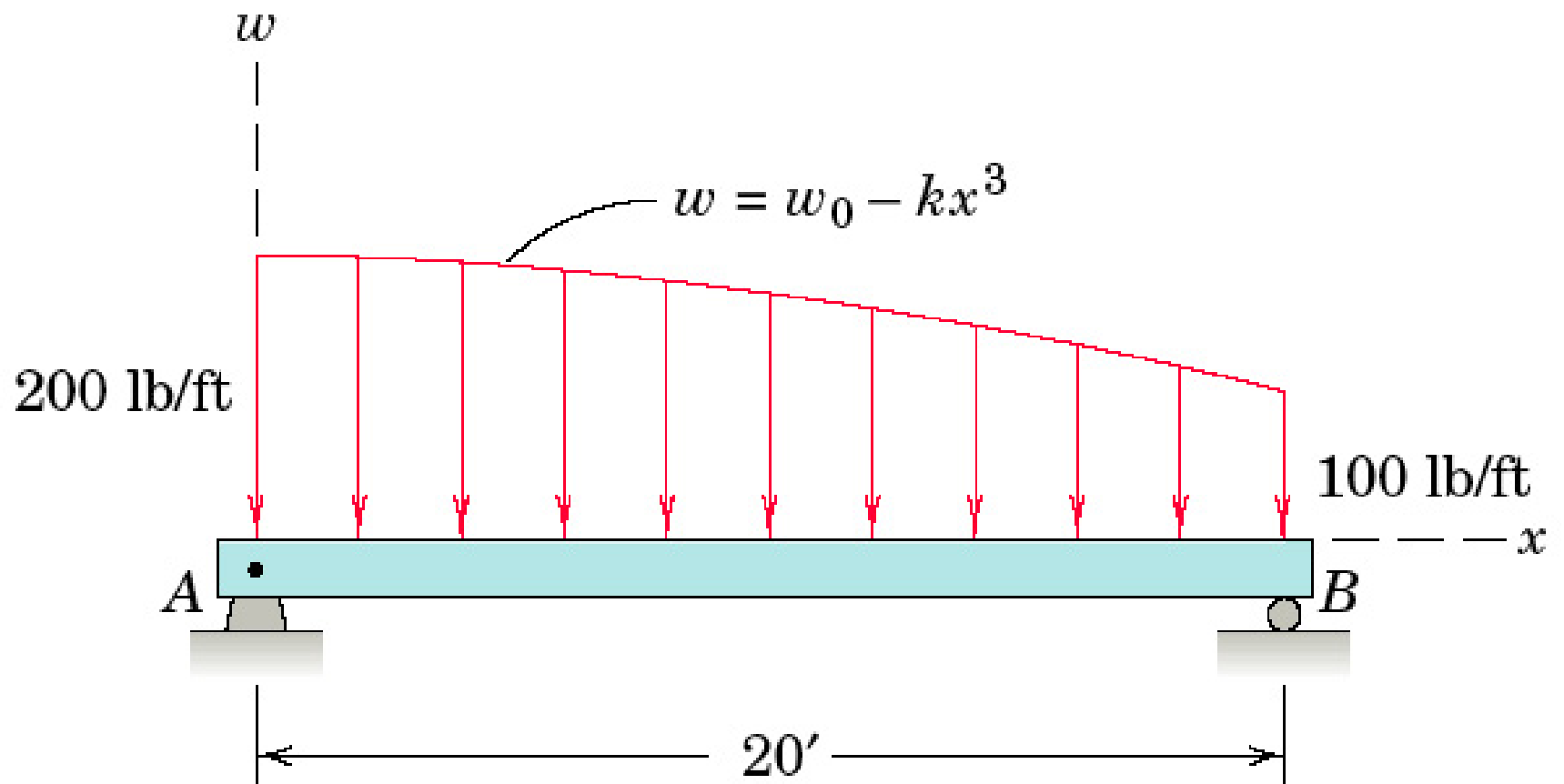


Problem 5/111



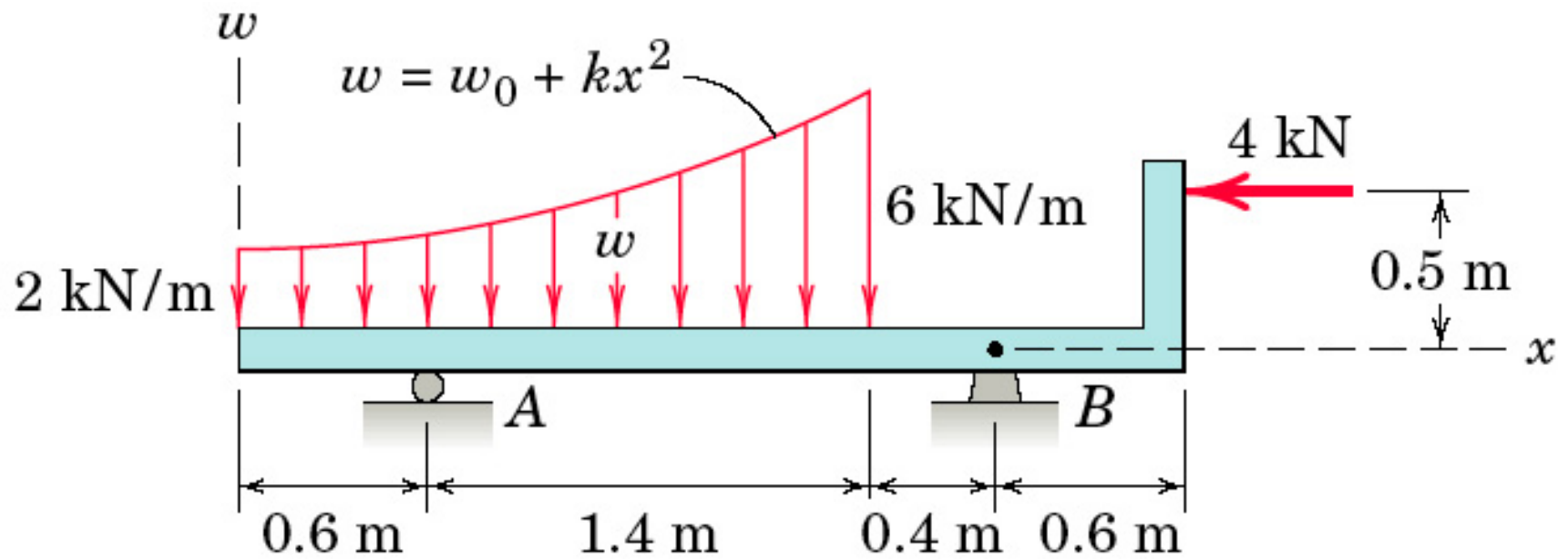
Problem 5/112

P_05_112



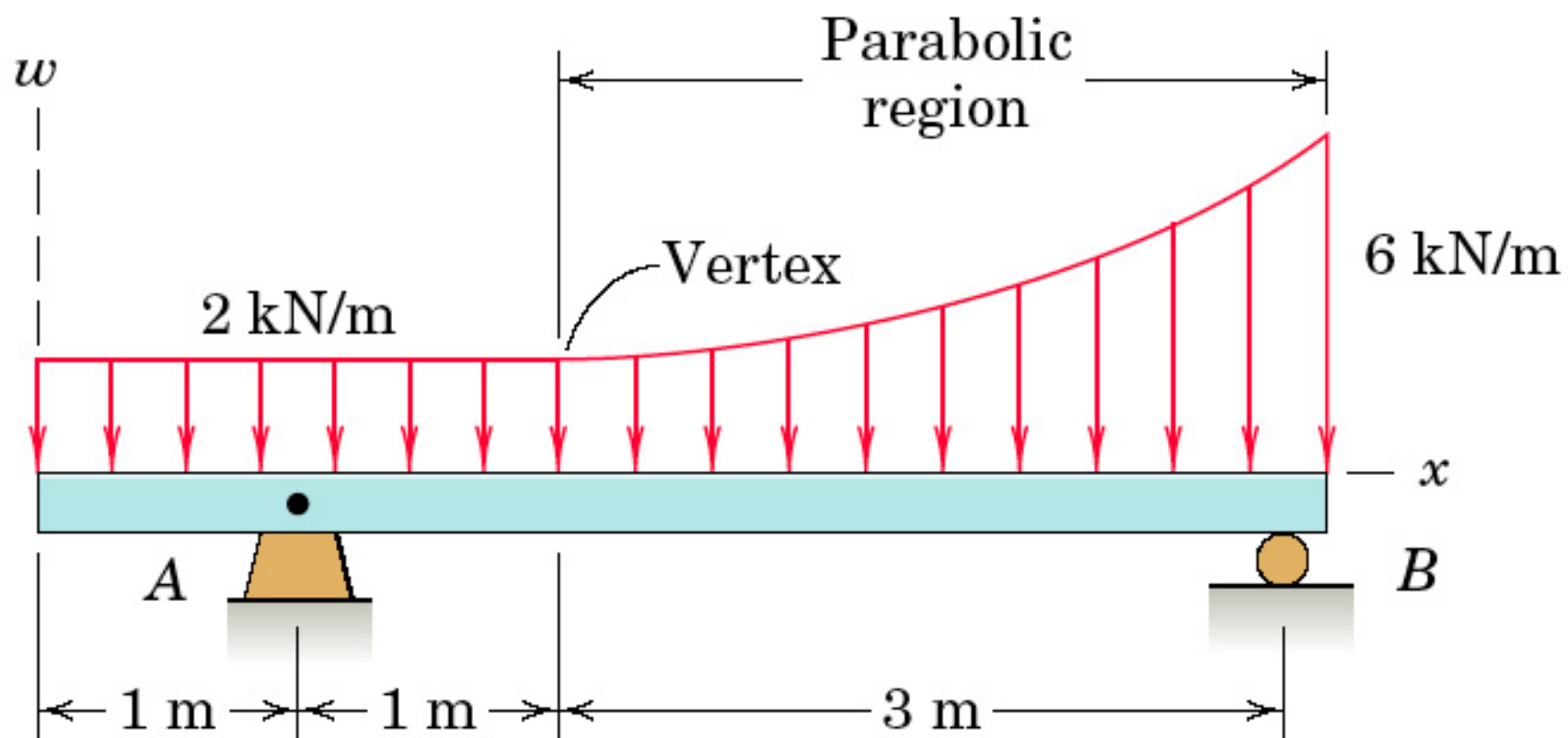
Problem 5/113

P_05_113



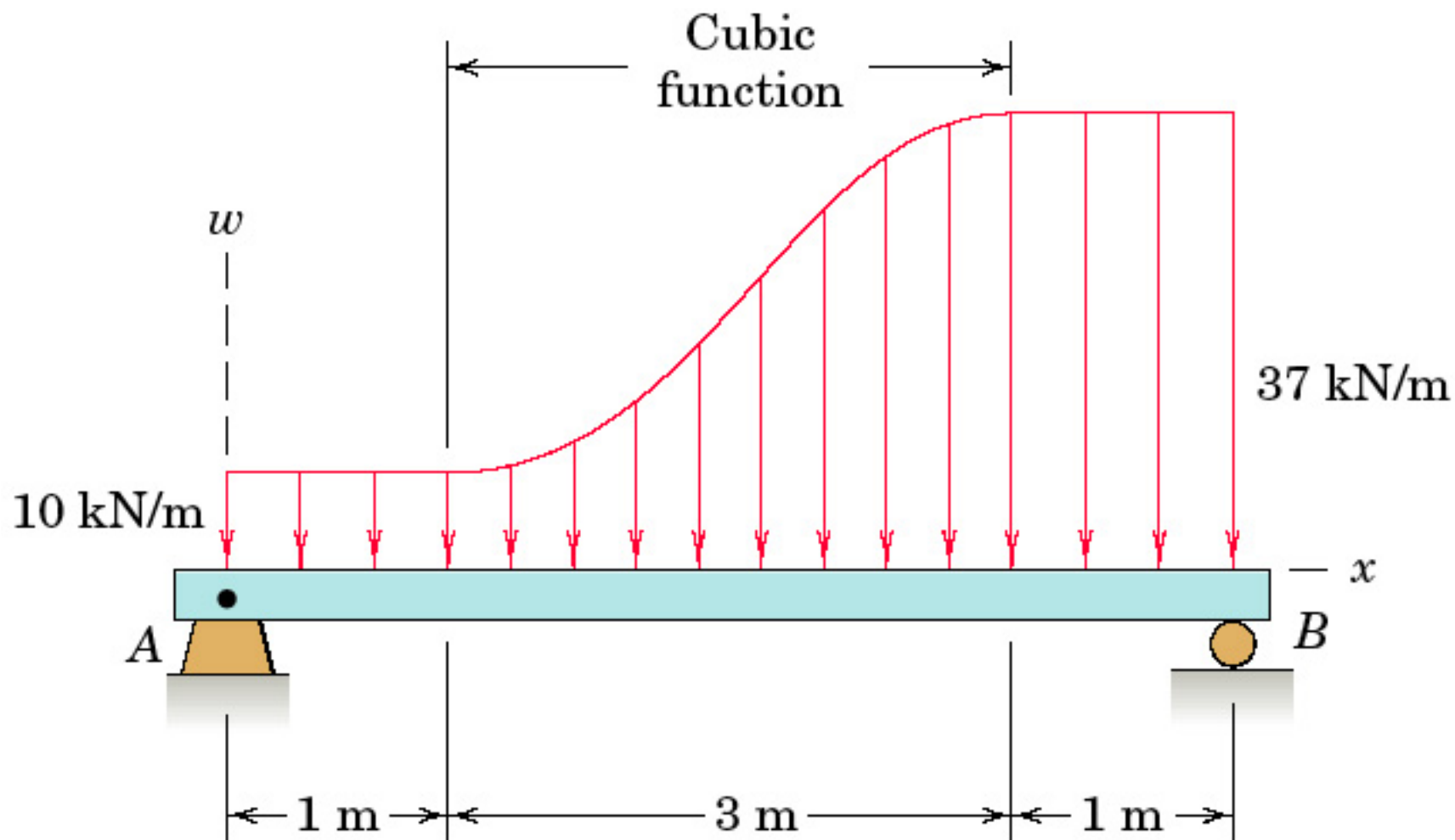
Problem 5/114

P_05_114

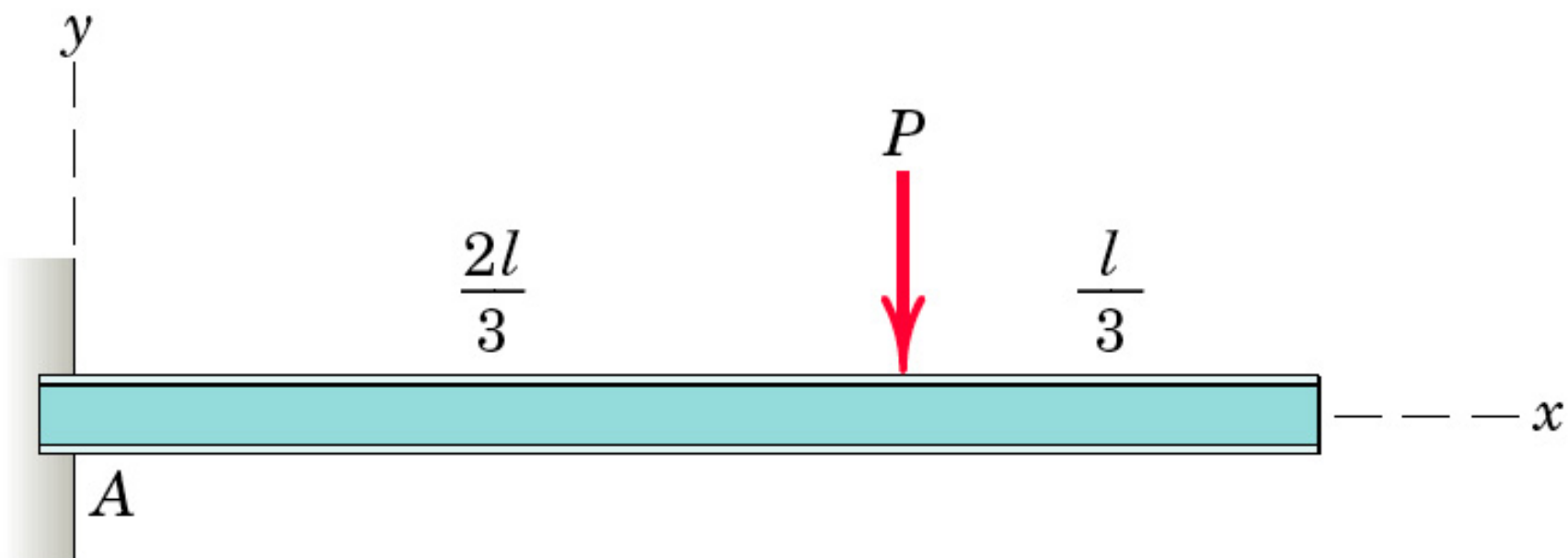


Problem 5/115

P_05_115

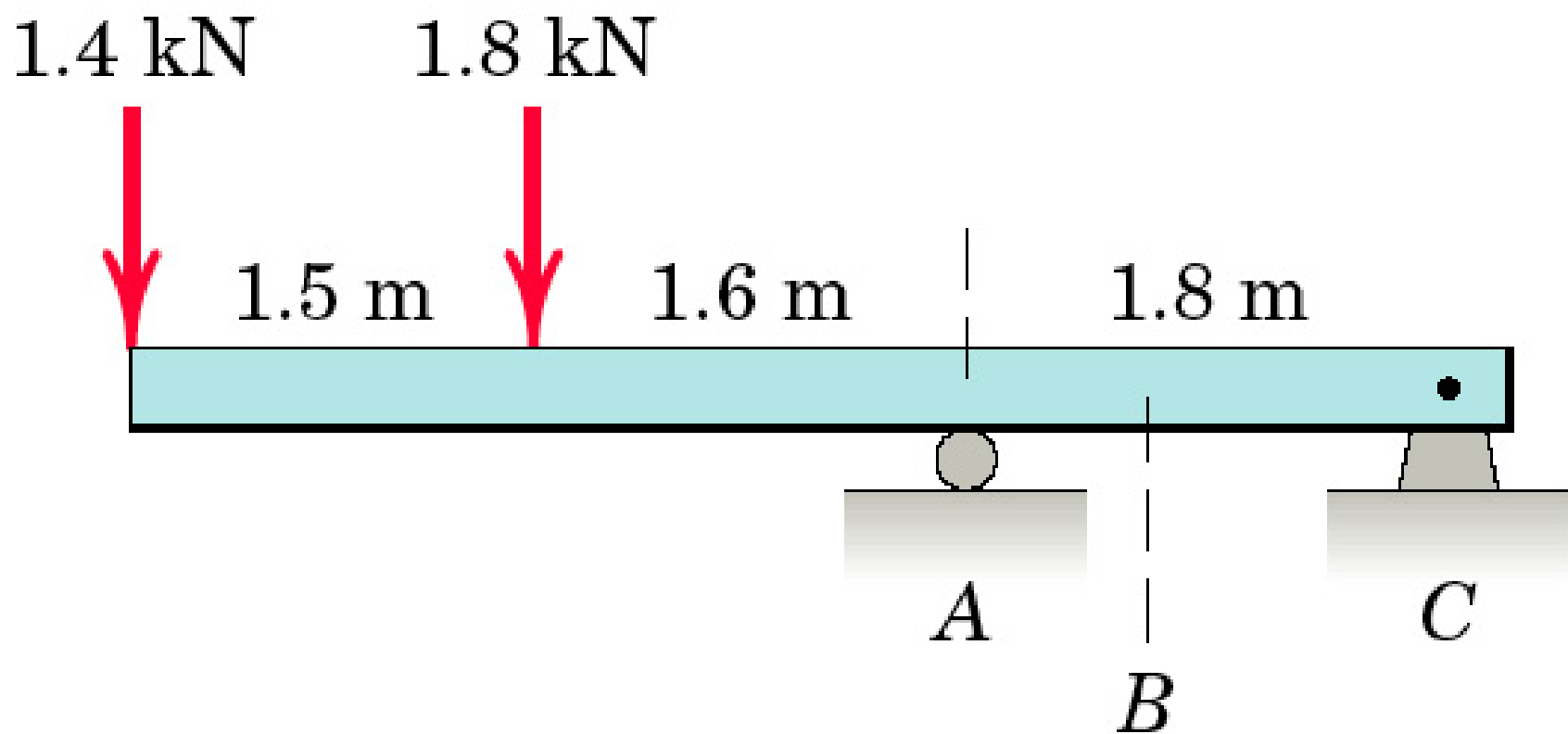


Problem 5/116



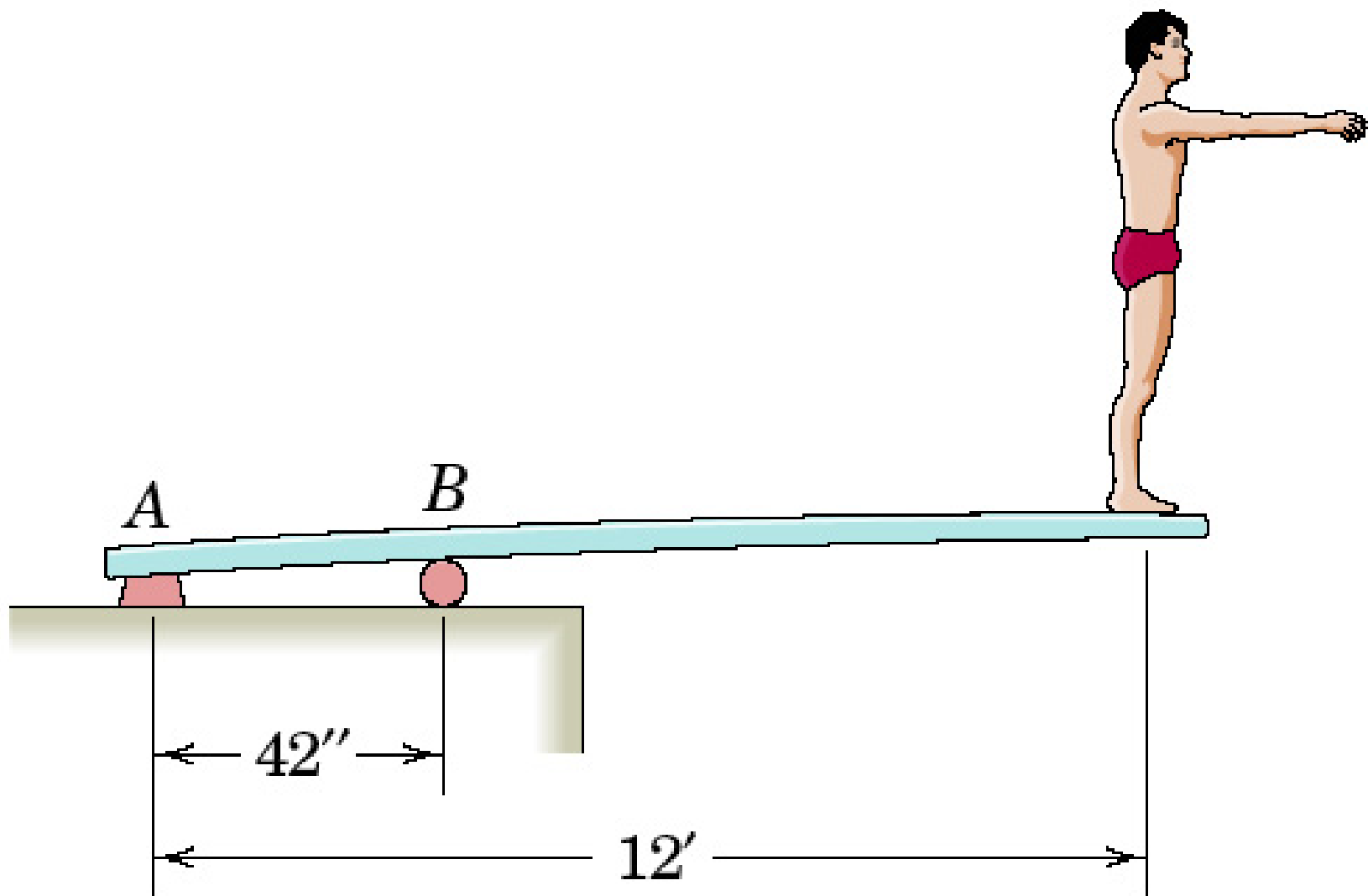
Problem 5/117

P_05_117

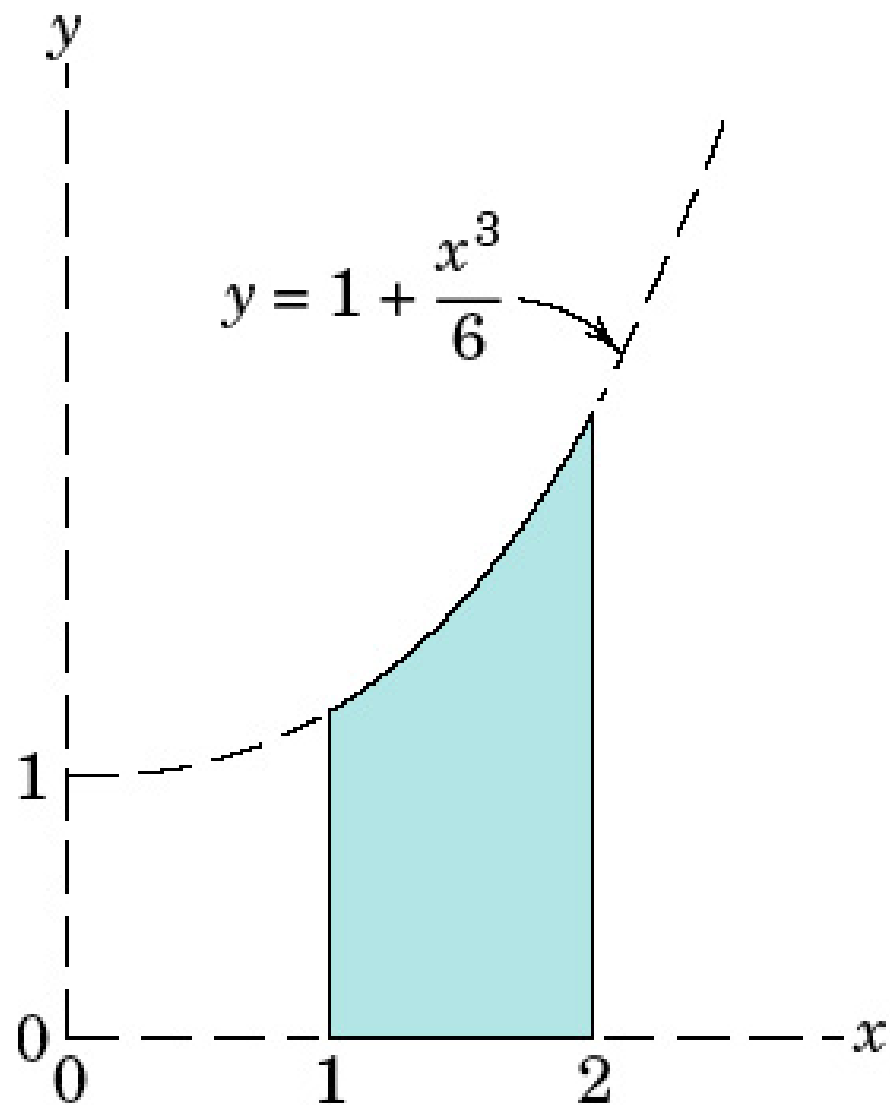


Problem 5/118

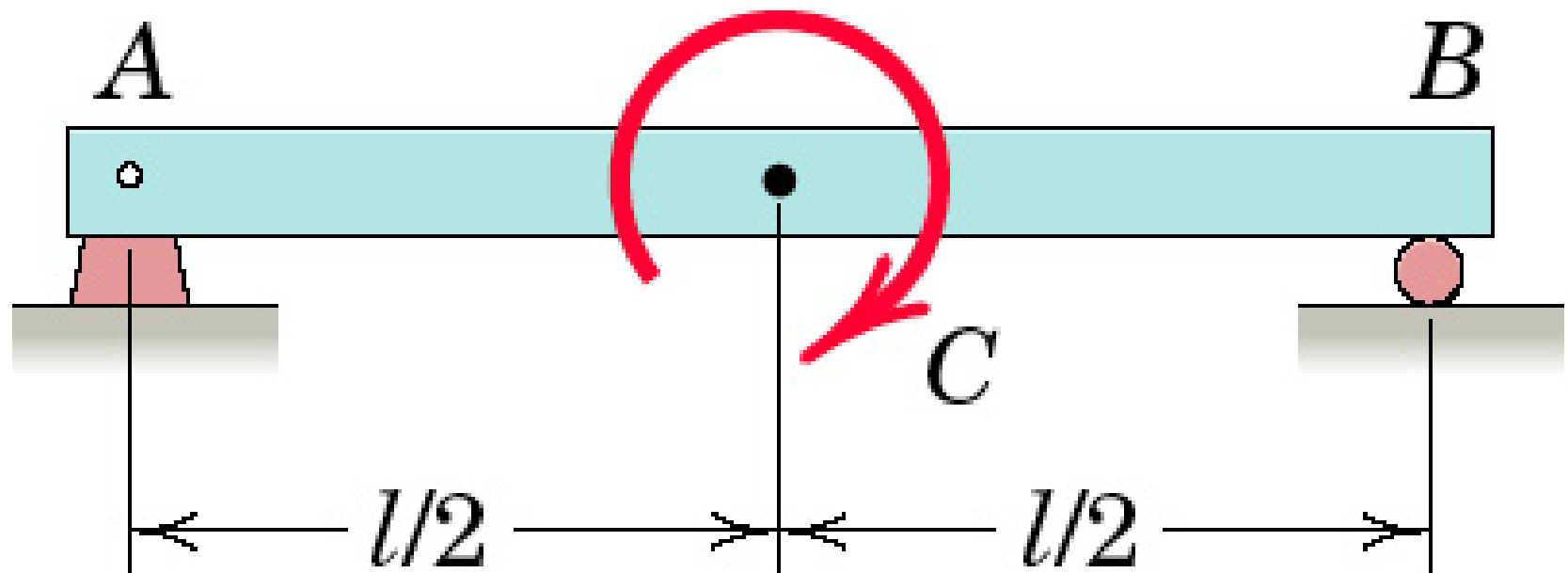
P_05_118



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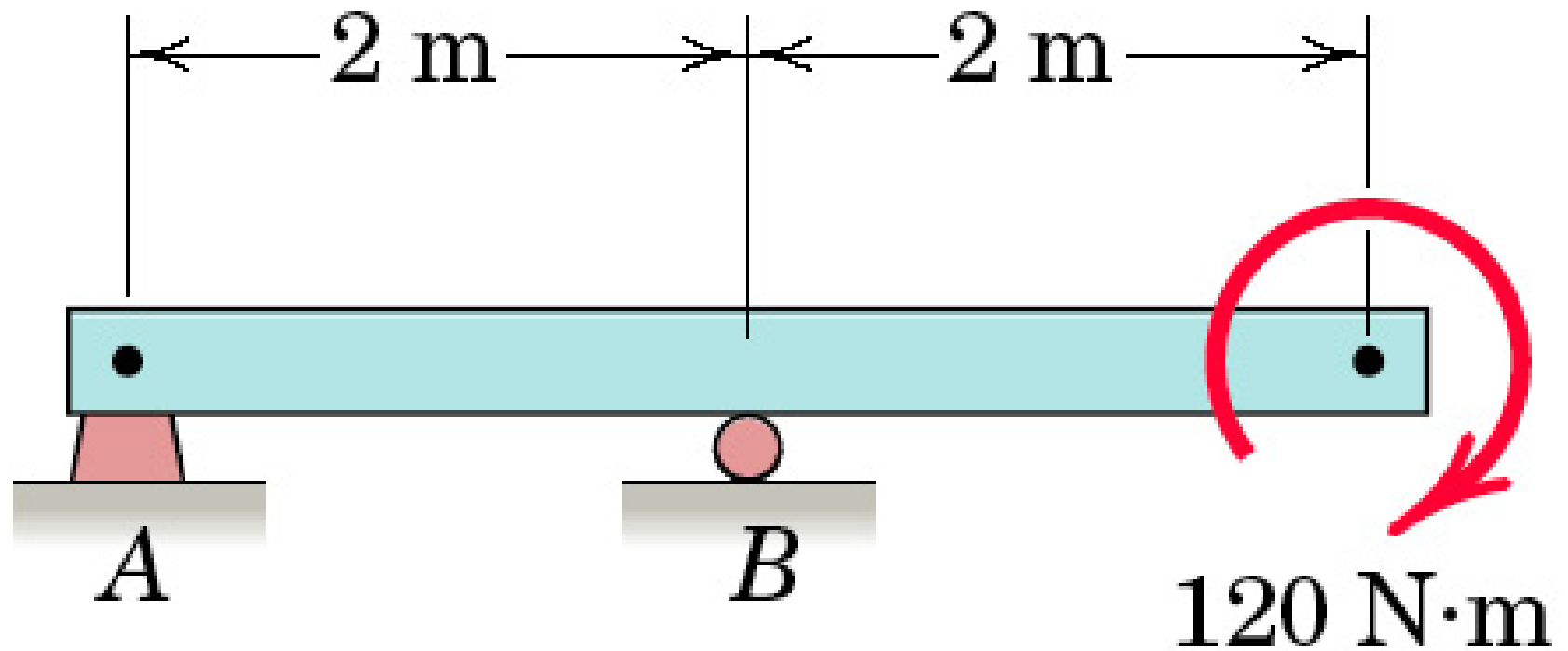


Problem 5/12



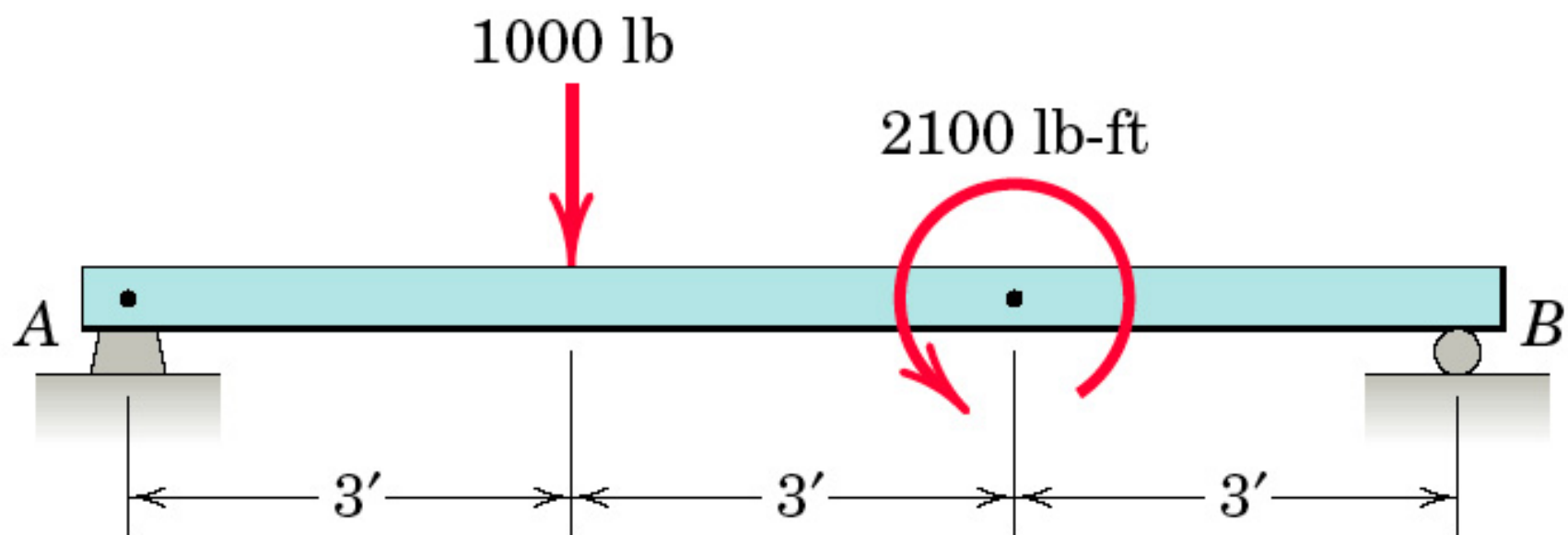
Problem 5/120

P_05_120



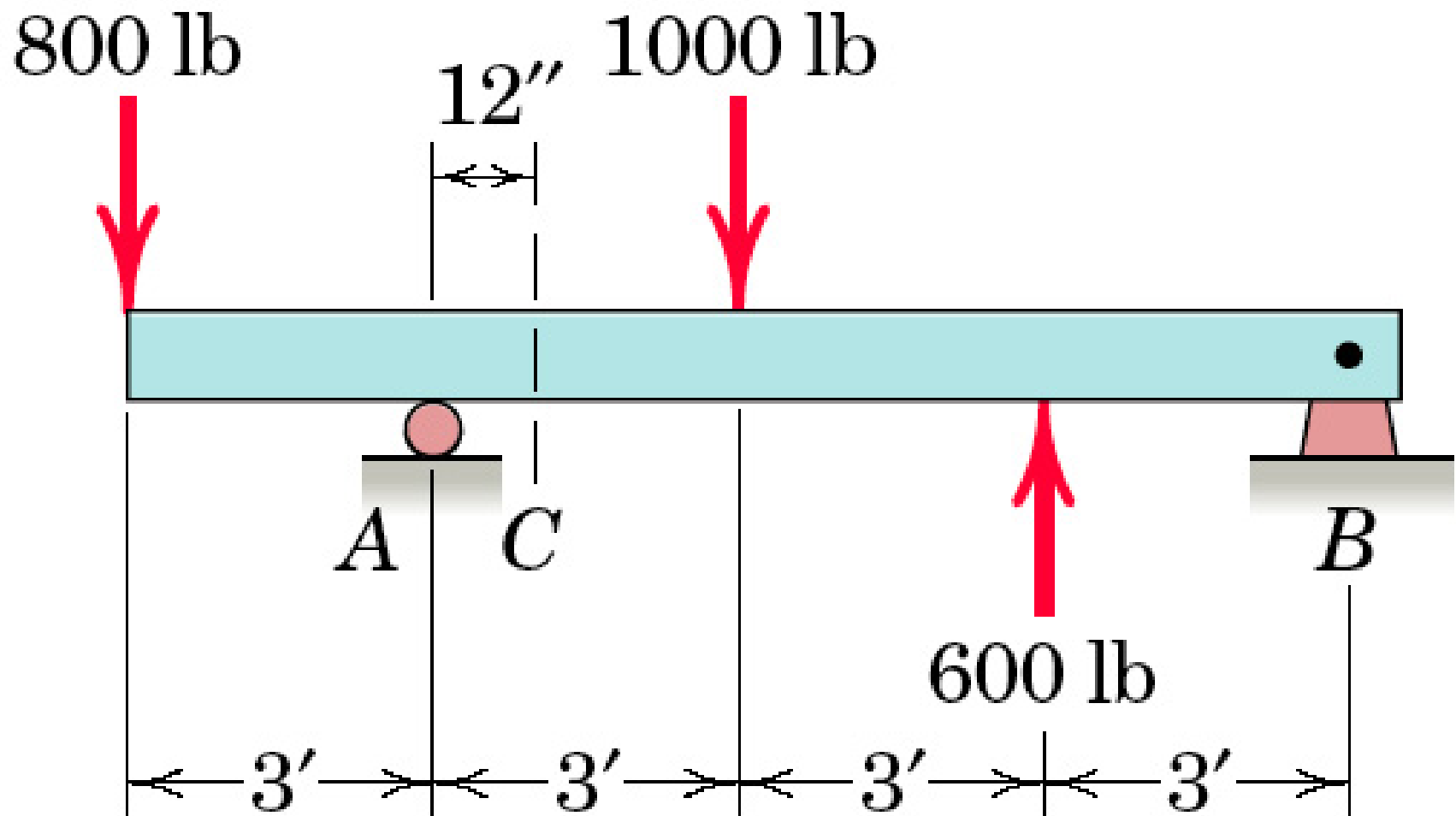
Problem 5/121

P_05_121

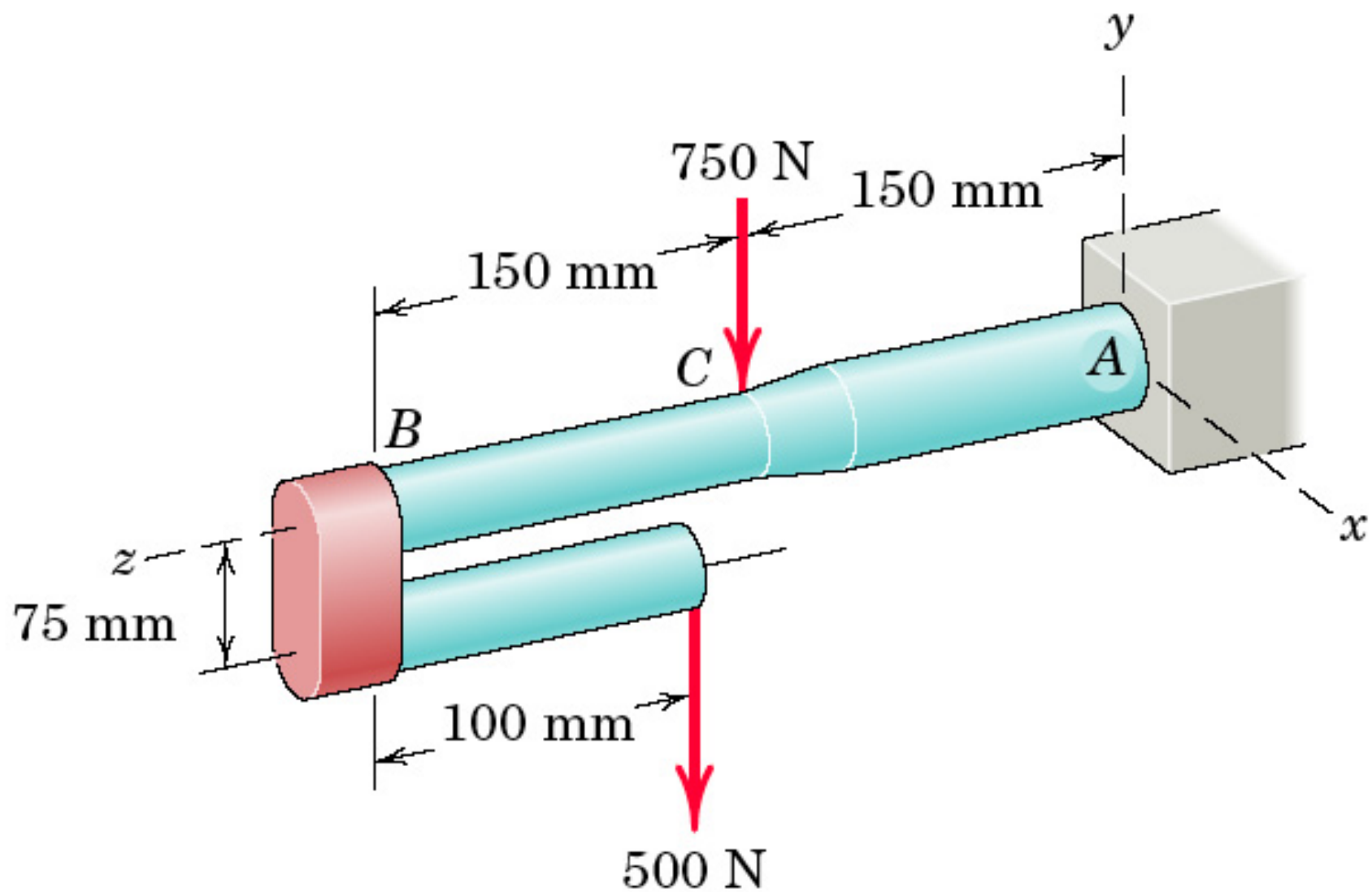


Problem 5/122

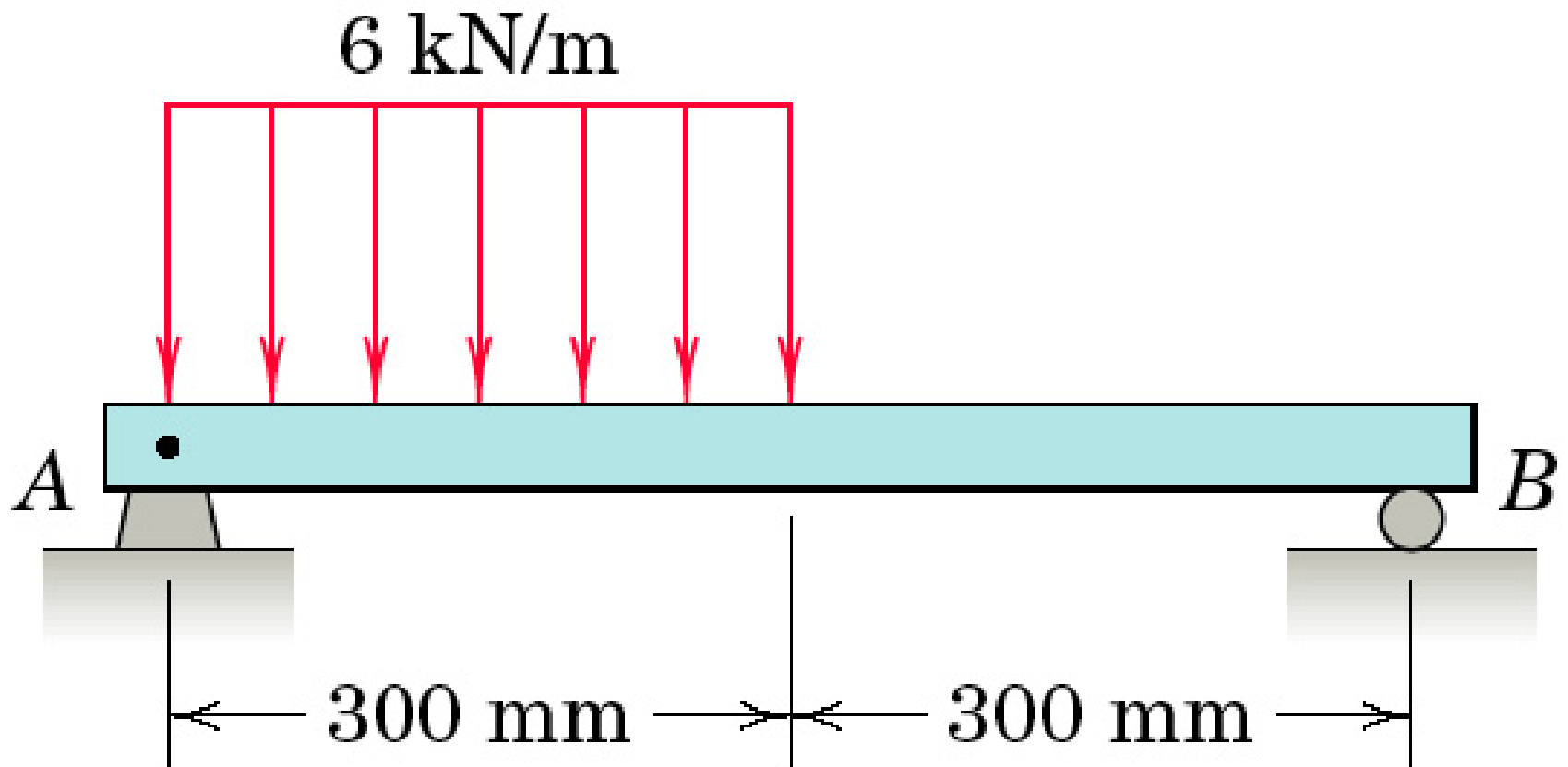
P_05_122



Problem 5/123

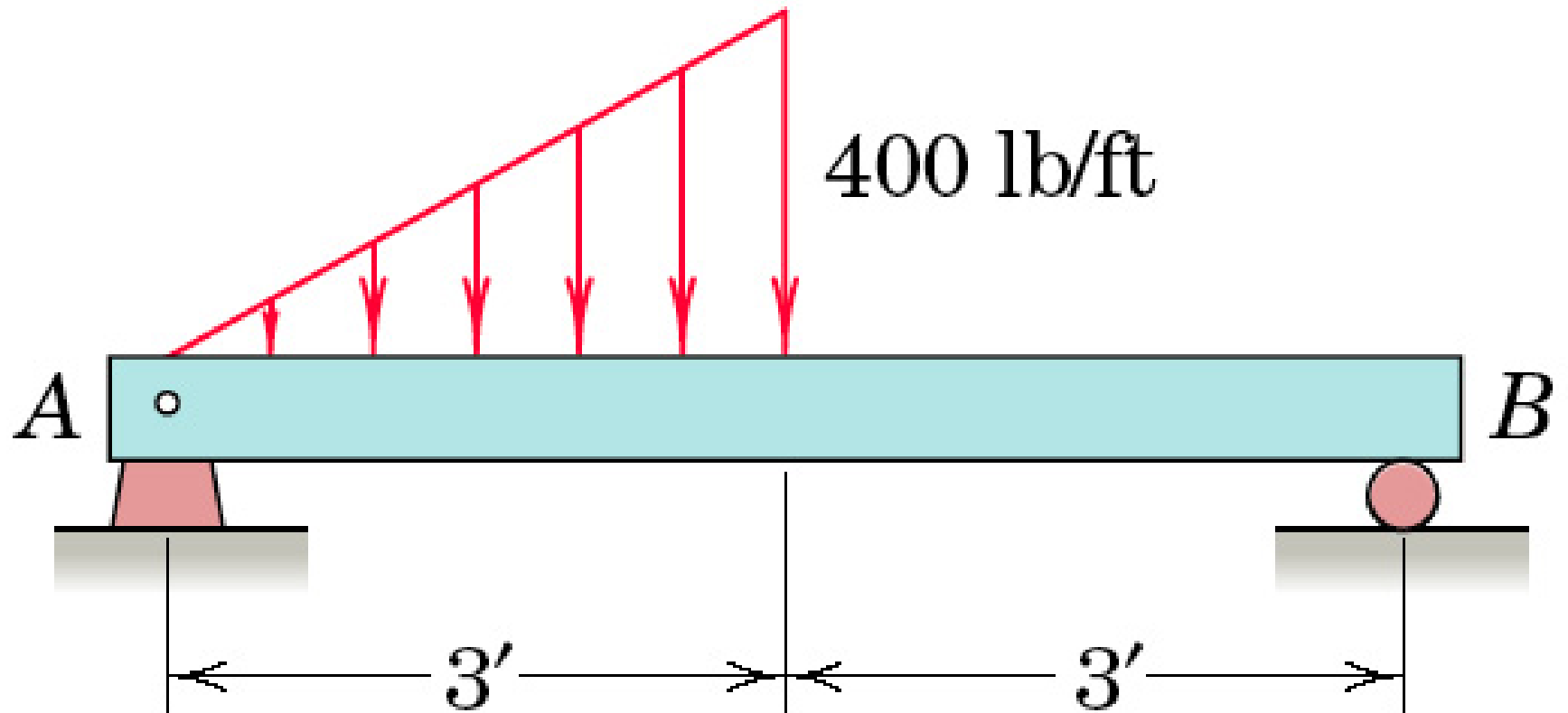


Problem 5/124



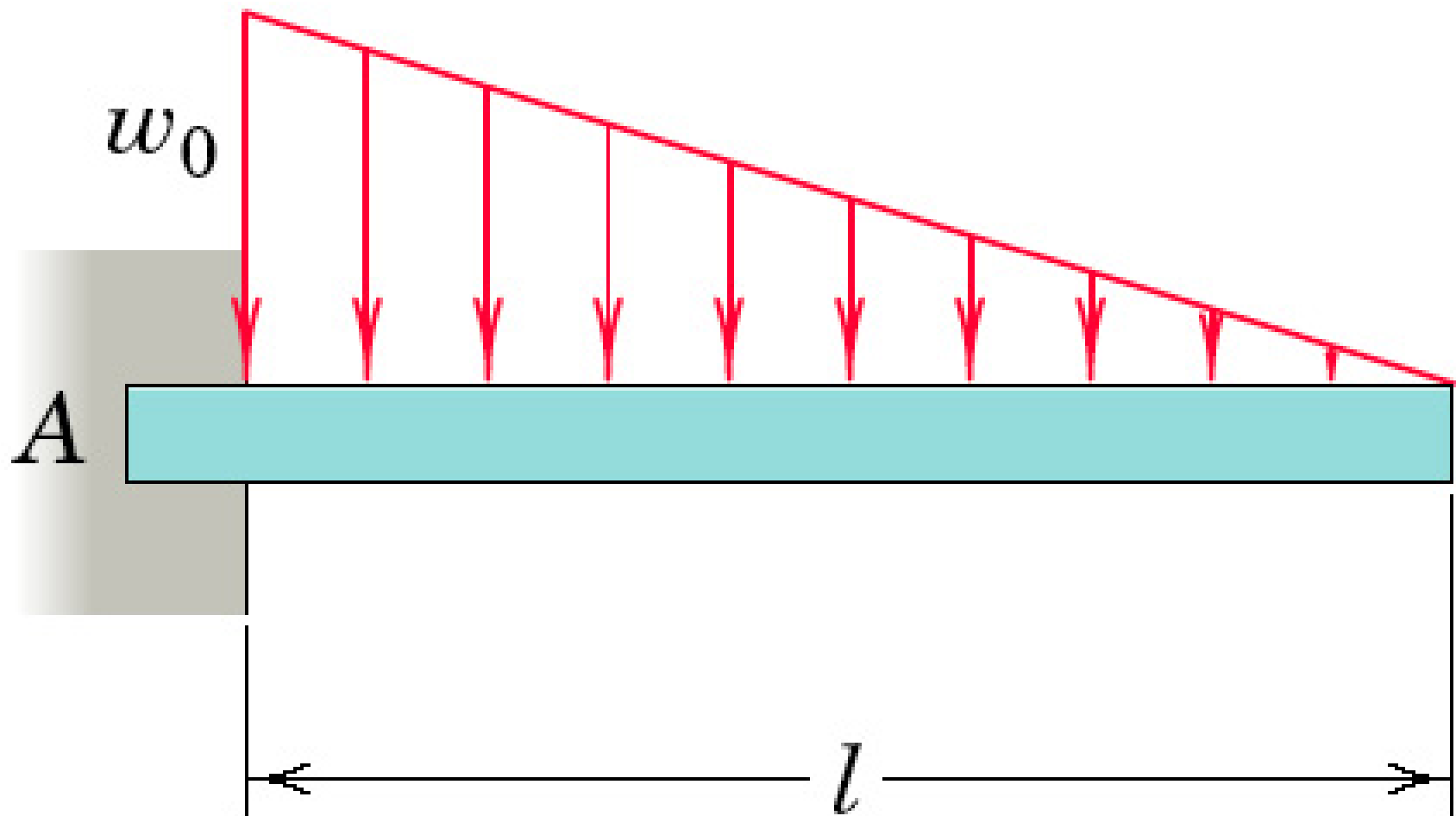
Problem 5/125

P_05_125

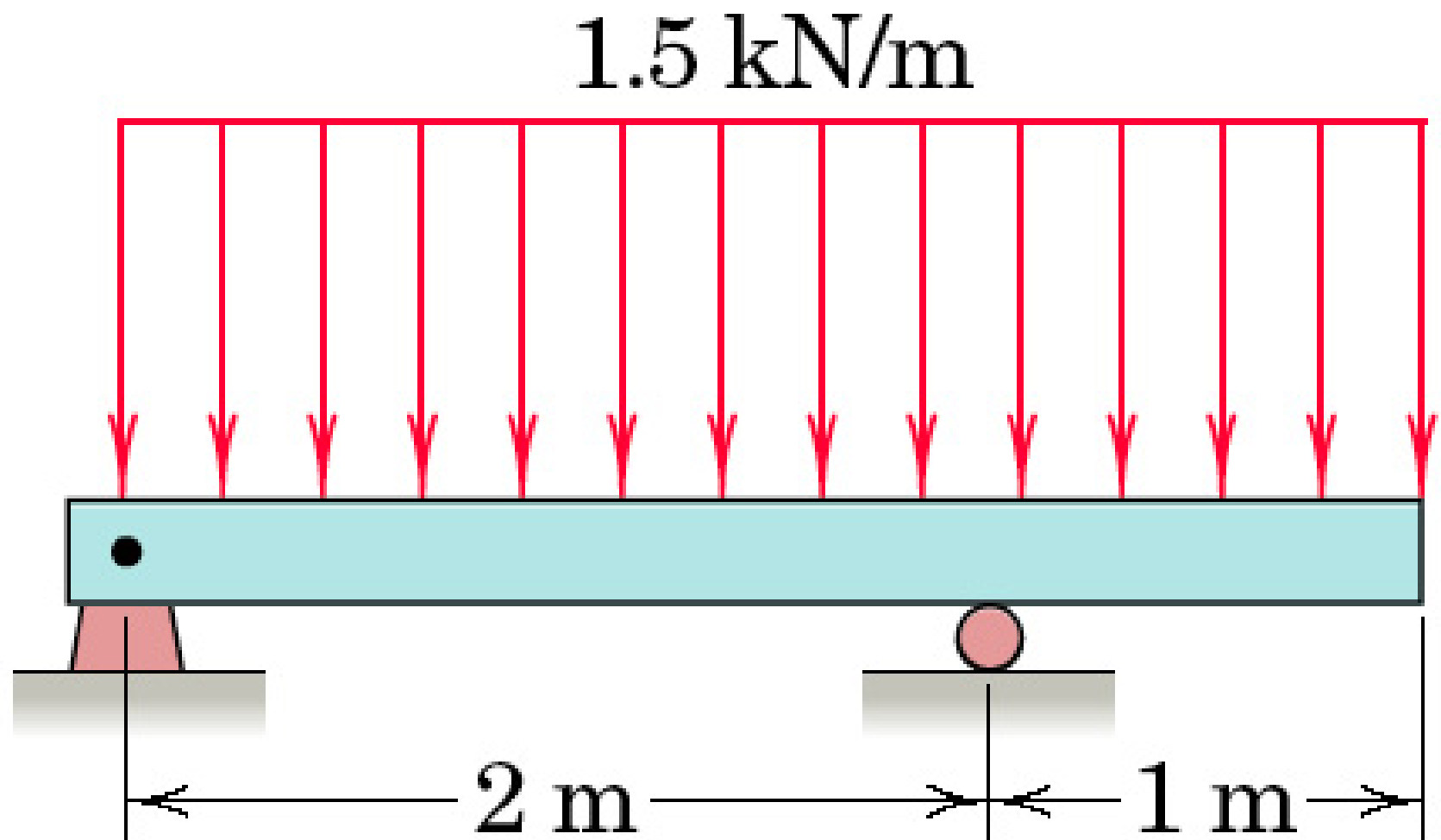


Problem 5/126

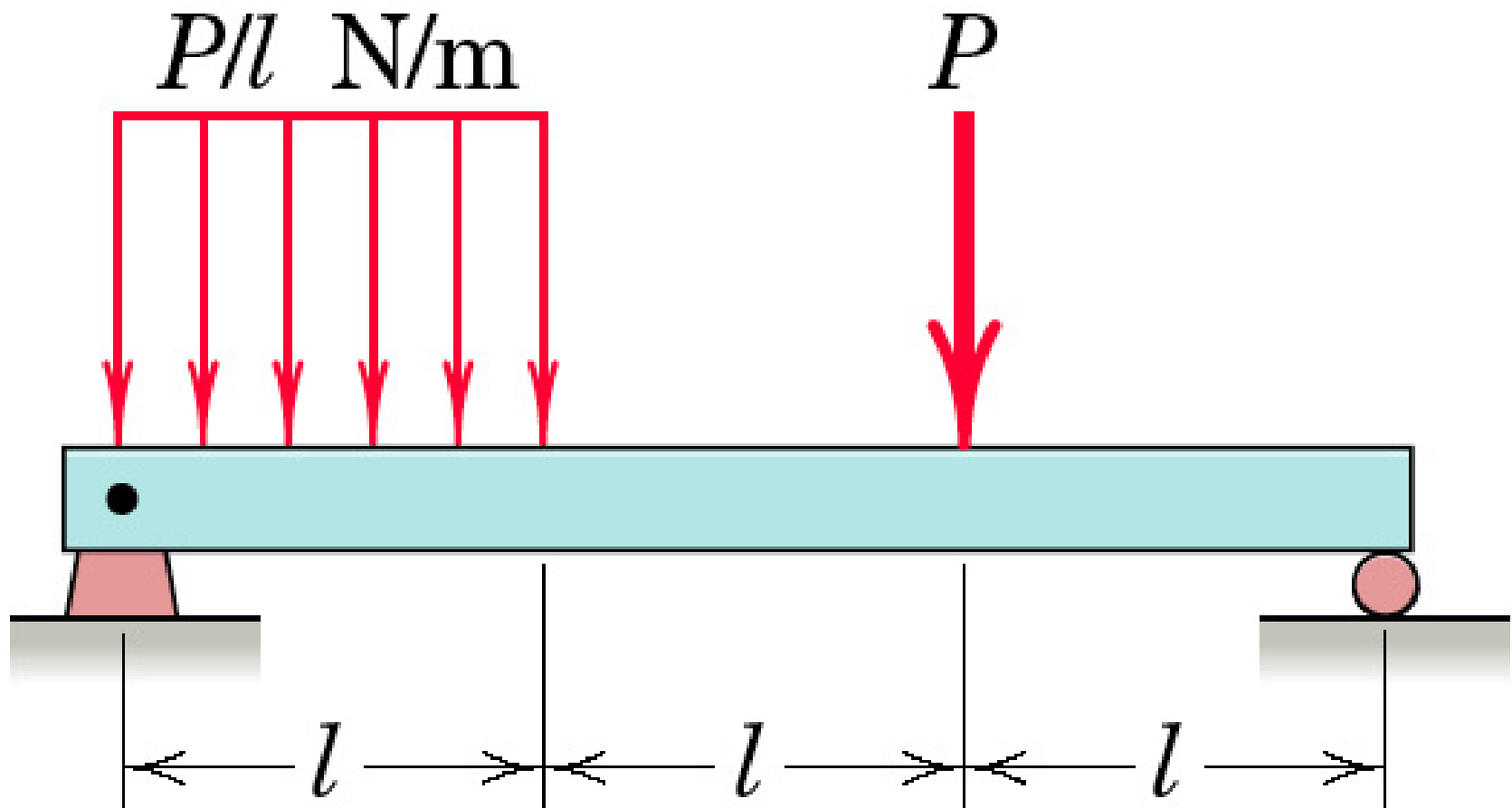
P_05_126



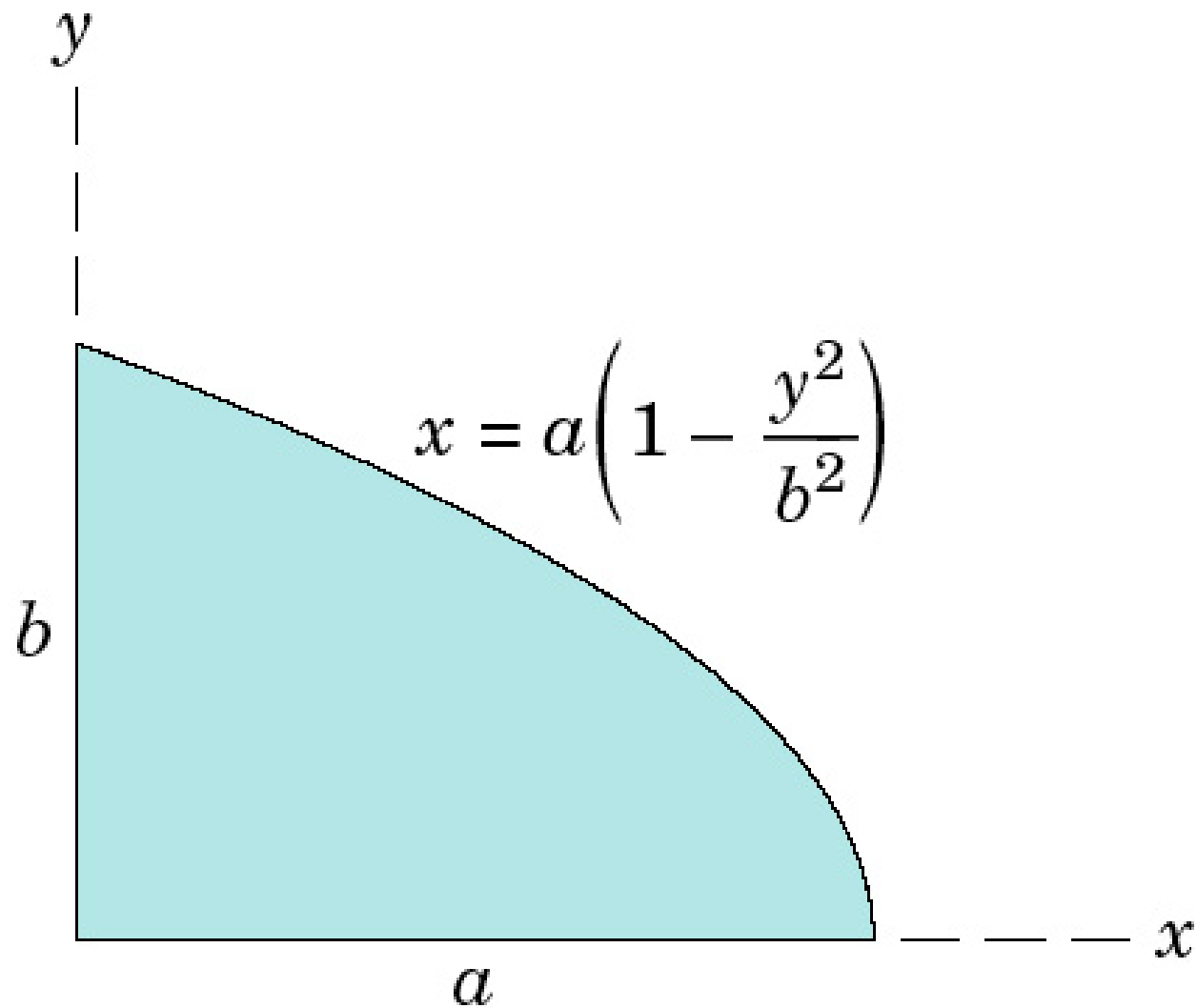
Problem 5/127



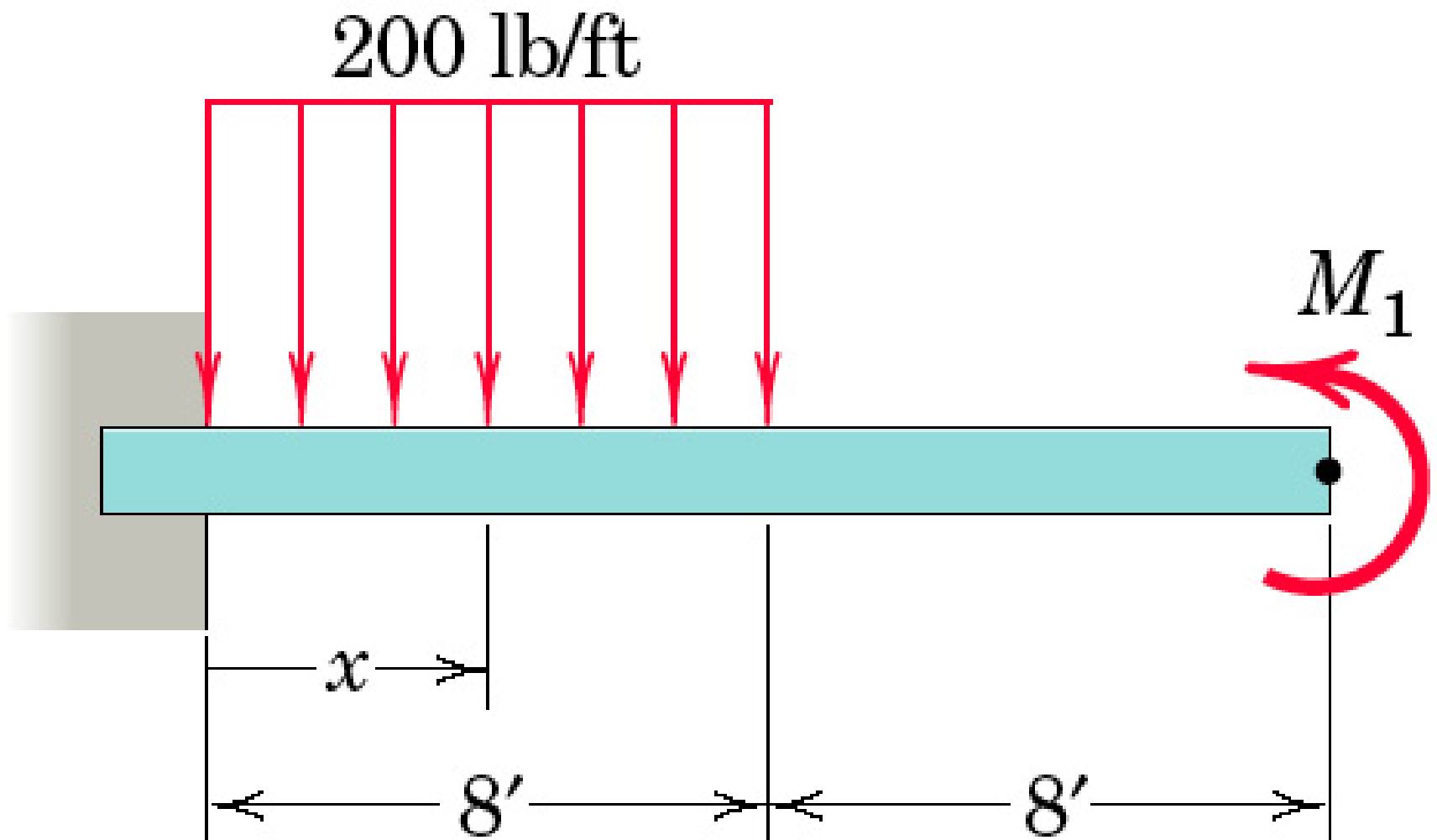
Problem 5/128



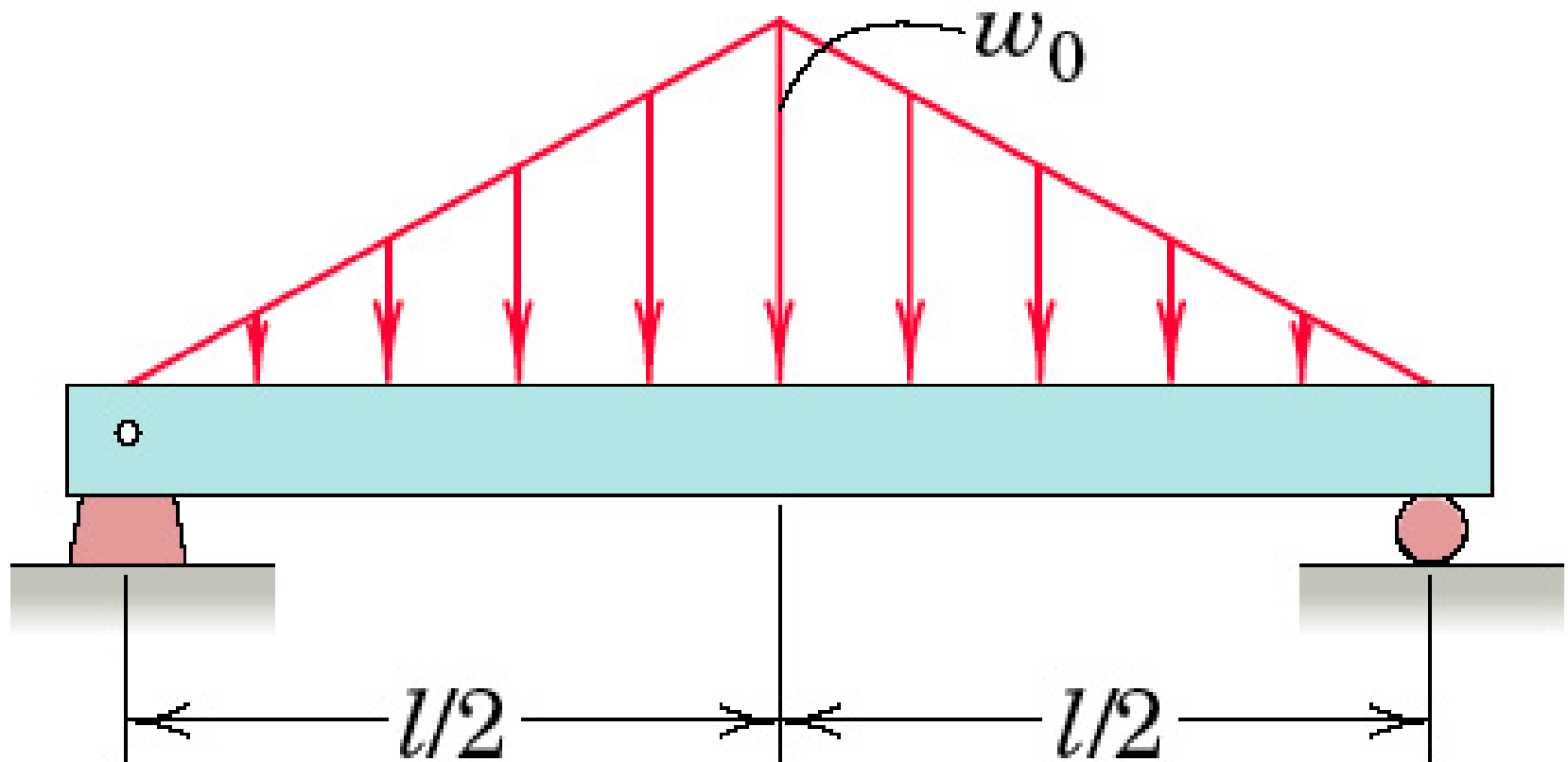
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Problem 5/13

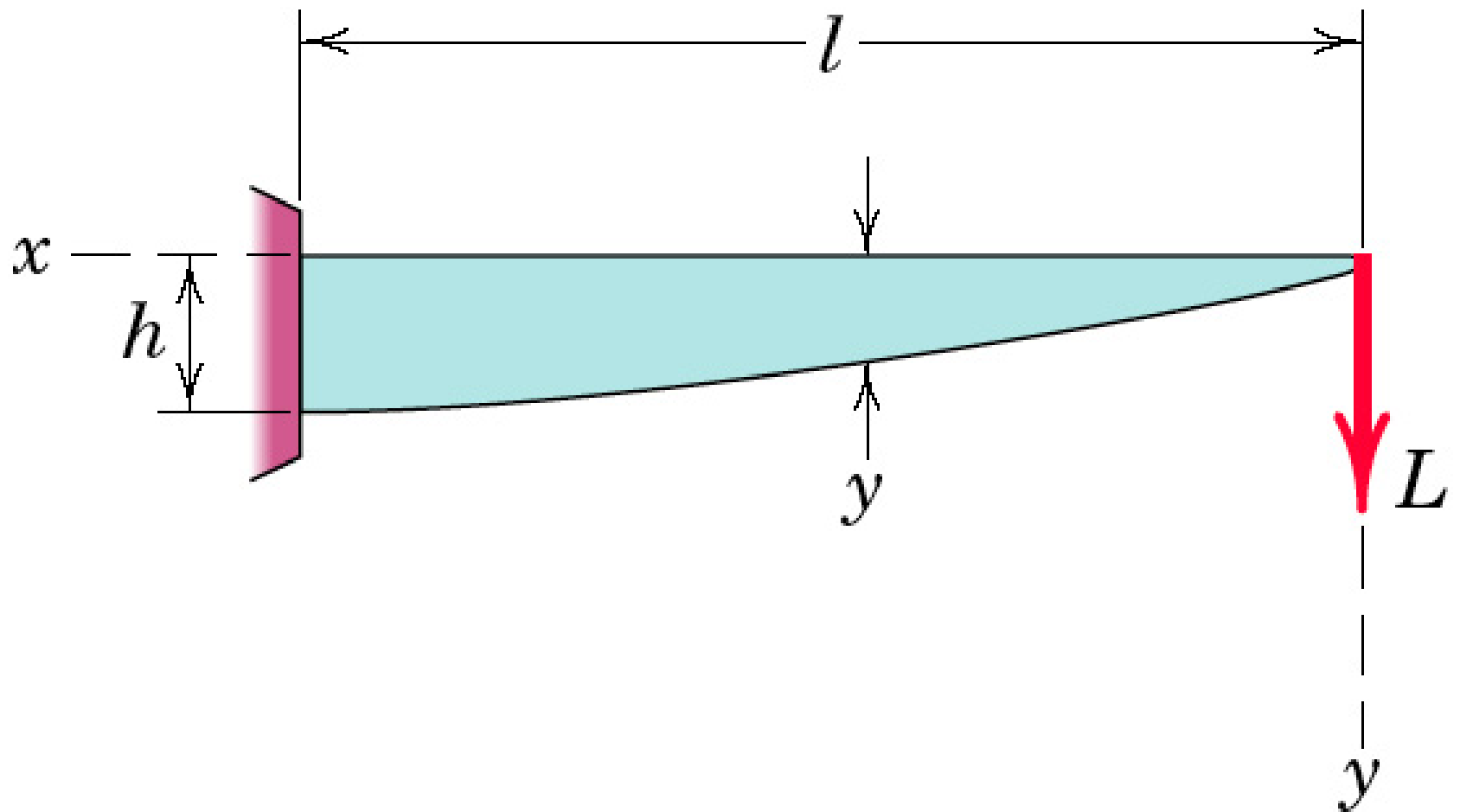


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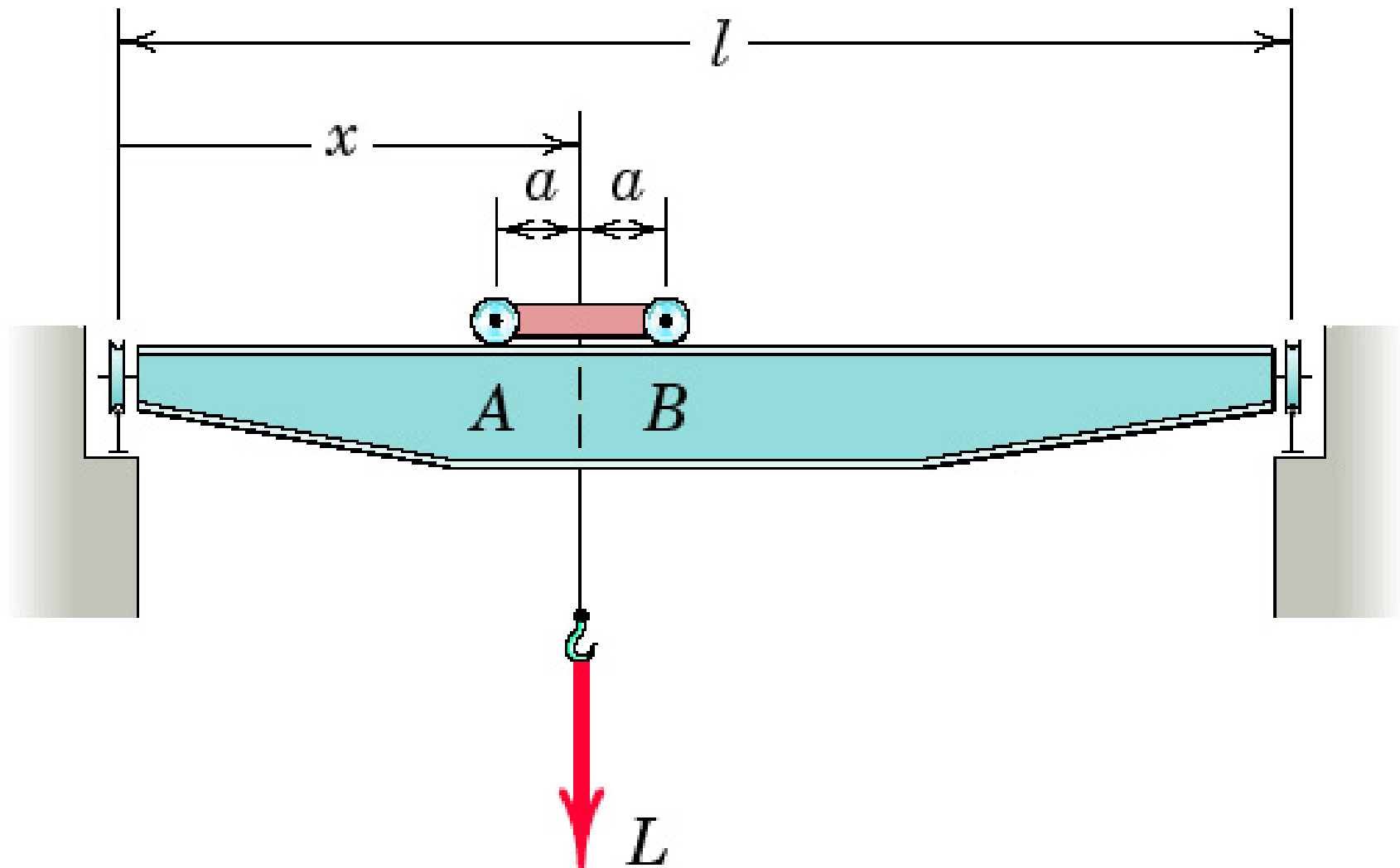


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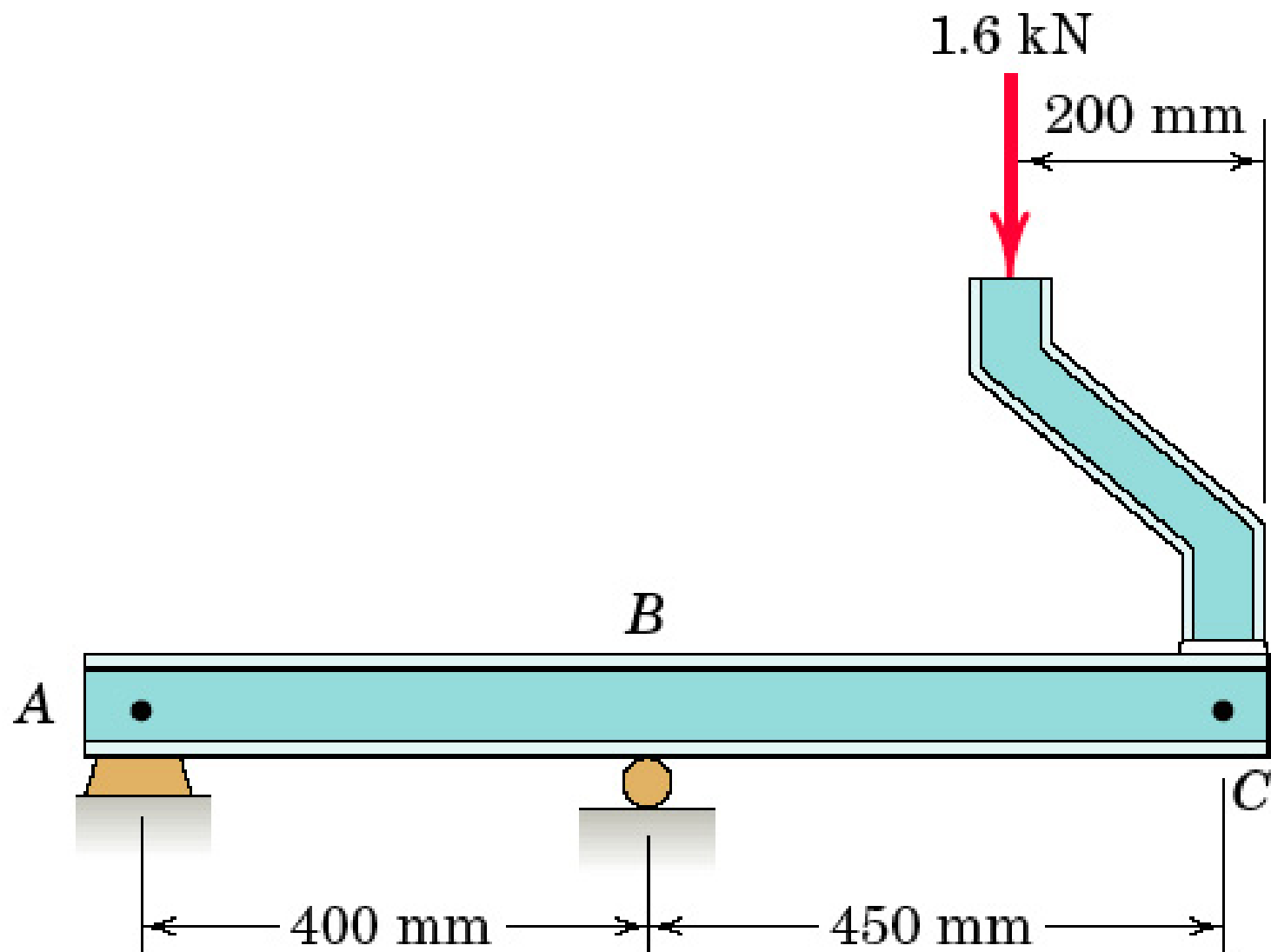
P_05_131



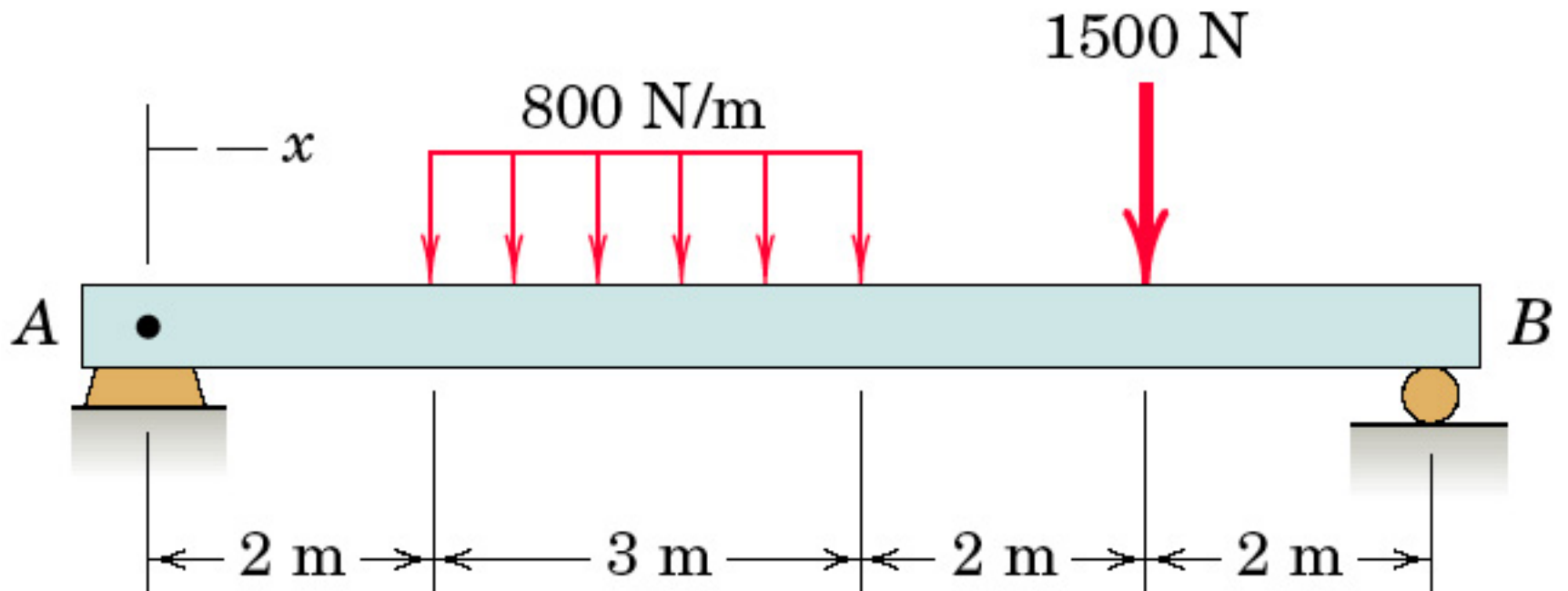
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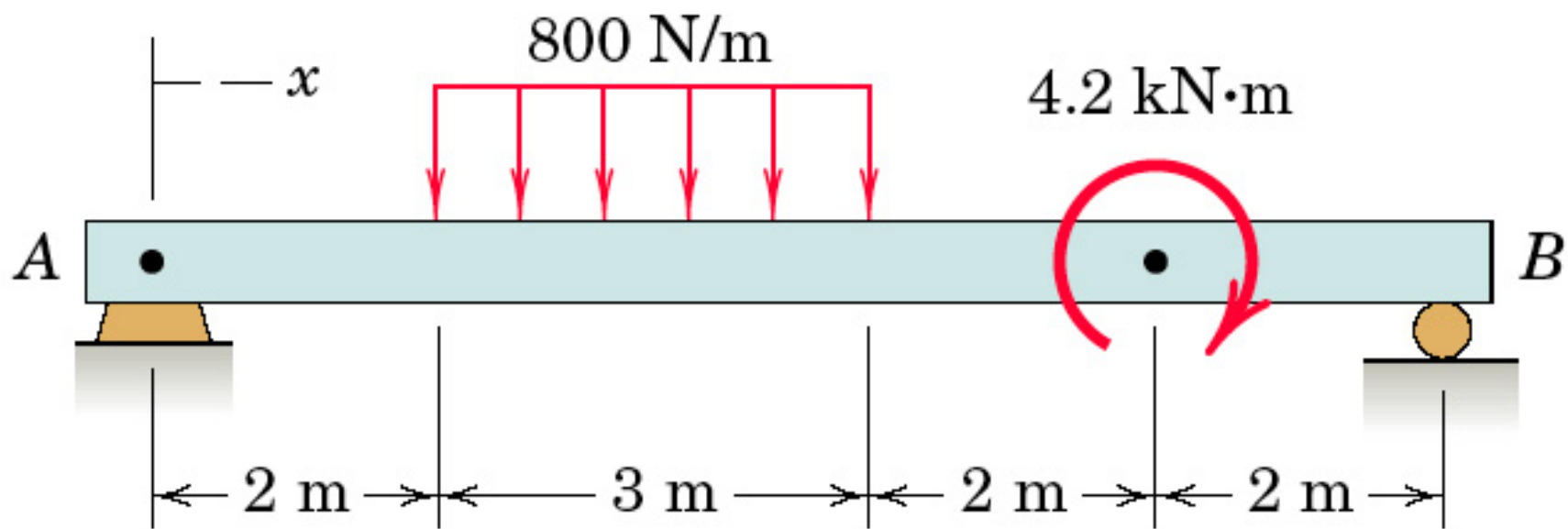


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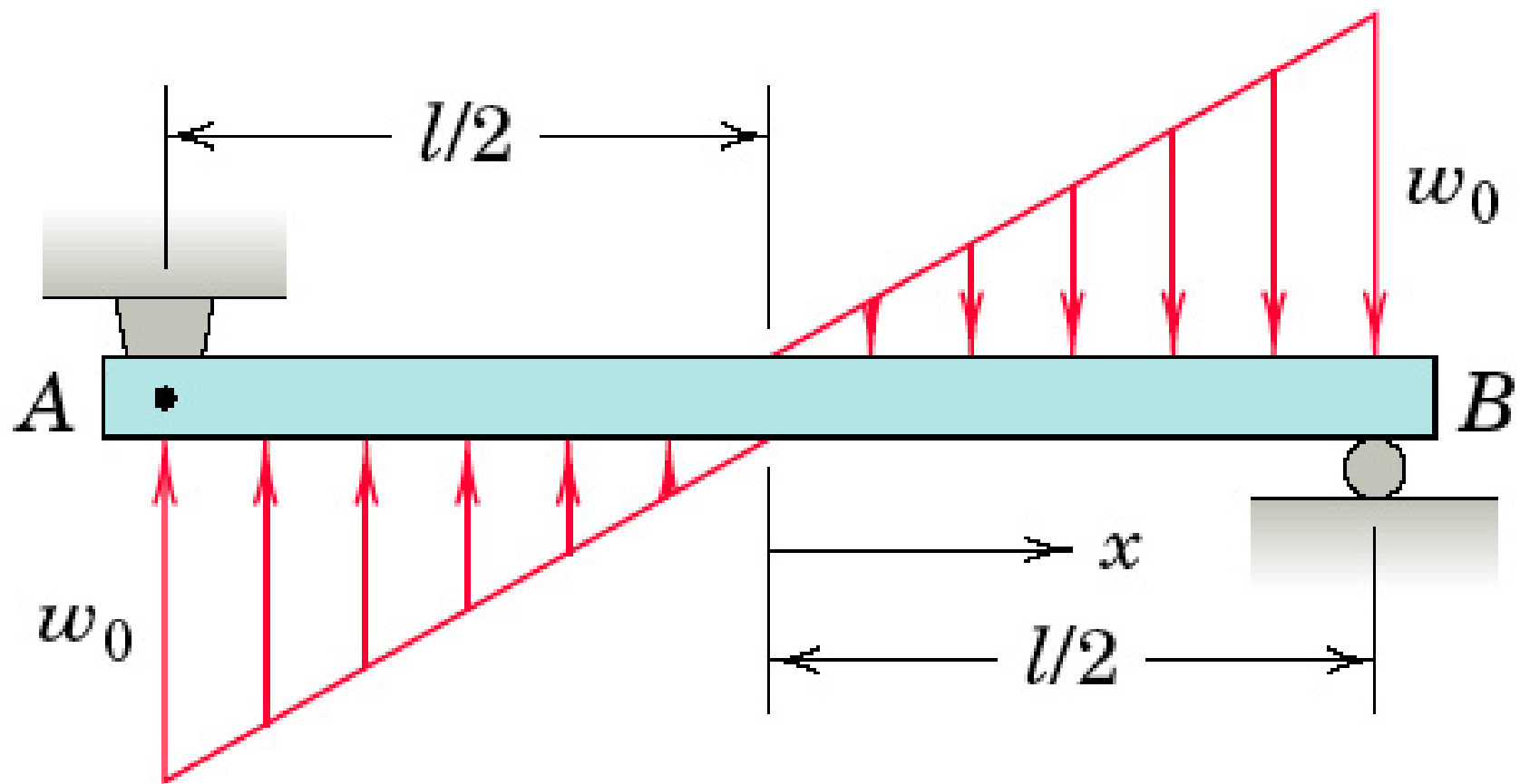
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P_05_136

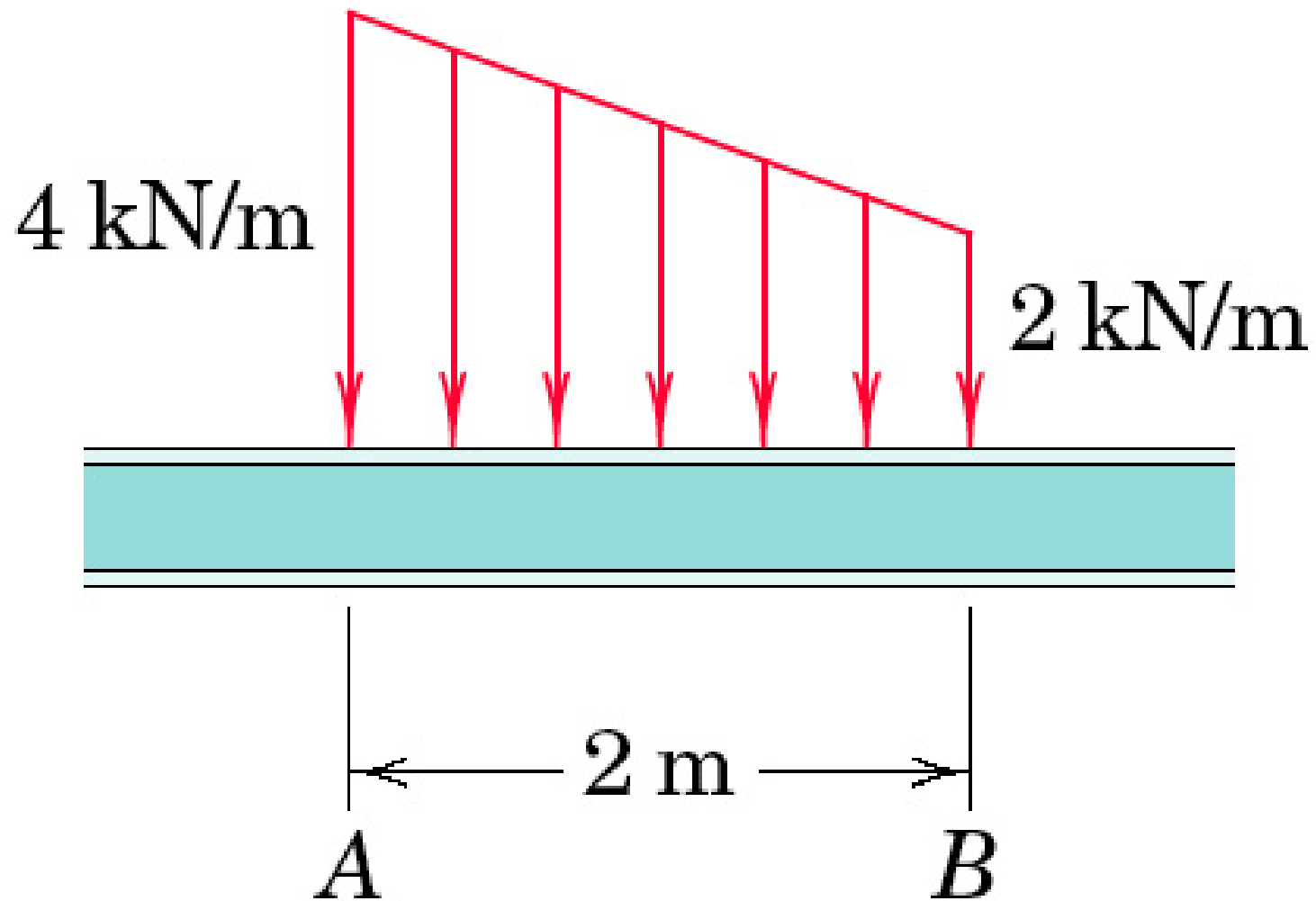


Problem 5/137

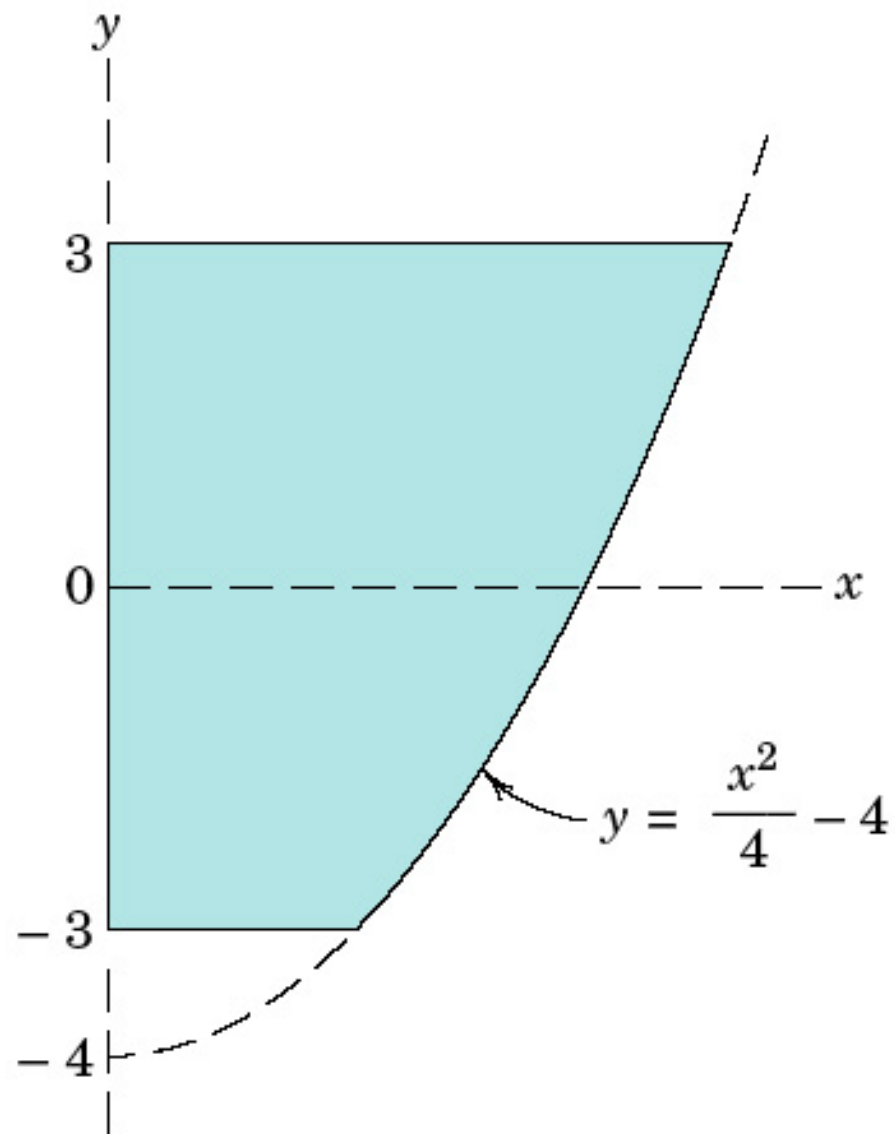
P_05_137



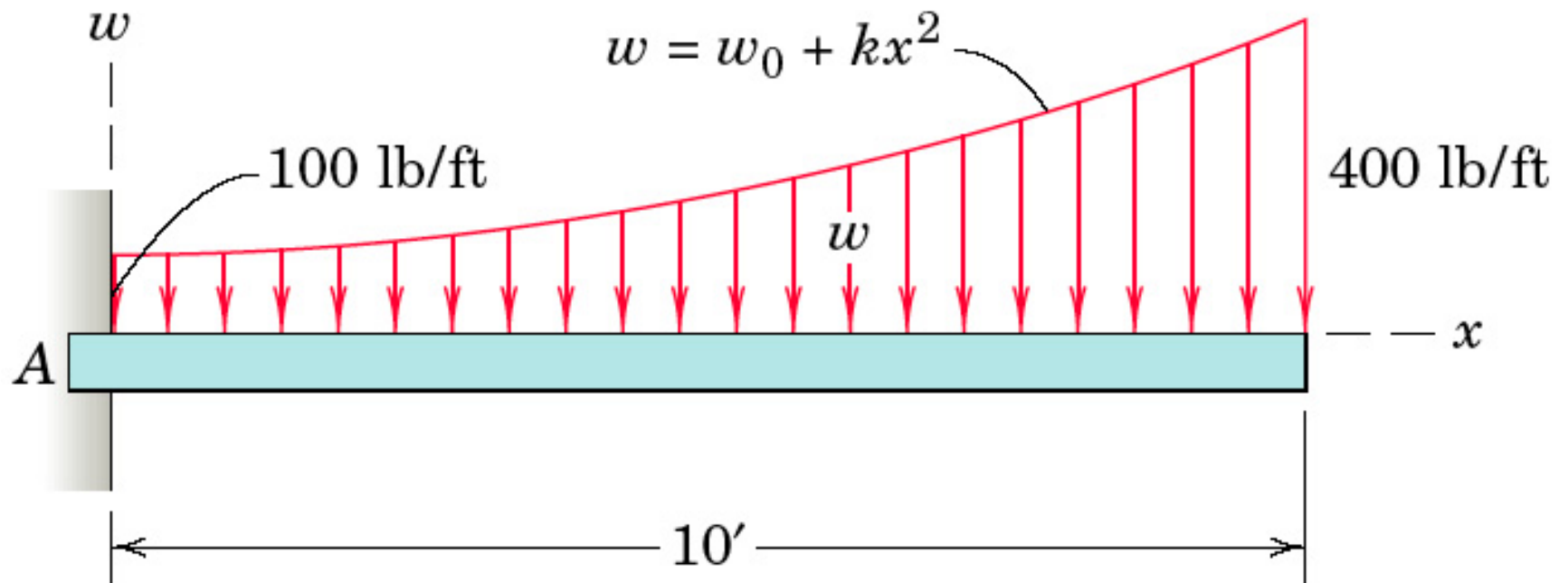
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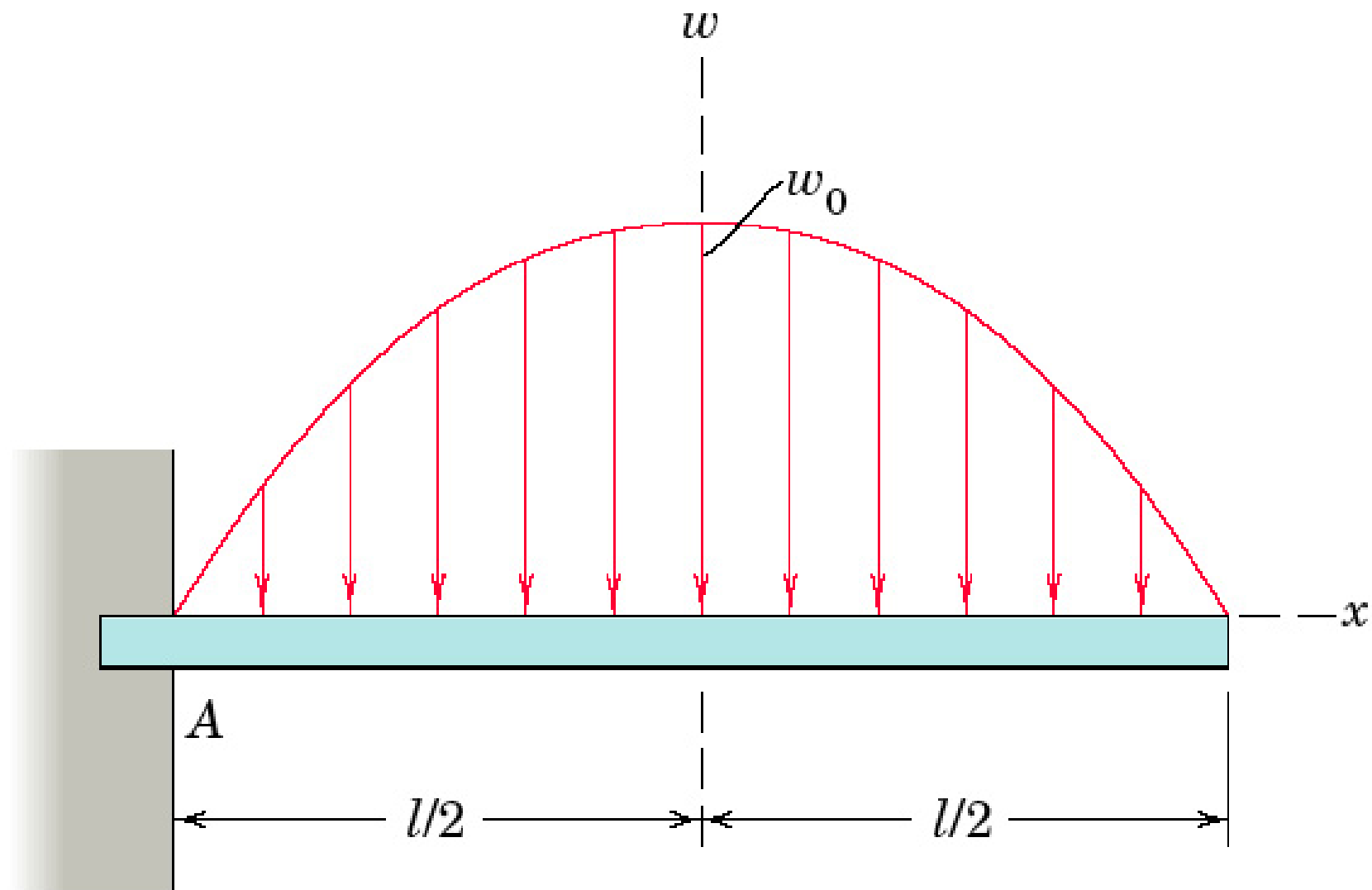


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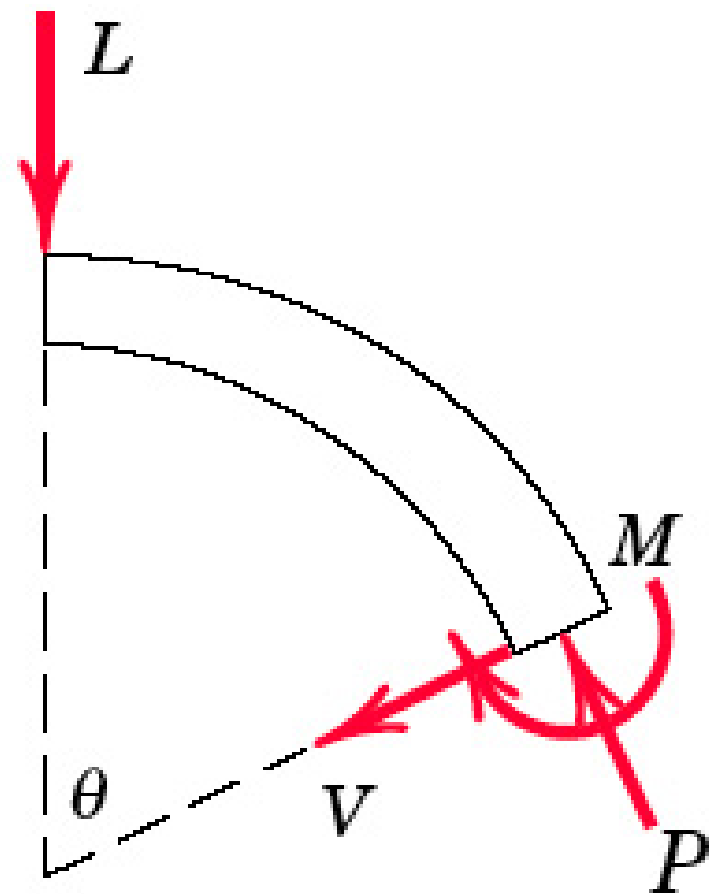
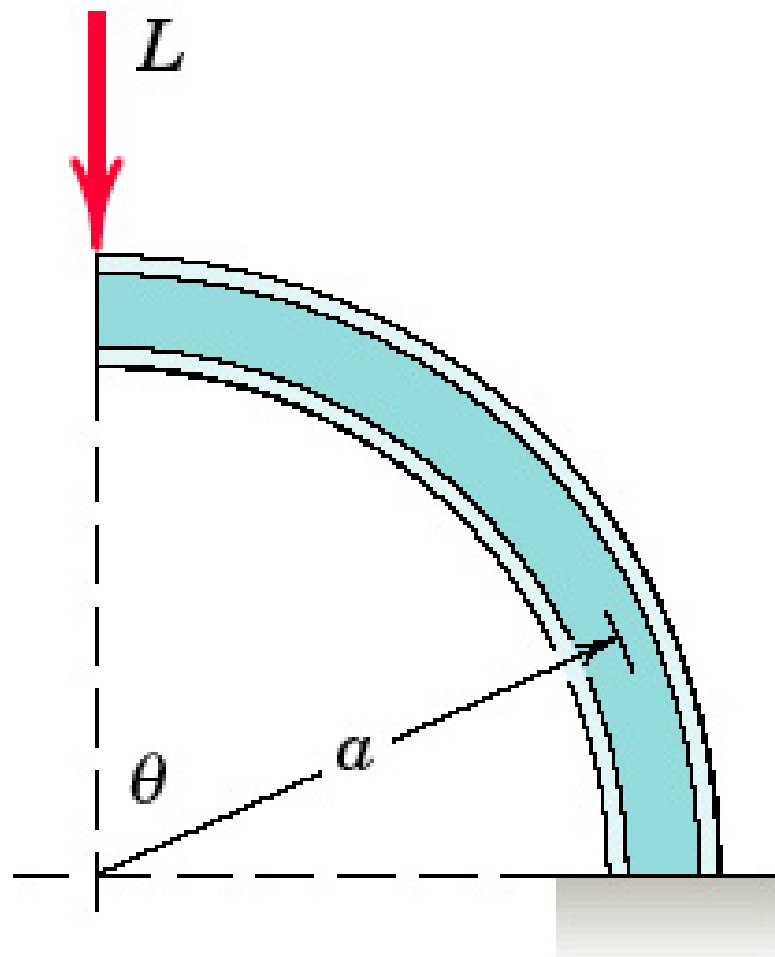


Problem 5/140

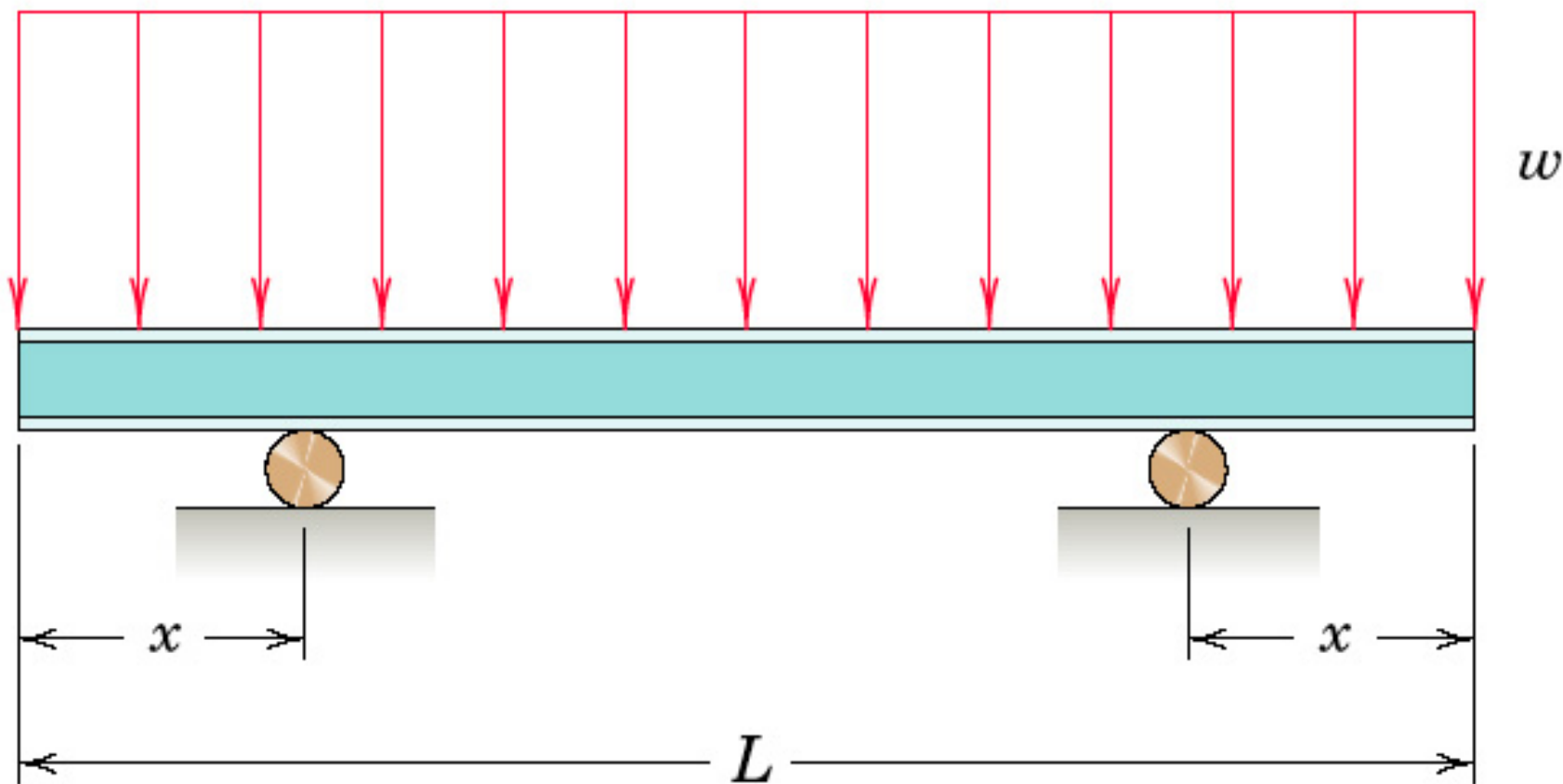
P_05_140



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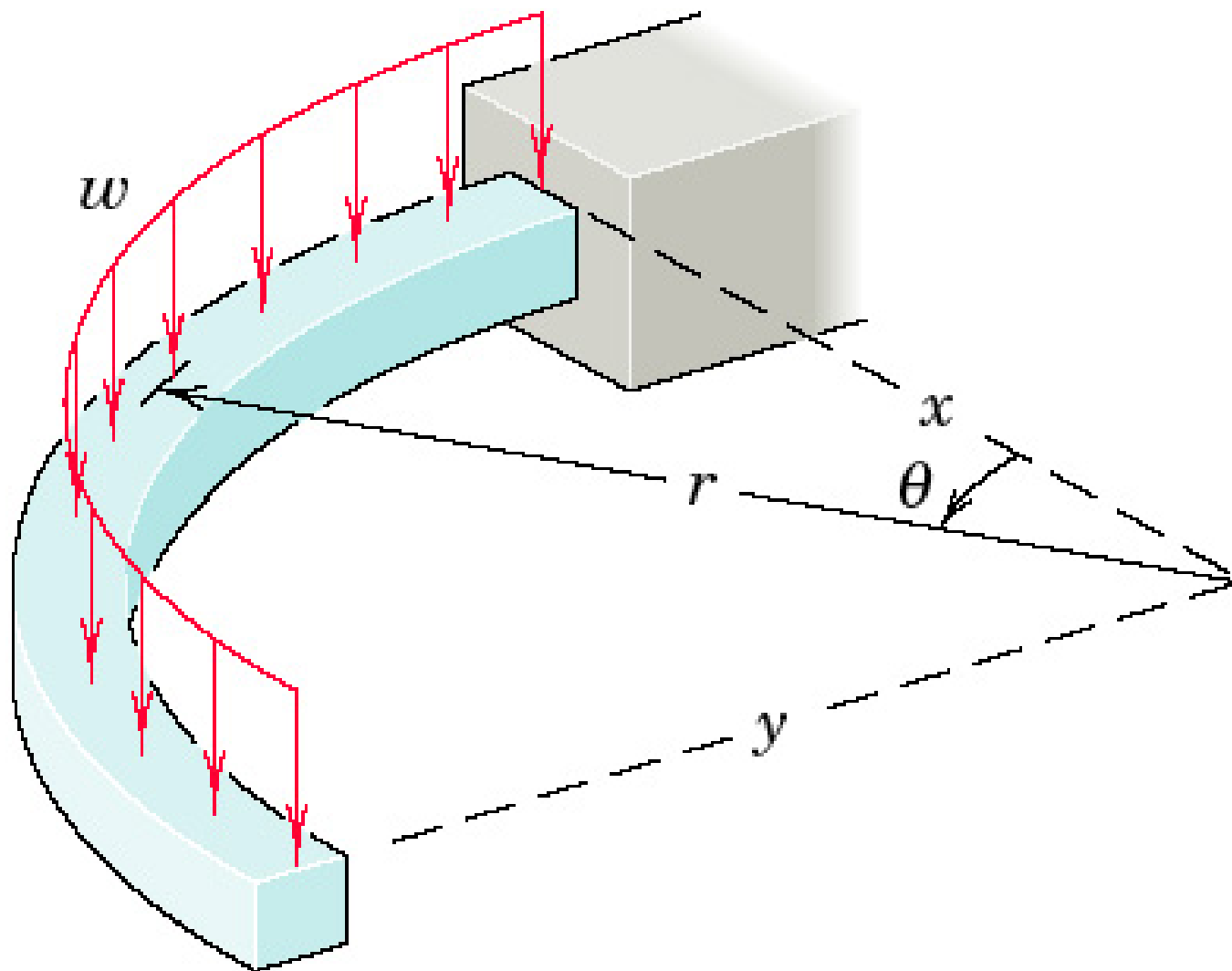


Problem 5/142

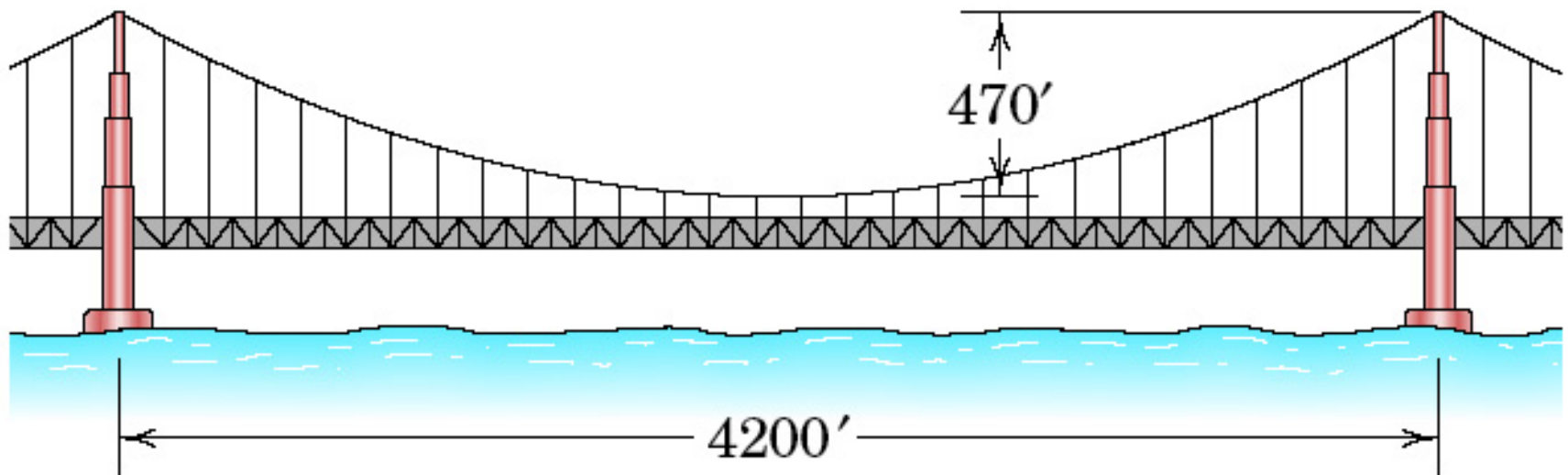


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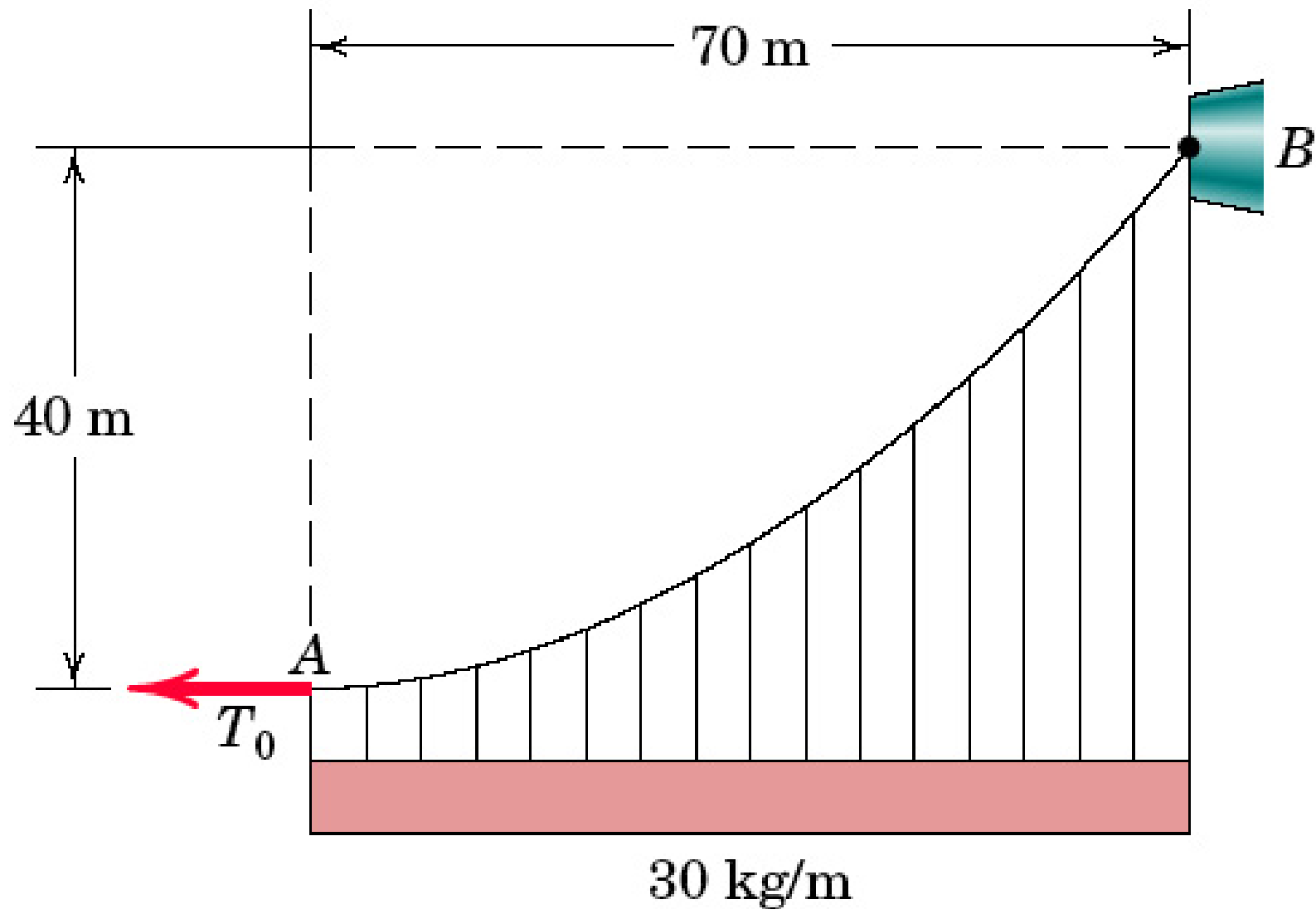


Problem 5/144

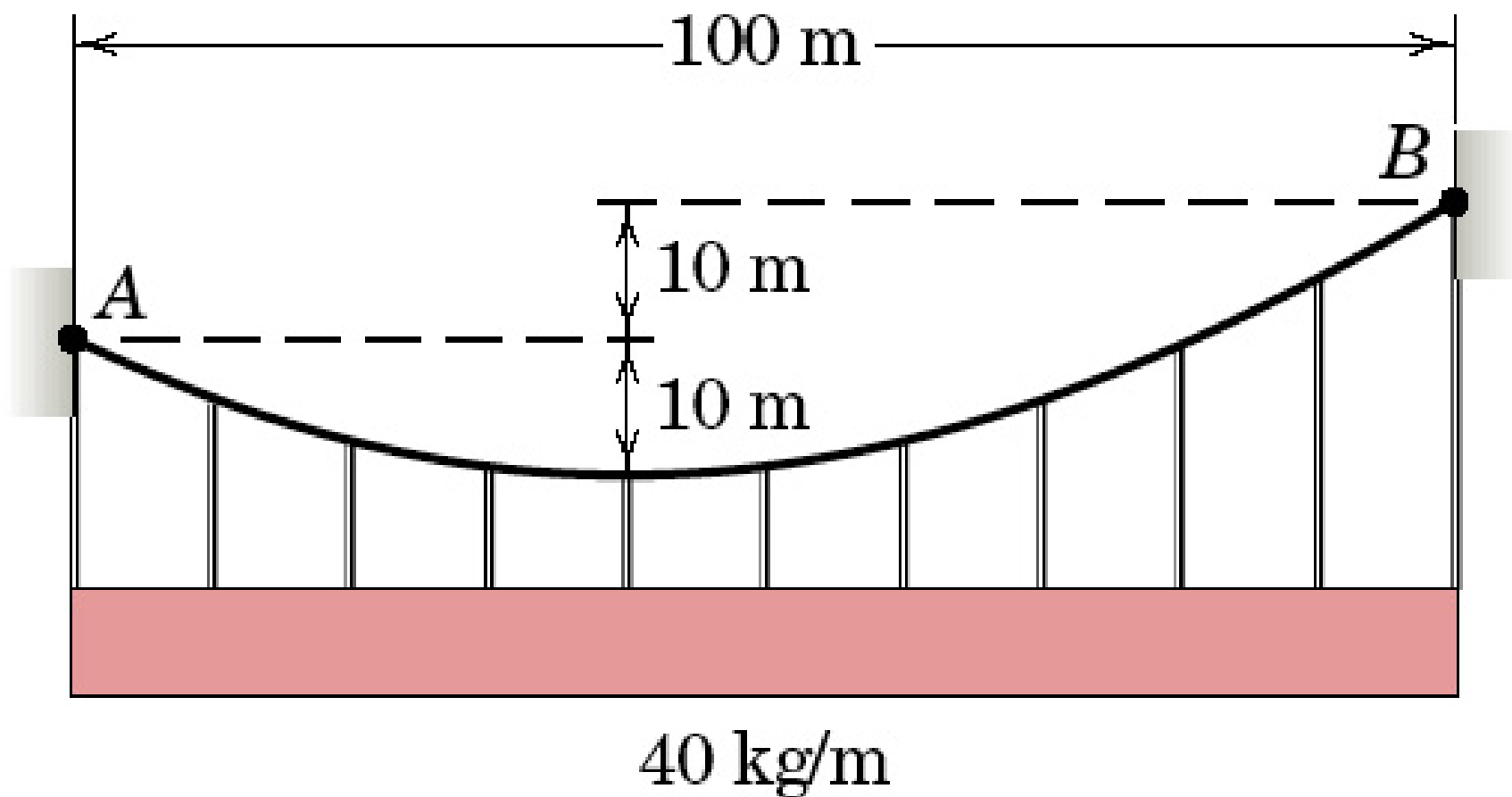


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P_05_146

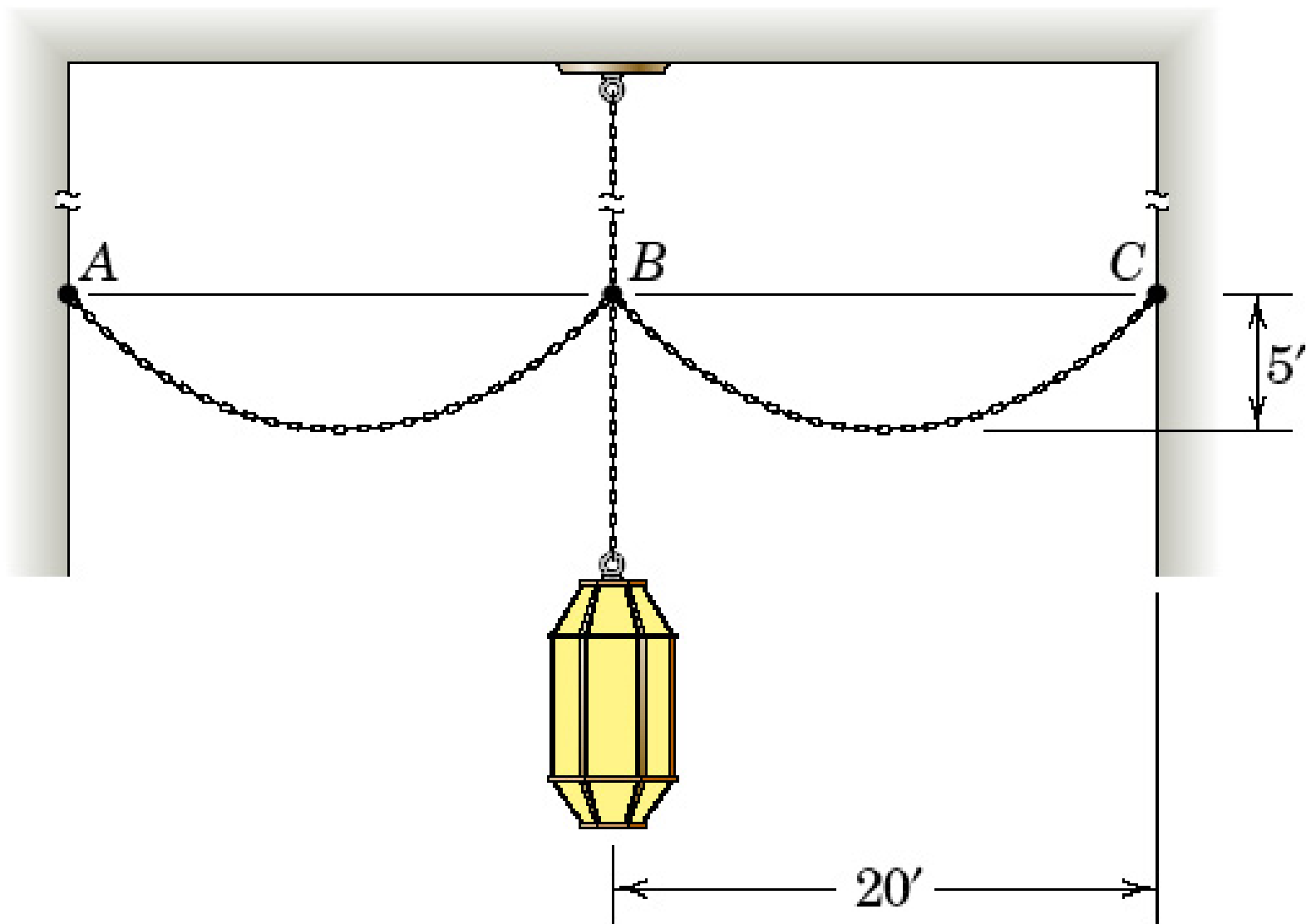


Problem 5/147

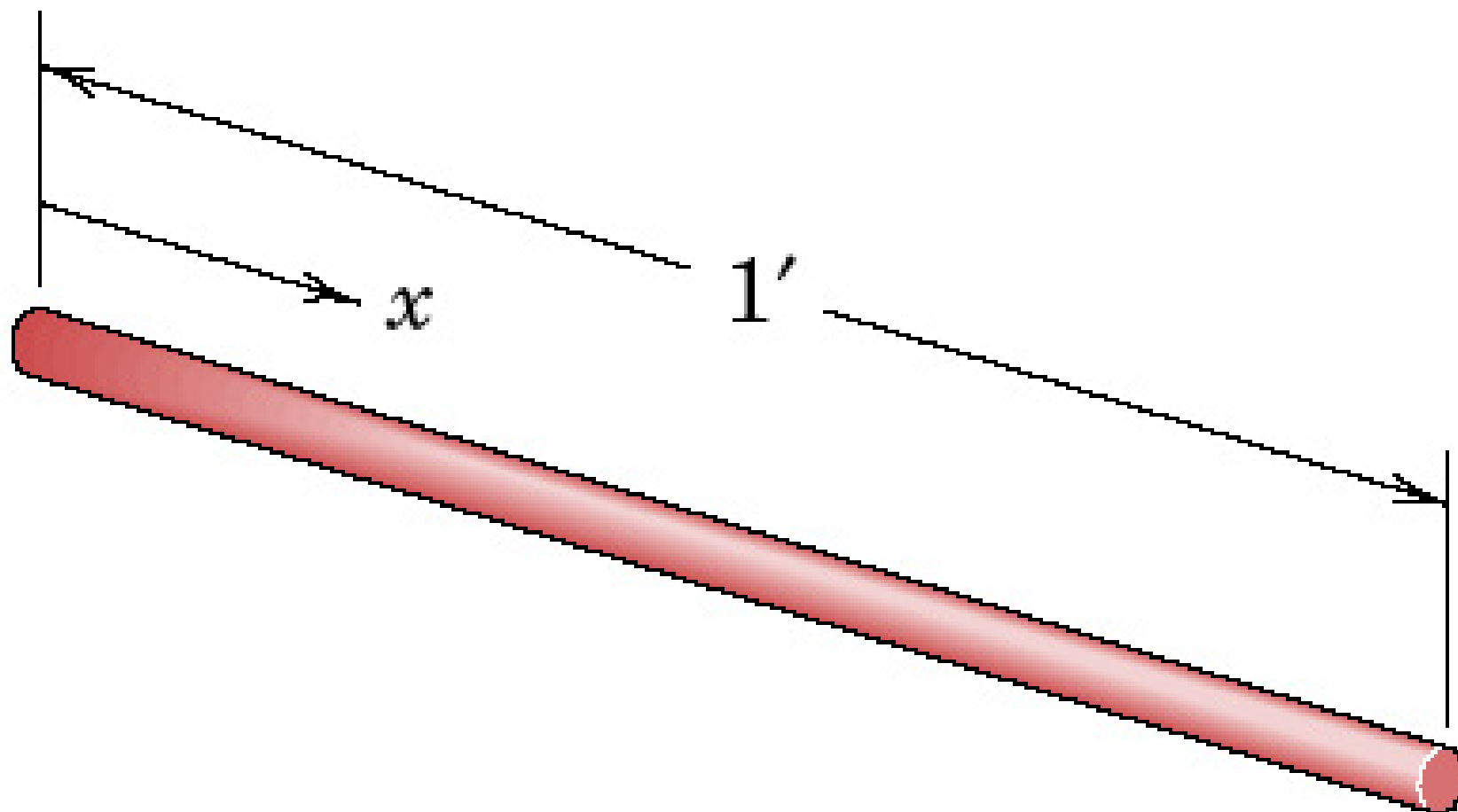


Problem 5/148

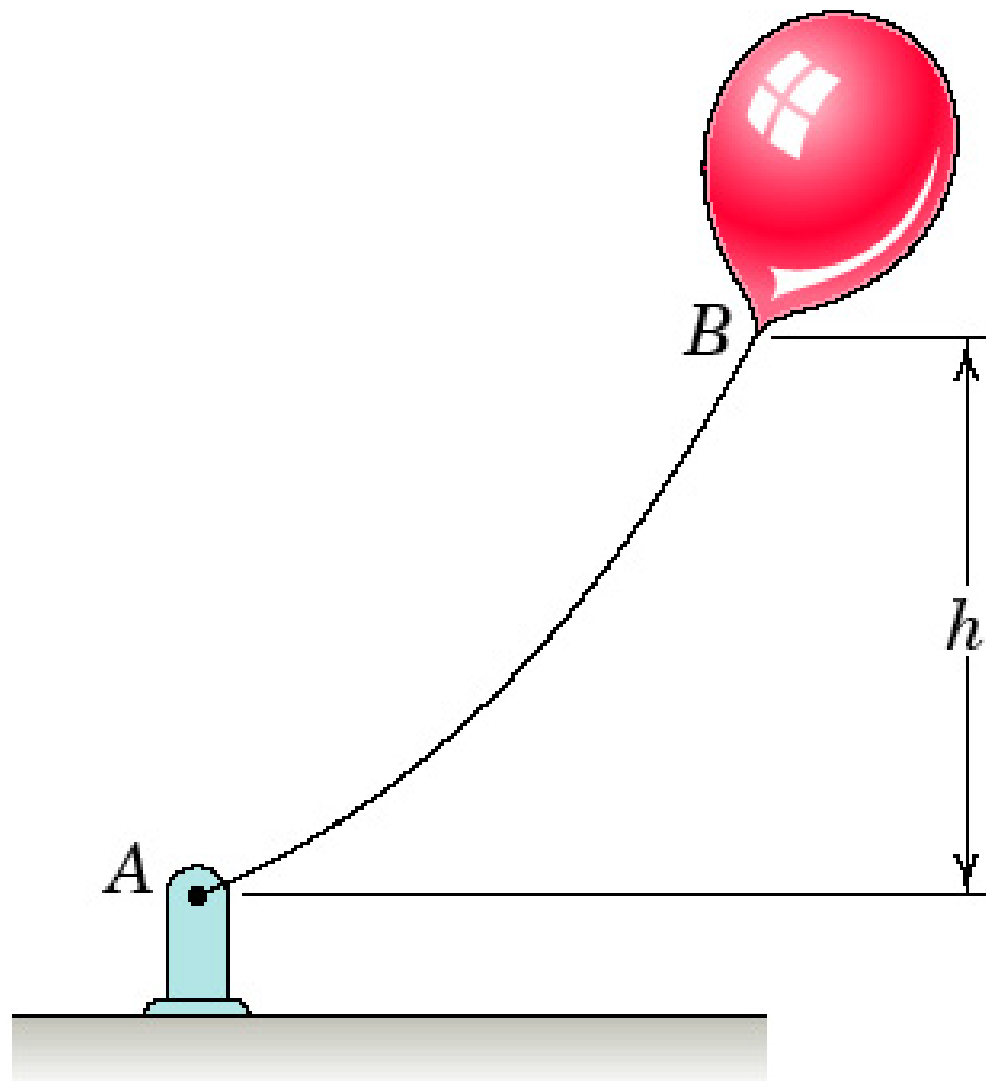
P_05_148



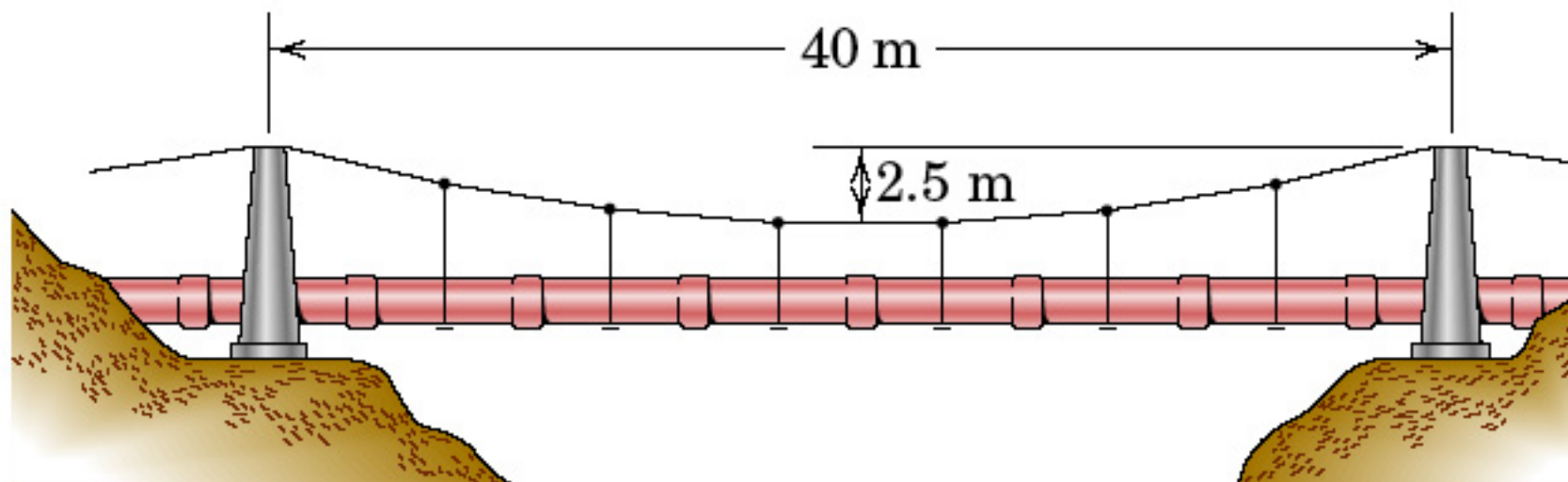
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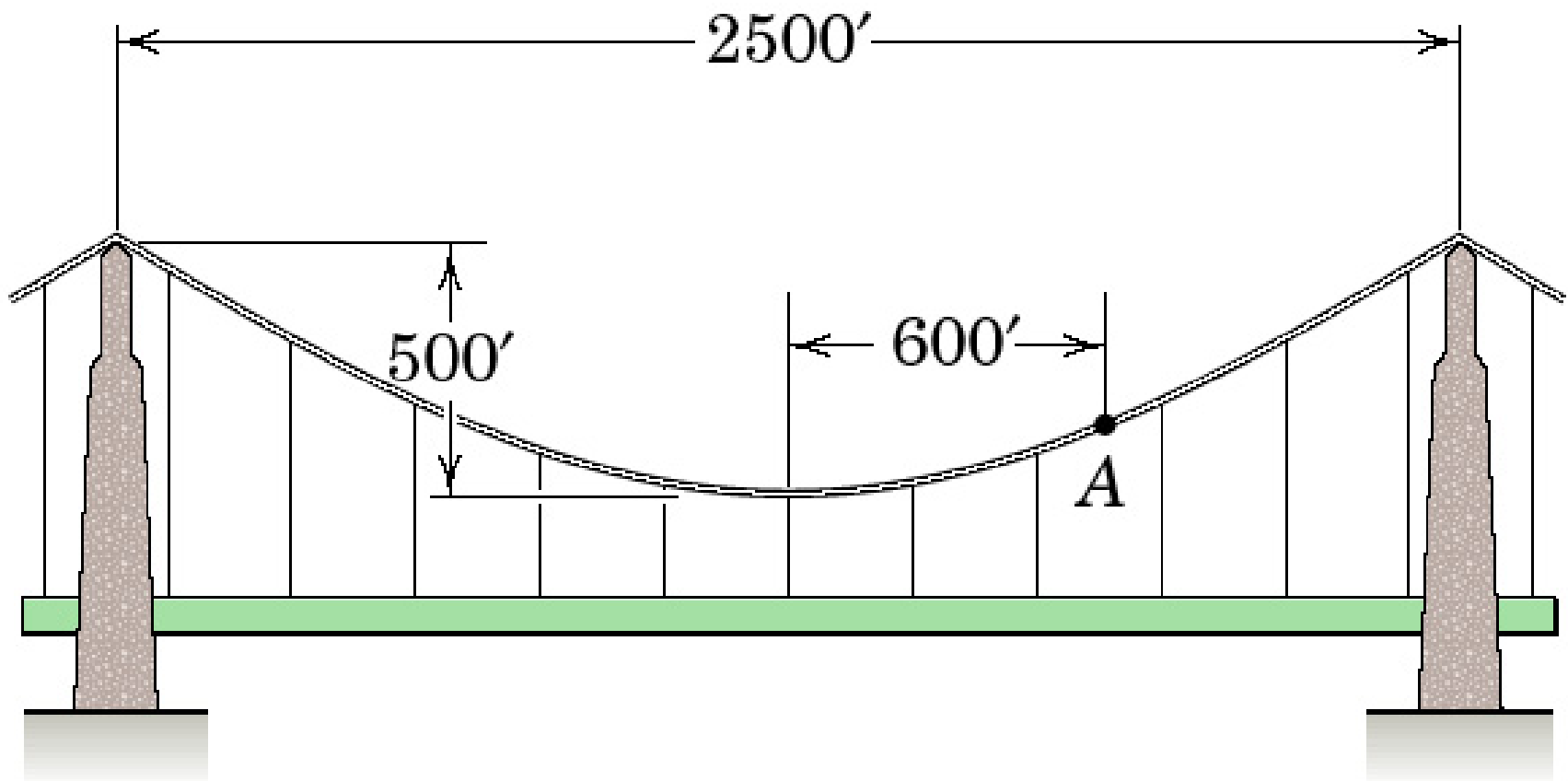


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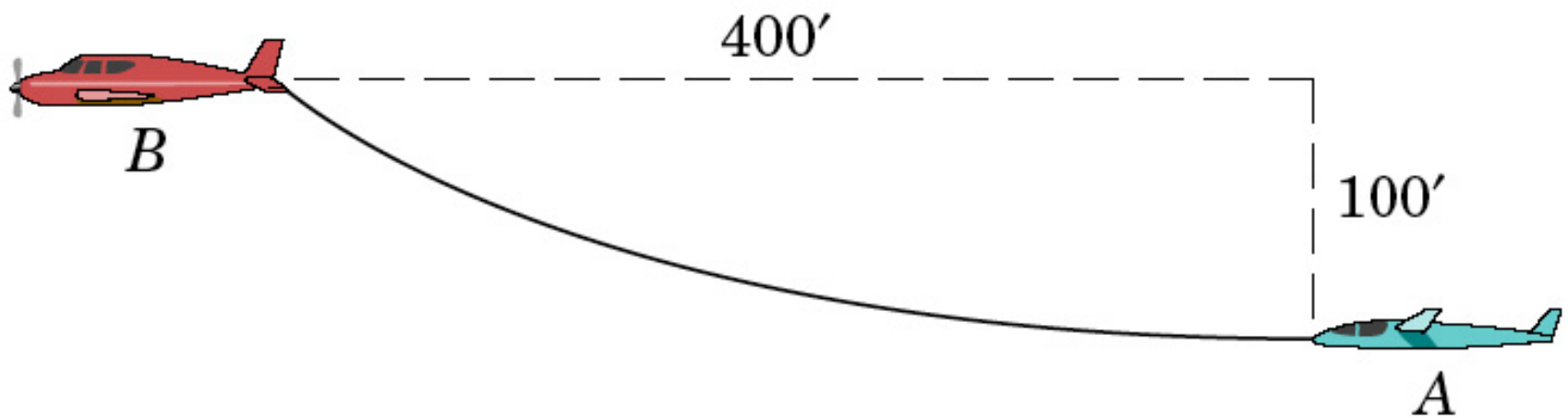
Problem 5/151

P_05_151



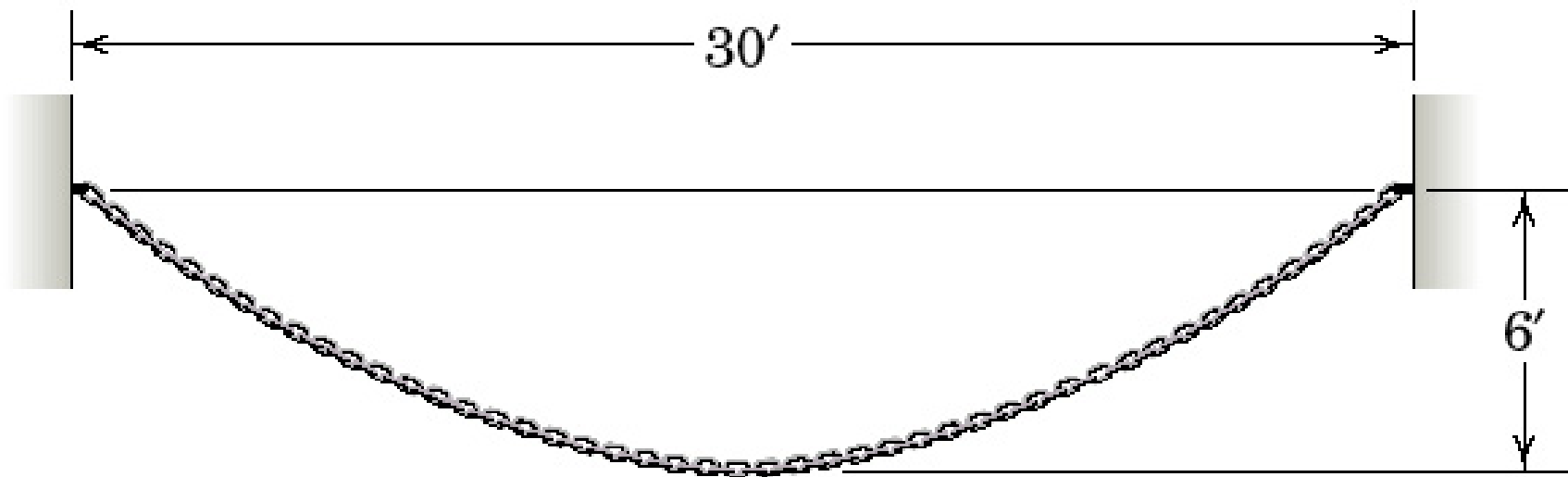
Problem 5/152

P_05_152



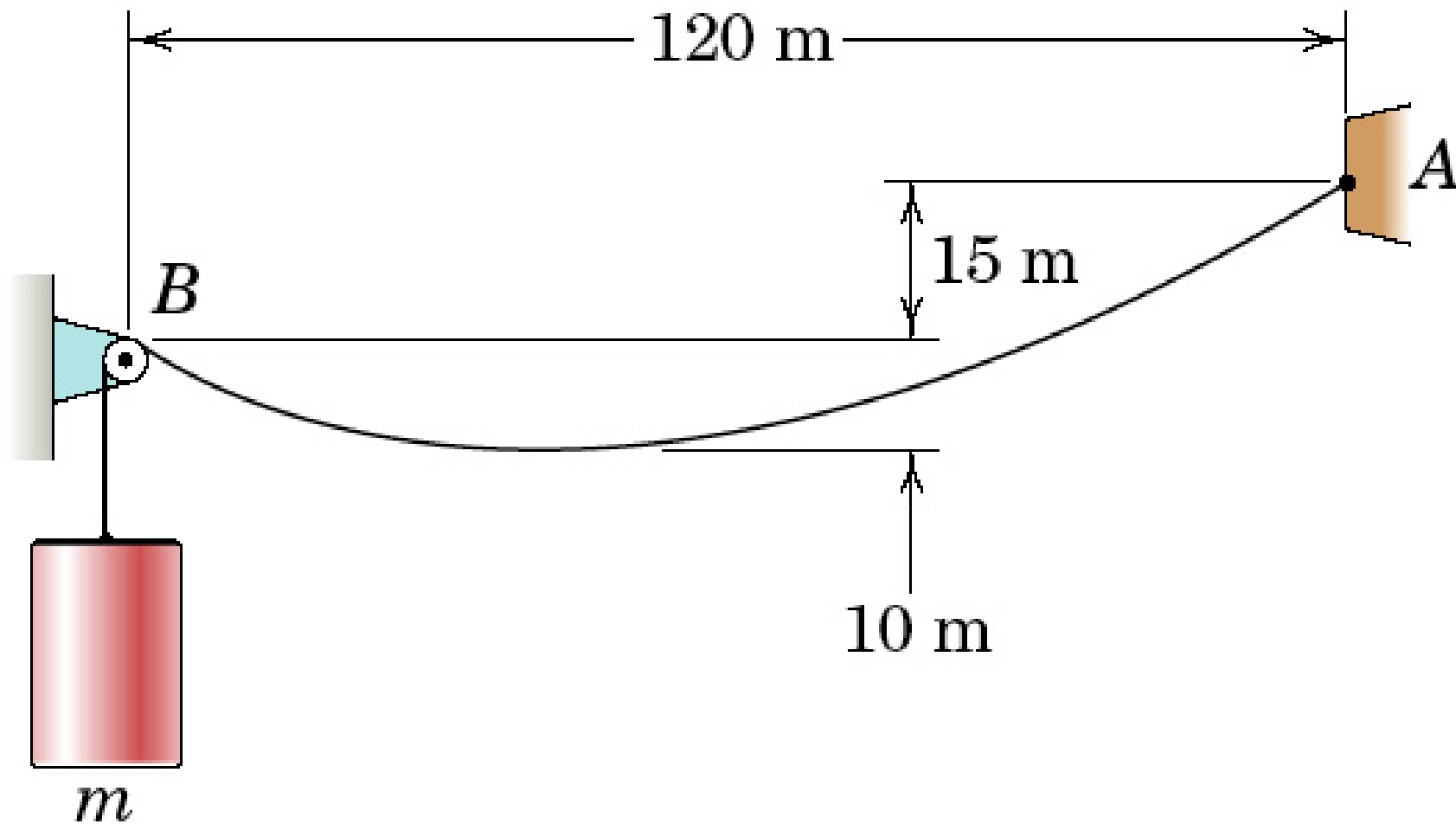
Problem 5/153

P_05_153



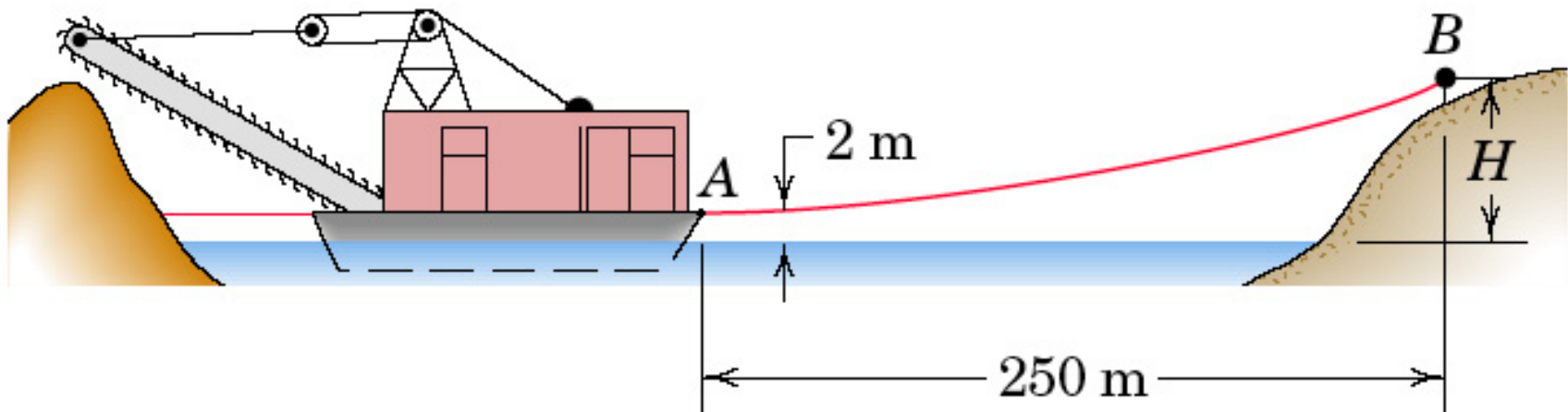
Problem 5/154

P_05_154



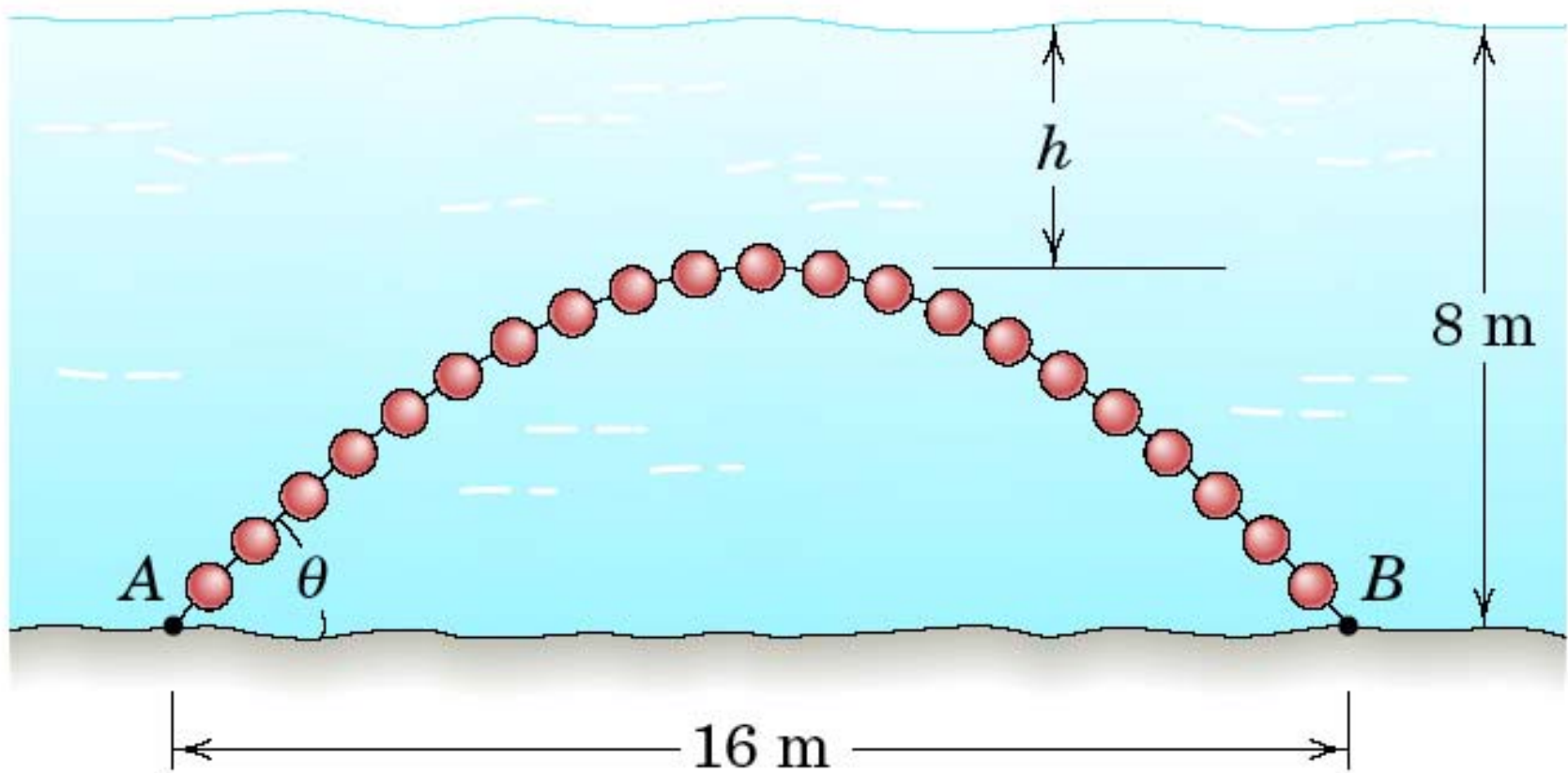
Problem 5/155

P_05_155



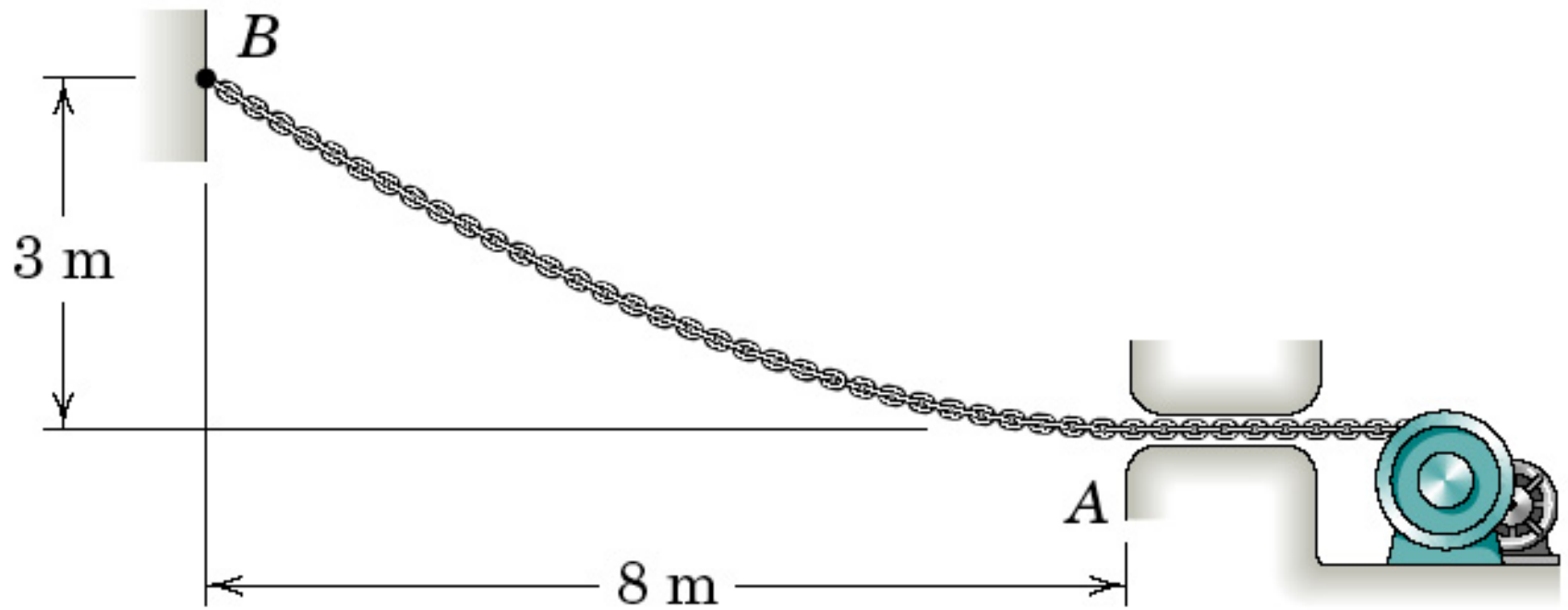
Problem 5/157

P_05_157



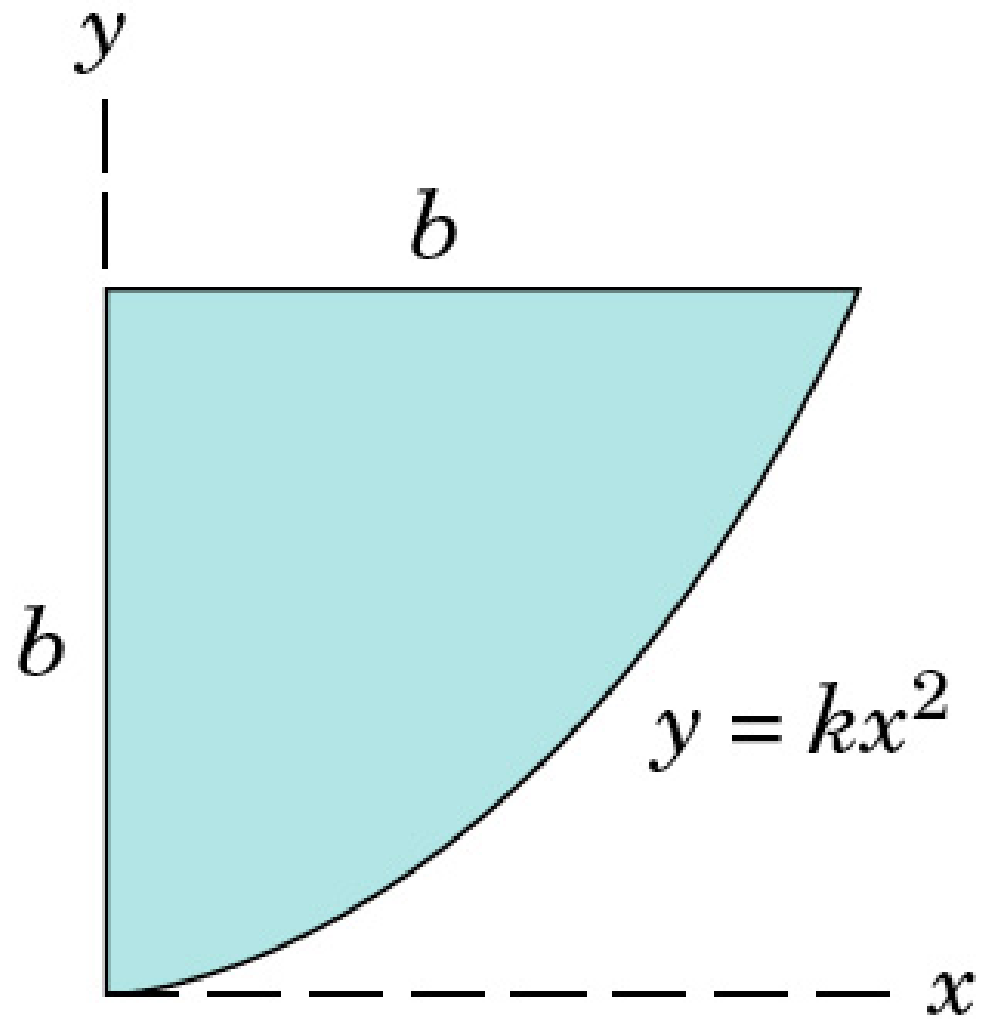
Problem 5/158

P_05_158

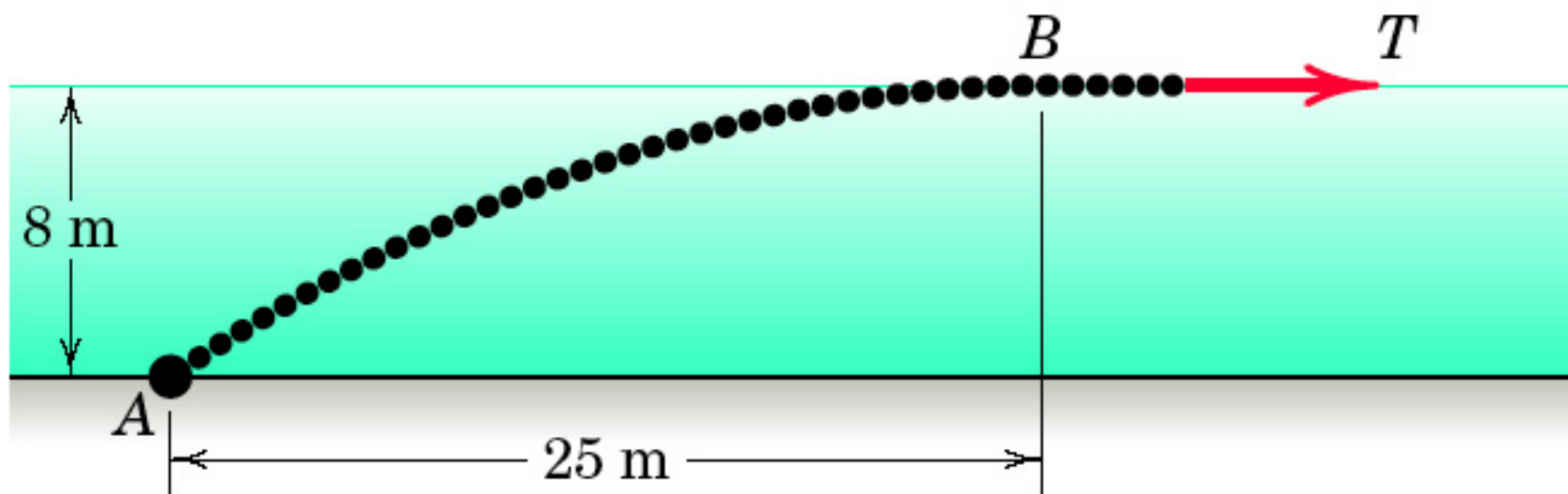


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P_05_159

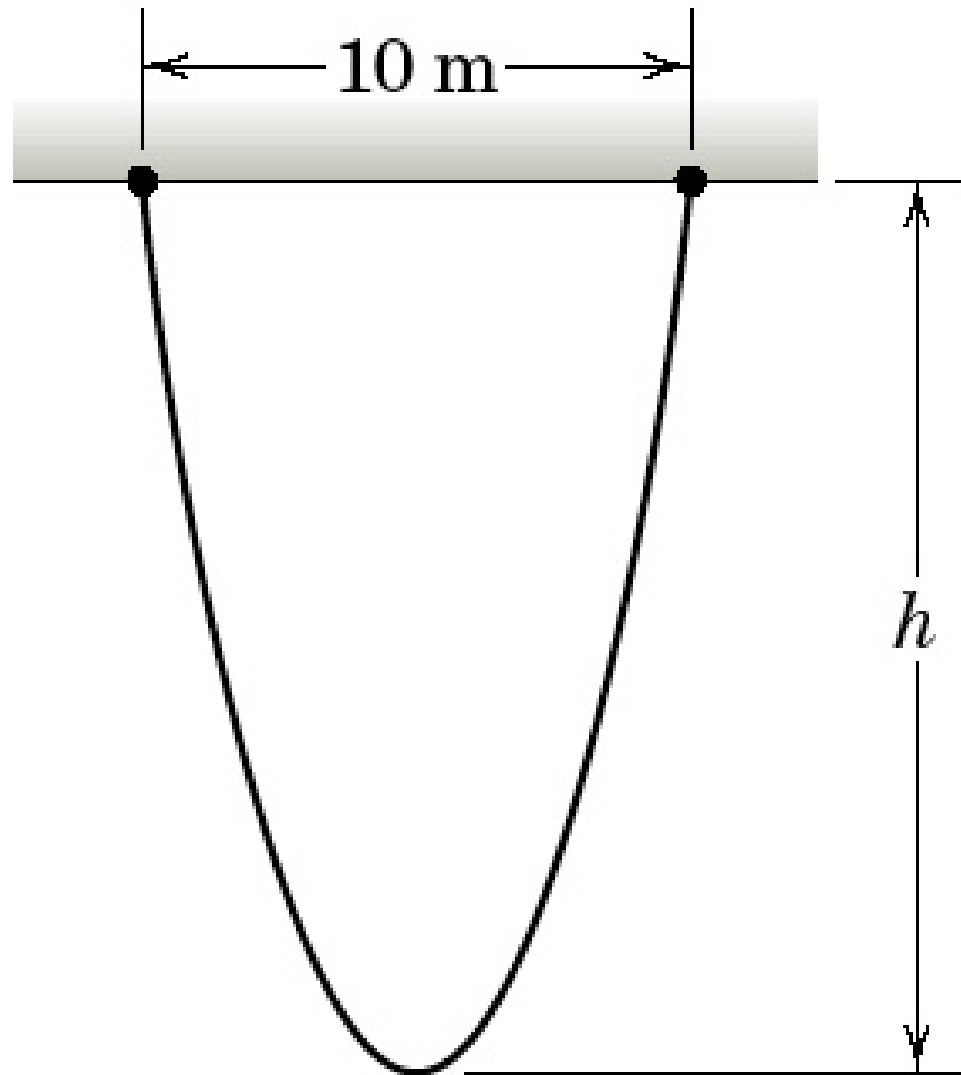


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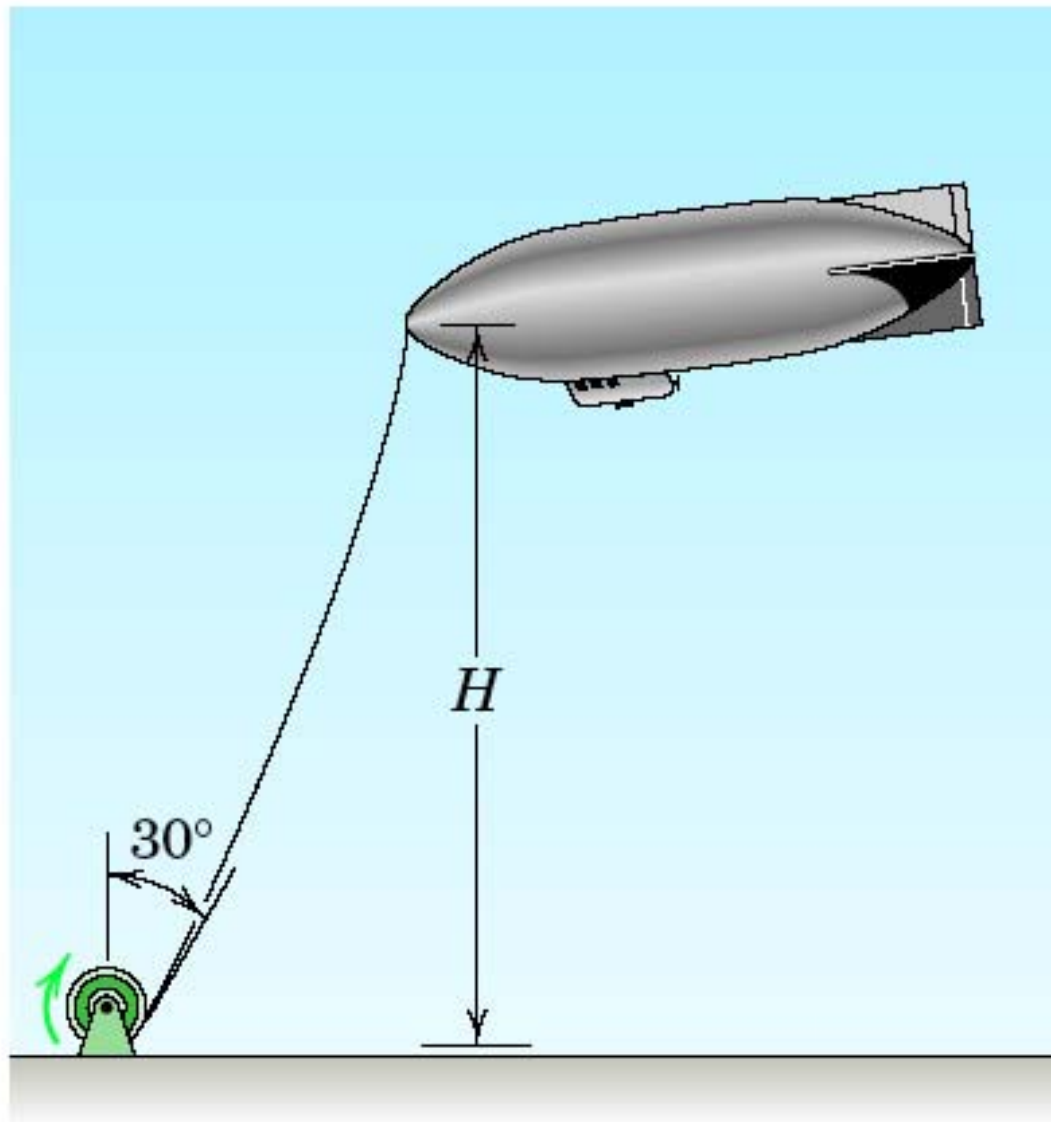


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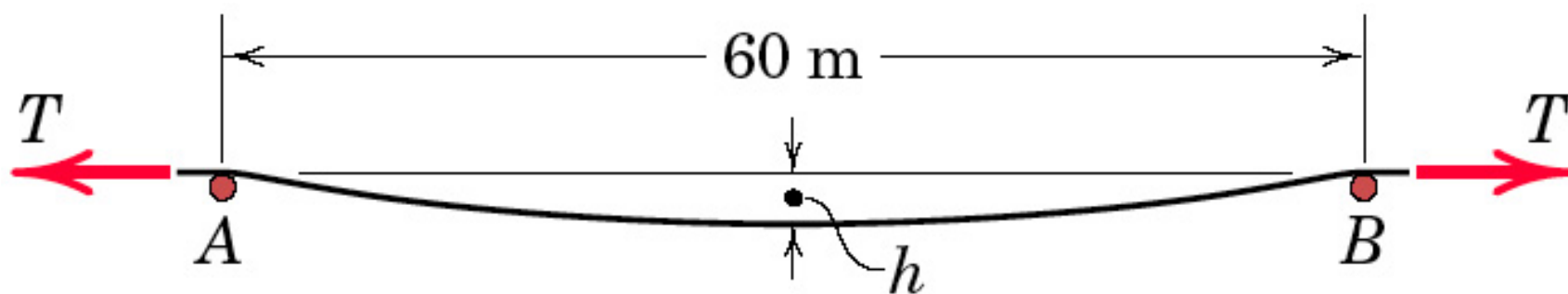
P_05_160



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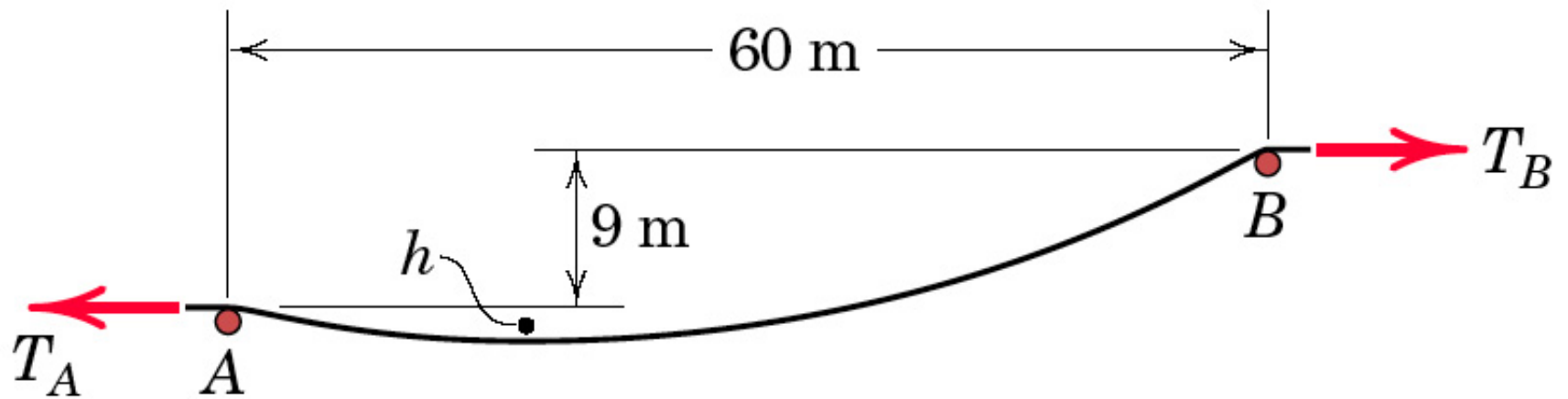


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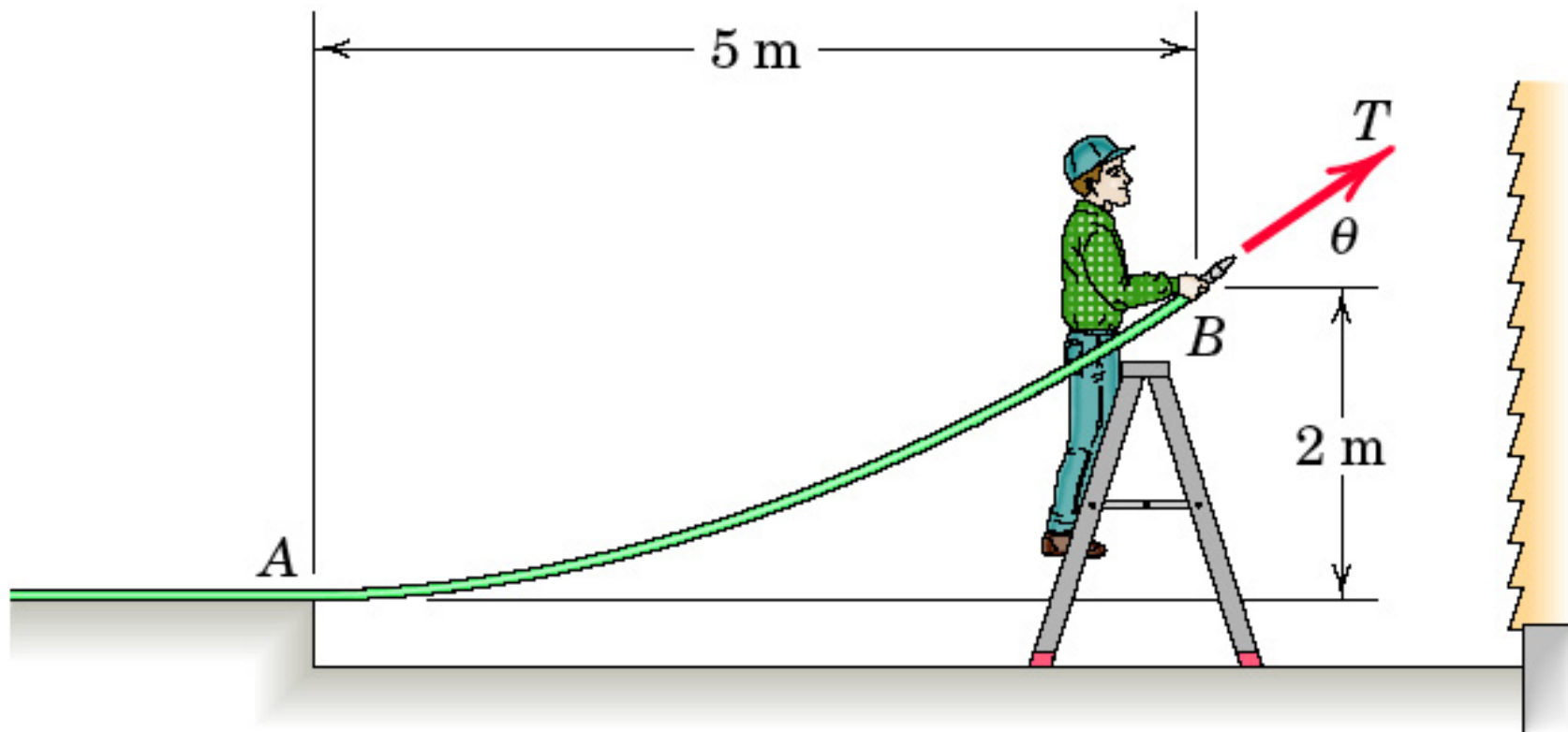
Problem 5/163

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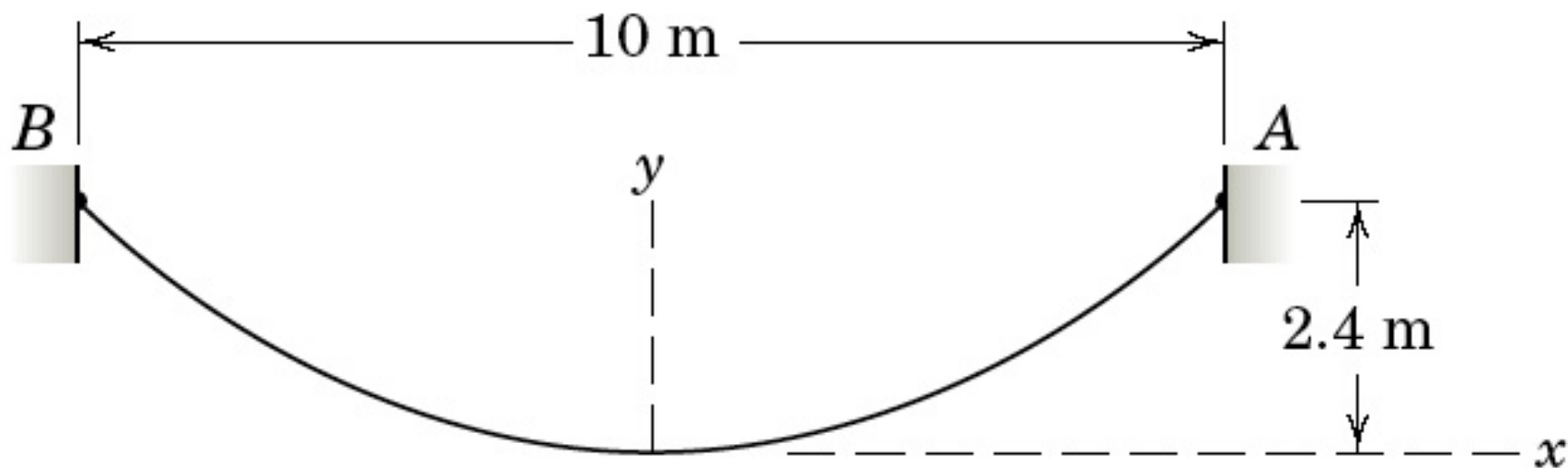
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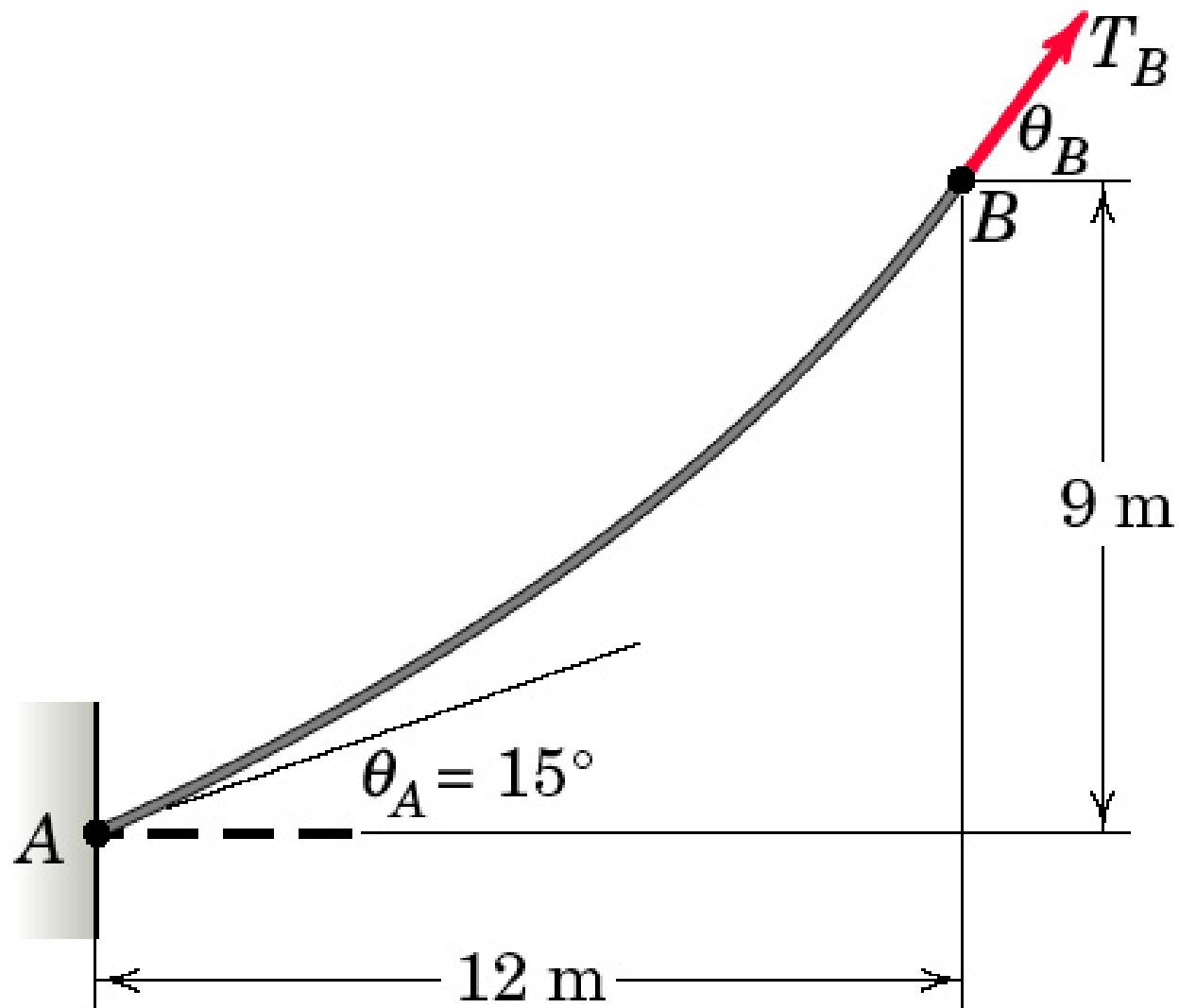
Problem 5/165

P_05_165

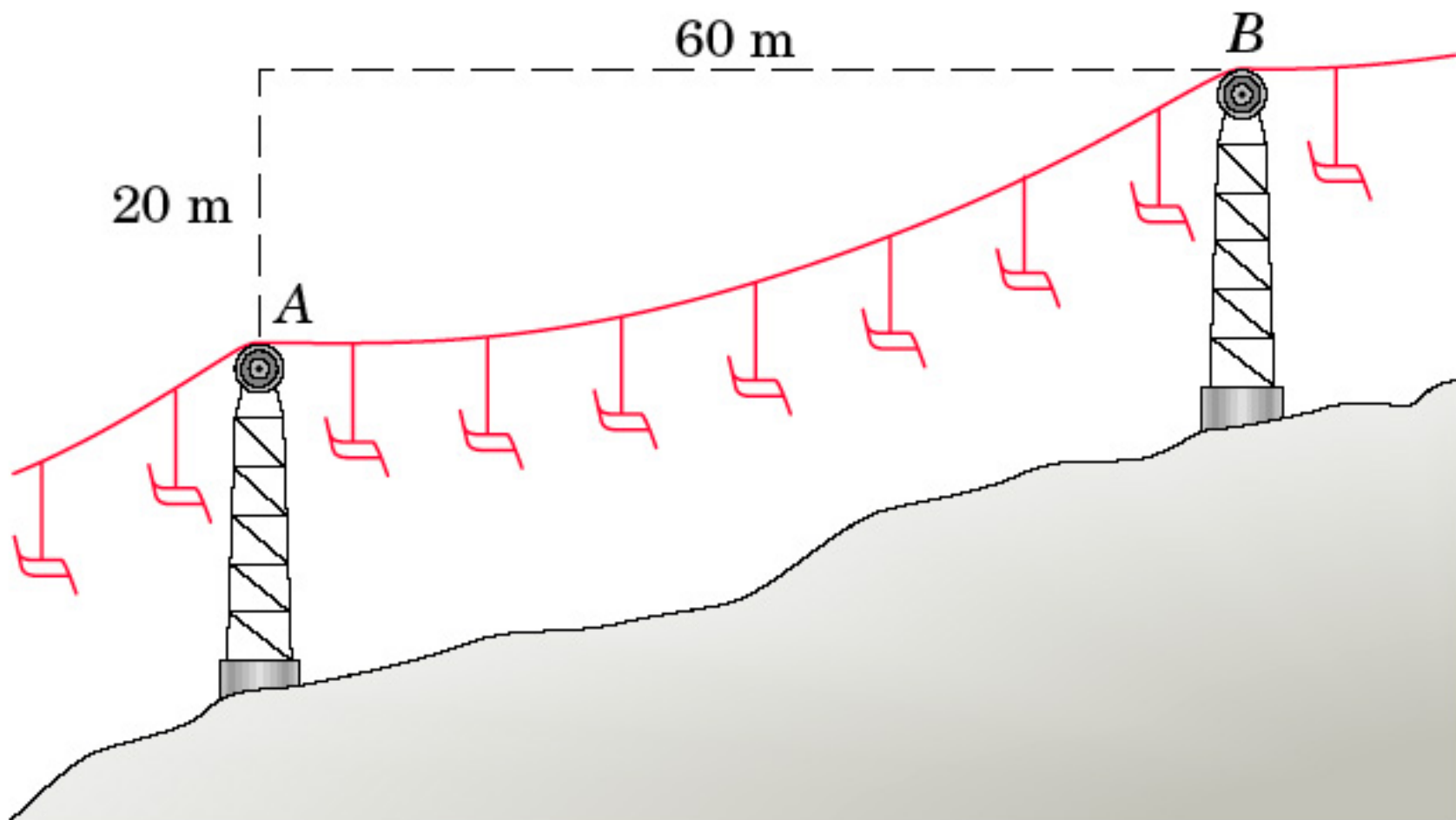


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P_05_166

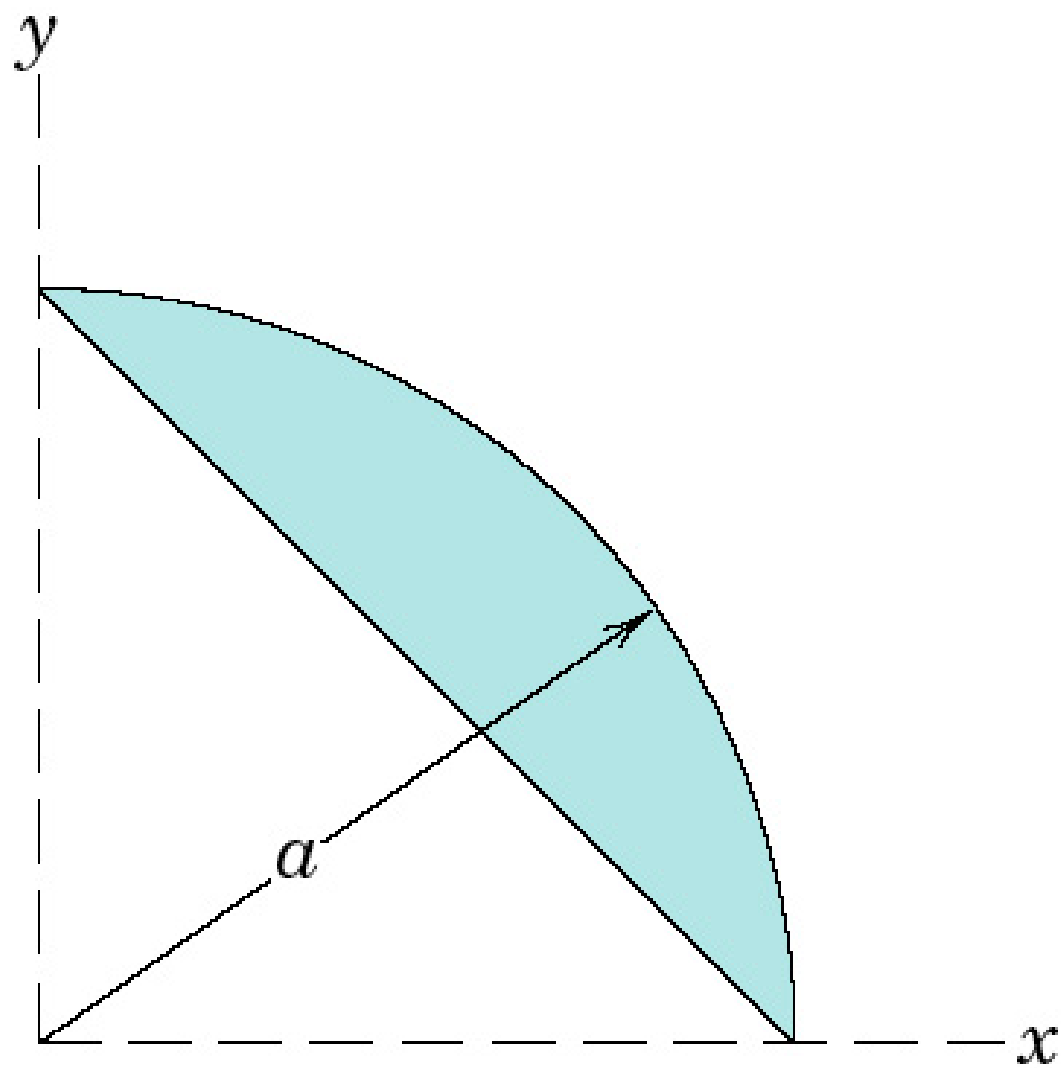


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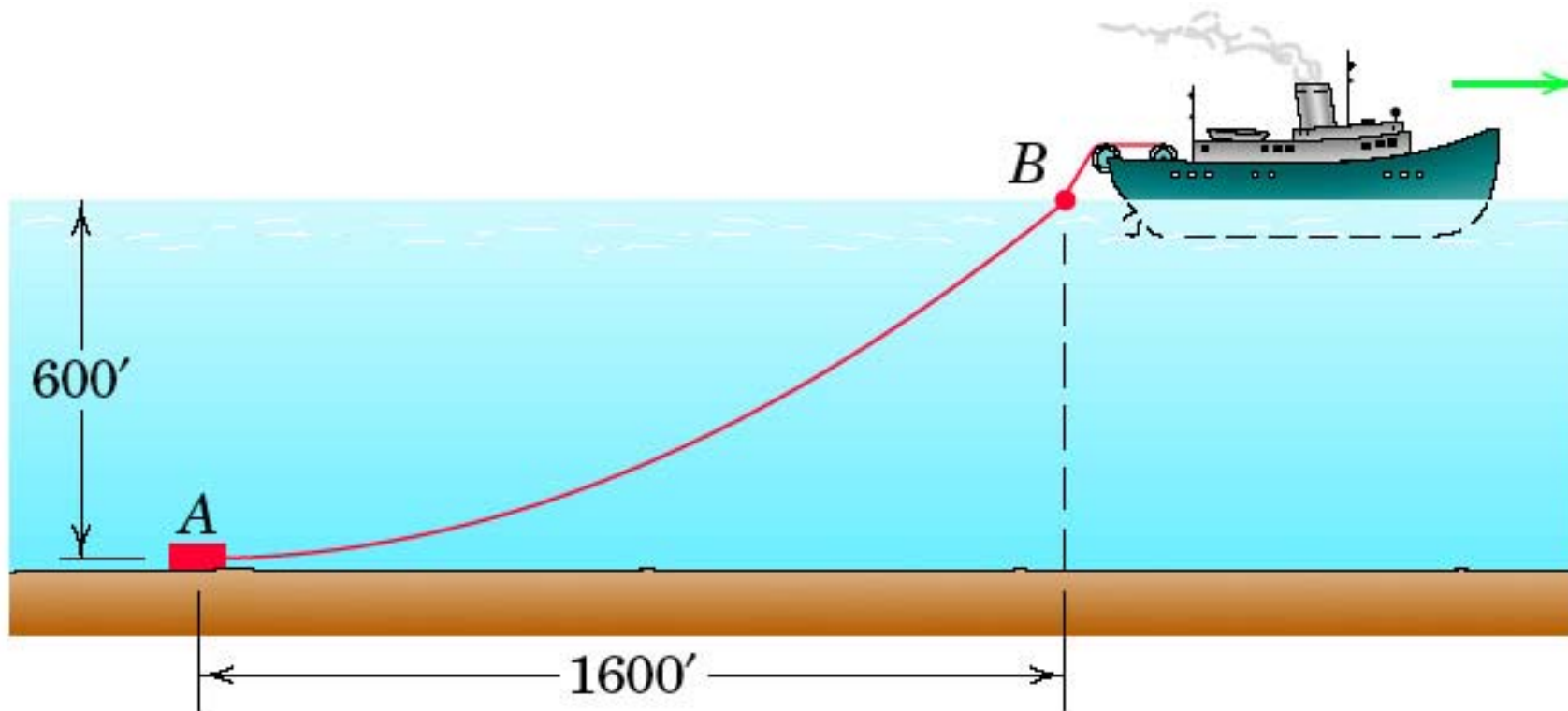


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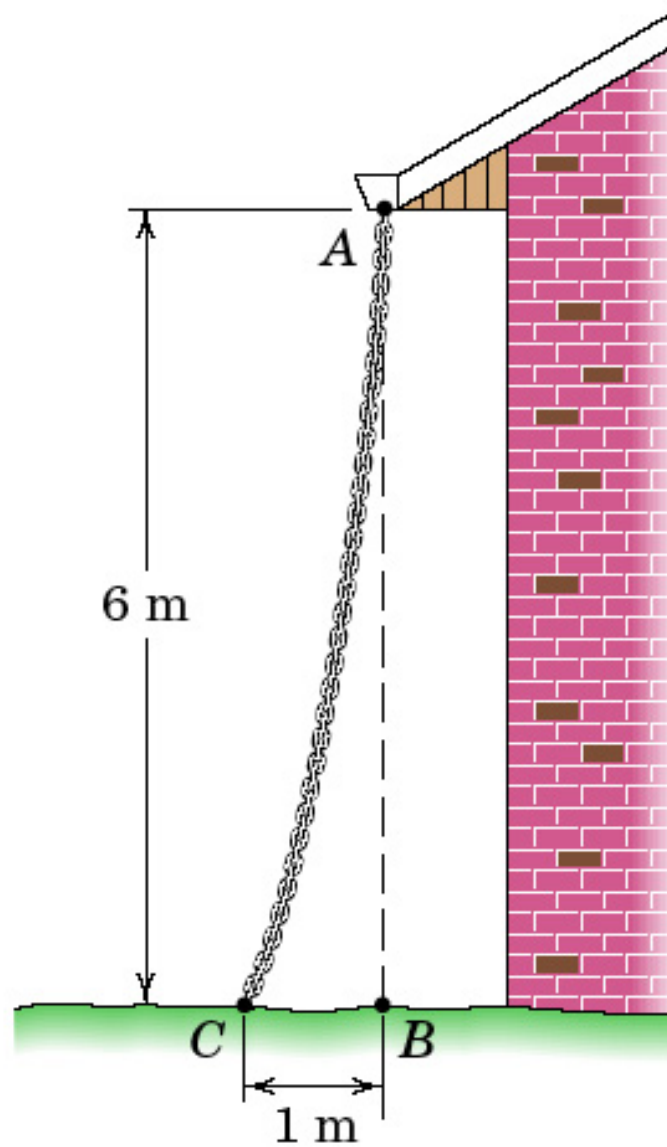


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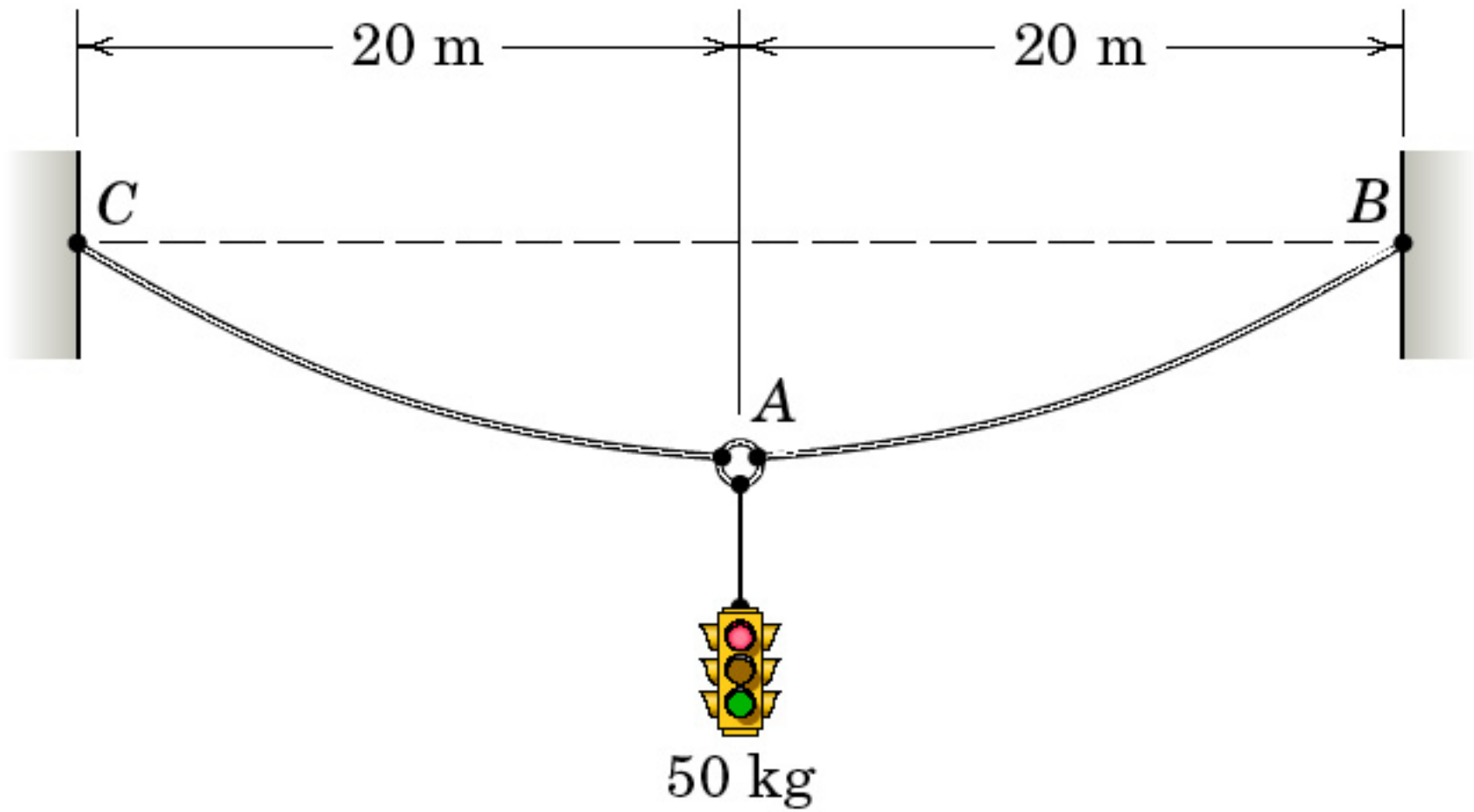


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P_05_170

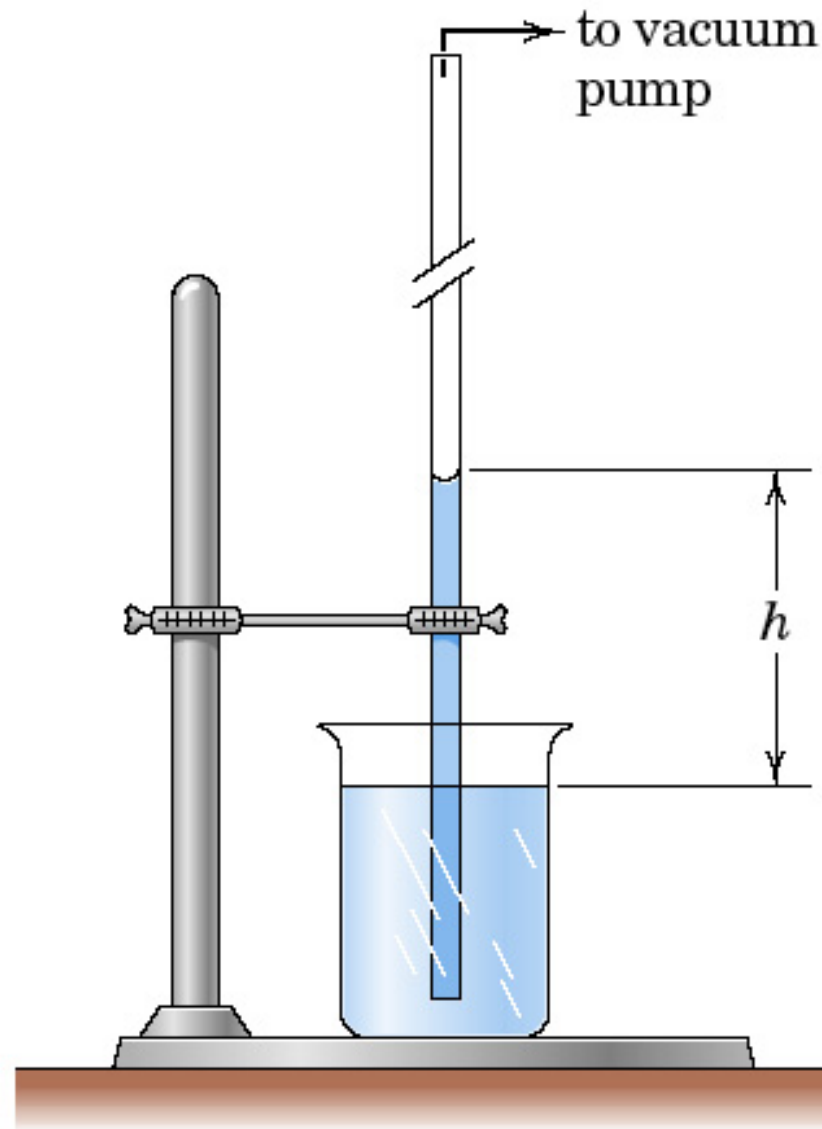


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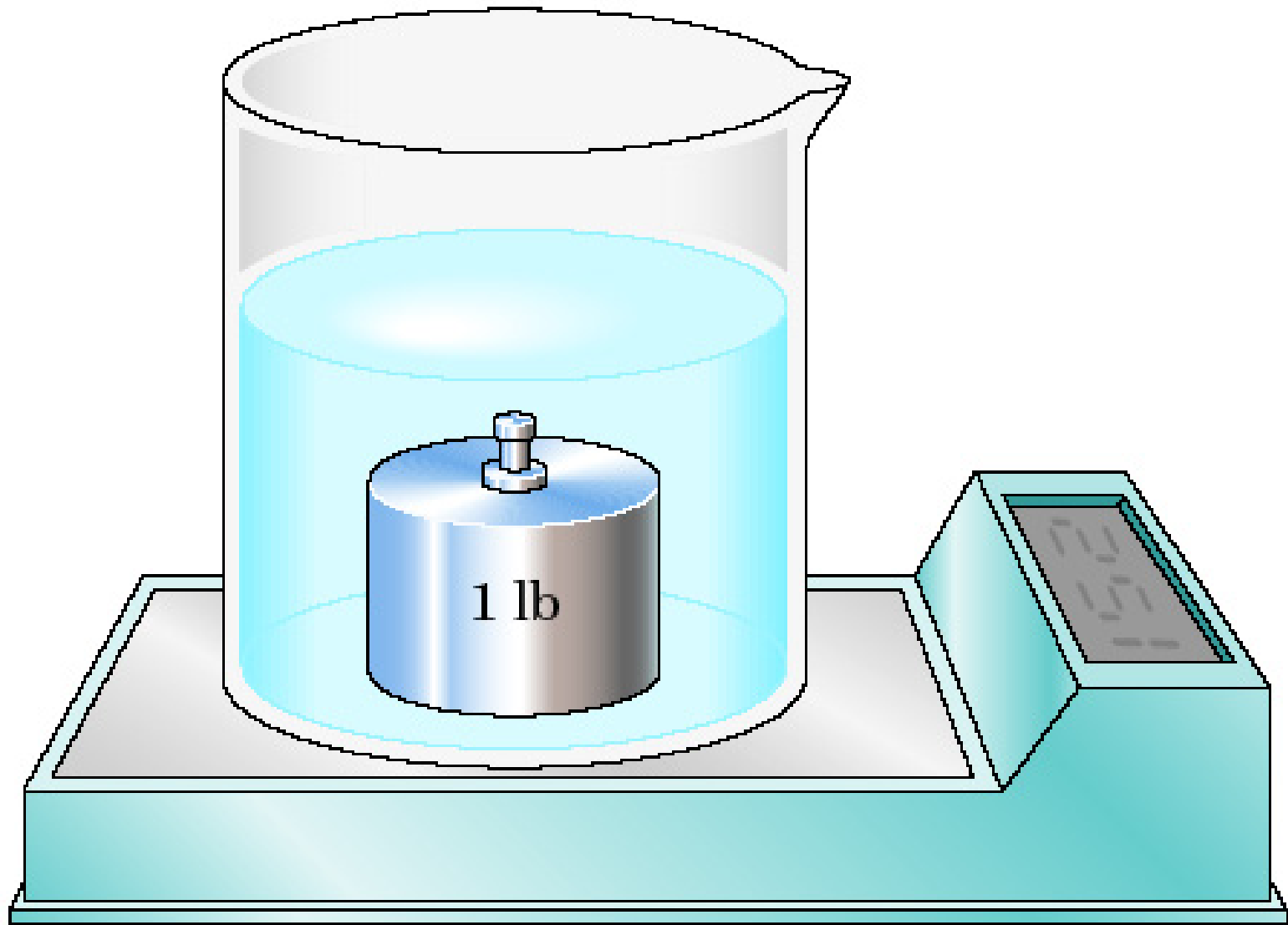


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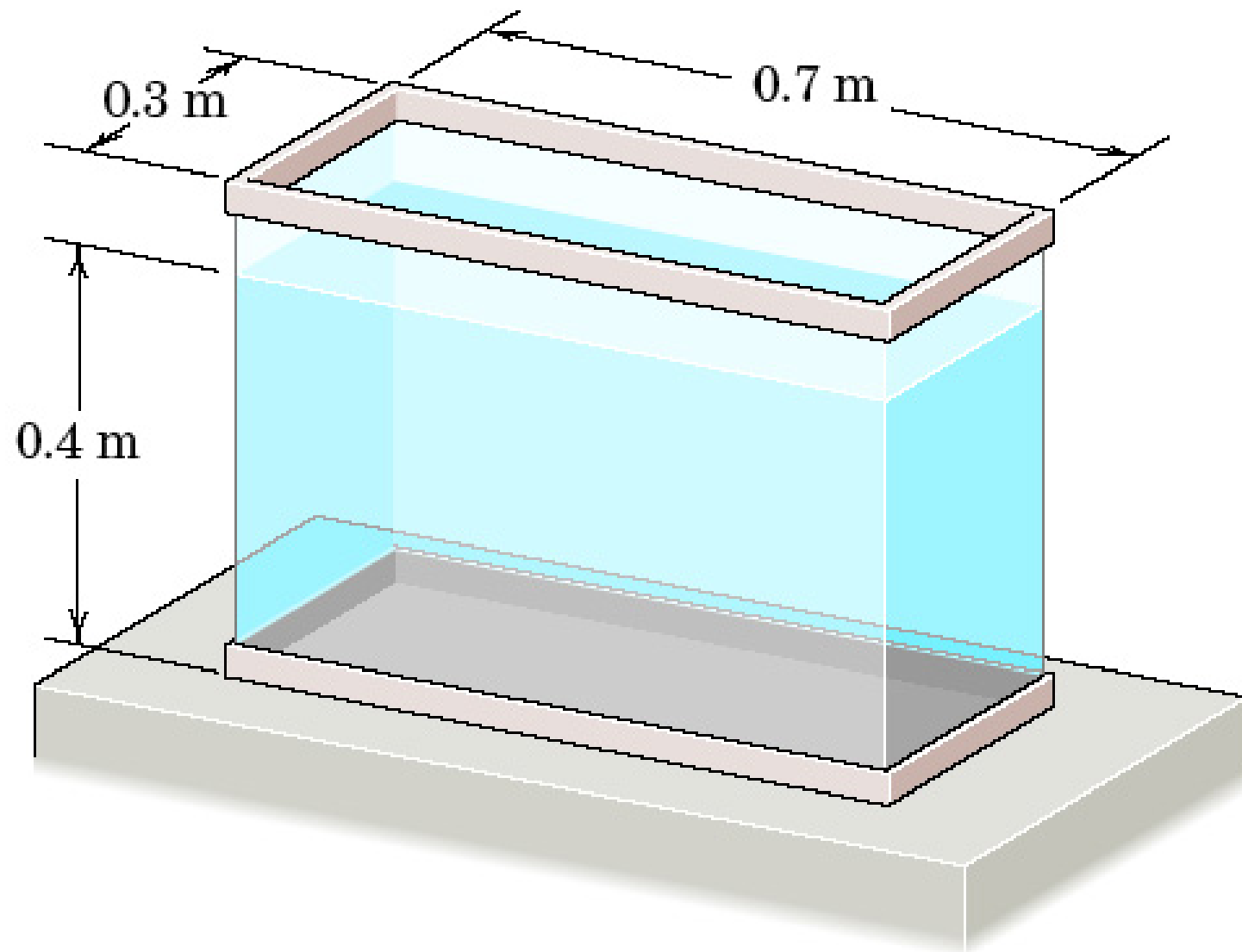
P_05_172



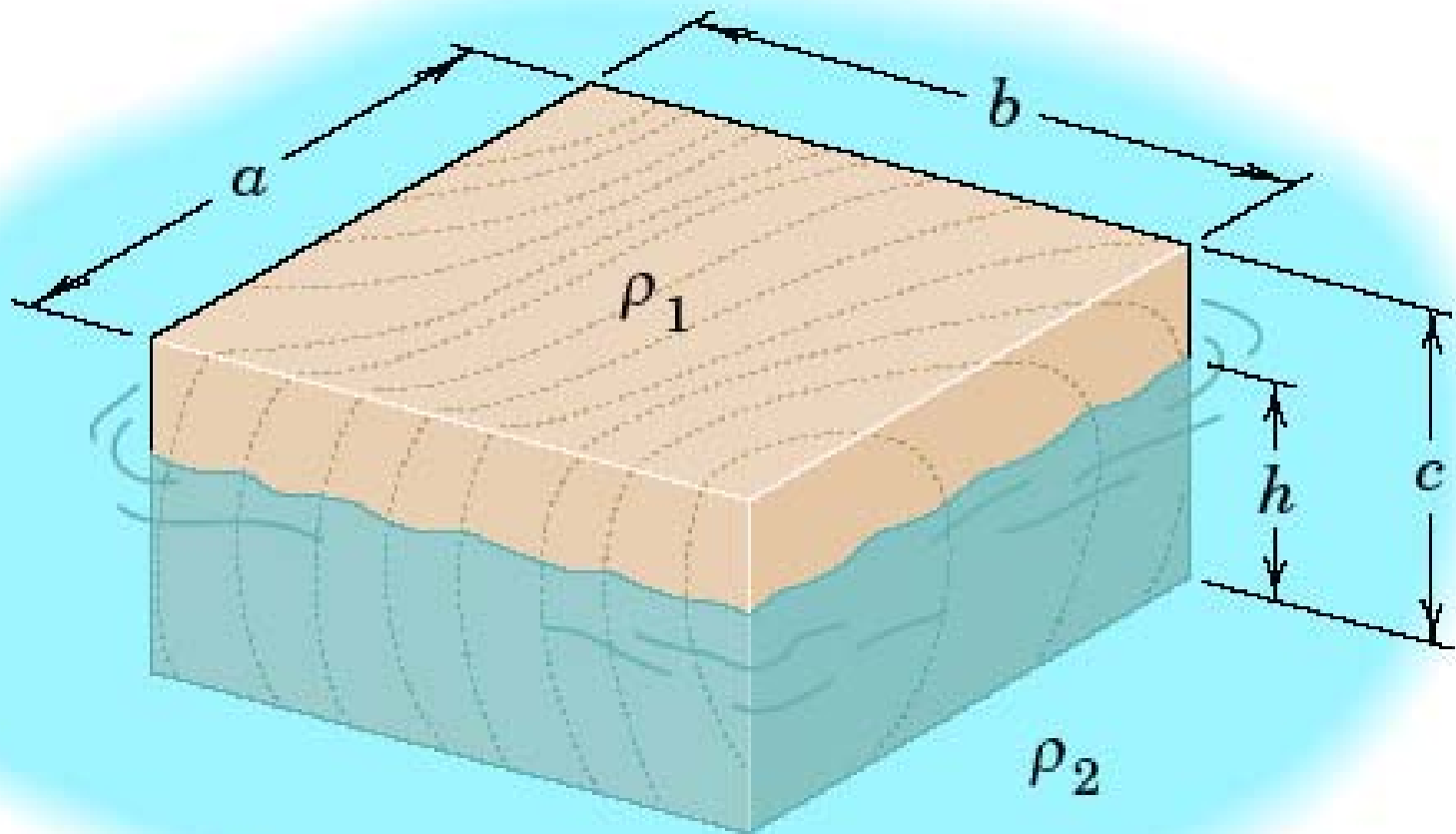
Problem 5/173



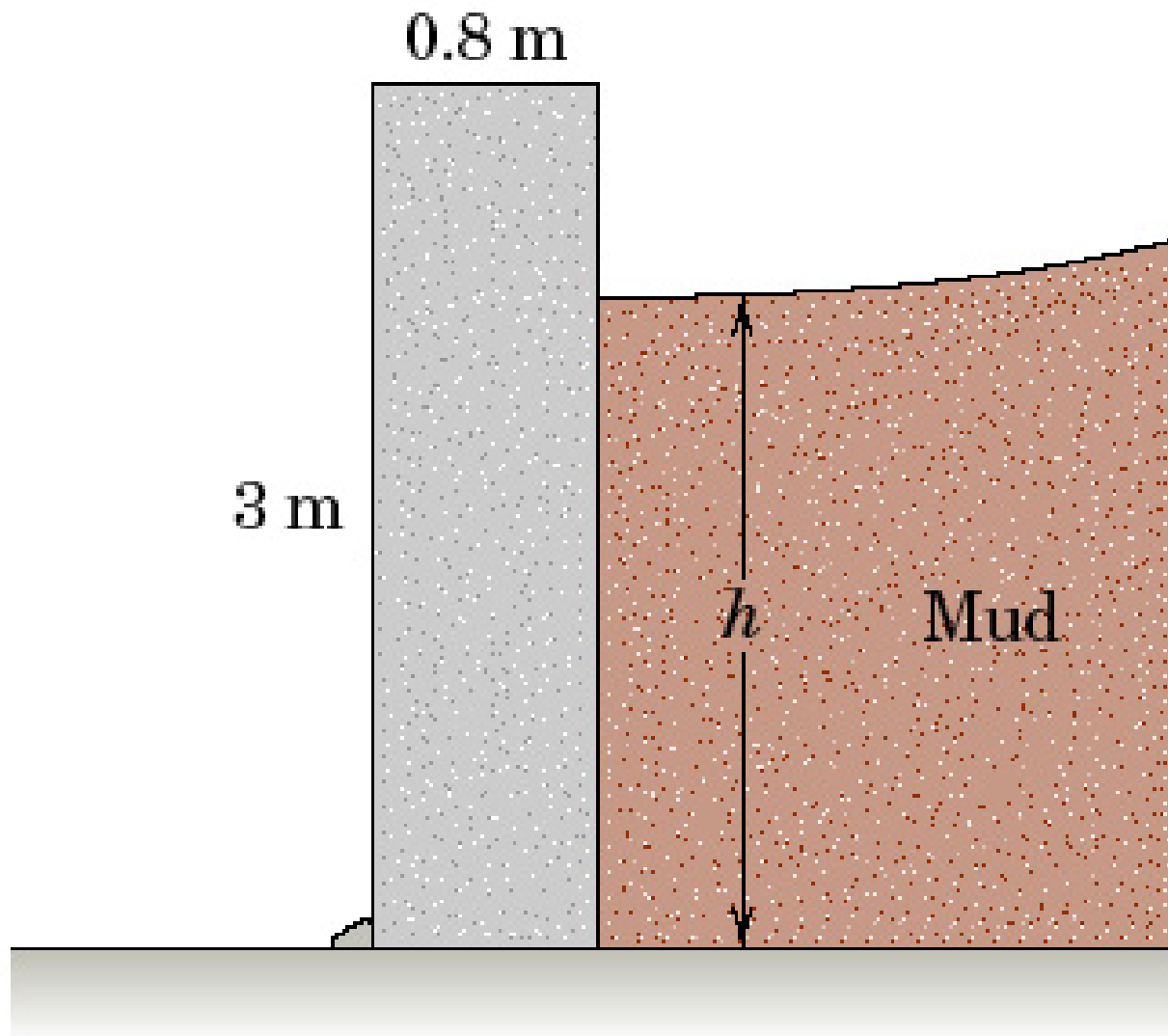
Problem 5/174



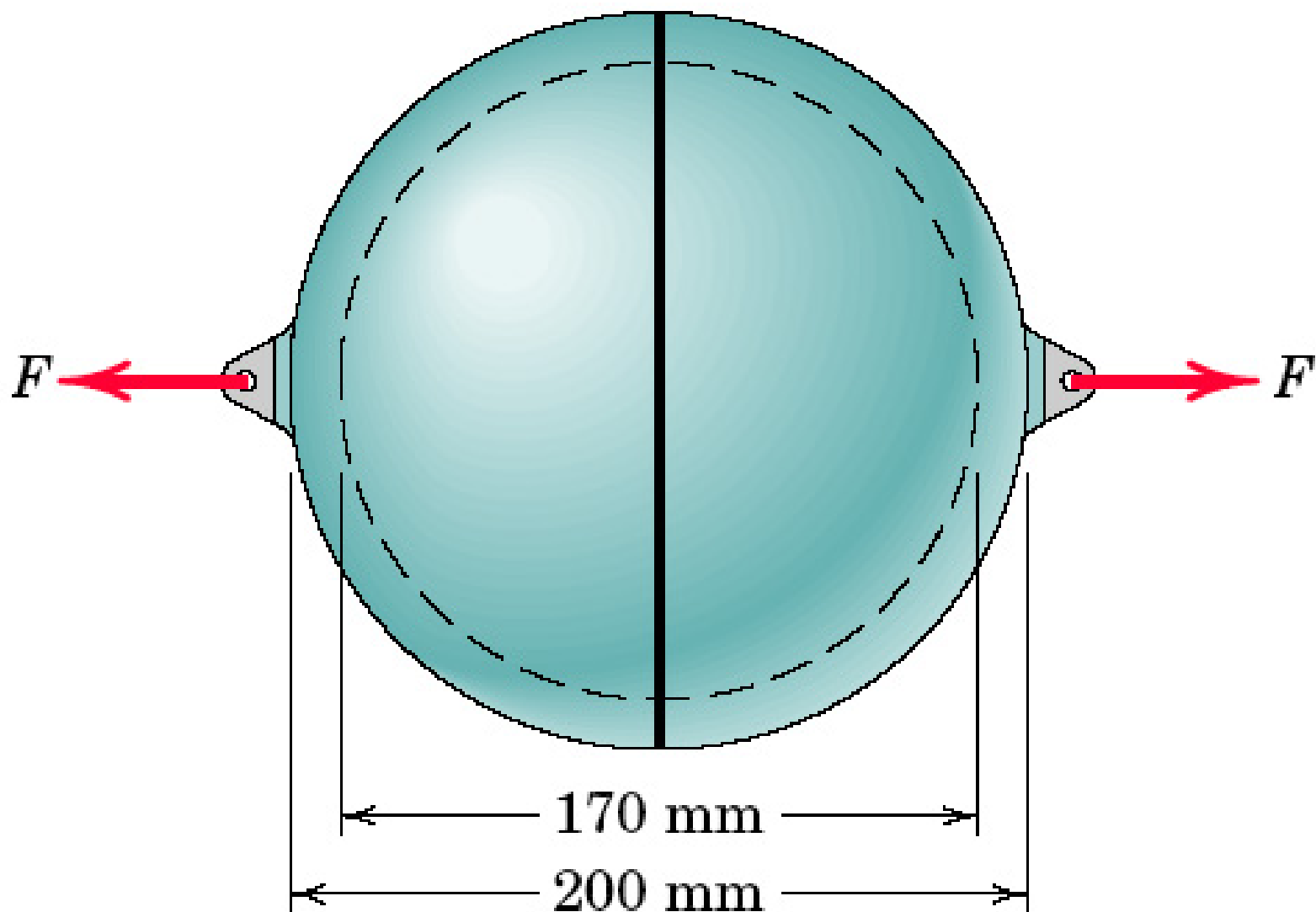
Problem 5/175



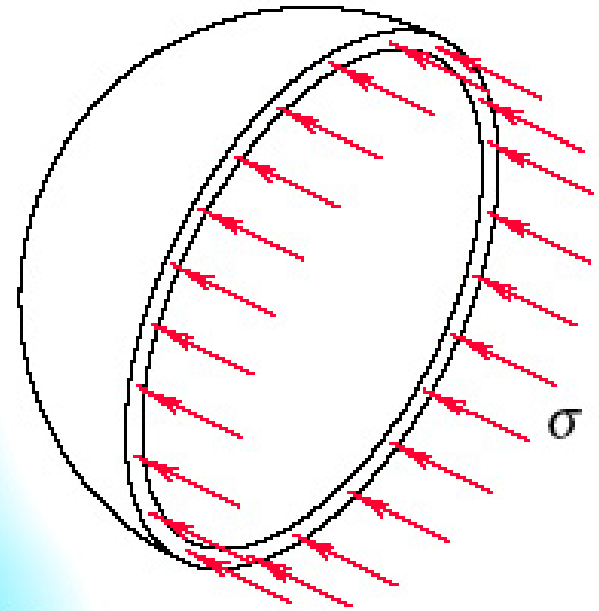
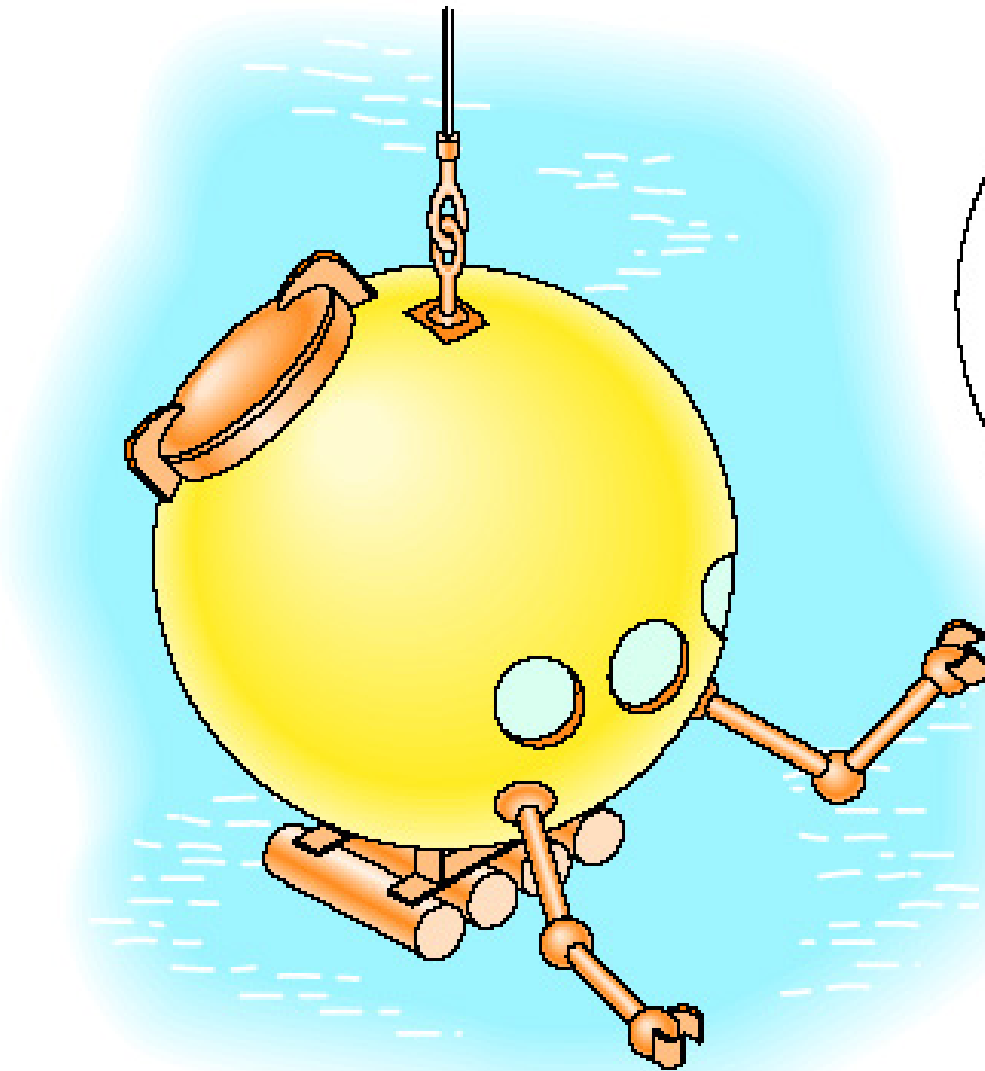
Problem 5/176



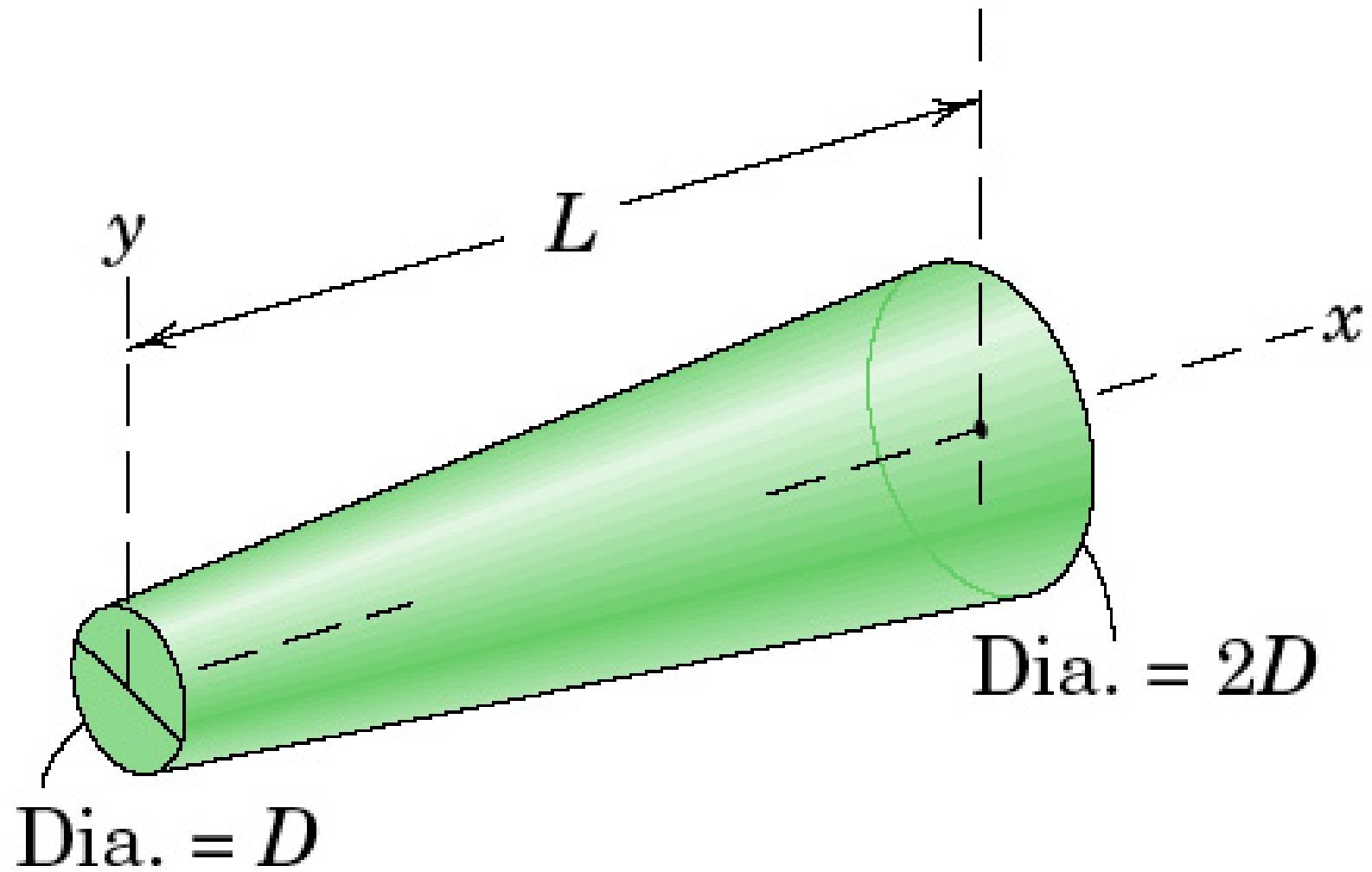
Problem 5/177



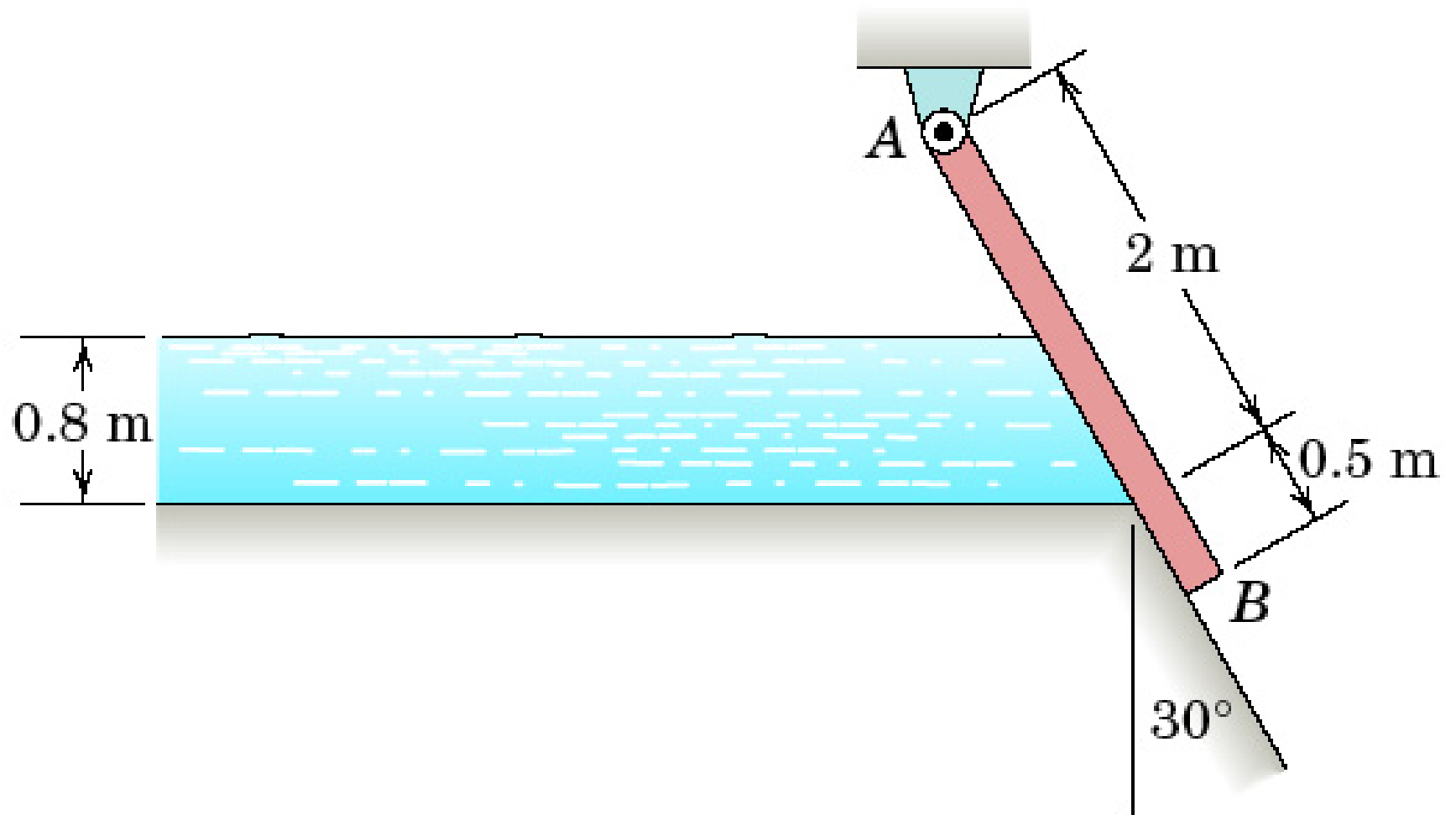
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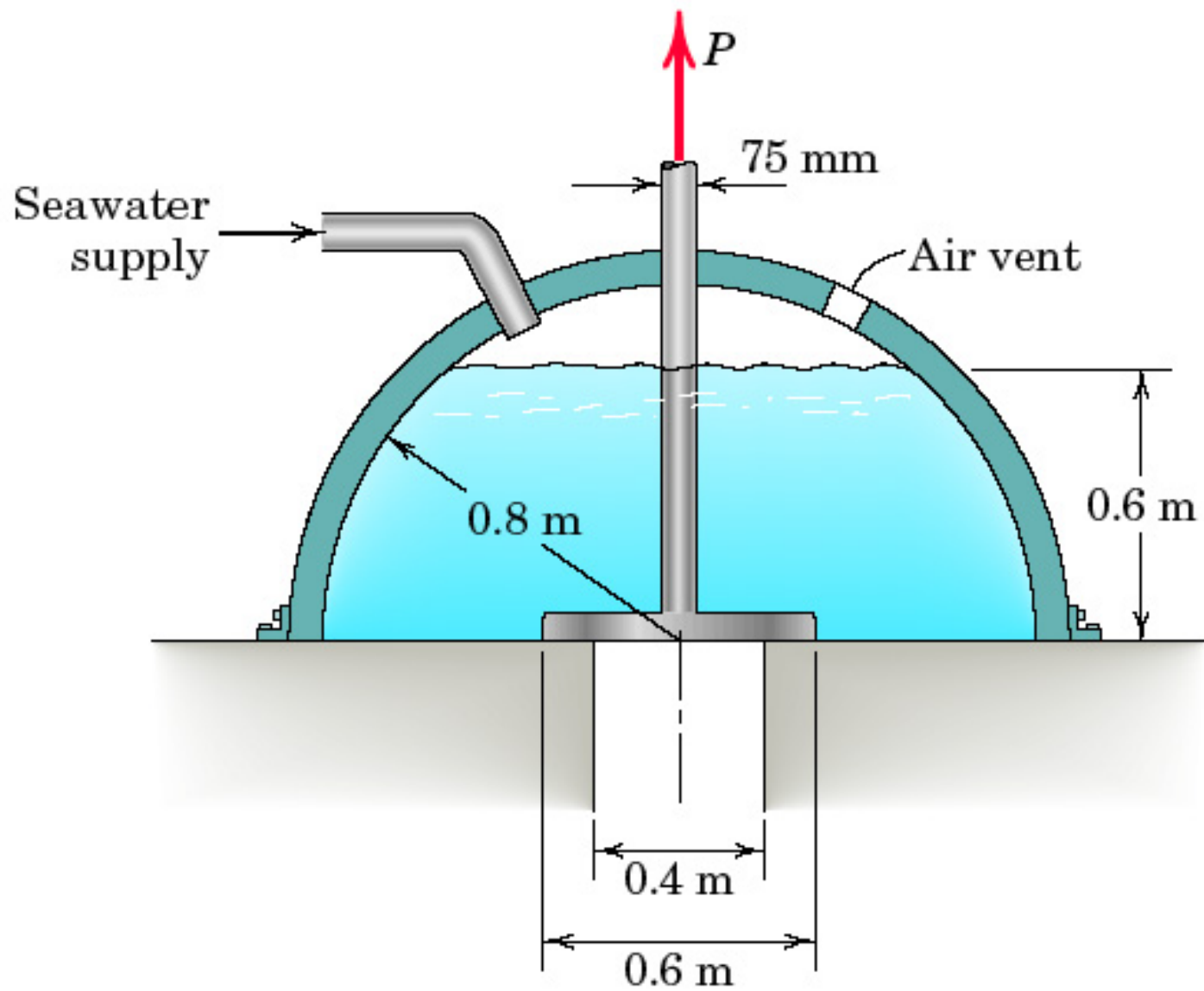


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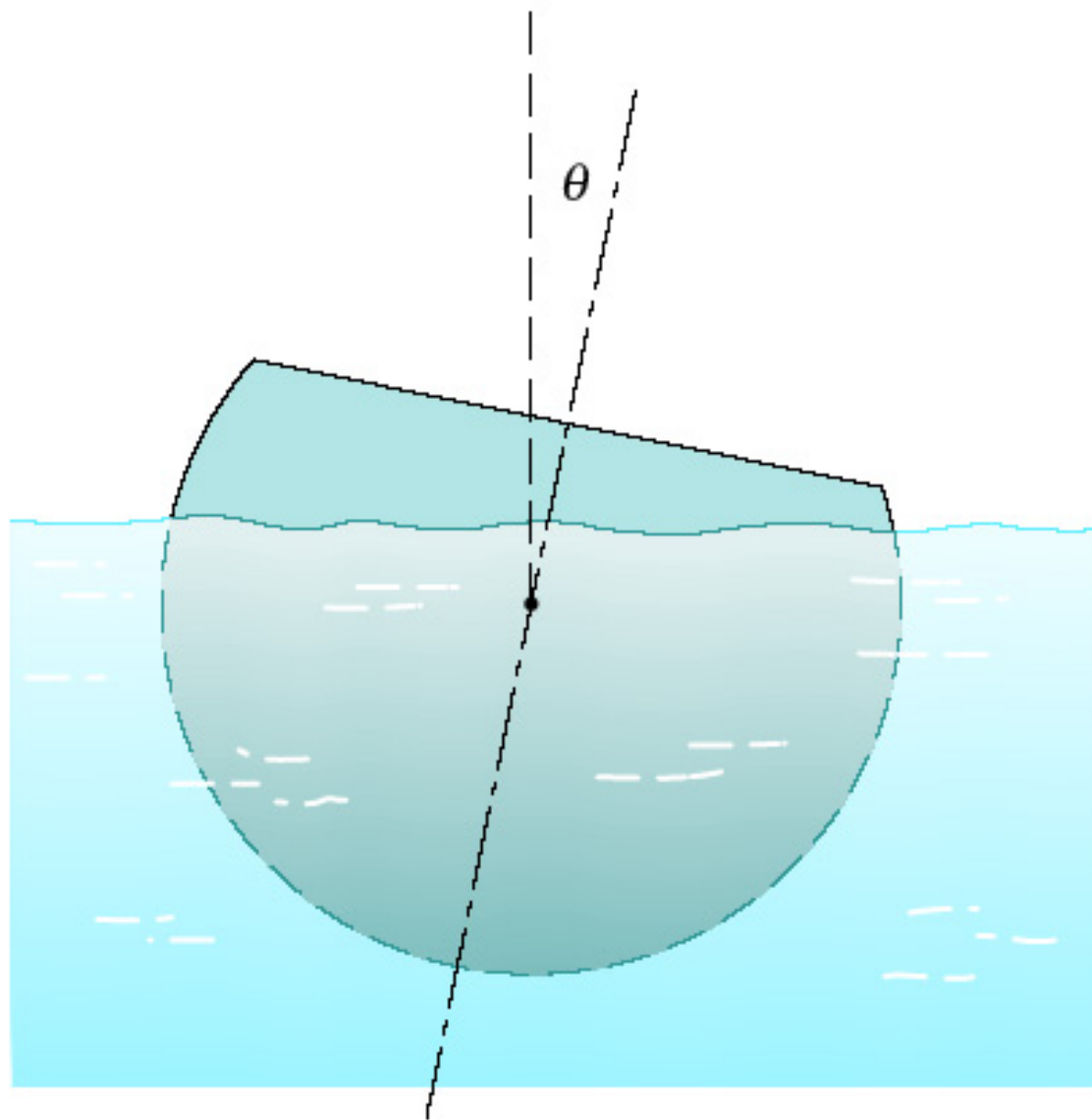


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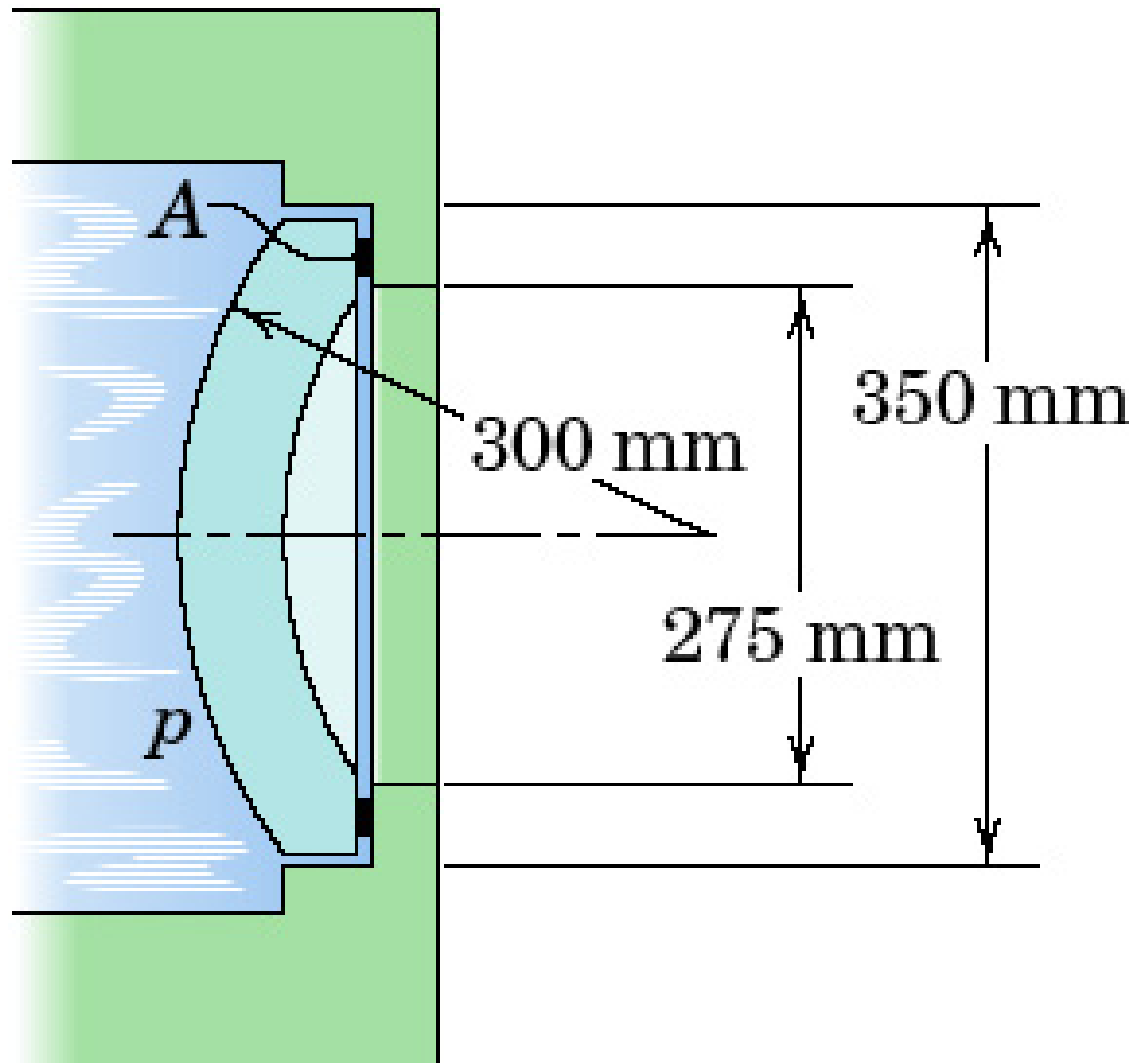
P_05_180



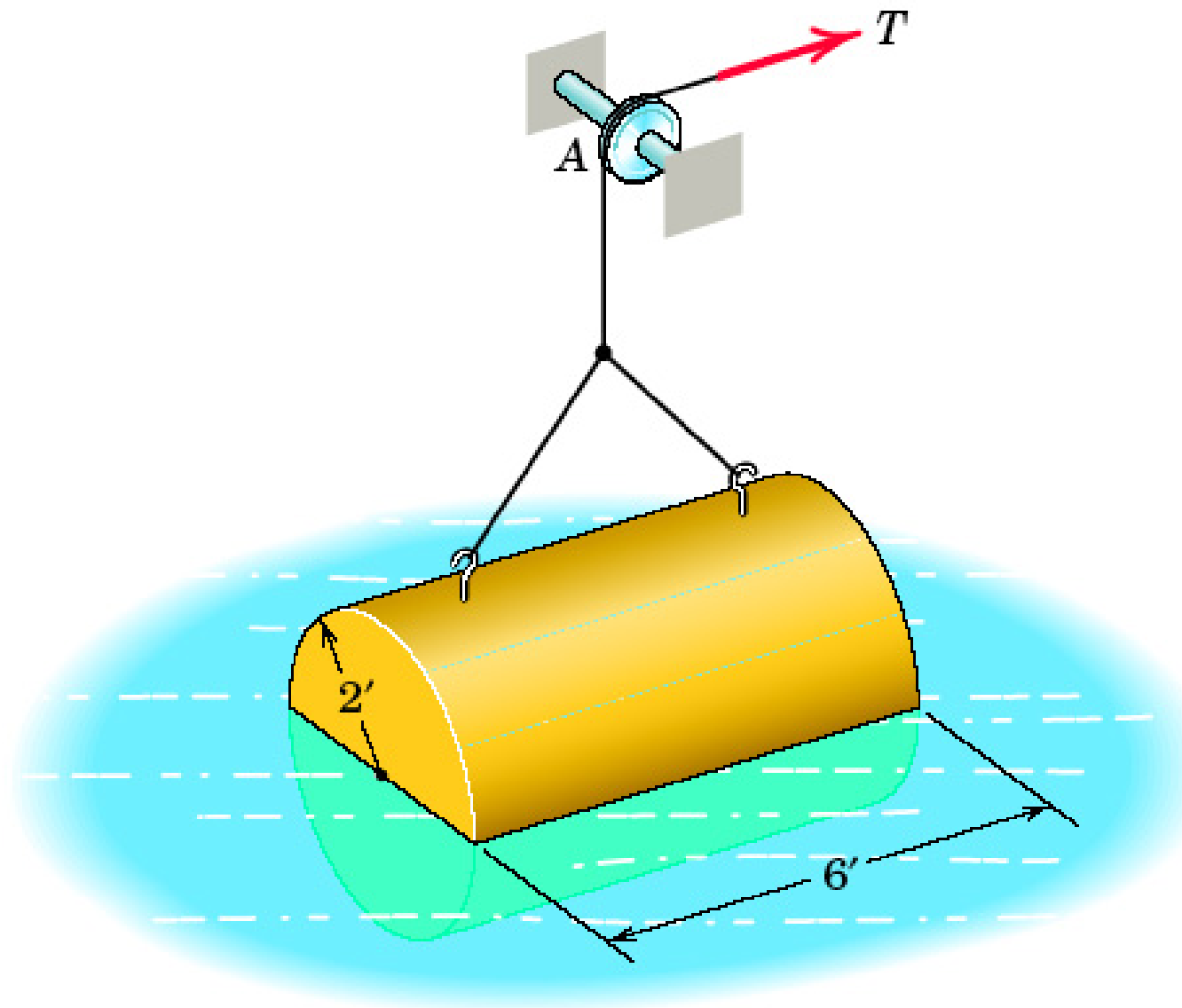
Problem 5/181



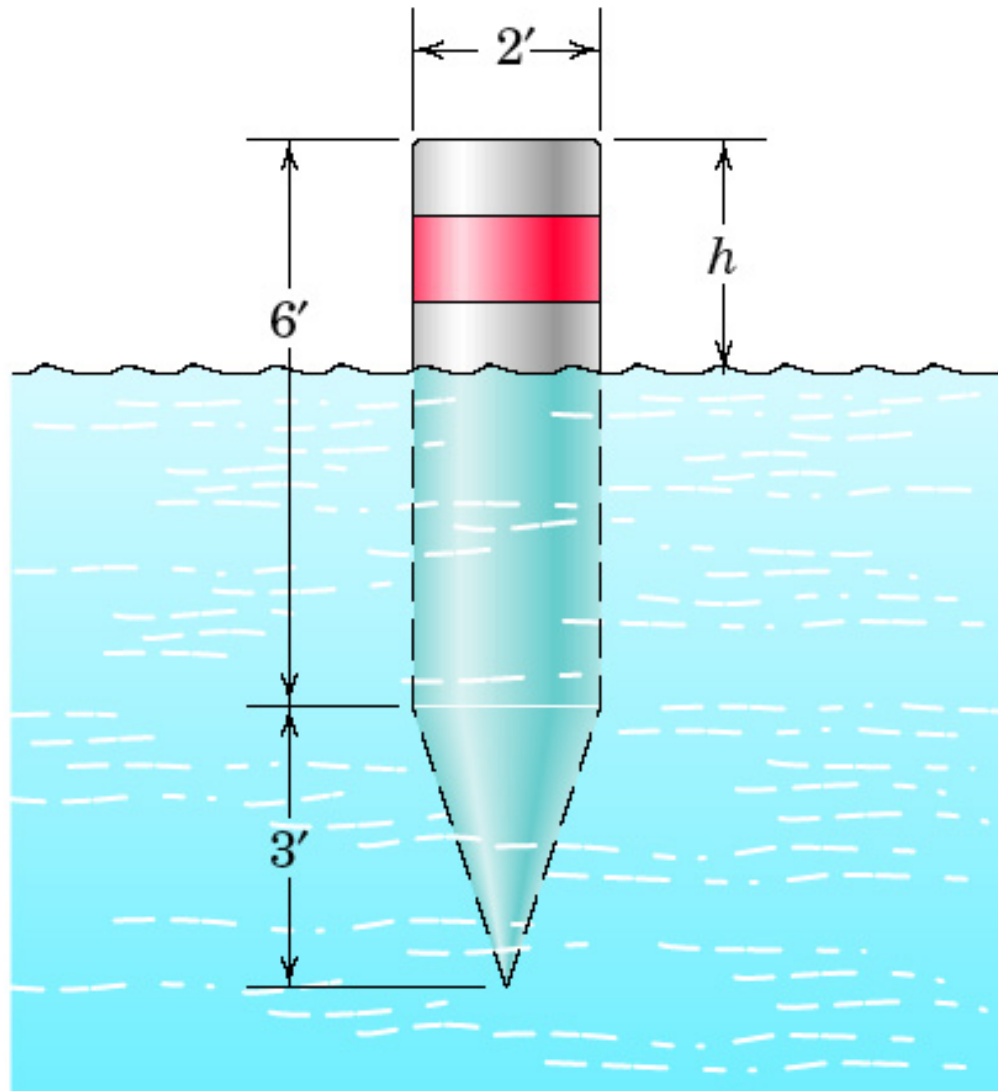
Problem 5/182



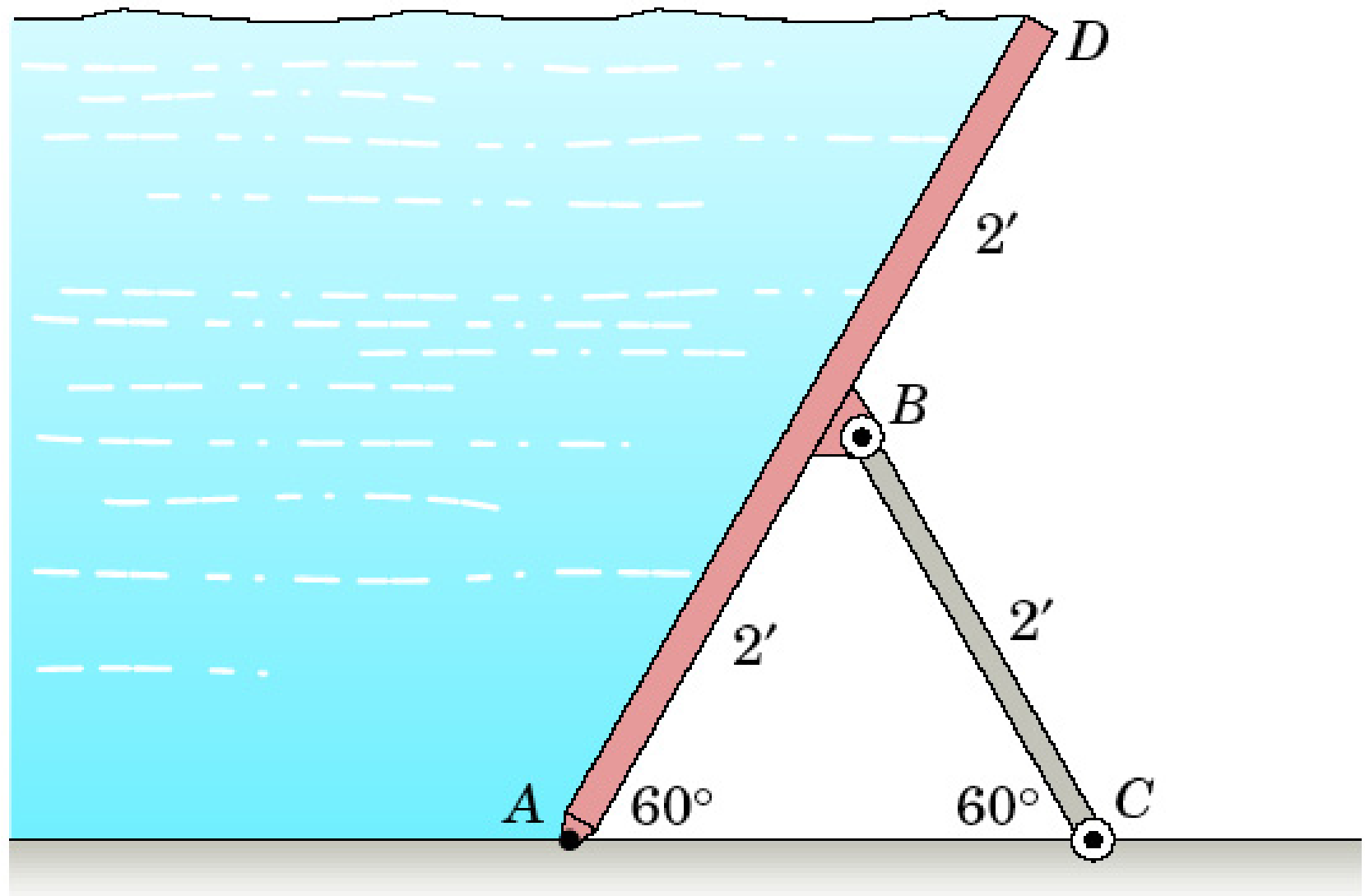
Problem 5/183



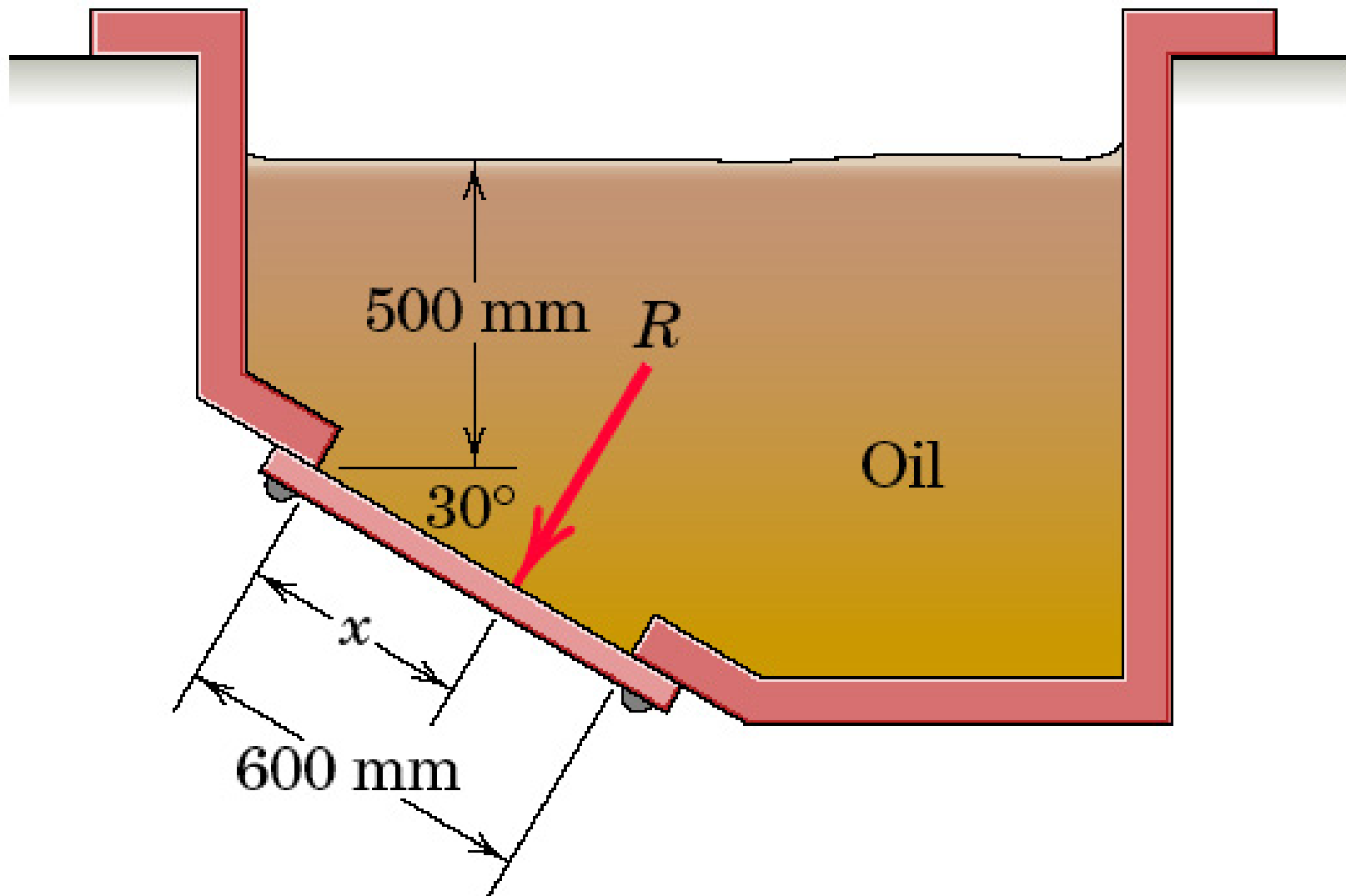
Problem 5/184



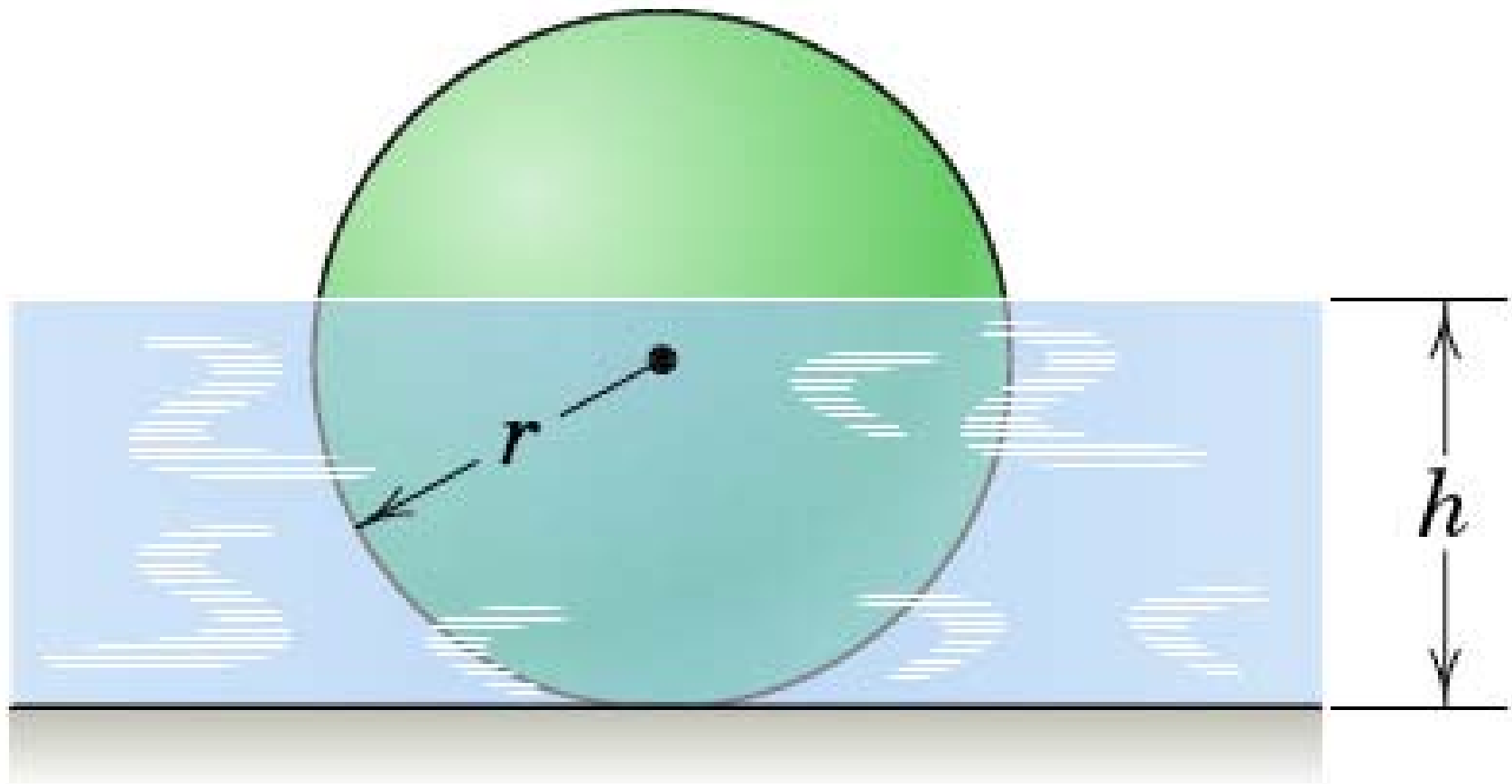
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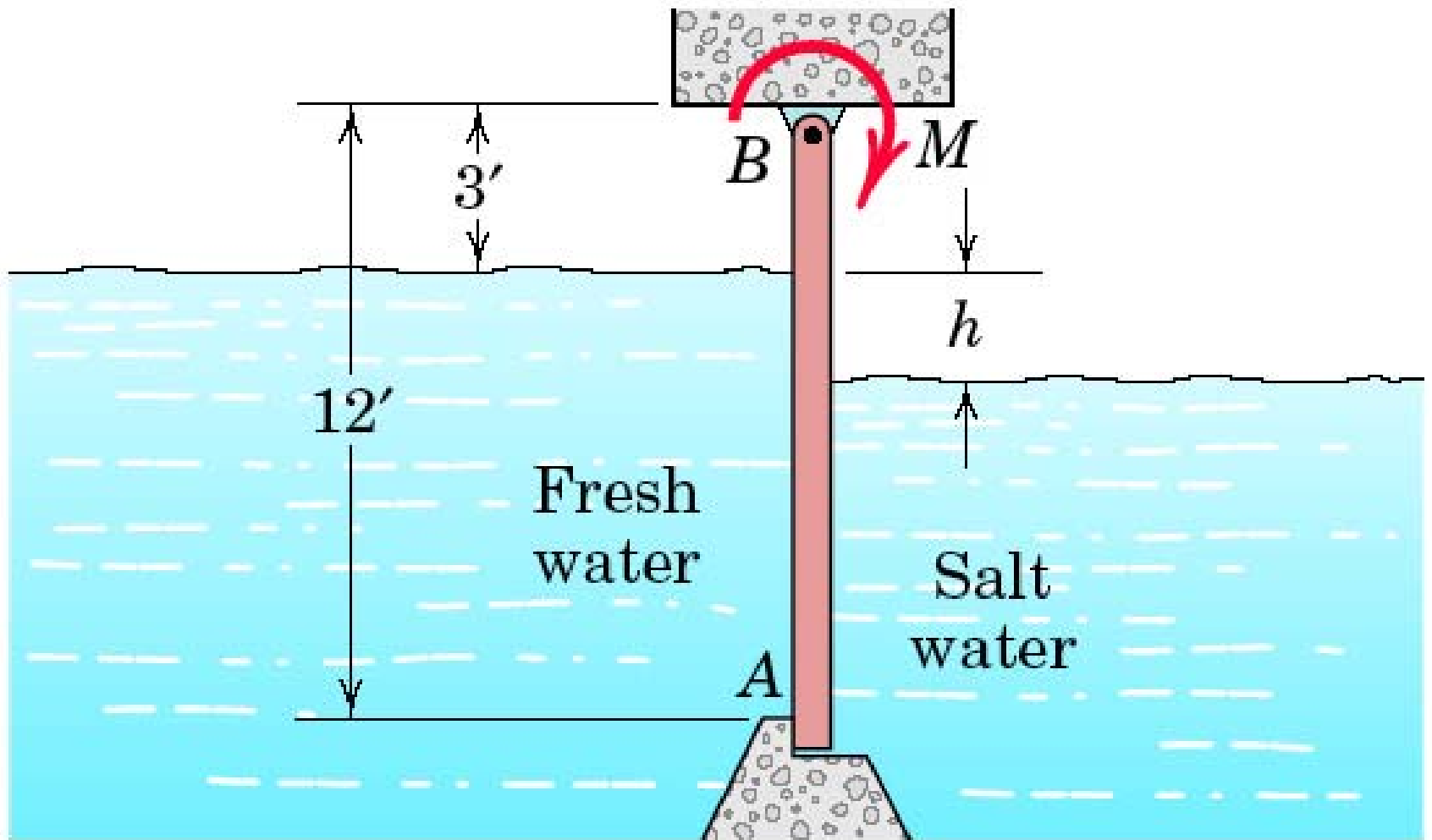


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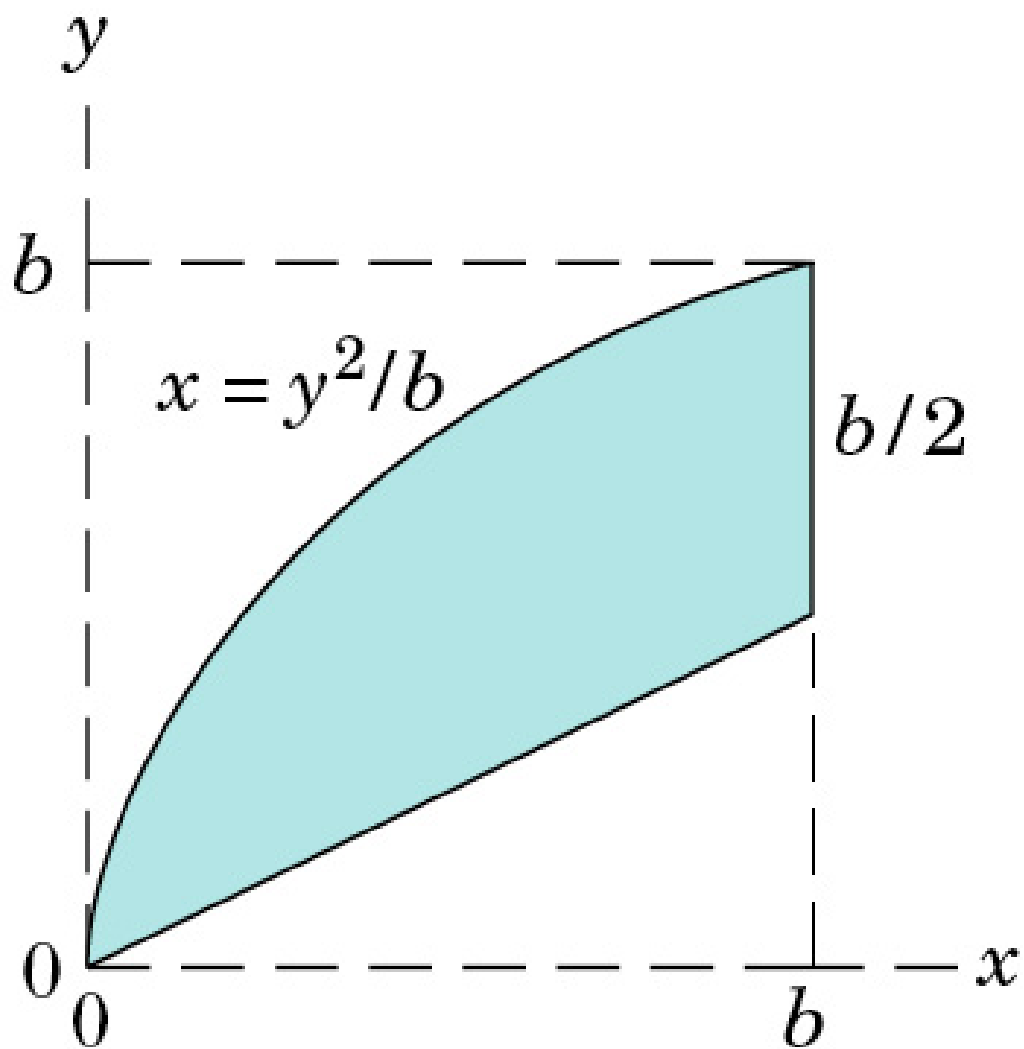


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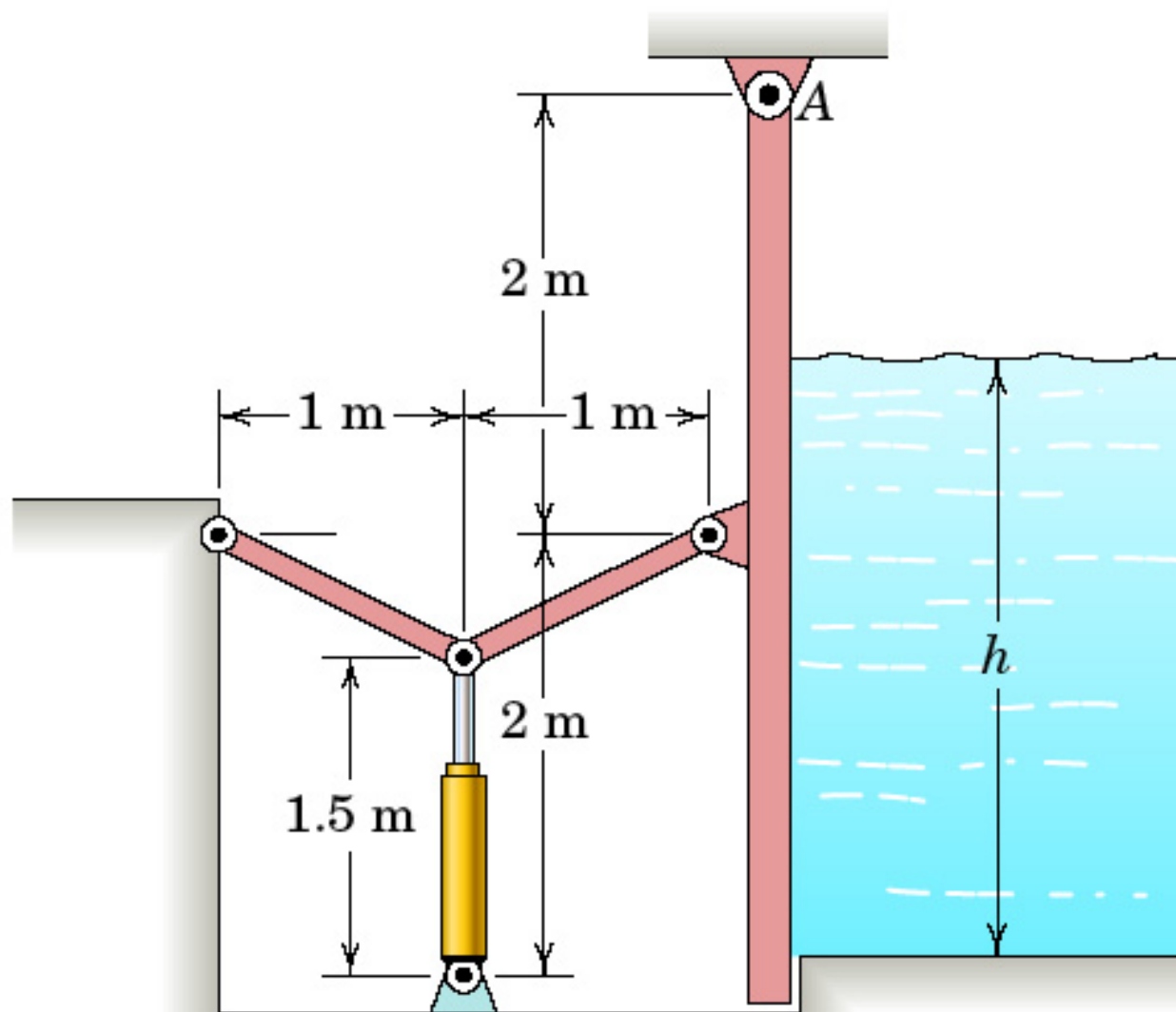
P_05_188



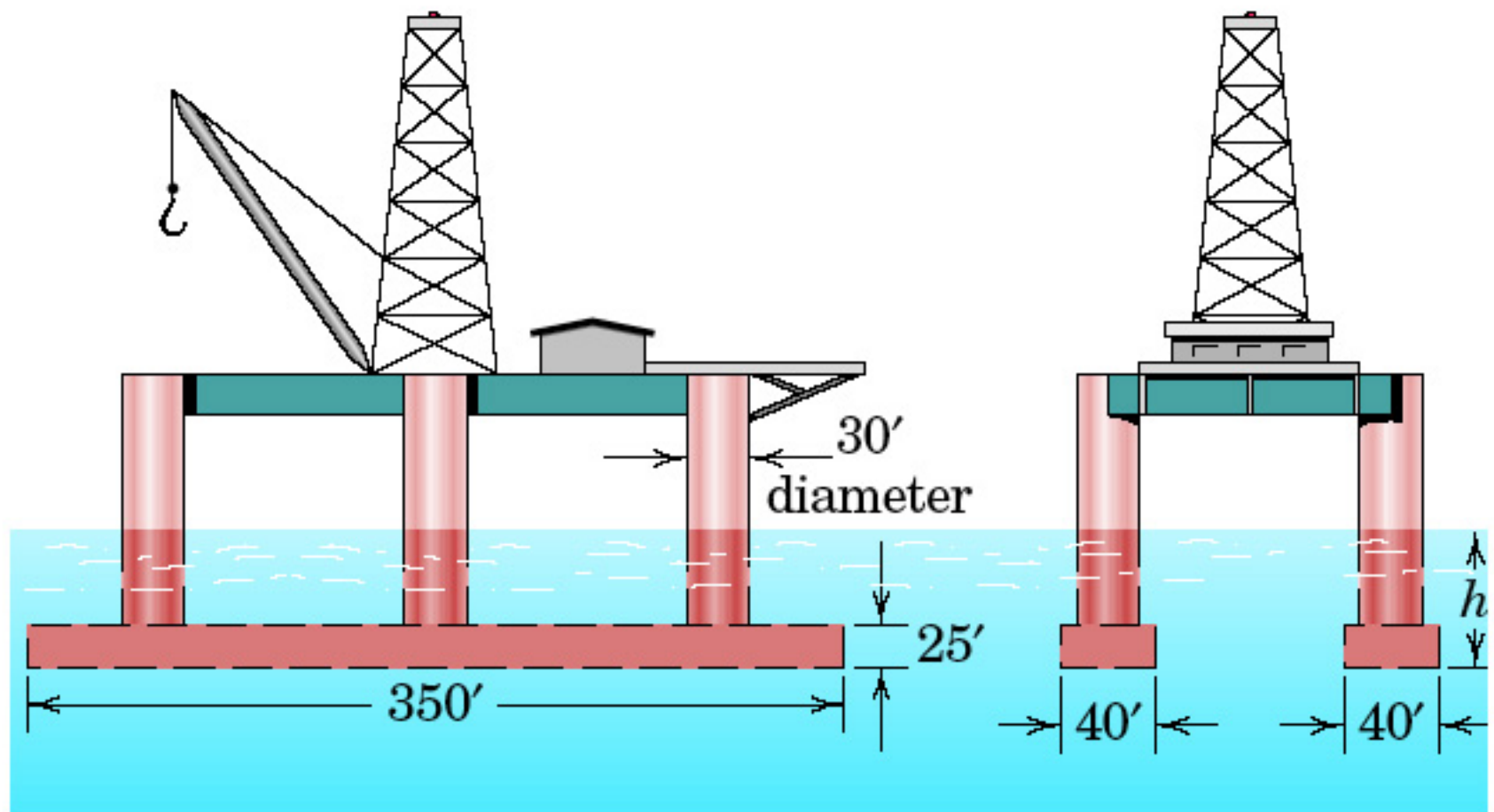
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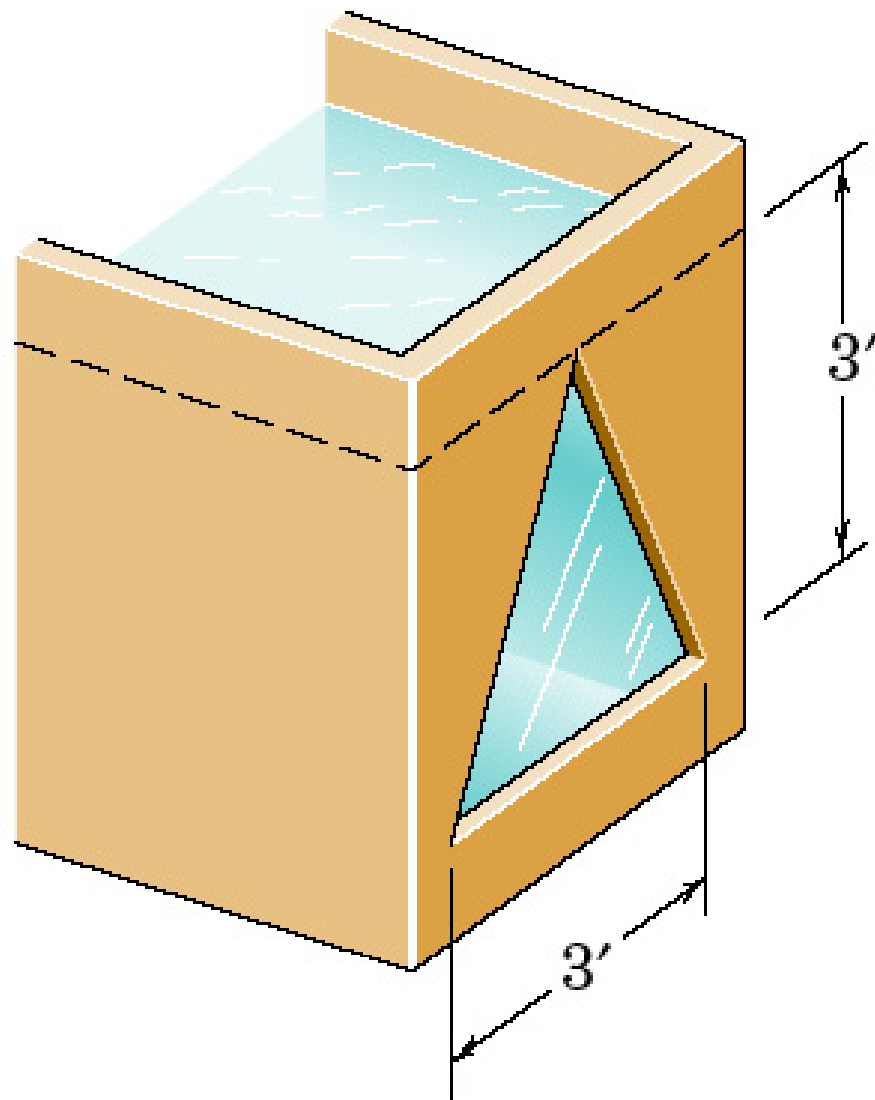
Problem 5/190



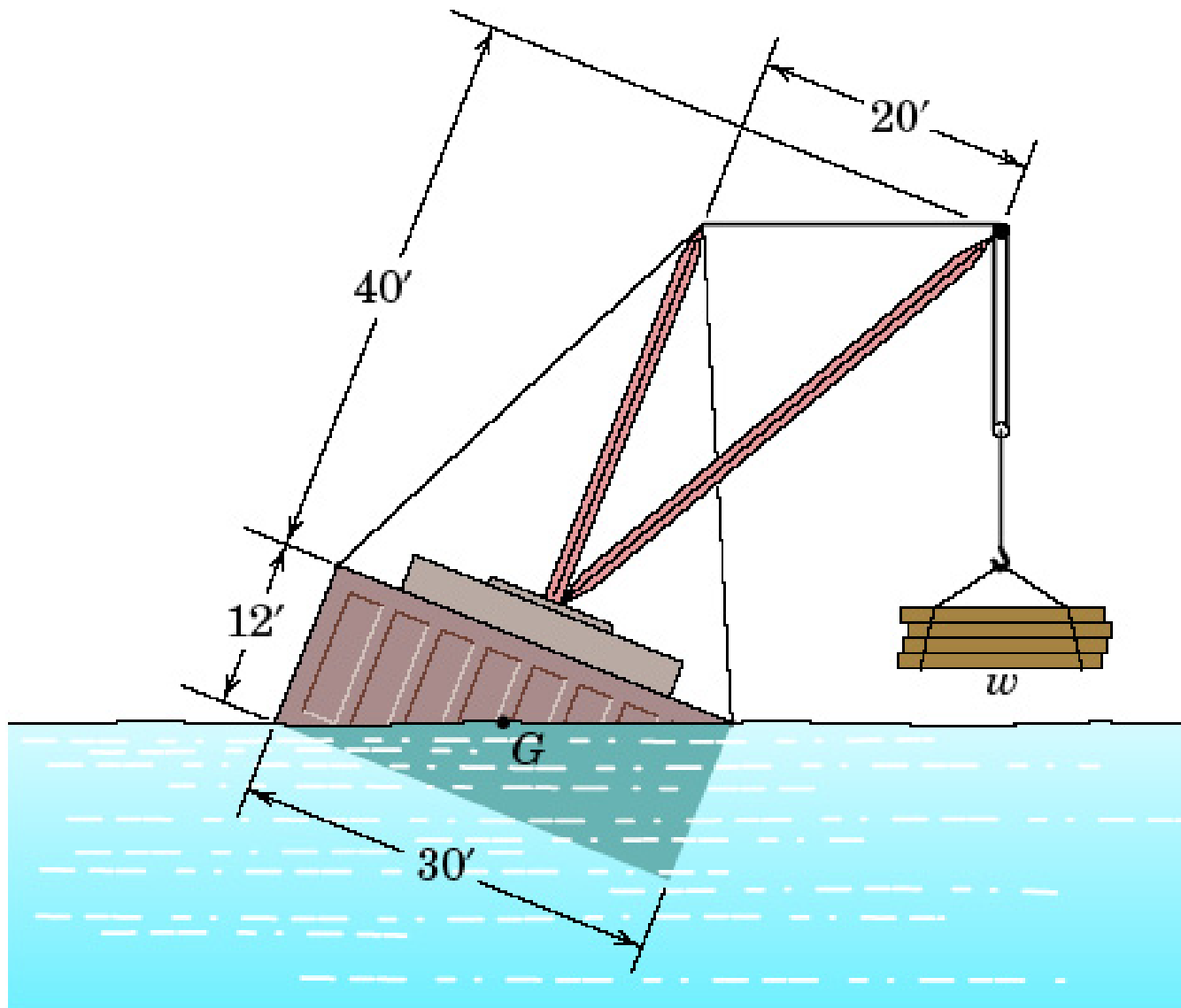
Side View

End View

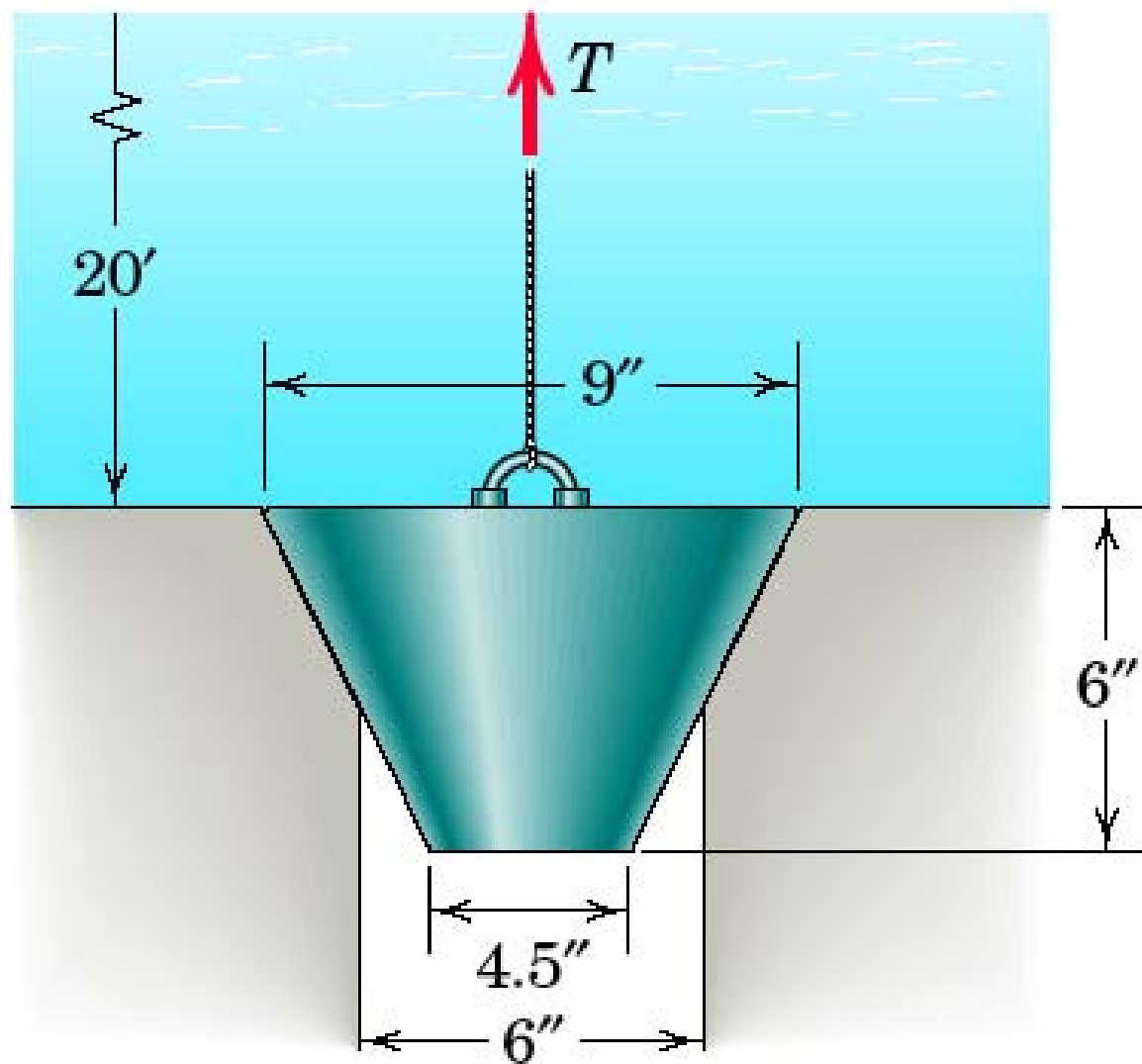
Problem 5/191



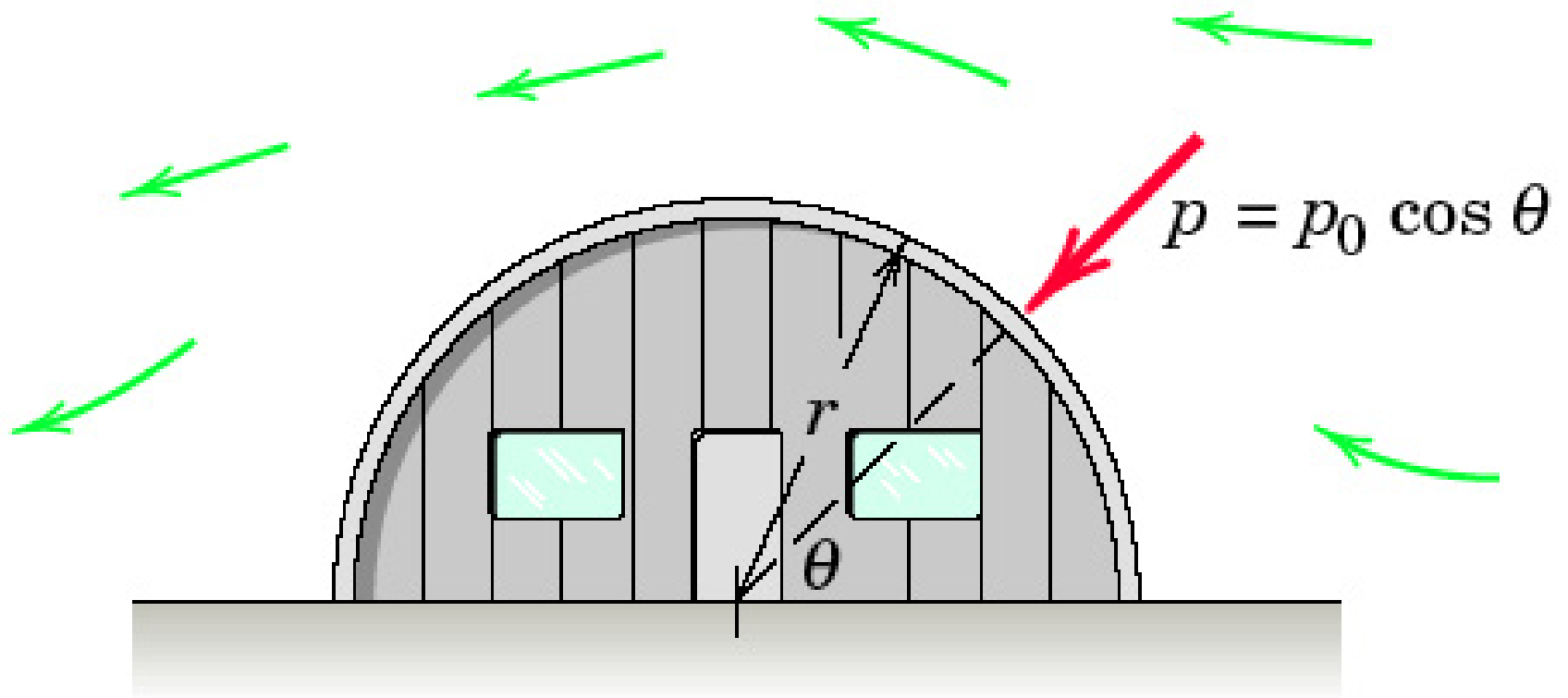
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Problem 5/193

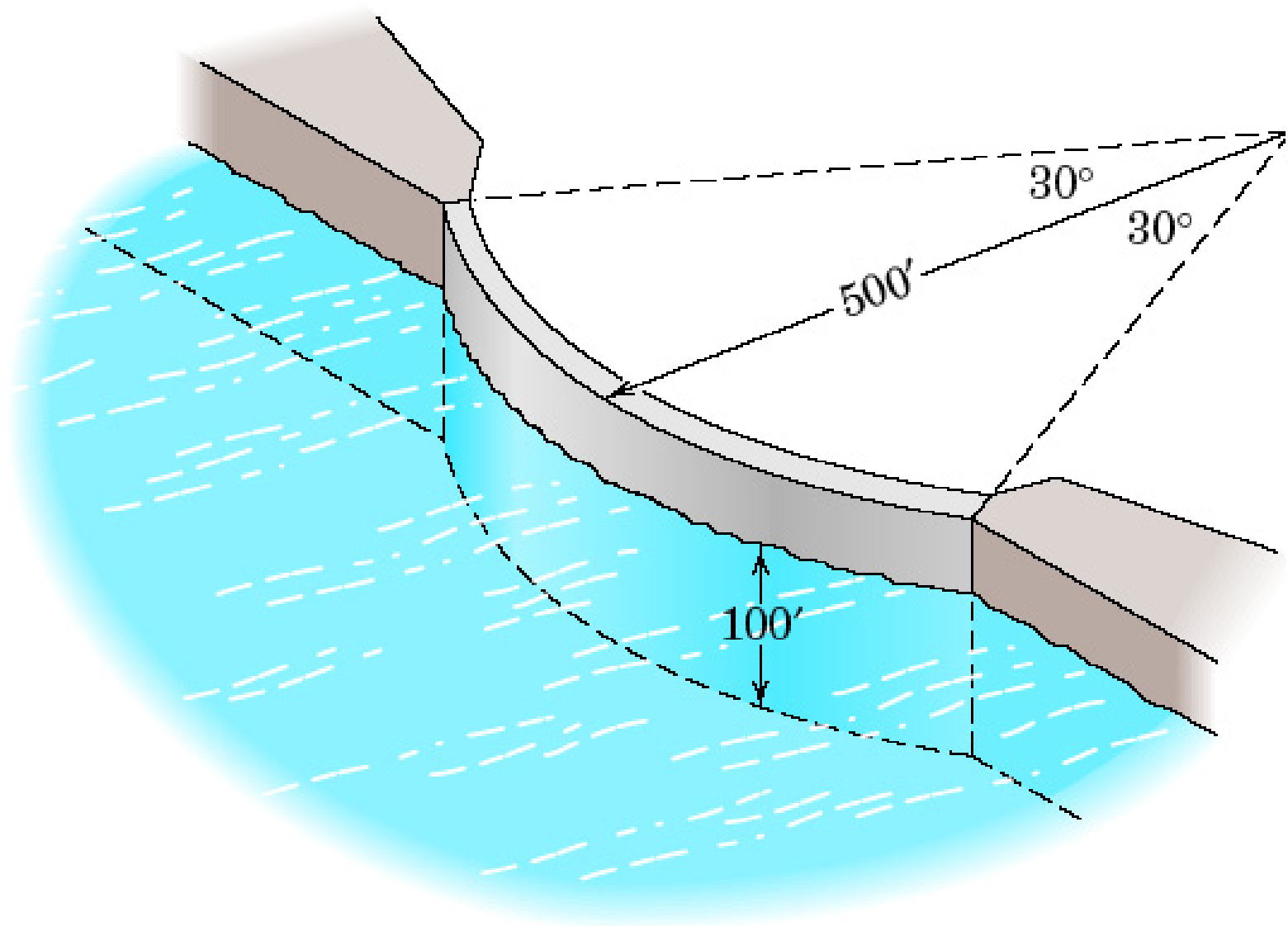


Problem 5/194

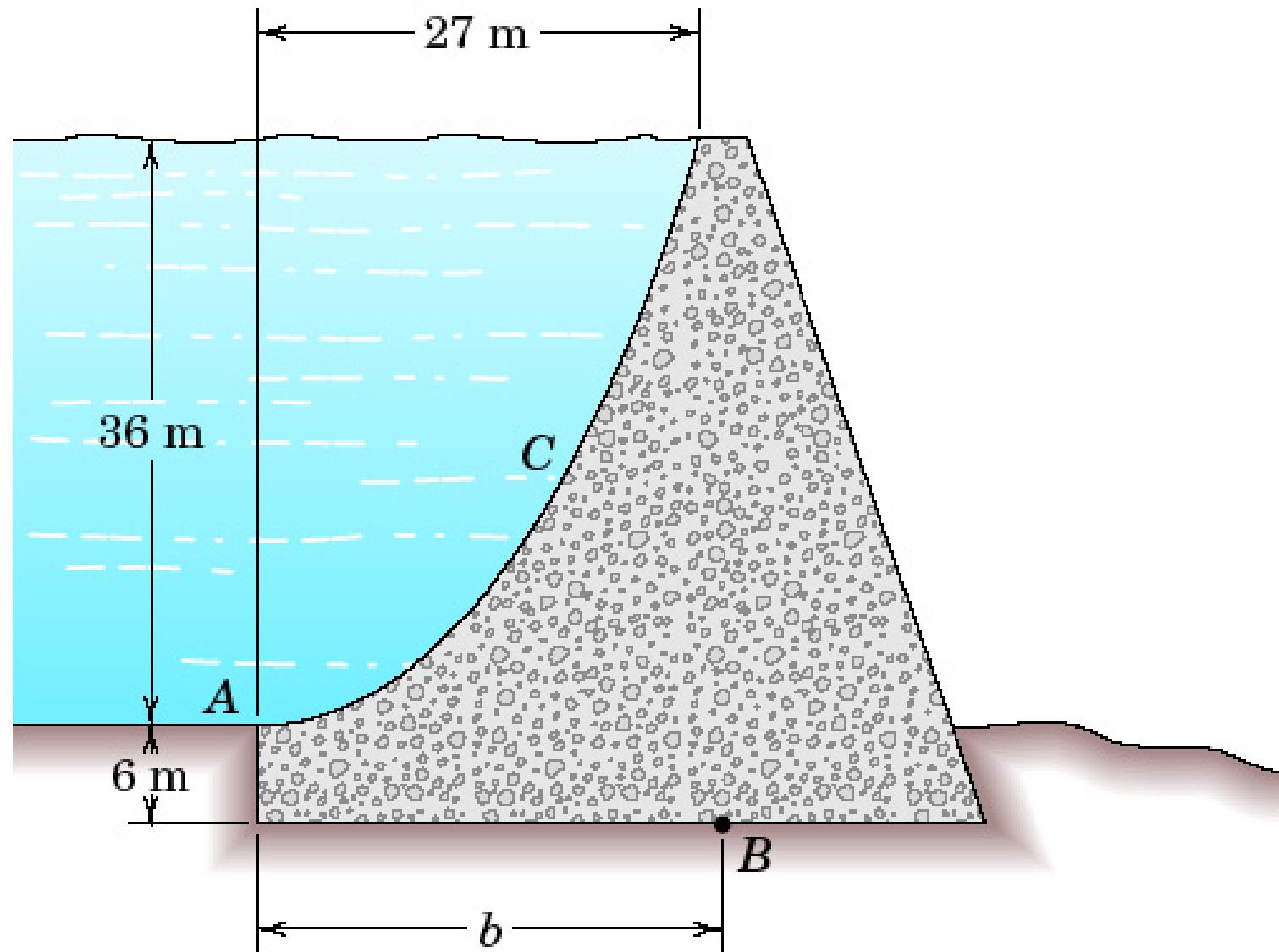


Problem 5/195

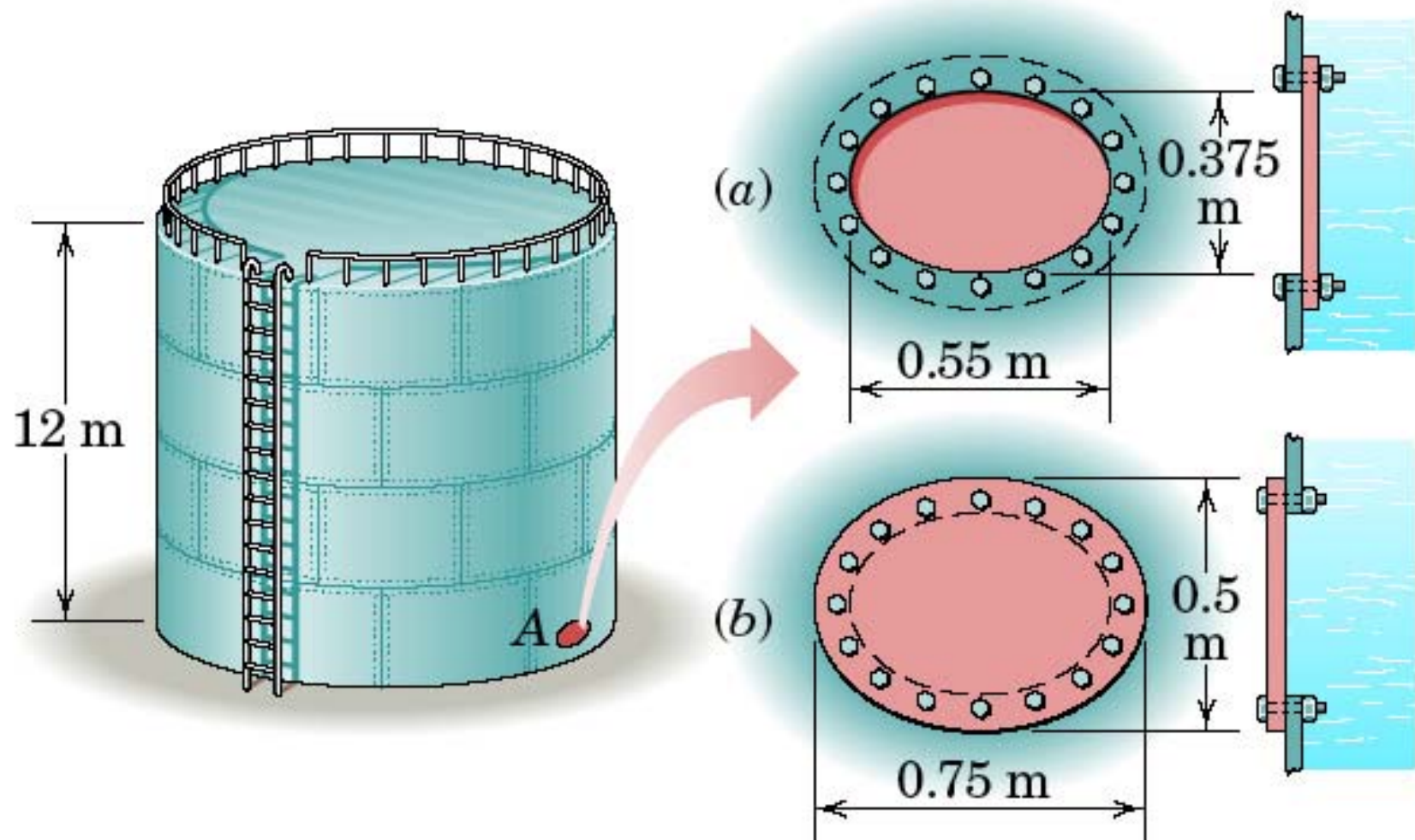
P_05_195



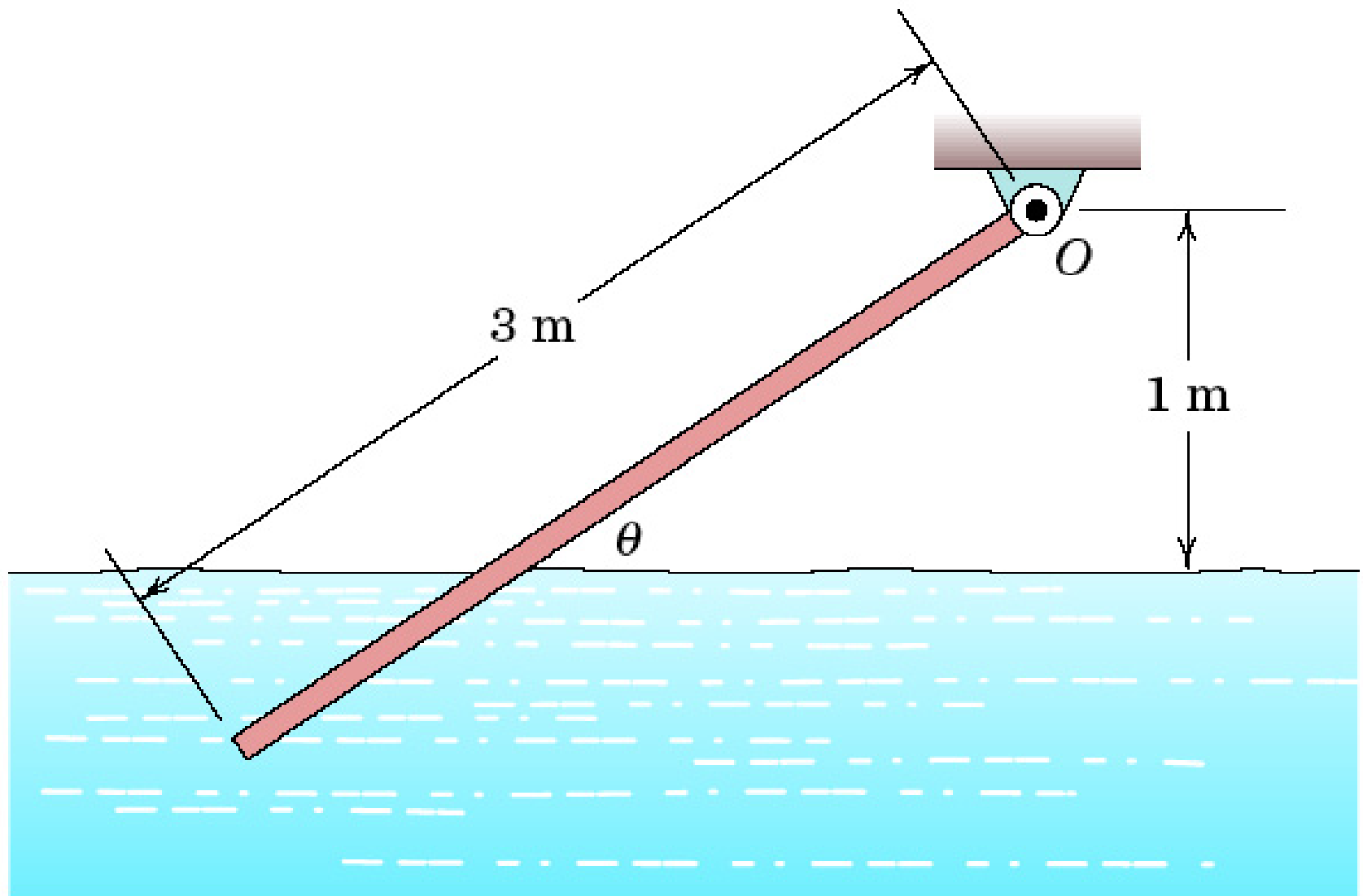
Problem 5/196



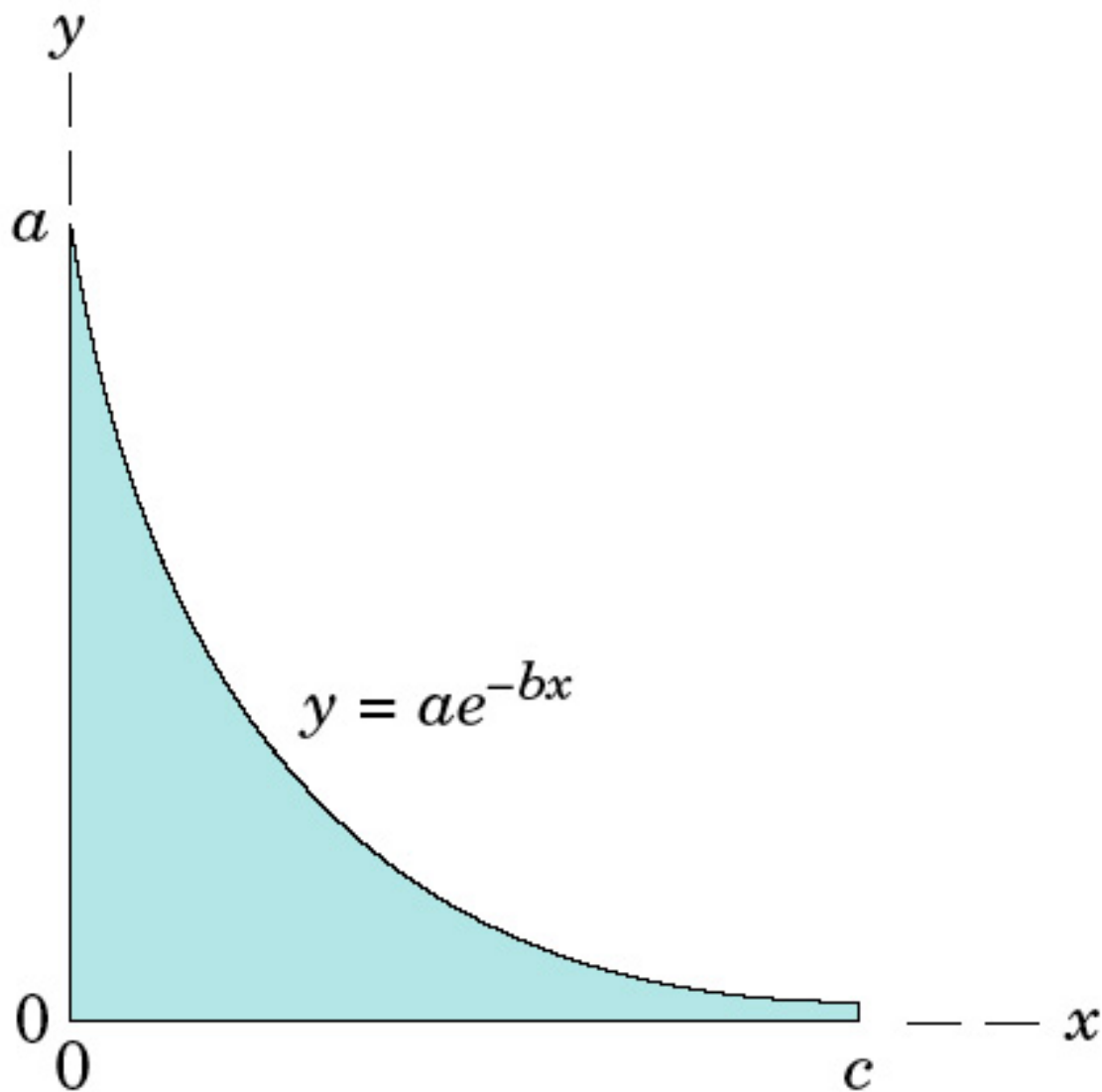
Problem 5/197



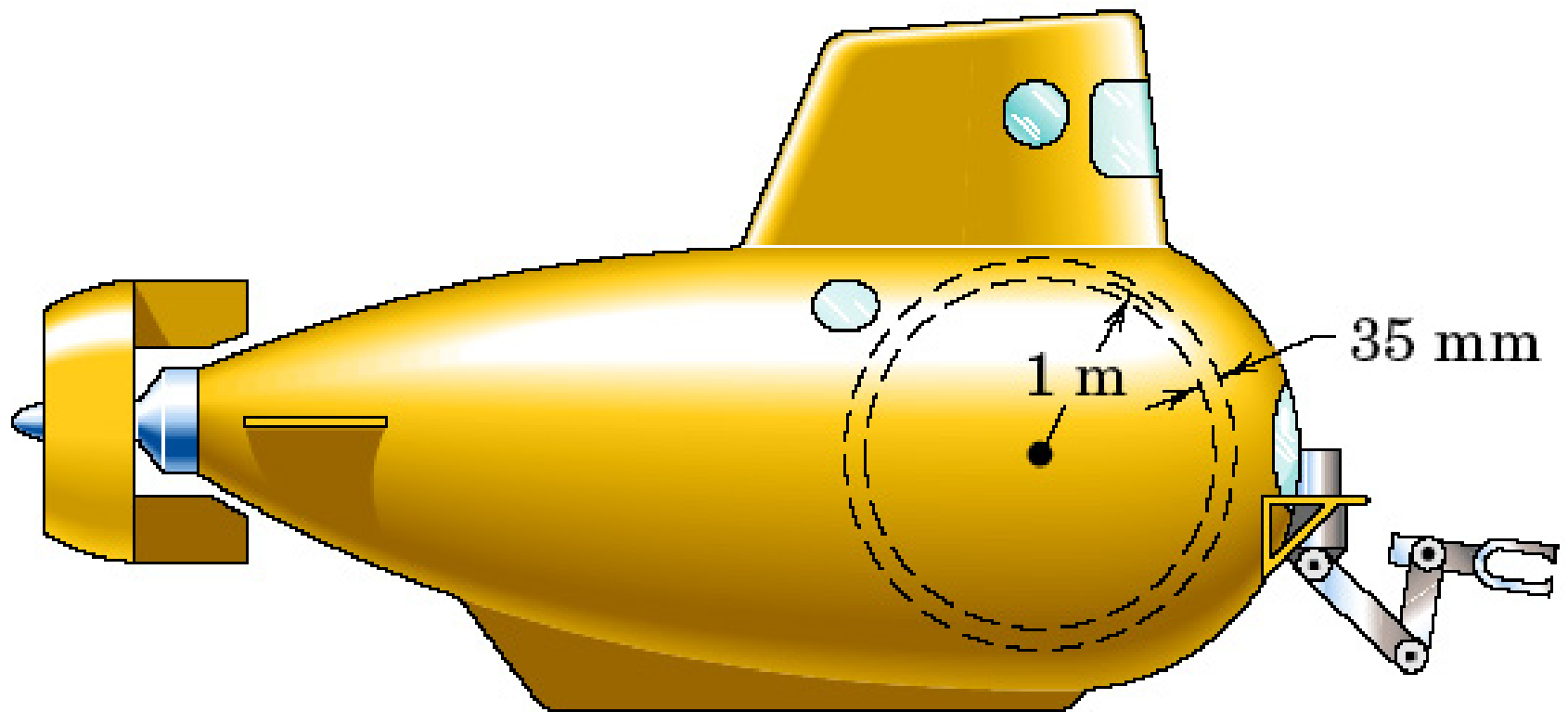
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Problem 5/199

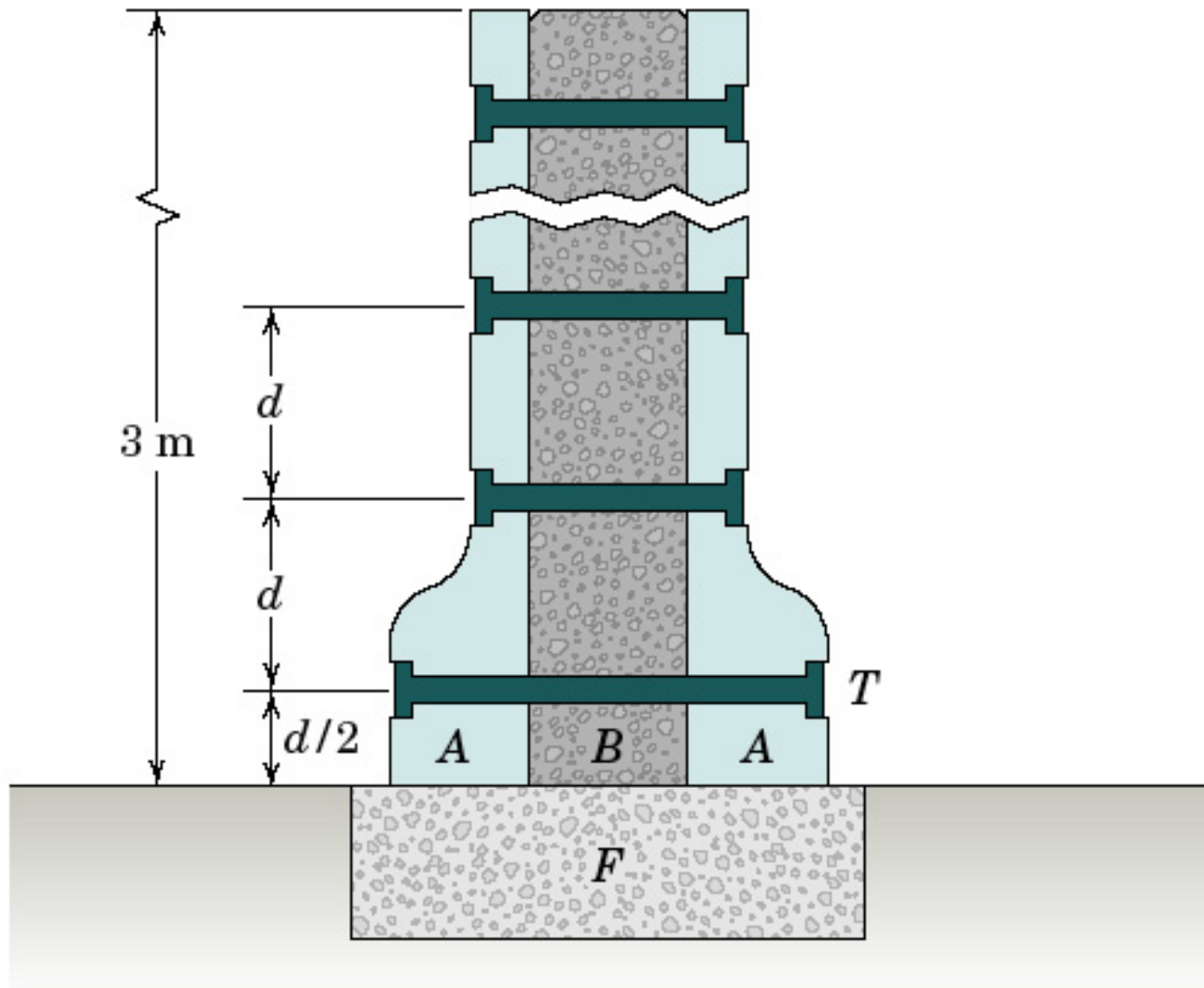


Problem 5/20

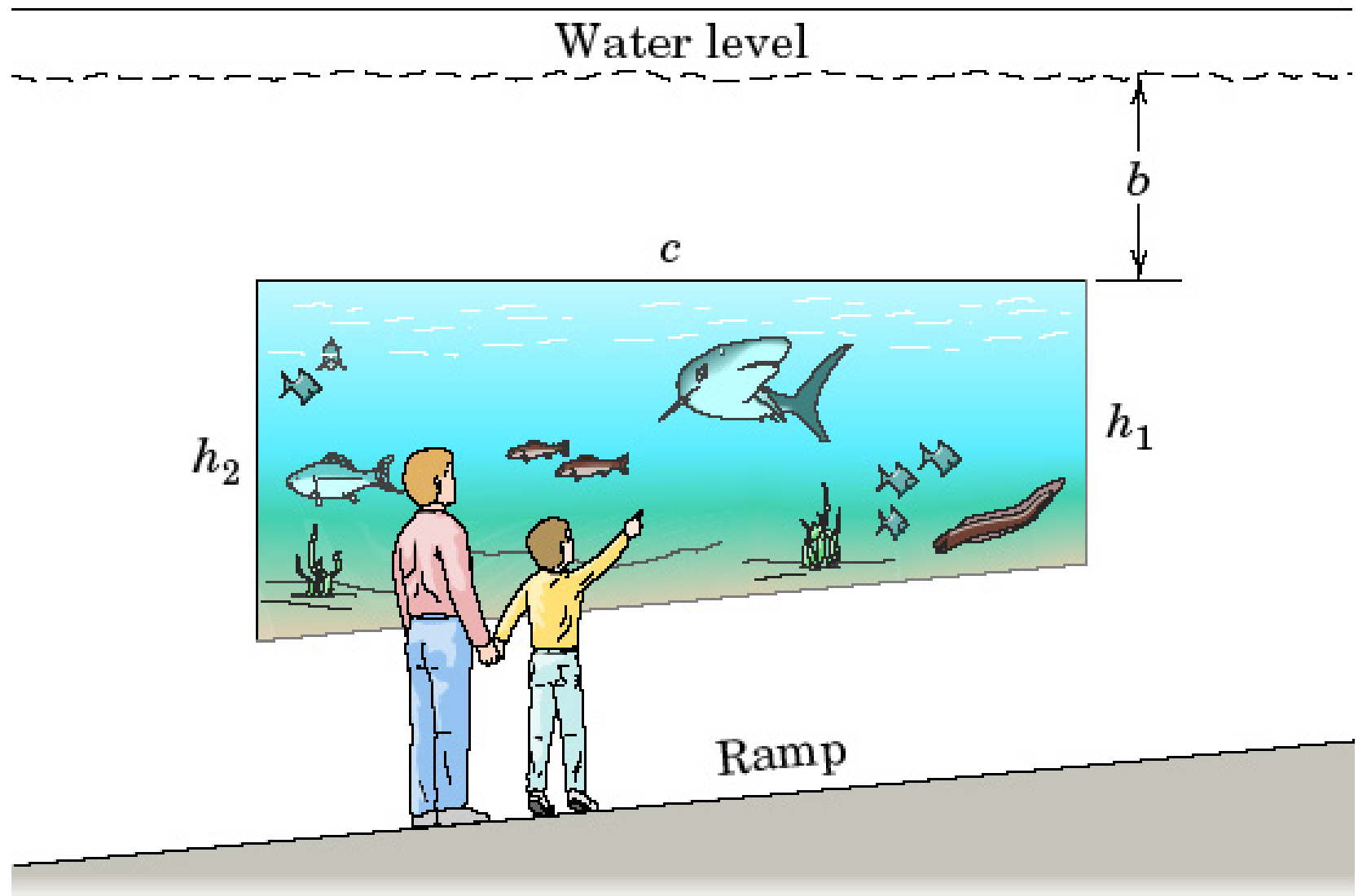


Problem 5/200

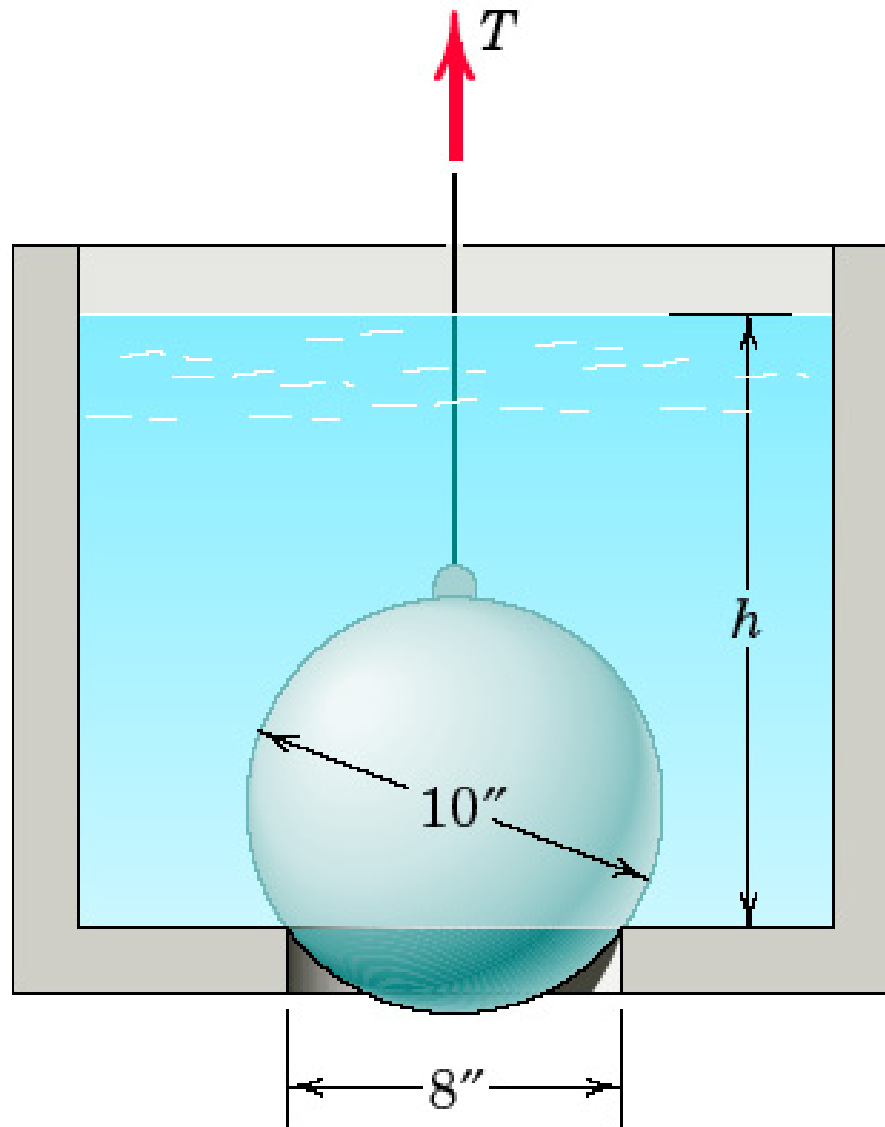
P_05_200



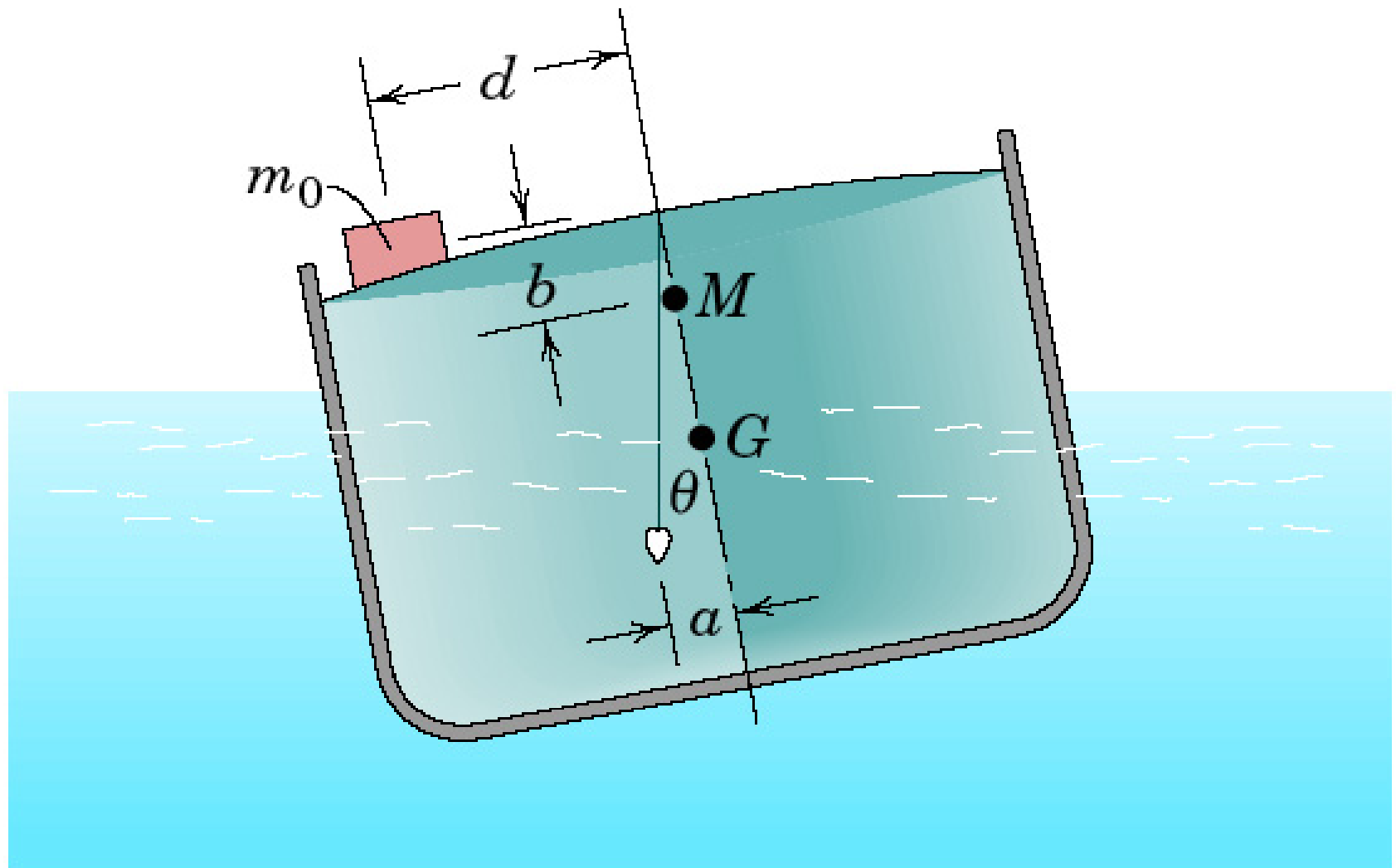
Problem 5/201



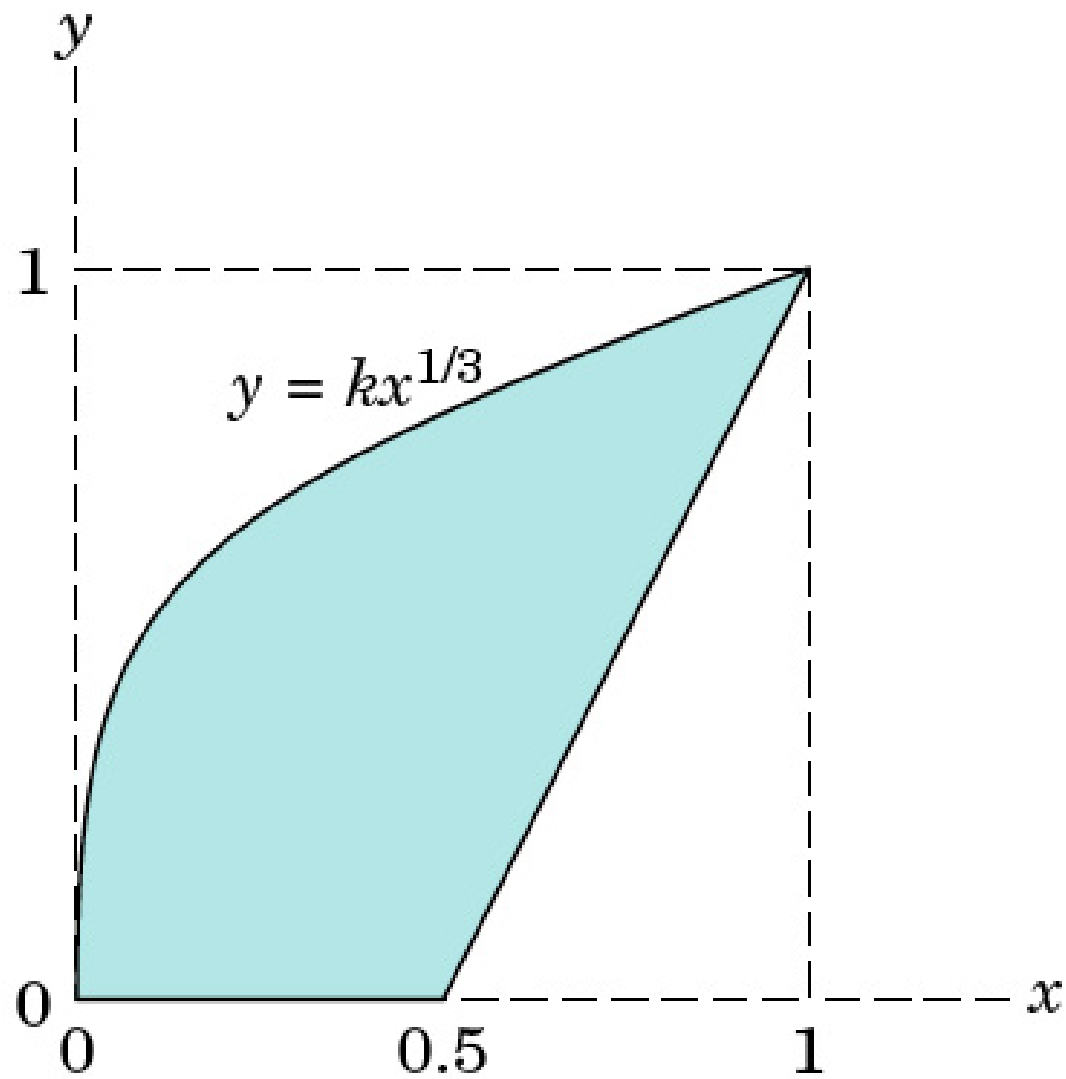
Problem 5/202



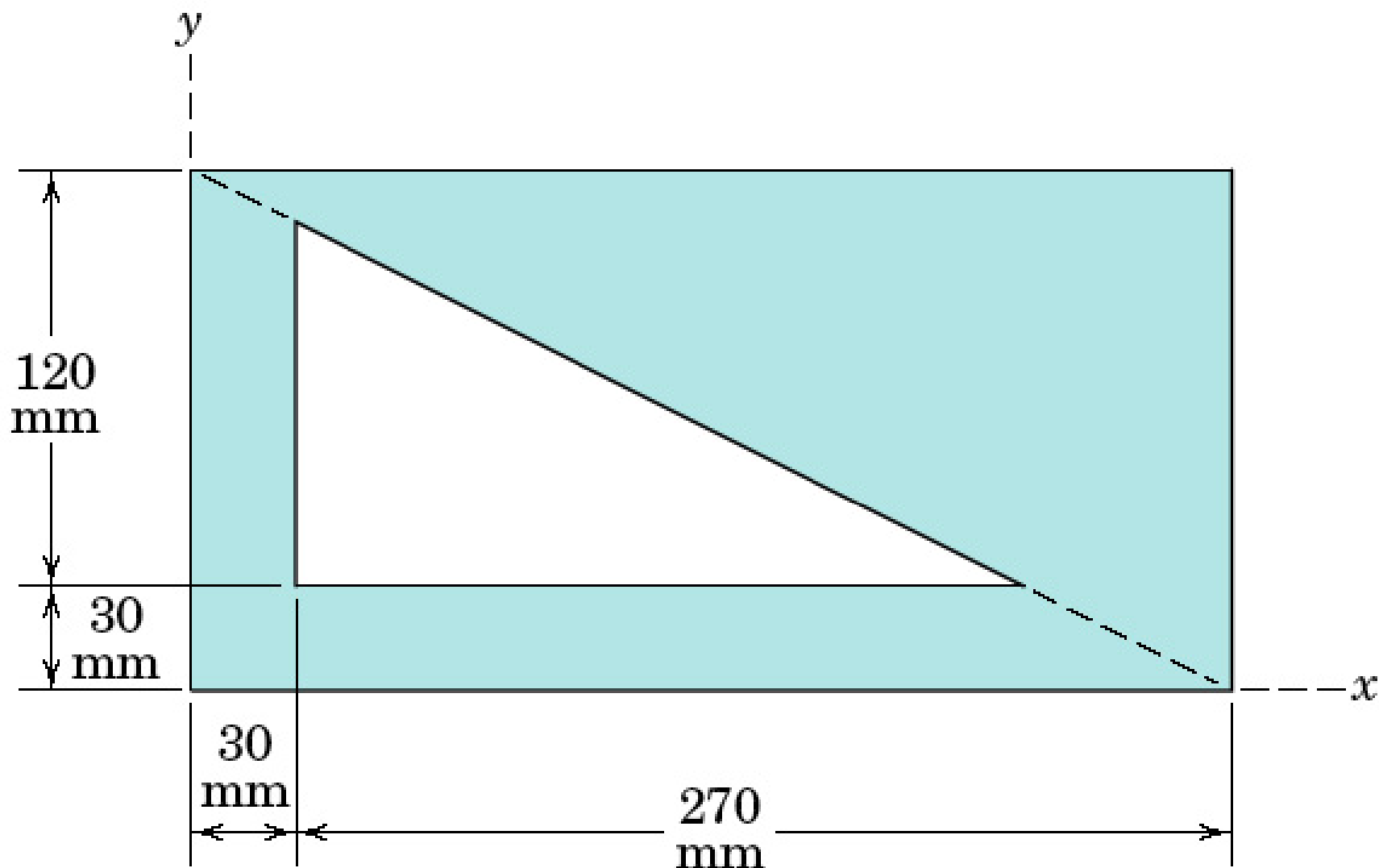
Problem 5/203



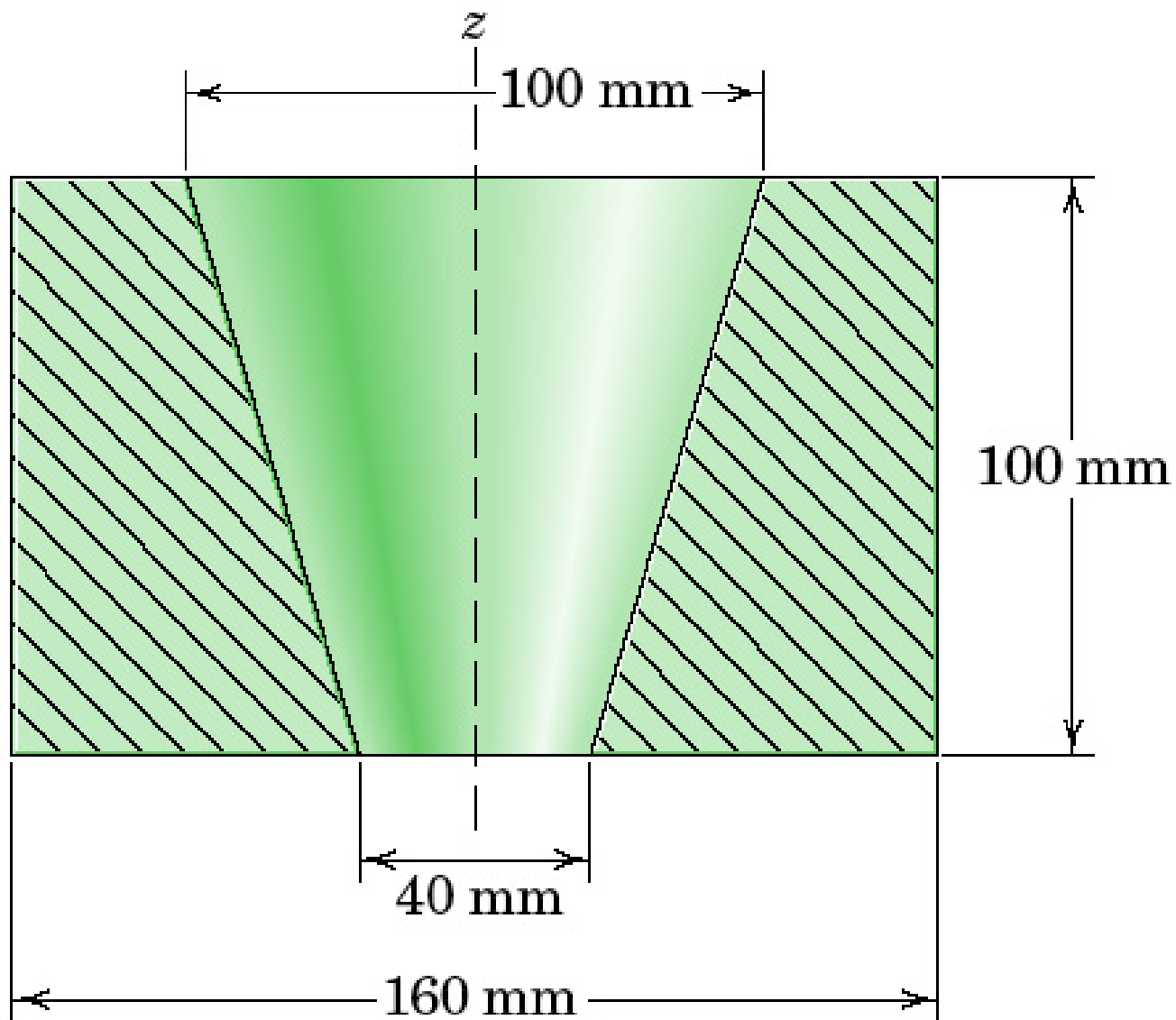
Problem 5/204



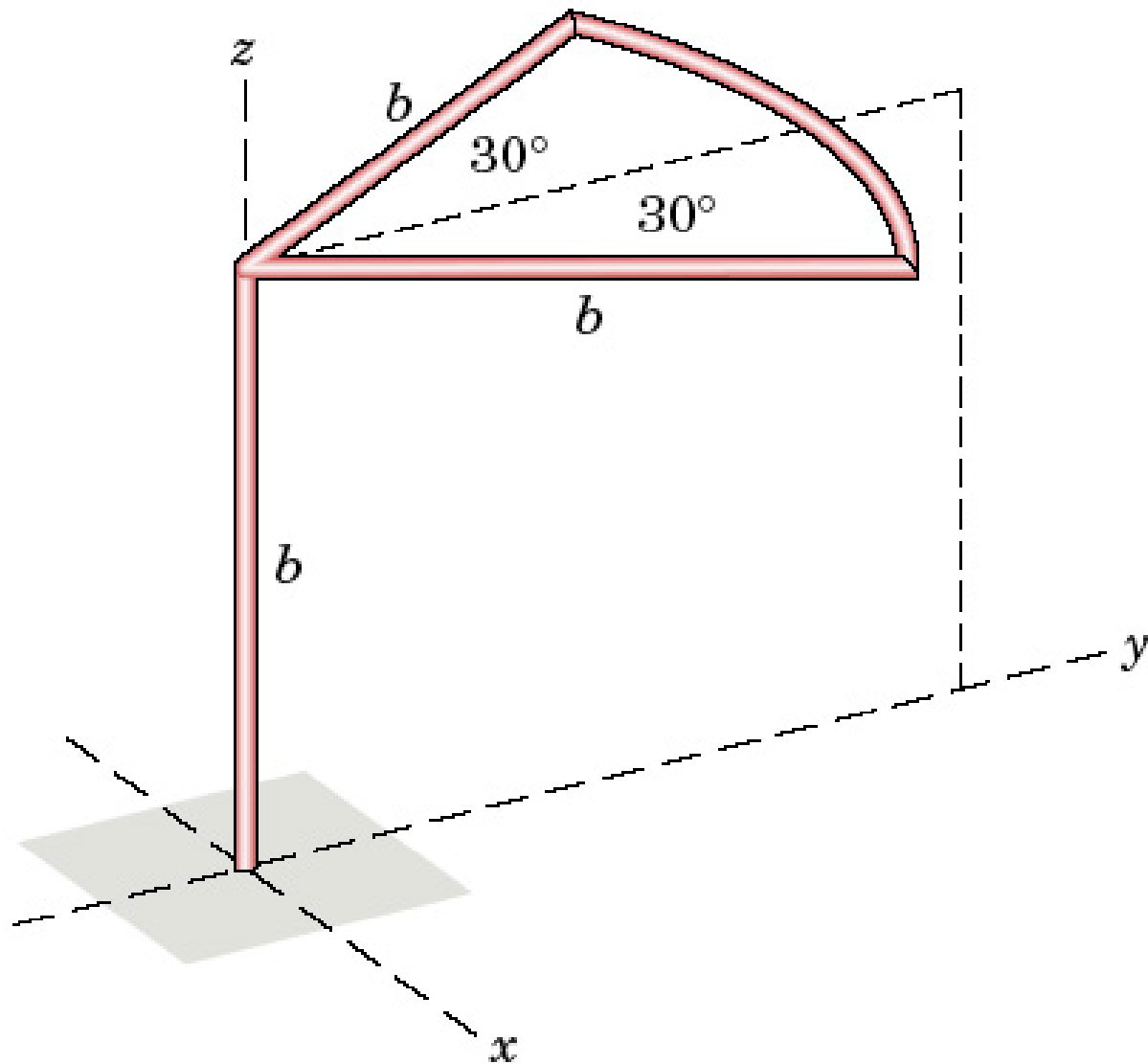
Problem 5/205



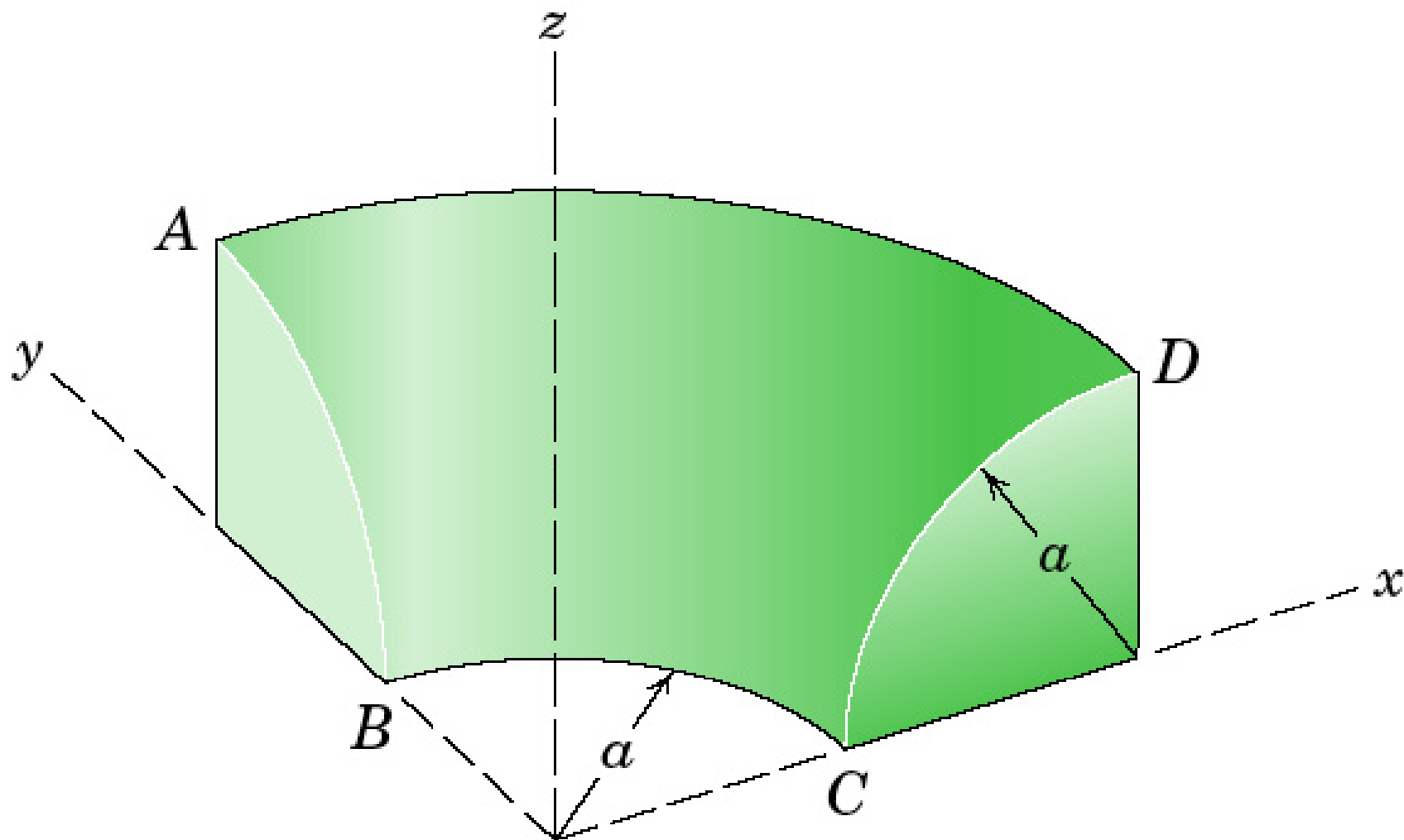
Problem 5/206



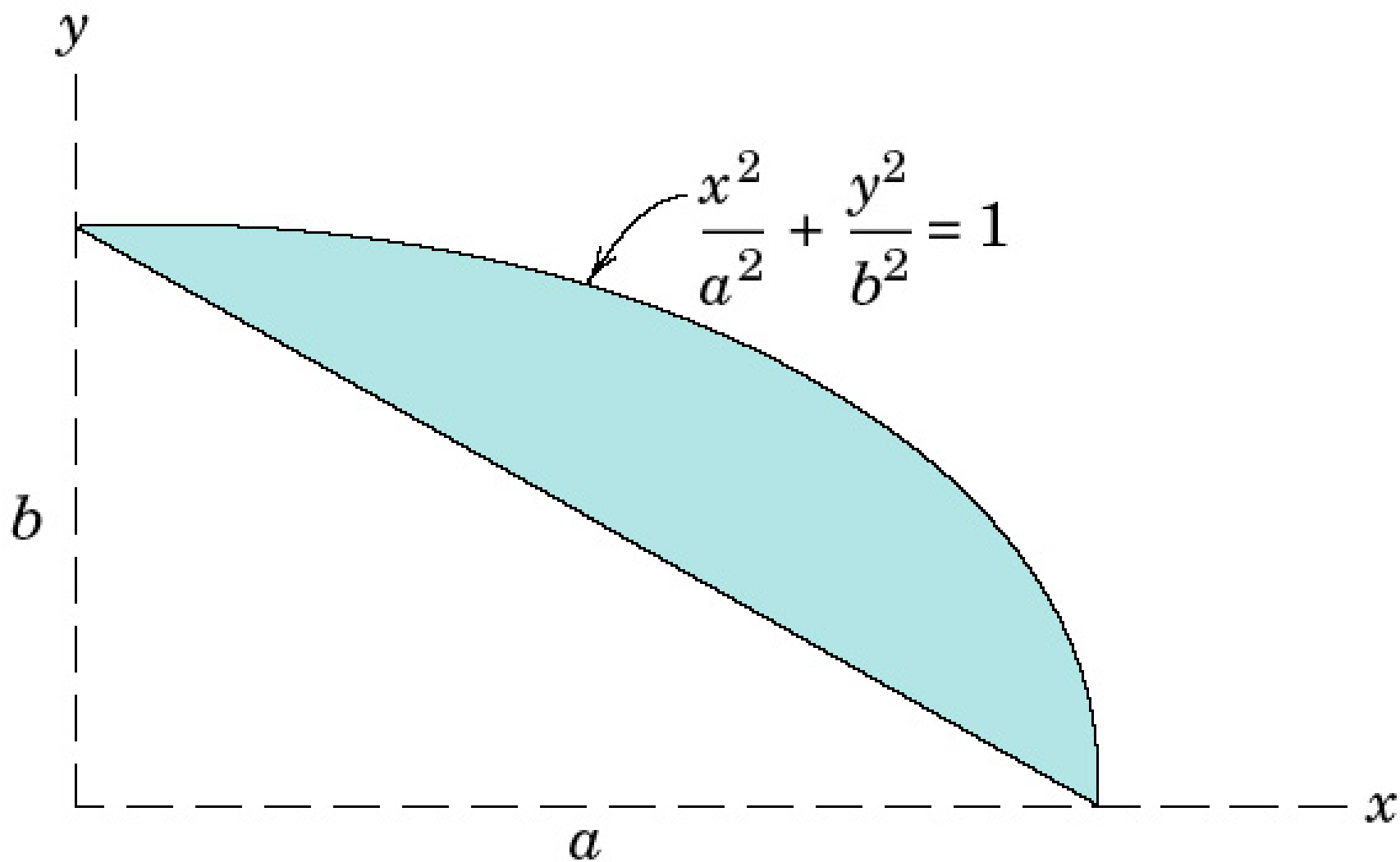
Problem 5/207



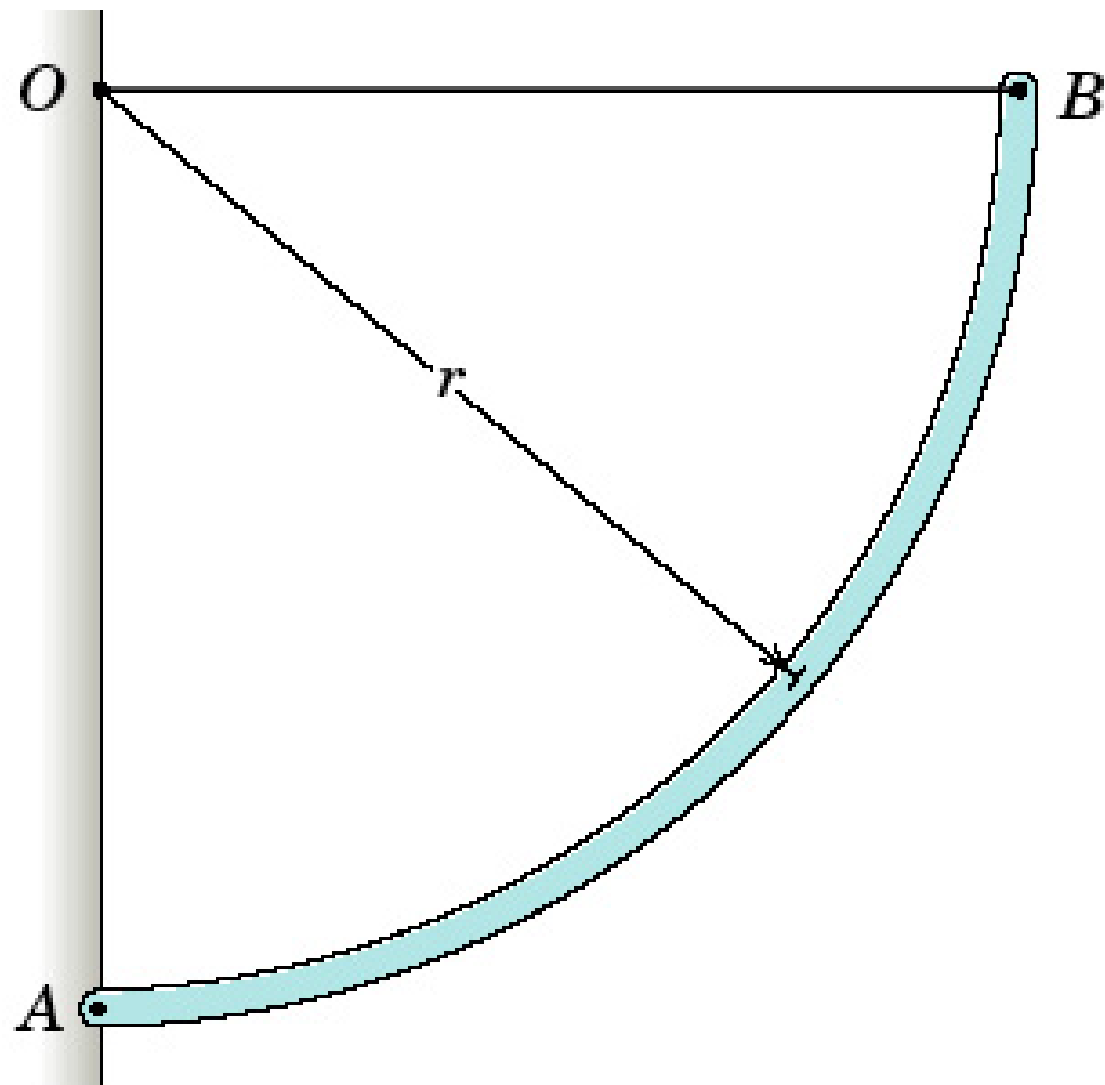
Problem 5/208



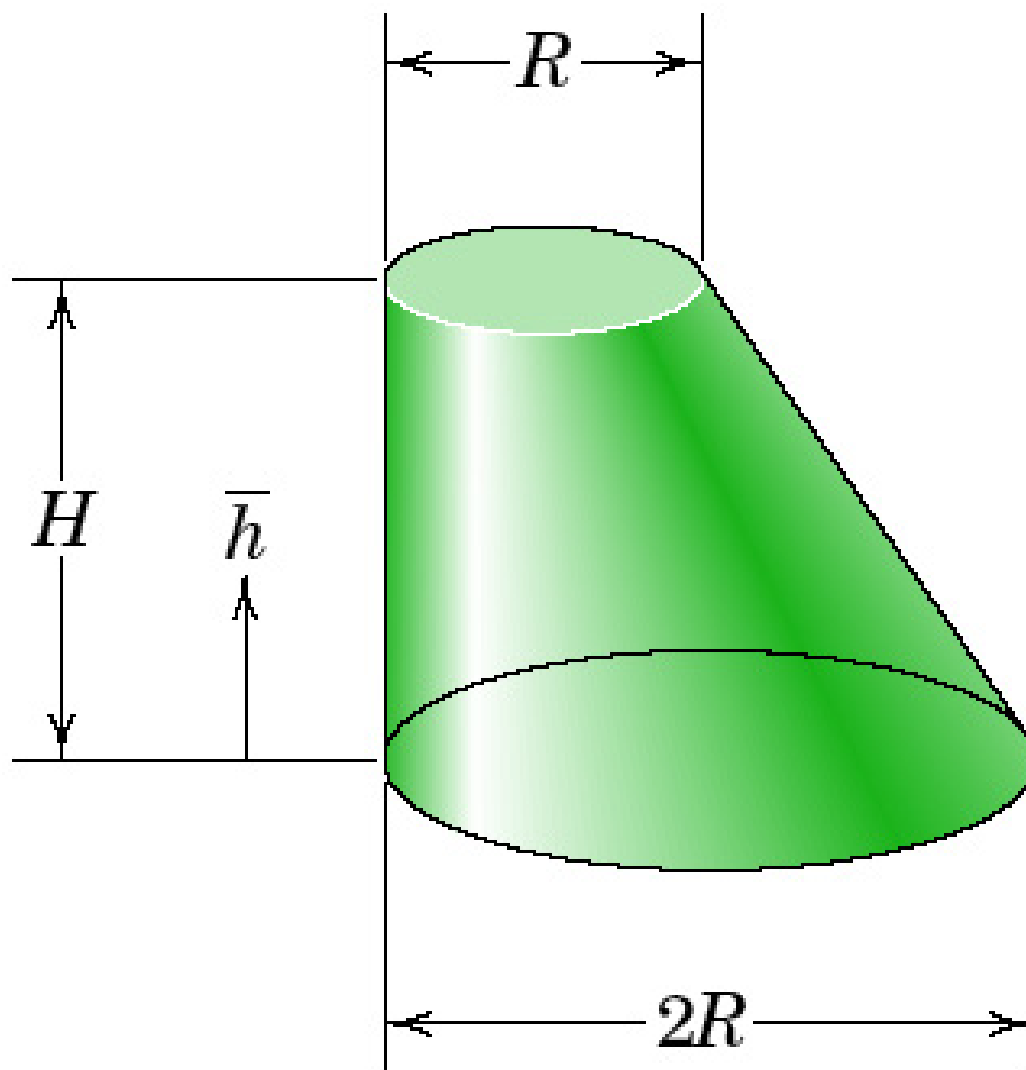
Problem 5/209



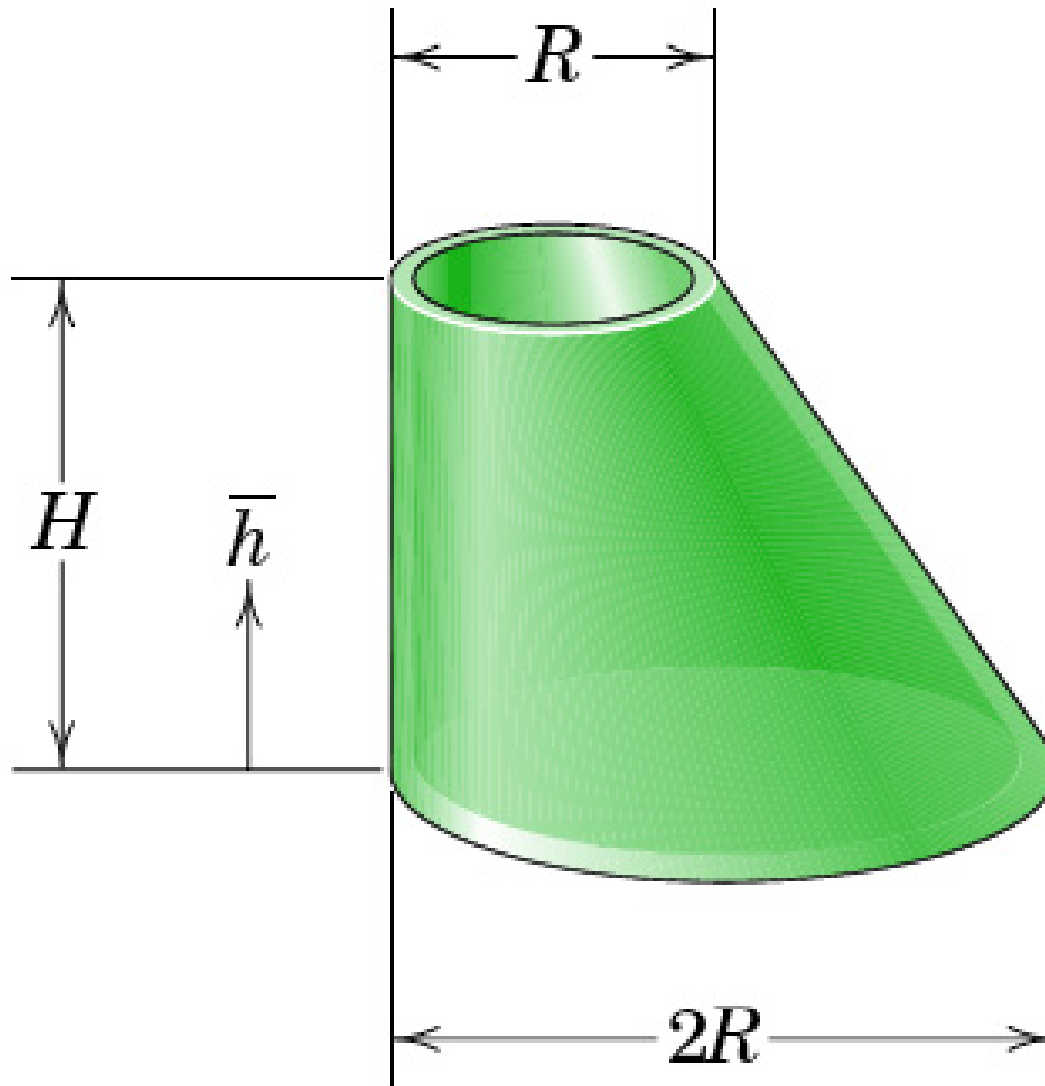
Problem 5/21



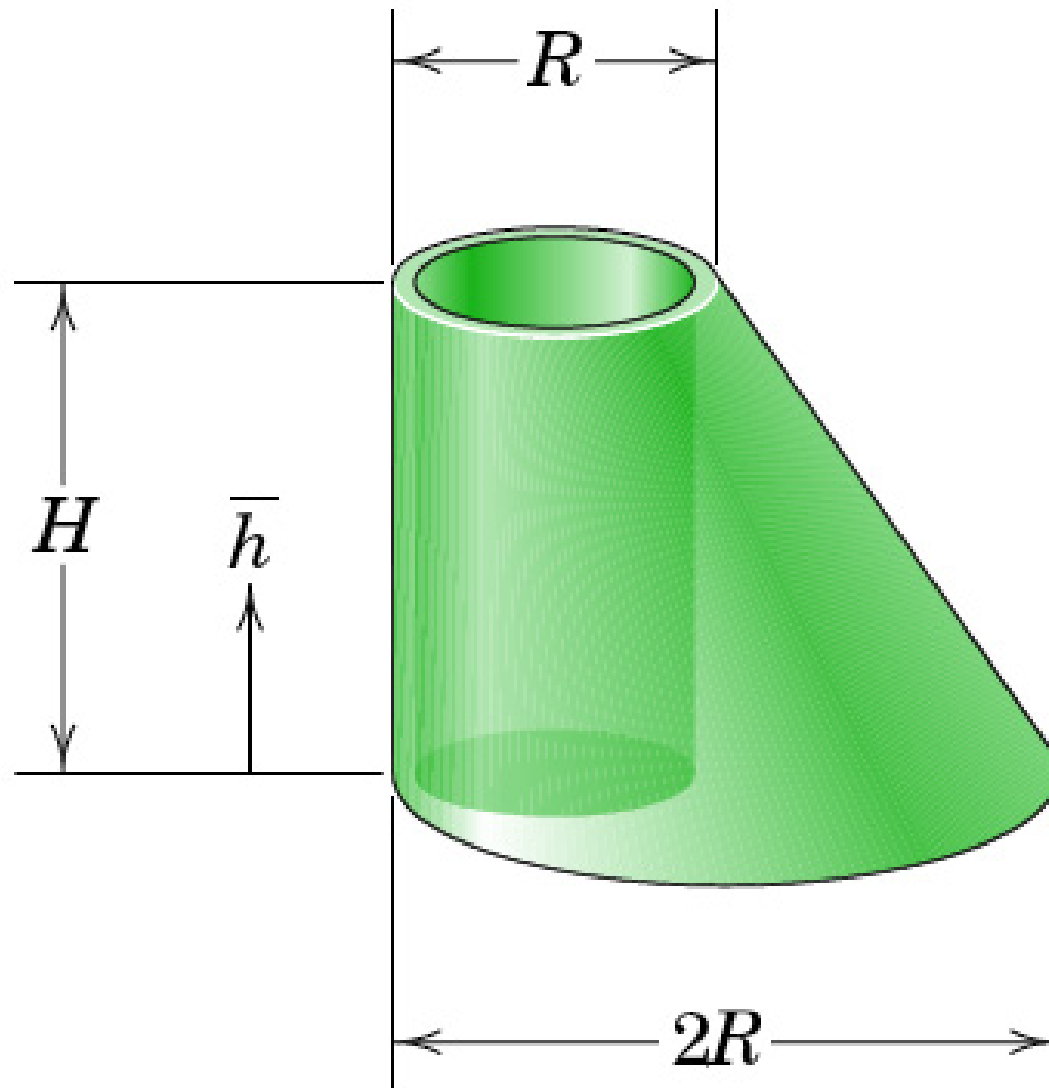
Problem 5/210



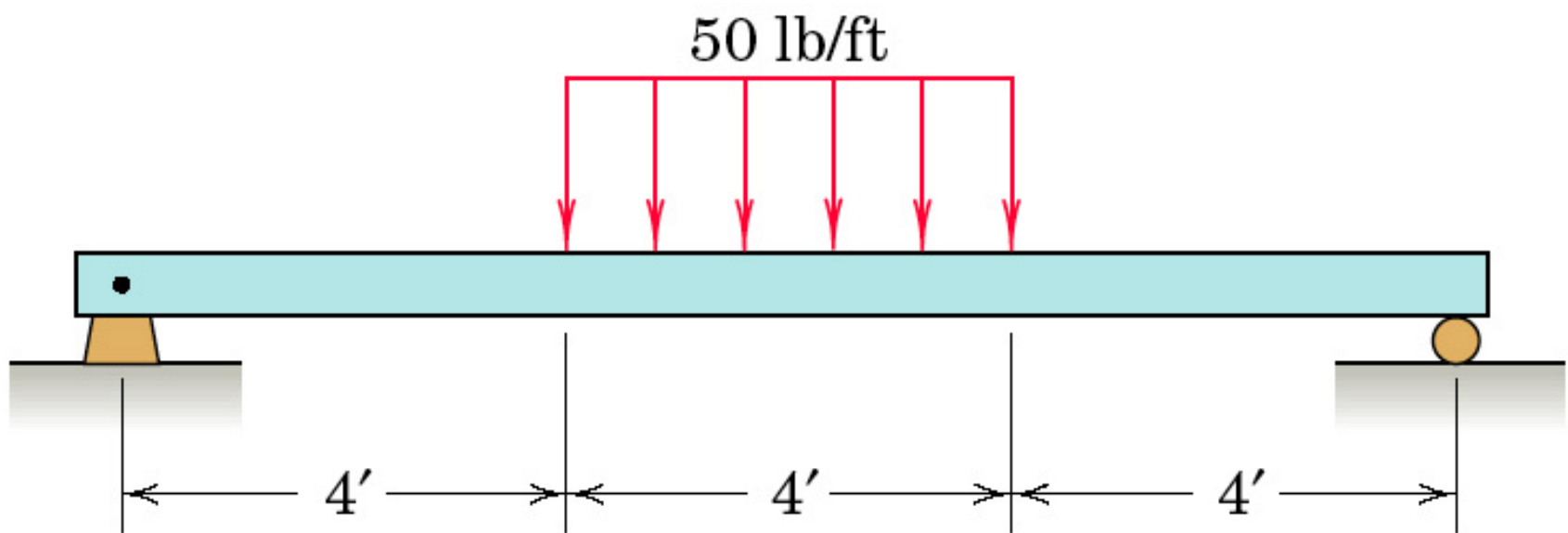
Problem 5/211



Problem 5/212

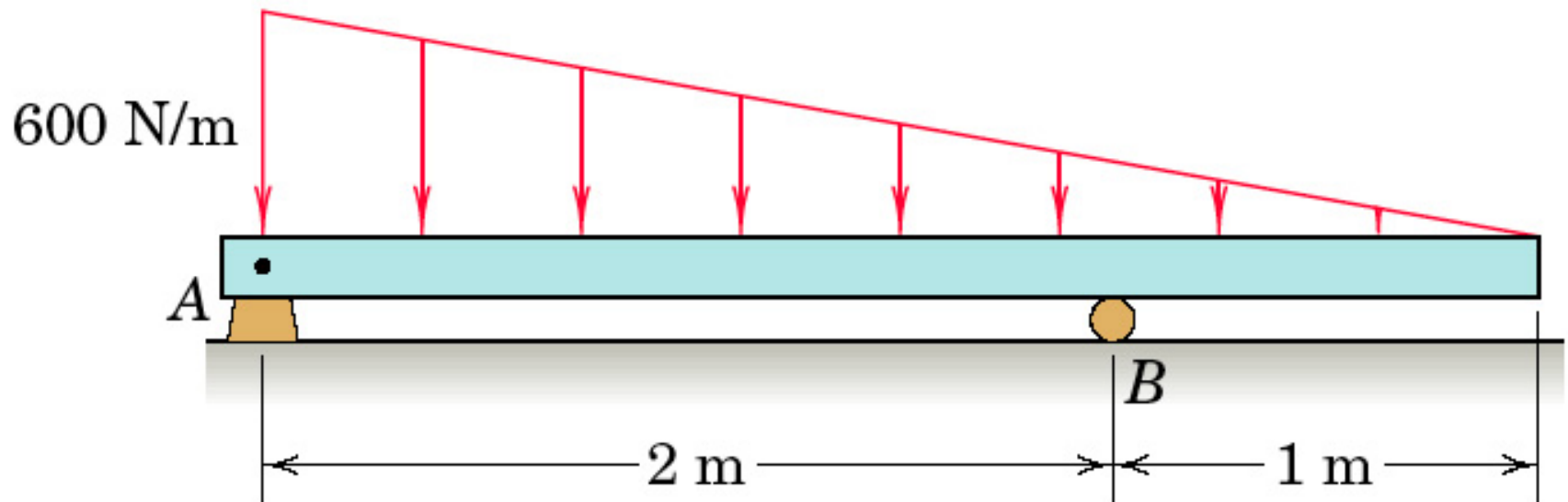


Problem 5/213



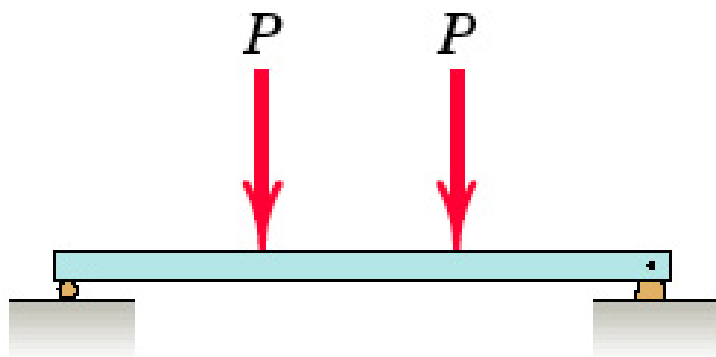
Problem 5/214

P_05_214

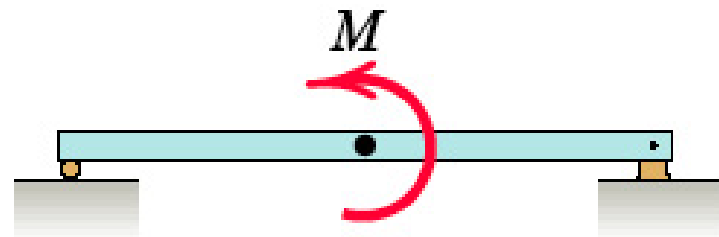


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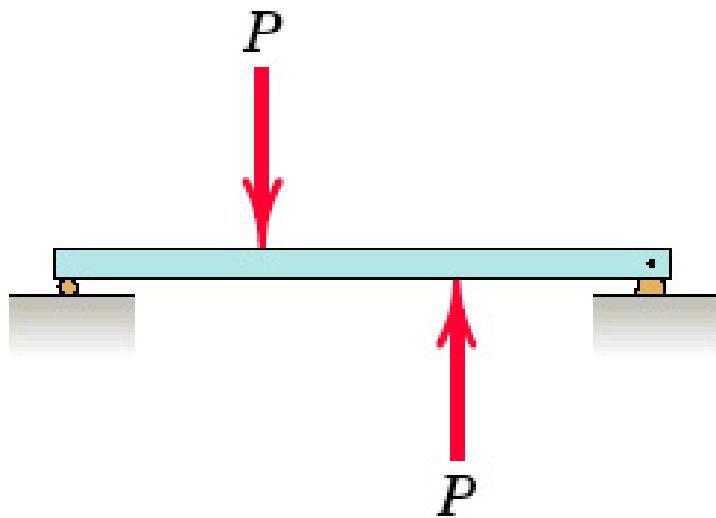
P_05_215



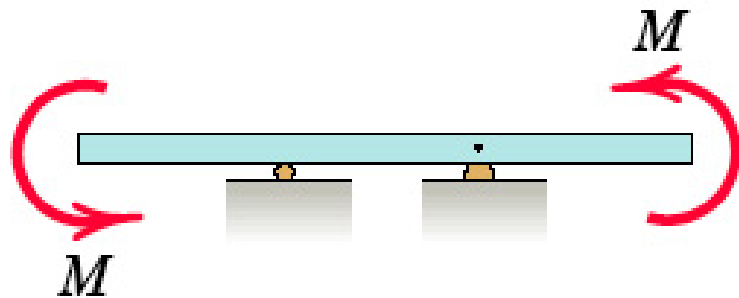
(a)



(c)

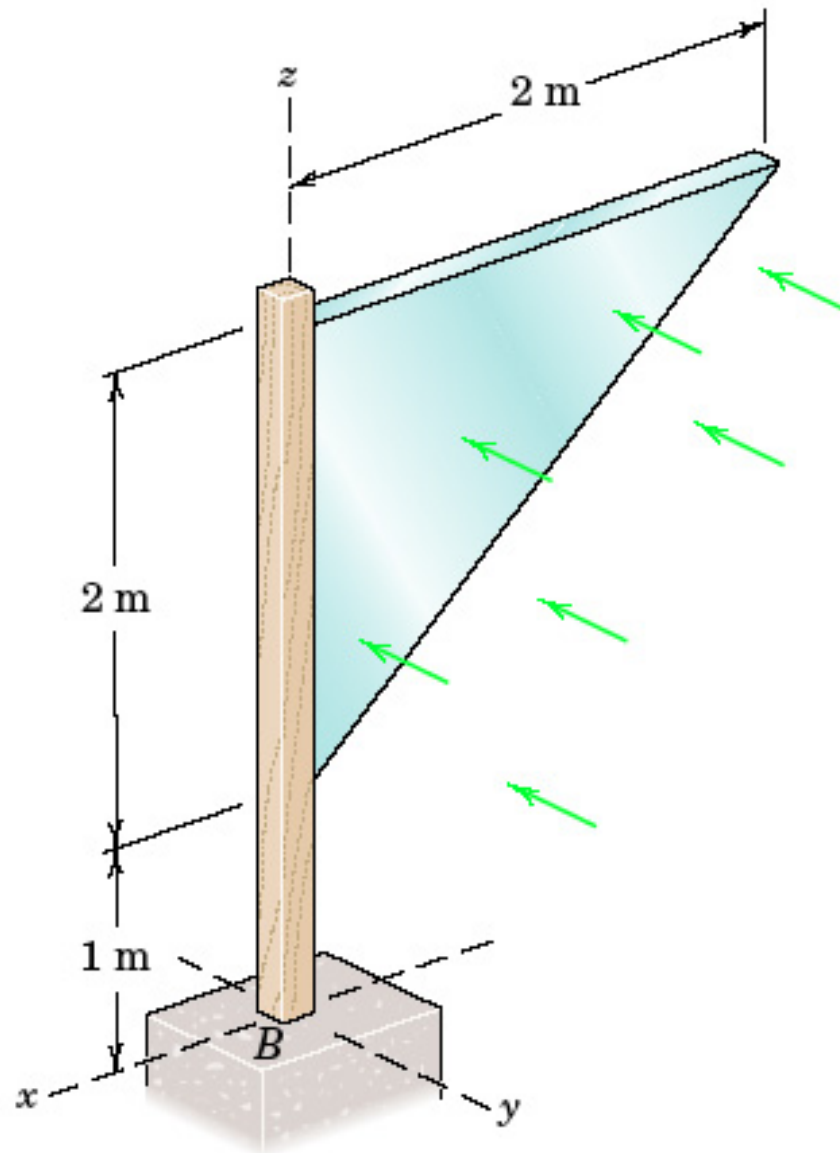


(b)

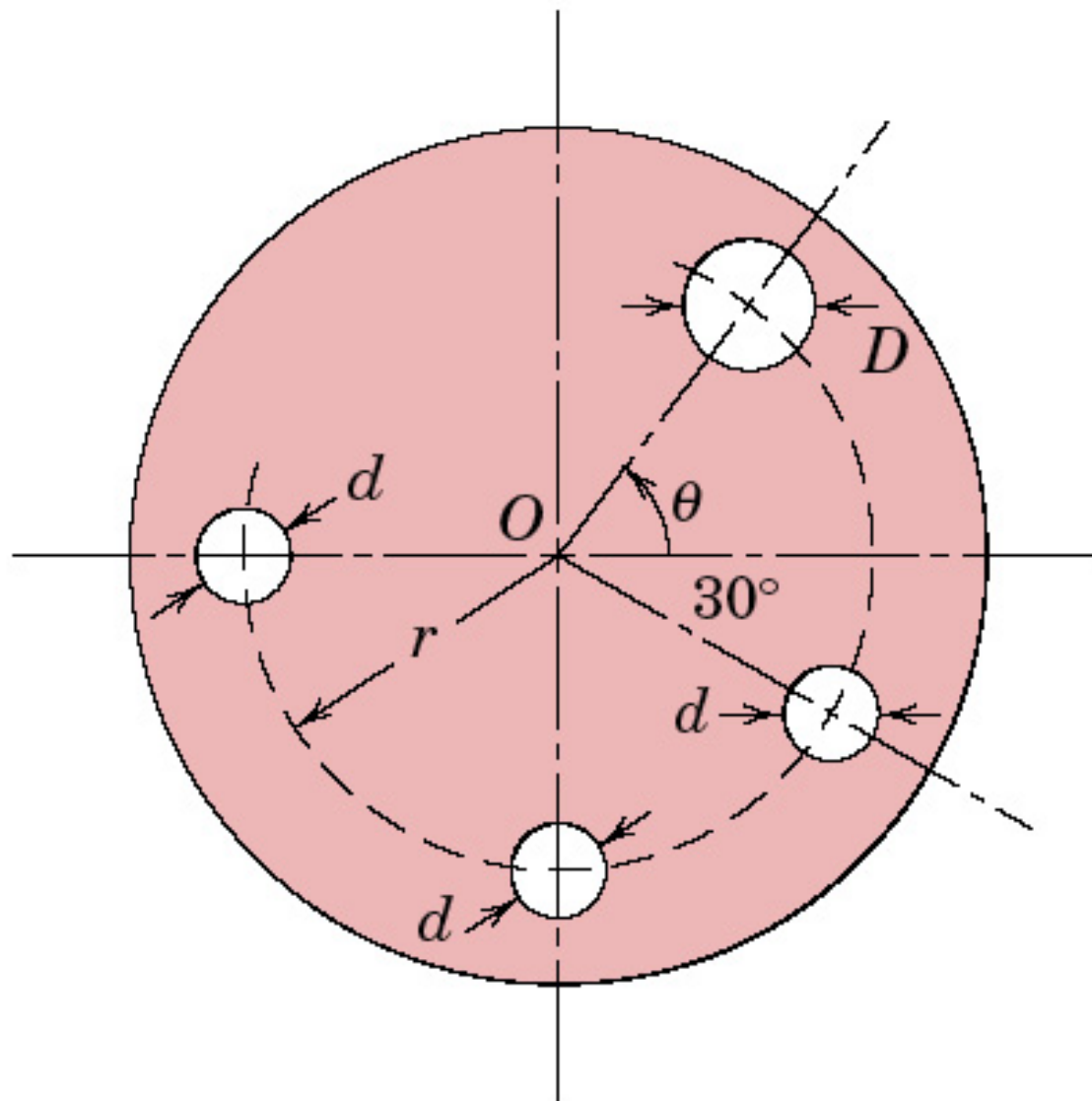


(d)

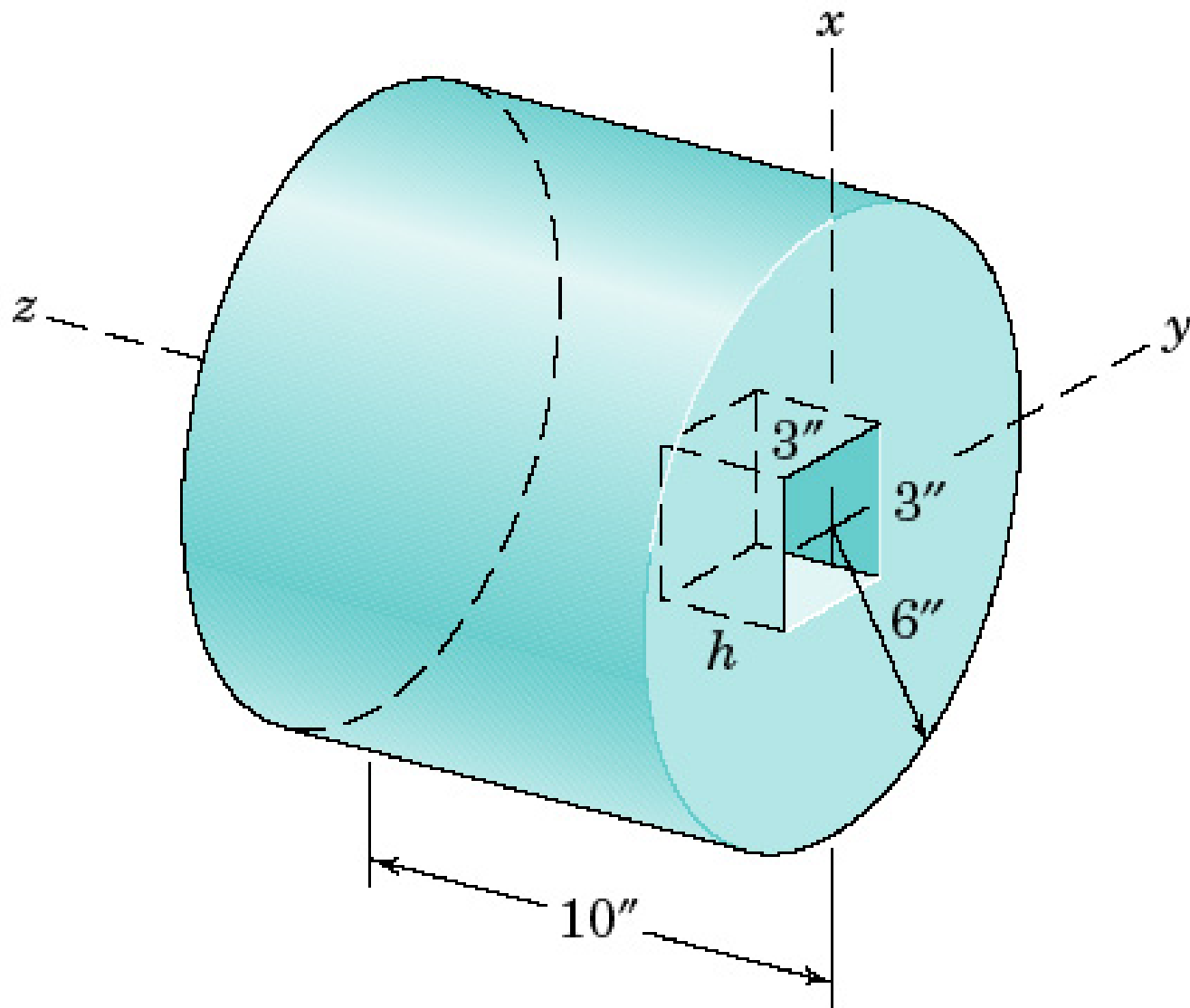
Problem 5/216



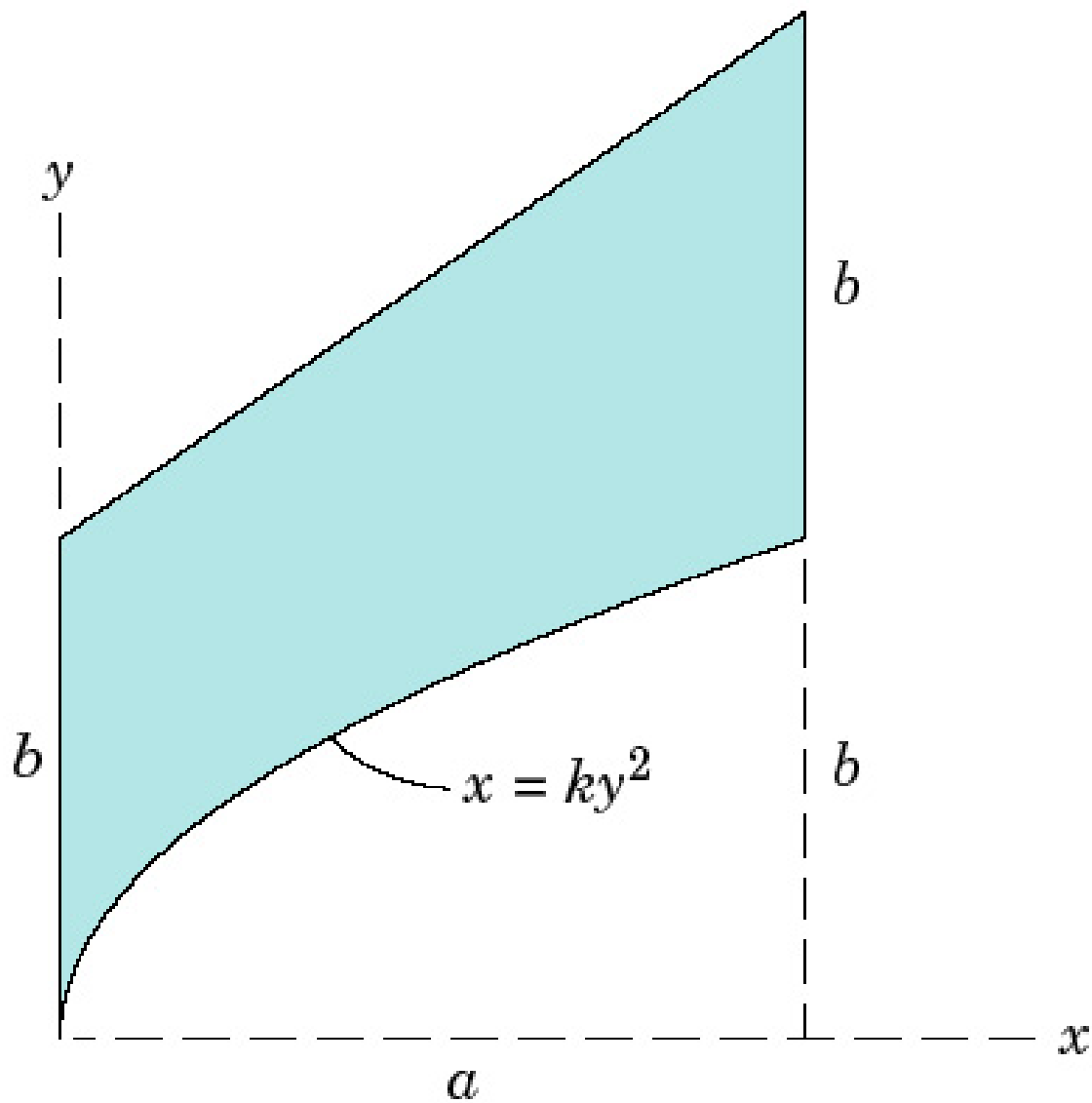
Problem 5/217



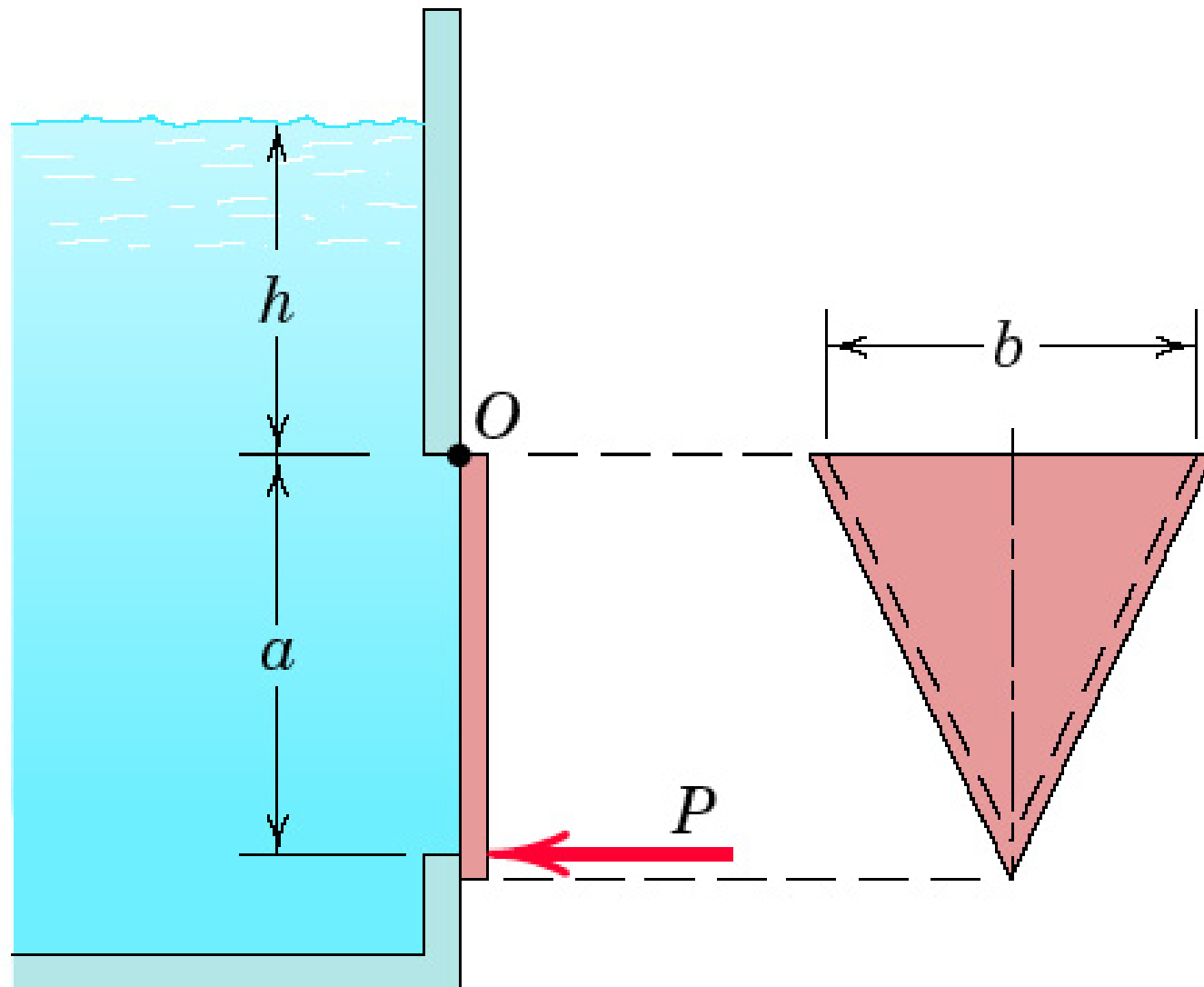
Problem 5/218



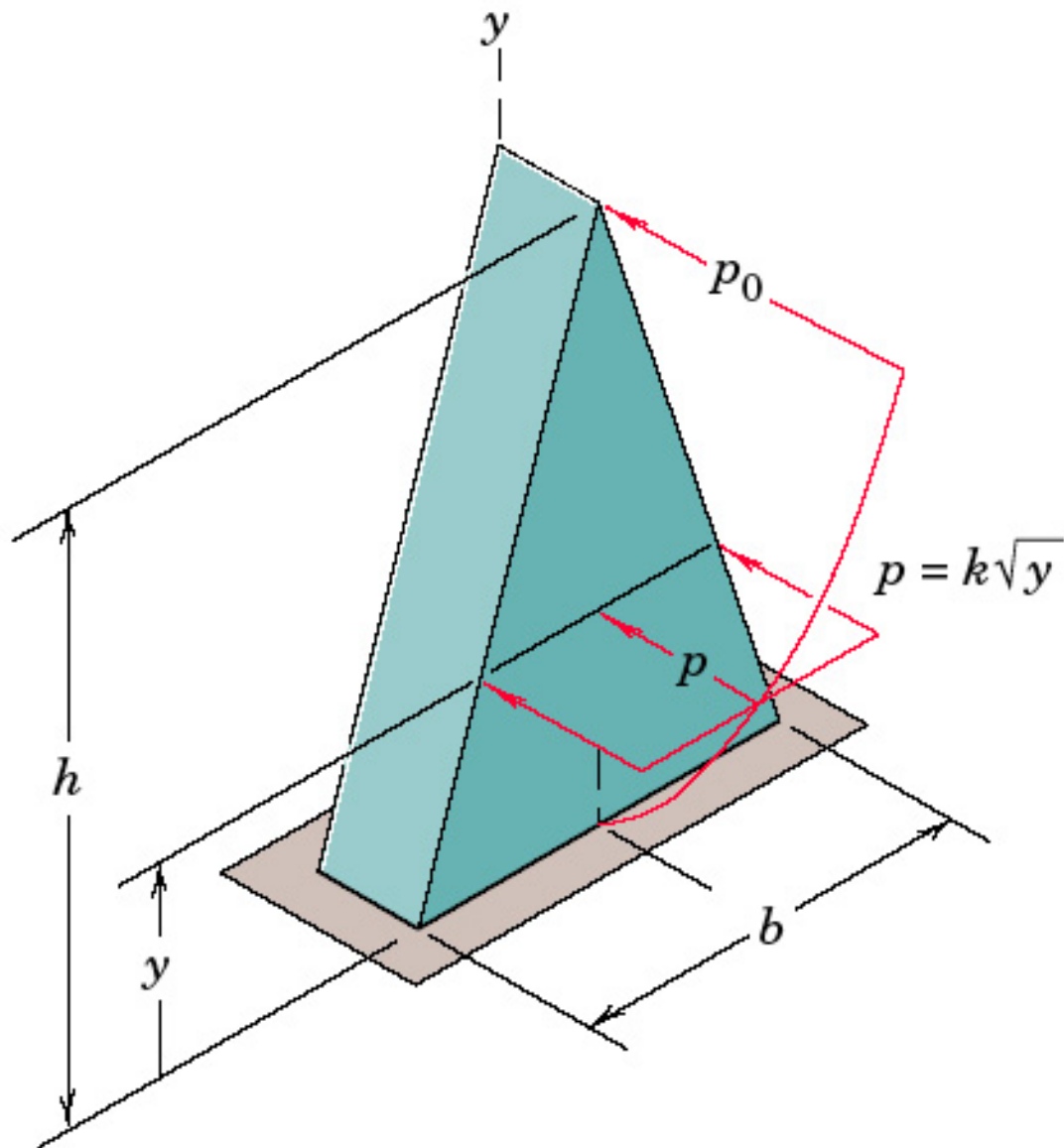
Problem 5/219



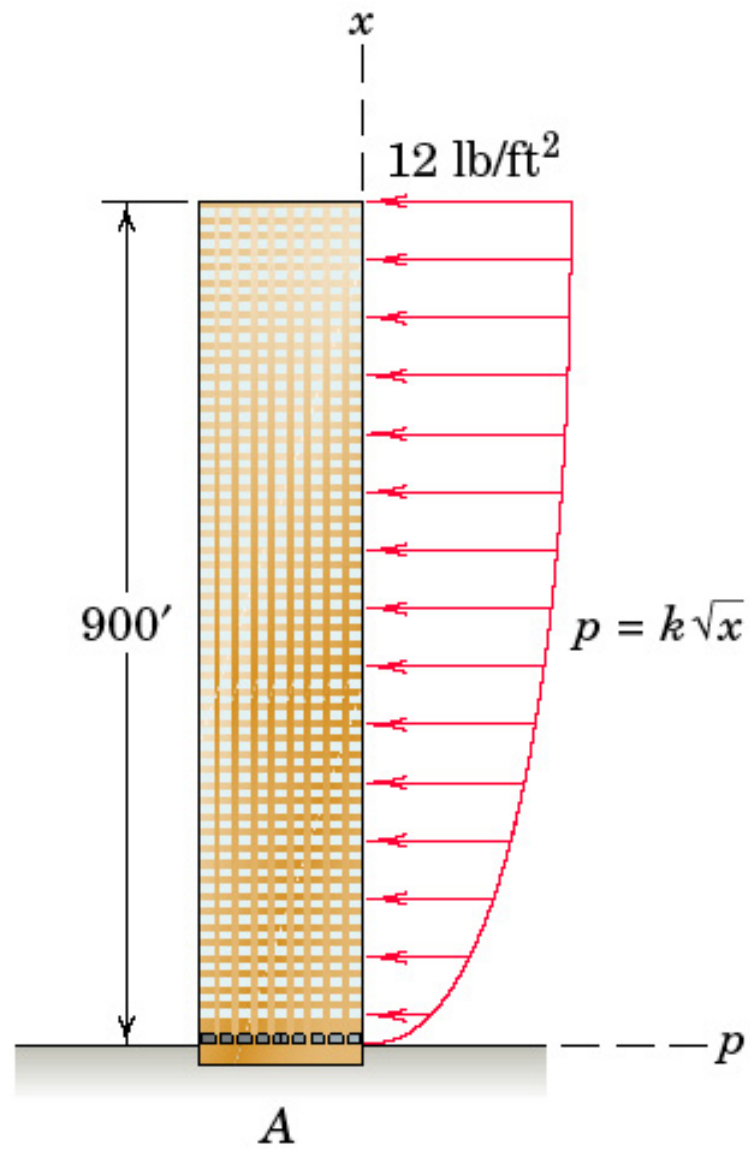
Problem 5/22



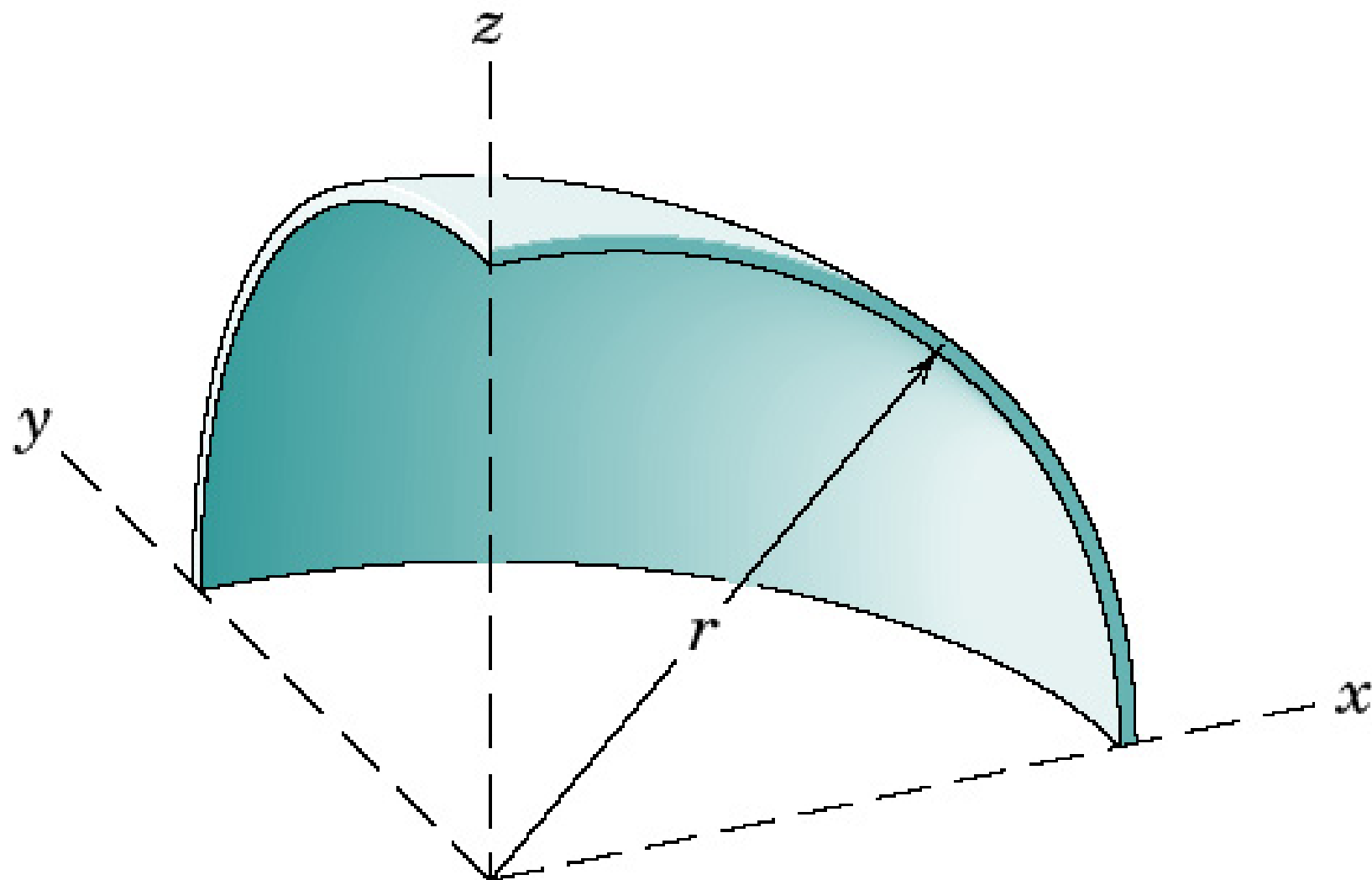
Problem 5/220



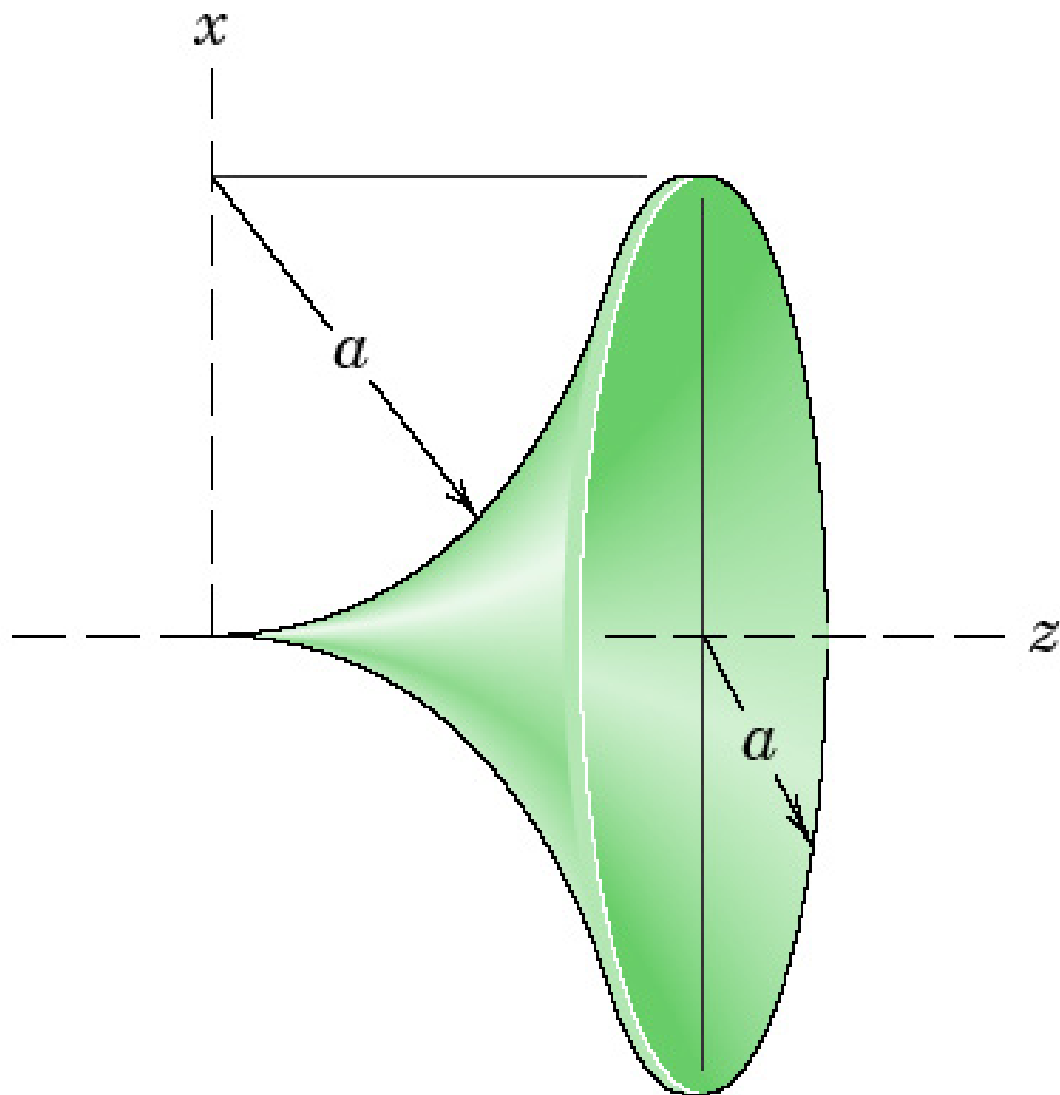
Problem 5/221



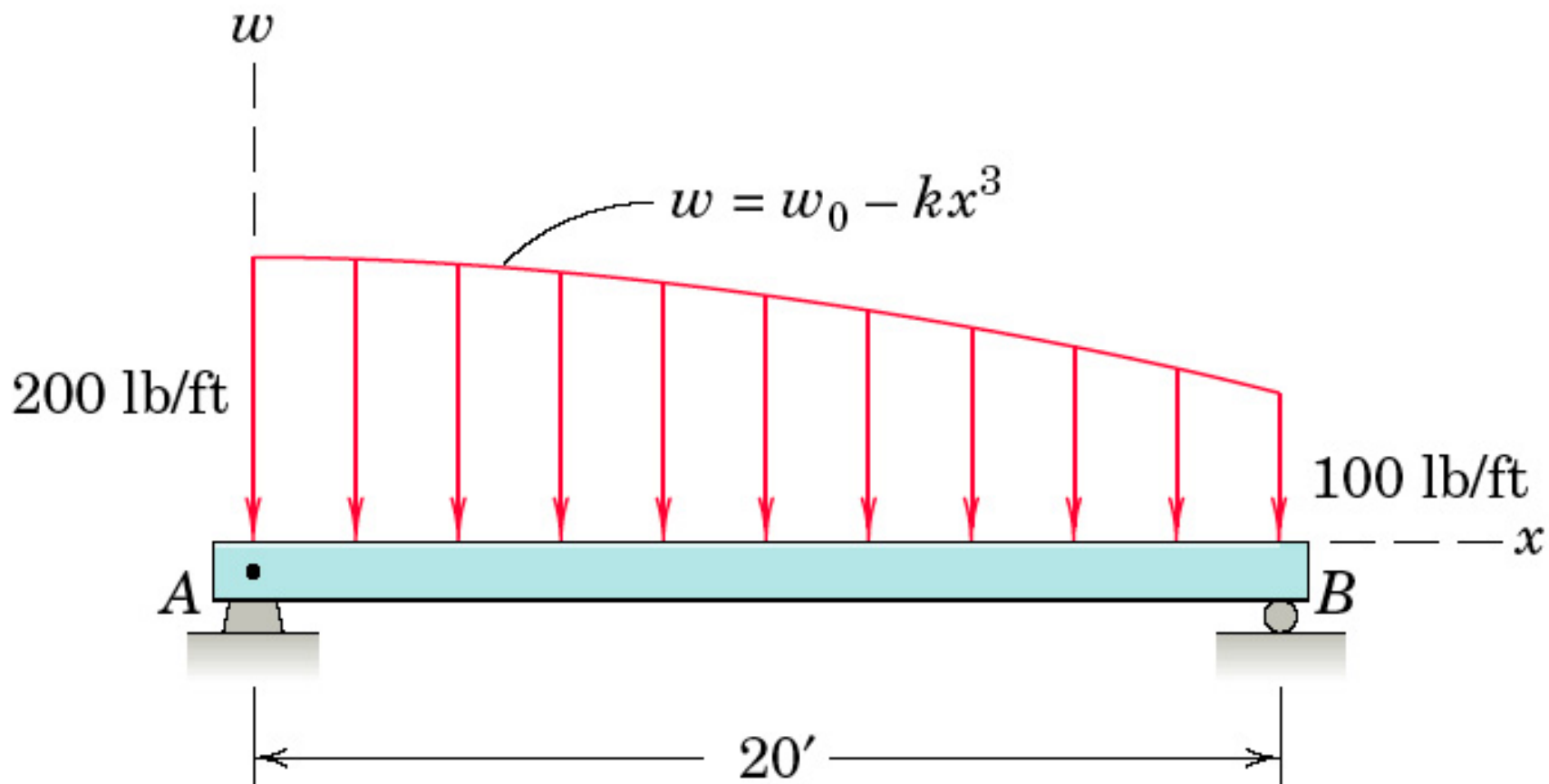
Problem 5/222



Problem 5/224

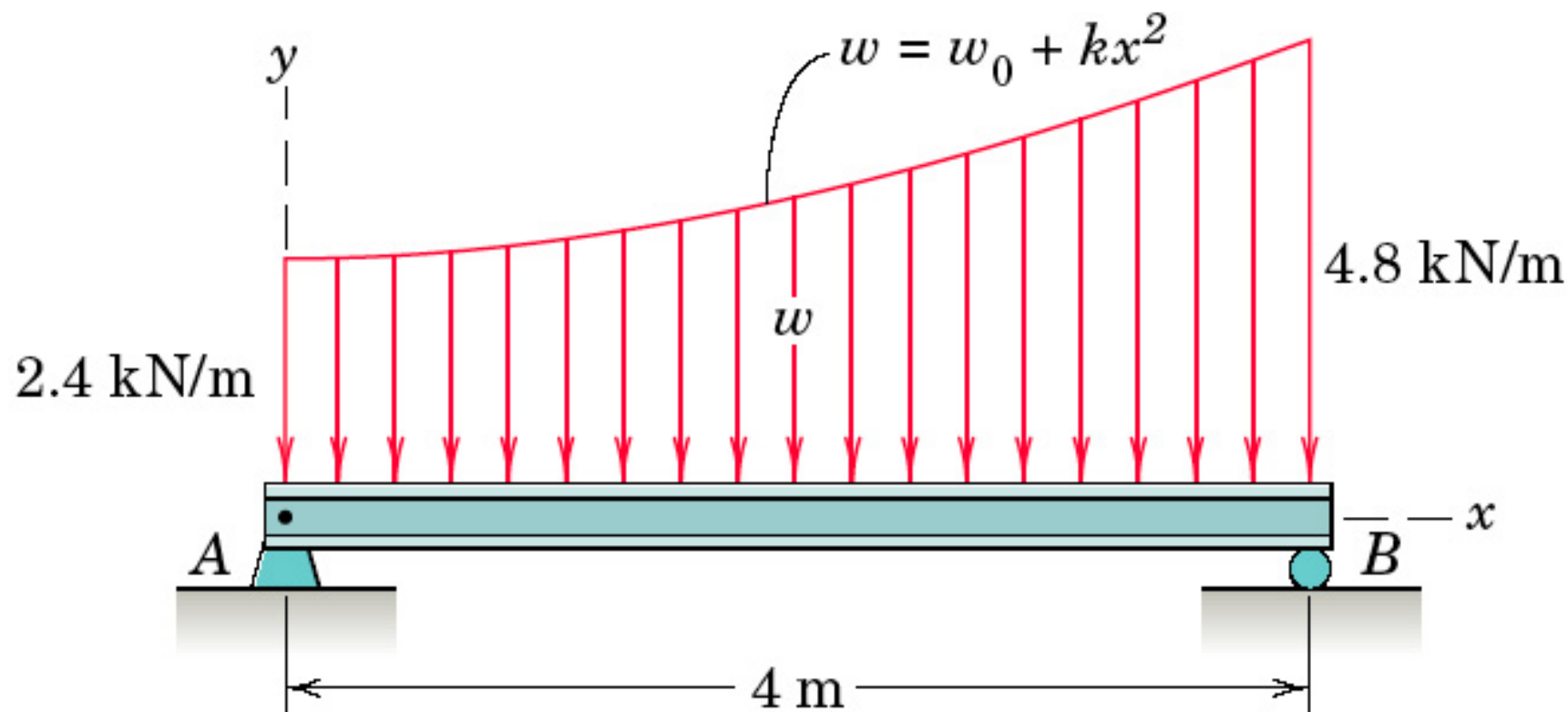


Problem 5/225



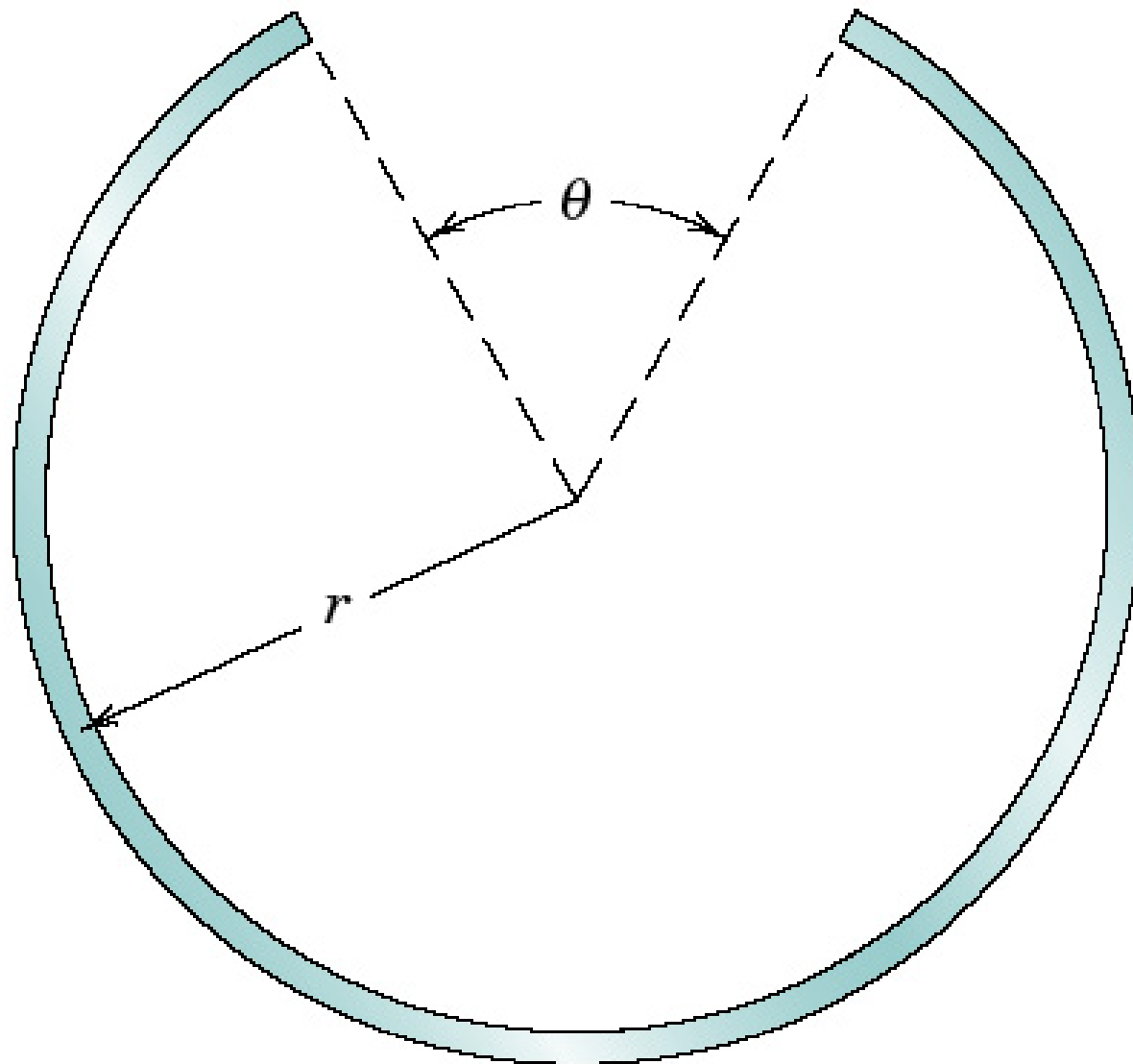
Problem 5/226

P_05_226

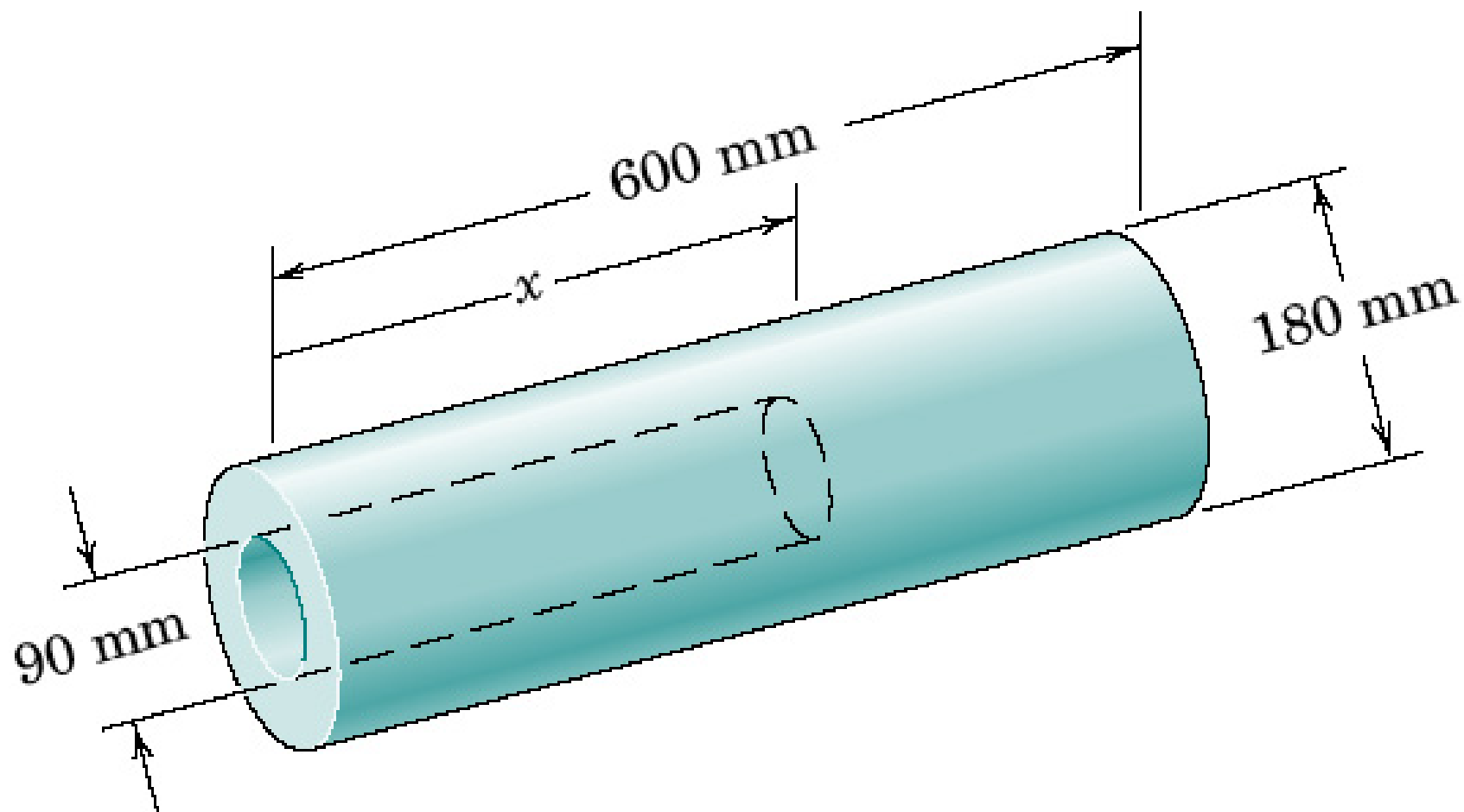


Problem 5/227

P_05_227

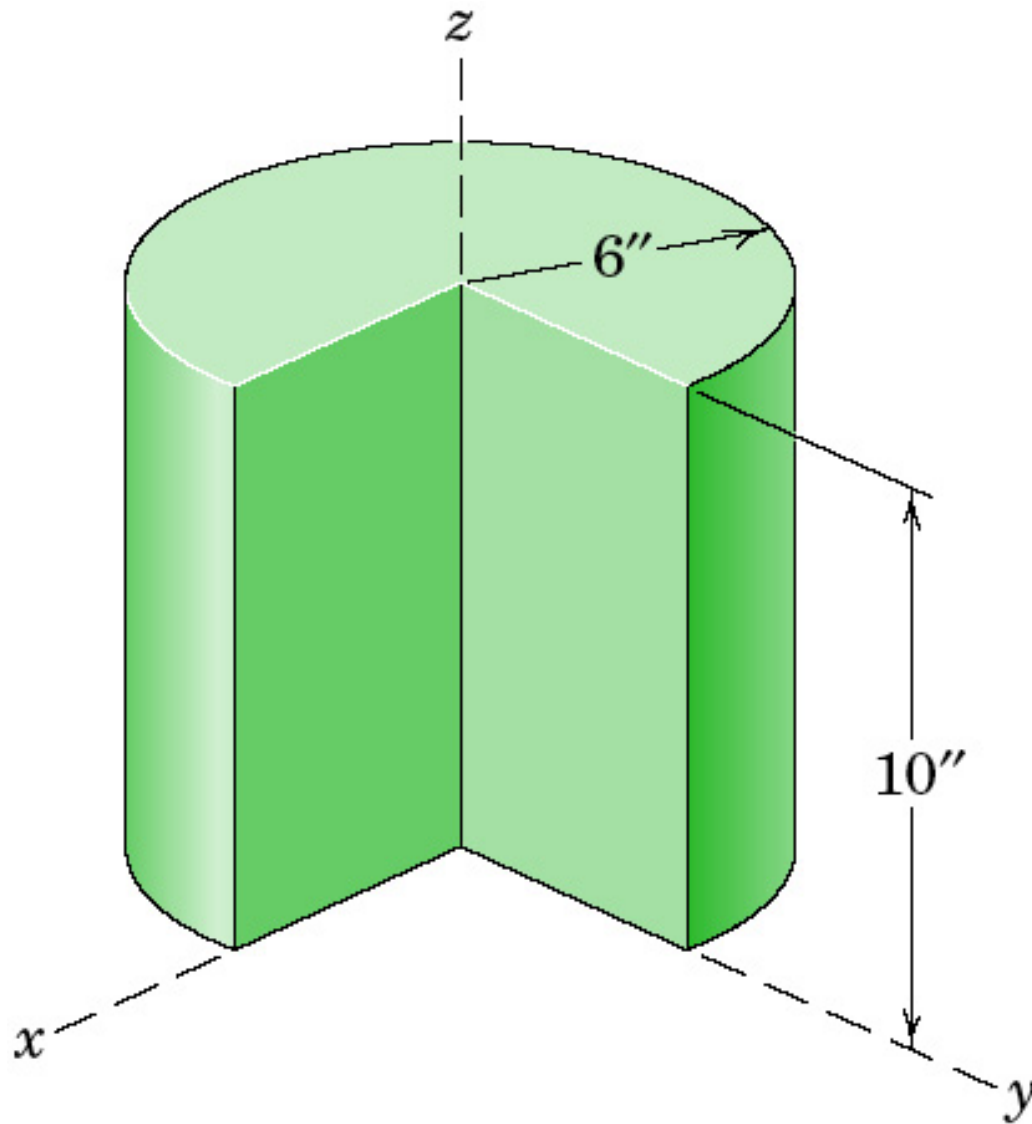


Problem 5/228

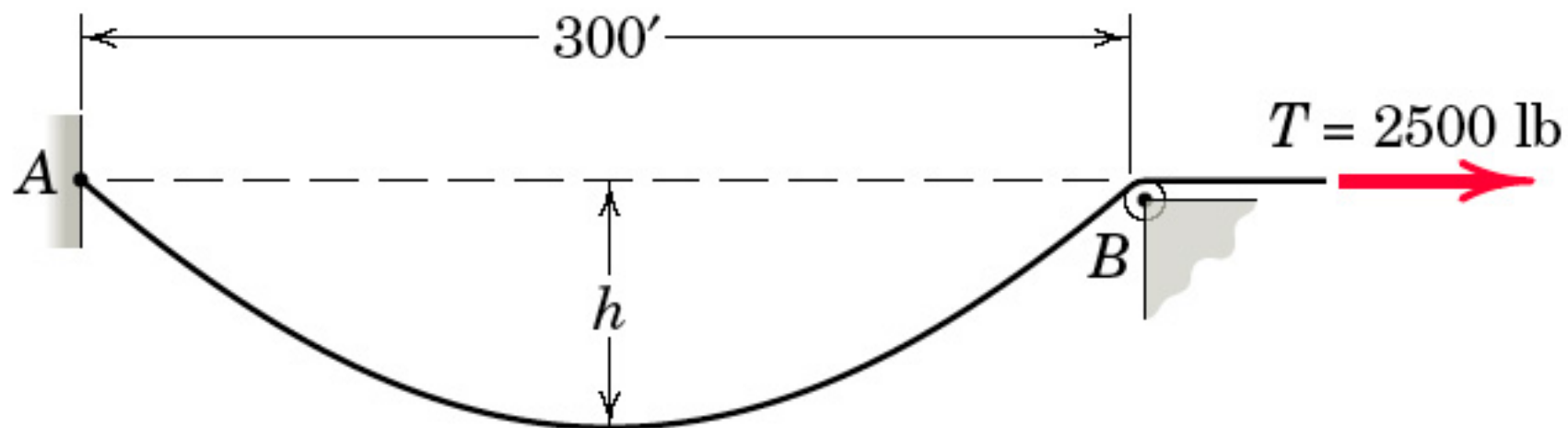


Problem 5/229

P_05_229

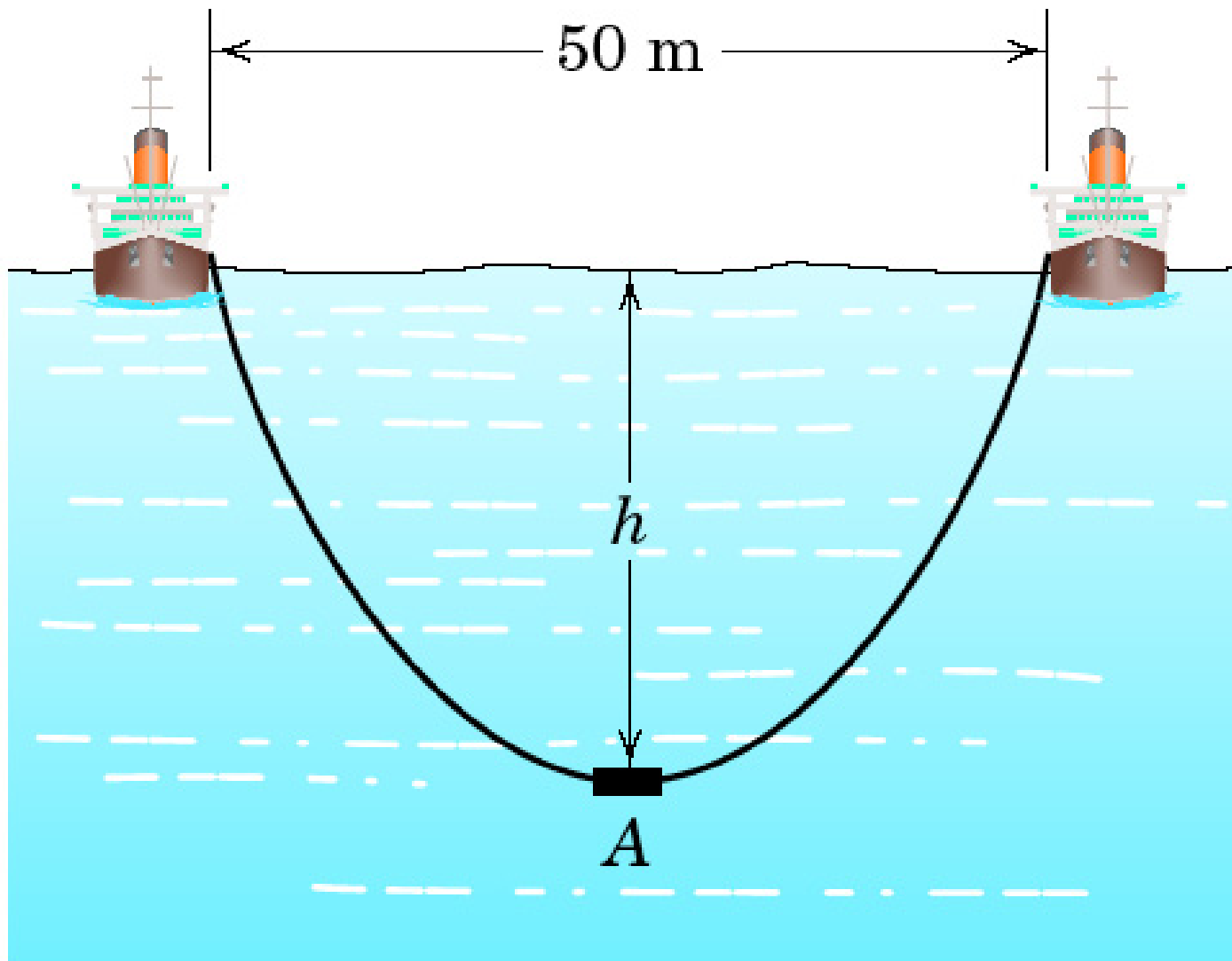


Problem 5/23

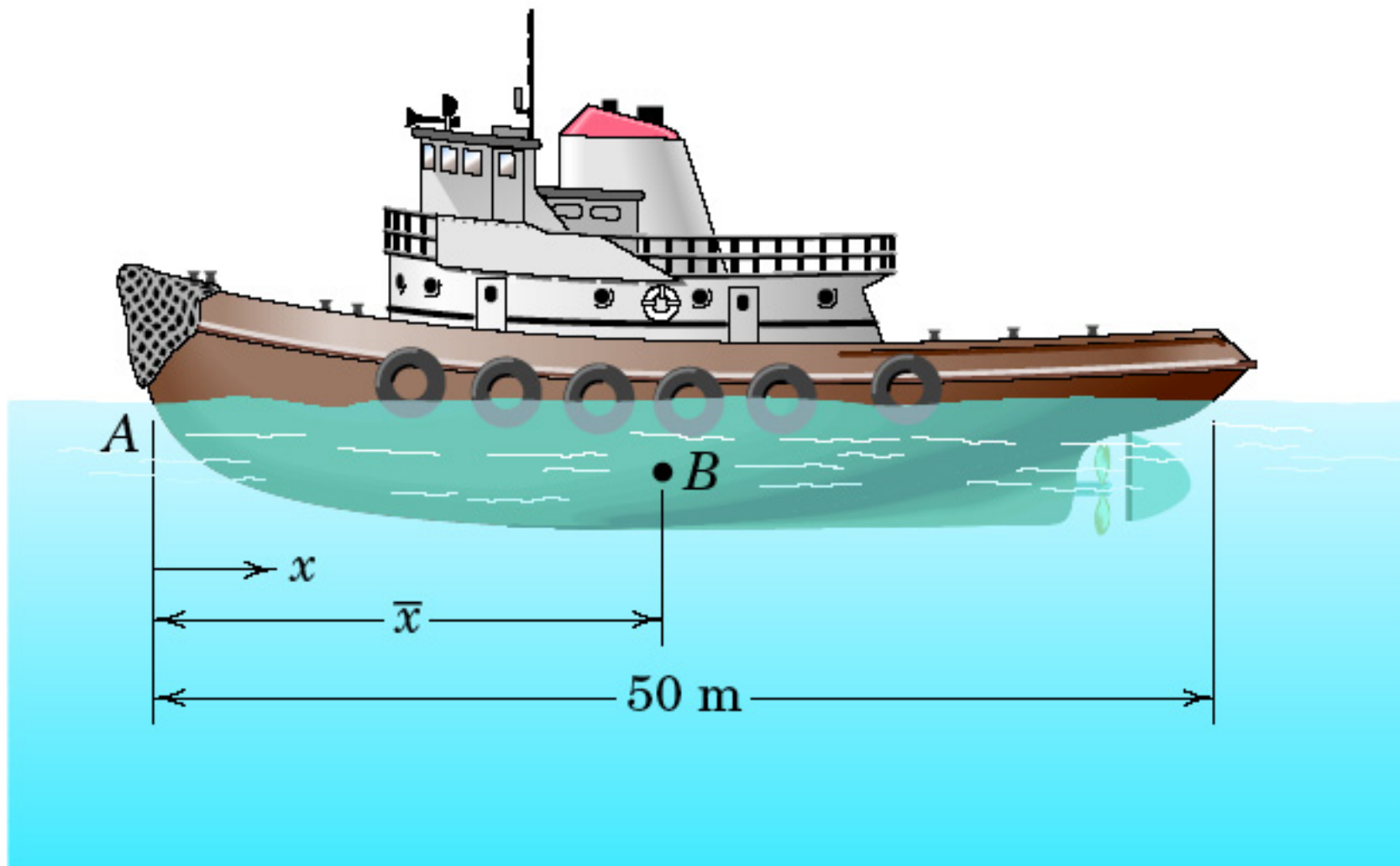


Problem 5/230

P_05_230

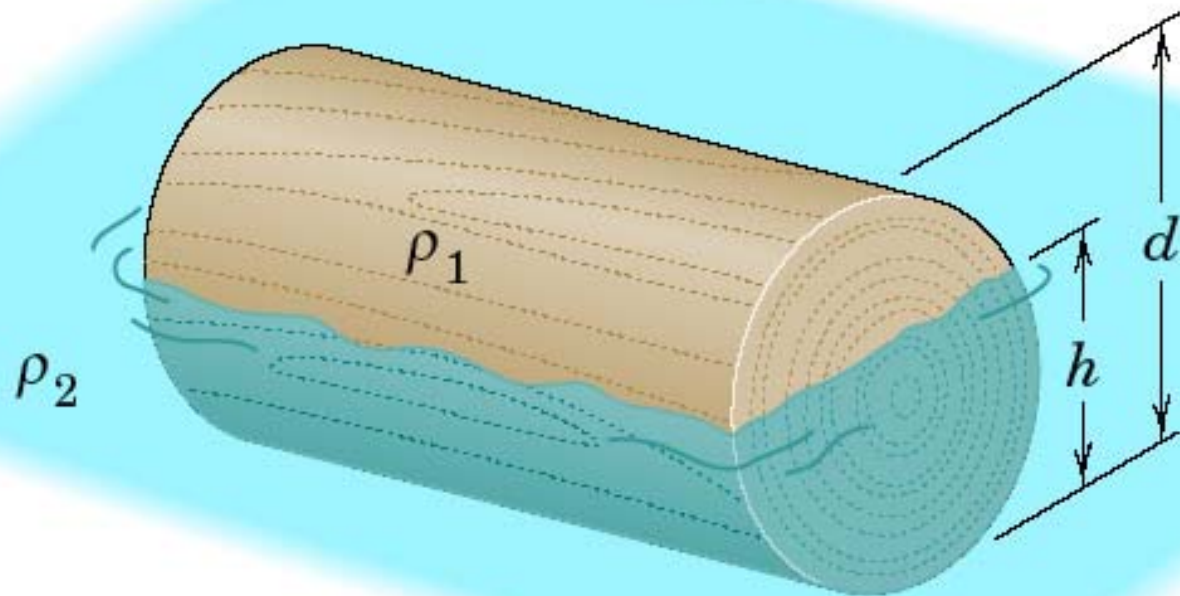


Problem 5/231



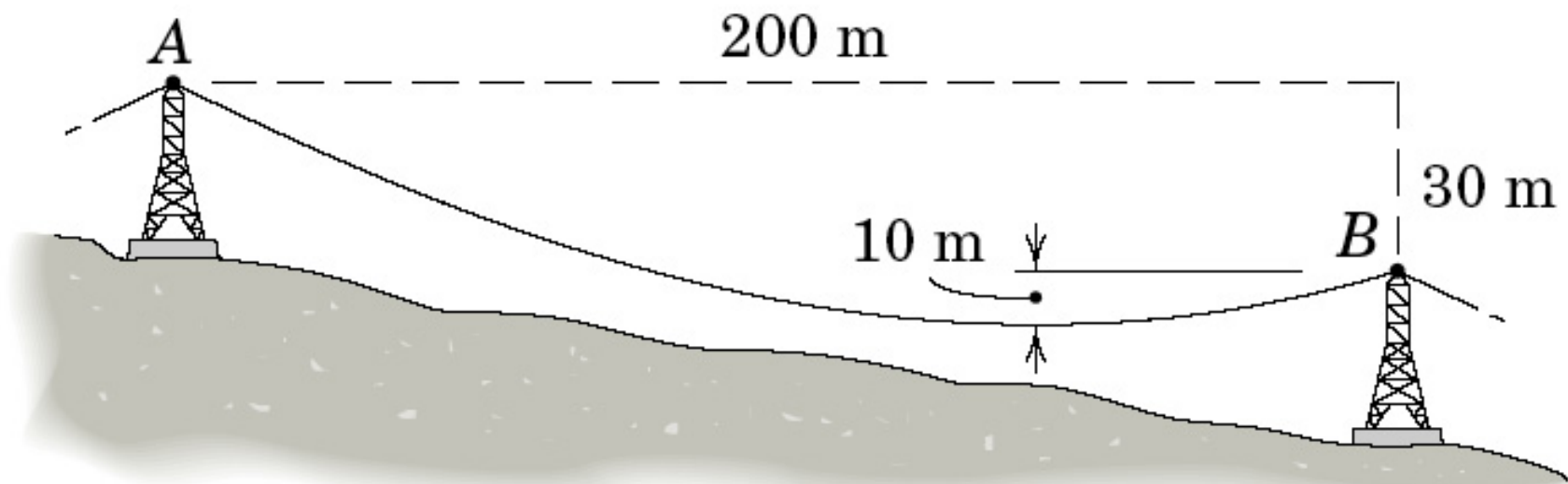
Problem 5/232

P_05_232



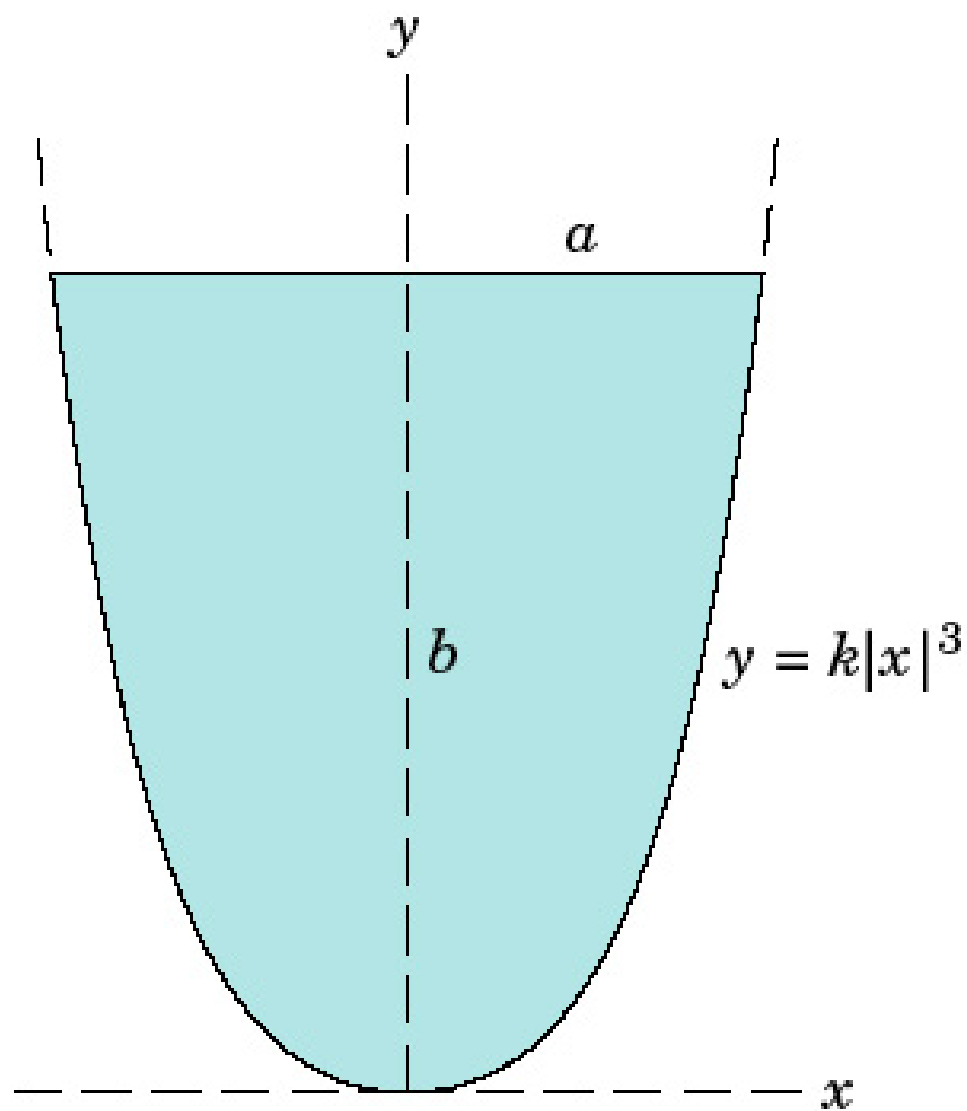
Problem 5/233

P_05_233

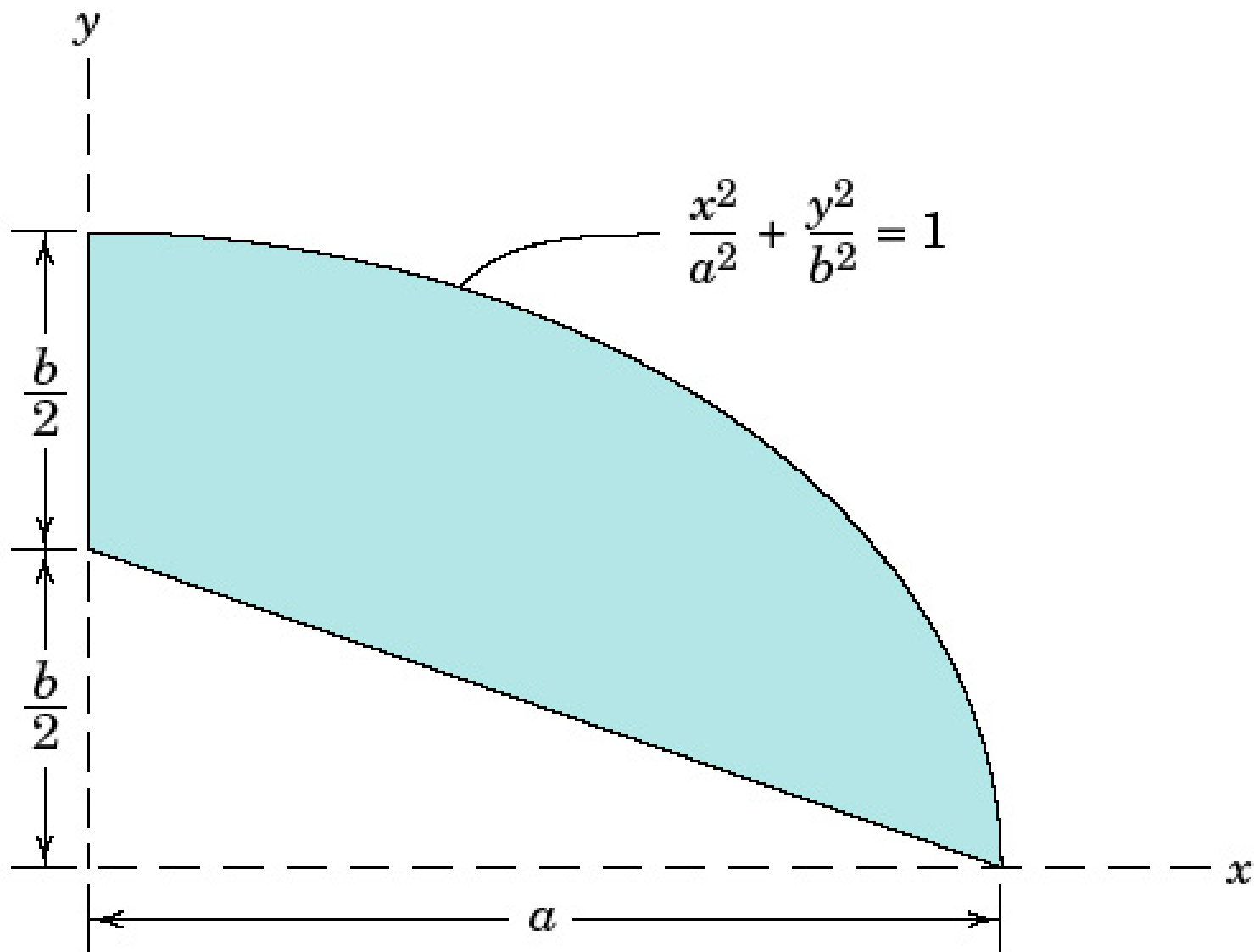


Problem 5/234

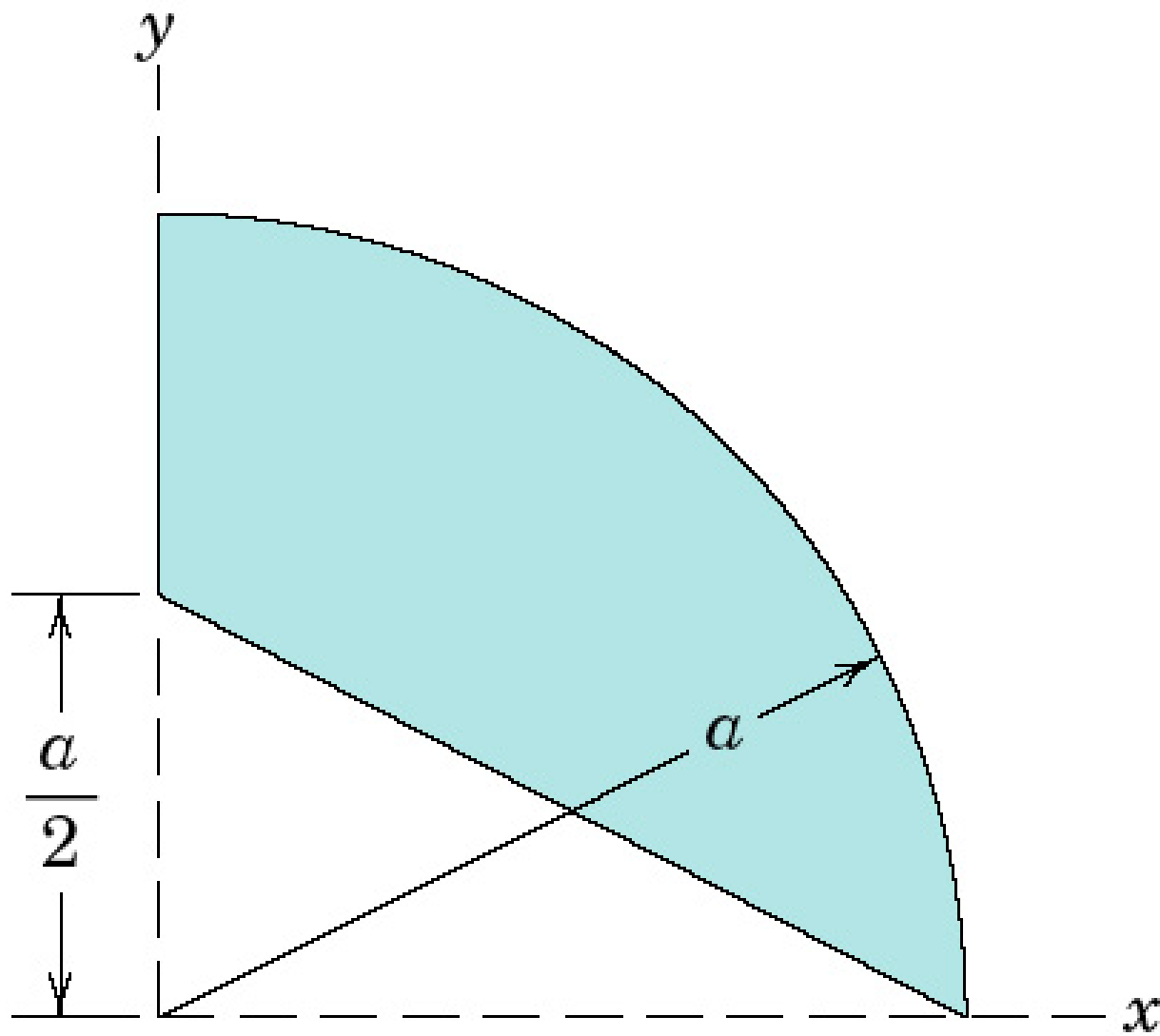
P_05_234



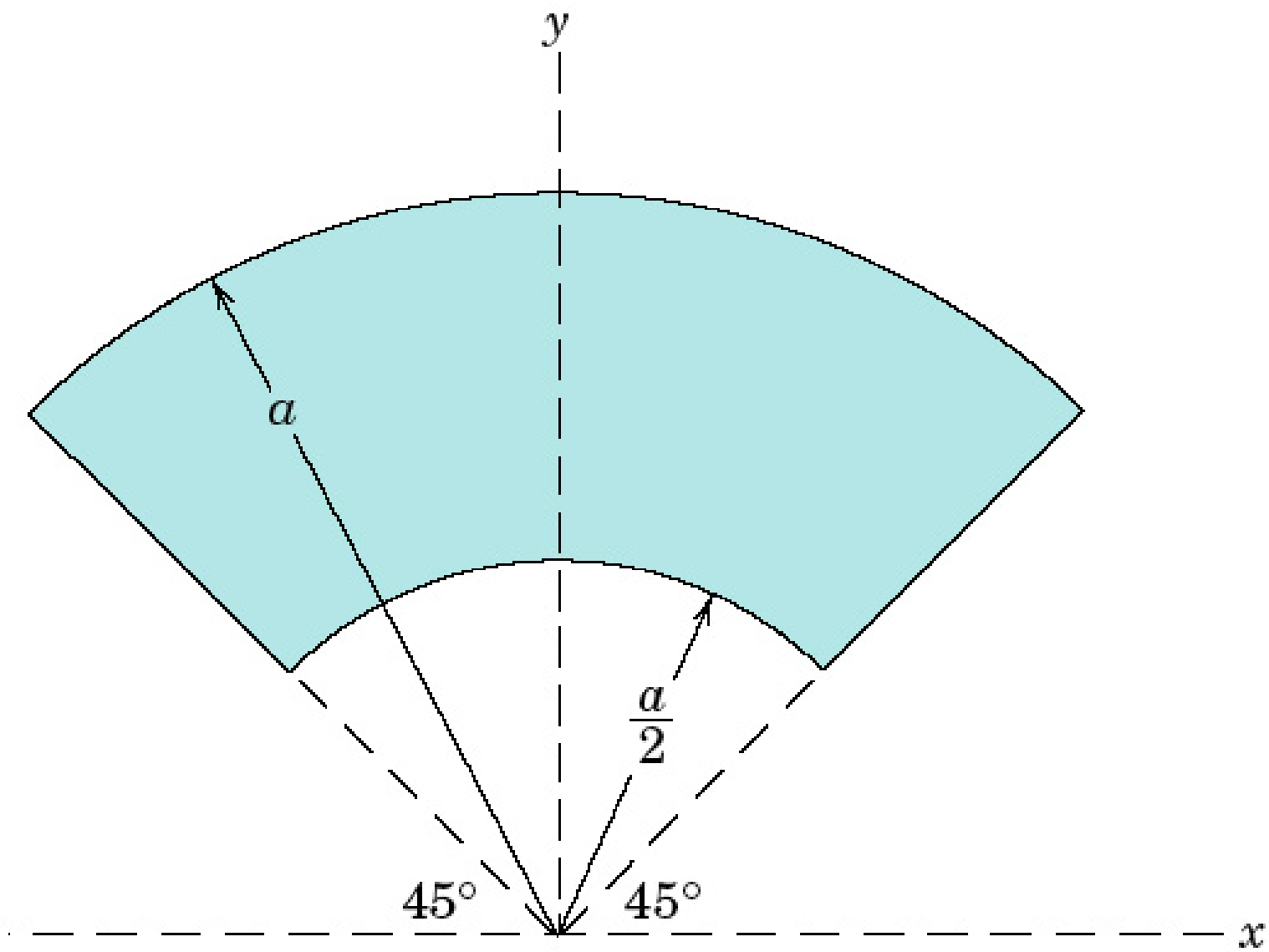
Problem 5/24



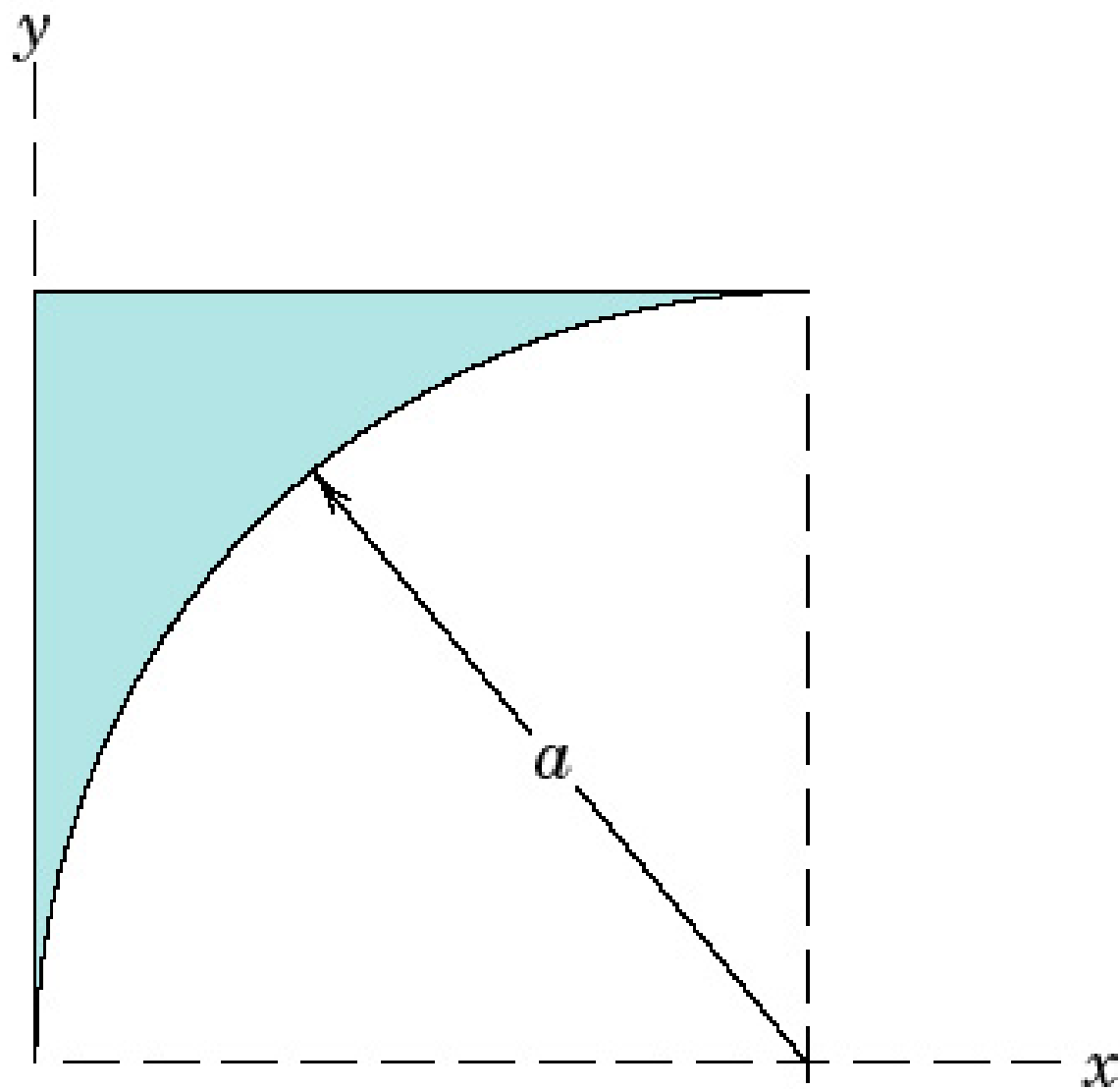
Problem 5/25



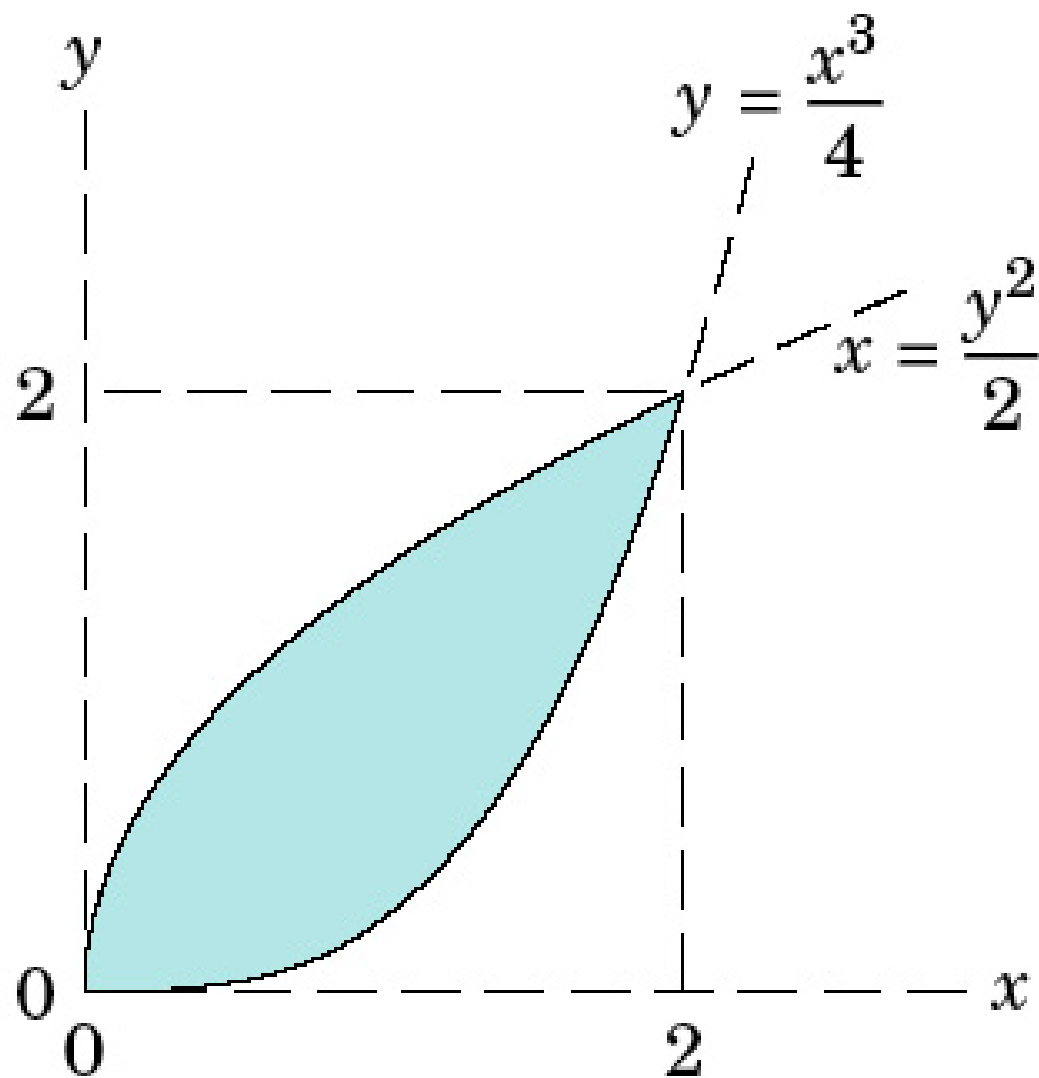
Problem 5/26



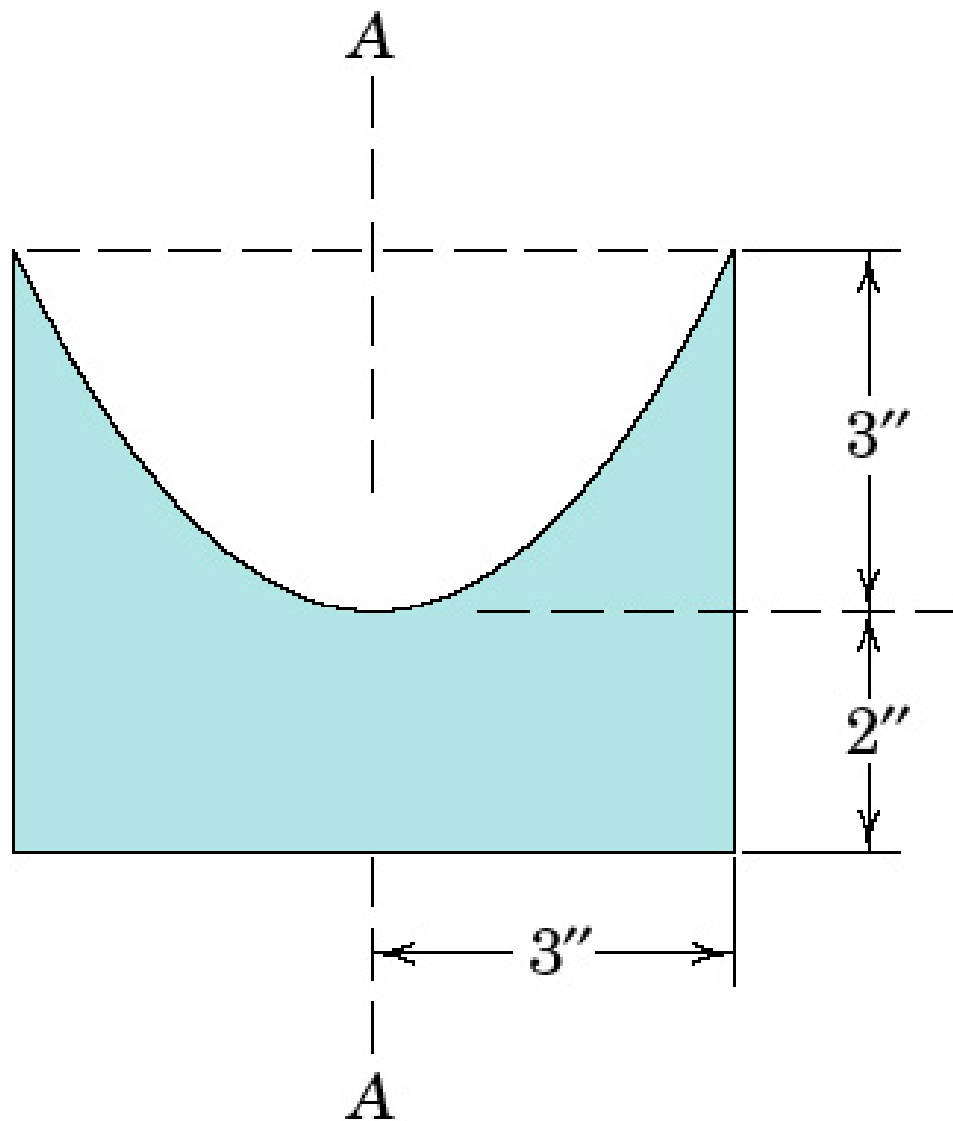
Problem 5/27



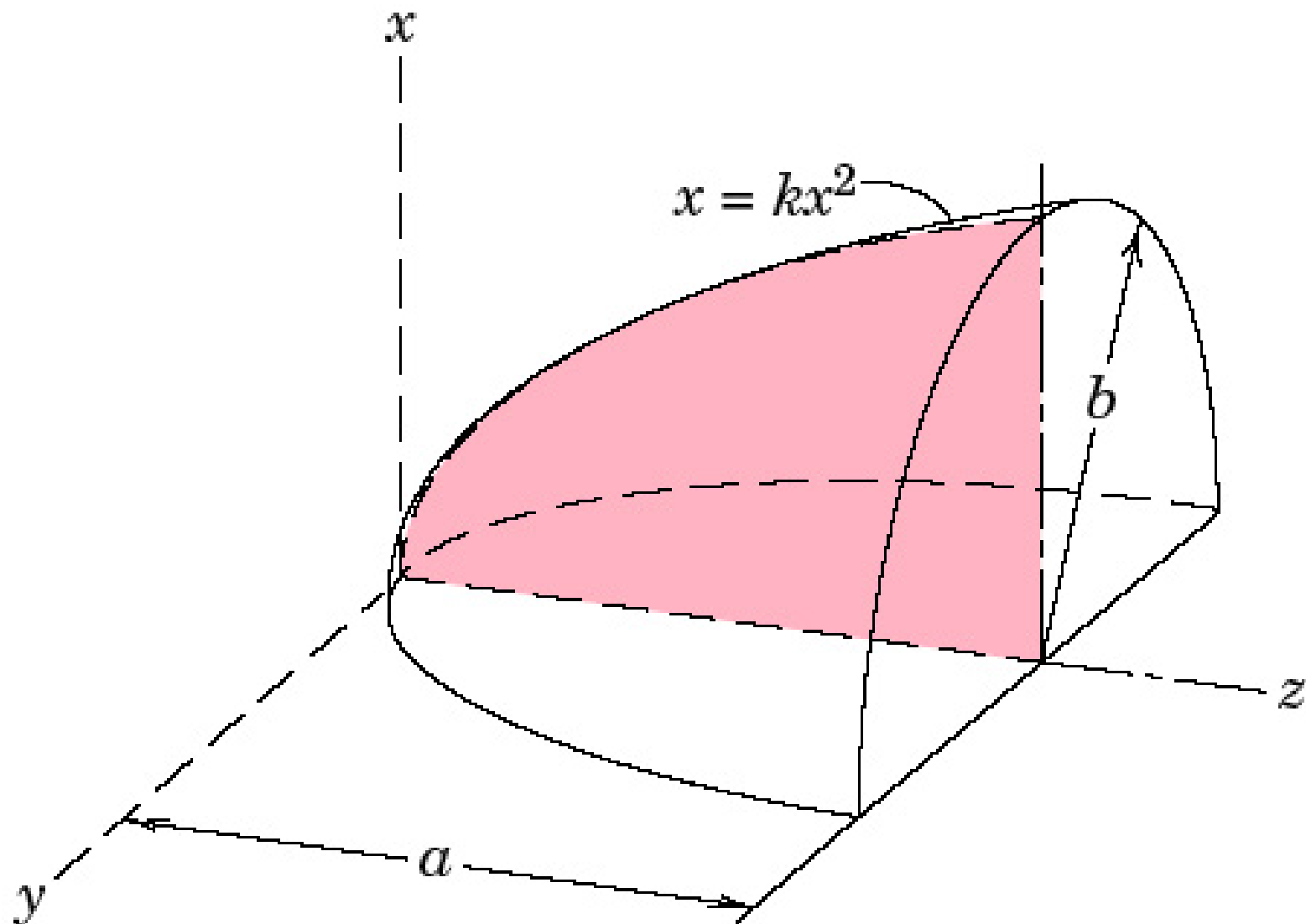
Problem 5/28



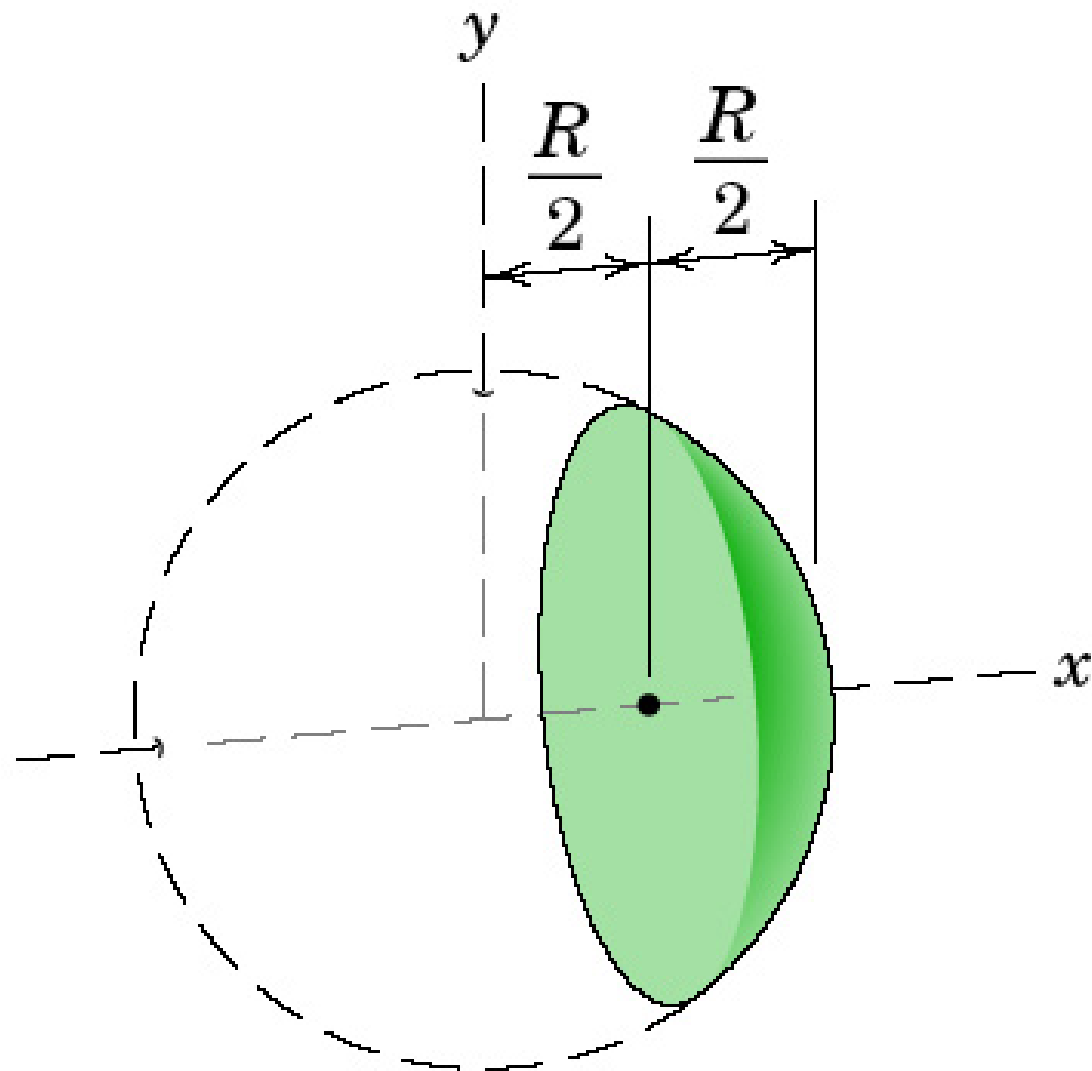
Problem 5/29



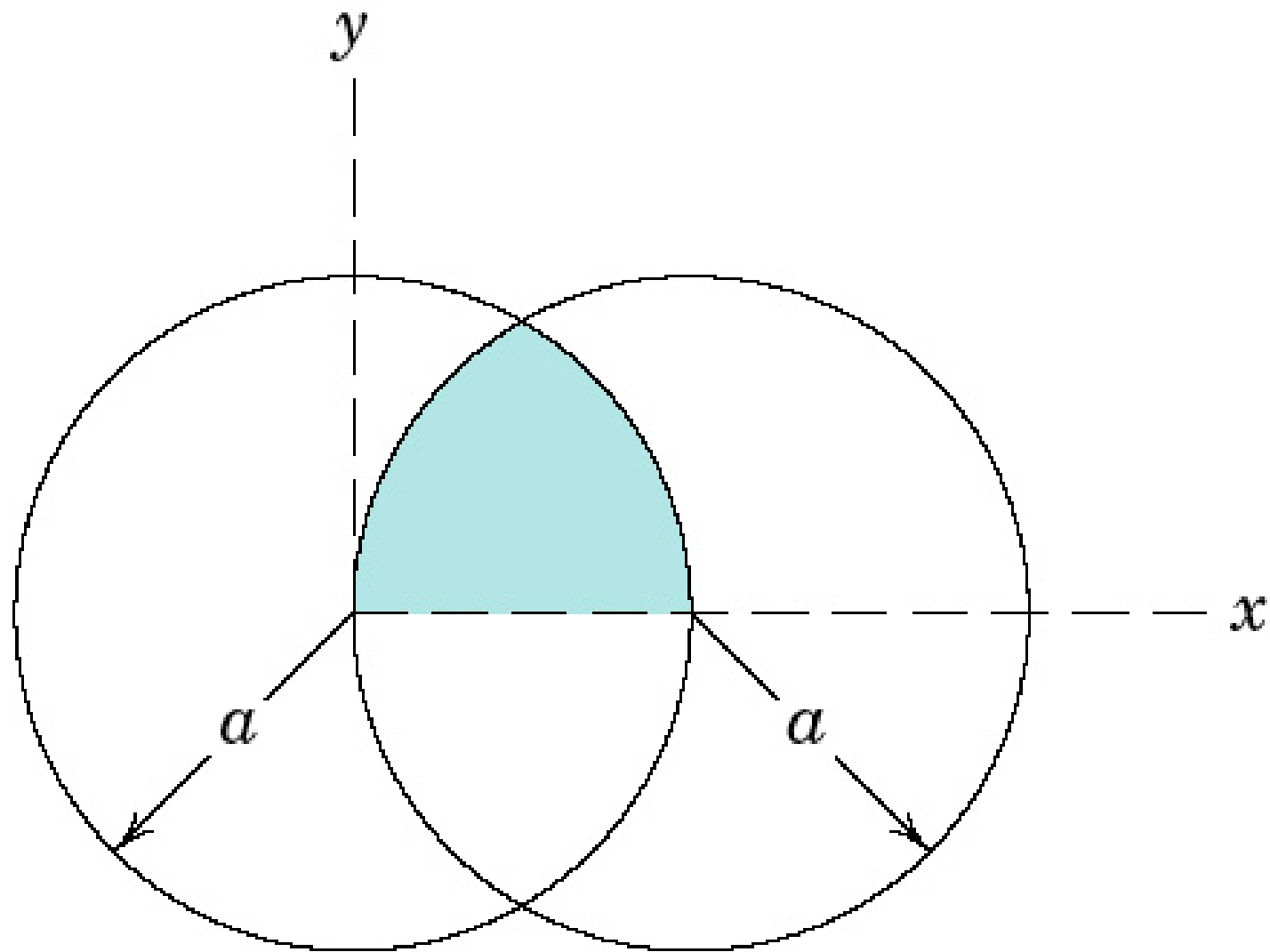
Problem 5/30



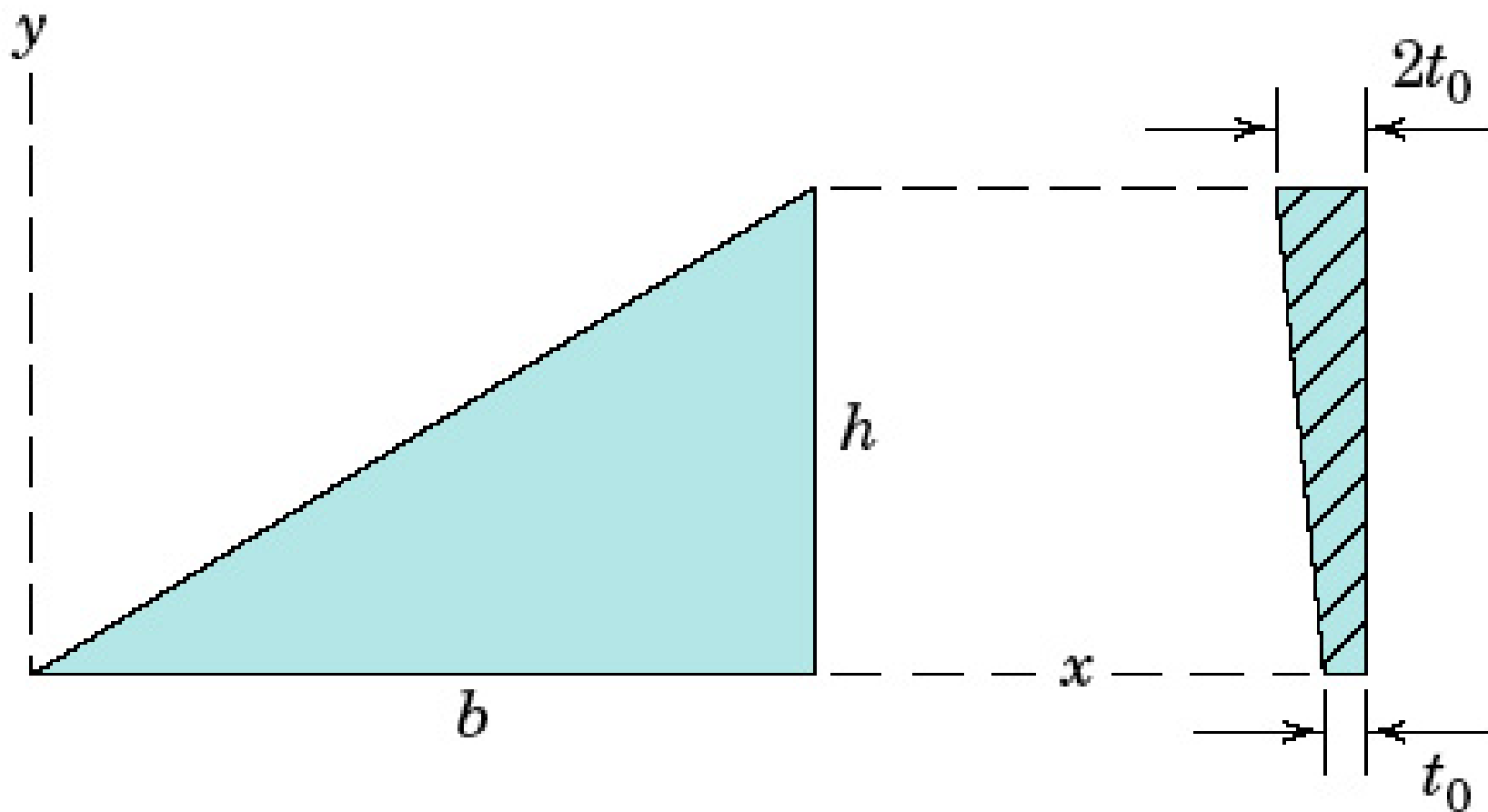
Problem 5/31



Problem 5/32

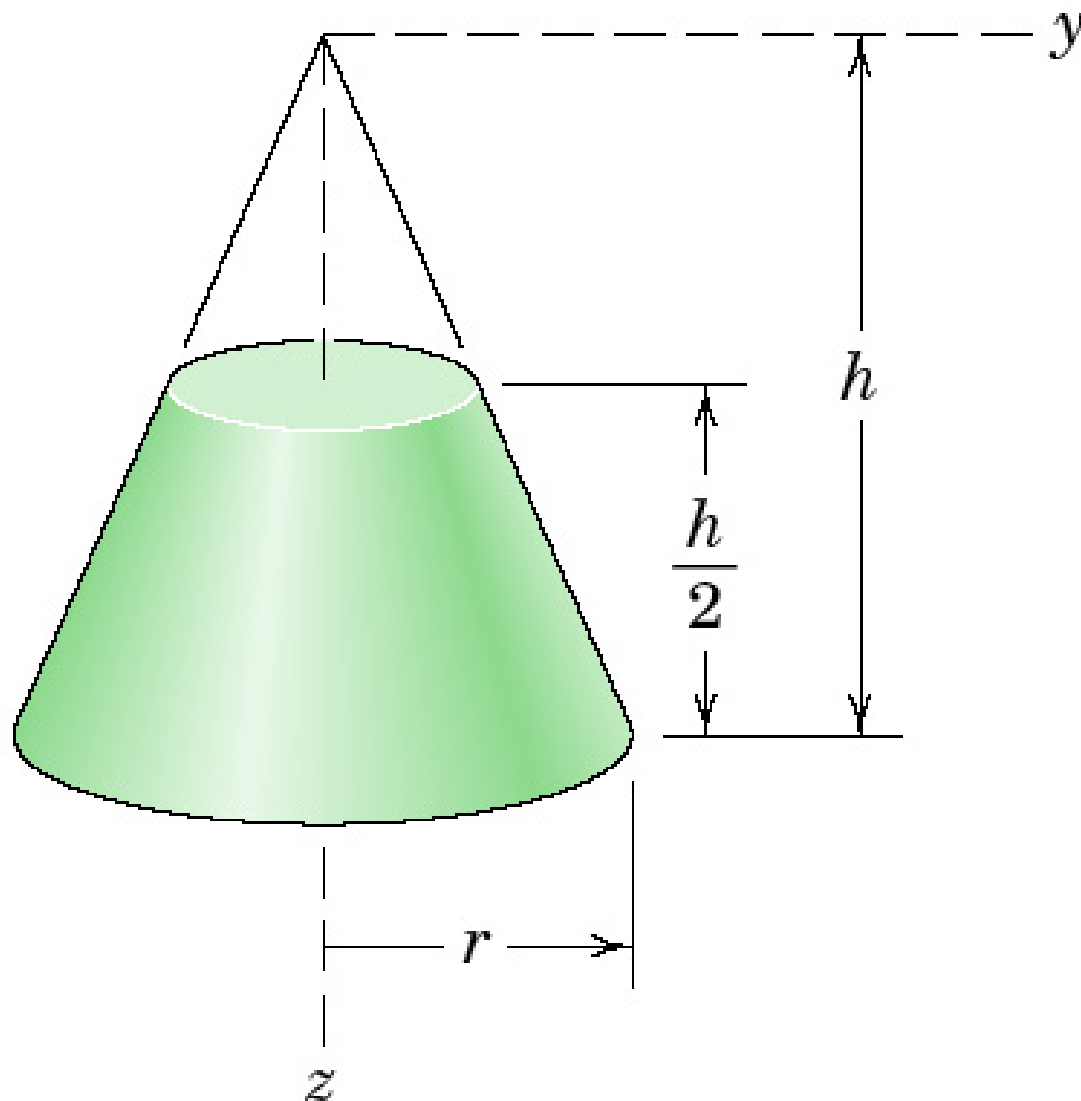


Problem 5/33

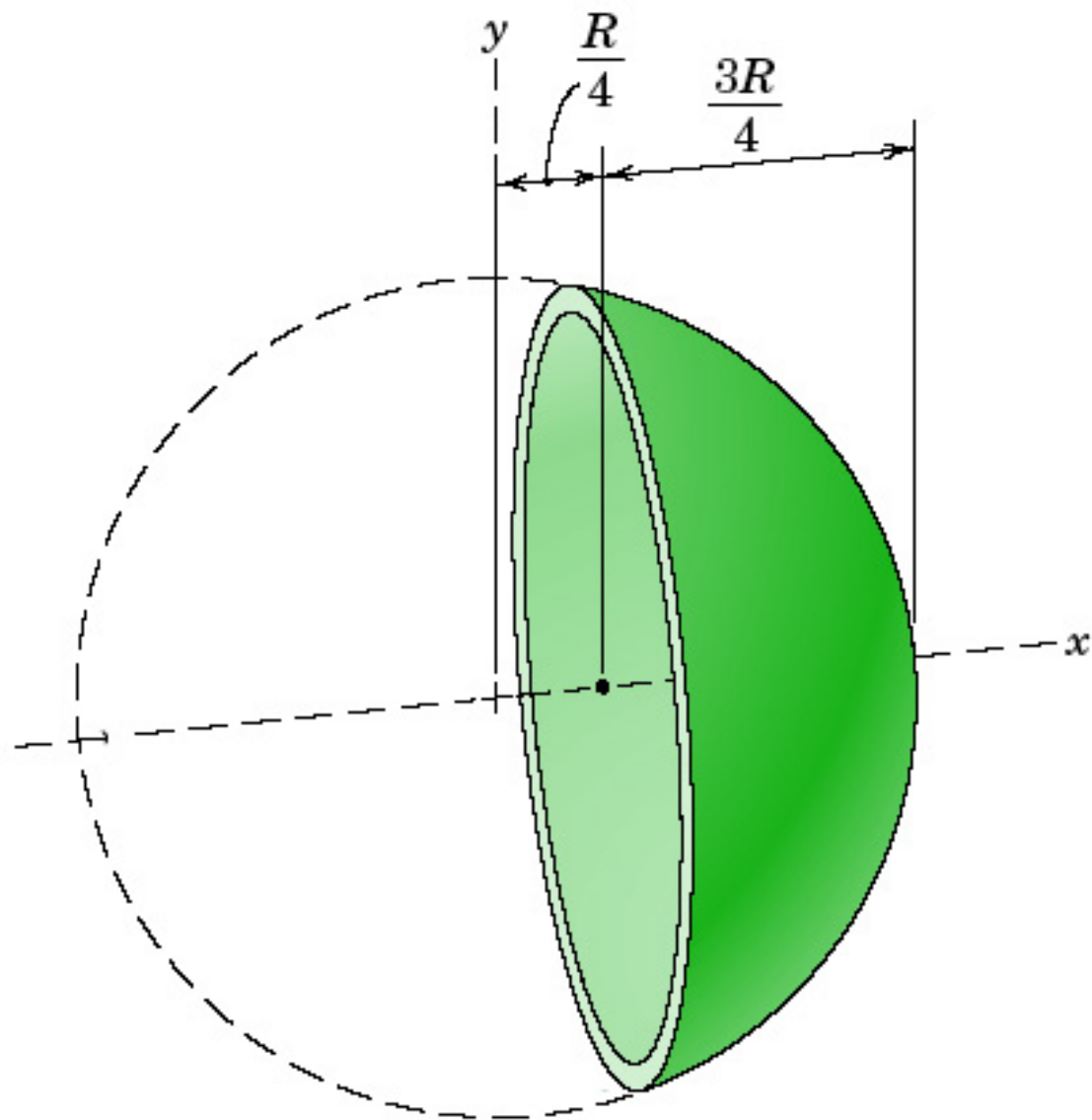


Problem 5/34

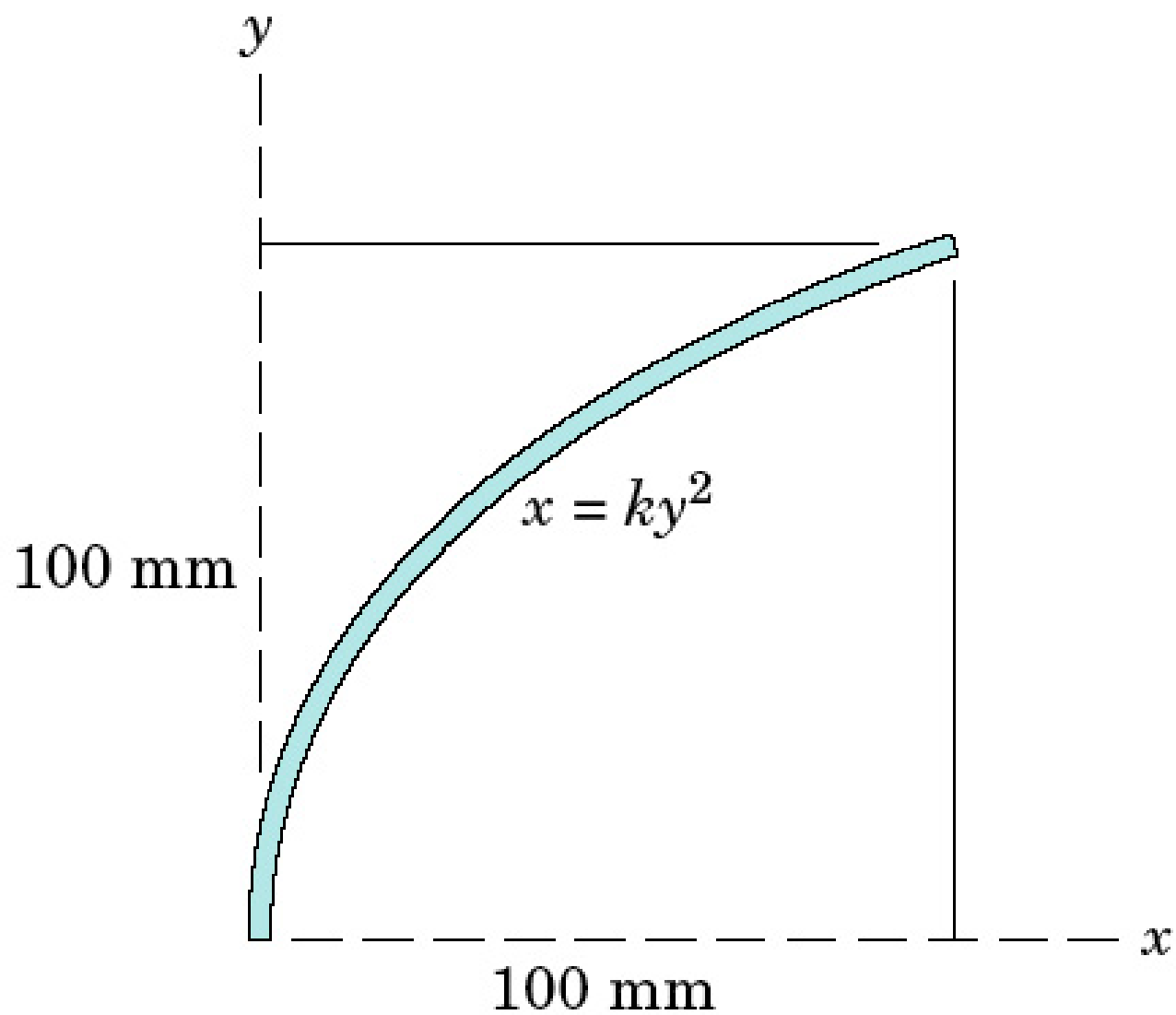
P_05_34



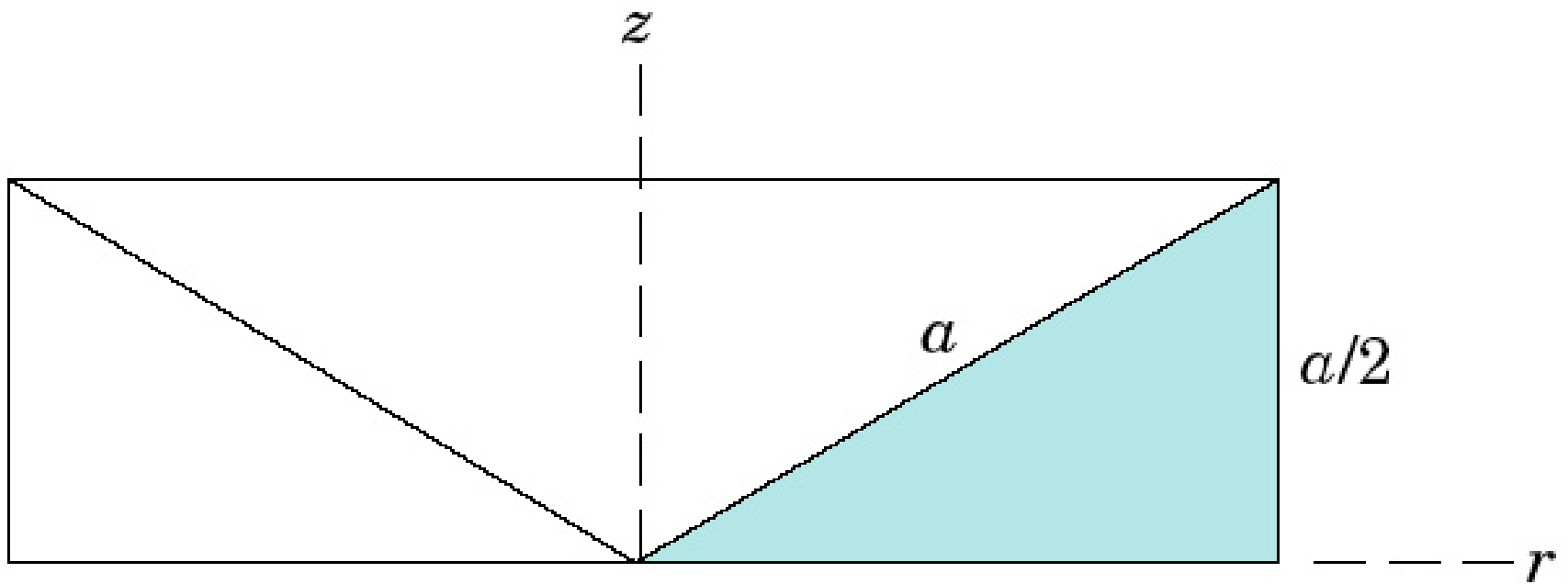
Problem 5/35



Problem 5/36

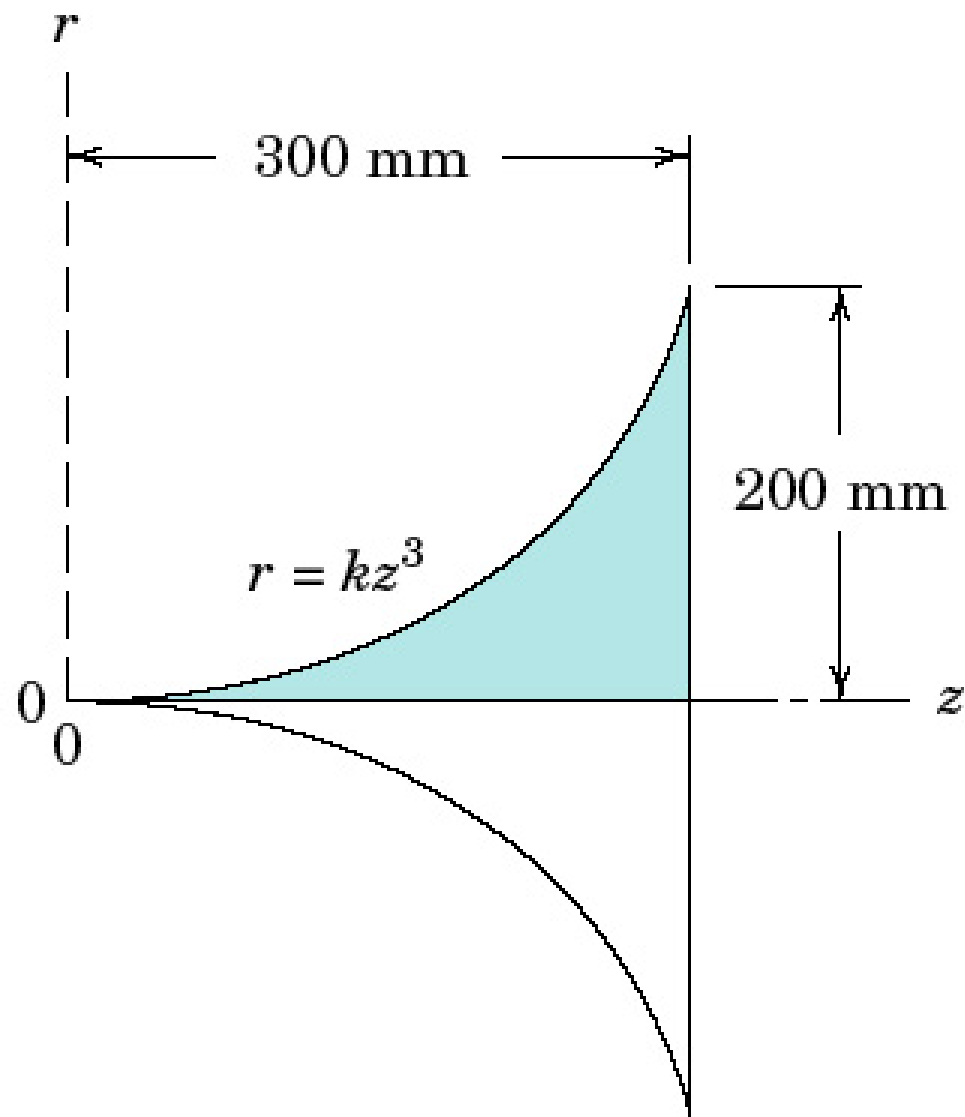


Problem 5/37

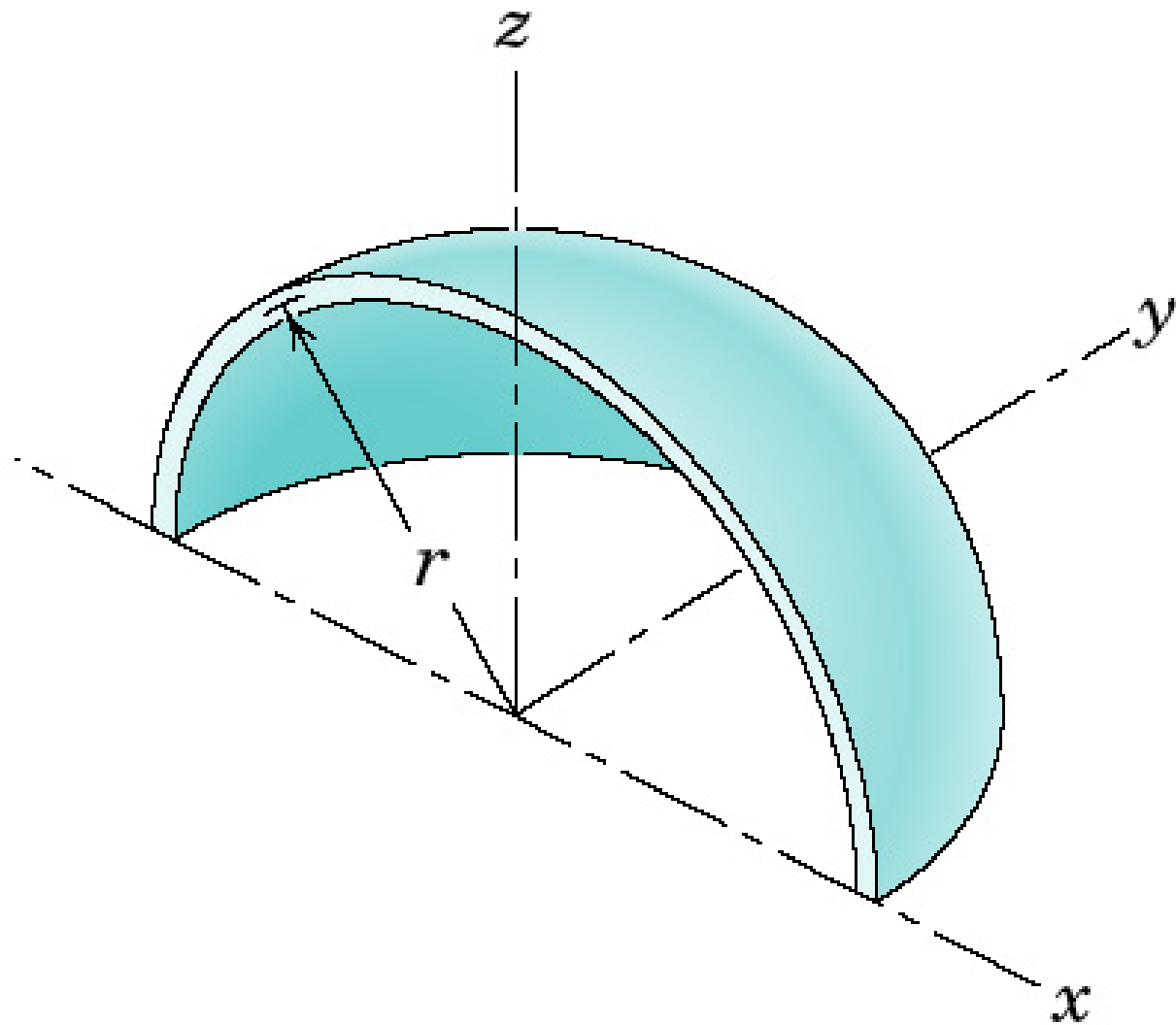


Problem 5/38

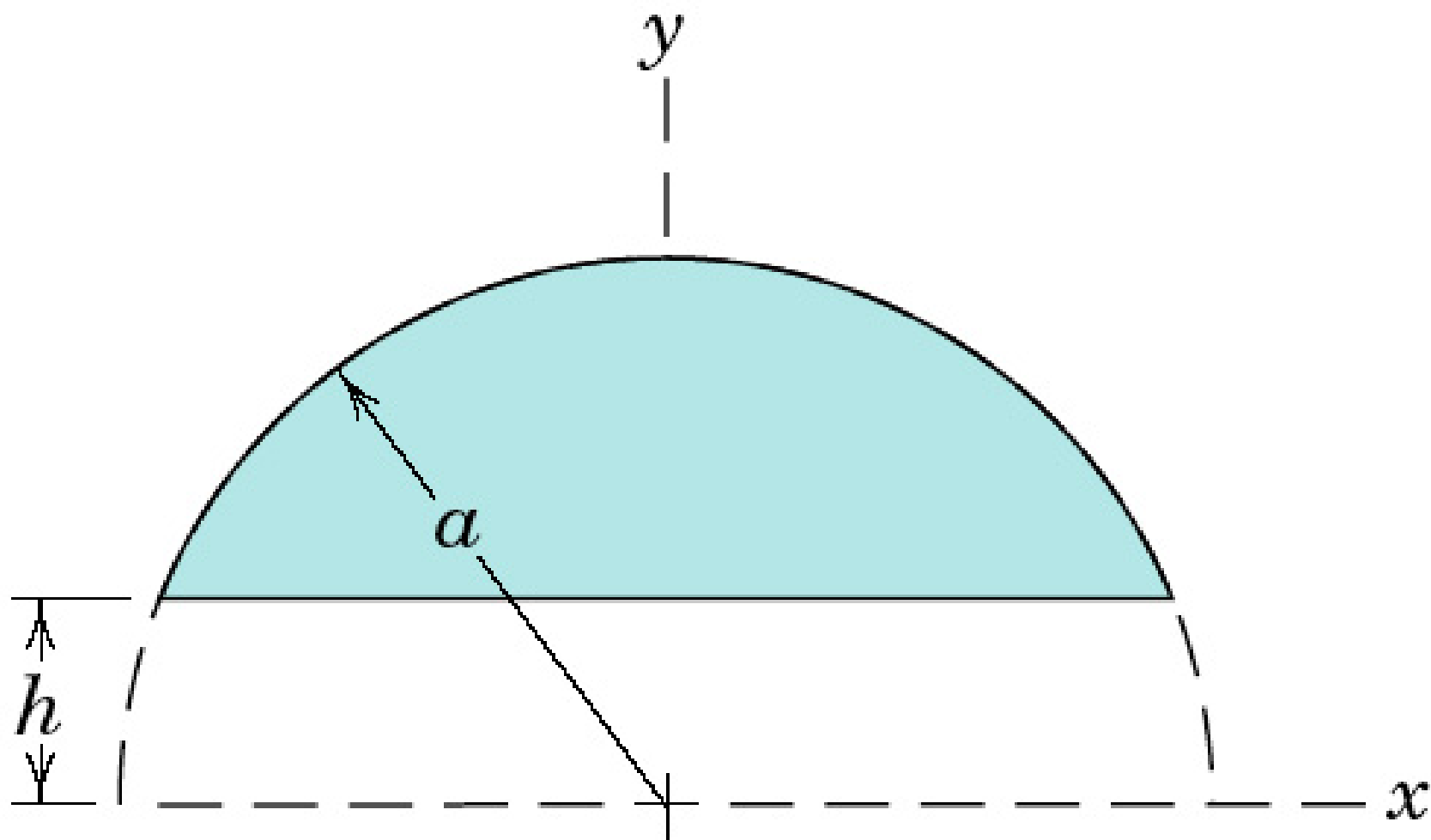
P_05_38



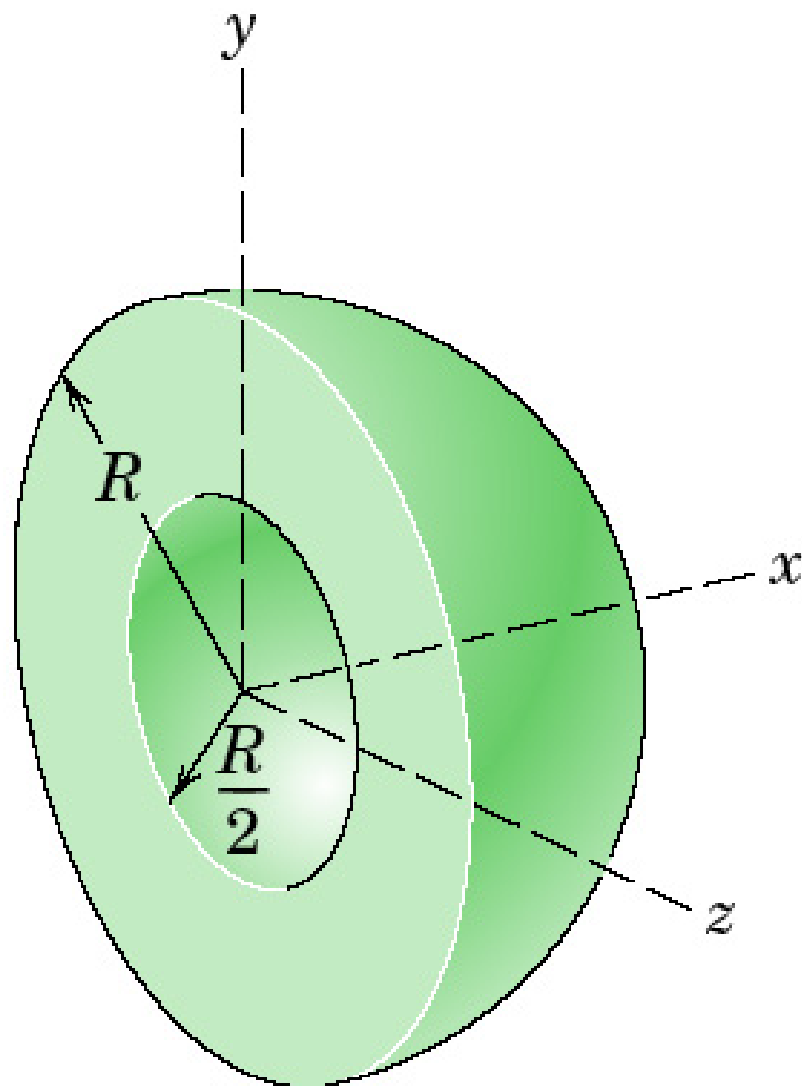
Problem 5/39



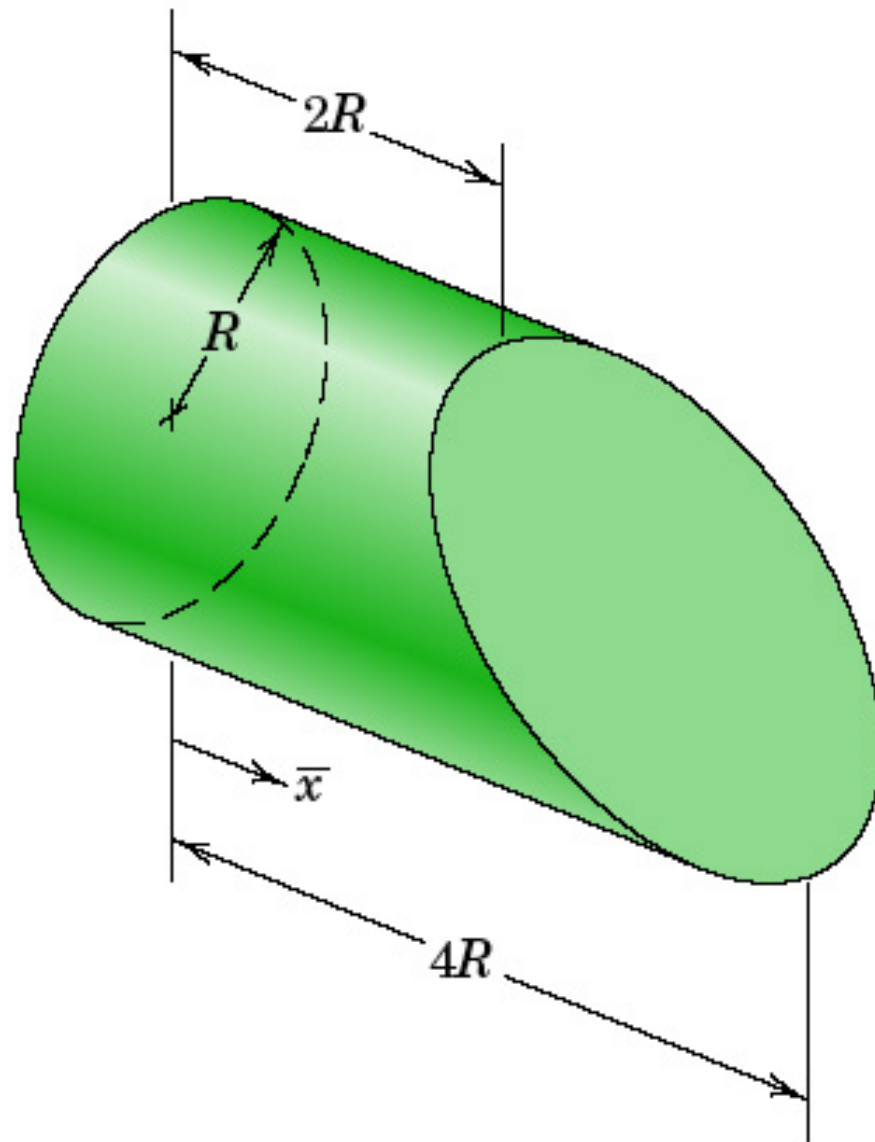
Problem 5/40



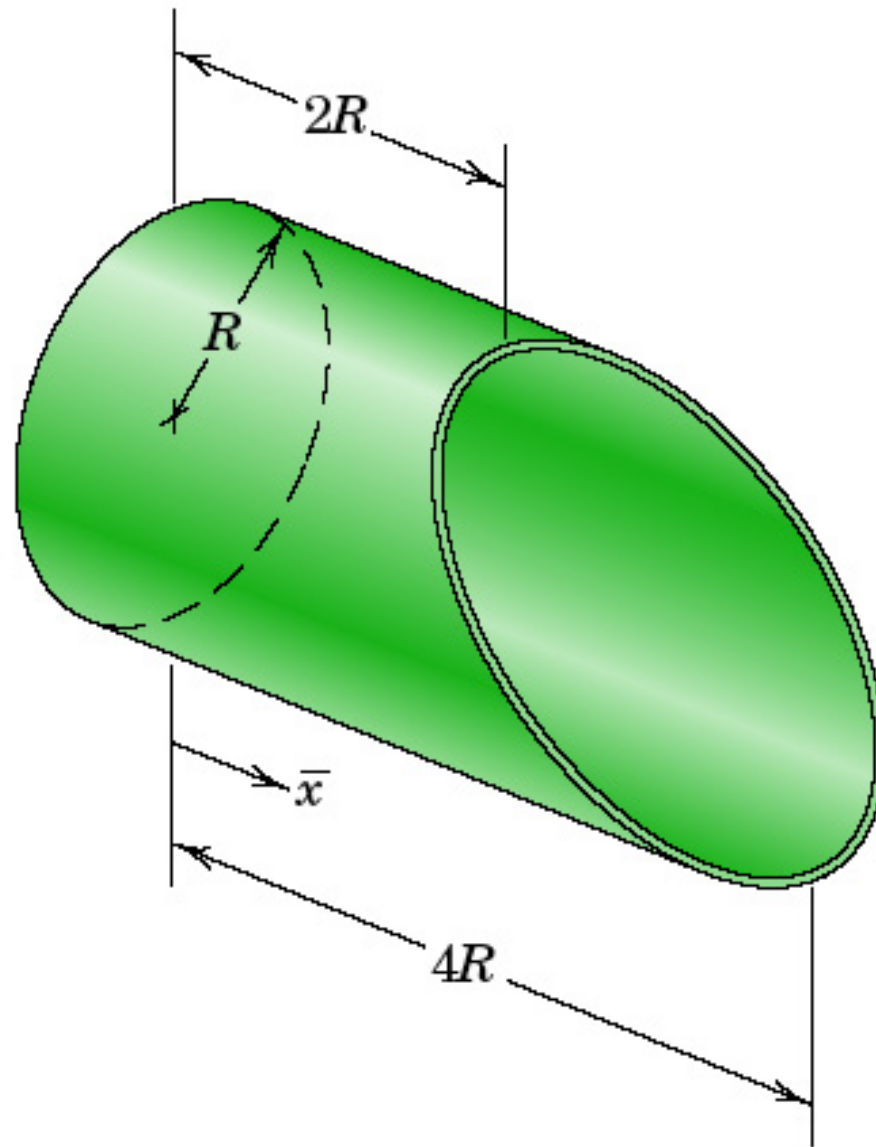
Problem 5/41



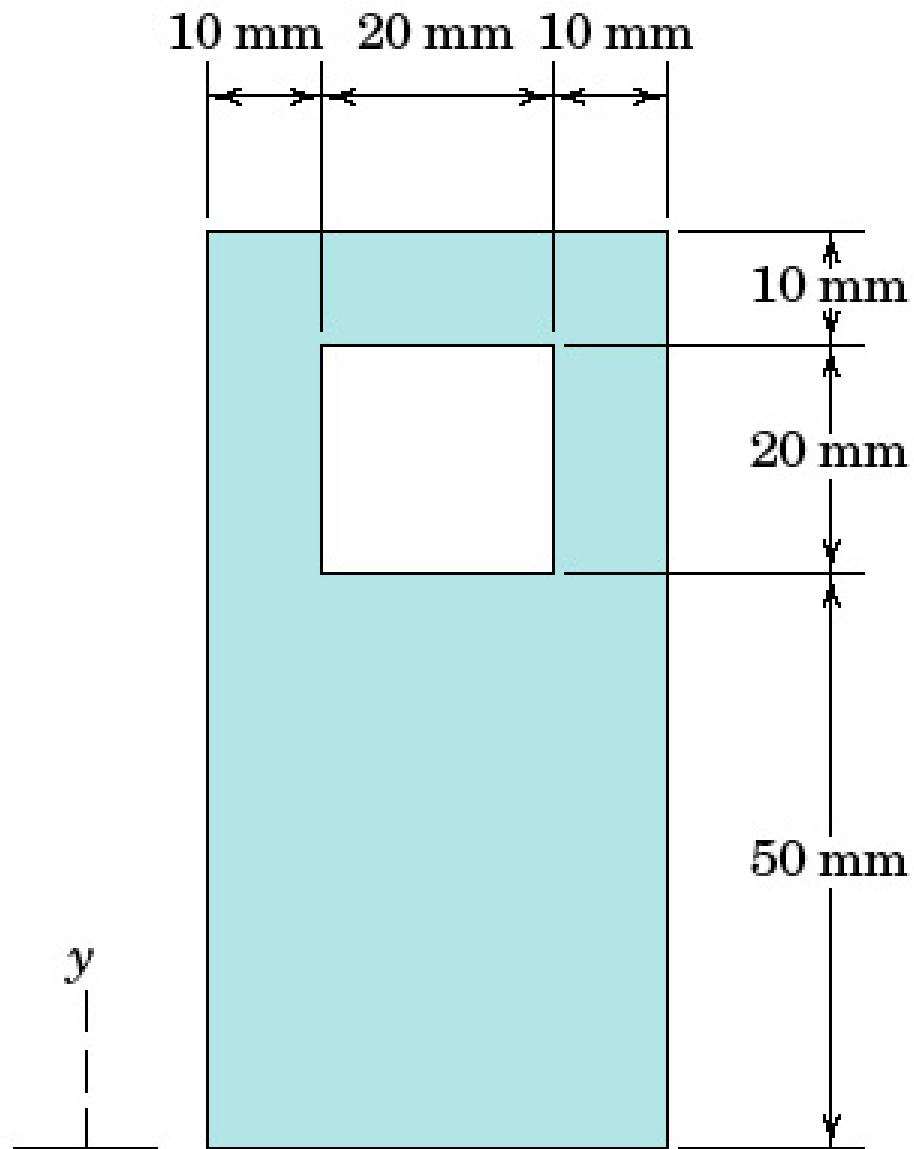
Problem 5/42



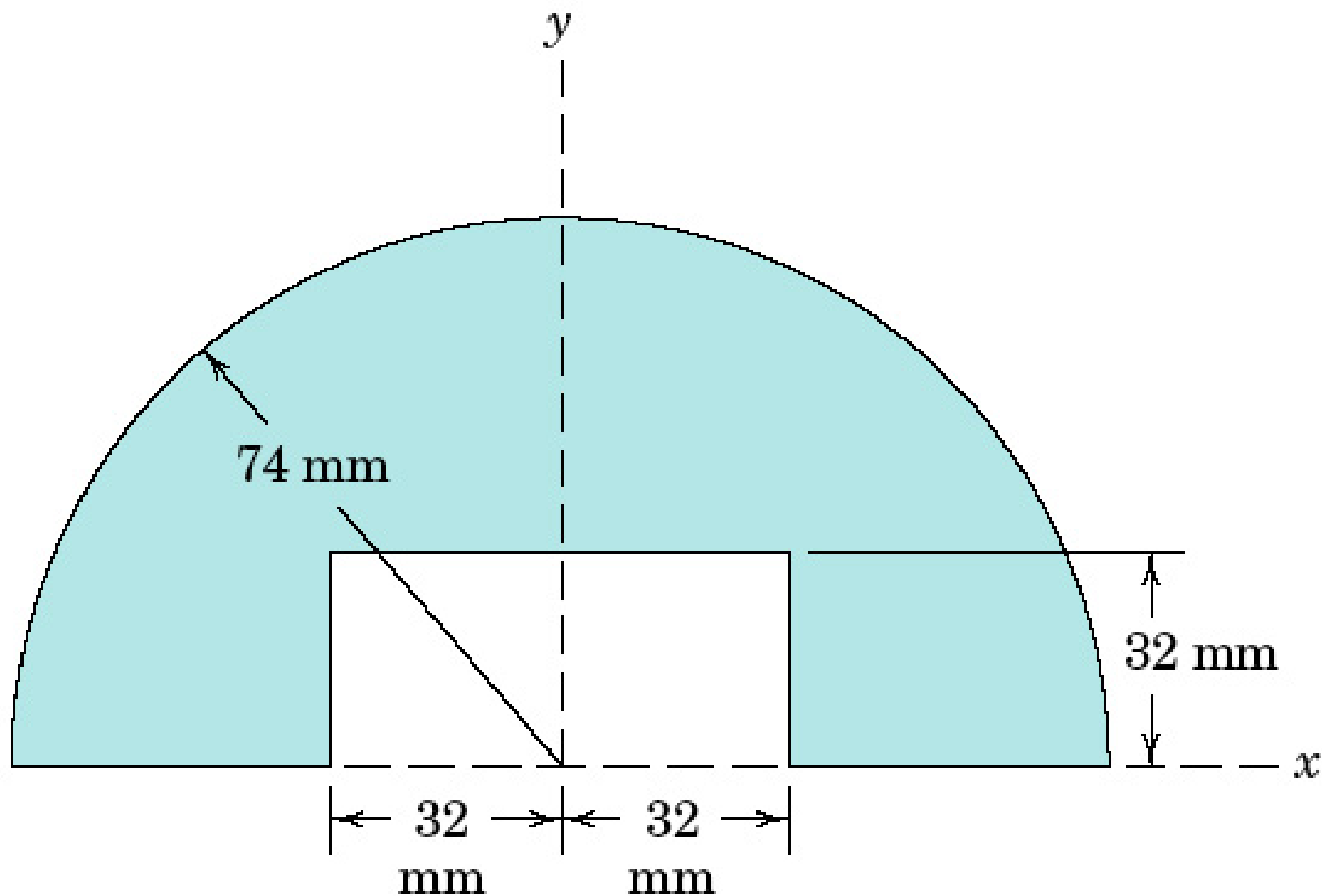
Problem 5/43



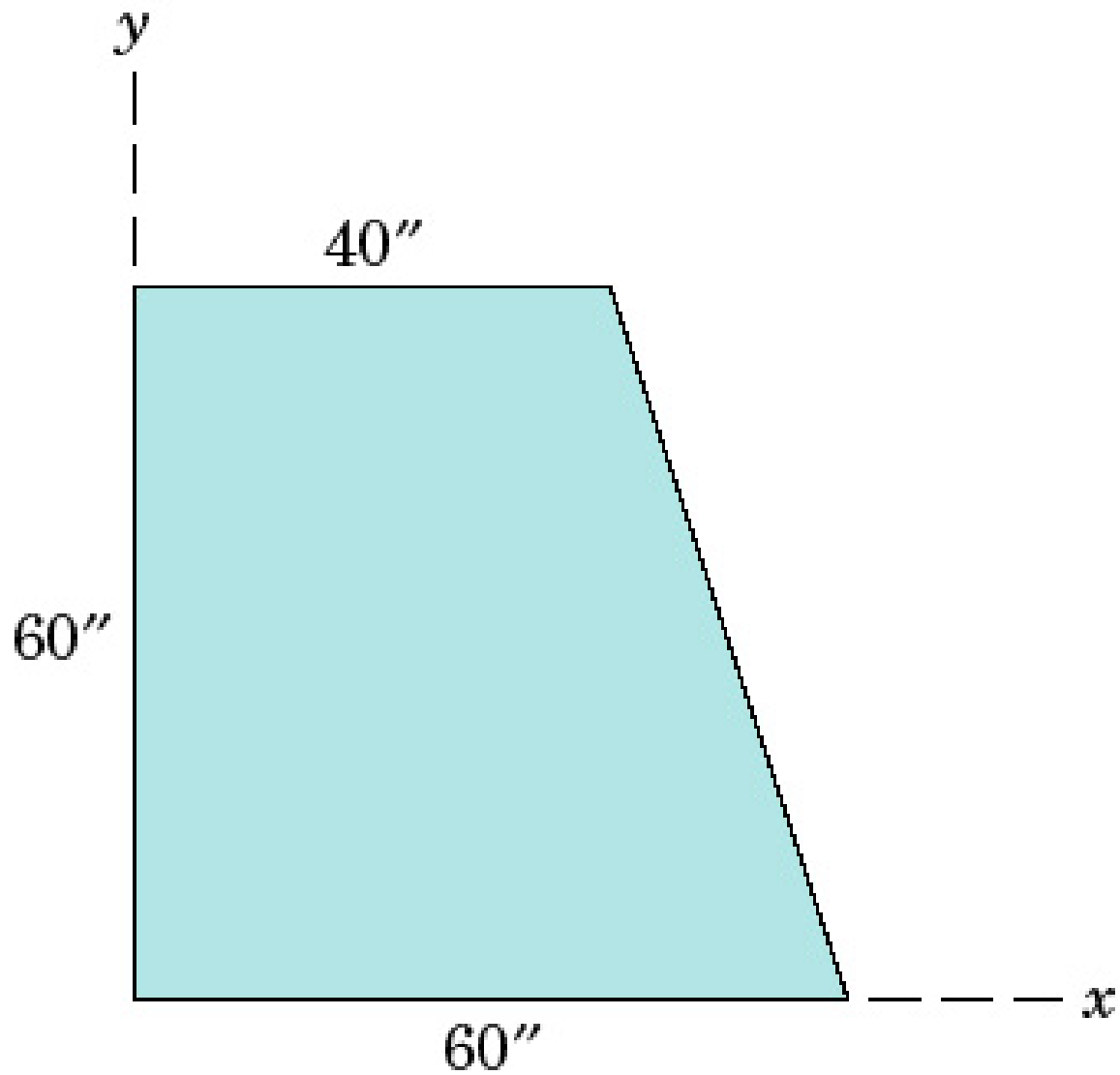
Problem 5/44



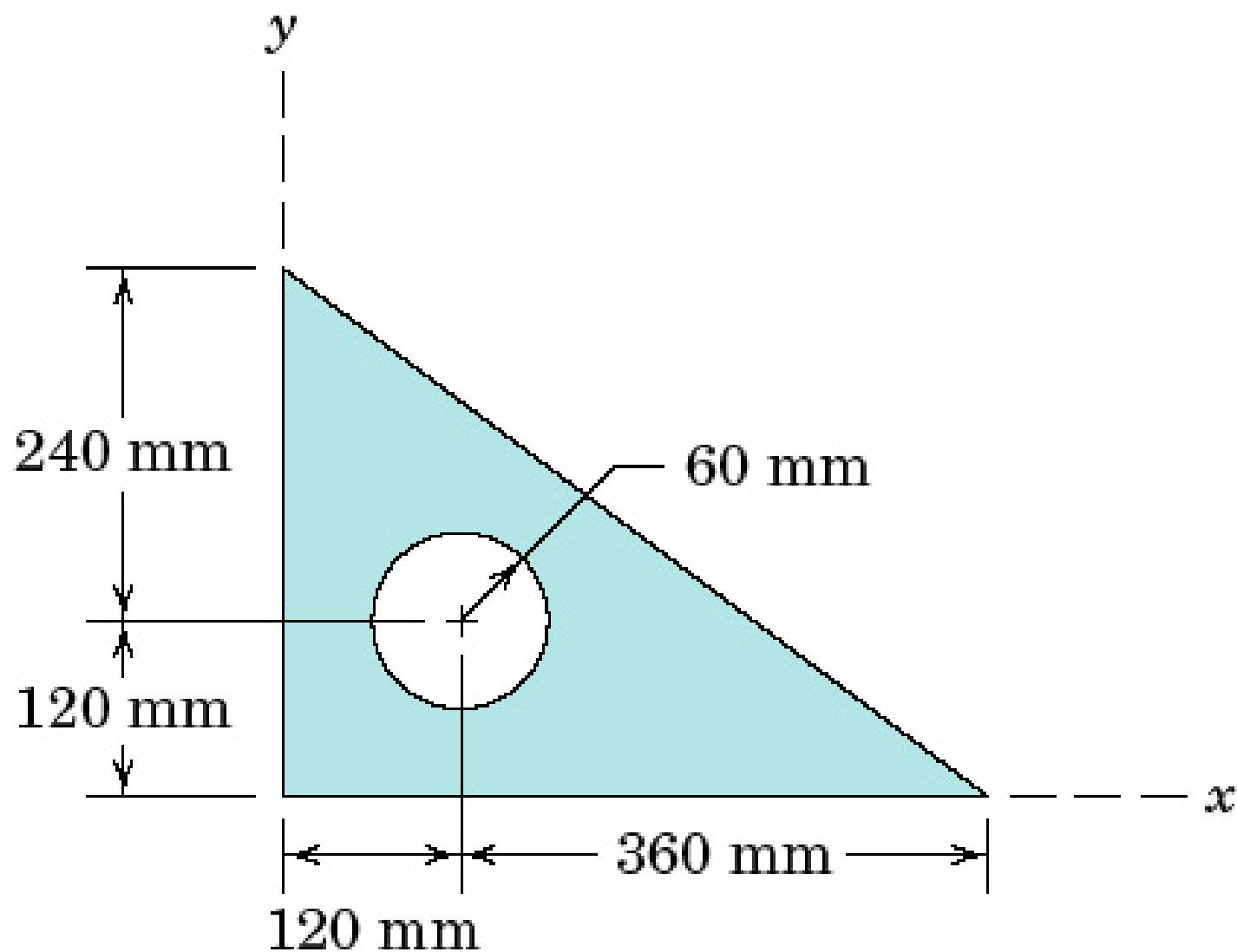
Problem 5/45



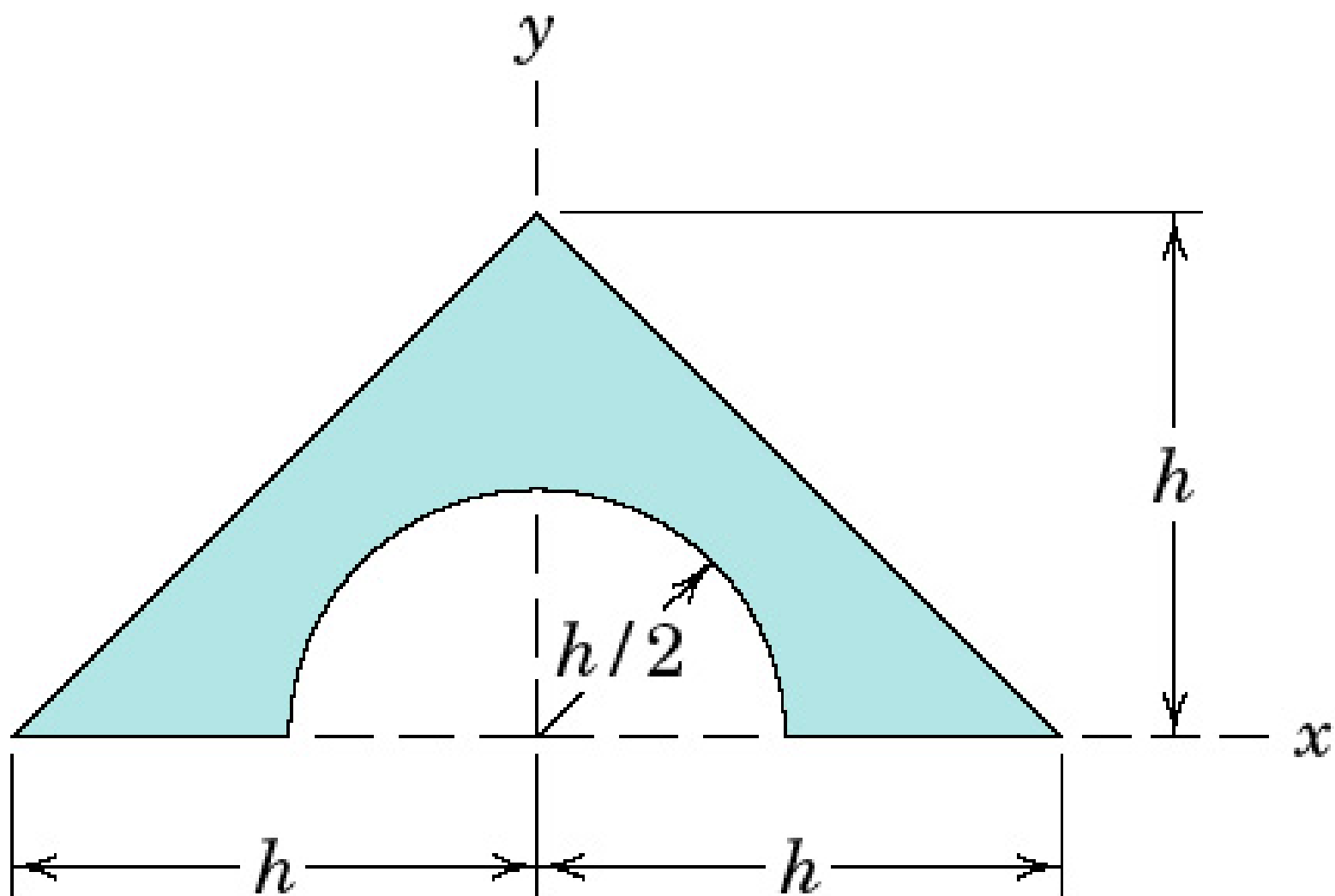
Problem 5/46



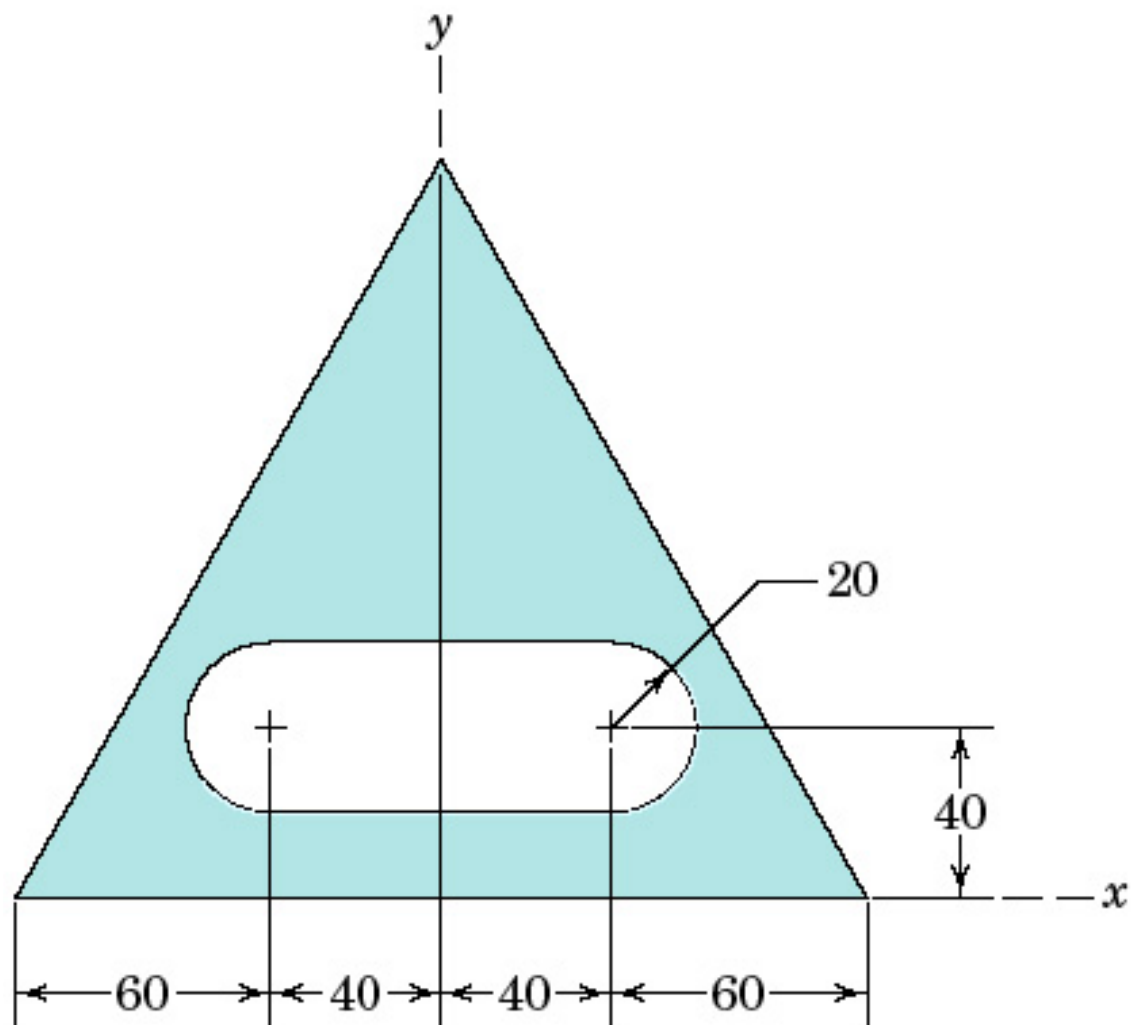
Problem 5/47



Problem 5/48

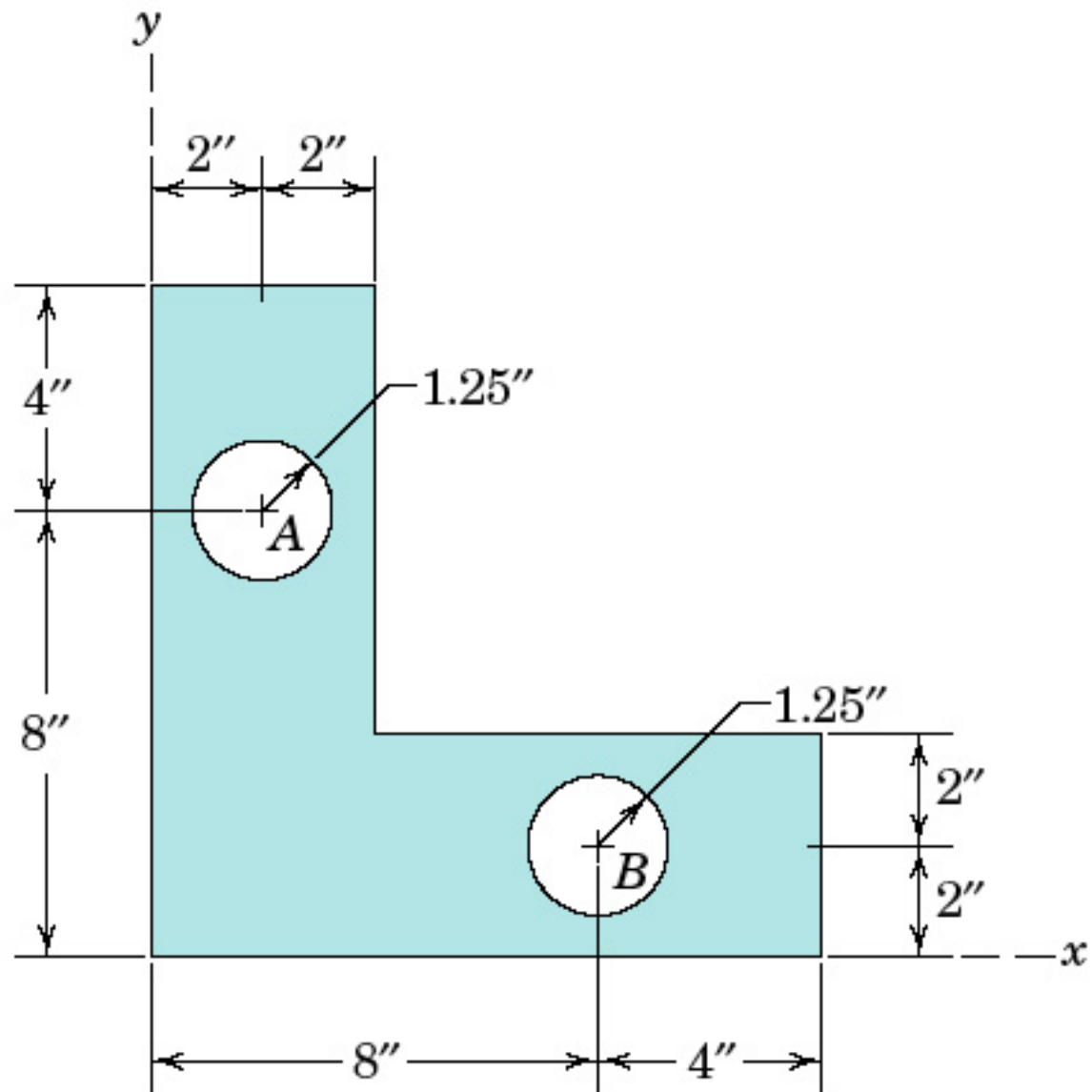


Problem 5/49

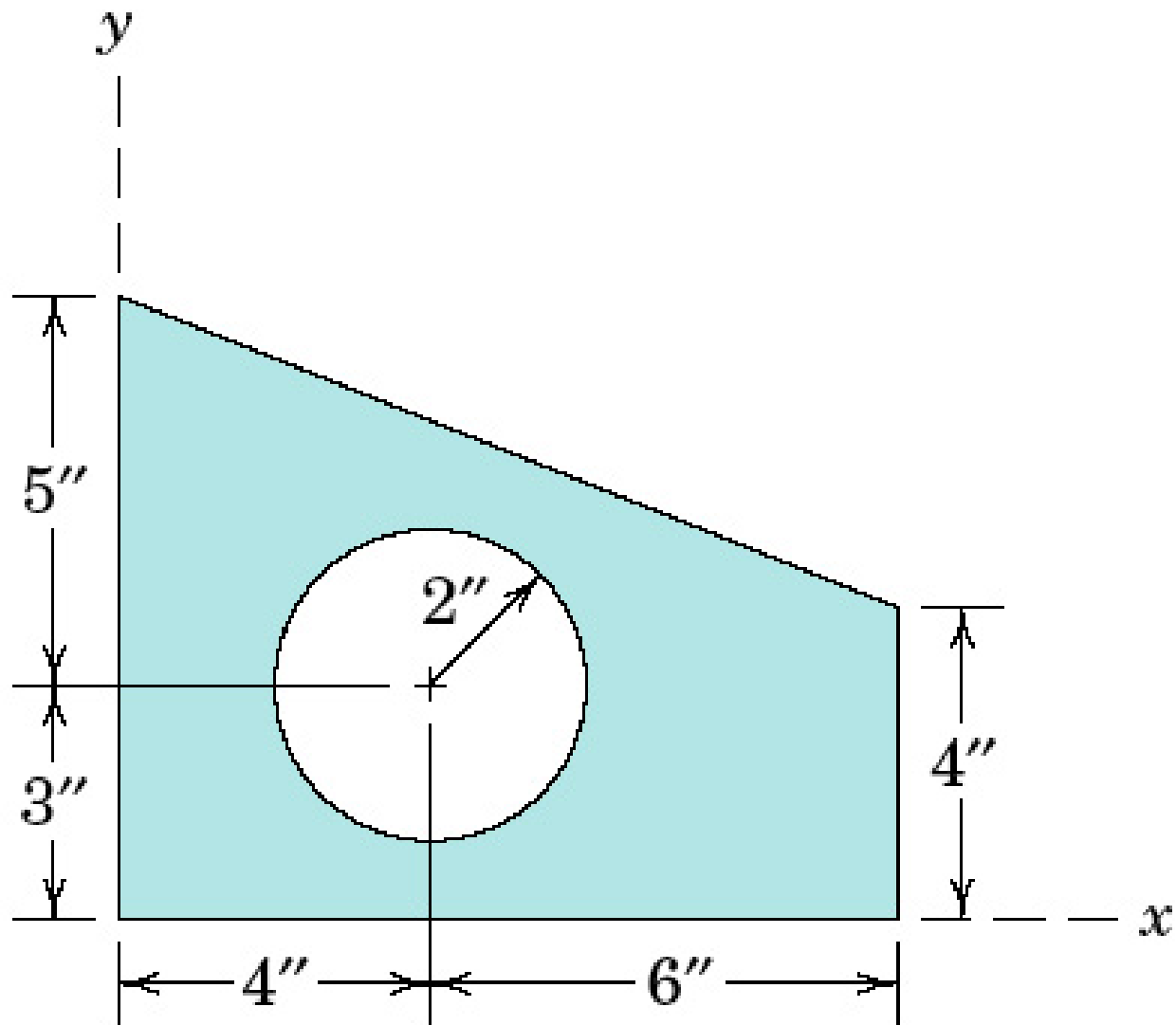


Dimensions in millimeters

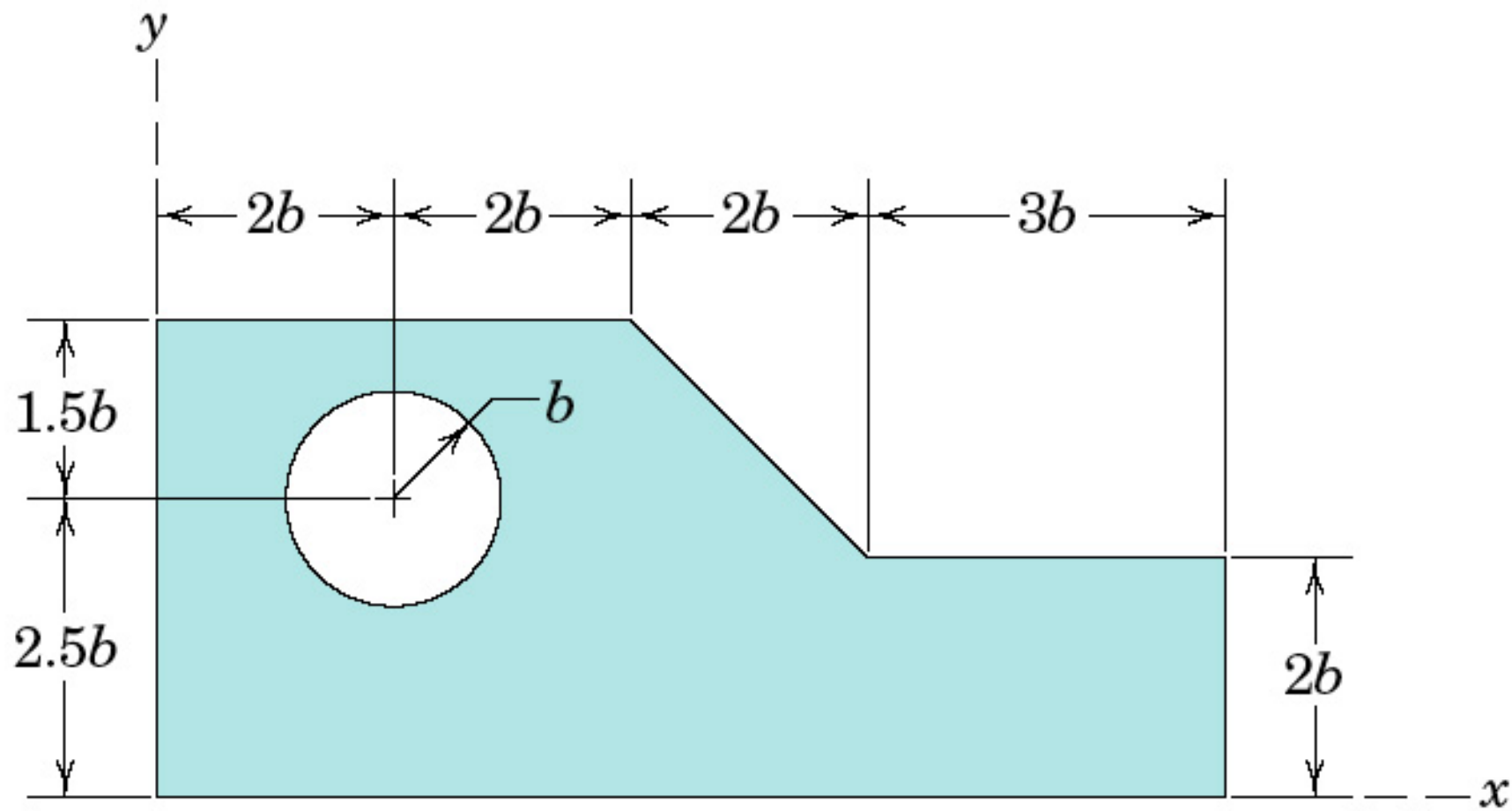
Problem 5/50



Problem 5/51

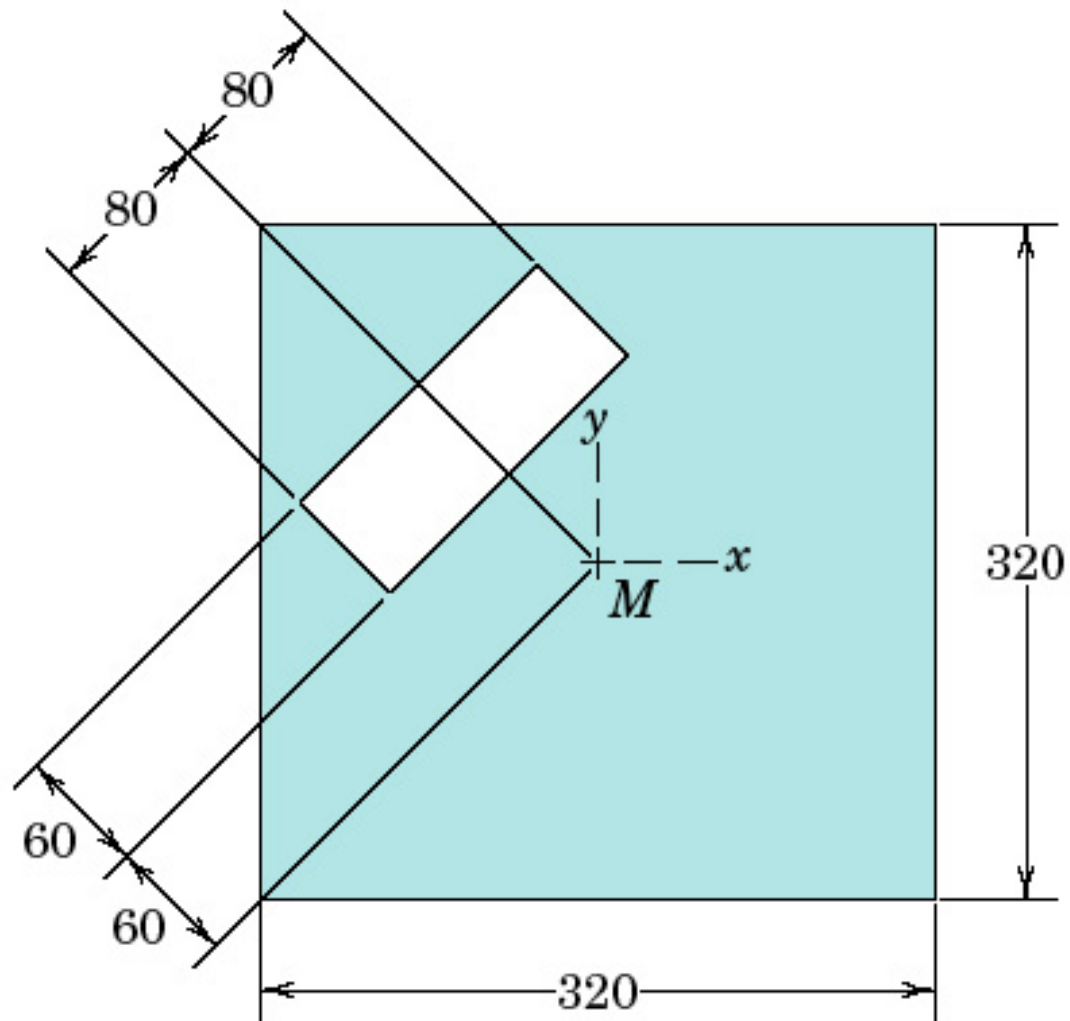


Problem 5/52



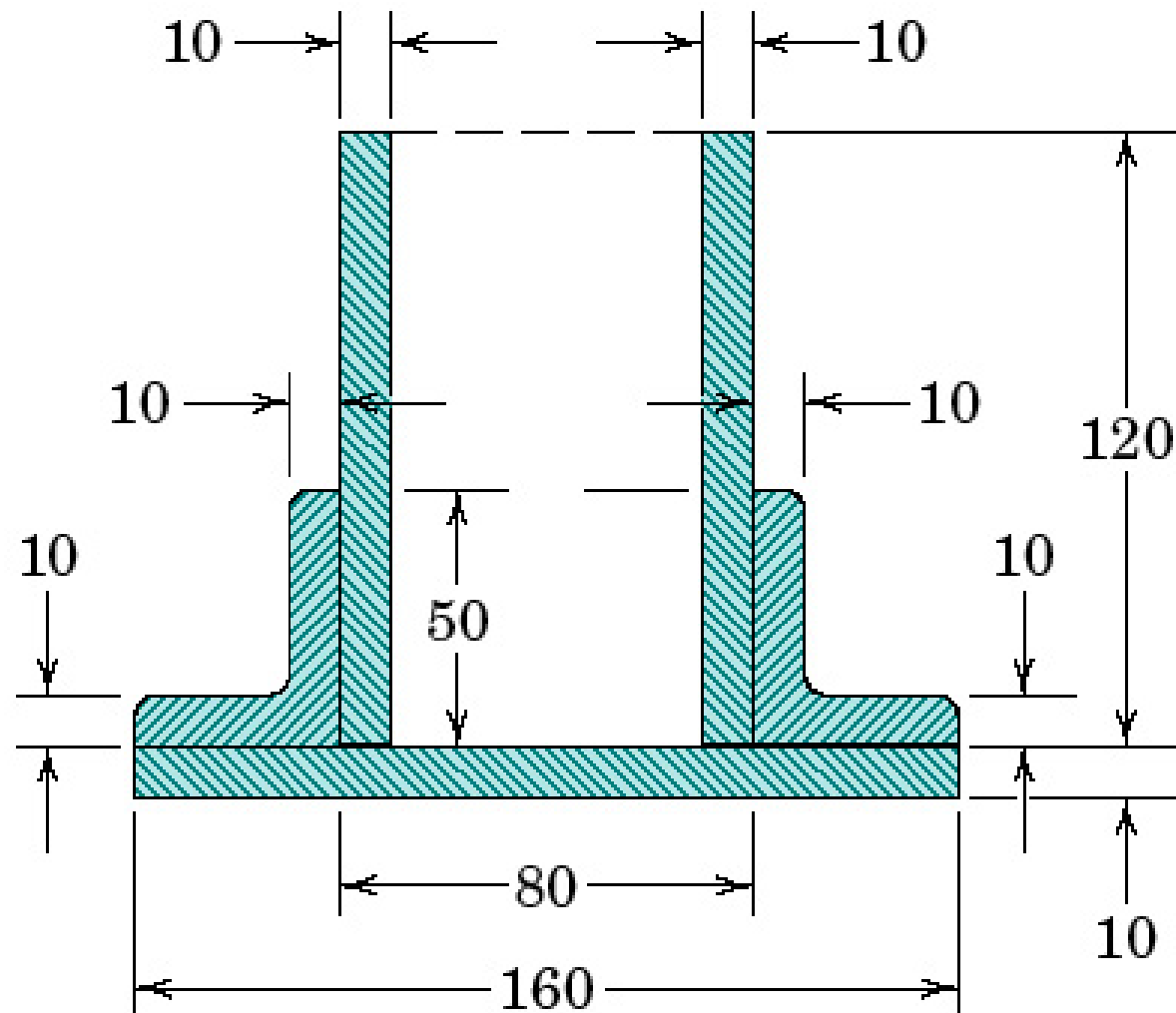
Problem 5/53

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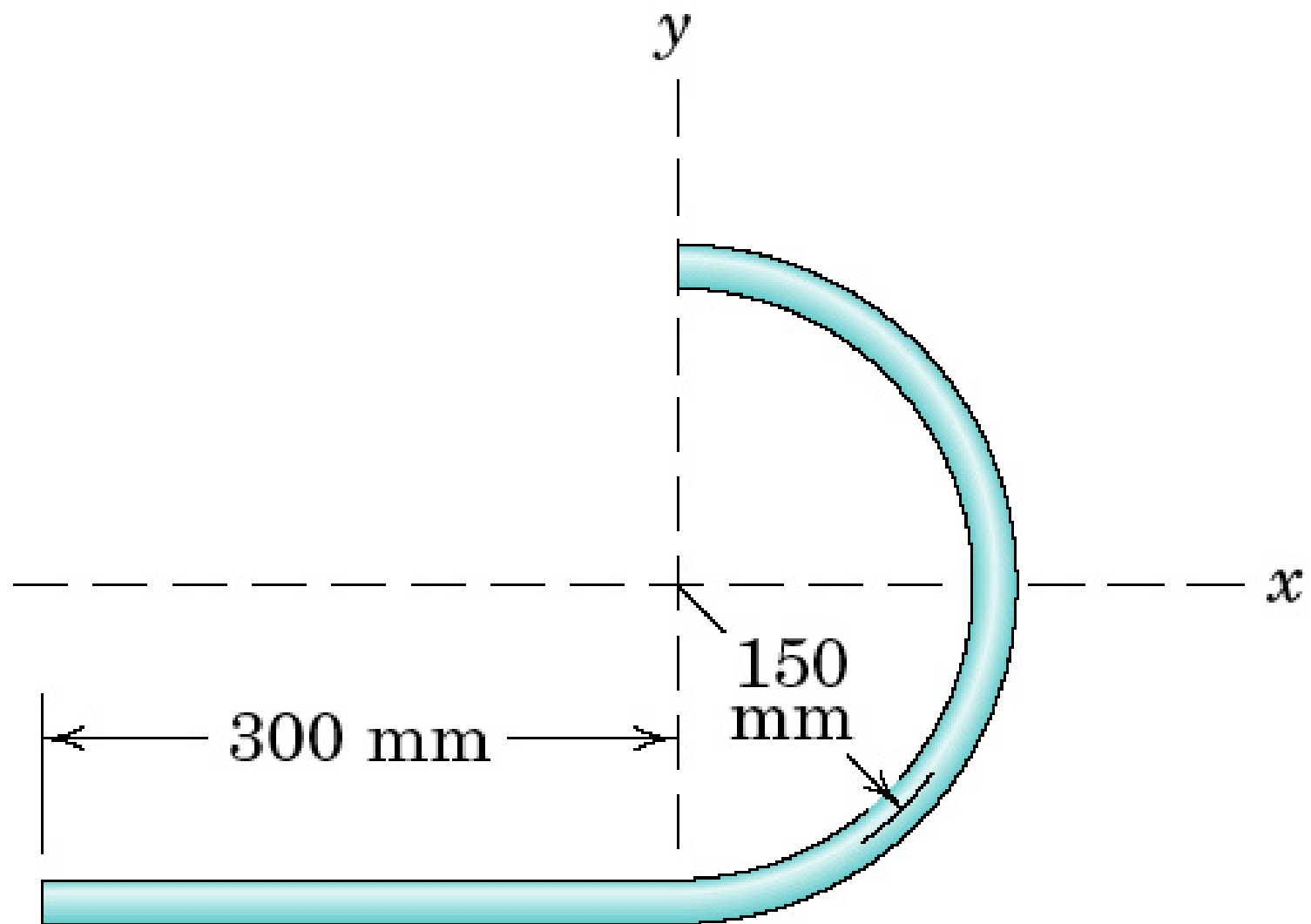
Dimensions in millimeters

Problem 5/54

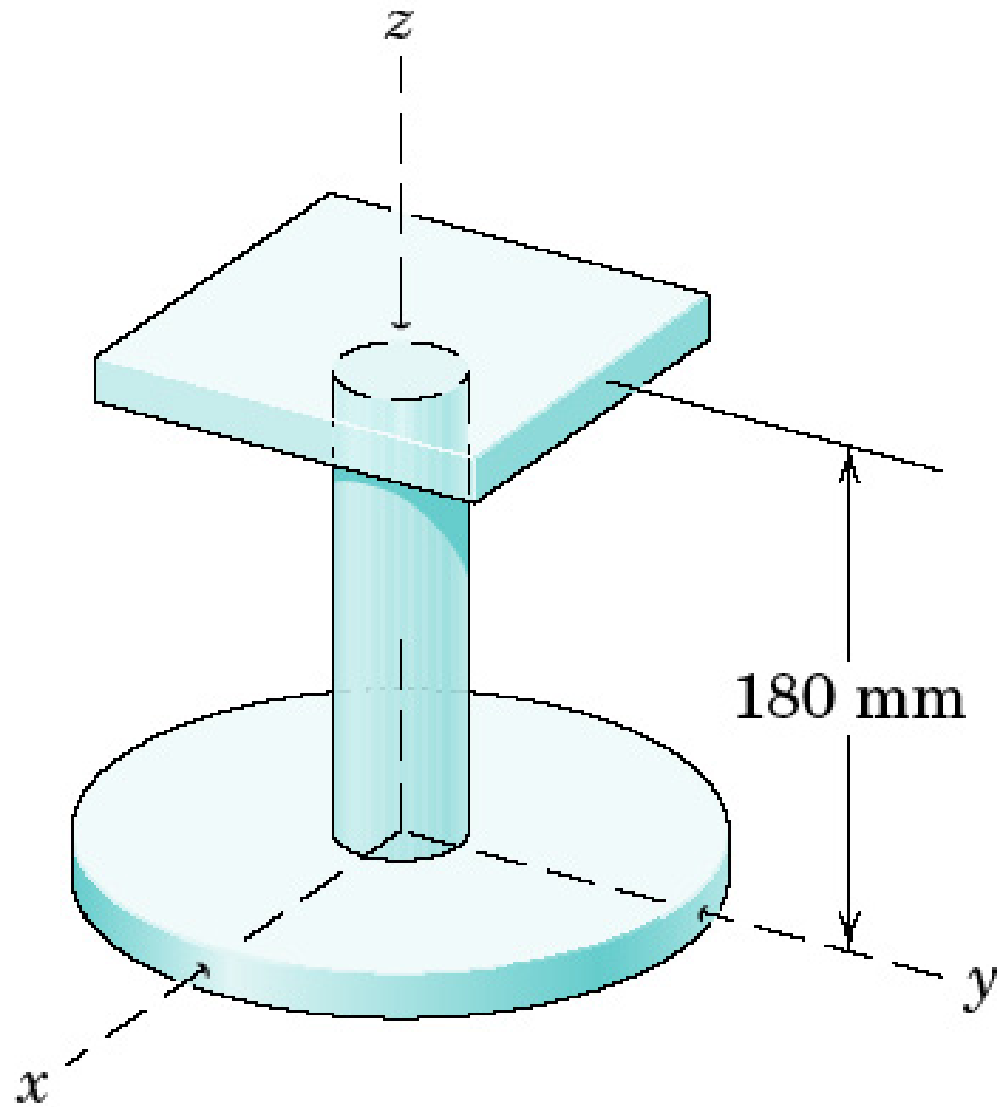


Dimensions in millimeters

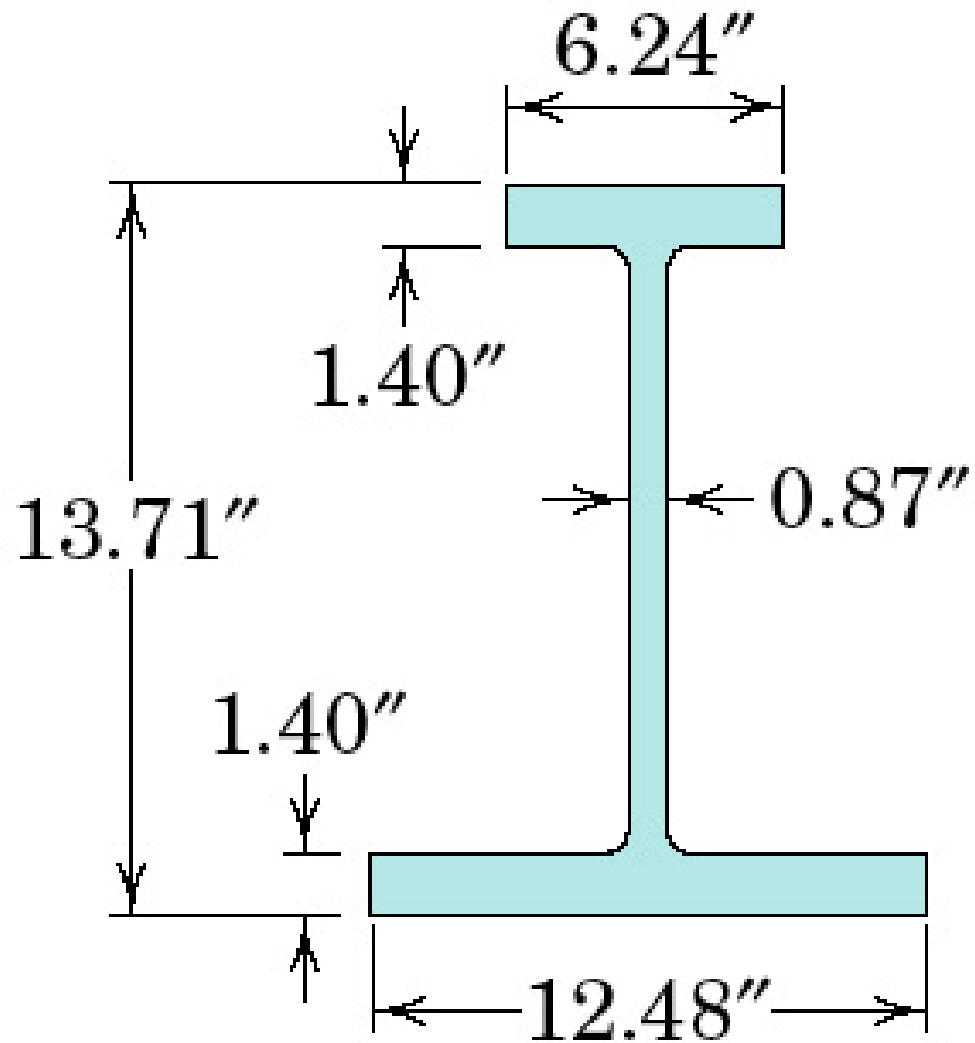
Problem 5/55



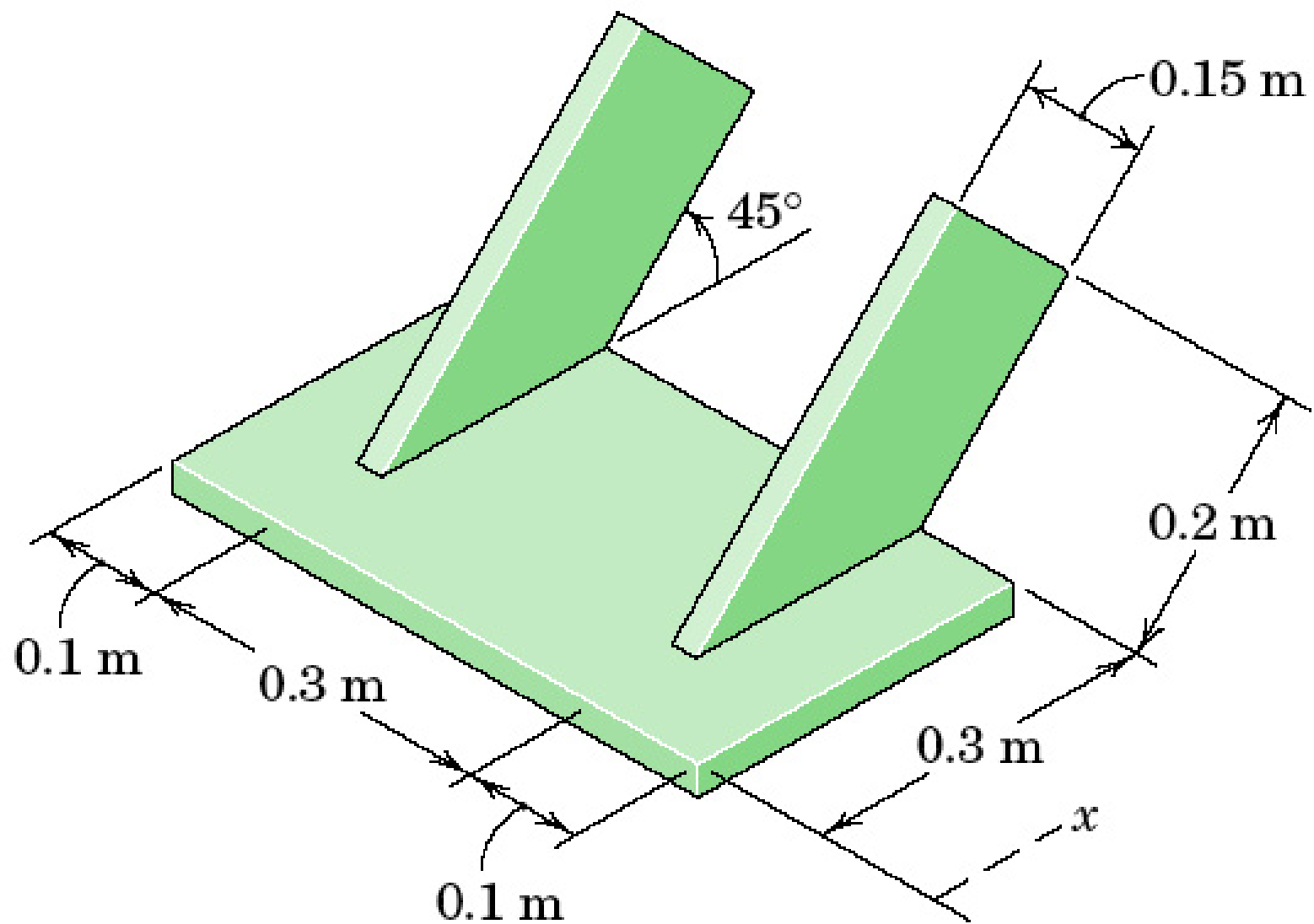
Problem 5/56



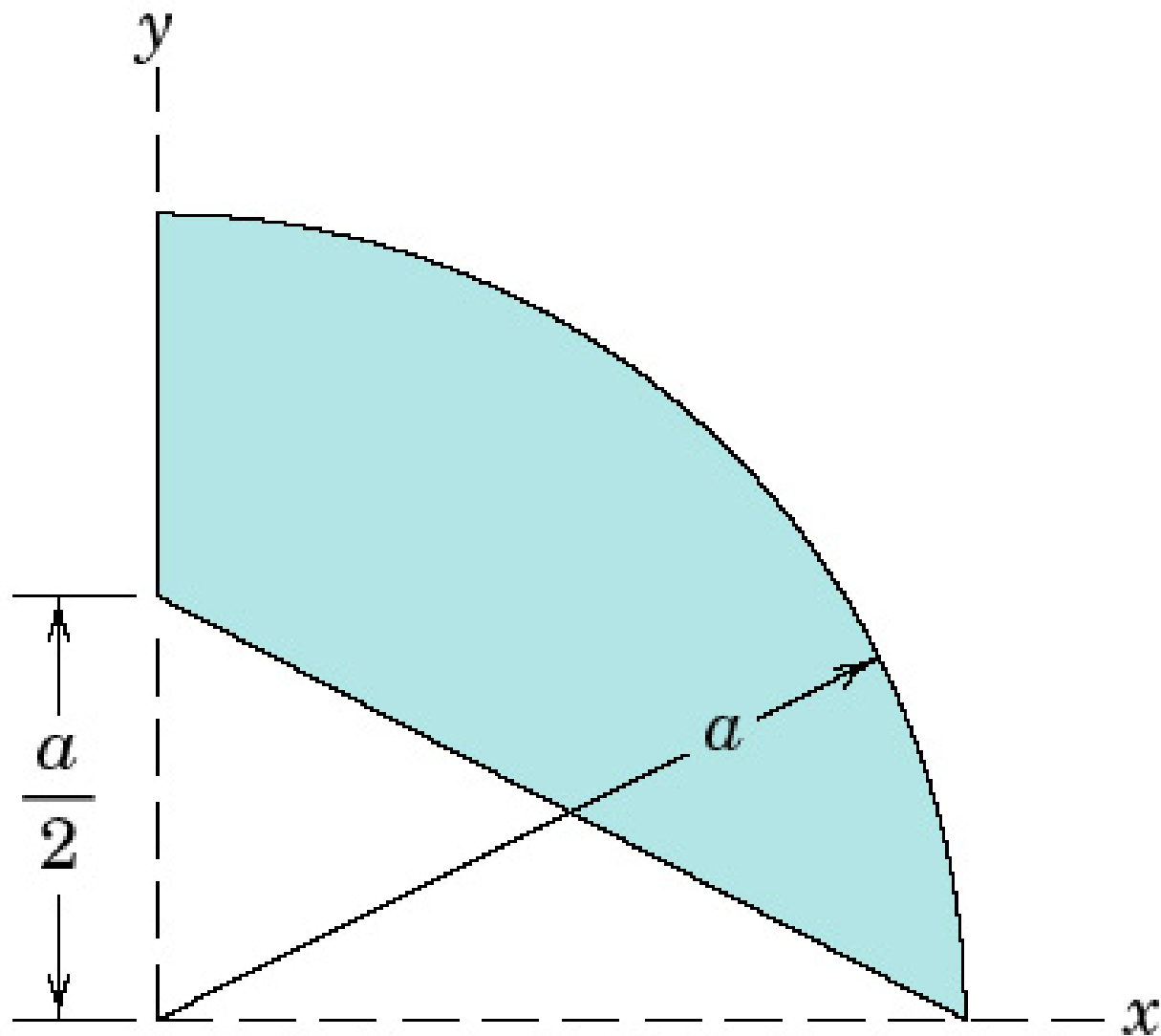
Problem 5/57



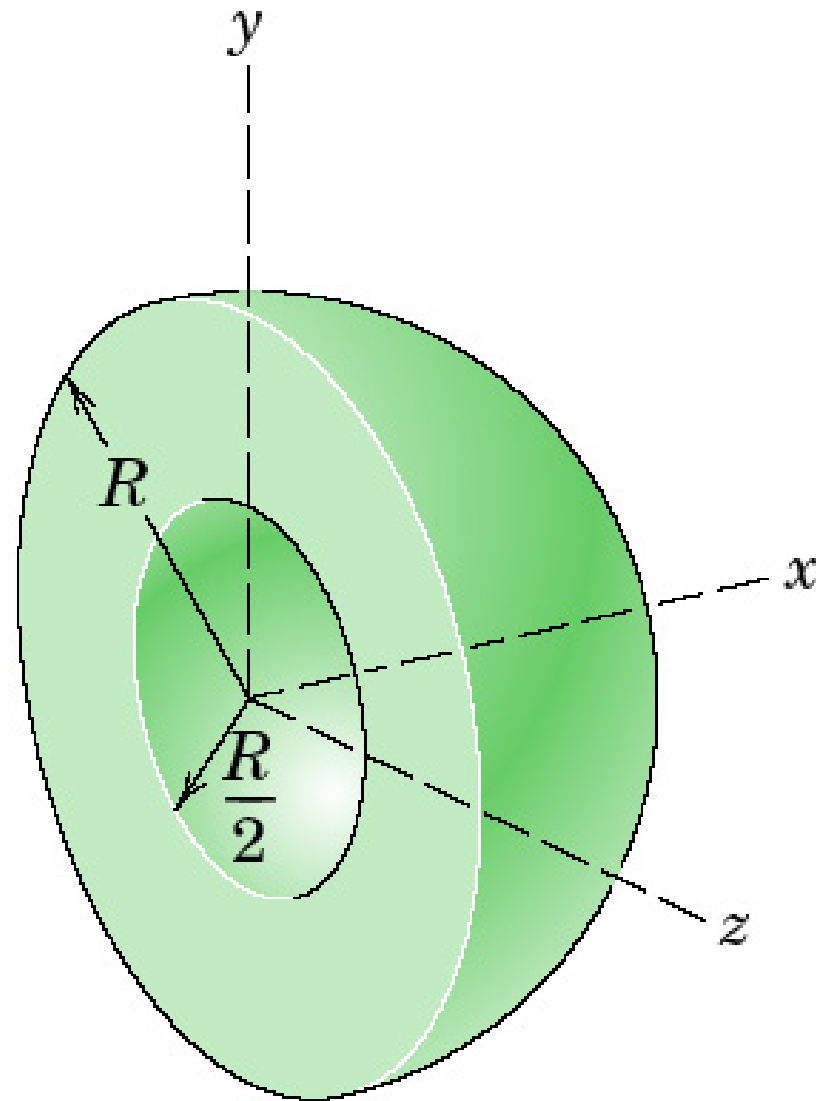
Problem 5/58



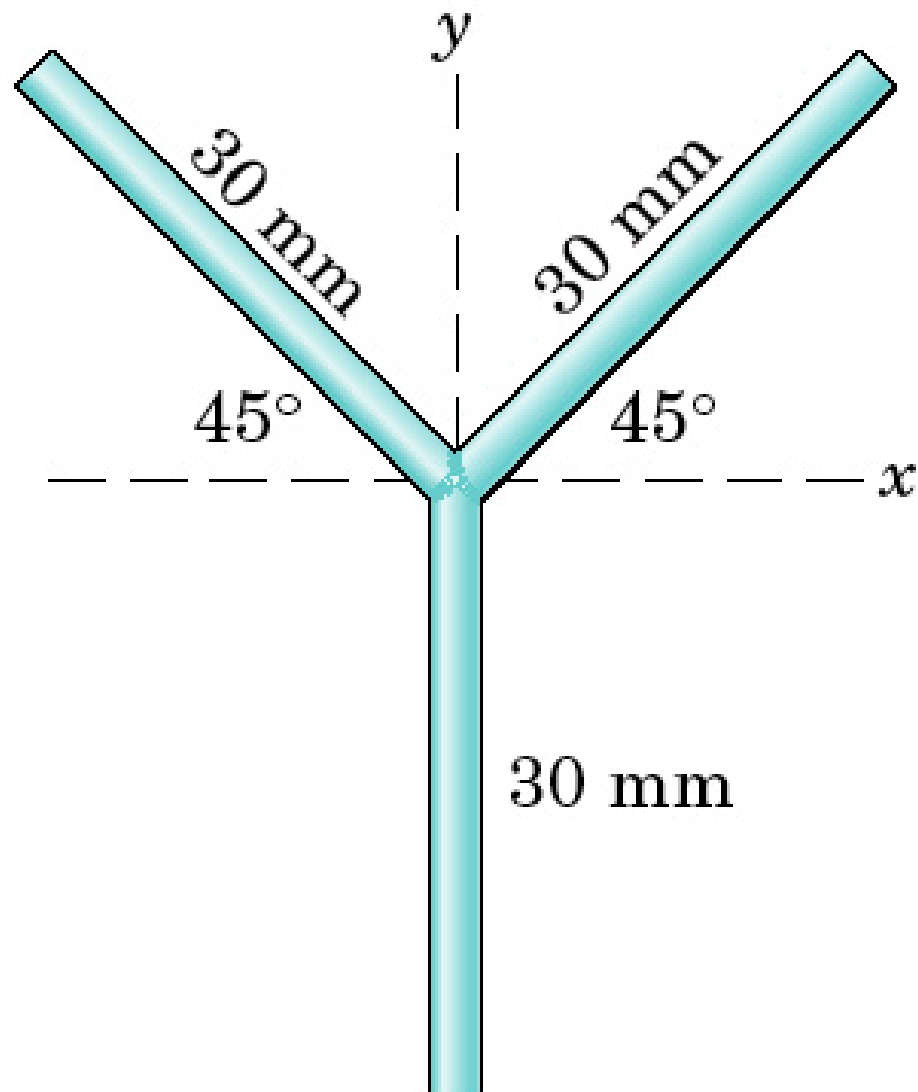
Problem 5/59



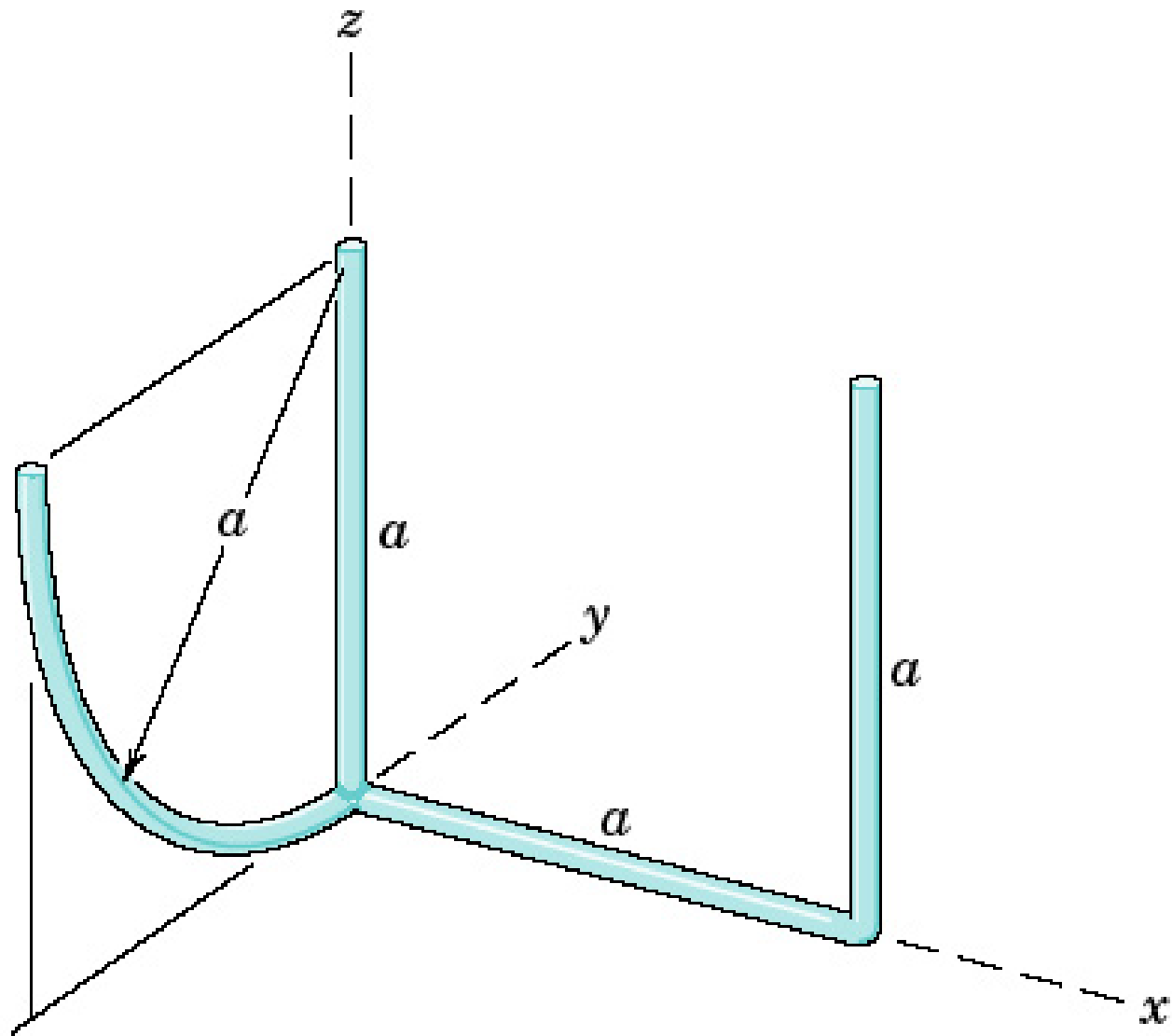
Problem 5/60



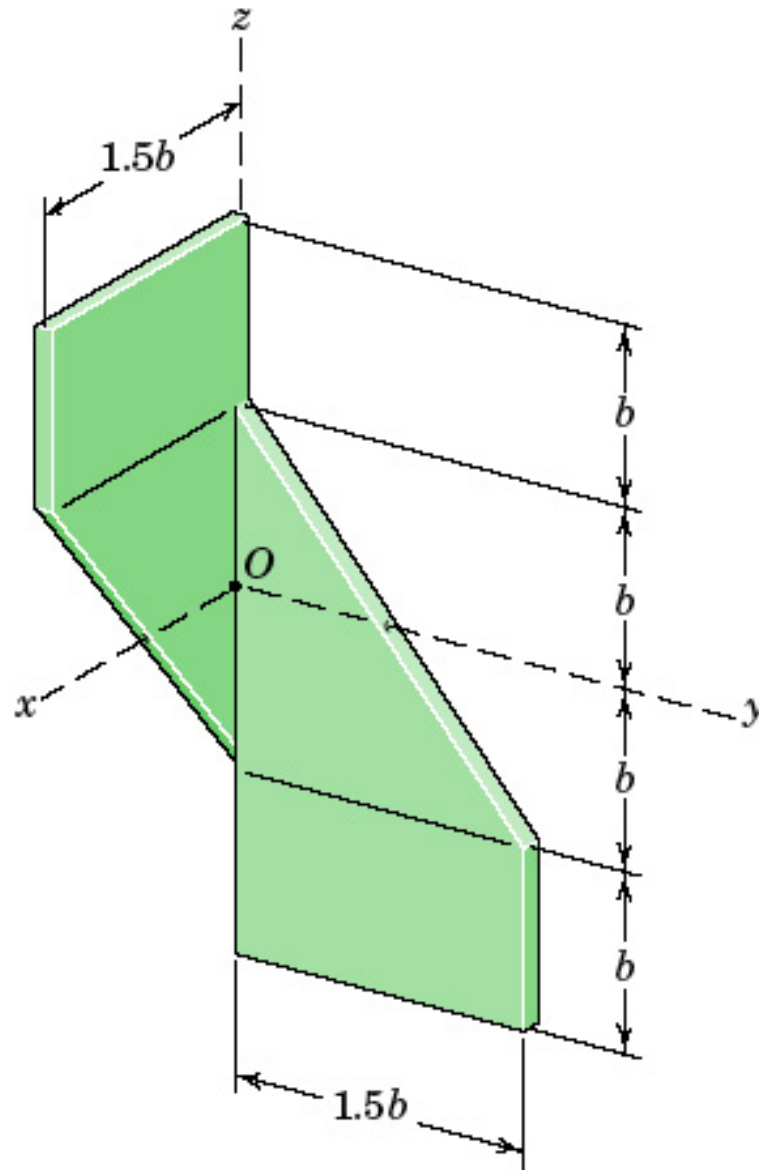
Problem 5/61



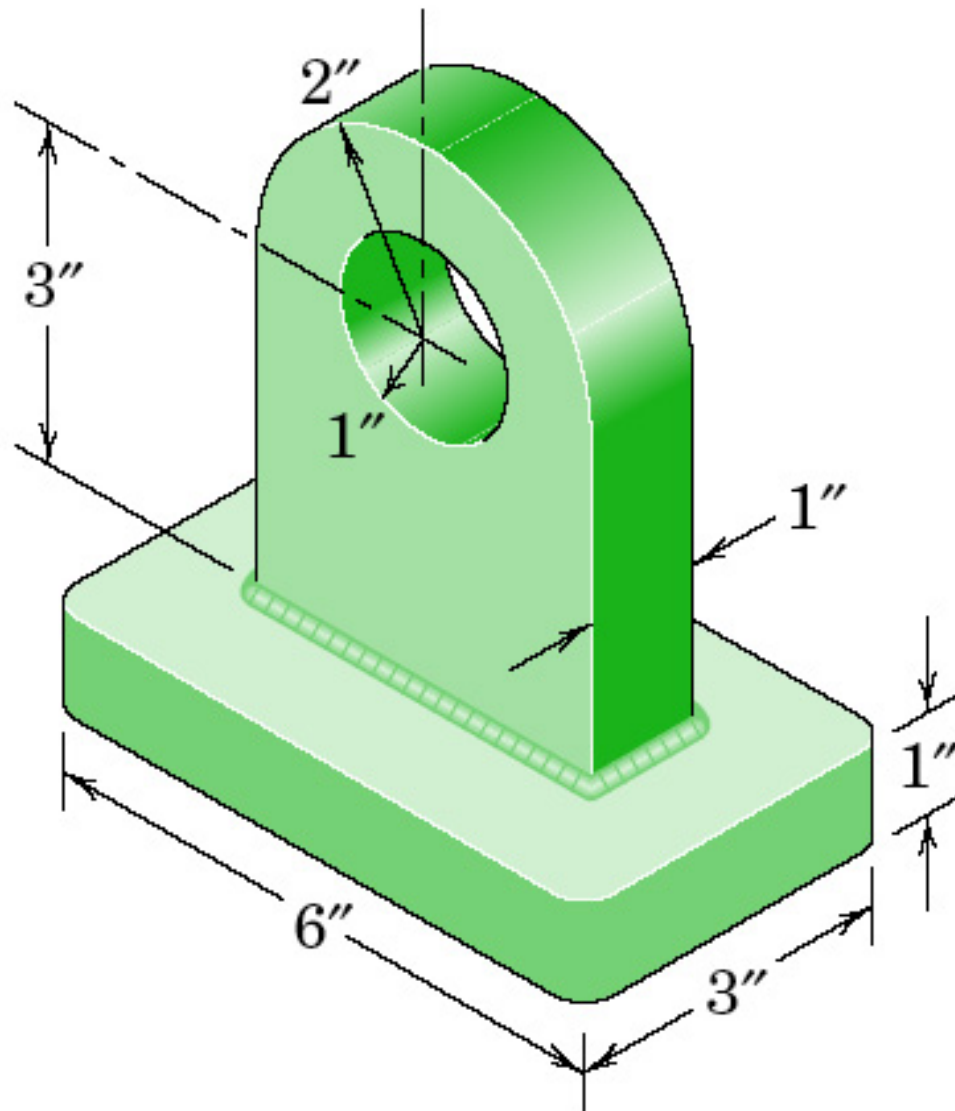
Problem 5/62



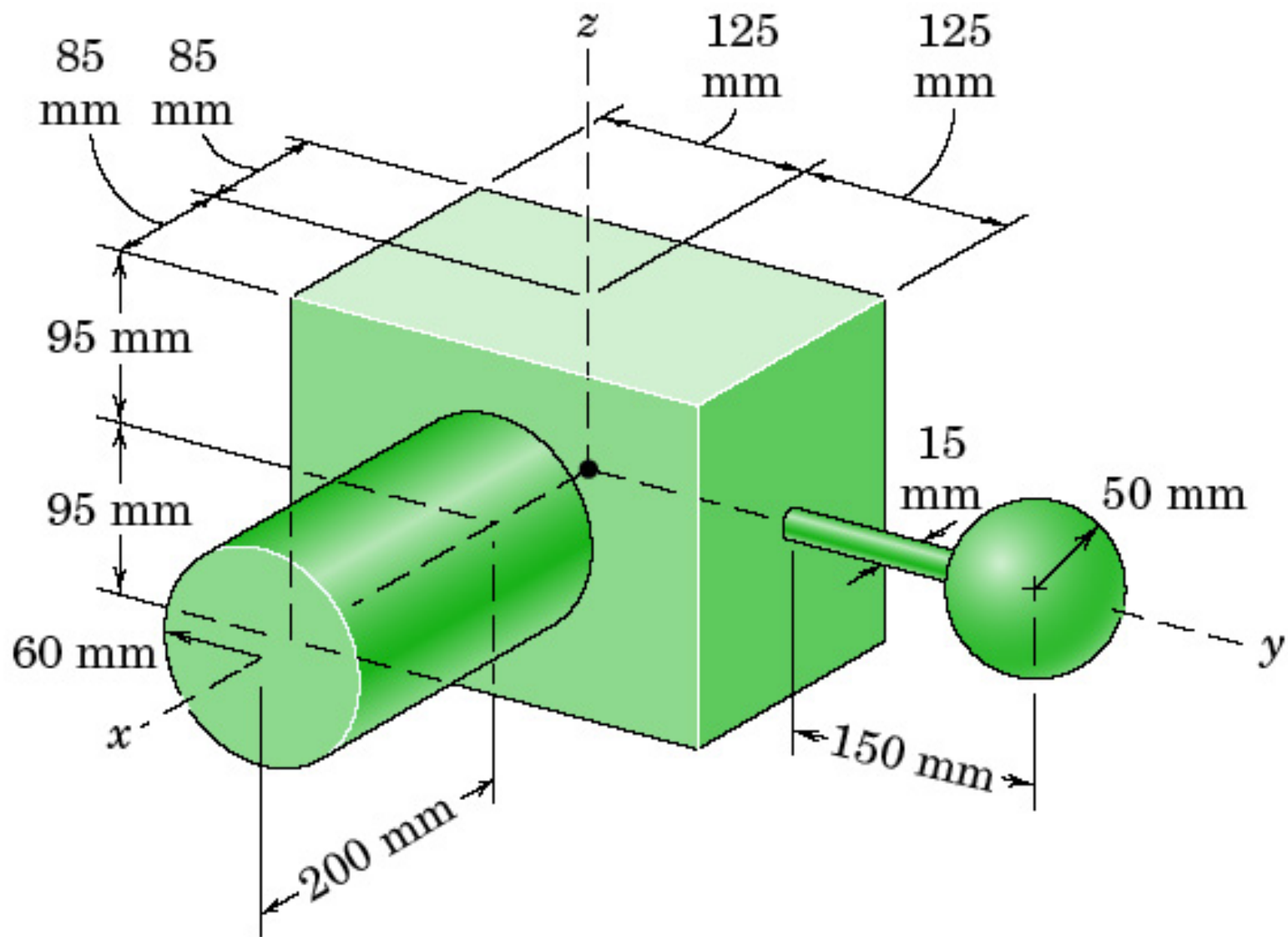
Problem 5/63



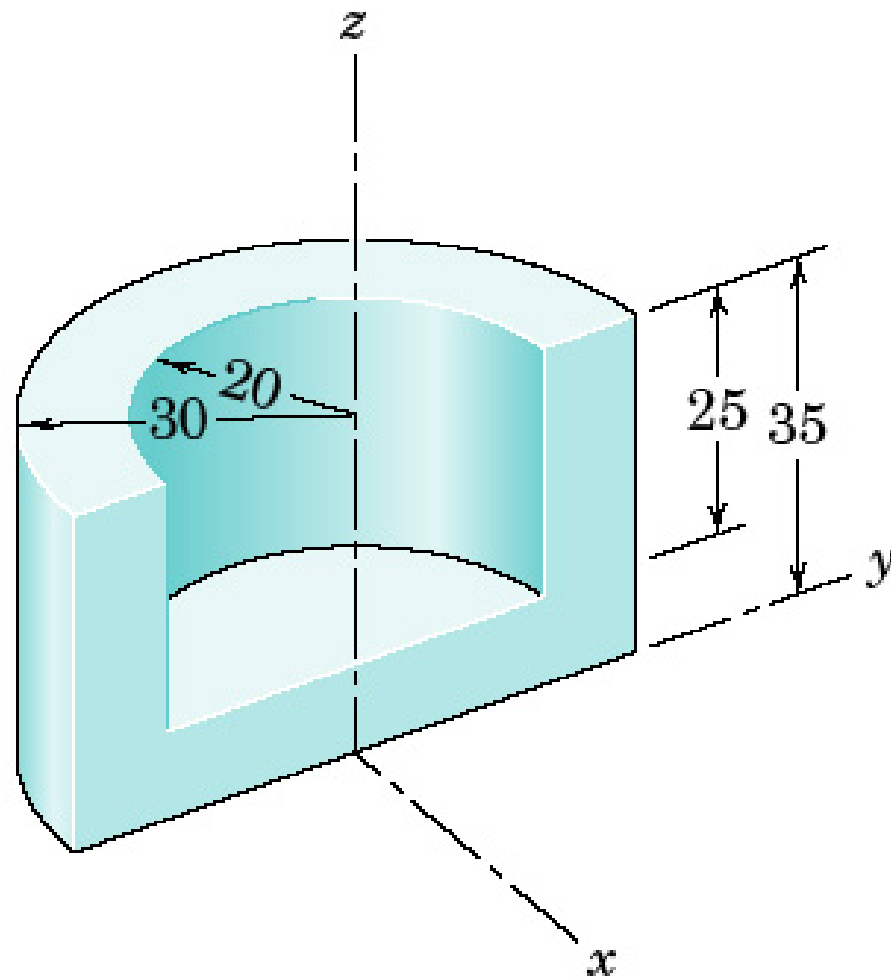
Problem 5/64



Problem 5/66

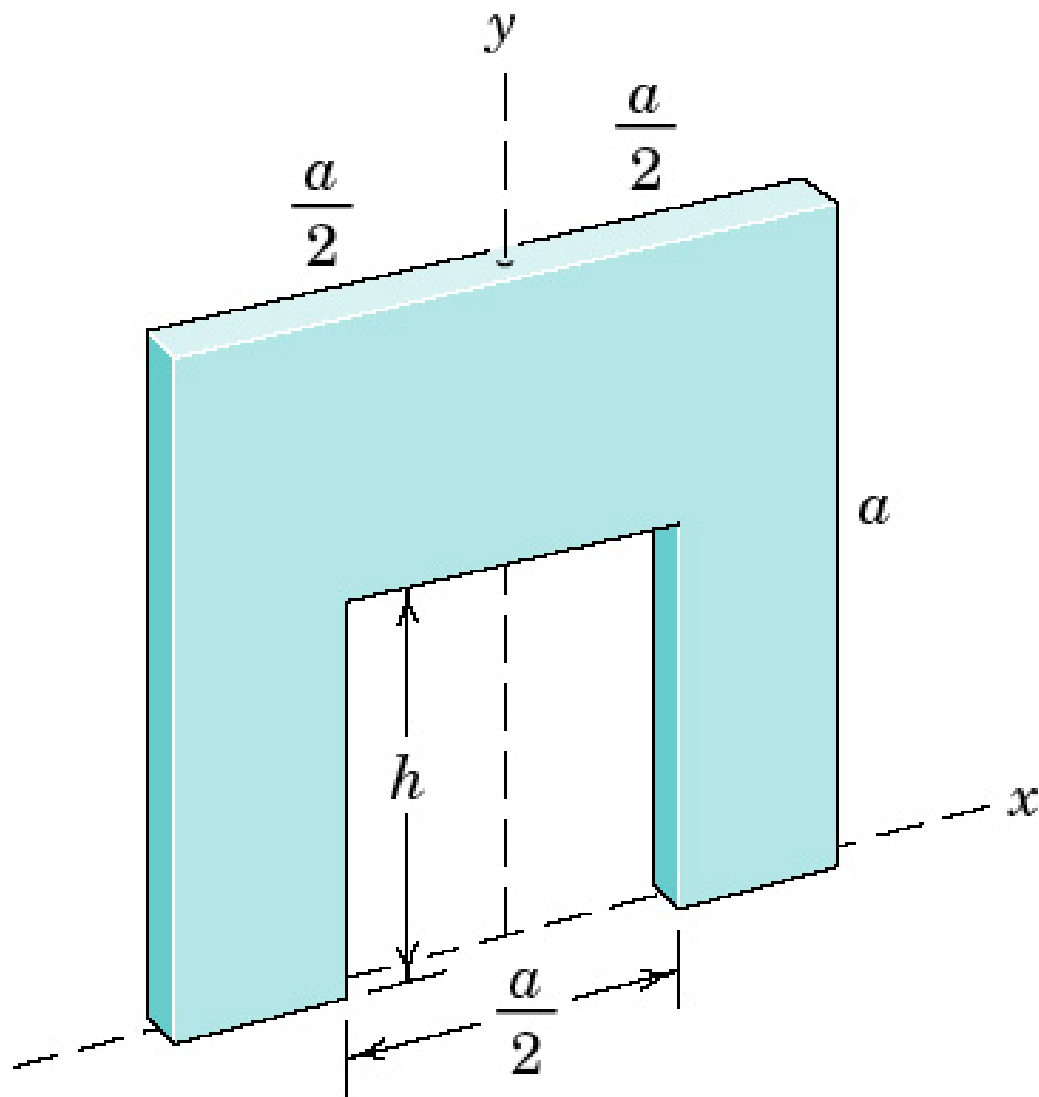


Problem 5/67

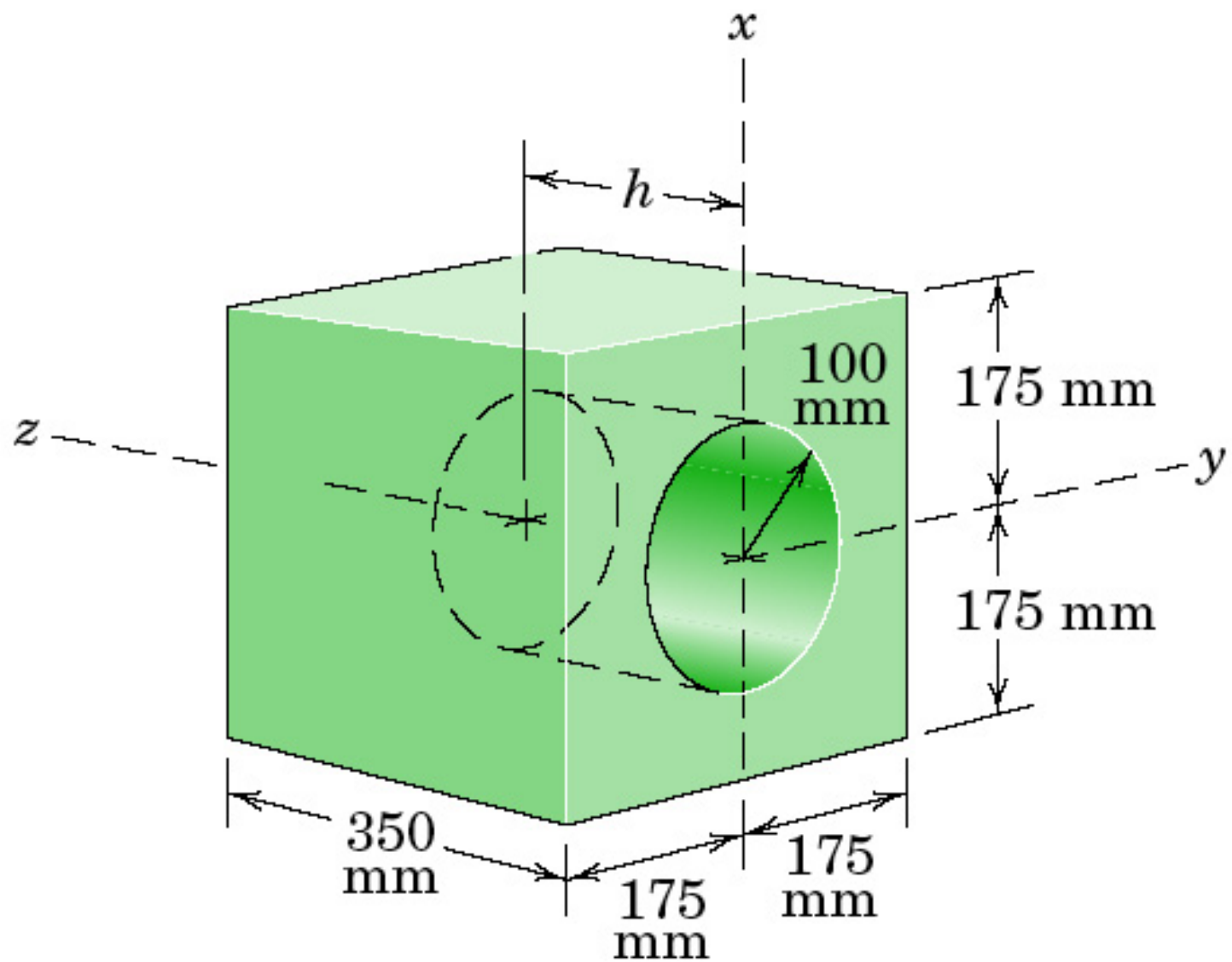


Dimensions in millimeters

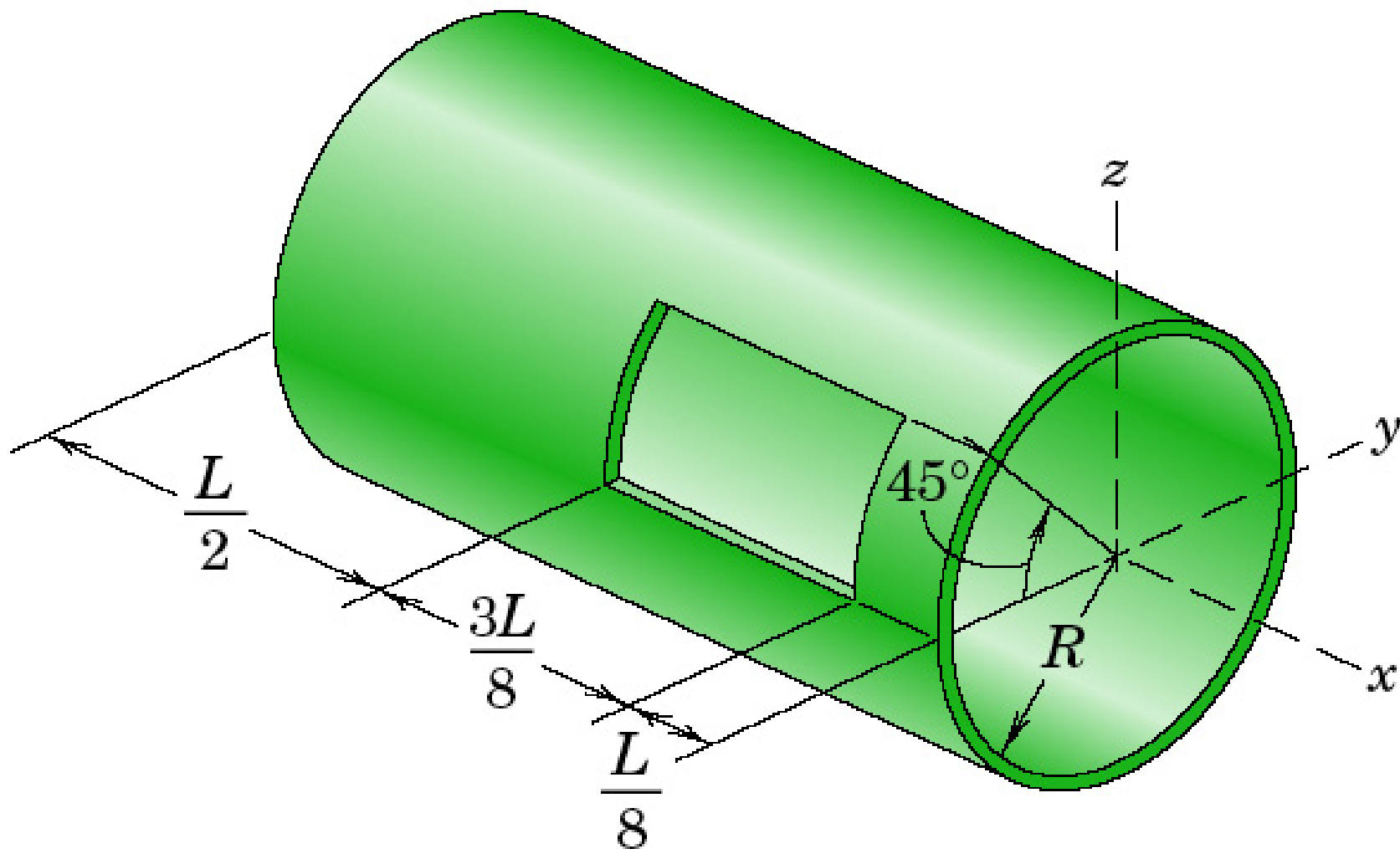
Problem 5/68



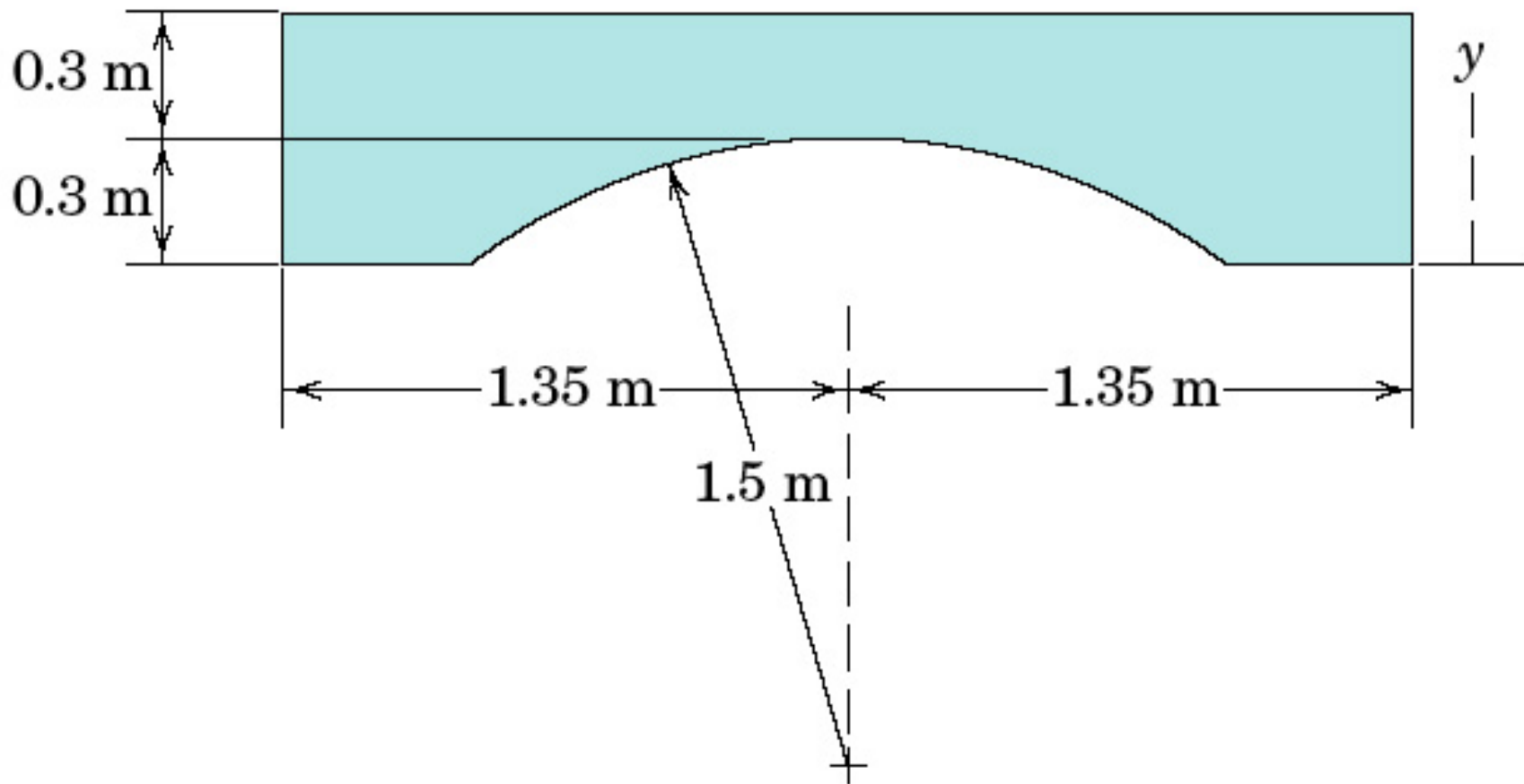
Problem 5/69



Problem 5/70

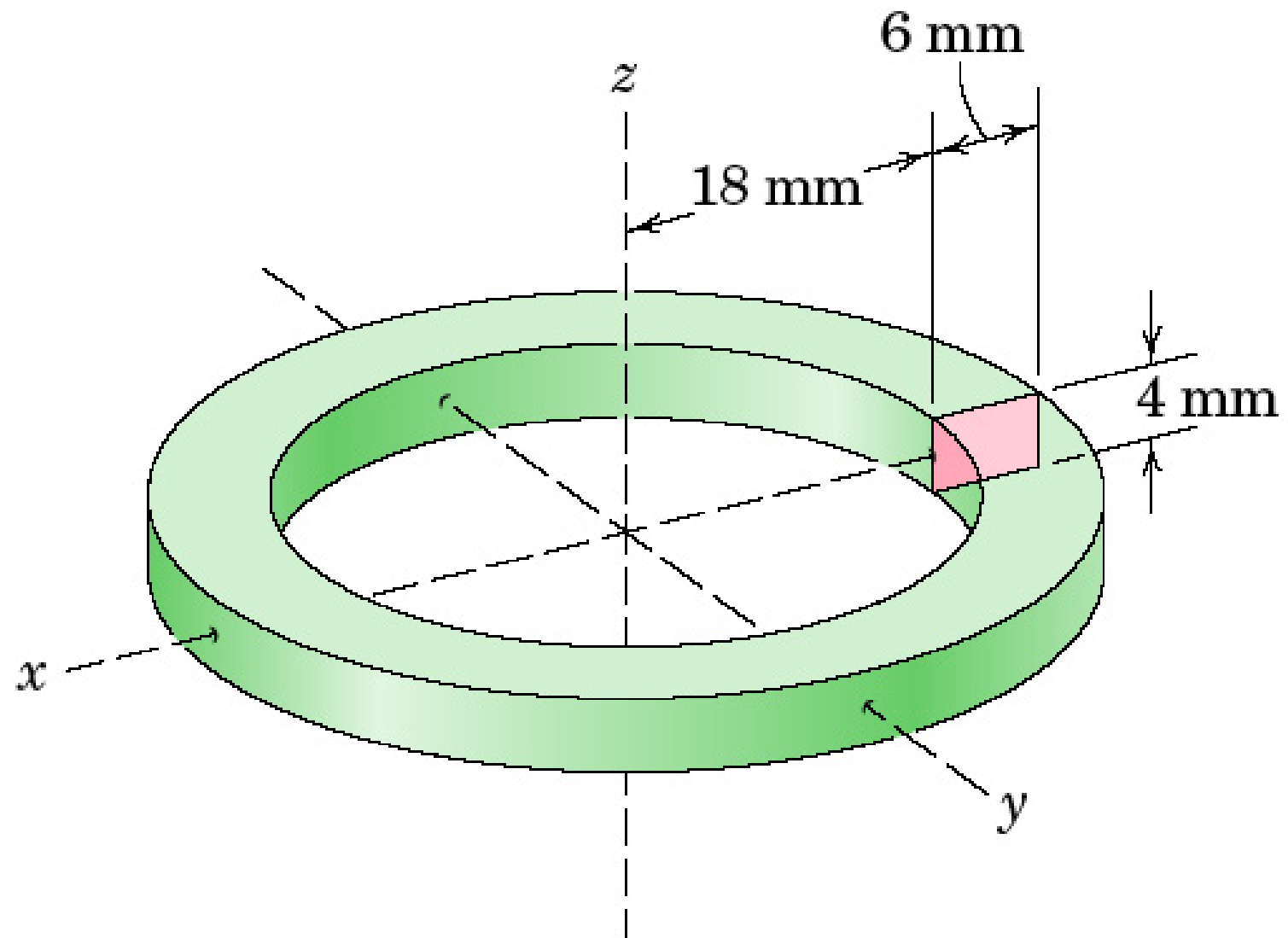


Problem 5/71

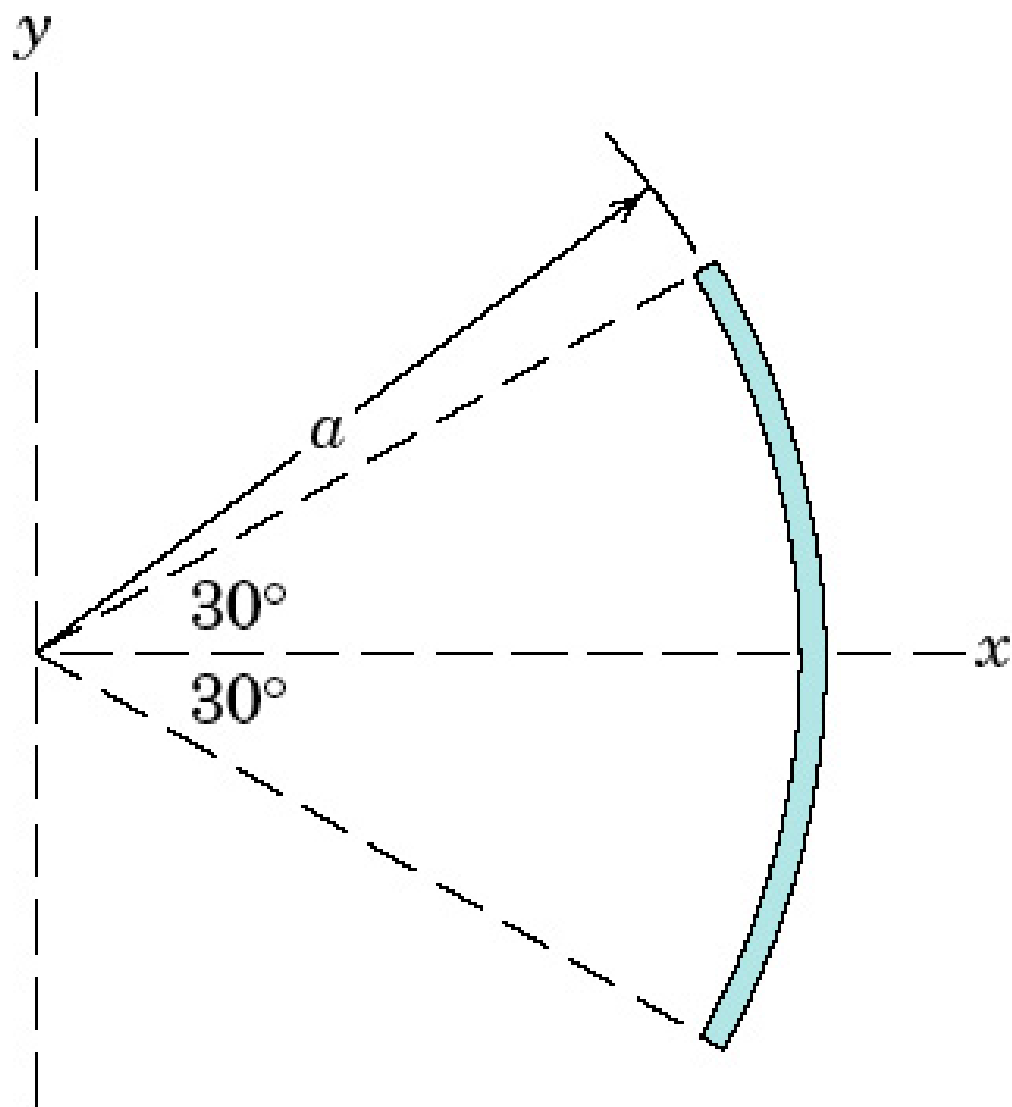


Problem 5/72

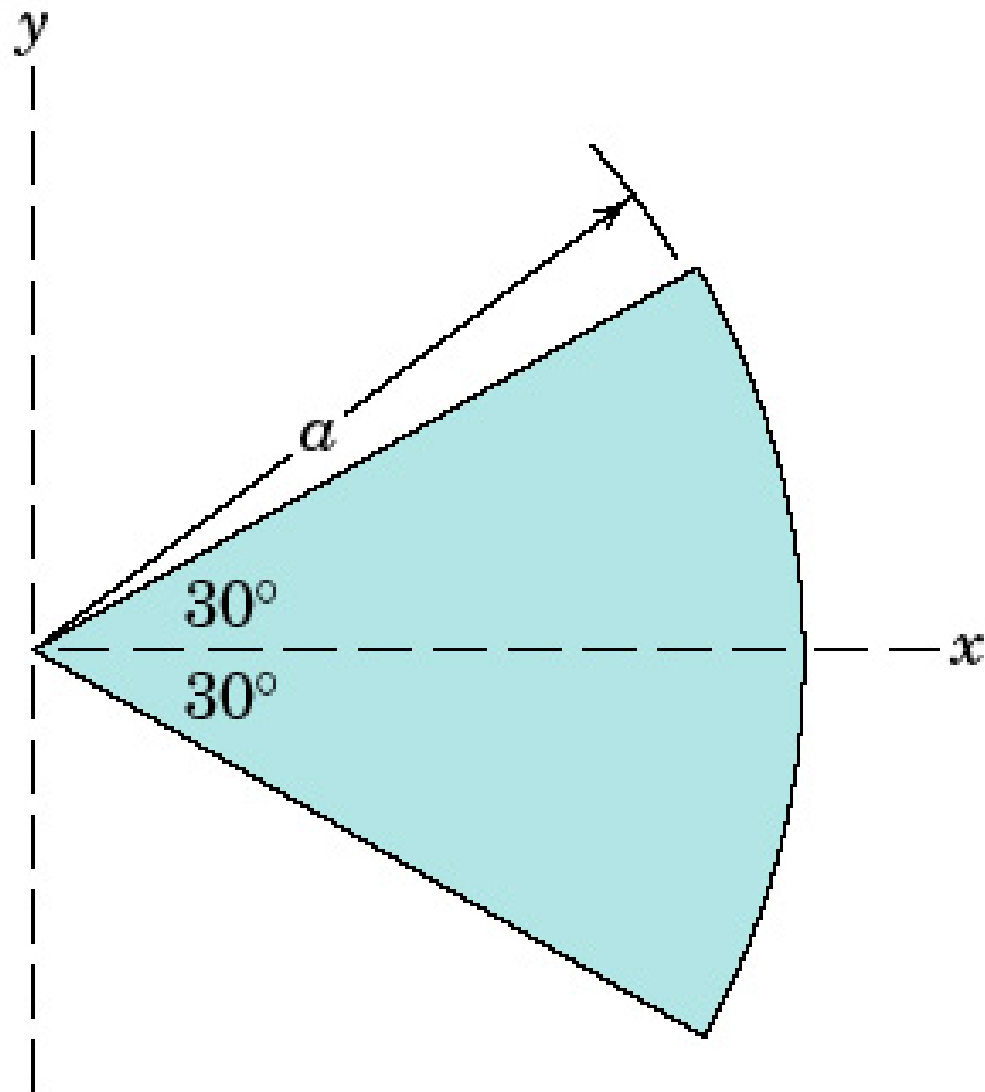
P_05_72



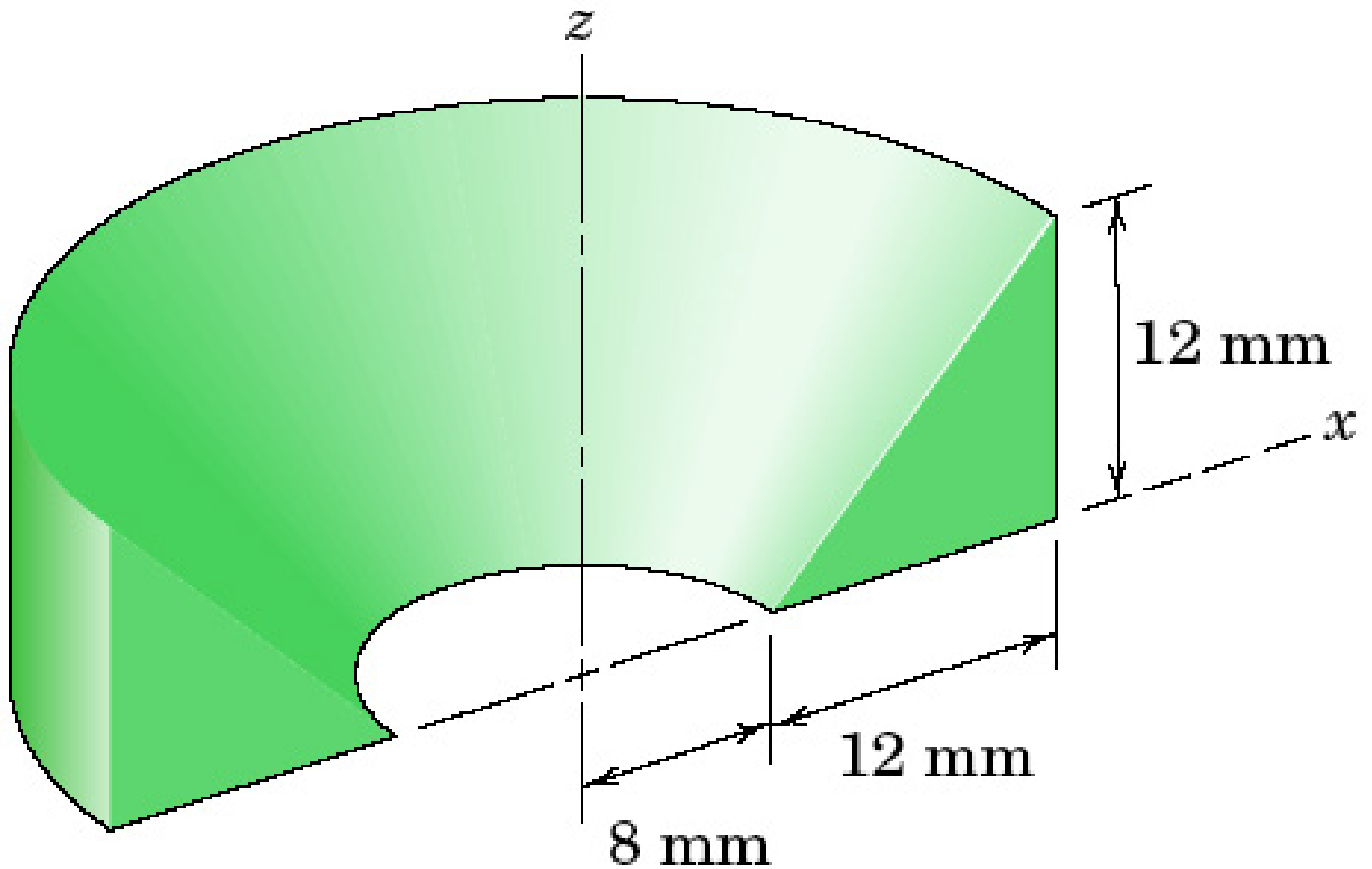
Problem 5/73



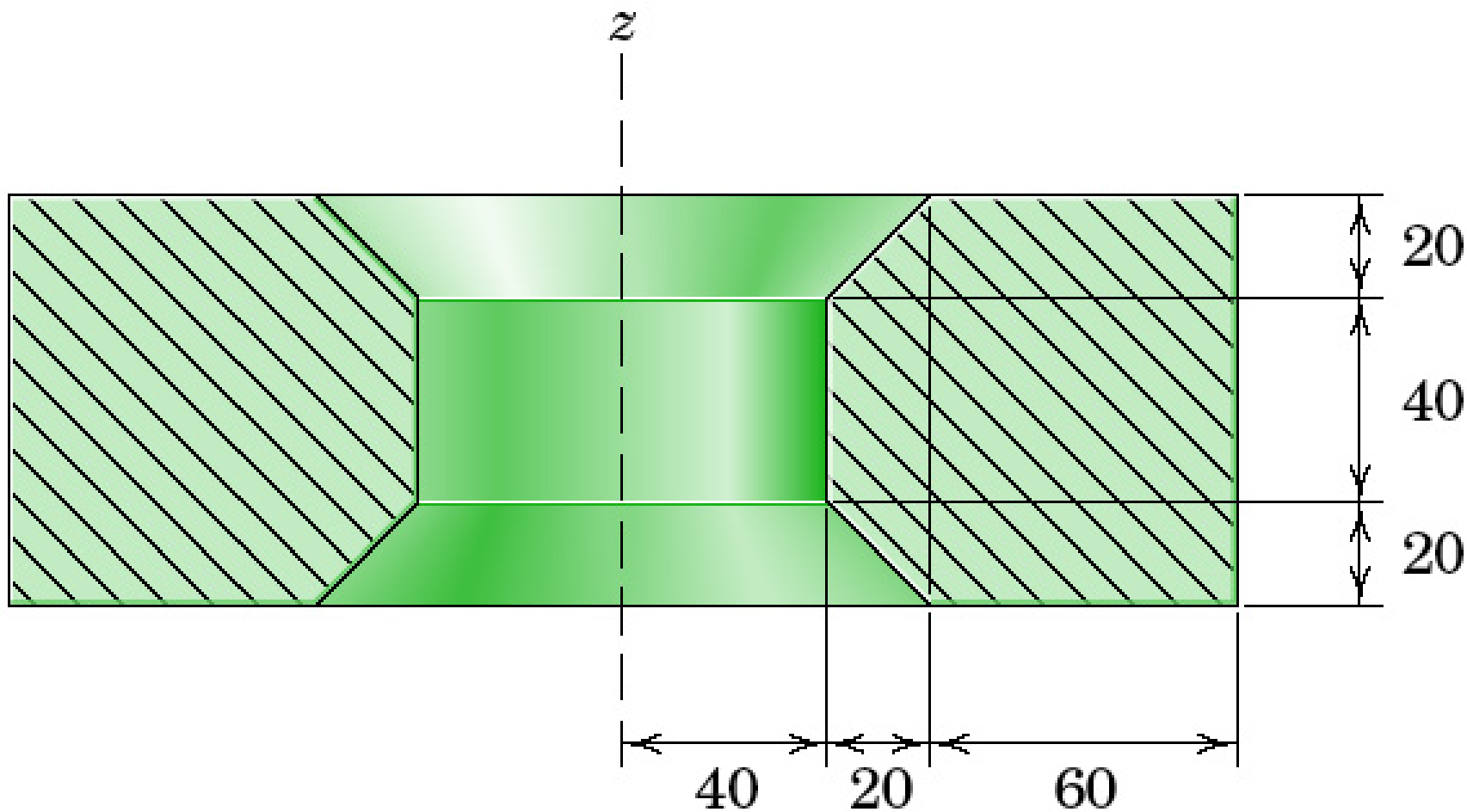
Problem 5/74



Problem 5/75

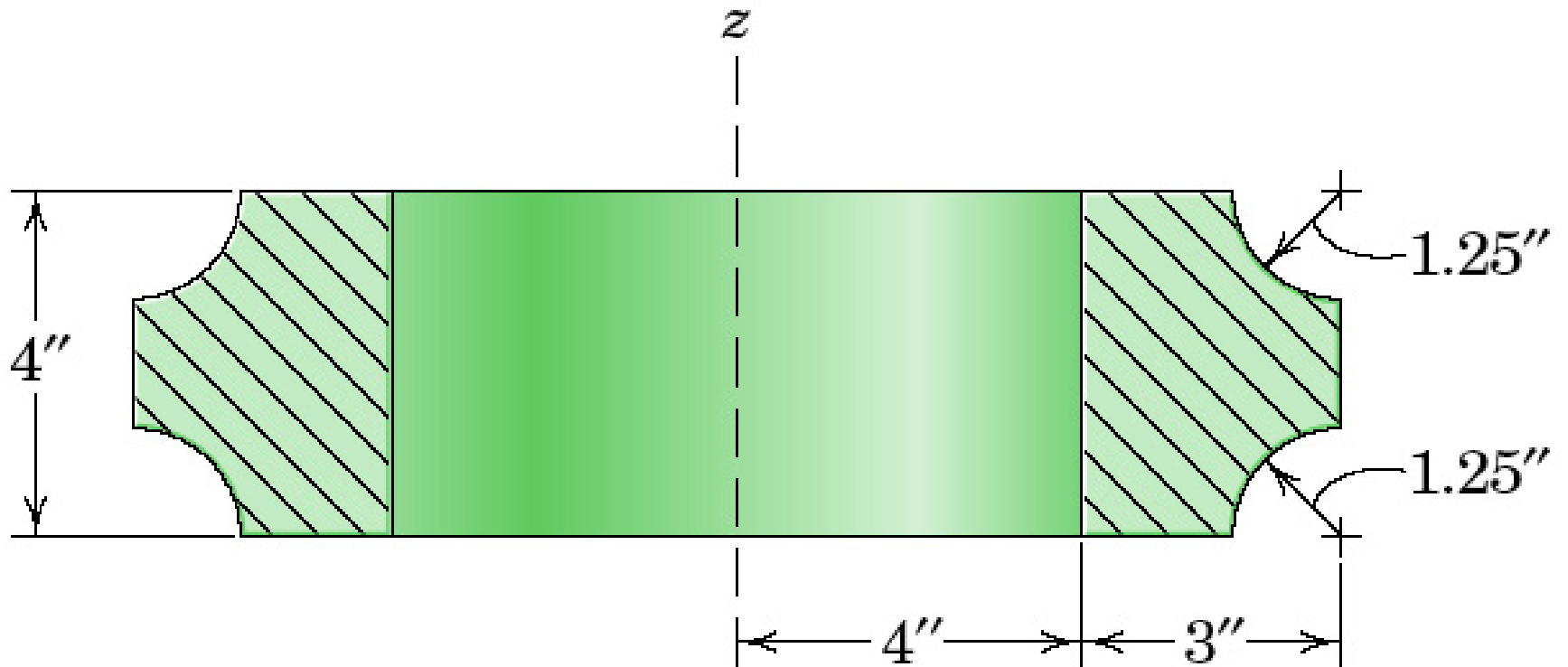


Problem 5/76



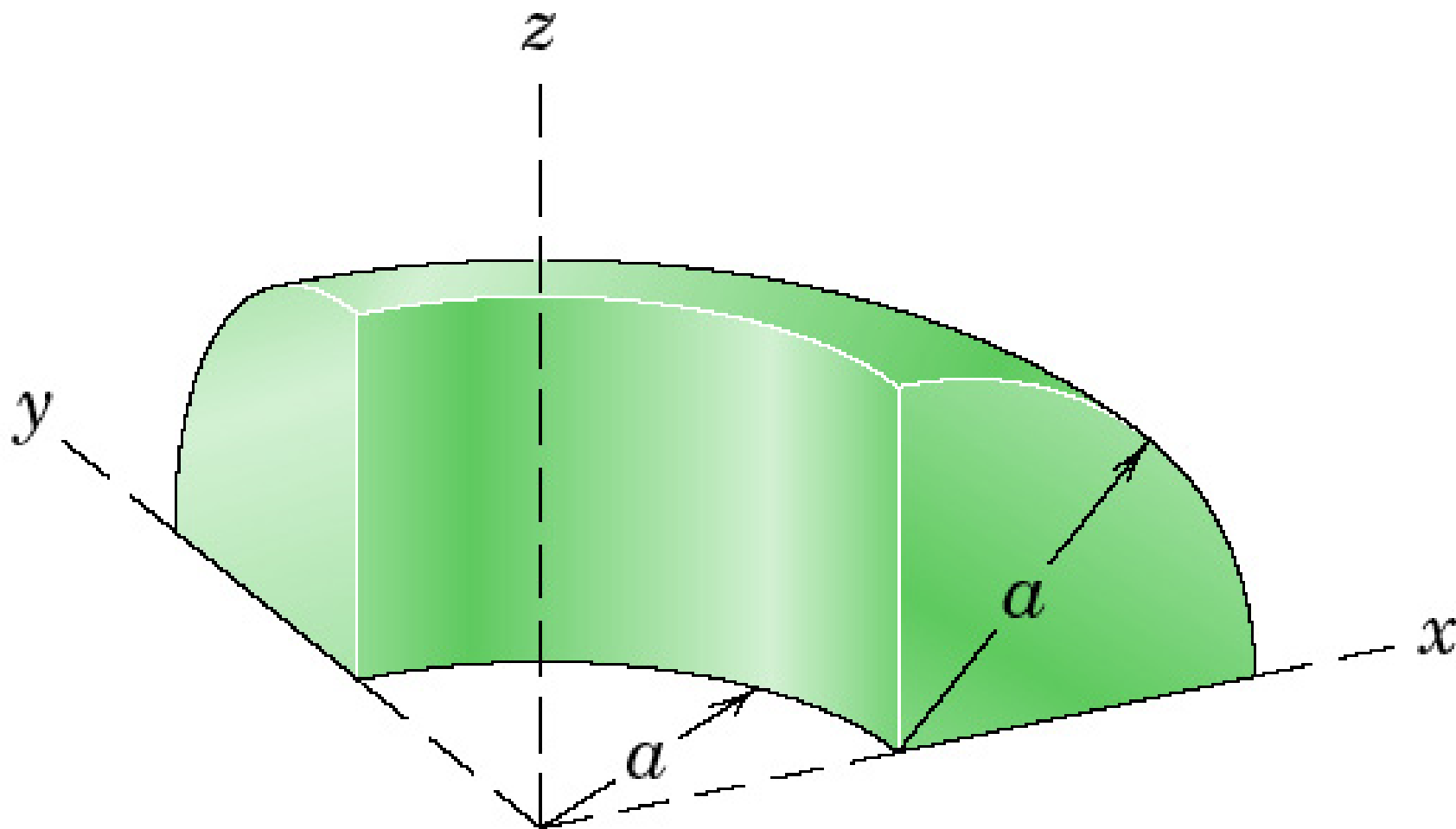
Dimensions in millimeters

Problem 5/77

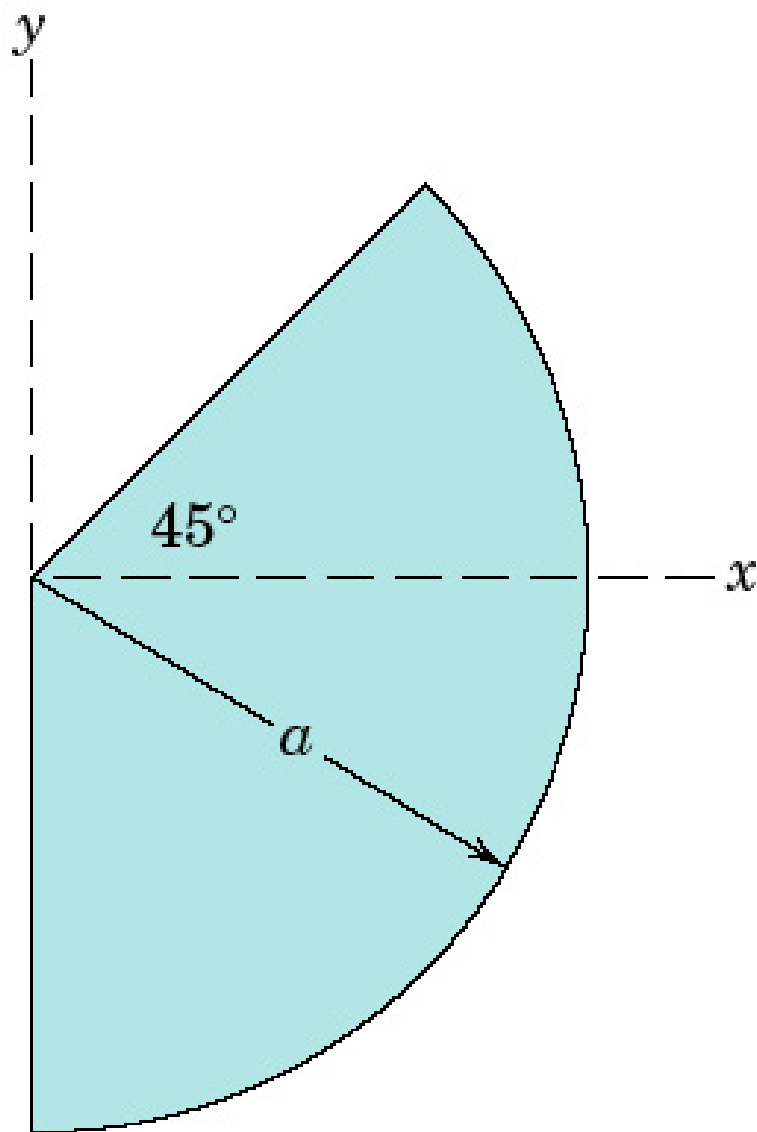


Problem 5/78

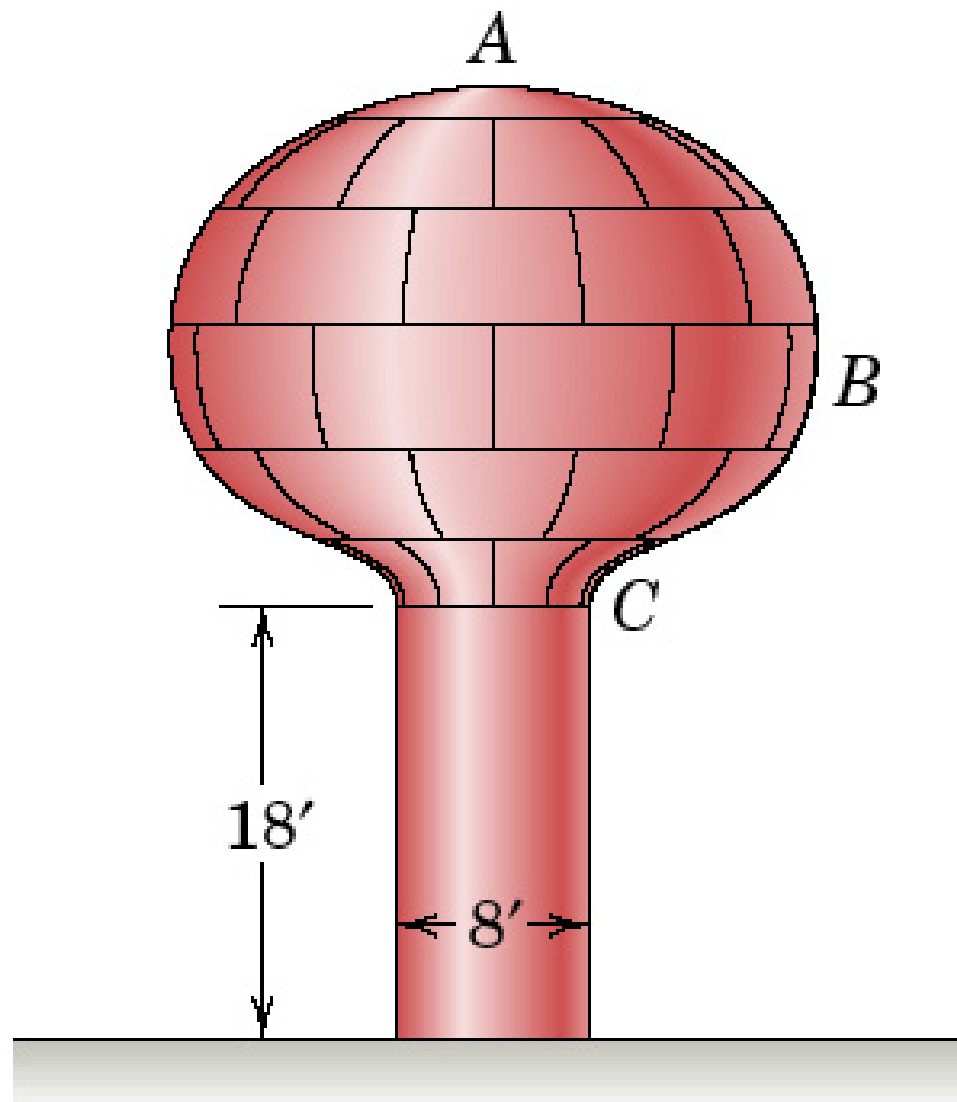
P_05_78



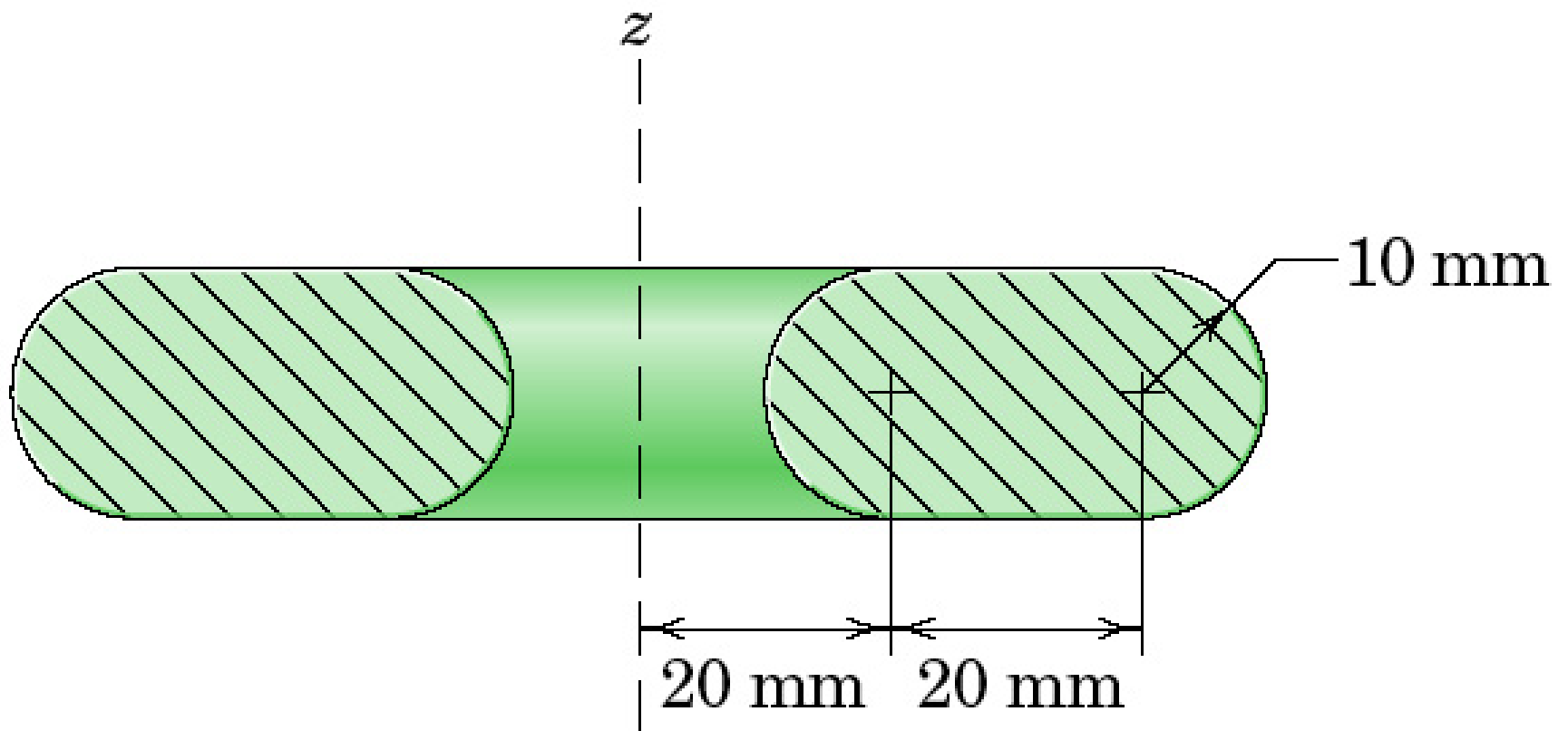
Problem 5/79



Problem 5/80

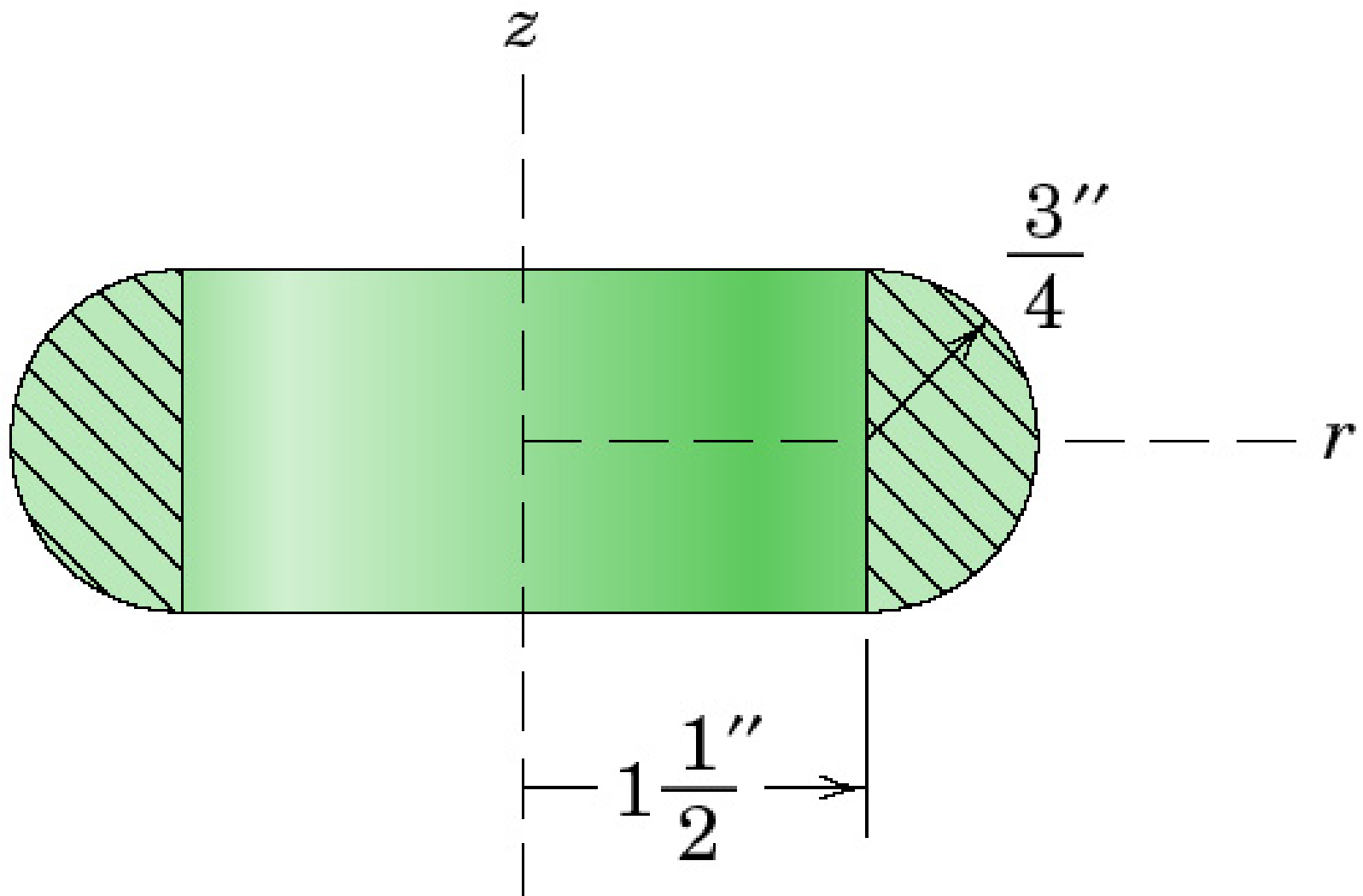


Problem 5/81

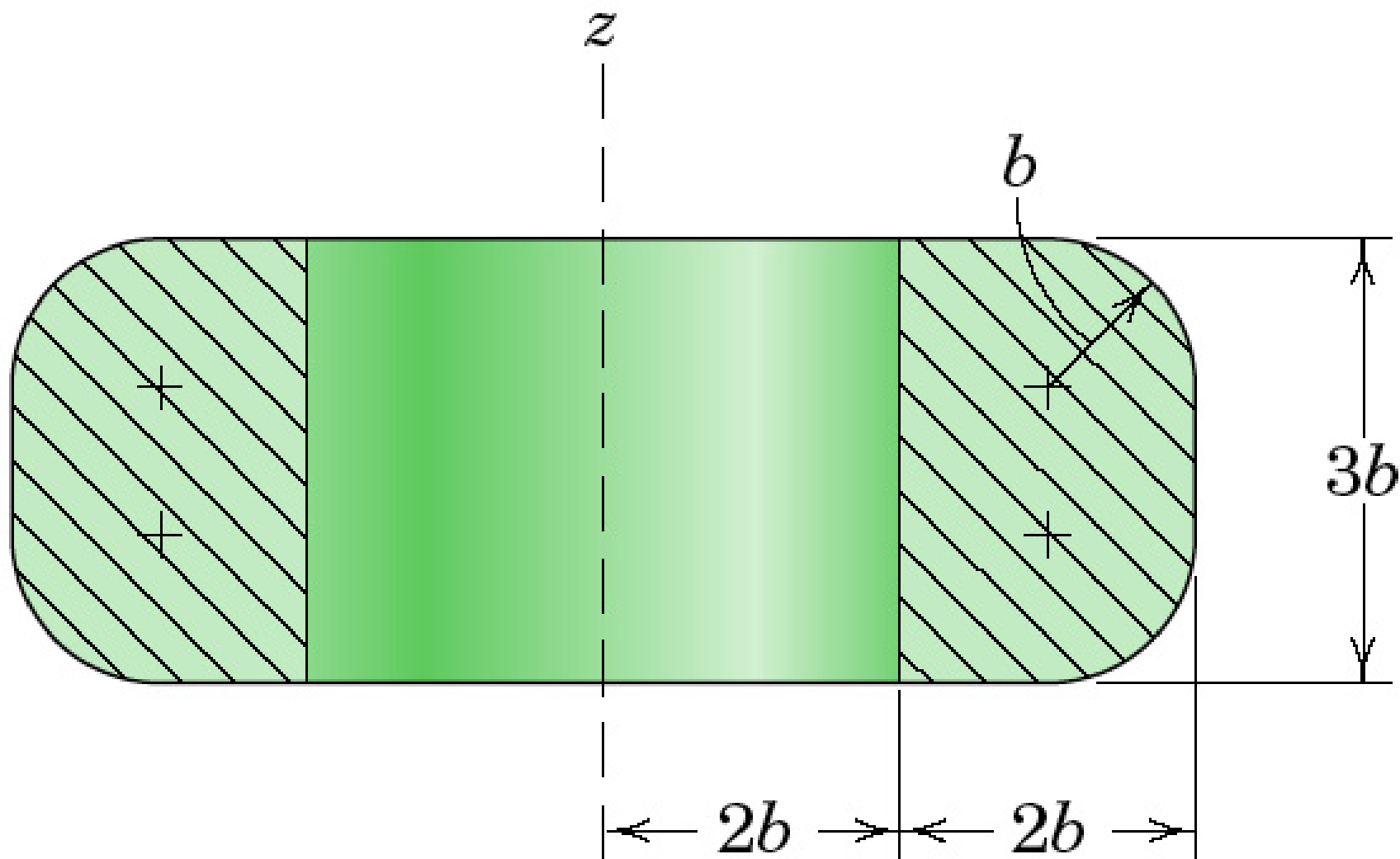


Problem 5/82

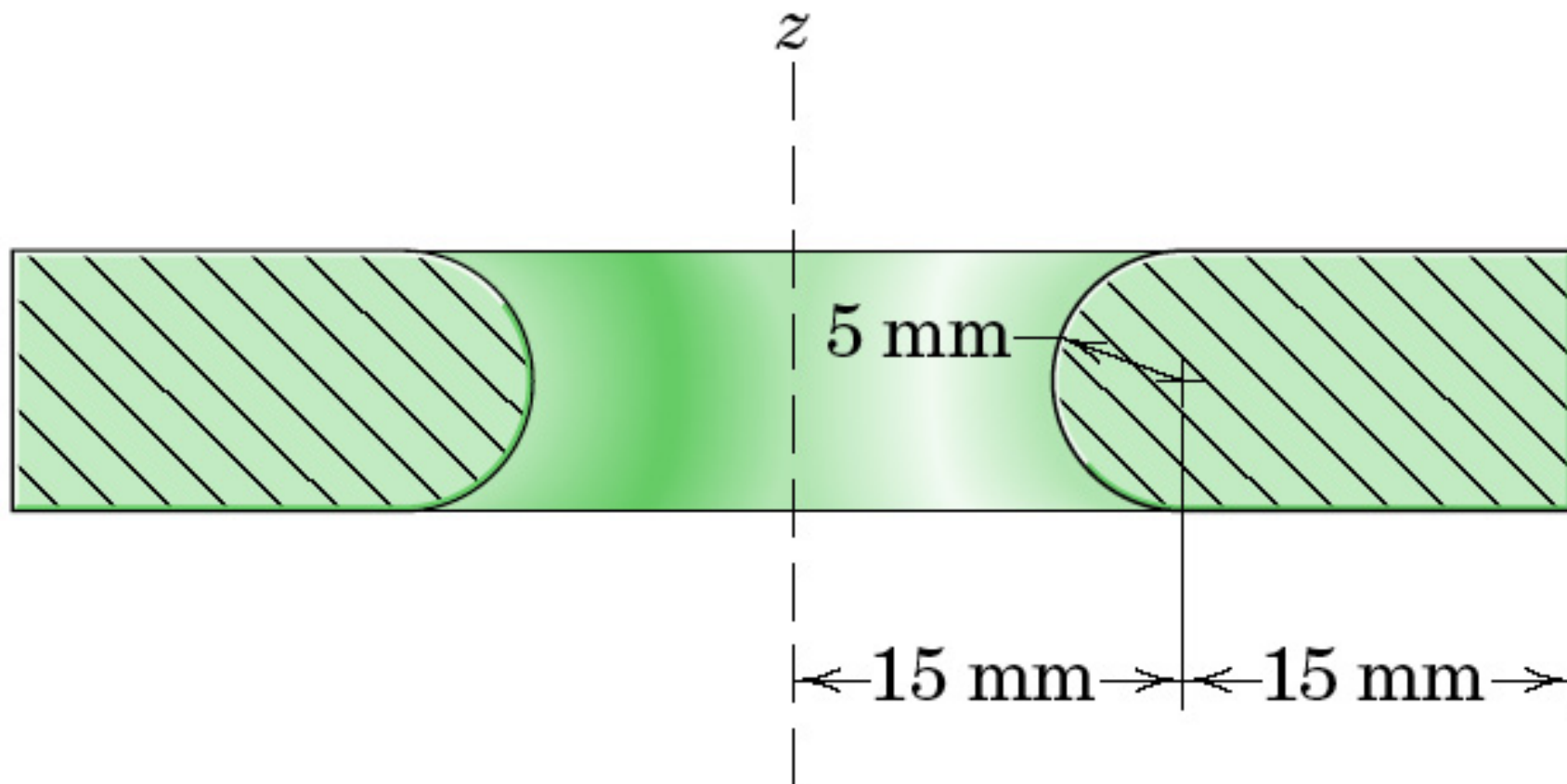
P_05_82



Problem 5/83

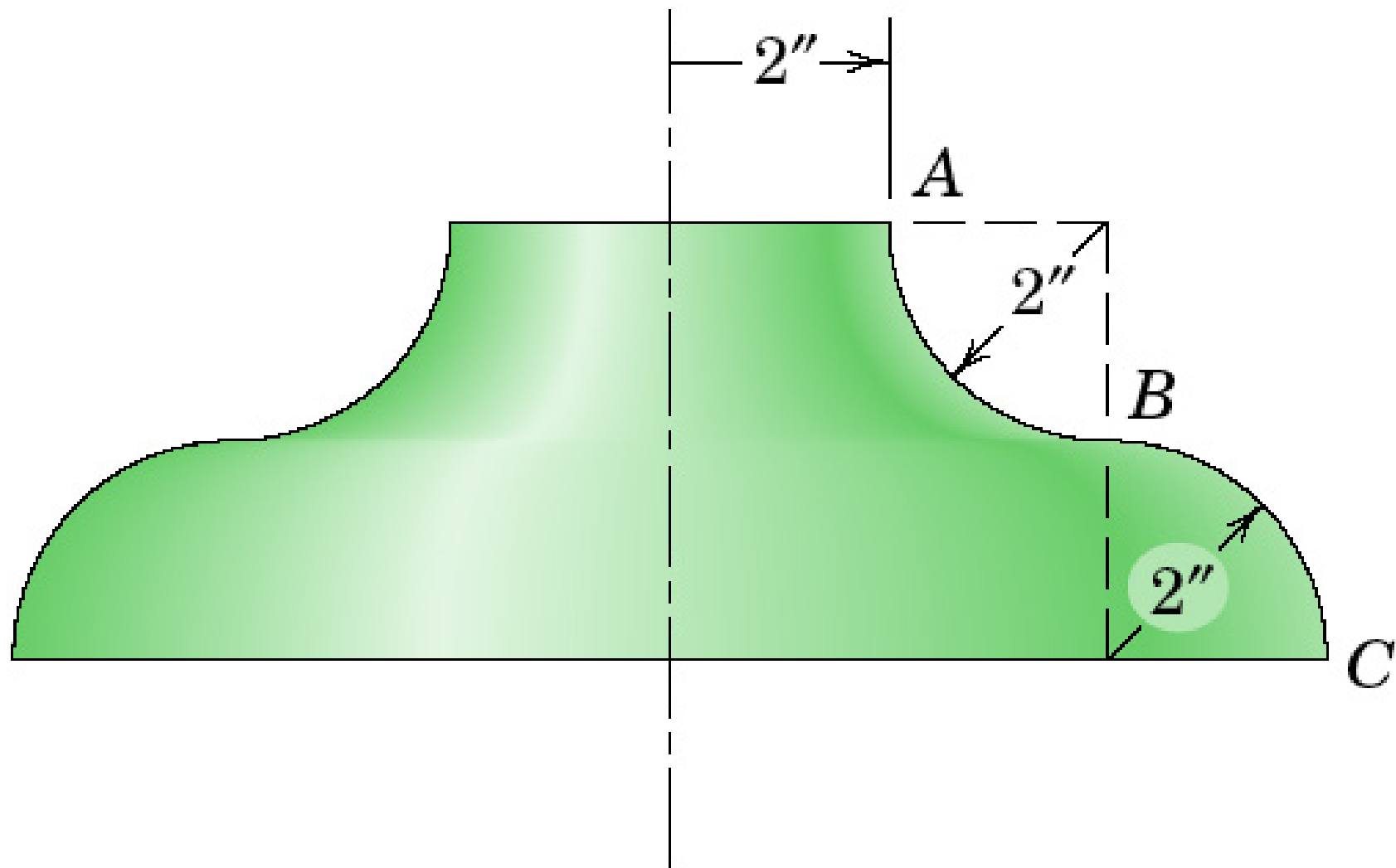


Problem 5/84

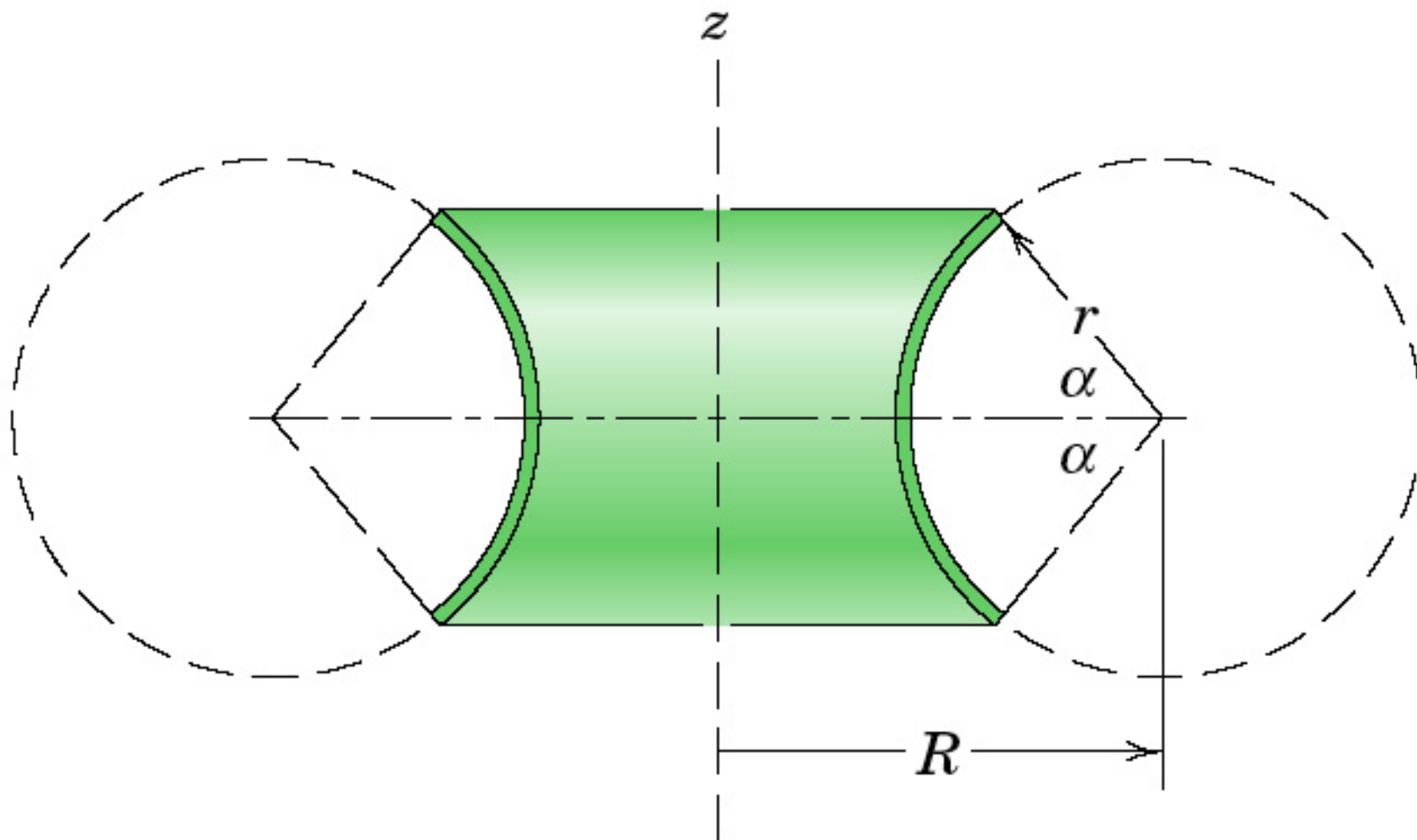


Problem 5/85

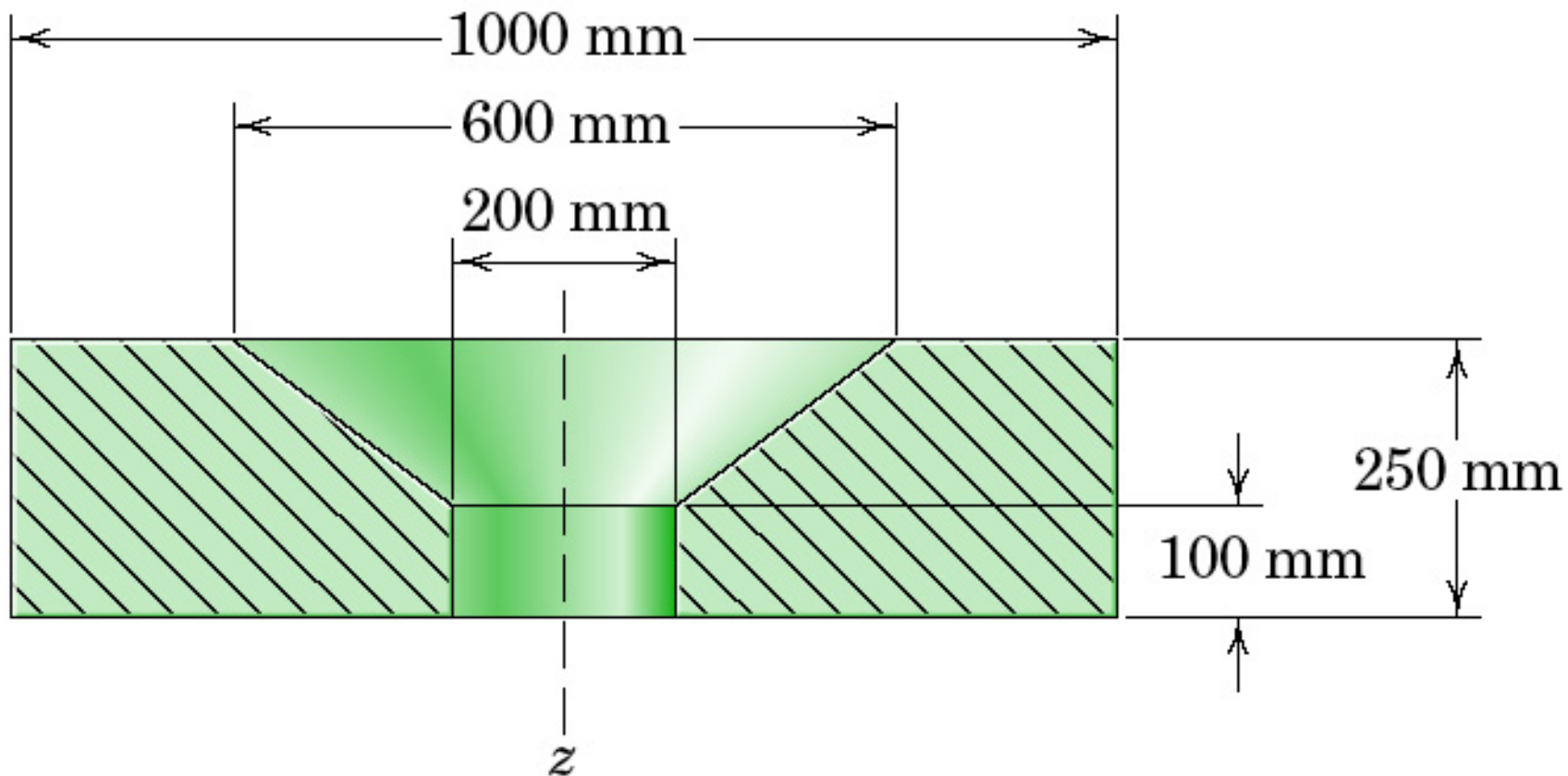
P_05_85



Problem 5/86

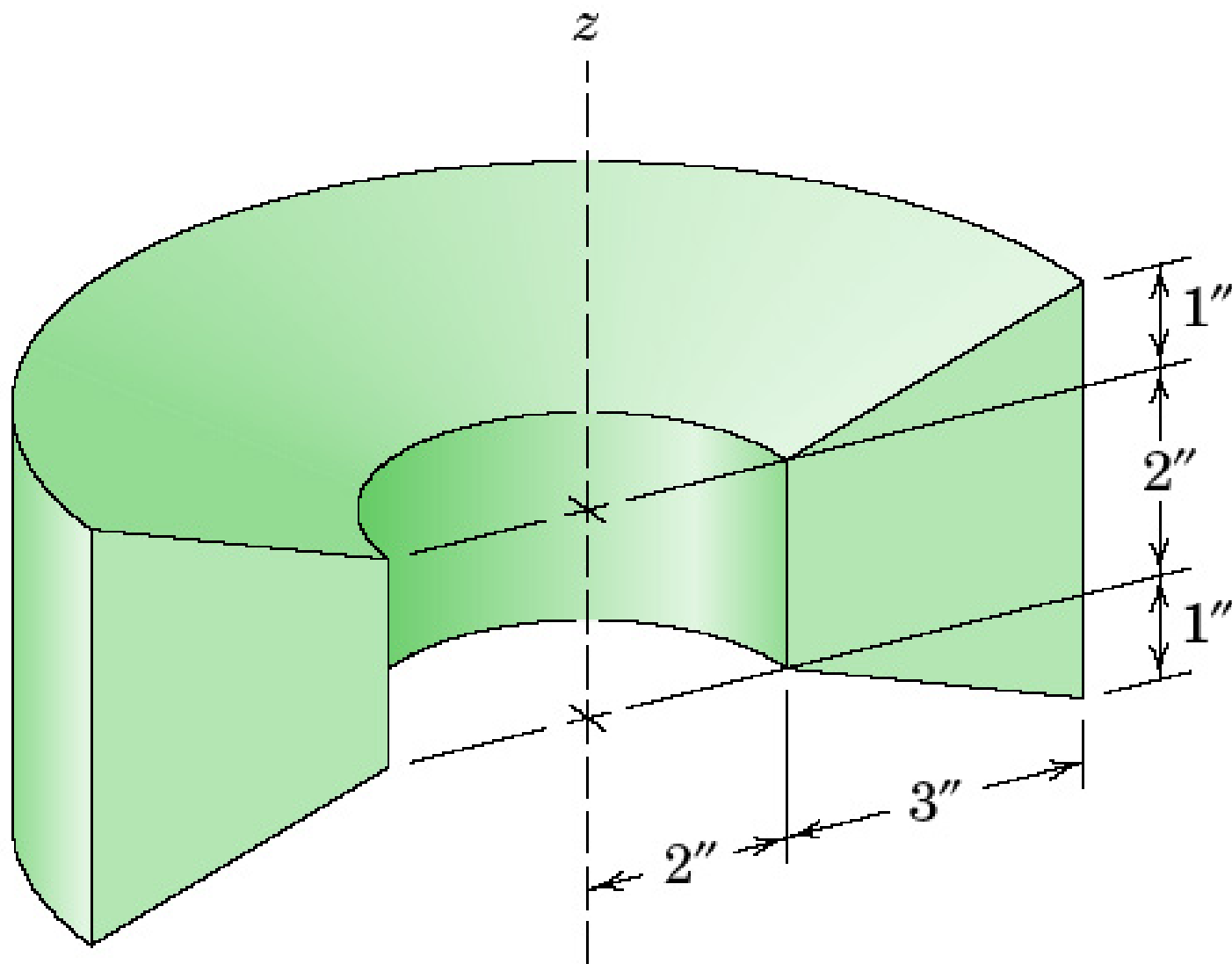


Problem 5/87

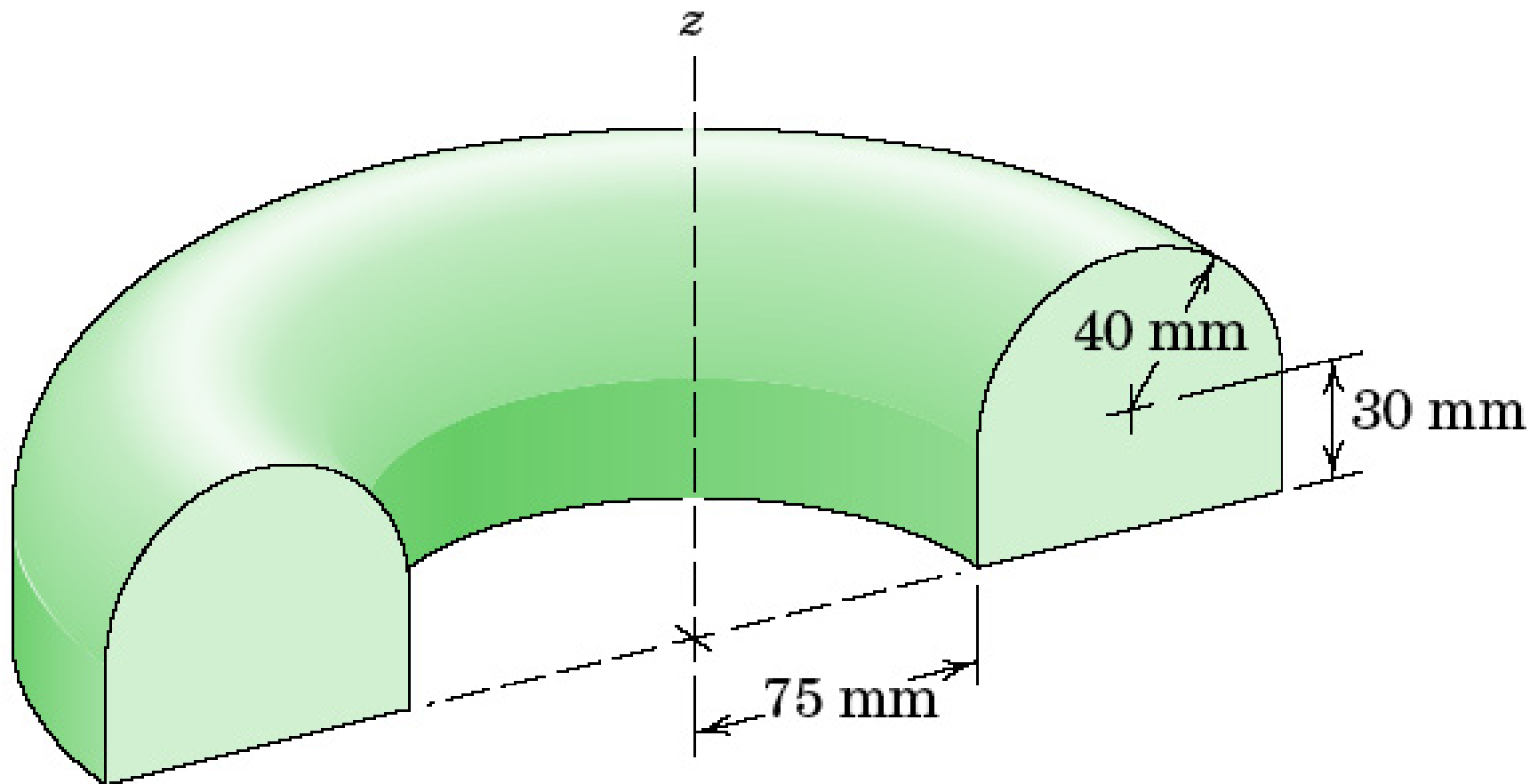


Problem 5/88

P_05_88

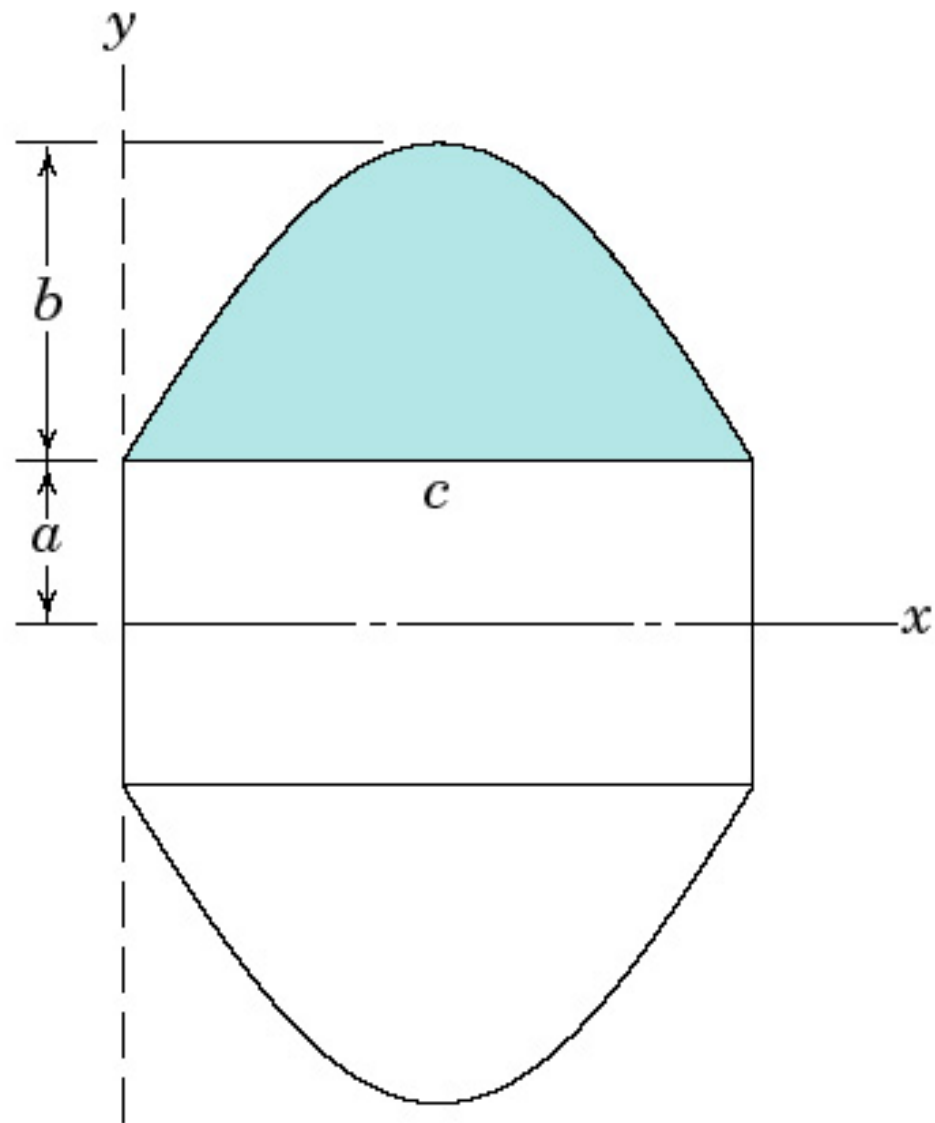


Problem 5/89

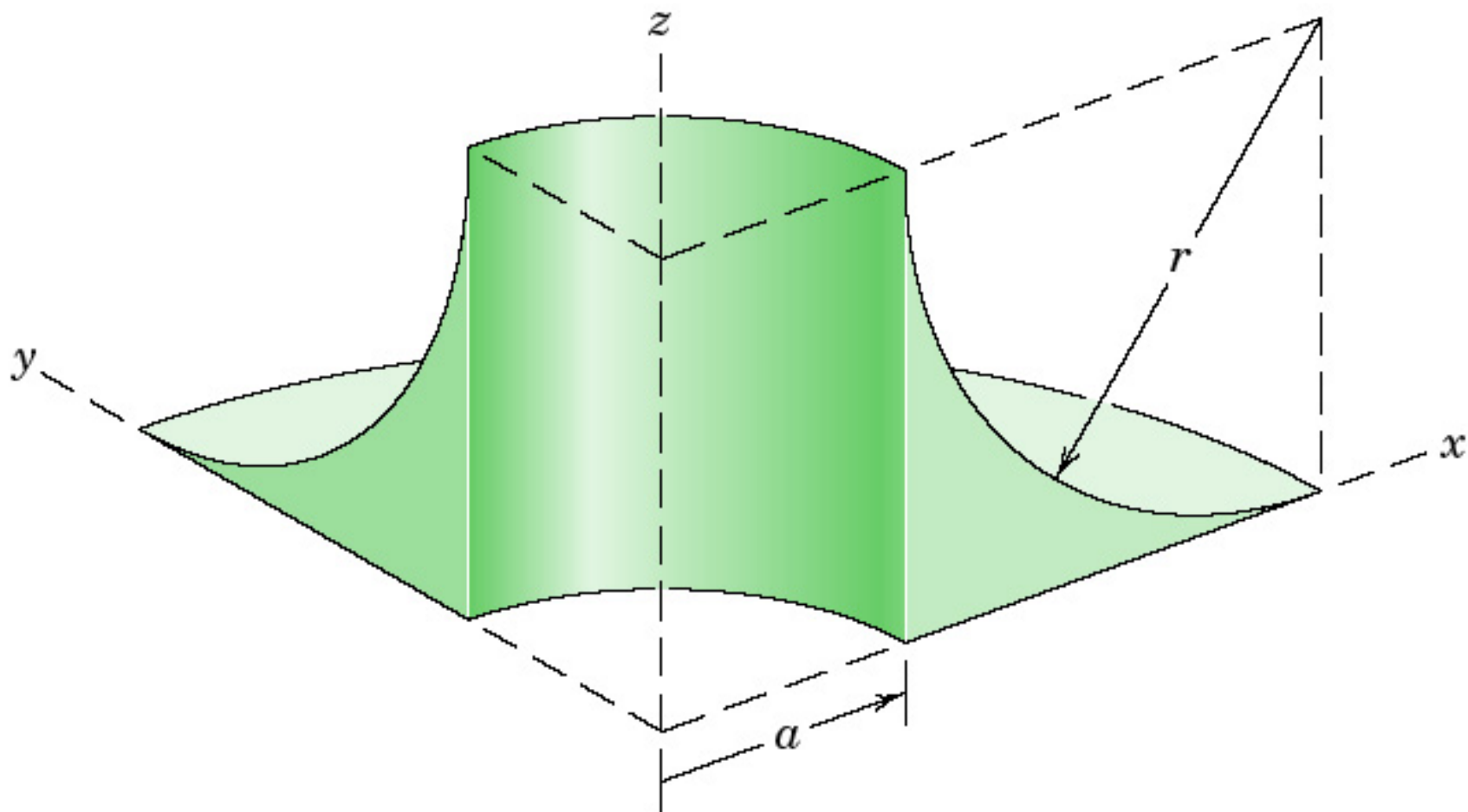


Problem 5/90

P_05_90

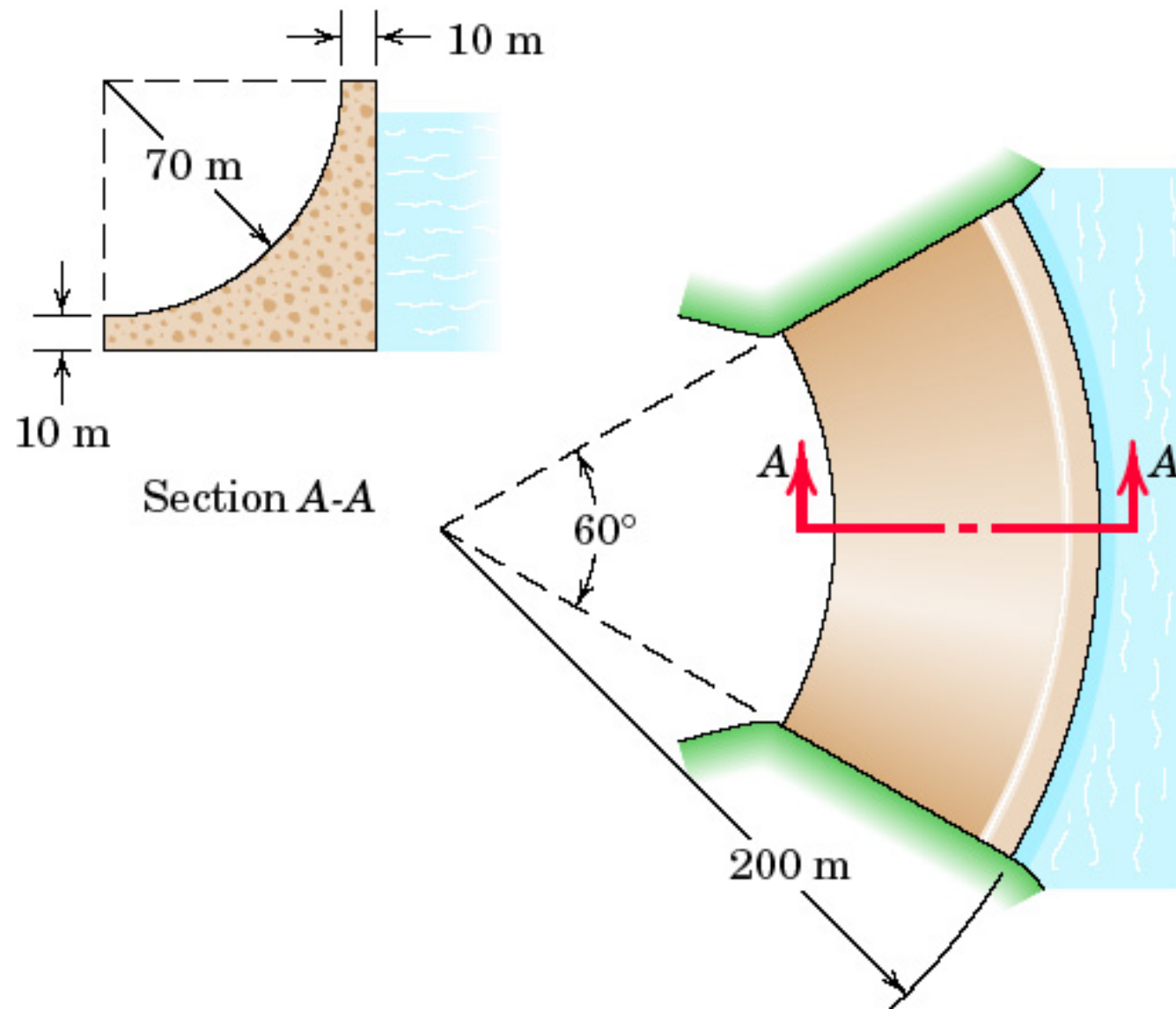


Problem 5/91

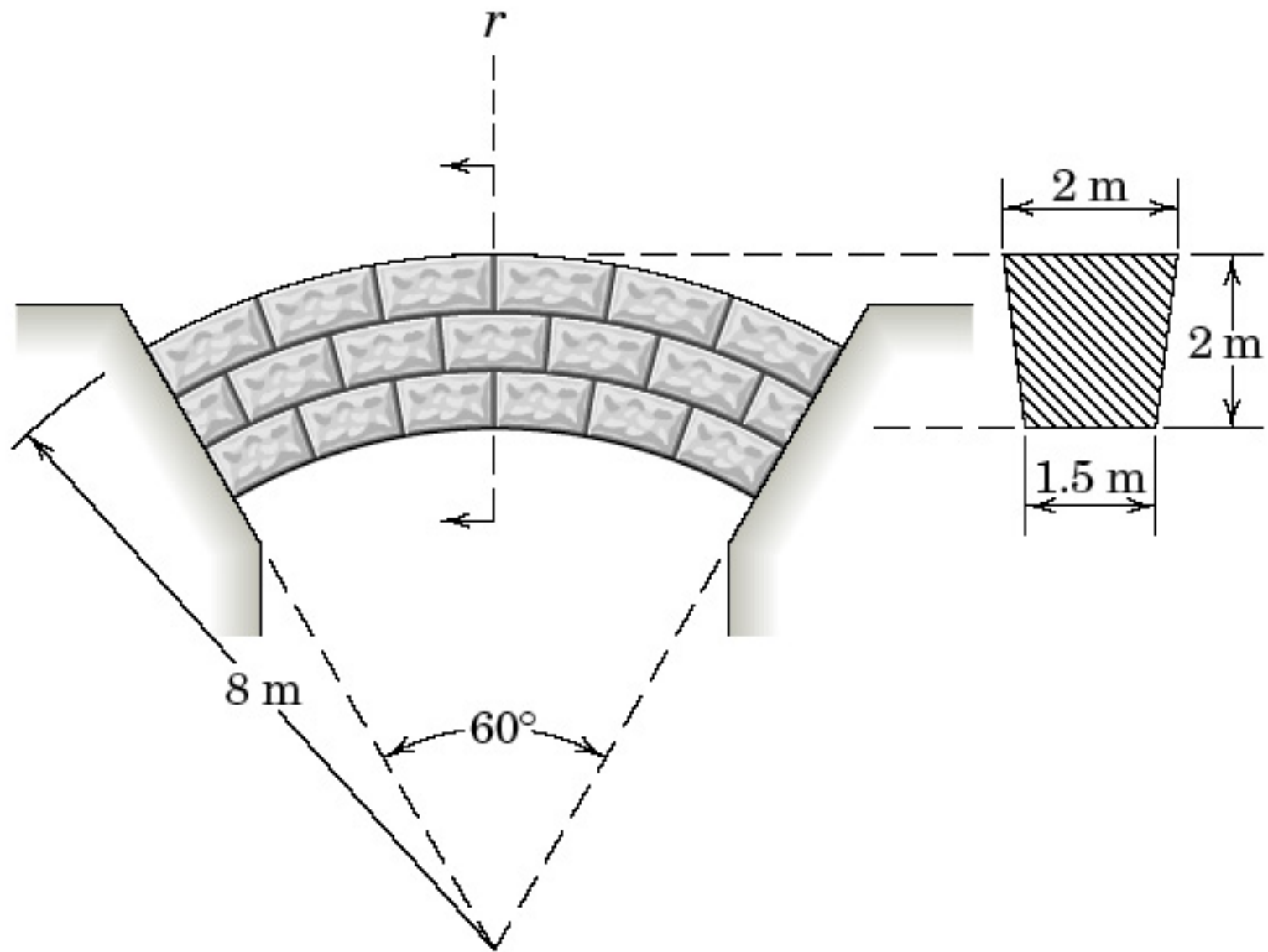


Problem 5/92

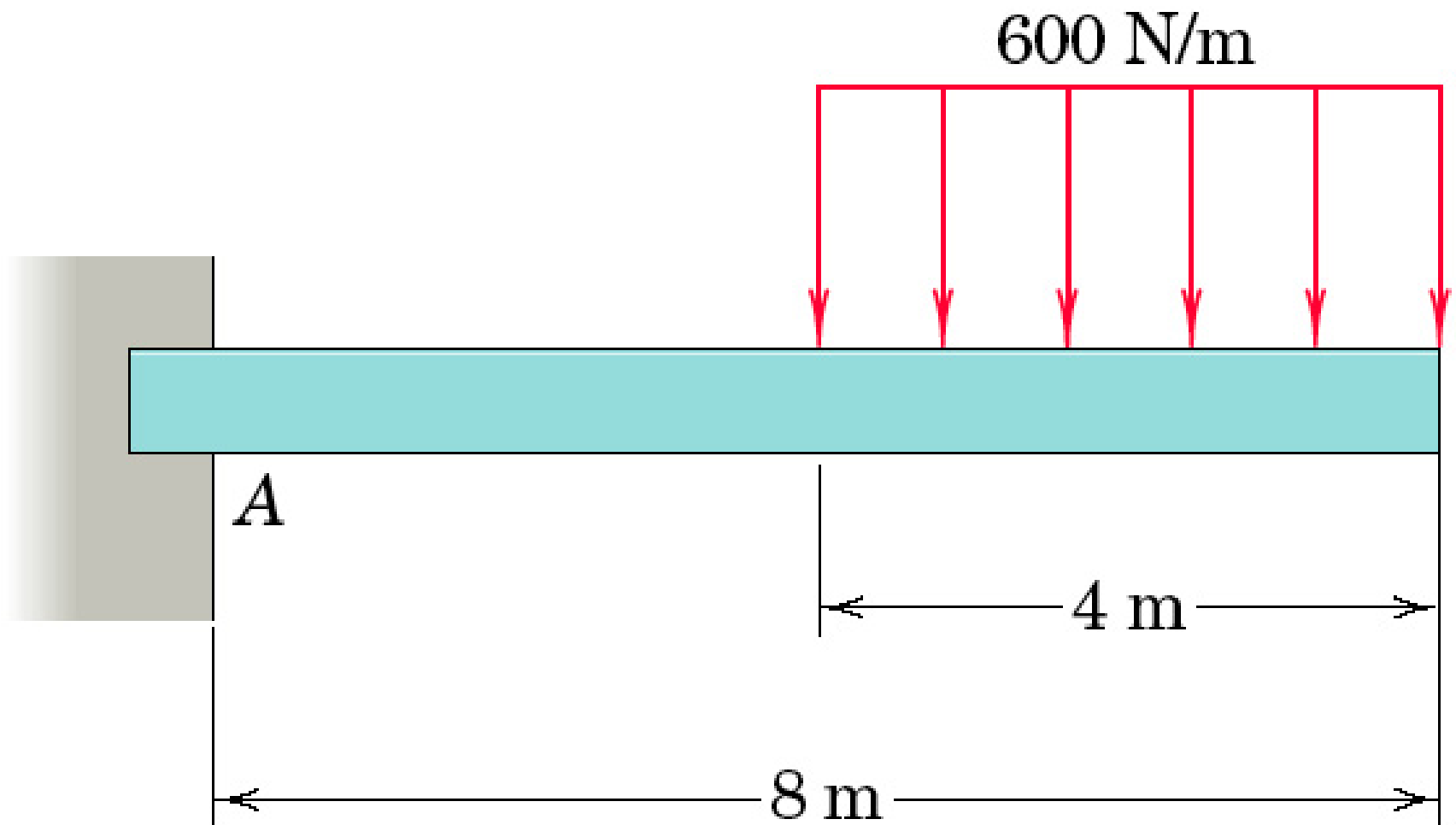
P_05_92



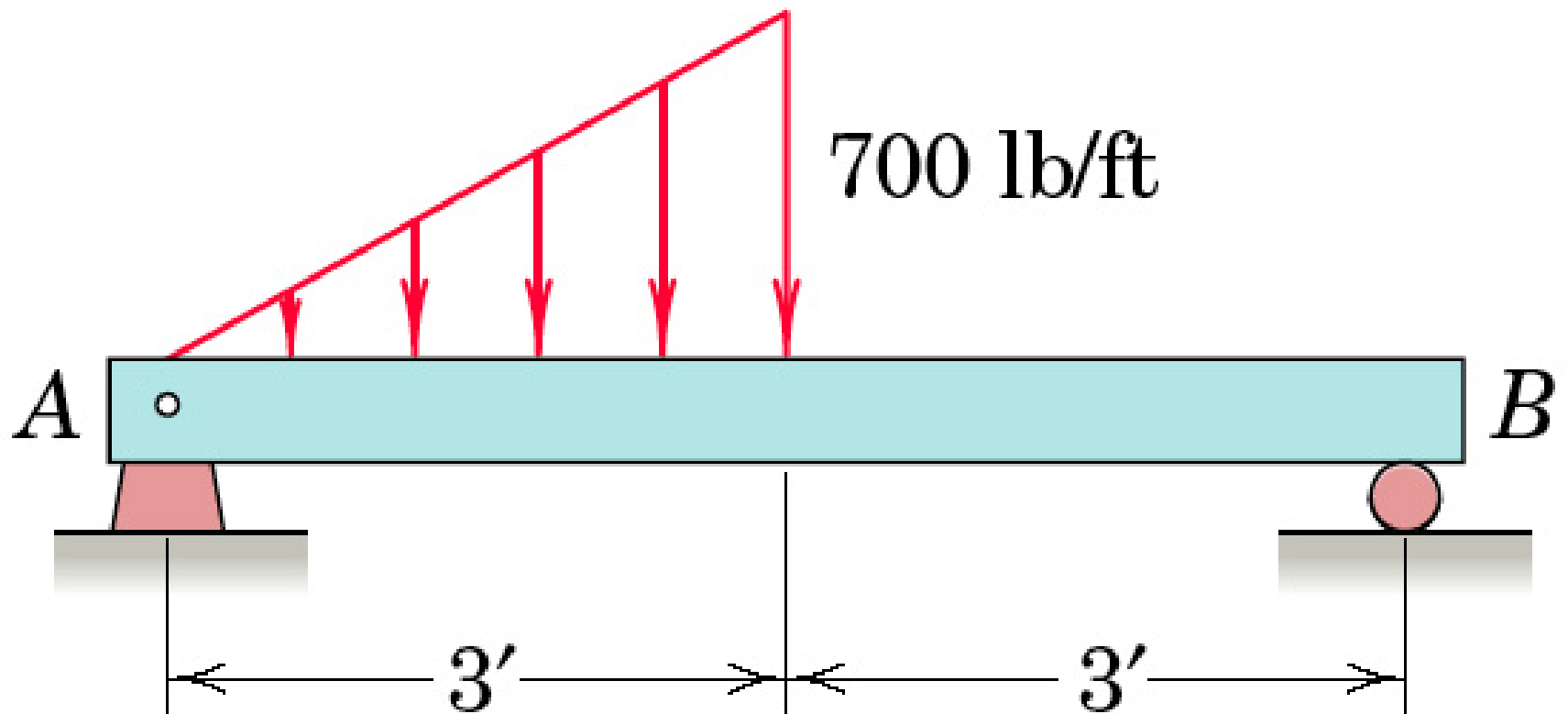
Problem 5/93



Problem 5/94

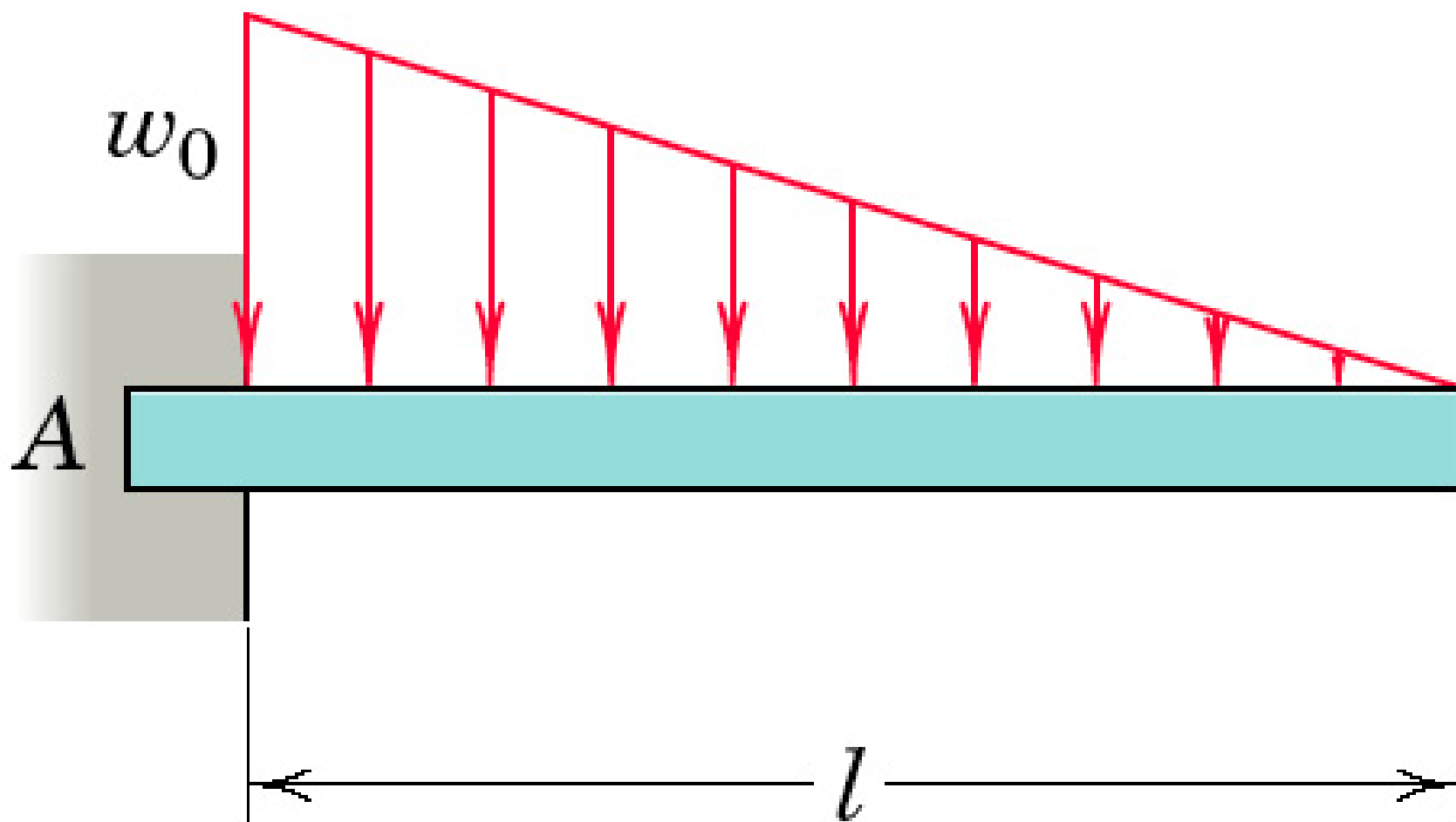


Problem 5/95

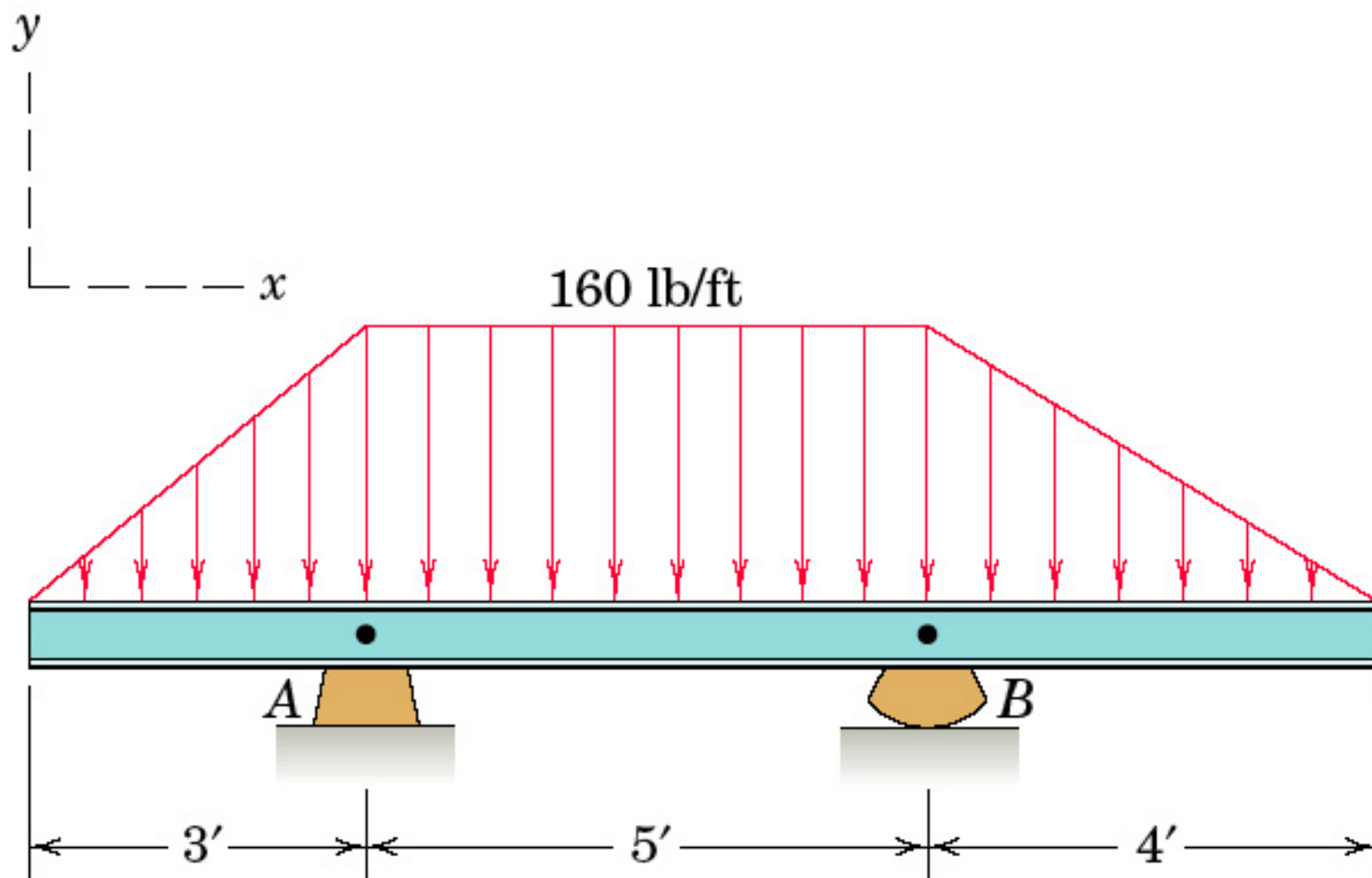


Problem 5/96

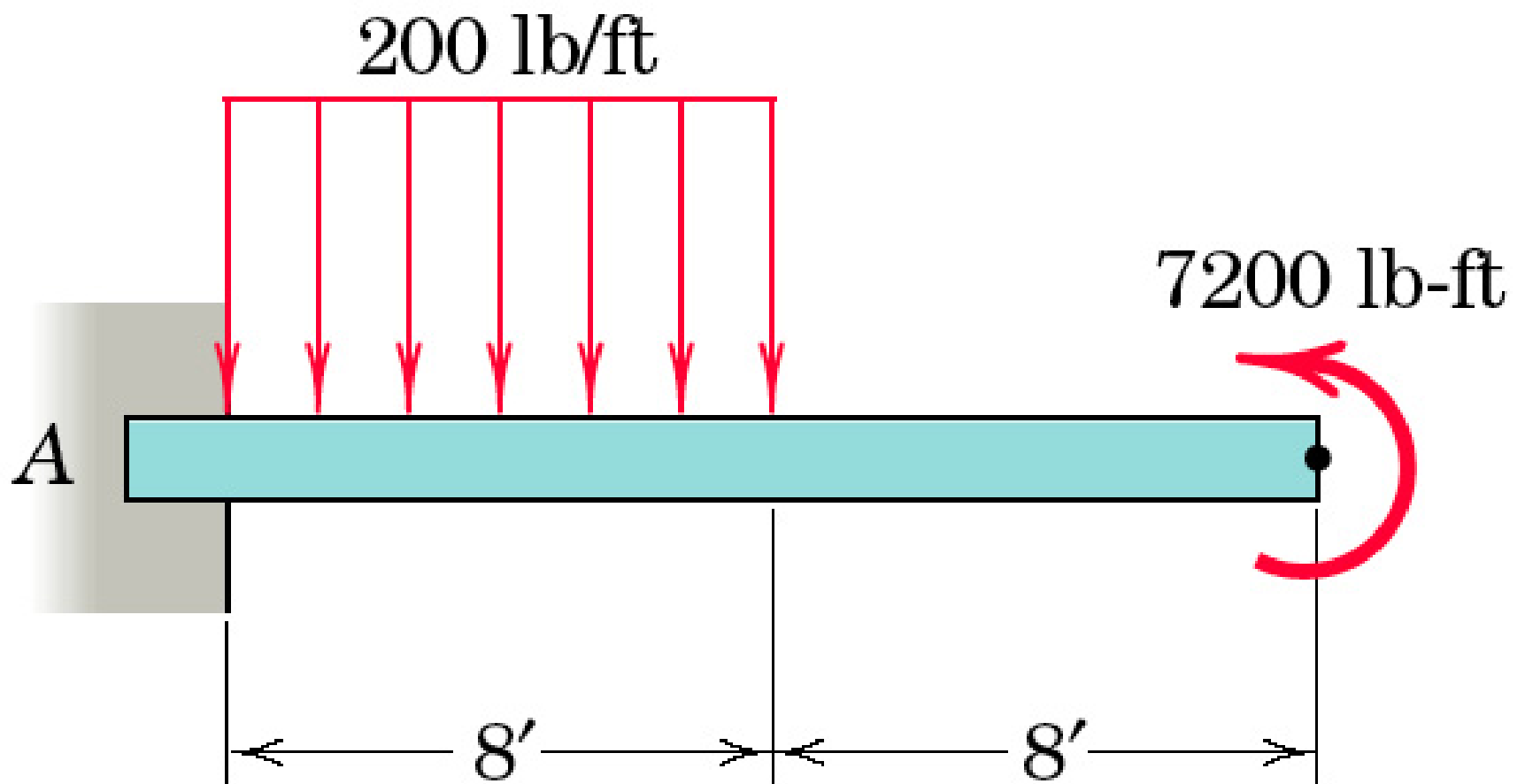
P_05_96



Problem 5/97



Problem 5/98



Problem 5/99

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 6 : Friction.

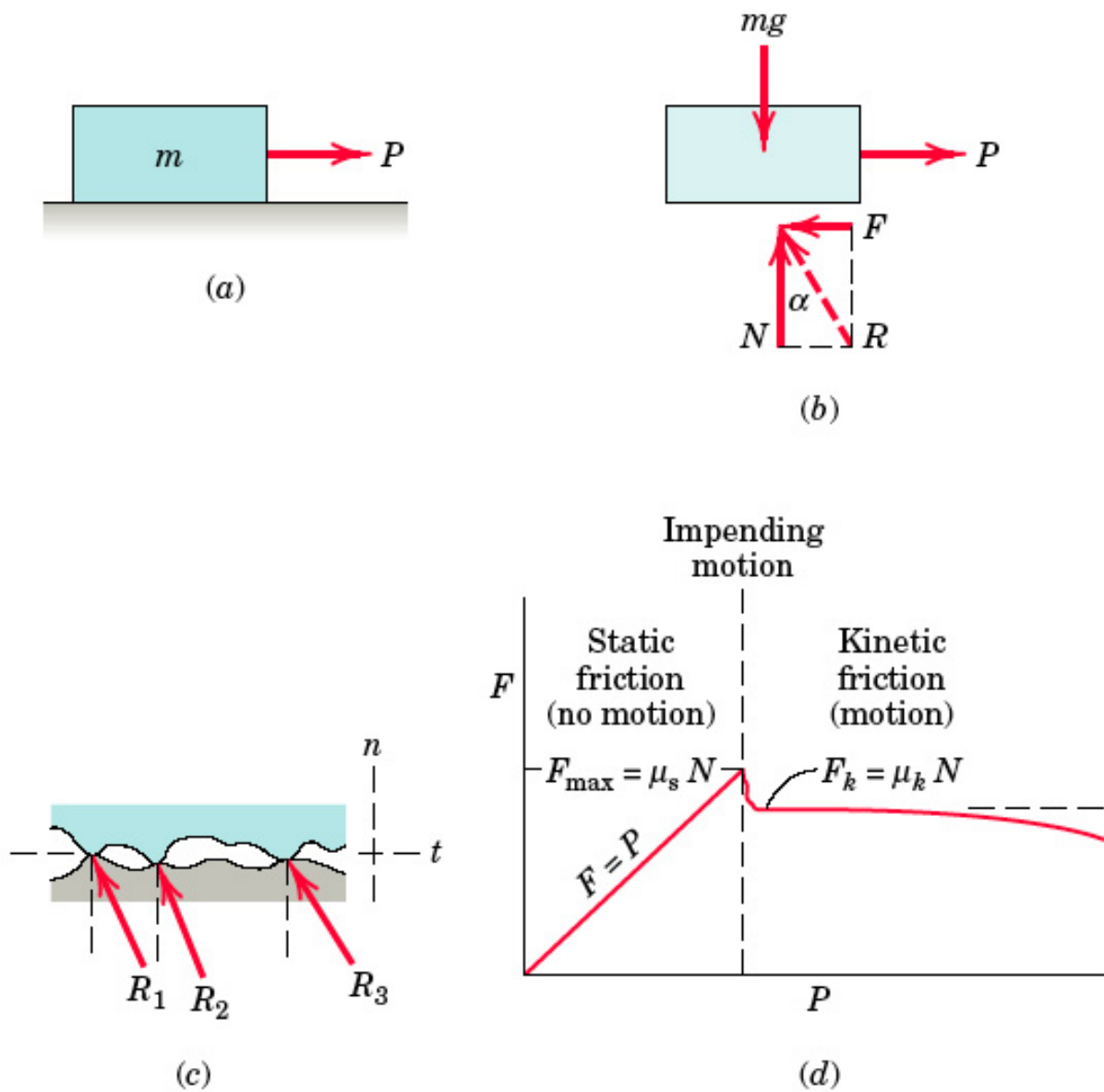


Figure 6/1

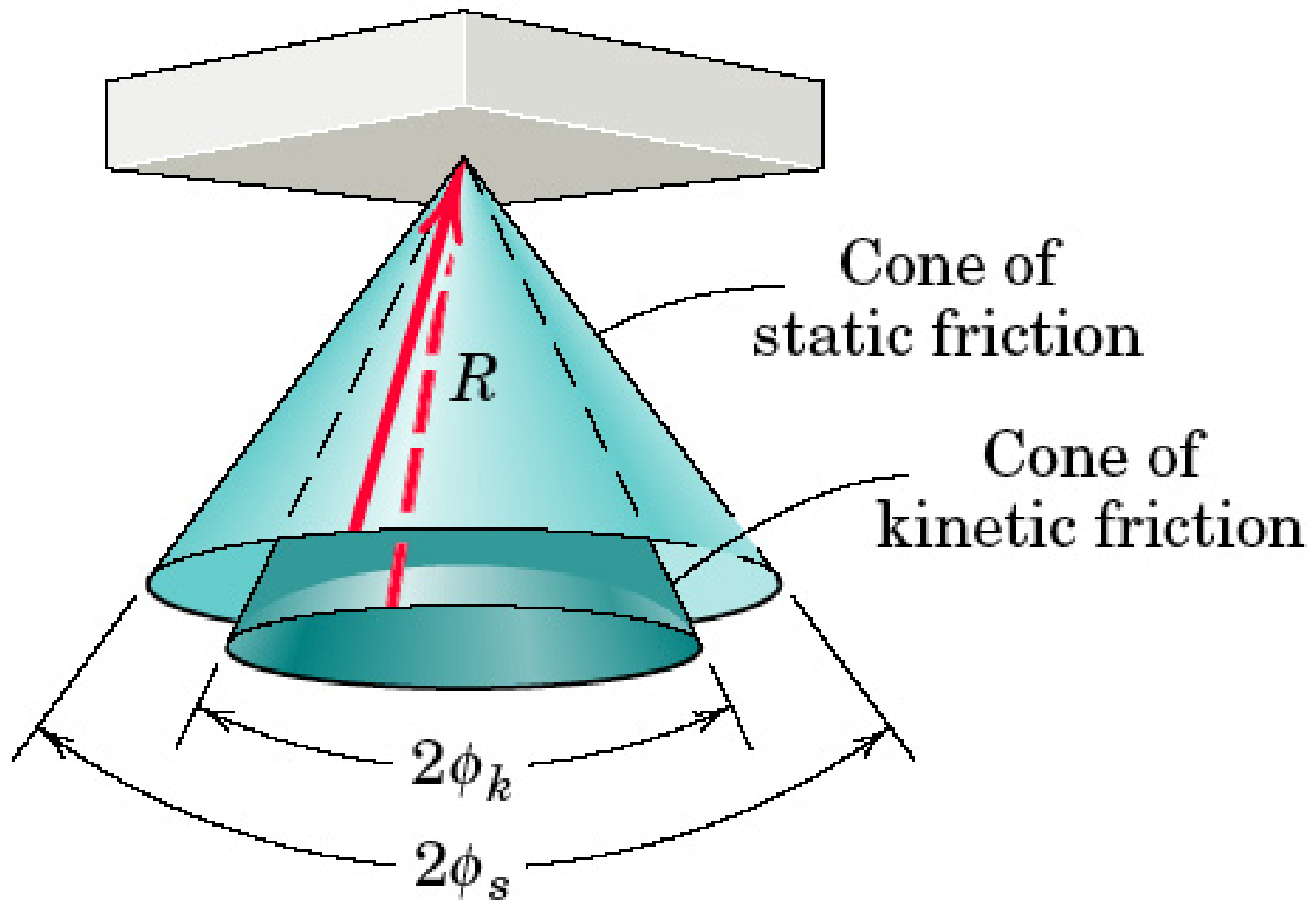
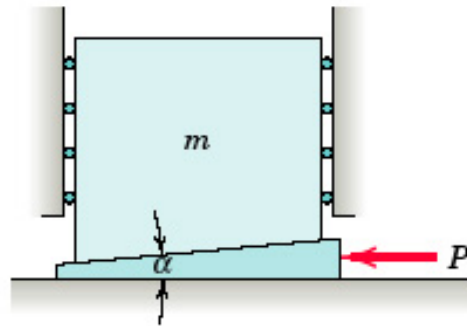
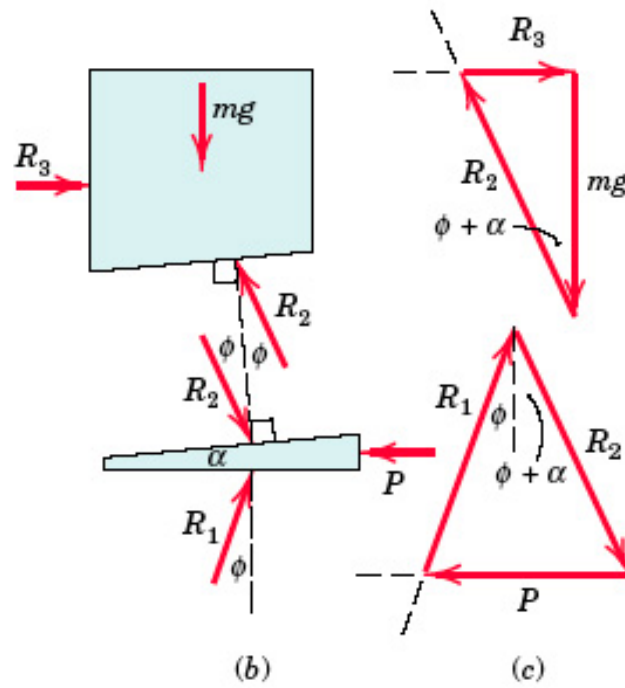


Figure 6/2

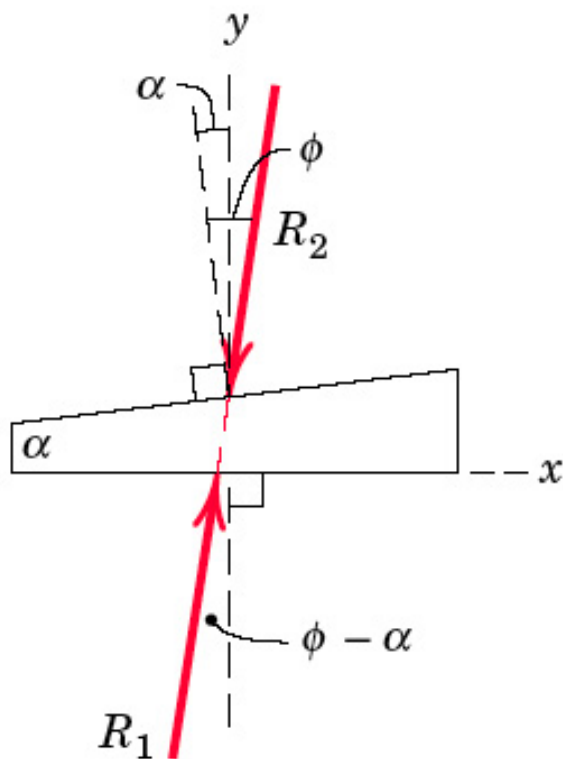


(a)

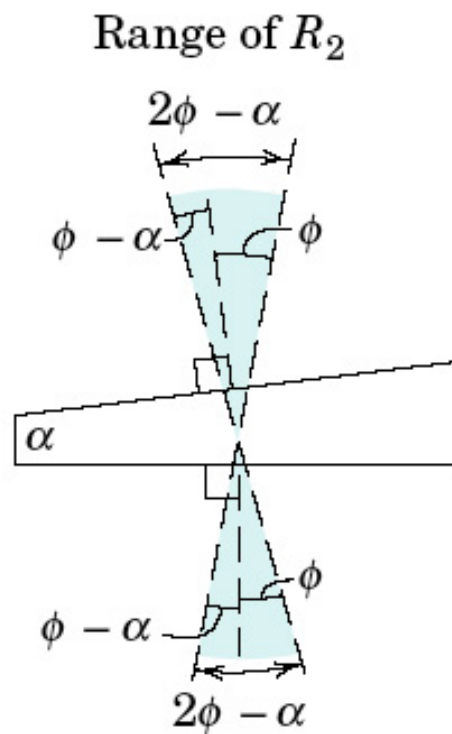


Forces to raise load

Figure 6/3



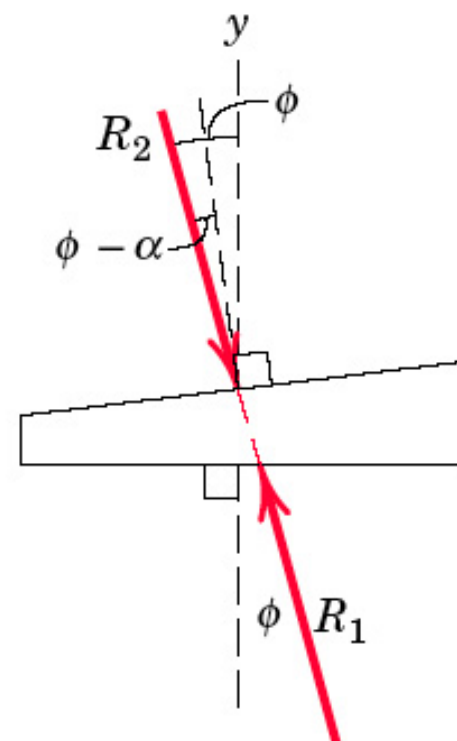
(a) Slipping impending at upper surface



Range of R_2

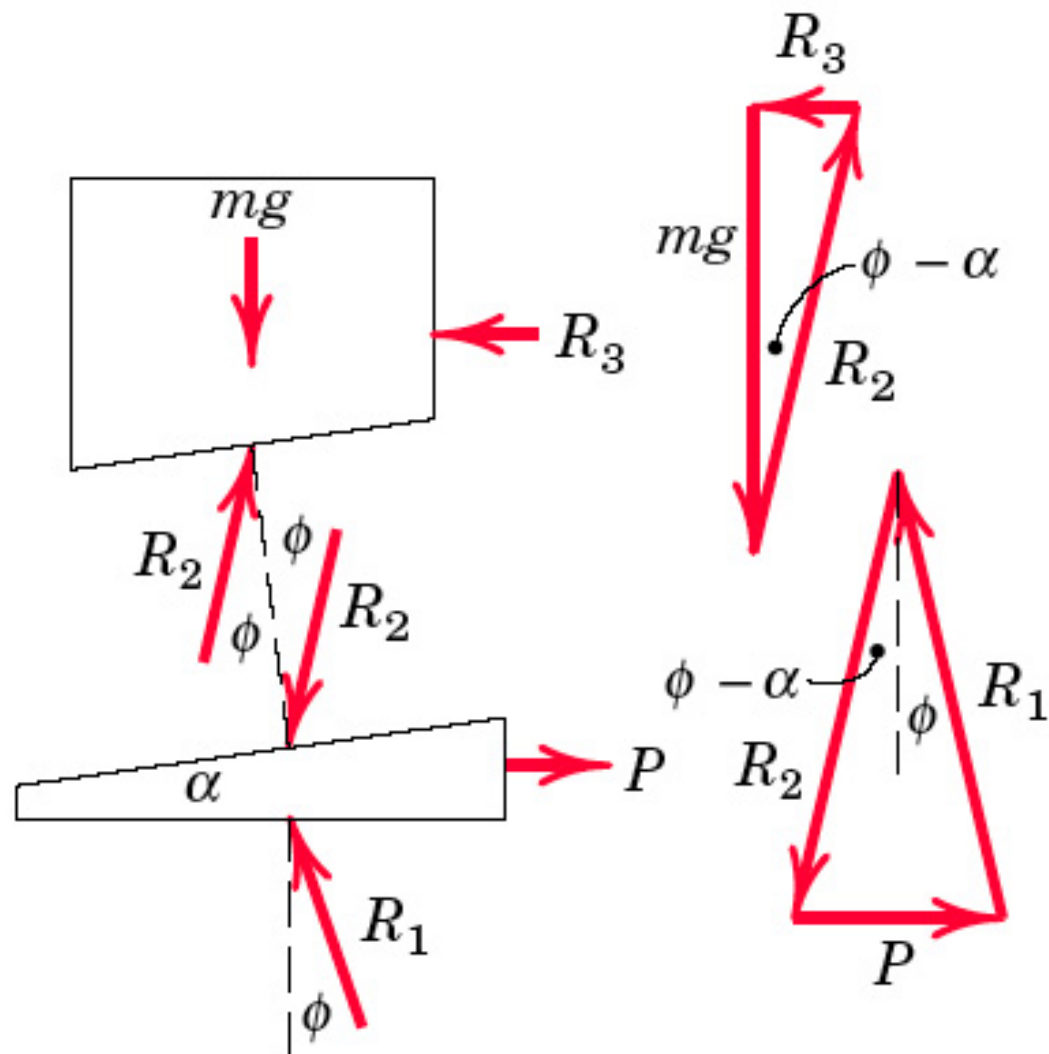
Range of R_1

(b) Range of $R_1 = R_2$ for no slip



(c) Slipping impending at lower surface

Figure 6/4



Forces to lower load

Figure 6/5

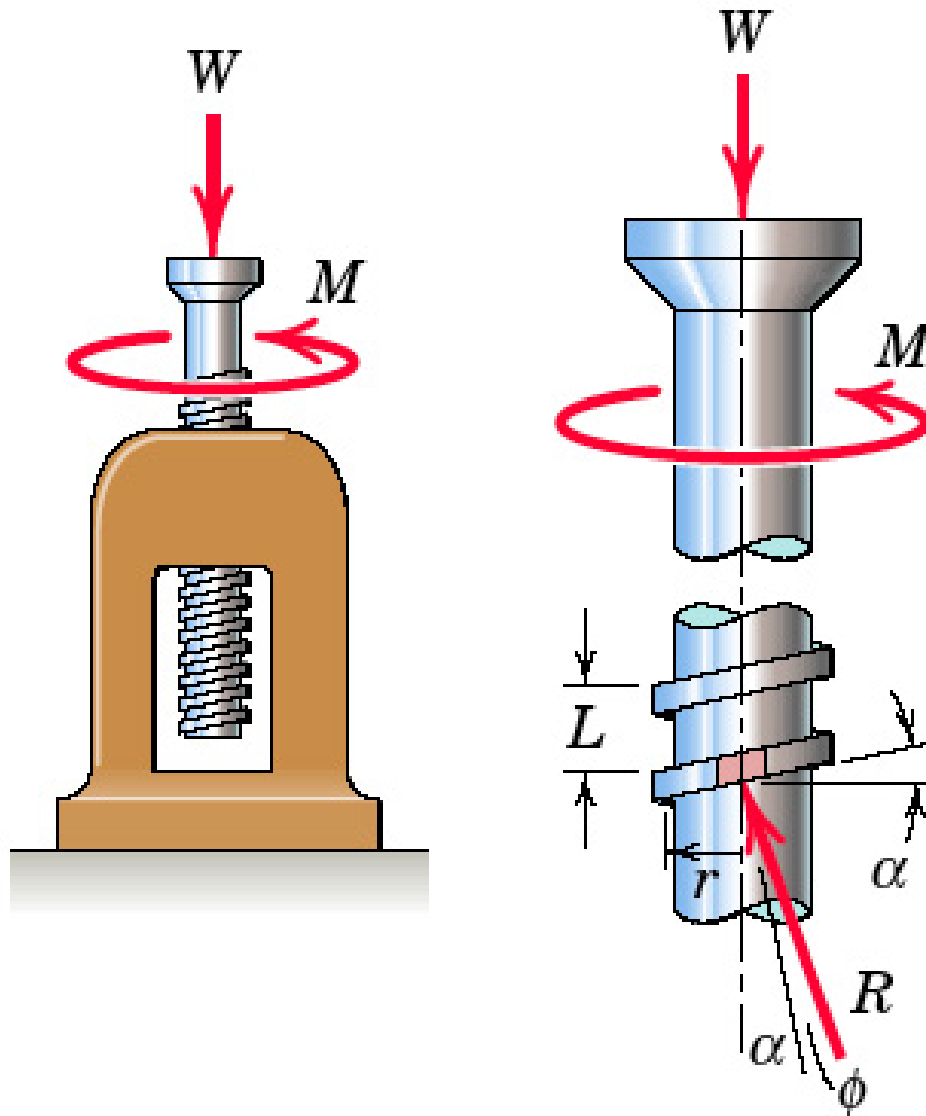
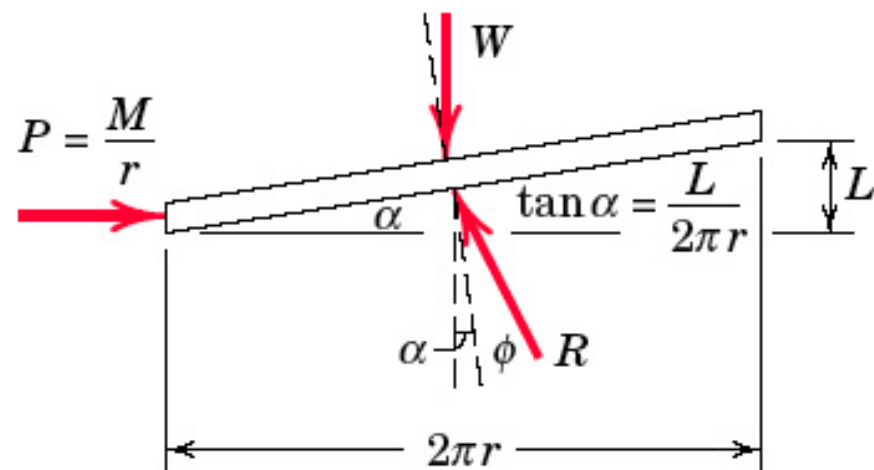
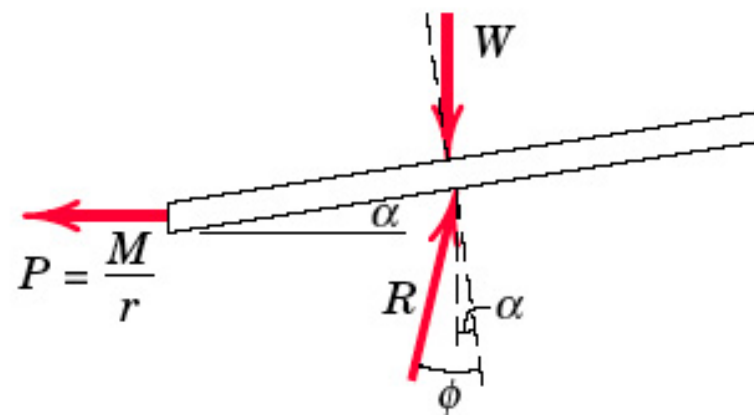


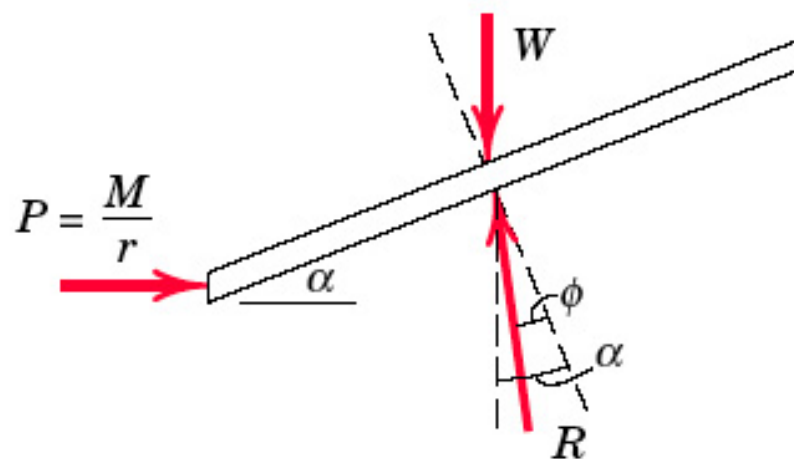
Figure 6/6



(a) To raise load



(b) To lower load ($\alpha < \phi$)



(c) To lower load ($\alpha > \phi$)

Figure 6/7

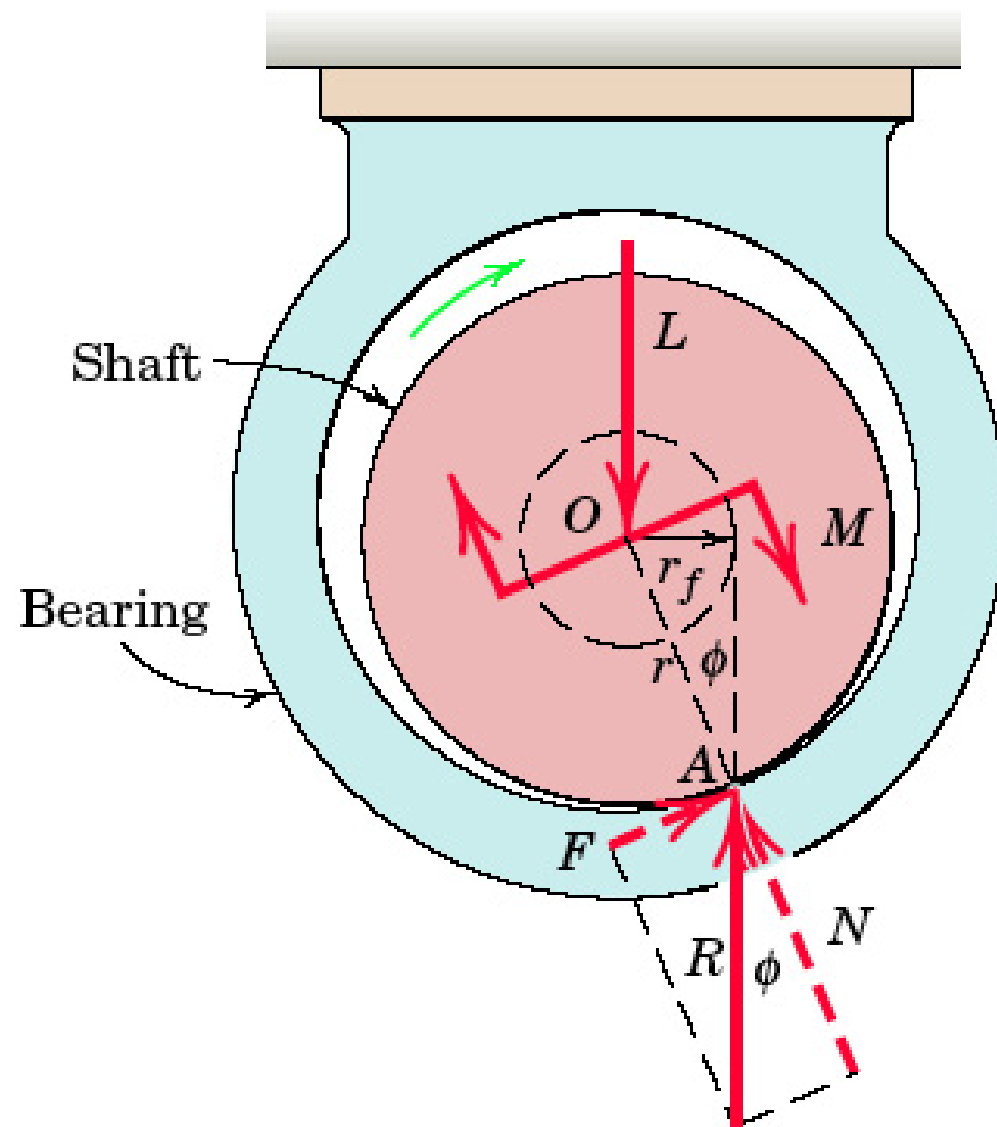


Figure 6/8

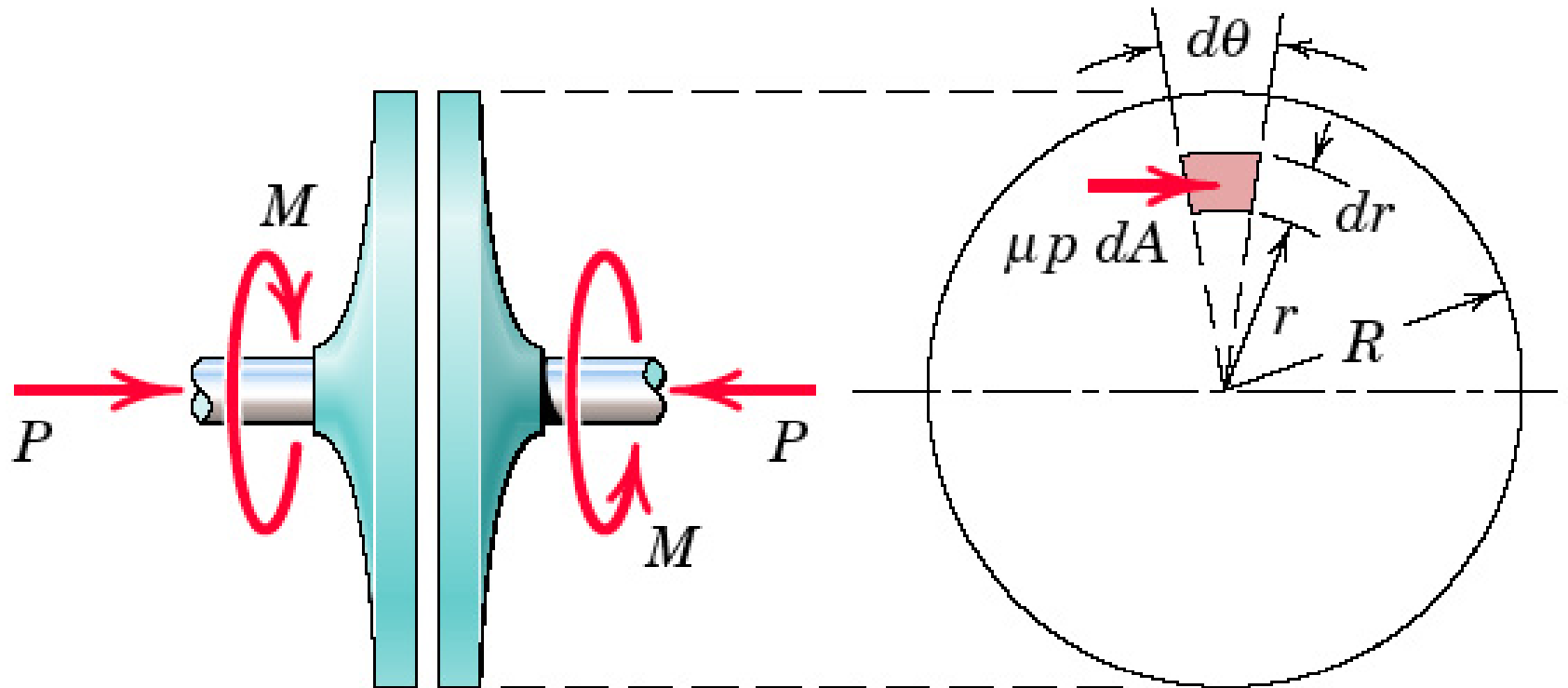


Figure 6/9

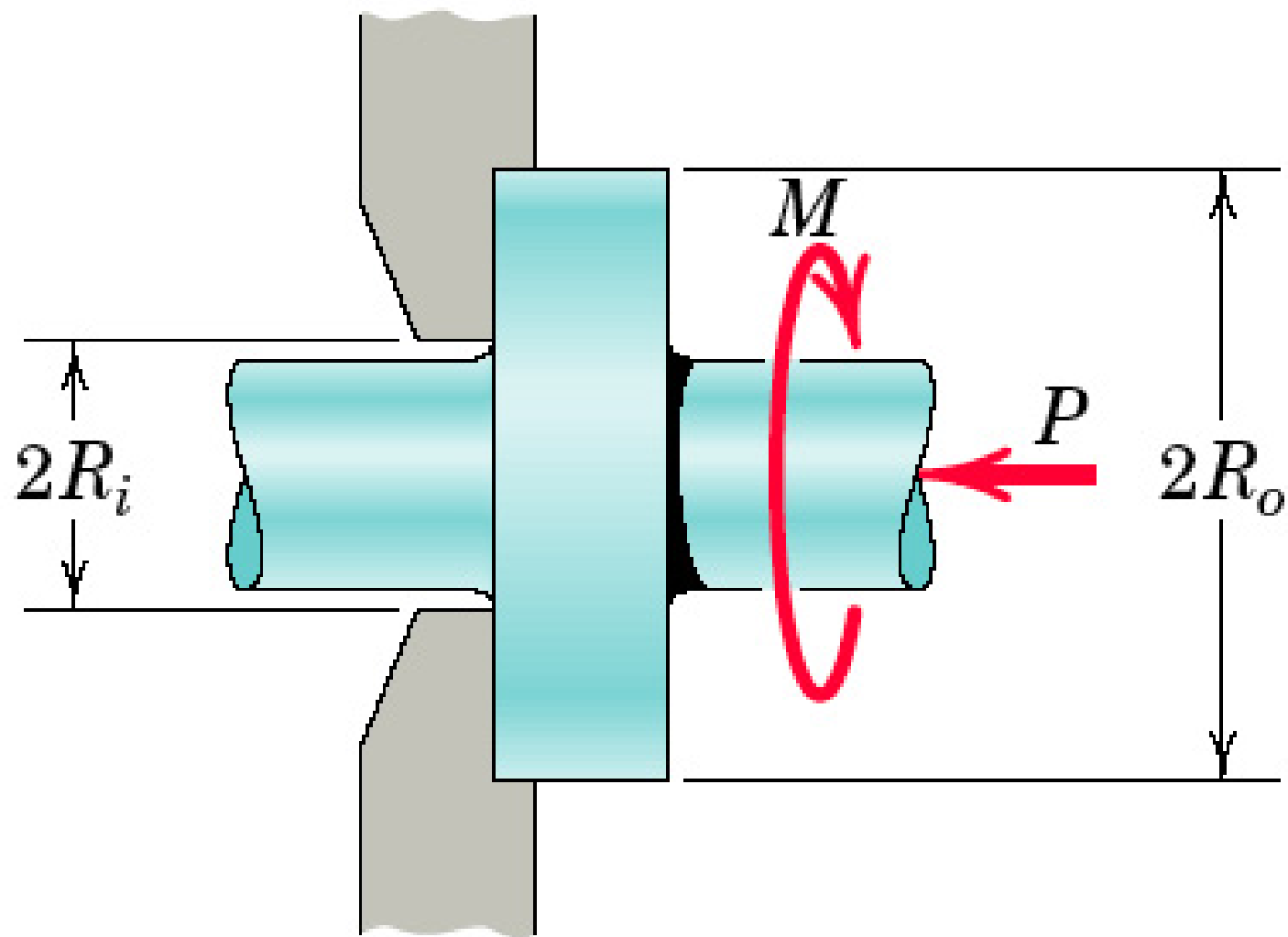
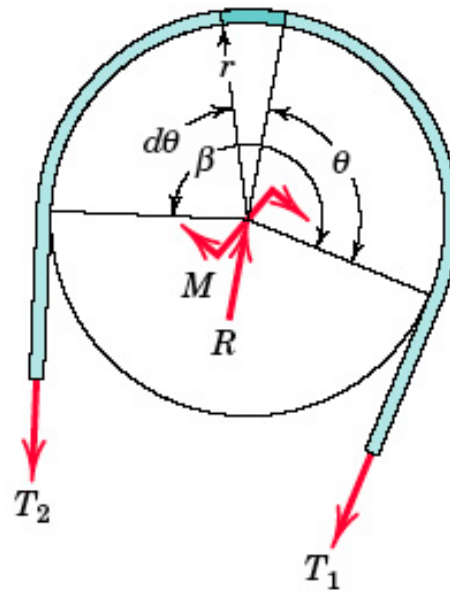
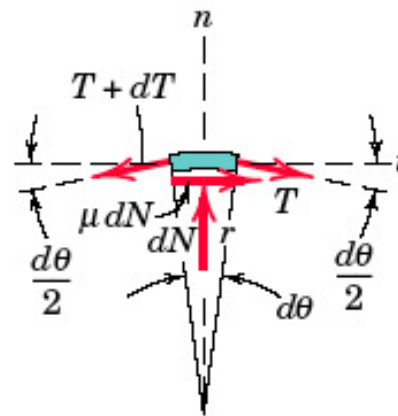


Figure 6/10



(a)



(b)

Figure 6/11

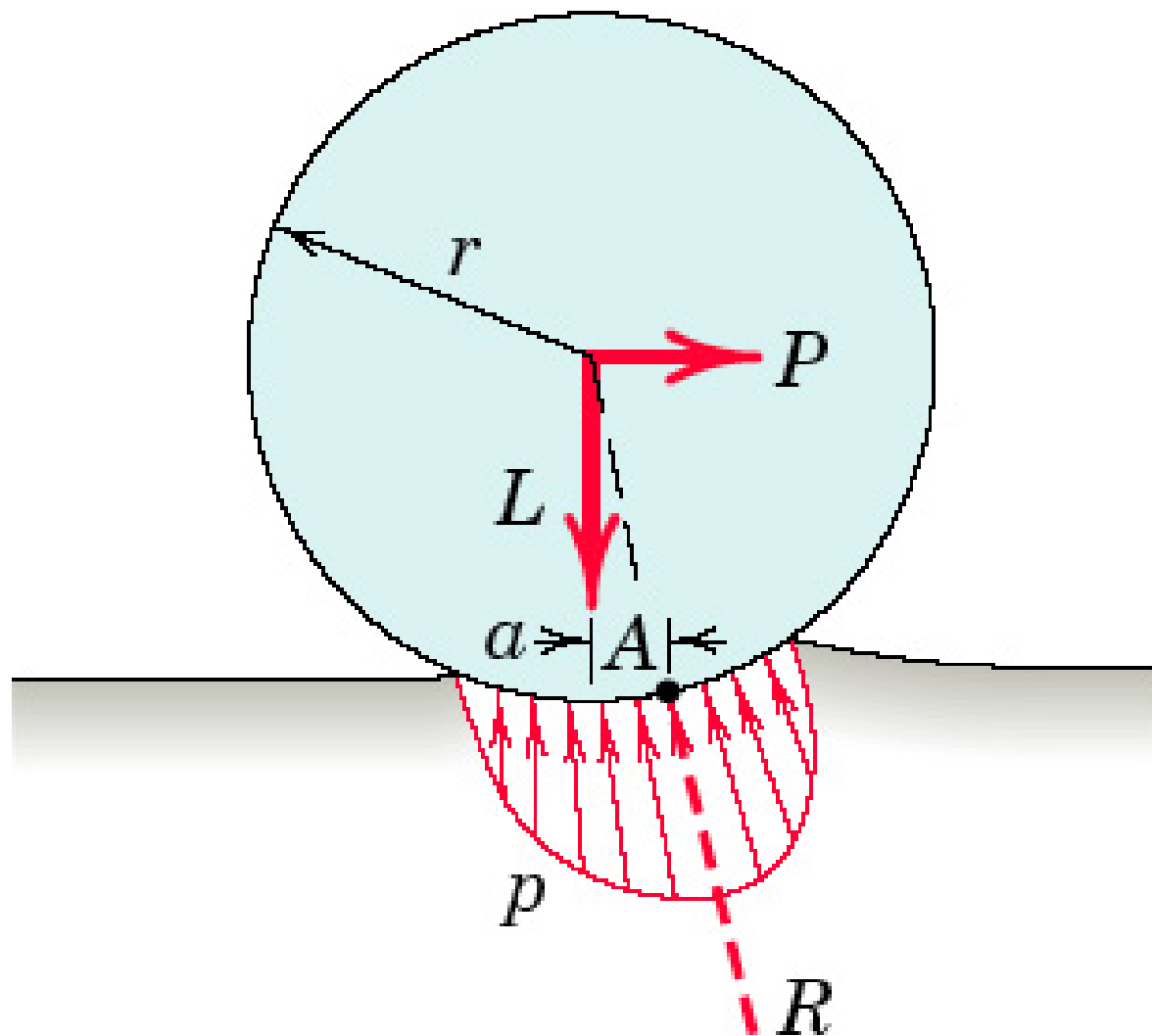
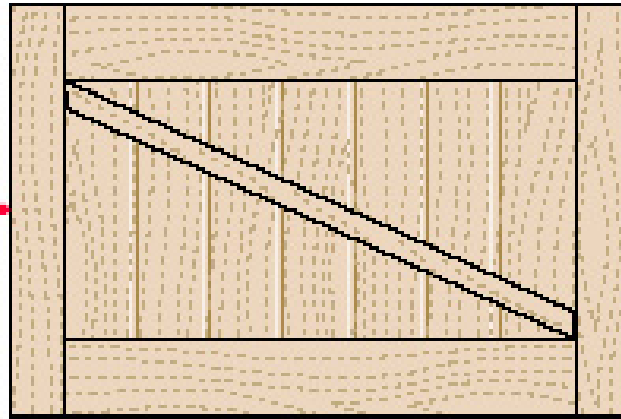


Figure 6/12

90 kg

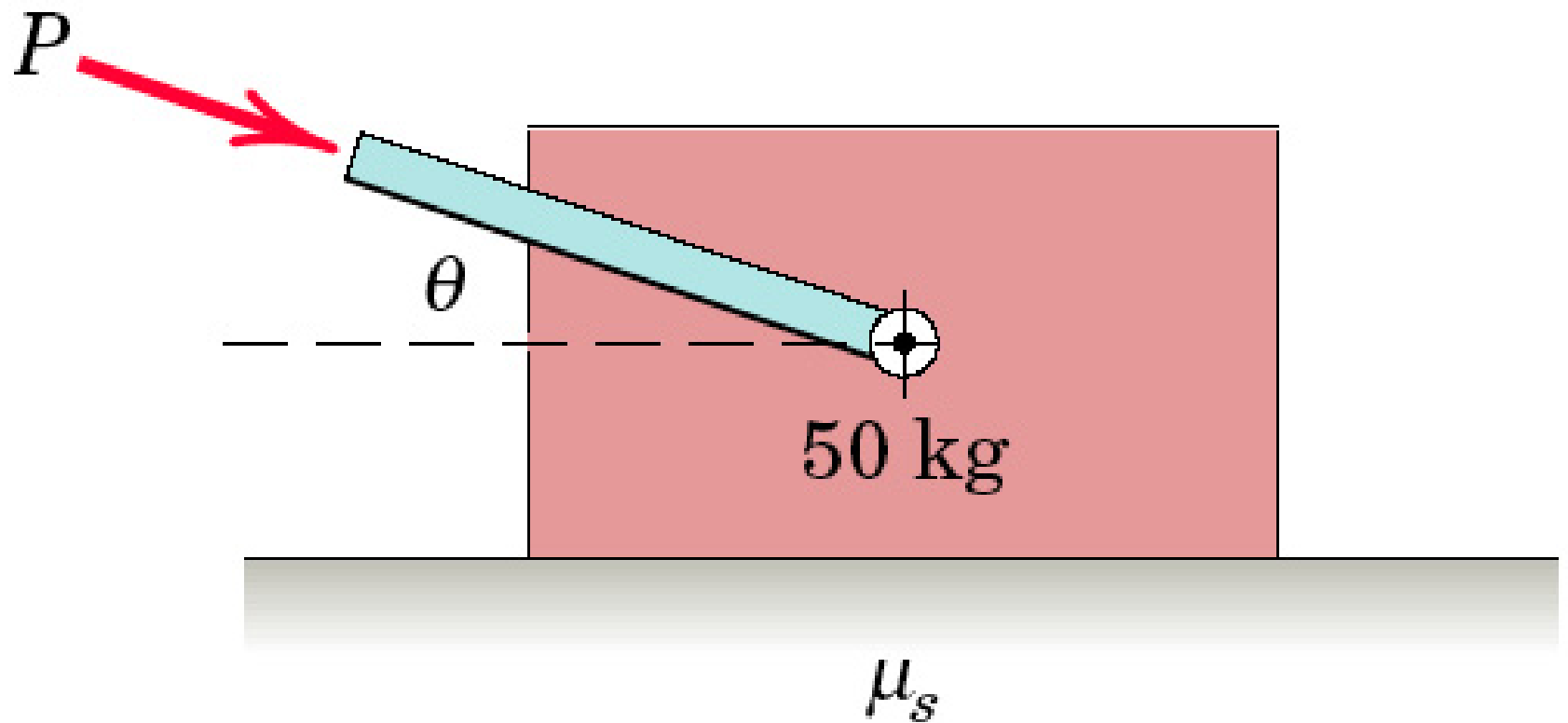
P



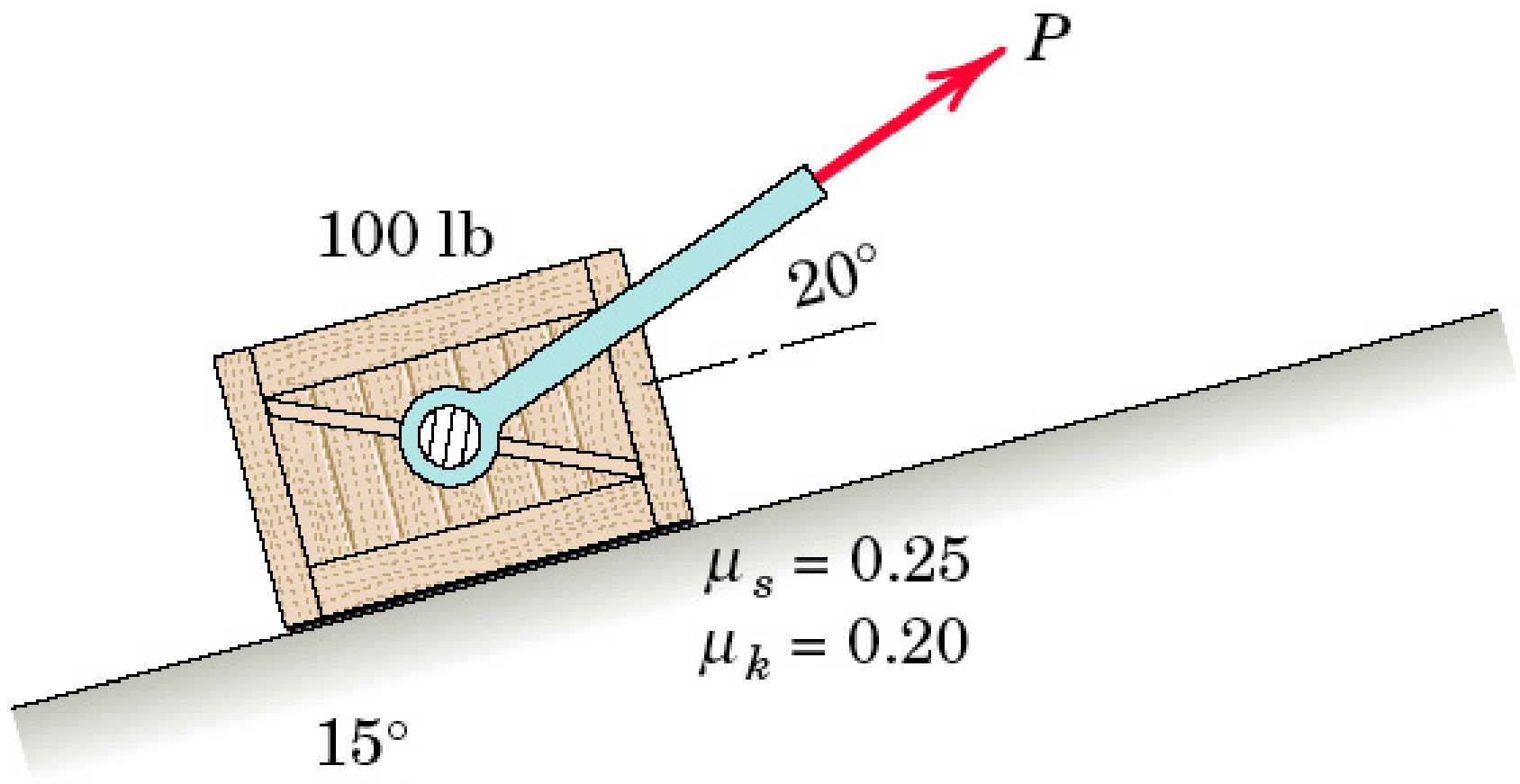
$$\begin{cases} \mu_s = 0.50 \\ \mu_k = 0.40 \end{cases}$$

Problem 6/1

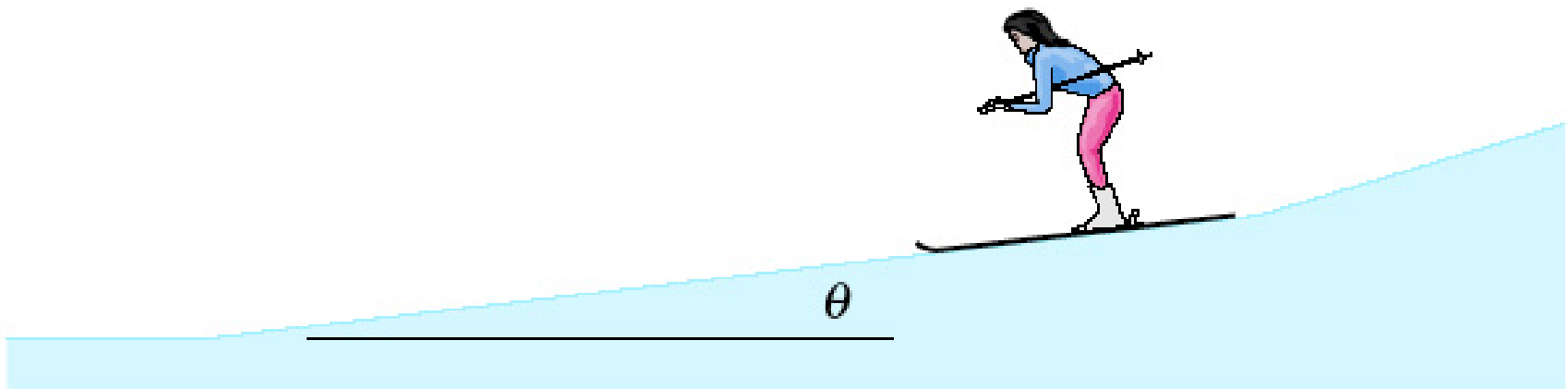
P_06_01



Problem 6/2

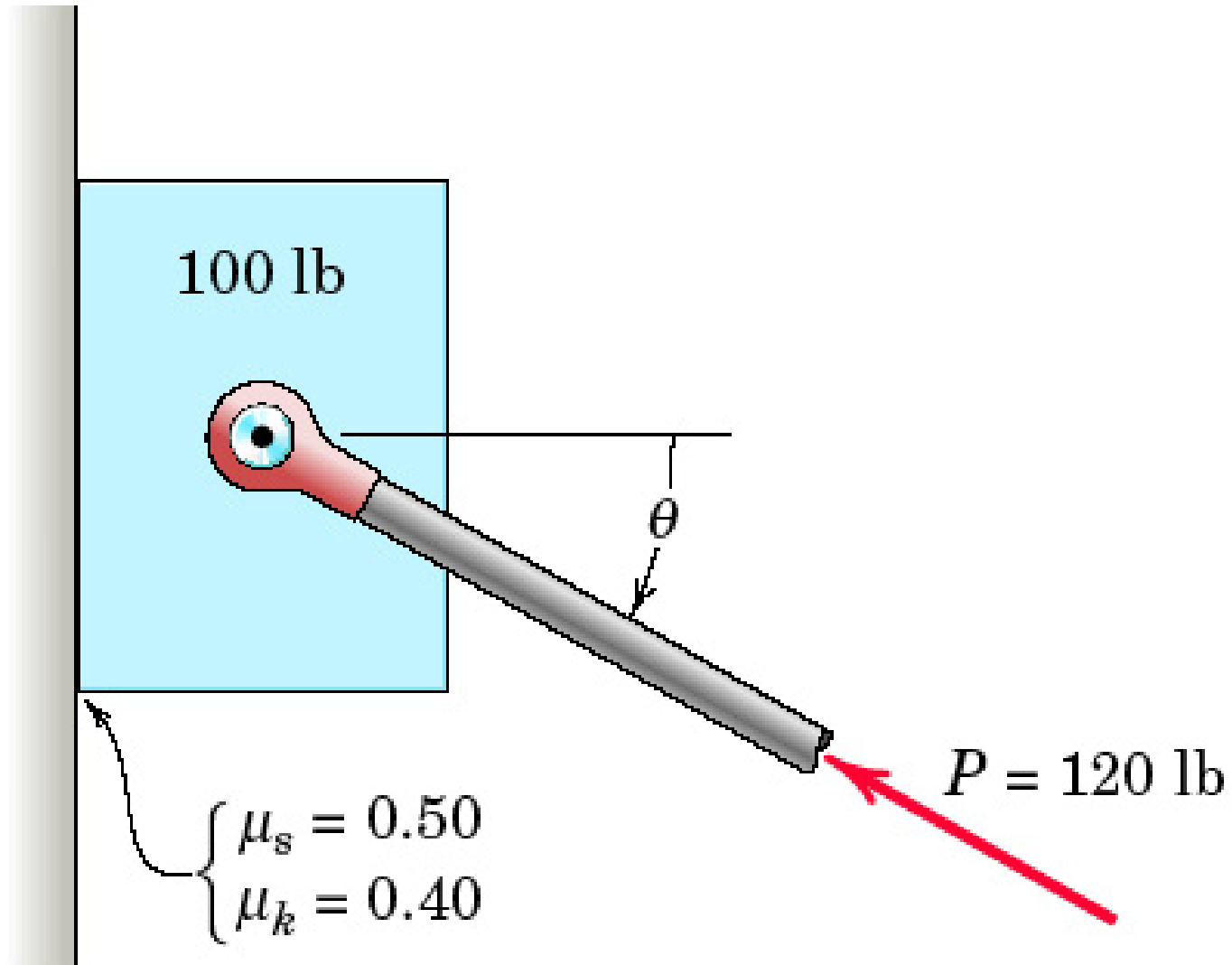


Problem 6/3

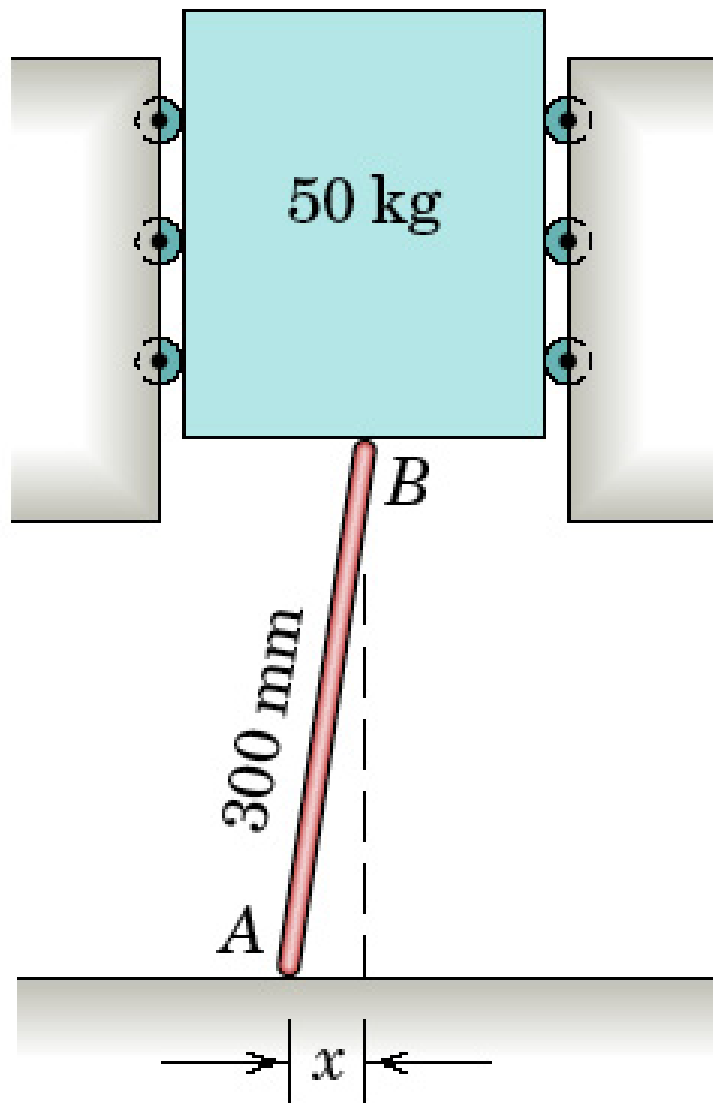


Problem 6/4

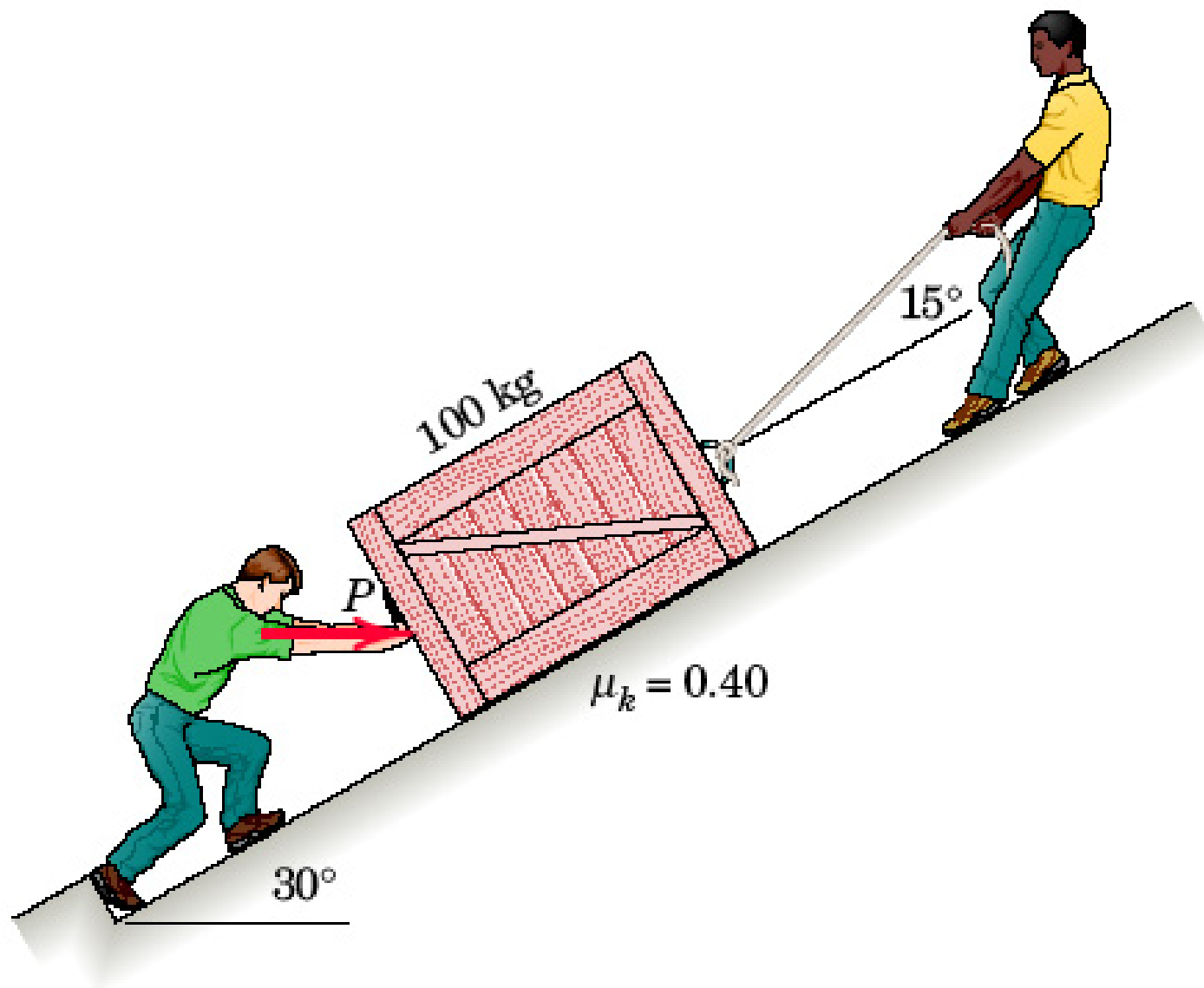
P_06_04



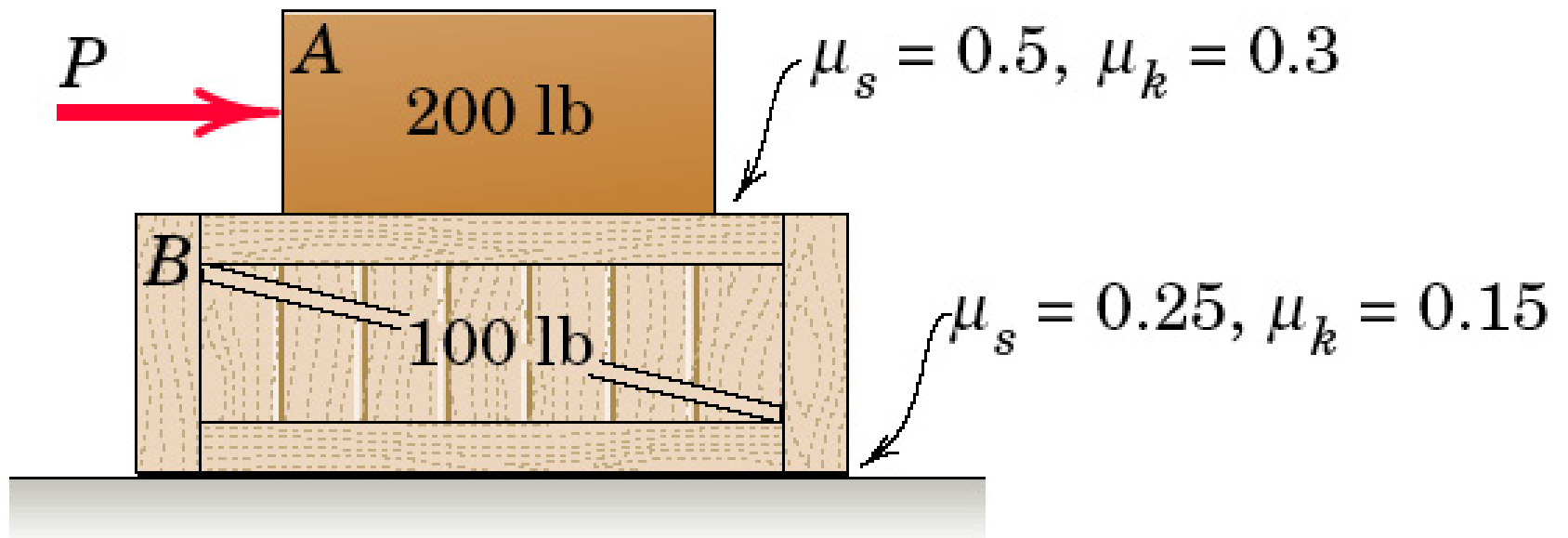
Problem 6/5



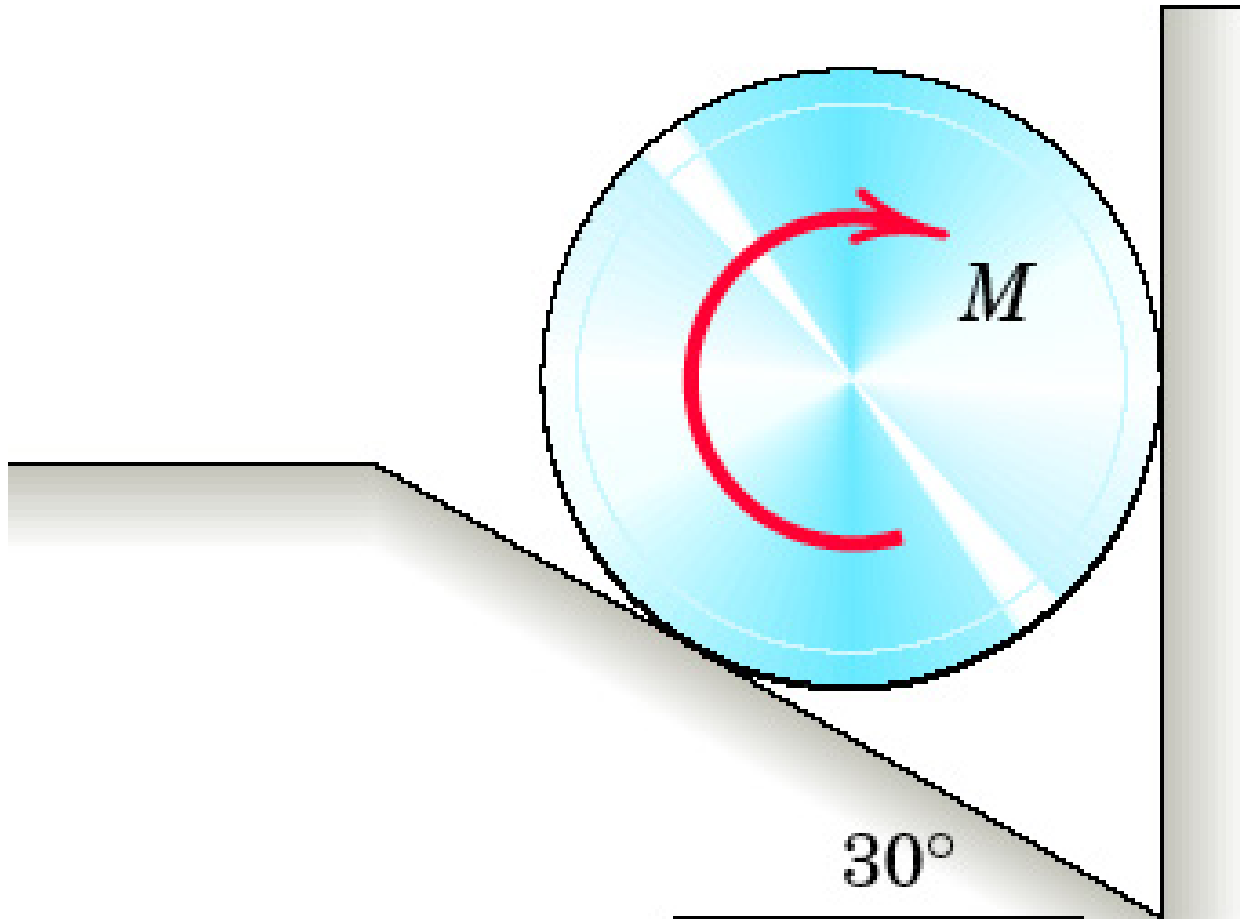
Problem 6/6



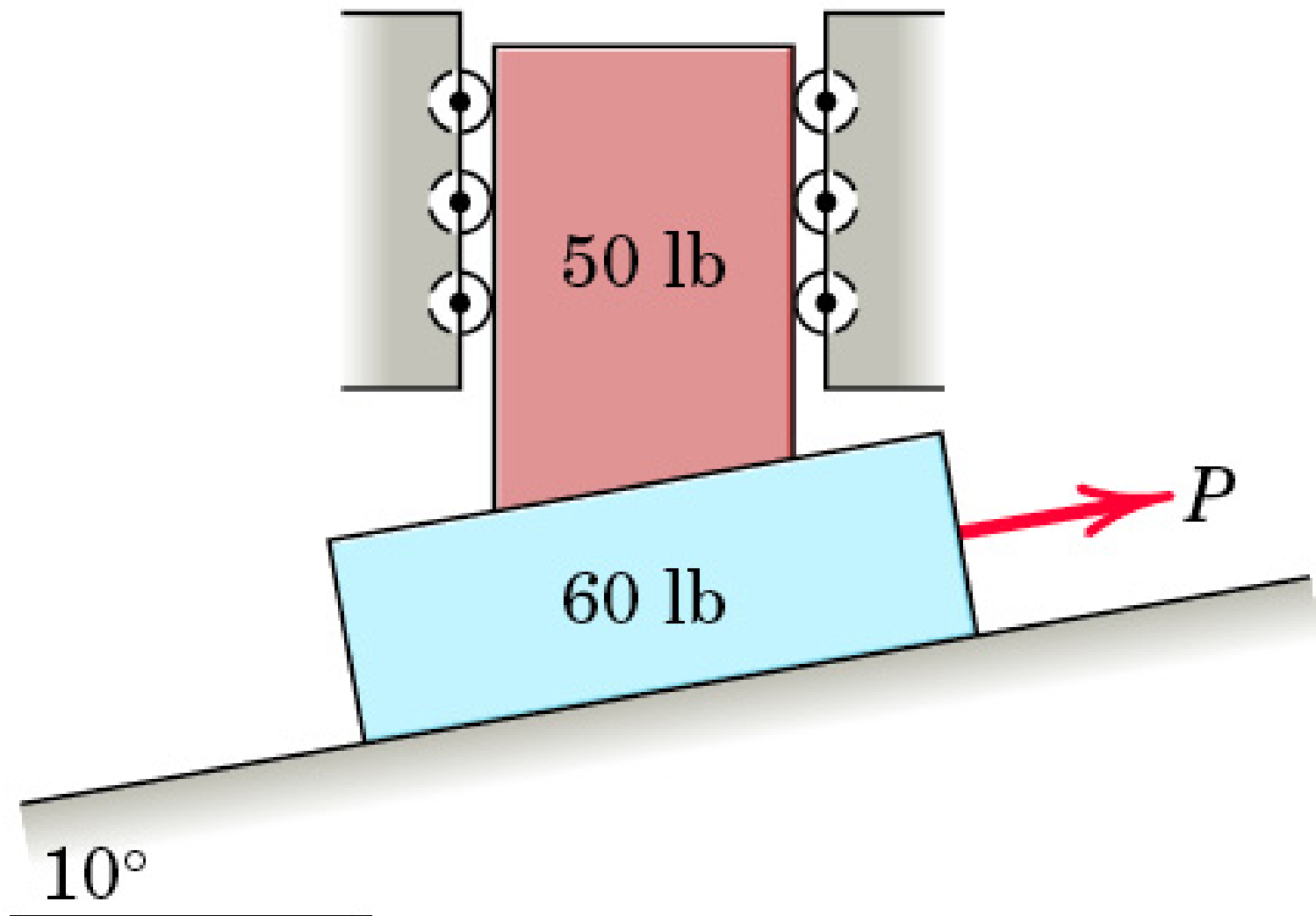
Problem 6/7



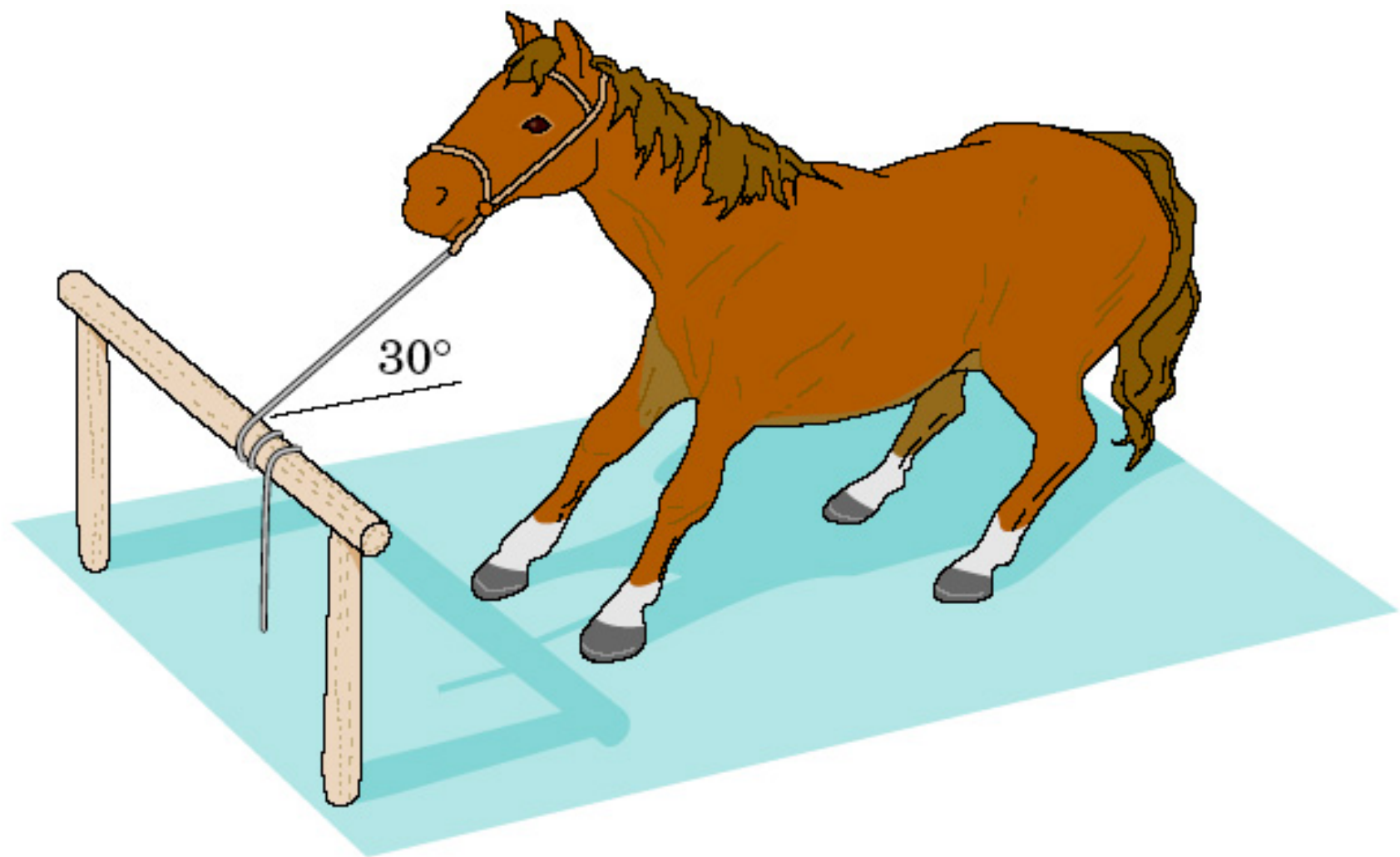
Problem 6/8



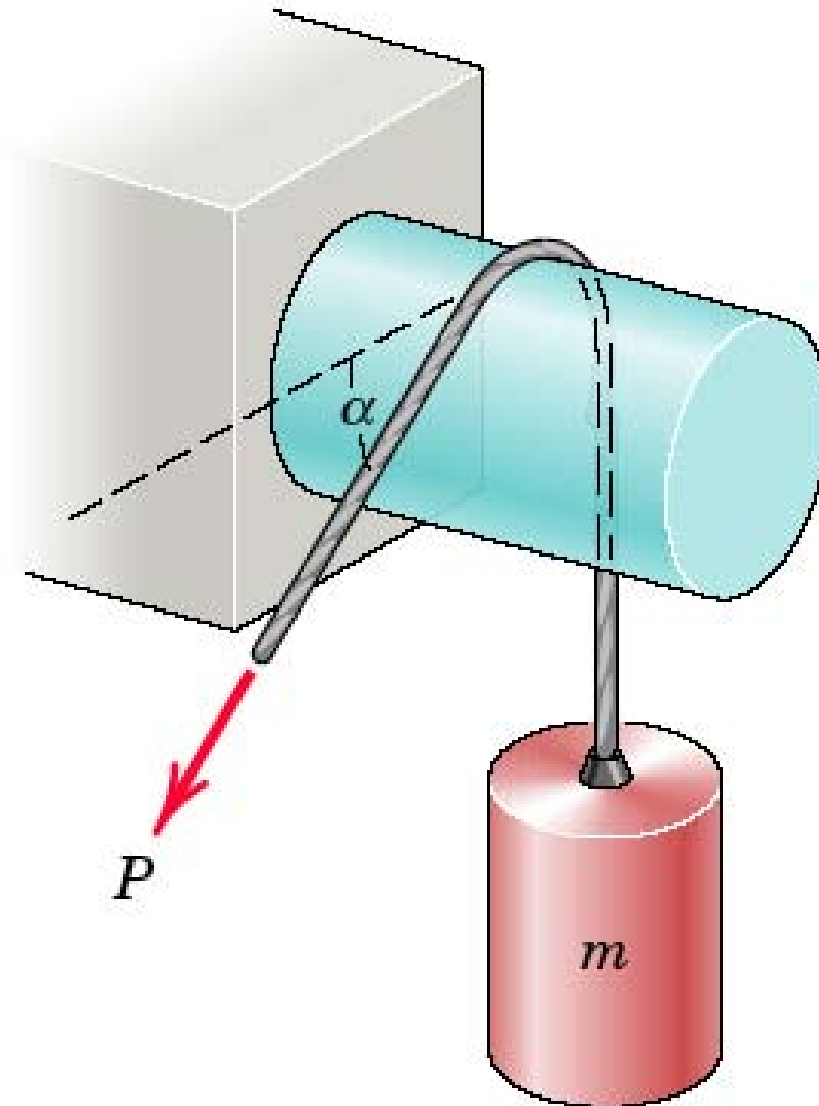
Problem 6/9



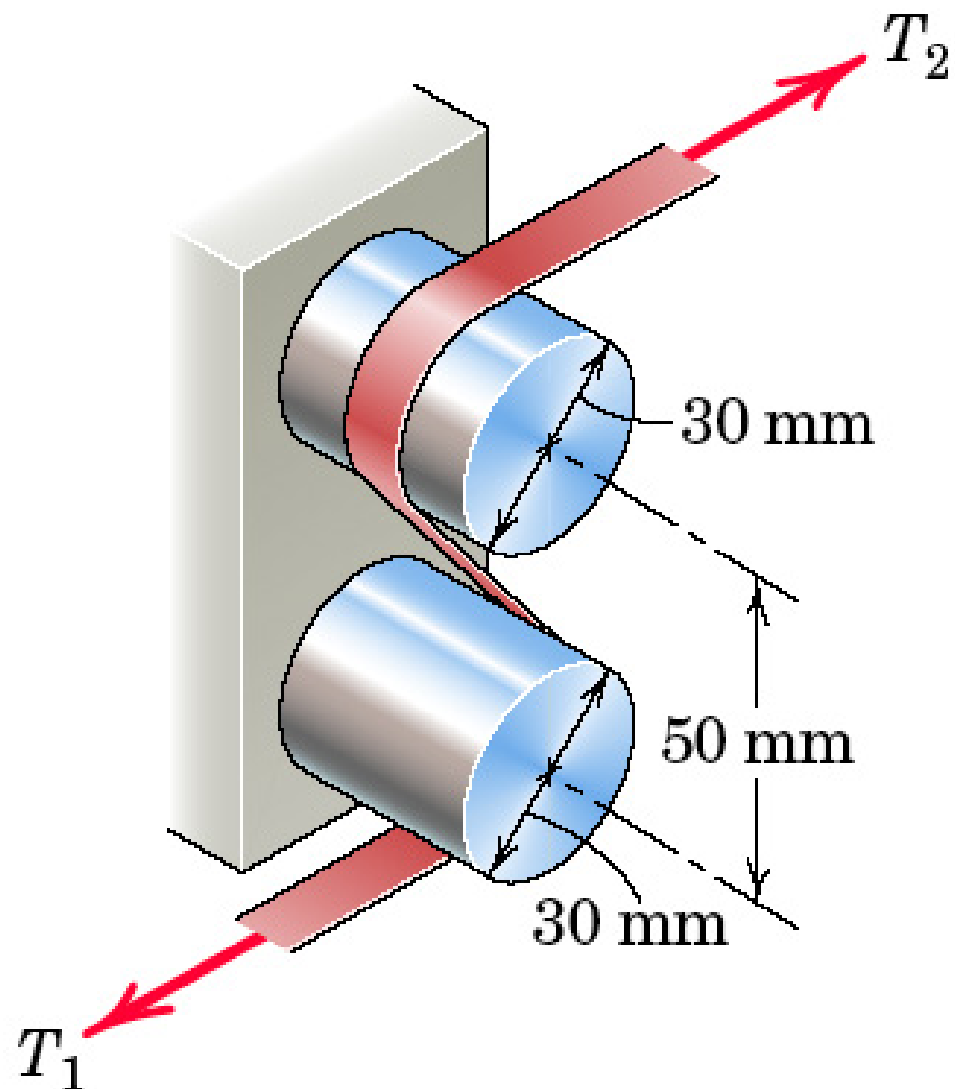
Problem 6/10



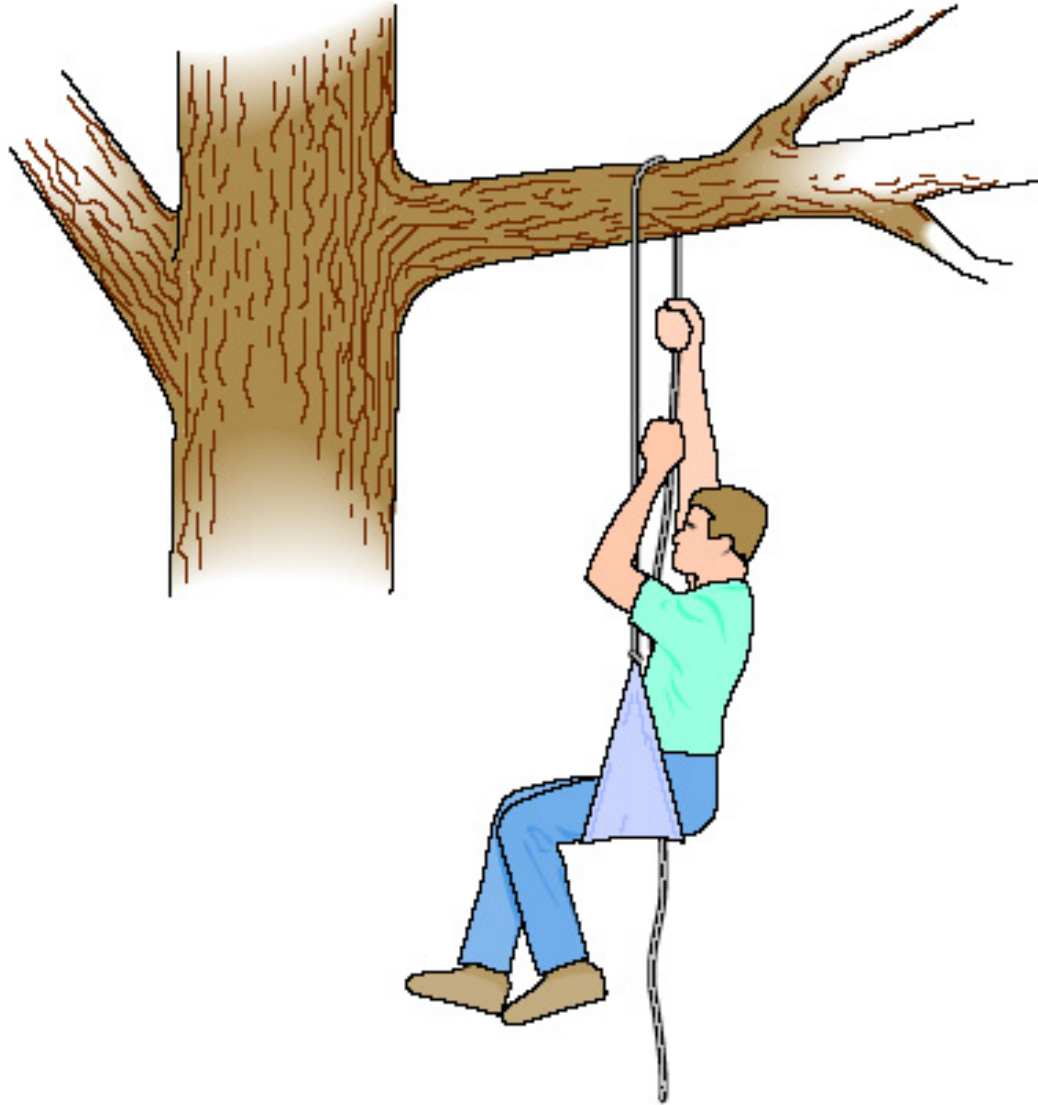
Problem 6/100



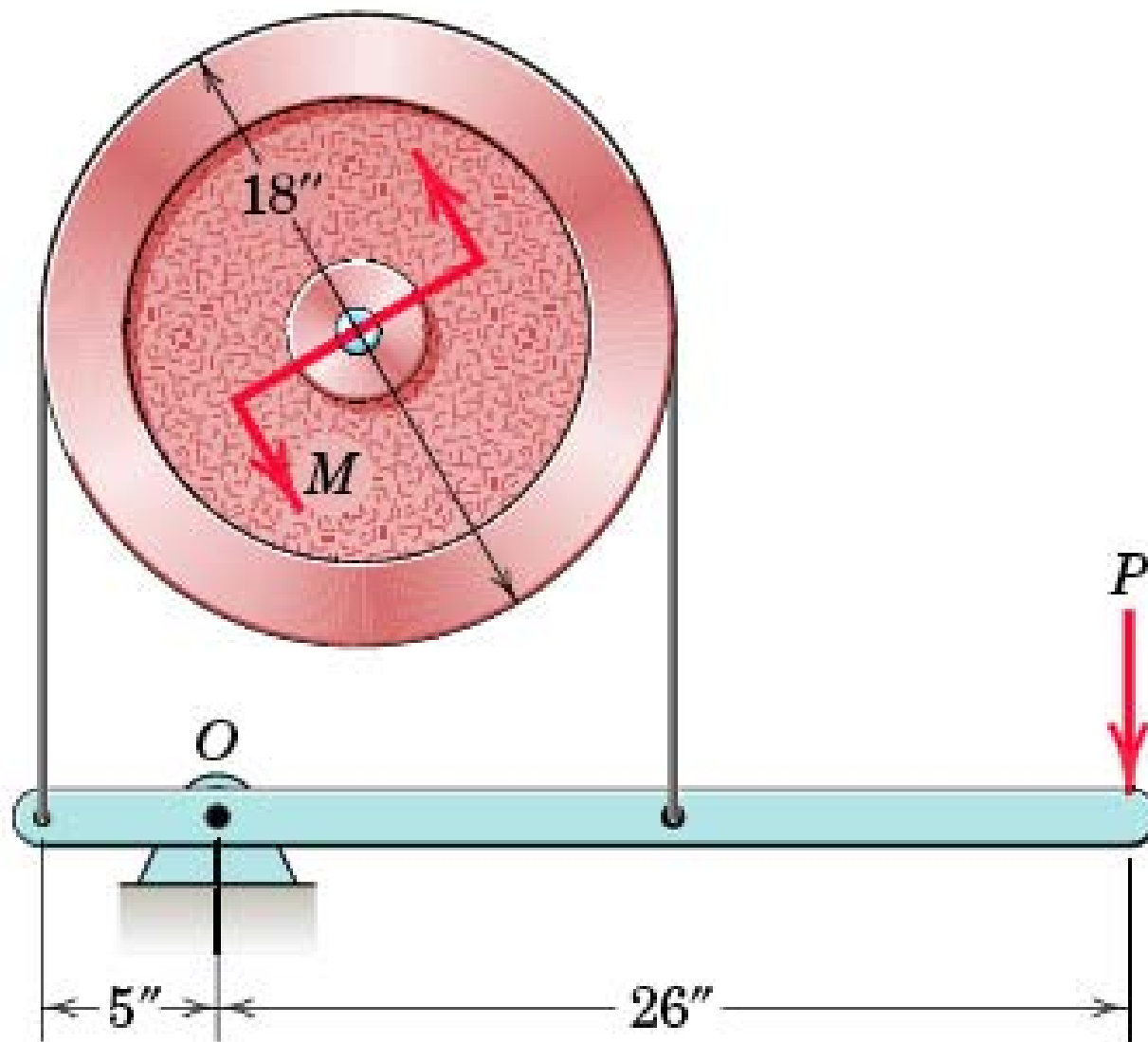
Problem 6/101



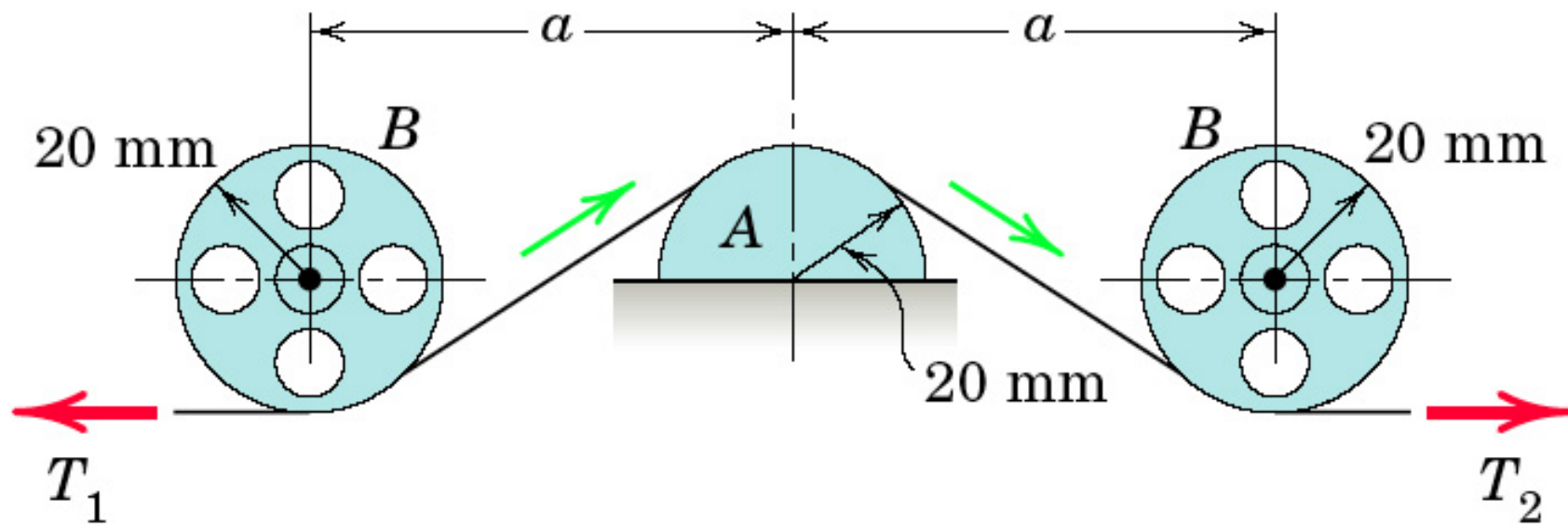
Problem 6/102



Problem 6/103

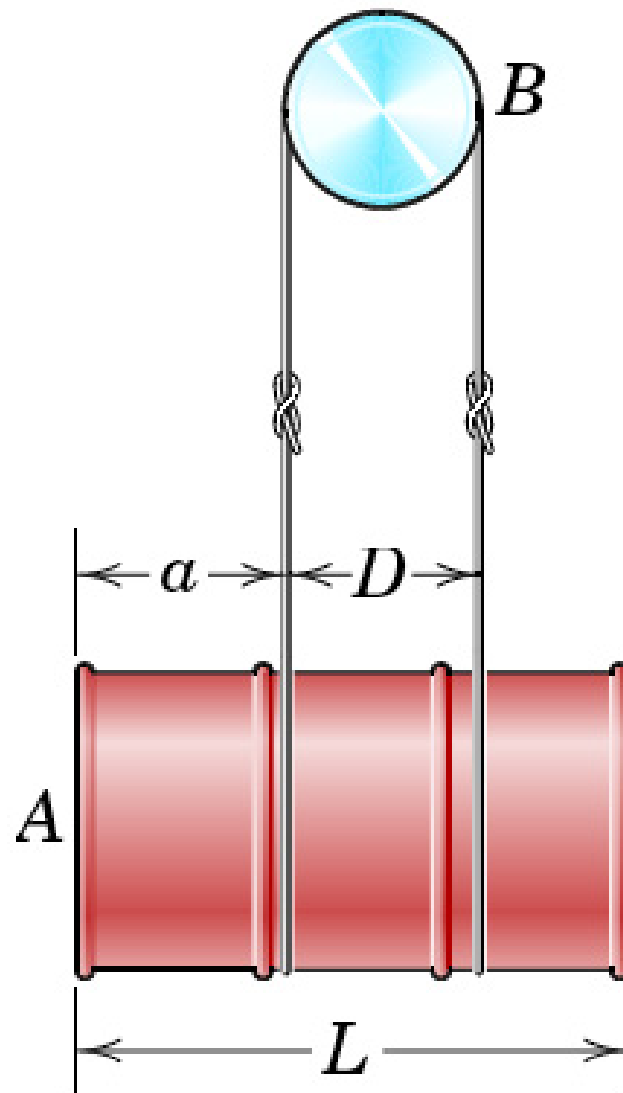


Problem 6/104

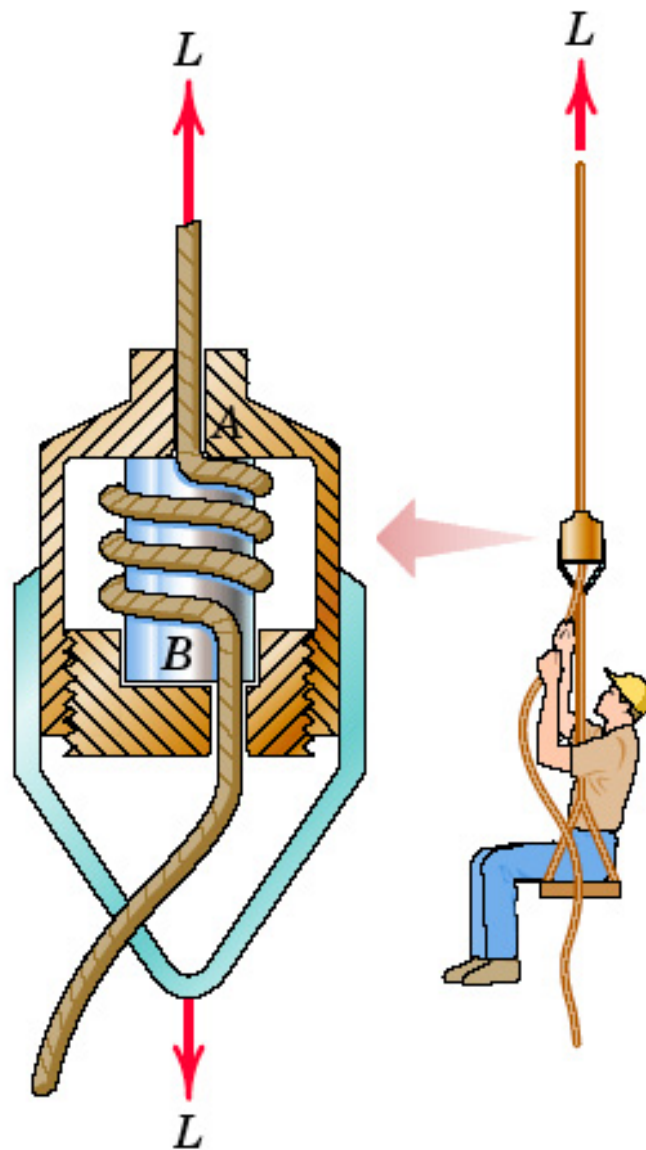


Problem 6/105

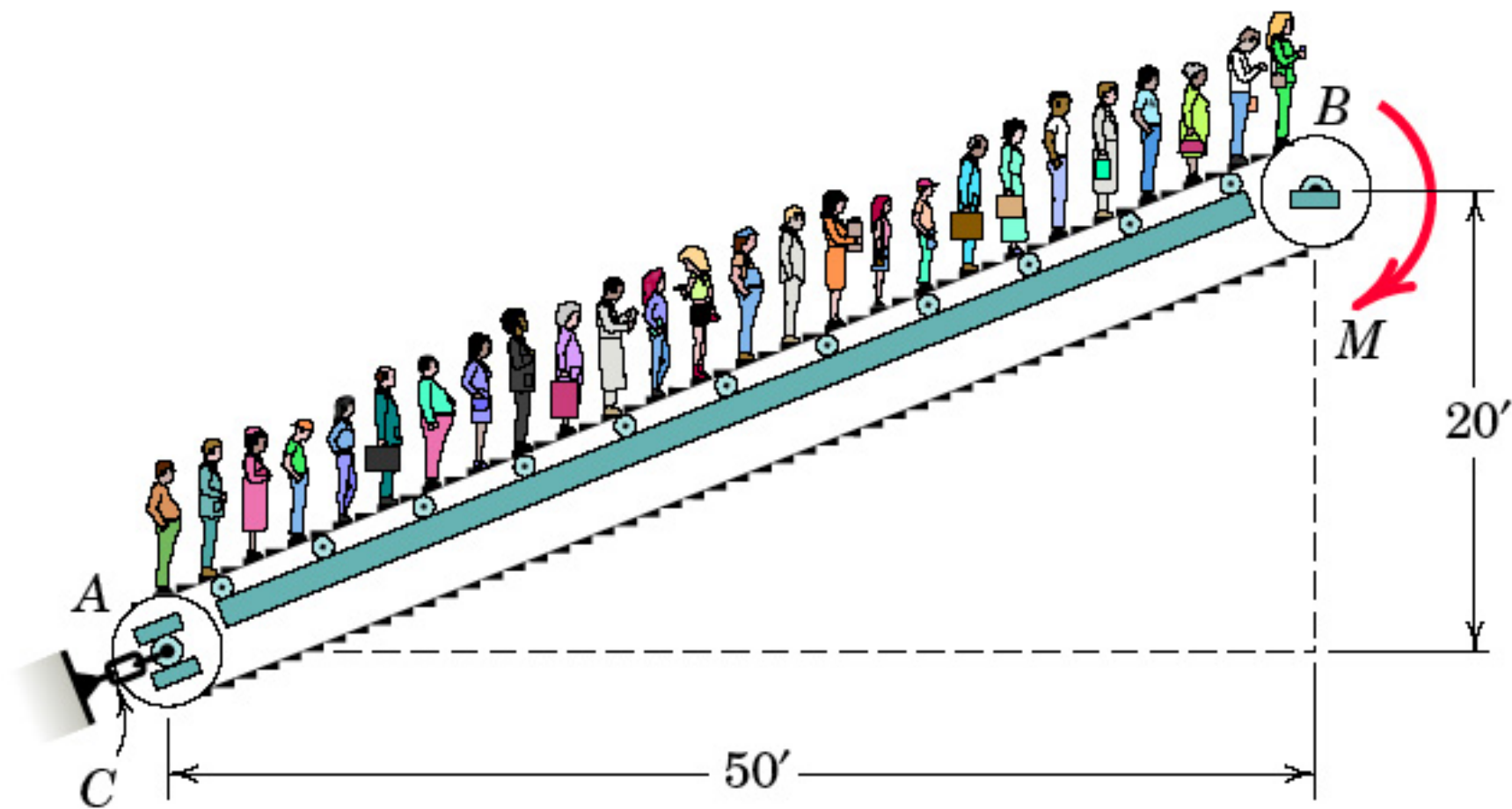
P_06_105



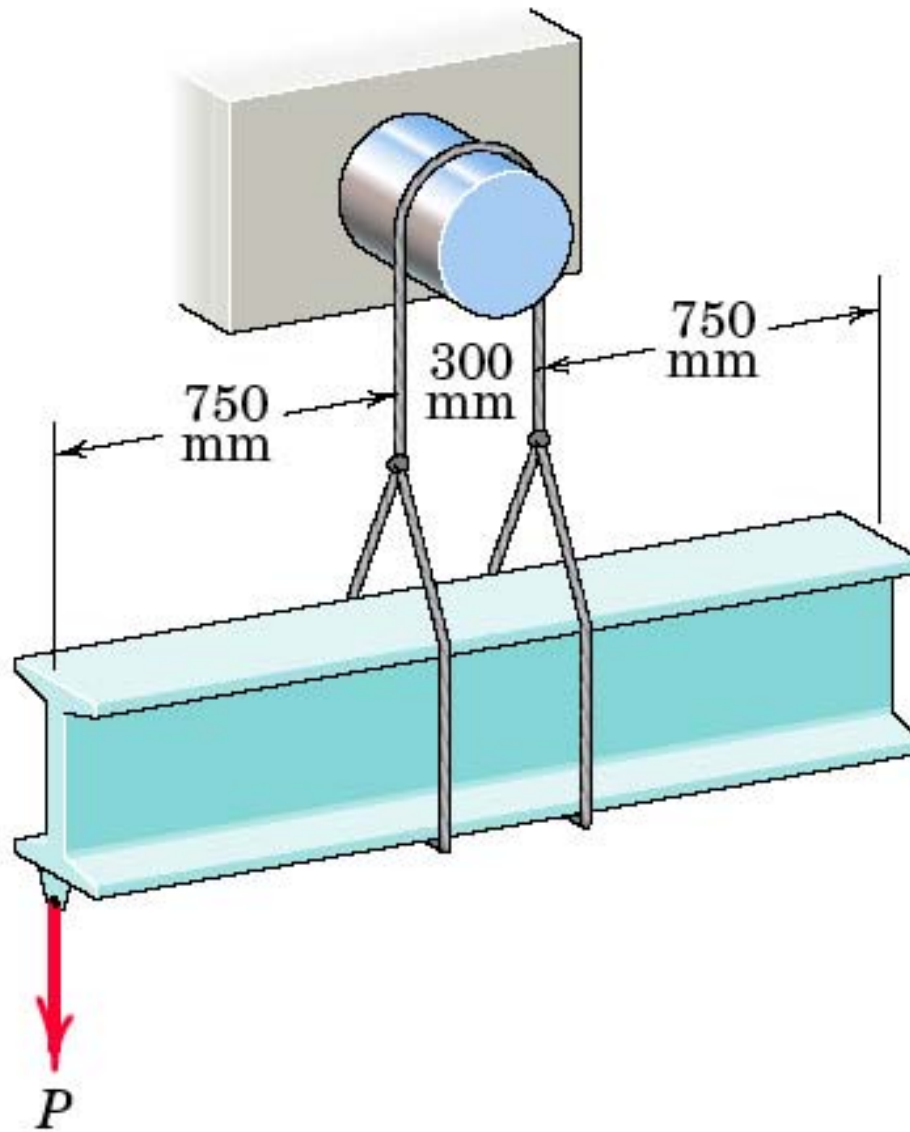
Problem 6/106



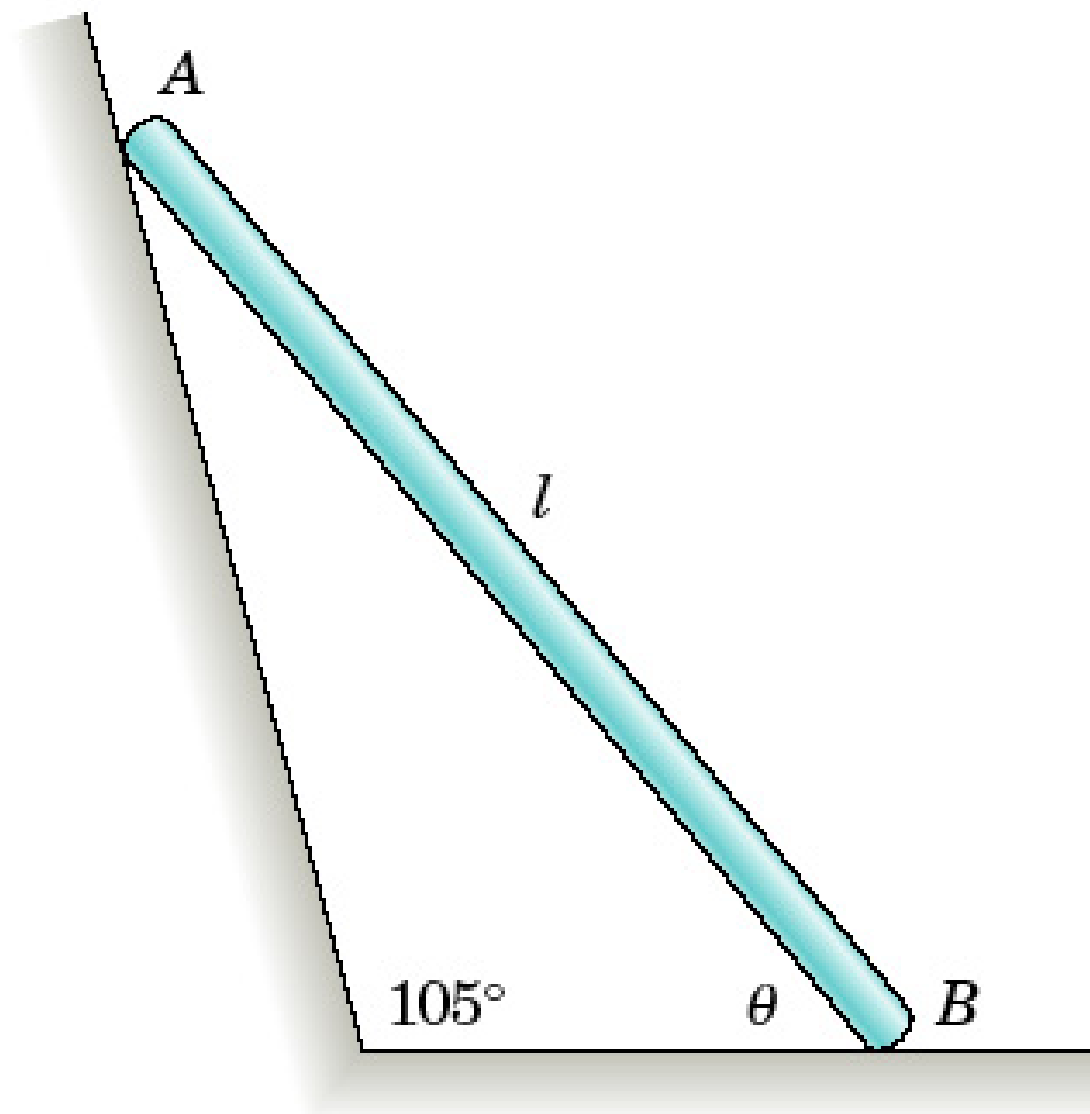
Problem 6/107



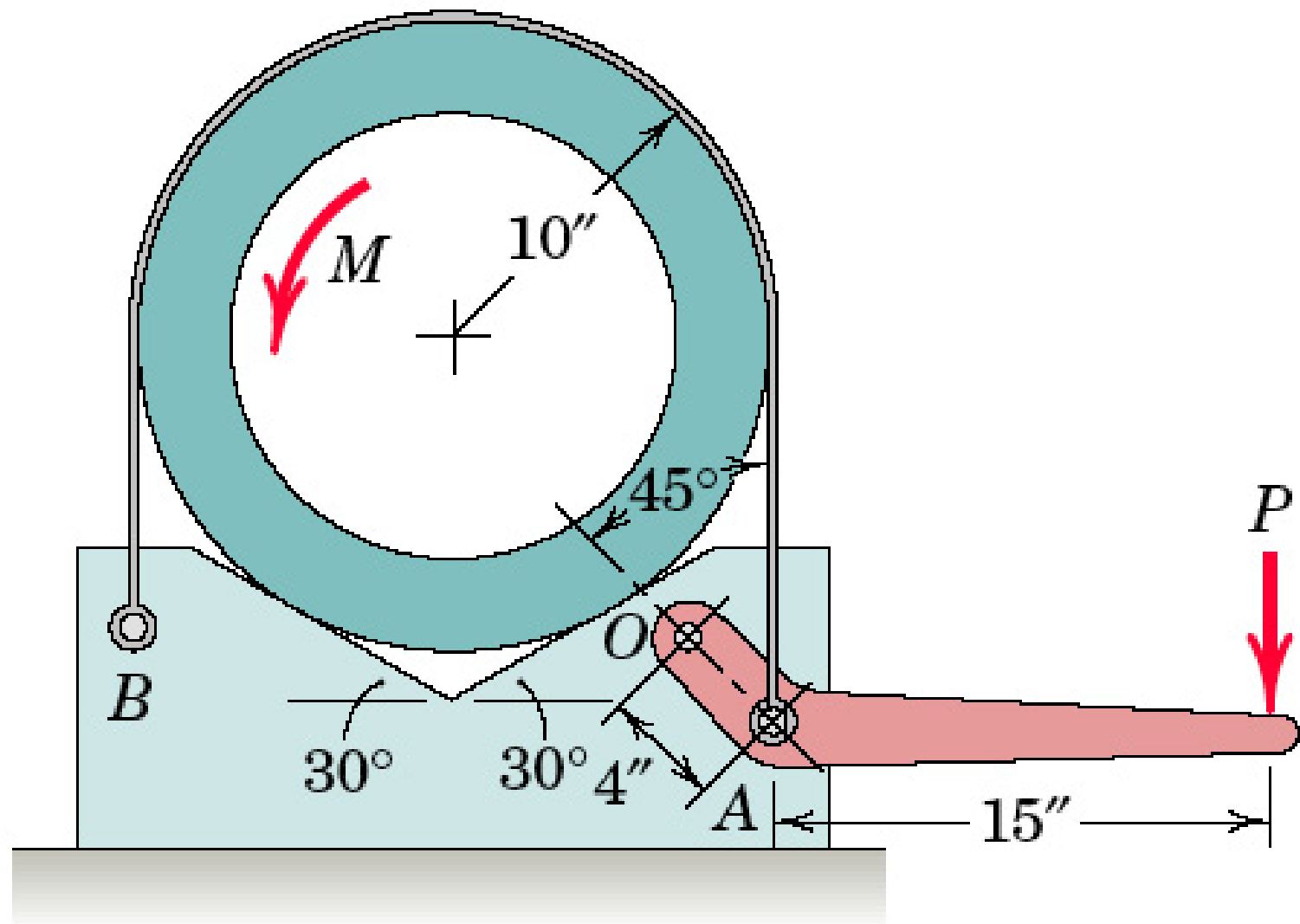
Problem 6/108



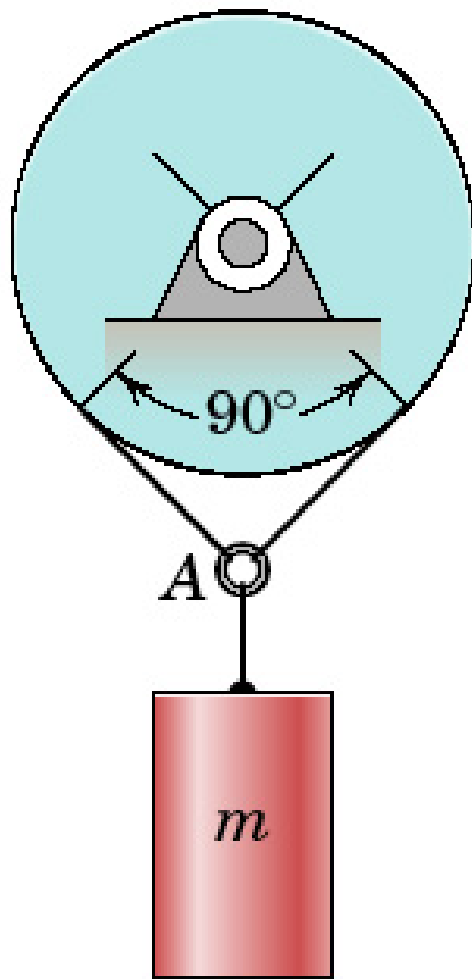
Problem 6/109



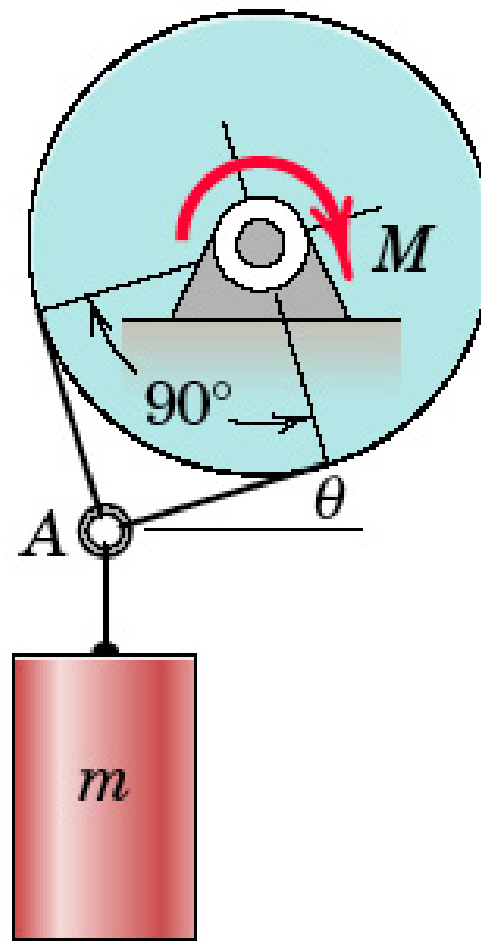
Problem 6/11



Problem 6/110

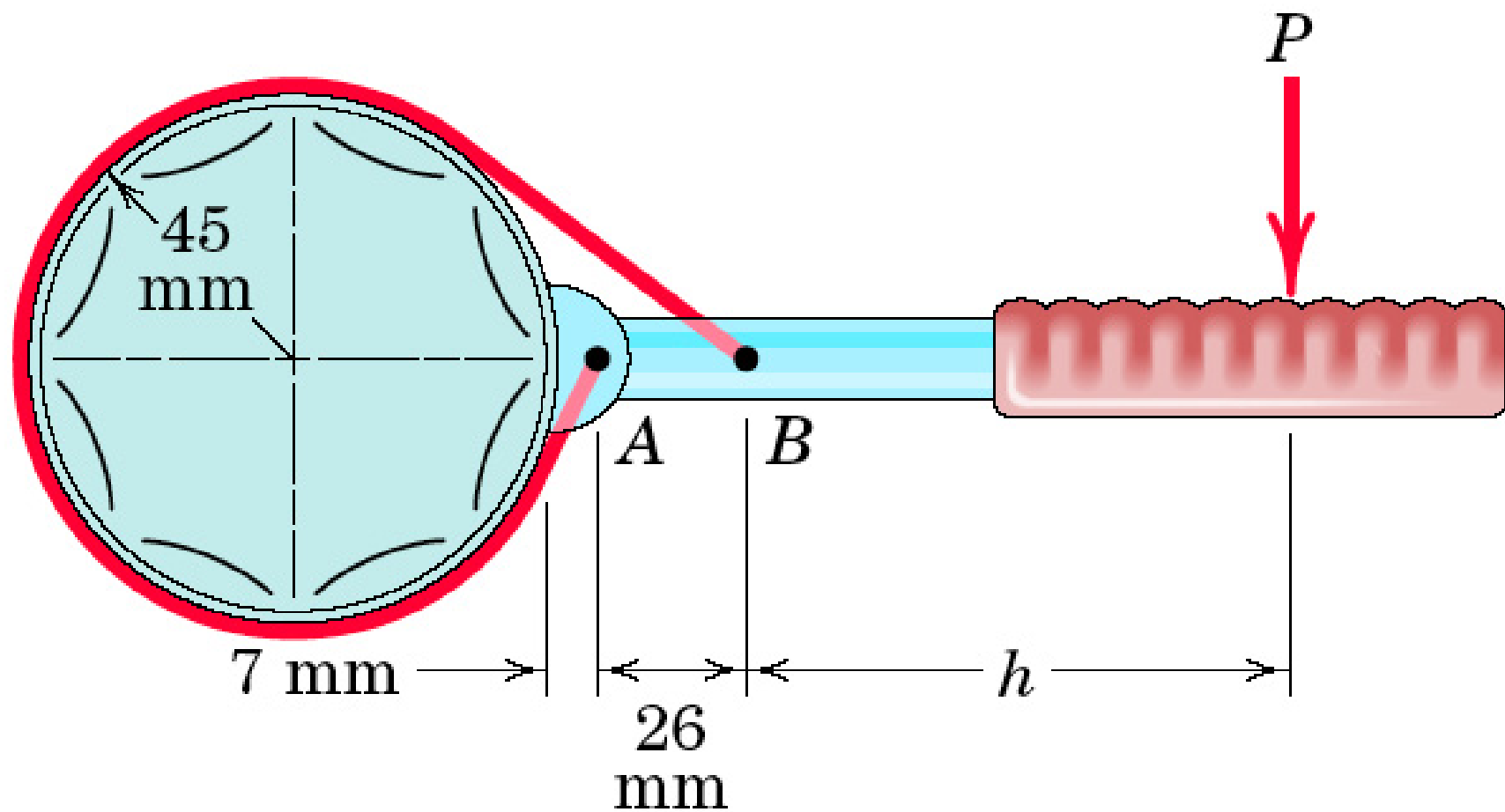


(a)

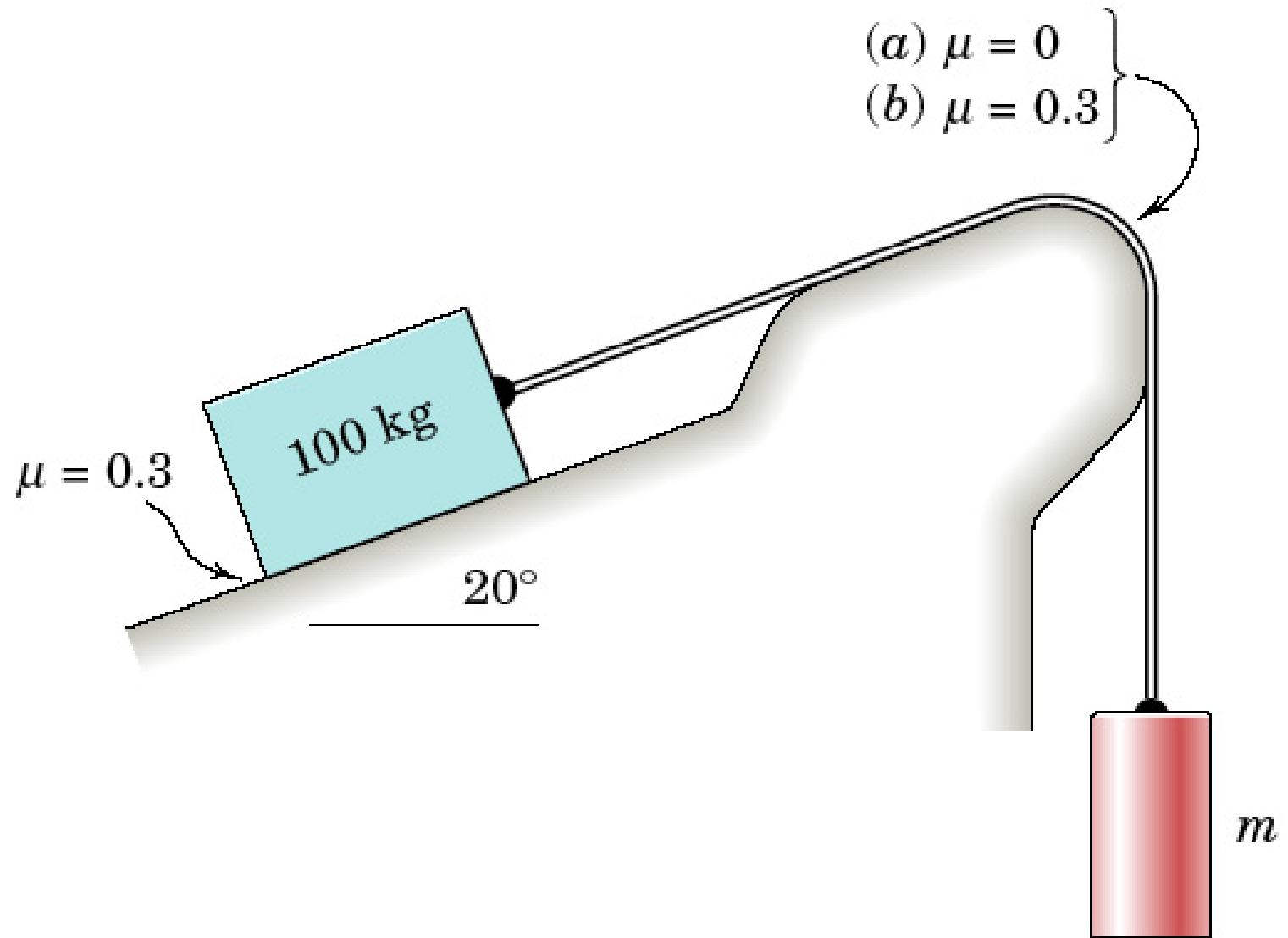


(b)

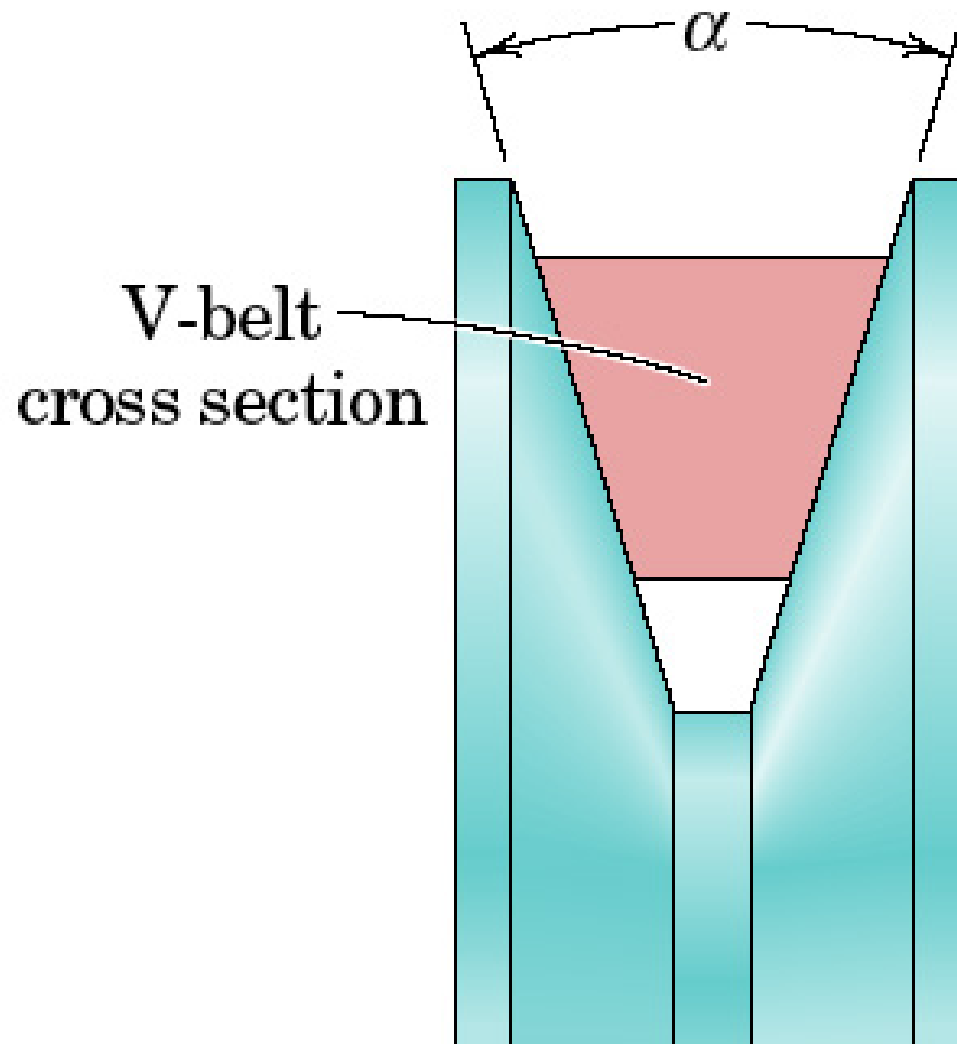
Problem 6/111



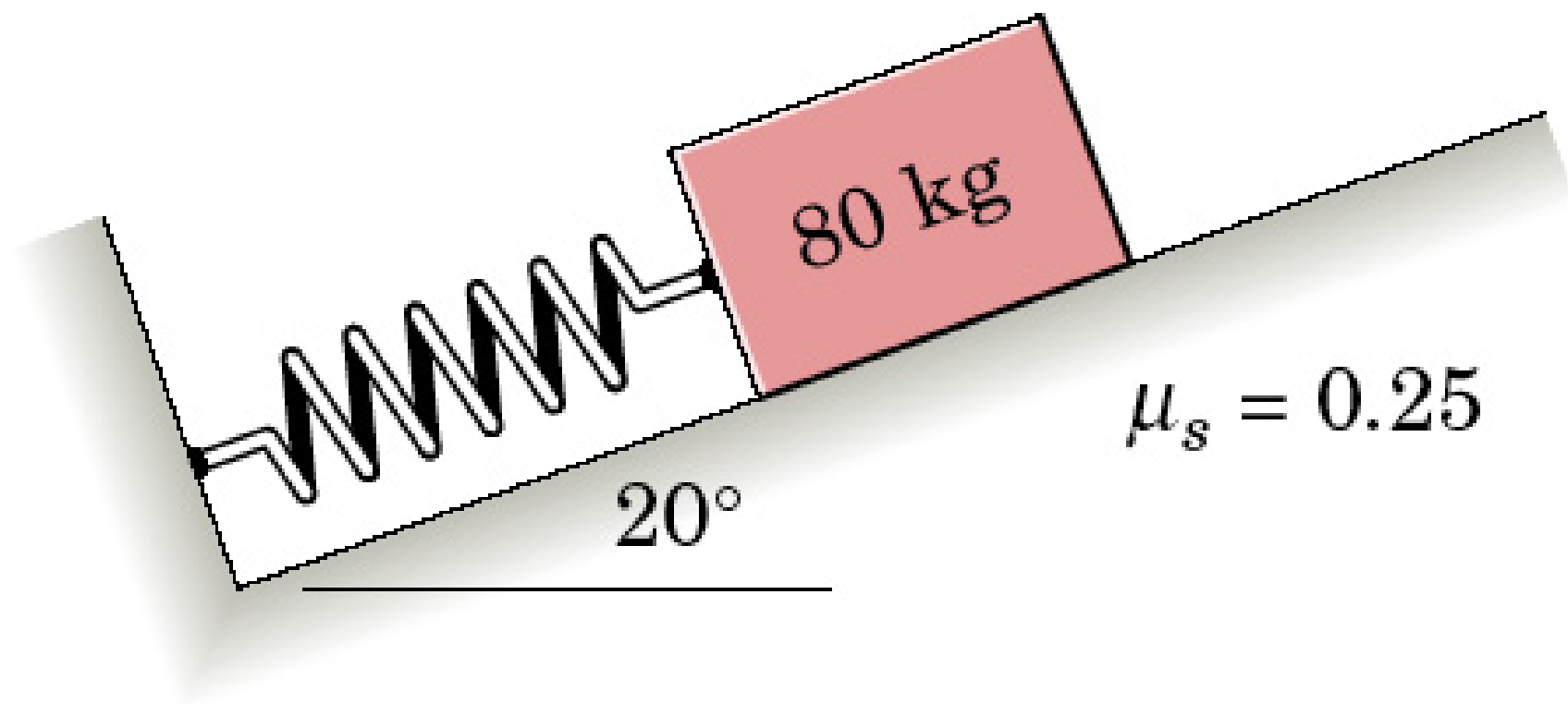
Problem 6/112



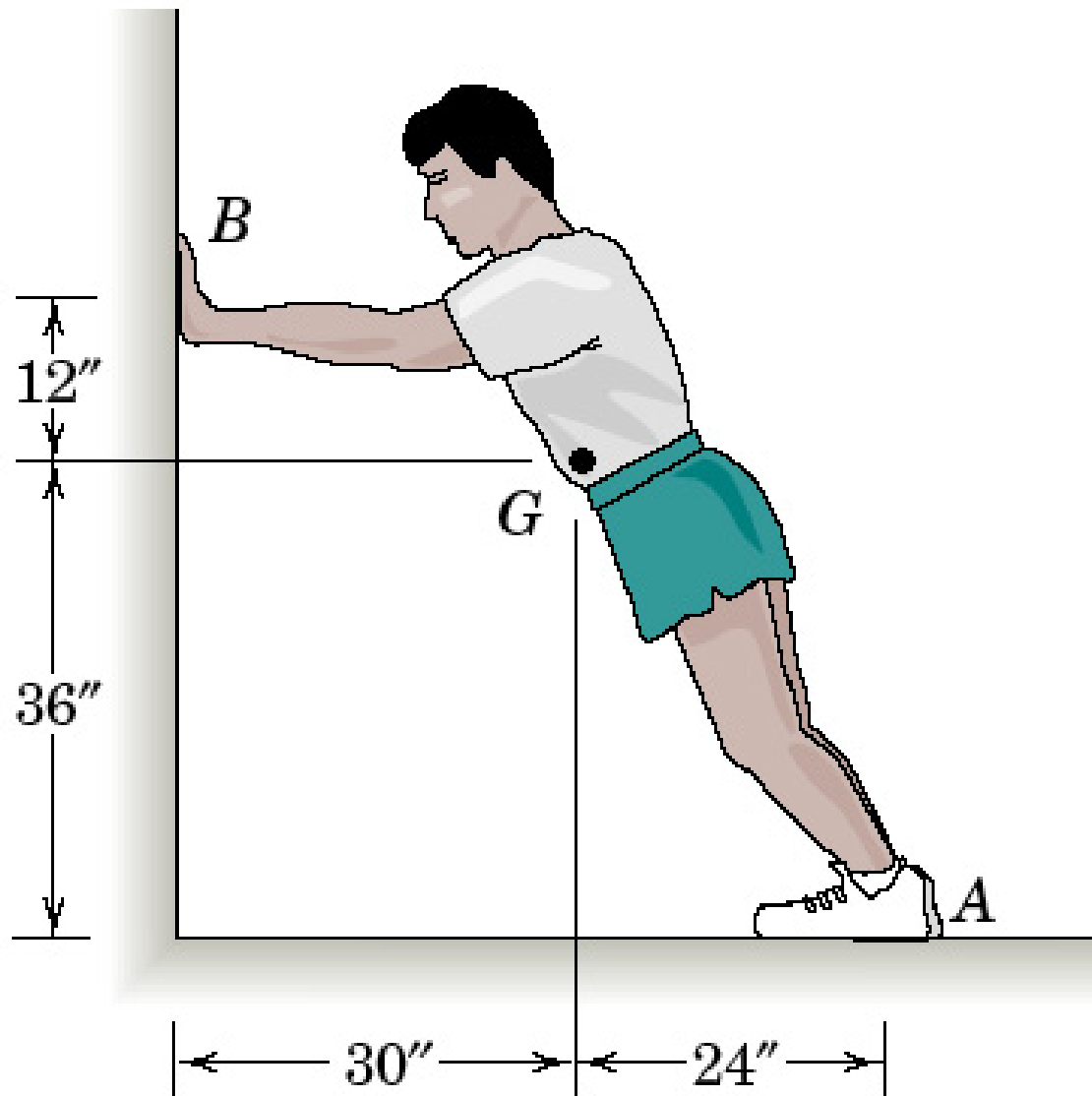
Problem 6/113



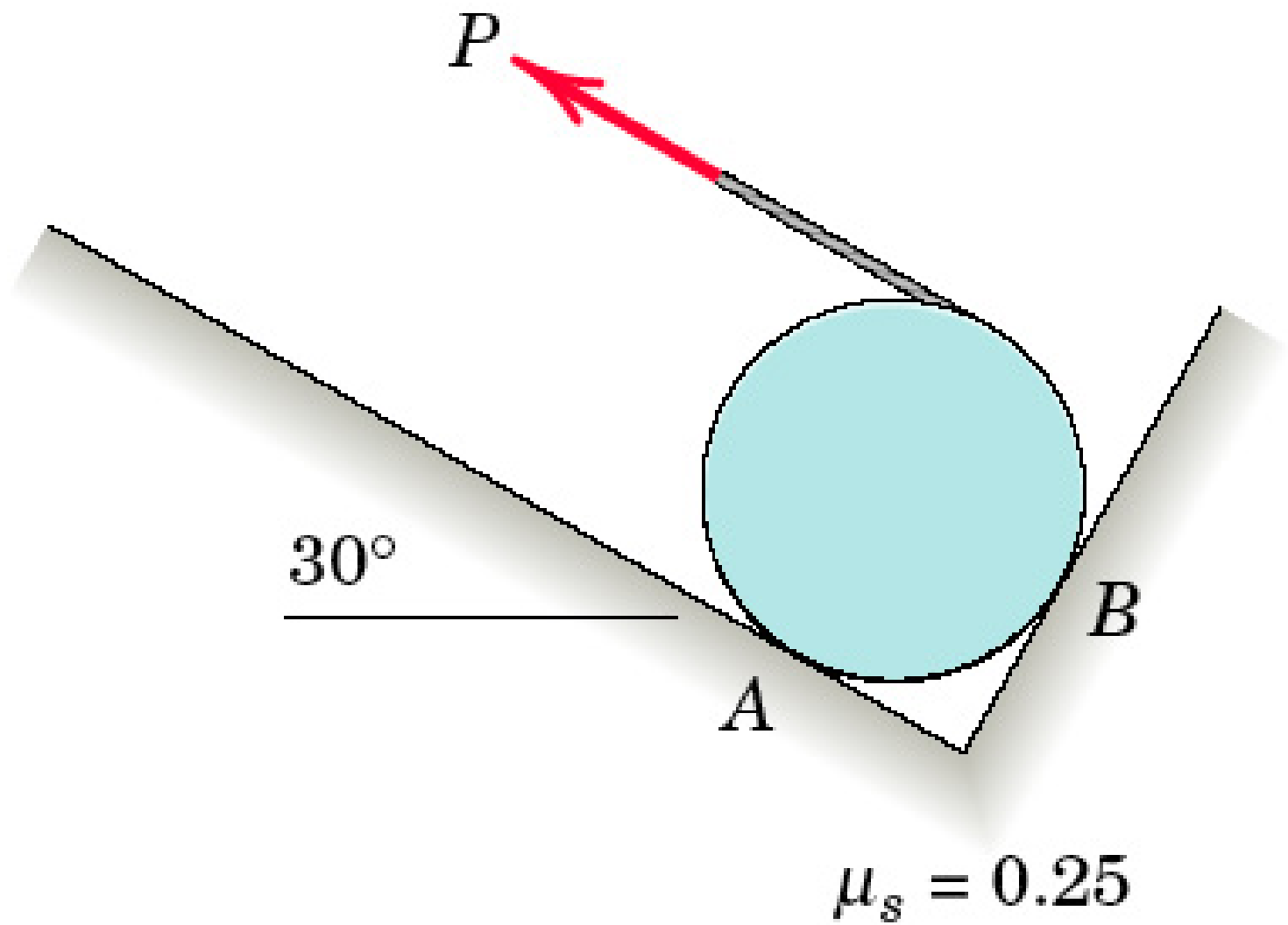
Problem 6/114



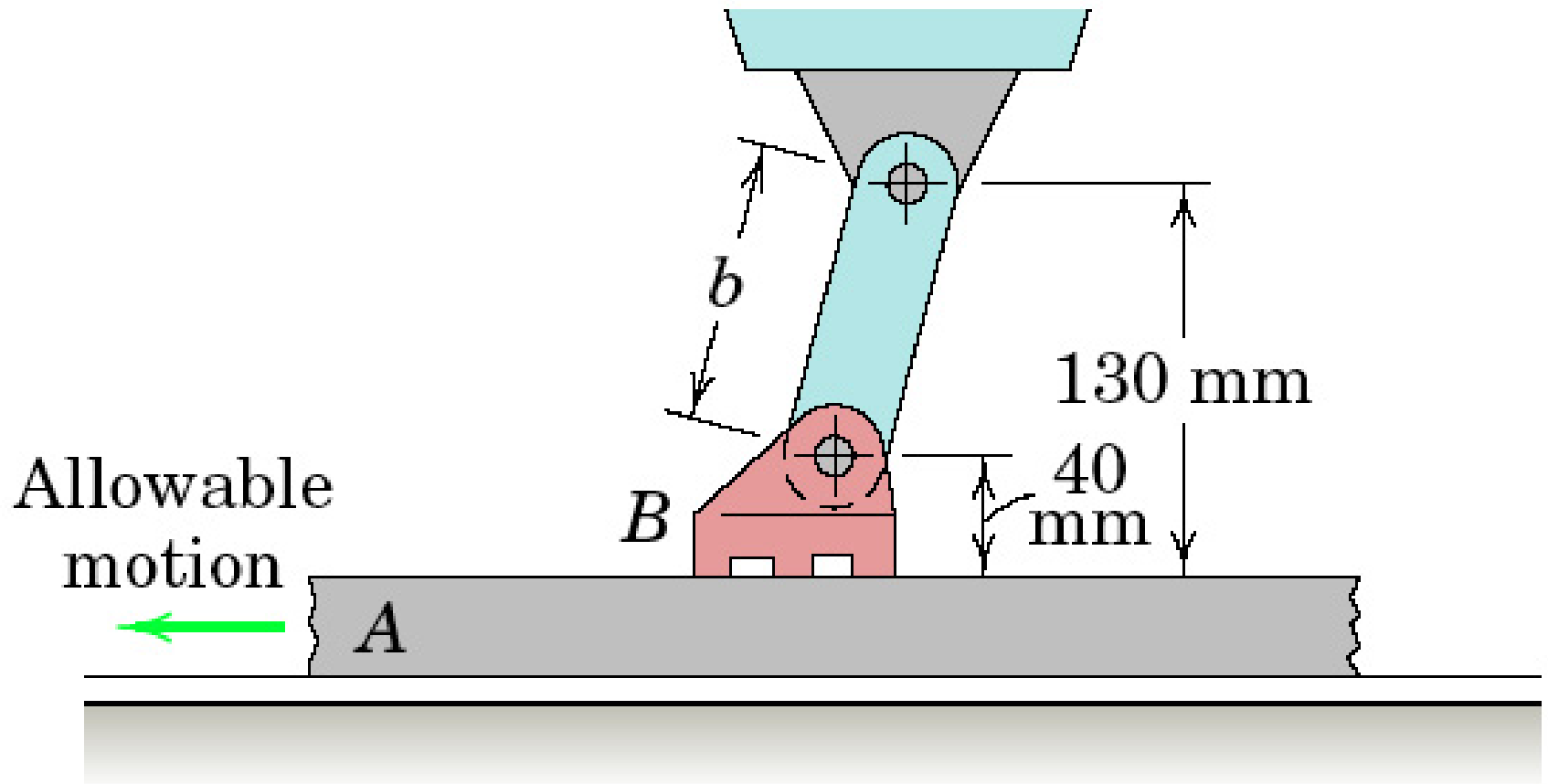
Problem 6/115



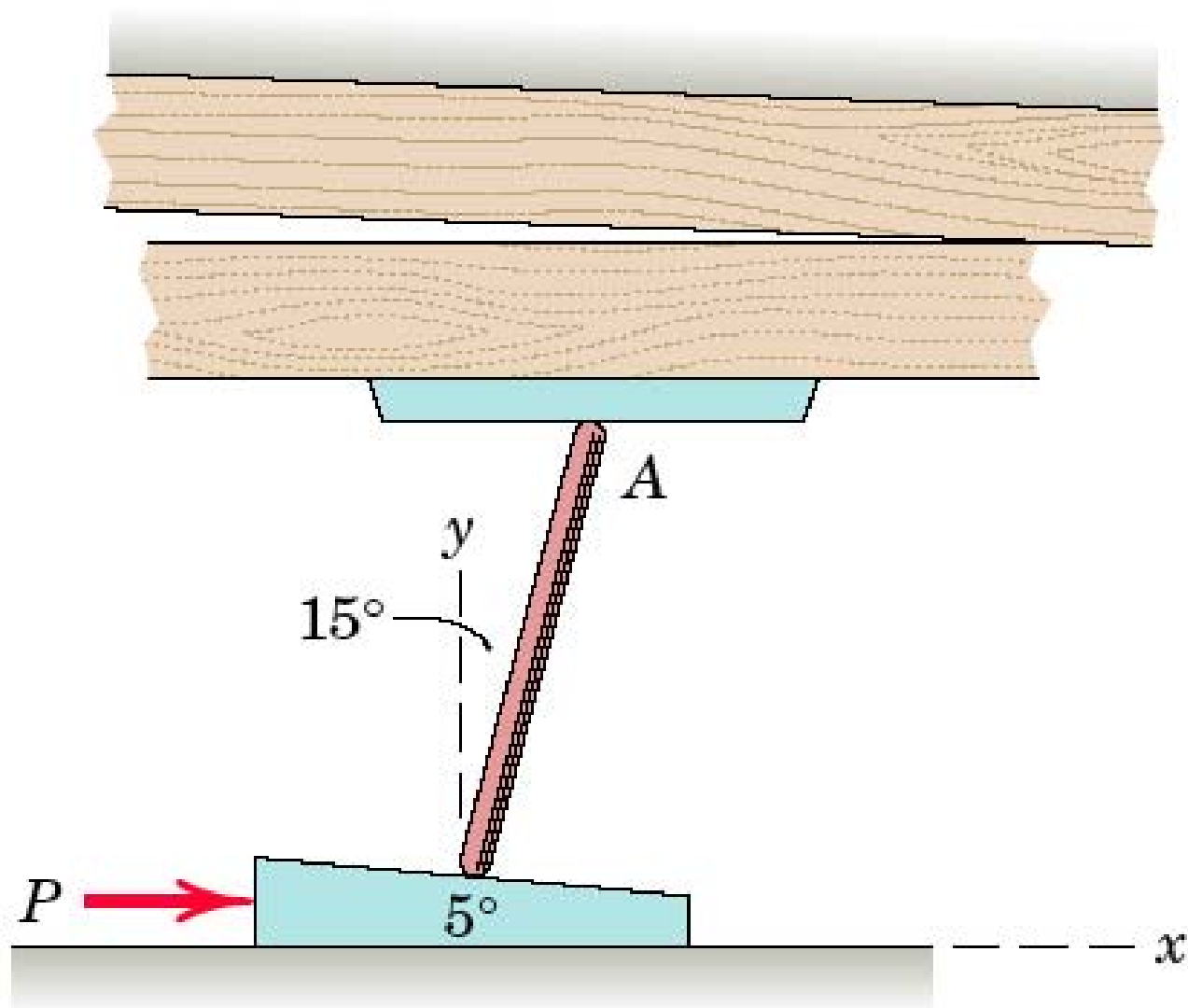
Problem 6/116



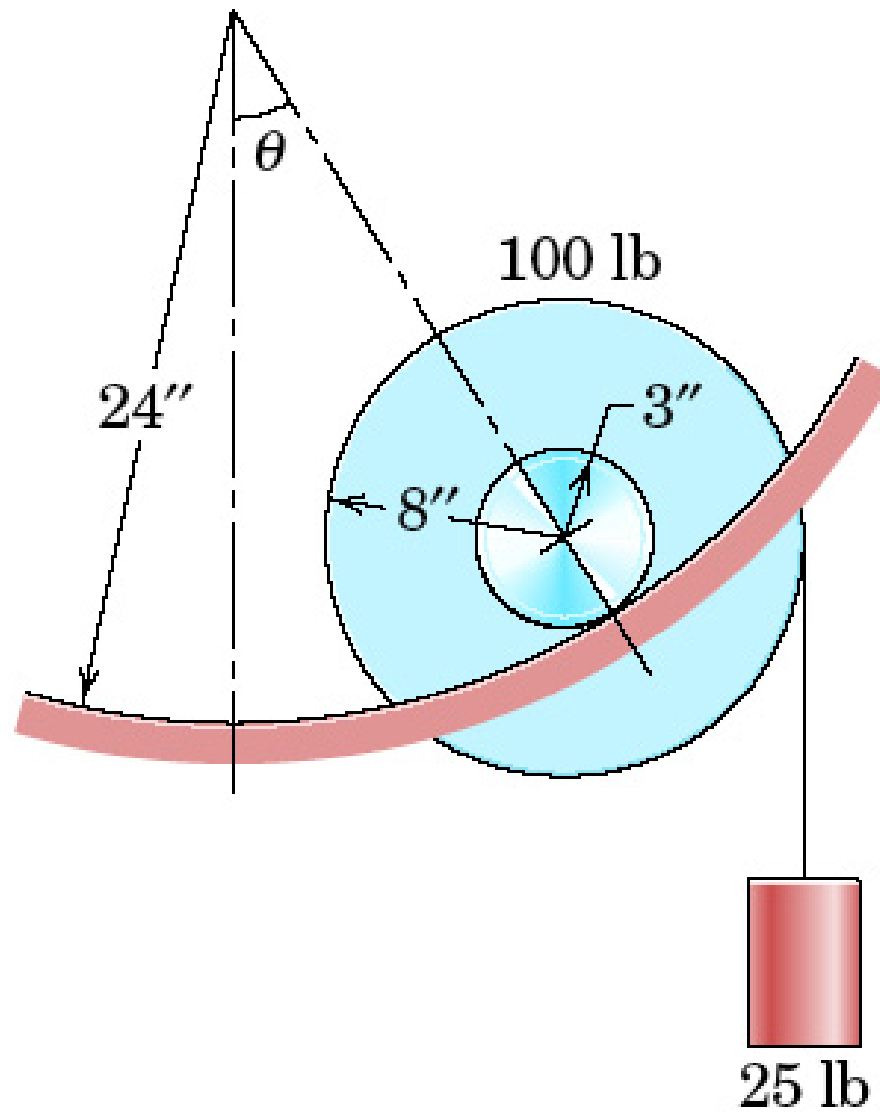
Problem 6/117



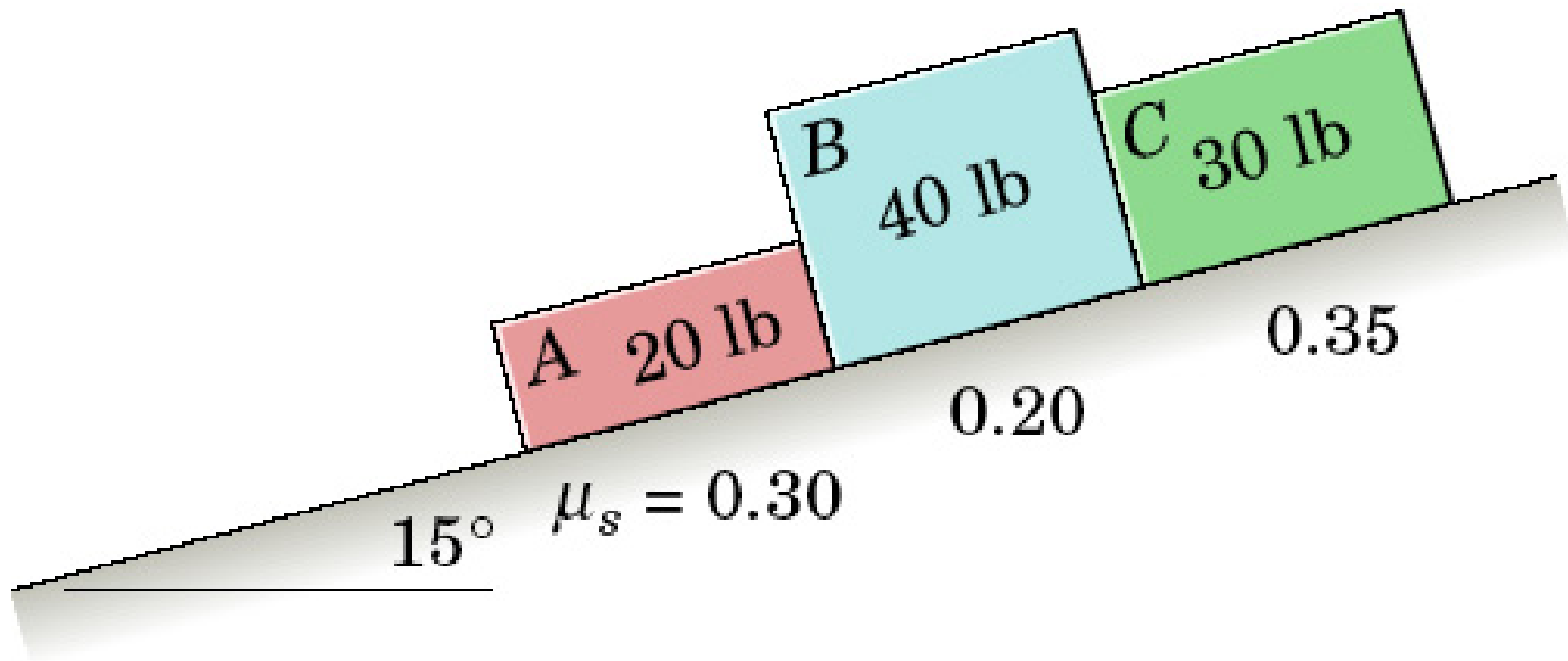
Problem 6/118



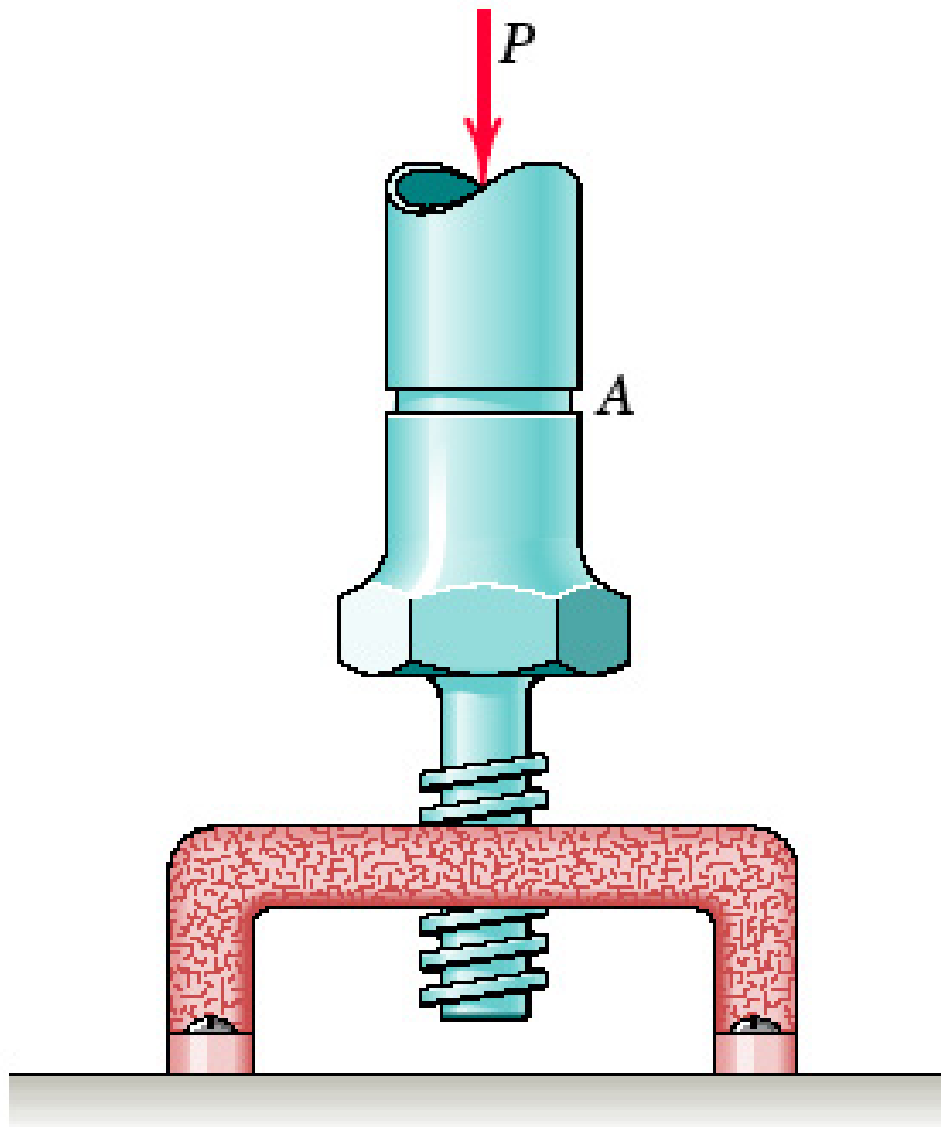
Problem 6/119



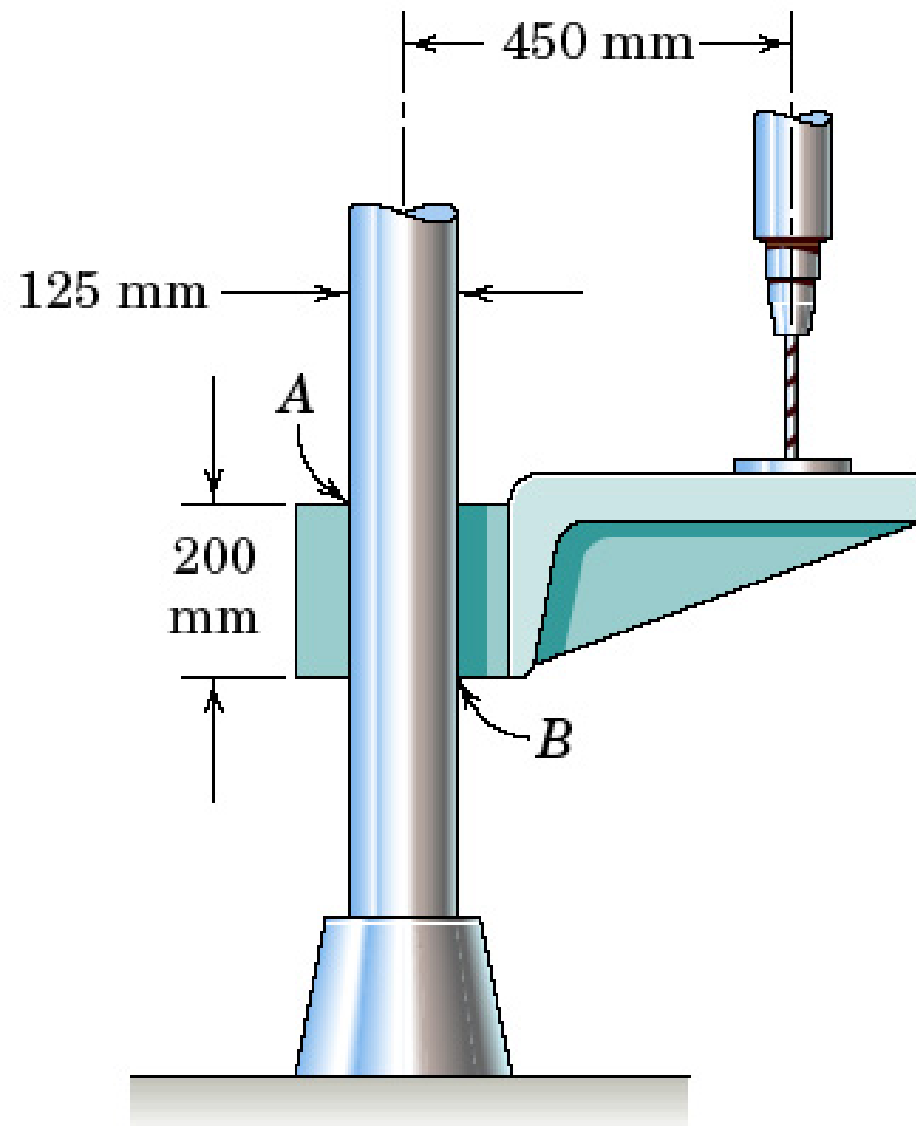
Problem 6/12



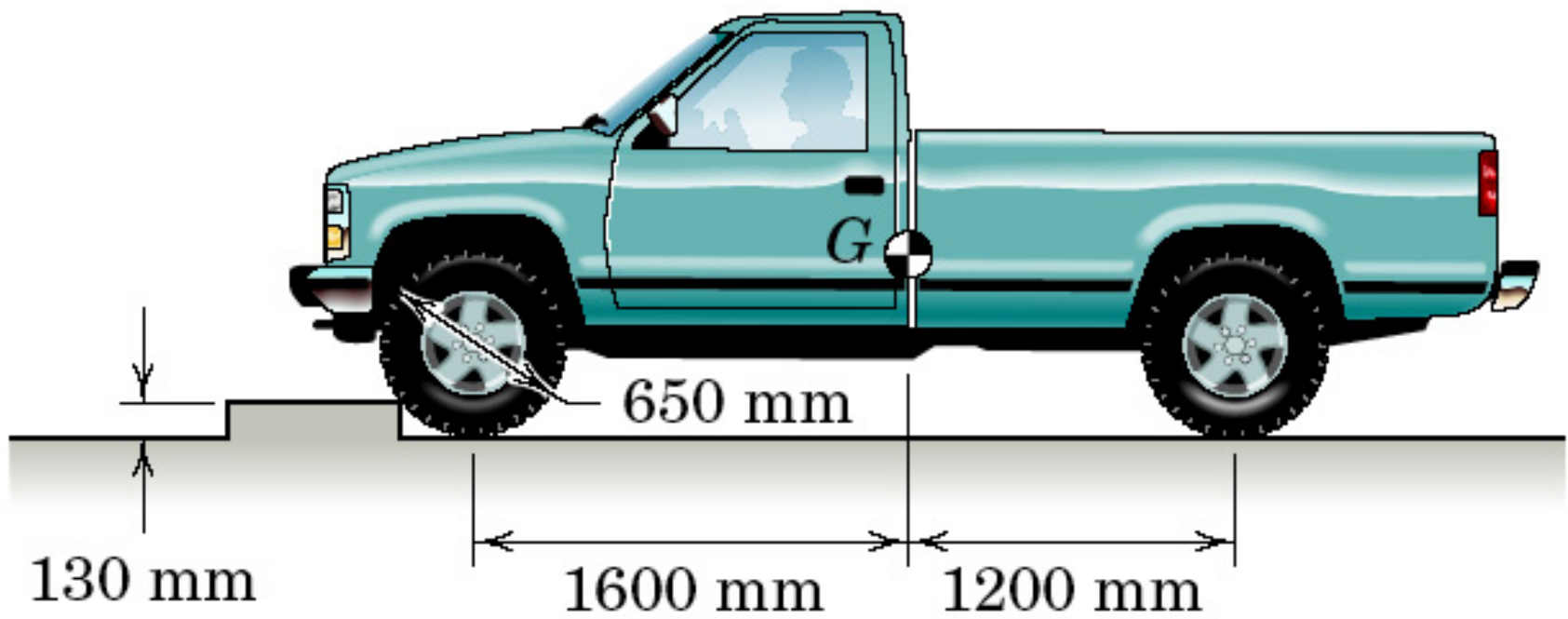
Problem 6/120



Problem 6/121

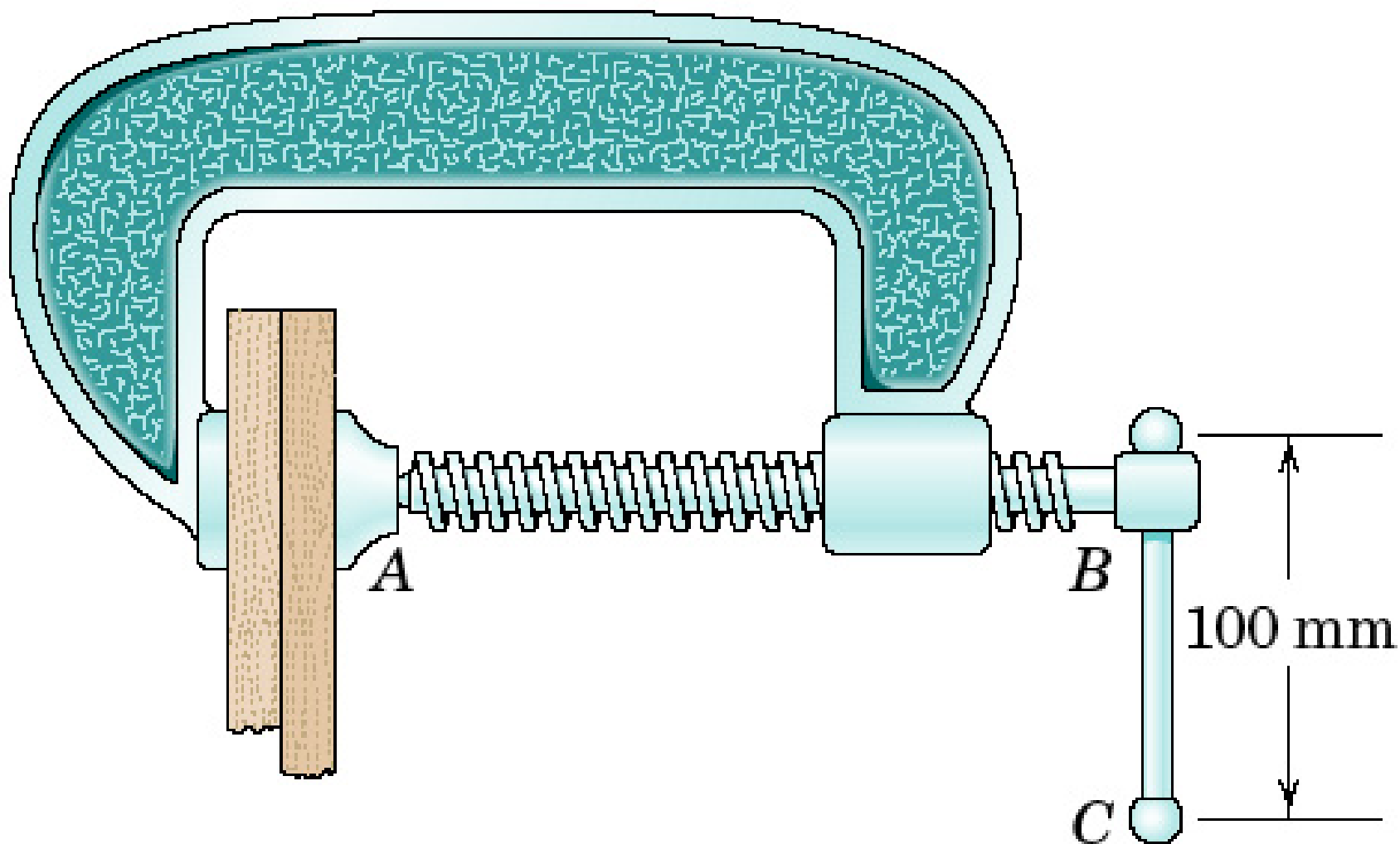


Problem 6/122

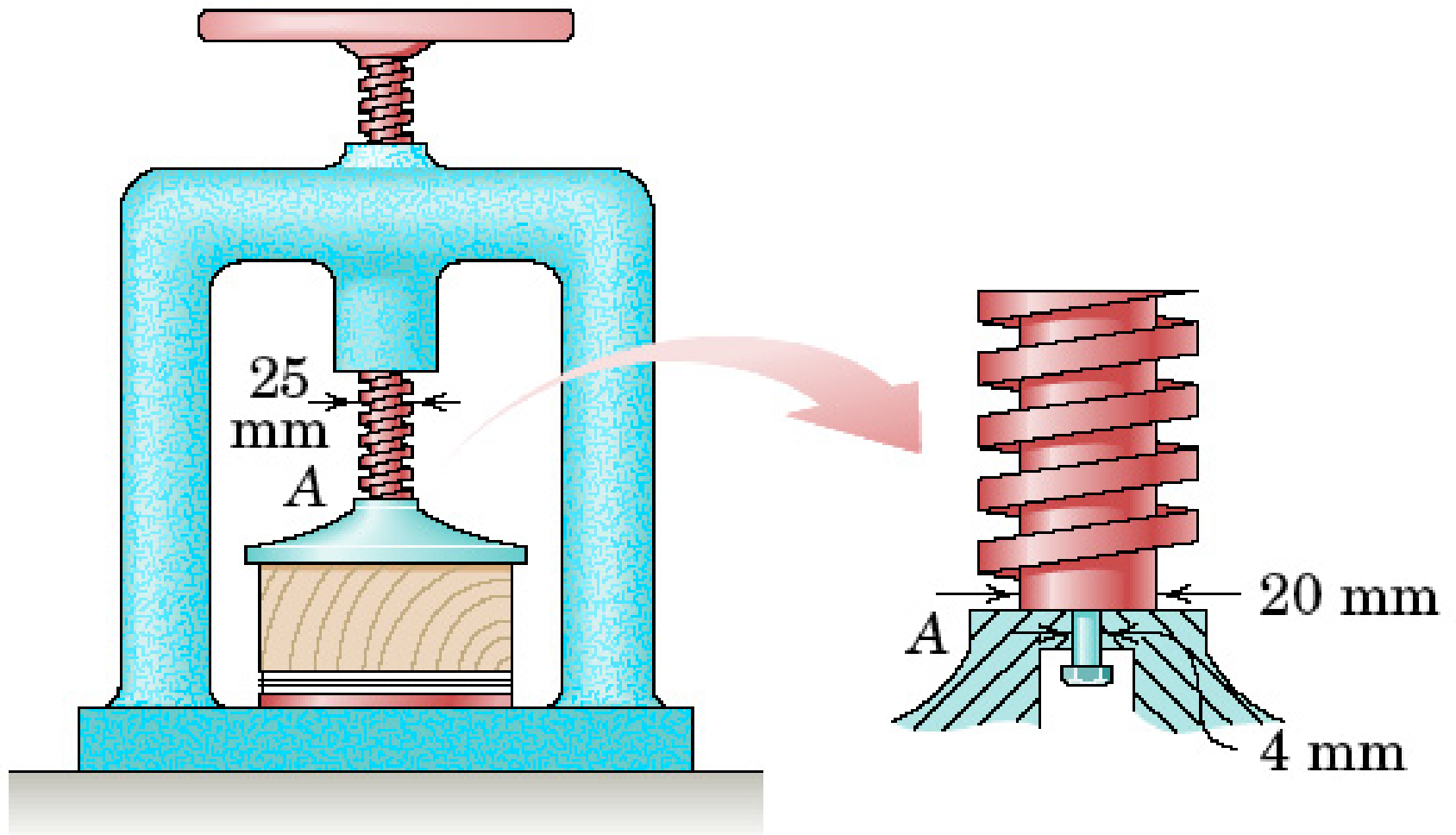


Problem 6/123

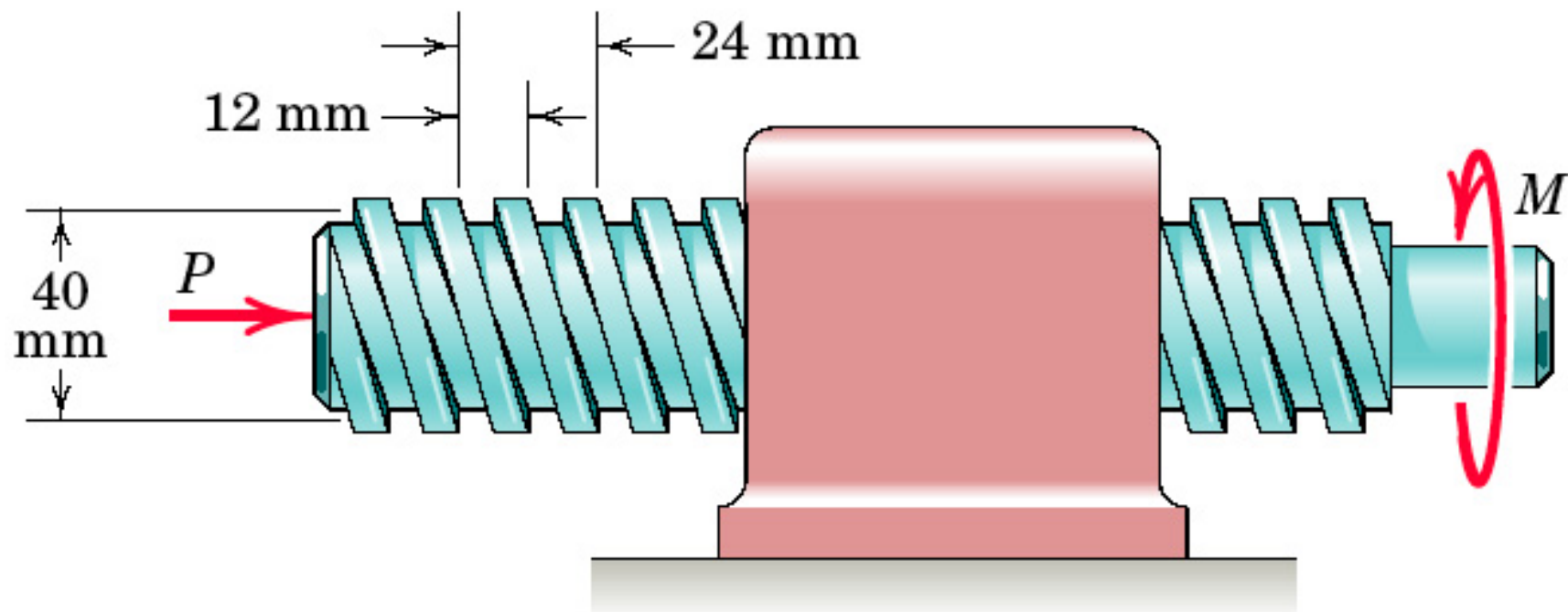
P_06_123



Problem 6/124

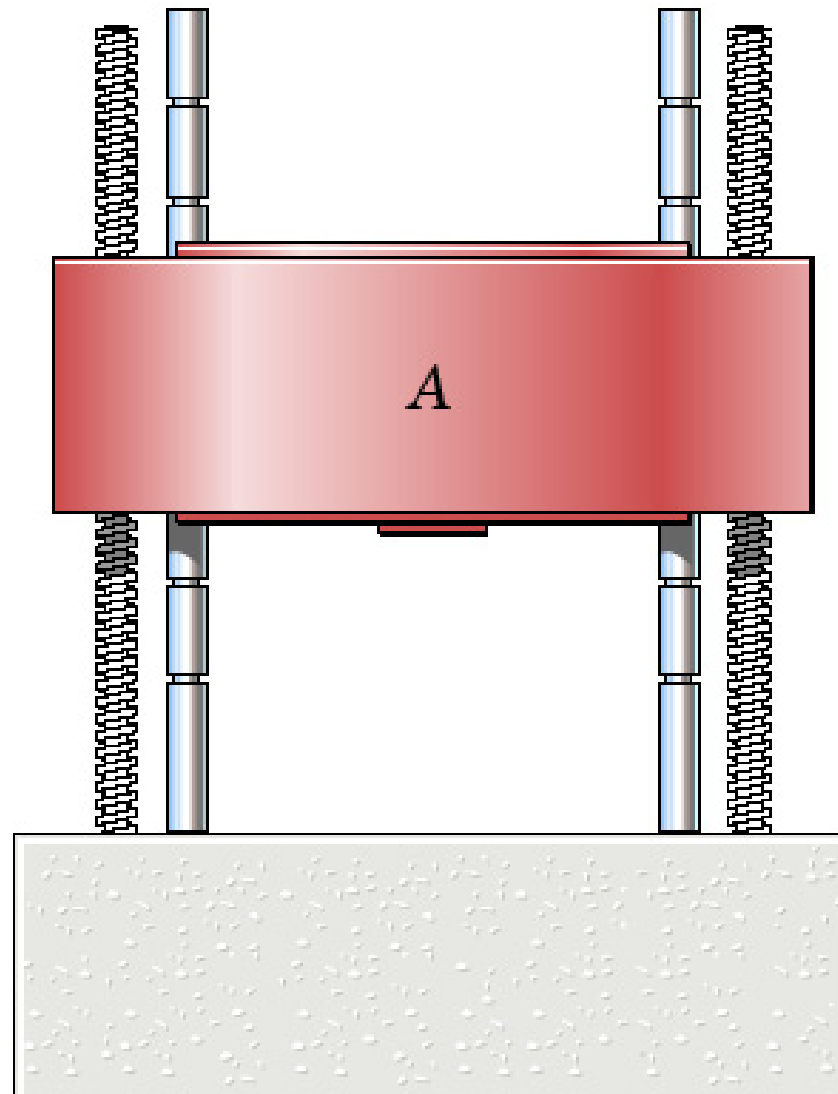


Problem 6/125

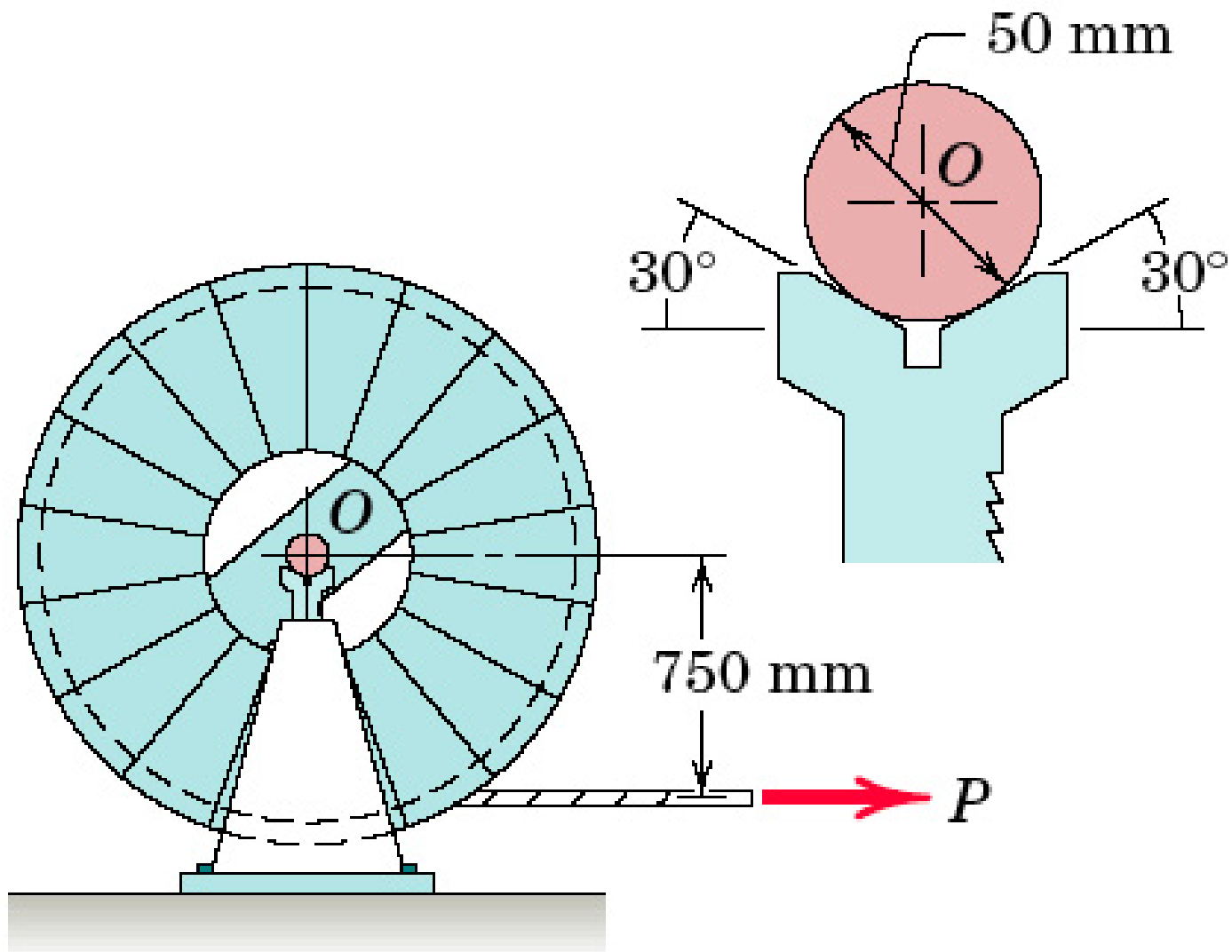


Problem 6/126

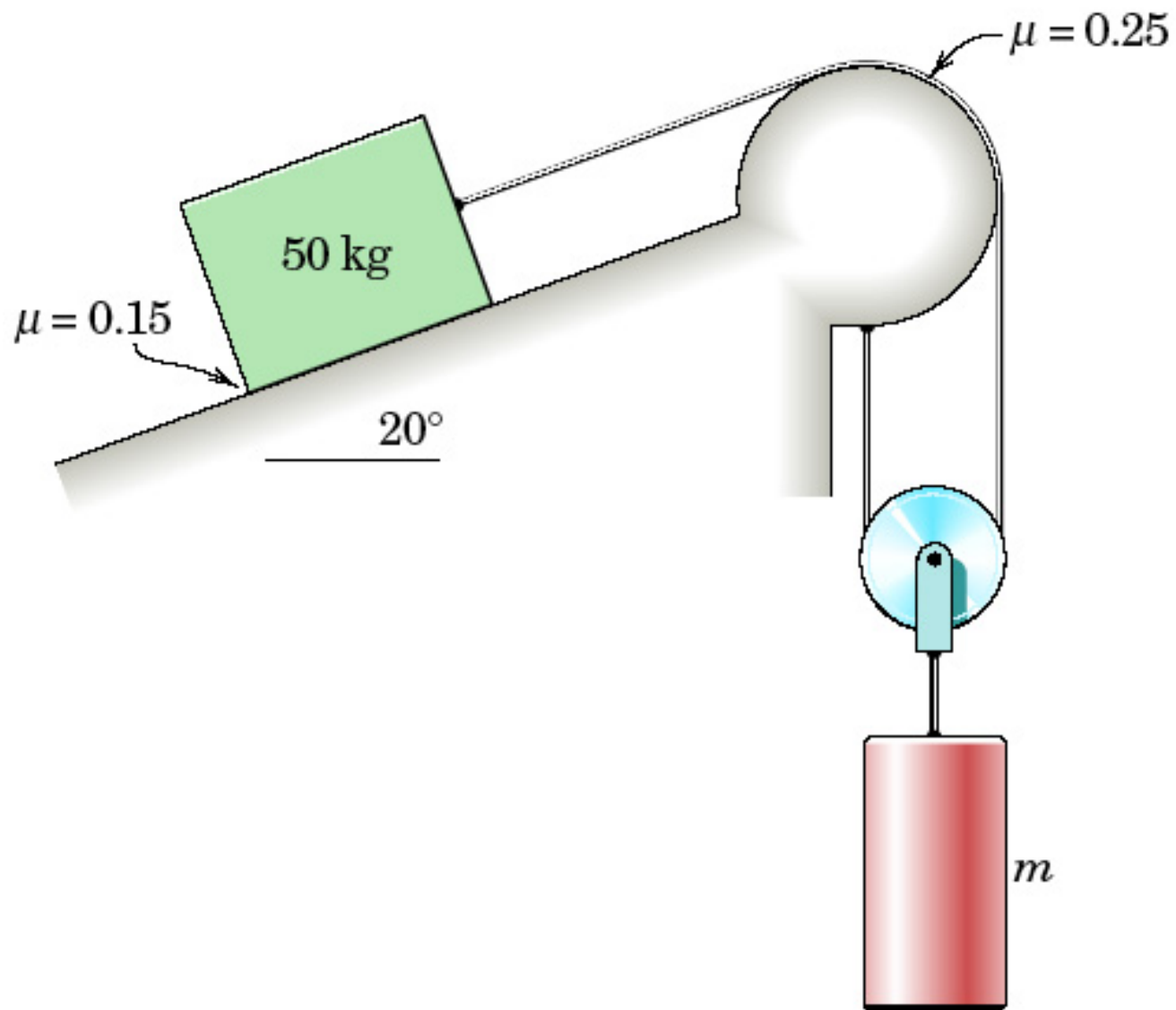
P_06_126



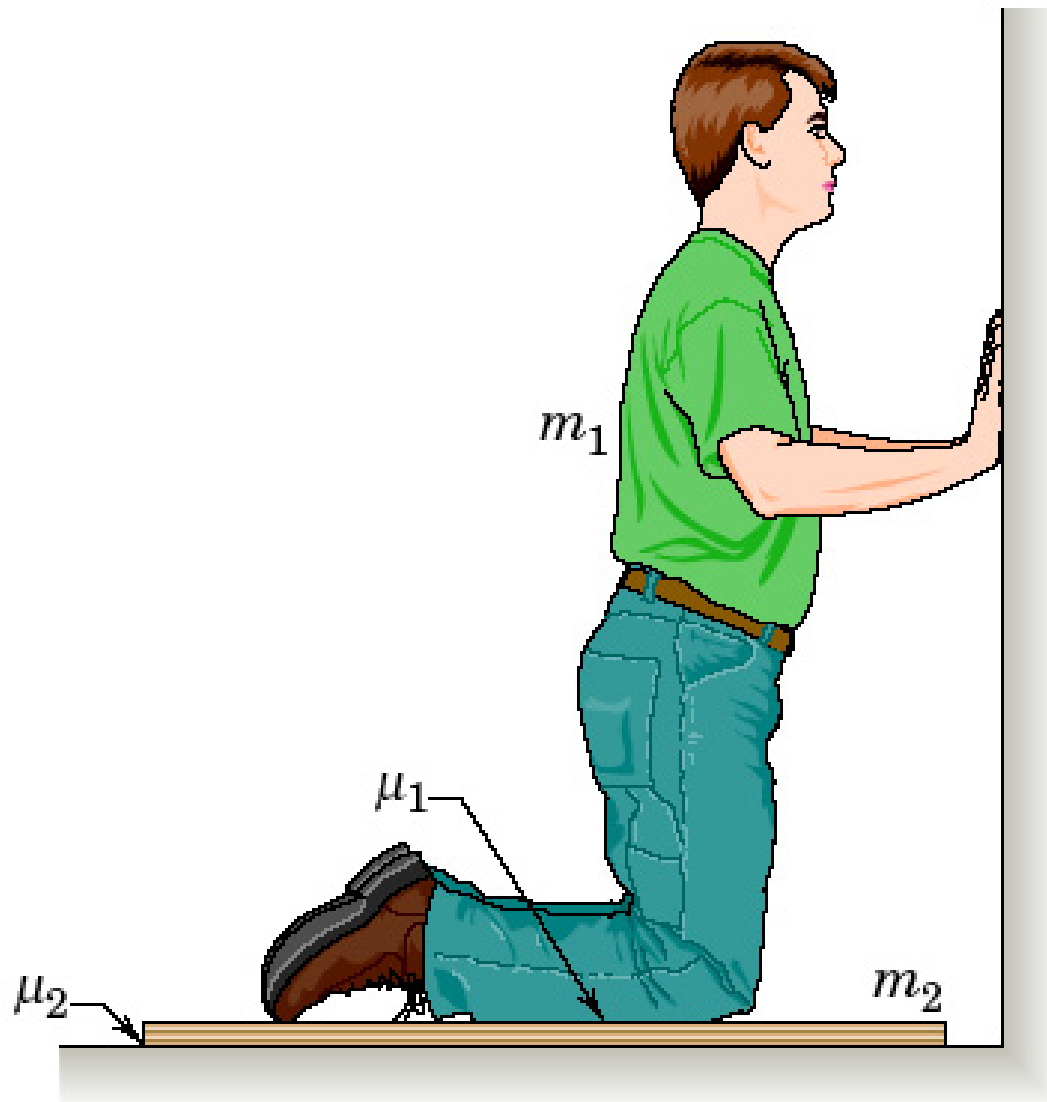
Problem 6/127



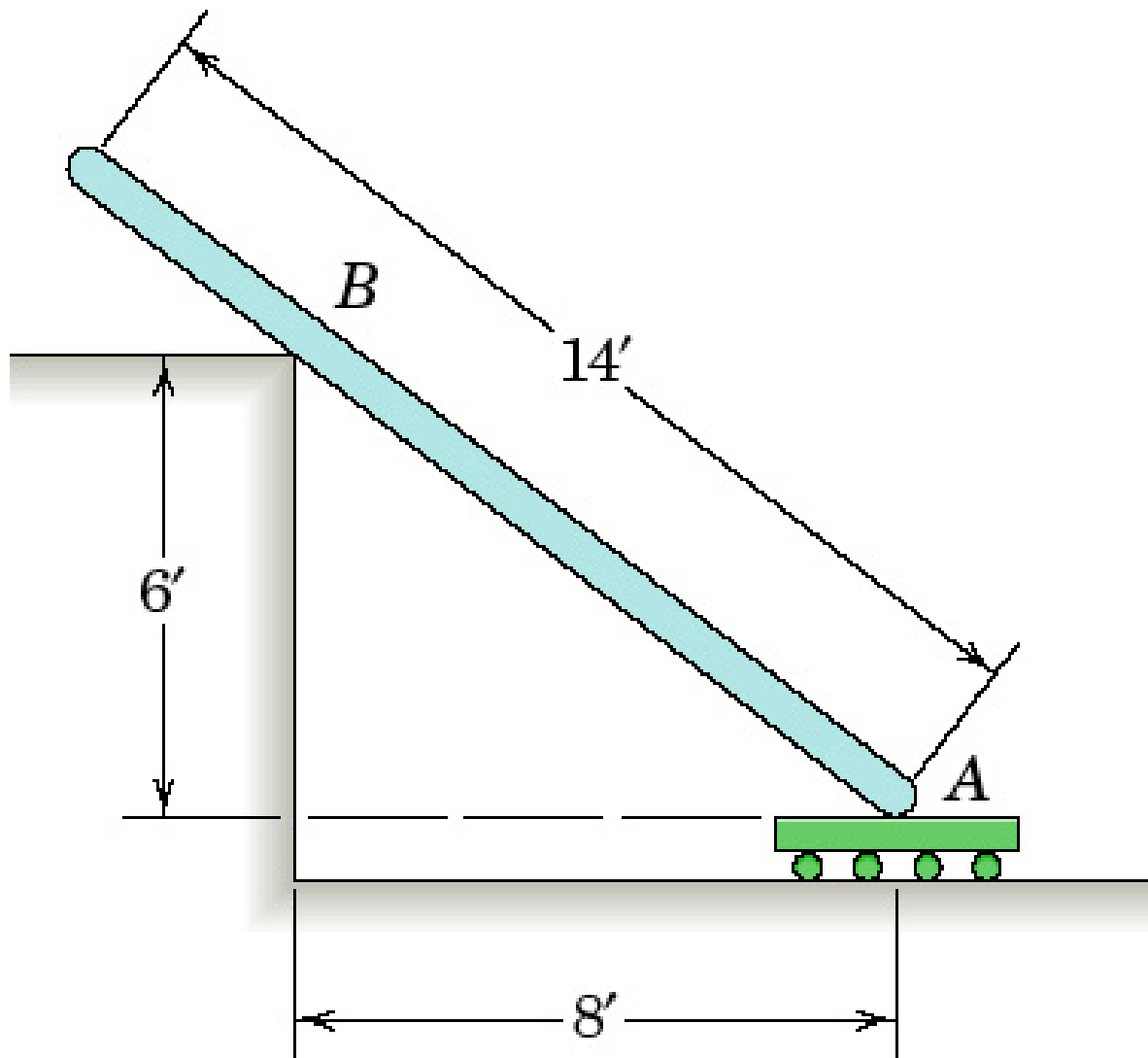
Problem 6/128



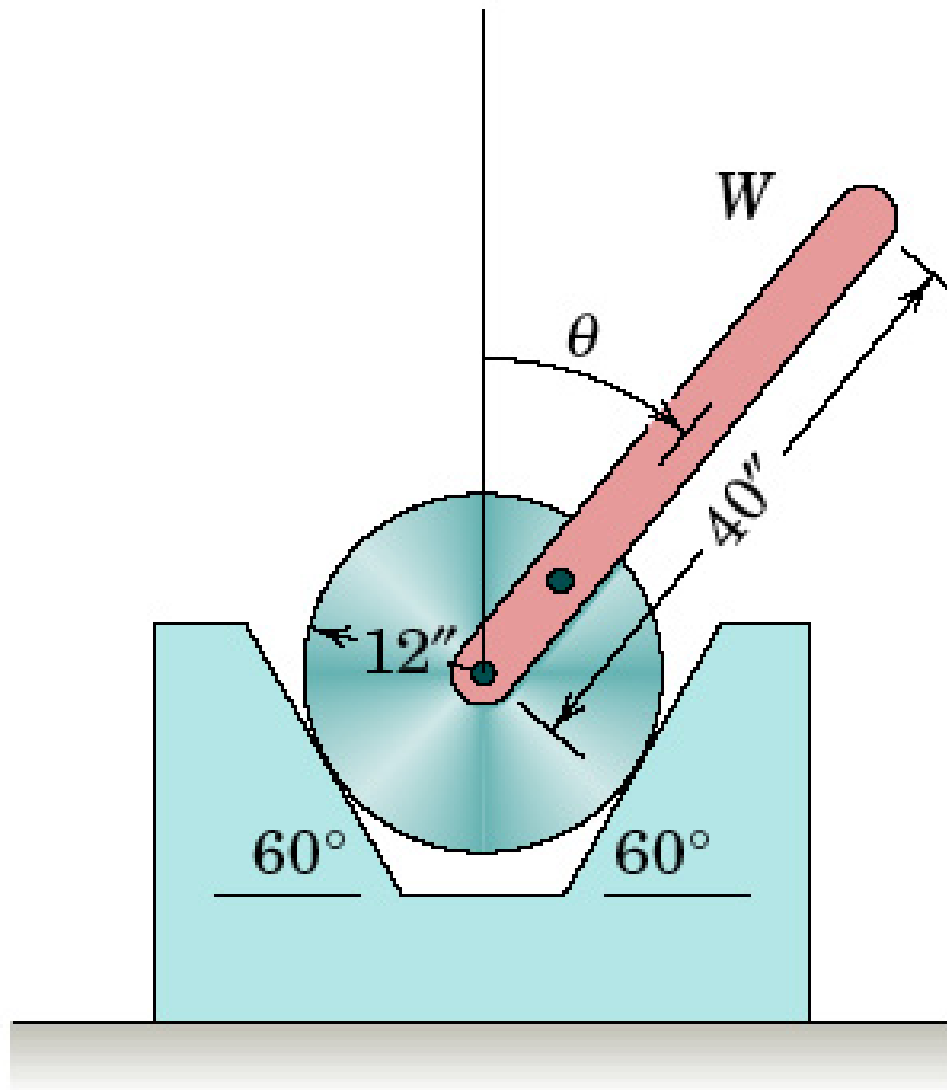
Problem 6/129



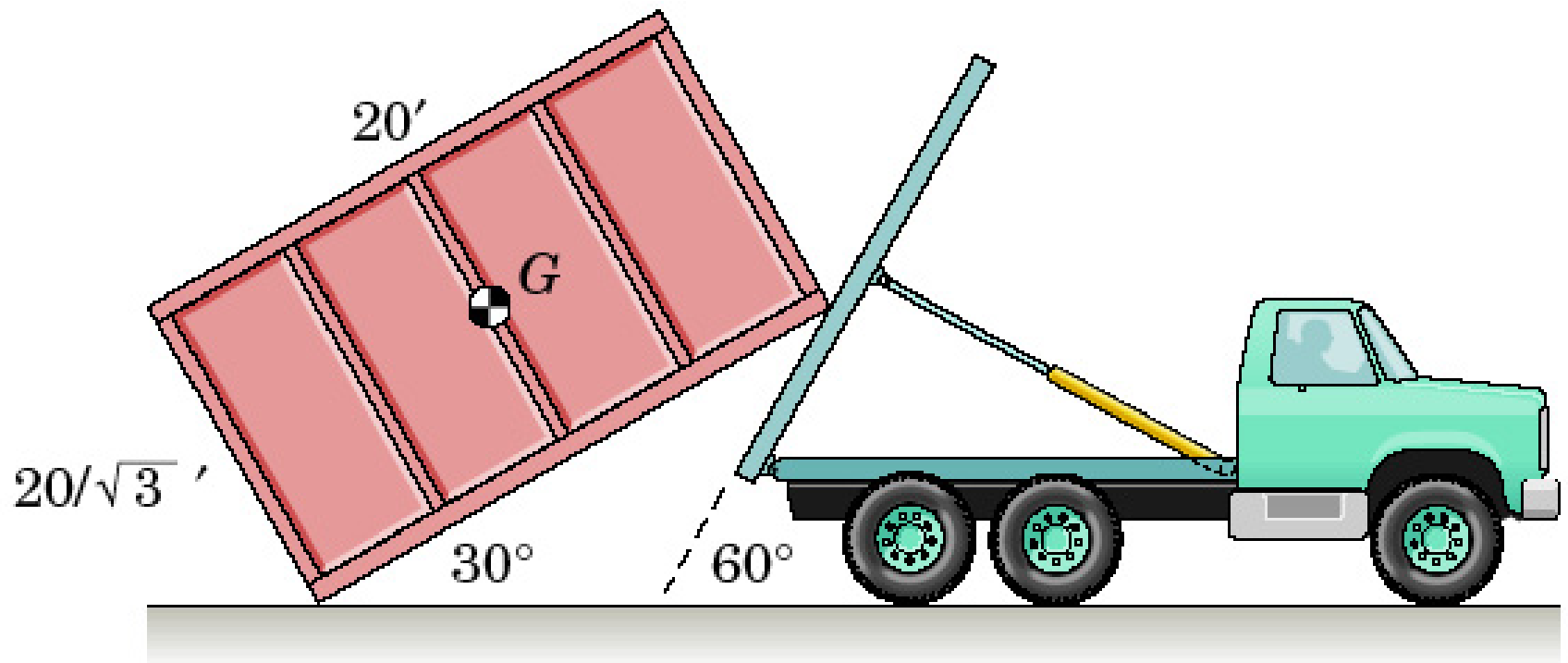
Problem 6/13



Problem 6/130

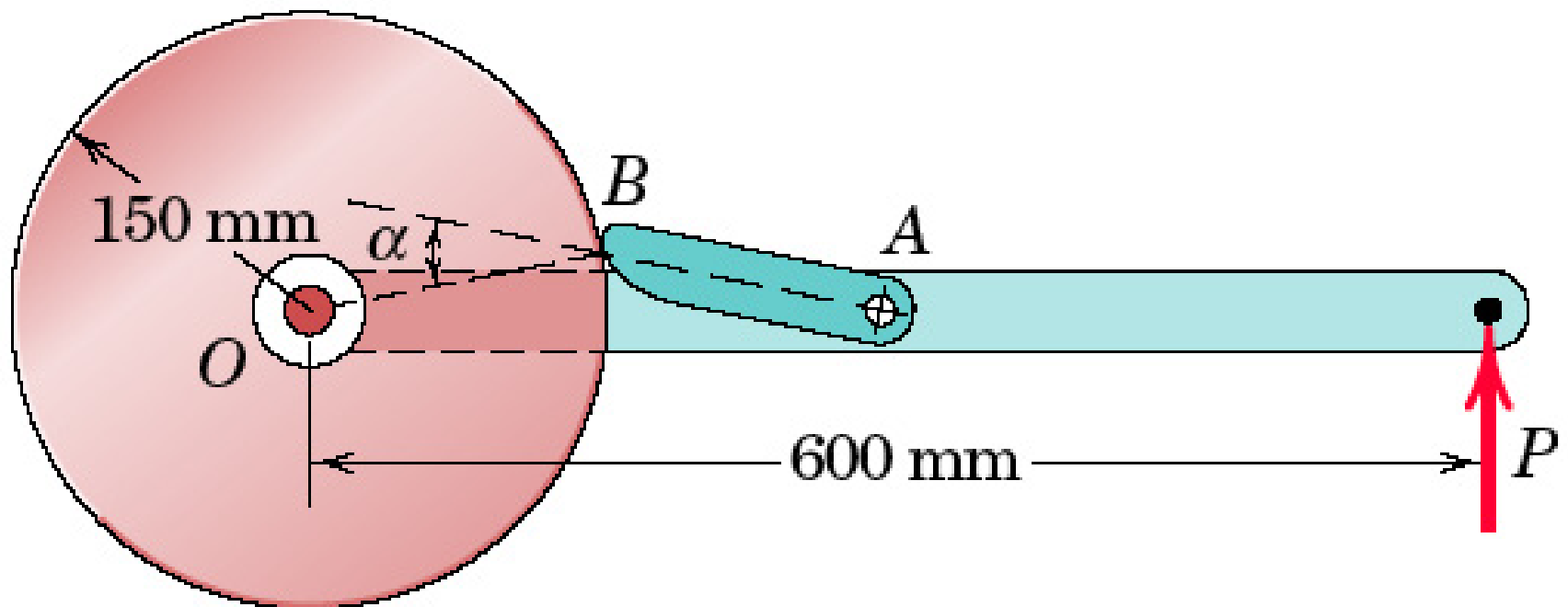


Problem 6/131



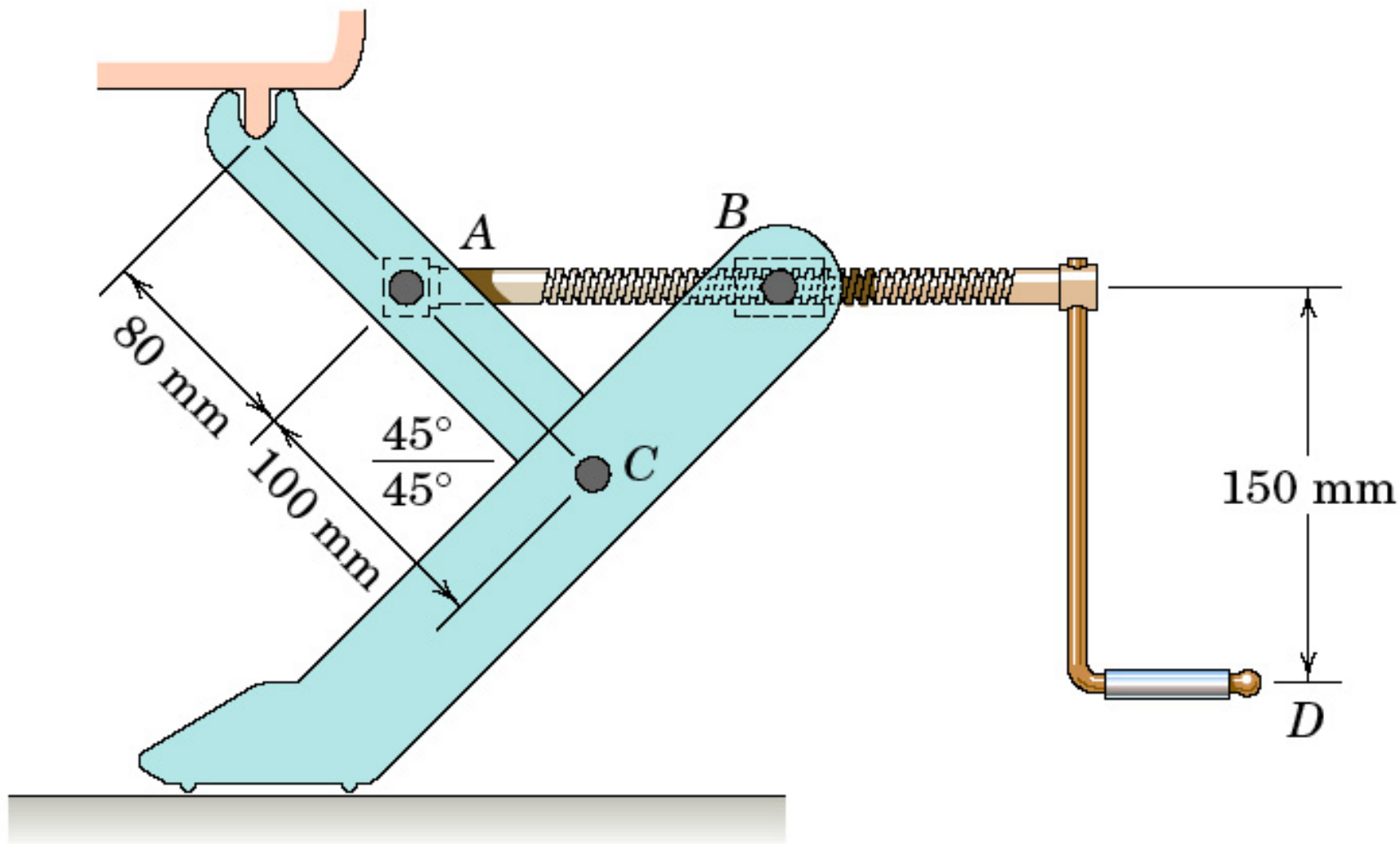
Problem 6/132

P_06_132

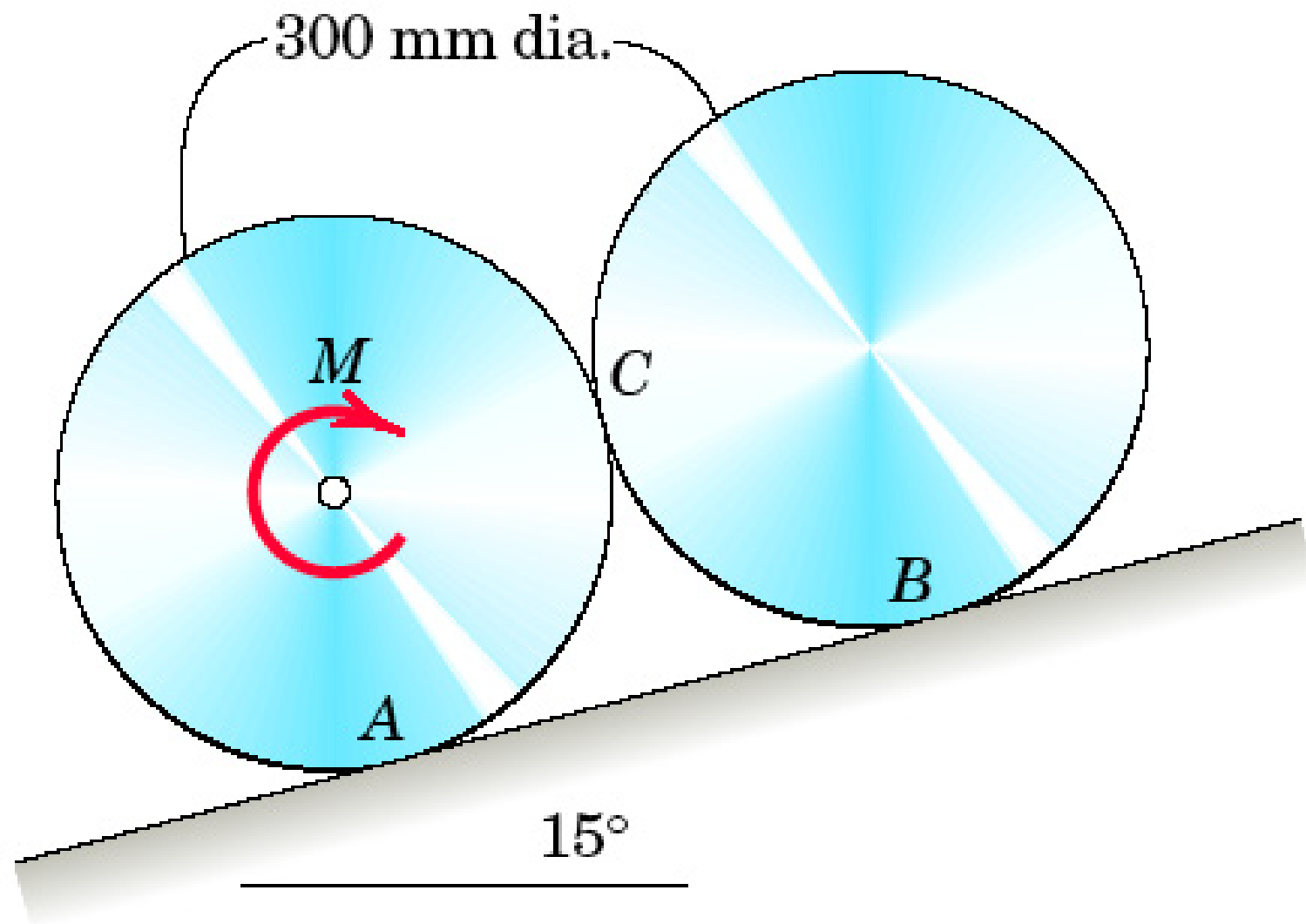


Problem 6/133

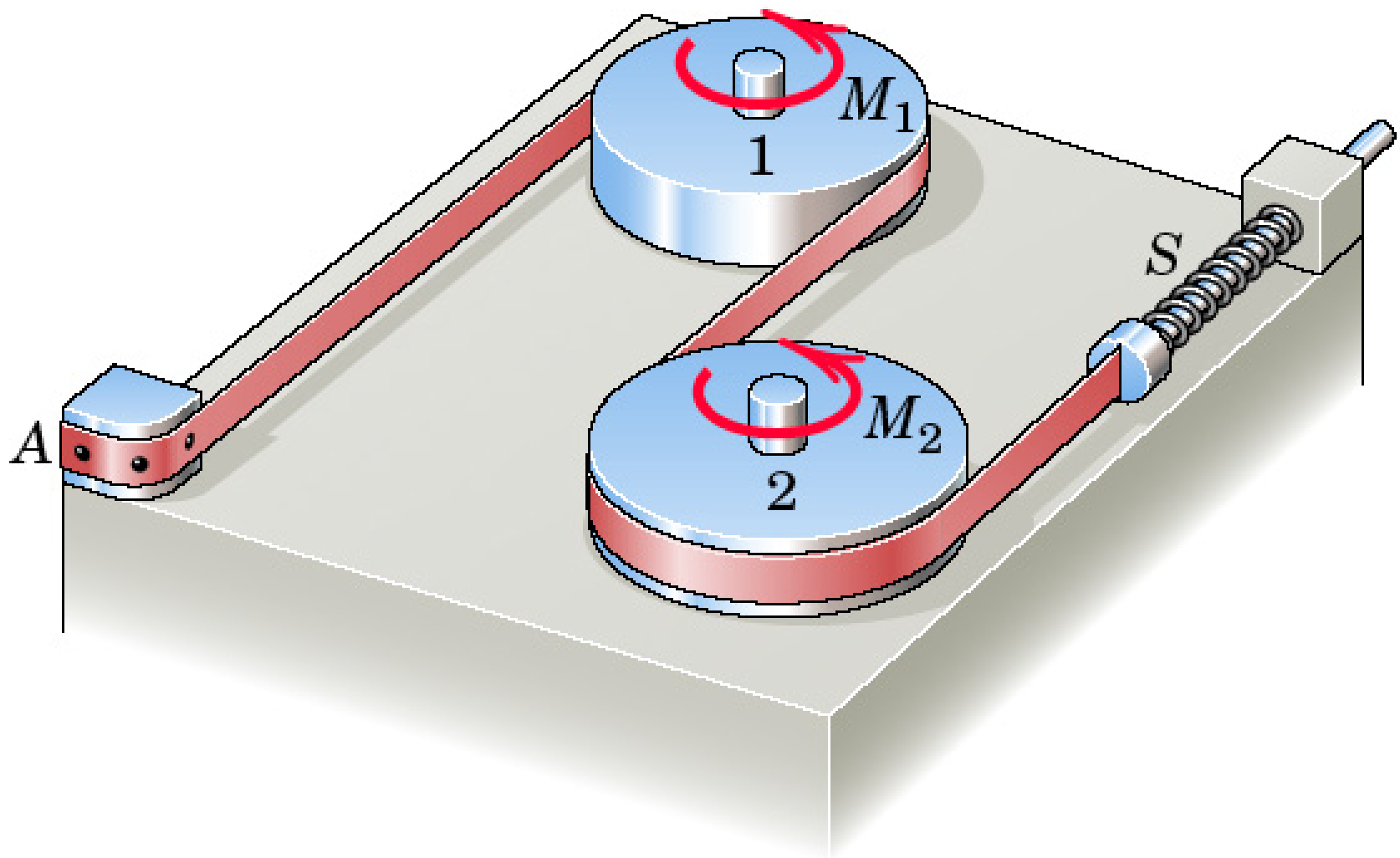
P_06_133



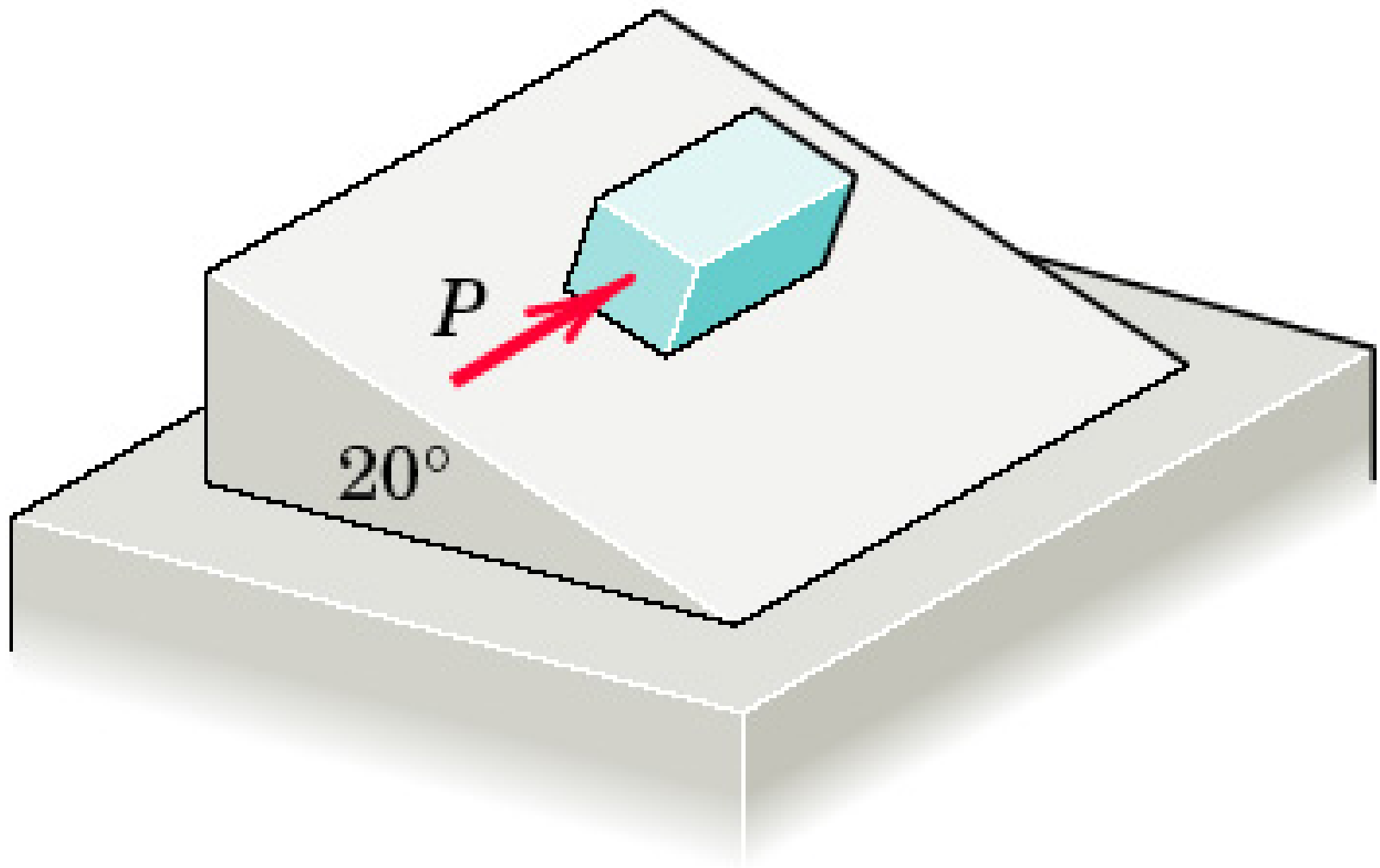
Problem 6/134



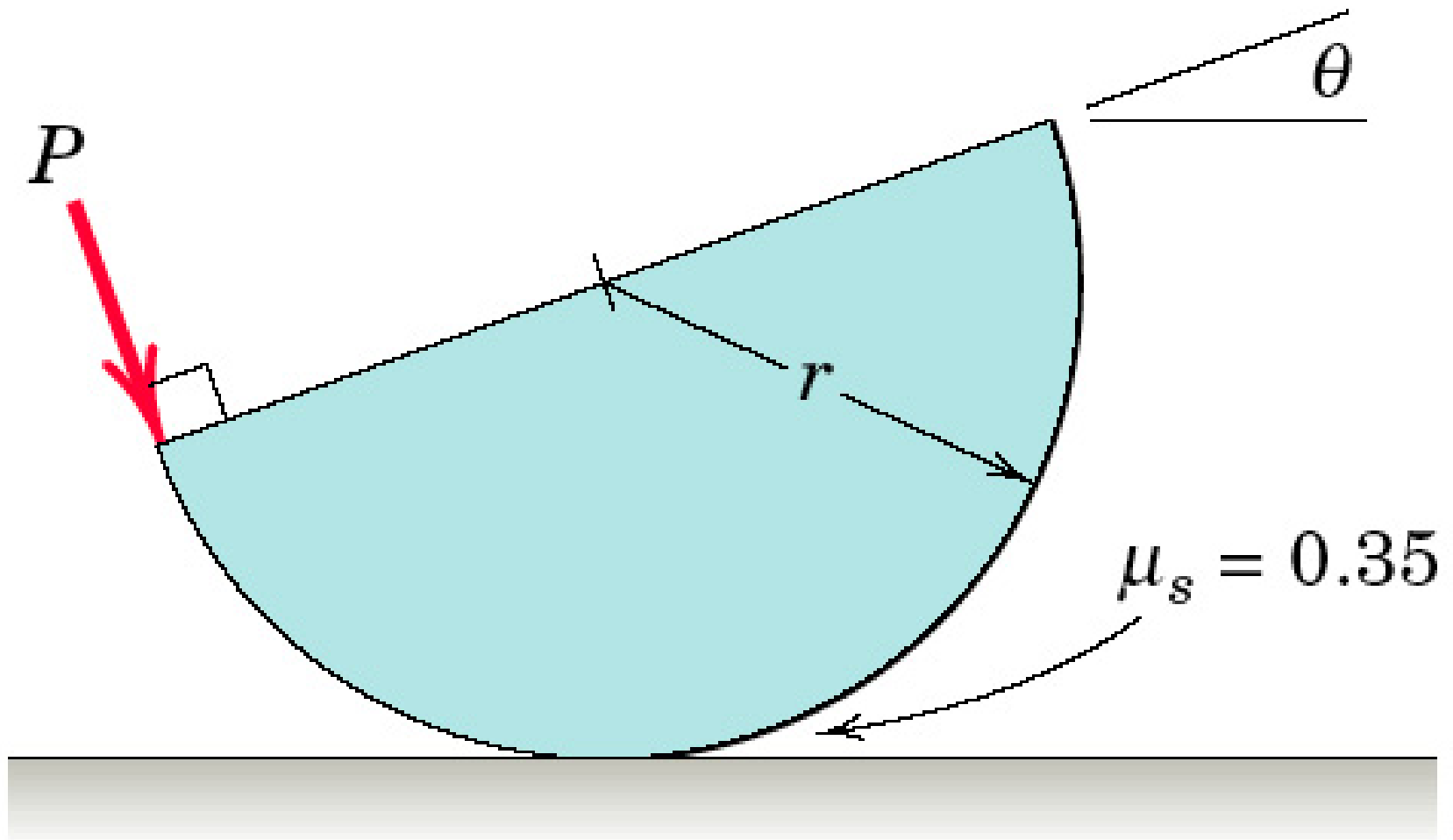
Problem 6/135



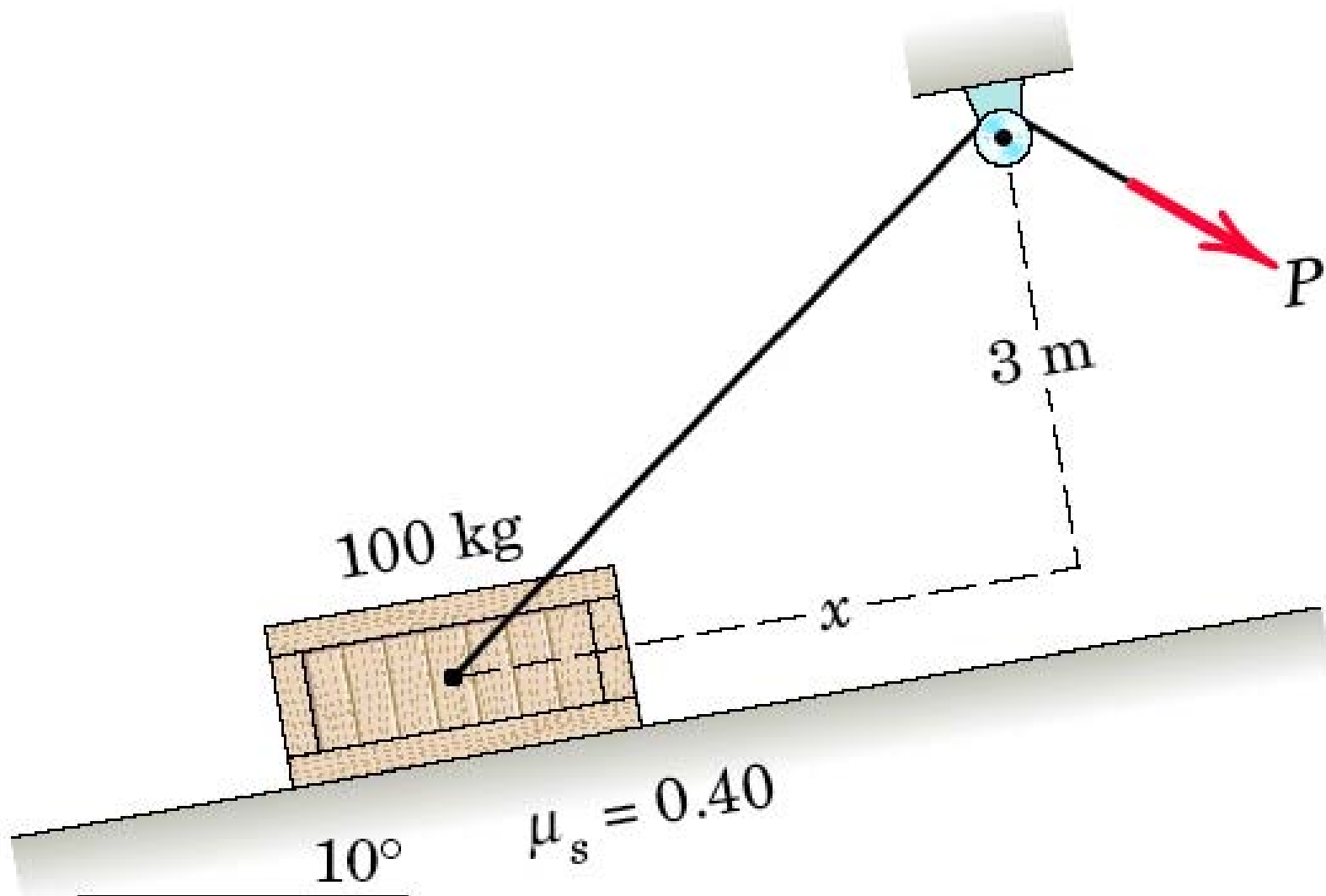
Problem 6/136



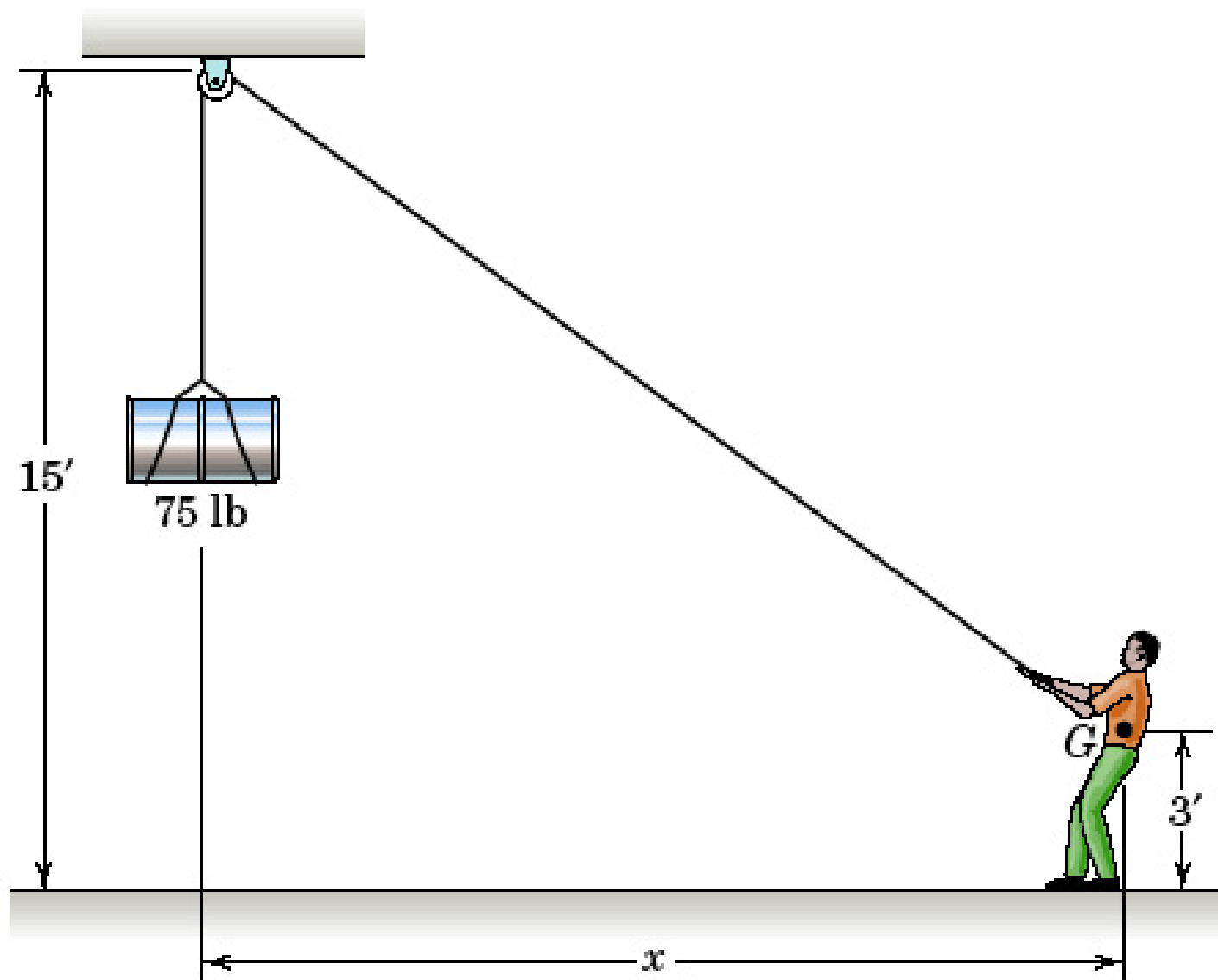
Problem 6/137



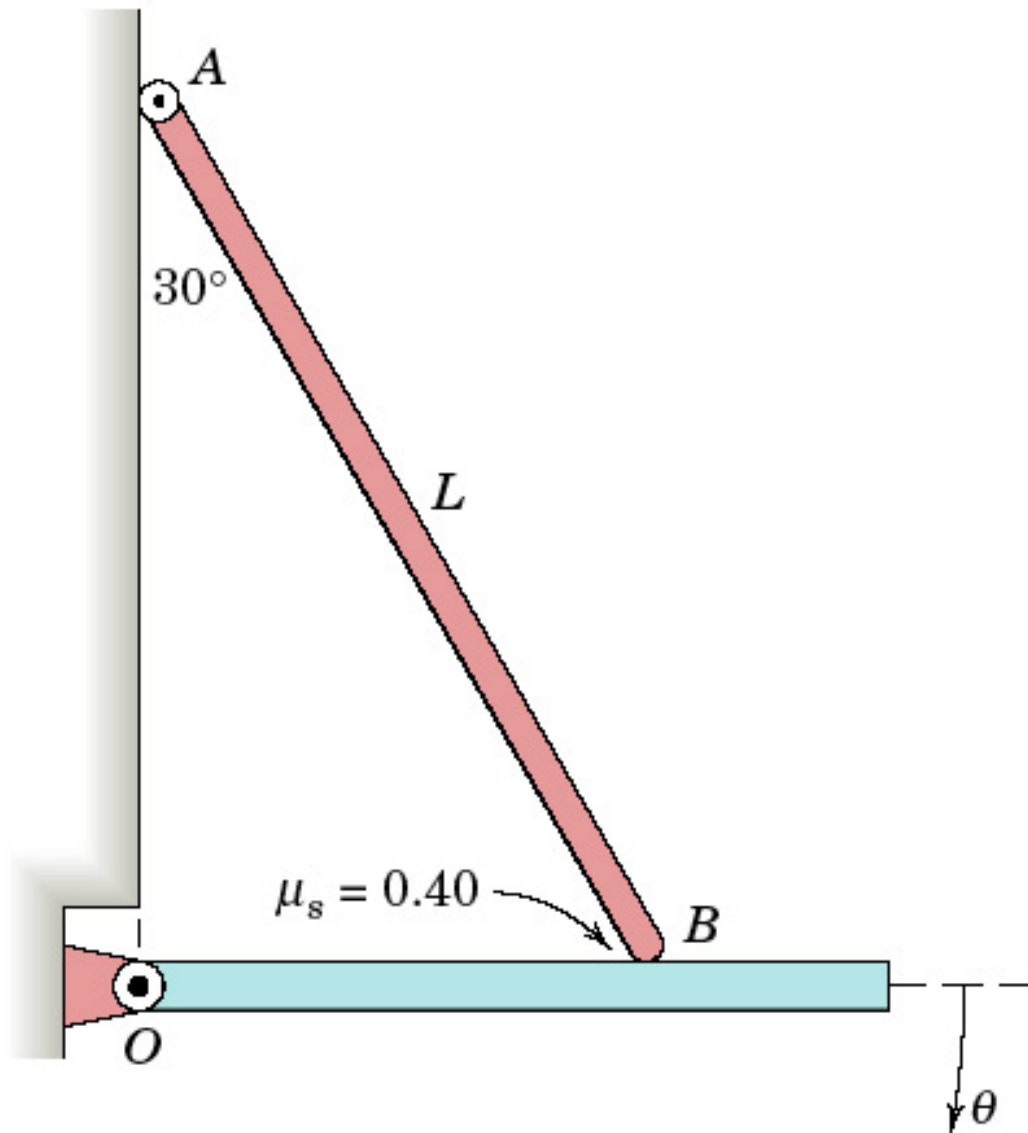
Problem 6/138



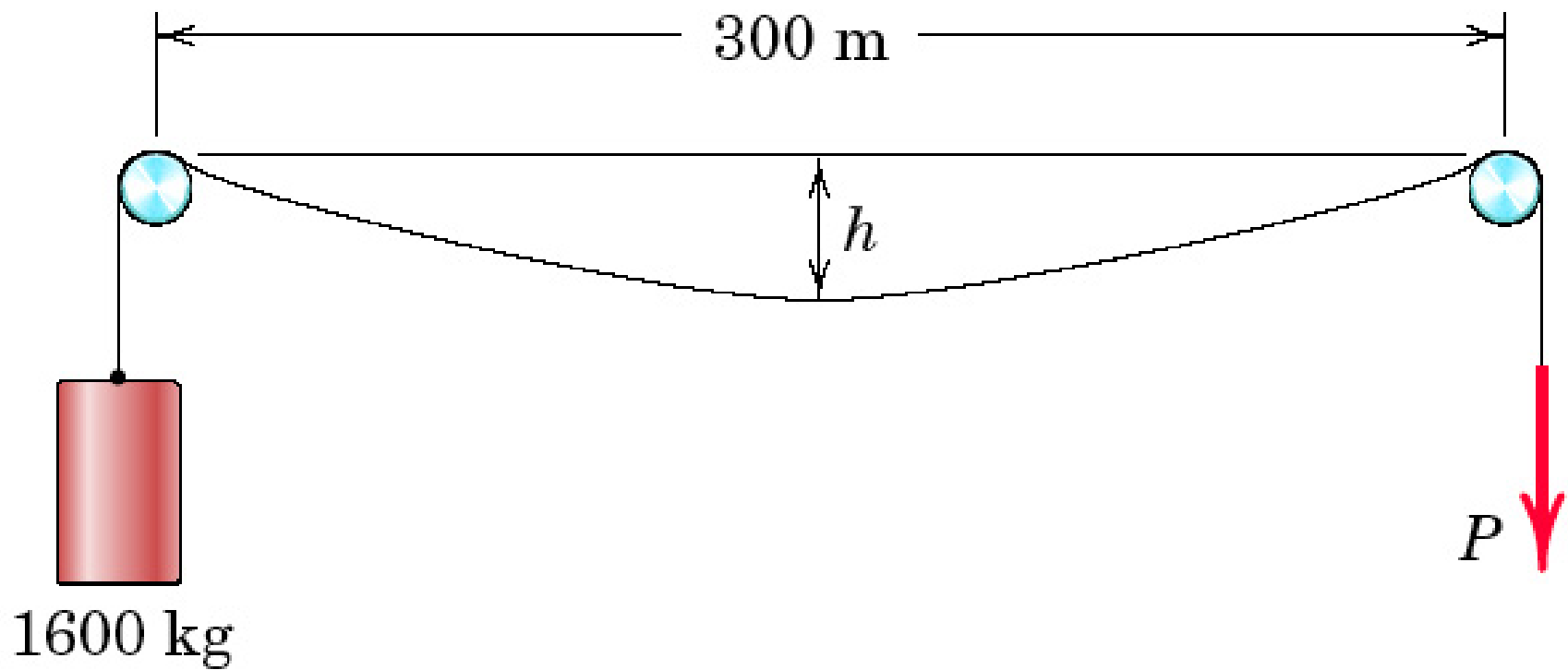
Problem 6/139



Problem 6/14

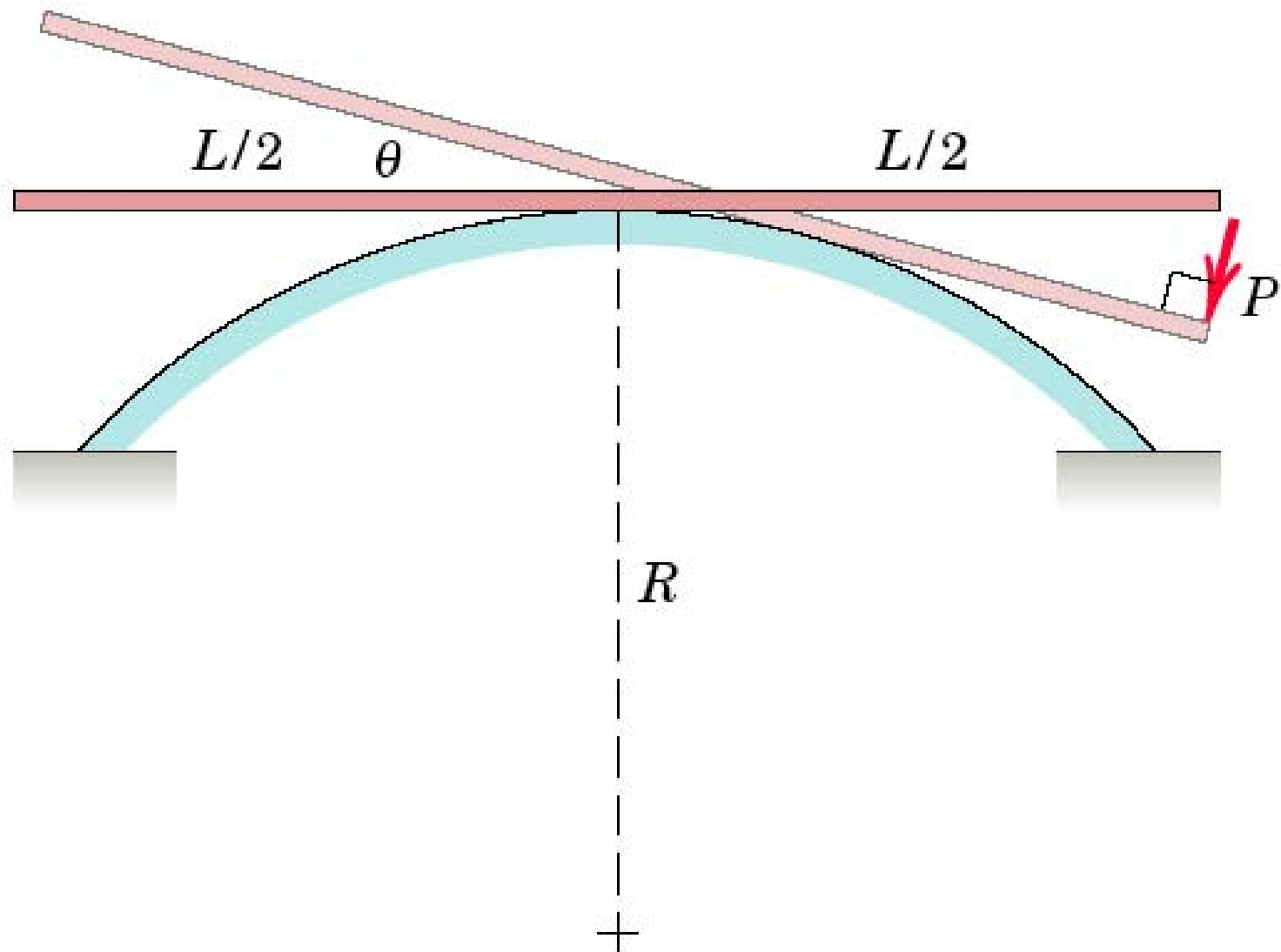


Problem 6/140

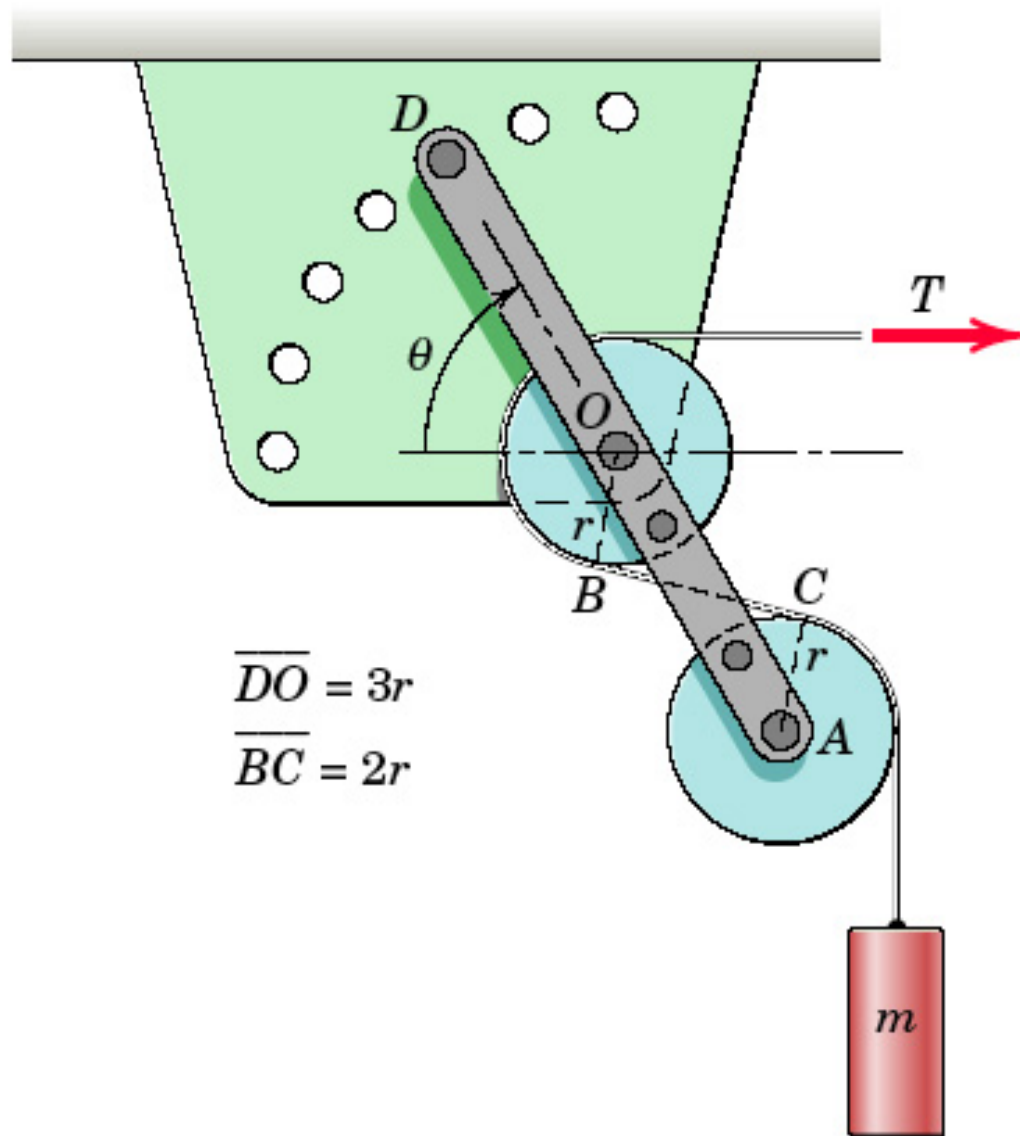


Problem 6/141

P_06_141

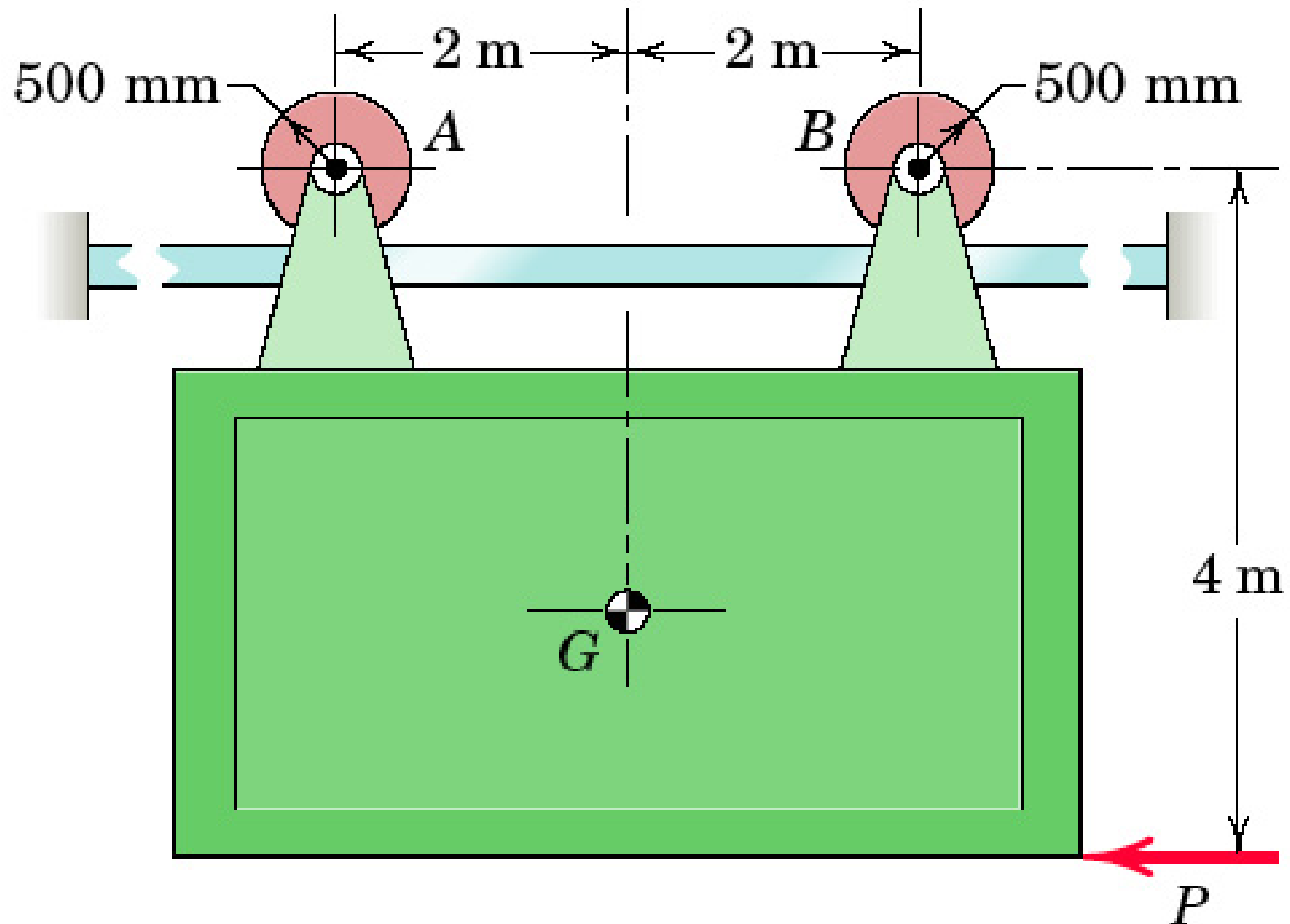


Problem 6/142

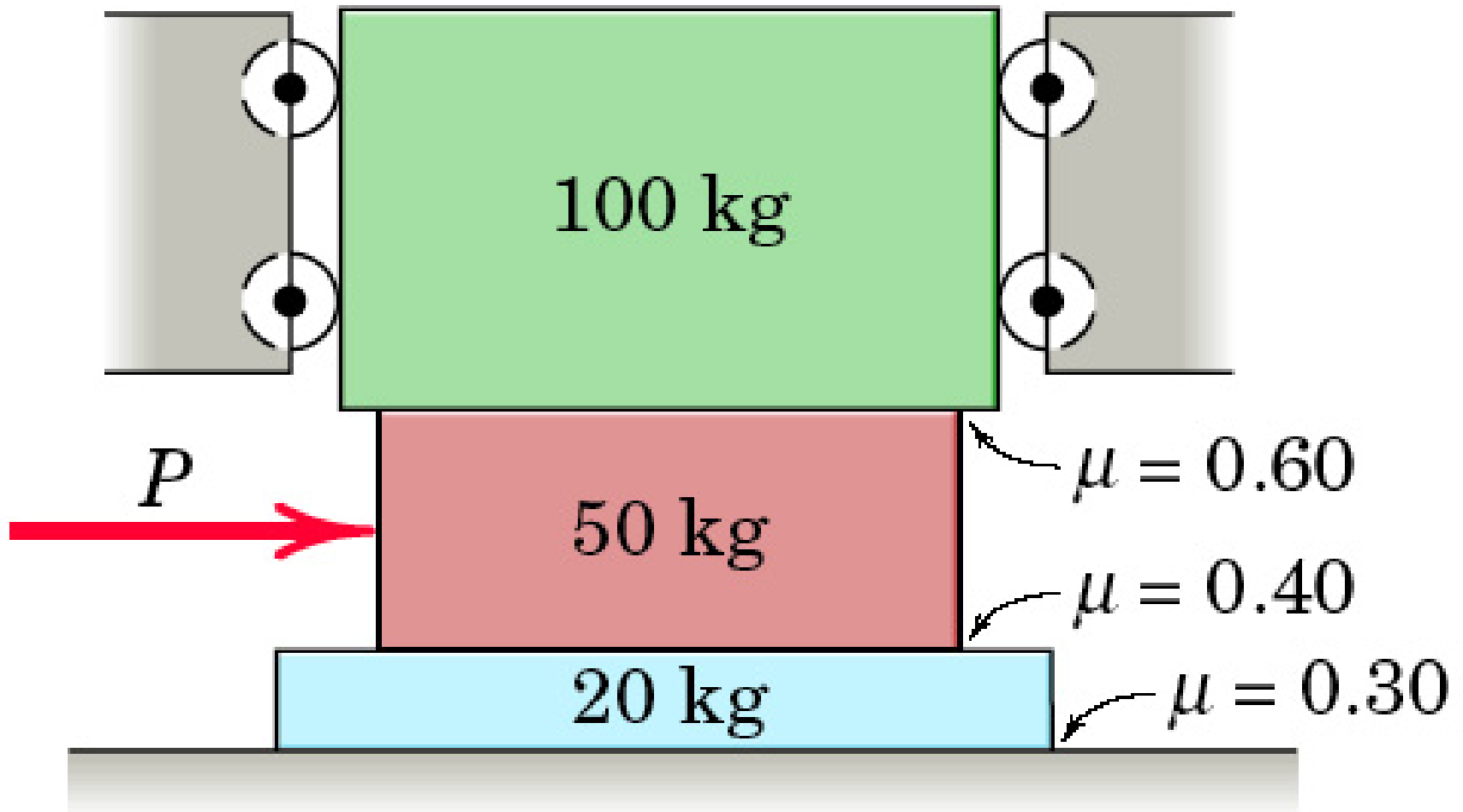


Problem 6/144

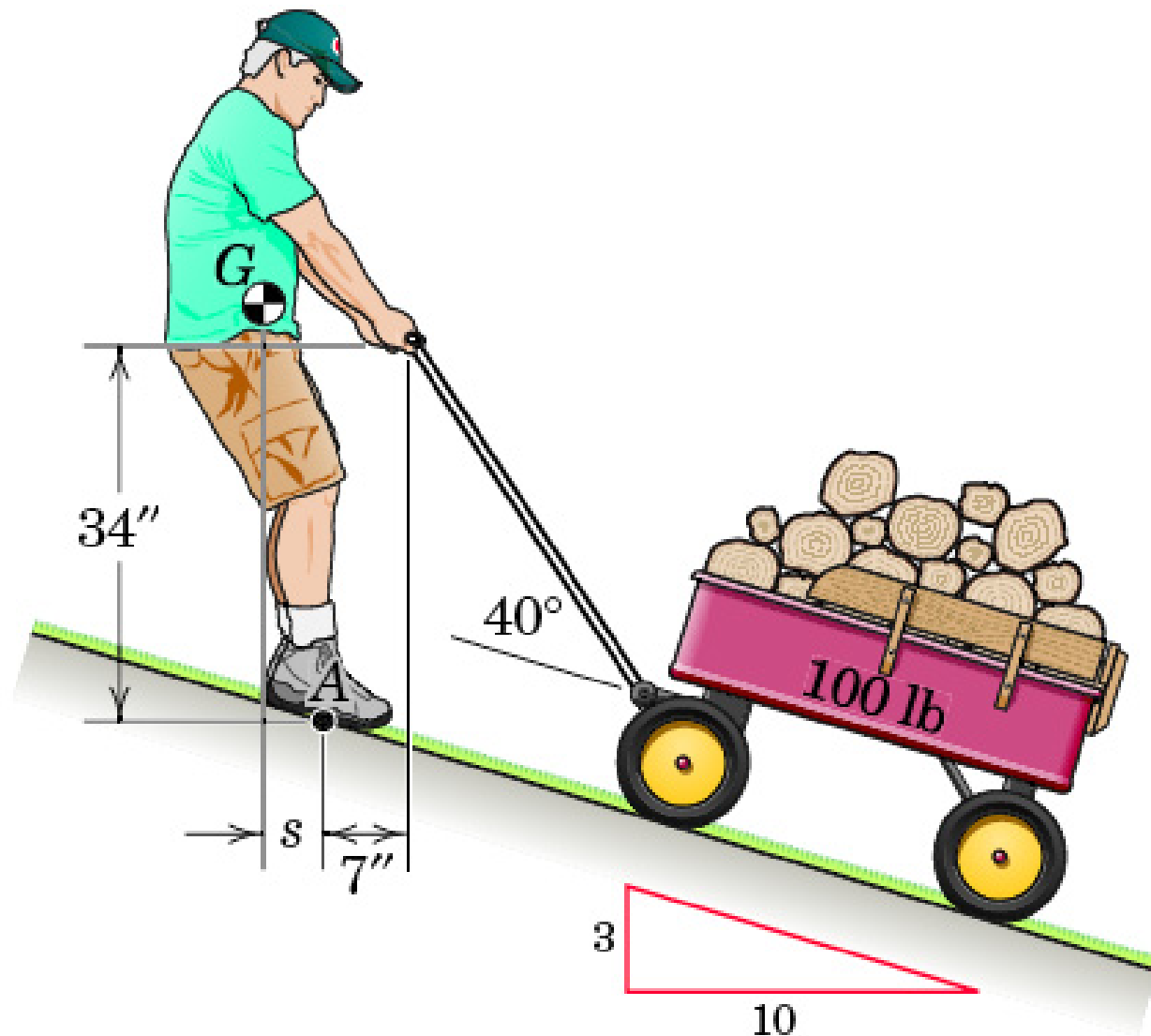




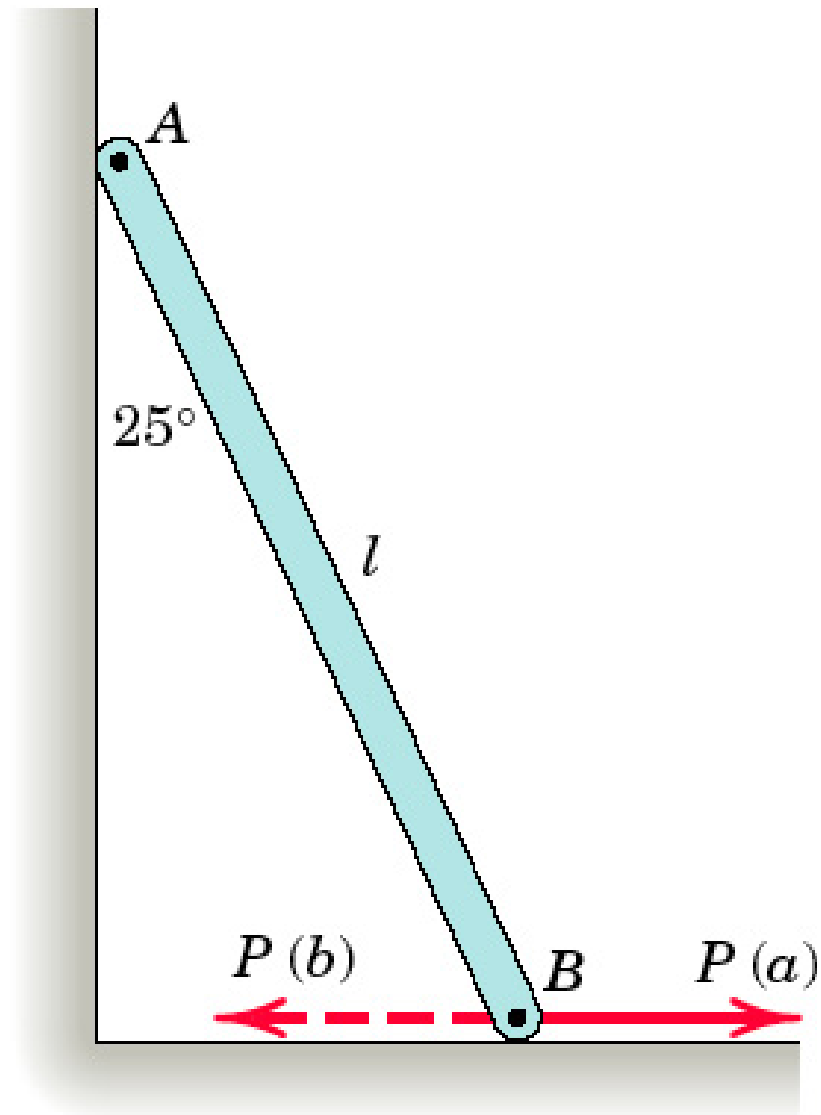
Problem 6/15



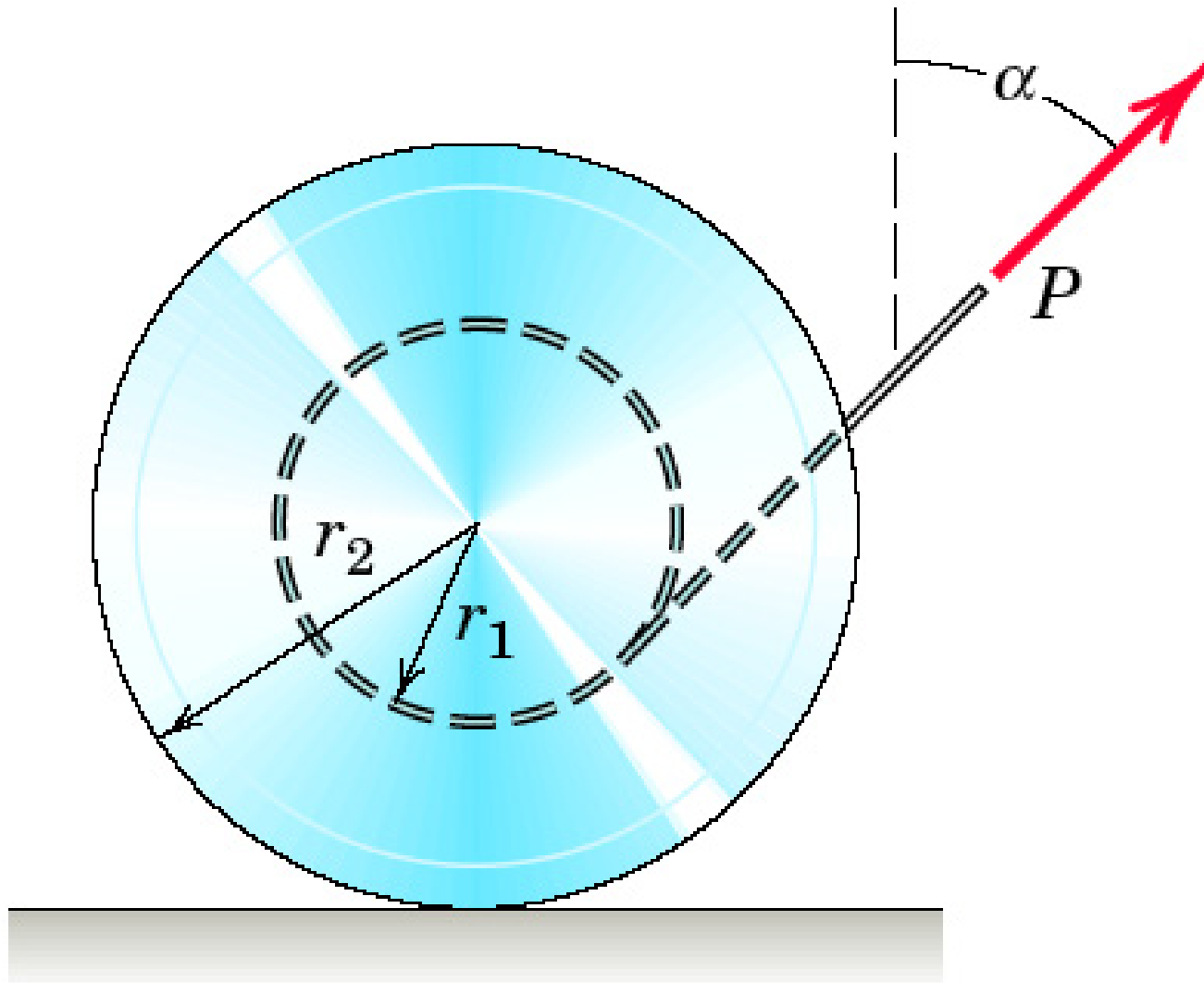
Problem 6/16



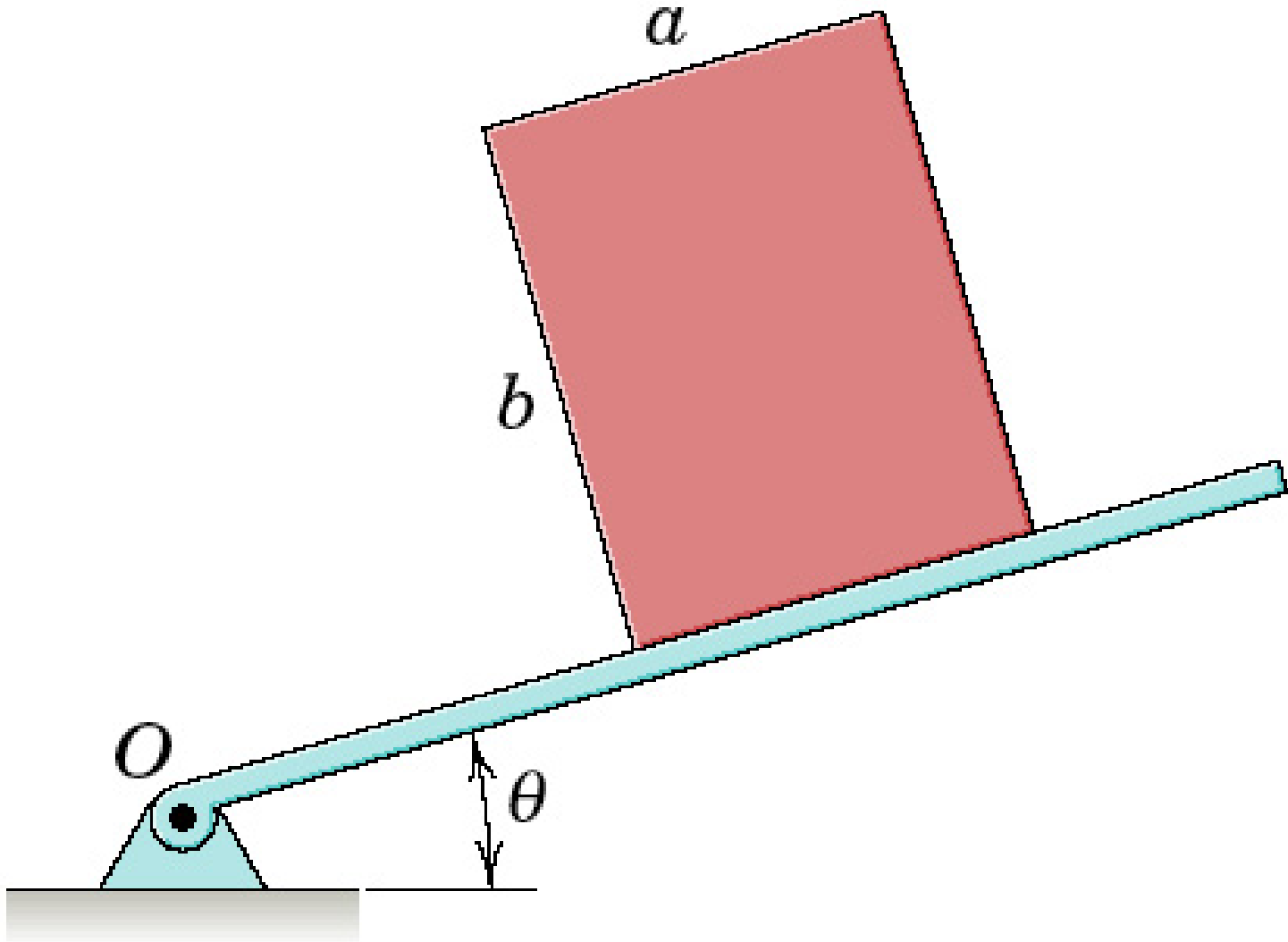
Problem 6/17



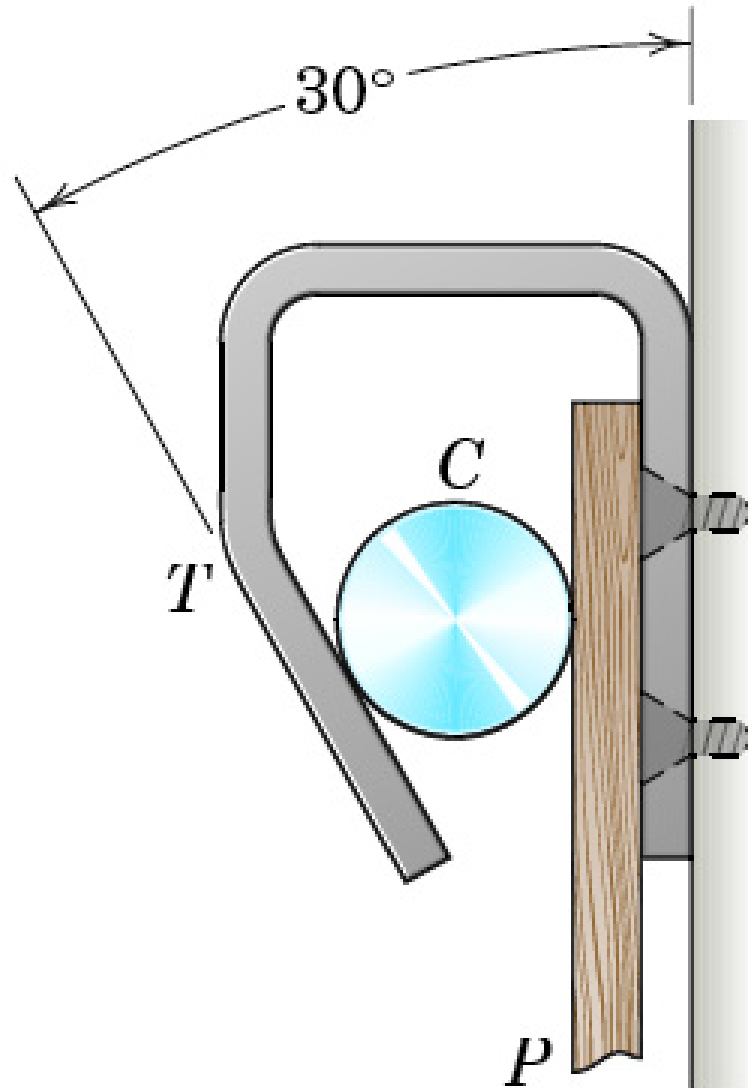
Problem 6/18



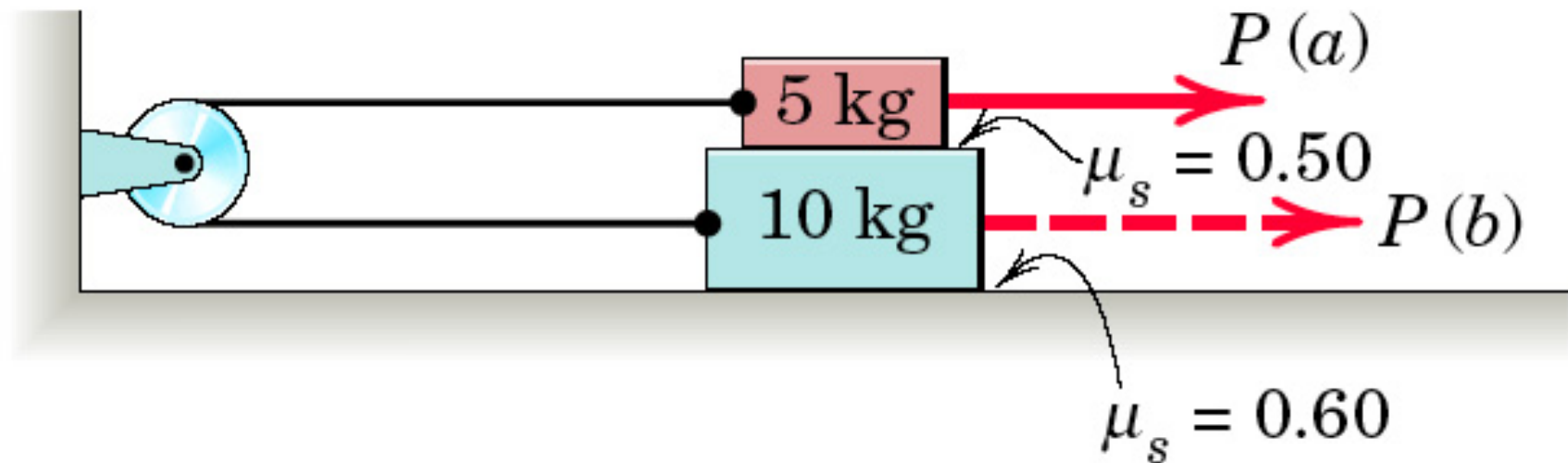
Problem 6/19



Problem 6/20

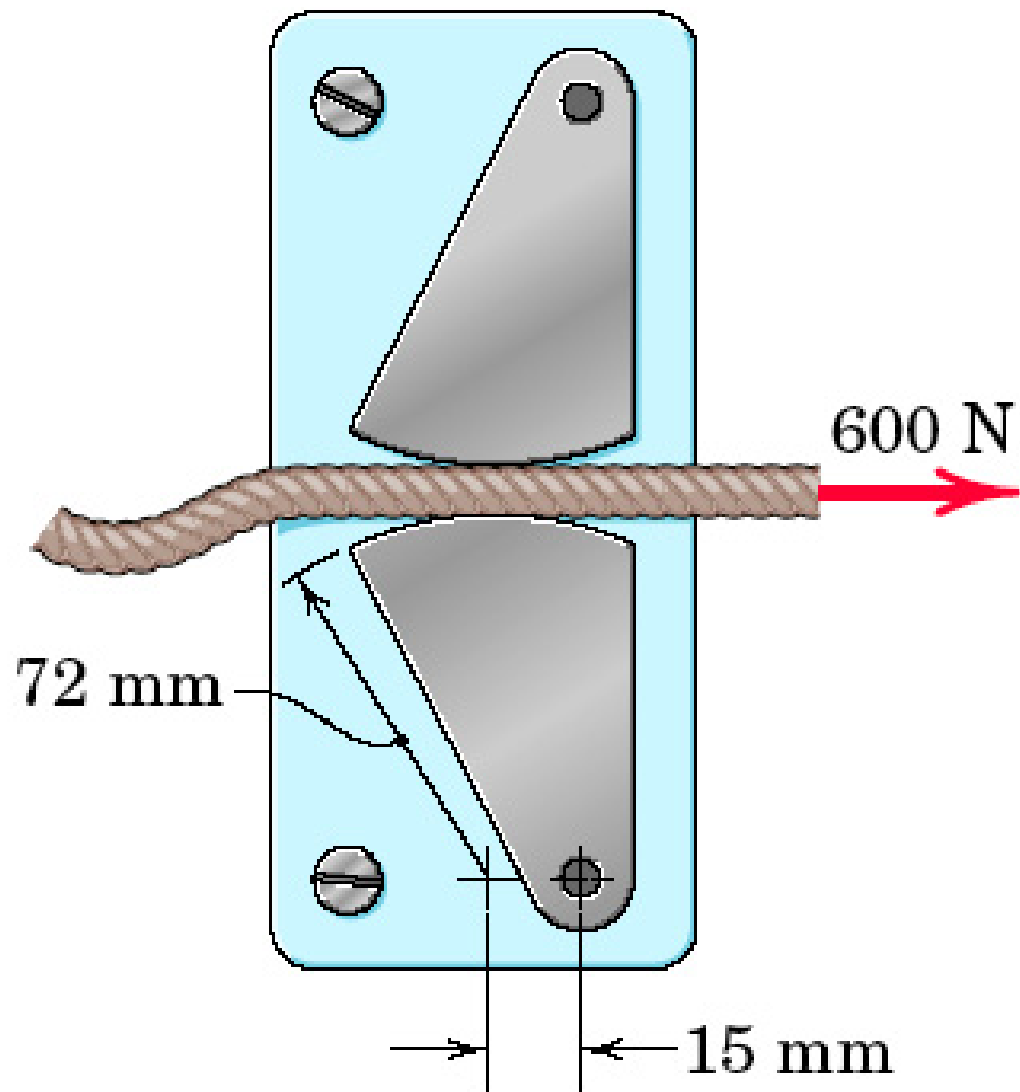


Problem 6/21

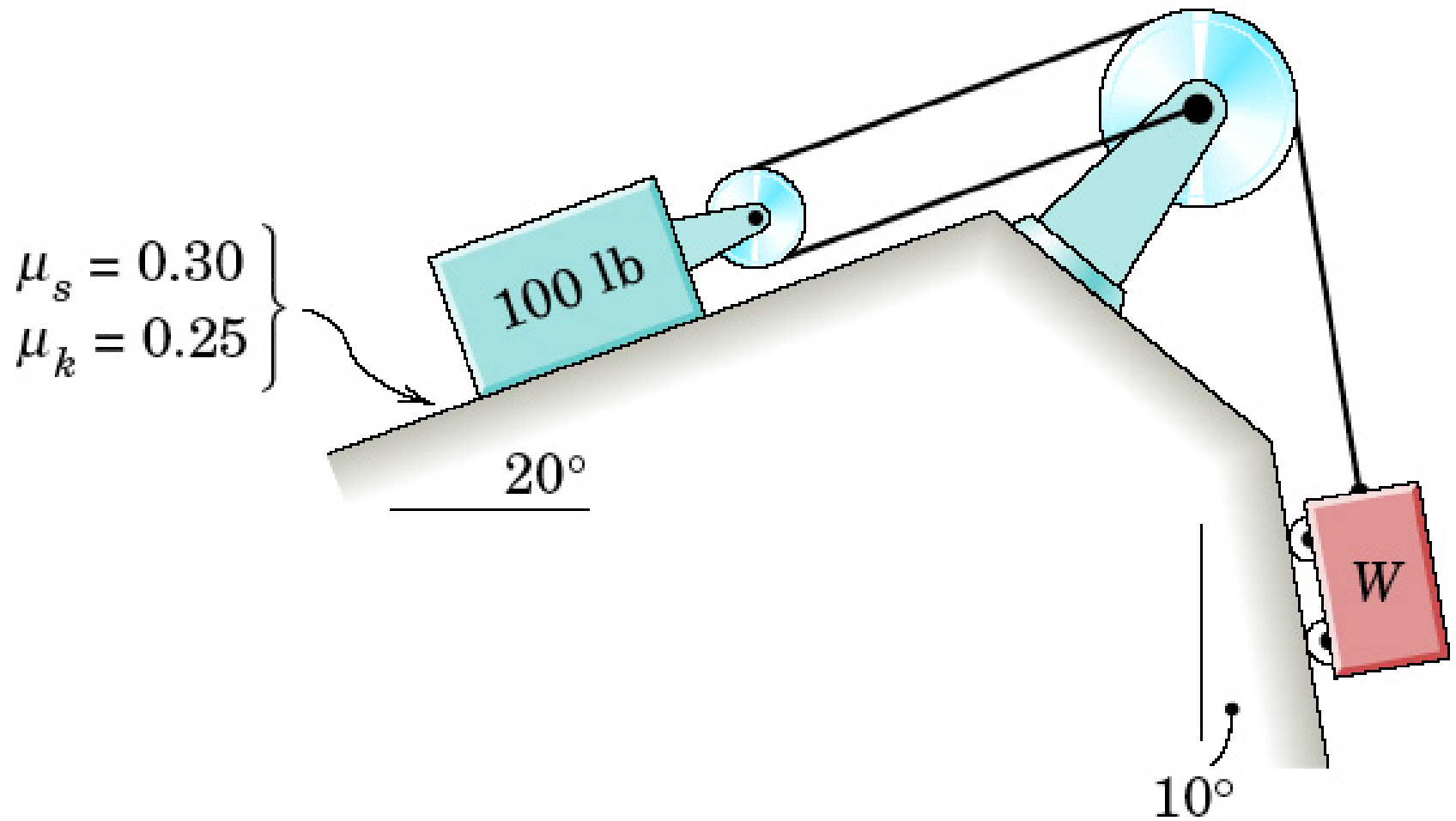


Problem 6/22

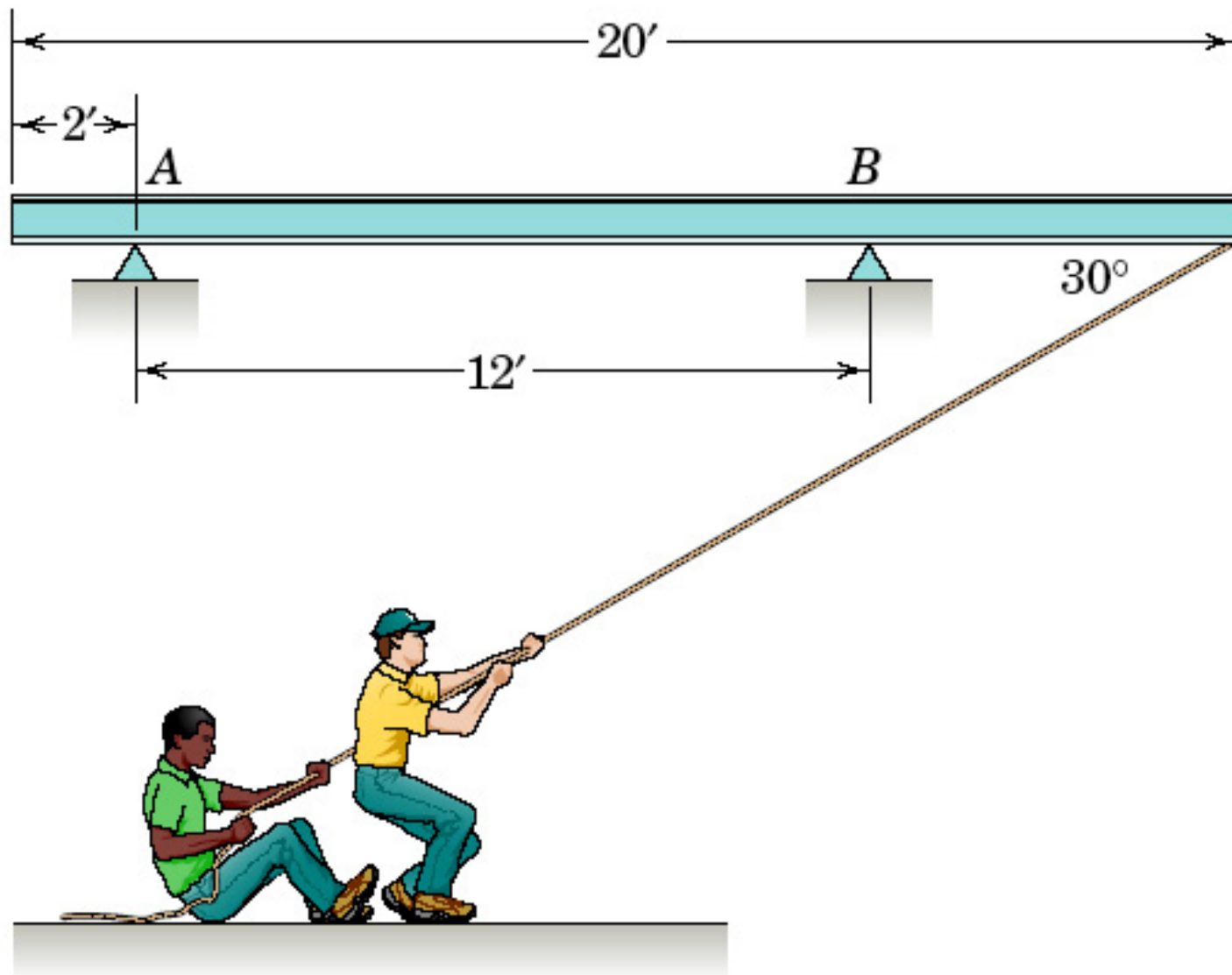
P_06_22



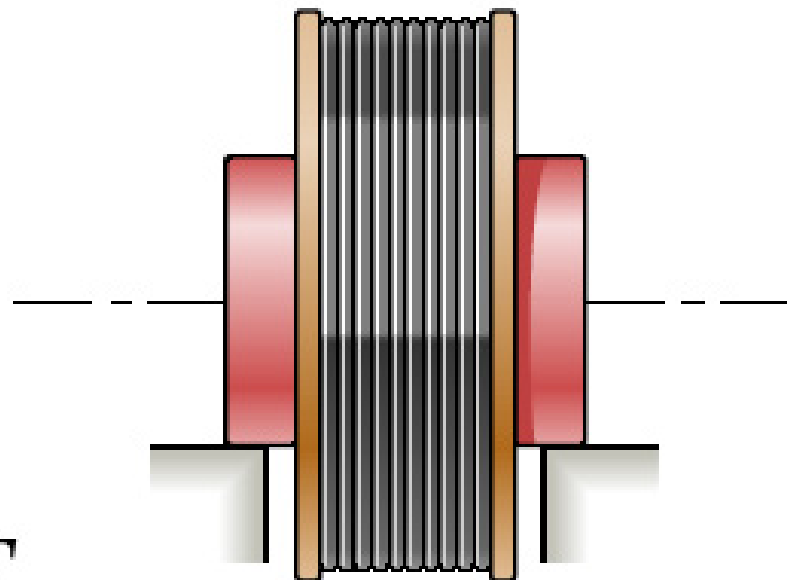
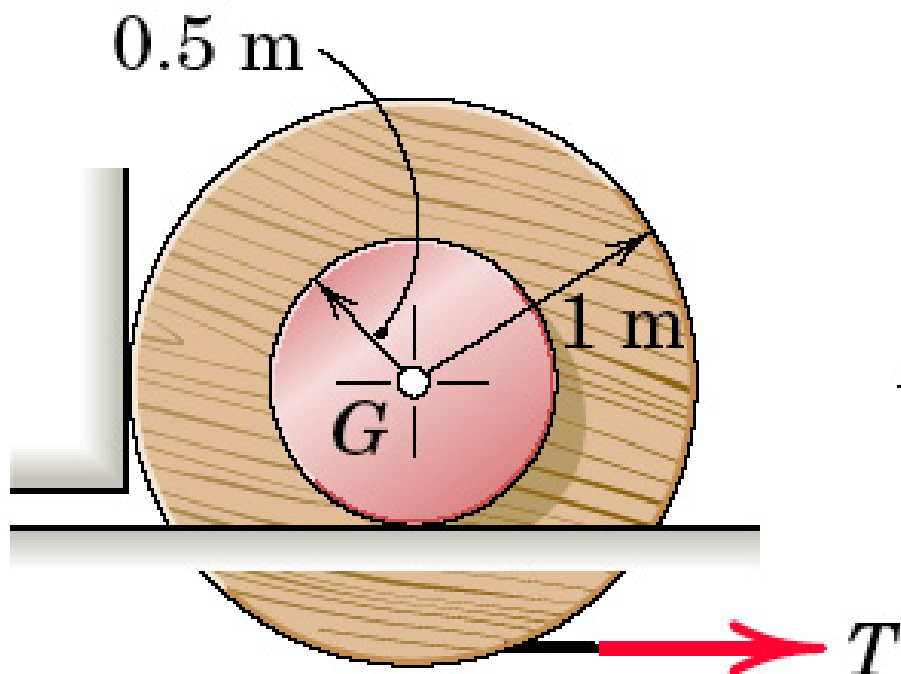
Problem 6/23



Problem 6/24

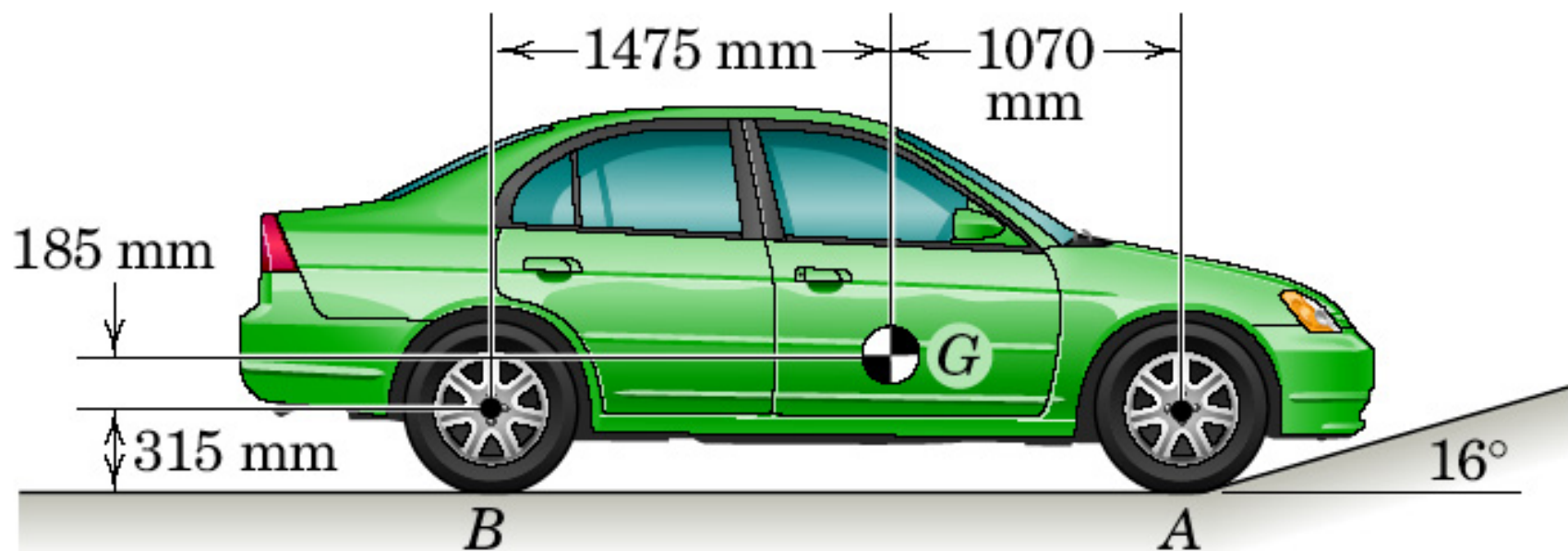


Problem 6/25



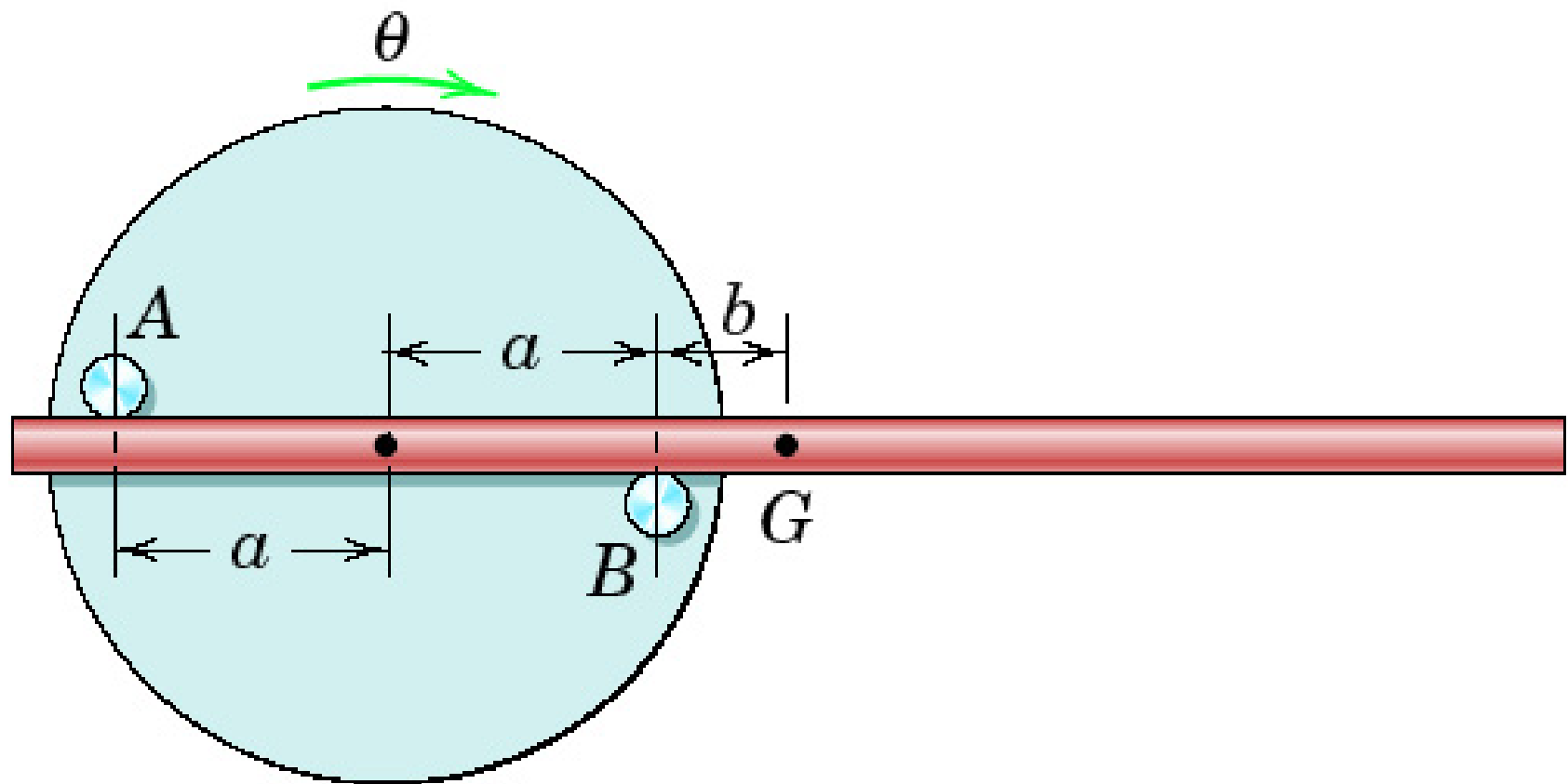
Problem 6/26

P_06_26

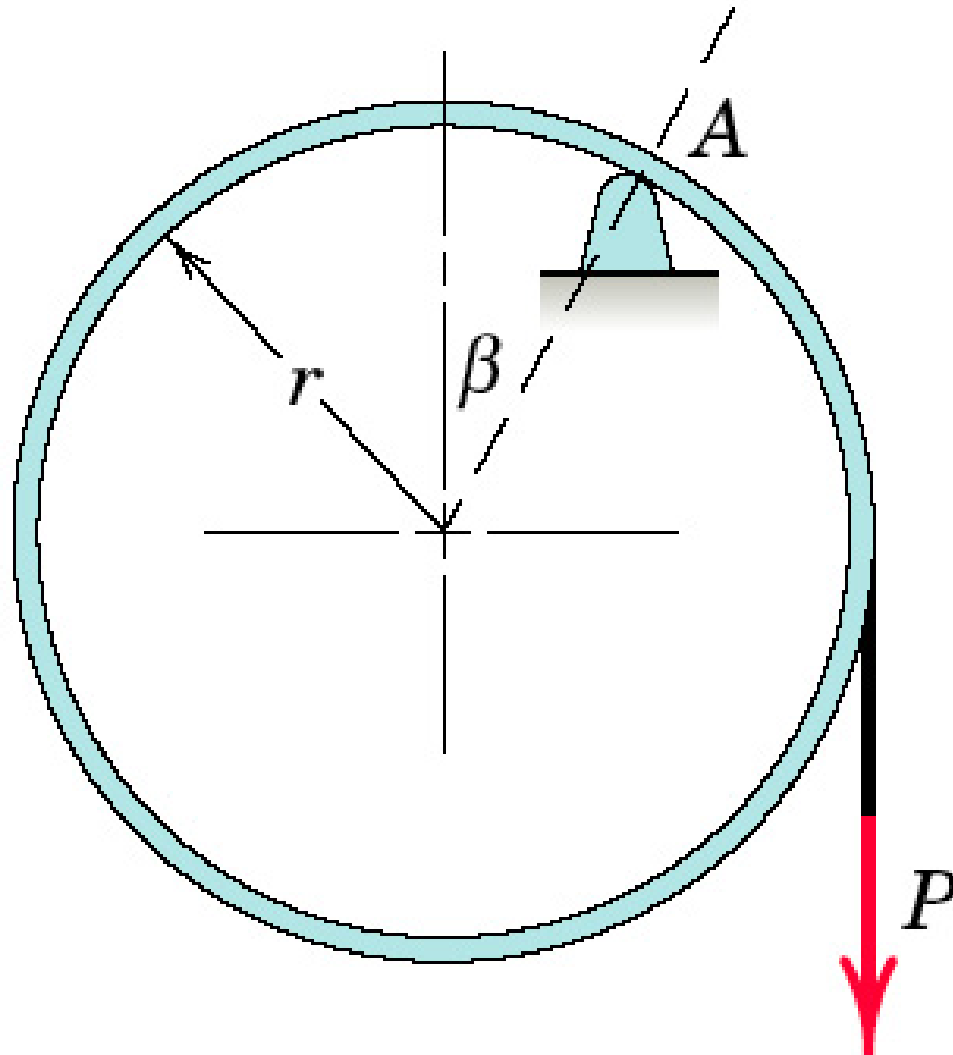


Problem 6/27

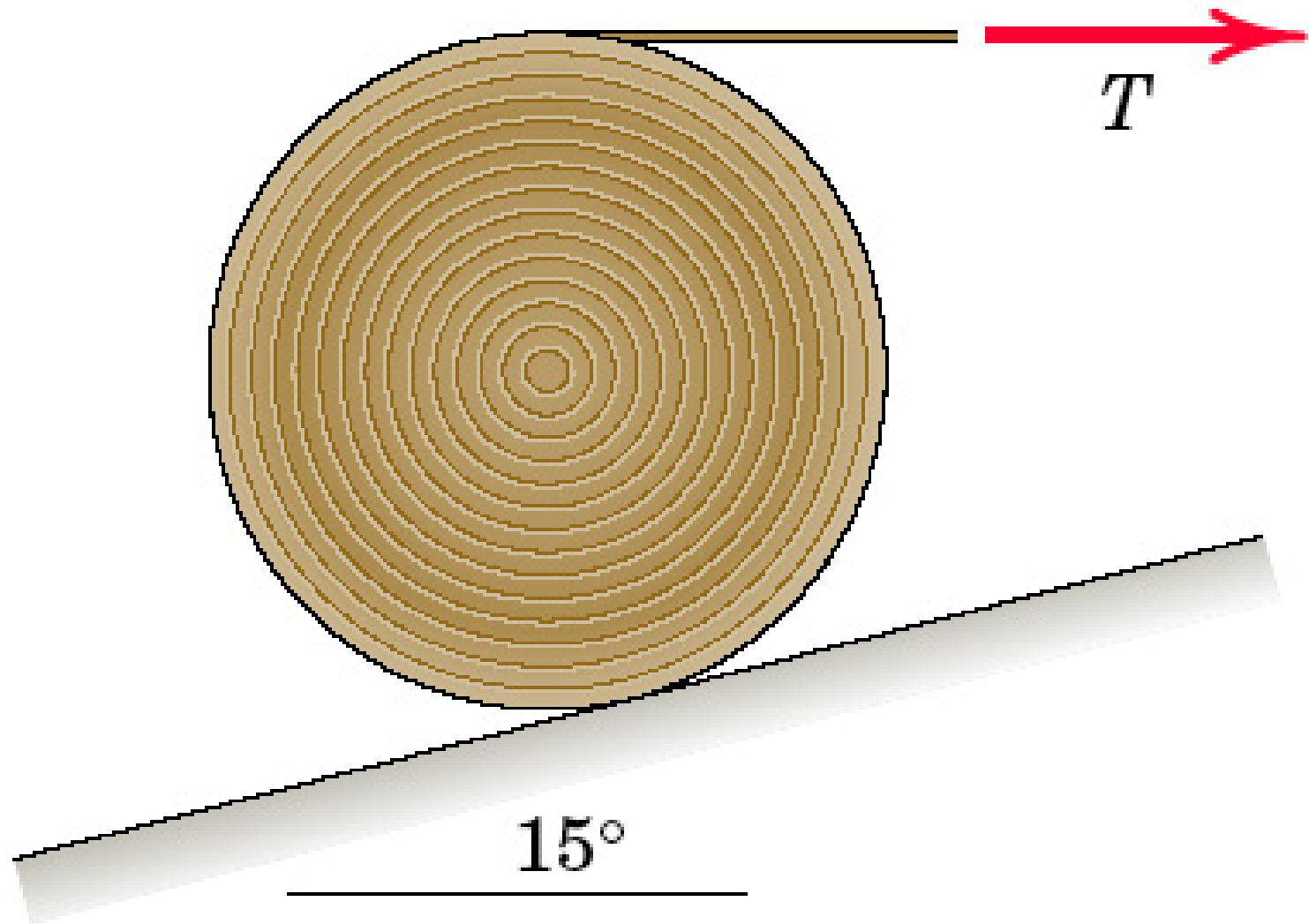
P_06_27



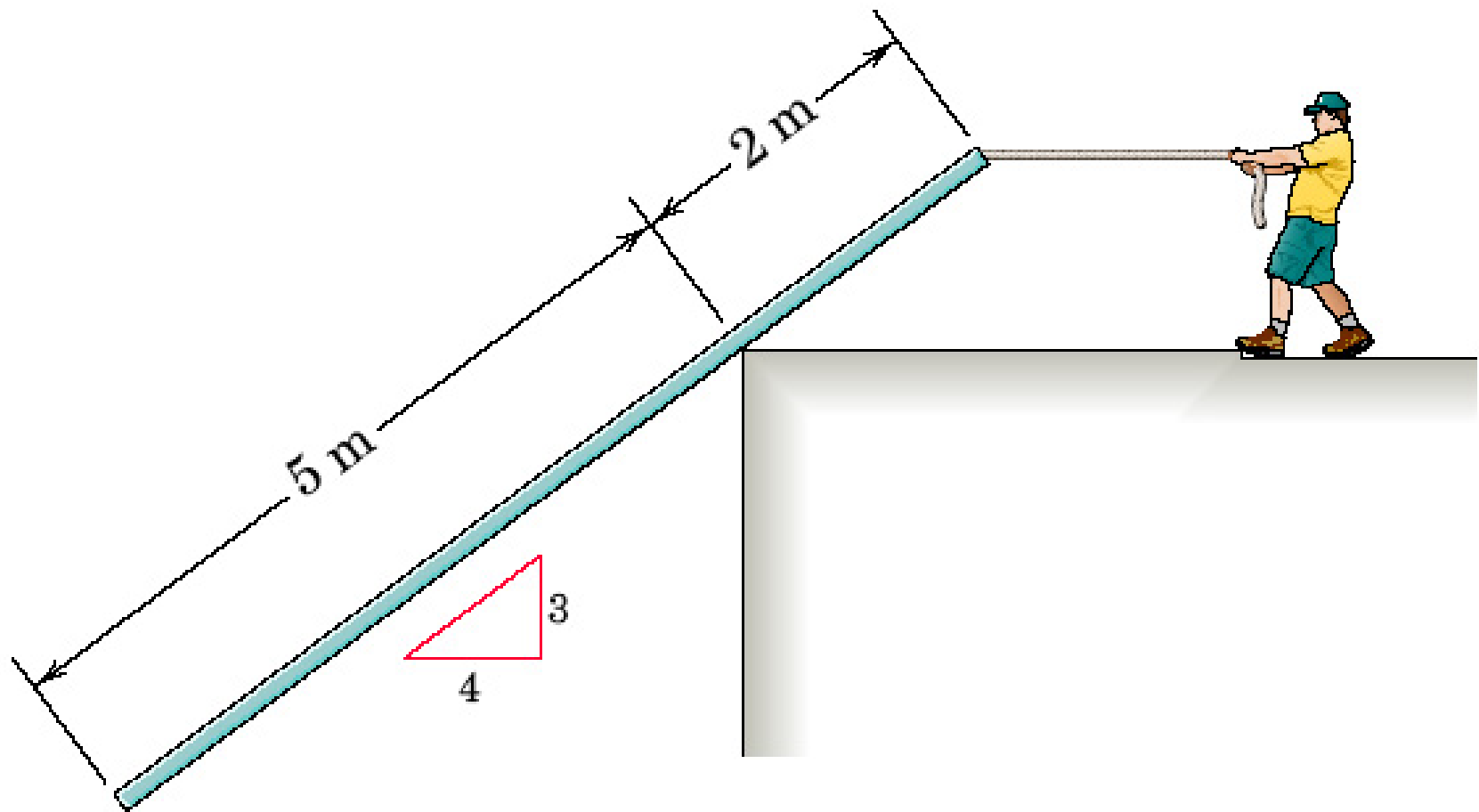
Problem 6/30



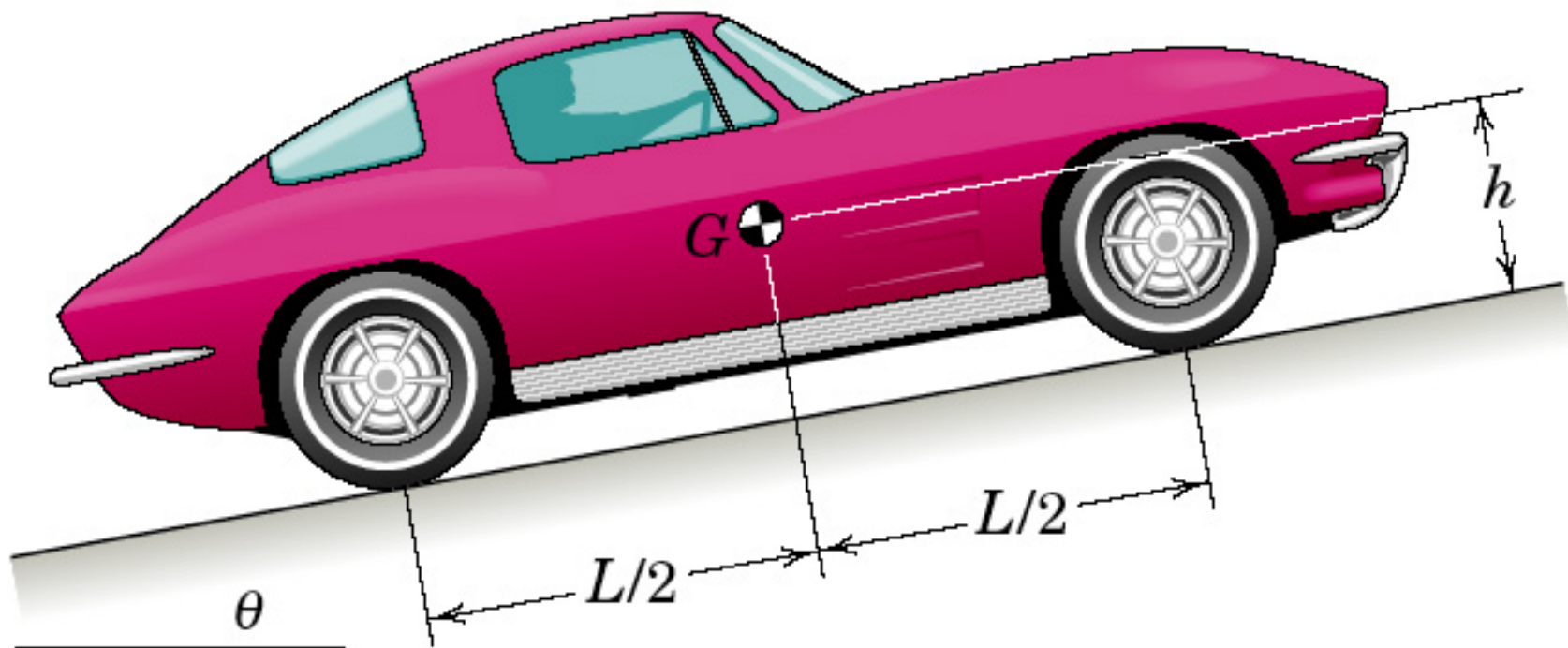
Problem 6/31



Problem 6/32

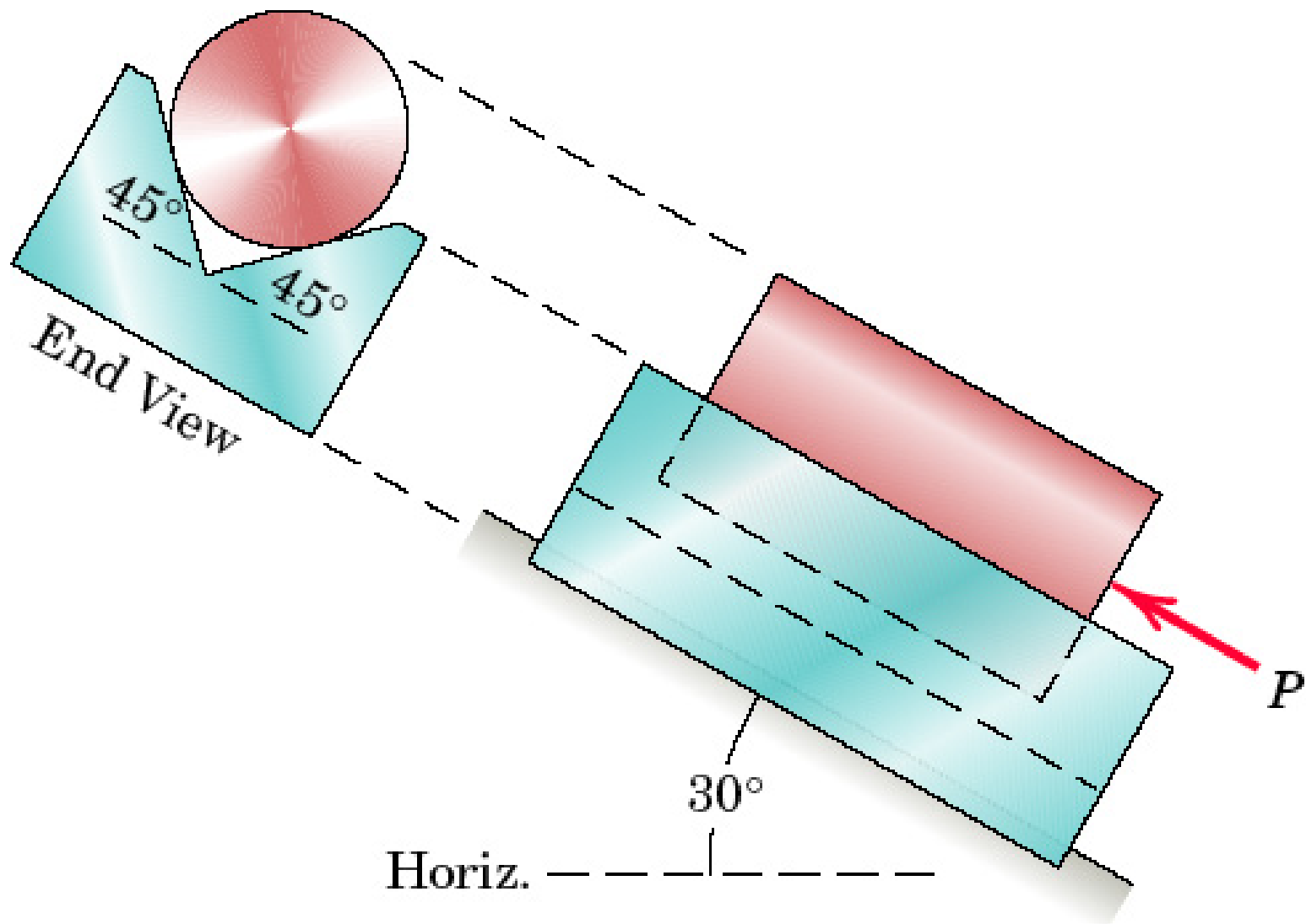


Problem 6/33

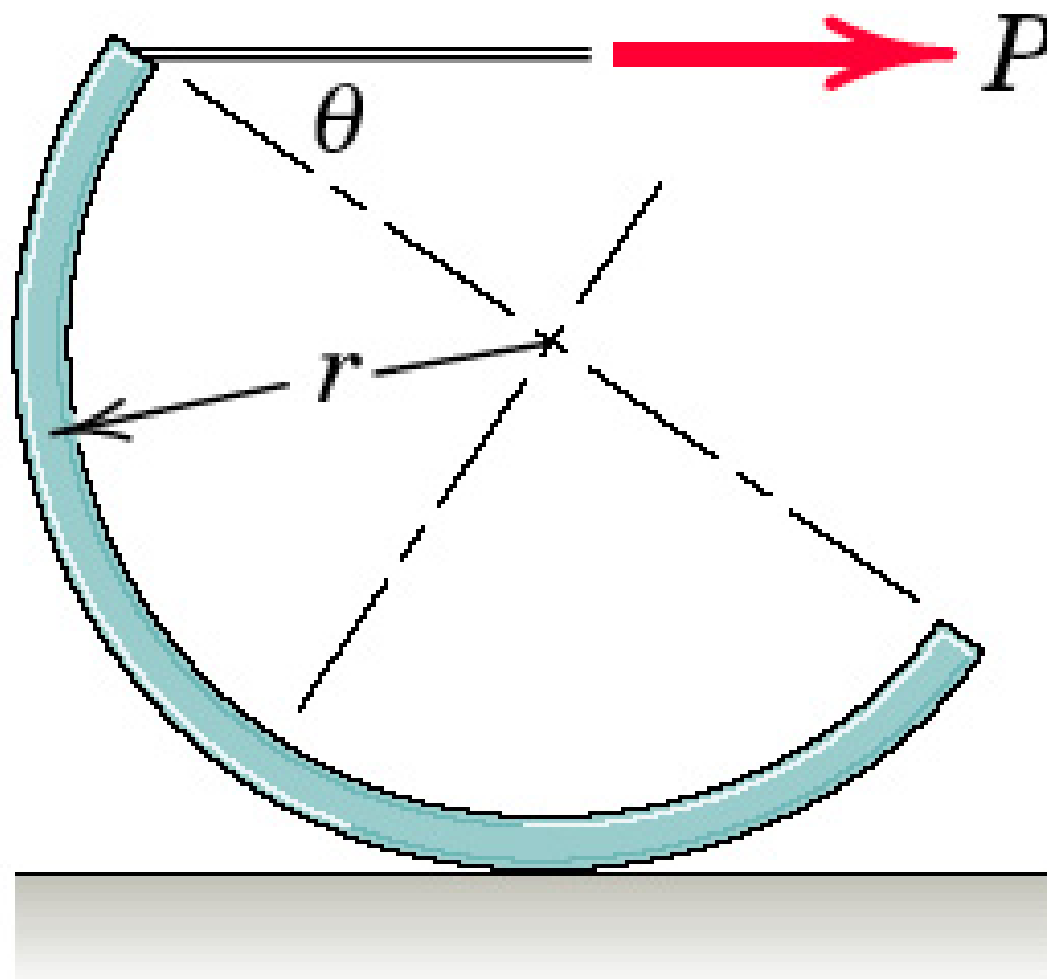


Problem 6/34

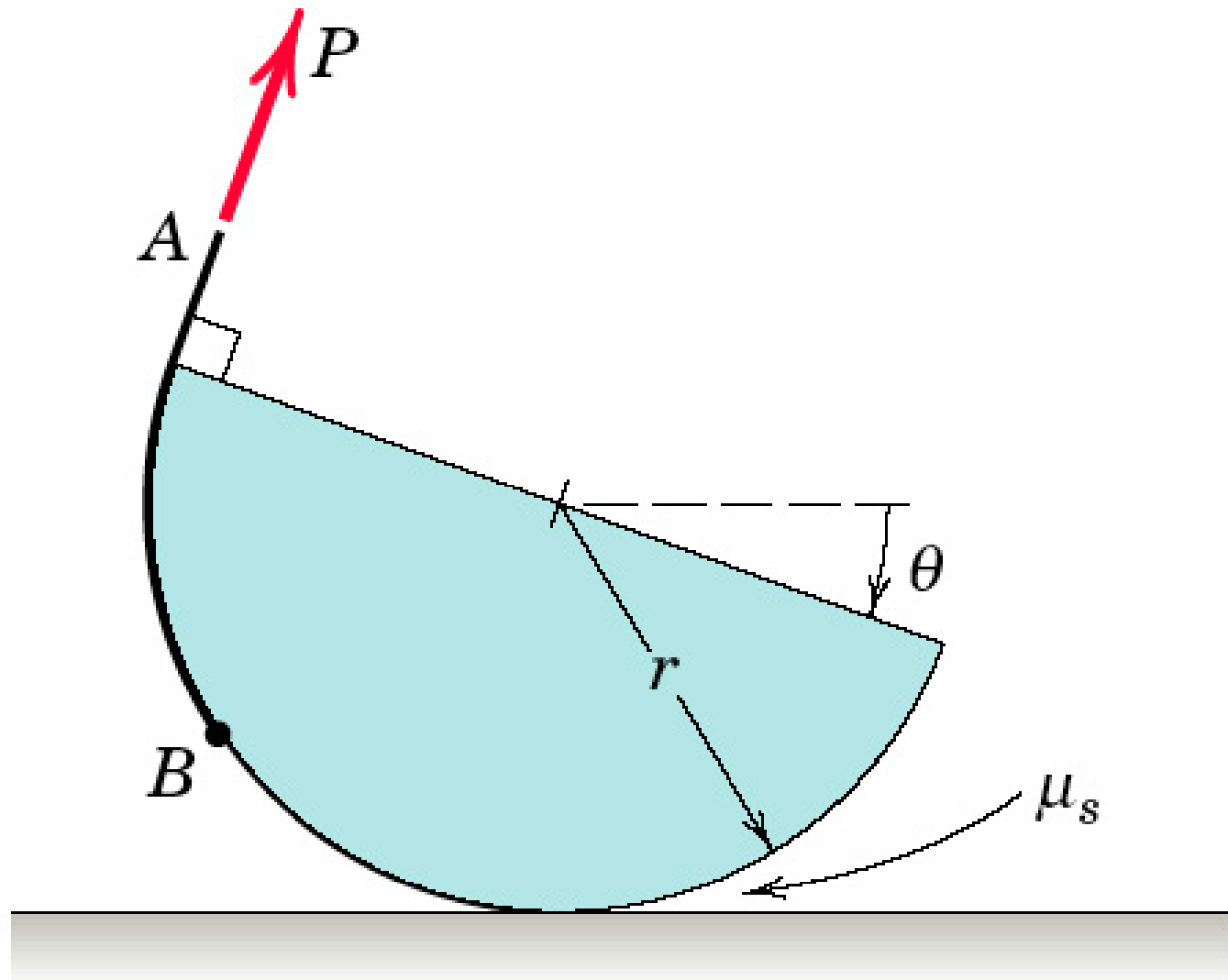
P_06_34



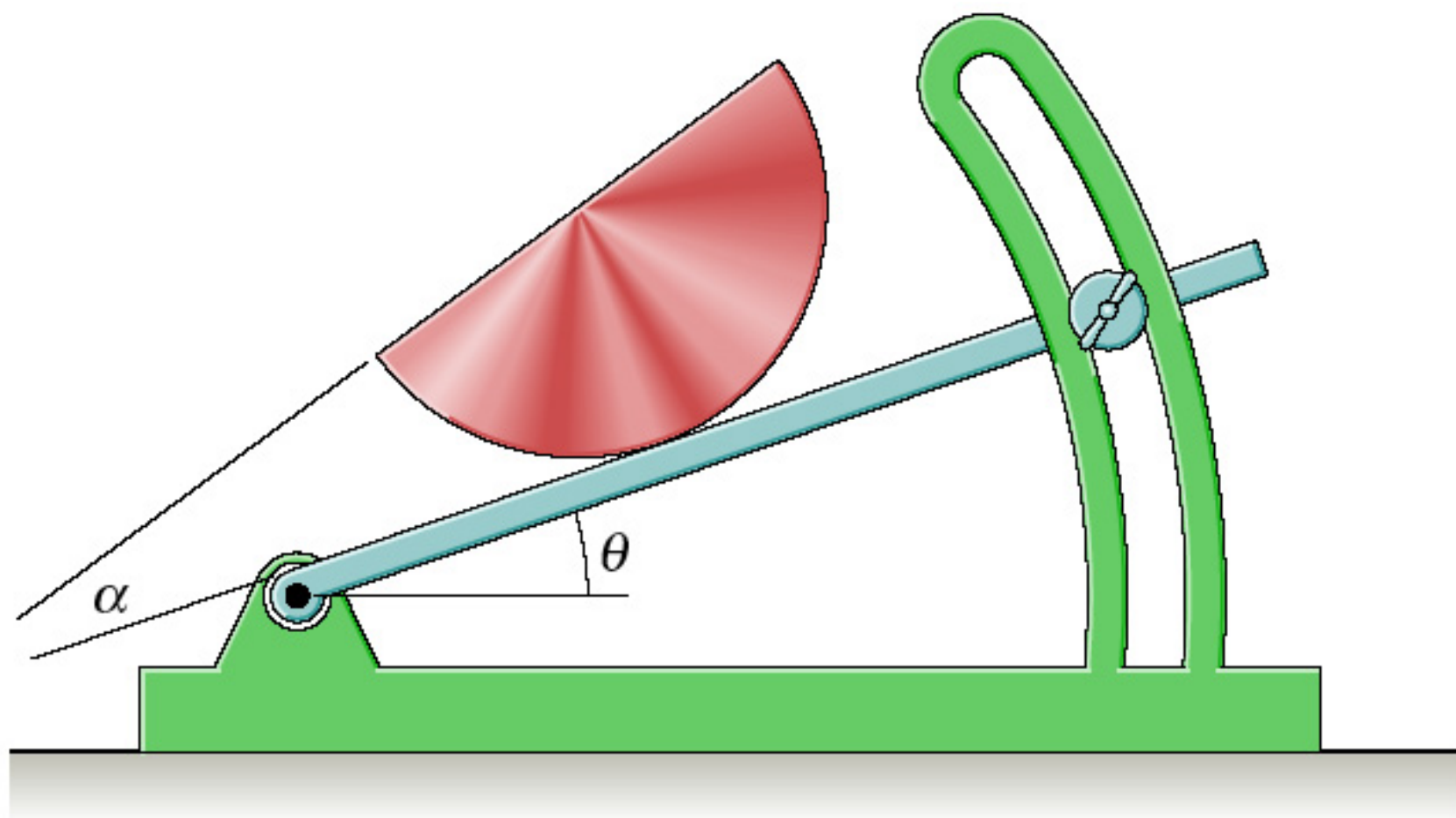
Problem 6/35



Problem 6/36

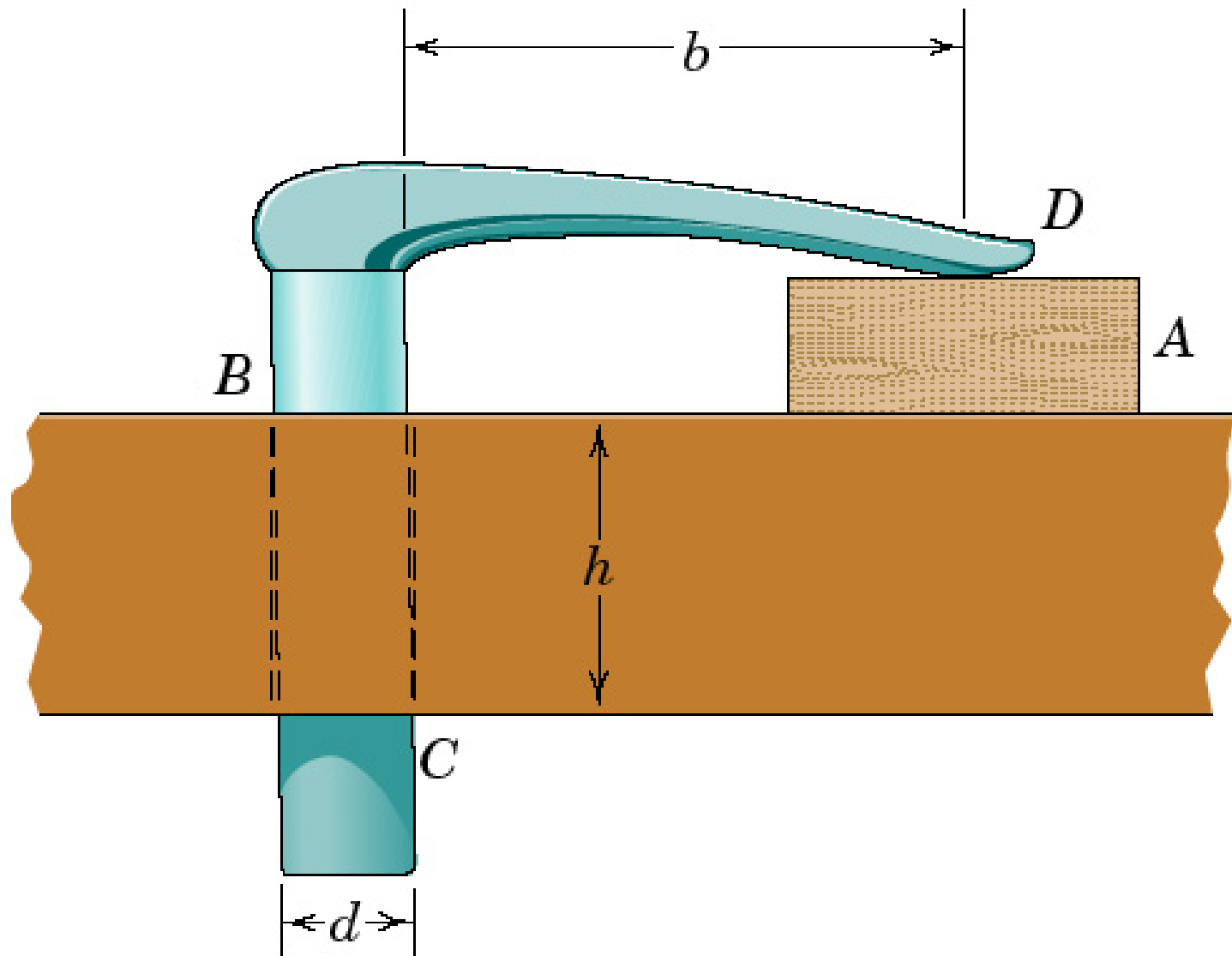


Problem 6/37

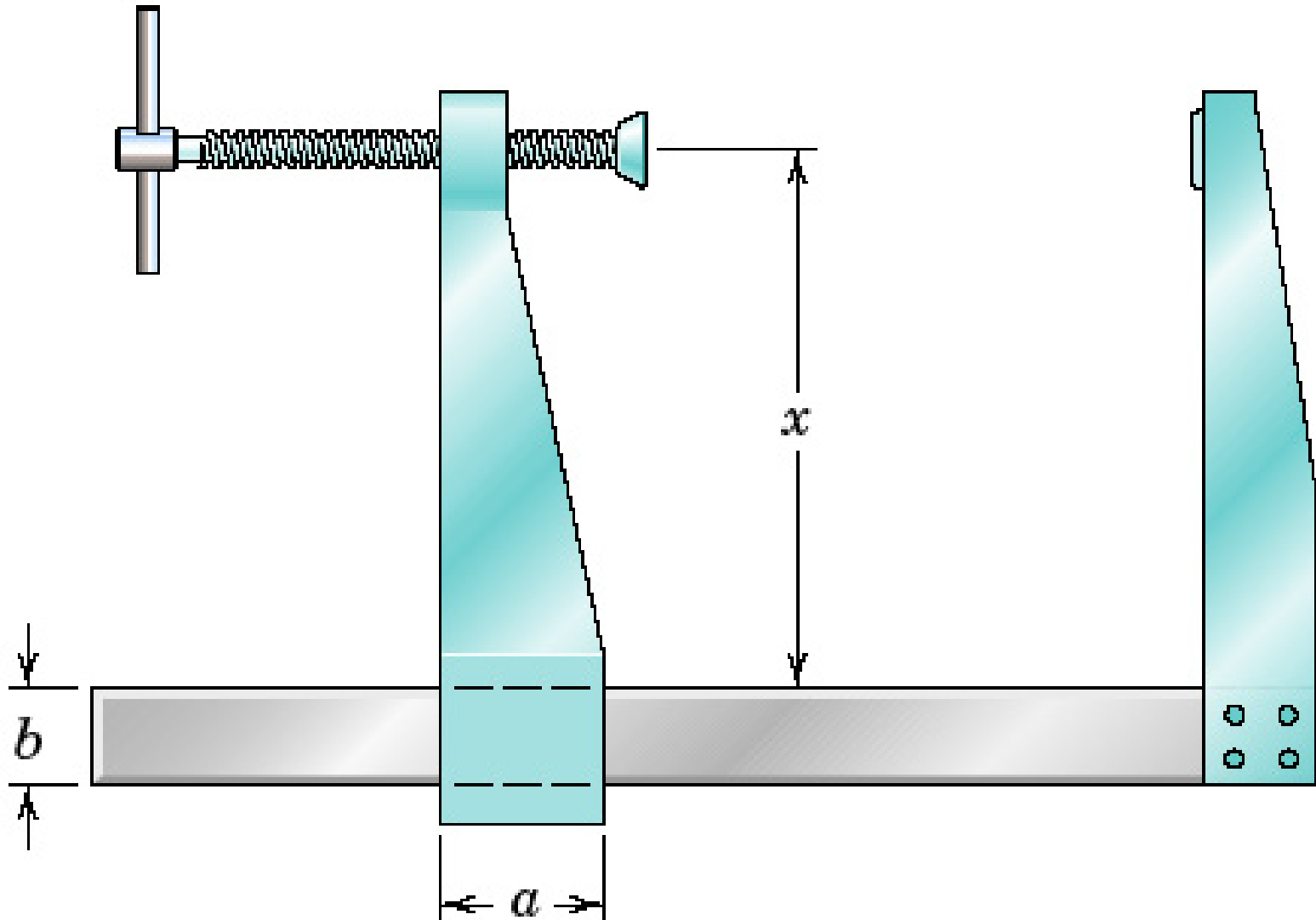


Problem 6/38

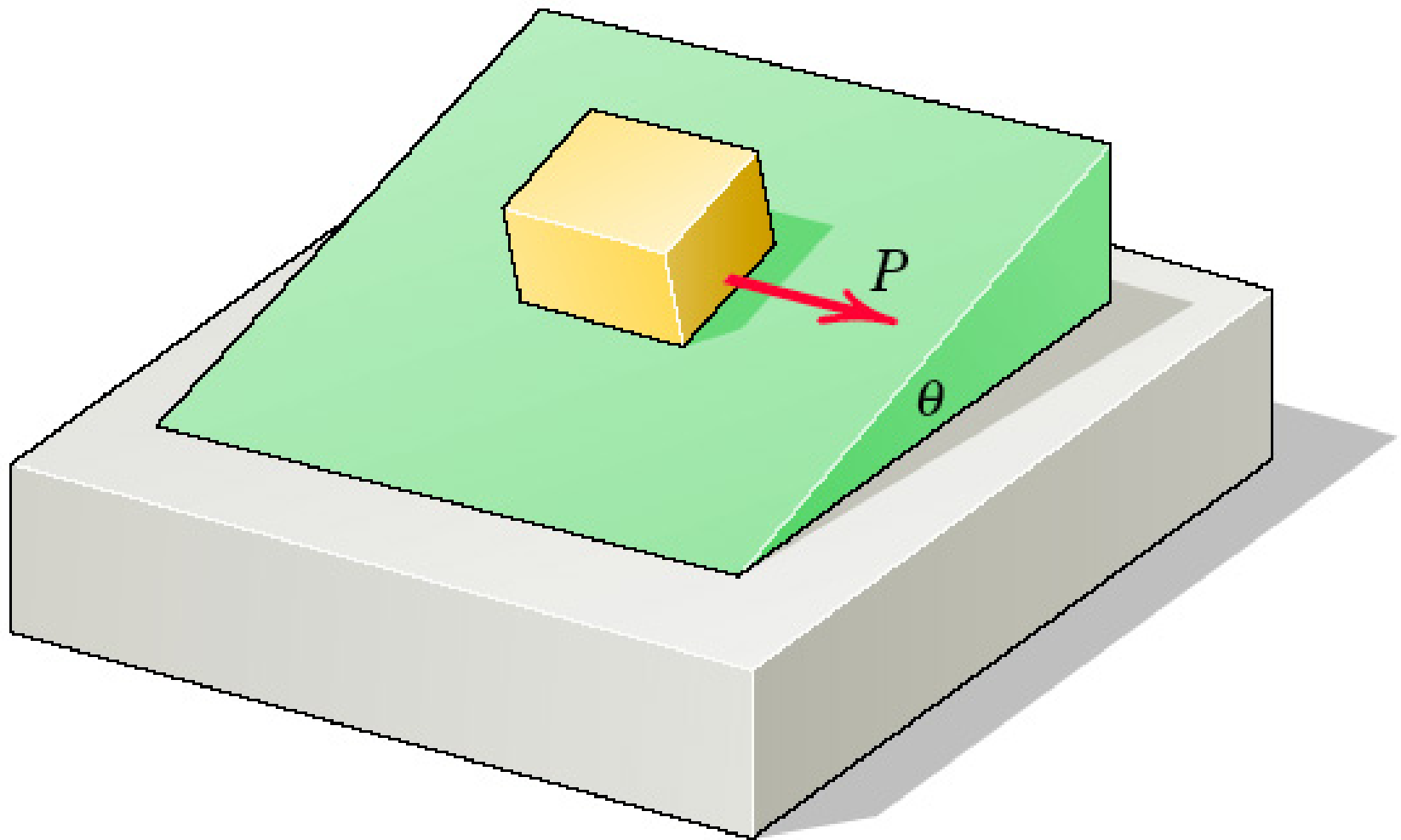
P_06_38



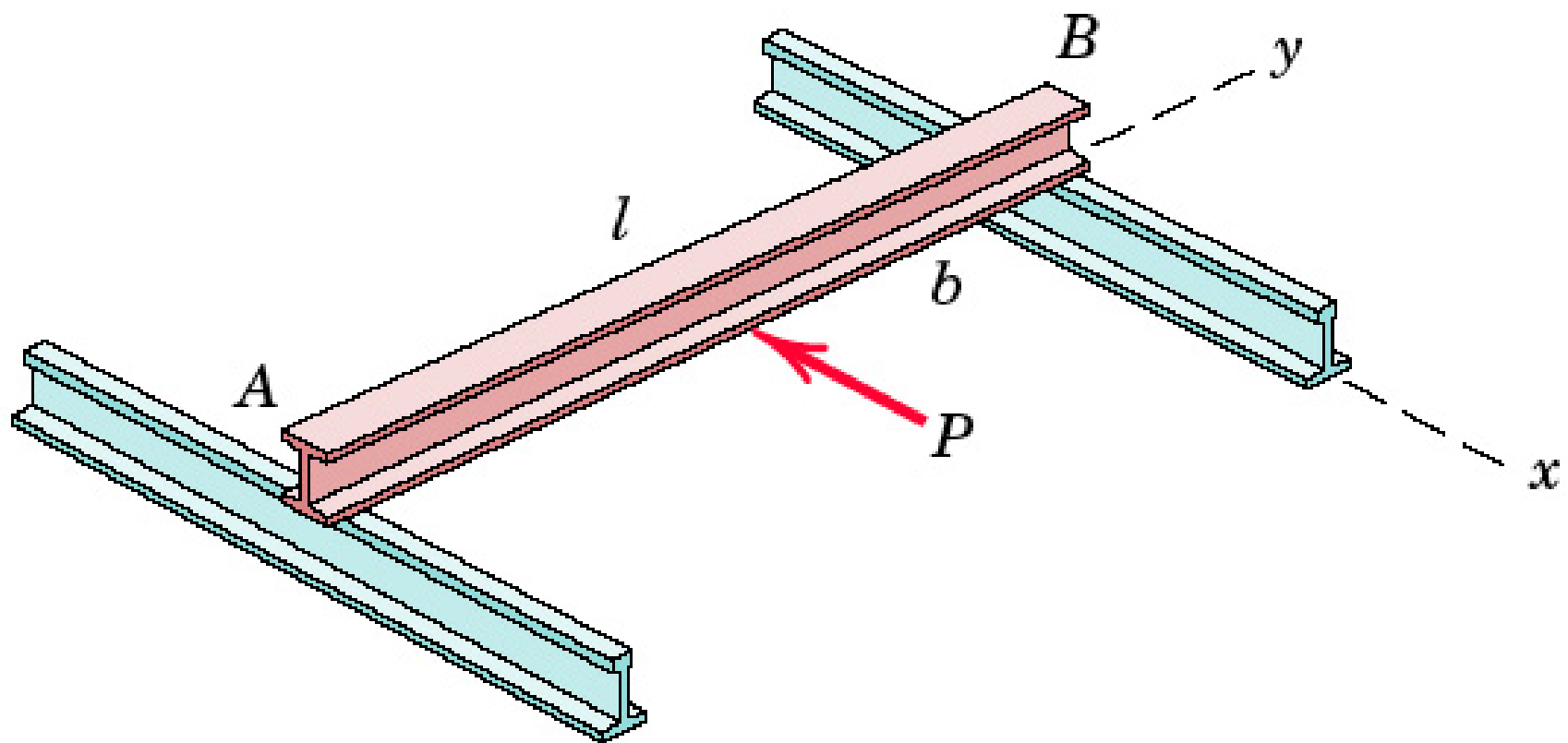
Problem 6/39



Problem 6/40

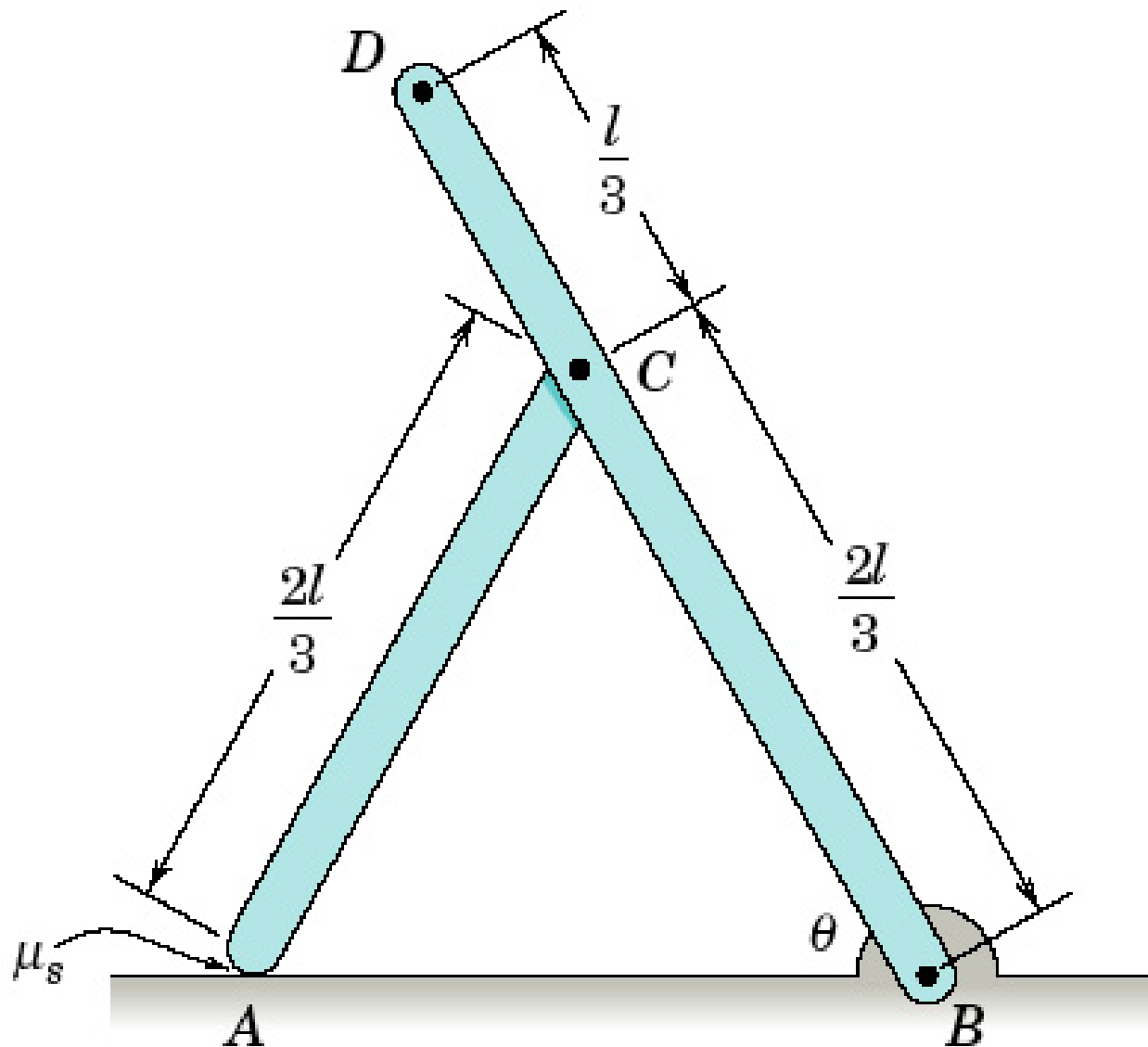


Problem 6/41

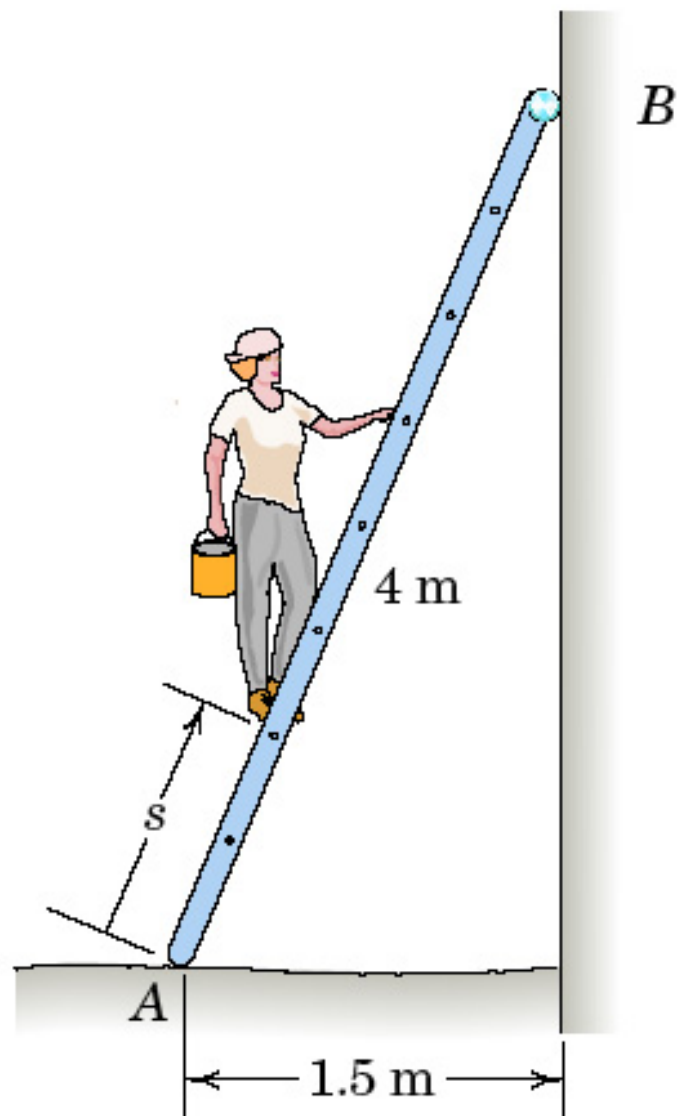


Problem 6/42

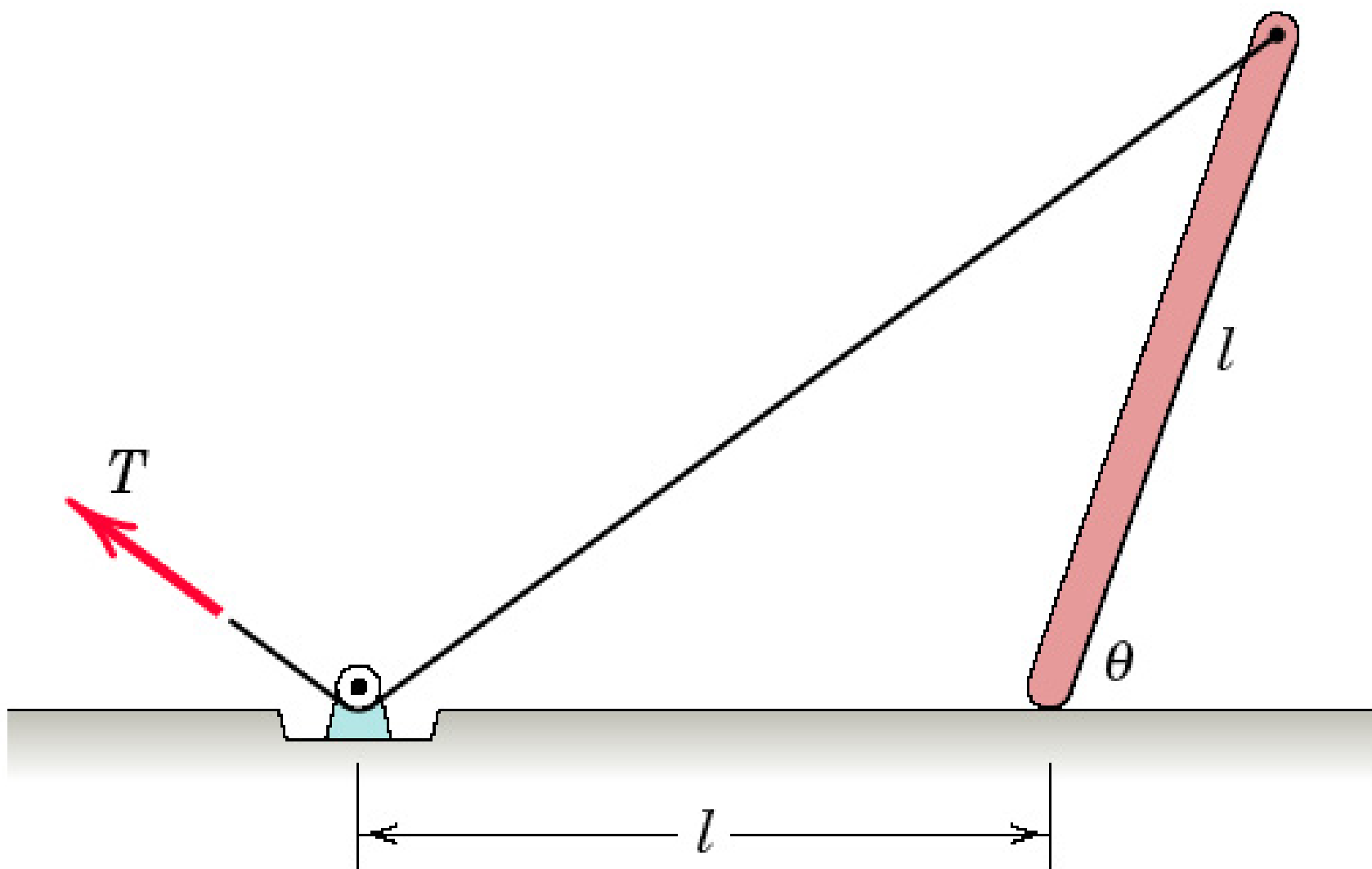
P_06_42



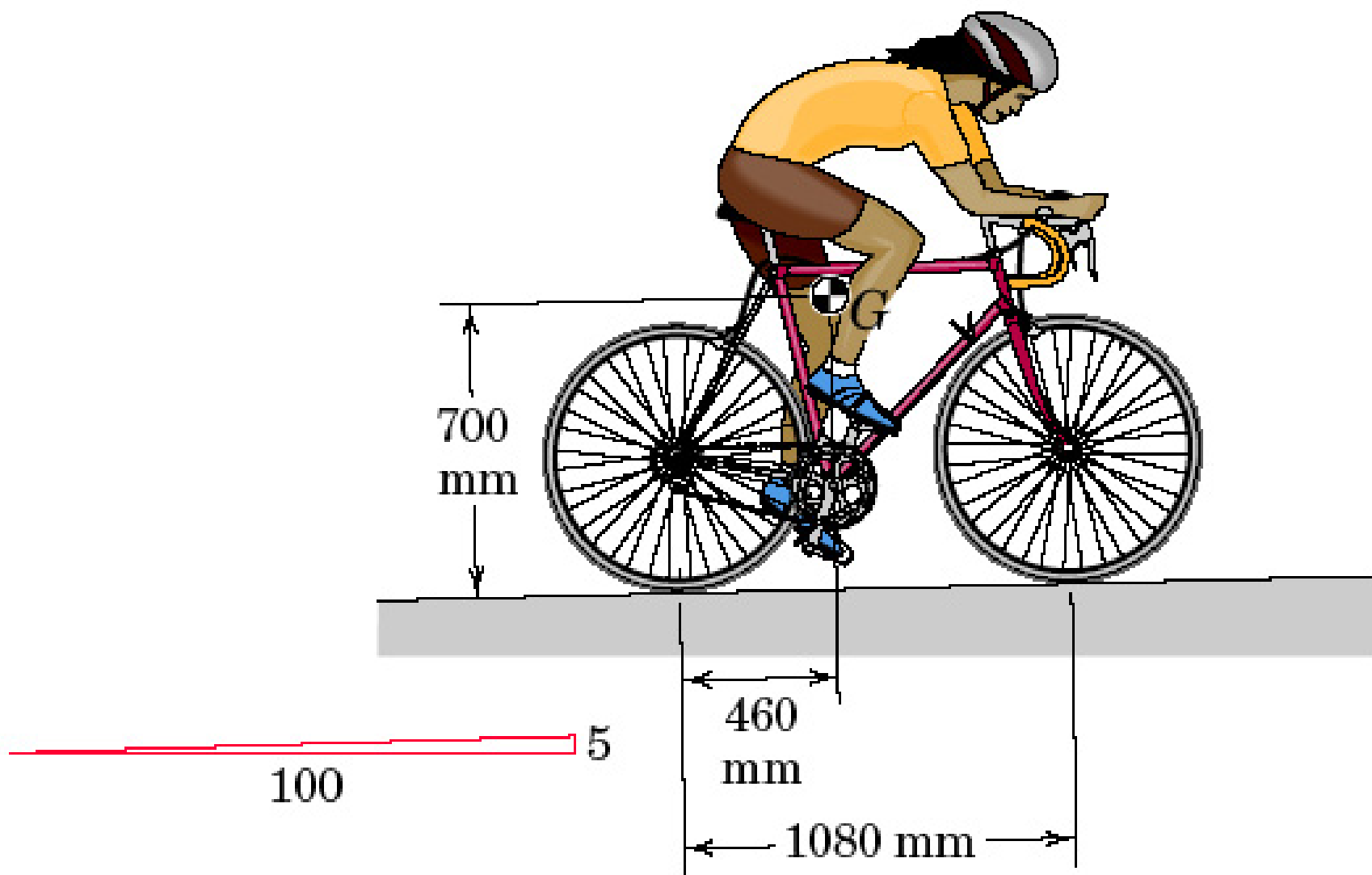
Problem 6/43



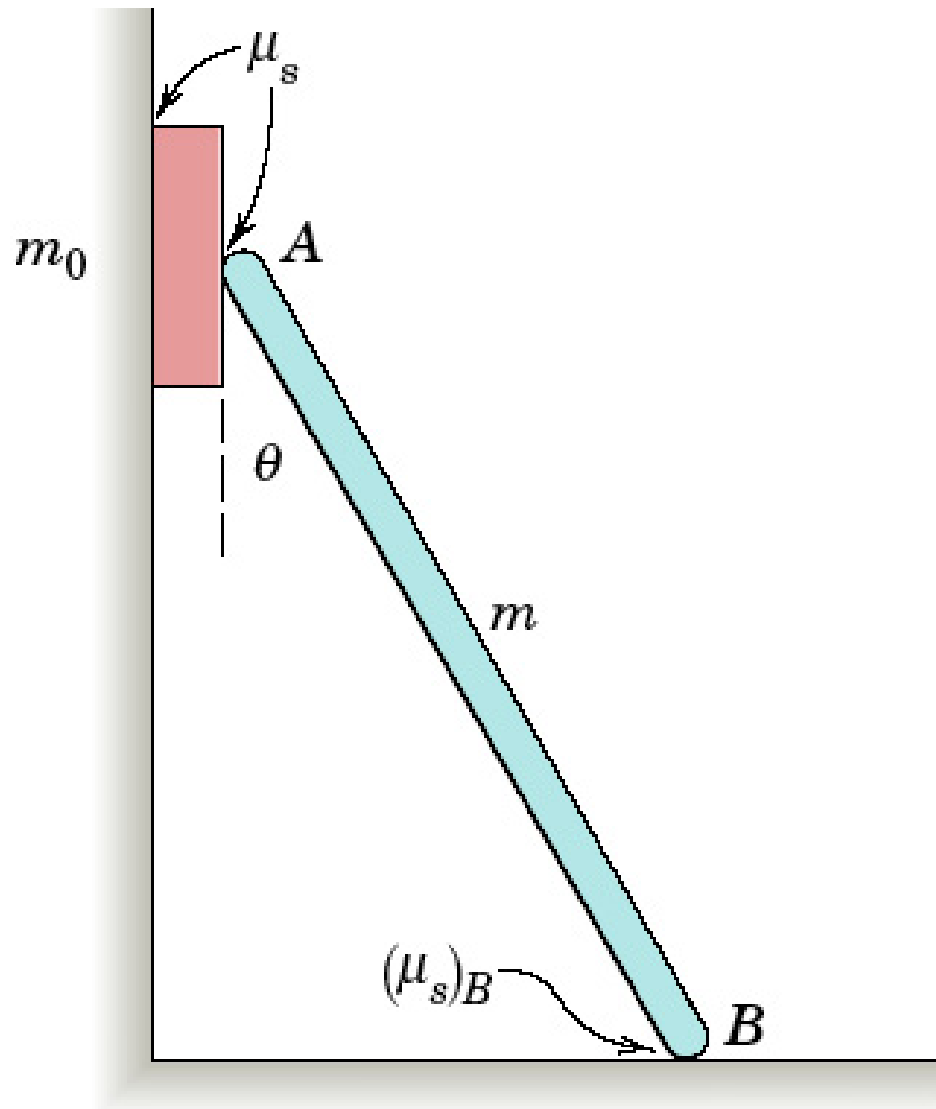
Problem 6/44



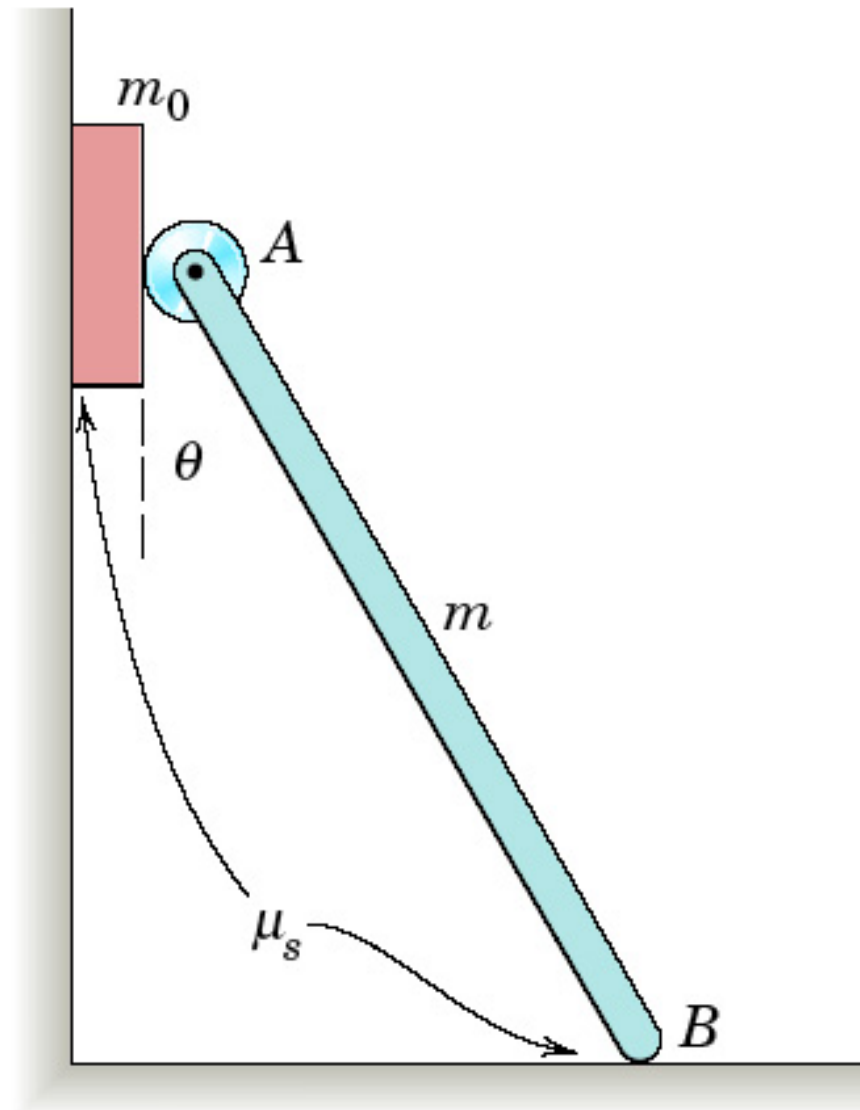
Problem 6/45



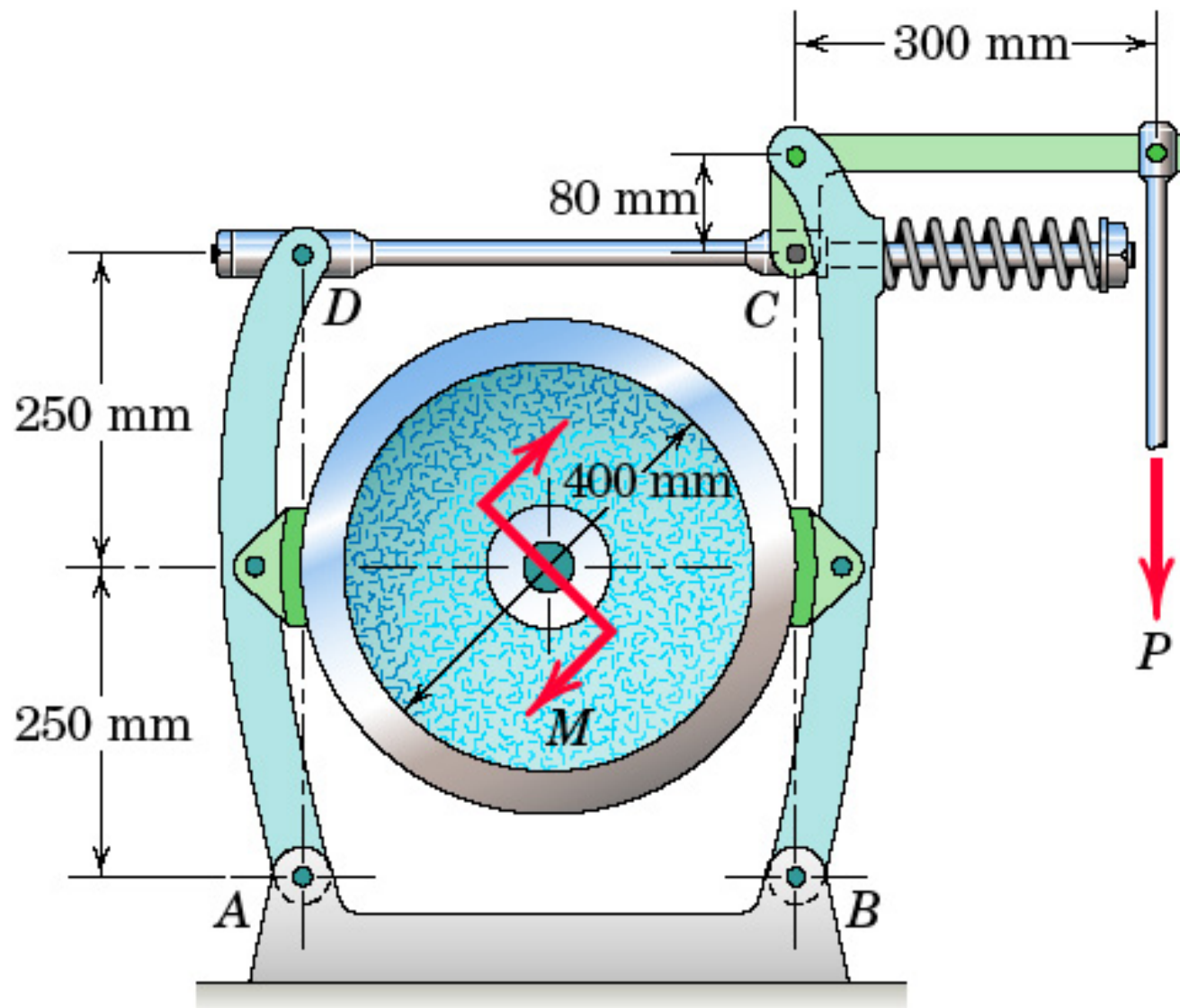
Problem 6/46



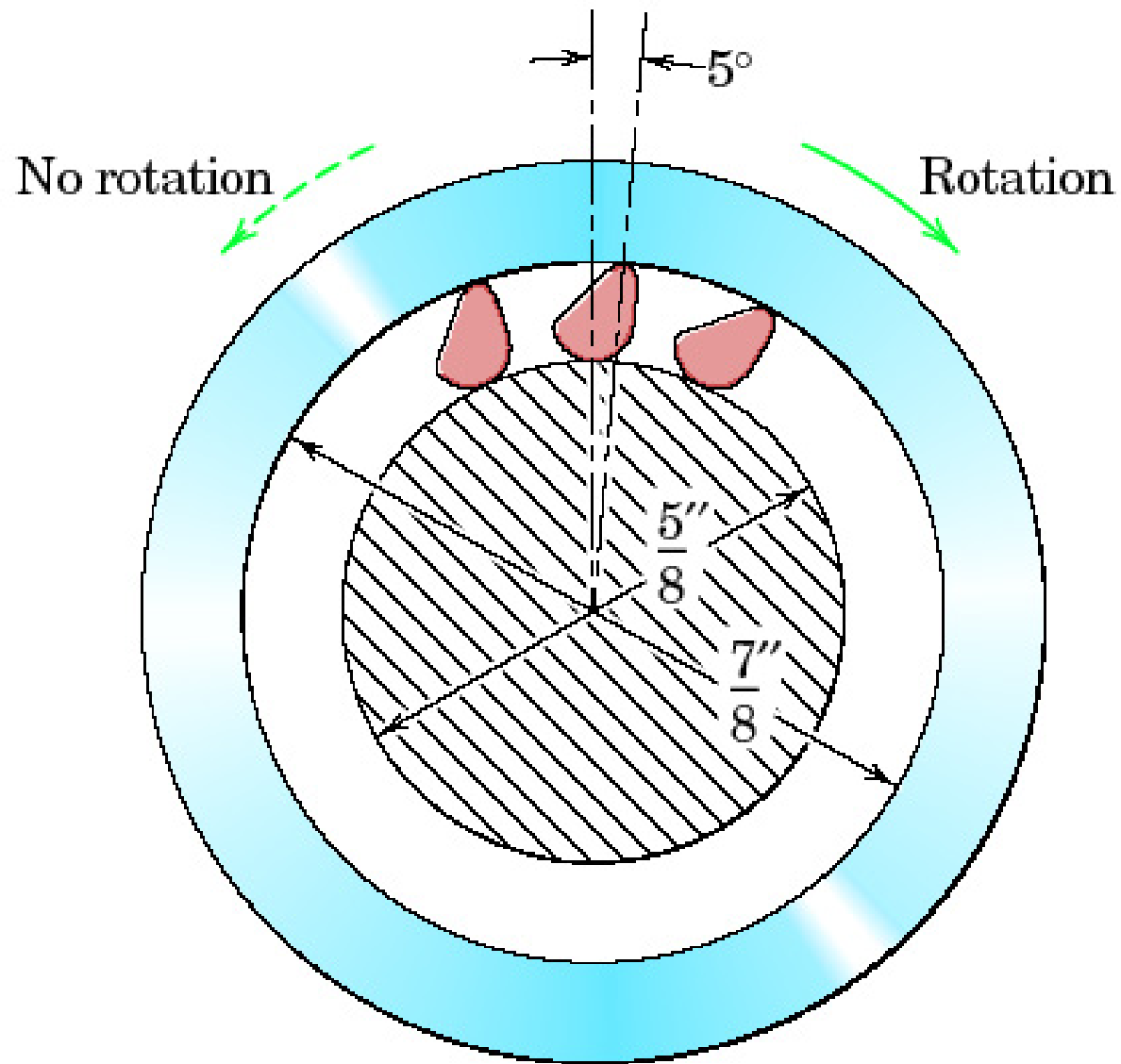
Problem 6/47



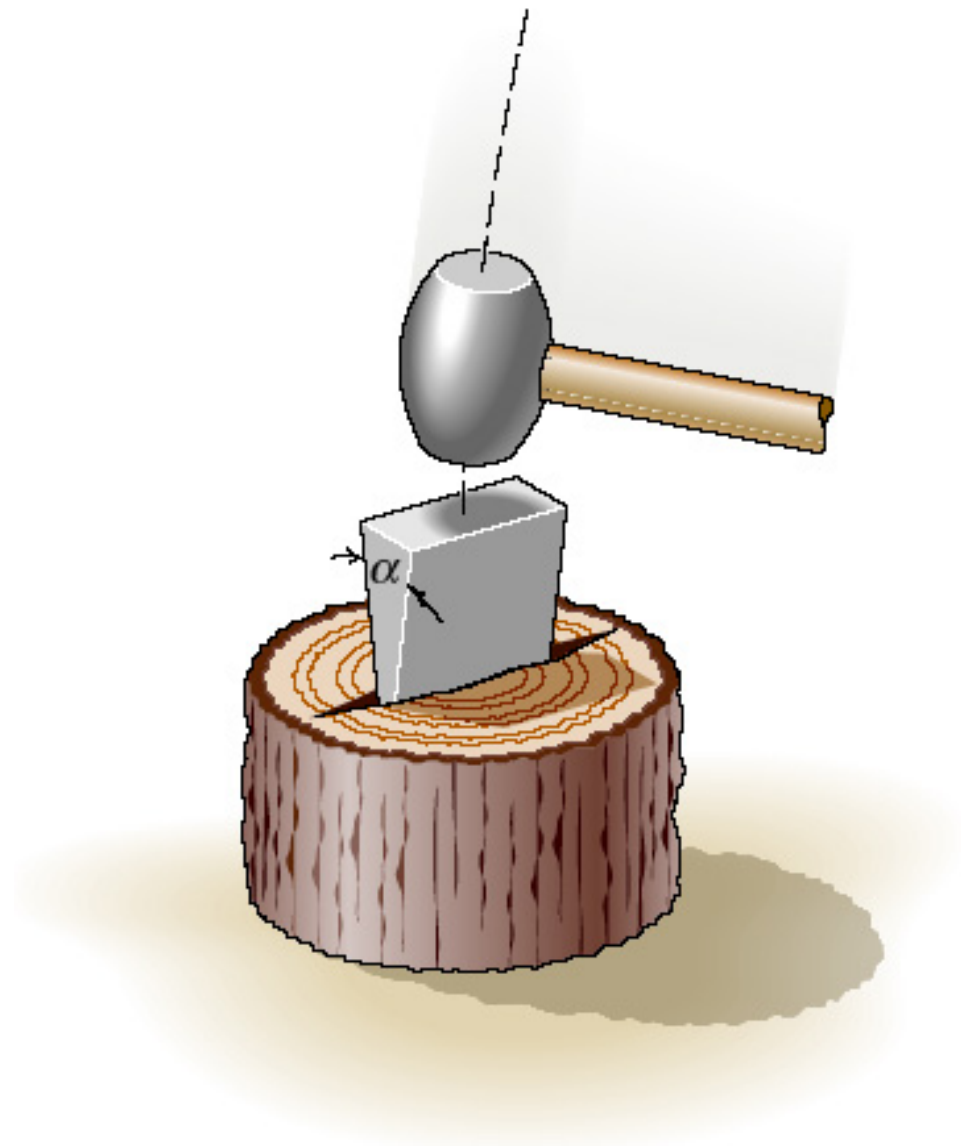
Problem 6/48



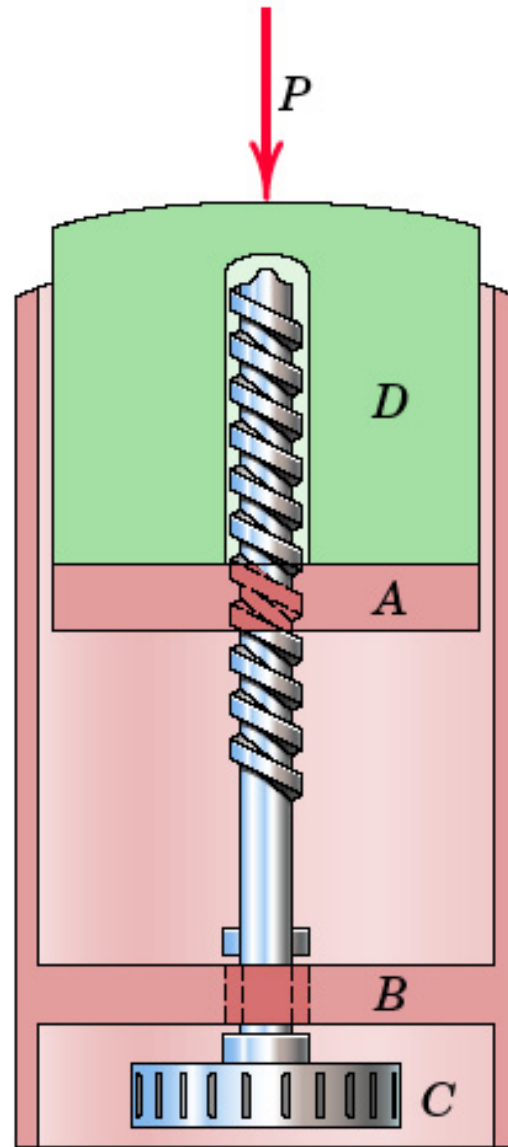
Problem 6/49



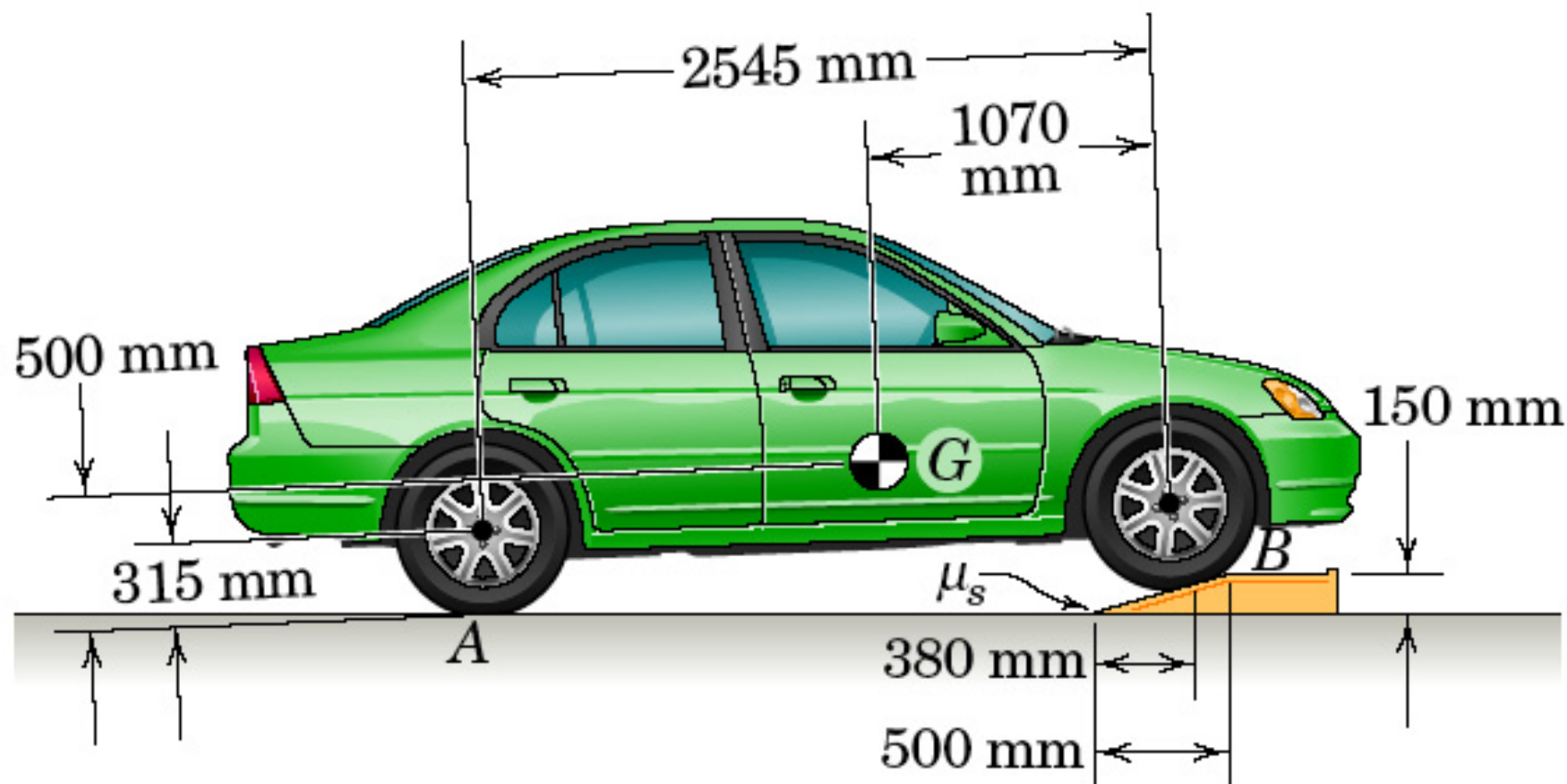
Problem 6/50



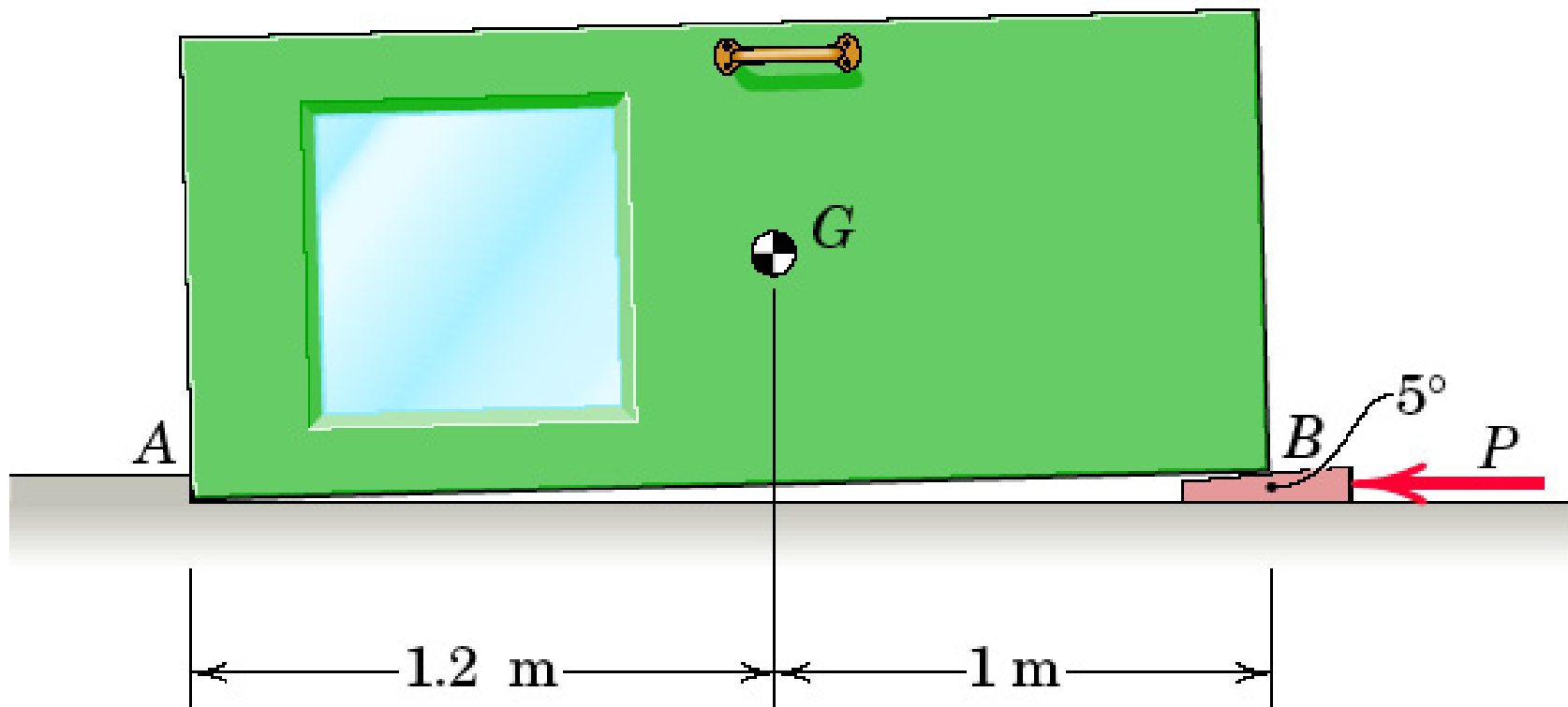
Problem 6/51



Problem 6/52

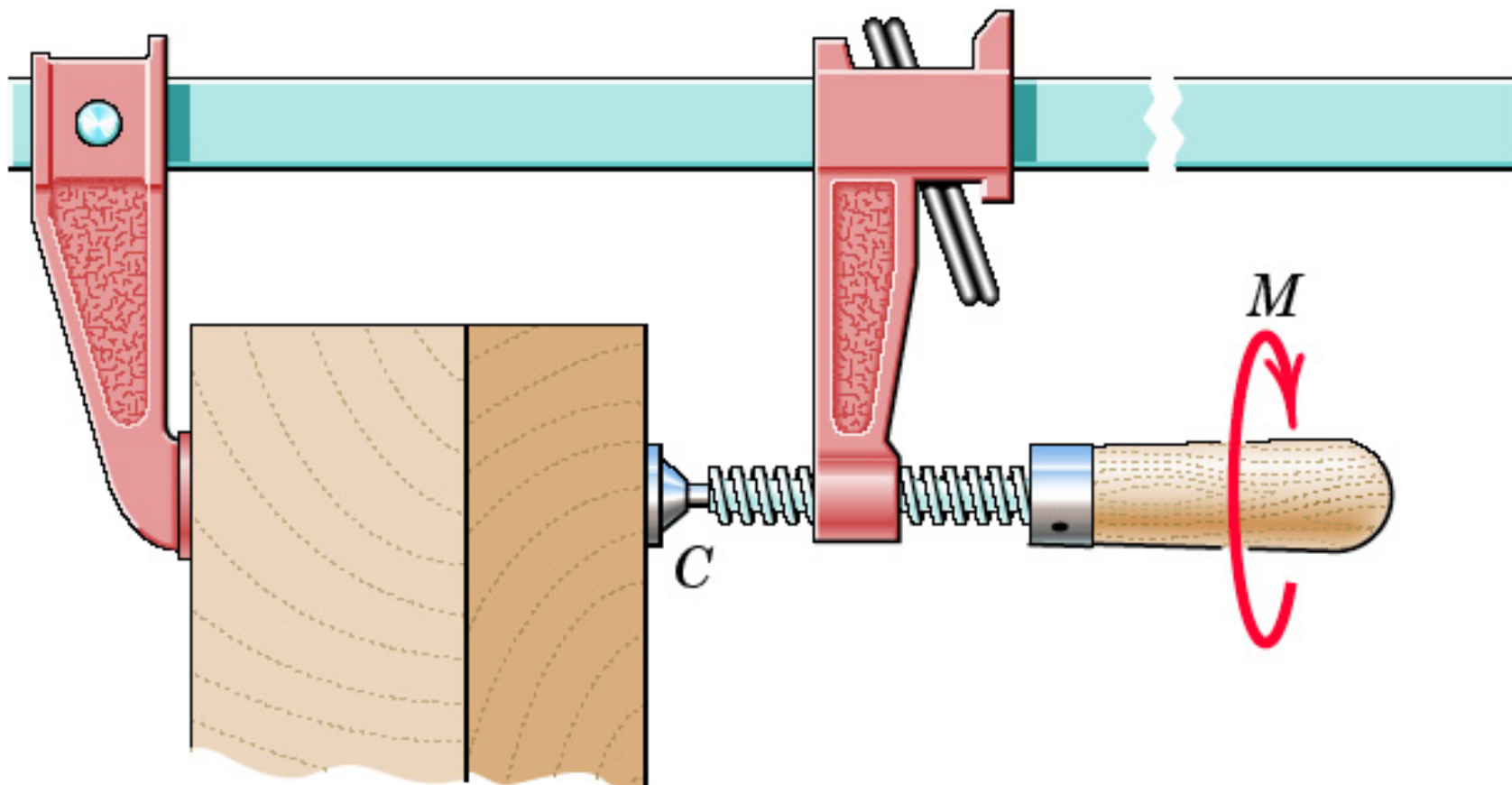


Problem 6/53



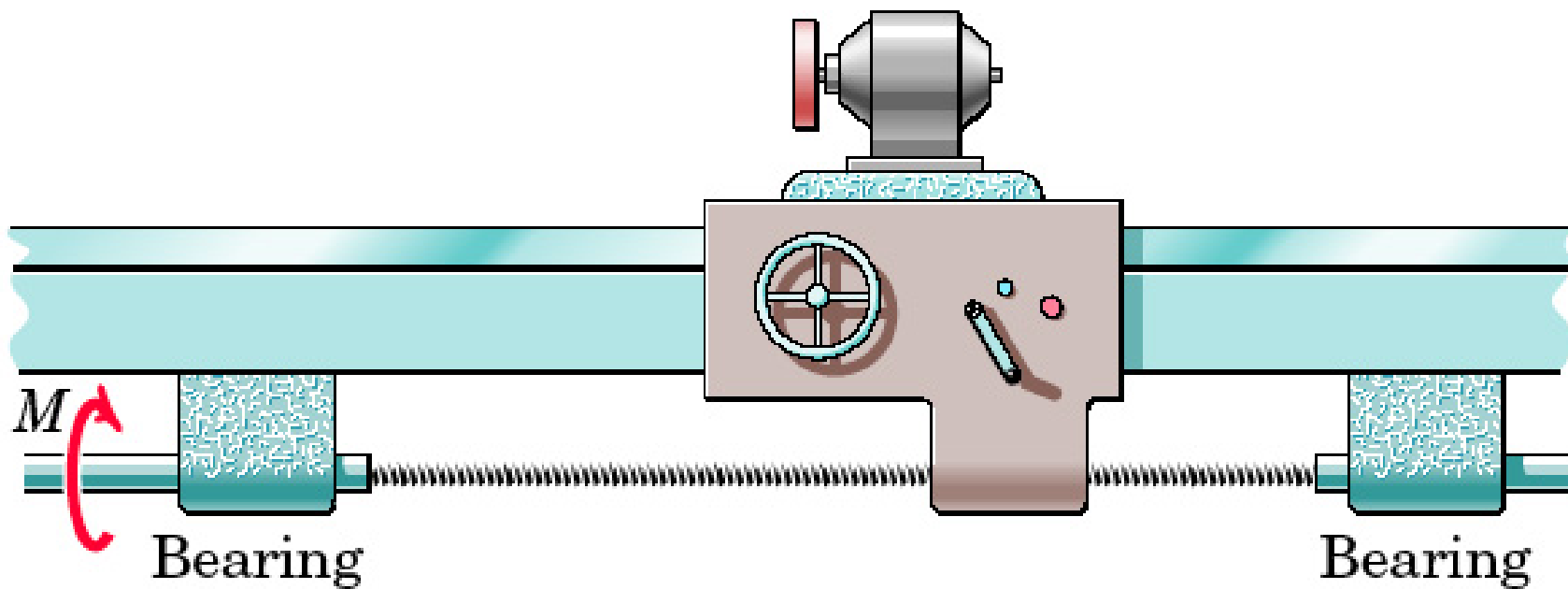
Problem 6/54

P_06_54



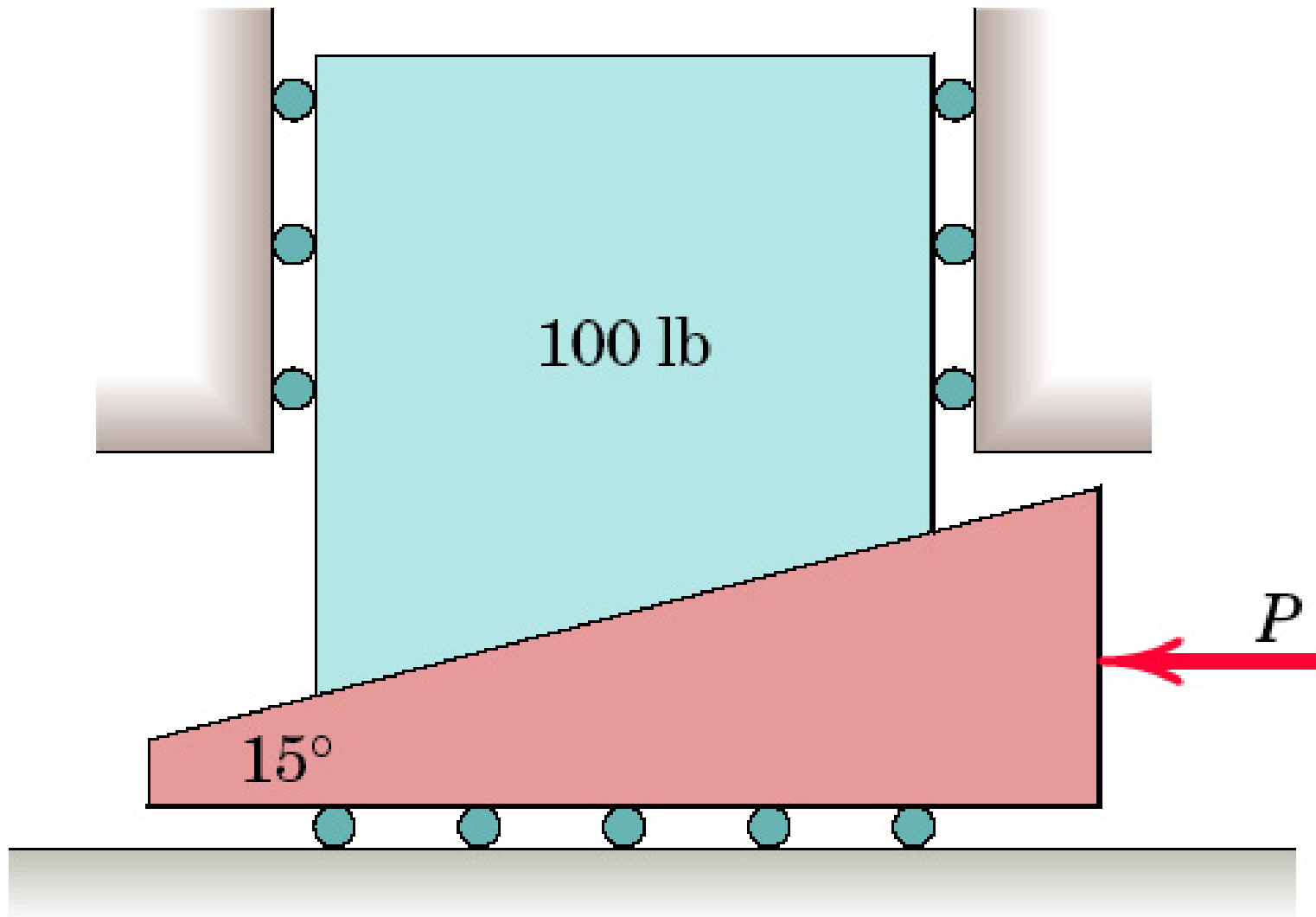
Problem 6/56

P_06_56

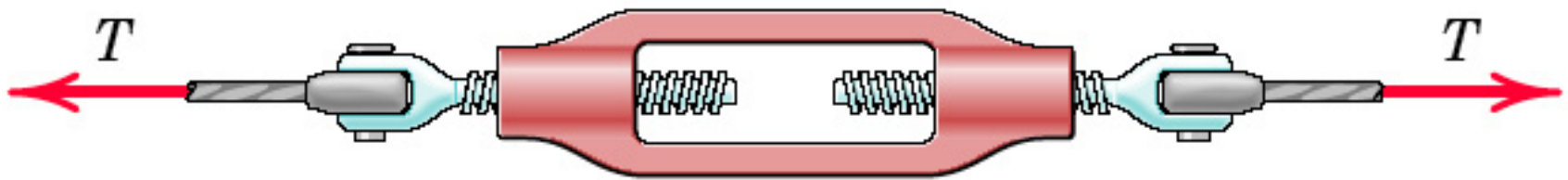


Problem 6/57

P_06_57

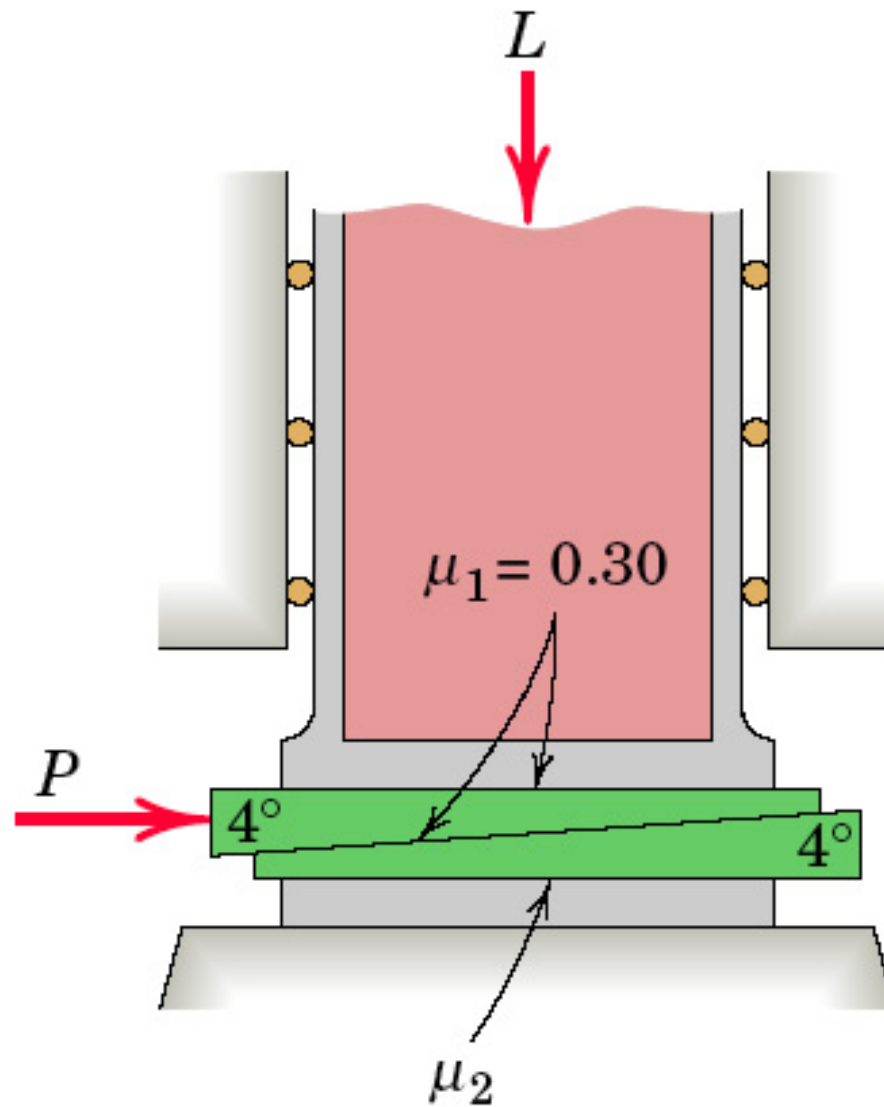


Problem 6/58

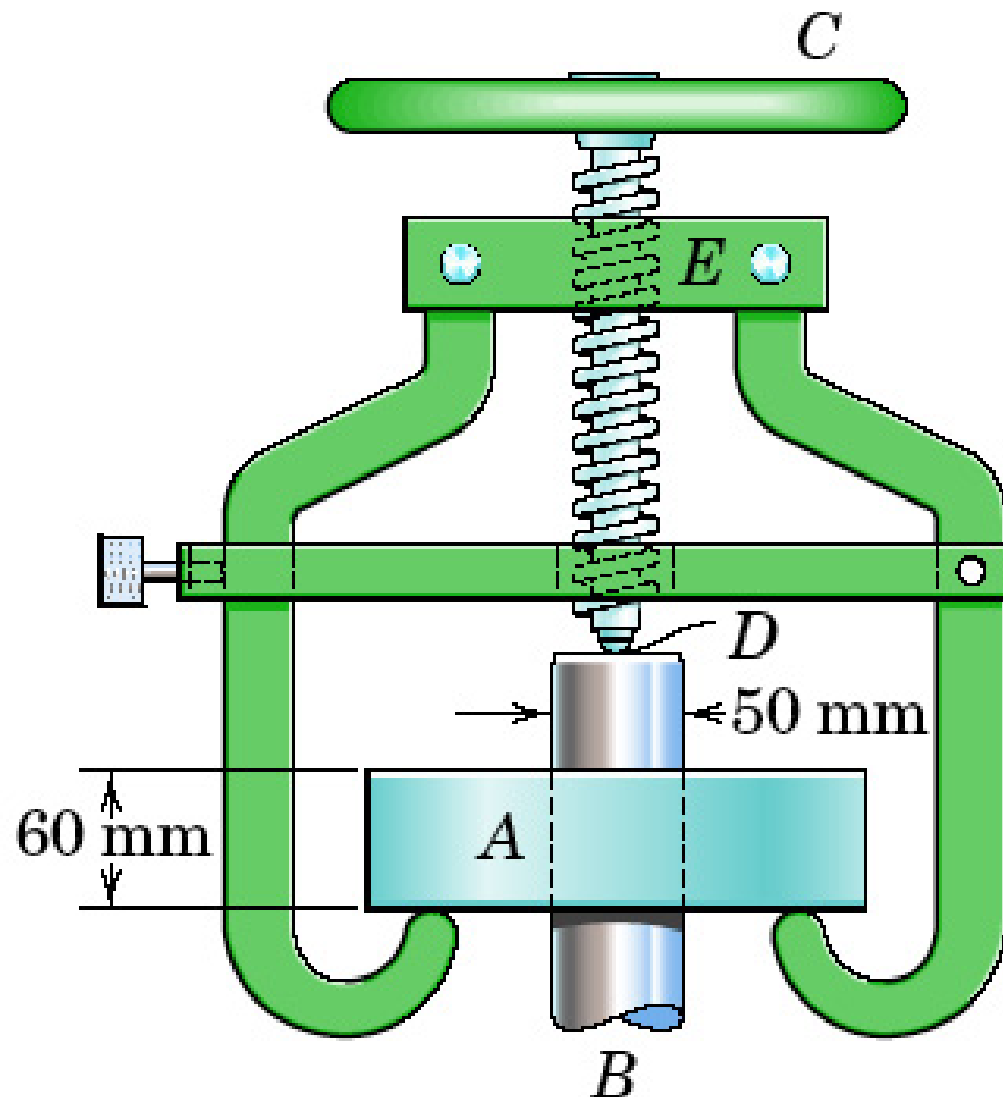


Problem 6/60

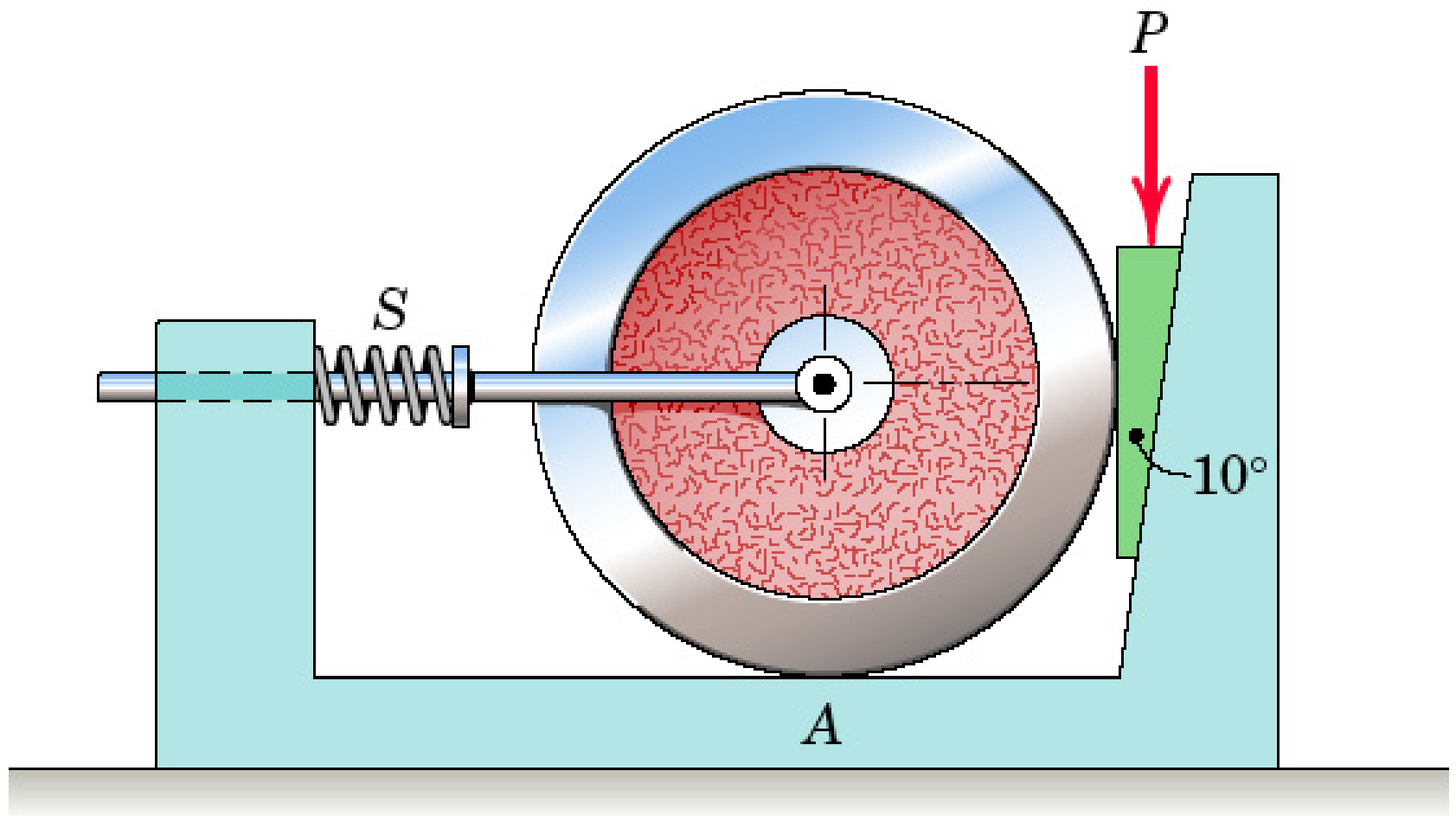
P_06_60



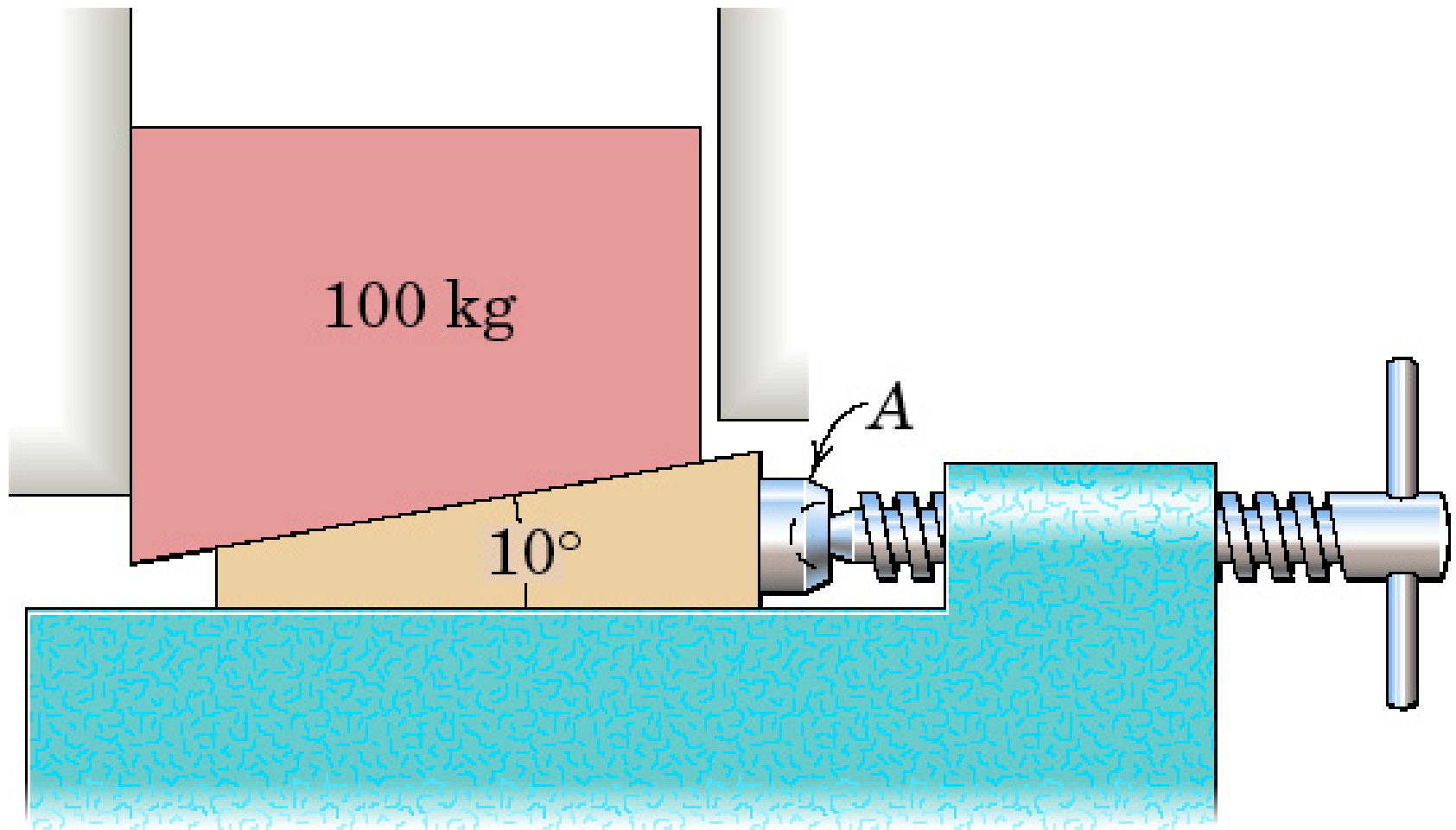
Problem 6/61



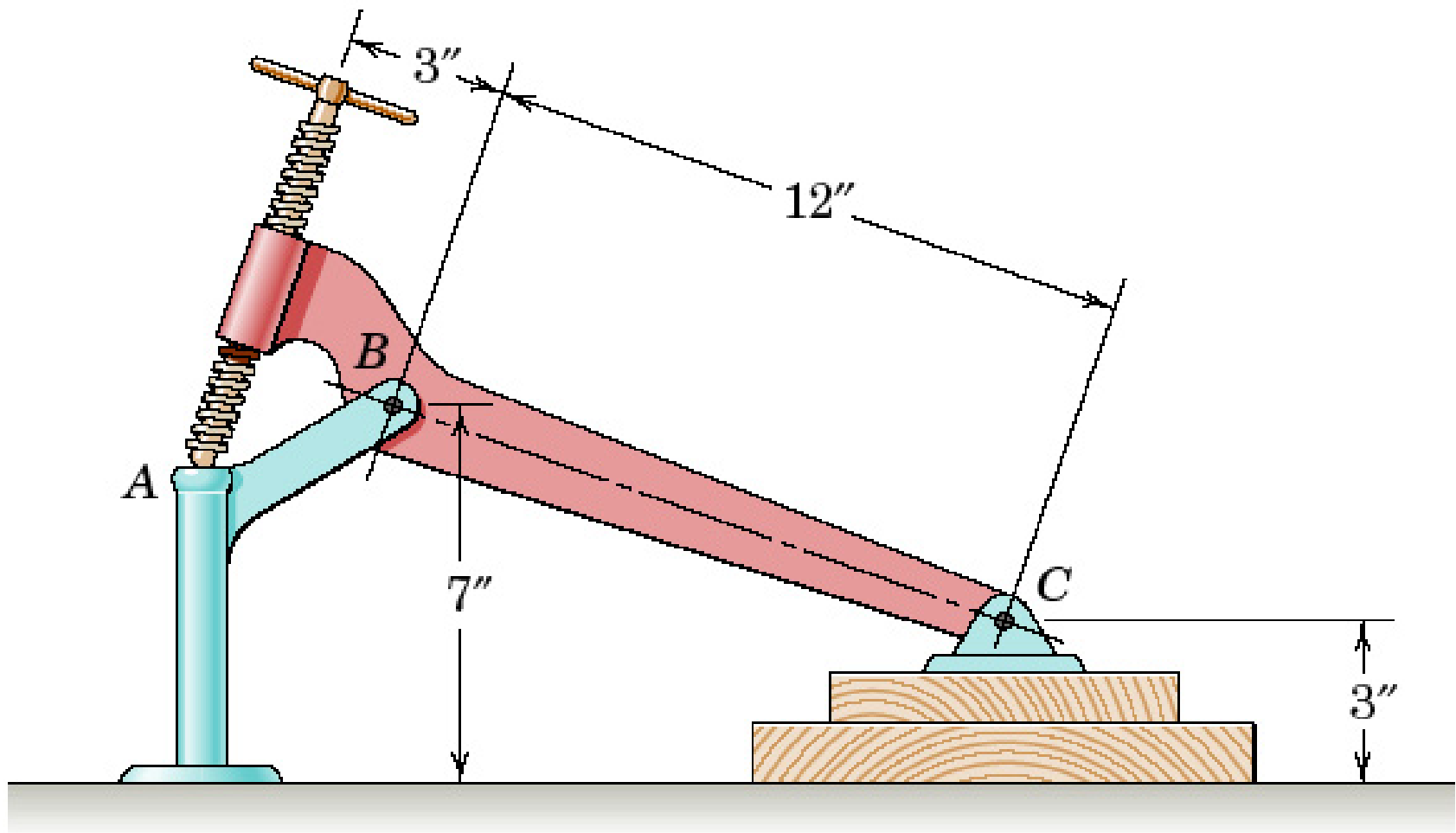
Problem 6/62



Problem 6/63

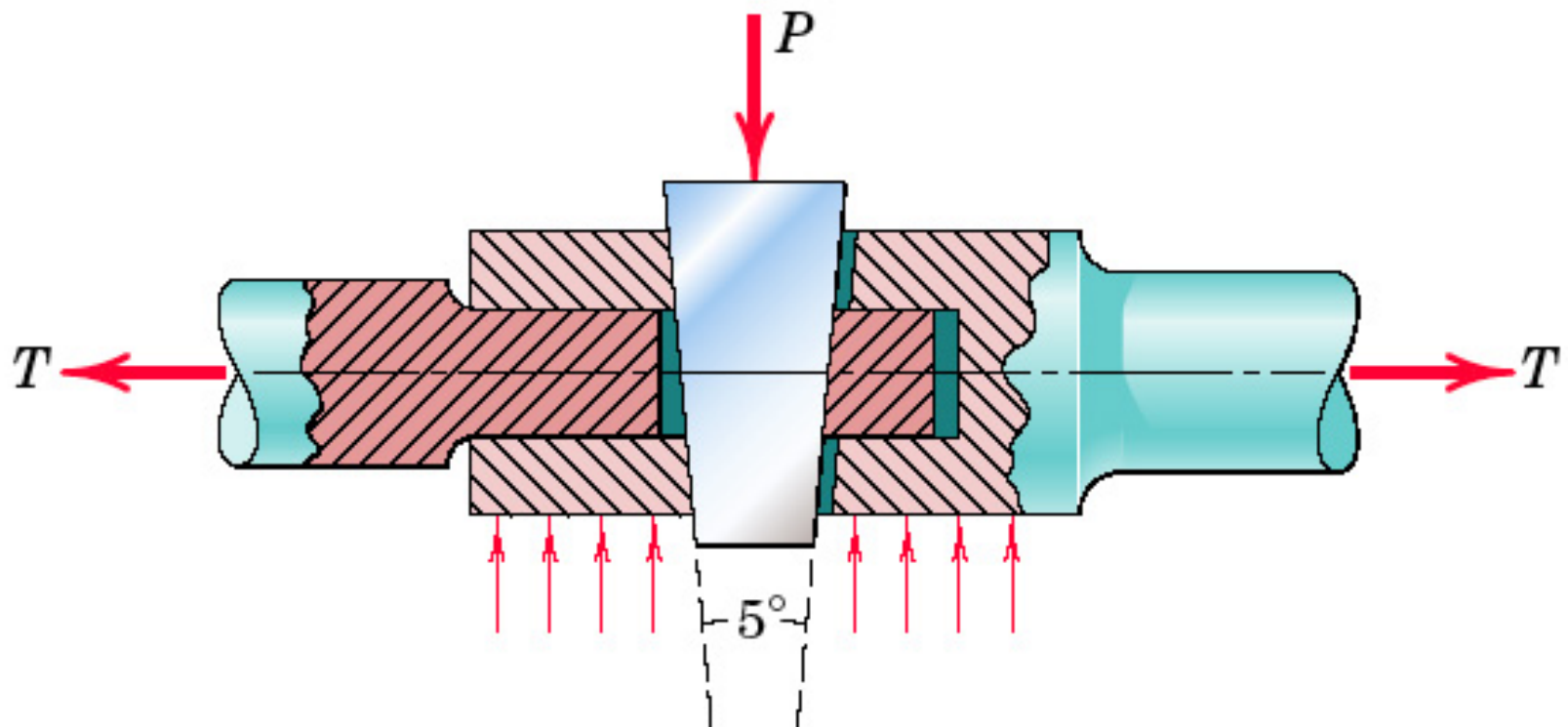
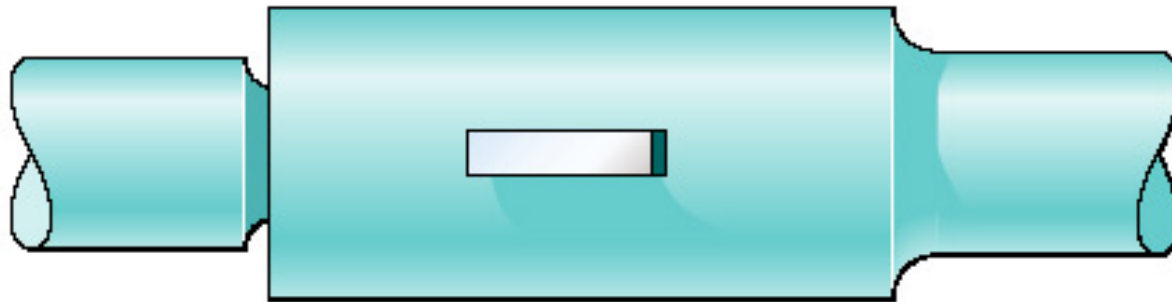


Problem 6/65

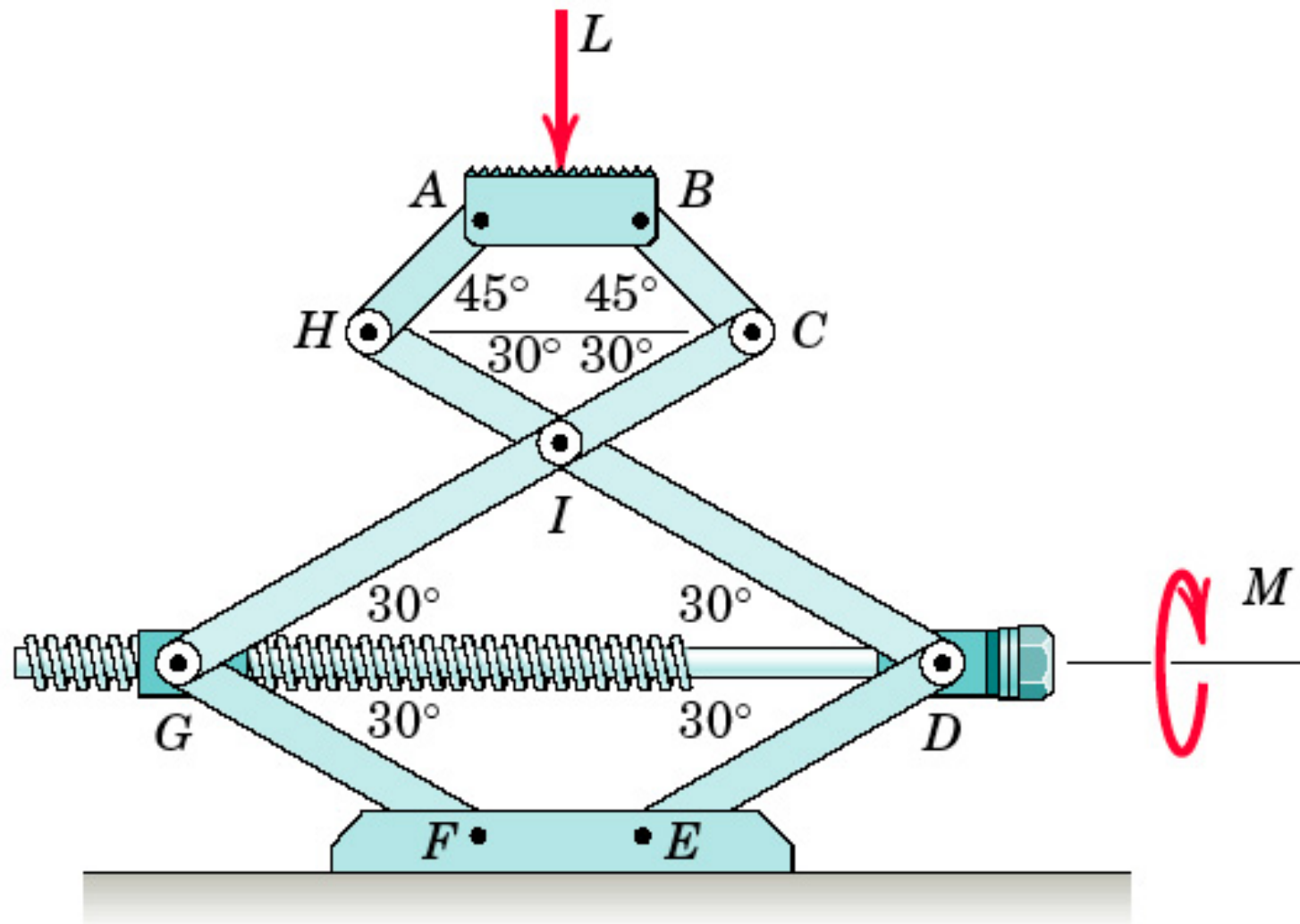


Problem 6/67

P_06_67

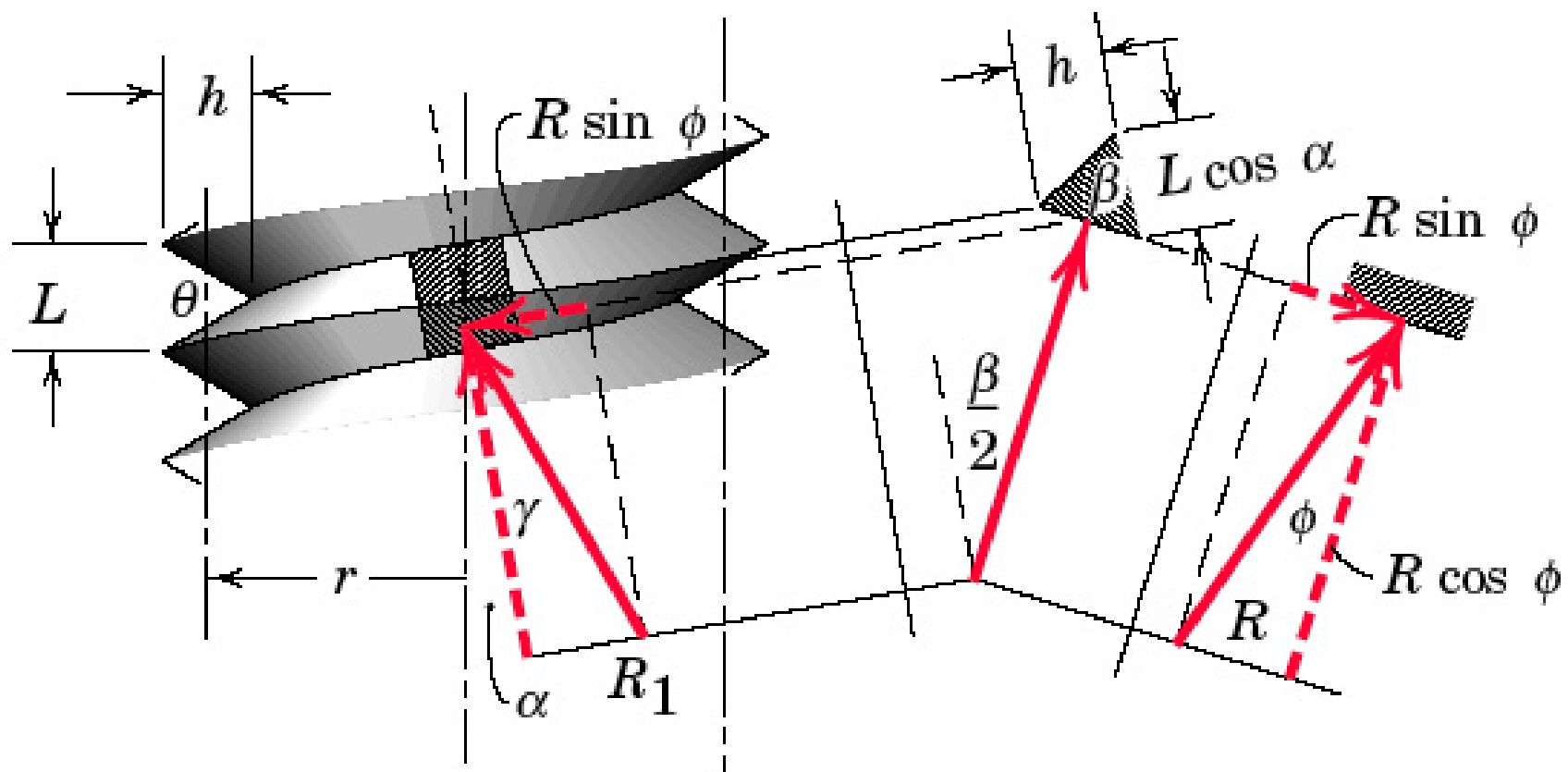


Problem 6/68

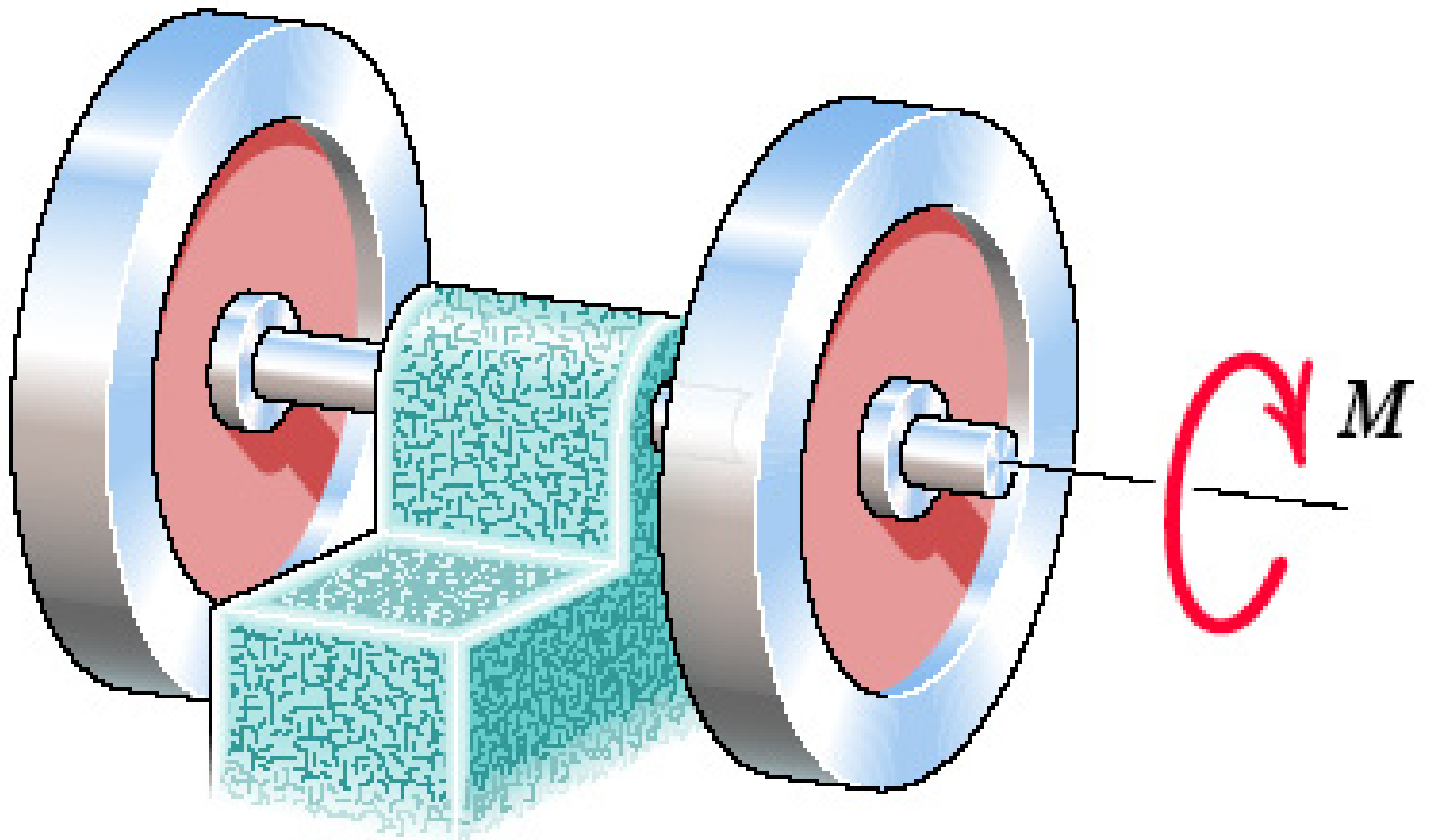


$$\overline{GI} = \overline{DI} = 160 \text{ mm}, \overline{HI} = \overline{CI} = 80 \text{ mm}$$

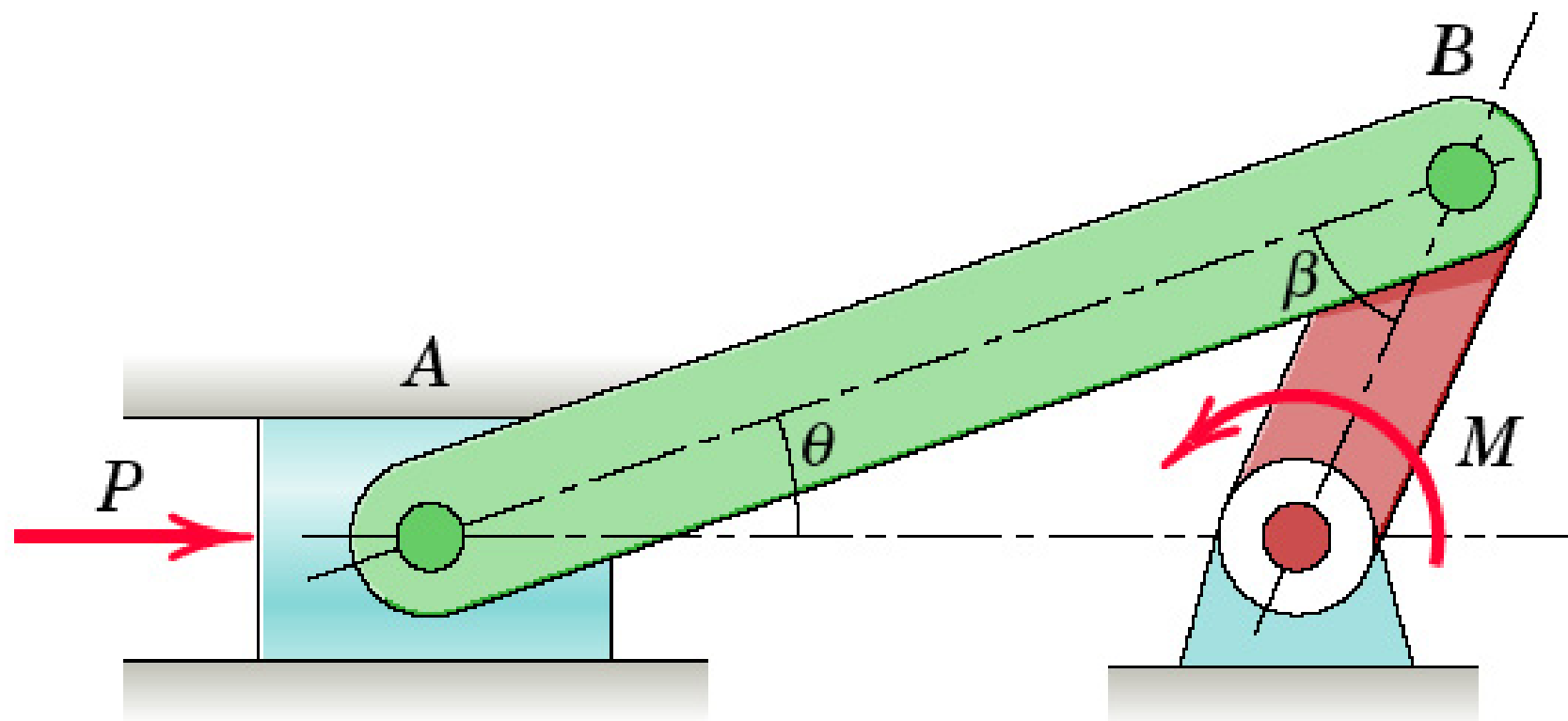
Problem 6/69



Problem 6/70

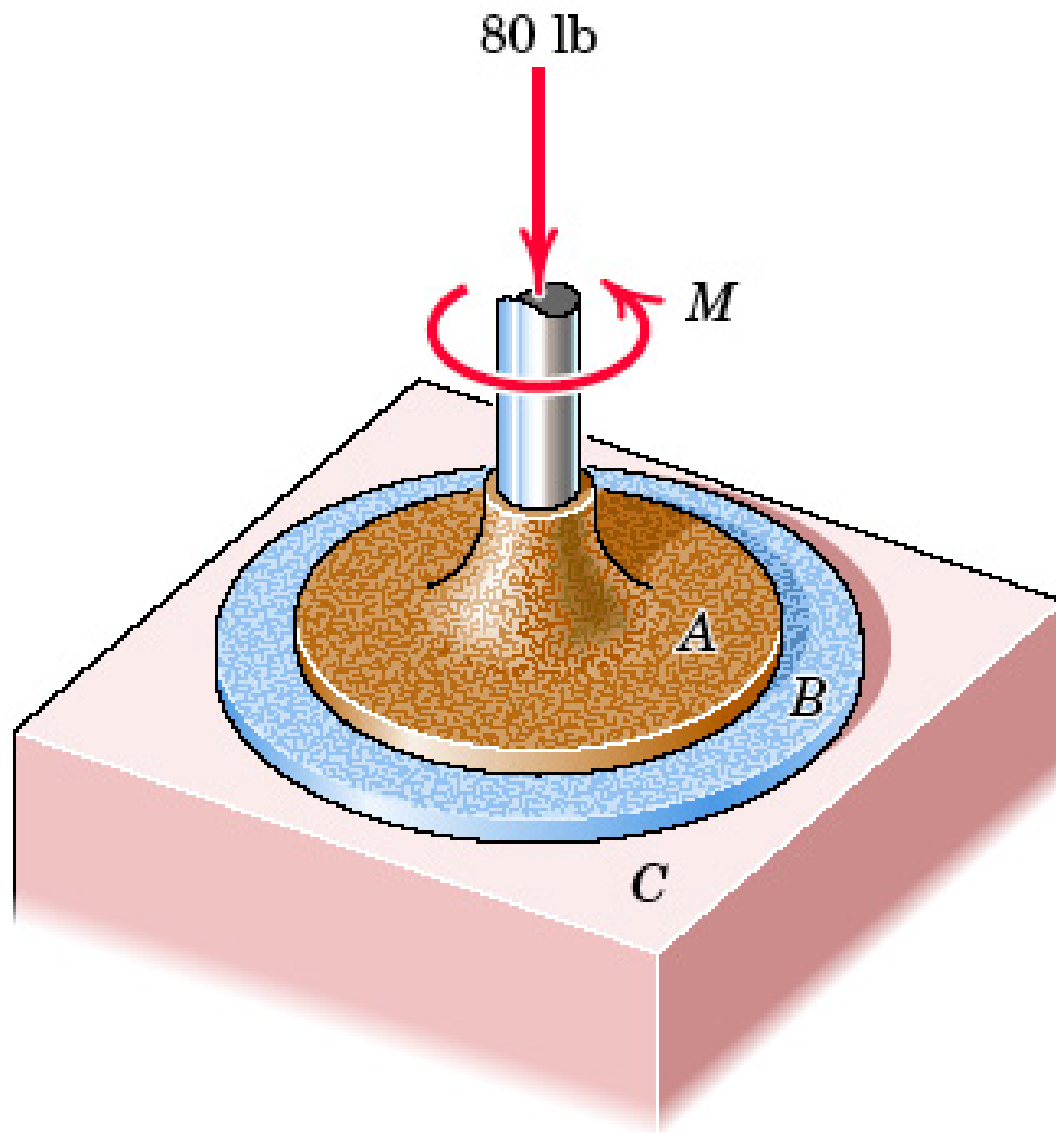


Problem 6/71

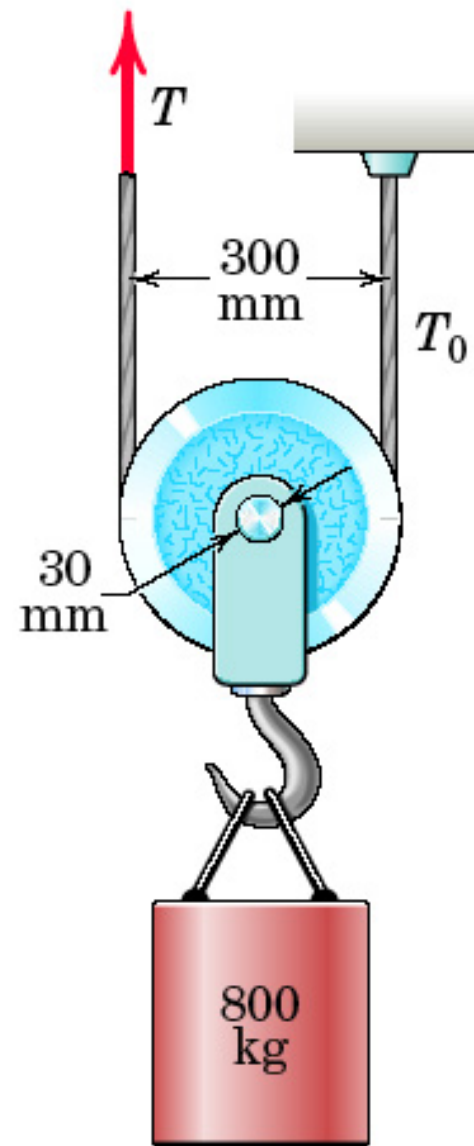


Problem 6/72

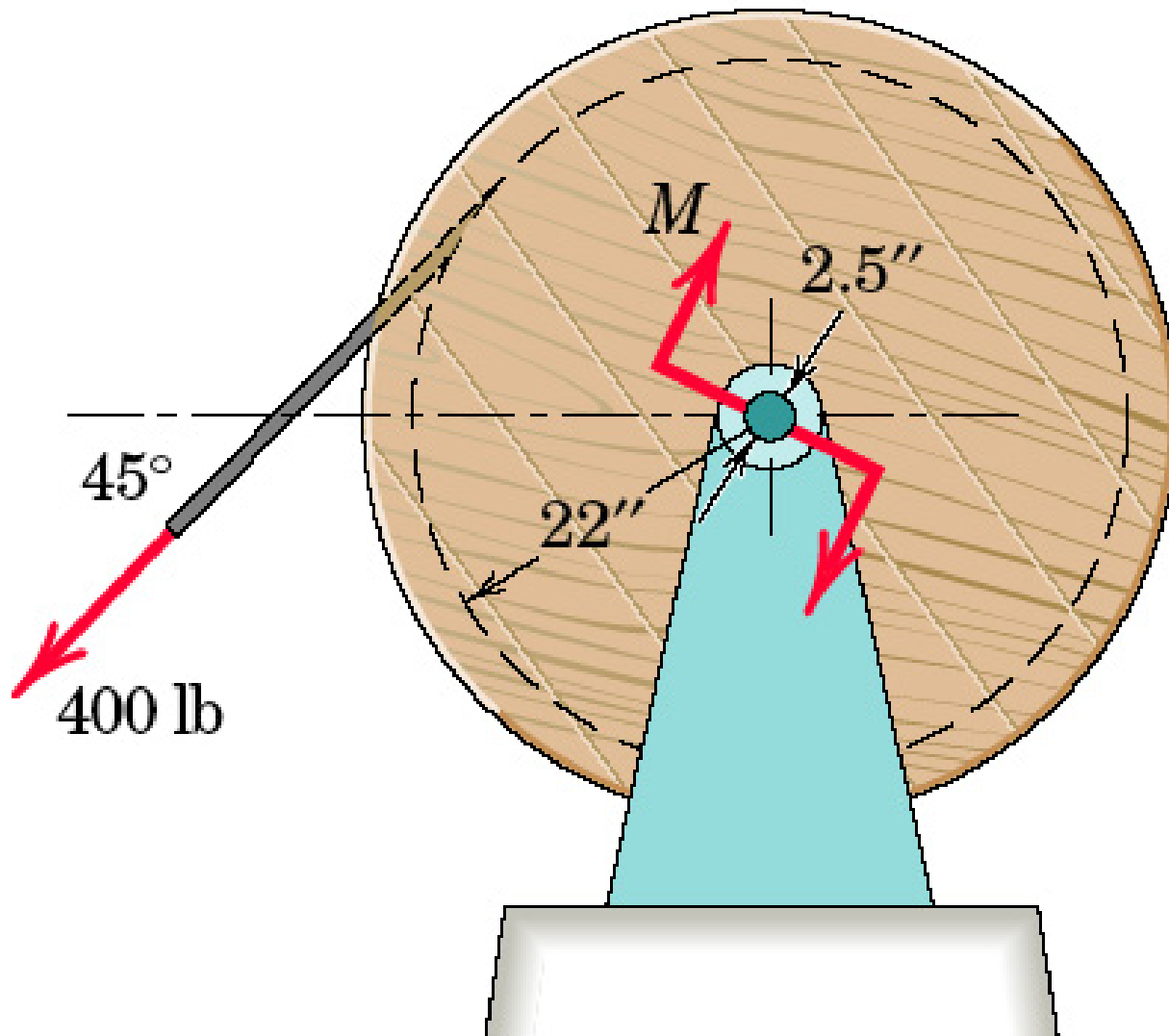
P_06_72



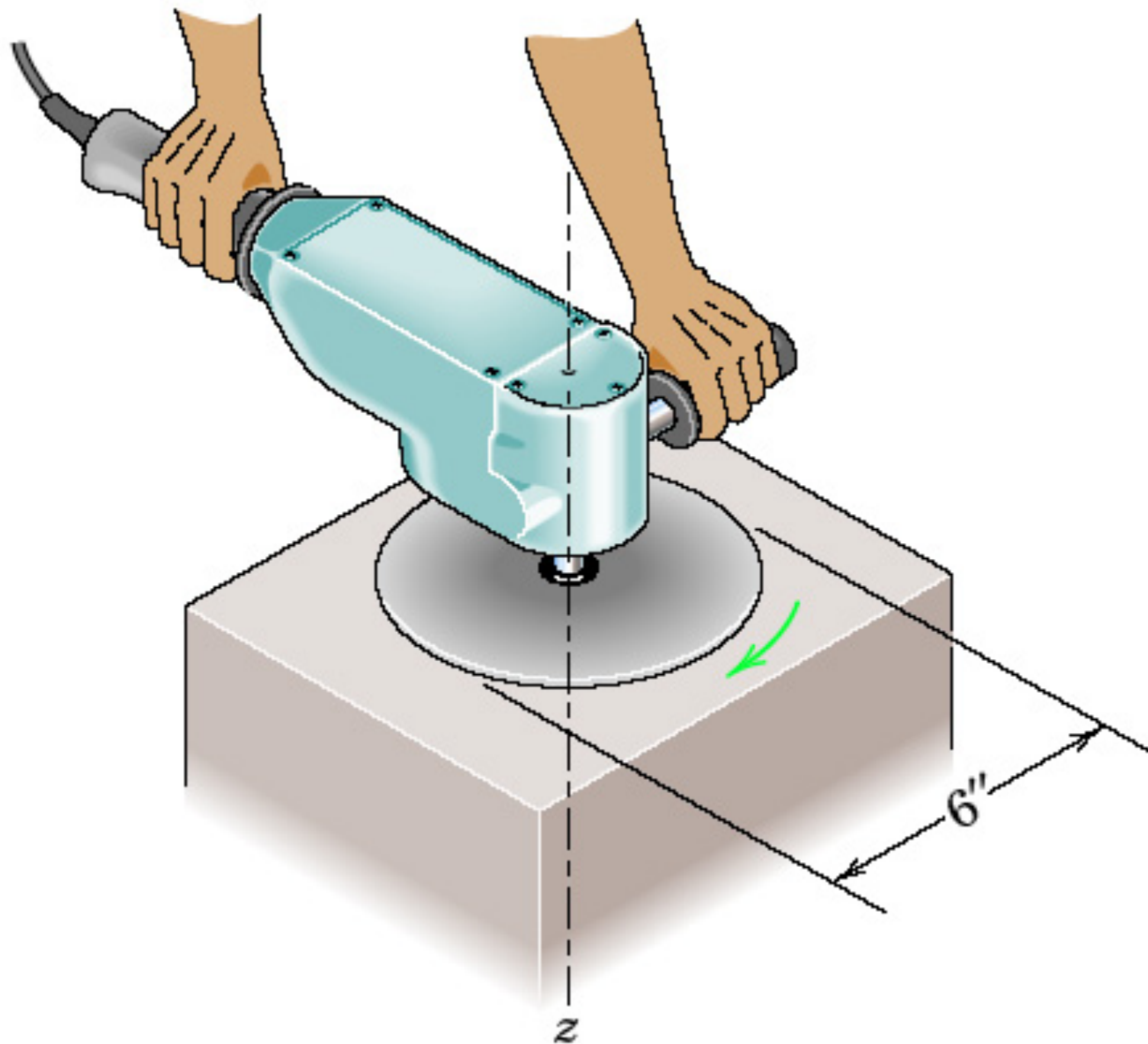
Problem 6/73



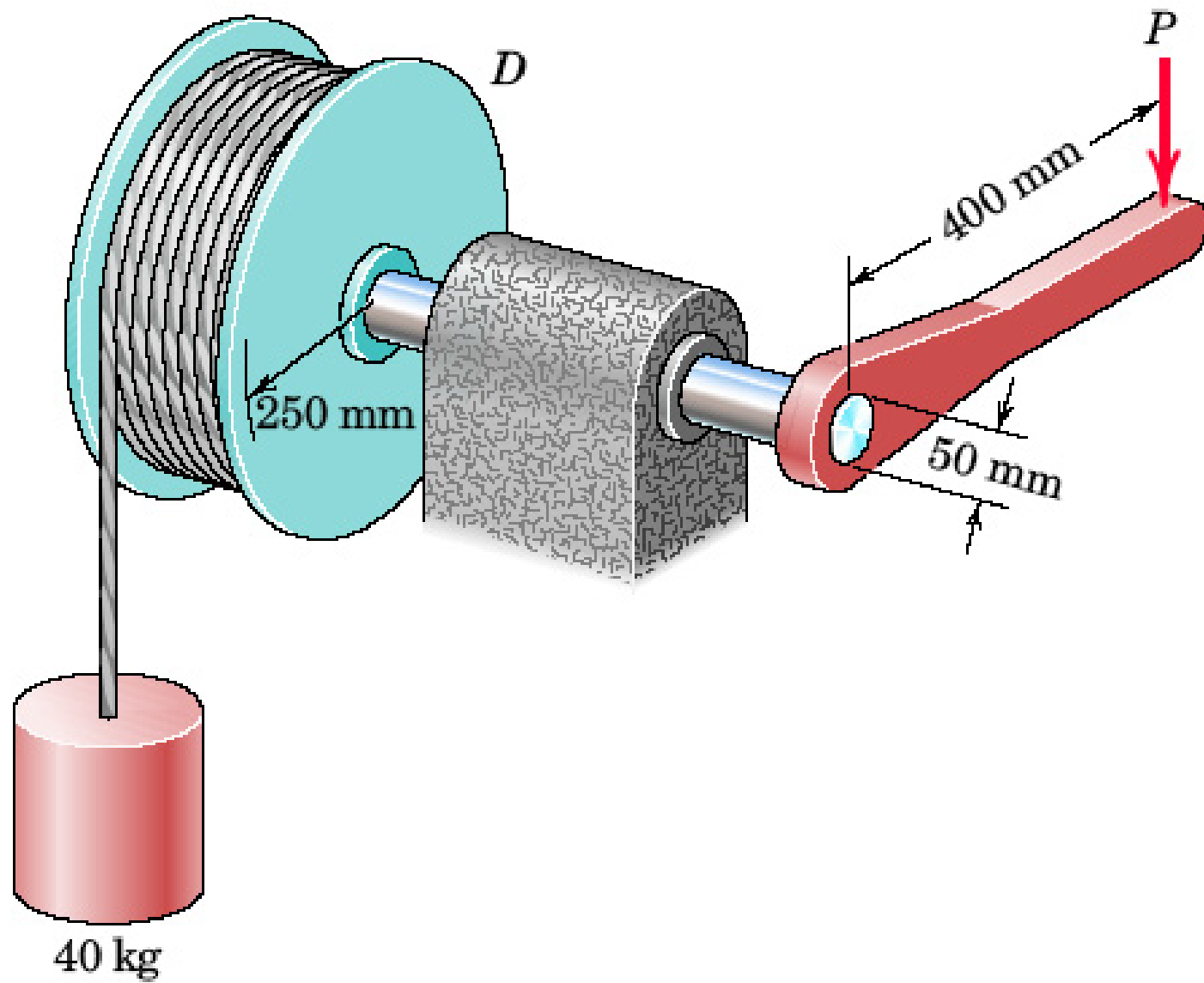
Problem 6/74



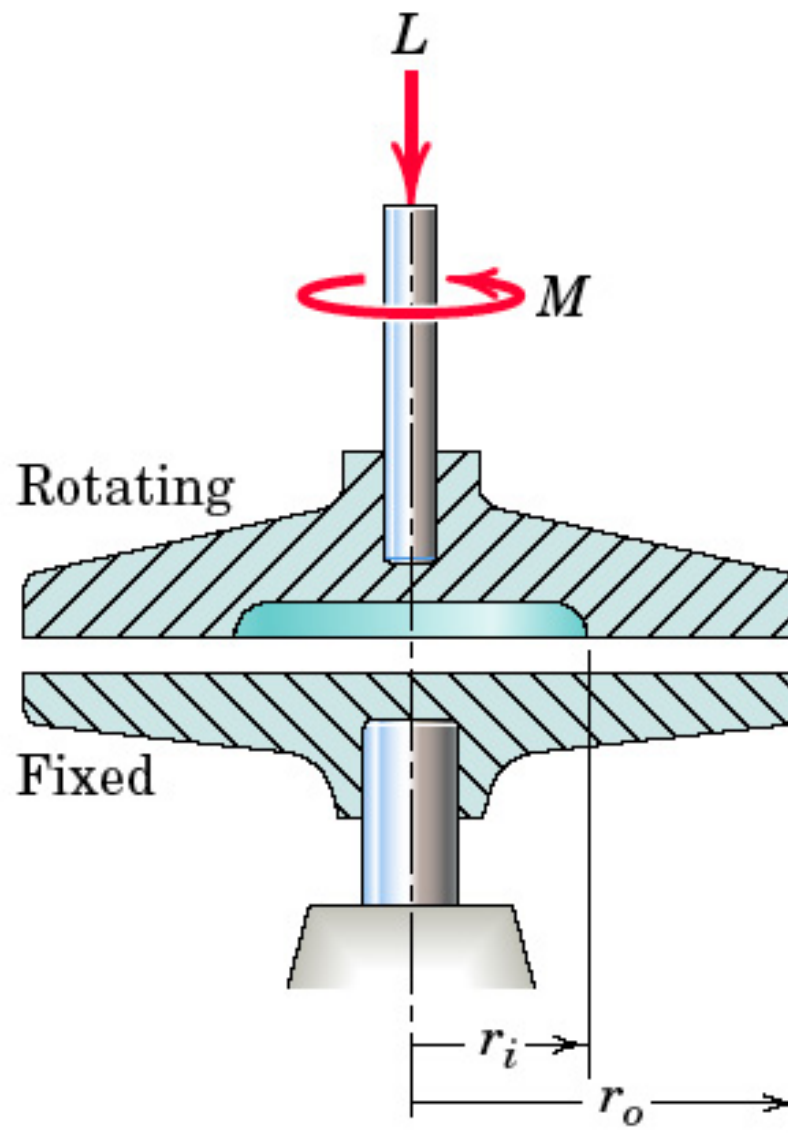
Problem 6/76



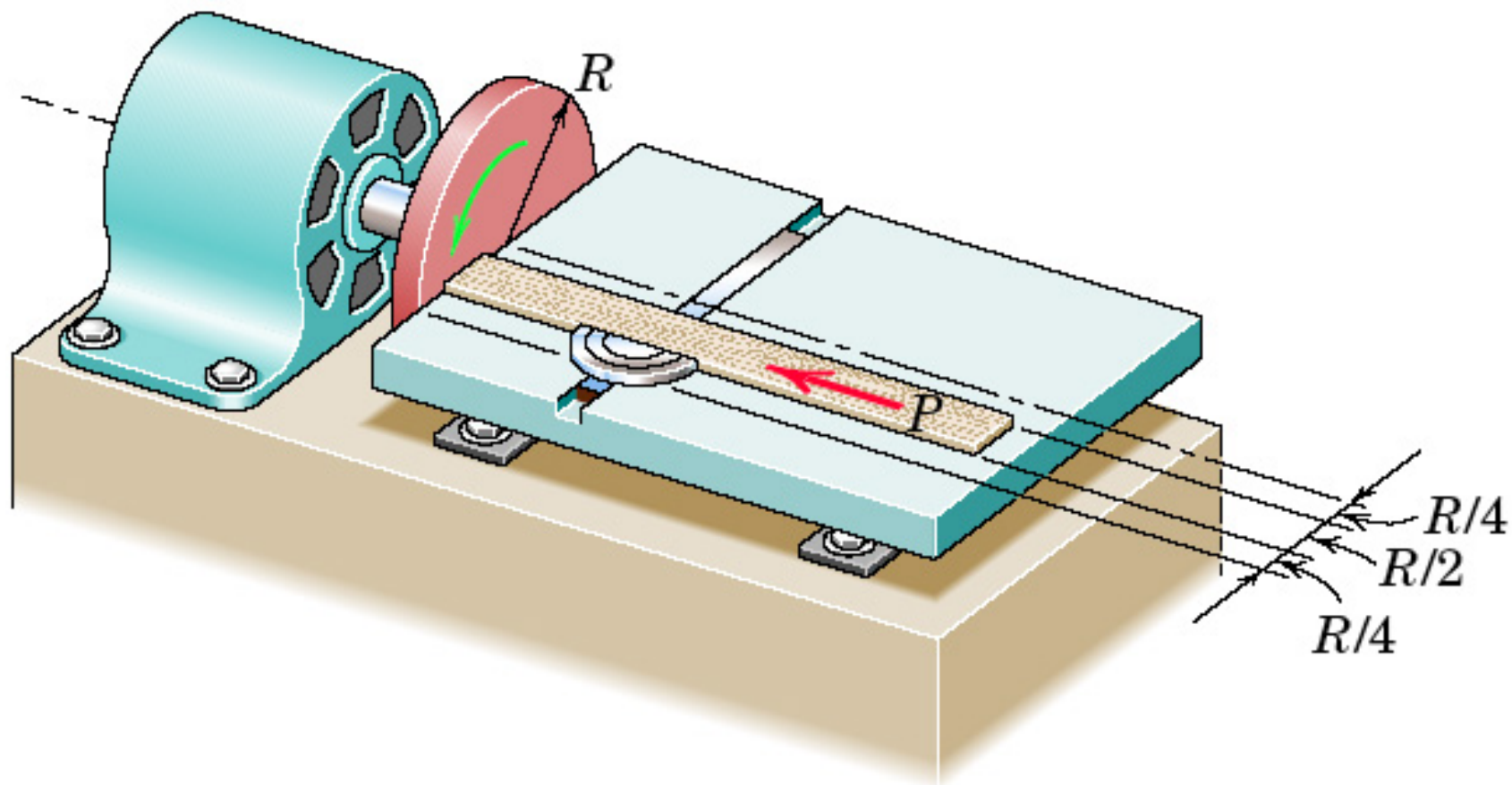
Problem 6/78



Problem 6/79

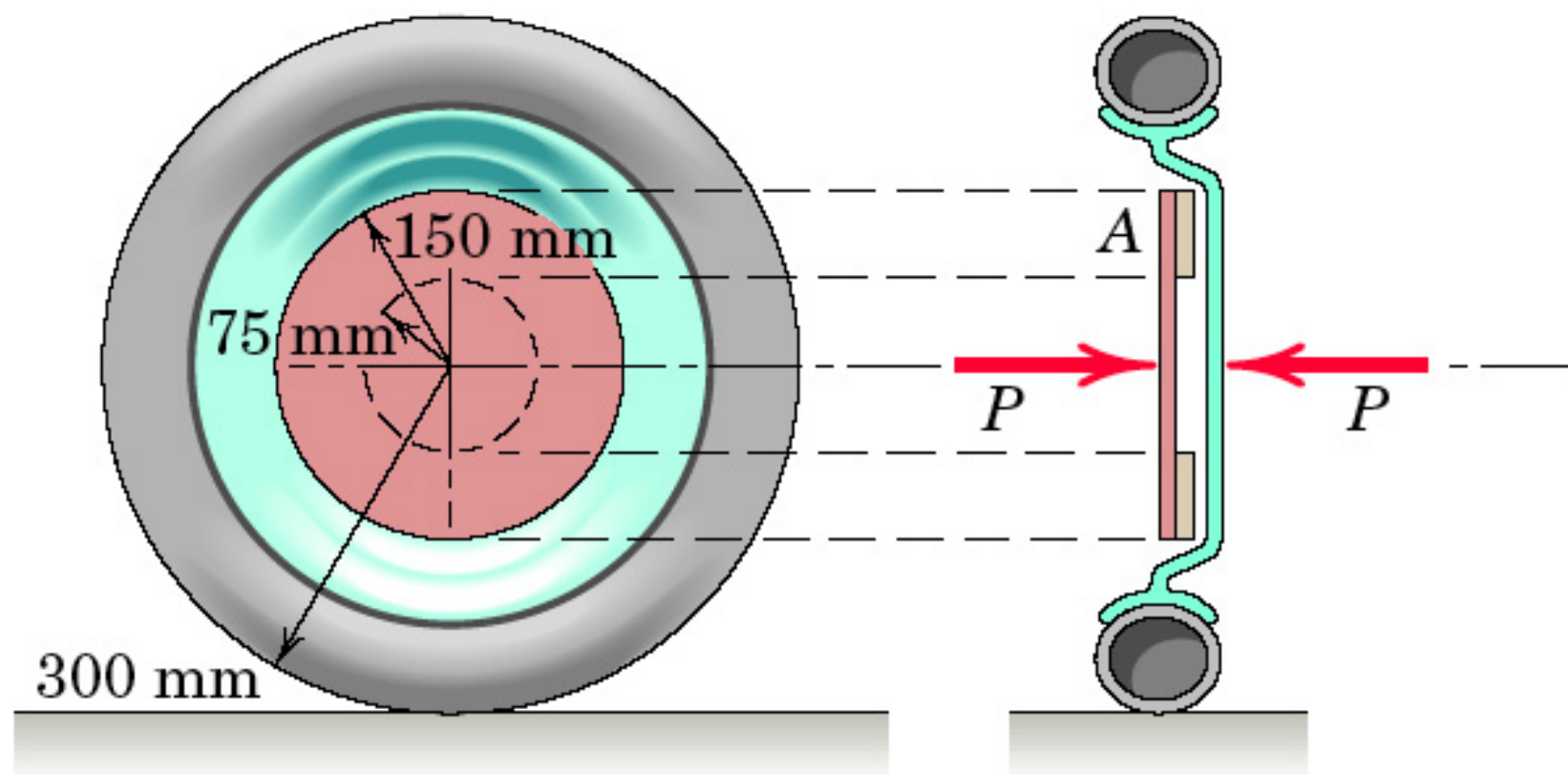


Problem 6/81



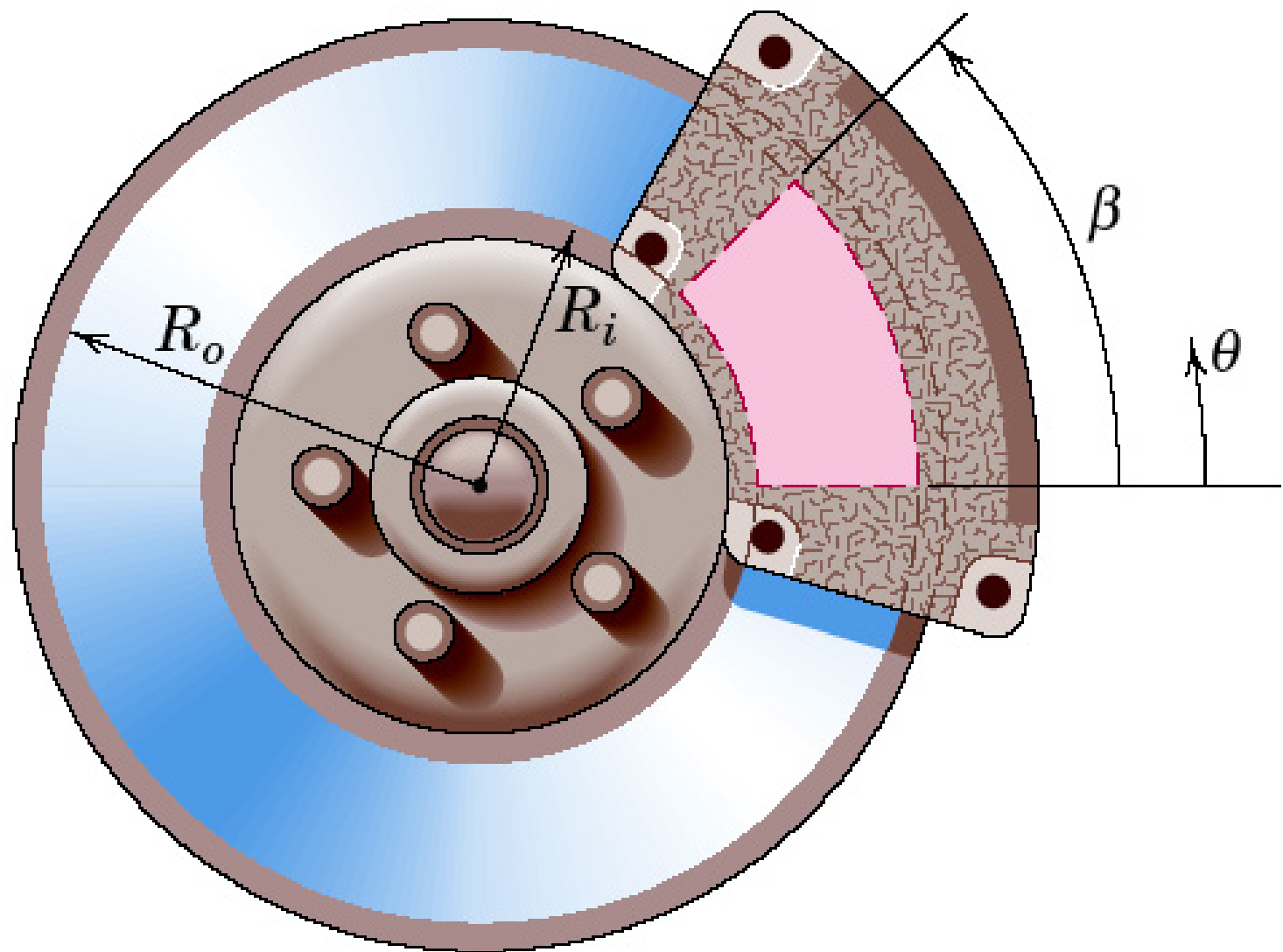
Problem 6/82

P_06_82

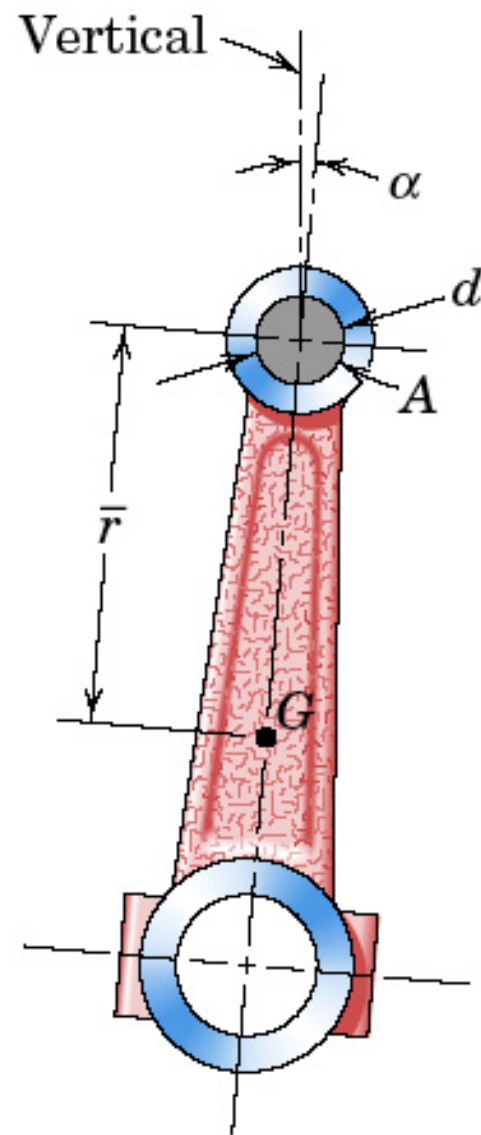


Problem 6/83

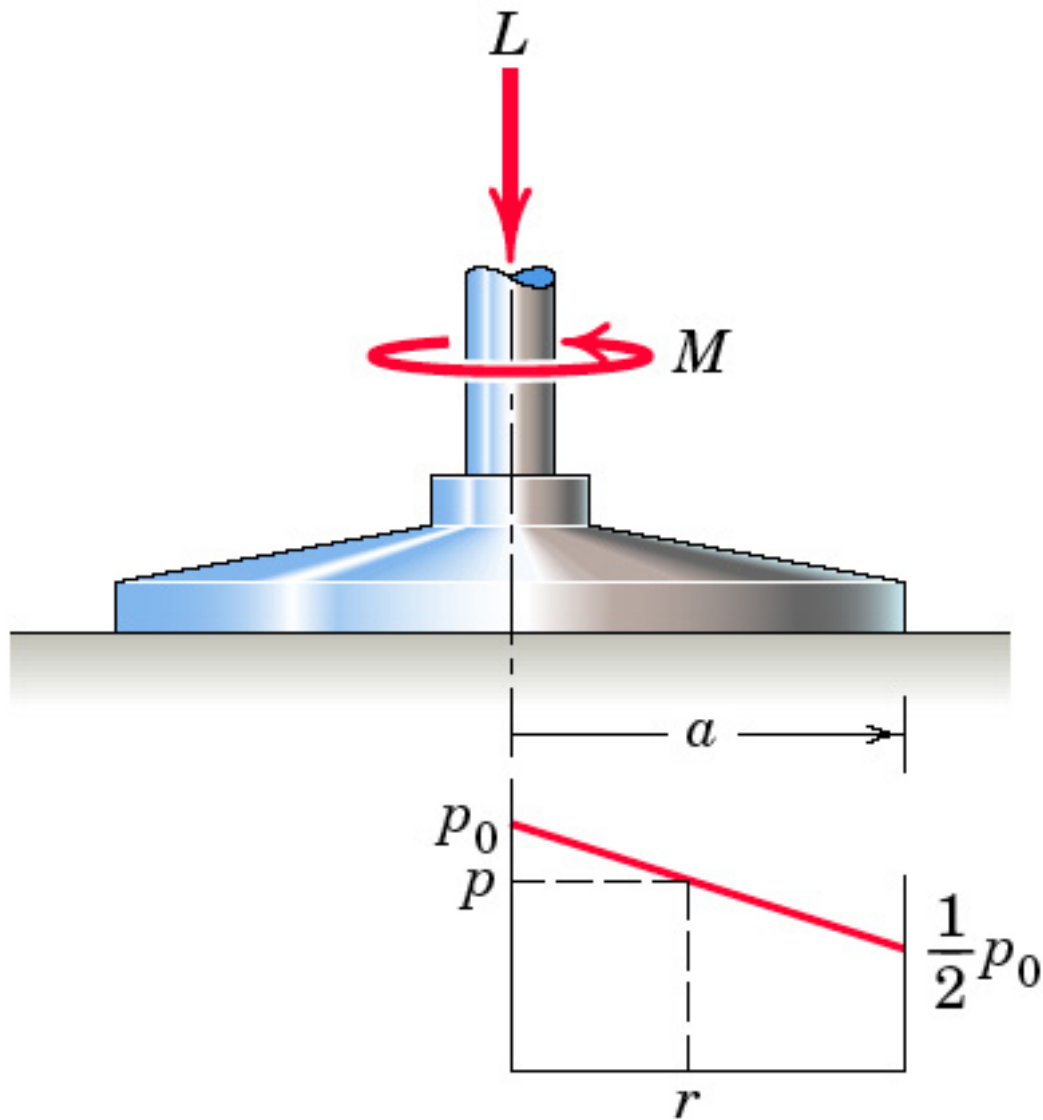
P_06_83



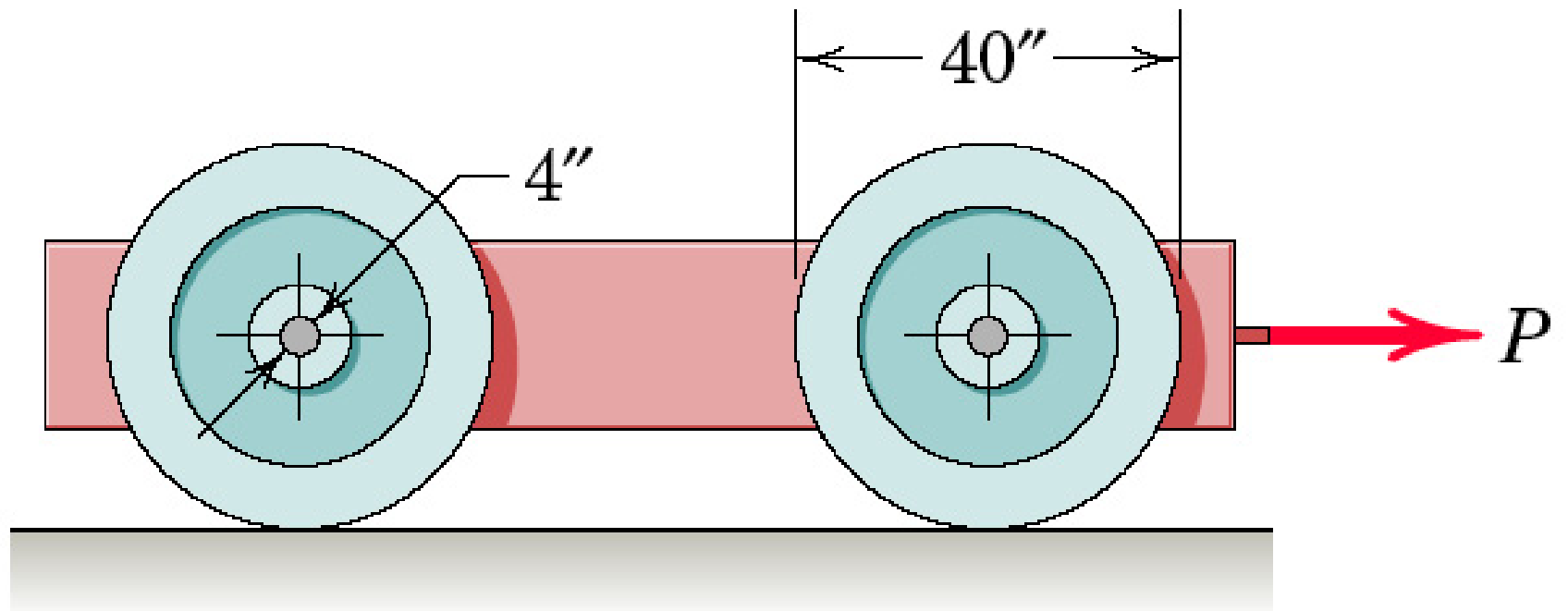
Problem 6/84



Problem 6/85

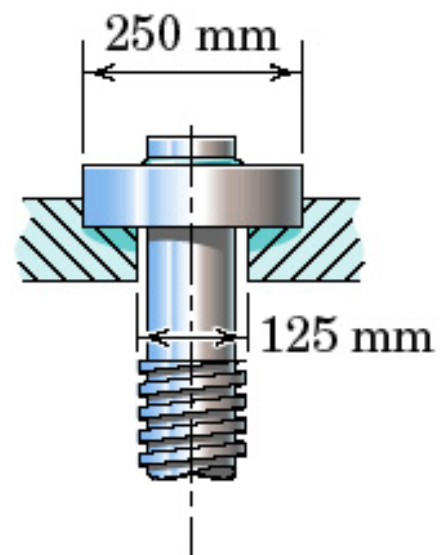
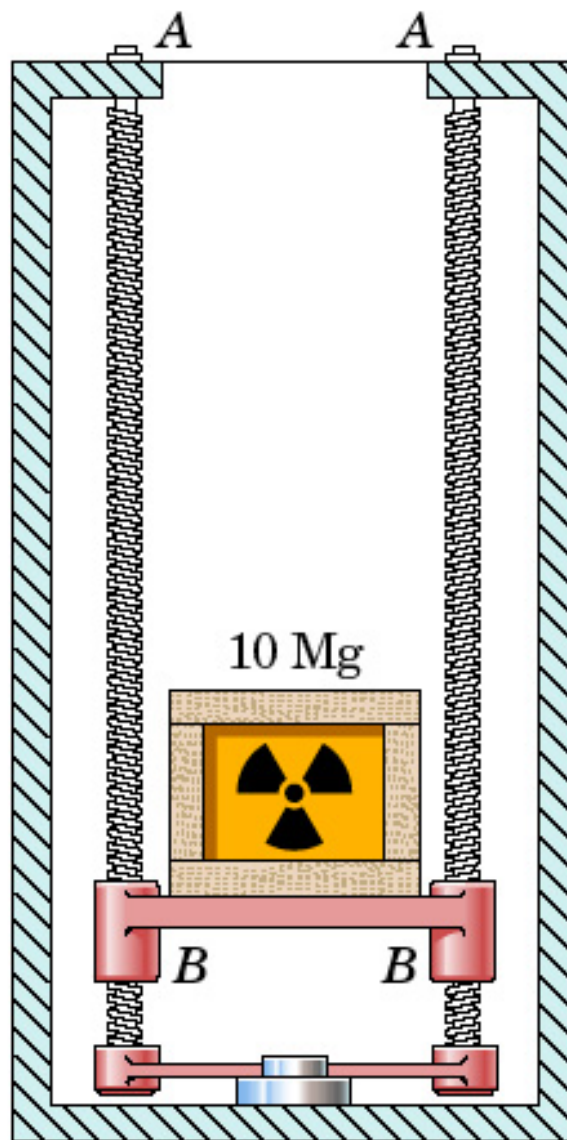


Problem 6/86



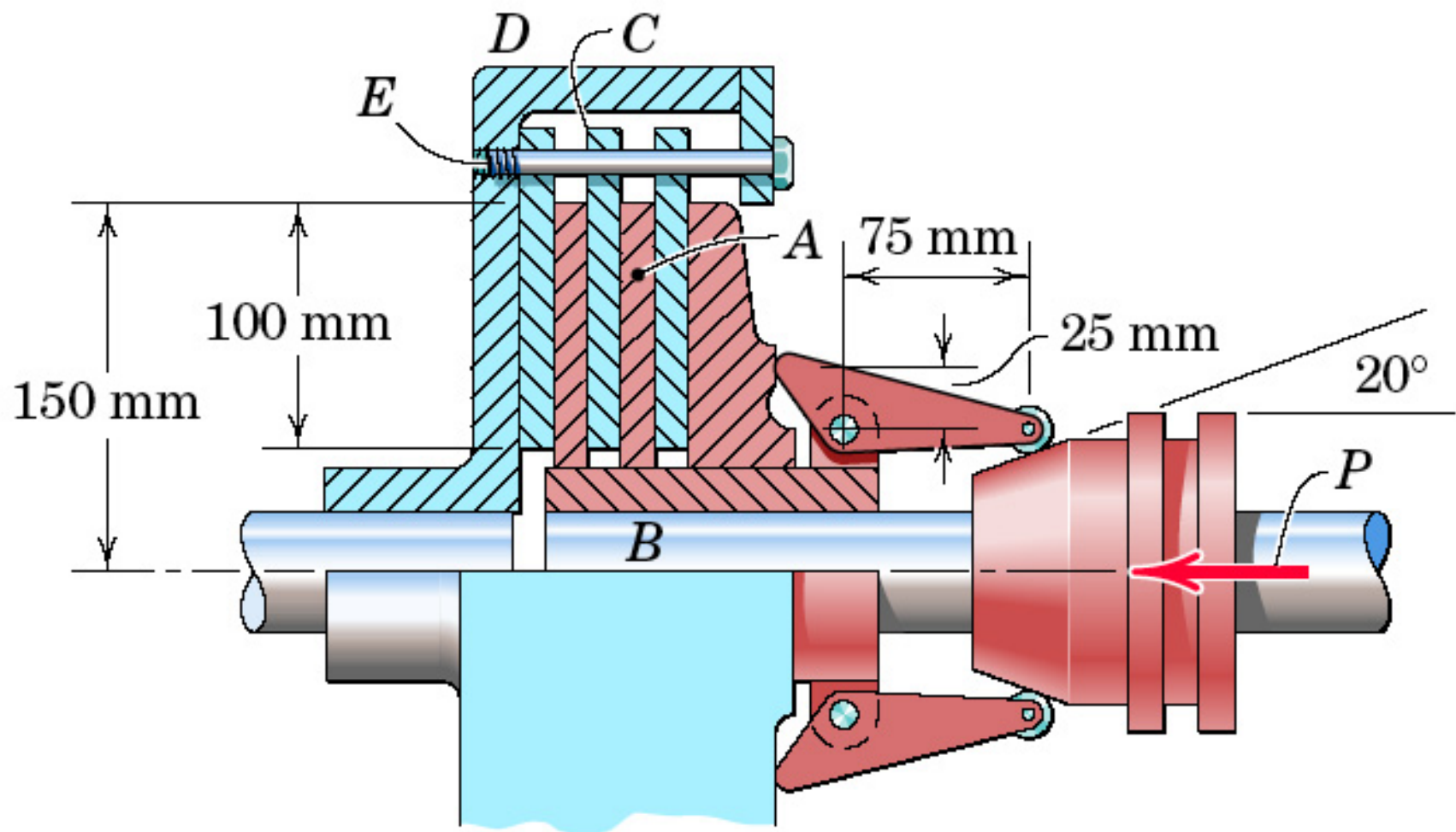
Problem 6/87

P_06_87

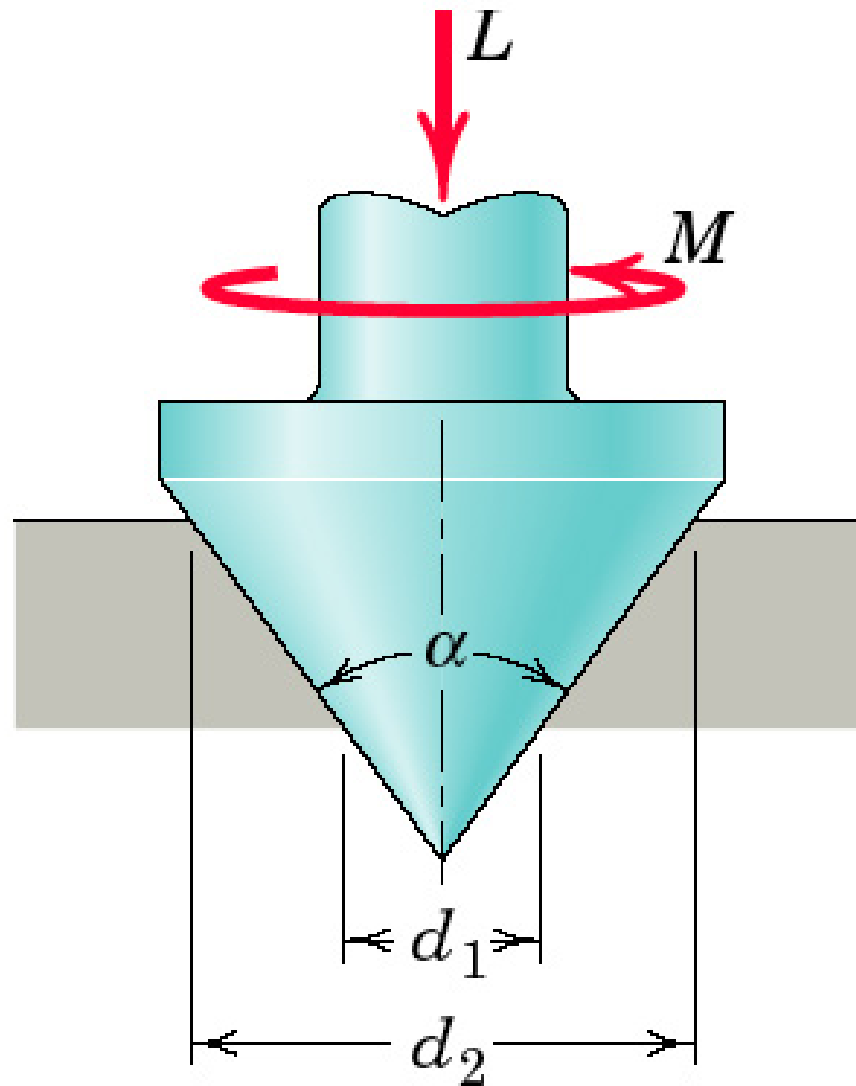


Detail of collar
bearing at A

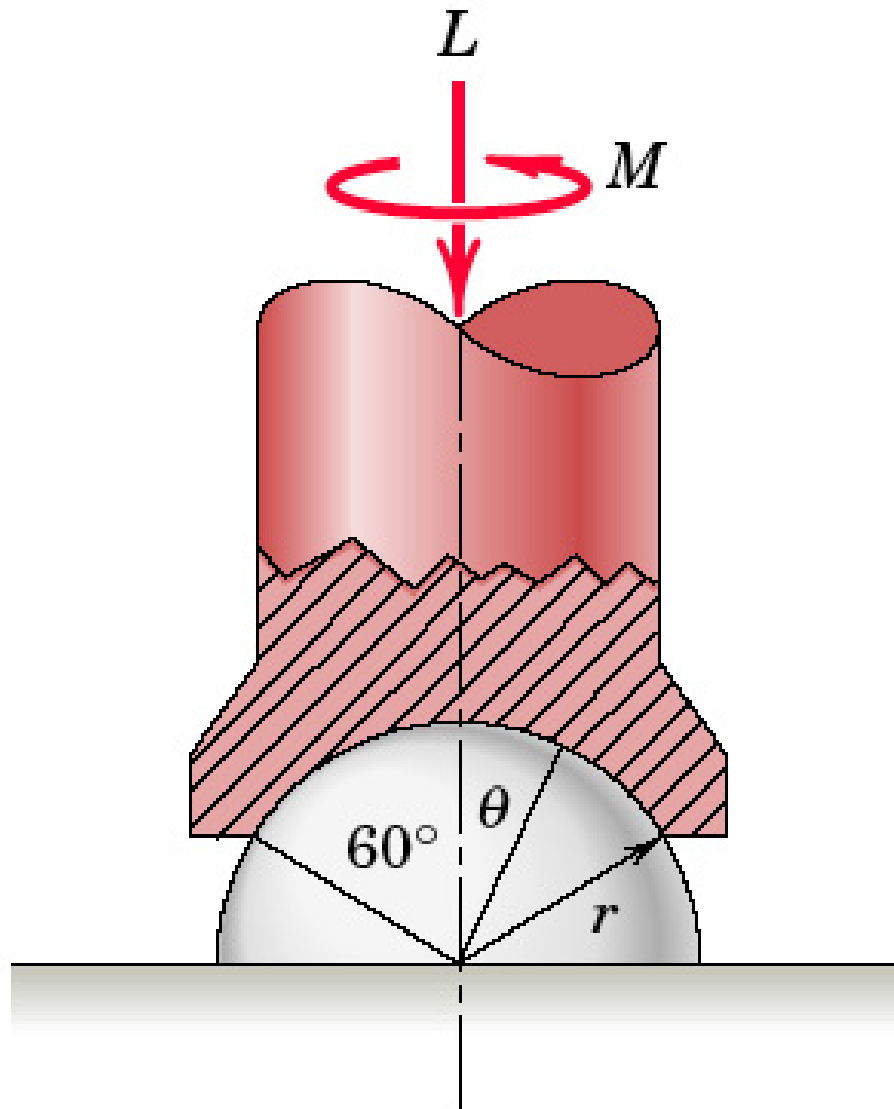
Problem 6/89



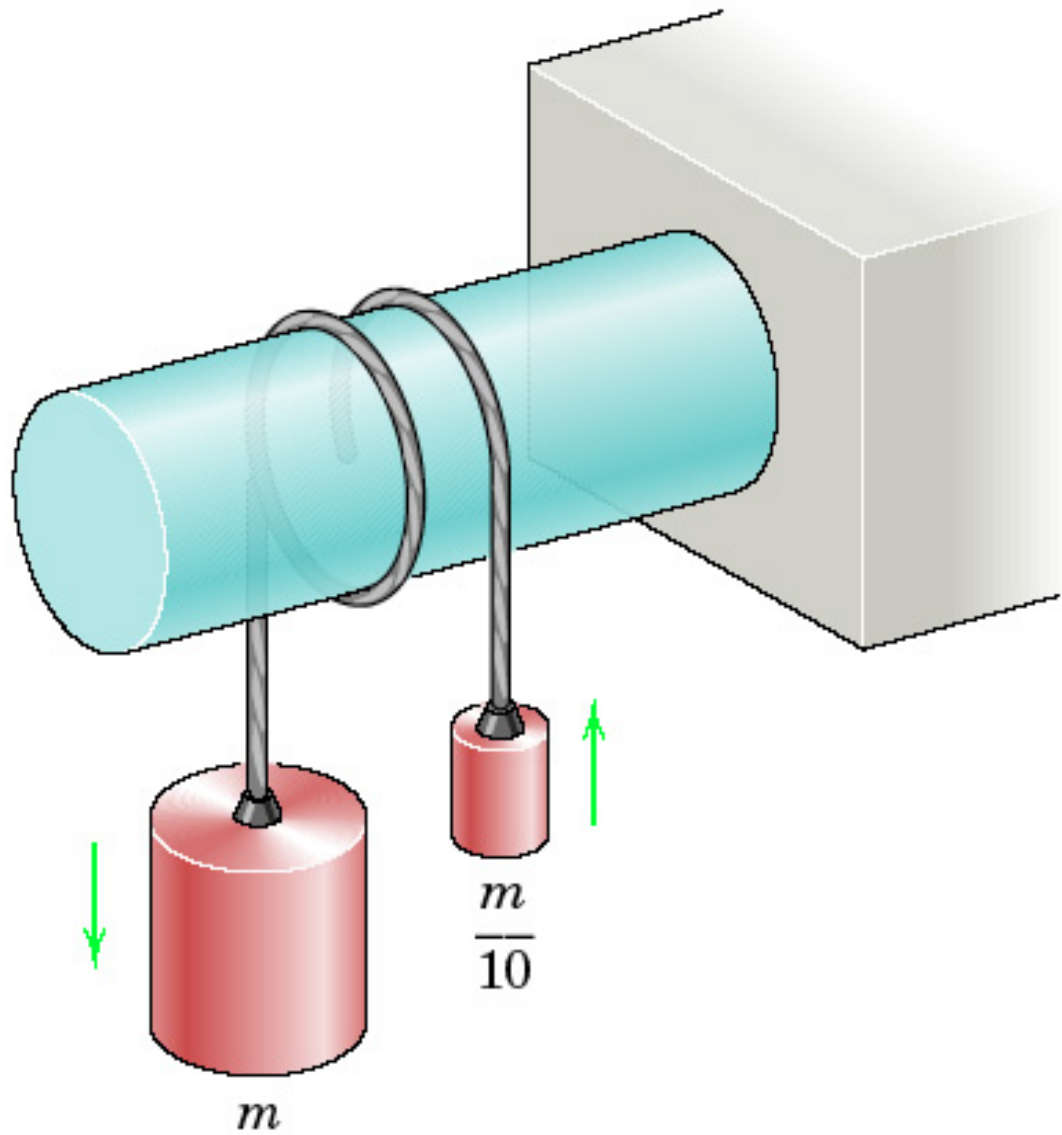
Problem 6/90



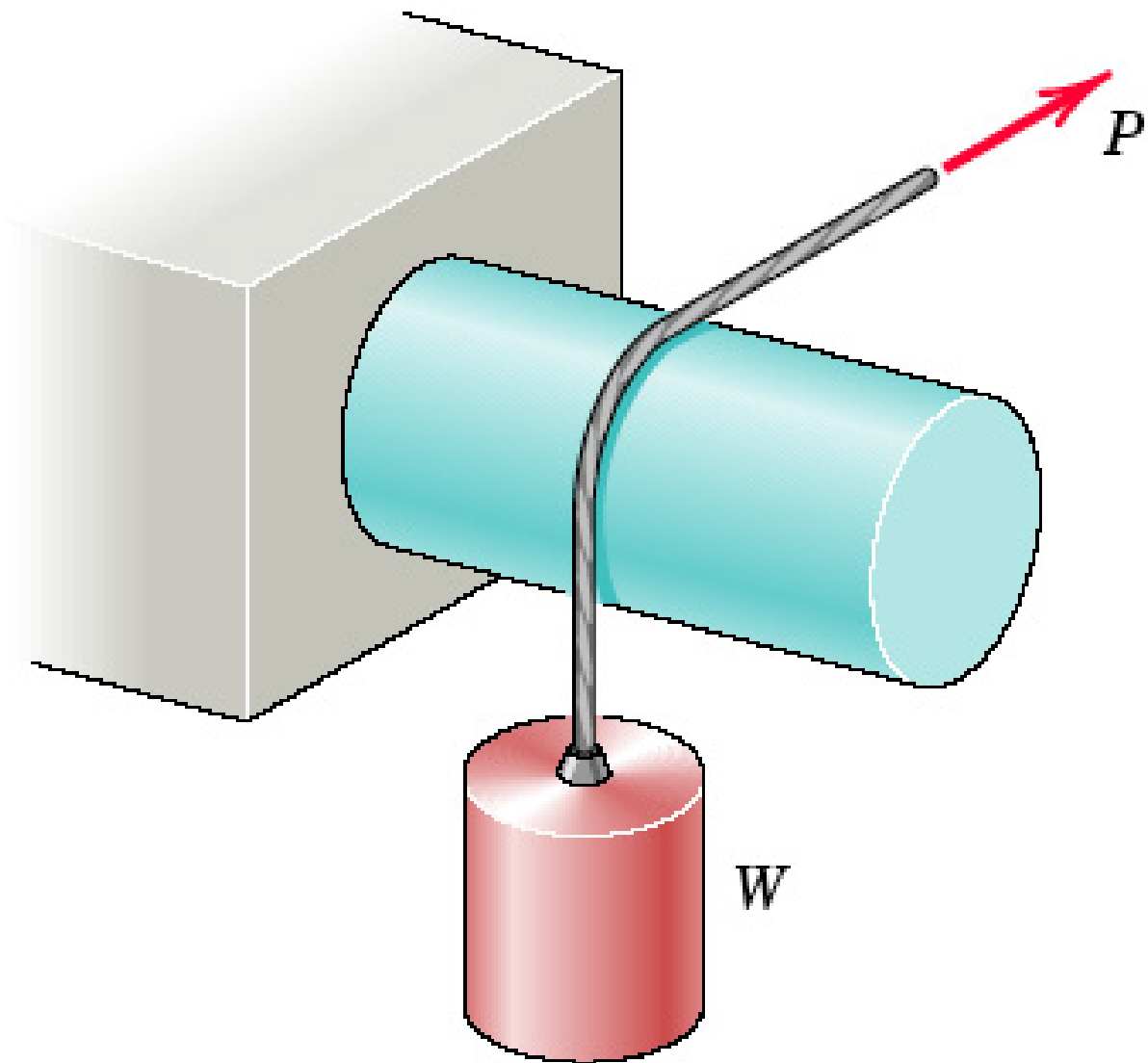
Problem 6/91



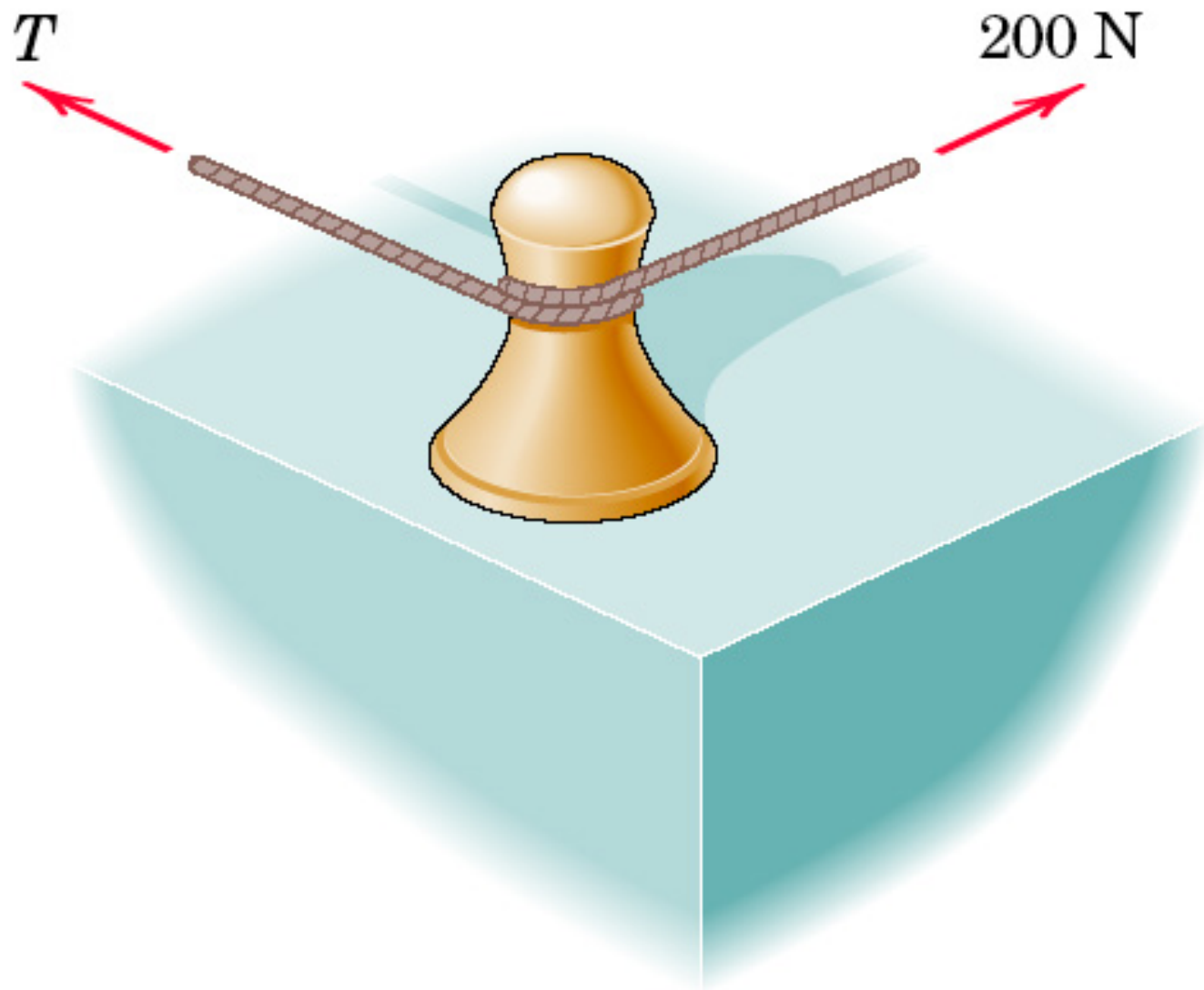
Problem 6/92



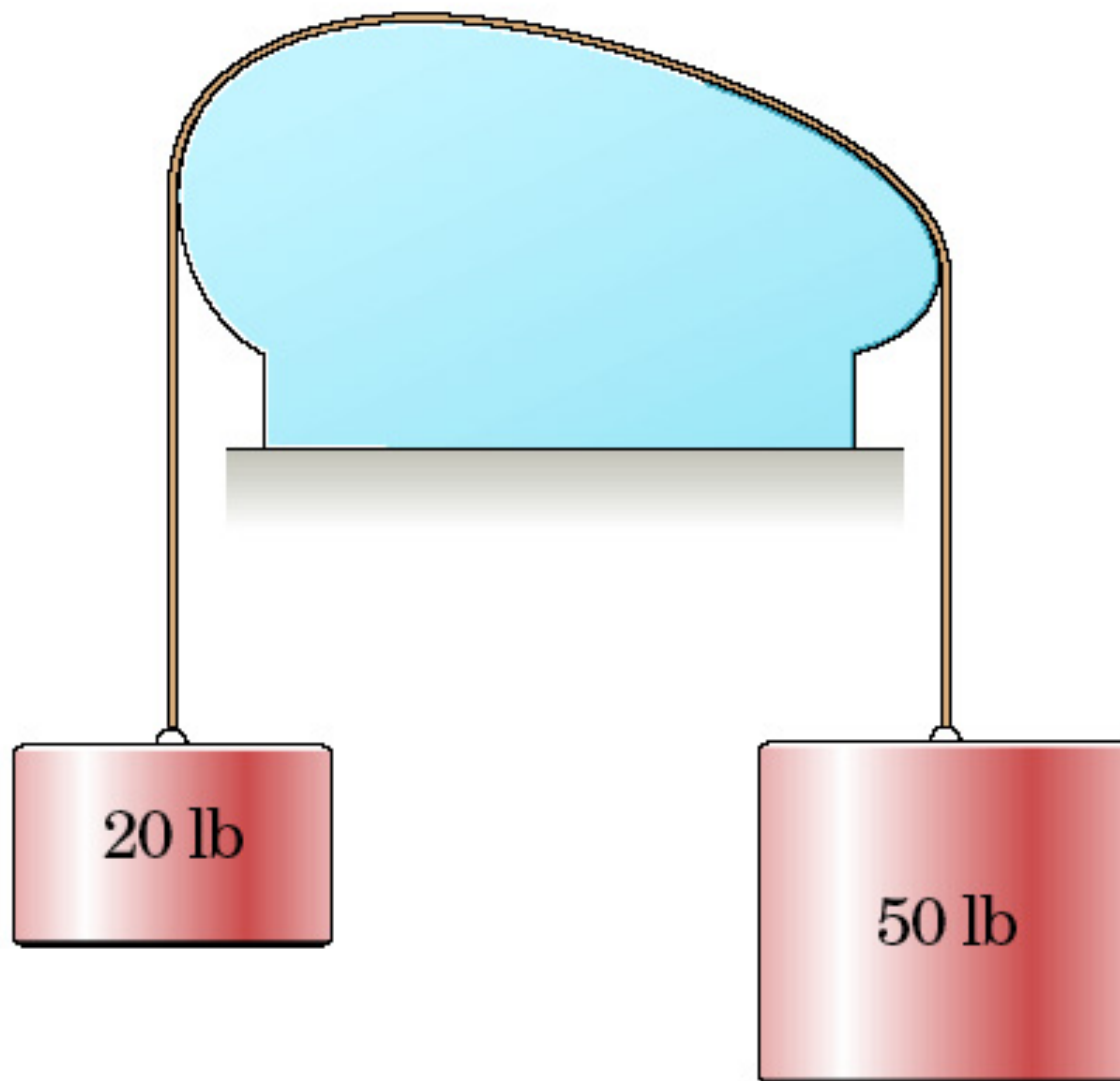
Problem 6/93



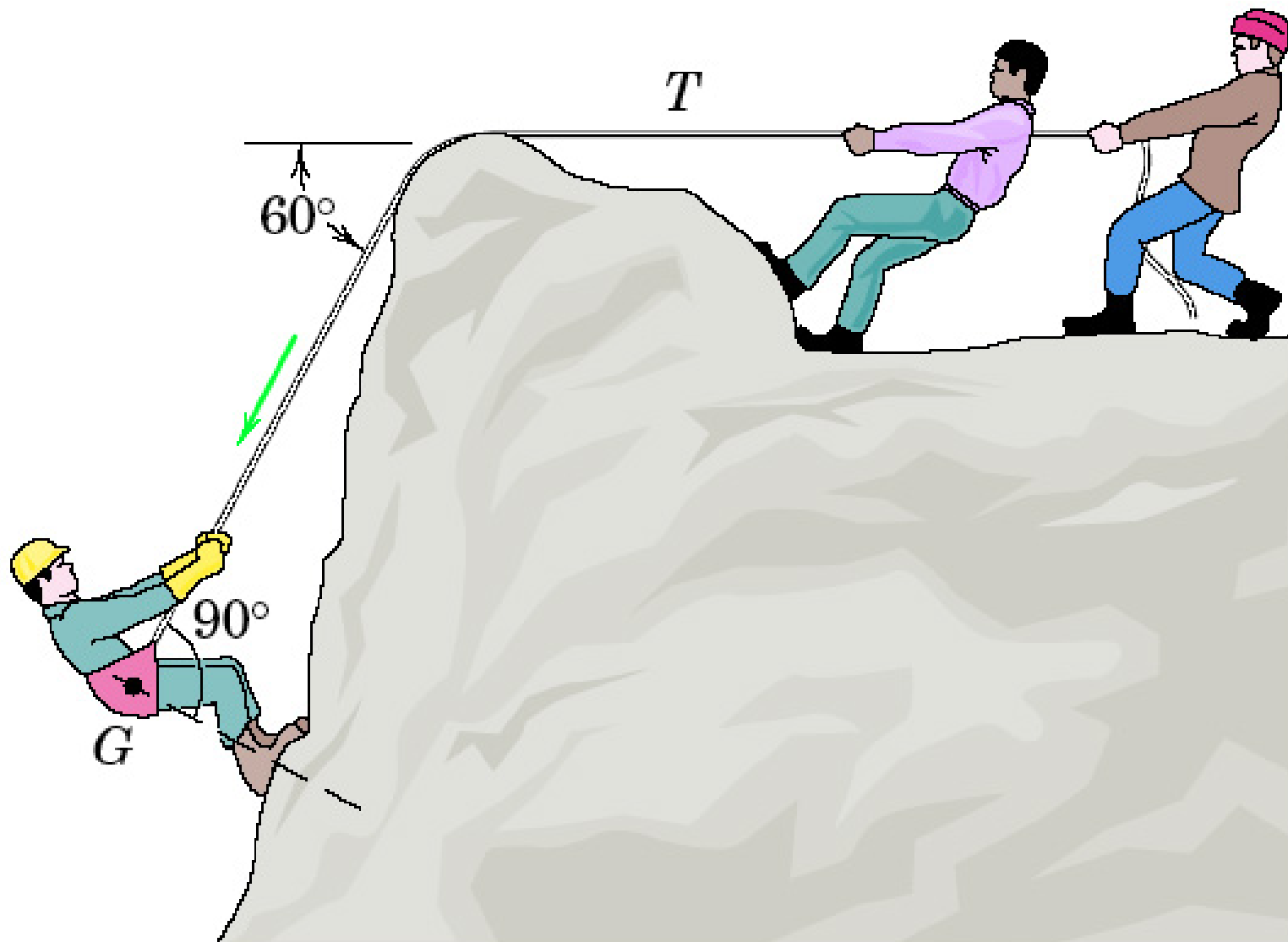
Problem 6/94



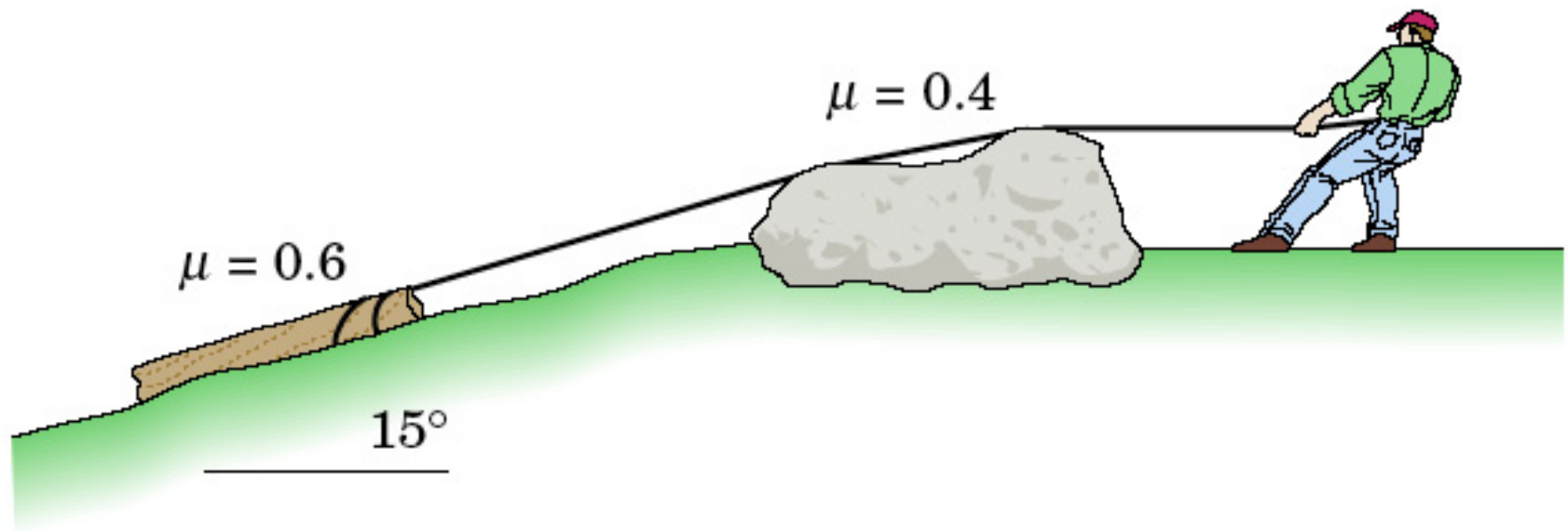
Problem 6/95



Problem 6/97



Problem 6/98



Problem 6/99

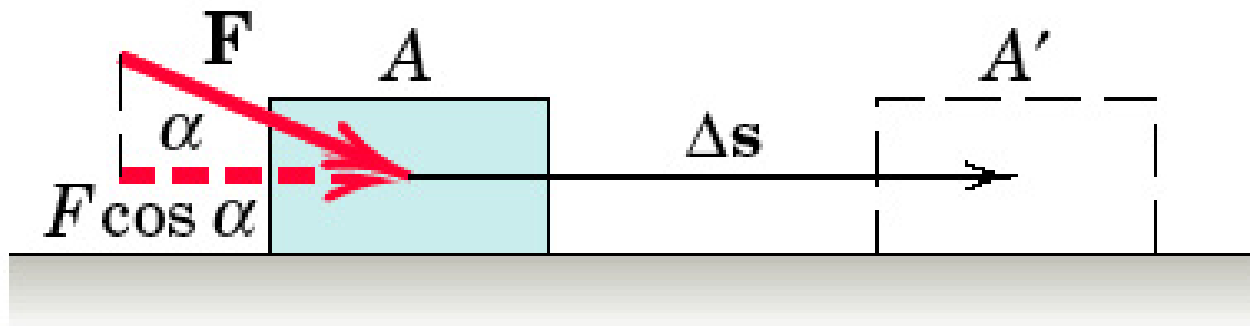
P_06_99

Text Illustrations

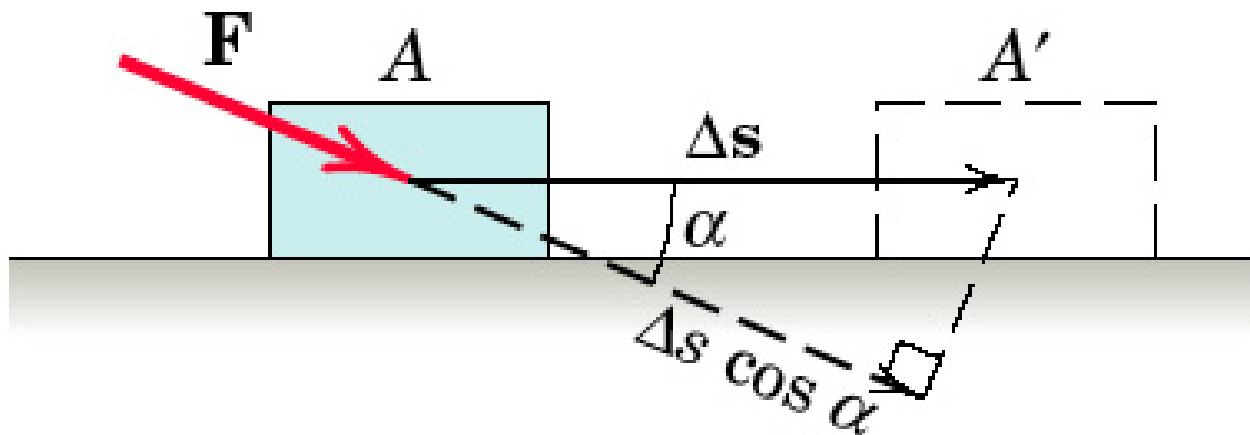
To Accompany

Meriam/Kraige – Statics 6e

Chapter 7 : Virtual Work.



(a)



(b)

Figure 7/1

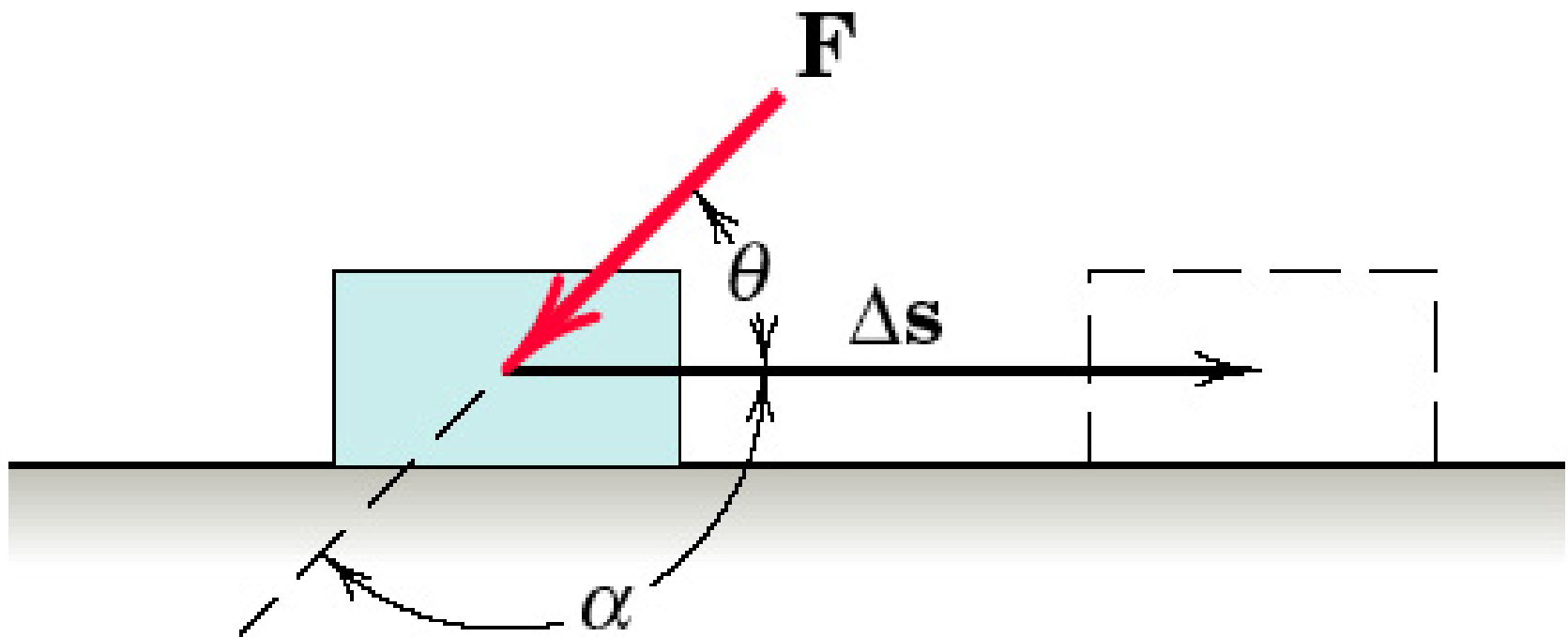


Figure 7/2

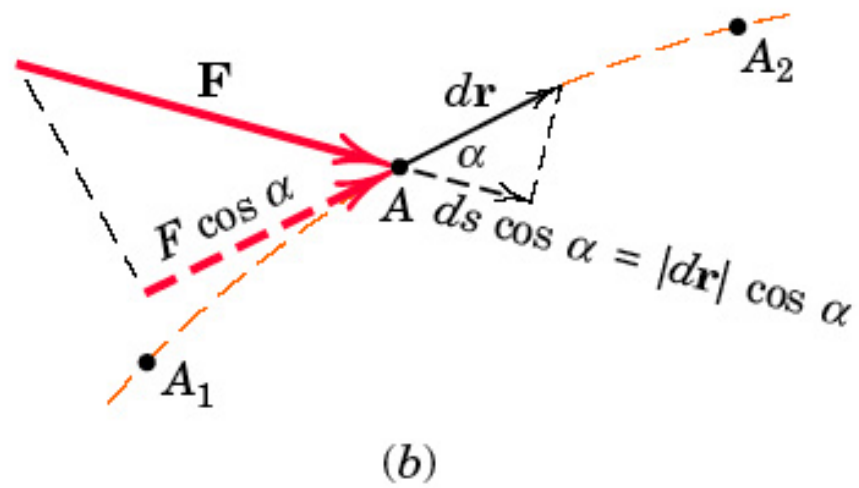
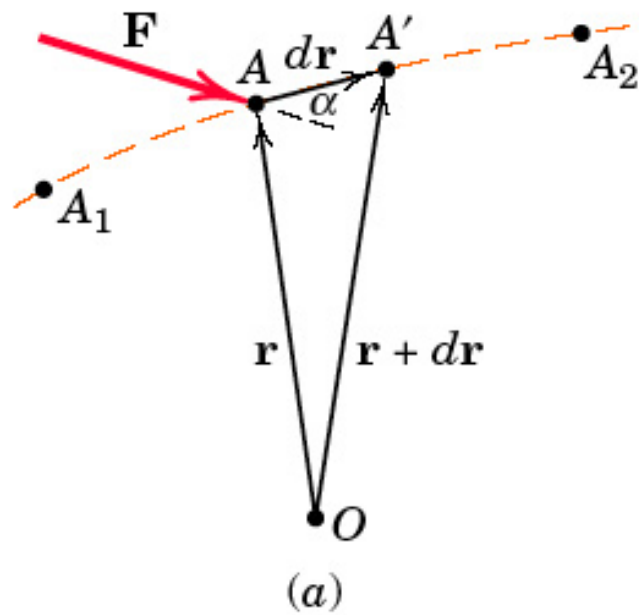
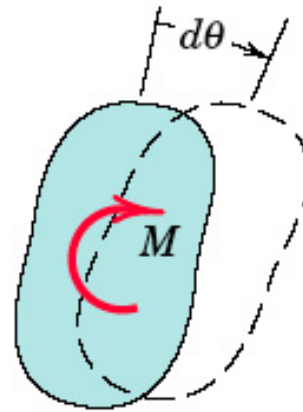
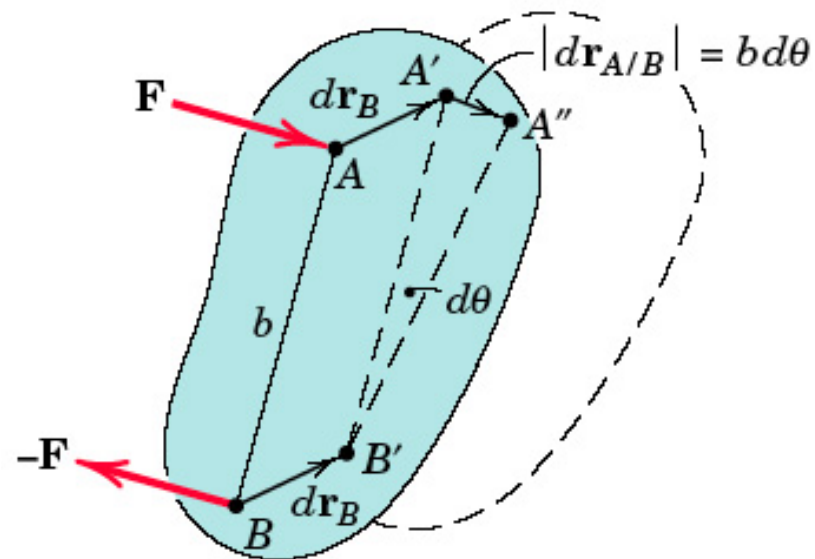


Figure 7/3



(a)



(b)

Figure 7/4

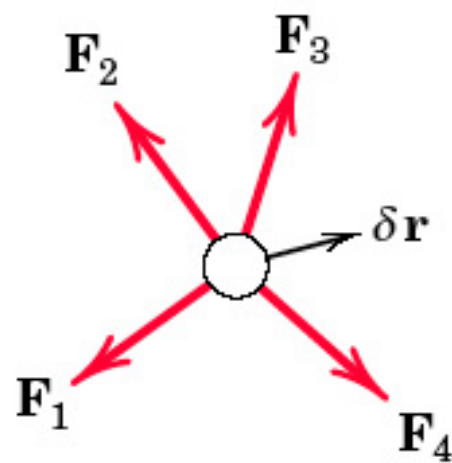
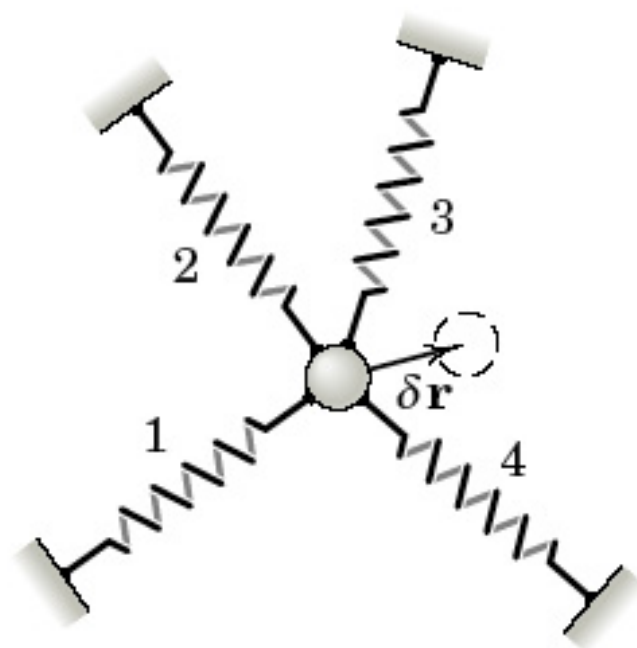


Figure 7/5

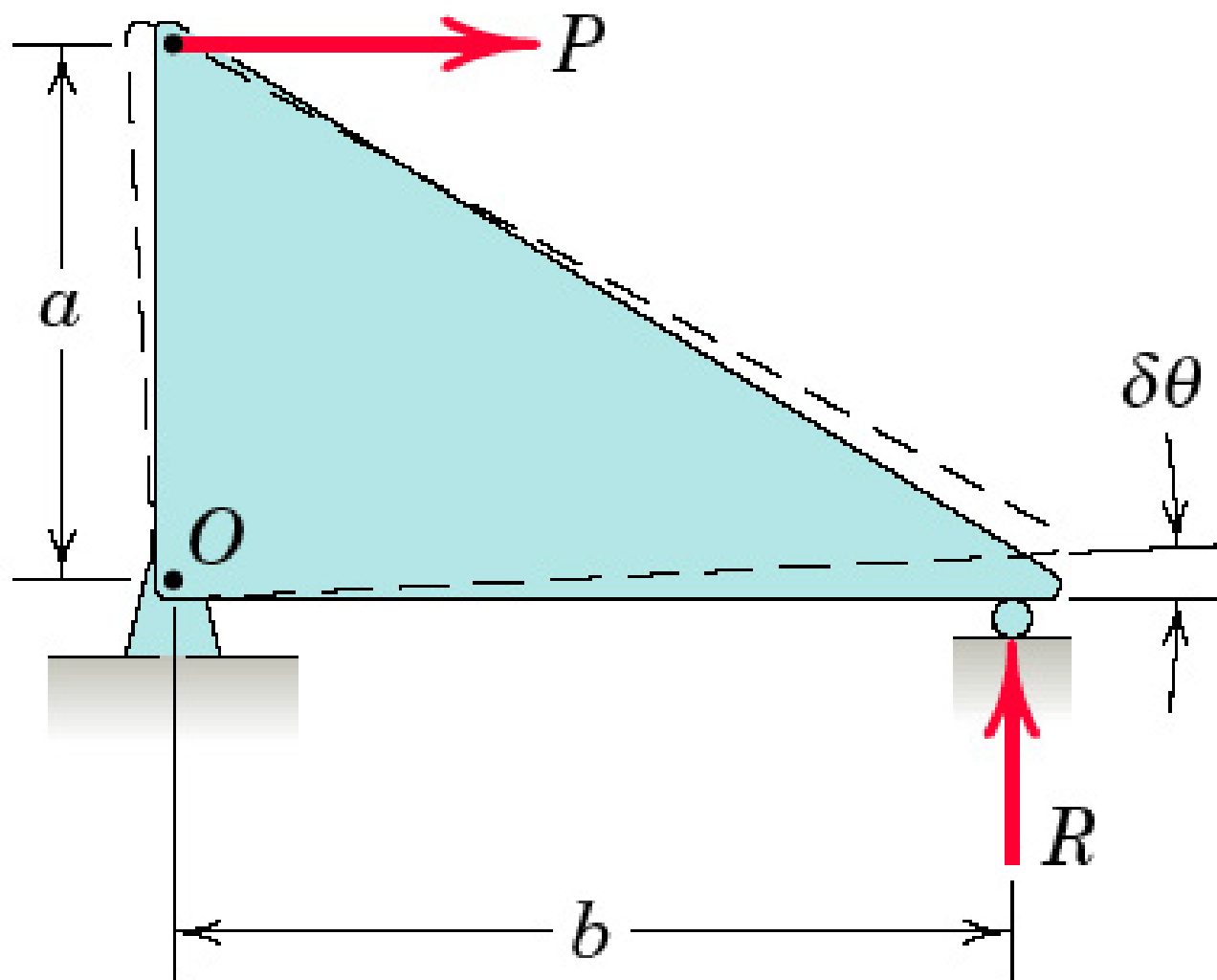
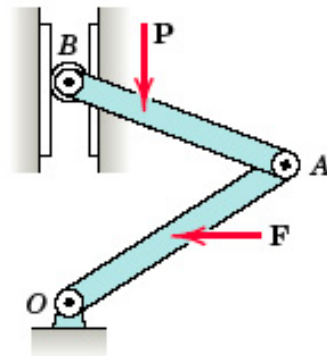
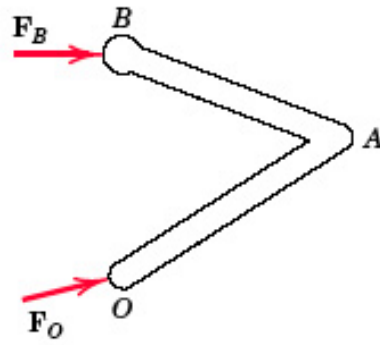


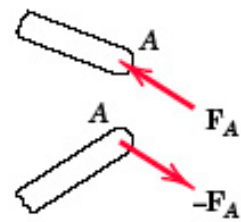
Figure 7/6



(a) Active forces

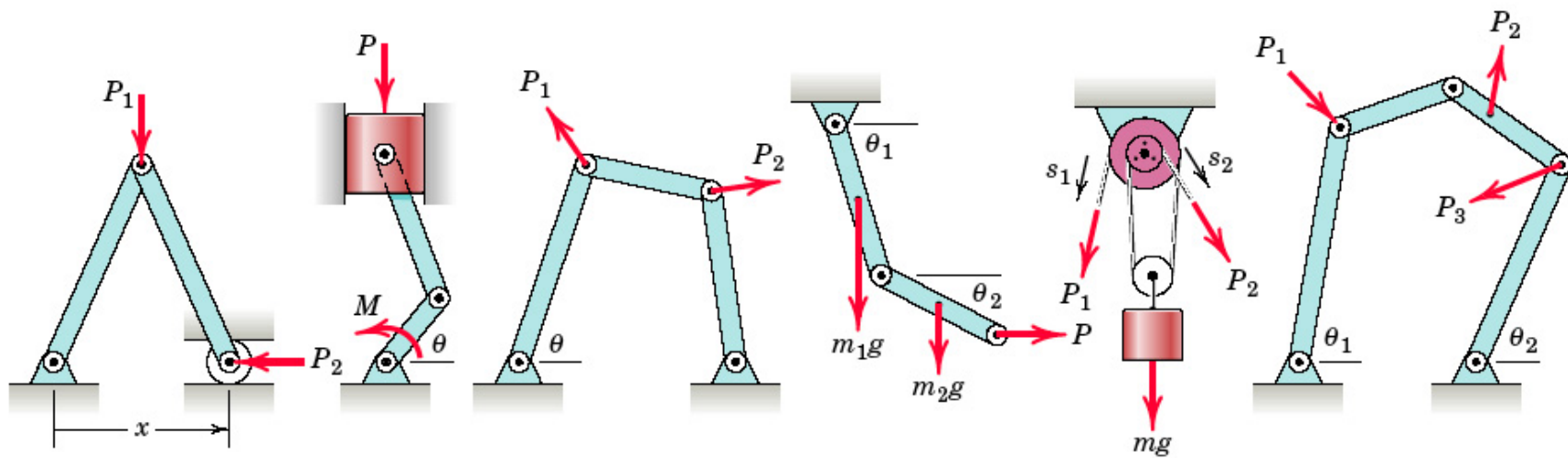


(b) Reactive forces



(c) Internal forces

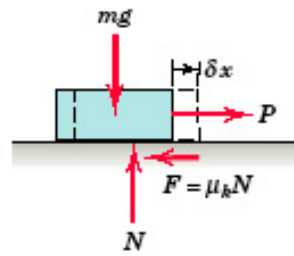
Figure 7/7



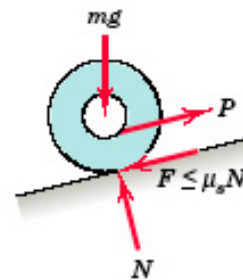
(a) Examples of one-degree-of-freedom systems

(b) Examples of two-degree-of-freedom systems

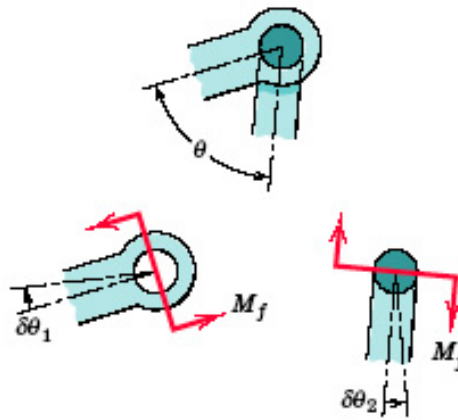
Figure 7/8



(a)



(b)



(c)

Figure 7/9

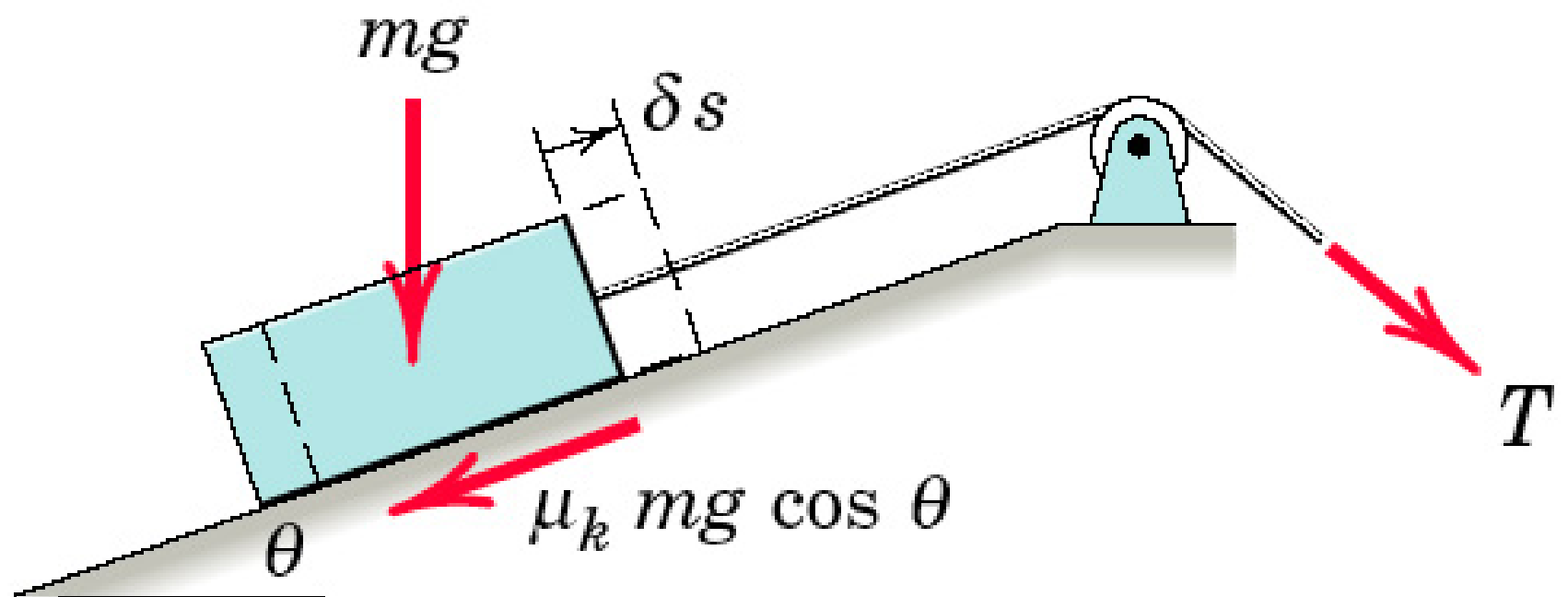


Figure 7/10

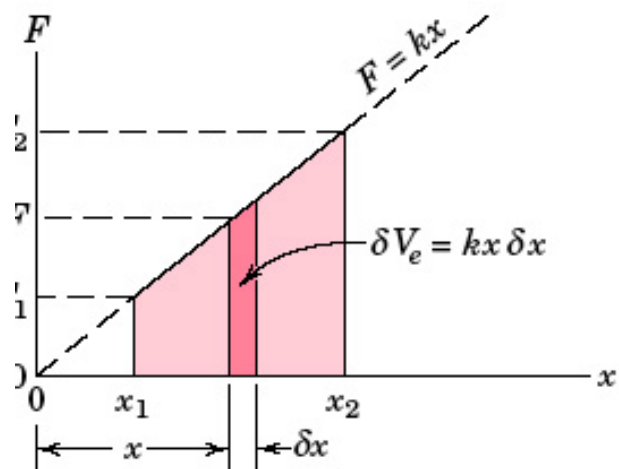
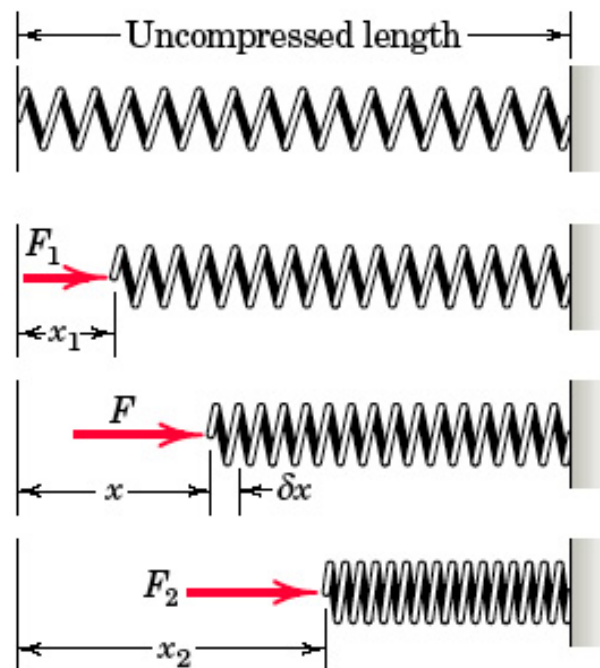


Figure 7/11

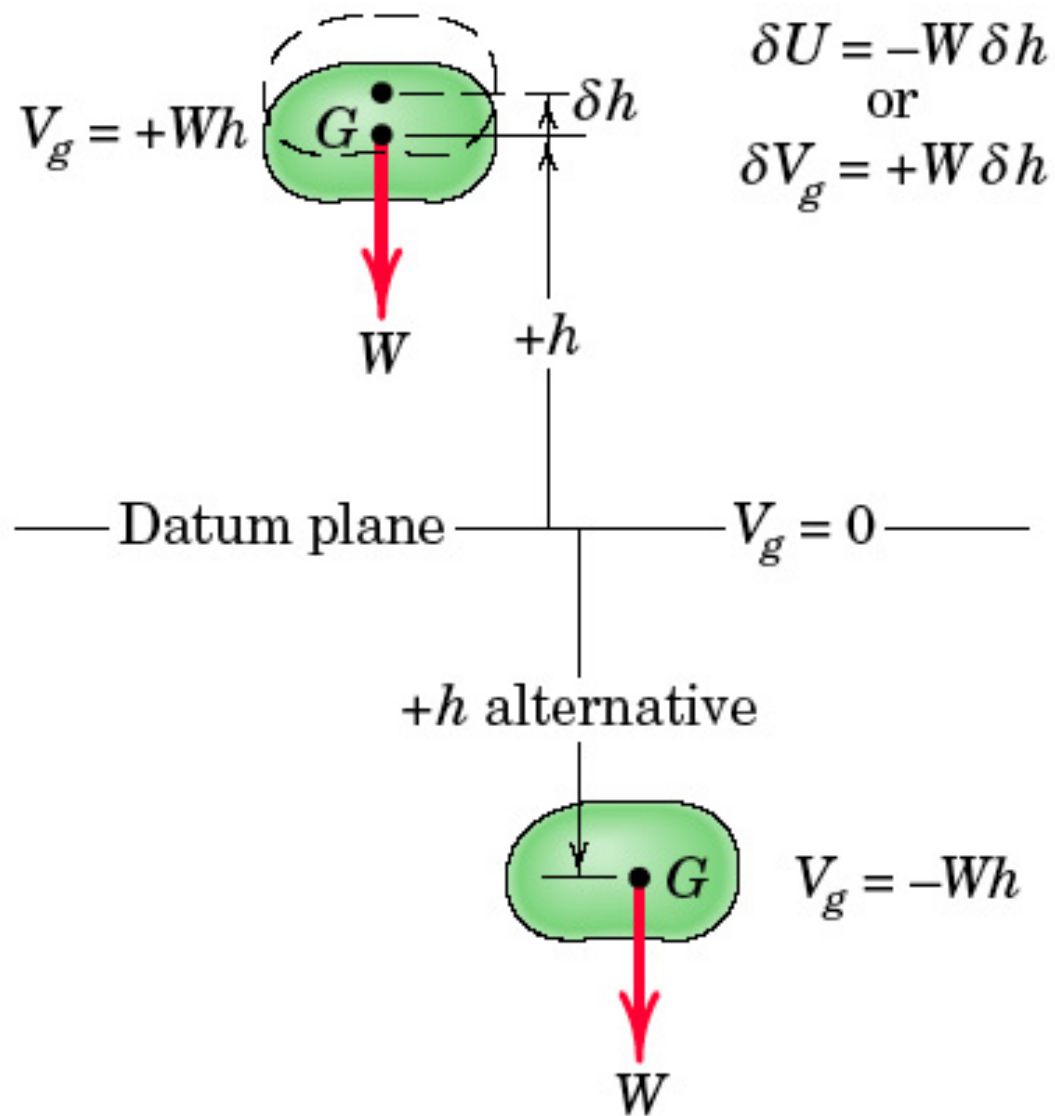


Figure 7/12

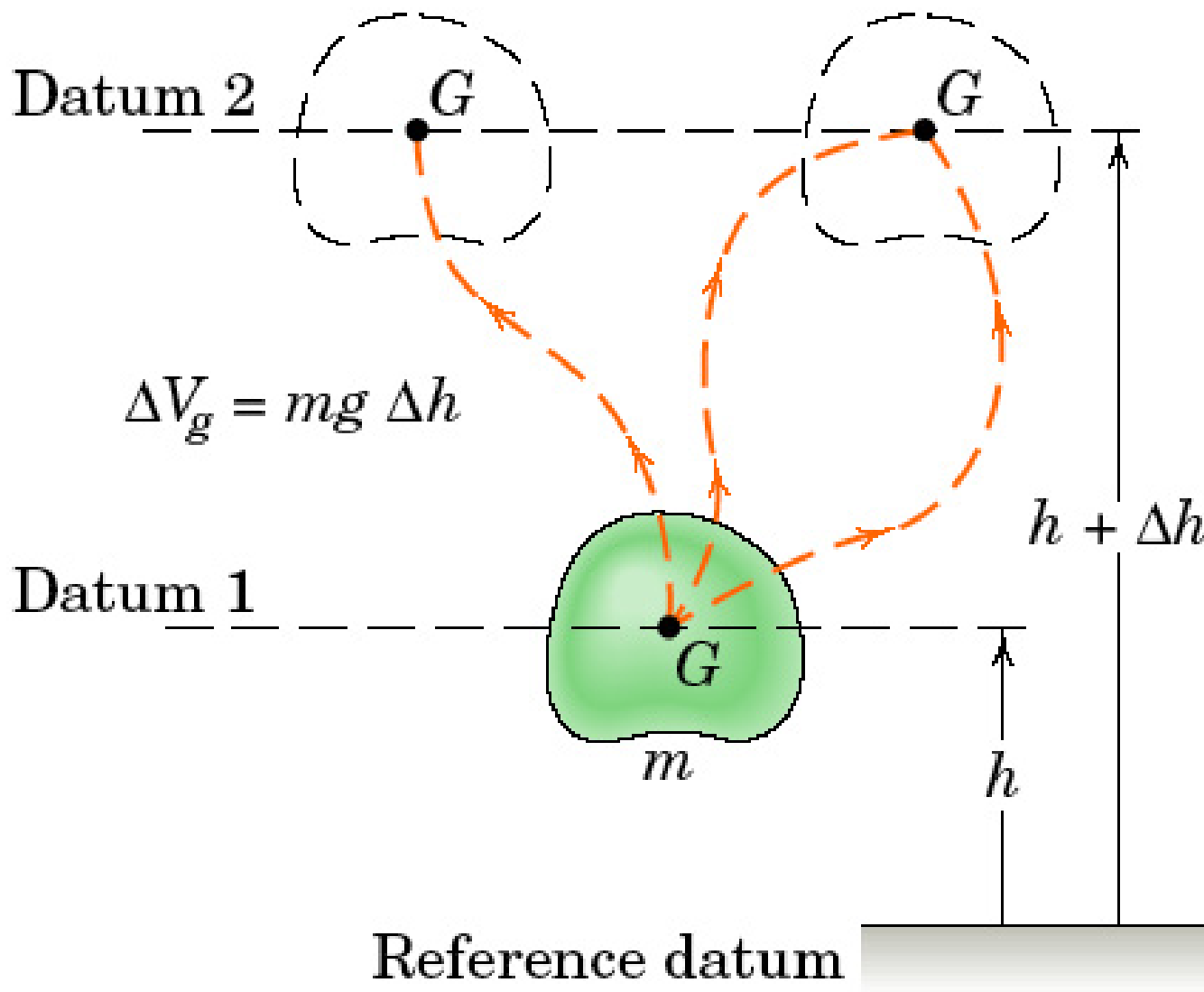
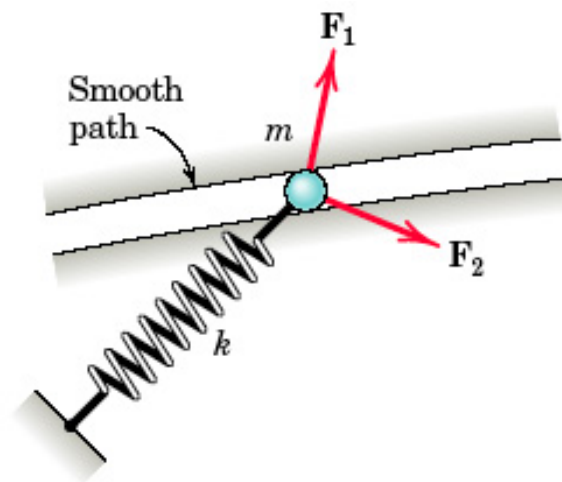
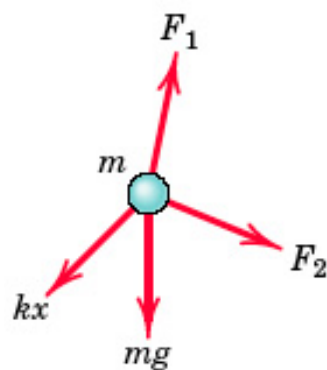


Figure 7/13

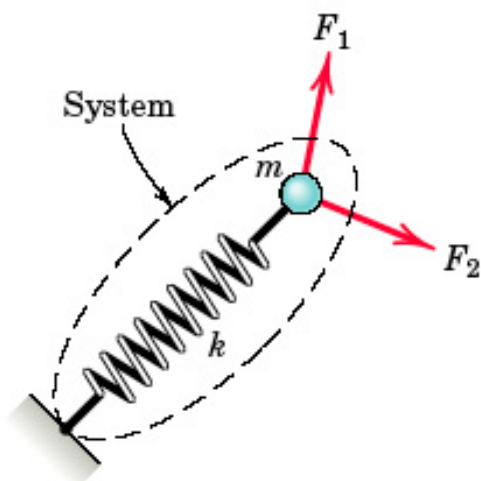


(a)



Eq. 7/3: $\delta U = 0$

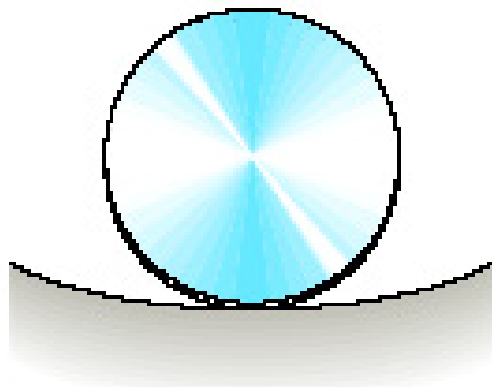
(b)



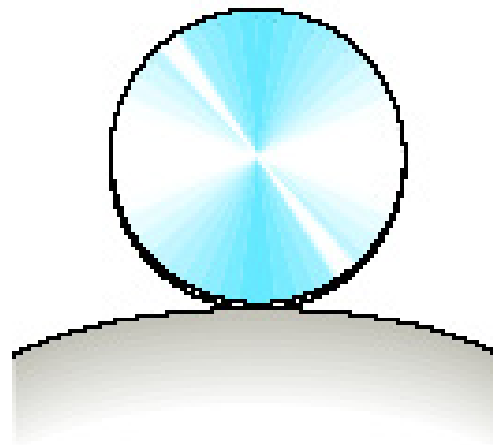
Eq. 7/6: $\delta U' = \delta V_e + \delta V_g = \delta V$

(c)

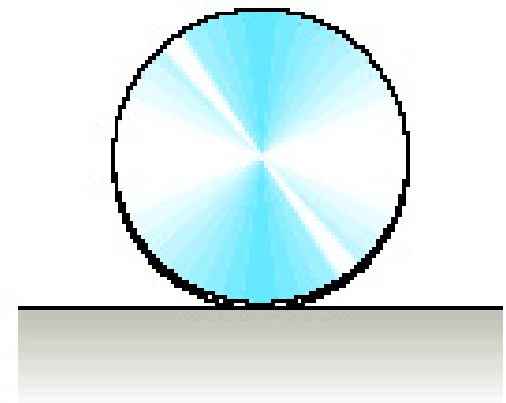
Figure 7/14



Stable

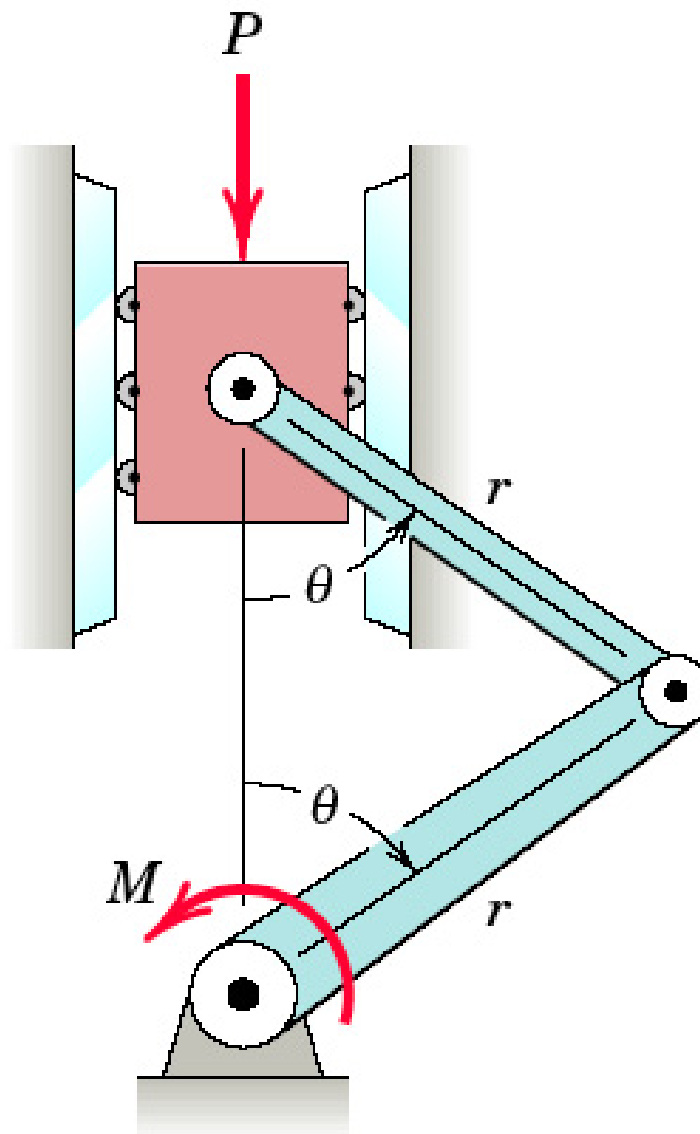


Unstable

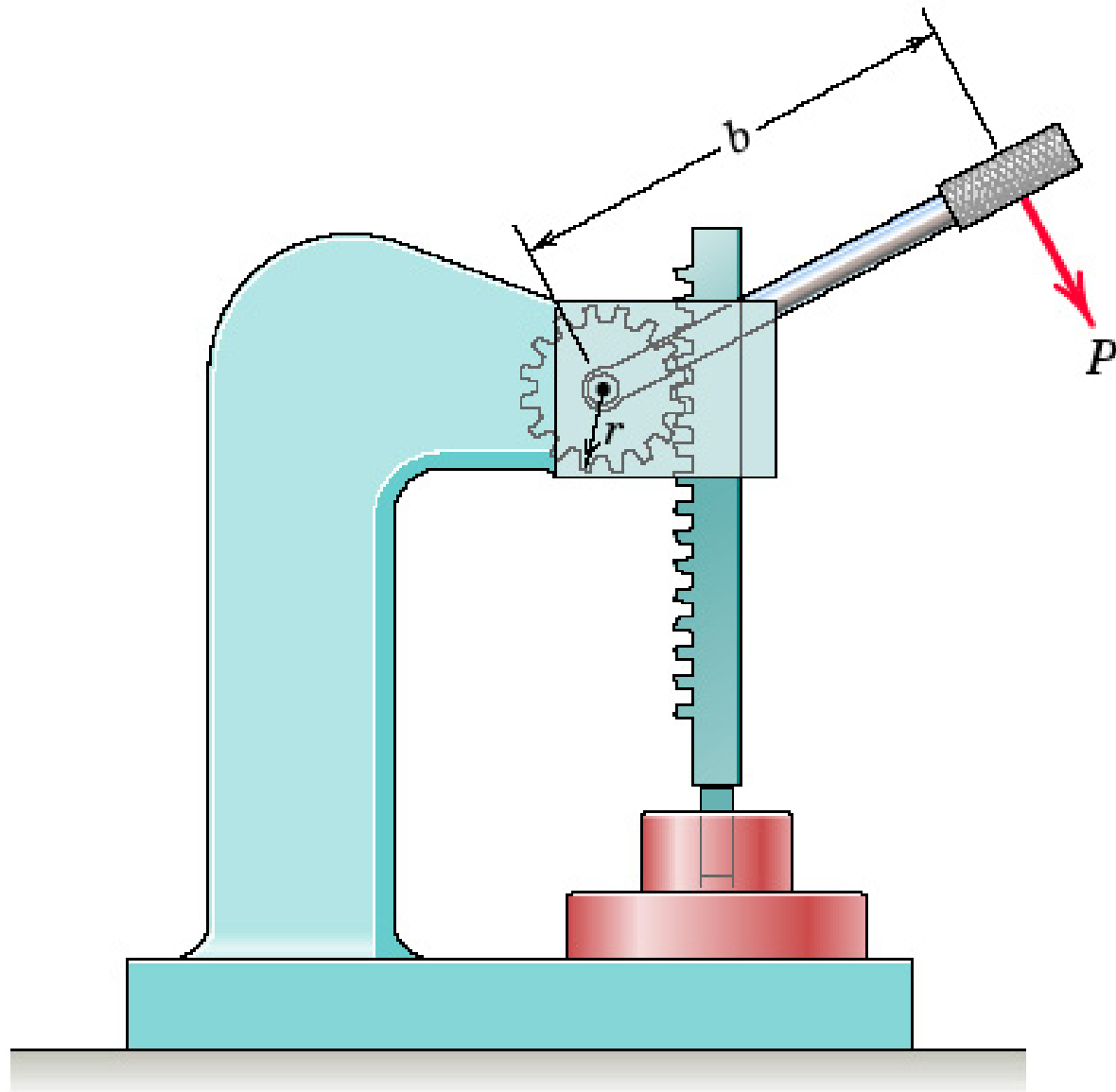


Neutral

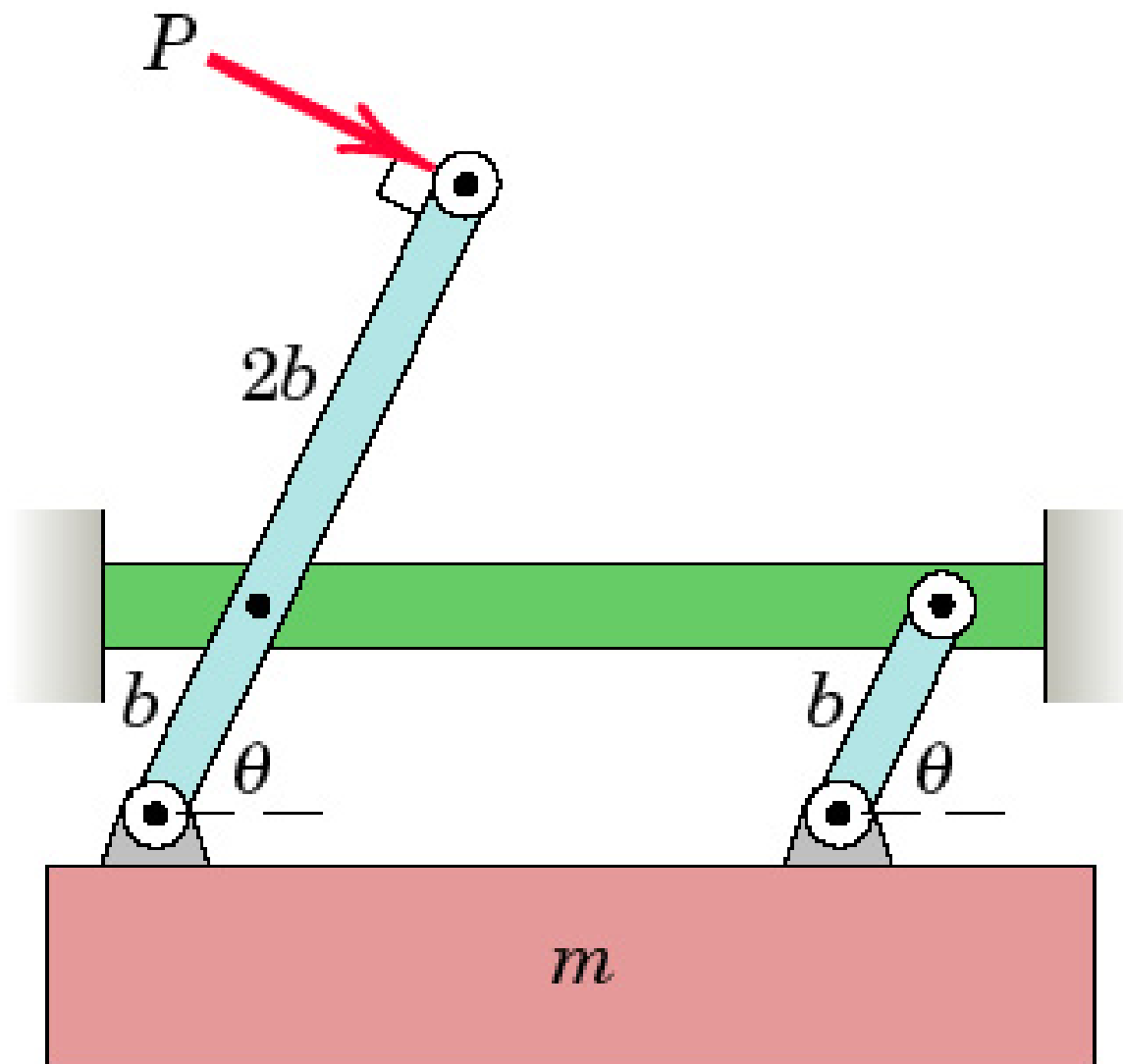
Figure 7/15



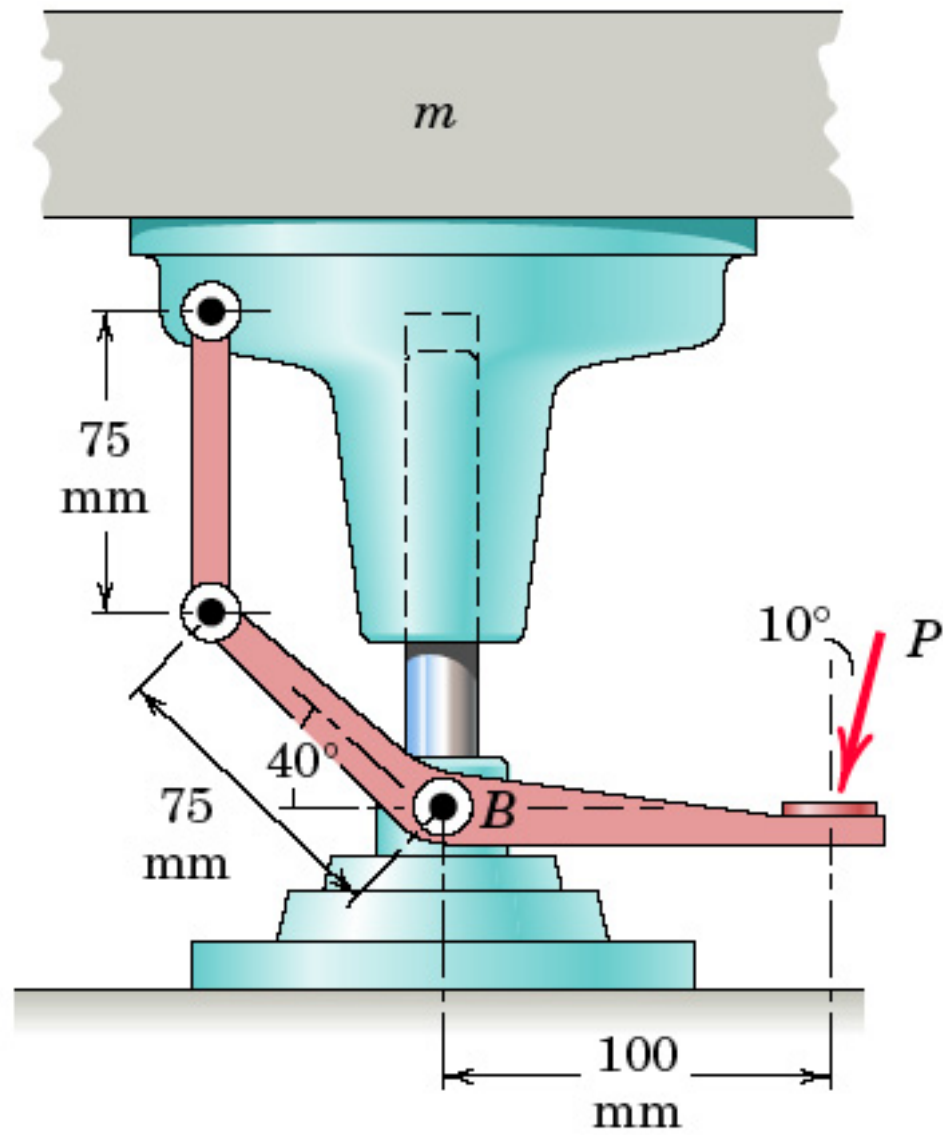
Problem 7/1



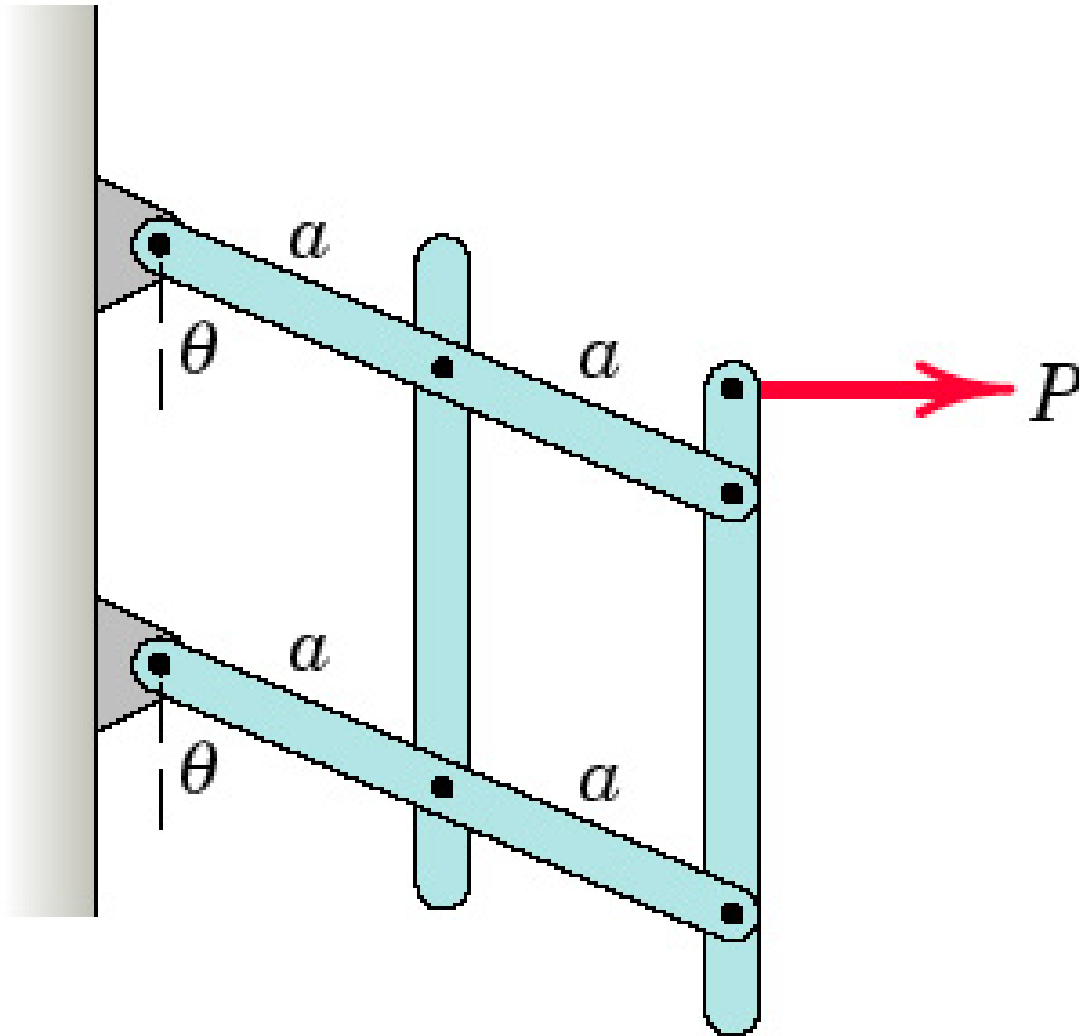
Problem 7/2



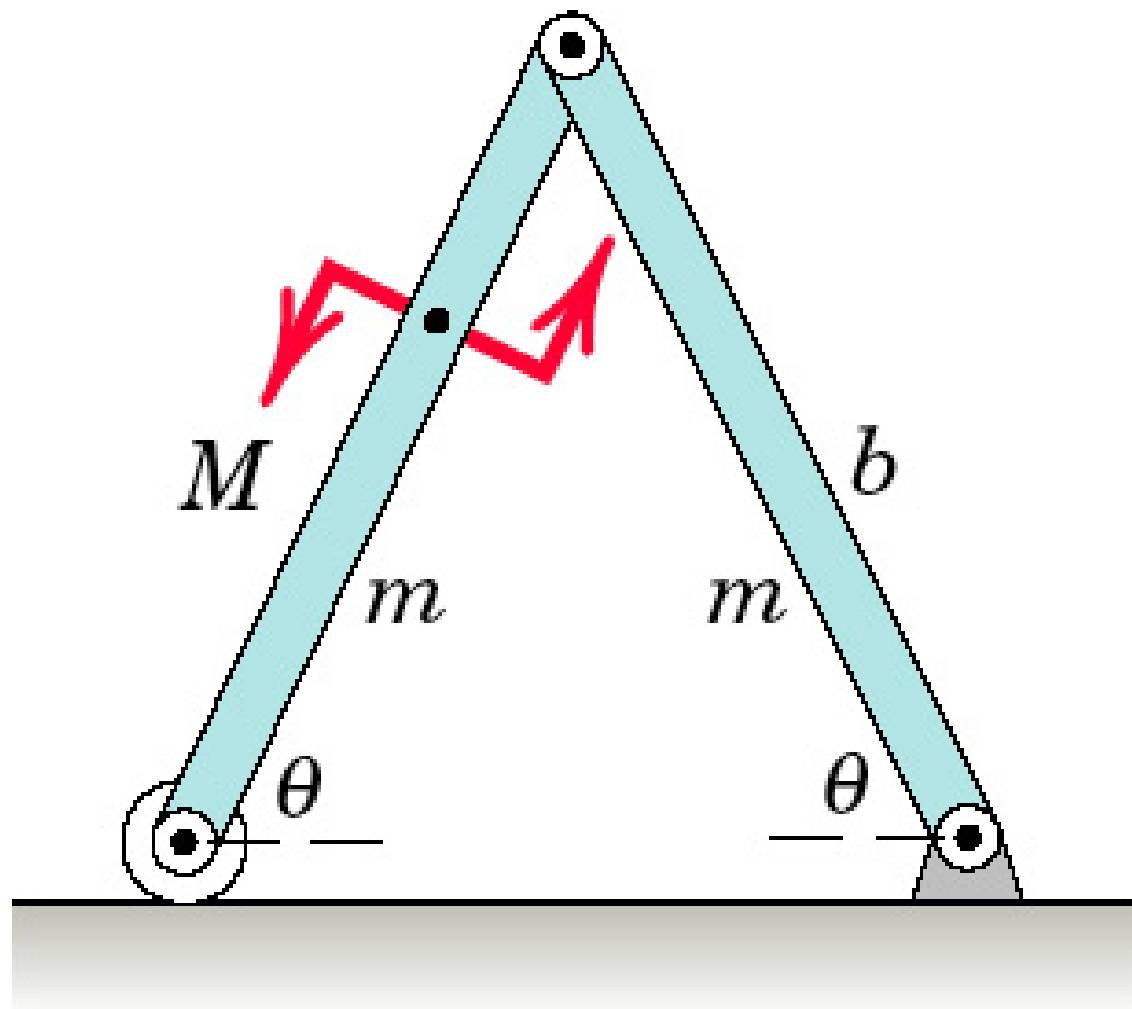
Problem 7/3



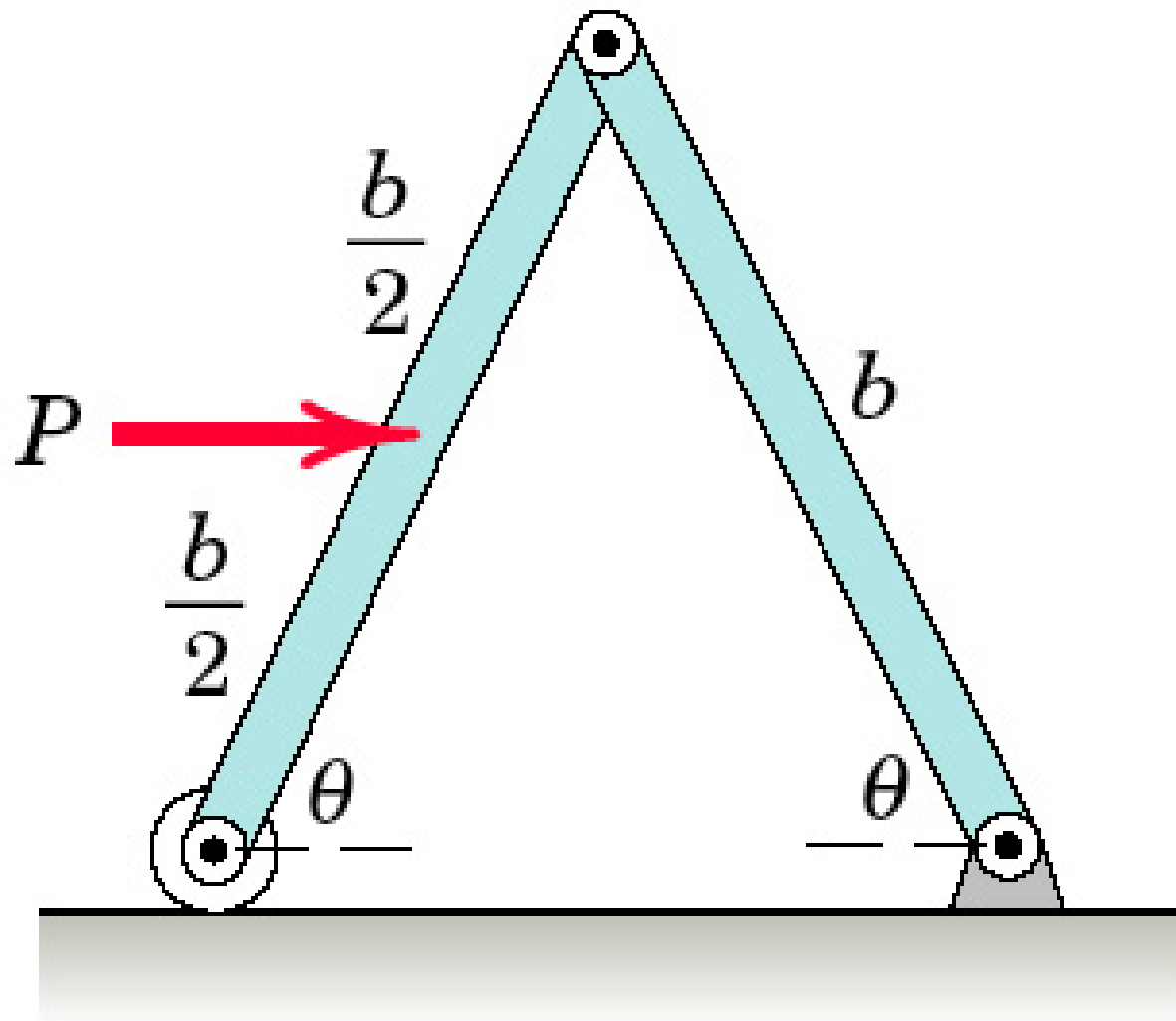
Problem 7/4



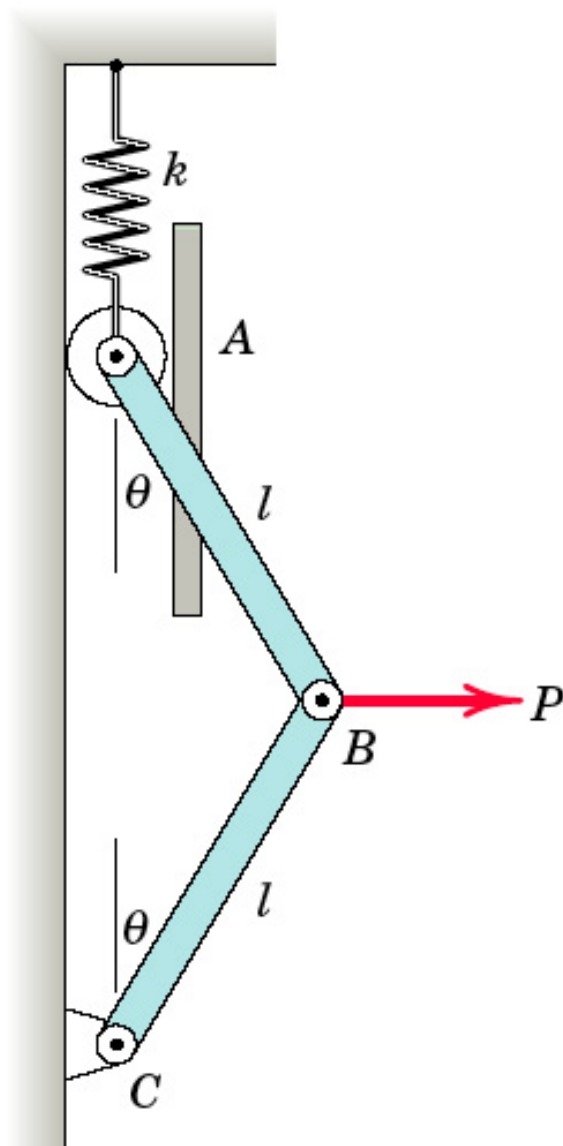
Problem 7/5



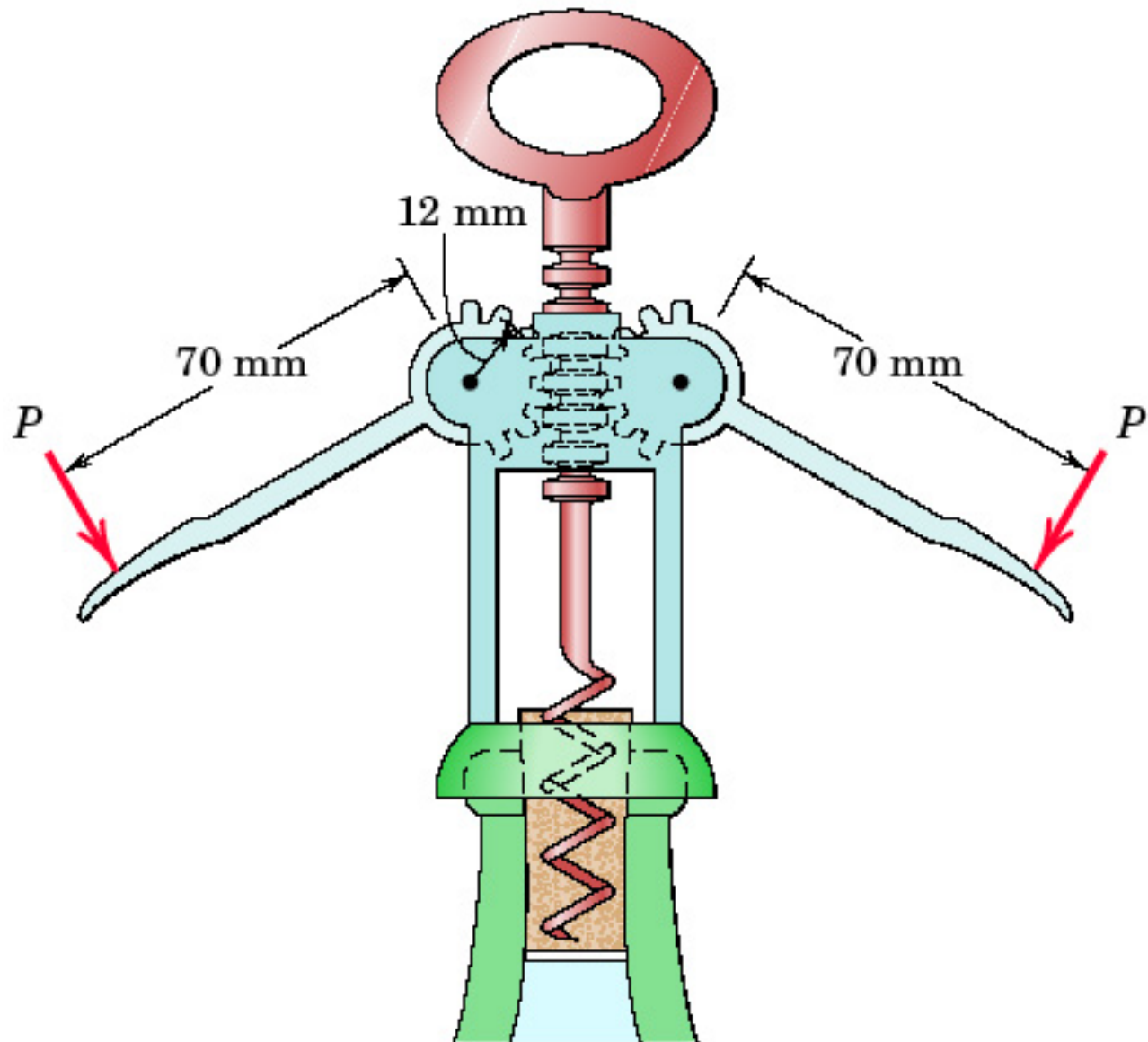
Problem 7/6



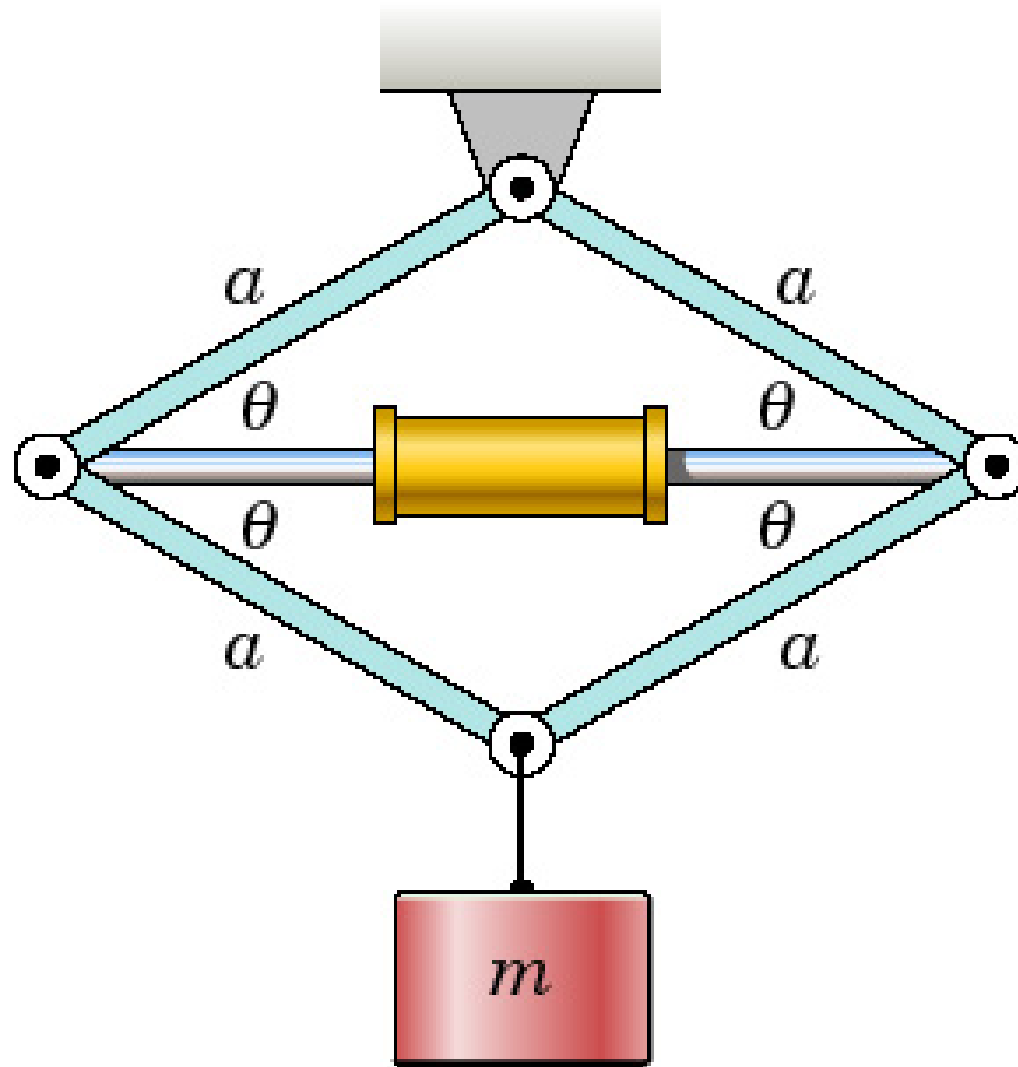
Problem 7/7



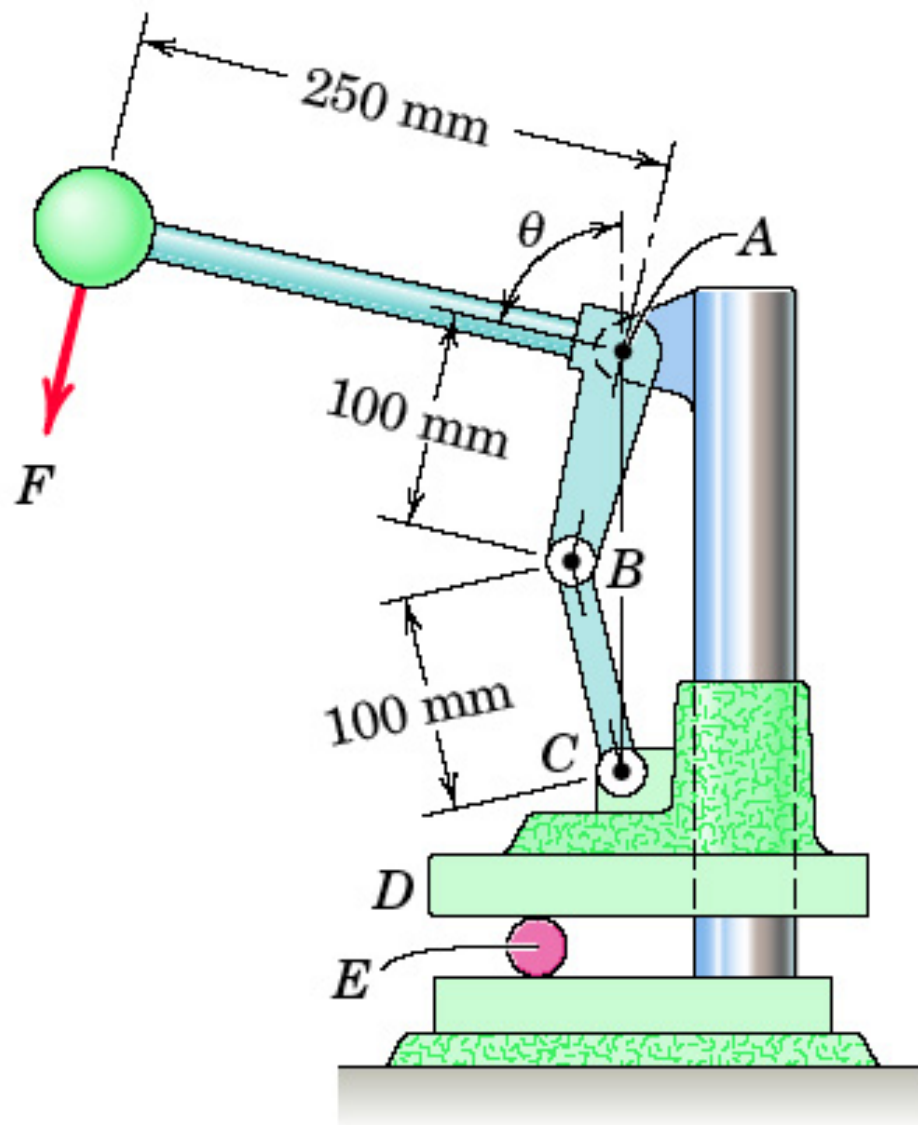
Problem 7/8



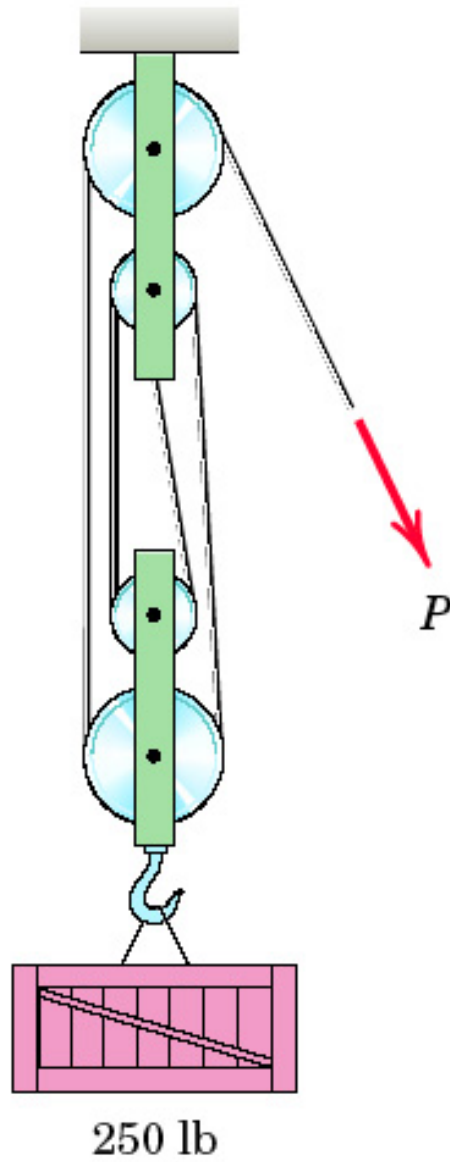
Problem 7/9



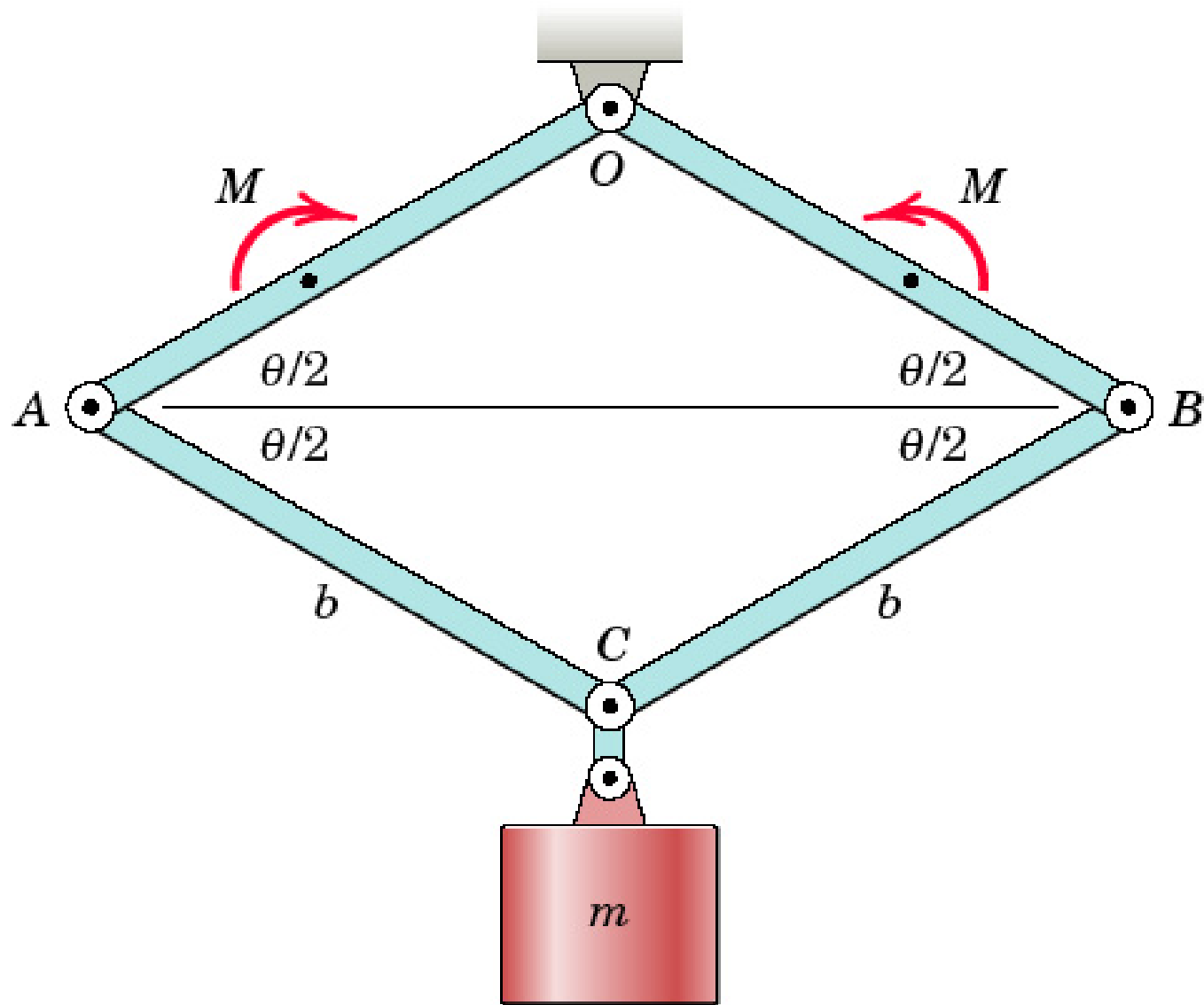
Problem 7/10



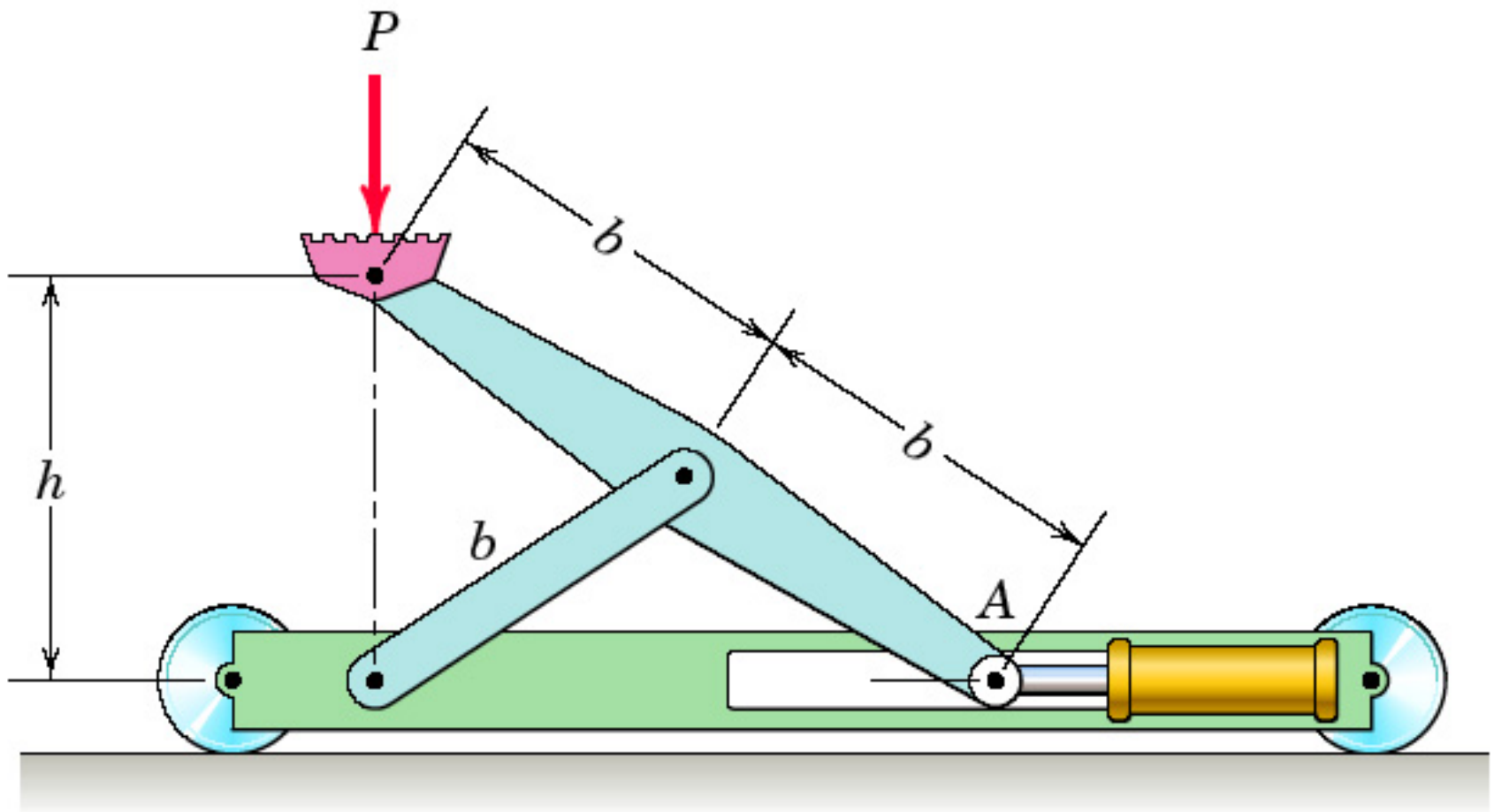
Problem 7/11



Problem 7/12

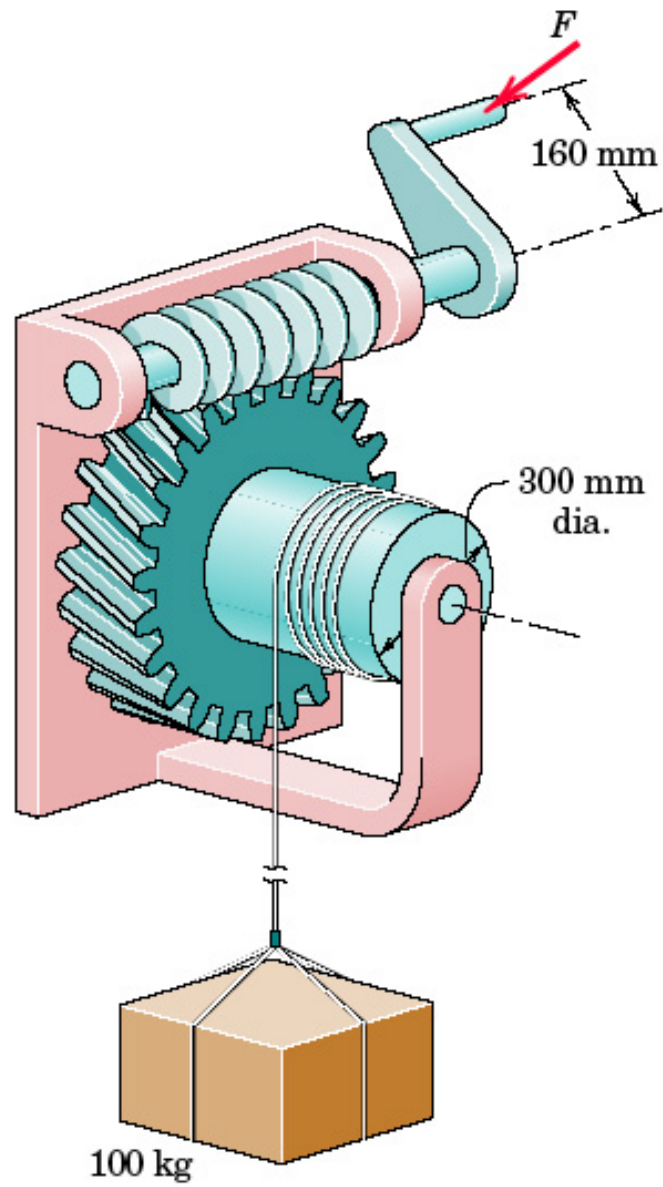


Problem 7/13

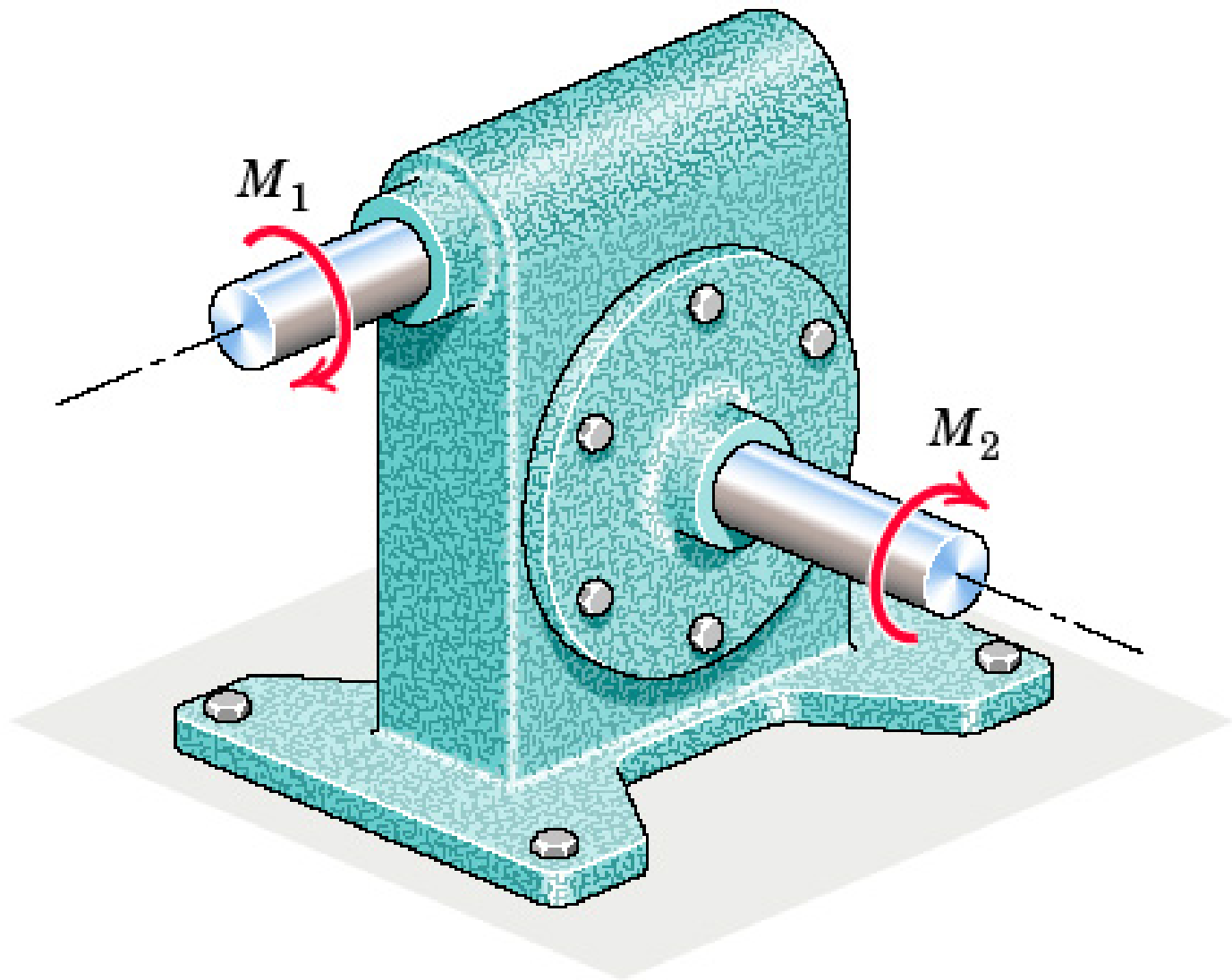


Problem 7/14

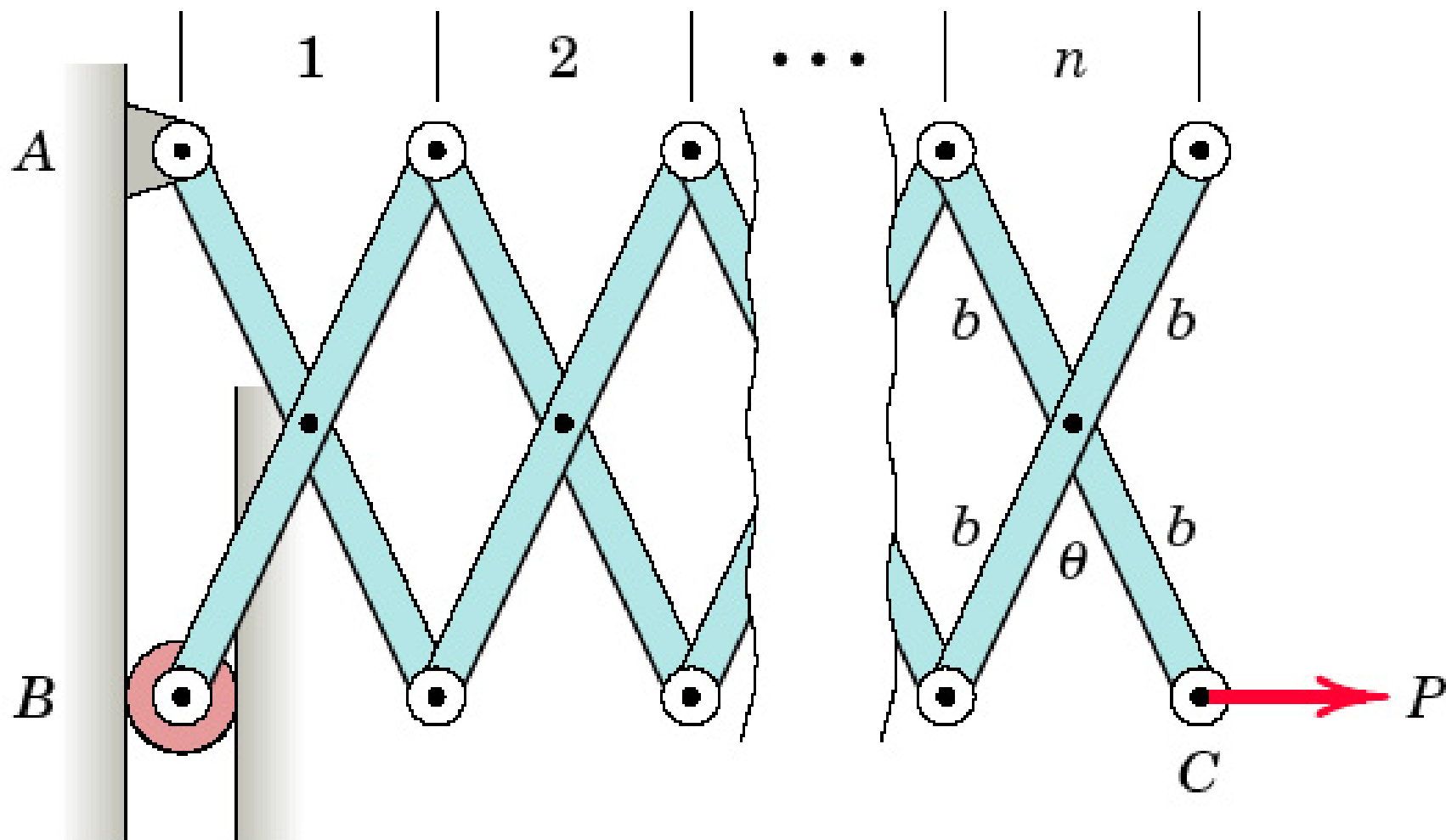
P_07_14



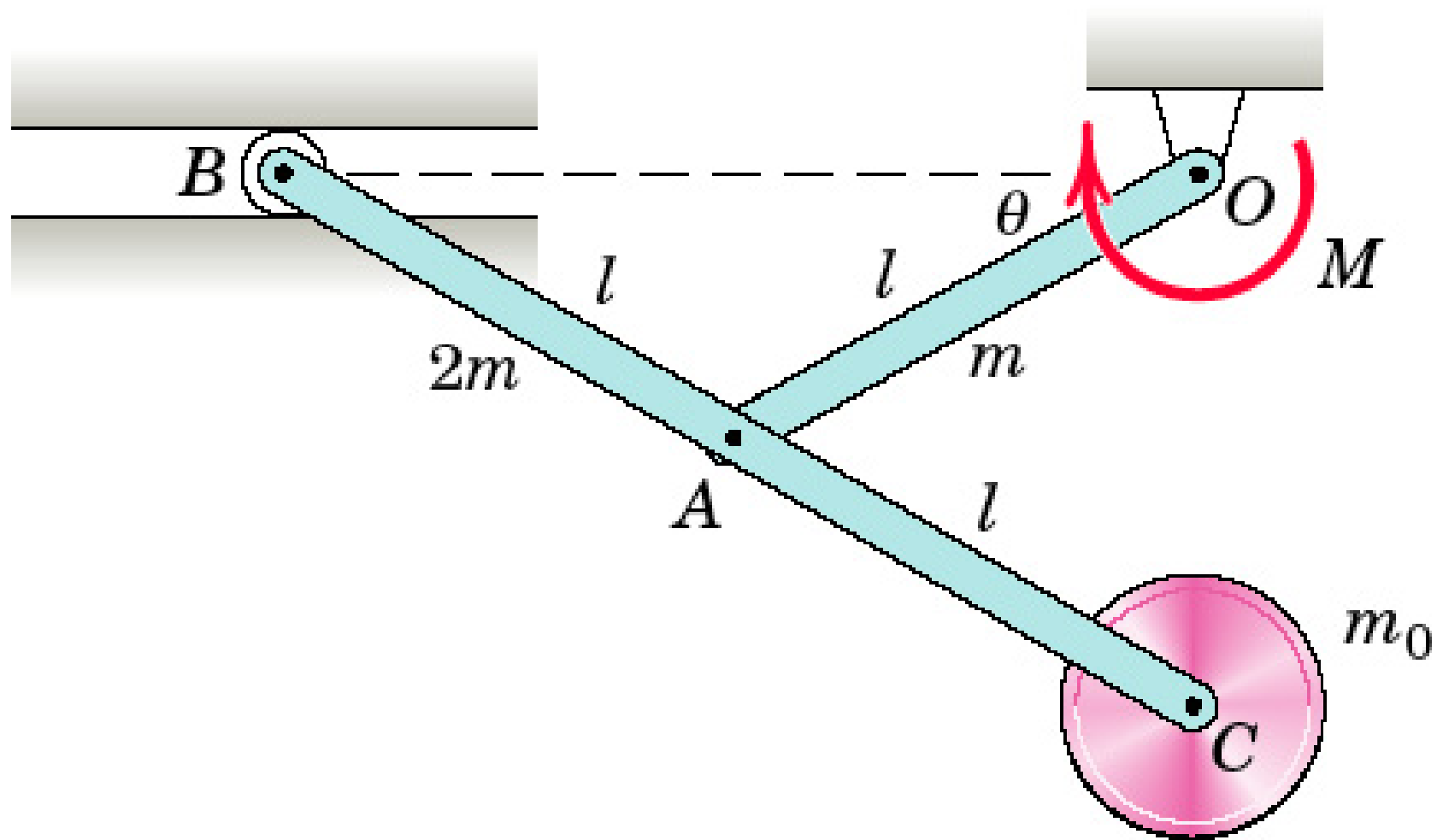
Problem 7/15



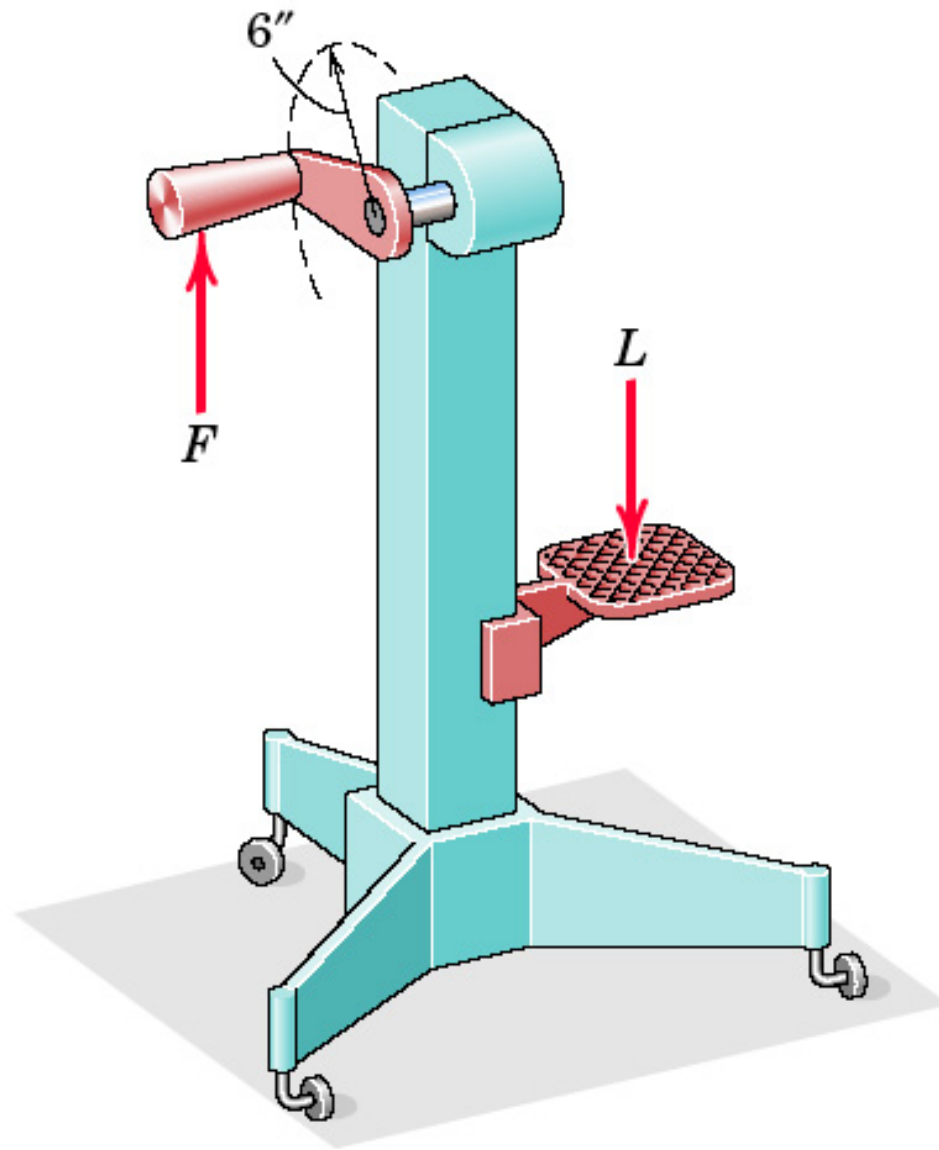
Problem 7/16



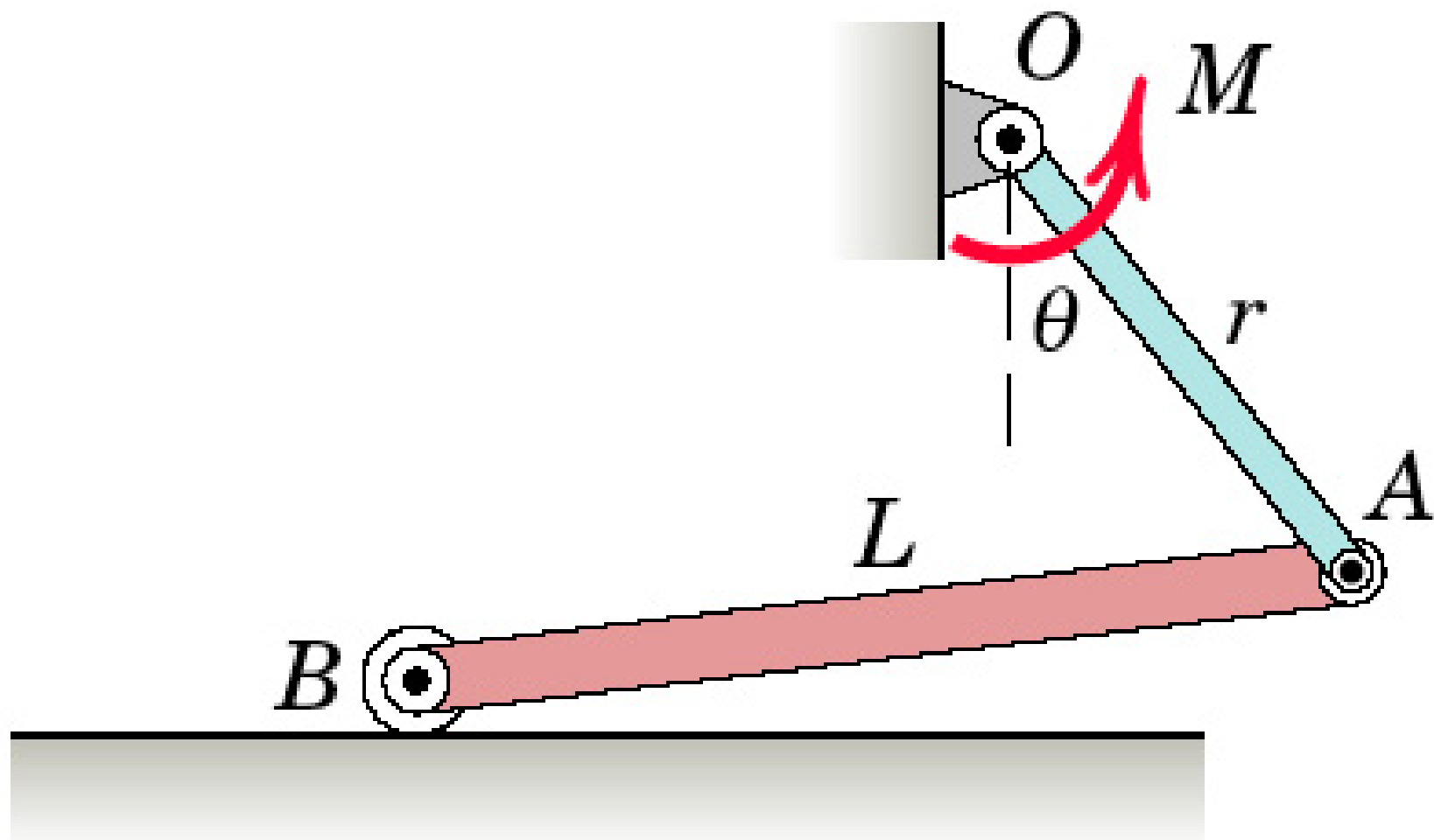
Problem 7/17



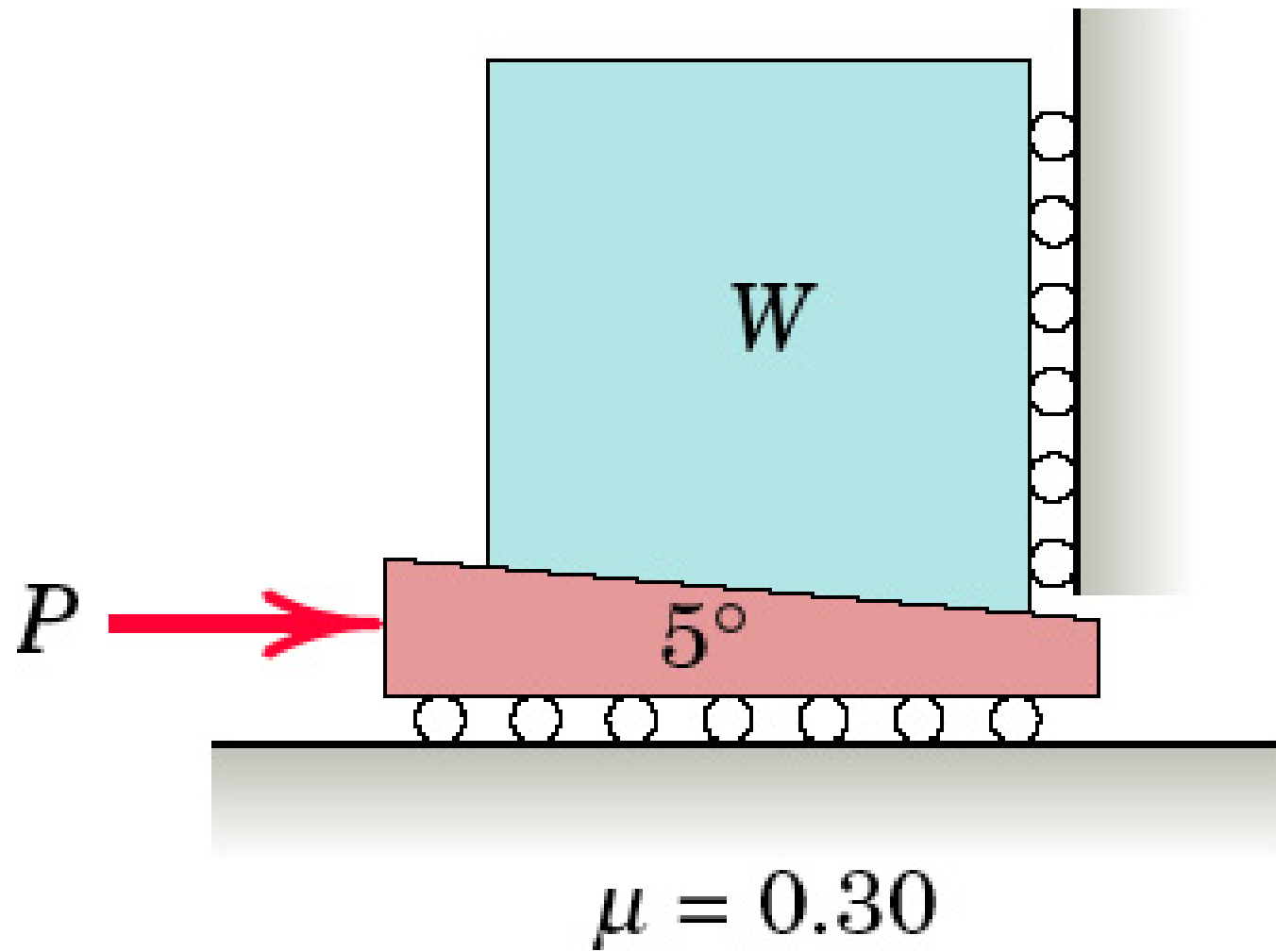
Problem 7/19



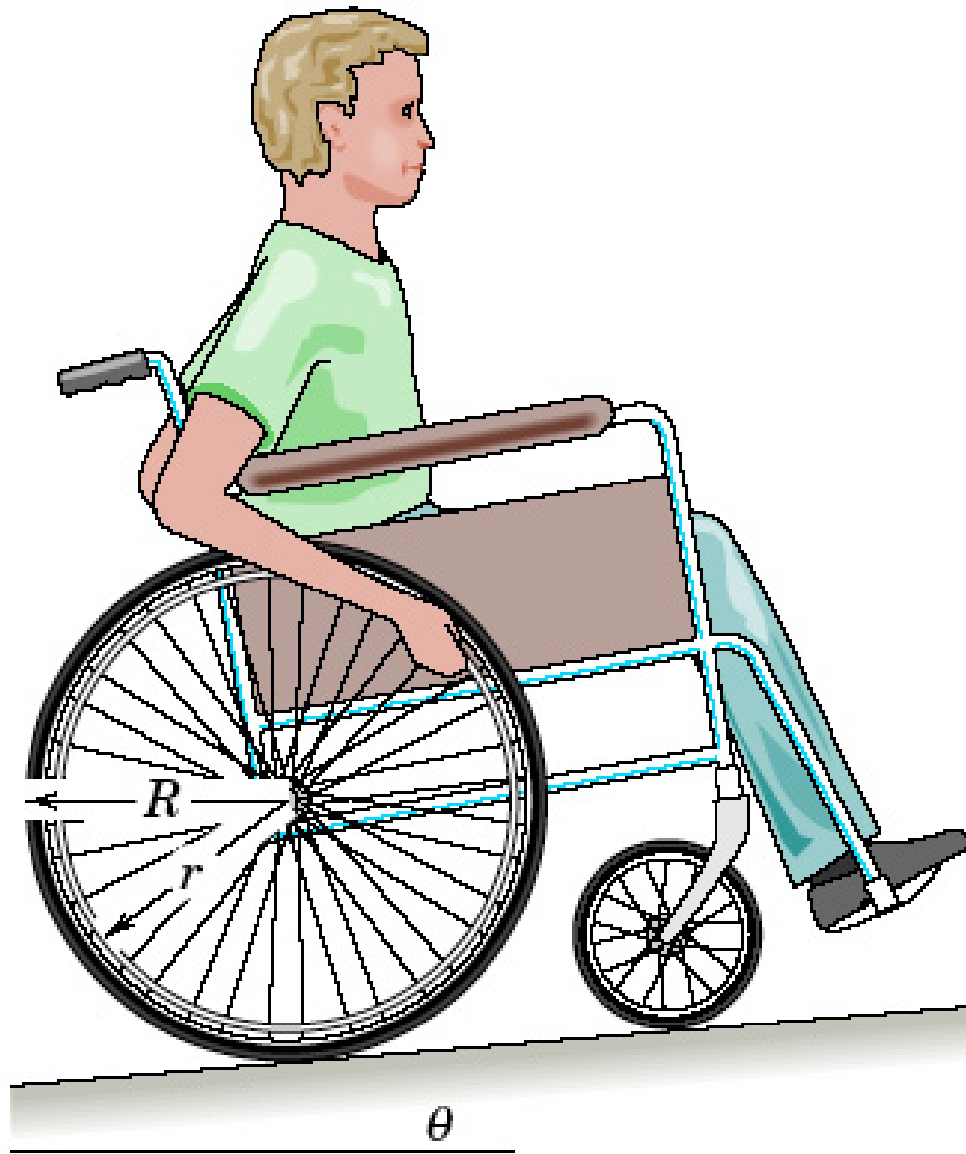
Problem 7/20



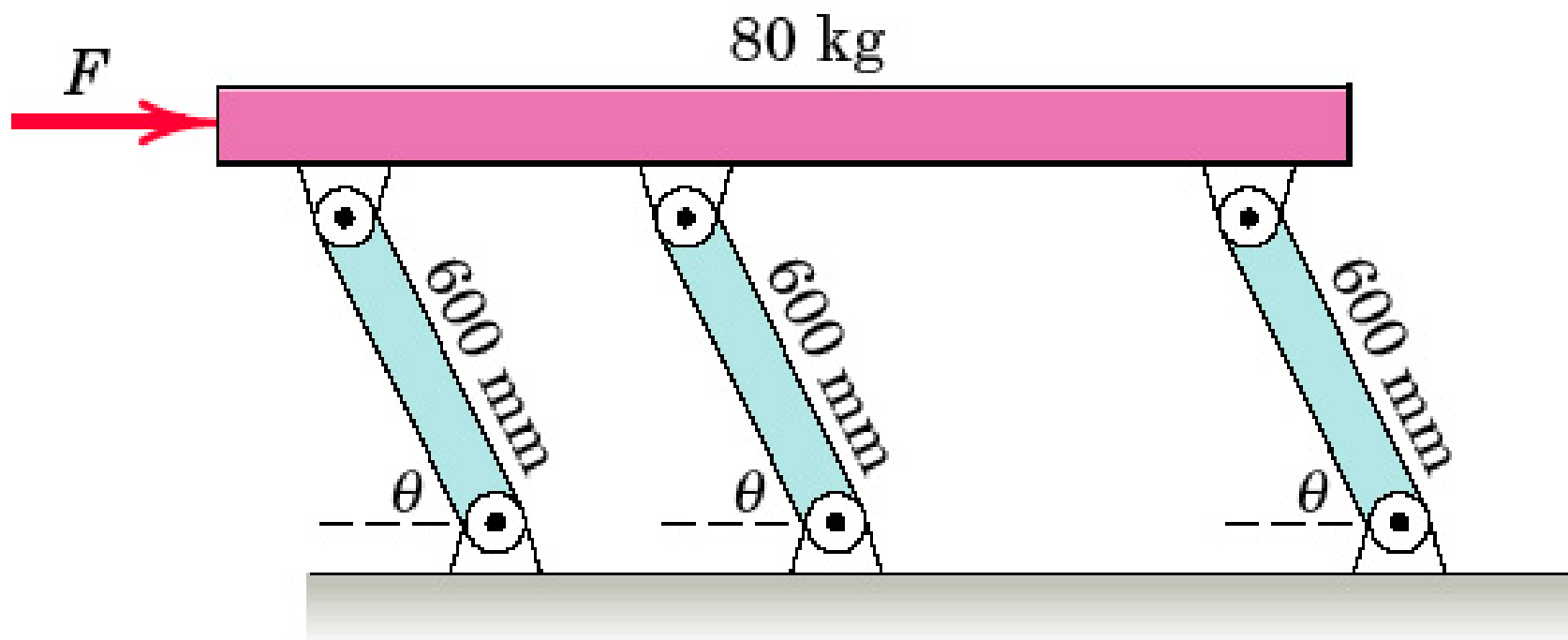
Problem 7/21



Problem 7/22

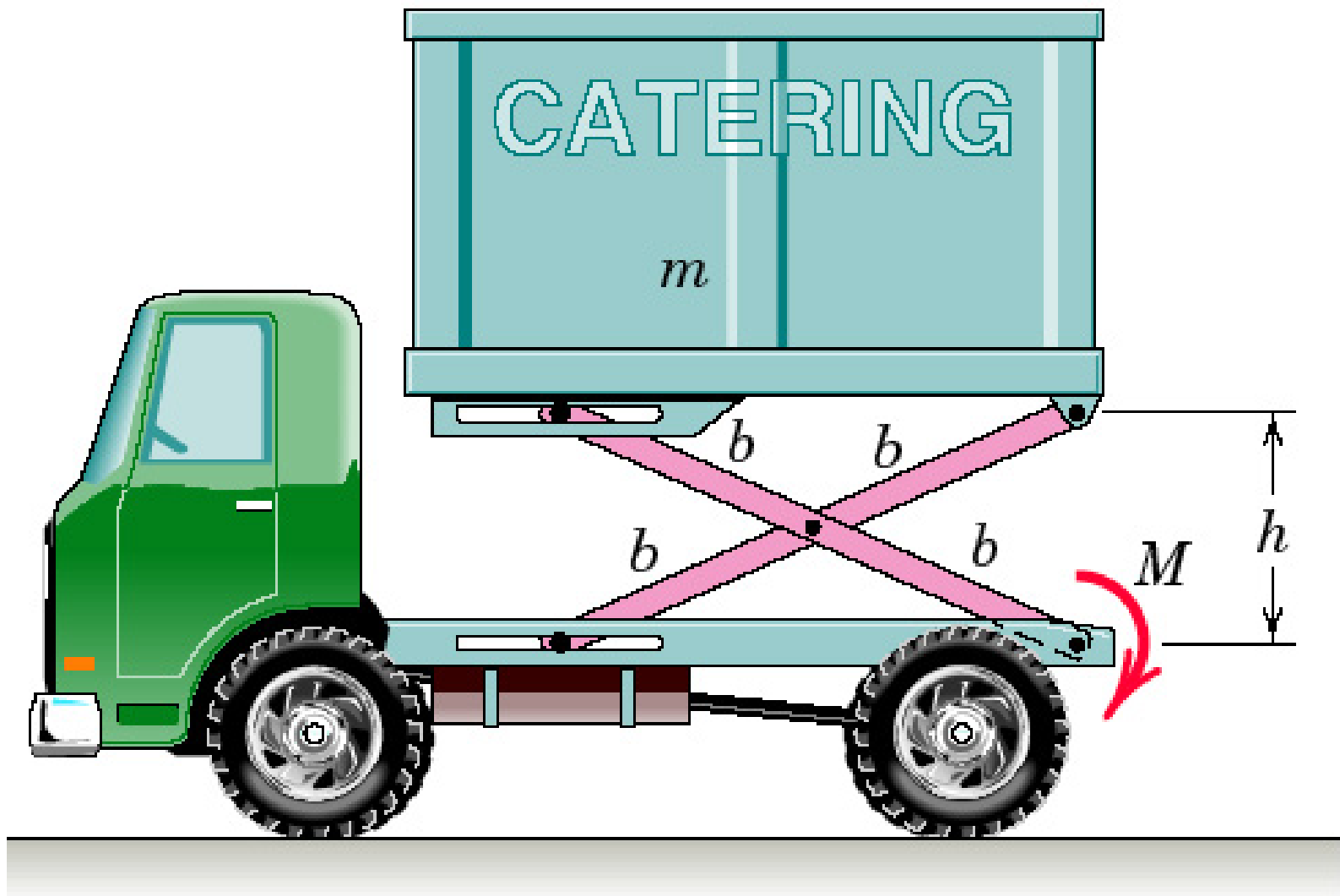


Problem 7/23

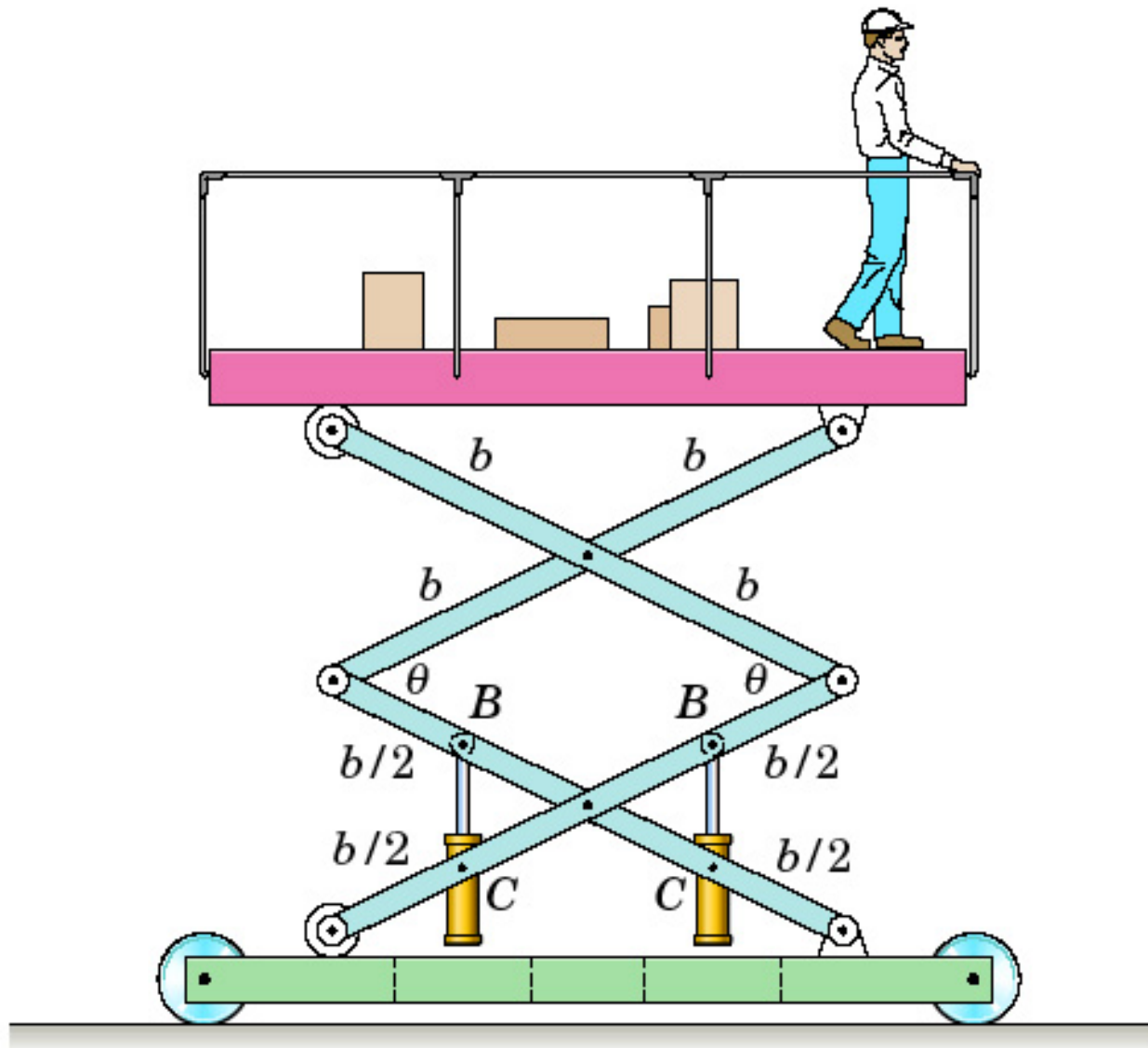


Problem 7/24

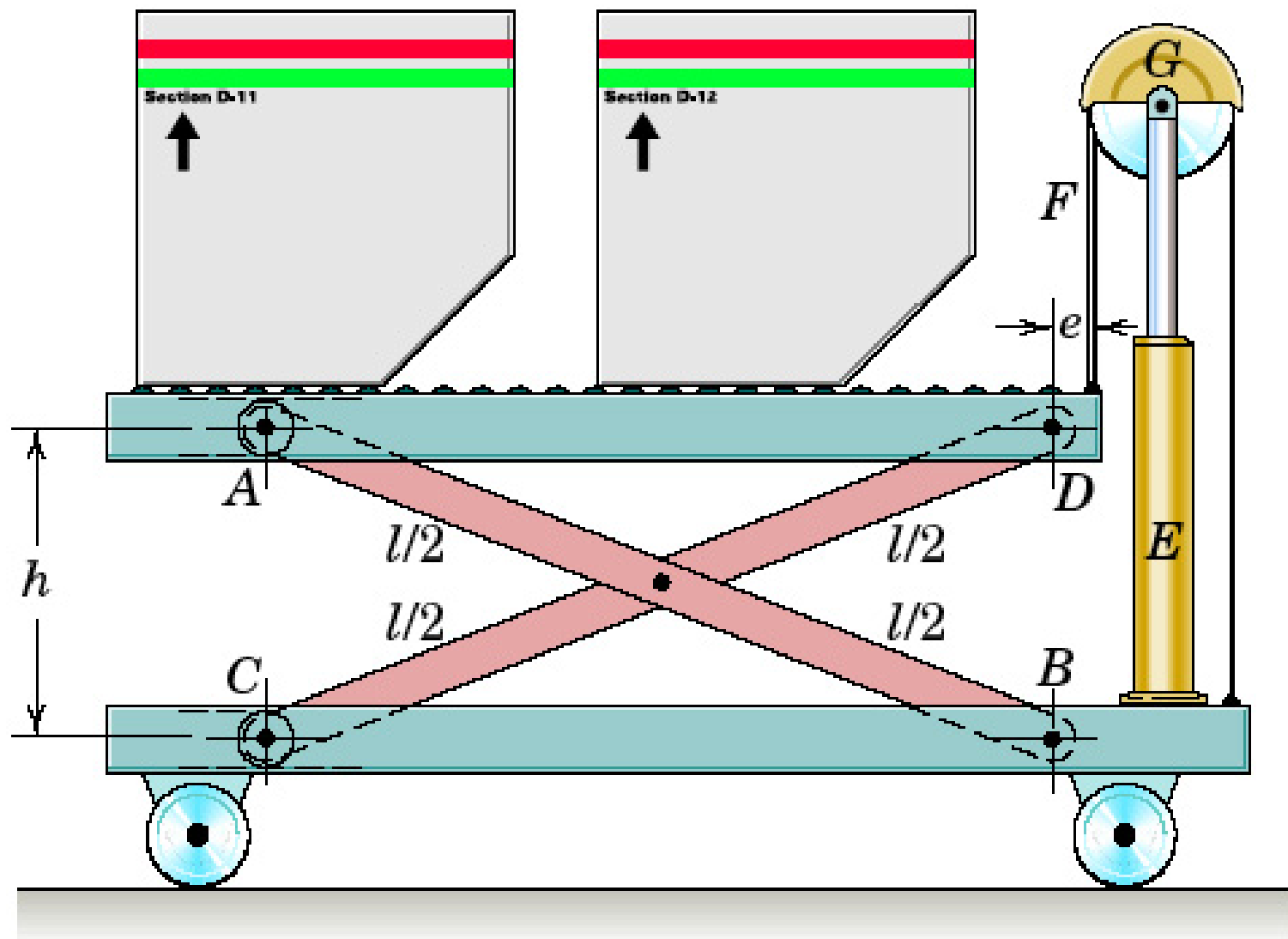
P_07_24



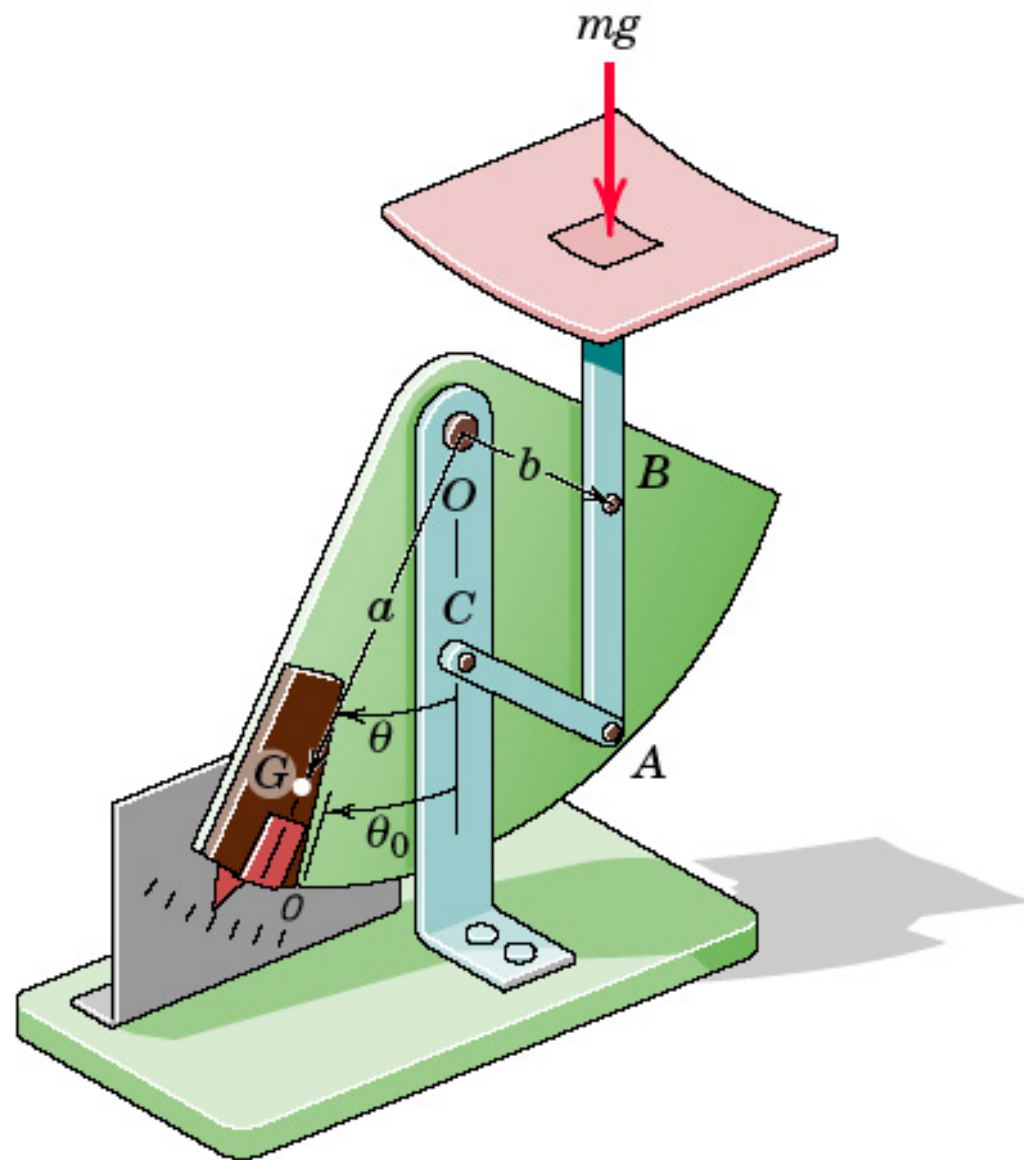
Problem 7/25



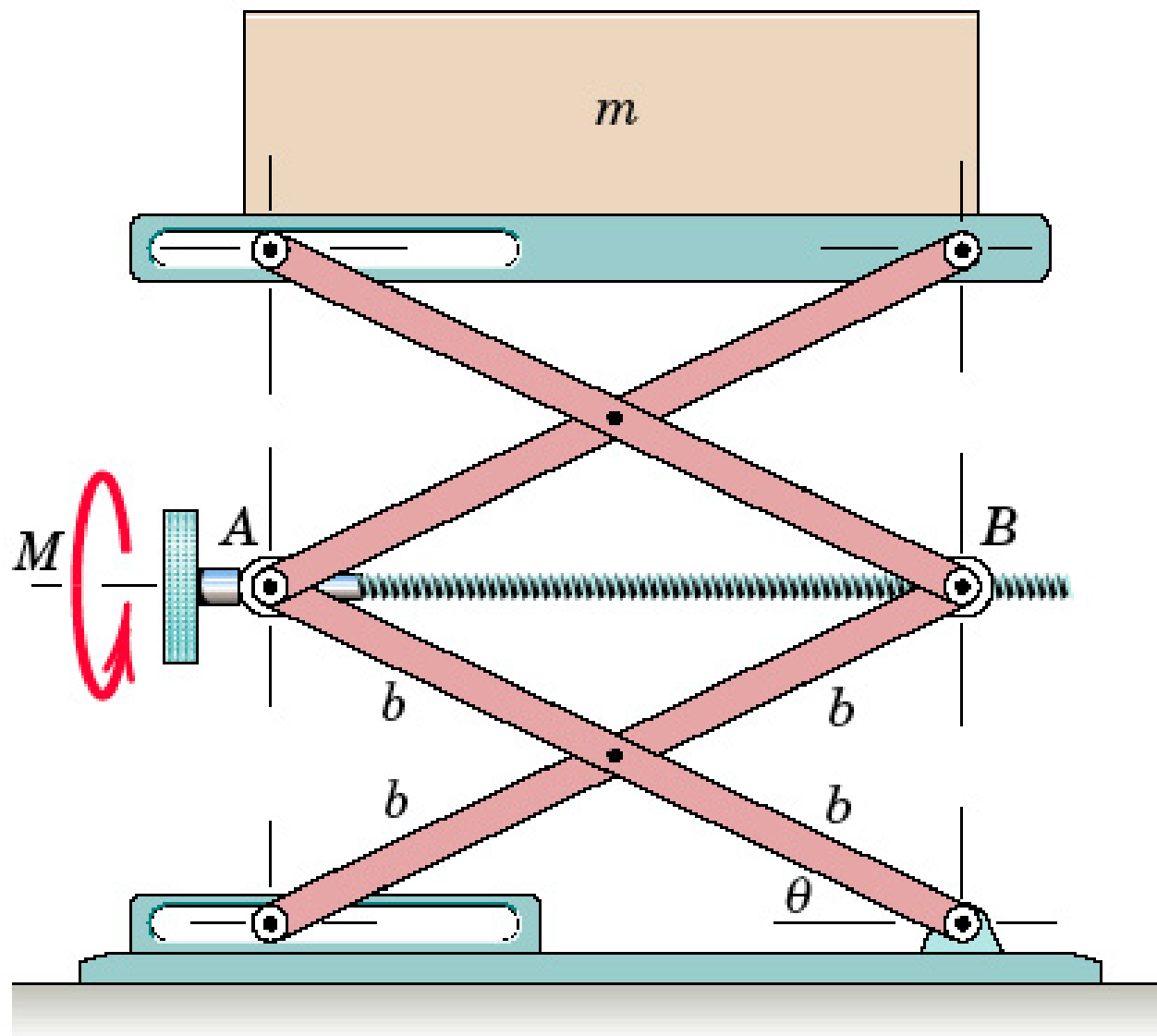
Problem 7/26



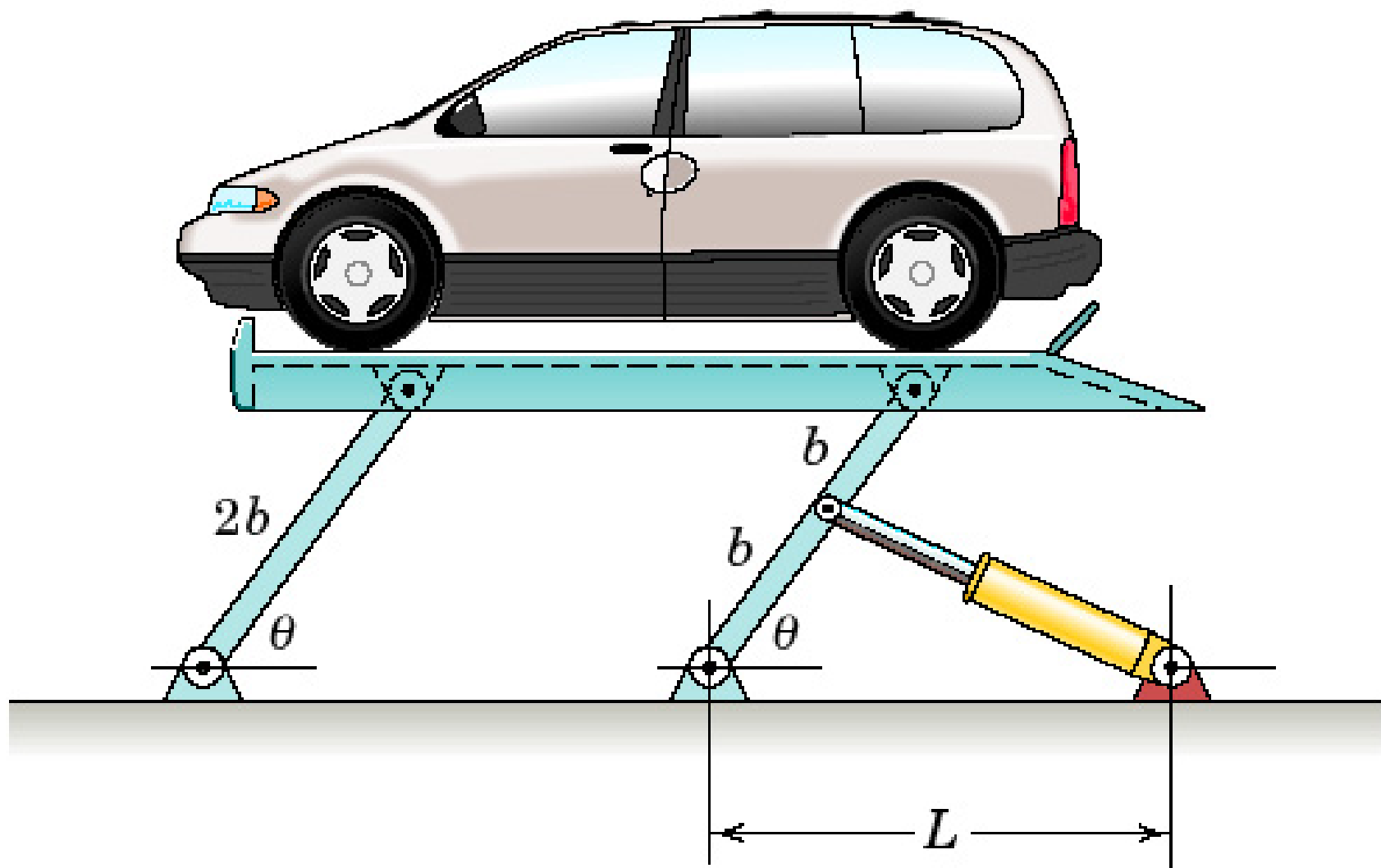
Problem 7/27



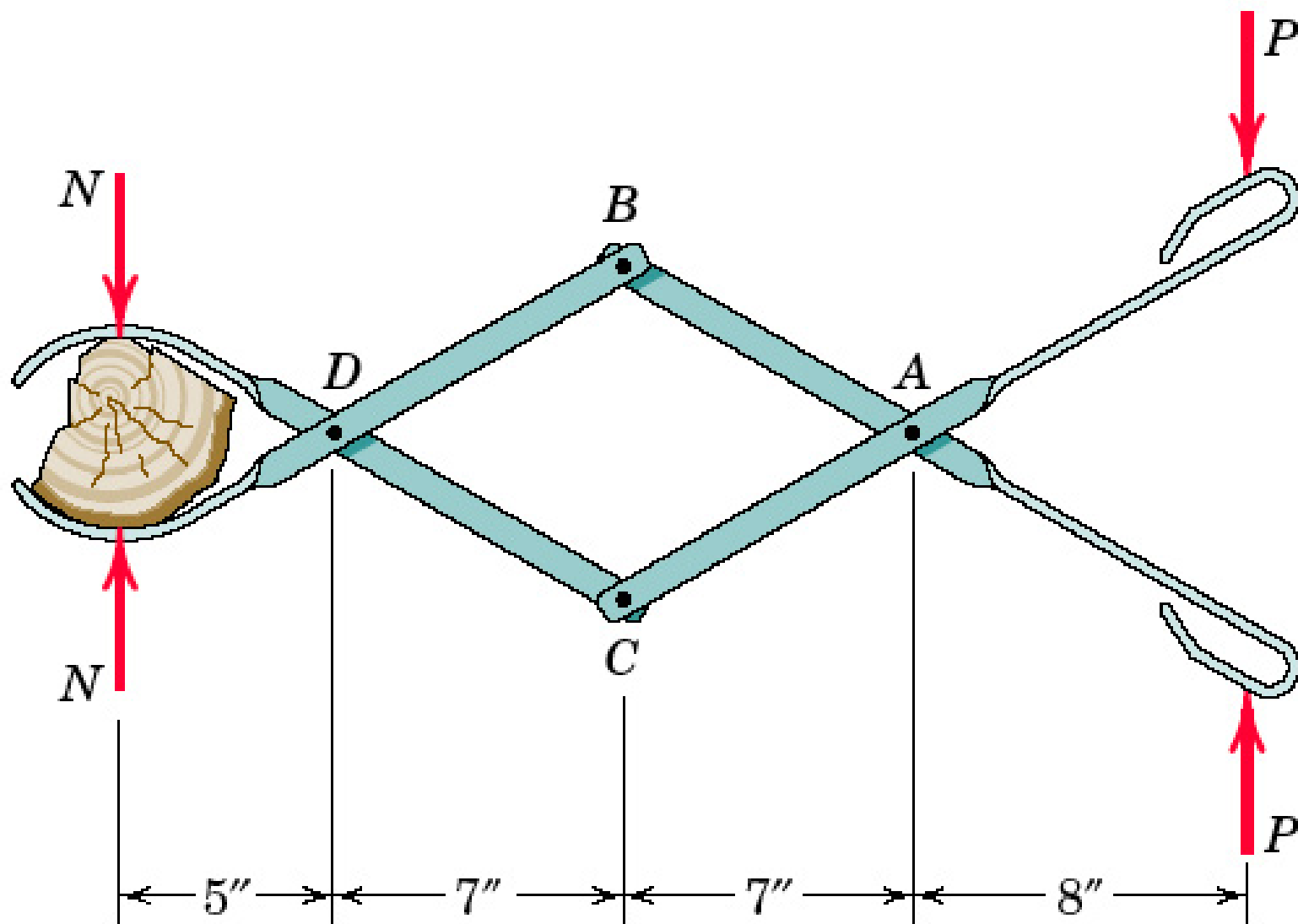
Problem 7/28



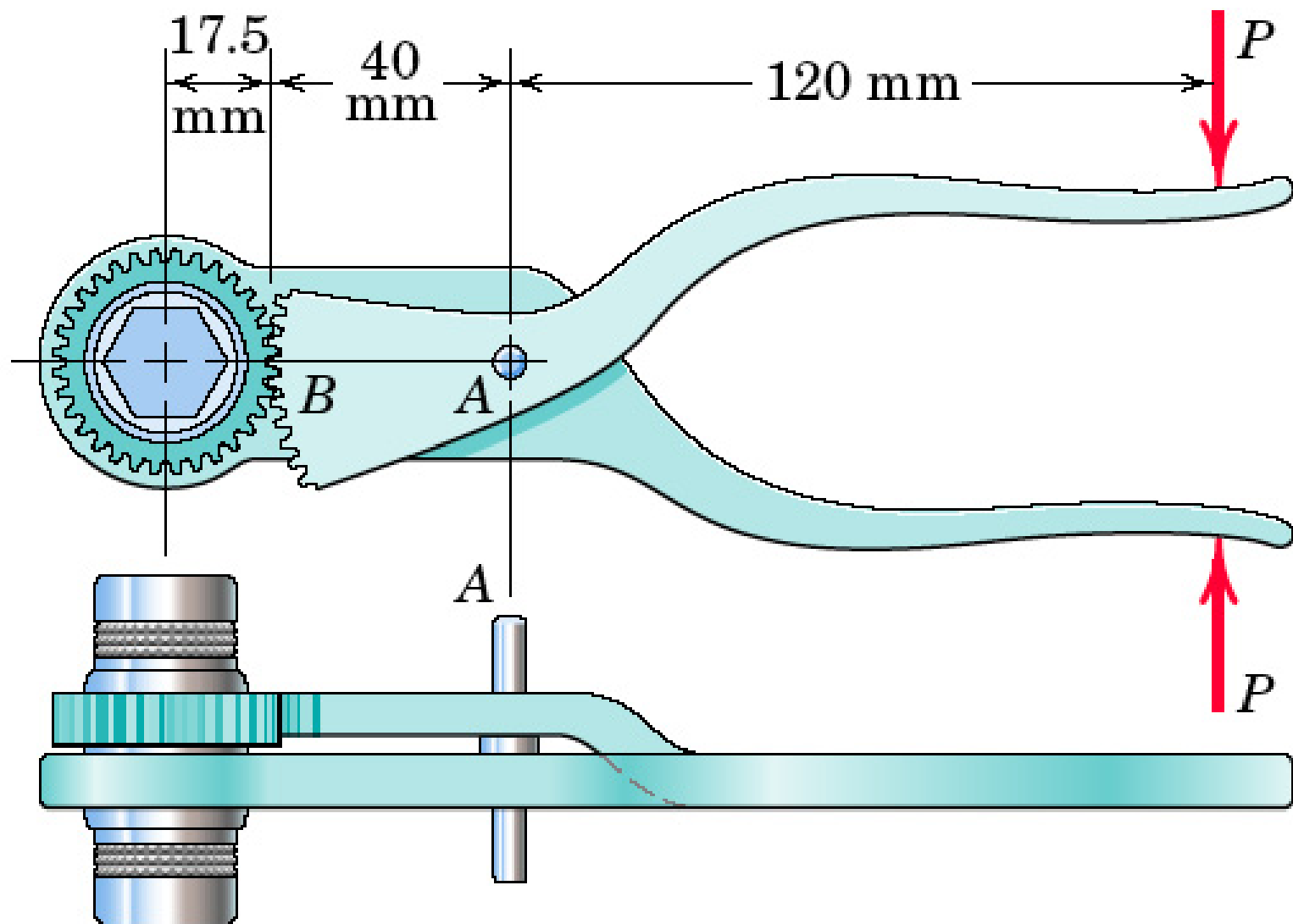
Problem 7/29



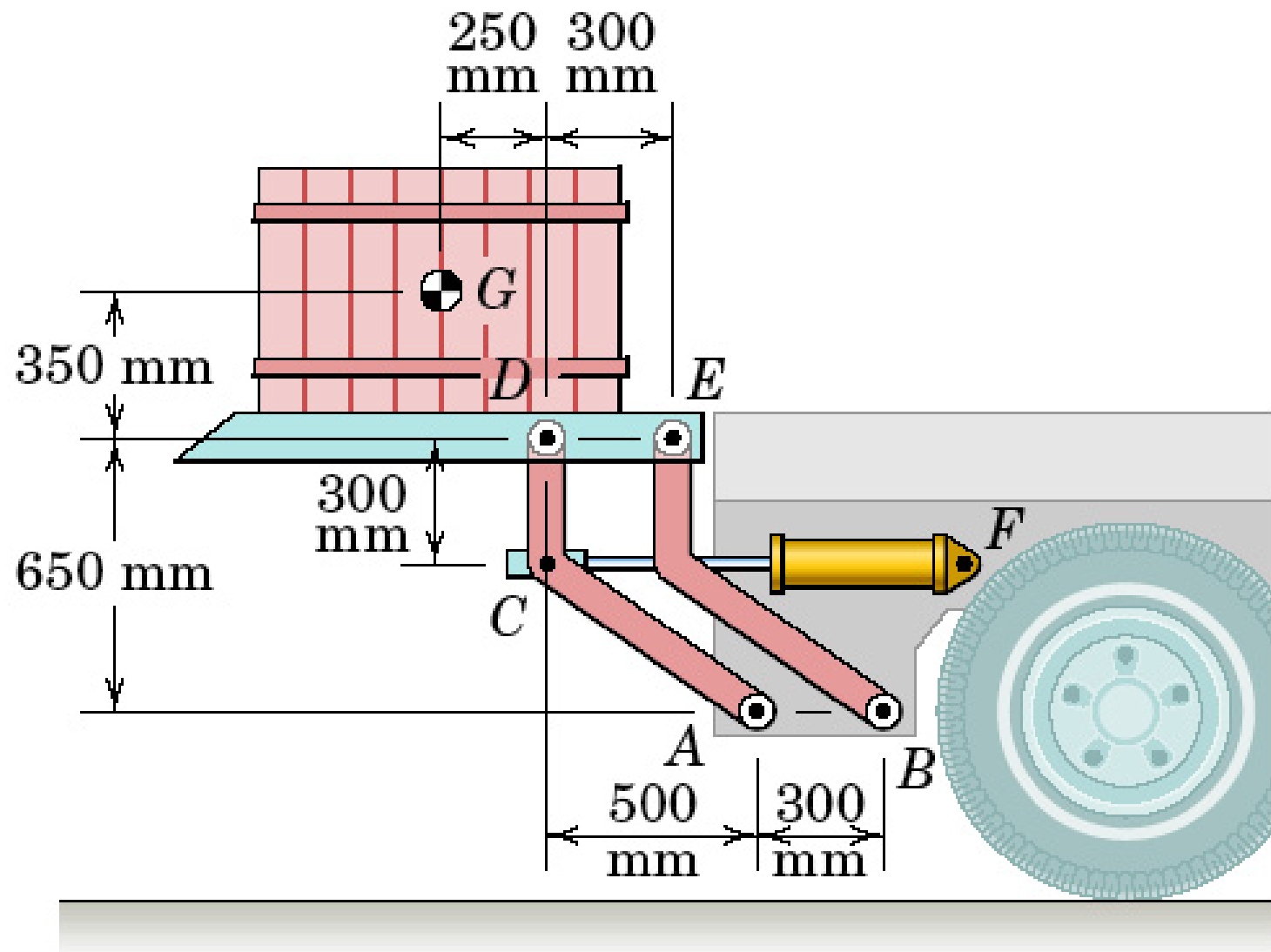
Problem 7/30



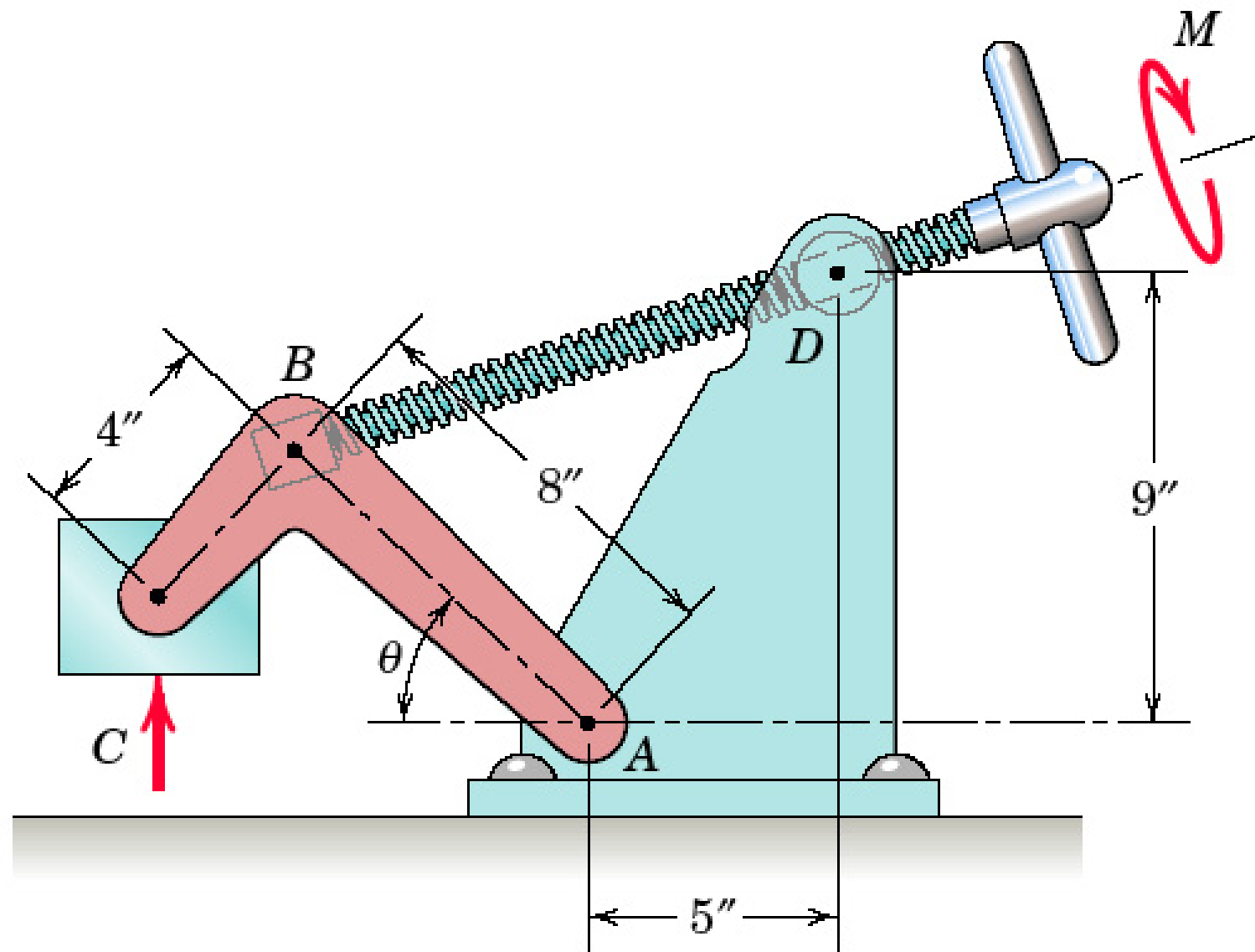
Problem 7/31



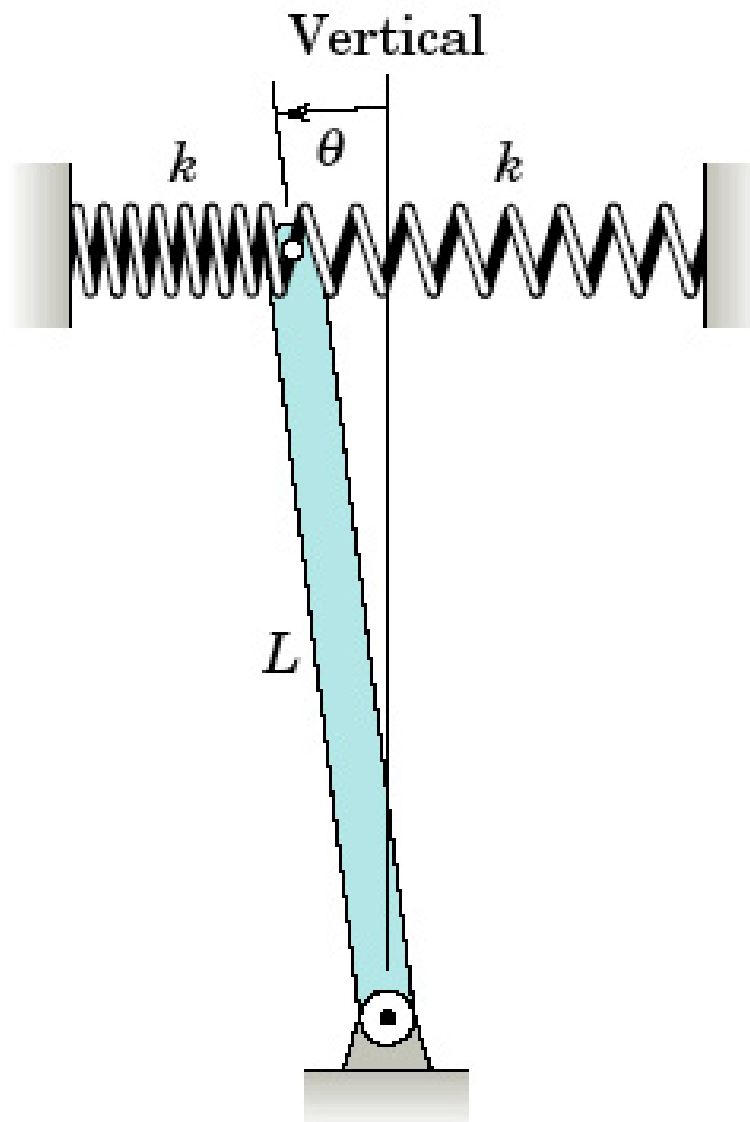
Problem 7/32



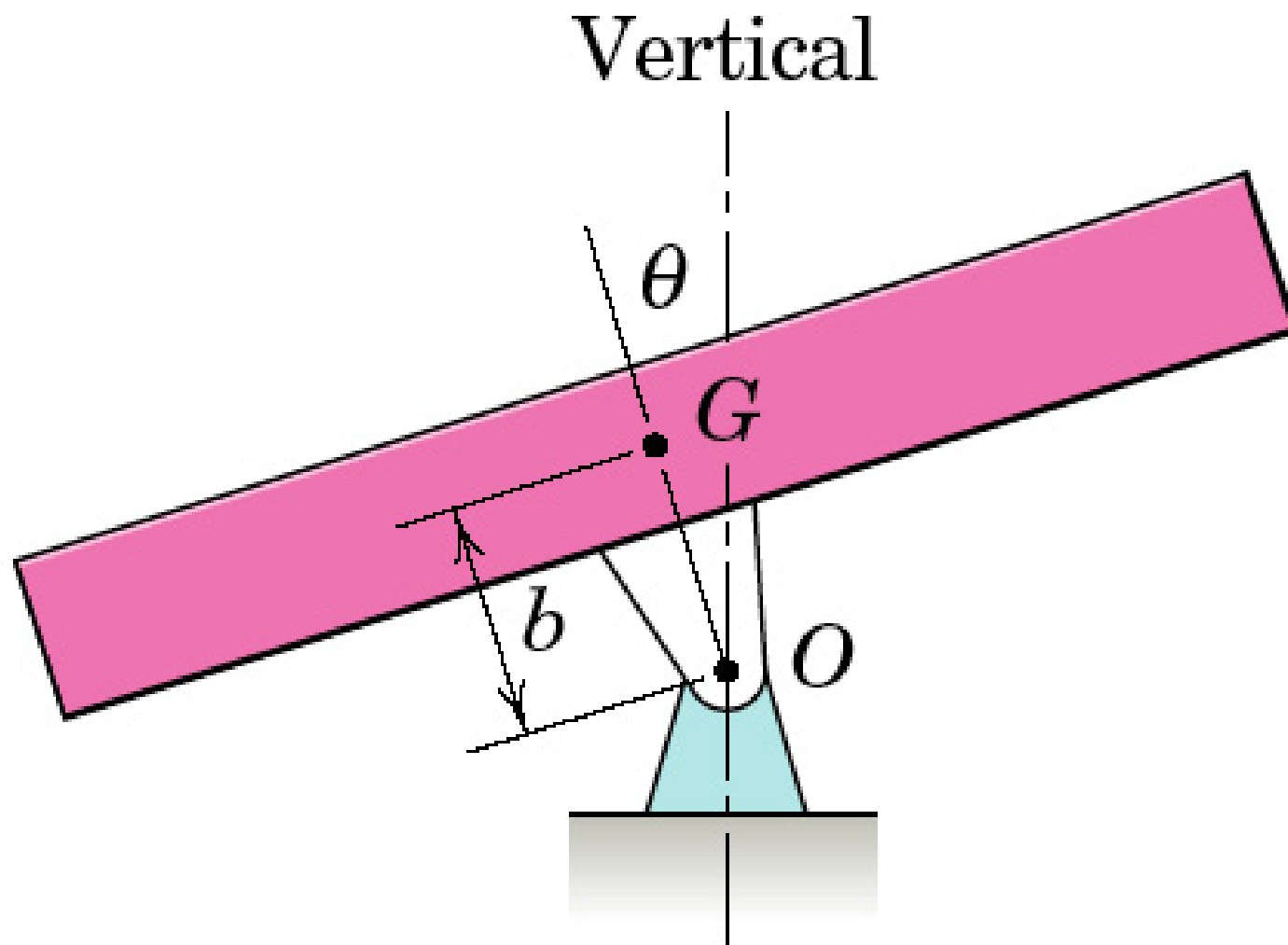
Problem 7/33



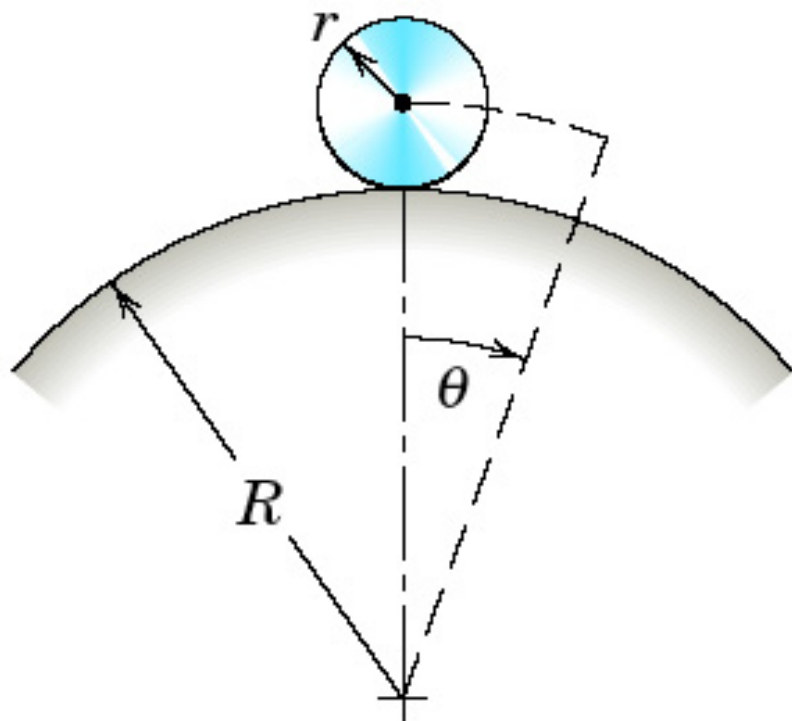
Problem 7/34



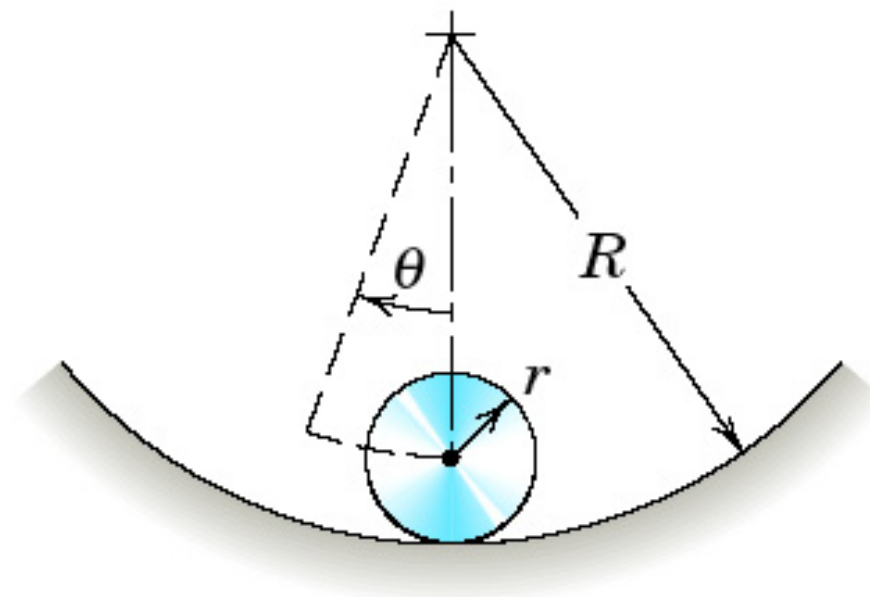
Problem 7/36



Problem 7/37

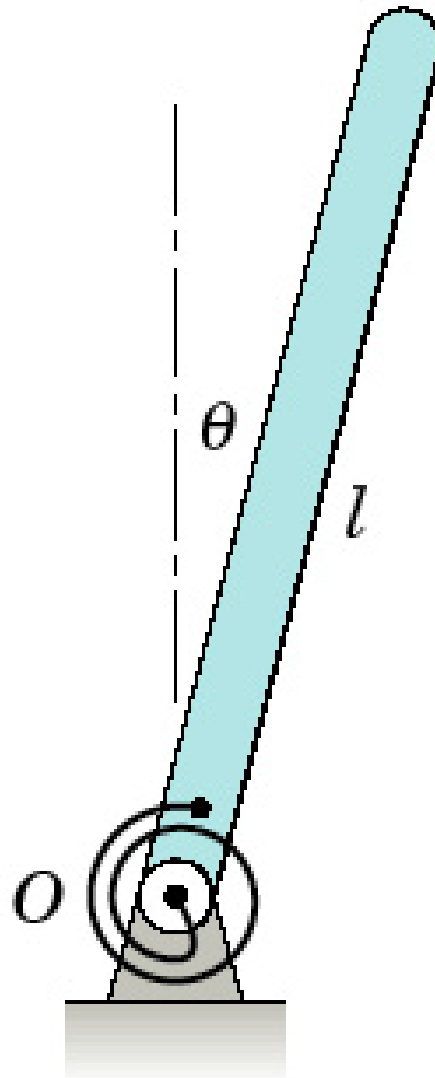


(a)

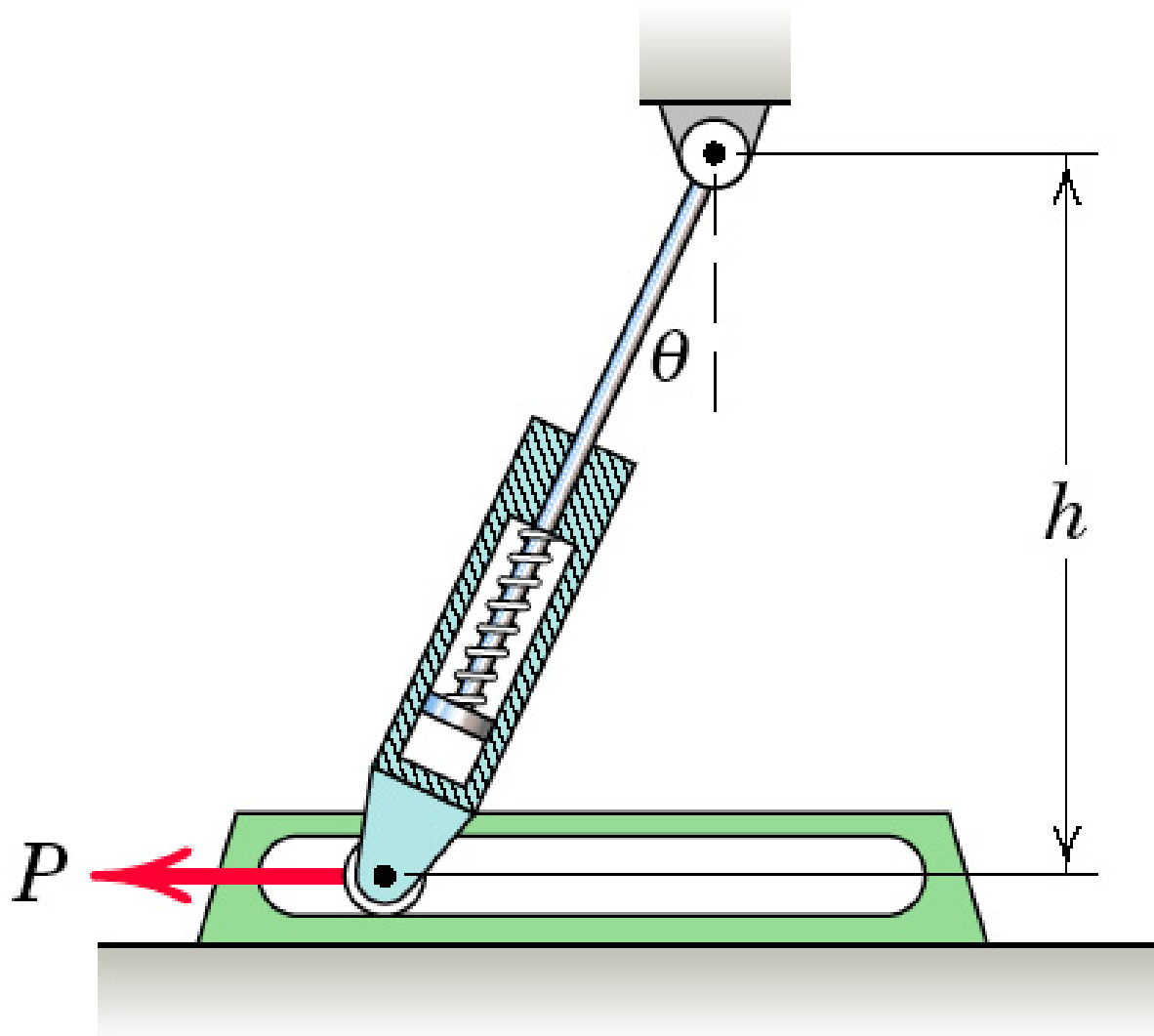


(b)

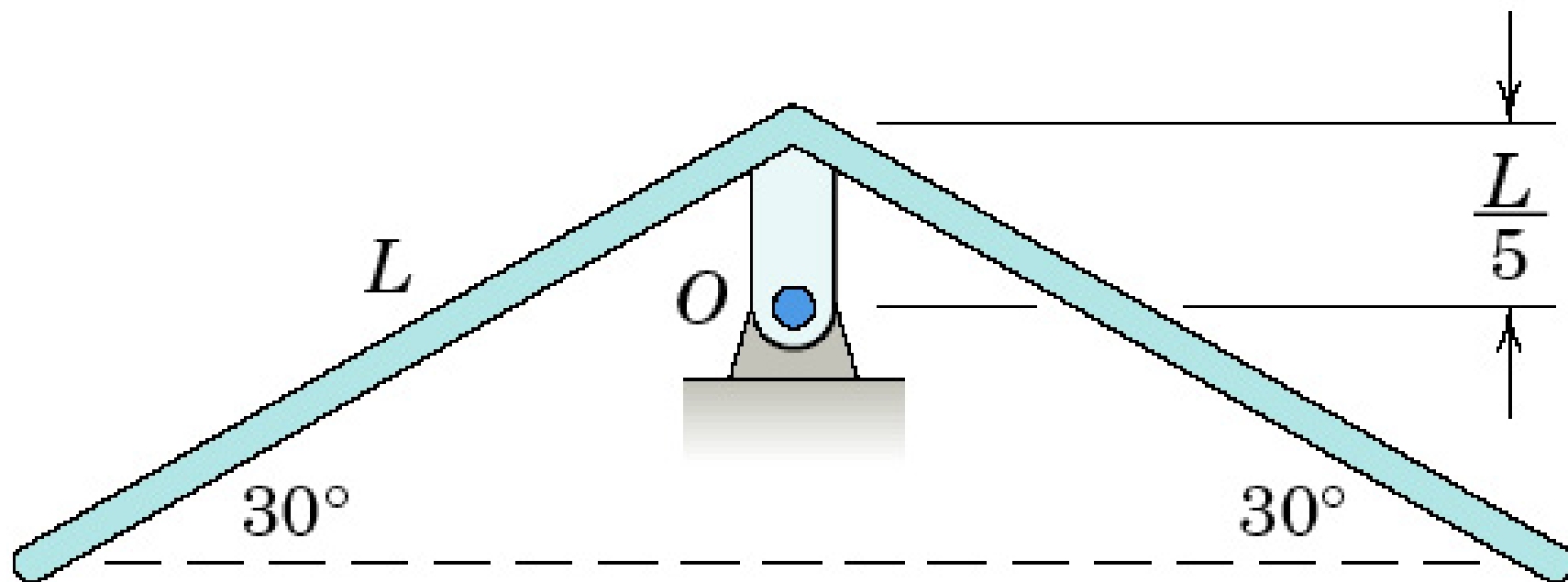
Problem 7/38



Problem 7/39

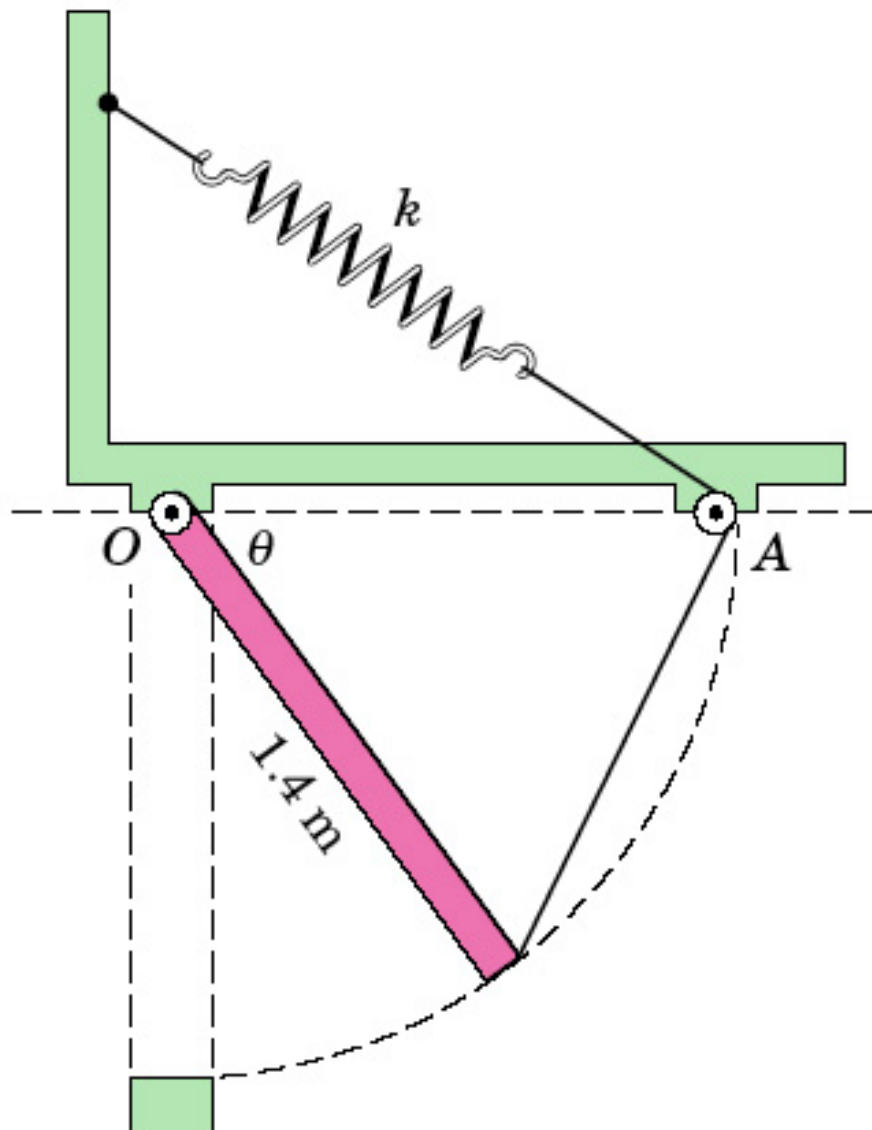


Problem 7/40

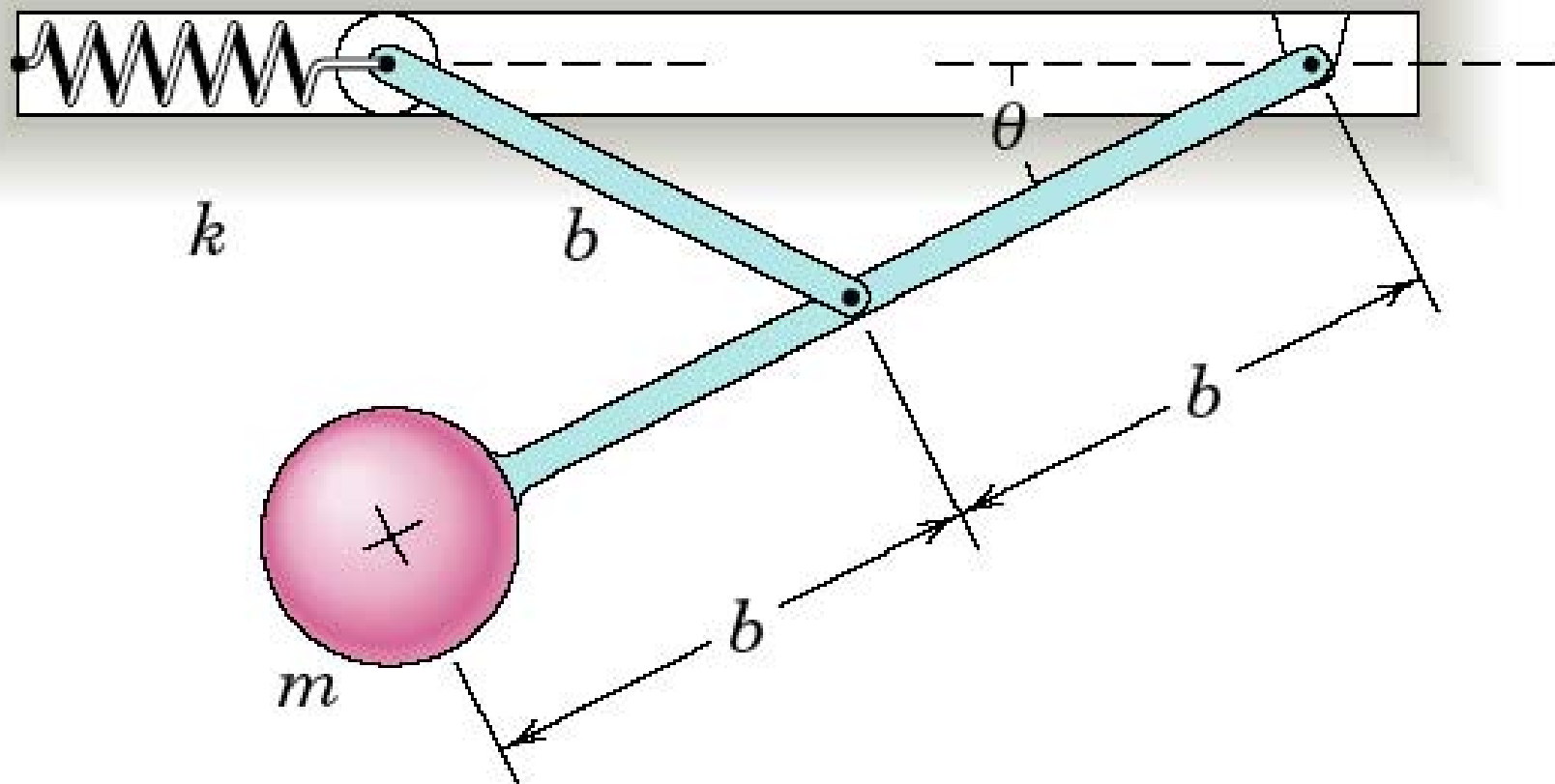


Problem 7/41

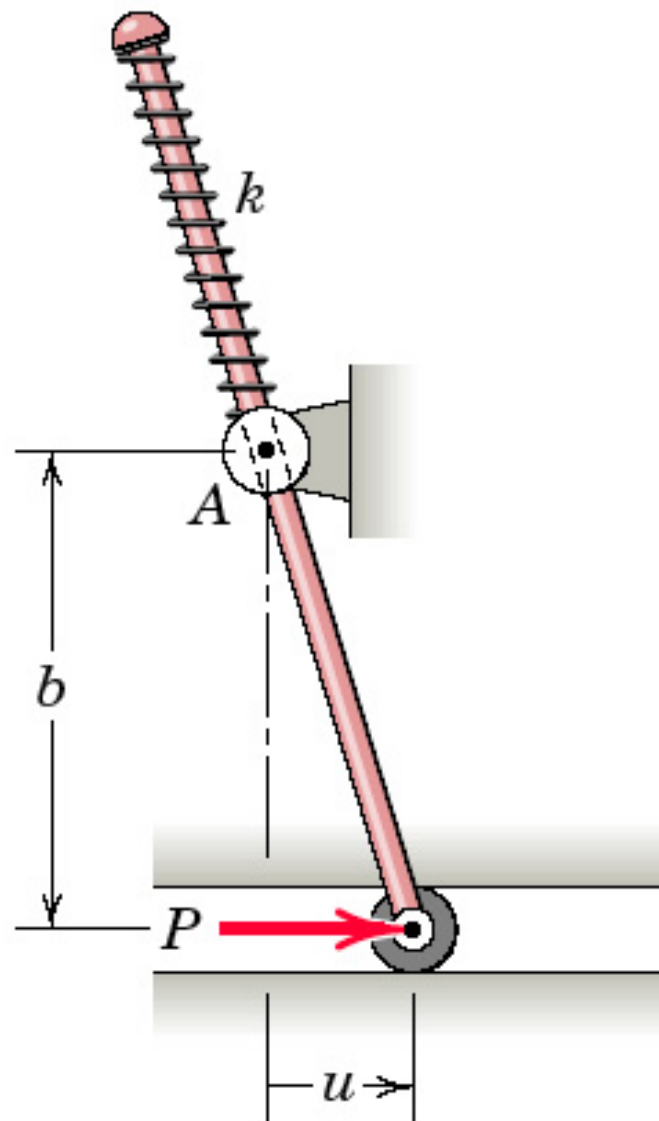
P_07_41



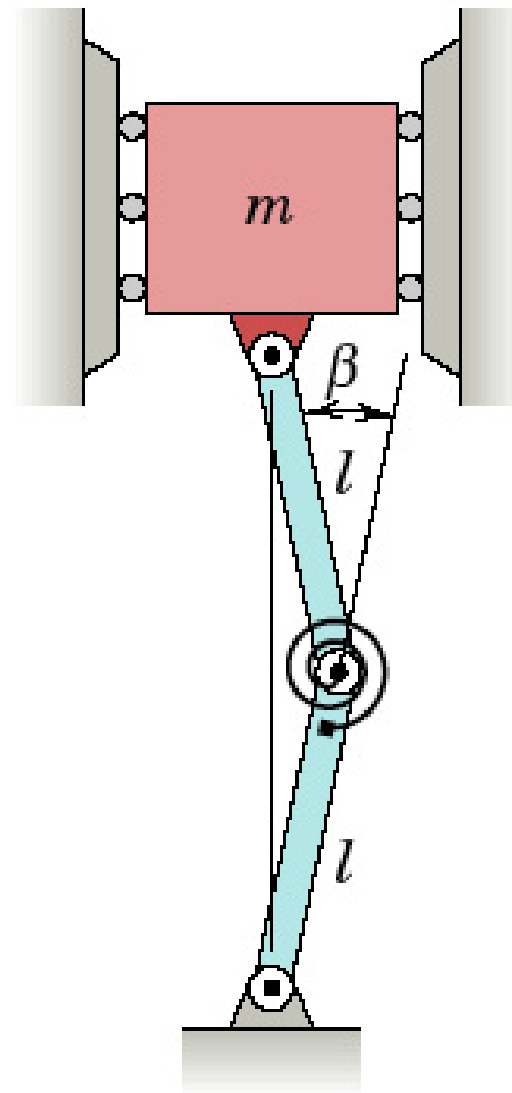
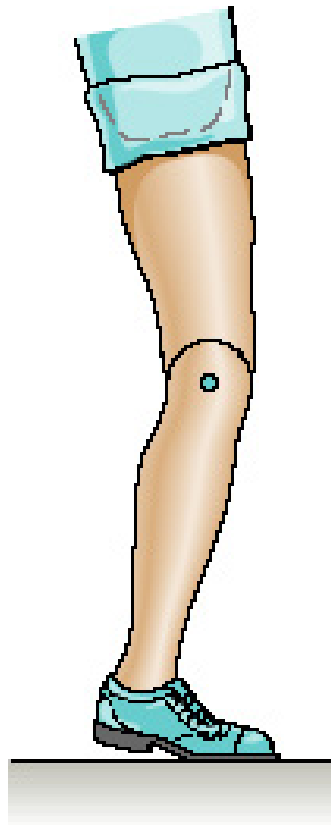
Problem 7/42



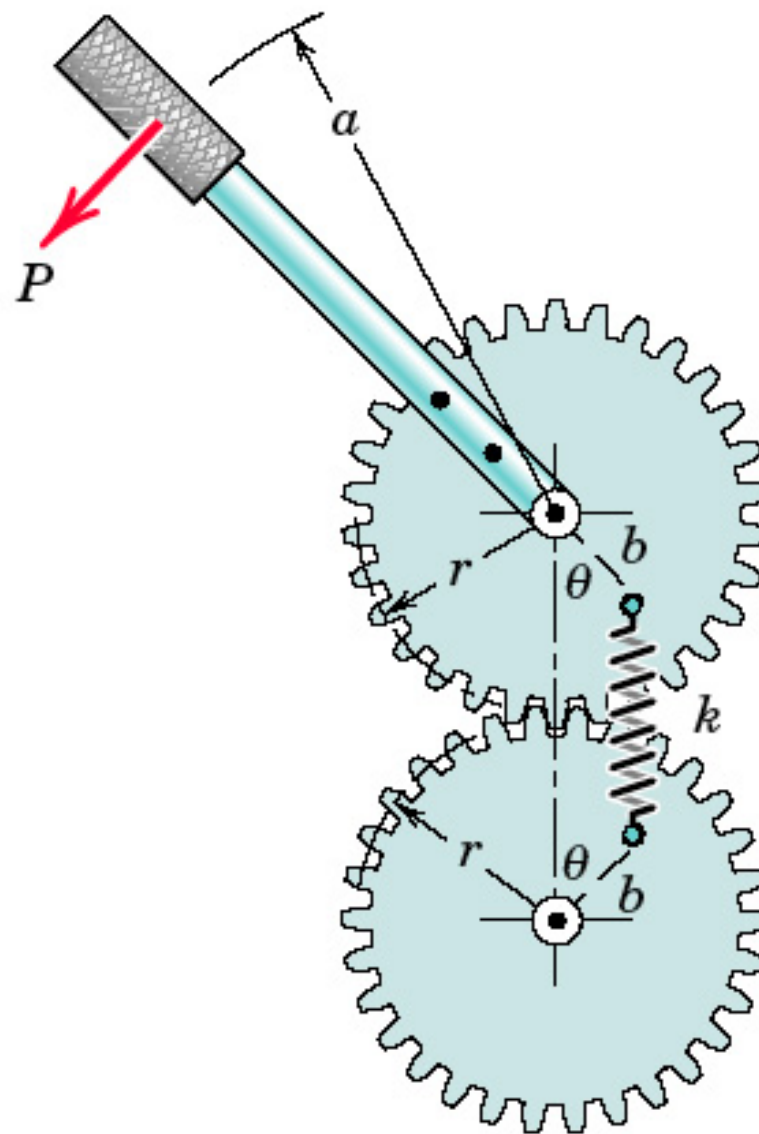
Problem 7/43



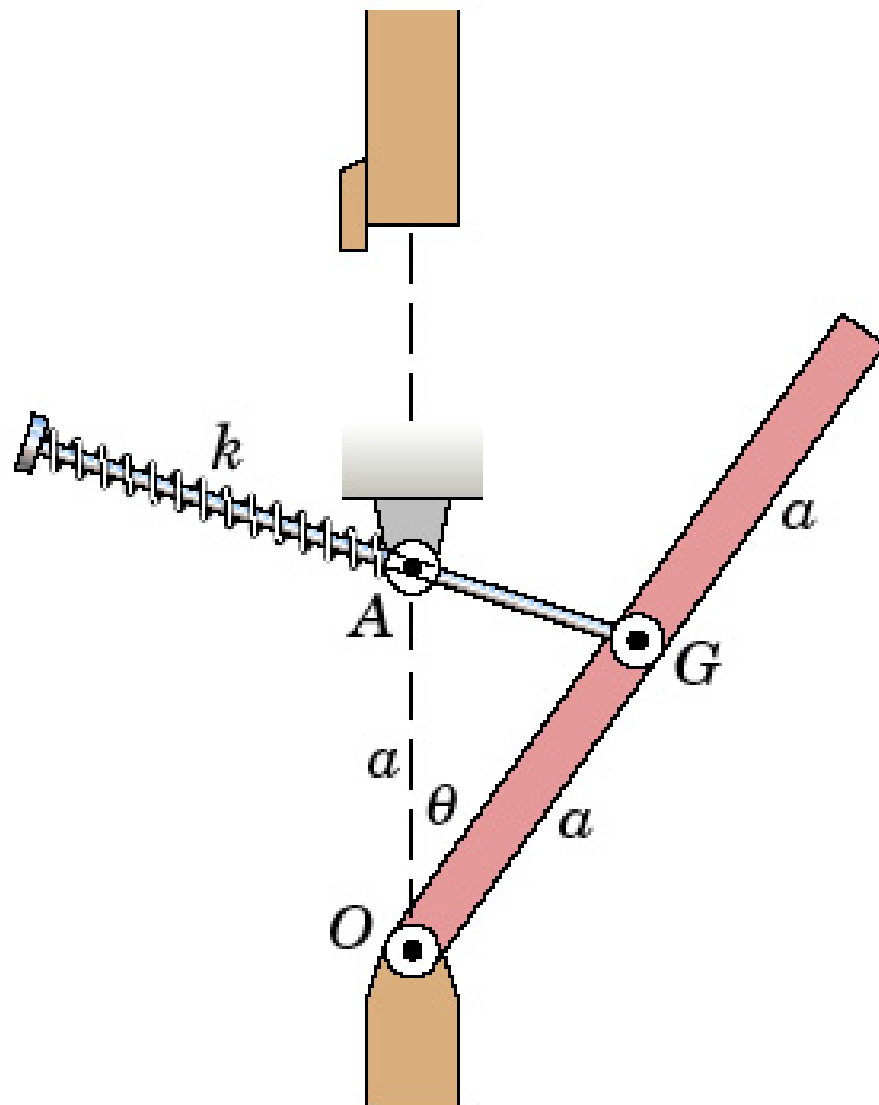
Problem 7/44



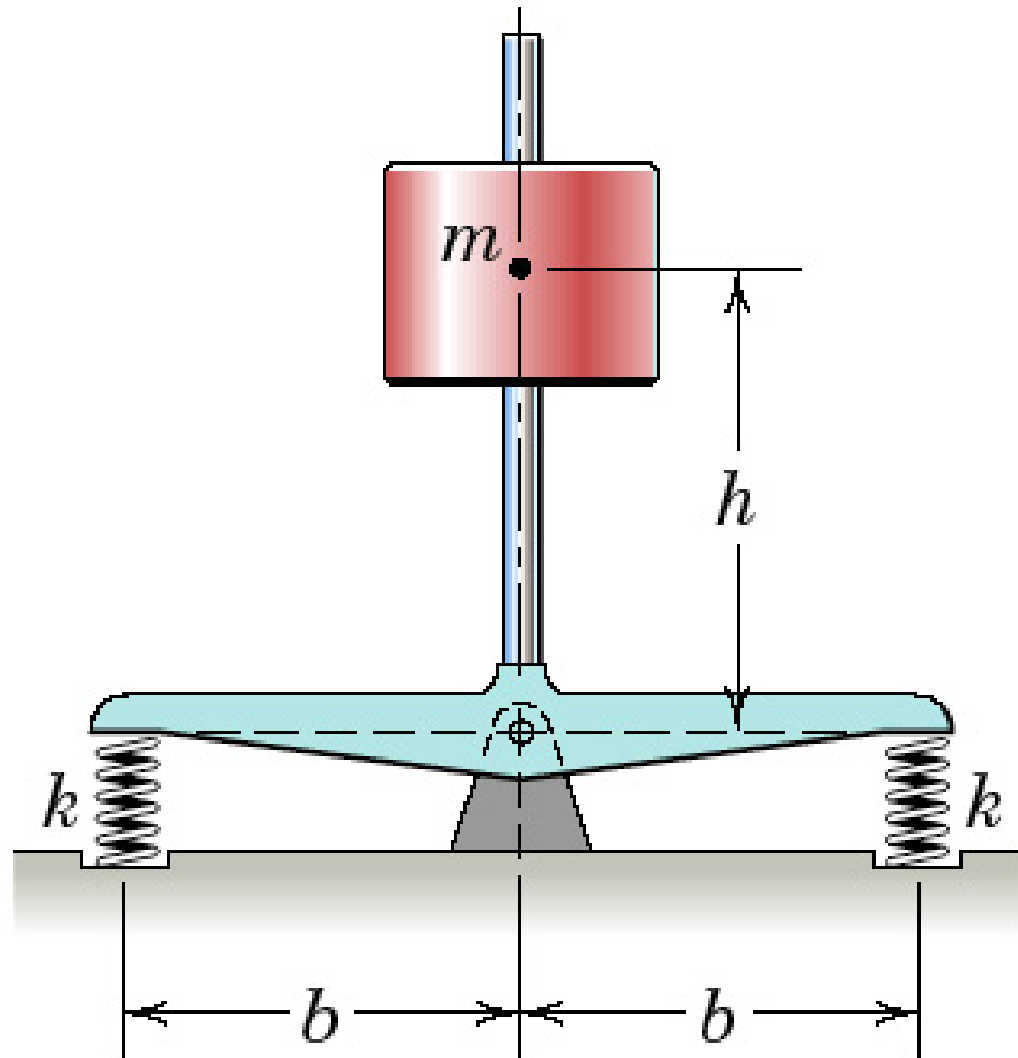
Problem 7/45



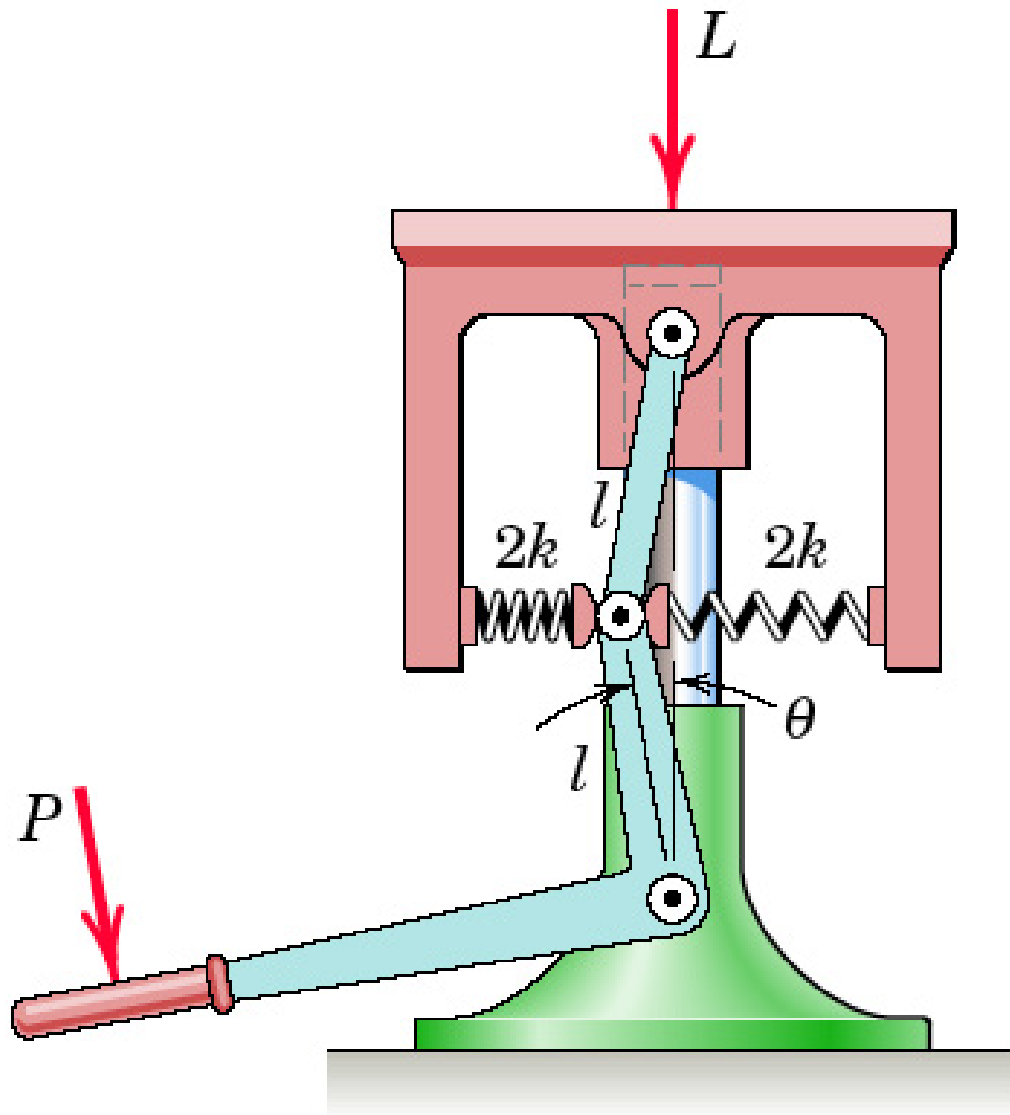
Problem 7/46



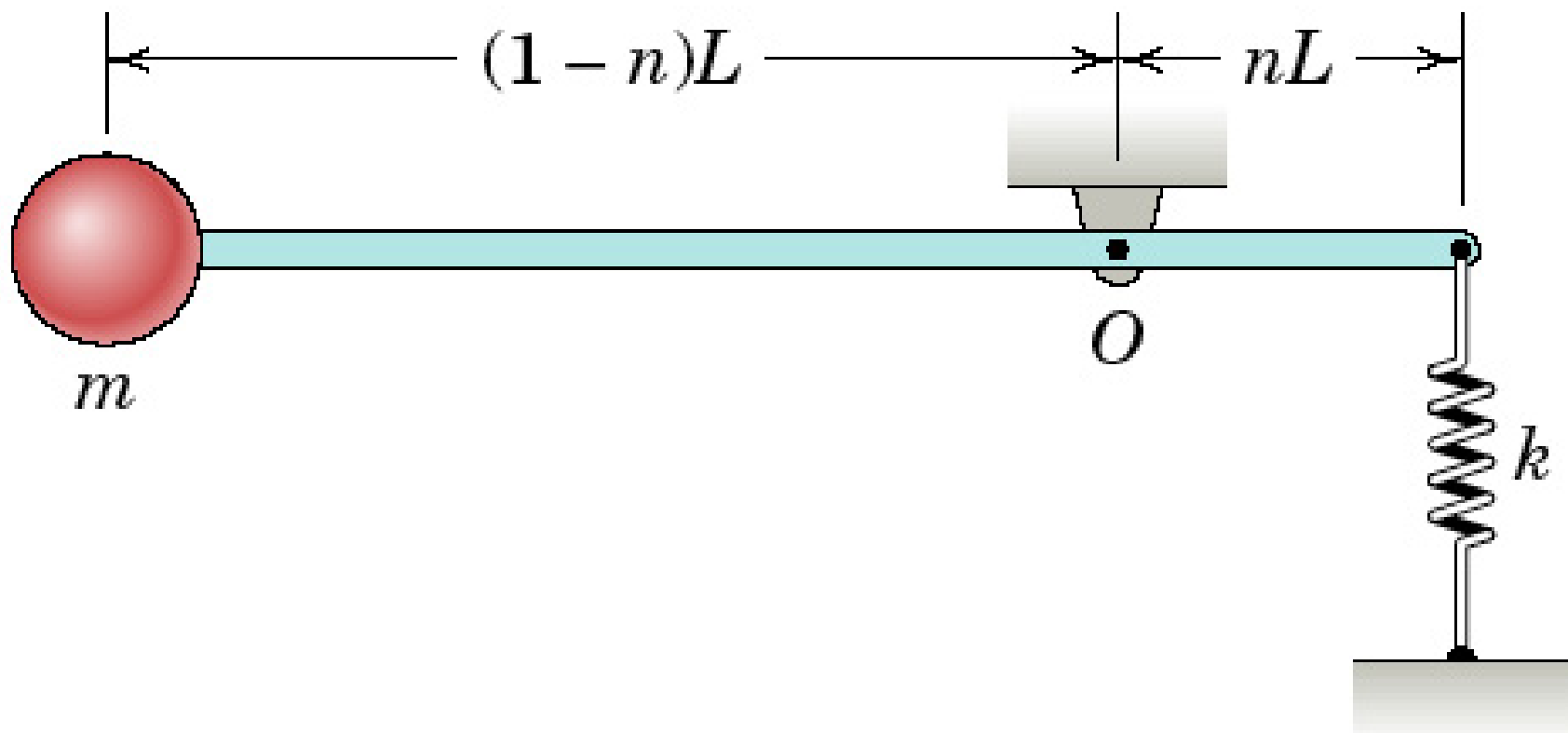
Problem 7/47



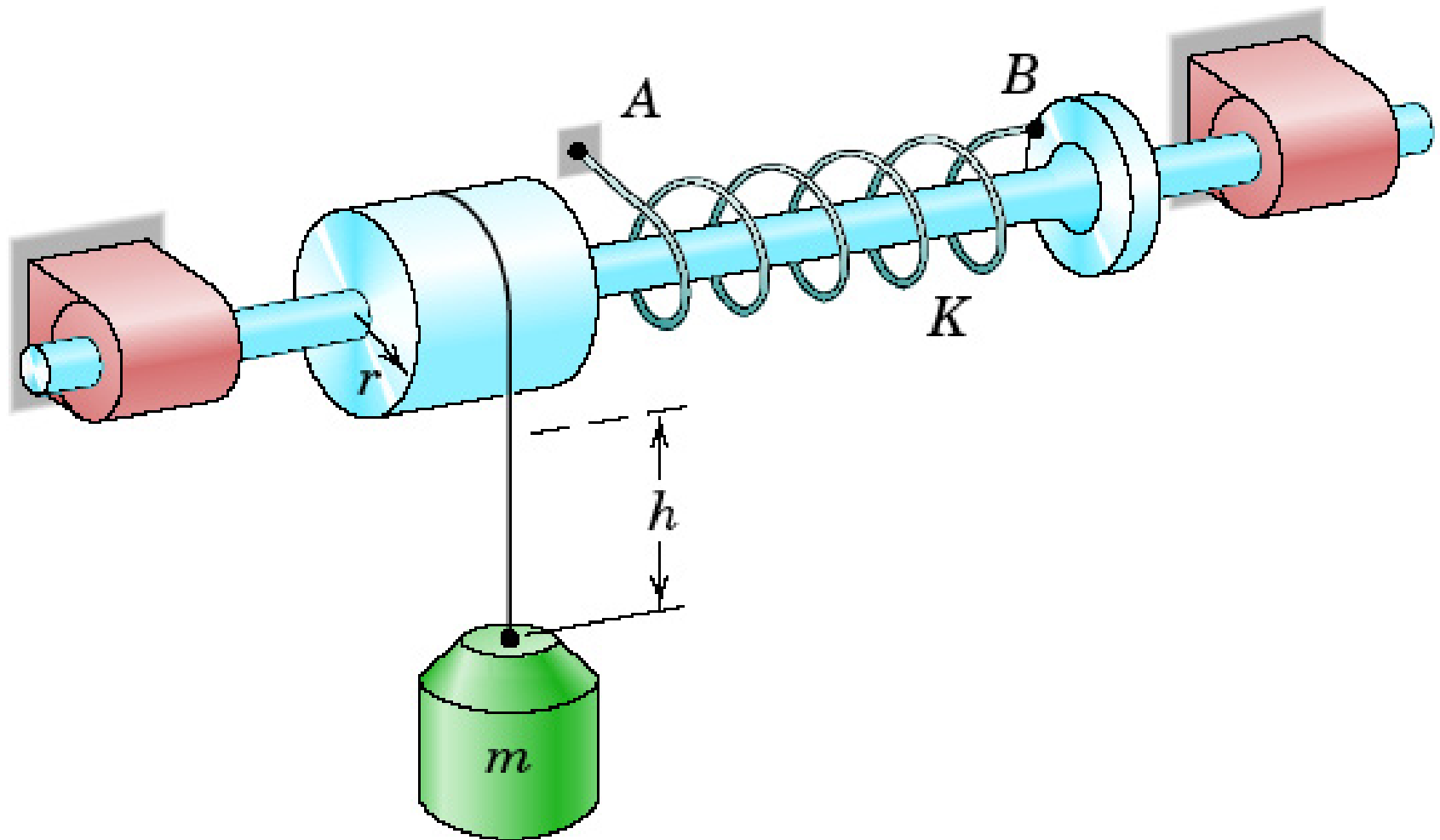
Problem 7/48



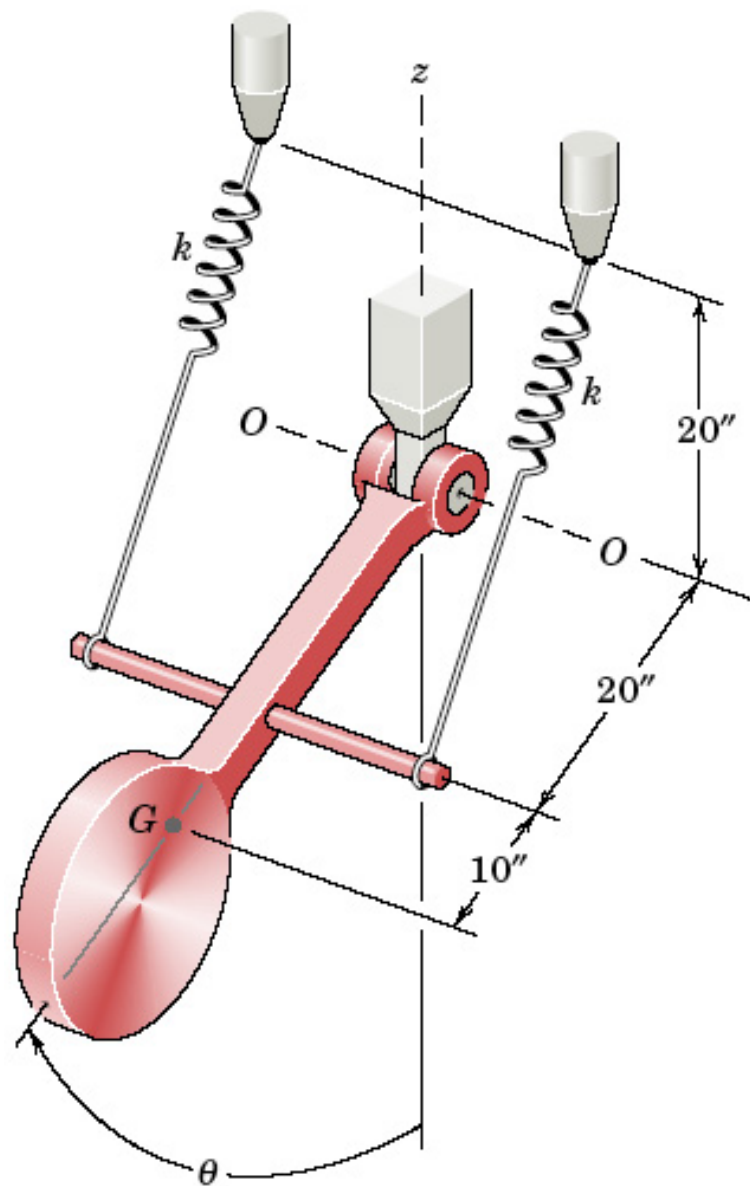
Problem 7/49



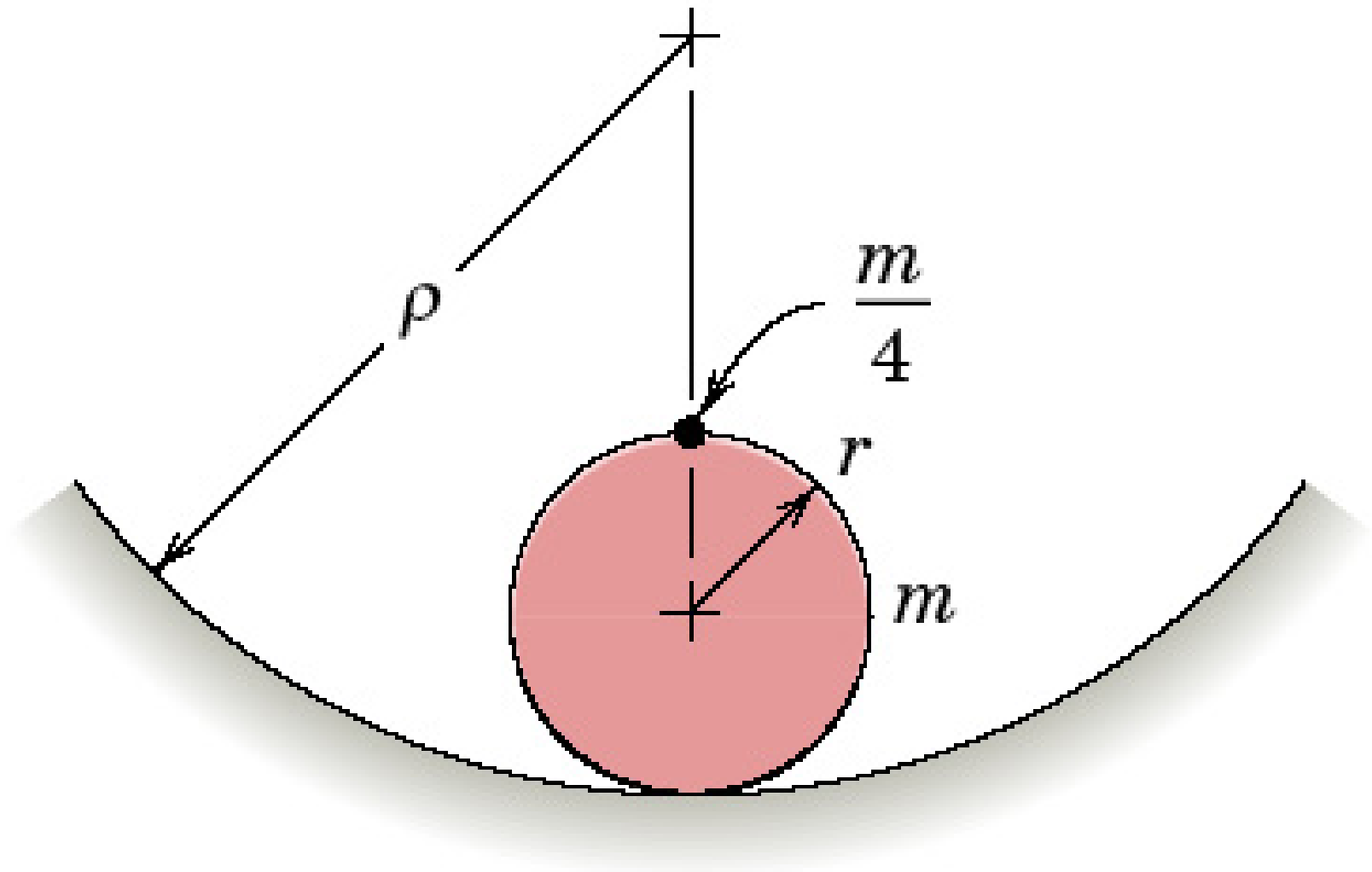
Problem 7/50



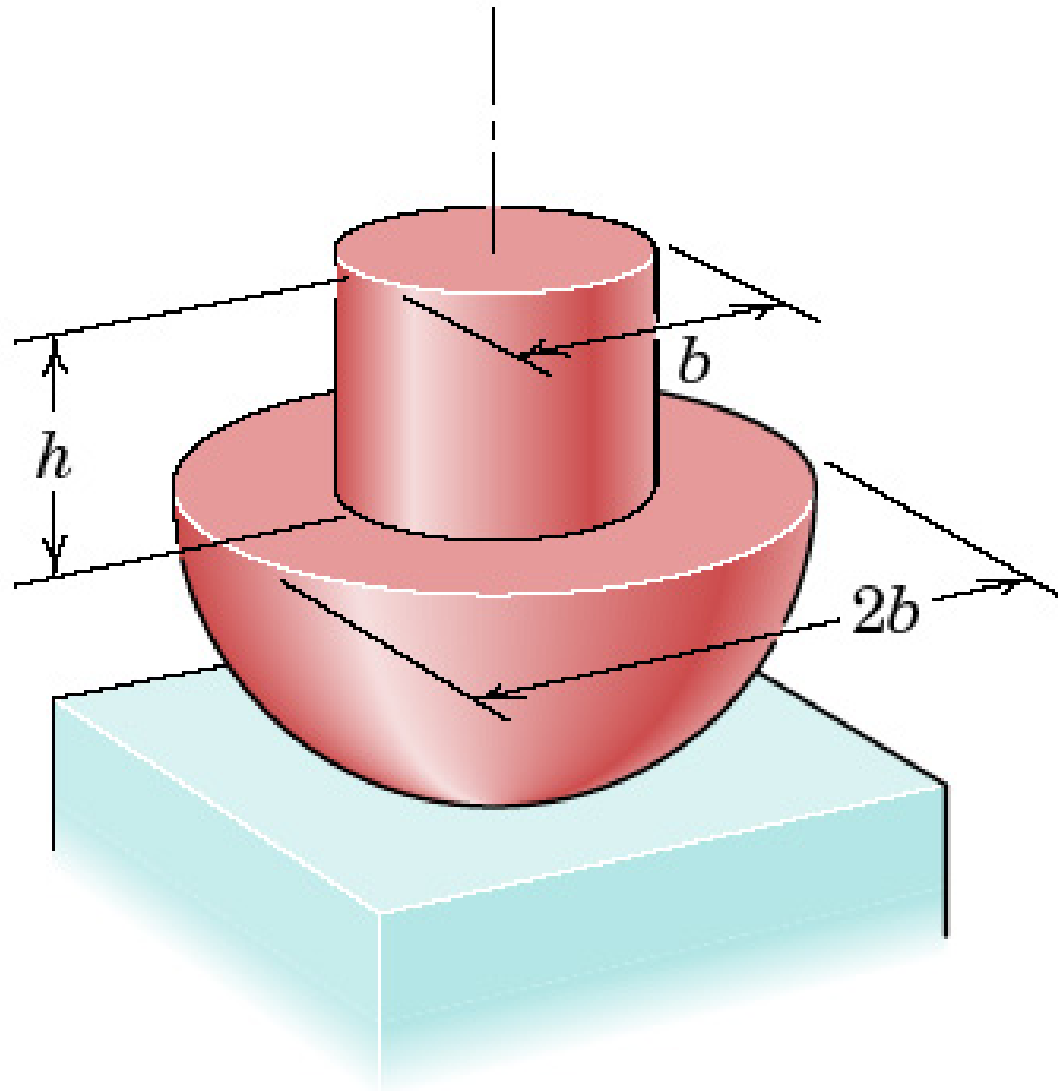
Problem 7/51



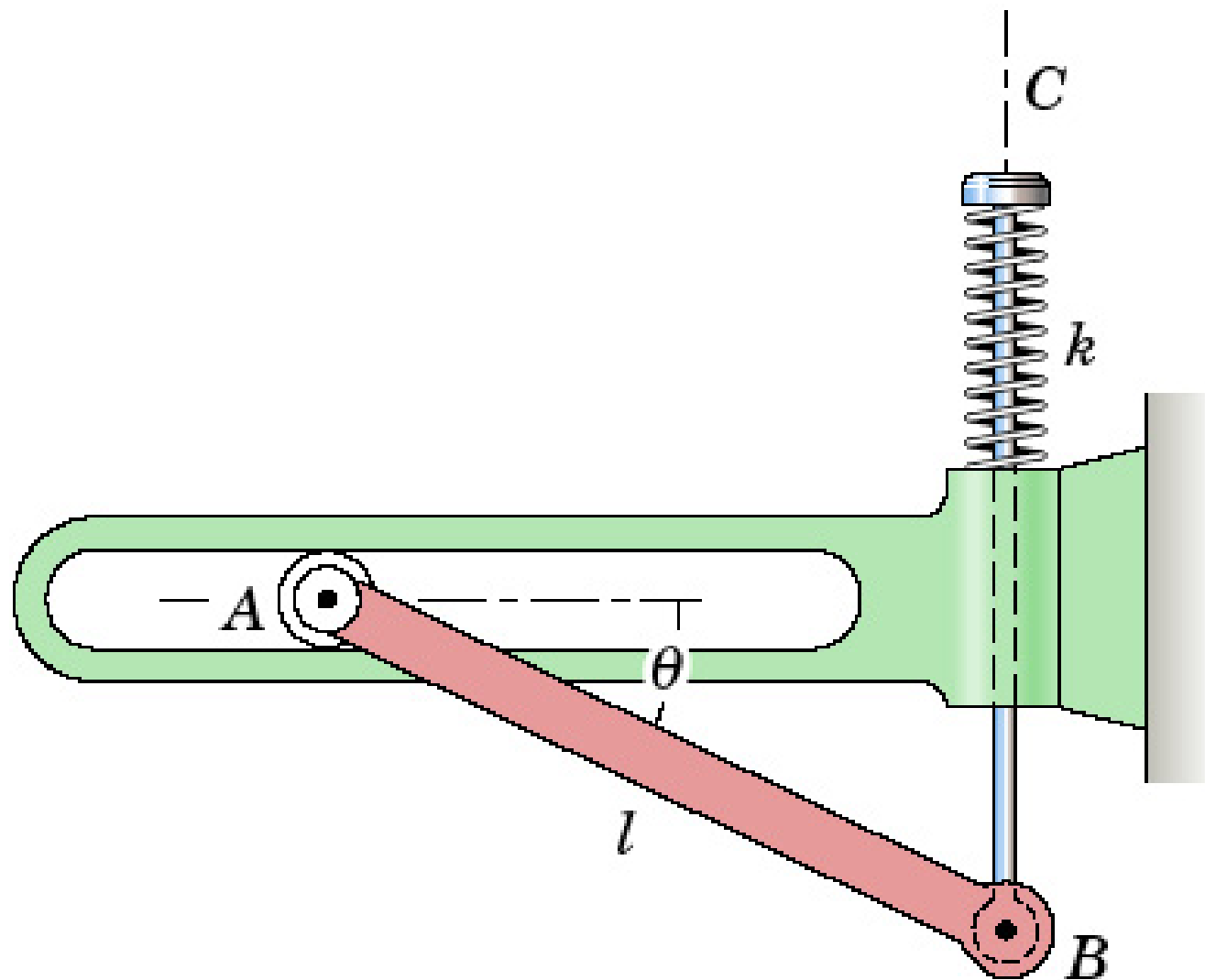
Problem 7/52



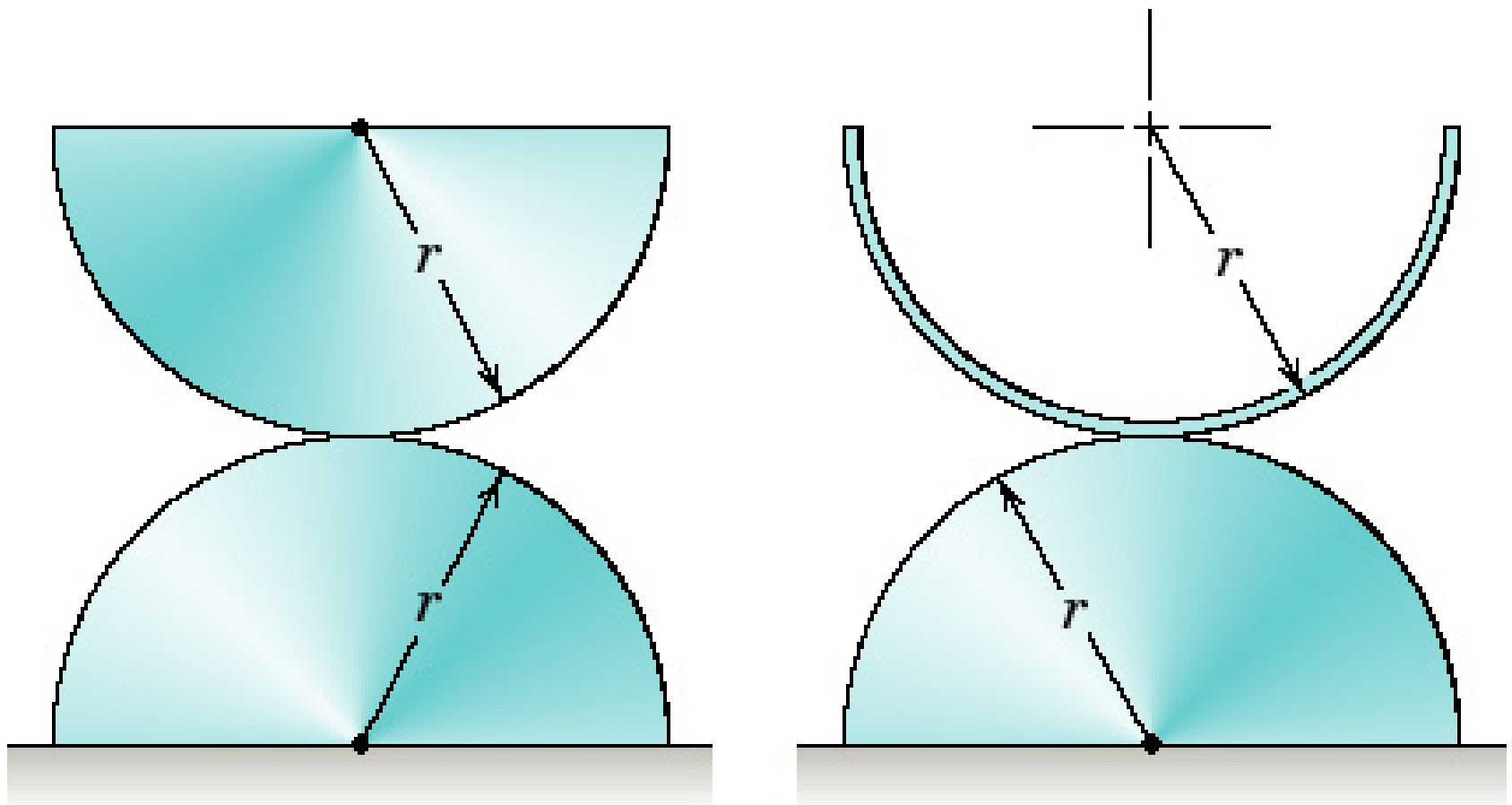
Problem 7/53



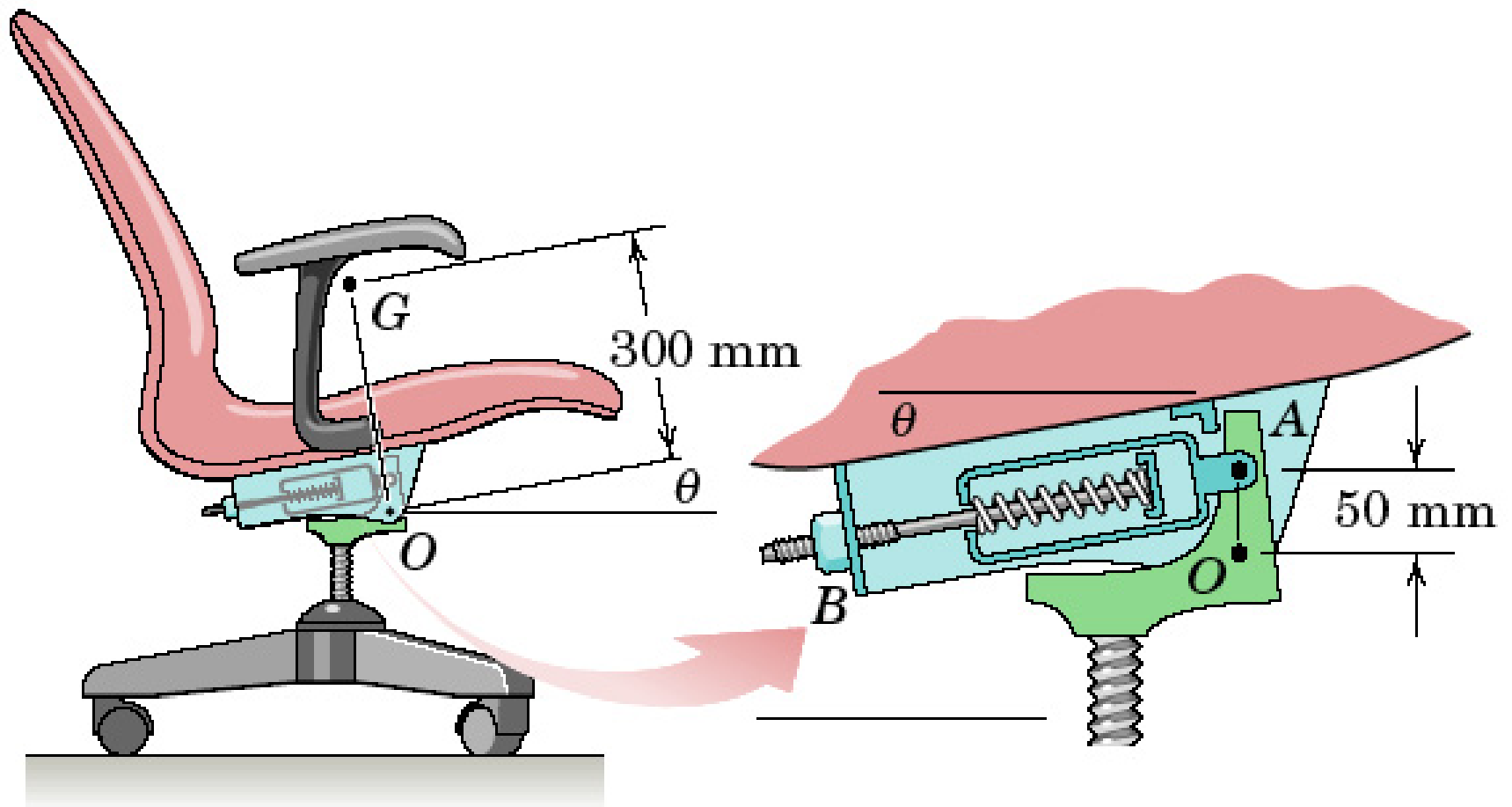
Problem 7/54



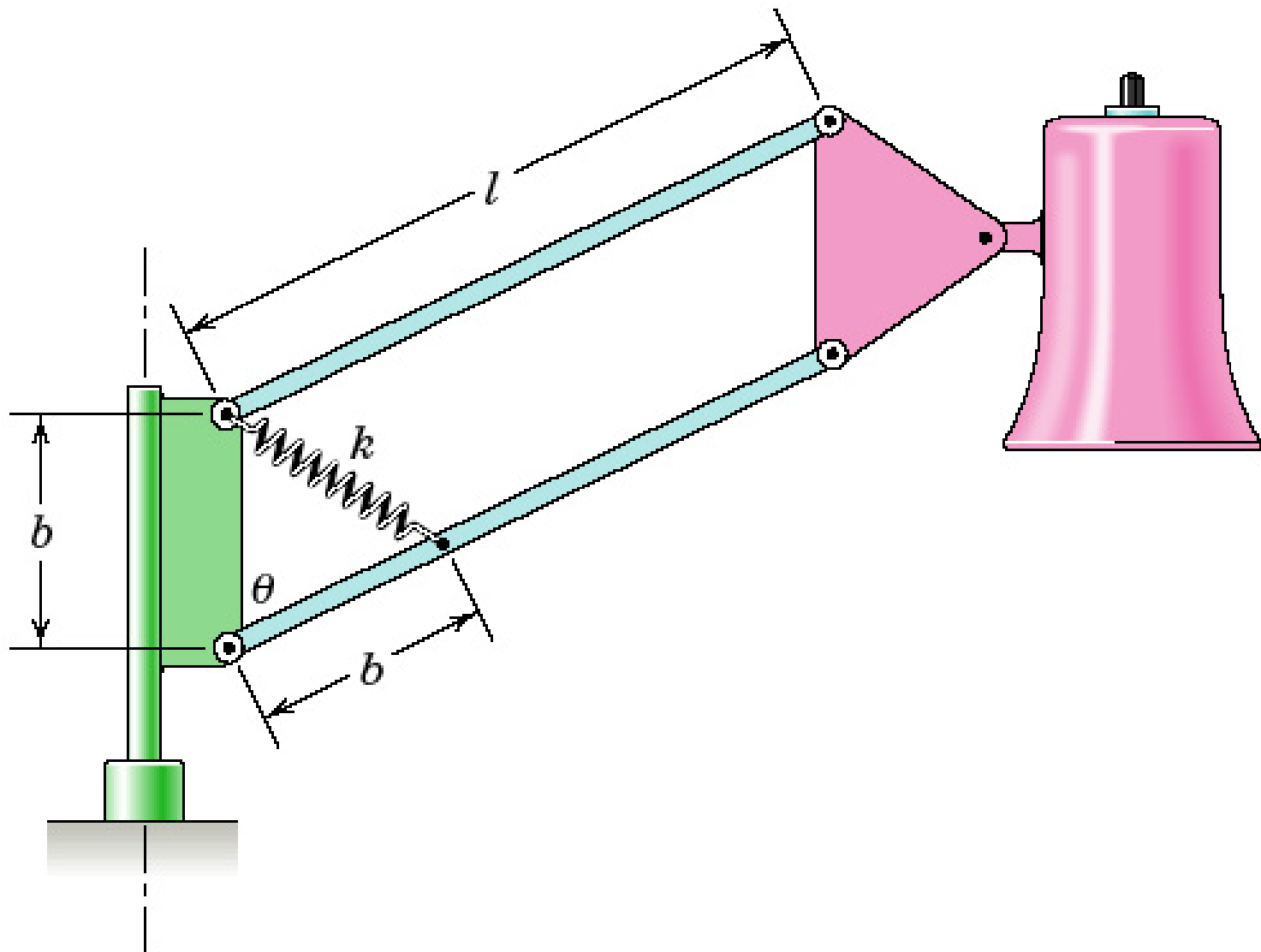
Problem 7/55



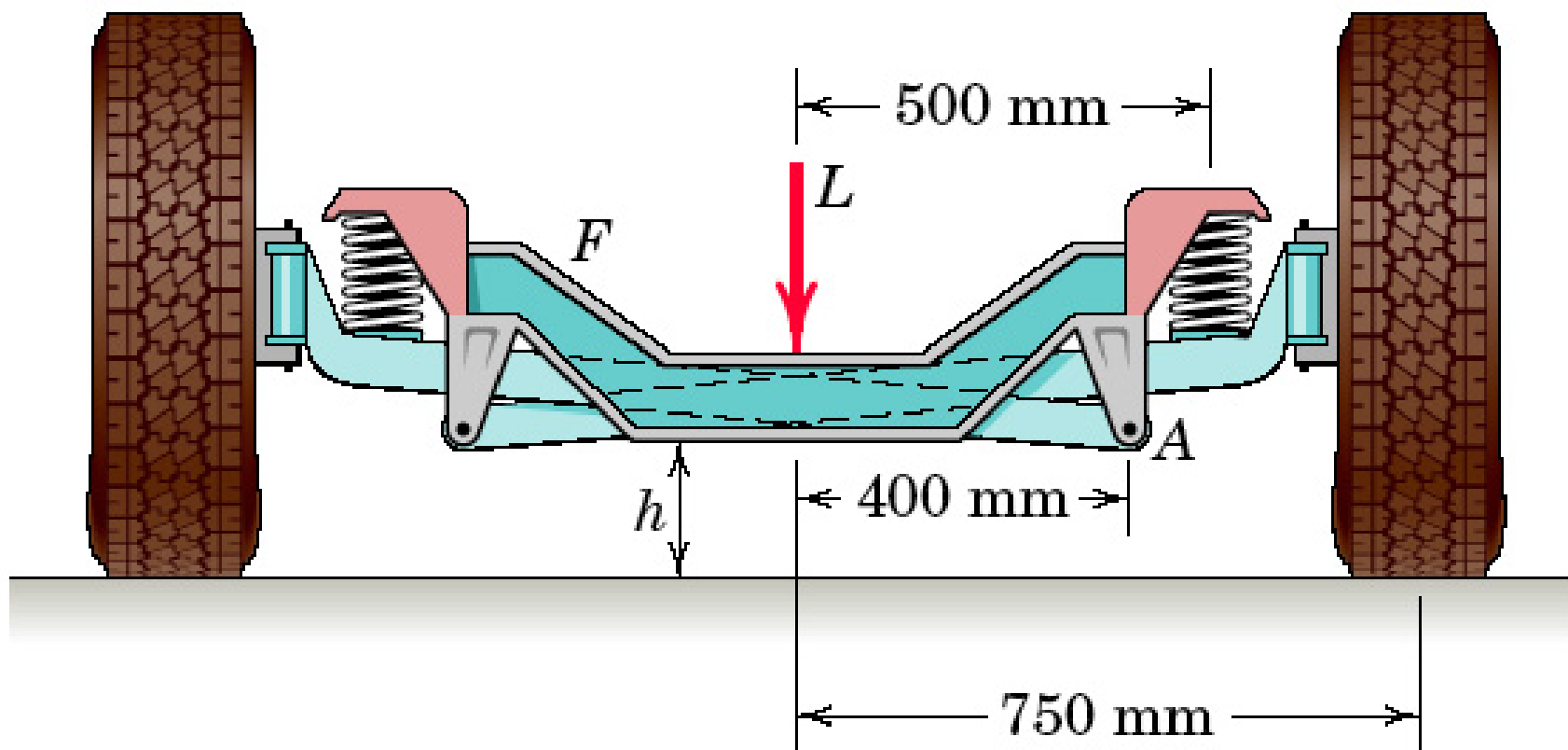
Problem 7/56



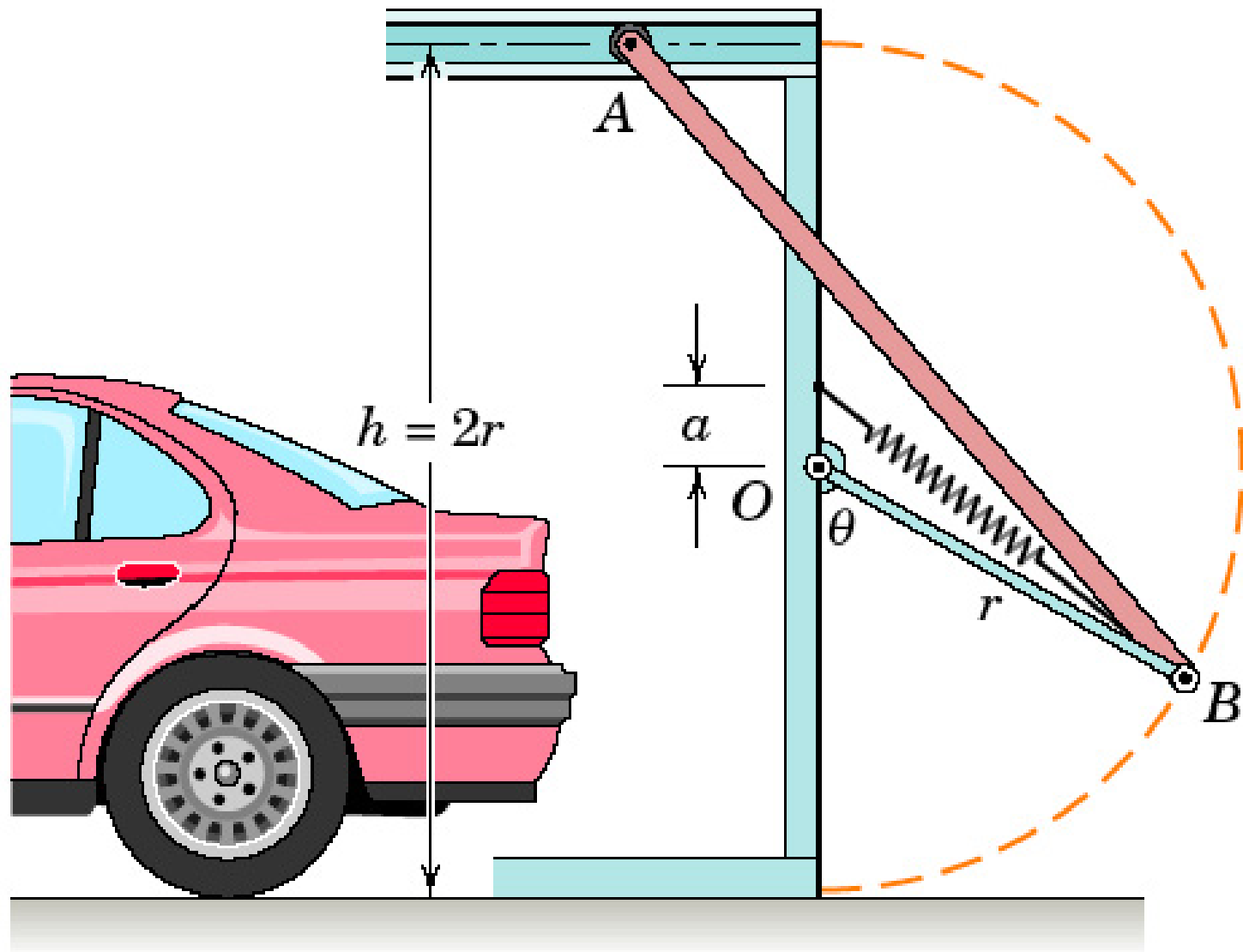
Problem 7/57



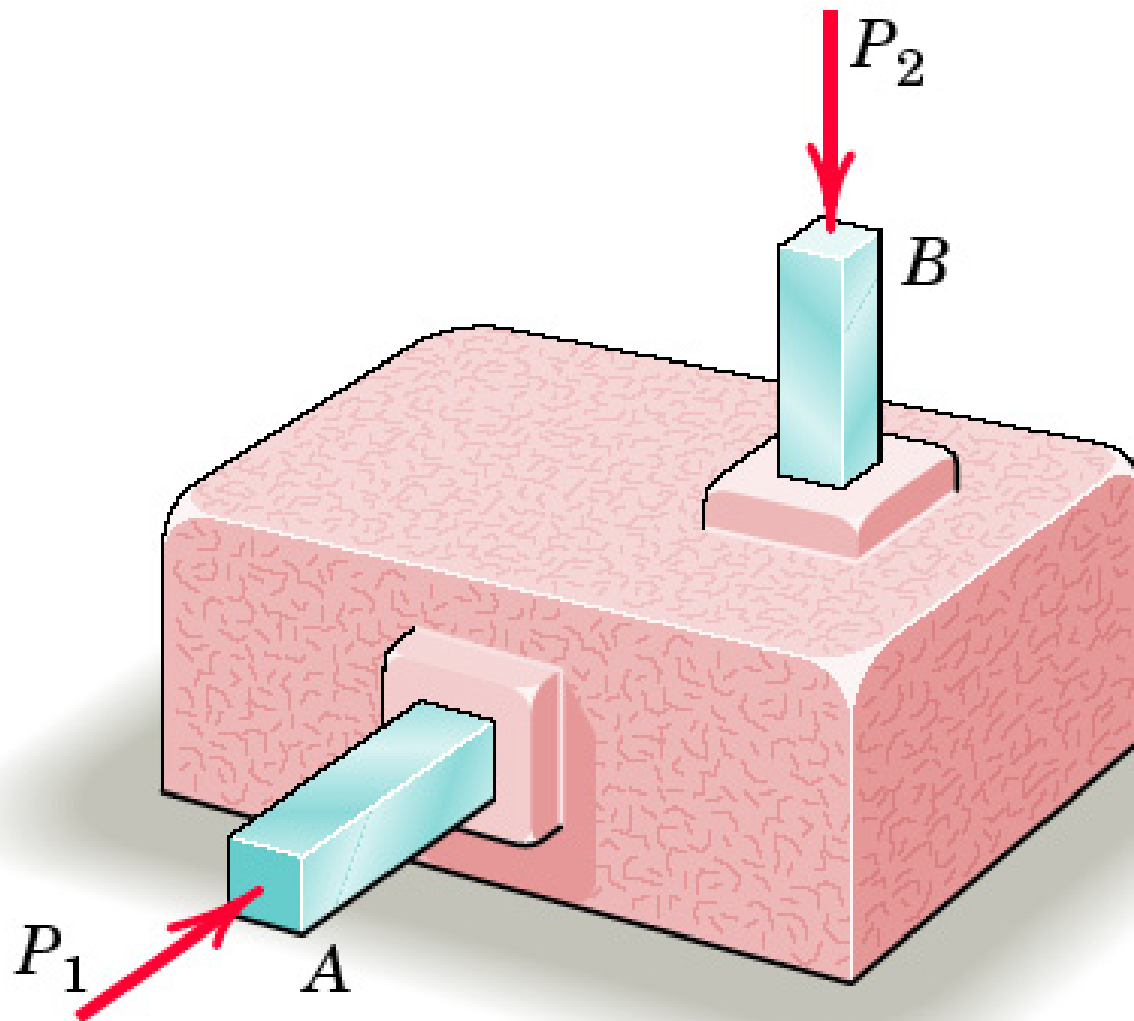
Problem 7/58



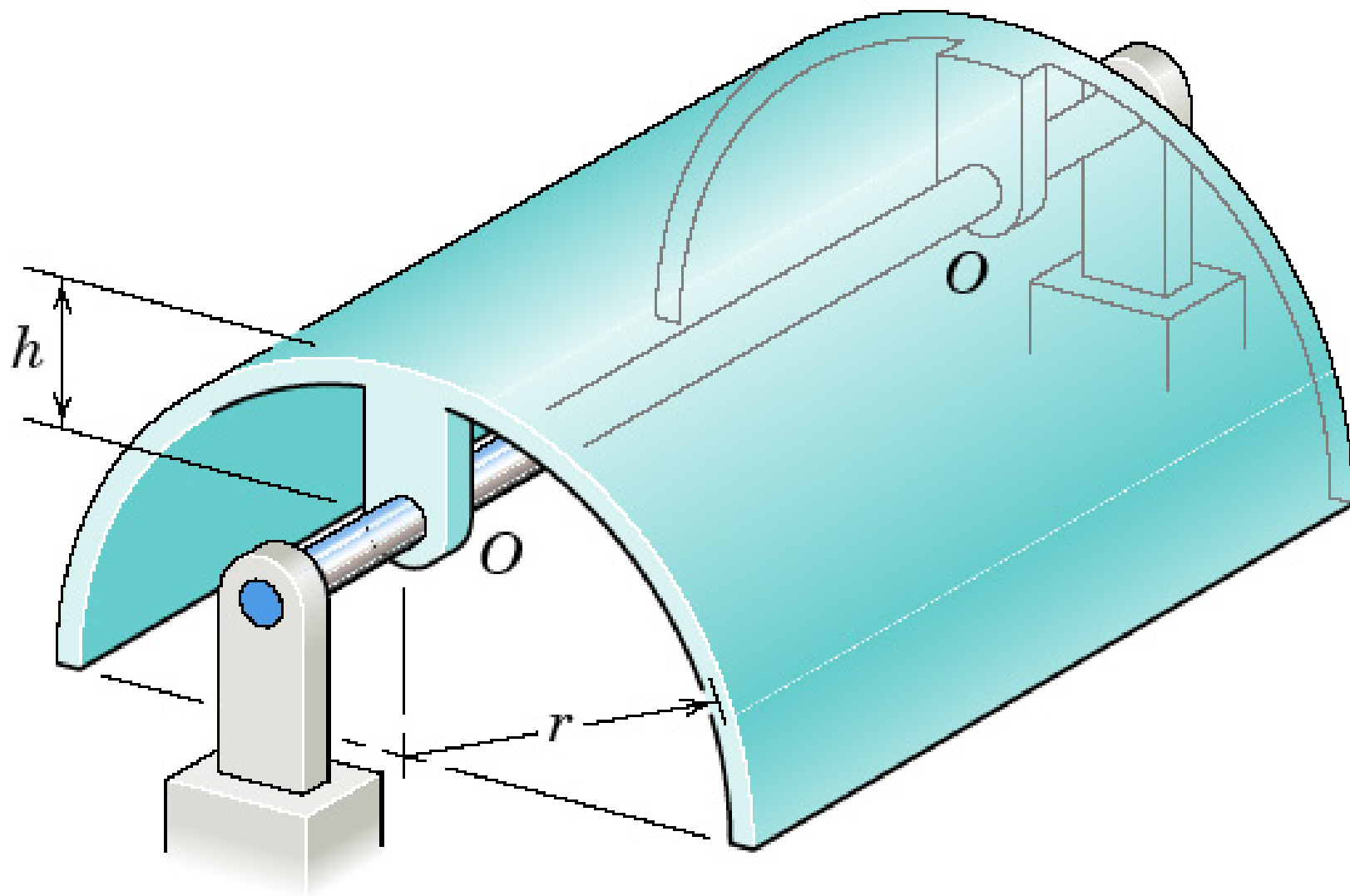
Problem 7/59



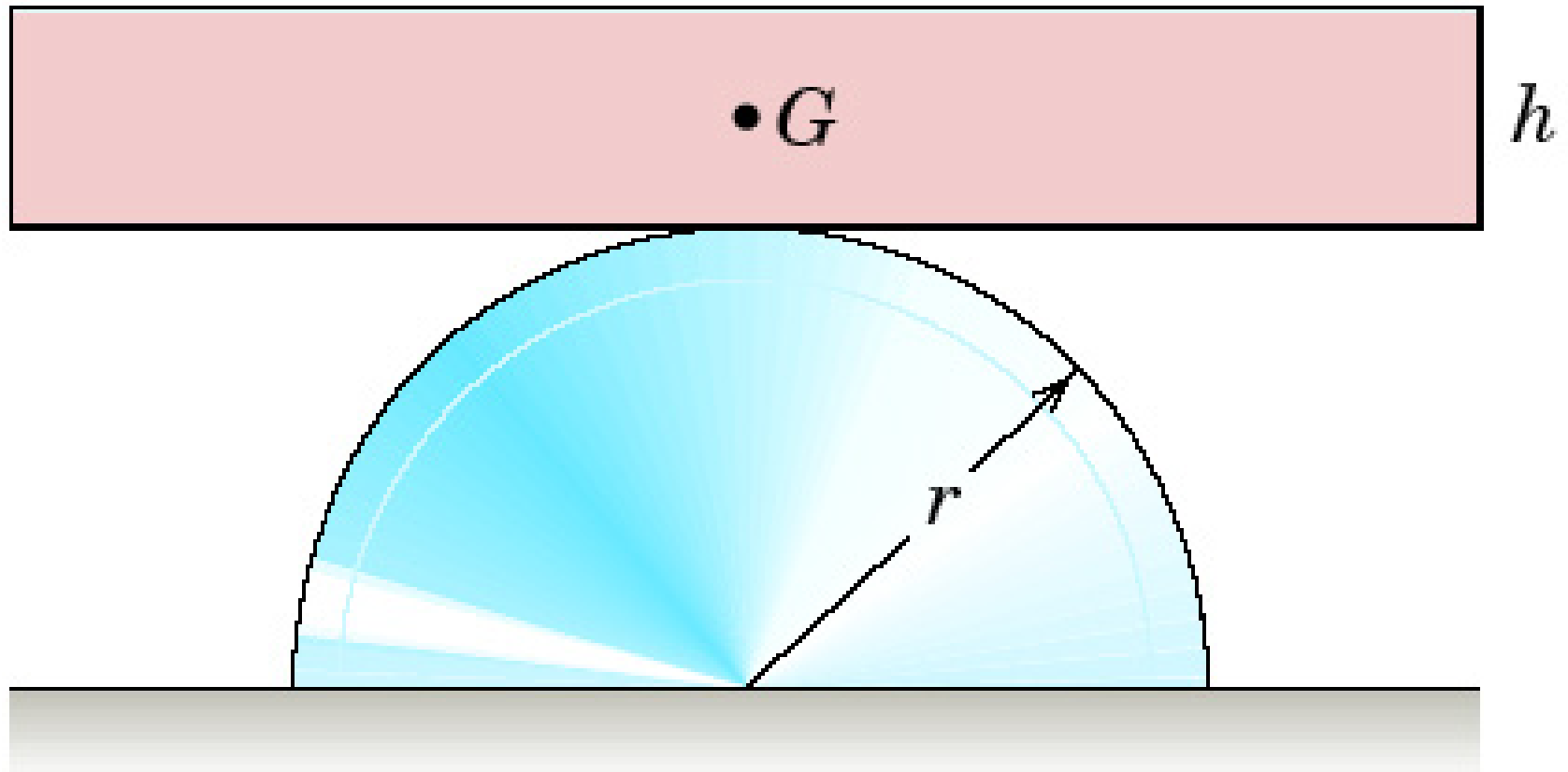
Problem 7/60



Problem 7/61

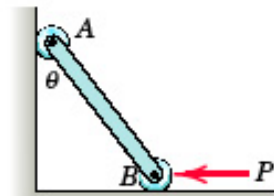


Problem 7/62

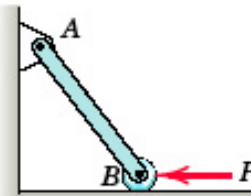


Problem 7/63

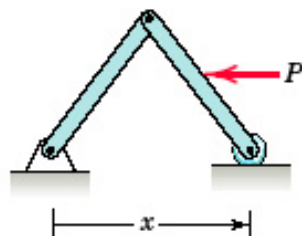
Plane of each figure is vertical. Size and mass of each member and applied force are known.



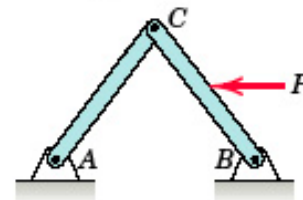
Find θ for equilibrium
(a)



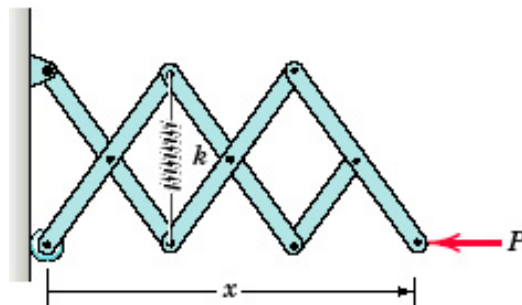
Find reactions at A and B
(b)



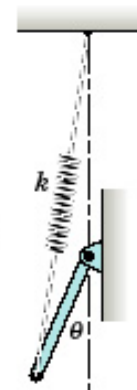
Find x for equilibrium
(c)



Find forces at A, B, and C
(d)

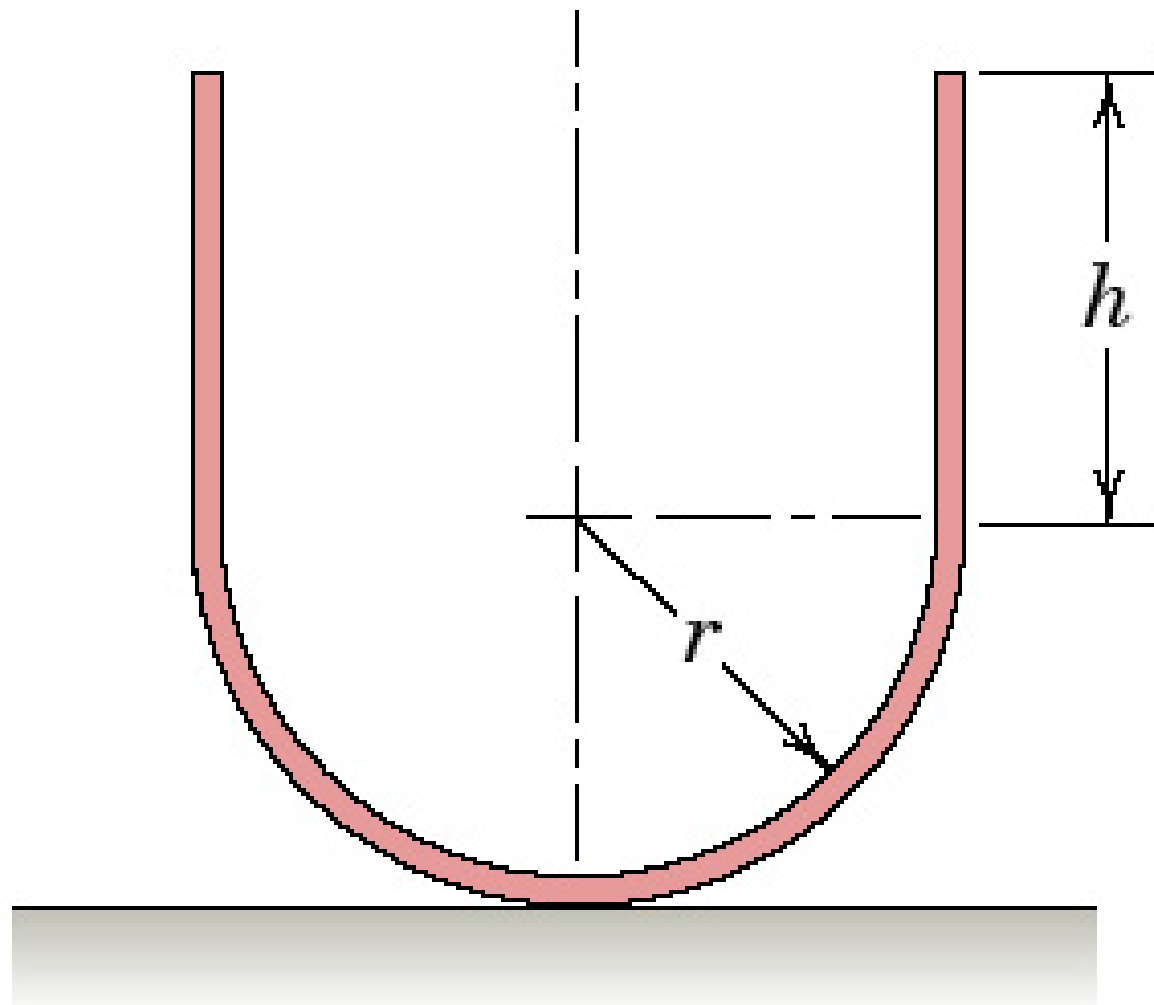


Find x for equilibrium
(e)

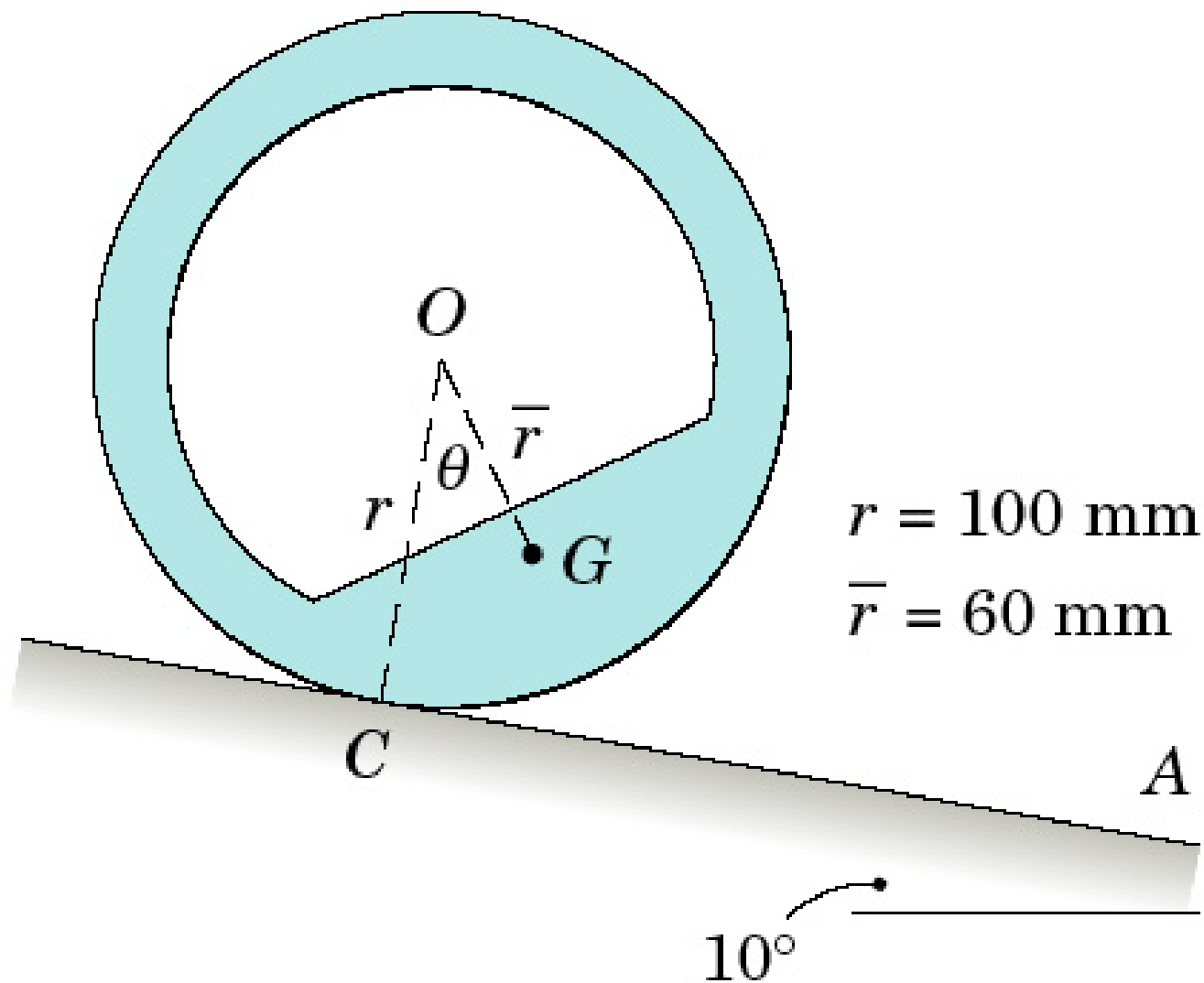


Determine maximum k for stable equilibrium at $\theta = 0$
(f)

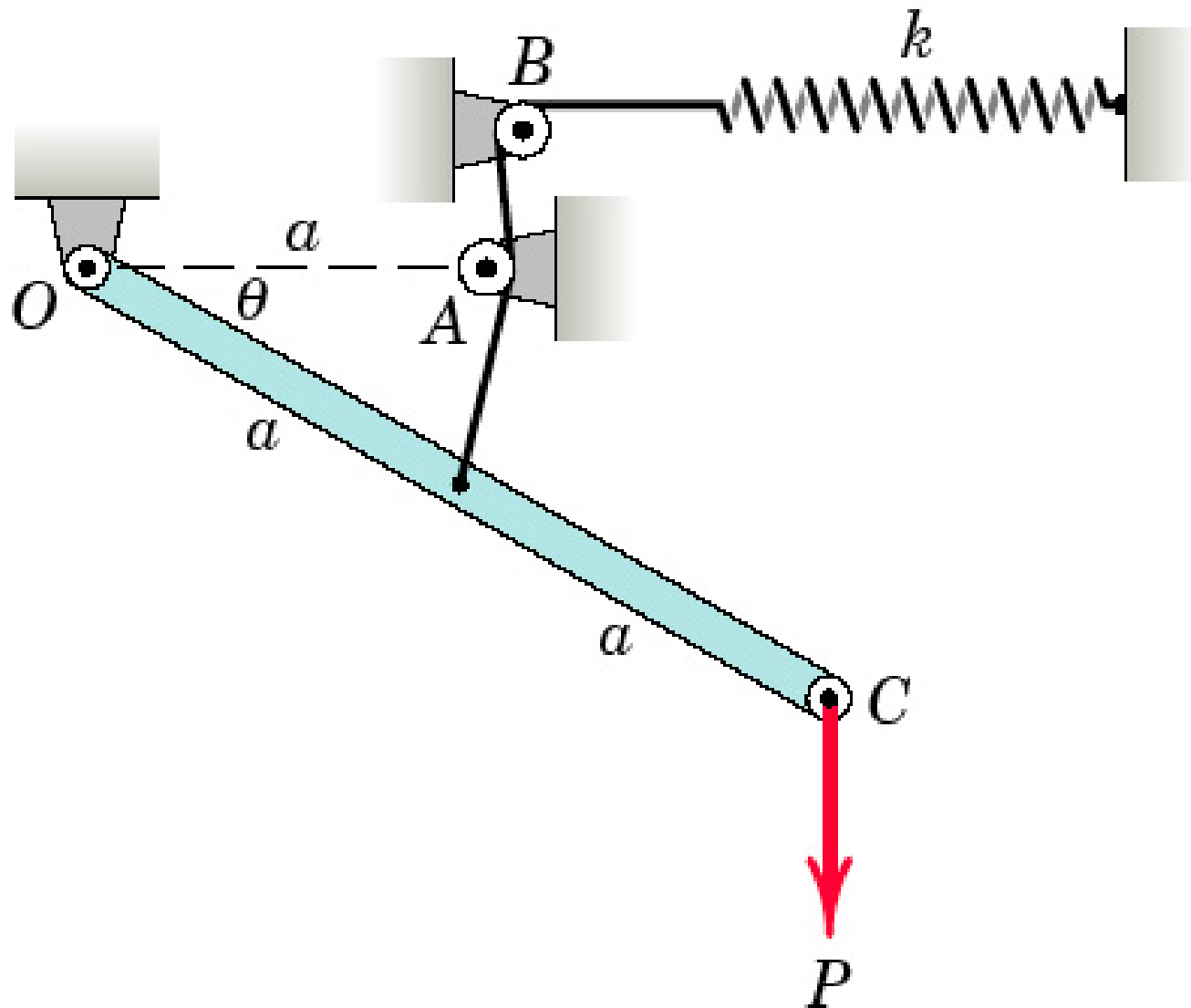
Problem 7/64



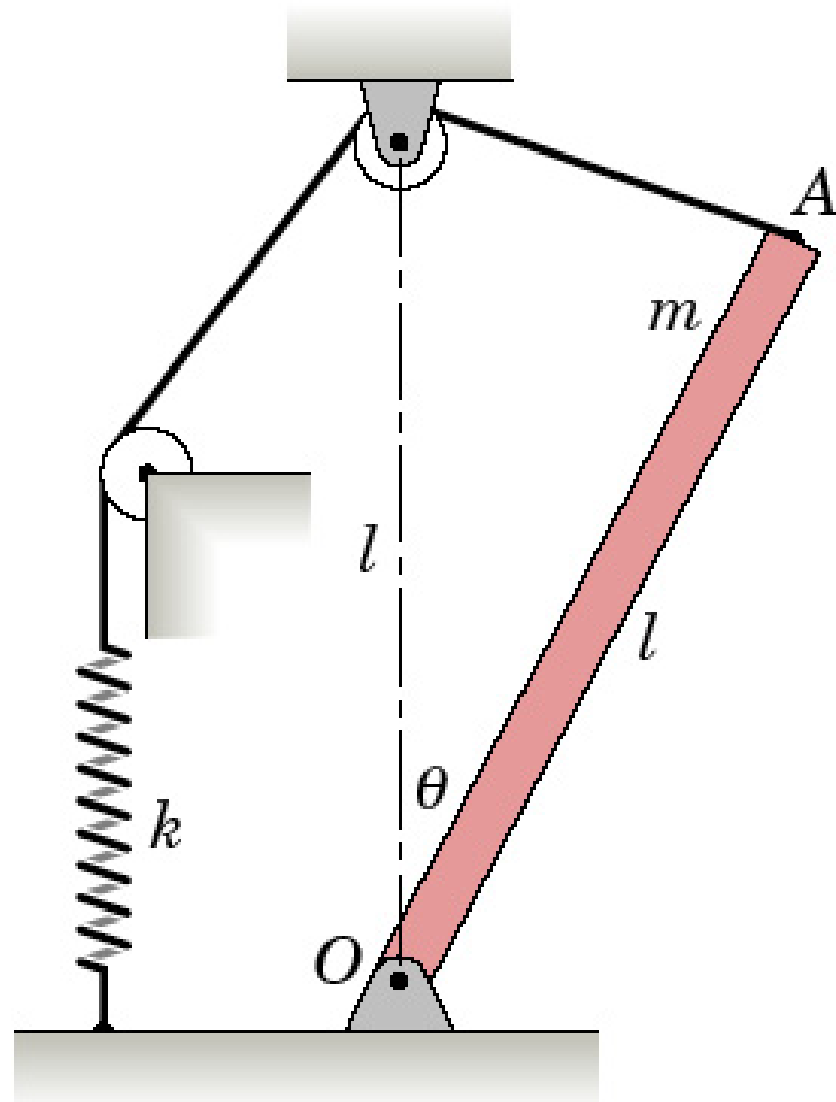
Problem 7/65



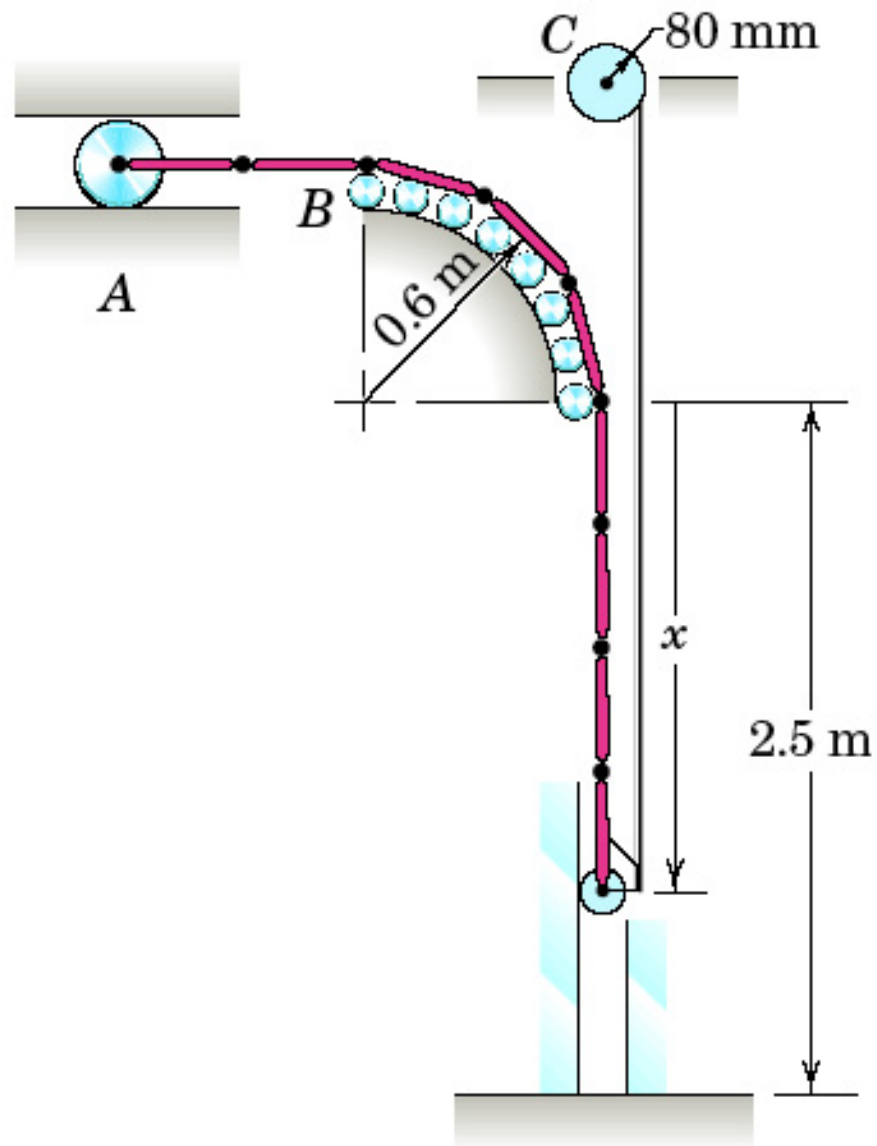
Problem 7/66



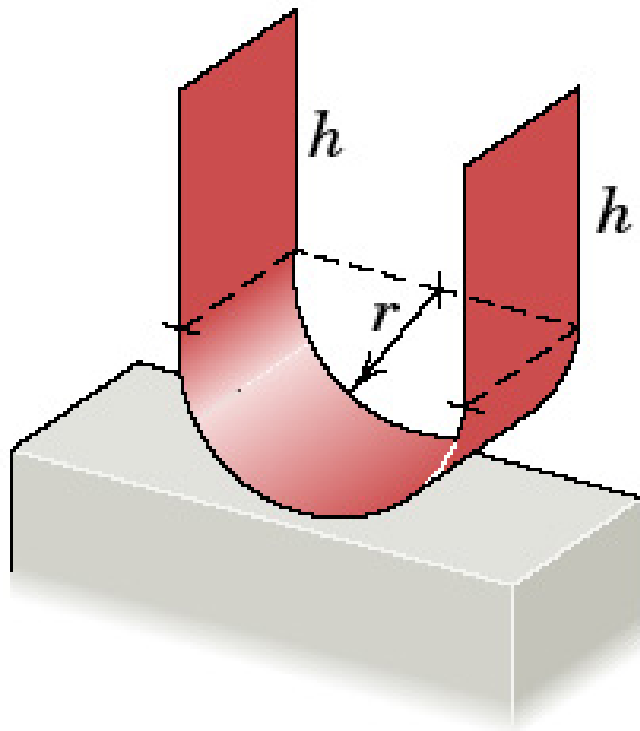
Problem 7/67



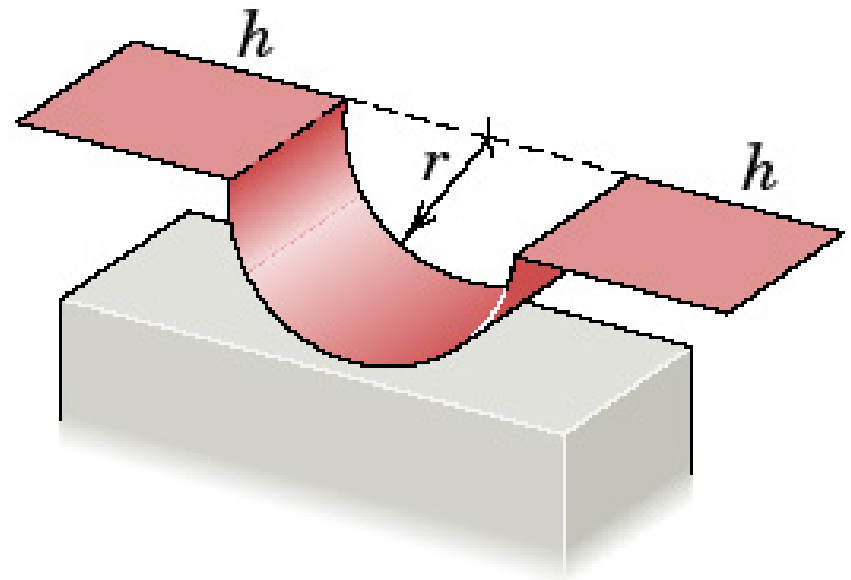
Problem 7/68



Problem 7/69

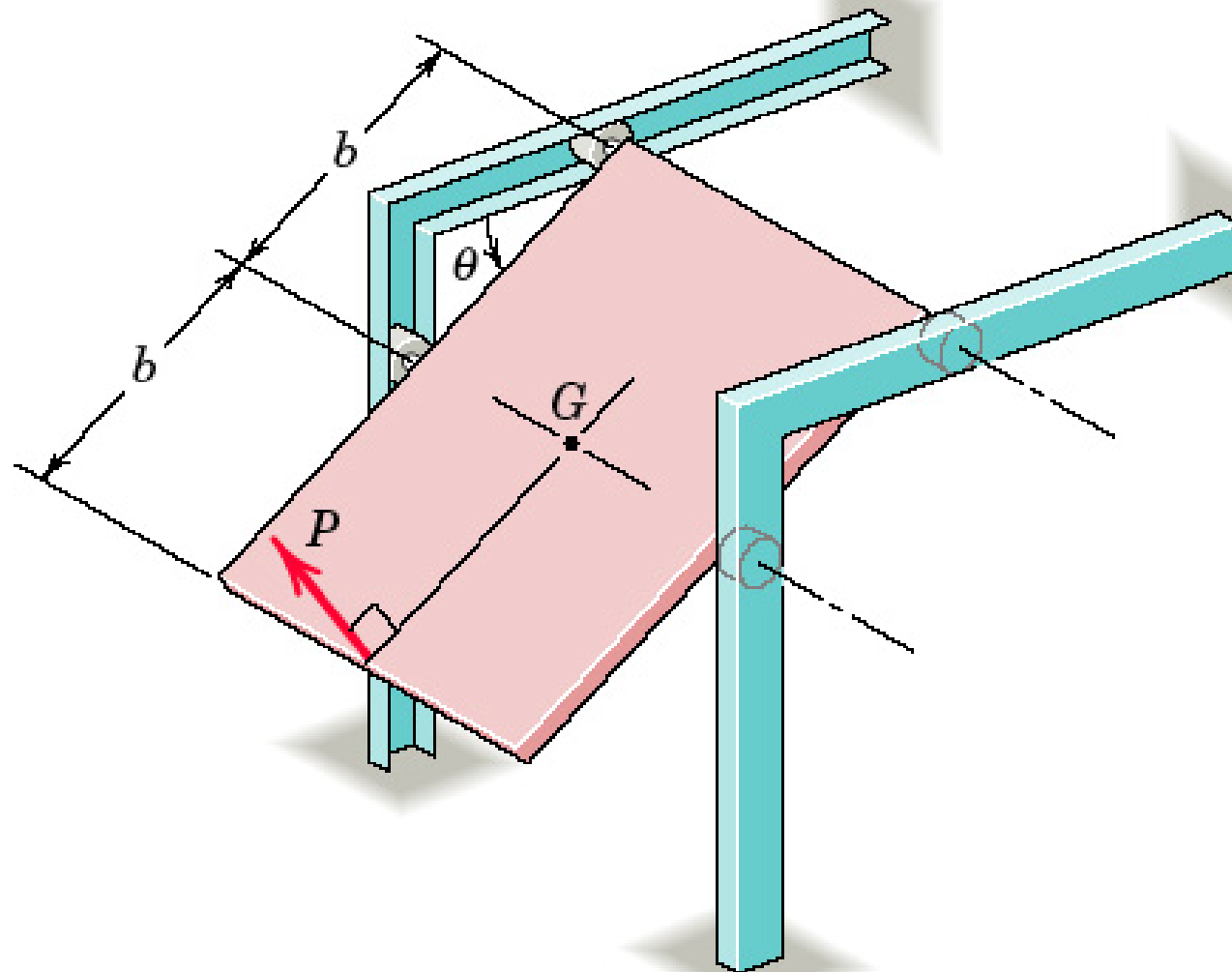


(a)

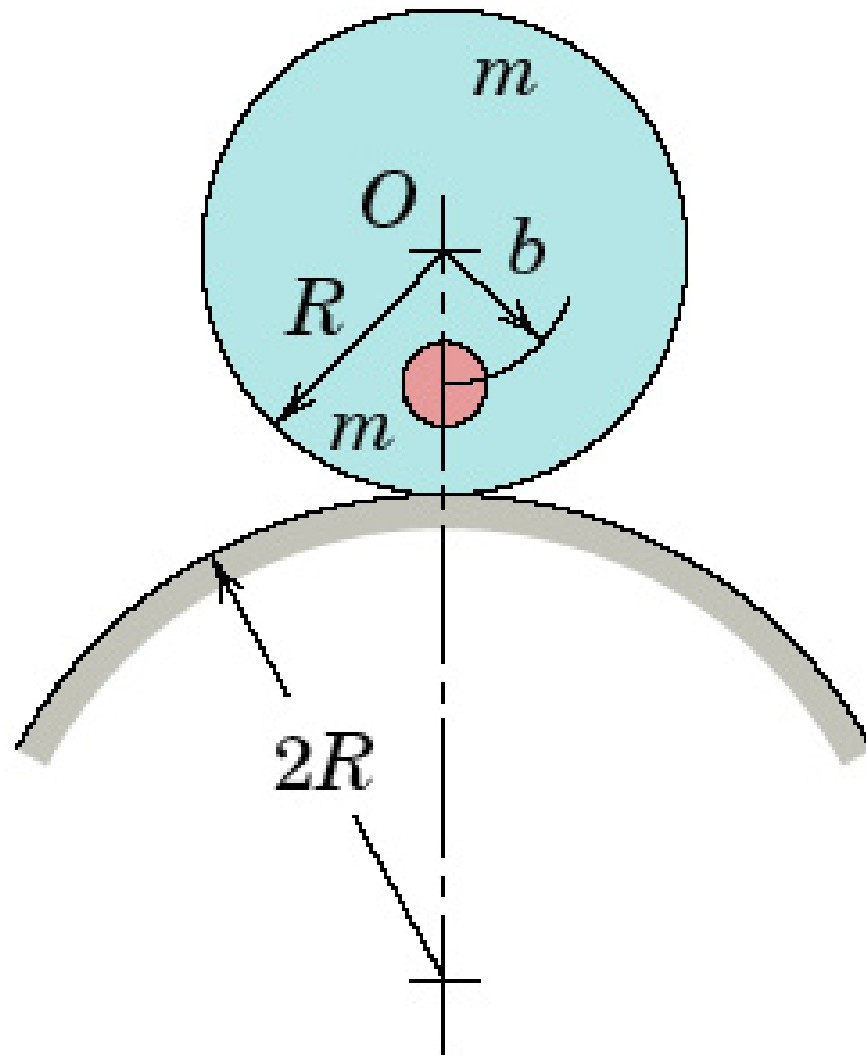


(b)

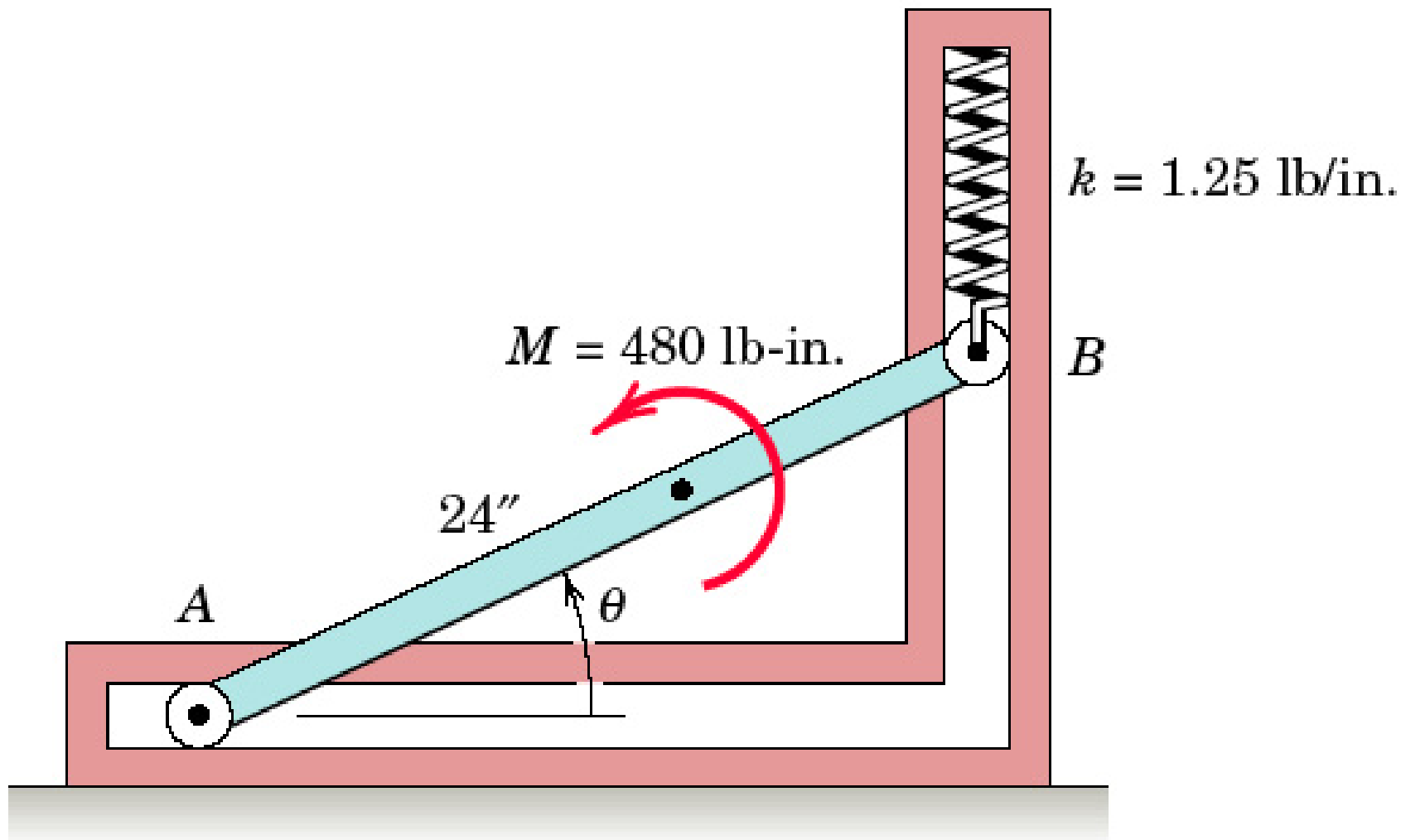
Problem 7/70



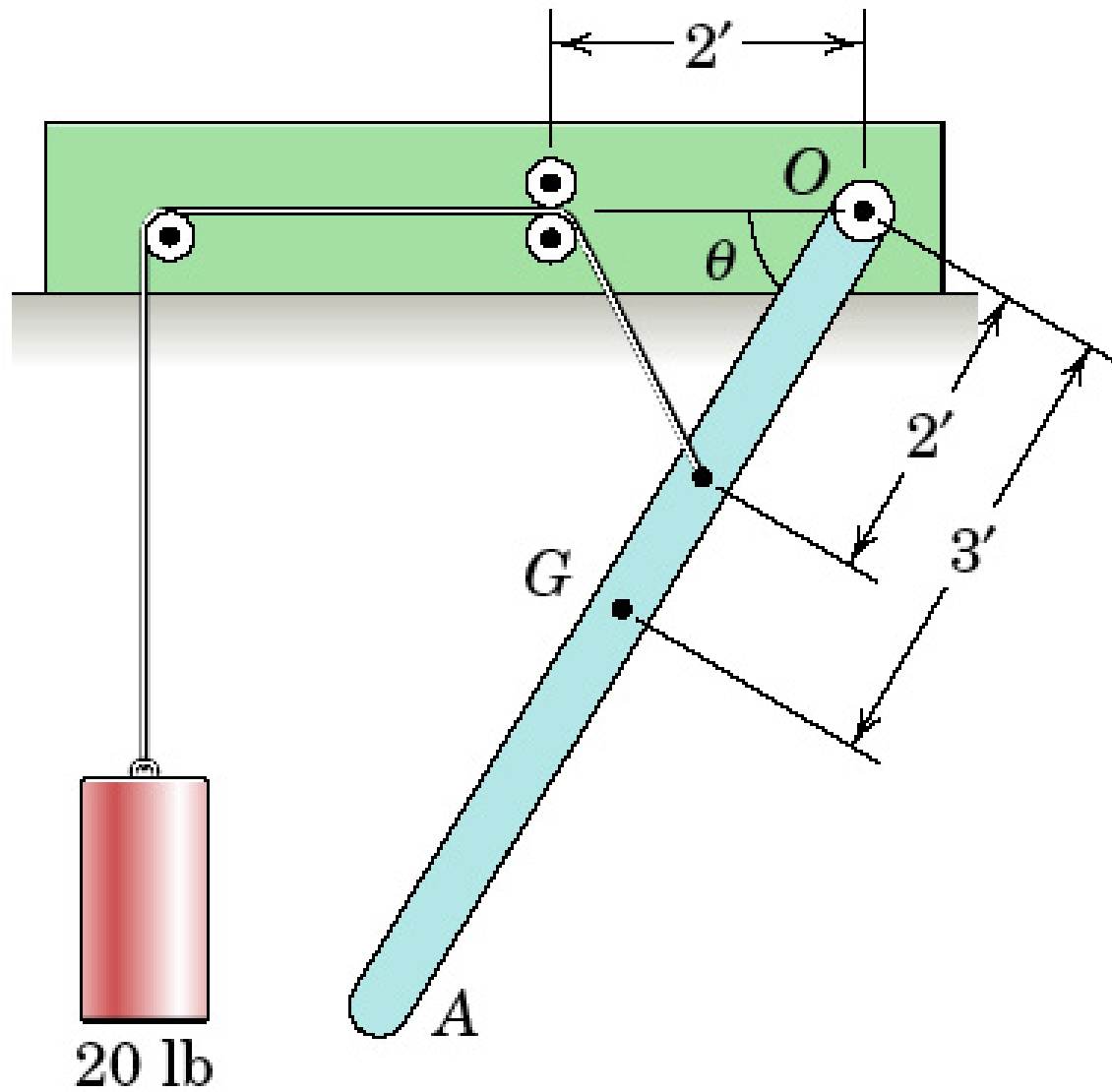
Problem 7/71



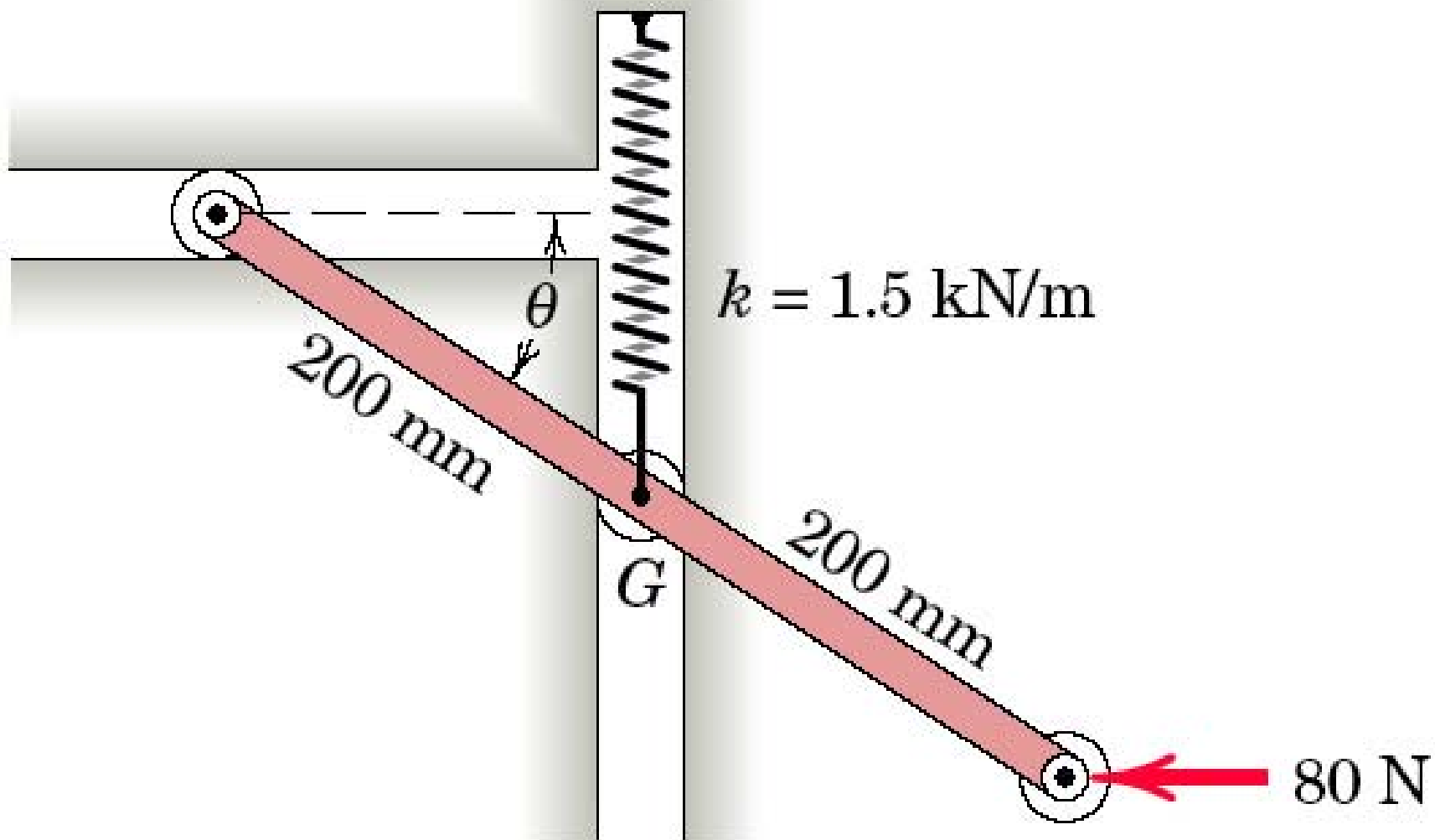
Problem 7/72



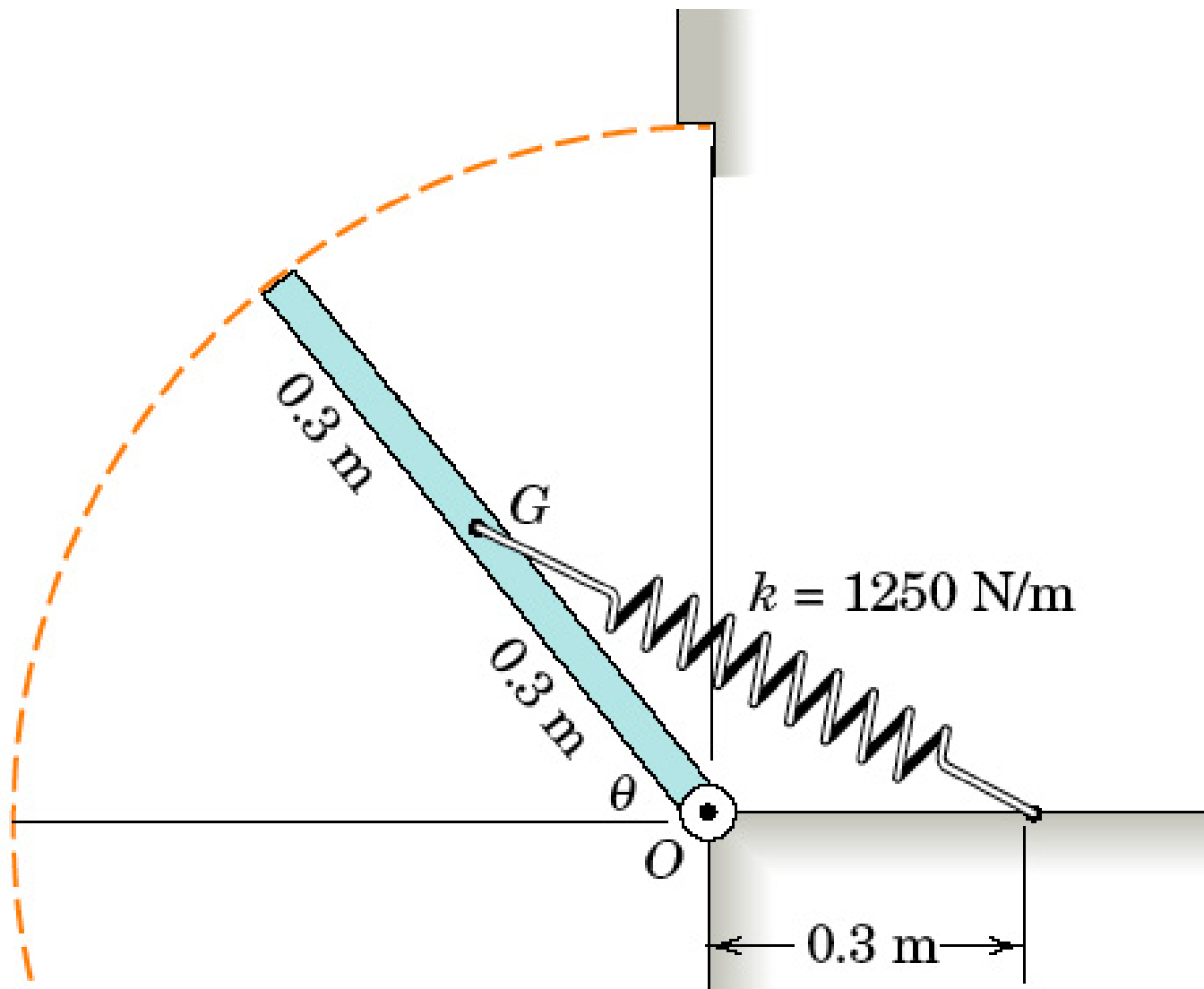
Problem 7/73



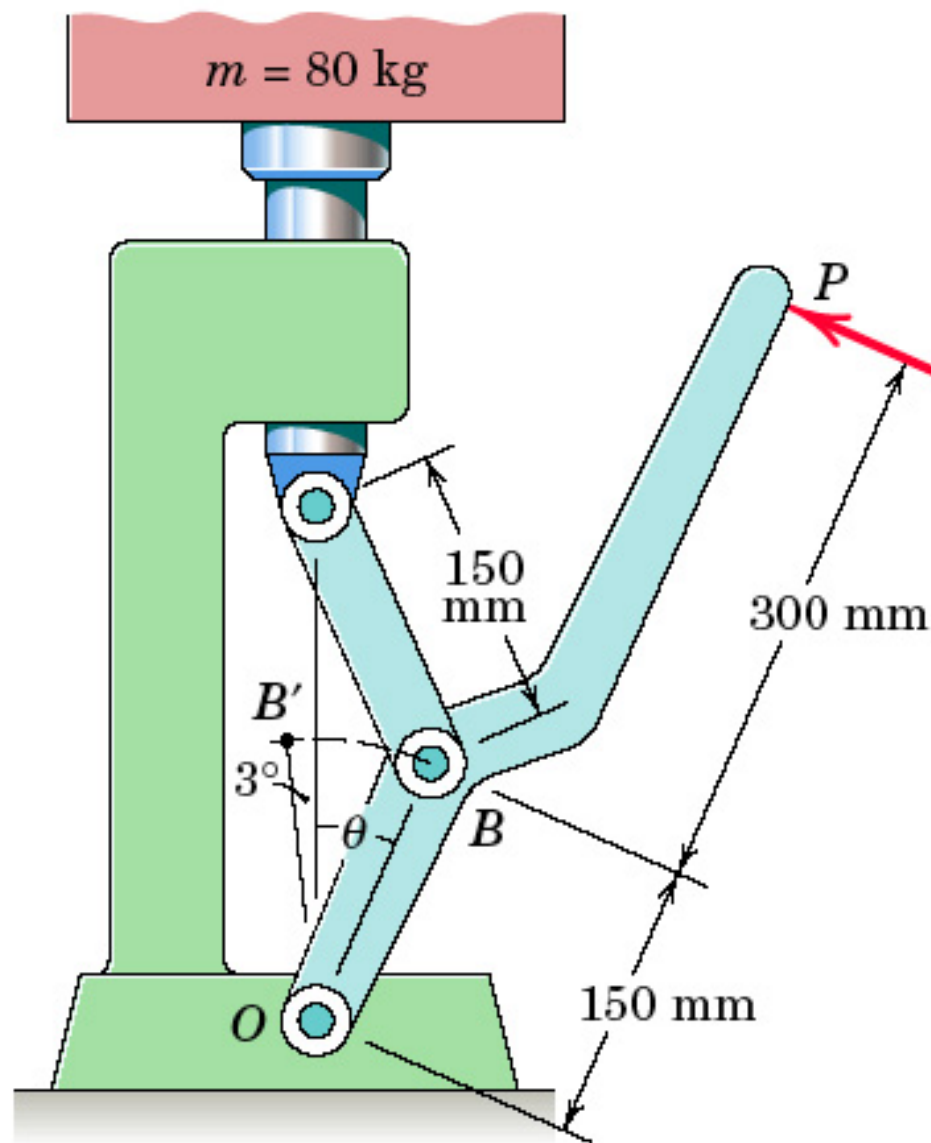
Problem 7/75



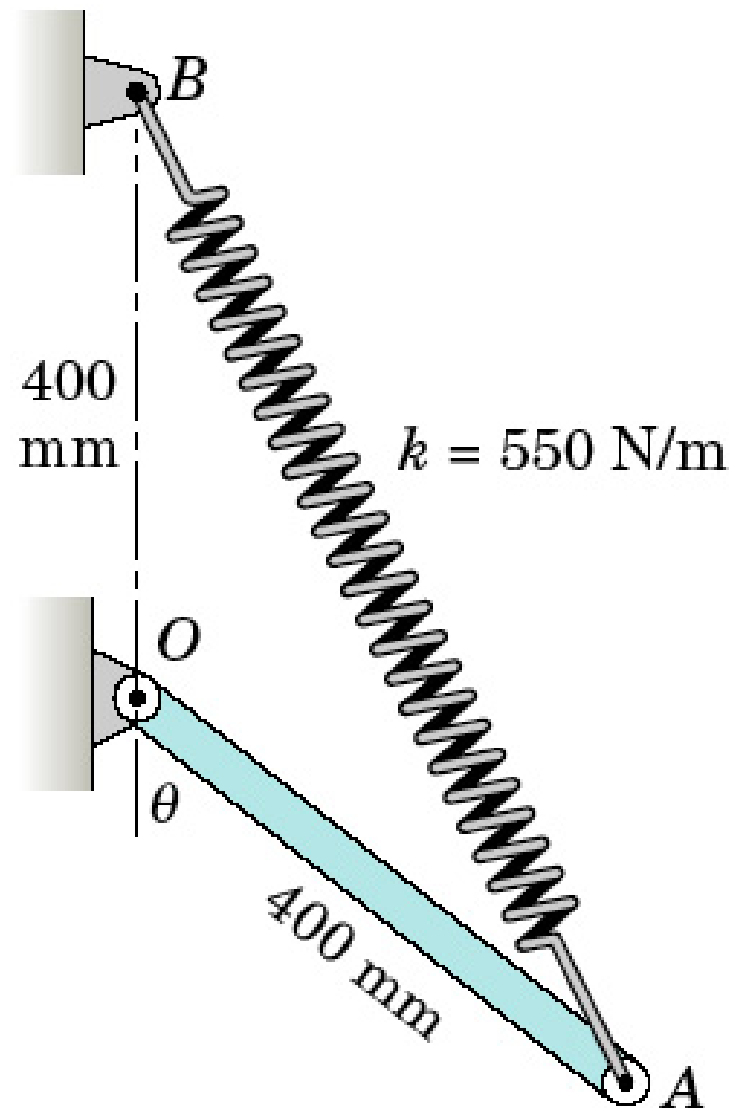
Problem 7/76



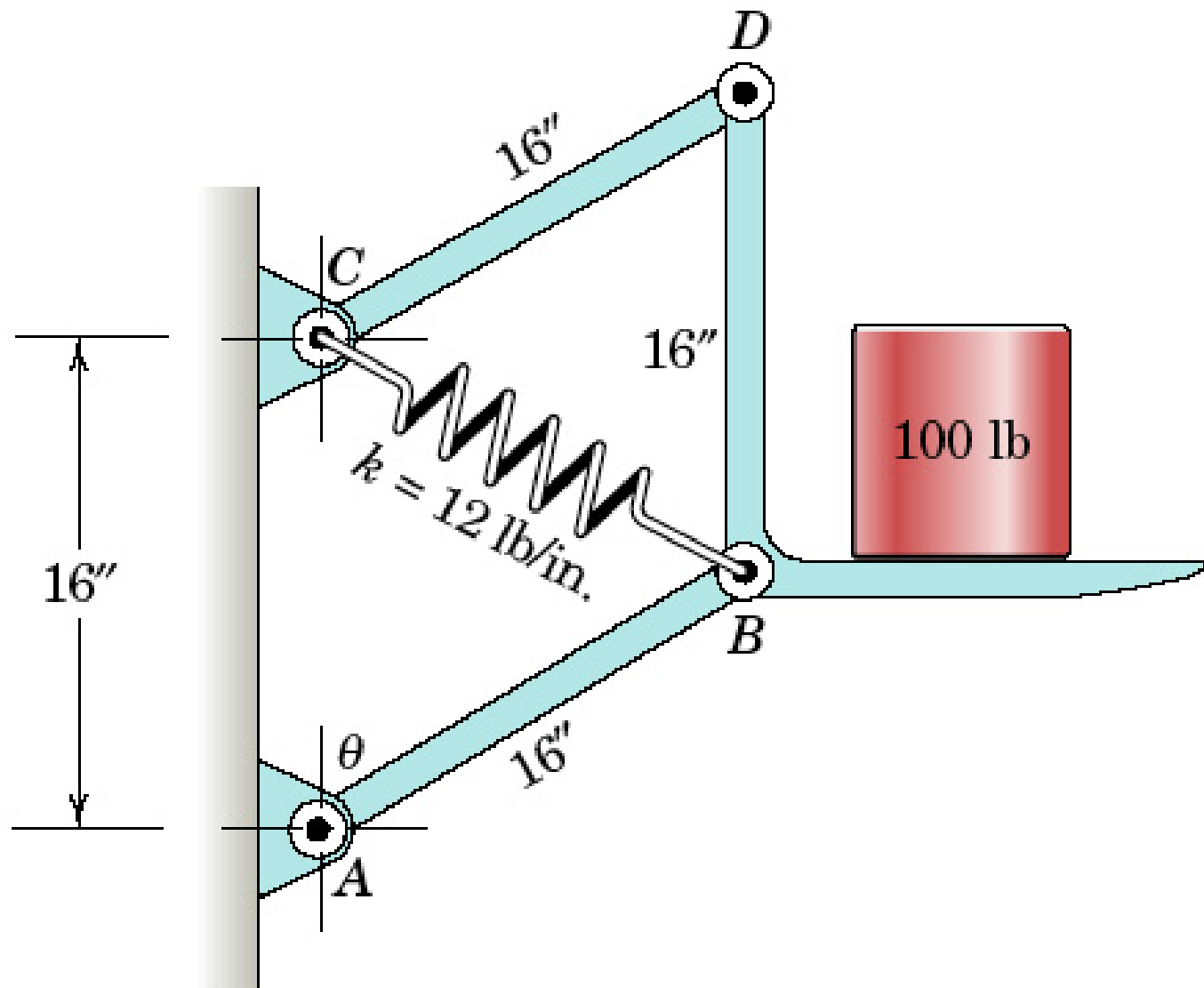
Problem 7/77



Problem 7/78



Problem 7/79



Problem 7/80